Intel® Math Kernel Library

Reference Manual

Document Number: 630813-045US

MKL 10.3 Update 8

Legal Information

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- Some Fast Fourier Transform (FFT) functions in this release of Intel© MKL have been generated by the SPIRAL software generation system (http://www.spiral.net/) under license from Carnegie Mellon University. The authors of SPIRAL are Markus Puschel, Jose Moura, Jeremy Johnson, David Padua, Manuela Veloso, Bryan Singer, Jianxin Xiong, Franz Franchetti, Aca Gacic, Yevgen Voronenko, Kang Chen, Robert W. Johnson, and Nick Rizzolo.

Introducing the Intel® Math Kernel Library

The Intel® Math Kernel Library (Intel® MKL) improves performance of scientific, engineering, and financial software that solves large computational problems. Among other functionality, Intel MKL provides linear algebra routines, fast Fourier transforms, as well as vectorized math and random number generation functions, all optimized for the latest Intel processors, including processors with multiple cores (see the *Intel® MKL Release Notes* for the full list of supported processors). Intel MKL also performs well on non-Intel processors.

Intel MKL is thread-safe and extensively threaded using the OpenMP* technology.

For more details about functionality provided by Intel MKL, see the Function Domains section.

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Getting Help and Support

Getting Help

The online version of the Intel® Math Kernel Library (Intel® MKL) Reference Manual integrates into the Microsoft Visual Studio* development system help on Windows* OS or into the Eclipse* development system help on Linux* OS. For information on how to use the online help, see the Intel MKL User's Guide.

Getting Technical Support

Intel MKL provides a product web site that offers timely and comprehensive product information, including product features, white papers, and technical articles. For the latest information, check: http://www.intel.com/software/products/support.

Intel also provides a support web site that contains a rich repository of self help information, including getting started tips, known product issues, product errata, license information, user forums, and more (visit http://www.intel.com/software/products/).

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What's New

This Reference Manual documents Intel® Math Kernel Library (Intel® MKL) 10.3 Update 8 release.

The following function domains were updated in Intel MKL 10.3 Update 8 with new functions, enhancements to the existing functionality, or improvements to the existing documentation:

- New data fitting functions provide spline-based interpolation capabilities that you can use to approximate functions, function derivatives or function integrals, and perform cell search operations. See Data Fitting Functions.
- The Fourier transform documentation has been updated and improved, especially in the descriptions of configuration settings that define the forward domain of the transform (see DFTI_FORWARD_DOMAIN), memory layout of the input/output data (see DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES), distances between consecutive data sets for computing multiple transforms (see DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE), and storage schemes (see DFTI_COMPLEX_STORAGE, DFTI_REAL_STORAGE).

Additionally, several minor updates have been made to correct errors in the manual.

Notational Conventions

This manual uses the following terms to refer to operating systems:

Windows* OS	This term refers to information that is valid on all supported Windows* operating systems.
Linux* OS	This term refers to information that is valid on all supported Linux* operating systems.
Mac OS* X	This term refers to information that is valid on Intel®-based systems running the Mac OS* X operating system.

This manual uses the following notational conventions:

- Routine name shorthand (for example, `?ungqr` instead of `cungqr`/`zungqr`).
- Font conventions used for distinction between the text and the code.

Routine Name Shorthand

For shorthand, names that contain a question mark "`?`" represent groups of routines with similar functionality. Each group typically consists of routines used with four basic data types: single-precision real, double-precision real, single-precision complex, and double-precision complex. The question mark is used to indicate any or all possible varieties of a function; for example:

`?swap`	Refers to all four data types of the vector-vector `?swap` routine: `sswap`, `dswap`, `cswap`, and `zswap`.

Font Conventions

The following font conventions are used:

`UPPERCASE COURIER`	Data type used in the description of input and output parameters for Fortran interface. For example, `CHARACTER*1`.
`lowercase courier`	Code examples: `a(k+i,j) = matrix(i,j)` and data types for C interface, for example, `const float*`
`lowercase courier mixed with UpperCase courier`	Function names for C interface, for example, `vmlSetMode`
`lowercase courier italic`	Variables in arguments and parameters description. For example, *`incx`*.
`*`	Used as a multiplication symbol in code examples and equations and where required by the Fortran syntax.

Function Domains

1

The Intel® Math Kernel Library includes Fortran routines and functions optimized for Intel® processor-based computers running operating systems that support multiprocessing. In addition to the Fortran interface, Intel MKL includes a C-language interface for the Discrete Fourier transform functions, as well as for the Vector Mathematical Library and Vector Statistical Library functions. For hardware and software requirements to use Intel MKL, see *Intel® MKL Release Notes*.

The Intel® Math Kernel Library includes the following groups of routines:

- Basic Linear Algebra Subprograms (BLAS):
  - vector operations
  - matrix-vector operations
  - matrix-matrix operations
- Sparse BLAS Level 1, 2, and 3 (basic operations on sparse vectors and matrices)
- LAPACK routines for solving systems of linear equations
- LAPACK routines for solving least squares problems, eigenvalue and singular value problems, and Sylvester's equations
- Auxiliary and utility LAPACK routines
- ScaLAPACK computational, driver and auxiliary routines (only in Intel MKL for Linux* and Windows* operating systems)
- PBLAS routines for distributed vector, matrix-vector, and matrix-matrix operation
- Direct and Iterative Sparse Solver routines
- Vector Mathematical Library (VML) functions for computing core mathematical functions on vector arguments (with Fortran and C interfaces)
- Vector Statistical Library (VSL) functions for generating vectors of pseudorandom numbers with different types of statistical distributions and for performing convolution and correlation computations
- General Fast Fourier Transform (FFT) Functions, providing fast computation of Discrete Fourier Transform via the FFT algorithms and having Fortran and C interfaces
- Cluster FFT functions (only in Intel MKL for Linux* and Windows* operating systems)
- Tools for solving partial differential equations - trigonometric transform routines and Poisson solver
- Optimization Solver routines for solving nonlinear least squares problems through the Trust-Region (TR) algorithms and computing Jacobi matrix by central differences
- Basic Linear Algebra Communication Subprograms (BLACS) that are used to support a linear algebra oriented message passing interface
- Data Fitting functions for spline-based approximation of functions, derivatives and integrals of functions, and search
- GMP arithmetic functions

For specific issues on using the library, also see the *Intel® MKL Release Notes*.

BLAS Routines

The BLAS routines and functions are divided into the following groups according to the operations they perform:

- BLAS Level 1 Routines perform operations of both addition and reduction on vectors of data. Typical operations include scaling and dot products.
- BLAS Level 2 Routines perform matrix-vector operations, such as matrix-vector multiplication, rank-1 and rank-2 matrix updates, and solution of triangular systems.
- BLAS Level 3 Routines perform matrix-matrix operations, such as matrix-matrix multiplication, rank-k update, and solution of triangular systems.

Starting from release 8.0, Intel® MKL also supports the Fortran 95 interface to the BLAS routines.

Starting from release 10.1, a number of BLAS-like Extensions are added to enable the user to perform certain data manipulation, including matrix in-place and out-of-place transposition operations combined with simple matrix arithmetic operations.

Sparse BLAS Routines

The Sparse BLAS Level 1 Routines and Functions and Sparse BLAS Level 2 and Level 3 Routines routines and functions operate on sparse vectors and matrices. These routines perform vector operations similar to the BLAS Level 1, 2, and 3 routines. The Sparse BLAS routines take advantage of vector and matrix sparsity: they allow you to store only non-zero elements of vectors and matrices. Intel MKL also supports Fortran 95 interface to Sparse BLAS routines.

LAPACK Routines

The Intel® Math Kernel Library fully supports LAPACK 3.1 set of computational, driver, auxiliary and utility routines.

The original versions of LAPACK from which that part of Intel MKL was derived can be obtained from http://www.netlib.org/lapack/index.html. The authors of LAPACK are E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen.

The LAPACK routines can be divided into the following groups according to the operations they perform:

- Routines for solving systems of linear equations, factoring and inverting matrices, and estimating condition numbers (see Chapter 3).
- Routines for solving least squares problems, eigenvalue and singular value problems, and Sylvester's equations (see Chapter 4).
- Auxiliary and utility routines used to perform certain subtasks, common low-level computation or related tasks (see Chapter 5).

Starting from release 8.0, Intel MKL also supports the Fortran 95 interface to LAPACK computational and driver routines. This interface provides an opportunity for simplified calls of LAPACK routines with fewer required arguments.

ScaLAPACK Routines

The ScaLAPACK package (included only with the Intel® MKL versions for Linux* and Windows* operating systems, see Chapter 6 and Chapter 7) runs on distributed-memory architectures and includes routines for solving systems of linear equations, solving linear least squares problems, eigenvalue and singular value problems, as well as performing a number of related computational tasks.

The original versions of ScaLAPACK from which that part of Intel MKL was derived can be obtained from http://www.netlib.org/scalapack/index.html. The authors of ScaLAPACK are L. Blackford, J. Choi, A.Cleary, E. D'Azevedo, J. Demmel, I. Dhillon, J. Dongarra, S. Hammarling, G. Henry, A. Petitet, K.Stanley, D. Walker, and R. Whaley.

The Intel MKL version of ScaLAPACK is optimized for Intel® processors and uses MPICH version of MPI as well as Intel MPI.

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PBLAS Routines

The PBLAS routines perform operations with distributed vectors and matrices.

- PBLAS Level 1 Routines perform operations of both addition and reduction on vectors of data. Typical operations include scaling and dot products.
- PBLAS Level 2 Routines perform distributed matrix-vector operations, such as matrix-vector multiplication, rank-1 and rank-2 matrix updates, and solution of triangular systems.
- PBLAS Level 3 Routines perform distributed matrix-matrix operations, such as matrix-matrix multiplication, rank-k update, and solution of triangular systems.

Intel MKL provides the PBLAS routines with interface similar to the interface used in the Netlib PBLAS (part of the ScaLAPACK package, see http://www.netlib.org/scalapack/html/pblas_qref.html).

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Sparse Solver Routines

Direct sparse solver routines in Intel MKL (see Chapter 8) solve symmetric and symmetrically-structured sparse matrices with real or complex coefficients. For symmetric matrices, these Intel MKL subroutines can solve both positive-definite and indefinite systems. Intel MKL includes the PARDISO* sparse solver interface as well as an alternative set of user callable direct sparse solver routines.

If you use the sparse solver PARDISO* from Intel MKL, please cite:

O.Schenk and K.Gartner. Solving unsymmetric sparse systems of linear equations with PARDISO. J. of Future Generation Computer Systems, 20(3):475-487, 2004.

Intel MKL provides also an iterative sparse solver (see Chapter 8) that uses Sparse BLAS level 2 and 3 routines and works with different sparse data formats.

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VML Functions

The Vector Mathematical Library (VML) functions (see Chapter 9) include a set of highly optimized implementations of certain computationally expensive core mathematical functions (power, trigonometric, exponential, hyperbolic, etc.) that operate on vectors of real and complex numbers.

Application programs that might significantly improve performance with VML include nonlinear programming software, integrals computation, and many others. VML provides interfaces both for Fortran and C languages.

Statistical Functions

The Vector Statistical Library (VSL) contains three sets of functions (see Chapter 10):

- The first set includes a collection of pseudo- and quasi-random number generator subroutines implementing basic continuous and discrete distributions. To provide best performance, the VSL subroutines use calls to highly optimized Basic Random Number Generators (BRNGs) and a library of vector mathematical functions.
- The second set includes a collection of routines that implement a wide variety of convolution and correlation operations.
- The third set includes a collection of routines for initial statistical analysis of raw single and double precision multi-dimensional datasets.

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Fourier Transform Functions

The Intel® MKL multidimensional Fast Fourier Transform (FFT) functions with mixed radix support (see Chapter 11) provide uniformity of discrete Fourier transform computation and combine functionality with ease of use. Both Fortran and C interface specification are given. There is also a cluster version of FFT functions, which runs on distributed-memory architectures and is provided only in Intel MKL versions for the Linux* and Windows* operating systems.

The FFT functions provide fast computation via the FFT algorithms for arbitrary lengths. See *the Intel© MKL User's Guide* for the specific radices supported.

Partial Differential Equations Support

Intel® MKL provides tools for solving Partial Differential Equations (PDE) (see Chapter 13). These tools are Trigonometric Transform interface routines and Poisson Library.

The Trigonometric Transform routines may be helpful to users who implement their own solvers similar to the solver that the Poisson Library provides. The users can improve performance of their solvers by using fast sine, cosine, and staggered cosine transforms implemented in the Trigonometric Transform interface.

The Poisson Library is designed for fast solving of simple Helmholtz, Poisson, and Laplace problems. The Trigonometric Transform interface, which underlies the solver, is based on the Intel MKL FFT interface (refer to Chapter 11), optimized for Intel® processors.

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Nonlinear Optimization Problem Solvers

Intel® MKL provides Nonlinear Optimization Problem Solver routines (see Chapter 14) that can be used to solve nonlinear least squares problems with or without linear (bound) constraints through the Trust-Region (TR) algorithms and compute Jacobi matrix by central differences.

Support Functions

The Intel® MKL support functions (see Chapter 15) are used to support the operation of the Intel MKL software and provide basic information on the library and library operation, such as the current library version, timing, setting and measuring of CPU frequency, error handling, and memory allocation.

Starting from release 10.0, the Intel MKL support functions provide additional threading control.

Starting from release 10.1, Intel MKL selectively supports a *Progress Routine* feature to track progress of a lengthy computation and/or interrupt the computation using a callback function mechanism. The user application can define a function called `mkl_progress` that is regularly called from the Intel MKL routine supporting the progress routine feature. See the Progress Routines section in Chapter 15 for reference. Refer to a specific LAPACK or DSS/PARDISO function description to see whether the function supports this feature or not.

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BLACS Routines

The Intel® Math Kernel Library implements routines from the BLACS (Basic Linear Algebra Communication Subprograms) package (see Chapter 16) that are used to support a linear algebra oriented message passing interface that may be implemented efficiently and uniformly across a large range of distributed memory platforms.

The original versions of BLACS from which that part of Intel MKL was derived can be obtained from http://www.netlib.org/blacs/index.html. The authors of BLACS are Jack Dongarra and R. Clint Whaley.

Data Fitting Functions

The Data Fitting component includes a set of highly-optimized implementations of algorithms for the following spline-based computations:

- spline construction
- interpolation including computation of derivatives and integration
- search

The algorithms operate on single and double vector-valued functions set in the points of the given partition. You can use Data Fitting algorithms in applications that are based on data approximation.

GMP Arithmetic Functions

Intel® MKL implementation of GMP* arithmetic functions includes arbitrary precision arithmetic operations on integer numbers. The interfaces of such functions fully match the GNU Multiple Precision (GMP*) Arithmetic Library.

NOTE GMP Arithmetic Functions are deprecated and will be removed in a future Intel MKL release.

Performance Enhancements

The Intel® Math Kernel Library has been optimized by exploiting both processor and system features and capabilities. Special care has been given to those routines that most profit from cache-management techniques. These especially include matrix-matrix operation routines such as `dgemm()`.

In addition, code optimization techniques have been applied to minimize dependencies of scheduling integer and floating-point units on the results within the processor.

The major optimization techniques used throughout the library include:

- Loop unrolling to minimize loop management costs
- Blocking of data to improve data reuse opportunities
- Copying to reduce chances of data eviction from cache
- Data prefetching to help hide memory latency
- Multiple simultaneous operations (for example, dot products in `dgemm`) to eliminate stalls due to arithmetic unit pipelines
- Use of hardware features such as the SIMD arithmetic units, where appropriate

These are techniques from which the arithmetic code benefits the most.

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Parallelism

In addition to the performance enhancements discussed above, Intel® MKL offers performance gains through parallelism provided by the symmetric multiprocessing performance (SMP) feature. You can obtain improvements from SMP in the following ways:

- One way is based on user-managed threads in the program and further distribution of the operations over the threads based on data decomposition, domain decomposition, control decomposition, or some other parallelizing technique. Each thread can use any of the Intel MKL functions (except for the deprecated `?lacon` LAPACK routine) because the library has been designed to be thread-safe.
- Another method is to use the FFT and BLAS level 3 routines. They have been parallelized and require no alterations of your application to gain the performance enhancements of multiprocessing. Performance using multiple processors on the level 3 BLAS shows excellent scaling. Since the threads are called and managed within the library, the application does not need to be recompiled thread-safe (see also Fortran 95 Interface Conventions in Chapter 2).
- Yet another method is to use *tuned LAPACK routines*. Currently these include the single- and double precision flavors of routines for *QR* factorization of general matrices, triangular factorization of general and symmetric positive-definite matrices, solving systems of equations with such matrices, as well as solving symmetric eigenvalue problems.

For instructions on setting the number of available processors for the BLAS level 3 and LAPACK routines, see *Intel® MKL User's Guide*.

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C Datatypes Specific to Intel MKL

The `mkl_types.h` file defines datatypes specific to Intel MKL.

C/C++ Type	Fortran Type	LP32 Equivalent (Size in Bytes)	LP64 Equivalent (Size in Bytes)	ILP64 Equivalent (Size in Bytes)
`MKL_INT` (MKL `integer`)	`INTEGER` (default `INTEGER`)	C/C++: `int` Fortran: `INTEGER*4` (4 bytes)	C/C++: `int` Fortran: `INTEGER*4` (4 bytes)	C/C++: `long long` (or define `MKL_ILP64` macros Fortran: `INTEGER*8` (8 bytes)
`MKL_UINT` (MKL `unsigned integer`)	N/A	C/C++: `unsigned int` (4 bytes)	C/C++: `unsigned int` (4 bytes)	C/C++: `unsigned long long` (8 bytes)
`MKL_LONG` (MKL `long integer`)	N/A	C/C++: `long` (4 bytes)	C/C++: `long` (Windows: 4 bytes) (Linux, Mac: 8 bytes)	C/C++: `long` (8 bytes)

C/C++ Type	Fortran Type	LP32 Equivalent (Size in Bytes)	LP64 Equivalent (Size in Bytes)	ILP64 Equivalent (Size in Bytes)
MKL_Complex8 (Like C99 complex float)	COMPLEX*8	(8 bytes)	(8 bytes)	(8 bytes)
MKL_Complex16 (Like C99 complex double)	COMPLEX*16	(16 bytes)	(16 bytes)	(16 bytes)

You can redefine datatypes specific to Intel MKL. One reason to do this is if you have your own types which are binary-compatible with Intel MKL datatypes, with the same representation or memory layout. To redefine a datatype, use one of these methods:

- Insert the `#define` statement redefining the datatype before the `mkl.h` header file `#include` statement. For example,

```
#define MKL_INT size_t
#include "mkl.h"
```

- Use the compiler `-D` option to redefine the datatype. For example,

```
...-DMKL_INT=size_t...
```

NOTE As the user, if you redefine Intel MKL datatypes you are responsible for making sure that your definition is compatible with that of Intel MKL. If not, it might cause unpredictable results or crash the application.

BLAS and Sparse BLAS Routines 2

This chapter describes the Intel® Math Kernel Library implementation of the BLAS and Sparse BLAS routines, and BLAS-like extensions. The routine descriptions are arranged in several sections:

- BLAS Level 1 Routines (vector-vector operations)
- BLAS Level 2 Routines (matrix-vector operations)
- BLAS Level 3 Routines (matrix-matrix operations)
- Sparse BLAS Level 1 Routines (vector-vector operations).
- Sparse BLAS Level 2 and Level 3 Routines (matrix-vector and matrix-matrix operations)
- BLAS-like Extensions

Each section presents the routine and function group descriptions in alphabetical order by routine or function group name; for example, the `?asum` group, the `?axpy` group. The question mark in the group name corresponds to different character codes indicating the data type (`s`, `d`, `c`, and `z` or their combination); see *Routine Naming Conventions*.

When BLAS or Sparse BLAS routines encounter an error, they call the error reporting routine `xerbla`.

In BLAS Level 1 groups `i?amax` and `i?amin`, an "`i`" is placed before the data-type indicator and corresponds to the index of an element in the vector. These groups are placed in the end of the BLAS Level 1 section.

BLAS Routines

Routine Naming Conventions

BLAS routine names have the following structure:

`<character> <name> <mod> ( )`

The `<character>` field indicates the data type:

`s`	real, single precision
`c`	complex, single precision
`d`	real, double precision
`z`	complex, double precision

Some routines and functions can have combined character codes, such as `sc` or `dz`.

For example, the function `scasum` uses a complex input array and returns a real value.

The `<name>` field, in BLAS level 1, indicates the operation type. For example, the BLAS level 1 routines `?dot`, `?rot`, `?swap` compute a vector dot product, vector rotation, and vector swap, respectively.

In BLAS level 2 and 3, `<name>` reflects the matrix argument type:

`ge`	general matrix
`gb`	general band matrix
`sy`	symmetric matrix
`sp`	symmetric matrix (packed storage)
`sb`	symmetric band matrix
`he`	Hermitian matrix
`hp`	Hermitian matrix (packed storage)

hb	Hermitian band matrix
tr	triangular matrix
tp	triangular matrix (packed storage)
tb	triangular band matrix.

The <*mod*> field, if present, provides additional details of the operation. BLAS level 1 names can have the following characters in the <*mod*> field:

c	conjugated vector
u	unconjugated vector
g	Givens rotation construction
m	modified Givens rotation
mg	modified Givens rotation construction

BLAS level 2 names can have the following characters in the <*mod*> field:

mv	matrix-vector product
sv	solving a system of linear equations with a single unknown vector
r	rank-1 update of a matrix
r2	rank-2 update of a matrix.

BLAS level 3 names can have the following characters in the <*mod*> field:

mm	matrix-matrix product
sm	solving a system of linear equations with multiple unknown vectors
rk	rank-*k* update of a matrix
r2k	rank-2*k* update of a matrix.

The examples below illustrate how to interpret BLAS routine names:

ddot	<d> <dot>: double-precision real vector-vector dot product
cdotc	<c> <dot> <c>: complex vector-vector dot product, conjugated
scasum	<sc> <asum>: sum of magnitudes of vector elements, single precision real output and single precision complex input
cdotu	<c> <dot> <u>: vector-vector dot product, unconjugated, complex
sgemv	<s> <ge> <mv>: matrix-vector product, general matrix, single precision
ztrmm	<z> <tr> <mm>: matrix-matrix product, triangular matrix, double-precision complex.

Sparse BLAS level 1 naming conventions are similar to those of BLAS level 1. For more information, see Naming Conventions.

Fortran 95 Interface Conventions

Fortran 95 interface to BLAS and Sparse BLAS Level 1 routines is implemented through wrappers that call respective FORTRAN 77 routines. This interface uses such features of Fortran 95 as assumed-shape arrays and optional arguments to provide simplified calls to BLAS and Sparse BLAS Level 1 routines with fewer parameters.

NOTE For BLAS, Intel MKL offers two types of Fortran 95 interfaces:

- using `mkl_blas.fi` only through `include 'mkl_blas_subroutine.fi'` statement. Such interfaces allow you to make use of the original LAPACK routines with all their arguments
- using `blas.f90` that includes improved interfaces. This file is used to generate the module files `blas95.mod` and `f95_precision.mod`. The module files `mkl95_blas.mod` and `mkl95_precision.mod` are also generated. See also section "Fortran 95 interfaces and wrappers to LAPACK and BLAS" of *Intel® MKL User's Guide* for details. The module files are used to process the FORTRAN use clauses referencing the BLAS interface: `use blas95` (or an equivalent `use mkl95_blas`) and `use f95_precision` (or an equivalent `use mkl95_precision`).

The main conventions used in Fortran 95 interface are as follows:

- The names of parameters used in Fortran 95 interface are typically the same as those used for the respective generic (FORTRAN 77) interface. In rare cases formal argument names may be different.
- Some input parameters such as array dimensions are not required in Fortran 95 and are skipped from the calling sequence. Array dimensions are reconstructed from the user data that must exactly follow the required array shape.
- A parameter can be skipped if its value is completely defined by the presence or absence of another parameter in the calling sequence, and the restored value is the only meaningful value for the skipped parameter.
- Parameters specifying the increment values *incx* and *incy* are skipped. In most cases their values are equal to 1. In Fortran 95 an increment with different value can be directly established in the corresponding parameter.
- Some generic parameters are declared as optional in Fortran 95 interface and may or may not be present in the calling sequence. A parameter can be declared optional if it satisfies one of the following conditions:

  1. It can take only a few possible values. The default value of such parameter typically is the first value in the list; all exceptions to this rule are explicitly stated in the routine description.
  2. It has a natural default value.

  Optional parameters are given in square brackets in Fortran 95 call syntax.

The particular rules used for reconstructing the values of omitted optional parameters are specific for each routine and are detailed in the respective "Fortran 95 Notes" subsection at the end of routine specification section. If this subsection is omitted, the Fortran 95 interface for the given routine does not differ from the corresponding FORTRAN 77 interface.

Note that this interface is not implemented in the current version of Sparse BLAS Level 2 and Level 3 routines.

Matrix Storage Schemes

Matrix arguments of BLAS routines can use the following storage schemes:

- *Full storage*: a matrix A is stored in a two-dimensional array a, with the matrix element a_{ij} stored in the array element $a(i,j)$.
- *Packed storage* scheme allows you to store symmetric, Hermitian, or triangular matrices more compactly: the upper or lower triangle of the matrix is packed by columns in a one-dimensional array.
- *Band storage*: a band matrix is stored compactly in a two-dimensional array: columns of the matrix are stored in the corresponding columns of the array, and *diagonals* of the matrix are stored in rows of the array.

For more information on matrix storage schemes, see Matrix Arguments in Appendix B.

BLAS Level 1 Routines and Functions

BLAS Level 1 includes routines and functions, which perform vector-vector operations. Table "BLAS Level 1 Routine Groups and Their Data Types" lists the BLAS Level 1 routine and function groups and the data types associated with them.

BLAS Level 1 Routine and Function Groups and Their Data Types

Routine or Function Group	Data Types	Description
?asum	s, d, sc, dz	Sum of vector magnitudes (functions)
?axpy	s, d, c, z	Scalar-vector product (routines)
?copy	s, d, c, z	Copy vector (routines)
?dot	s, d	Dot product (functions)
?sdot	sd, d	Dot product with extended precision (functions)
?dotc	c, z	Dot product conjugated (functions)
?dotu	c, z	Dot product unconjugated (functions)
?nrm2	s, d, sc, dz	Vector 2-norm (Euclidean norm) (functions)
?rot	s, d, cs, zd	Plane rotation of points (routines)
?rotg	s, d, c, z	Generate Givens rotation of points (routines)
?rotm	s, d	Modified Givens plane rotation of points (routines)
?rotmg	s, d	Generate modified Givens plane rotation of points (routines)
?scal	s, d, c, z, cs, zd	Vector-scalar product (routines)
?swap	s, d, c, z	Vector-vector swap (routines)
i?amax	s, d, c, z	Index of the maximum absolute value element of a vector (functions)
i?amin	s, d, c, z	Index of the minimum absolute value element of a vector (functions)
?cabs1	s, d	Auxiliary functions, compute the absolute value of a complex number of single or double precision

?asum

Computes the sum of magnitudes of the vector elements.

Syntax

Fortran 77:

```
res = sasum(n, x, incx)
res = scasum(n, x, incx)
res = dasum(n, x, incx)
res = dzasum(n, x, incx)
```

Fortran 95:

```
res = asum(x)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`

- C: `mkl_blas.h`

Description

The `?asum` routine computes the sum of the magnitudes of elements of a real vector, or the sum of magnitudes of the real and imaginary parts of elements of a complex vector:

```
res = |Re x(1)| + |Im x(1)| + |Re  x(2)| + |Im  x(2)|+ ... + |Re  x(n)| + |Im x(n)|,
```

where *x* is a vector with a number of elements that equals *n*.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vector *x*.
x	`REAL` for `sasum` `DOUBLE PRECISION` for `dasum` `COMPLEX` for `scasum` `DOUBLE COMPLEX` for `dzasum` Array, `DIMENSION` at least `(1 + (n-1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for indexing vector *x*.

Output Parameters

res	`REAL` for `sasum` `DOUBLE PRECISION` for `dasum` `REAL` for `scasum` `DOUBLE PRECISION` for `dzasum` Contains the sum of magnitudes of real and imaginary parts of all elements of the vector.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `asum` interface are the following:

x	Holds the array of size *n*.

?axpy

Computes a vector-scalar product and adds the result to a vector.

Syntax

Fortran 77:

```
call saxpy(n, a, x, incx, y, incy)
call daxpy(n, a, x, incx, y, incy)
call caxpy(n, a, x, incx, y, incy)
call zaxpy(n, a, x, incx, y, incy)
```

Fortran 95:

```
call axpy(x, y [,a])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?axpy` routines perform a vector-vector operation defined as

```
y := a*x + y
```

where:

a is a scalar

x and *y* are vectors each with a number of elements that equals *n*.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vectors *x* and *y*.
a	`REAL` for `saxpy` `DOUBLE PRECISION` for `daxpy` `COMPLEX` for `caxpy` `DOUBLE COMPLEX` for `zaxpy` Specifies the scalar *a*.
x	`REAL` for `saxpy` `DOUBLE PRECISION` for `daxpy` `COMPLEX` for `caxpy` `DOUBLE COMPLEX` for `zaxpy` Array, `DIMENSION` at least `(1 + (n-1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for the elements of *x*.
y	`REAL` for `saxpy` `DOUBLE PRECISION` for `daxpy` `COMPLEX` for `caxpy` `DOUBLE COMPLEX` for `zaxpy` Array, `DIMENSION` at least `(1 + (n-1)*abs(incy))`.
incy	`INTEGER`. Specifies the increment for the elements of *y*.

Output Parameters

y	Contains the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `axpy` interface are the following:

x	Holds the array of size *n*.
y	Holds the array of size *n*.
a	The default value is 1.

?copy

Copies vector to another vector.

Syntax

Fortran 77:

```
call scopy(n, x, incx, y, incy)
call dcopy(n, x, incx, y, incy)
call ccopy(n, x, incx, y, incy)
call zcopy(n, x, incx, y, incy)
```

Fortran 95:

```
call copy(x, y)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?copy` routines perform a vector-vector operation defined as

$y = x$,

where x and y are vectors.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vectors x and y.
x	`REAL` for `scopy` `DOUBLE PRECISION` for `dcopy` `COMPLEX` for `ccopy` `DOUBLE COMPLEX` for `zcopy` Array, `DIMENSION` at least `(1 + (n-1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for the elements of x.
y	`REAL` for `scopy` `DOUBLE PRECISION` for `dcopy` `COMPLEX` for `ccopy` `DOUBLE COMPLEX` for `zcopy` Array, `DIMENSION` at least `(1 + (n-1)*abs(incy))`.
incy	`INTEGER`. Specifies the increment for the elements of y.

Output Parameters

y	Contains a copy of the vector x if n is positive. Otherwise, parameters are unaltered.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `copy` interface are the following:

x	Holds the vector with the number of elements n.
y	Holds the vector with the number of elements n.

?dot

Computes a vector-vector dot product.

Syntax

Fortran 77:

```
res = sdot(n, x, incx, y, incy)
res = ddot(n, x, incx, y, incy)
```

Fortran 95:

```
res = dot(x, y)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?dot` routines perform a vector-vector reduction operation defined as

$$res = \sum_{i=1}^{n} x_i * y_i,$$

where x_i and y_i are elements of vectors x and y.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vectors *x* and *y*.
x	`REAL` for `sdot` `DOUBLE PRECISION` for `ddot` Array, `DIMENSION` at least `(1+(n-1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for the elements of *x*.
y	`REAL` for `sdot` `DOUBLE PRECISION` for `ddot` Array, `DIMENSION` at least `(1+(n-1)*abs(incy))`.
incy	`INTEGER`. Specifies the increment for the elements of *y*.

Output Parameters

res	`REAL` for `sdot` `DOUBLE PRECISION` for `ddot` Contains the result of the dot product of *x* and *y*, if *n* is positive. Otherwise, *res* contains 0.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `dot` interface are the following:

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.

?sdot

Computes a vector-vector dot product with extended precision.

Syntax

Fortran 77:

```
res = sdsdot(n, sb, sx, incx, sy, incy)
res = dsdot(n, sx, incx, sy, incy)
```

Fortran 95:

```
res = sdot(sx, sy)
res = sdot(sx, sy, sb)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?sdot` routines compute the inner product of two vectors with extended precision. Both routines use extended precision accumulation of the intermediate results, but the `sdsdot` routine outputs the final result in single precision, whereas the `dsdot` routine outputs the double precision result. The function `sdsdot` also adds scalar value *sb* to the inner product.

Input Parameters

n	`INTEGER`. Specifies the number of elements in the input vectors *sx* and *sy*.
sb	`REAL`. Single precision scalar to be added to inner product (for the function `sdsdot` only).
sx, *sy*	`REAL`. Arrays, `DIMENSION` at least `(1+(n –1)*abs(incx))` and `(1+(n–1)*abs(incy))`, respectively. Contain the input single precision vectors.
incx	`INTEGER`. Specifies the increment for the elements of *sx*.
incy	`INTEGER`. Specifies the increment for the elements of *sy*.

Output Parameters

res	`REAL` for `sdsdot` `DOUBLE PRECISION` for `dsdot` Contains the result of the dot product of *sx* and *sy* (with *sb* added for `sdsdot`), if *n* is positive. Otherwise, *res* contains *sb* for `sdsdot` and `0` for `dsdot`.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `sdot` interface are the following:

sx	Holds the vector with the number of elements *n*.
sy	Holds the vector with the number of elements *n*.

NOTE Note that scalar parameter *sb* is declared as a required parameter in Fortran 95 interface for the function `sdot` to distinguish between function flavors that output final result in different precision.

?dotc

Computes a dot product of a conjugated vector with another vector.

Syntax

Fortran 77:

```
res = cdotc(n, x, incx, y, incy)
res = zdotc(n, x, incx, y, incy)
```

Fortran 95:

```
res = dotc(x, y)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?dotc` routines perform a vector-vector operation defined as:

$$res = \sum_{i=1}^{n} conjg(x_i) * y_i,$$

where x_i and y_i are elements of vectors x and y.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vectors *x* and *y*.
x	`COMPLEX` for `cdotc` `DOUBLE COMPLEX` for `zdotc` Array, `DIMENSION` at least `(1 + (n -1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for the elements of *x*.
y	`COMPLEX` for `cdotc` `DOUBLE COMPLEX` for `zdotc` Array, `DIMENSION` at least `(1 + (n -1)*abs(incy))`.
incy	`INTEGER`. Specifies the increment for the elements of *y*.

Output Parameters

res	COMPLEX for cdotc DOUBLE COMPLEX for zdotc Contains the result of the dot product of the conjugated *x* and unconjugated *y*, if *n* is positive. Otherwise, *res* contains 0.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine dotc interface are the following:

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.

?dotu

Computes a vector-vector dot product.

Syntax

Fortran 77:

```
res = cdotu(n, x, incx, y, incy)
res = zdotu(n, x, incx, y, incy)
```

Fortran 95:

```
res = dotu(x, y)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?dotu routines perform a vector-vector reduction operation defined as

$$res = \sum_{i=1}^{n} x_i * y_i,$$

where x_i and y_i are elements of complex vectors *x* and *y*.

Input Parameters

n	INTEGER. Specifies the number of elements in vectors *x* and *y*.
x	COMPLEX for cdotu DOUBLE COMPLEX for zdotu Array, DIMENSION at least (1 + (*n* –1)*abs(*incx*)).
incx	INTEGER. Specifies the increment for the elements of *x*.
y	COMPLEX for cdotu DOUBLE COMPLEX for zdotu

	Array, DIMENSION at least (1 + (*n* -1)*abs(*incy*)).
incy	INTEGER. Specifies the increment for the elements of *y*.

Output Parameters

res	COMPLEX for cdotu DOUBLE COMPLEX for zdotu Contains the result of the dot product of *x* and *y*, if *n* is positive. Otherwise, *res* contains 0.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine dotu interface are the following:

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.

?nrm2

Computes the Euclidean norm of a vector.

Syntax

Fortran 77:

res = snrm2(*n*, *x*, *incx*)

res = dnrm2(*n*, *x*, *incx*)

res = scnrm2(*n*, *x*, *incx*)

res = dznrm2(*n*, *x*, *incx*)

Fortran 95:

res = nrm2(*x*)

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?nrm2 routines perform a vector reduction operation defined as

```
res = ||x||,
```

where:

x is a vector,

res is a value containing the Euclidean norm of the elements of *x*.

Input Parameters

n	INTEGER. Specifies the number of elements in vector *x*.
x	REAL for snrm2

DOUBLE PRECISION for dnrm2
COMPLEX for scnrm2
DOUBLE COMPLEX for dznrm2
Array, DIMENSION at least (1 + (*n* -1)*abs (*incx*)).

incx INTEGER. Specifies the increment for the elements of *x*.

Output Parameters

res REAL for snrm2
DOUBLE PRECISION for dnrm2
REAL for scnrm2
DOUBLE PRECISION for dznrm2
Contains the Euclidean norm of the vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine nrm2 interface are the following:

x Holds the vector with the number of elements *n*.

?rot

Performs rotation of points in the plane.

Syntax

Fortran 77:

```
call srot(n, x, incx, y, incy, c, s)
call drot(n, x, incx, y, incy, c, s)
call csrot(n, x, incx, y, incy, c, s)
call zdrot(n, x, incx, y, incy, c, s)
```

Fortran 95:

```
call rot(x, y, c, s)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

Given two complex vectors *x* and *y*, each vector element of these vectors is replaced as follows:

```
x(i) = c*x(i) + s*y(i)
y(i) = c*y(i) - s*x(i)
```

Input Parameters

n INTEGER. Specifies the number of elements in vectors *x* and *y*.

x REAL for srot

	DOUBLE PRECISION for drot COMPLEX for csrot DOUBLE COMPLEX for zdrot Array, DIMENSION at least (1 + (*n*-1)*abs(*incx*)).
incx	INTEGER. Specifies the increment for the elements of *x*.
y	REAL for srot DOUBLE PRECISION for drot COMPLEX for csrot DOUBLE COMPLEX for zdrot Array, DIMENSION at least (1 + (*n* -1)*abs(*incy*)).
incy	INTEGER. Specifies the increment for the elements of *y*.
c	REAL for srot DOUBLE PRECISION for drot REAL for csrot DOUBLE PRECISION for zdrot A scalar.
s	REAL for srot DOUBLE PRECISION for drot REAL for csrot DOUBLE PRECISION for zdrot A scalar.

Output Parameters

x	Each element is replaced by *c***x* + *s***y*.
y	Each element is replaced by *c***y* - *s***x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine rot interface are the following:

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.

?rotg

Computes the parameters for a Givens rotation.

Syntax

Fortran 77:

```
call srotg(a, b, c, s)
call drotg(a, b, c, s)
call crotg(a, b, c, s)
call zrotg(a, b, c, s)
```

Fortran 95:

```
call rotg(a, b, c, s)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

Given the Cartesian coordinates `(a, b)` of a point, these routines return the parameters `c`, `s`, `r`, and `z` associated with the Givens rotation. The parameters `c` and `s` define a unitary matrix such that:

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

The parameter `z` is defined such that if |`a`| > |`b`|, `z` is `s`; otherwise if `c` is not 0 `z` is 1/`c`; otherwise `z` is 1.

See a more accurate LAPACK version `?lartg`.

Input Parameters

`a`	`REAL` for `srotg` `DOUBLE PRECISION` for `drotg` `COMPLEX` for `crotg` `DOUBLE COMPLEX` for `zrotg` Provides the `x`-coordinate of the point `p`.
`b`	`REAL` for `srotg` `DOUBLE PRECISION` for `drotg` `COMPLEX` for `crotg` `DOUBLE COMPLEX` for `zrotg` Provides the `y`-coordinate of the point `p`.

Output Parameters

`a`	Contains the parameter `r` associated with the Givens rotation.
`b`	Contains the parameter `z` associated with the Givens rotation.
`c`	`REAL` for `srotg` `DOUBLE PRECISION` for `drotg` `REAL` for `crotg` `DOUBLE PRECISION` for `zrotg` Contains the parameter `c` associated with the Givens rotation.
`s`	`REAL` for `srotg` `DOUBLE PRECISION` for `drotg` `COMPLEX` for `crotg` `DOUBLE COMPLEX` for `zrotg` Contains the parameter `s` associated with the Givens rotation.

?rotm

Performs modified Givens rotation of points in the plane.

Syntax

Fortran 77:

```
call srotm(n, x, incx, y, incy, param)
```

```
call drotm(n, x, incx, y, incy, param)
```

Fortran 95:

```
call rotm(x, y, param)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

Given two vectors *x* and *y*, each vector element of these vectors is replaced as follows:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = H \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

for *i*=1 to *n*, where *H* is a modified Givens transformation matrix whose values are stored in the *param(2)* through *param(5)* array. See discussion on the *param* argument.

Input Parameters

n	INTEGER. Specifies the number of elements in vectors *x* and *y*.
x	REAL for srotm DOUBLE PRECISION for drotm Array, DIMENSION at least (1 + (*n* -1)*abs(*incx*)).
incx	INTEGER. Specifies the increment for the elements of *x*.
y	REAL for srotm DOUBLE PRECISION for drotm Array, DIMENSION at least (1 + (*n* -1)*abs(*incy*)).
incy	INTEGER. Specifies the increment for the elements of *y*.
param	REAL for srotm DOUBLE PRECISION for drotm Array, DIMENSION 5. The elements of the *param* array are: *param(1)* contains a switch, *flag*. *param(2-5)* contain *h11*, *h21*, *h12*, and *h22*, respectively, the components of the array *H*. Depending on the values of *flag*, the components of *H* are set as follows:

$$flag = -1. : H = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix}$$

$$flag = 1. : H = \begin{bmatrix} h11 & 1. \\ -1 & h22 \end{bmatrix}$$

In the last three cases, the matrix entries of 1., -1., and 0. are assumed based on the value of *flag* and are not required to be set in the *param* vector.

Output Parameters

x	Each element `x(i)` is replaced by *h11*`*x(i) + `*h12*`*y(i).
y	Each element `y(i)` is replaced by *h21*`*x(i) + `*h22*`*y(i).

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `rotm` interface are the following:

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.

?rotmg

Computes the parameters for a modified Givens rotation.

Syntax

Fortran 77:

```
call srotmg(d1, d2, x1, y1, param)
call drotmg(d1, d2, x1, y1, param)
```

Fortran 95:

```
call rotmg(d1, d2, x1, y1, param)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

Given Cartesian coordinates (*x1*, *y1*) of an input vector, these routines compute the components of a modified Givens transformation matrix *H* that zeros the *y*-component of the resulting vector:

$$\begin{bmatrix} x1 \\ 0 \end{bmatrix} = H \begin{bmatrix} x1\sqrt{d1} \\ y1\sqrt{d1} \end{bmatrix}$$

Input Parameters

d1
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the scaling factor for the *x*-coordinate of the input vector.

d2
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the scaling factor for the *y*-coordinate of the input vector.

x1
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the *x*-coordinate of the input vector.

y1
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the *y*-coordinate of the input vector.

Output Parameters

d1
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the first diagonal element of the updated matrix.

d2
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the second diagonal element of the updated matrix.

x1
REAL for srotmg
DOUBLE PRECISION for drotmg
Provides the *x*-coordinate of the rotated vector before scaling.

param
REAL for srotmg
DOUBLE PRECISION for drotmg
Array, DIMENSION 5.
The elements of the *param* array are:
param(1) contains a switch, *flag*. *param(2-5)* contain *h11*, *h21*, *h12*, and *h22*, respectively, the components of the array *H*.
Depending on the values of *flag*, the components of *H* are set as follows:

$$flag = -1. : H = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix}$$

$$flag = 1. : H = \begin{bmatrix} h11 & 1. \\ -1 & h22 \end{bmatrix}$$

In the last three cases, the matrix entries of 1., -1., and 0. are assumed based on the value of *flag* and are not required to be set in the *param* vector.

?scal

Computes the product of a vector by a scalar.

Syntax

Fortran 77:

```
call sscal(n, a, x, incx)
call dscal(n, a, x, incx)
call cscal(n, a, x, incx)
call zscal(n, a, x, incx)
call csscal(n, a, x, incx)
call zdscal(n, a, x, incx)
```

Fortran 95:

```
call scal(x, a)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?scal` routines perform a vector operation defined as

```
x = a*x
```

where:

a is a scalar, *x* is an *n*-element vector.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vector *x*.
a	`REAL` for `sscal` and `csscal` `DOUBLE PRECISION` for `dscal` and `zdscal` `COMPLEX` for `cscal` `DOUBLE COMPLEX` for `zscal` Specifies the scalar *a*.
x	`REAL` for `sscal` `DOUBLE PRECISION` for `dscal` `COMPLEX` for `cscal` and `csscal` `DOUBLE COMPLEX` for `zscal` and `zdscal` Array, `DIMENSION` at least `(1 + (n -1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for the elements of *x*.

Output Parameters

x	Updated vector x.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine scal interface are the following:

x	Holds the vector with the number of elements *n*.

?swap

Swaps a vector with another vector.

Syntax

Fortran 77:

```
call sswap(n, x, incx, y, incy)
call dswap(n, x, incx, y, incy)
call cswap(n, x, incx, y, incy)
call zswap(n, x, incx, y, incy)
```

Fortran 95:

```
call swap(x, y)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

Given two vectors *x* and *y*, the ?swap routines return vectors *y* and *x* swapped, each replacing the other.

Input Parameters

n	INTEGER. Specifies the number of elements in vectors *x* and *y*.
x	REAL for sswap DOUBLE PRECISION for dswap COMPLEX for cswap DOUBLE COMPLEX for zswap Array, DIMENSION at least (1 + (*n*-1)*abs(*incx*)).
incx	INTEGER. Specifies the increment for the elements of *x*.
y	REAL for sswap DOUBLE PRECISION for dswap COMPLEX for cswap DOUBLE COMPLEX for zswap Array, DIMENSION at least (1 + (*n*-1)*abs(*incy*)).
incy	INTEGER. Specifies the increment for the elements of *y*.

Output Parameters

x	Contains the resultant vector *x*, that is, the input vector *y*.
y	Contains the resultant vector *y*, that is, the input vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `swap` interface are the following:

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.

i?amax

Finds the index of the element with maximum absolute value.

Syntax

Fortran 77:

```
index = isamax(n, x, incx)
index = idamax(n, x, incx)
index = icamax(n, x, incx)
index = izamax(n, x, incx)
```

Fortran 95:

```
index = iamax(x)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

This function is declared in `mkl_blas.fi` for FORTRAN 77 interface, in `blas.f90` for Fortran 95 interface, and in `mkl_blas.h` for C interface.

Given a vector *x*, the `i?amax` functions return the position of the vector element `x(i)` that has the largest absolute value for real flavors, or the largest sum `|Re(x(i))|+|Im(x(i))|` for complex flavors.

If *n* is not positive, `0` is returned.

If more than one vector element is found with the same largest absolute value, the index of the first one encountered is returned.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vector *x*.
x	`REAL` for `isamax` `DOUBLE PRECISION` for `idamax` `COMPLEX` for `icamax`

	DOUBLE COMPLEX for izamax Array, DIMENSION at least (1+(*n*-1)*abs(*incx*)).
incx	INTEGER. Specifies the increment for the elements of *x*.

Output Parameters

index	INTEGER. Contains the position of vector element *x* that has the largest absolute value.

Fortran 95 Interface Notes

Functions and routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the function iamax interface are the following:

x	Holds the vector with the number of elements *n*.

i?amin

Finds the index of the element with the smallest absolute value.

Syntax

Fortran 77:

```
index = isamin(n, x, incx)
index = idamin(n, x, incx)
index = icamin(n, x, incx)
index = izamin(n, x, incx)
```

Fortran 95:

```
index = iamin(x)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

This function is declared in mkl_blas.fi for FORTRAN 77 interface, in blas.f90 for Fortran 95 interface, and in mkl_blas.h for C interface.

Given a vector *x*, the i?amin functions return the position of the vector element *x*(*i*) that has the smallest absolute value for real flavors, or the smallest sum |Re(*x*(*i*))|+|Im(*x*(*i*))| for complex flavors.

If *n* is not positive, 0 is returned.

If more than one vector element is found with the same smallest absolute value, the index of the first one encountered is returned.

Input Parameters

n	INTEGER. On entry, *n* specifies the number of elements in vector *x*.
x	REAL for isamin

	DOUBLE PRECISION for idamin COMPLEX for icamin DOUBLE COMPLEX for izamin Array, DIMENSION at least (1+(*n*-1)*abs(*incx*)).
incx	INTEGER. Specifies the increment for the elements of *x*.

Output Parameters

index	INTEGER. Contains the position of vector element *x* that has the smallest absolute value.

Fortran 95 Interface Notes

Functions and routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the function iamin interface are the following:

x	Holds the vector with the number of elements *n*.

?cabs1

Computes absolute value of complex number.

Syntax

Fortran 77:

```
res = scabs1(z)
res = dcabs1(z)
```

Fortran 95:

```
res = cabs1(z)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?cabs1 is an auxiliary routine for a few BLAS Level 1 routines. This routine performs an operation defined as

```
res=|Re(z)|+|Im(z)|,
```

where *z* is a scalar, and *res* is a value containing the absolute value of a complex number *z*.

Input Parameters

z	COMPLEX scalar for scabs1. DOUBLE COMPLEX scalar for dcabs1.

Output Parameters

res	REAL for scabs1. DOUBLE PRECISION for dcabs1. Contains the absolute value of a complex number *z*.

BLAS Level 2 Routines

This section describes BLAS Level 2 routines, which perform matrix-vector operations. Table "BLAS Level 2 Routine Groups and Their Data Types" lists the BLAS Level 2 routine groups and the data types associated with them.

BLAS Level 2 Routine Groups and Their Data Types

Routine Groups	Data Types	Description
?gbmv	s, d, c, z	Matrix-vector product using a general band matrix
gemv	s, d, c, z	Matrix-vector product using a general matrix
?ger	s, d	Rank-1 update of a general matrix
?gerc	c, z	Rank-1 update of a conjugated general matrix
?geru	c, z	Rank-1 update of a general matrix, unconjugated
?hbmv	c, z	Matrix-vector product using a Hermitian band matrix
?hemv	c, z	Matrix-vector product using a Hermitian matrix
?her	c, z	Rank-1 update of a Hermitian matrix
?her2	c, z	Rank-2 update of a Hermitian matrix
?hpmv	c, z	Matrix-vector product using a Hermitian packed matrix
?hpr	c, z	Rank-1 update of a Hermitian packed matrix
?hpr2	c, z	Rank-2 update of a Hermitian packed matrix
?sbmv	s, d	Matrix-vector product using symmetric band matrix
?spmv	s, d	Matrix-vector product using a symmetric packed matrix
?spr	s, d	Rank-1 update of a symmetric packed matrix
?spr2	s, d	Rank-2 update of a symmetric packed matrix
?symv	s, d	Matrix-vector product using a symmetric matrix
?syr	s, d	Rank-1 update of a symmetric matrix
?syr2	s, d	Rank-2 update of a symmetric matrix
?tbmv	s, d, c, z	Matrix-vector product using a triangular band matrix
?tbsv	s, d, c, z	Solution of a linear system of equations with a triangular band matrix
?tpmv	s, d, c, z	Matrix-vector product using a triangular packed matrix
?tpsv	s, d, c, z	Solution of a linear system of equations with a triangular packed matrix
?trmv	s, d, c, z	Matrix-vector product using a triangular matrix
?trsv	s, d, c, z	Solution of a linear system of equations with a triangular matrix

?gbmv

Computes a matrix-vector product using a general band matrix

Syntax

Fortran 77:

```
call sgbmv(trans, m, n, kl, ku, alpha, a, lda, x, incx, beta, y, incy)
call dgbmv(trans, m, n, kl, ku, alpha, a, lda, x, incx, beta, y, incy)
call cgbmv(trans, m, n, kl, ku, alpha, a, lda, x, incx, beta, y, incy)
call zgbmv(trans, m, n, kl, ku, alpha, a, lda, x, incx, beta, y, incy)
```

Fortran 95:

```
call gbmv(a, x, y [,kl] [,m] [,alpha] [,beta] [,trans])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?gbmv` routines perform a matrix-vector operation defined as

`y := alpha*A*x + beta*y`,

or

`y := alpha*A'*x + beta*y`,

or

`y := alpha *conjg(A')*x + beta*y`,

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*n* band matrix, with *kl* sub-diagonals and *ku* super-diagonals.

Input Parameters

trans	CHARACTER*1. Specifies the operation: If *trans*= 'N' or 'n', then *y* := *alpha**A*x + beta*y* If *trans*= 'T' or 't', then *y* := *alpha**A'*x + beta*y* If *trans*= 'C' or 'c', then *y* := *alpha* *conjg(A')*x + beta*y*
m	INTEGER. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	INTEGER. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
kl	INTEGER. Specifies the number of sub-diagonals of the matrix *A*. The value of *kl* must satisfy $0 \le kl$.
ku	INTEGER. Specifies the number of super-diagonals of the matrix *A*. The value of *ku* must satisfy $0 \le ku$.

alpha

REAL for sgbmv
DOUBLE PRECISION for dgbmv
COMPLEX for cgbmv
DOUBLE COMPLEX for zgbmv
Specifies the scalar *alpha*.

a

REAL for sgbmv
DOUBLE PRECISION for dgbmv
COMPLEX for cgbmv
DOUBLE COMPLEX for zgbmv
Array, DIMENSION (*lda*, *n*).
Before entry, the leading (*kl* + *ku* + 1) by *n* part of the array *a* must contain the matrix of coefficients. This matrix must be supplied column-by-column, with the leading diagonal of the matrix in row (*ku* + 1) of the array, the first super-diagonal starting at position 2 in row *ku*, the first sub-diagonal starting at position 1 in row (*ku* + 2), and so on. Elements in the array *a* that do not correspond to elements in the band matrix (such as the top left *ku* by *ku* triangle) are not referenced.
The following program segment transfers a band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
        k = ku + 1 - j
        do 10, i = max(1, j-ku), min(m, j+kl)
           a(k+i, j) = matrix(i,j)
10       continue
20    continue
```

lda

INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least (*kl* + *ku* + 1).

x

REAL for sgbmv
DOUBLE PRECISION for dgbmv
COMPLEX for cgbmv
DOUBLE COMPLEX for zgbmv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)) when *trans* = 'N' or 'n', and at least (1 + (*m* – 1)*abs(*incx*)) otherwise. Before entry, the array *x* must contain the vector *x*.

incx

INTEGER. Specifies the increment for the elements of *x*. *incx* must not be zero.

beta

REAL for sgbmv
DOUBLE PRECISION for dgbmv
COMPLEX for cgbmv
DOUBLE COMPLEX for zgbmv
Specifies the scalar beta. When *beta* is equal to zero, then *y* need not be set on input.

y

REAL for sgbmv
DOUBLE PRECISION for dgbmv
COMPLEX for cgbmv
DOUBLE COMPLEX for zgbmv
Array, DIMENSION at least (1 +(*m* – 1)*abs(*incy*)) when *trans* = 'N' or 'n' and at least (1 +(*n* – 1)*abs(*incy*)) otherwise. Before entry, the incremented array *y* must contain the vector *y*.

incy

INTEGER. Specifies the increment for the elements of *y*.
The value of *incy* must not be zero.

Output Parameters

y	Updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `gbmv` interface are the following:

a	Holds the array *a* of size (*kl+ku+1*, *n*). Contains a banded matrix *m***n*with *kl* lower diagonal and *ku* upper diagonal.
x	Holds the vector with the number of elements *rx*, where *rx = n* if *trans = 'N'*, *rx = m* otherwise.
y	Holds the vector with the number of elements *ry*, where *ry = m* if *trans = 'N'*, *ry = n* otherwise.
trans	Must be `'N'`, `'C'`, or `'T'`. The default value is `'N'`.
kl	If omitted, assumed *kl = ku*, that is, the number of lower diagonals equals the number of the upper diagonals.
ku	Restored as *ku = lda-kl-1*, where *lda* is the leading dimension of matrix *A*.
m	If omitted, assumed *m = n*, that is, a square matrix.
alpha	The default value is 1.
beta	The default value is 0.

?gemv

Computes a matrix-vector product using a general matrix

Syntax

Fortran 77:

```
call sgemv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call dgemv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call cgemv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call zgemv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call scgemv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call dzgemv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
```

Fortran 95:

```
call gemv(a, x, y [,alpha][,beta] [,trans])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The ?gemv routines perform a matrix-vector operation defined as

y := *alpha***A***x* + *beta***y*,

or

y := *alpha***A*'**x* + *beta***y*,

or

y := *alpha**conjg(*A*')**x* + *beta***y*,

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*n* matrix.

Input Parameters

trans	CHARACTER*1. Specifies the operation: if *trans*= 'N' or 'n', then *y* := *alpha***A***x* + *beta***y*; if *trans*= 'T' or 't', then *y* := *alpha***A*'**x* + *beta***y*; if *trans*= 'C' or 'c', then *y* := *alpha* *conjg(*A*')**x* + *beta***y*.
m	INTEGER. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	INTEGER. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
alpha	REAL for sgemv DOUBLE PRECISION for dgemv COMPLEX for cgemv, scgemv DOUBLE COMPLEX for zgemv, dzgemv Specifies the scalar *alpha*.
a	REAL for sgemv, scgemv DOUBLE PRECISION for dgemv, dzgemv COMPLEX for cgemv DOUBLE COMPLEX for zgemv Array, DIMENSION (*lda*, *n*). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).
x	REAL for sgemv DOUBLE PRECISION for dgemv COMPLEX for cgemv, scgemv DOUBLE COMPLEX for zgemv, dzgemv Array, DIMENSION at least (1+(*n*–1)*abs(*incx*)) when *trans* = 'N' or 'n' and at least (1+(*m* – 1)*abs(*incx*)) otherwise. Before entry, the incremented array *x* must contain the vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
beta	REAL for sgemv DOUBLE PRECISION for dgemv COMPLEX for cgemv, scgemv

	DOUBLE COMPLEX for zgemv, dzgemv Specifies the scalar *beta*. When *beta* is set to zero, then *y* need not be set on input.
y	REAL for sgemv DOUBLE PRECISION for dgemv COMPLEX for cgemv, scgemv DOUBLE COMPLEX for zgemv, dzgemv Array, DIMENSION at least (1 +(*m* - 1)*abs(*incy*)) when *trans* = 'N' or 'n' and at least (1 +(*n* - 1)*abs(*incy*)) otherwise. Before entry with non-zero *beta*, the incremented array *y* must contain the vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gemv interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
x	Holds the vector with the number of elements *rx* where *rx* = *n* if *trans* = 'N', *rx* = *m* otherwise.
y	Holds the vector with the number of elements *ry* where *ry* = *m* if *trans* = 'N', *ry* = *n* otherwise.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
alpha	The default value is 1.
beta	The default value is 0.

?ger

Performs a rank-1 update of a general matrix.

Syntax

Fortran 77:

```
call sger(m, n, alpha, x, incx, y, incy, a, lda)
call dger(m, n, alpha, x, incx, y, incy, a, lda)
```

Fortran 95:

```
call ger(a, x, y [,alpha])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?ger routines perform a matrix-vector operation defined as

```
A := alpha*x*y'+ A,
```

where:

alpha is a scalar,

x is an *m*-element vector,

y is an *n*-element vector,

A is an *m*-by-*n* general matrix.

Input Parameters

m	INTEGER. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	INTEGER. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
alpha	REAL for sger DOUBLE PRECISION for dger Specifies the scalar *alpha*.
x	REAL for sger DOUBLE PRECISION for dger Array, DIMENSION at least (1 + (*m* - 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *m*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	REAL for sger DOUBLE PRECISION for dger Array, DIMENSION at least (1 + (*n* - 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.
a	REAL for sger DOUBLE PRECISION for dger Array, DIMENSION (*lda*, *n*). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).

Output Parameters

a	Overwritten by the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ger interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).

x	Holds the vector with the number of elements *m*.
y	Holds the vector with the number of elements *n*.
alpha	The default value is 1.

?gerc

Performs a rank-1 update (conjugated) of a general matrix.

Syntax

Fortran 77:

```
call cgerc(m, n, alpha, x, incx, y, incy, a, lda)
call zgerc(m, n, alpha, x, incx, y, incy, a, lda)
```

Fortran 95:

```
call gerc(a, x, y [,alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?gerc` routines perform a matrix-vector operation defined as

```
A := alpha*x*conjg(y') + A,
```

where:

alpha is a scalar,

x is an *m*-element vector,

y is an *n*-element vector,

A is an *m*-by-*n* matrix.

Input Parameters

m	`INTEGER`. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	`INTEGER`. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
alpha	`COMPLEX` for `cgerc` `DOUBLE COMPLEX` for `zgerc` Specifies the scalar *alpha*.
x	`COMPLEX` for `cgerc` `DOUBLE COMPLEX` for `zgerc` Array, `DIMENSION` at least `(1 + (m - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *m*-element vector *x*.
incx	`INTEGER`. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	`COMPLEX` for `cgerc`

	DOUBLE COMPLEX for zgerc Array, DIMENSION at least (1 + (*n* - 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.
a	COMPLEX for cgerc DOUBLE COMPLEX for zgerc Array, DIMENSION (*lda*, *n*). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).

Output Parameters

a	Overwritten by the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gerc interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
x	Holds the vector with the number of elements *m*.
y	Holds the vector with the number of elements *n*.
alpha	The default value is 1.

?geru

Performs a rank-1 update (unconjugated) of a general matrix.

Syntax

Fortran 77:

```
call cgeru(m, n, alpha, x, incx, y, incy, a, lda)
call zgeru(m, n, alpha, x, incx, y, incy, a, lda)
```

Fortran 95:

```
call geru(a, x, y [,alpha])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?geru routines perform a matrix-vector operation defined as

```
A := alpha*x*y ' + A,
```

where:

alpha is a scalar,

x is an *m*-element vector,

y is an *n*-element vector,

A is an *m*-by-*n* matrix.

Input Parameters

m	INTEGER. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	INTEGER. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
alpha	COMPLEX for cgeru DOUBLE COMPLEX for zgeru Specifies the scalar *alpha*.
x	COMPLEX for cgeru DOUBLE COMPLEX for zgeru Array, DIMENSION at least (1 + (*m* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *m*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	COMPLEX for cgeru DOUBLE COMPLEX for zgeru Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.
a	COMPLEX for cgeru DOUBLE COMPLEX for zgeru Array, DIMENSION (*lda*, *n*). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).

Output Parameters

a	Overwritten by the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine geru interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
x	Holds the vector with the number of elements *m*.
y	Holds the vector with the number of elements *n*.
alpha	The default value is 1.

?hbmv

Computes a matrix-vector product using a Hermitian band matrix.

Syntax

Fortran 77:

```
call chbmv(uplo, n, k, alpha, a, lda, x, incx, beta, y, incy)
call zhbmv(uplo, n, k, alpha, a, lda, x, incx, beta, y, incy)
```

Fortran 95:

```
call hbmv(a, x, y [,uplo][,alpha] [,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?hbmv` routines perform a matrix-vector operation defined as `y := alpha*A*x + beta*y`,

where:

alpha and *beta* are scalars,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian band matrix, with *k* super-diagonals.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the Hermitian band matrix *A* is used: If *uplo* = `'U'` or `'u'`, then the upper triangular part of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangular part of the matrix *A* is used.
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
k	`INTEGER`. Specifies the number of super-diagonals of the matrix *A*. The value of *k* must satisfy $0 \le k$.
alpha	`COMPLEX` for `chbmv` `DOUBLE COMPLEX` for `zhbmv` Specifies the scalar *alpha*.
a	`COMPLEX` for `chbmv` `DOUBLE COMPLEX` for `zhbmv` Array, `DIMENSION (lda, n)`. Before entry with *uplo* = `'U'` or `'u'`, the leading `(k + 1)` by *n* part of the array *a* must contain the upper triangular band part of the Hermitian matrix. The matrix must be supplied column-by-column, with the leading diagonal of the matrix in row `(k + 1)` of the array, the first super-diagonal starting at position 2 in row *k*, and so on. The top left *k* by *k* triangle of the array *a* is not referenced.

The following program segment transfers the upper triangular part of a Hermitian band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
         m = k + 1 - j
         do 10, i = max(1, j - k), j
            a(m + i, j) = matrix(i, j)
10       continue
20    continue
```

Before entry with *uplo* = 'L' or 'l', the leading (*k* + 1) by *n* part of the array *a* must contain the lower triangular band part of the Hermitian matrix, supplied column-by-column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right *k* by *k* triangle of the array *a* is not referenced.
The following program segment transfers the lower triangular part of a Hermitian band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
         m = 1 - j
         do 10, i = j, min( n, j + k )
            a( m + i, j ) = matrix( i, j )
10       continue
20    continue
```

The imaginary parts of the diagonal elements need not be set and are assumed to be zero.

lda — INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least (*k* + 1).

x — COMPLEX for chbmv
DOUBLE COMPLEX for zhbmv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the vector *x*.

incx — INTEGER. Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

beta — COMPLEX for chbmv
DOUBLE COMPLEX for zhbmv
Specifies the scalar *beta*.

y — COMPLEX for chbmv
DOUBLE COMPLEX for zhbmv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the vector *y*.

incy — INTEGER. Specifies the increment for the elements of *y*.
The value of *incy* must not be zero.

Output Parameters

y — Overwritten by the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbmv interface are the following:

a — Holds the array *a* of size (*k+1*,*n*).

x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?hemv

Computes a matrix-vector product using a Hermitian matrix.

Syntax

Fortran 77:

```
call chemv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
call zhemv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
```

Fortran 95:

```
call hemv(a, x, y [,uplo][,alpha] [,beta])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?hemv routines perform a matrix-vector operation defined as

```
y := alpha*A*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian matrix.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *a* is used. If *uplo* = 'U' or 'u', then the upper triangular of the array *a* is used. If *uplo* = 'L' or 'l', then the low triangular of the array *a* is used.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	COMPLEX for chemv DOUBLE COMPLEX for zhemv Specifies the scalar *alpha*.
a	COMPLEX for chemv DOUBLE COMPLEX for zhemv Array, DIMENSION (*lda*, *n*).

	Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *a* is not referenced. The imaginary parts of the diagonal elements need not be set and are assumed to be zero.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).
x	COMPLEX for chemv DOUBLE COMPLEX for zhemv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
beta	COMPLEX for chemv DOUBLE COMPLEX for zhemv Specifies the scalar *beta*. When *beta* is supplied as zero then *y* need not be set on input.
y	COMPLEX for chemv DOUBLE COMPLEX for zhemv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Overwritten by the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hemv interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?her

Performs a rank-1 update of a Hermitian matrix.

Syntax

Fortran 77:

```
call cher(uplo, n, alpha, x, incx, a, lda)
```

```
call zher(uplo, n, alpha, x, incx, a, lda)
```

Fortran 95:

```
call her(a, x [,uplo] [, alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?her` routines perform a matrix-vector operation defined as

```
A := alpha*x*conjg(x') + A,
```

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* Hermitian matrix.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the array *a* is used. If *uplo* = `'U'` or `'u'`, then the upper triangular of the array *a* is used. If *uplo* = `'L'` or `'l'`, then the low triangular of the array *a* is used.
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`REAL` for `cher` `DOUBLE PRECISION` for `zher` Specifies the scalar *alpha*.
x	`COMPLEX` for `cher` `DOUBLE COMPLEX` for `zher` Array, dimension at least `(1 + (n - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	`INTEGER`. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
a	`COMPLEX` for `cher` `DOUBLE COMPLEX` for `zher` Array, `DIMENSION (lda, n)`. Before entry with *uplo* = `'U'` or `'u'`, the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = `'L'` or `'l'`, the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *a* is not referenced. The imaginary parts of the diagonal elements need not be set and are assumed to be zero.
lda	`INTEGER`. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least `max(1, n)`.

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix. The imaginary parts of the diagonal elements are set to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine her interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.

?her2

Performs a rank-2 update of a Hermitian matrix.

Syntax

Fortran 77:

```
call cher2(uplo, n, alpha, x, incx, y, incy, a, lda)
call zher2(uplo, n, alpha, x, incx, y, incy, a, lda)
```

Fortran 95:

```
call her2(a, x, y [,uplo][,alpha])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?her2 routines perform a matrix-vector operation defined as

```
A := alpha *x*conjg(y') + conjg(alpha)*y *conjg(x') + A,
```

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian matrix.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *a* is used. If *uplo* = 'U' or 'u', then the upper triangular of the array *a* is used.

	If *uplo* = 'L' or 'l', then the low triangular of the array *a* is used.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	COMPLEX for cher2 DOUBLE COMPLEX for zher2 Specifies the scalar *alpha*.
x	COMPLEX for cher2 DOUBLE COMPLEX for zher2 Array, DIMENSION at least (1 + (*n* - 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	COMPLEX for cher2 DOUBLE COMPLEX for zher2 Array, DIMENSION at least (1 + (*n* - 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.
a	COMPLEX for cher2 DOUBLE COMPLEX for zher2 Array, DIMENSION (*lda*, *n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *a* is not referenced. The imaginary parts of the diagonal elements need not be set and are assumed to be zero.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix. The imaginary parts of the diagonal elements are set to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine her2 interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.

?hpmv

Computes a matrix-vector product using a Hermitian packed matrix.

Syntax

Fortran 77:

```
call chpmv(uplo, n, alpha, ap, x, incx, beta, y, incy)
call zhpmv(uplo, n, alpha, ap, x, incx, beta, y, incy)
```

Fortran 95:

```
call hpmv(ap, x, y [,uplo][,alpha] [,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?hpmv` routines perform a matrix-vector operation defined as

```
y := alpha*A*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian matrix, supplied in packed form.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*. If *uplo* = `'U'` or `'u'`, then the upper triangular part of the matrix *A* is supplied in the packed array *ap* . If *uplo* = `'L'` or `'l'`, then the low triangular part of the matrix *A* is supplied in the packed array *ap* .
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`COMPLEX` for `chpmv` `DOUBLE COMPLEX` for `zhpmv` Specifies the scalar *alpha*.
ap	`COMPLEX` for `chpmv` `DOUBLE COMPLEX` for `zhpmv` Array, `DIMENSION` at least `((n*(n + 1))/2)`. Before entry with *uplo* = `'U'` or `'u'`, the array *ap* must contain the upper triangular part of the Hermitian matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1, 1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(1, 2)` and *a*`(2, 2)` respectively, and so on. Before entry with *uplo* = `'L'` or `'l'`, the array *ap* must contain the lower triangular part of the Hermitian matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1, 1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(2, 1)` and *a*`(3, 1)` respectively, and so on.

	The imaginary parts of the diagonal elements need not be set and are assumed to be zero.
x	COMPLEX for chpmv DOUBLE PRECISION COMPLEX for zhpmv Array, DIMENSION at least (1 +(*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
beta	COMPLEX for chpmv DOUBLE COMPLEX for zhpmv Specifies the scalar *beta*. When *beta* is equal to zero then *y* need not be set on input.
y	COMPLEX for chpmv DOUBLE COMPLEX for zhpmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Overwritten by the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpmv interface are the following:

ap	Holds the array *ap* of size (*n*(n+1)/2*).
x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?hpr

Performs a rank-1 update of a Hermitian packed matrix.

Syntax

Fortran 77:

```
call chpr(uplo, n, alpha, x, incx, ap)
call zhpr(uplo, n, alpha, x, incx, ap)
```

Fortran 95:

```
call hpr(ap, x [,uplo] [, alpha])
```

Include Files

- FORTRAN 77: mkl_blas.fi

- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?hpr` routines perform a matrix-vector operation defined as

```
A := alpha*x*conjg(x') + A,
```

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* Hermitian matrix, supplied in packed form.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*. If *uplo* = `'U'` or `'u'`, the upper triangular part of the matrix *A* is supplied in the packed array *ap* . If *uplo* = `'L'` or `'l'`, the low triangular part of the matrix *A* is supplied in the packed array *ap* .
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`REAL` for `chpr` `DOUBLE PRECISION` for `zhpr` Specifies the scalar *alpha*.
x	`COMPLEX` for `chpr` `DOUBLE COMPLEX` for `zhpr` Array, `DIMENSION` at least `(1 + (n - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	`INTEGER`. Specifies the increment for the elements of *x*. *incx* must not be zero.
ap	`COMPLEX` for `chpr` `DOUBLE COMPLEX` for `zhpr` Array, `DIMENSION` at least `((n*(n + 1))/2)`. Before entry with *uplo* = `'U'` or `'u'`, the array *ap* must contain the upper triangular part of the Hermitian matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1, 1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(1, 2)` and *a*`(2, 2)` respectively, and so on. Before entry with *uplo* = `'L'` or `'l'`, the array *ap* must contain the lower triangular part of the Hermitian matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1, 1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(2, 1)` and *a*`(3, 1)` respectively, and so on. The imaginary parts of the diagonal elements need not be set and are assumed to be zero.

Output Parameters

ap	With *uplo* = `'U'` or `'u'`, overwritten by the upper triangular part of the updated matrix. With *uplo* = `'L'` or `'l'`, overwritten by the lower triangular part of the updated matrix. The imaginary parts of the diagonal elements are set to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `hpr` interface are the following:

ap	Holds the array *ap* of size (*n*(n+1)/2*).
x	Holds the vector with the number of elements *n*.
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.
alpha	The default value is 1.

?hpr2

Performs a rank-2 update of a Hermitian packed matrix.

Syntax

Fortran 77:

```
call chpr2(uplo, n, alpha, x, incx, y, incy, ap)
call zhpr2(uplo, n, alpha, x, incx, y, incy, ap)
```

Fortran 95:

```
call hpr2(ap, x, y [,uplo][,alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?hpr2` routines perform a matrix-vector operation defined as

```
A := alpha*x*conjg(y') + conjg(alpha)*y*conjg(x') + A,
```

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian matrix, supplied in packed form.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*. If *uplo* = `'U'` or `'u'`, then the upper triangular part of the matrix *A* is supplied in the packed array *ap* . If *uplo* = `'L'` or `'l'`, then the low triangular part of the matrix *A* is supplied in the packed array *ap* .
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`COMPLEX` for `chpr2`

DOUBLE COMPLEX for zhpr2
Specifies the scalar *alpha*.

x
COMPLEX for chpr2
DOUBLE COMPLEX for zhpr2
Array, dimension at least (1 +(*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.

incx
INTEGER. Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

y
COMPLEX for chpr2
DOUBLE COMPLEX for zhpr2
Array, DIMENSION at least (1 +(*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.

incy
INTEGER. Specifies the increment for the elements of *y*.
The value of *incy* must not be zero.

ap
COMPLEX for chpr2
DOUBLE COMPLEX for zhpr2
Array, DIMENSION at least ((*n**(*n* + 1))/2). Before entry with *uplo* = 'U' or 'u', the array *ap* must contain the upper triangular part of the Hermitian matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(1,2) and *a*(2,2) respectively, and so on.
Before entry with *uplo* = 'L' or 'l', the array *ap* must contain the lower triangular part of the Hermitian matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(2,1) and *a*(3,1) respectively, and so on.
The imaginary parts of the diagonal elements need not be set and are assumed to be zero.

Output Parameters

ap
With *uplo* = 'U' or 'u', overwritten by the upper triangular part of the updated matrix.
With *uplo* = 'L' or 'l', overwritten by the lower triangular part of the updated matrix.
The imaginary parts of the diagonal elements need are set to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpr2 interface are the following:

ap
Holds the array *ap* of size (*n**(*n*+1)/2).

x
Holds the vector with the number of elements *n*.

y
Holds the vector with the number of elements *n*.

uplo
Must be 'U' or 'L'. The default value is 'U'.

alpha
The default value is 1.

?sbmv

Computes a matrix-vector product using a symmetric band matrix.

Syntax

Fortran 77:

```
call ssbmv(uplo, n, k, alpha, a, lda, x, incx, beta, y, incy)
call dsbmv(uplo, n, k, alpha, a, lda, x, incx, beta, y, incy)
```

Fortran 95:

```
call sbmv(a, x, y [,uplo][,alpha] [,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?sbmv` routines perform a matrix-vector operation defined as

y := *alpha***A***x* + *beta***y*,

where:

alpha and *beta* are scalars,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* symmetric band matrix, with *k* super-diagonals.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the band matrix *A* is used: if *uplo* = `'U'` or `'u'` - upper triangular part; if *uplo* = `'L'` or `'l'` - low triangular part.
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
k	`INTEGER`. Specifies the number of super-diagonals of the matrix *A*. The value of *k* must satisfy $0 \le k$.
alpha	`REAL` for `ssbmv` `DOUBLE PRECISION` for `dsbmv` Specifies the scalar *alpha*.
a	`REAL` for `ssbmv` `DOUBLE PRECISION` for `dsbmv` Array, `DIMENSION (lda, n)`. Before entry with *uplo* = `'U'` or `'u'`, the leading `(k + 1)` by *n* part of the array *a* must contain the upper triangular band part of the symmetric matrix, supplied column-by-column, with the leading diagonal of the matrix in row `(k + 1)` of the array, the first super-diagonal starting at position 2 in row *k*, and so on. The top left *k* by *k* triangle of the array *a* is not referenced. The following program segment transfers the upper triangular part of a symmetric band matrix from conventional full matrix storage to band storage: `do 20, j = 1, n` `m = k + 1 - j` `do 10, i = max( 1, j - k ), j`

```
            a( m + i, j ) = matrix( i, j )
10        continue
20     continue
```

Before entry with *uplo* = 'L' or 'l', the leading (*k* + 1) by *n* part of the array *a* must contain the lower triangular band part of the symmetric matrix, supplied column-by-column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right *k* by *k* triangle of the array *a* is not referenced. The following program segment transfers the lower triangular part of a symmetric band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
         m = 1 - j
         do 10, i = j, min( n, j + k )
            a( m + i, j ) = matrix( i, j )
10        continue
20     continue
```

lda — INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least (*k* + 1).

x — REAL for ssbmv
DOUBLE PRECISION for dsbmv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the vector *x*.

incx — INTEGER. Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

beta — REAL for ssbmv
DOUBLE PRECISION for dsbmv
Specifies the scalar *beta*.

y — REAL for ssbmv
DOUBLE PRECISION for dsbmv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the vector *y*.

incy — INTEGER. Specifies the increment for the elements of *y*.
The value of *incy* must not be zero.

Output Parameters

y — Overwritten by the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbmv interface are the following:

a — Holds the array *a* of size (*k+1*,*n*).

x — Holds the vector with the number of elements *n*.

y — Holds the vector with the number of elements *n*.

uplo — Must be 'U' or 'L'. The default value is 'U'.

alpha — The default value is 1.

beta — The default value is 0.

?spmv

Computes a matrix-vector product using a symmetric packed matrix.

Syntax

Fortran 77:

```
call sspmv(uplo, n, alpha, ap, x, incx, beta, y, incy)
call dspmv(uplo, n, alpha, ap, x, incx, beta, y, incy)
```

Fortran 95:

```
call spmv(ap, x, y [,uplo][,alpha] [,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?spmv` routines perform a matrix-vector operation defined as

```
y := alpha*A*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* symmetric matrix, supplied in packed form.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*. If *uplo* = `'U'` or `'u'`, then the upper triangular part of the matrix *A* is supplied in the packed array *ap* . If *uplo* = `'L'` or `'l'`, then the low triangular part of the matrix *A* is supplied in the packed array *ap* .
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`REAL` for `sspmv` `DOUBLE PRECISION` for `dspmv` Specifies the scalar *alpha*.
ap	`REAL` for `sspmv` `DOUBLE PRECISION` for `dspmv` Array, `DIMENSION` at least `((n*(n + 1))/2)`. Before entry with *uplo* = `'U'` or `'u'`, the array *ap* must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1,1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(1,2)` and *a*`(2, 2)` respectively, and so on. Before entry with *uplo* = `'L'` or `'l'`, the array *ap* must contain the lower triangular part of the symmetric

	matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(2,1) and *a*(3,1) respectively, and so on.
x	REAL for sspmv DOUBLE PRECISION for dspmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
beta	REAL for sspmv DOUBLE PRECISION for dspmv Specifies the scalar *beta*. When *beta* is supplied as zero, then *y* need not be set on input.
y	REAL for sspmv DOUBLE PRECISION for dspmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Overwritten by the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spmv interface are the following:

ap	Holds the array *ap* of size (*n**(n+1)/2).
x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?spr

Performs a rank-1 update of a symmetric packed matrix.

Syntax

Fortran 77:

```
call sspr(uplo, n, alpha, x, incx, ap)
call dspr(uplo, n, alpha, x, incx, ap)
```

Fortran 95:

```
call spr(ap, x [,uplo] [, alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?spr` routines perform a matrix-vector operation defined as

```
a:= alpha*x*x'+ A,
```

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* symmetric matrix, supplied in packed form.

Input Parameters

uplo
`CHARACTER*1`. Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*.
If *uplo* = `'U'` or `'u'`, then the upper triangular part of the matrix *A* is supplied in the packed array *ap* .
If *uplo* = `'L'` or `'l'`, then the low triangular part of the matrix *A* is supplied in the packed array *ap* .

n
`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.

alpha
`REAL` for `sspr`
`DOUBLE PRECISION` for `dspr`
Specifies the scalar *alpha*.

x
`REAL` for `sspr`
`DOUBLE PRECISION` for `dspr`
Array, `DIMENSION` at least `(1 + (n - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *n*-element vector *x*.

incx
`INTEGER`. Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

ap
`REAL` for `sspr`
`DOUBLE PRECISION` for `dspr`
Array, `DIMENSION` at least `((n*(n + 1))/2)`. Before entry with *uplo* = `'U'` or `'u'`, the array *ap* must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1,1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(1,2)` and *a*`(2,2)` respectively, and so on.
Before entry with *uplo* = `'L'` or `'l'`, the array *ap* must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *a*`(1,1)`, *ap*`(2)` and *ap*`(3)` contain *a*`(2,1)` and *a*`(3,1)` respectively, and so on.

Output Parameters

ap
With *uplo* = `'U'` or `'u'`, overwritten by the upper triangular part of the updated matrix.

With *uplo* = 'L' or 'l', overwritten by the lower triangular part of the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spr interface are the following:

ap	Holds the array *ap* of size (*n*(n+1)/2*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.

?spr2

Performs a rank-2 update of a symmetric packed matrix.

Syntax

Fortran 77:

```
call sspr2(uplo, n, alpha, x, incx, y, incy, ap)
call dspr2(uplo, n, alpha, x, incx, y, incy, ap)
```

Fortran 95:

```
call spr2(ap, x, y [,uplo][,alpha])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?spr2 routines perform a matrix-vector operation defined as

```
A:= alpha*x*y'+ alpha*y*x' + A,
```

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* symmetric matrix, supplied in packed form.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*. If *uplo* = 'U' or 'u', then the upper triangular part of the matrix *A* is supplied in the packed array *ap* . If *uplo* = 'L' or 'l', then the low triangular part of the matrix *A* is supplied in the packed array *ap* .

n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	REAL for sspr2 DOUBLE PRECISION for dspr2 Specifies the scalar *alpha*.
x	REAL for sspr2 DOUBLE PRECISION for dspr2 Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	REAL for sspr2 DOUBLE PRECISION for dspr2 Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.
ap	REAL for sspr2 DOUBLE PRECISION for dspr2 Array, DIMENSION at least ((*n**(*n* + 1))/2). Before entry with *uplo* = 'U' or 'u', the array *ap* must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(1,2) and *a*(2,2) respectively, and so on. Before entry with *uplo* = 'L' or 'l', the array *ap* must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a* (2,1) and *a*(3,1) respectively, and so on.

Output Parameters

ap	With *uplo* = 'U' or 'u', overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', overwritten by the lower triangular part of the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spr2 interface are the following:

ap	Holds the array *ap* of size (*n*(n+1)/2*).
x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.

?symv

Computes a matrix-vector product for a symmetric matrix.

Syntax

Fortran 77:

```
call ssymv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
call dsymv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
```

Fortran 95:

```
call symv(a, x, y [,uplo][,alpha] [,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?symv` routines perform a matrix-vector operation defined as

```
y := alpha*A*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* symmetric matrix.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the array *a* is used. If *uplo* = `'U'` or `'u'`, then the upper triangular part of the array *a* is used. If *uplo* = `'L'` or `'l'`, then the low triangular part of the array *a* is used.
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`REAL` for `ssymv` `DOUBLE PRECISION` for `dsymv` Specifies the scalar *alpha*.
a	`REAL` for `ssymv` `DOUBLE PRECISION` for `dsymv` Array, `DIMENSION (lda, n)`. Before entry with *uplo* = `'U'` or `'u'`, the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix *A* and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = `'L'` or `'l'`, the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix *A* and the strictly upper triangular part of *a* is not referenced.
lda	`INTEGER`. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least `max(1, n)`.
x	`REAL` for `ssymv` `DOUBLE PRECISION` for `dsymv` Array, `DIMENSION` at least `(1 + (n - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *n*-element vector *x*.

incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
beta	REAL for ssymv DOUBLE PRECISION for dsymv Specifies the scalar *beta*. When *beta* is supplied as zero, then *y* need not be set on input.
y	REAL for ssymv DOUBLE PRECISION for dsymv Array, DIMENSION at least (1 + (*n* - 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Overwritten by the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine symv interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
y	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?syr

Performs a rank-1 update of a symmetric matrix.

Syntax

Fortran 77:

```
call ssyr(uplo, n, alpha, x, incx, a, lda)
call dsyr(uplo, n, alpha, x, incx, a, lda)
```

Fortran 95:

```
call syr(a, x [,uplo] [, alpha])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?syr routines perform a matrix-vector operation defined as

*A := alpha*x*x' + A*,

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* symmetric matrix.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *a* is used. If *uplo* = 'U' or 'u', then the upper triangular part of the array *a* is used. If *uplo* = 'L' or 'l', then the low triangular part of the array *a* is used.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	REAL for ssyr DOUBLE PRECISION for dsyr Specifies the scalar *alpha*.
x	REAL for ssyr DOUBLE PRECISION for dsyr Array, DIMENSION at least (1 + (*n*-1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
a	REAL for ssyr DOUBLE PRECISION for dsyr Array, DIMENSION (*lda*, *n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix *A* and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix *A* and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syr interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.

?syr2

Performs a rank-2 update of symmetric matrix.

Syntax

Fortran 77:

```
call ssyr2(uplo, n, alpha, x, incx, y, incy, a, lda)
call dsyr2(uplo, n, alpha, x, incx, y, incy, a, lda)
```

Fortran 95:

```
call syr2(a, x, y [,uplo][,alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?syr2` routines perform a matrix-vector operation defined as

```
A := alpha*x*y'+ alpha*y*x' + A,
```

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* symmetric matrix.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the array *a* is used. If *uplo* = `'U'` or `'u'`, then the upper triangular part of the array *a* is used. If *uplo* = `'L'` or `'l'`, then the low triangular part of the array *a* is used.
n	`INTEGER`. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
alpha	`REAL` for `ssyr2` `DOUBLE PRECISION` for `dsyr2` Specifies the scalar *alpha*.
x	`REAL` for `ssyr2` `DOUBLE PRECISION` for `dsyr2` Array, `DIMENSION` at least `(1 + (n - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	`INTEGER`. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	`REAL` for `ssyr2` `DOUBLE PRECISION` for `dsyr2` Array, `DIMENSION` at least `(1 + (n - 1)*abs(incy))`. Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	`INTEGER`. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

a	REAL for ssyr2 DOUBLE PRECISION for dsyr2 Array, DIMENSION (*lda*, *n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syr2 interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector *x* of length *n*.
y	Holds the vector *y* of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.

?tbmv

Computes a matrix-vector product using a triangular band matrix.

Syntax

Fortran 77:

```
call stbmv(uplo, trans, diag, n, k, a, lda, x, incx)
call dtbmv(uplo, trans, diag, n, k, a, lda, x, incx)
call ctbmv(uplo, trans, diag, n, k, a, lda, x, incx)
call ztbmv(uplo, trans, diag, n, k, a, lda, x, incx)
```

Fortran 95:

```
call tbmv(a, x [,uplo] [, trans] [,diag])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?tbmv routines perform one of the matrix-vector operations defined as

x := *A***x*, or *x* := *A*'**x*, or *x* := conj(*A*')**x*,

where:

x is an *n*-element vector,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular band matrix, with (*k* +1) diagonals.

Input Parameters

uplo — CHARACTER*1. Specifies whether the matrix *A* is an upper or lower triangular matrix:
if *uplo* = 'U' or 'u', then the matrix is upper triangular;
if *uplo* = 'L' or 'l', then the matrix is low triangular.

trans — CHARACTER*1. Specifies the operation:
if *trans* = 'N' or 'n', then *x* := *A***x*;
if *trans* = 'T' or 't', then *x* := *A*'**x*;
if *trans* = 'C' or 'c', then *x* := conj(*A*')**x*.

diag — CHARACTER*1. Specifies whether the matrix *A* is unit triangular:
if *diag* = 'U' or 'u' then the matrix is unit triangular;
if *diag* = 'N' or 'n', then the matrix is not unit triangular.

n — INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.

k — INTEGER. On entry with *uplo* = 'U' or 'u', *k* specifies the number of super-diagonals of the matrix *A*. On entry with *uplo* = 'L' or 'l', *k* specifies the number of sub-diagonals of the matrix *a*.
The value of *k* must satisfy $0 \le k$.

a — REAL for stbmv
DOUBLE PRECISION for dtbmv
COMPLEX for ctbmv
DOUBLE COMPLEX for ztbmv
Array, DIMENSION (*lda*, *n*).
Before entry with *uplo* = 'U' or 'u', the leading (*k* + 1) by *n* part of the array *a* must contain the upper triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row (*k* + 1) of the array, the first super-diagonal starting at position 2 in row *k*, and so on. The top left *k* by *k* triangle of the array *a* is not referenced. The following program segment transfers an upper triangular band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
       m = k + 1 - j
      do 10, i = max(1, j - k), j
        a(m + i, j) = matrix(i, j)
10    continue
20    continue
```

Before entry with *uplo* = 'L' or 'l', the leading (*k* + 1) by *n* part of the array *a* must contain the lower triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row1 of the array, the first sub-diagonal starting at position 1 in

row 2, and so on. The bottom right *k* by *k* triangle of the array *a* is not referenced. The following program segment transfers a lower triangular band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
        m = 1 - j
        do 10, i = j, min(n, j + k)
          a(m + i, j) = matrix (i, j)
10      continue
20    continue
```

Note that when *diag* = 'U' or 'u', the elements of the array *a* corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity.

lda — INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least (*k* + 1).

x — REAL for stbmv
DOUBLE PRECISION for dtbmv
COMPLEX for ctbmv
DOUBLE COMPLEX for ztbmv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.

incx — INTEGER. Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

Output Parameters

x — Overwritten with the transformed vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tbmv interface are the following:

a — Holds the array *a* of size (*k+1,n*).

x — Holds the vector with the number of elements *n*.

uplo — Must be 'U' or 'L'. The default value is 'U'.

trans — Must be 'N', 'C', or 'T'.
The default value is 'N'.

diag — Must be 'N' or 'U'. The default value is 'N'.

?tbsv

Solves a system of linear equations whose coefficients are in a triangular band matrix.

Syntax

Fortran 77:

```
call stbsv(uplo, trans, diag, n, k, a, lda, x, incx)
call dtbsv(uplo, trans, diag, n, k, a, lda, x, incx)
call ctbsv(uplo, trans, diag, n, k, a, lda, x, incx)
call ztbsv(uplo, trans, diag, n, k, a, lda, x, incx)
```

Fortran 95:

```
call tbsv(a, x [,uplo] [, trans] [,diag])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?tbsv routines solve one of the following systems of equations:

*A***x* = *b*, or *A*'**x* = *b*, or conjg(*A*')**x* = *b*,

where:

b and *x* are *n*-element vectors,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular band matrix, with (*k* + 1) diagonals.

The routine does not test for singularity or near-singularity.

Such tests must be performed before calling this routine.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix *A* is an upper or lower triangular matrix: if *uplo* = 'U' or 'u' the matrix is upper triangular; if *uplo* = 'L' or 'l', the matrix is low triangular.
trans	CHARACTER*1. Specifies the system of equations: if *trans* = 'N' or 'n', then *A***x* = *b*; if *trans* = 'T' or 't', then *A*'**x* = *b*; if *trans* = 'C' or 'c', then conjg(*A*')**x* = *b*.
diag	CHARACTER*1. Specifies whether the matrix *A* is unit triangular: if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
k	INTEGER. On entry with *uplo* = 'U' or 'u', *k* specifies the number of super-diagonals of the matrix *A*. On entry with *uplo* = 'L' or 'l', *k* specifies the number of sub-diagonals of the matrix *A*. The value of *k* must satisfy 0 ≤ *k*.
a	REAL for stbsv DOUBLE PRECISION for dtbsv COMPLEX for ctbsv DOUBLE COMPLEX for ztbsv Array, DIMENSION (*lda*, *n*). Before entry with *uplo* = 'U' or 'u', the leading (*k* + 1) by *n* part of the array *a* must contain the upper triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row (*k* + 1) of the array, the first super-diagonal starting at position 2 in row *k*, and so on. The top left *k* by *k* triangle of the array *a* is not referenced.

The following program segment transfers an upper triangular band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
        m = k + 1 - j
        do 10, i = max(1, j - k), jl
          a(m + i, j) = matrix (i, j)
         10 continue
      20 continue
```

Before entry with *uplo* = 'L' or 'l', the leading (*k* + 1) by *n* part of the array *a* must contain the lower triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row 1 of the array, the first sub-diagonal starting at position 1 in row 2, and so on. The bottom right *k* by *k* triangle of the array *a* is not referenced.
The following program segment transfers a lower triangular band matrix from conventional full matrix storage to band storage:

```
      do 20, j = 1, n
         m = 1 - j
         do 10, i = j, min(n, j + k)
          a(m + i, j) = matrix (i, j)
        10 continue
      20 continue
```

When *diag* = 'U' or 'u', the elements of the array *a* corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity.

lda — INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least (*k* + 1).

x — REAL for stbsv
DOUBLE PRECISION for dtbsv
COMPLEX for ctbsv
DOUBLE COMPLEX for ztbsv
Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element right-hand side vector *b*.

incx — INTEGER. Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

Output Parameters

x — Overwritten with the solution vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tbsv interface are the following:

a — Holds the array *a* of size (*k+1*,*n*).

x — Holds the vector with the number of elements *n*.

uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

?tpmv

Computes a matrix-vector product using a triangular packed matrix.

Syntax

Fortran 77:

```
call stpmv(uplo, trans, diag, n, ap, x, incx)
call dtpmv(uplo, trans, diag, n, ap, x, incx)
call ctpmv(uplo, trans, diag, n, ap, x, incx)
call ztpmv(uplo, trans, diag, n, ap, x, incx)
```

Fortran 95:

```
call tpmv(ap, x [,uplo] [, trans] [,diag])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?tpmv routines perform one of the matrix-vector operations defined as

x := *A***x*, or *x* := *A*'**x*, or *x* := conjg(*A*')**x*,

where:

x is an *n*-element vector,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix *A* is upper or lower triangular: if *uplo* = 'U' or 'u', then the matrix is upper triangular; if *uplo* = 'L' or 'l', then the matrix is low triangular.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then *x* := *A***x*; if *trans* = 'T' or 't', then *x* := *A*'**x*; if *trans* = 'C' or 'c', then *x* := conjg(*A*')**x*.
diag	CHARACTER*1. Specifies whether the matrix *A* is unit triangular: if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
ap	REAL for stpmv

	DOUBLE PRECISION for dtpmv COMPLEX for ctpmv DOUBLE COMPLEX for ztpmv Array, DIMENSION at least ((*n**(*n* + 1))/2). Before entry with *uplo* = 'U' or 'u', the array *ap* must contain the upper triangular matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(1,2) and *a*(2,2) respectively, and so on. Before entry with *uplo* = 'L' or 'l', the array *ap* must contain the lower triangular matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(2,1) and *a*(3,1) respectively, and so on. When *diag* = 'U' or 'u', the diagonal elements of *a* are not referenced, but are assumed to be unity.
x	REAL for stpmv DOUBLE PRECISION for dtpmv COMPLEX for ctpmv DOUBLE COMPLEX for ztpmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.

Output Parameters

x	Overwritten with the transformed vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tpmv interface are the following:

ap	Holds the array *ap* of size (*n*(n+1)/2*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

?tpsv

Solves a system of linear equations whose coefficients are in a triangular packed matrix.

Syntax

Fortran 77:

```
call stpsv(uplo, trans, diag, n, ap, x, incx)
call dtpsv(uplo, trans, diag, n, ap, x, incx)
call ctpsv(uplo, trans, diag, n, ap, x, incx)
call ztpsv(uplo, trans, diag, n, ap, x, incx)
```

Fortran 95:

```
call tpsv(ap, x [,uplo] [, trans] [,diag])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?tpsv` routines solve one of the following systems of equations

`A*x = b`, or `A'*x = b`, or `conjg(A')*x = b`,

where:

`b` and `x` are `n`-element vectors,

`A` is an `n`-by-`n` unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

This routine does not test for singularity or near-singularity.

Such tests must be performed before calling this routine.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the matrix `A` is upper or lower triangular: if `uplo = 'U'` or `'u'`, then the matrix is upper triangular; if `uplo = 'L'` or `'l'`, then the matrix is low triangular.
trans	`CHARACTER*1`. Specifies the system of equations: if `trans = 'N'` or `'n'`, then `A*x = b`; if `trans = 'T'` or `'t'`, then `A'*x = b`; if `trans = 'C'` or `'c'`, then `conjg(A')*x = b`.
diag	`CHARACTER*1`. Specifies whether the matrix `A` is unit triangular: if `diag = 'U'` or `'u'` then the matrix is unit triangular; if `diag = 'N'` or `'n'`, then the matrix is not unit triangular.
n	`INTEGER`. Specifies the order of the matrix `A`. The value of `n` must be at least zero.
ap	`REAL` for `stpsv` `DOUBLE PRECISION` for `dtpsv` `COMPLEX` for `ctpsv` `DOUBLE COMPLEX` for `ztpsv` Array, `DIMENSION` at least `((n*(n + 1))/2)`. Before entry with `uplo = 'U'` or `'u'`, the array `ap` must contain the upper triangular matrix packed sequentially, column-by-column, so that `ap(1)` contains `a(1, +1)`, `ap(2)` and `ap(3)` contain `a(1, 2)` and `a(2, 2)` respectively, and so on. Before entry with `uplo = 'L'` or `'l'`, the array `ap` must contain the lower triangular matrix packed sequentially, column-by-column, so that `ap(1)` contains `a(1, +1)`, `ap(2)` and `ap(3)` contain `a(2, +1)` and `a(3, +1)` respectively, and so on. When `diag = 'U'` or `'u'`, the diagonal elements of `a` are not referenced, but are assumed to be unity.
x	`REAL` for `stpsv` `DOUBLE PRECISION` for `dtpsv` `COMPLEX` for `ctpsv`

	DOUBLE COMPLEX for ztpsv Array, DIMENSION at least (1 + (*n* - 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element right-hand side vector *b*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.

Output Parameters

x	Overwritten with the solution vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tpsv interface are the following:

ap	Holds the array *ap* of size (*n*(n+1)/2*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

?trmv

Computes a matrix-vector product using a triangular matrix.

Syntax

Fortran 77:

```
call strmv(uplo, trans, diag, n, a, lda, x, incx)
call dtrmv(uplo, trans, diag, n, a, lda, x, incx)
call ctrmv(uplo, trans, diag, n, a, lda, x, incx)
call ztrmv(uplo, trans, diag, n, a, lda, x, incx)
```

Fortran 95:

```
call trmv(a, x [,uplo] [, trans] [,diag])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?trmv routines perform one of the following matrix-vector operations defined as

x := *A*x*, or *x* := A'*x*, or *x* := conjg(*A*')*x*,

where:

x is an *n*-element vector,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular matrix.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix *A* is upper or lower triangular: if *uplo* = 'U' or 'u', then the matrix is upper triangular; if *uplo* = 'L' or 'l', then the matrix is low triangular.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then *x* := *A***x*; if *trans* = 'T' or 't', then *x* := *A*'**x*; if *trans* = 'C' or 'c', then *x* := conjg(*A*')**x*.
diag	CHARACTER*1. Specifies whether the matrix *A* is unit triangular: if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
a	REAL for strmv DOUBLE PRECISION for dtrmv COMPLEX for ctrmv DOUBLE COMPLEX for ztrmv Array, DIMENSION (*lda*,*n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular matrix and the strictly upper triangular part of *a* is not referenced. When *diag* = 'U' or 'u', the diagonal elements of *a* are not referenced either, but are assumed to be unity.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).
x	REAL for strmv DOUBLE PRECISION for dtrmv COMPLEX for ctrmv DOUBLE COMPLEX for ztrmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.

Output Parameters

x	Overwritten with the transformed vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trmv interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'.

	The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

?trsv

Solves a system of linear equations whose coefficients are in a triangular matrix.

Syntax

Fortran 77:

```
call strsv(uplo, trans, diag, n, a, lda, x, incx)
call dtrsv(uplo, trans, diag, n, a, lda, x, incx)
call ctrsv(uplo, trans, diag, n, a, lda, x, incx)
call ztrsv(uplo, trans, diag, n, a, lda, x, incx)
```

Fortran 95:

```
call trsv(a, x [,uplo] [, trans] [,diag])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?trsv routines solve one of the systems of equations:

*A*x = b*, or *A'*x = b*, or conj(*A'*)*x = b*,

where:

b and *x* are *n*-element vectors,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular matrix.

The routine does not test for singularity or near-singularity.

Such tests must be performed before calling this routine.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix *A* is upper or lower triangular: if *uplo* = 'U' or 'u', then the matrix is upper triangular; if *uplo* = 'L' or 'l', then the matrix is low triangular.
trans	CHARACTER*1. Specifies the systems of equations: if *trans* = 'N' or 'n', then *A*x = b*; if *trans* = 'T' or 't', then *A'*x = b*; if *trans* = 'C' or 'c', then oconj(*A'*)*x = b*.
diag	CHARACTER*1. Specifies whether the matrix *A* is unit triangular: if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
n	INTEGER. Specifies the order of the matrix *A*. The value of *n* must be at least zero.
a	REAL for strsv

	DOUBLE PRECISION for dtrsv COMPLEX for ctrsv DOUBLE COMPLEX for ztrsv Array, DIMENSION (*lda*,*n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular matrix and the strictly upper triangular part of *a* is not referenced. When *diag* = 'U' or 'u', the diagonal elements of *a* are not referenced either, but are assumed to be unity.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).
x	REAL for strsv DOUBLE PRECISION for dtrsv COMPLEX for ctrsv DOUBLE COMPLEX for ztrsv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element right-hand side vector *b*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.

Output Parameters

x	Overwritten with the solution vector *x*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trsv interface are the following:

a	Holds the matrix *a* of size (*n*,*n*).
x	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

BLAS Level 3 Routines

BLAS Level 3 routines perform matrix-matrix operations. Table "BLAS Level 3 Routine Groups and Their Data Types" lists the BLAS Level 3 routine groups and the data types associated with them.

BLAS Level 3 Routine Groups and Their Data Types

Routine Group	Data Types	Description
?gemm	s, d, c, z	Matrix-matrix product of general matrices
?hemm	c, z	Matrix-matrix product of Hermitian matrices
?herk	c, z	Rank-k update of Hermitian matrices

Routine Group	Data Types	Description
?her2k	c, z	Rank-2k update of Hermitian matrices
?symm	s, d, c, z	Matrix-matrix product of symmetric matrices
?syrk	s, d, c, z	Rank-k update of symmetric matrices
?syr2k	s, d, c, z	Rank-2k update of symmetric matrices
?trmm	s, d, c, z	Matrix-matrix product of triangular matrices
?trsm	s, d, c, z	Linear matrix-matrix solution for triangular matrices

Symmetric Multiprocessing Version of Intel® MKL

Many applications spend considerable time executing BLAS routines. This time can be scaled by the number of processors available on the system through using the symmetric multiprocessing (SMP) feature built into the Intel MKL Library. The performance enhancements based on the parallel use of the processors are available without any programming effort on your part.

To enhance performance, the library uses the following methods:

- The BLAS functions are blocked where possible to restructure the code in a way that increases the localization of data reference, enhances cache memory use, and reduces the dependency on the memory bus.
- The code is distributed across the processors to maximize parallelism.

?gemm

Computes a scalar-matrix-matrix product and adds the result to a scalar-matrix product.

Syntax

Fortran 77:

```
call sgemm(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call dgemm(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call cgemm(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call zgemm(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call scgemm(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call dzgemm(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
```

Fortran 95:

```
call gemm(a, b, c [,transa][,transb] [,alpha][,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?gemm` routines perform a matrix-matrix operation with general matrices. The operation is defined as

```
C := alpha*op(A)*op(B) + beta*C,
```

where:

`op(x)` is one of `op(x) = x`, or `op(x) = x'`, or `op(x) = conjg(x')`,

alpha and *beta* are scalars,

A, *B* and *C* are matrices:

`op(A)` is an *m*-by-*k* matrix,

`op(B)` is a *k*-by-*n* matrix,

C is an *m*-by-*n* matrix.

See also `?gemm3m`, BLAS-like extension routines, that use matrix multiplication for similar matrix-matrix operations.

Input Parameters

transa	`CHARACTER*1`. Specifies the form of `op(A)` used in the matrix multiplication: if *transa* = `'N'` or `'n'`, then `op(A) = A`; if *transa* = `'T'` or `'t'`, then `op(A) = A'`; if *transa* = `'C'` or `'c'`, then `op(A) = conjg(A')`.
transb	`CHARACTER*1`. Specifies the form of `op(B)` used in the matrix multiplication: if *transb* = `'N'` or `'n'`, then `op(B) = B`; if *transb* = `'T'` or `'t'`, then `op(B) = B'`; if *transb* = `'C'` or `'c'`, then `op(B) = conjg(B')`.
m	`INTEGER`. Specifies the number of rows of the matrix `op(A)` and of the matrix *C*. The value of *m* must be at least zero.
n	`INTEGER`. Specifies the number of columns of the matrix `op(B)` and the number of columns of the matrix *C*. The value of *n* must be at least zero.
k	`INTEGER`. Specifies the number of columns of the matrix `op(A)` and the number of rows of the matrix `op(B)`. The value of *k* must be at least zero.
alpha	`REAL` for `sgemm` `DOUBLE PRECISION` for `dgemm` `COMPLEX` for `cgemm`, `scgemm` `DOUBLE COMPLEX` for `zgemm`, `dzgemm` Specifies the scalar *alpha*.
a	`REAL` for `sgemm`, `scgemm` `DOUBLE PRECISION` for `dgemm`, `dzgemm` `COMPLEX` for `cgemm` `DOUBLE COMPLEX` for `zgemm`

Array, DIMENSION (*lda*, *ka*), where *ka* is *k* when *transa* = 'N' or 'n', and is *m* otherwise. Before entry with *transa* = 'N' or 'n', the leading *m*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*m* part of the array *a* must contain the matrix *A*.

lda — INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *transa* = 'N' or 'n', then *lda* must be at least max(1, *m*), otherwise *lda* must be at least max(1, *k*).

b — REAL for sgemm
DOUBLE PRECISION for dgemm
COMPLEX for cgemm, scgemm
DOUBLE COMPLEX for zgemm, dzgemm
Array, DIMENSION (*ldb*, *kb*), where *kb* is *n* when *transb* = 'N' or 'n', and is *k* otherwise. Before entry with *transb* = 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *n*-by-*k* part of the array *b* must contain the matrix *B*.

ldb — INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. When *transb* = 'N' or 'n', then *ldb* must be at least max(1, *k*), otherwise *ldb* must be at least max(1, *n*).

beta — REAL for sgemm
DOUBLE PRECISION for dgemm
COMPLEX for cgemm, scgemm
DOUBLE COMPLEX for zgemm, dzgemm
Specifies the scalar *beta*.
When *beta* is equal to zero, then *c* need not be set on input.

c — REAL for sgemm
DOUBLE PRECISION for dgemm
COMPLEX for cgemm, scgemm
DOUBLE COMPLEX for zgemm, dzgemm
Array, DIMENSION (*ldc*, *n*).
Before entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, except when *beta* is equal to zero, in which case *c* need not be set on entry.

ldc — INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *m*).

Output Parameters

c — Overwritten by the *m*-by-*n* matrix (*alpha**op(*A*)*op(*B*) + *beta***C*).

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gemm interface are the following:

a — Holds the matrix *A* of size (*ma*,*ka*) where
ka = *k* if *transa*= 'N',
ka = *m* otherwise,
ma = *m* if *transa*= 'N',
ma = *k* otherwise.

b — Holds the matrix *B* of size (*mb*,*kb*) where

	kb = *n* if *transb* = 'N', *kb* = *k* otherwise, *mb* = *k* if *transb* = 'N', *mb* = *n* otherwise.
c	Holds the matrix *C* of size (*m*,*n*).
transa	Must be 'N', 'C', or 'T'. The default value is 'N'.
transb	Must be 'N', 'C', or 'T'. The default value is 'N'.
alpha	The default value is 1.
beta	The default value is 0.

?hemm

Computes a scalar-matrix-matrix product (either one of the matrices is Hermitian) and adds the result to scalar-matrix product.

Syntax

Fortran 77:

```
call chemm(side, uplo, m, n, alpha, a, lda, b, ldb, beta, c, ldc)
call zhemm(side, uplo, m, n, alpha, a, lda, b, ldb, beta, c, ldc)
```

Fortran 95:

```
call hemm(a, b, c [,side][,uplo] [,alpha][,beta])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?hemm routines perform a matrix-matrix operation using Hermitian matrices. The operation is defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*B*A + beta*C,
```

where:

alpha and *beta* are scalars,

A is an Hermitian matrix,

B and *C* are *m*-by-*n* matrices.

Input Parameters

side	CHARACTER*1. Specifies whether the Hermitian matrix *A* appears on the left or right in the operation as follows: if *side* = 'L' or 'l', then *C* := *alpha*A*B* + *beta*C*; if *side* = 'R' or 'r', then *C* := *alpha*B*A* + *beta*C*.

uplo CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian matrix *A* is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the Hermitian matrix *A* is used.
If *uplo* = 'L' or 'l', then the low triangular part of the Hermitian matrix *A* is used.

m INTEGER. Specifies the number of rows of the matrix *C*.
The value of *m* must be at least zero.

n INTEGER. Specifies the number of columns of the matrix *C*.
The value of *n* must be at least zero.

alpha COMPLEX for chemm
DOUBLE COMPLEX for zhemm
Specifies the scalar *alpha*.

a COMPLEX for chemm
DOUBLE COMPLEX for zhemm
Array, DIMENSION (*lda*,*ka*), where *ka* is *m* when *side* = 'L' or 'l' and is *n* otherwise. Before entry with *side* = 'L' or 'l', the *m*-by-*m* part of the array *a* must contain the Hermitian matrix, such that when *uplo* = 'U' or 'u', the leading *m*-by-*m* upper triangular part of the array *a* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *a* is not referenced, and when *uplo* = 'L' or 'l', the leading *m*-by-*m* lower triangular part of the array *a* must contain the lower triangular part of the Hermitian matrix, and the strictly upper triangular part of *a* is not referenced.
Before entry with *side* = 'R' or 'r', the *n*-by-*n* part of the array *a* must contain the Hermitian matrix, such that when *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *a* is not referenced, and when *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the Hermitian matrix, and the strictly upper triangular part of *a* is not referenced. The imaginary parts of the diagonal elements need not be set, they are assumed to be zero.

lda INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub) program. When *side* = 'L' or 'l' then *lda* must be at least max(1, *m*), otherwise *lda* must be at least max(1,*n*).

b COMPLEX for chemm
DOUBLE COMPLEX for zhemm
Array, DIMENSION (*ldb*,*n*).
Before entry, the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. The value of *ldb* must be at least max(1, *m*).

beta COMPLEX for chemm
DOUBLE COMPLEX for zhemm
Specifies the scalar *beta*.
When *beta* is supplied as zero, then *c* need not be set on input.

c COMPLEX for chemm
DOUBLE COMPLEX for zhemm

	Array, DIMENSION (*c*, *n*). Before entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, except when *beta* is zero, in which case *c* need not be set on entry.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *m*).

Output Parameters

c	Overwritten by the *m*-by-*n* updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hemm interface are the following:

a	Holds the matrix *A* of size (*k*,*k*) where *k* = *m* if *side* = 'L', *k* = *n* otherwise.
b	Holds the matrix *B* of size (*m*,*n*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?herk

Performs a rank-k update of a Hermitian matrix.

Syntax

Fortran 77:

```
call cherk(uplo, trans, n, k, alpha, a, lda, beta, c, ldc)
call zherk(uplo, trans, n, k, alpha, a, lda, beta, c, ldc)
```

Fortran 95:

```
call herk(a, c [,uplo] [, trans] [,alpha][,beta])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?herk routines perform a matrix-matrix operation using Hermitian matrices. The operation is defined as

```
C := alpha*A*conjg(A') + beta*C,
```

or

```
C := alpha*conjg(A')*A + beta*C,
```

where:

alpha and *beta* are real scalars,

C is an *n*-by-*n* Hermitian matrix,

A is an *n*-by-*k* matrix in the first case and a *k*-by-*n* matrix in the second case.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *c* is used. If *uplo* = 'U' or 'u', then the upper triangular part of the array *c* is used. If *uplo* = 'L' or 'l', then the low triangular part of the array *c* is used.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then *C*:= *alpha***A**conjg(*A*')+*beta***C*; if *trans* = 'C' or 'c', then *C*:= *alpha**conjg(*A*')**A*+*beta***C*.
n	INTEGER. Specifies the order of the matrix *C*. The value of *n* must be at least zero.
k	INTEGER. With *trans* = 'N' or 'n', *k* specifies the number of columns of the matrix *A*, and with *trans* = 'C' or 'c', *k* specifies the number of rows of the matrix *A*. The value of *k* must be at least zero.
alpha	REAL for cherk DOUBLE PRECISION for zherk Specifies the scalar *alpha*.
a	COMPLEX for cherk DOUBLE COMPLEX for zherk Array, DIMENSION (*lda*, *ka*), where *ka* is *k* when *trans* = 'N' or 'n', and is *n* otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *a* must contain the matrix *a*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *lda* must be at least max(1, *n*), otherwise *lda* must be at least max(1, *k*).
beta	REAL for cherk DOUBLE PRECISION for zherk Specifies the scalar *beta*.
c	COMPLEX for cherk DOUBLE COMPLEX for zherk Array, DIMENSION (*ldc*,*n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *c* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *c* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *c* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *c* is not referenced. The imaginary parts of the diagonal elements need not be set, they are assumed to be zero.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *n*).

Output Parameters

c	With *uplo* = 'U' or 'u', the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix. The imaginary parts of the diagonal elements are set to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine herk interface are the following:

a	Holds the matrix *A* of size (*ma*,*ka*) where *ka* = *k* if *transa*= 'N', *ka* = *n* otherwise, *ma* = *n* if *transa*= 'N', *ma* = *k* otherwise.
c	Holds the matrix *C* of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N' or 'C'. The default value is 'N'.
alpha	The default value is 1.
beta	The default value is 0.

?her2k

Performs a rank-2k update of a Hermitian matrix.

Syntax

Fortran 77:

```
call cher2k(uplo, trans, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call zher2k(uplo, trans, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
```

Fortran 95:

```
call her2k(a, b, c [,uplo][,trans] [,alpha][,beta])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?her2k routines perform a rank-2k matrix-matrix operation using Hermitian matrices. The operation is defined as

```
C := alpha*A*conjg(B') + conjg(alpha)*B*conjg(A') + beta*C,
```

or

```
C := alpha *conjg(B')*A  + conjg(alpha) *conjg(A')*B + beta*C,
```

where:

alpha is a scalar and *beta* is a real scalar,

C is an *n*-by-*n* Hermitian matrix,

A and *B* are *n*-by-*k* matrices in the first case and *k*-by-*n* matrices in the second case.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *c* is used. If *uplo* = 'U' or 'u', then the upper triangular of the array *c* is used. If *uplo* = 'L' or 'l', then the low triangular of the array *c* is used.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then *C*:=*alpha***A**conjg(*B*') + *alpha***B**conjg(*A*') + *beta***C*; if *trans* = 'C' or 'c', then *C*:=*alpha**conjg(*A*')**B* + *alpha**conjg(*B*')**A* + *beta***C*.
n	INTEGER. Specifies the order of the matrix *C*. The value of *n* must be at least zero.
k	INTEGER. With *trans* = 'N' or 'n', *k* specifies the number of columns of the matrix *A*, and with *trans* = 'C' or 'c', *k* specifies the number of rows of the matrix *A*. The value of *k* must be at least equal to zero.
alpha	COMPLEX for cher2k DOUBLE COMPLEX for zher2k Specifies the scalar *alpha*.
a	COMPLEX for cher2k DOUBLE COMPLEX for zher2k Array, DIMENSION (*lda*, *ka*), where *ka* is *k* when *trans* = 'N' or 'n', and is *n* otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *lda* must be at least max(1, *n*), otherwise *lda* must be at least max(1, *k*).
beta	REAL for cher2k DOUBLE PRECISION for zher2k Specifies the scalar *beta*.
b	COMPLEX for cher2k DOUBLE COMPLEX for zher2k Array, DIMENSION (*ldb*, *kb*), where *kb* is *k* when *trans* = 'N' or 'n', and is *n* otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *b* must contain the matrix *B*, otherwise the leading *k*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *ldb* must be at least max(1, *n*), otherwise *ldb* must be at least max(1, *k*).
c	COMPLEX for cher2k DOUBLE COMPLEX for zher2k Array, DIMENSION (*ldc*,*n*).

	Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *c* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *c* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *c* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *c* is not referenced. The imaginary parts of the diagonal elements need not be set, they are assumed to be zero.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *n*).

Output Parameters

c	With *uplo* = 'U' or 'u', the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix. The imaginary parts of the diagonal elements are set to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine her2k interface are the following:

a	Holds the matrix *A* of size (*ma*,*ka*) where *ka* = *k* if *trans* = 'N', *ka* = *n* otherwise, *ma* = *n* if *trans* = 'N', *ma* = *k* otherwise.
b	Holds the matrix *B* of size (*mb*,*kb*) where *kb* = *k* if *trans* = 'N', *kb* = *n* otherwise, *mb* = *n* if *trans* = 'N', *mb* = *k* otherwise.
c	Holds the matrix *C* of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N' or 'C'. The default value is 'N'.
alpha	The default value is 1.
beta	The default value is 0.

?symm

Performs a scalar-matrix-matrix product (one matrix operand is symmetric) and adds the result to a scalar-matrix product.

Syntax

Fortran 77:

```
call ssymm(side, uplo, m, n, alpha, a, lda, b, ldb, beta, c, ldc)
call dsymm(side, uplo, m, n, alpha, a, lda, b, ldb, beta, c, ldc)
call csymm(side, uplo, m, n, alpha, a, lda, b, ldb, beta, c, ldc)
```

```
call zsymm(side, uplo, m, n, alpha, a, lda, b, ldb, beta, c, ldc)
```

Fortran 95:

```
call symm(a, b, c [,side][,uplo] [,alpha][,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?symm` routines perform a matrix-matrix operation using symmetric matrices. The operation is defined as

```
C := alpha*A*B  + beta*C,
```

or

```
C := alpha*B*A  + beta*C,
```

where:

alpha and *beta* are scalars,

A is a symmetric matrix,

B and *C* are *m*-by-*n* matrices.

Input Parameters

side
`CHARACTER*1`. Specifies whether the symmetric matrix *A* appears on the left or right in the operation:
if *side* = `'L'` or `'l'`, then *C* := *alpha**A*B + *beta**C*;
if *side* = `'R'` or `'r'`, then *C* := *alpha**B*A + *beta**C*.

uplo
`CHARACTER*1`. Specifies whether the upper or lower triangular part of the symmetric matrix *A* is used:
if *uplo* = `'U'` or `'u'`, then the upper triangular part is used;
if *uplo* = `'L'` or `'l'`, then the lower triangular part is used.

m
`INTEGER`. Specifies the number of rows of the matrix *C*.
The value of *m* must be at least zero.

n
`INTEGER`. Specifies the number of columns of the matrix *C*.
The value of *n* must be at least zero.

alpha
`REAL` for `ssymm`
`DOUBLE PRECISION` for `dsymm`
`COMPLEX` for `csymm`
`DOUBLE COMPLEX` for `zsymm`
Specifies the scalar *alpha*.

a
`REAL` for `ssymm`
`DOUBLE PRECISION` for `dsymm`
`COMPLEX` for `csymm`
`DOUBLE COMPLEX` for `zsymm`
Array, `DIMENSION (lda, ka)`, where *ka* is *m* when *side* = `'L'` or `'l'` and is *n* otherwise.
Before entry with *side* = `'L'` or `'l'`, the *m*-by-*m* part of the array *a* must contain the symmetric matrix, such that when *uplo* = `'U'` or `'u'`, the leading *m*-by-*m* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part

	of *a* is not referenced, and when *uplo* = 'L' or 'l', the leading *m*-by-*m* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *a* is not referenced. Before entry with *side* = 'R' or 'r', the *n*-by-*n* part of the array *a* must contain the symmetric matrix, such that when *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of *a* is not referenced, and when *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *side* = 'L' or 'l' then *lda* must be at least max(1, *m*), otherwise *lda* must be at least max(1, *n*).
b	REAL for ssymm DOUBLE PRECISION for dsymm COMPLEX for csymm DOUBLE COMPLEX for zsymm Array, DIMENSION (*ldb*, *n*). Before entry, the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. The value of *ldb* must be at least max(1, *m*).
beta	REAL for ssymm DOUBLE PRECISION for dsymm COMPLEX for csymm DOUBLE COMPLEX for zsymm Specifies the scalar *beta*. When *beta* is set to zero, then *c* need not be set on input.
c	REAL for ssymm DOUBLE PRECISION for dsymm COMPLEX for csymm DOUBLE COMPLEX for zsymm Array, DIMENSION (*ldc*,*n*). Before entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, except when beta is zero, in which case *c* need not be set on entry.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *m*).

Output Parameters

c	Overwritten by the *m*-by-*n* updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine symm interface are the following:

a	Holds the matrix *A* of size (*k*,*k*) where *k* = *m* if *side* = 'L', *k* = *n* otherwise.

b	Holds the matrix *B* of size (*m*,*n*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
alpha	The default value is 1.
beta	The default value is 0.

?syrk

Performs a rank-n update of a symmetric matrix.

Syntax

Fortran 77:

```
call ssyrk(uplo, trans, n, k, alpha, a, lda, beta, c, ldc)
call dsyrk(uplo, trans, n, k, alpha, a, lda, beta, c, ldc)
call csyrk(uplo, trans, n, k, alpha, a, lda, beta, c, ldc)
call zsyrk(uplo, trans, n, k, alpha, a, lda, beta, c, ldc)
```

Fortran 95:

```
call syrk(a, c [,uplo] [, trans] [,alpha][,beta])
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?syrk routines perform a matrix-matrix operation using symmetric matrices. The operation is defined as

```
C := alpha*A*A' + beta*C,
```

or

```
C := alpha*A'*A + beta*C,
```

where:

alpha and *beta* are scalars,

C is an *n*-by-*n* symmetric matrix,

A is an *n*-by-*k* matrix in the first case and a *k*-by-*n* matrix in the second case.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *c* is used. If *uplo* = 'U' or 'u', then the upper triangular part of the array *c* is used. If *uplo* = 'L' or 'l', then the low triangular part of the array *c* is used.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then *C* := *alpha*A*A'* + *beta*C*; if *trans* = 'T' or 't', then *C* := *alpha*A'*A* + *beta*C*; if *trans* = 'C' or 'c', then *C* := *alpha*A'*A* + *beta*C*.

n	INTEGER. Specifies the order of the matrix *C*. The value of *n* must be at least zero.
k	INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the matrix *a*, and on entry with *trans* = 'T' or 't' or 'C' or 'c', *k* specifies the number of rows of the matrix *a*. The value of *k* must be at least zero.
alpha	REAL for ssyrk DOUBLE PRECISION for dsyrk COMPLEX for csyrk DOUBLE COMPLEX for zsyrk Specifies the scalar *alpha*.
a	REAL for ssyrk DOUBLE PRECISION for dsyrk COMPLEX for csyrk DOUBLE COMPLEX for zsyrk Array, DIMENSION (*lda*,*ka*), where *ka* is *k* when *trans* = 'N' or 'n', and is *n* otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *lda* must be at least max(1,*n*), otherwise *lda* must be at least max(1, *k*).
beta	REAL for ssyrk DOUBLE PRECISION for dsyrk COMPLEX for csyrk DOUBLE COMPLEX for zsyrk Specifies the scalar *beta*.
c	REAL for ssyrk DOUBLE PRECISION for dsyrk COMPLEX for csyrk DOUBLE COMPLEX for zsyrk Array, DIMENSION (*ldc*,*n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *c* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of *c* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *c* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *c* is not referenced.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *n*).

Output Parameters

c	With *uplo* = 'U' or 'u', the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `syrk` interface are the following:

a	Holds the matrix *A* of size (*ma*,*ka*) where *ka* = *k* if *transa*= 'N', *ka* = *n* otherwise, *ma* = *n* if *transa*= 'N', *ma* = *k* otherwise.
c	Holds the matrix *C* of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
alpha	The default value is 1.
beta	The default value is 0.

?syr2k

Performs a rank-2k update of a symmetric matrix.

Syntax

Fortran 77:

```
call ssyr2k(uplo, trans, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call dsyr2k(uplo, trans, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call csyr2k(uplo, trans, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call zsyr2k(uplo, trans, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
```

Fortran 95:

```
call syr2k(a, b, c [,uplo][,trans] [,alpha][,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?syr2k` routines perform a rank-2k matrix-matrix operation using symmetric matrices. The operation is defined as

```
C := alpha*A*B' + alpha*B*A' + beta*C,
```

or

```
C := alpha*A'*B + alpha*B'*A + beta*C,
```

where:

alpha and *beta* are scalars,

C is an *n*-by-*n* symmetric matrix,

A and *B* are *n*-by-*k* matrices in the first case, and *k*-by-*n* matrices in the second case.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *c* is used. If *uplo* = 'U' or 'u', then the upper triangular part of the array *c* is used. If *uplo* = 'L' or 'l', then the low triangular part of the array *c* is used.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then *C* := *alpha**A*B'+alpha*B*A'+beta*C*; if *trans* = 'T' or 't', then *C* := *alpha**A'*B +alpha*B'*A +beta*C*; if *trans* = 'C' or 'c', then *C* := *alpha**A'*B +alpha*B'*A +beta*C*.
n	INTEGER. Specifies the order of the matrix *C*. The value of *n* must be at least zero.
k	INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the matrices *A* and *B*, and on entry with *trans* = 'T' or 't' or 'C' or 'c', *k* specifies the number of rows of the matrices *A* and *B*. The value of *k* must be at least zero.
alpha	REAL for ssyr2k DOUBLE PRECISION for dsyr2k COMPLEX for csyr2k DOUBLE COMPLEX for zsyr2k Specifies the scalar *alpha*.
a	REAL for ssyr2k DOUBLE PRECISION for dsyr2k COMPLEX for csyr2k DOUBLE COMPLEX for zsyr2k Array, DIMENSION (*lda*, *ka*), where *ka* is *k* when *trans* = 'N' or 'n', and is *n* otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *lda* must be at least max(1, *n*), otherwise *lda* must be at least max(1, *k*).
b	REAL for ssyr2k DOUBLE PRECISION for dsyr2k COMPLEX for csyr2k DOUBLE COMPLEX for zsyr2k Array, DIMENSION (*ldb, kb)* where *kb* is *k* when *trans* = 'N' or 'n' and is 'n' otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *b* must contain the matrix *B*, otherwise the leading *k*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *ldb* must be at least max(1, *n*), otherwise *ldb* must be at least max(1, *k*).
beta	REAL for ssyr2k DOUBLE PRECISION for dsyr2k COMPLEX for csyr2k DOUBLE COMPLEX for zsyr2k Specifies the scalar *beta*.
c	REAL for ssyr2k DOUBLE PRECISION for dsyr2k

COMPLEX for csyr2k
DOUBLE COMPLEX for zsyr2k
Array, DIMENSION (*ldc*,*n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *c* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of *c* is not referenced.
Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *c* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *c* is not referenced.

ldc — INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *n*).

Output Parameters

c — With *uplo* = 'U' or 'u', the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix.
With *uplo* = 'L' or 'l', the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syr2k interface are the following:

a — Holds the matrix *A* of size (*ma*,*ka*) where
ka = *k* if *trans* = 'N',
ka = *n* otherwise,
ma = *n* if *trans* = 'N',
ma = *k* otherwise.

b — Holds the matrix *B* of size (*mb*,*kb*) where
kb = *k* if *trans* = 'N',
kb = *n* otherwise,
mb = *n* if *trans* = 'N',
mb = *k* otherwise.

c — Holds the matrix *C* of size (*n*,*n*).

uplo — Must be 'U' or 'L'. The default value is 'U'.

trans — Must be 'N', 'C', or 'T'.
The default value is 'N'.

alpha — The default value is 1.

beta — The default value is 0.

?trmm

Computes a scalar-matrix-matrix product (one matrix operand is triangular).

Syntax

Fortran 77:

```
call strmm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
call dtrmm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
```

```
call ctrmm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
call ztrmm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
```

Fortran 95:

```
call trmm(a, b [,side] [, uplo] [,transa][,diag] [,alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?trmm` routines perform a matrix-matrix operation using triangular matrices. The operation is defined as

```
B := alpha*op(A)*B
```

or

```
B := alpha*B*op(A)
```

where:

alpha is a scalar,

B is an *m*-by-*n* matrix,

A is a unit, or non-unit, upper or lower triangular matrix

`op(A)` is one of `op(A) = A`, or `op(A) = A'`, or `op(A) = conjg(A')`.

Input Parameters

side	`CHARACTER*1`. Specifies whether `op(A)` appears on the left or right of *B* in the operation: if *side* = `'L'` or `'l'`, then *B* := *alpha**op(*A*)**B*; if *side* = `'R'` or `'r'`, then *B* := *alpha***B**op(*A*).
uplo	`CHARACTER*1`. Specifies whether the matrix *A* is upper or lower triangular: if *uplo* = `'U'` or `'u'`, then the matrix is upper triangular; if *uplo* = `'L'` or `'l'`, then the matrix is low triangular.
transa	`CHARACTER*1`. Specifies the form of `op(A)` used in the matrix multiplication: if *transa* = `'N'` or `'n'`, then `op(A) = A`; if *transa* = `'T'` or `'t'`, then `op(A) = A'`; if *transa* = `'C'` or `'c'`, then `op(A) = conjg(A')`.
diag	`CHARACTER*1`. Specifies whether the matrix *A* is unit triangular: if *diag* = `'U'` or `'u'` then the matrix is unit triangular; if *diag* = `'N'` or `'n'`, then the matrix is not unit triangular.
m	`INTEGER`. Specifies the number of rows of *B*. The value of *m* must be at least zero.
n	`INTEGER`. Specifies the number of columns of *B*. The value of *n* must be at least zero.
alpha	`REAL` for `strmm` `DOUBLE PRECISION` for `dtrmm` `COMPLEX` for `ctrmm` `DOUBLE COMPLEX` for `ztrmm` Specifies the scalar *alpha*.

When *alpha* is zero, then *a* is not referenced and *b* need not be set before entry.

a
REAL for strmm
DOUBLE PRECISION for dtrmm
COMPLEX for ctrmm
DOUBLE COMPLEX for ztrmm
Array, DIMENSION (*lda*,*k*), where *k* is *m* when *side* = 'L' or 'l' and is *n* when *side* = 'R' or 'r'. Before entry with *uplo* = 'U' or 'u', the leading *k* by *k* upper triangular part of the array *a* must contain the upper triangular matrix and the strictly lower triangular part of *a* is not referenced.
Before entry with *uplo* = 'L' or 'l', the leading *k* by *k* lower triangular part of the array *a* must contain the lower triangular matrix and the strictly upper triangular part of *a* is not referenced.
When *diag* = 'U' or 'u', the diagonal elements of *a* are not referenced either, but are assumed to be unity.

lda
INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *side* = 'L' or 'l', then *lda* must be at least max(1, *m*), when *side* = 'R' or 'r', then *lda* must be at least max(1, *n*).

b
REAL for strmm
DOUBLE PRECISION for dtrmm
COMPLEX for ctrmm
DOUBLE COMPLEX for ztrmm
Array, DIMENSION (*ldb*,*n*).
Before entry, the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb
INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. The value of *ldb* must be at least max(1, *m*).

Output Parameters

b
Overwritten by the transformed matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trmm interface are the following:

a
Holds the matrix *A* of size (*k*,*k*) where
k = *m* if *side* = 'L',
k = *n* otherwise.

b
Holds the matrix *B* of size (*m*,*n*).

side
Must be 'L' or 'R'. The default value is 'L'.

uplo
Must be 'U' or 'L'. The default value is 'U'.

transa
Must be 'N', 'C', or 'T'.
The default value is 'N'.

diag
Must be 'N' or 'U'. The default value is 'N'.

alpha
The default value is 1.

?trsm

Solves a matrix equation (one matrix operand is triangular).

Syntax

Fortran 77:

```
call strsm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
call dtrsm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
call ctrsm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
call ztrsm(side, uplo, transa, diag, m, n, alpha, a, lda, b, ldb)
```

Fortran 95:

```
call trsm(a, b [,side] [, uplo] [,transa][,diag] [,alpha])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?trsm` routines solve one of the following matrix equations:

```
op(A)*X = alpha*B,
```

or

```
X*op(A) = alpha*B,
```

where:

alpha is a scalar,

X and *B* are *m*-by-*n* matrices,

A is a unit, or non-unit, upper or lower triangular matrix

`op(A)` is one of `op(A) = A`, or `op(A) = A'`, or `op(A) = conjg(A')`.

The matrix *B* is overwritten by the solution matrix *X*.

Input Parameters

side	`CHARACTER*1`. Specifies whether `op(A)` appears on the left or right of *X* in the equation: if *side* = `'L'` or `'l'`, then `op(A)*X = alpha*B`; if *side* = `'R'` or `'r'`, then `X*op(A) = alpha*B`.
uplo	`CHARACTER*1`. Specifies whether the matrix *A* is upper or lower triangular: if *uplo* = `'U'` or `'u'`, then the matrix is upper triangular; if *uplo* = `'L'` or `'l'`, then the matrix is low triangular.
transa	`CHARACTER*1`. Specifies the form of `op(A)` used in the matrix multiplication: if *transa* = `'N'` or `'n'`, then `op(A) = A`; if *transa* = `'T'` or `'t'`, then `op(A) = A'`; if *transa* = `'C'` or `'c'`, then `op(A) = conjg(A')`.
diag	`CHARACTER*1`. Specifies whether the matrix *A* is unit triangular:

	if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
m	INTEGER. Specifies the number of rows of *B*. The value of *m* must be at least zero.
n	INTEGER. Specifies the number of columns of *B*. The value of *n* must be at least zero.
alpha	REAL for strsm DOUBLE PRECISION for dtrsm COMPLEX for ctrsm DOUBLE COMPLEX for ztrsm Specifies the scalar *alpha*. When *alpha* is zero, then *a* is not referenced and *b* need not be set before entry.
a	REAL for strsm DOUBLE PRECISION for dtrsm COMPLEX for ctrsm DOUBLE COMPLEX for ztrsm Array, DIMENSION (*lda*, *k*), where *k* is *m* when *side* = 'L' or 'l' and is *n* when *side* = 'R' or 'r'. Before entry with *uplo* = 'U' or 'u', the leading *k* by *k* upper triangular part of the array *a* must contain the upper triangular matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *k* by *k* lower triangular part of the array *a* must contain the lower triangular matrix and the strictly upper triangular part of *a* is not referenced. When *diag* = 'U' or 'u', the diagonal elements of *a* are not referenced either, but are assumed to be unity.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *side* = 'L' or 'l', then *lda* must be at least max(1, *m*), when *side* = 'R' or 'r', then *lda* must be at least max(1, *n*).
b	REAL for strsm DOUBLE PRECISION for dtrsm COMPLEX for ctrsm DOUBLE COMPLEX for ztrsm Array, DIMENSION (*ldb*, *n*). Before entry, the leading *m*-by-*n* part of the array *b* must contain the right-hand side matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. The value of *ldb* must be at least max(1, +*m*).

Output Parameters

b	Overwritten by the solution matrix *X*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trsm interface are the following:

a	Holds the matrix *A* of size (*k*,*k*) where *k* = *m* if *side* = 'L', *k* = *n* otherwise.

b	Holds the matrix *B* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
transa	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.
alpha	The default value is 1.

Sparse BLAS Level 1 Routines

This section describes Sparse BLAS Level 1, an extension of BLAS Level 1 included in the Intel® Math Kernel Library beginning with the Intel MKL release 2.1. Sparse BLAS Level 1 is a group of routines and functions that perform a number of common vector operations on sparse vectors stored in compressed form.

Sparse vectors are those in which the majority of elements are zeros. Sparse BLAS routines and functions are specially implemented to take advantage of vector sparsity. This allows you to achieve large savings in computer time and memory. If *nz* is the number of non-zero vector elements, the computer time taken by Sparse BLAS operations will be *O*(*nz*).

Vector Arguments

Compressed sparse vectors. Let *a* be a vector stored in an array, and assume that the only non-zero elements of *a* are the following:

$a(k_1), a(k_2), a(k_3) \ldots a(k_{nz}),$

where *nz* is the total number of non-zero elements in *a*.

In Sparse BLAS, this vector can be represented in compressed form by two FORTRAN arrays, *x* (values) and *indx* (indices). Each array has *nz* elements:

$x(1)=a(k_1), x(2)=a(k_2), \ldots x(nz)= a(k_{nz}),$

$indx(1)=k_1, indx(2)=k_2, \ldots indx(nz)= k_{nz}.$

Thus, a sparse vector is fully determined by the triple (*nz*, *x*, *indx*). If you pass a negative or zero value of *nz* to Sparse BLAS, the subroutines do not modify any arrays or variables.

Full-storage vectors. Sparse BLAS routines can also use a vector argument fully stored in a single FORTRAN array (a full-storage vector). If *y* is a full-storage vector, its elements must be stored contiguously: the first element in *y*(1), the second in *y*(2), and so on. This corresponds to an increment *incy* = 1 in BLAS Level 1. No increment value for full-storage vectors is passed as an argument to Sparse BLAS routines or functions.

Naming Conventions

Similar to BLAS, the names of Sparse BLAS subprograms have prefixes that determine the data type involved: *s* and *d* for single- and double-precision real; *c* and *z* for single- and double-precision complex respectively.

If a Sparse BLAS routine is an extension of a "dense" one, the subprogram name is formed by appending the suffix *i* (standing for *indexed*) to the name of the corresponding "dense" subprogram. For example, the Sparse BLAS routine saxpyi corresponds to the BLAS routine saxpy, and the Sparse BLAS function cdotci corresponds to the BLAS function cdotc.

Routines and Data Types

Routines and data types supported in the Intel MKL implementation of Sparse BLAS are listed in Table "Sparse BLAS Routines and Their Data Types".

Sparse BLAS Routines and Their Data Types

Routine/ Function	Data Types	Description
?axpyi	s, d, c, z	Scalar-vector product plus vector (routines)
?doti	s, d	Dot product (functions)
?dotci	c, z	Complex dot product conjugated (functions)
?dotui	c, z	Complex dot product unconjugated (functions)
?gthr	s, d, c, z	Gathering a full-storage sparse vector into compressed form *nz*, *x*, *indx* (routines)
?gthrz	s, d, c, z	Gathering a full-storage sparse vector into compressed form and assigning zeros to gathered elements in the full-storage vector (routines)
?roti	s, d	Givens rotation (routines)
?sctr	s, d, c, z	Scattering a vector from compressed form to full-storage form (routines)

BLAS Level 1 Routines That Can Work With Sparse Vectors

The following BLAS Level 1 routines will give correct results when you pass to them a compressed-form array *x*(with the increment *incx*=1):

?asum	sum of absolute values of vector elements
?copy	copying a vector
?nrm2	Euclidean norm of a vector
?scal	scaling a vector
i?amax	index of the element with the largest absolute value for real flavors, or the largest sum |Re(*x*(*i*))|+|Im(*x*(*i*))| for complex flavors.
i?amin	index of the element with the smallest absolute value for real flavors, or the smallest sum |Re(*x*(*i*))|+|Im(*x*(*i*))| for complex flavors.

The result *i* returned by i?amax and i?amin should be interpreted as index in the compressed-form array, so that the largest (smallest) value is *x*(*i*); the corresponding index in full-storage array is *indx*(*i*).

You can also call ?rotg to compute the parameters of Givens rotation and then pass these parameters to the Sparse BLAS routines ?roti.

?axpyi

Adds a scalar multiple of compressed sparse vector to a full-storage vector.

Syntax

Fortran 77:

```
call saxpyi(nz, a, x, indx, y)
```

```
call daxpyi(nz, a, x, indx, y)
call caxpyi(nz, a, x, indx, y)
call zaxpyi(nz, a, x, indx, y)
```

Fortran 95:

```
call axpyi(x, indx, y [, a])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?axpyi` routines perform a vector-vector operation defined as

```
y := a*x + y
```

where:

a is a scalar,

x is a sparse vector stored in compressed form,

y is a vector in full storage form.

The `?axpyi` routines reference or modify only the elements of *y* whose indices are listed in the array *indx*.

The values in *indx* must be distinct.

Input Parameters

nz	`INTEGER`. The number of elements in *x* and *indx*.
a	`REAL` for `saxpyi` `DOUBLE PRECISION` for `daxpyi` `COMPLEX` for `caxpyi` `DOUBLE COMPLEX` for `zaxpyi` Specifies the scalar *a*.
x	`REAL` for `saxpyi` `DOUBLE PRECISION` for `daxpyi` `COMPLEX` for `caxpyi` `DOUBLE COMPLEX` for `zaxpyi` Array, `DIMENSION` at least *nz*.
indx	`INTEGER`. Specifies the indices for the elements of *x*. Array, `DIMENSION` at least *nz*.
y	`REAL` for `saxpyi` `DOUBLE PRECISION` for `daxpyi` `COMPLEX` for `caxpyi` `DOUBLE COMPLEX` for `zaxpyi` Array, `DIMENSION` at least `max(`*indx*`(i))`.

Output Parameters

y	Contains the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `axpyi` interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.
a	The default value is 1.

?doti

Computes the dot product of a compressed sparse real vector by a full-storage real vector.

Syntax

Fortran 77:

```
res = sdoti(nz, x, indx, y )
res = ddoti(nz, x, indx, y )
```

Fortran 95:

```
res = doti(x, indx, y)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?doti` routines return the dot product of *x* and *y* defined as

```
res = x(1)*y(indx(1)) + x(2)*y(indx(2)) +...+ x(nz)*y(indx(nz))
```

where the triple (*nz*, *x*, *indx*) defines a sparse real vector stored in compressed form, and *y* is a real vector in full storage form. The functions reference only the elements of *y* whose indices are listed in the array *indx*. The values in *indx* must be distinct.

Input Parameters

nz	`INTEGER`. The number of elements in *x* and *indx* .
x	`REAL` for `sdoti` `DOUBLE PRECISION` for `ddoti` Array, `DIMENSION` at least *nz*.
indx	`INTEGER`. Specifies the indices for the elements of *x*. Array, `DIMENSION` at least *nz*.
y	`REAL` for `sdoti` `DOUBLE PRECISION` for `ddoti` Array, `DIMENSION` at least max(*indx(i)*).

Output Parameters

res	REAL for sdoti DOUBLE PRECISION for ddoti Contains the dot product of *x* and *y*, if *nz* is positive. Otherwise, *res* contains 0.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine doti interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.

?dotci

Computes the conjugated dot product of a compressed sparse complex vector with a full-storage complex vector.

Syntax

Fortran 77:

```
res = cdotci(nz, x, indx, y )
res = zdotci(nz, x, indx, y )
```

Fortran 95:

```
res = dotci(x, indx, y)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?dotci routines return the dot product of *x* and *y* defined as

```
conjg(x(1))*y(indx(1)) + ... + conjg(x(nz))*y(indx(nz))
```

where the triple (*nz*, *x*, *indx*) defines a sparse complex vector stored in compressed form, and *y* is a real vector in full storage form. The functions reference only the elements of *y* whose indices are listed in the array *indx*. The values in *indx* must be distinct.

Input Parameters

nz	INTEGER. The number of elements in *x* and *indx* .
x	COMPLEX for cdotci DOUBLE COMPLEX for zdotci Array, DIMENSION at least *nz*.
indx	INTEGER. Specifies the indices for the elements of *x*.

	Array, DIMENSION at least *nz*.
y	COMPLEX for cdotci DOUBLE COMPLEX for zdotci Array, DIMENSION at least max(*indx(i)*).

Output Parameters

res	COMPLEX for cdotci DOUBLE COMPLEX for zdotci Contains the conjugated dot product of *x* and *y*, if *nz* is positive. Otherwise, *res* contains 0.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine dotci interface are the following:

x	Holds the vector with the number of elements (*nz*).
indx	Holds the vector with the number of elements (*nz*).
y	Holds the vector with the number of elements (*nz*).

?dotui

Computes the dot product of a compressed sparse complex vector by a full-storage complex vector.

Syntax

Fortran 77:

```
res = cdotui(nz, x, indx, y )
res = zdotui(nz, x, indx, y )
```

Fortran 95:

```
res = dotui(x, indx, y)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?dotui routines return the dot product of *x* and *y* defined as

```
res = x(1)*y(indx(1)) + x(2)*y(indx(2)) +...+ x(nz)*y(indx(nz))
```

where the triple (*nz*, *x*, *indx*) defines a sparse complex vector stored in compressed form, and *y* is a real vector in full storage form. The functions reference only the elements of *y* whose indices are listed in the array *indx*. The values in *indx* must be distinct.

Input Parameters

nz	INTEGER. The number of elements in *x* and *indx* .

x	COMPLEX for cdotui DOUBLE COMPLEX for zdotui Array, DIMENSION at least *nz*.
indx	INTEGER. Specifies the indices for the elements of *x*. Array, DIMENSION at least *nz*.
y	COMPLEX for cdotui DOUBLE COMPLEX for zdotui Array, DIMENSION at least max(*indx(i)*).

Output Parameters

res	COMPLEX for cdotui DOUBLE COMPLEX for zdotui Contains the dot product of *x* and *y*, if *nz* is positive. Otherwise, *res* contains 0.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine dotui interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.

?gthr

Gathers a full-storage sparse vector's elements into compressed form.

Syntax

Fortran 77:

```
call sgthr(nz, y, x, indx )
call dgthr(nz, y, x, indx )
call cgthr(nz, y, x, indx )
call zgthr(nz, y, x, indx )
```

Fortran 95:

```
res = gthr(x, indx, y)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?gthr routines gather the specified elements of a full-storage sparse vector *y* into compressed form(*nz*, *x*, *indx*). The routines reference only the elements of *y* whose indices are listed in the array *indx*:

`x(i) = y(indx(i))`, for `i=1,2,... +nz`.

Input Parameters

nz	INTEGER. The number of elements of *y* to be gathered.
indx	INTEGER. Specifies indices of elements to be gathered. Array, DIMENSION at least *nz*.
y	REAL for sgthr DOUBLE PRECISION for dgthr COMPLEX for cgthr DOUBLE COMPLEX for zgthr Array, DIMENSION at least max(*indx(i)*).

Output Parameters

x	REAL for sgthr DOUBLE PRECISION for dgthr COMPLEX for cgthr DOUBLE COMPLEX for zgthr Array, DIMENSION at least *nz*. Contains the vector converted to the compressed form.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gthr interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.

?gthrz

Gathers a sparse vector's elements into compressed form, replacing them by zeros.

Syntax

Fortran 77:

```
call sgthrz(nz, y, x, indx )
call dgthrz(nz, y, x, indx )
call cgthrz(nz, y, x, indx )
call zgthrz(nz, y, x, indx )
```

Fortran 95:

```
res = gthrz(x, indx, y)
```

Include Files

- FORTRAN 77: mkl_blas.fi
- Fortran 95: blas.f90
- C: mkl_blas.h

Description

The ?gthrz routines gather the elements with indices specified by the array *indx* from a full-storage vector *y* into compressed form (*nz*, *x*, *indx*) and overwrite the gathered elements of *y* by zeros. Other elements of *y* are not referenced or modified (see also ?gthr).

Input Parameters

nz	INTEGER. The number of elements of *y* to be gathered.
indx	INTEGER. Specifies indices of elements to be gathered. Array, DIMENSION at least *nz*.
y	REAL for sgthrz DOUBLE PRECISION for dgthrz COMPLEX for cgthrz DOUBLE COMPLEX for zgthrz Array, DIMENSION at least max(*indx(i)*).

Output Parameters

x	REAL for sgthrz DOUBLE PRECISION for d gthrz COMPLEX for cgthrz DOUBLE COMPLEX for zgthrz Array, DIMENSION at least *nz*. Contains the vector converted to the compressed form.
y	The updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gthrz interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.

?roti

Applies Givens rotation to sparse vectors one of which is in compressed form.

Syntax

Fortran 77:

```
call sroti(nz, x, indx, y, c, s)
call droti(nz, x, indx, y, c, s)
```

Fortran 95:

```
call roti(x, indx, y, c, s)
```

Include Files

- FORTRAN 77: mkl_blas.fi

- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?roti` routines apply the Givens rotation to elements of two real vectors, *x* (in compressed form *nz*, *x*, *indx*) and *y* (in full storage form):

```
x(i) = c*x(i) + s*y(indx(i))
```

```
y(indx(i)) = c*y(indx(i))- s*x(i)
```

The routines reference only the elements of *y* whose indices are listed in the array *indx*. The values in *indx* must be distinct.

Input Parameters

nz	`INTEGER`. The number of elements in *x* and *indx*.
x	`REAL` for `sroti` `DOUBLE PRECISION` for `droti` Array, `DIMENSION` at least *nz*.
indx	`INTEGER`. Specifies the indices for the elements of *x*. Array, `DIMENSION` at least *nz*.
y	`REAL` for `sroti` `DOUBLE PRECISION` for `droti` Array, `DIMENSION` at least max(*indx(i)*).
c	A scalar: `REAL` for `sroti` `DOUBLE PRECISION` for `droti`.
s	A scalar: `REAL` for `sroti` `DOUBLE PRECISION` for `droti`.

Output Parameters

x and *y*	The updated arrays.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `roti` interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.

?sctr

Converts compressed sparse vectors into full storage form.

Syntax

Fortran 77:

```
call ssctr(nz, x, indx, y )
```

```
call dsctr(nz, x, indx, y )
```

```
call csctr(nz, x, indx, y )
call zsctr(nz, x, indx, y )
```

Fortran 95:

```
call sctr(x, indx, y)
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?sctr` routines scatter the elements of the compressed sparse vector (*nz*, *x*, *indx*) to a full-storage vector *y*. The routines modify only the elements of *y* whose indices are listed in the array *indx*:

y(*indx*(*i*)) = *x*(*i*), for *i*=1,2,... +*nz*.

Input Parameters

nz	`INTEGER`. The number of elements of *x* to be scattered.
indx	`INTEGER`. Specifies indices of elements to be scattered. Array, `DIMENSION` at least *nz*.
x	`REAL` for `ssctr` `DOUBLE PRECISION` for `dsctr` `COMPLEX` for `csctr` `DOUBLE COMPLEX` for `zsctr` Array, `DIMENSION` at least *nz*. Contains the vector to be converted to full-storage form.

Output Parameters

y	`REAL` for `ssctr` `DOUBLE PRECISION` for `dsctr` `COMPLEX` for `csctr` `DOUBLE COMPLEX` for `zsctr` Array, `DIMENSION` at least max(*indx(i)*). Contains the vector *y* with updated elements.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `sctr` interface are the following:

x	Holds the vector with the number of elements *nz*.
indx	Holds the vector with the number of elements *nz*.
y	Holds the vector with the number of elements *nz*.

Sparse BLAS Level 2 and Level 3 Routines

This section describes Sparse BLAS Level 2 and Level 3 routines included in the Intel® Math Kernel Library (Intel® MKL) . Sparse BLAS Level 2 is a group of routines and functions that perform operations between a sparse matrix and dense vectors. Sparse BLAS Level 3 is a group of routines and functions that perform operations between a sparse matrix and dense matrices.

The terms and concepts required to understand the use of the Intel MKL Sparse BLAS Level 2 and Level 3 routines are discussed in the Linear Solvers Basics appendix.

The Sparse BLAS routines can be useful to implement iterative methods for solving large sparse systems of equations or eigenvalue problems. For example, these routines can be considered as building blocks for Iterative Sparse Solvers based on Reverse Communication Interface (RCI ISS) described in the Chapter 8 of the manual.

Intel MKL provides Sparse BLAS Level 2 and Level 3 routines with typical (or conventional) interface similar to the interface used in the NIST* Sparse BLAS library [Rem05].

Some software packages and libraries (the PARDISO* Solver used in Intel MKL, *Sparskit 2* [Saad94], the Compaq* Extended Math Library (CXML)[CXML01]) use different (early) variation of the compressed sparse row (CSR) format and support only Level 2 operations with simplified interfaces. Intel MKL provides an additional set of Sparse BLAS Level 2 routines with similar simplified interfaces. Each of these routines operates only on a matrix of the fixed type.

The routines described in this section support both one-based indexing and zero-based indexing of the input data (see details in the section One-based and Zero-based Indexing).

Naming Conventions in Sparse BLAS Level 2 and Level 3

Each Sparse BLAS Level 2 and Level 3 routine has a six- or eight-character base name preceded by the prefix `mkl_` or `mkl_cspblas_` .

The routines with typical (conventional) interface have six-character base names in accordance with the template:

```
mkl_<character > <data> <operation>( )
```

The routines with simplified interfaces have eight-character base names in accordance with the templates:

```
mkl_<character > <data> <mtype> <operation>( )
```

for routines with one-based indexing; and

```
mkl_cspblas_<character> <data>  <mtype> <operation>( )
```

for routines with zero-based indexing.

The *<character>* field indicates the data type:

`s`	real, single precision
`c`	complex, single precision
`d`	real, double precision
`z`	complex, double precision

The *<data>* field indicates the sparse matrix storage format (see section Sparse Matrix Storage Formats):

`coo`	coordinate format
`csr`	compressed sparse row format and its variations
`csc`	compressed sparse column format and its variations
`dia`	diagonal format
`sky`	skyline storage format
`bsr`	block sparse row format and its variations

The *<operation>* field indicates the type of operation:

mv	matrix-vector product (Level 2)
mm	matrix-matrix product (Level 3)
sv	solving a single triangular system (Level 2)
sm	solving triangular systems with multiple right-hand sides (Level 3)

The field <mtype> indicates the matrix type:

ge	sparse representation of a general matrix
sy	sparse representation of the upper or lower triangle of a symmetric matrix
tr	sparse representation of a triangular matrix

Sparse Matrix Storage Formats

The current version of Intel MKL Sparse BLAS Level 2 and Level 3 routines support the following point entry [Duff86] storage formats for sparse matrices:

- *compressed sparse row* format (CSR) and its variations;
- *compressed sparse column* format (CSC);
- *coordinate* format;
- *diagonal* format;
- *skyline* storage format;

and one block entry storage format:

- *block sparse row* format (BSR) and its variations.

For more information see "Sparse Matrix Storage Formats" in Appendix A.

Intel MKL provides auxiliary routines - matrix converters - that convert sparse matrix from one storage format to another.

Routines and Supported Operations

This section describes operations supported by the Intel MKL Sparse BLAS Level 2 and Level 3 routines. The following notations are used here:

A is a sparse matrix;
B and *C* are dense matrices;
D is a diagonal scaling matrix;
x and *y* are dense vectors;
alpha and *beta* are scalars;

op(*A*) is one of the possible operations:

op(*A*) = *A*;
op(*A*) = *A*' - transpose of *A*;
op(*A*) = conj(*A*') - conjugated transpose of *A*.

inv(op(*A*)) denotes the inverse of op(*A*).

The Intel MKL Sparse BLAS Level 2 and Level 3 routines support the following operations:

- computing the vector product between a sparse matrix and a dense vector:
  ```
  y := alpha*op(A)*x + beta*y
  ```
- solving a single triangular system:
  ```
  y := alpha*inv(op(A))*x
  ```

- computing a product between sparse matrix and dense matrix:

```
C := alpha*op(A)*B + beta*C
```

- solving a sparse triangular system with multiple right-hand sides:

```
C := alpha*inv(op(A))*B
```

Intel MKL provides an additional set of the Sparse BLAS Level 2 routines with *simplified interfaces*. Each of these routines operates on a matrix of the fixed type. The following operations are supported:

- computing the vector product between a sparse matrix and a dense vector (for general and symmetric matrices):

```
y := op(A)*x
```

- solving a single triangular system (for triangular matrices):

```
y := inv(op(A))*x
```

Matrix type is indicated by the field <*mtype*> in the routine name (see section Naming Conventions in Sparse BLAS Level 2 and Level 3).

NOTE The routines with simplified interfaces support only four sparse matrix storage formats, specifically:

CSR format in the 3-array variation accepted in the direct sparse solvers and in the CXML;
diagonal format accepted in the CXML;
coordinate format;
BSR format in the 3-array variation.

Note that routines with both typical (conventional) and simplified interfaces use the same computational kernels that work with certain internal data structures.

The Intel MKL Sparse BLAS Level 2 and Level 3 routines do not support in-place operations.

Complete list of all routines is given in the "Sparse BLAS Level 2 and Level 3 Routines".

Interface Consideration

One-Based and Zero-Based Indexing

The Intel MKL Sparse BLAS Level 2 and Level 3 routines support one-based and zero-based indexing of data arrays.

Routines with typical interfaces support zero-based indexing for the following sparse data storage formats: CSR, CSC, BSR, and COO. Routines with simplified interfaces support zero based indexing for the following sparse data storage formats: CSR, BSR, and COO. See the complete list of Sparse BLAS Level 2 and Level 3 Routines.

The one-based indexing uses the convention of starting array indices at 1. The zero-based indexing uses the convention of starting array indices at 0. For example, indices of the 5-element array x can be presented in case of one-based indexing as follows:

Element index: `1 2 3 4 5`

Element value: `1.0 5.0 7.0 8.0 9.0`

and in case of zero-based indexing as follows:

Element index: `0 1 2 3 4`

Element value: `1.0 5.0 7.0 8.0 9.0`

The detailed descriptions of the one-based and zero-based variants of the sparse data storage formats are given in the "Sparse Matrix Storage Formats" in Appendix A.

Most parameters of the routines are identical for both one-based and zero-based indexing, but some of them have certain differences. The following table lists all these differences.

Parameter	One-based Indexing	Zero-based Indexing
val	Array containing non-zero elements of the matrix *A*, its length is *pntre*(m) – *pntrb*(1).	Array containing non-zero elements of the matrix *A*, its length is *pntre*(m–1) – *pntrb*(0).
pntrb	Array of length *m*. This array contains row indices, such that *pntrb*(i) – *pntrb*(1)+1 is the first index of row i in the arrays *val* and *indx*	Array of length *m*. This array contains row indices, such that *pntrb*(i) – *pntrb*(0) is the first index of row i in the arrays *val* and *indx*.
pntre	Array of length *m*. This array contains row indices, such that *pntre*(*I*) – *pntrb*(1) is the last index of row i in the arrays *val* and *indx*.	Array of length *m*. This array contains row indices, such that *pntre*(i) – *pntrb*(0)–1 is the last index of row i in the arrays *val* and *indx*.
ia	Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia*(i) is the index in the array *a* of the first non-zero element from the row i. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros plus one.	Array of length *m*+1, containing indices of elements in the array *a*, such that *ia*(i) is the index in the array *a* of the first non-zero element from the row i. The value of the last element *ia*(*m*) is equal to the number of non-zeros.
ldb	Specifies the leading dimension of *b* as declared in the calling (sub)program.	Specifies the second dimension of *b* as declared in the calling (sub)program.
ldc	Specifies the leading dimension of *c* as declared in the calling (sub)program.	Specifies the second dimension of *c* as declared in the calling (sub)program.

Difference Between Fortran and C Interfaces

Intel MKL provides both Fortran and C interfaces to all Sparse BLAS Level 2 and Level 3 routines. Parameter descriptions are common for both interfaces with the exception of data types that refer to the FORTRAN 77 standard types. Correspondence between data types specific to the Fortran and C interfaces are given below:

Fortran	C
REAL*4	float
REAL*8	double
INTEGER*4	int
INTEGER*8	long long int
CHARACTER	char

For routines with C interfaces all parameters (including scalars) must be passed by references.

Another difference is how two-dimensional arrays are represented. In Fortran the column-major order is used, and in C - row-major order. This changes the meaning of the parameters *ldb* and *ldc* (see the table above).

Differences Between Intel MKL and NIST* Interfaces

The Intel MKL Sparse BLAS Level 3 routines have the following conventional interfaces:

`mkl_xyyymm(transa, m, n, k, alpha, matdescra, arg(A), b, ldb, beta, c, ldc)`, for matrix-matrix product;

`mkl_xyyysm(transa, m, n, alpha, matdescra, arg(A), b, ldb, c, ldc)`, for triangular solvers with multiple right-hand sides.

Here `x` denotes data type, and *yyy* - sparse matrix data structure (storage format).

The analogous NIST* Sparse BLAS (NSB) library routines have the following interfaces:

`xyyymm(transa, m, n, k, alpha, descra, arg(A), b, ldb, beta, c, ldc, work, lwork)`, for matrix-matrix product;

`xyyysm(transa, m, n, unitd, dv, alpha, descra, arg(A), b, ldb, beta, c, ldc, work, lwork)`, for triangular solvers with multiple right-hand sides.

Some similar arguments are used in both libraries. The argument *transa* indicates what operation is performed and is slightly different in the NSB library (see Table "Parameter *transa*"). The arguments *m* and *k* are the number of rows and column in the matrix *A*, respectively, *n* is the number of columns in the matrix *C*. The arguments *alpha* and *beta* are scalar *alpha* and *beta* respectively (*beta* is not used in the Intel MKL triangular solvers.) The arguments *b* and *c* are rectangular arrays with the leading dimension *ldb* and *ldc*, respectively. `arg(A)` denotes the list of arguments that describe the sparse representation of *A*.

Parameter *transa*

	MKL interface	NSB interface	Operation
data type	CHARACTER*1	INTEGER	
value	N or n	0	`op(A) = A`
	T or t	1	`op(A) = A'`
	C or c	2	`op(A) = A'`

Parameter matdescra

The parameter *matdescra* describes the relevant characteristic of the matrix *A*. This manual describes *matdescra* as an array of six elements in line with the NIST* implementation. However, only the first four elements of the array are used in the current versions of the Intel MKL Sparse BLAS routines. Elements `matdescra(5)` and `matdescra(6)` are reserved for future use. Note that whether *matdescra* is described in your application as an array of length 6 or 4 is of no importance because the array is declared as a pointer in the Intel MKL routines. To learn more about declaration of the *matdescra* array, see Sparse BLAS examples located in the following subdirectory of the Intel MKL installation directory: `examples/spblas/`. The table below lists elements of the parameter *matdescra*, their values and meanings. The parameter *matdescra* corresponds to the argument *descra* from NSB library.

Possible Values of the Parameter *matdescra* (*descra*)

	MKL interface		NSB interface	Matrix characteristics
	one-based indexing	zero-based indexing		
data type	CHARACTER	Char	INTEGER	
1st element	`matdescra(1)`	`matdescra(0)`	`descra(1)`	matrix structure
value	G	G	0	general
	S	S	1	symmetric (`A = A'`)

	MKL interface		NSB interface	Matrix characteristics
	H	H	2	Hermitian (A=conjg(A'))
	T	T	3	triangular
	A	A	4	skew(anti)-symmetric (A=-A')
	D	D	5	diagonal
2nd element	matdescra(2)	matdescra(1)	descra(2)	upper/lower triangular indicator
value	L	L	1	lower
	U	U	2	upper
3rd element	matdescra(3)	matdescra(2)	descra(3)	main diagonal type
value	N	N	0	non-unit
	U	U	1	unit
4th element	matdescra(4)	matdescra(3)		type of indexing
value	F			one-based indexing
		C		zero-based indexing

In some cases possible element values of the parameter matdescra depend on the values of other elements. The Table "Possible Combinations of Element Values of the Parameter *matdescra*" lists all possible combinations of element values for both multiplication routines and triangular solvers.

Possible Combinations of Element Values of the Parameter *matdescra*

Routines	matdescra(1)	matdescra(2)	matdescra(3)	matdescra(4)
Multiplication Routines	G	ignored	ignored	**F** (default) or C
	S or H	**L** (default)	**N** (default)	**F** (default) or C
	S or H	**L** (default)	U	**F** (default) or C
	S or H	U	**N** (default)	**F** (default) or C
	S or H	U	U	**F** (default) or C
	A	**L** (default)	ignored	**F** (default) or C
	A	U	ignored	**F** (default) or C
Multiplication Routines and Triangular Solvers	T	L	U	**F** (default) or C
	T	L	N	**F** (default) or C
	T	U	U	**F** (default) or C
	T	U	N	**F** (default) or C
	D	ignored	**N** (default)	**F** (default) or C
	D	ignored	U	**F** (default) or C

For a matrix in the skyline format with the main diagonal declared to be a unit, diagonal elements must be stored in the sparse representation even if they are zero. In all other formats, diagonal elements can be stored (if needed) in the sparse representation if they are not zero.

Operations with Partial Matrices

One of the distinctive feature of the Intel MKL Sparse BLAS routines is a possibility to perform operations only on partial matrices composed of certain parts (triangles and the main diagonal) of the input sparse matrix. It can be done by setting properly first three elements of the parameter *matdescra*.

An arbitrary sparse matrix *A* can be decomposed as

```
A = L + D + U
```

where *L* is the strict lower triangle of *A*, *U* is the strict upper triangle of *A*, *D* is the main diagonal.

Table "Output Matrices for Multiplication Routines" shows correspondence between the output matrices and values of the parameter *matdescra* for the sparse matrix *A* for multiplication routines.

Output Matrices for Multiplication Routines

matdescra(1)	matdescra(2)	matdescra(3)	Output Matrix
G	ignored	ignored	*alpha**op(*A*)**x* + *beta***y* *alpha**op(*A*)**B* + *beta***C*
S or H	L	N	*alpha**op(*L+D+L'*)**x* + *beta***y* *alpha**op(*L+D+L'*)**B* + *beta***C*
S or H	L	U	*alpha**op(*L+I+L'*)**x* + *beta***y* *alpha**op(*L+I+L'*)**B* + *beta***C*
S or H	U	N	*alpha**op(*U'+D+U*)**x* + *beta***y* *alpha**op(*U'+D+U*)**B* + *beta***C*
S or H	U	U	*alpha**op(*U'+I+U*)**x* + *beta***y* *alpha**op(*U'+I+U*)**B* + *beta***C*
T	L	U	*alpha**op(*L+I*)**x* + *beta***y* *alpha**op(*L+I*)**B* + *beta***C*
T	L	N	*alpha**op(*L+D*)**x* + *beta***y* *alpha**op(*L+D*)**B* + *beta***C*
T	U	U	*alpha**op(*U+I*)**x* + *beta***y* *alpha**op(*U+I*)**B* + *beta***C*
T	U	N	*alpha**op(*U+D*)**x* + *beta***y* *alpha**op(*U+D*)**B* + *beta***C*
A	L	ignored	*alpha**op(*L-L'*)**x* + *beta***y* *alpha**op(*L-L'*)**B* + *beta***C*
A	U	ignored	*alpha**op(*U-U'*)**x* + *beta***y* *alpha**op(*U-U'*)**B* + *beta***C*
D	ignored	N	*alpha***D***x* + *beta***y* *alpha***D***B* + *beta***C*
D	ignored	U	*alpha***x* + *beta***y* *alpha***B* + *beta***C*

Table "Output Matrices for Triangular Solvers" shows correspondence between the output matrices and values of the parameter *matdescra* for the sparse matrix *A* for triangular solvers.

Output Matrices for Triangular Solvers

matdescra(1)	matdescra(2)	matdescra(3)	Output Matrix
T	L	N	*alpha**inv(op(*L*+*L*))**x* *alpha**inv(op(*L*+*L*))**B*
T	L	U	*alpha**inv(op(*L*+*L*))**x* *alpha**inv(op(*L*+*L*))**B*
T	U	N	*alpha**inv(op(*U*+*U*))**x* *alpha**inv(op(*U*+*U*))**B*
T	U	U	*alpha**inv(op(*U*+*U*))**x* *alpha**inv(op(*U*+*U*))**B*
D	ignored	N	*alpha**inv(*D*)**x* *alpha**inv(*D*)**B*
D	ignored	U	*alpha***x* *alpha***B*

Sparse BLAS Level 2 and Level 3 Routines.

Table "Sparse BLAS Level 2 and Level 3 Routines" lists the sparse BLAS Level 2 and Level 3 routines described in more detail later in this section.

Sparse BLAS Level 2 and Level 3 Routines

Routine/Function	Description
Simplified interface, one-based indexing	
mkl_?csrgemv	Computes matrix - vector product of a sparse general matrix in the CSR format (3-array variation)
mkl_?bsrgemv	Computes matrix - vector product of a sparse general matrix in the BSR format (3-array variation).
mkl_?coogemv	Computes matrix - vector product of a sparse general matrix in the coordinate format.
mkl_?diagemv	Computes matrix - vector product of a sparse general matrix in the diagonal format.
mkl_?csrsymv	Computes matrix - vector product of a sparse symmetrical matrix in the CSR format (3-array variation)
mkl_?bsrsymv	Computes matrix - vector product of a sparse symmetrical matrix in the BSR format (3-array variation).
mkl_?coosymv	Computes matrix - vector product of a sparse symmetrical matrix in the coordinate format.
mkl_?diasymv	Computes matrix - vector product of a sparse symmetrical matrix in the diagonal format.
mkl_?csrtrsv	Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation).

Routine/Function	Description
mkl_?bsrtrsv	Triangular solver with simplified interface for a sparse matrix in the BSR format (3-array variation).
mkl_?cootrsv	Triangular solvers with simplified interface for a sparse matrix in the coordinate format.
mkl_?diatrsv	Triangular solvers with simplified interface for a sparse matrix in the diagonal format.

Simplified interface, zero-based indexing

mkl_cspblas_?csrgemv	Computes matrix - vector product of a sparse general matrix in the CSR format (3-array variation) with zero-based indexing.
mkl_cspblas_?bsrgemv	Computes matrix - vector product of a sparse general matrix in the BSR format (3-array variation)with zero-based indexing.
mkl_cspblas_?coogemv	Computes matrix - vector product of a sparse general matrix in the coordinate format with zero-based indexing.
mkl_cspblas_?csrsymv	Computes matrix - vector product of a sparse symmetrical matrix in the CSR format (3-array variation) with zero-based indexing
mkl_cspblas_?bsrsymv	Computes matrix - vector product of a sparse symmetrical matrix in the BSR format (3-array variation) with zero-based indexing.
mkl_cspblas_?coosymv	Computes matrix - vector product of a sparse symmetrical matrix in the coordinate format with zero-based indexing.
mkl_cspblas_?csrtrsv	Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation) with zero-based indexing.
mkl_cspblas_?bsrtrsv	Triangular solver with simplified interface for a sparse matrix in the BSR format (3-array variation) with zero-based indexing.
mkl_cspblas_?cootrsv	Triangular solver with simplified interface for a sparse matrix in the coordinate format with zero-based indexing.

Typical (conventional) interface, one-based and zero-based indexing

mkl_?csrmv	Computes matrix - vector product of a sparse matrix in the CSR format.
mkl_?bsrmv	Computes matrix - vector product of a sparse matrix in the BSR format.
mkl_?cscmv	Computes matrix - vector product for a sparse matrix in the CSC format.
mkl_?coomv	Computes matrix - vector product for a sparse matrix in the coordinate format.
mkl_?csrsv	Solves a system of linear equations for a sparse matrix in the CSR format.

Routine/Function	Description
mkl_?bsrsv	Solves a system of linear equations for a sparse matrix in the BSR format.
mkl_?cscsv	Solves a system of linear equations for a sparse matrix in the CSC format.
mkl_?coosv	Solves a system of linear equations for a sparse matrix in the coordinate format.
mkl_?csrmm	Computes matrix - matrix product of a sparse matrix in the CSR format
mkl_?bsrmm	Computes matrix - matrix product of a sparse matrix in the BSR format.
mkl_?cscmm	Computes matrix - matrix product of a sparse matrix in the CSC format
mkl_?coomm	Computes matrix - matrix product of a sparse matrix in the coordinate format.
mkl_?csrsm	Solves a system of linear matrix equations for a sparse matrix in the CSR format.
mkl_?bsrsm	Solves a system of linear matrix equations for a sparse matrix in the BSR format.
mkl_?cscsm	Solves a system of linear matrix equations for a sparse matrix in the CSC format.
mkl_?coosm	Solves a system of linear matrix equations for a sparse matrix in the coordinate format.
Typical (conventional) interface, one-based indexing	
mkl_?diamv	Computes matrix - vector product of a sparse matrix in the diagonal format.
mkl_?skymv	Computes matrix - vector product for a sparse matrix in the skyline storage format.
mkl_?diasv	Solves a system of linear equations for a sparse matrix in the diagonal format.
mkl_?skysv	Solves a system of linear equations for a sparse matrix in the skyline format.
mkl_?diamm	Computes matrix - matrix product of a sparse matrix in the diagonal format.
mkl_?skymm	Computes matrix - matrix product of a sparse matrix in the skyline storage format.
mkl_?diasm	Solves a system of linear matrix equations for a sparse matrix in the diagonal format.
mkl_?skysm	Solves a system of linear matrix equations for a sparse matrix in the skyline storage format.
Auxiliary routines	
Matrix converters	

Routine/Function	Description
mkl_?dnscsr	Converts a sparse matrix in the dense representation to the CSR format (3-array variation).
mkl_?csrcoo	Converts a sparse matrix in the CSR format (3-array variation) to the coordinate format and vice versa.
mkl_?csrbsr	Converts a sparse matrix in the CSR format to the BSR format (3-array variations) and vice versa.
mkl_?csrcsc	Converts a sparse matrix in the CSR format to the CSC and vice versa (3-array variations).
mkl_?csrdia	Converts a sparse matrix in the CSR format (3-array variation) to the diagonal format and vice versa.
mkl_?csrsky	Converts a sparse matrix in the CSR format (3-array variation) to the sky line format and vice versa.
Operations on sparse matrices	
mkl_?csradd	Computes the sum of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing.
mkl_?csrmultcsr	Computes the product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing.
mkl_?csrmultd	Computes product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing. The result is stored in the dense matrix.

mkl_?csrgemv

Computes matrix - vector product of a sparse general matrix stored in the CSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_scsrgemv(transa, m, a, ia, ja, x, y)
call mkl_dcsrgemv(transa, m, a, ia, ja, x, y)
call mkl_ccsrgemv(transa, m, a, ia, ja, x, y)
call mkl_zcsrgemv(transa, m, a, ia, ja, x, y)
```

C:

```
mkl_scsrgemv(&transa, &m, a, ia, ja, x, y);
mkl_dcsrgemv(&transa, &m, a, ia, ja, x, y);
mkl_ccsrgemv(&transa, &m, a, ia, ja, x, y);
mkl_zcsrgemv(&transa, &m, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The mkl_?csrgemv routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and *y* are vectors,

A is an *m*-by-*m* sparse square matrix in the CSR format (3-array variation), *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then as *y* := *A*x* If *transa* = 'T' or 't' or 'C' or 'c', then *y* := *A'*x*,
m	INTEGER. Number of rows of the matrix *A*.
a	REAL for mkl_scsrgemv. DOUBLE PRECISION for mkl_dcsrgemv. COMPLEX for mkl_ccsrgemv. DOUBLE COMPLEX for mkl_zcsrgemv. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ia	INTEGER. Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia(i)* is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia(m* + 1*)* is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
ja	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
x	REAL for mkl_scsrgemv. DOUBLE PRECISION for mkl_dcsrgemv. COMPLEX for mkl_ccsrgemv. DOUBLE COMPLEX for mkl_zcsrgemv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_scsrgemv. DOUBLE PRECISION for mkl_dcsrgemv. COMPLEX for mkl_ccsrgemv.

DOUBLE COMPLEX for mkl_zcsrgemv.
Array, DIMENSION at least *m*.
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   REAL       a(*), x(*), y(*)
SUBROUTINE mkl_dcsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_ccsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_zcsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER           m
   INTEGER           ia(*), ja(*)
   DOUBLE COMPLEX    a(*), x(*), y(*)
```

C:

```
void mkl_scsrgemv(char *transa, int *m, float *a,
int *ia, int *ja, float *x, float *y);
void mkl_dcsrgemv(char *transa, int *m, double *a,
int *ia, int *ja, double *x, double *y);
void mkl_ccsrgemv(char *transa, int *m, MKL_Complex8 *a,
int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zcsrgemv(char *transa, int *m, MKL_Complex16 *a,
int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?bsrgemv

Computes matrix - vector product of a sparse general matrix stored in the BSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_sbsrgemv(transa, m, lb, a, ia, ja, x, y)
call mkl_dbsrgemv(transa, m, lb, a, ia, ja, x, y)
call mkl_cbsrgemv(transa, m, lb, a, ia, ja, x, y)
call mkl_zbsrgemv(transa, m, lb, a, ia, ja, x, y)
```

C:

```
mkl_sbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
mkl_dbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
mkl_cbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
mkl_zbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrgemv` routine performs a matrix-vector operation defined as

`y := A*x`

or

`y := A'*x`,

where:

x and y are vectors,

A is an m-by-m block sparse square matrix in the BSR format (3-array variation), A' is the transpose of A.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-vector product is computed as *y* := *A*x* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *y* := *A'*x*,
m	INTEGER. Number of block rows of the matrix *A*.

lb	INTEGER. Size of the block in the matrix *A*.
a	REAL for mkl_sbsrgemv. DOUBLE PRECISION for mkl_dbsrgemv. COMPLEX for mkl_cbsrgemv. DOUBLE COMPLEX for mkl_zbsrgemv. Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to *values* array description in BSR Format for more details.
ia	INTEGER. Array of length *(m + 1)*, containing indices of block in the array *a*, such that *ia(i)* is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia(m + 1)* is equal to the number of non-zero blocks plus one. Refer to *rowIndex* array description in BSR Format for more details.
ja	INTEGER. Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.
x	REAL for mkl_sbsrgemv. DOUBLE PRECISION for mkl_dbsrgemv. COMPLEX for mkl_cbsrgemv. DOUBLE COMPLEX for mkl_zbsrgemv. Array, DIMENSION *(m*lb)*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_sbsrgemv. DOUBLE PRECISION for mkl_dbsrgemv. COMPLEX for mkl_cbsrgemv. DOUBLE COMPLEX for mkl_zbsrgemv. Array, DIMENSION at least *(m*lb)*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)
SUBROUTINE mkl_dbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION     a(*), x(*), y(*)
```

```
SUBROUTINE mkl_cbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_zbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX       a(*), x(*), y(*)
```

C:

```
void mkl_dbsrgemv(char *transa, int *m, int *lb, double *a,
int *ia, int *ja, double *x, double *y);
void mkl_sbsrgemv(char *transa, int *m, int *lb, float *a,
int *ia, int *ja, float *x, float *y);
void mkl_cbsrgemv(char *transa, int *m, int *lb, MKL_Complex8 *a,
int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zbsrgemv(char *transa, int *m, int *lb, MKL_Complex16 *a,
int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?coogemv

Computes matrix-vector product of a sparse general matrix stored in the coordinate format with one-based indexing.

Syntax

Fortran:

```
call mkl_scoogemv(transa, m, val, rowind, colind, nnz, x, y)
call mkl_dcoogemv(transa, m, val, rowind, colind, nnz, x, y)
call mkl_ccoogemv(transa, m, val, rowind, colind, nnz, x, y)
call mkl_zcoogemv(transa, m, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_scoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
mkl_dcoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
mkl_ccoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
mkl_zcoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The mkl_?coogemv routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and *y* are vectors,

A is an *m*-by-*m* sparse square matrix in the coordinate format, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-vector product is computed as *y* := *A*x* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *y* := *A'*x*,
m	INTEGER. Number of rows of the matrix *A*.
val	REAL for mkl_scoogemv. DOUBLE PRECISION for mkl_dcoogemv. COMPLEX for mkl_ccoogemv. DOUBLE COMPLEX for mkl_zcoogemv. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_scoogemv. DOUBLE PRECISION for mkl_dcoogemv. COMPLEX for mkl_ccoogemv. DOUBLE COMPLEX for mkl_zcoogemv. Array, DIMENSION is *m*. One entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_scoogemv. DOUBLE PRECISION for mkl_dcoogemv.

COMPLEX for mkl_ccoogemv.
DOUBLE COMPLEX for mkl_zcoogemv.
Array, DIMENSION at least *m*.
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  REAL            val(*), x(*), y(*)
SUBROUTINE mkl_dcoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION      val(*), x(*), y(*)
SUBROUTINE mkl_ccoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX     val(*), x(*), y(*)
SUBROUTINE mkl_zcoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_scoogemv(char *transa, int *m, float  *val, int *rowind,
int *colind, int *nnz, float *x, float *y);
void mkl_dcoogemv(char *transa, int *m, double  *val, int *rowind,
int *colind, int *nnz, double *x, double *y);
void mkl_ccoogemv(char *transa, int *m, MKL_Complex8  *val, int *rowind,
int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zcoogemv(char *transa, int *m, MKL_Complex16  *val, int *rowind,
int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?diagemv

Computes matrix - vector product of a sparse general matrix stored in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiagemv(transa, m, val, lval, idiag, ndiag, x, y)
call mkl_ddiagemv(transa, m, val, lval, idiag, ndiag, x, y)
call mkl_cdiagemv(transa, m, val, lval, idiag, ndiag, x, y)
call mkl_zdiagemv(transa, m, val, lval, idiag, ndiag, x, y)
```

C:

```
mkl_sdiagemv(&transa, &m, val, &lval, idiag, &ndiag, x, y);
mkl_ddiagemv(&transa, &m, val, &lval, idiag, &ndiag, x, y);
mkl_cdiagemv(&transa, &m, val, &lval, idiag, &ndiag, x, y);
mkl_zdiagemv(&transa, &m, val, &lval, idiag, &ndiag, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?diagemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and *y* are vectors,

A is an *m*-by-*m* sparse square matrix in the diagonal storage format, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then *y* := *A*x* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *y* := *A'*x*,
m	`INTEGER`. Number of rows of the matrix *A*.
val	`REAL` for `mkl_sdiagemv`. `DOUBLE PRECISION` for `mkl_ddiagemv`.

	COMPLEX for mkl_ccsrgemv. DOUBLE COMPLEX for mkl_zdiagemv. Two-dimensional array of size *lval***ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.
lval	INTEGER. Leading dimension of *val* *lval*≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.
idiag	INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*. Refer to *distance* array description in Diagonal Storage Scheme for more details.
ndiag	INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.
x	REAL for mkl_sdiagemv. DOUBLE PRECISION for mkl_ddiagemv. COMPLEX for mkl_ccsrgemv. DOUBLE COMPLEX for mkl_zdiagemv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_sdiagemv. DOUBLE PRECISION for mkl_ddiagemv. COMPLEX for mkl_ccsrgemv. DOUBLE COMPLEX for mkl_zdiagemv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiagemv(transa, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  REAL           val(lval,*), x(*), y(*)

SUBROUTINE mkl_ddiagemv(transa, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE PRECISION       val(lval,*), x(*), y(*)

SUBROUTINE mkl_cdiagemv(transa, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  COMPLEX    val(lval,*), x(*), y(*)
```

```
SUBROUTINE mkl_zdiagemv(transa, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE COMPLEX        val(lval,*), x(*), y(*)
```

C:

```
void mkl_sdiagemv(char *transa, int *m, float  *val, int *lval,
int *idiag, int *ndiag, float *x, float *y);
void mkl_ddiagemv(char *transa, int *m, double  *val, int *lval,
int *idiag, int *ndiag, double *x, double *y);
void mkl_cdiagemv(char *transa, int *m, MKL_Complex8  *val, int *lval,
int *idiag, int *ndiag, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zdiagemv(char *transa, int *m, MKL_Complex16  *val, int *lval,
int *idiag, int *ndiag, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?csrsymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the CSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_scsrsymv(uplo, m, a, ia, ja, x, y)
call mkl_dcsrsymv(uplo, m, a, ia, ja, x, y)
call mkl_ccsrsymv(uplo, m, a, ia, ja, x, y)
call mkl_zcsrsymv(uplo, m, a, ia, ja, x, y)
```

C:

```
mkl_scsrsymv(&uplo, &m, a, ia, ja, x, y);
mkl_dcsrsymv(&uplo, &m, a, ia, ja, x, y);
mkl_ccsrsymv(&uplo, &m, a, ia, ja, x, y);
mkl_zcsrsymv(&uplo, &m, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrsymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the CSR format (3-array variation).

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	CHARACTER*1. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used. If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.
m	INTEGER. Number of rows of the matrix *A*.
a	REAL for mkl_scsrsymv. DOUBLE PRECISION for mkl_dcsrsymv. COMPLEX for mkl_ccsrsymv. DOUBLE COMPLEX for mkl_zcsrsymv. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ia	INTEGER. Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia*(*i*) is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
ja	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
x	REAL for mkl_scsrsymv. DOUBLE PRECISION for mkl_dcsrsymv. COMPLEX for mkl_ccsrsymv. DOUBLE COMPLEX for mkl_zcsrsymv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_scsrsymv. DOUBLE PRECISION for mkl_dcsrsymv. COMPLEX for mkl_ccsrsymv. DOUBLE COMPLEX for mkl_zcsrsymv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m
   INTEGER      ia(*), ja(*)
   REAL           a(*), x(*), y(*)
SUBROUTINE mkl_dcsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_ccsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_zcsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX       a(*), x(*), y(*)
```

C:

```
void mkl_scsrsymv(char *uplo, int *m, float *a,
 int *ia, int *ja, float *x, float *y);
void mkl_dcsrsymv(char *uplo, int *m, double *a,
 int *ia, int *ja, double *x, double *y);
void mkl_ccsrsymv(char *uplo, int *m, MKL_Complex8 *a,
 int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zcsrsymv(char *uplo, int *m, MKL_Complex16 *a,
 int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?bsrsymv

Computes matrix-vector product of a sparse symmetrical matrix stored in the BSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_sbsrsymv(uplo, m, lb, a, ia, ja, x, y)
call mkl_dbsrsymv(uplo, m, lb, a, ia, ja, x, y)
call mkl_cbsrsymv(uplo, m, lb, a, ia, ja, x, y)
call mkl_zbsrsymv(uplo, m, lb, a, ia, ja, x, y)
```

C:

```
mkl_sbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
mkl_dbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
mkl_cbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
mkl_zbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrsymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and *y* are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the BSR format (3-array variation).

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	`CHARACTER*1`. Specifies whether the upper or low triangle of the matrix *A* is considered. If *uplo* = `'U'` or `'u'`, then the upper triangle of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangle of the matrix *A* is used.
m	`INTEGER`. Number of block rows of the matrix *A*.
lb	`INTEGER`. Size of the block in the matrix *A*.
a	`REAL` for `mkl_sbsrsymv`. `DOUBLE PRECISION` for `mkl_dbsrsymv`. `COMPLEX` for `mkl_cbsrsymv`. `DOUBLE COMPLEX` for `mkl_zcsrgemv`.

Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to *values* array description in BSR Format for more details.

ia INTEGER. Array of length (*m* + 1), containing indices of block in the array *a*, such that *ia*(*i*) is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zero blocks plus one. Refer to *rowIndex* array description in BSR Format for more details.

ja INTEGER. Array containing the column indices for each non-zero block in the matrix *A*.
Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.

x REAL for mkl_sbsrsymv.
DOUBLE PRECISION for mkl_dbsrsymv.
COMPLEX for mkl_cbsrsymv.
DOUBLE COMPLEX for mkl_zcsrgemv.
Array, DIMENSION (*m*lb*).
On entry, the array *x* must contain the vector *x*.

Output Parameters

y REAL for mkl_sbsrsymv.
DOUBLE PRECISION for mkl_dbsrsymv.
COMPLEX for mkl_cbsrsymv.
DOUBLE COMPLEX for mkl_zcsrgemv.
Array, DIMENSION at least (*m*lb*).
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)
SUBROUTINE mkl_dbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_cbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   COMPLEX    a(*), x(*), y(*)
```

```
SUBROUTINE mkl_zbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX       a(*), x(*), y(*)
```

C:

```
void mkl_sbsrsymv(char *uplo, int *m, int *lb,
float *a, int *ia, int *ja, float *x, float *y);

void mkl_dbsrsymv(char *uplo, int *m, int *lb,
double *a, int *ia, int *ja, double *x, double *y);

void mkl_cbsrsymv(char *uplo, int *m, int *lb,
MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_zbsrsymv(char *uplo, int *m, int *lb,
MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?coosymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the coordinate format with one-based indexing.

Syntax

Fortran:

```
call mkl_scoosymv(uplo, m, val, rowind, colind, nnz, x, y)
call mkl_dcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
call mkl_ccoosymv(uplo, m, val, rowind, colind, nnz, x, y)
call mkl_zcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_scoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
mkl_dcoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
mkl_ccoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
mkl_zcoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?coosymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the coordinate format.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	CHARACTER*1. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used. If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.
m	INTEGER. Number of rows of the matrix *A*.
val	REAL for mkl_scoosymv. DOUBLE PRECISION for mkl_dcoosymv. COMPLEX for mkl_ccoosymv. DOUBLE COMPLEX for mkl_zcoosymv. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_scoosymv. DOUBLE PRECISION for mkl_dcoosymv. COMPLEX for mkl_ccoosymv. DOUBLE COMPLEX for mkl_zcoosymv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_scoosymv. DOUBLE PRECISION for mkl_dcoosymv. COMPLEX for mkl_ccoosymv. DOUBLE COMPLEX for mkl_zcoosymv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  REAL           val(*), x(*), y(*)
SUBROUTINE mkl_dcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION     val(*), x(*), y(*)
SUBROUTINE mkl_cdcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX     val(*), x(*), y(*)
SUBROUTINE mkl_zcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_scoosymv(char *uplo, int *m, float  *val, int *rowind,
int *colind, int *nnz, float *x, float *y);
void mkl_dcoosymv(char *uplo, int *m, double  *val, int *rowind,
int *colind, int *nnz, double *x, double *y);
void mkl_ccoosymv(char *uplo, int *m, MKL_Complex8  *val, int *rowind,
int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zcoosymv(char *uplo, int *m, MKL_Complex16  *val, int *rowind,
int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?diasymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
call mkl_ddiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
call mkl_cdiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
call mkl_zdiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
```

C:

```
mkl_sdiasymv(&uplo, &m, val, &lval, idiag, &ndiag, x, y);
mkl_ddiasymv(&uplo, &m, val, &lval, idiag, &ndiag, x, y);
mkl_cdiasymv(&uplo, &m, val, &lval, idiag, &ndiag, x, y);
mkl_zdiasymv(&uplo, &m, val, &lval, idiag, &ndiag, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?diasymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and *y* are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	`CHARACTER*1`. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = `'U'` or `'u'`, then the upper triangle of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangle of the matrix *A* is used.
m	`INTEGER`. Number of rows of the matrix *A*.
val	`REAL` for `mkl_sdiasymv`. `DOUBLE PRECISION` for `mkl_ddiasymv`. `COMPLEX` for `mkl_cdiasymv`. `DOUBLE COMPLEX` for `mkl_zdiasymv`. Two-dimensional array of size *lval* by *ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.
lval	`INTEGER`. Leading dimension of *val*, *lval* ≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.

idiag	INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*. Refer to *distance* array description in Diagonal Storage Scheme for more details.
ndiag	INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.
x	REAL for mkl_sdiasymv. DOUBLE PRECISION for mkl_ddiasymv. COMPLEX for mkl_cdiasymv. DOUBLE COMPLEX for mkl_zdiasymv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_sdiasymv. DOUBLE PRECISION for mkl_ddiasymv. COMPLEX for mkl_cdiasymv. DOUBLE COMPLEX for mkl_zdiasymv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  REAL           val(lval,*), x(*), y(*)

SUBROUTINE mkl_ddiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE PRECISION      val(lval,*), x(*), y(*)

SUBROUTINE mkl_cdiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  COMPLEX    val(lval,*), x(*), y(*)

SUBROUTINE mkl_zdiasymv(uplo, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo
  INTEGER       m, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE COMPLEX        val(lval,*), x(*), y(*)
```

C:

```
void mkl_sdiasymv(char *uplo, int *m, float  *val, int *lval,
int *idiag, int *ndiag, float *x, float *y);
void mkl_ddiasymv(char *uplo, int *m, double  *val, int *lval,
int *idiag, int *ndiag, double *x, double *y);
void mkl_cdiasymv(char *uplo, int *m, MKL_Complex8  *val, int *lval,
int *idiag, int *ndiag, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zdiasymv(char *uplo, int *m, MKL_Complex16  *val, int *lval,
int *idiag, int *ndiag, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?csrtrsv

Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_scsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
call mkl_dcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
call mkl_ccsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
call mkl_zcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
```

C:

```
mkl_scsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
mkl_dcsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
mkl_ccsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
mkl_zcsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the CSR format (3 array variation):

```
A*y = x
```

or

```
A'*y = x,
```

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A' is the transpose of A.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo — CHARACTER*1. Specifies whether the upper or low triangle of the matrix *A* is used.
If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used.
If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.

transa — CHARACTER*1. Specifies the system of linear equations.
If *transa* = 'N' or 'n', then *A*y = x*
If *transa* = 'T' or 't' or 'C' or 'c', then *A'*y = x*,

diag — CHARACTER*1. Specifies whether *A* is unit triangular.
If *diag* = 'U' or 'u', then *A* is a unit triangular.
If *diag* = 'N' or 'n', then *A* is not unit triangular.

m — INTEGER. Number of rows of the matrix *A*.

a — REAL for mkl_scsrtrmv.
DOUBLE PRECISION for mkl_dcsrtrmv.
COMPLEX for mkl_ccsrtrmv.
DOUBLE COMPLEX for mkl_zcsrtrmv.
Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia — INTEGER. Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia(i)* is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia(m* + 1*)* is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

ja — INTEGER. Array containing the column indices for each non-zero element of the matrix *A*.
Its length is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

NOTE Column indices must be sorted in increasing order for each row.

x — REAL for mkl_scsrtrmv.
DOUBLE PRECISION for mkl_dcsrtrmv.
COMPLEX for mkl_ccsrtrmv.
DOUBLE COMPLEX for mkl_zcsrtrmv.
Array, DIMENSION is *m*.
On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_scsrtrmv. DOUBLE PRECISION for mkl_dcsrtrmv. COMPLEX for mkl_ccsrtrmv. DOUBLE COMPLEX for mkl_zcsrtrmv. Array, DIMENSION at least *m*. Contains the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)
SUBROUTINE mkl_dcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION     a(*), x(*), y(*)
SUBROUTINE mkl_ccsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   COMPLEX     a(*), x(*), y(*)
SUBROUTINE mkl_zcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX      a(*), x(*), y(*)
```

C:

```
void mkl_scsrtrsv(char *uplo, char *transa, char *diag, int *m,
float *a, int *ia, int *ja, float *x, float *y);
void mkl_dcsrtrsv(char *uplo, char *transa, char *diag, int *m,
double *a, int *ia, int *ja, double *x, double *y);
void mkl_ccsrtrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zcsrtrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?bsrtrsv

Triangular solver with simplified interface for a sparse matrix stored in the BSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_sbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
call mkl_dbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
call mkl_cbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
call mkl_zbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
```

C:

```
mkl_sbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
mkl_dbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
mkl_cbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
mkl_zbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the BSR format (3-array variation) :

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	CHARACTER*1. Specifies the upper or low triangle of the matrix *A* is used. If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used. If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.
transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-vector product is computed as *y* := *A*x*

If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *y* := *A*'**x*.

diag CHARACTER*1. Specifies whether *A* is a unit triangular matrix.
If *diag* = 'U' or 'u', then *A* is a unit triangular.
If *diag* = 'N' or 'n', then *A* is not a unit triangular.

m INTEGER. Number of block rows of the matrix *A*.

lb INTEGER. Size of the block in the matrix *A*.

a REAL for mkl_sbsrtrsv.
DOUBLE PRECISION for mkl_dbsrtrsv.
COMPLEX for mkl_cbsrtrsv.
DOUBLE COMPLEX for mkl_zbsrtrsv.
Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to *values* array description in BSR Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia INTEGER. Array of length (*m* + 1), containing indices of block in the array *a*, such that *ia*(*I*) is the index in the array *a* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zero blocks plus one. Refer to *rowIndex* array description in BSR Format for more details.

ja INTEGER.
Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.

x REAL for mkl_sbsrtrsv.
DOUBLE PRECISION for mkl_dbsrtrsv.
COMPLEX for mkl_cbsrtrsv.
DOUBLE COMPLEX for mkl_zbsrtrsv.
Array, DIMENSION (*m***lb*).
On entry, the array *x* must contain the vector *x*.

Output Parameters

y REAL for mkl_sbsrtrsv.
DOUBLE PRECISION for mkl_dbsrtrsv.
COMPLEX for mkl_cbsrtrsv.
DOUBLE COMPLEX for mkl_zbsrtrsv.
Array, DIMENSION at least (*m***lb*).
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)
SUBROUTINE mkl_dbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_cbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_zbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX       a(*), x(*), y(*)
```

C:

```
void mkl_sbsrtrsv(char *uplo, char *transa, char *diag, int *m,
 int *lb, float *a, int *ia, int *ja, float *x, float *y);
void mkl_dbsrtrsv(char *uplo, char *transa, char *diag, int *m,
 int *lb, double *a, int *ia, int *ja, double *x, double *y);
void mkl_cbsrtrsv(char *uplo, char *transa, char *diag, int *m,
 int *lb, MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zbsrtrsv(char *uplo, char *transa, char *diag, int *m,
 int *lb, MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?cootrsv

Triangular solvers with simplified interface for a sparse matrix in the coordinate format with one-based indexing.

Syntax

Fortran:

```
call mkl_scootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
call mkl_dcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
call mkl_ccootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
call mkl_zcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_scootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
mkl_dcootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
mkl_ccootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
mkl_zcootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?cootrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the coordinate format:

```
A*y = x
```

or

```
A'*y = x,
```

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	CHARACTER*1. Specifies whether the upper or low triangle of the matrix *A* is considered. If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used. If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.
transa	CHARACTER*1. Specifies the system of linear equations. If *transa* = 'N' or 'n', then *A*y = x* If *transa* = 'T' or 't' or 'C' or 'c', then *A'*y = x*,
diag	CHARACTER*1. Specifies whether *A* is unit triangular. If *diag* = 'U' or 'u', then *A* is unit triangular. If *diag* = 'N' or 'n', then *A* is not unit triangular.

m	INTEGER. Number of rows of the matrix *A*.
val	REAL for mkl_scootrsv. DOUBLE PRECISION for mkl_dcootrsv. COMPLEX for mkl_ccootrsv. DOUBLE COMPLEX for mkl_zcootrsv. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_scootrsv. DOUBLE PRECISION for mkl_dcootrsv. COMPLEX for mkl_ccootrsv. DOUBLE COMPLEX for mkl_zcootrsv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_scootrsv. DOUBLE PRECISION for mkl_dcootrsv. COMPLEX for mkl_ccootrsv. DOUBLE COMPLEX for mkl_zcootrsv. Array, DIMENSION at least *m*. Contains the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  REAL            val(*), x(*), y(*)
SUBROUTINE mkl_dcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION        val(*), x(*), y(*)
```

```
SUBROUTINE mkl_ccootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        val(*), x(*), y(*)
SUBROUTINE mkl_zcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_scootrsv(char *uplo, char *transa, char *diag, int *m,
float  *val, int *rowind,   int *colind, int *nnz, float *x, double *y);
void mkl_dcootrsv(char *uplo, char *transa, char *diag, int *m,
double  *val, int *rowind,   int *colind, int *nnz, double *x, double *y);
void mkl_ccootrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex8  *val, int *rowind,  int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zcootrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex16  *val, int *rowind,  int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?diatrsv

Triangular solvers with simplified interface for a sparse matrix in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
call mkl_ddiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
call mkl_cdiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
call mkl_zdiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
```

C:

```
mkl_sdiatrsv(&uplo, &transa, &diag, &m, val, &lval, idiag, &ndiag, x, y);
mkl_ddiatrsv(&uplo, &transa, &diag, &m, val, &lval, idiag, &ndiag, x, y);
mkl_cdiatrsv(&uplo, &transa, &diag, &m, val, &lval, idiag, &ndiag, x, y);
mkl_zdiatrsv(&uplo, &transa, &diag, &m, val, &lval, idiag, &ndiag, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The mkl_?diatrsv routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the diagonal format:

```
A*y = x
```

or

```
A'*y = x,
```

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	CHARACTER*1. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used. If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.
transa	CHARACTER*1. Specifies the system of linear equations. If *transa* = 'N' or 'n', then *A*y* = x* If *transa* = 'T' or 't' or 'C' or 'c', then *A'*y* = x*,
diag	CHARACTER*1. Specifies whether *A* is unit triangular. If *diag* = 'U' or 'u', then *A* is unit triangular. If *diag* = 'N' or 'n', then *A* is not unit triangular.
m	INTEGER. Number of rows of the matrix *A*.
val	REAL for mkl_sdiatrsv. DOUBLE PRECISION for mkl_ddiatrsv. COMPLEX for mkl_cdiatrsv. DOUBLE COMPLEX for mkl_zdiatrsv. Two-dimensional array of size *lval* by *ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.
lval	INTEGER. Leading dimension of *val*, *lval*≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.
idiag	INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*. **NOTE** All elements of this array must be sorted in increasing order. Refer to *distance* array description in Diagonal Storage Scheme for more details.
ndiag	INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.
x	REAL for mkl_sdiatrsv.

DOUBLE PRECISION for mkl_ddiatrsv.
COMPLEX for mkl_cdiatrsv.
DOUBLE COMPLEX for mkl_zdiatrsv.
Array, DIMENSION is *m*.
On entry, the array *x* must contain the vector *x*.

Output Parameters

y

REAL for mkl_sdiatrsv.
DOUBLE PRECISION for mkl_ddiatrsv.
COMPLEX for mkl_cdiatrsv.
DOUBLE COMPLEX for mkl_zdiatrsv.
Array, DIMENSION at least *m*.
Contains the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  REAL            val(lval,*), x(*), y(*)

SUBROUTINE mkl_ddiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  DOUBLE PRECISION       val(lval,*), x(*), y(*)

SUBROUTINE mkl_cdiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  COMPLEX     val(lval,*), x(*), y(*)

SUBROUTINE mkl_zdiatrsv(uplo, transa, diag, m, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  DOUBLE COMPLEX        val(lval,*), x(*), y(*)
```

C:

```
void mkl_sdiatrsv(char *uplo, char *transa, char *diag, int *m, float
  *val, int *lval, int *idiag, int *ndiag, float *x, float *y);

void mkl_ddiatrsv(char *uplo, char *transa, char *diag, int *m, double
       *val, int *lval, int *idiag, int *ndiag, double *x, double *y);
```

```
void mkl_cdiatrsv(char *uplo, char *transa, char *diag, int *m, MKL_Complex8
        *val, int *lval, int *idiag, int *ndiag, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zdiatrsv(char *uplo, char *transa, char *diag, int *m, MKL_Complex16
        *val, int *lval, int *idiag, int *ndiag, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?csrgemv

Computes matrix - vector product of a sparse general matrix stored in the CSR format (3-array variation) with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_scsrgemv(transa, m, a, ia, ja, x, y)
call mkl_cspblas_dcsrgemv(transa, m, a, ia, ja, x, y)
call mkl_cspblas_ccsrgemv(transa, m, a, ia, ja, x, y)
call mkl_cspblas_zcsrgemv(transa, m, a, ia, ja, x, y)
```

C:

```
mkl_cspblas_scsrgemv(&transa, &m, a, ia, ja, x, y);
mkl_cspblas_dcsrgemv(&transa, &m, a, ia, ja, x, y);
mkl_cspblas_ccsrgemv(&transa, &m, a, ia, ja, x, y);
mkl_cspblas_zcsrgemv(&transa, &m, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?csrgemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and y are vectors,

A is an m-by-m sparse square matrix in the CSR format (3-array variation) with zero-based indexing, A' is the transpose of A.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-vector product is computed as *y* := *A*x* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *y* := *A'*x*,
m	INTEGER. Number of rows of the matrix *A*.
a	REAL for mkl_cspblas_scsrgemv. DOUBLE PRECISION for mkl_cspblas_dcsrgemv. COMPLEX for mkl_cspblas_ccsrgemv. DOUBLE COMPLEX for mkl_cspblas_zcsrgemv. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ia	INTEGER. Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia*(*I*) is the index in the array *a* of the first non-zero element from the row *I*. The value of the last element *ia*(*m*) is equal to the number of non-zeros. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
ja	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
x	REAL for mkl_cspblas_scsrgemv. DOUBLE PRECISION for mkl_cspblas_dcsrgemv. COMPLEX for mkl_cspblas_ccsrgemv. DOUBLE COMPLEX for mkl_cspblas_zcsrgemv. Array, DIMENSION is *m*. One entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_cspblas_scsrgemv. DOUBLE PRECISION for mkl_cspblas_dcsrgemv. COMPLEX for mkl_cspblas_ccsrgemv. DOUBLE COMPLEX for mkl_cspblas_zcsrgemv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_scsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   REAL           a(*), x(*), y(*)
```

```
SUBROUTINE mkl_cspblas_dcsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_ccsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zcsrgemv(transa, m, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX      a(*), x(*), y(*)
```

C:

```
void mkl_cspblas_scsrgemv(char *transa, int *m, float *a,
int *ia, int *ja, float *x, float *y);
void mkl_cspblas_dcsrgemv(char *transa, int *m, double *a,
int *ia, int *ja, double *x, double *y);
void mkl_cspblas_ccsrgemv(char *transa, int *m, MKL_Complex8 *a,
int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_cspblas_zcsrgemv(char *transa, int *m, MKL_Complex16 *a,
int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?bsrgemv

Computes matrix - vector product of a sparse general matrix stored in the BSR format (3-array variation) with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_sbsrgemv(transa, m, lb, a, ia, ja, x, y)
call mkl_cspblas_dbsrgemv(transa, m, lb, a, ia, ja, x, y)
call mkl_cspblas_cbsrgemv(transa, m, lb, a, ia, ja, x, y)
call mkl_cspblas_zbsrgemv(transa, m, lb, a, ia, ja, x, y)
```

C:

```
mkl_cspblas_sbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_dbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
```

```
mkl_cspblas_cbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_zbsrgemv(&transa, &m, &lb, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?bsrgemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and *y* are vectors,

A is an *m*-by-*m* block sparse square matrix in the BSR format (3-array variation) with zero-based indexing, *A'* is the transpose of *A*.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then the matrix-vector product is computed as *y* := *A*x* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then the matrix-vector product is computed as *y* := *A'*x*,
m	`INTEGER`. Number of block rows of the matrix *A*.
lb	`INTEGER`. Size of the block in the matrix *A*.
a	`REAL` for `mkl_cspblas_sbsrgemv`. `DOUBLE PRECISION` for `mkl_cspblas_dbsrgemv`. `COMPLEX` for `mkl_cspblas_cbsrgemv`. `DOUBLE COMPLEX` for `mkl_cspblas_zbsrgemv`. Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to *values* array description in BSR Format for more details.
ia	`INTEGER`. Array of length *(m + 1)*, containing indices of block in the array *a*, such that *ia(i)* is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia(m + 1)* is equal to the number of non-zero blocks. Refer to *rowIndex* array description in BSR Format for more details.
ja	`INTEGER`. Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.

x	REAL for mkl_cspblas_sbsrgemv. DOUBLE PRECISION for mkl_cspblas_dbsrgemv. COMPLEX for mkl_cspblas_cbsrgemv. DOUBLE COMPLEX for mkl_cspblas_zbsrgemv. Array, DIMENSION (*m***lb*). On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_cspblas_sbsrgemv. DOUBLE PRECISION for mkl_cspblas_dbsrgemv. COMPLEX for mkl_cspblas_cbsrgemv. DOUBLE COMPLEX for mkl_cspblas_zbsrgemv. Array, DIMENSION at least (*m***lb*). On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_sbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_dbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_cbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zbsrgemv(transa, m, lb, a, ia, ja, x, y)
   CHARACTER*1  transa
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX       a(*), x(*), y(*)
```

C:

```
void mkl_cspblas_sbsrgemv(char *transa, int *m, int *lb,
float *a, int *ia, int *ja, float *x, float *y);
```

```
void mkl_cspblas_dbsrgemv(char *transa, int *m, int *lb,
double *a, int *ia, int *ja, double *x, double *y);

void mkl_cspblas_cbsrgemv(char *transa, int *m, int *lb,
MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_cspblas_zbsrgemv(char *transa, int *m, int *lb,
MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?coogemv

Computes matrix - vector product of a sparse general matrix stored in the coordinate format with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_scoogemv(transa, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_dcoogemv(transa, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_ccoogemv(transa, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_zcoogemv(transa, m, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_cspblas_scoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_dcoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_ccoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_zcoogemv(&transa, &m, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_dcoogemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and y are vectors,

A is an m-by-m sparse square matrix in the coordinate format with zero-based indexing, A' is the transpose of A.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-vector product is computed as *y* := *A*x* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *y* := *A'*x*.
m	INTEGER. Number of rows of the matrix *A*.
val	REAL for mkl_cspblas_scoogemv. DOUBLE PRECISION for mkl_cspblas_dcoogemv. COMPLEX for mkl_cspblas_ccoogemv. DOUBLE COMPLEX for mkl_cspblas_zcoogemv. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_cspblas_scoogemv. DOUBLE PRECISION for mkl_cspblas_dcoogemv. COMPLEX for mkl_cspblas_ccoogemv. DOUBLE COMPLEX for mkl_cspblas_zcoogemv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_cspblas_scoogemv. DOUBLE PRECISION for mkl_cspblas_dcoogemv. COMPLEX for mkl_cspblas_ccoogemv. DOUBLE COMPLEX for mkl_cspblas_zcoogemv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_scoogemv(transa, m, val, rowind, colind, nnz, x, y)

  CHARACTER*1   transa

  INTEGER       m, nnz

  INTEGER       rowind(*), colind(*)

  REAL            val(*), x(*), y(*)
```

```
SUBROUTINE mkl_cspblas_dcoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION      val(*), x(*), y(*)
SUBROUTINE mkl_cspblas_ccoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        val(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zcoogemv(transa, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX       val(*), x(*), y(*)
```

C:

```
void mkl_cspblas_scoogemv(char *transa, int *m, float  *val, int *rowind,
int *colind, int *nnz, float *x, float *y);
void mkl_cspblas_dcoogemv(char *transa, int *m, double  *val, int *rowind,
int *colind, int *nnz, double *x, double *y);
void mkl_cspblas_ccoogemv(char *transa, int *m, MKL_Complex8  *val, int *rowind,
int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_cspblas_zcoogemv(char *transa, int *m, MKL_Complex16  *val, int *rowind,
int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?csrsymv

Computes matrix-vector product of a sparse symmetrical matrix stored in the CSR format (3-array variation) with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_scsrsymv(uplo, m, a, ia, ja, x, y)
call mkl_cspblas_dcsrsymv(uplo, m, a, ia, ja, x, y)
call mkl_cspblas_ccsrsymv(uplo, m, a, ia, ja, x, y)
call mkl_cspblas_zcsrsymv(uplo, m, a, ia, ja, x, y)
```

C:

```
mkl_cspblas_scsrsymv(&uplo, &m, a, ia, ja, x, y);
mkl_cspblas_dcsrsymv(&uplo, &m, a, ia, ja, x, y);
```

```
mkl_cspblas_ccsrsymv(&uplo, &m, a, ia, ja, x, y);
mkl_cspblas_zcsrsymv(&uplo, &m, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?csrsymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and *y* are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the CSR format (3-array variation) with zero-based indexing.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	`CHARACTER*1`. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = `'U'` or `'u'`, then the upper triangle of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangle of the matrix *A* is used.
m	`INTEGER`. Number of rows of the matrix *A*.
a	`REAL` for `mkl_cspblas_scsrsymv`. `DOUBLE PRECISION` for `mkl_cspblas_dcsrsymv`. `COMPLEX` for `mkl_cspblas_ccsrsymv`. `DOUBLE COMPLEX` for `mkl_cspblas_zcsrsymv`. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ia	`INTEGER`. Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia*(*i*) is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
ja	`INTEGER`. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
x	`REAL` for `mkl_cspblas_scsrsymv`. `DOUBLE PRECISION` for `mkl_cspblas_dcsrsymv`. `COMPLEX` for `mkl_cspblas_ccsrsymv`.

DOUBLE COMPLEX for mkl_cspblas_zcsrsymv.
Array, DIMENSION is *m*.
On entry, the array *x* must contain the vector *x*.

Output Parameters

y REAL for mkl_cspblas_scsrsymv.
DOUBLE PRECISION for mkl_cspblas_dcsrsymv.
COMPLEX for mkl_cspblas_ccsrsymv.
DOUBLE COMPLEX for mkl_cspblas_zcsrsymv.
Array, DIMENSION at least *m*.
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_scsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m
   INTEGER      ia(*), ja(*)
   REAL         a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_dcsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1        uplo
   INTEGER            m
   INTEGER            ia(*), ja(*)
   DOUBLE PRECISION   a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_ccsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m
   INTEGER      ia(*), ja(*)
   COMPLEX      a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zcsrsymv(uplo, m, a, ia, ja, x, y)
   CHARACTER*1      uplo
   INTEGER          m
   INTEGER          ia(*), ja(*)
   DOUBLE COMPLEX   a(*), x(*), y(*)
```

C:

```
void mkl_cspblas_scsrsymv(char *uplo, int *m, float *a,
int *ia, int *ja, float *x, float *y);
void mkl_cspblas_dcsrsymv(char *uplo, int *m, double *a,
int *ia, int *ja, double *x, double *y);
```

```
void mkl_cspblas_ccsrsymv(char *uplo, int *m, MKL_Complex8 *a,
int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_cspblas_zcsrsymv(char *uplo, int *m, MKL_Complex16 *a,
int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?bsrsymv

Computes matrix-vector product of a sparse symmetrical matrix stored in the BSR format (3-arrays variation) with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_sbsrsymv(uplo, m, lb, a, ia, ja, x, y)
call mkl_cspblas_dbsrsymv(uplo, m, lb, a, ia, ja, x, y)
call mkl_cspblas_cbsrsymv(uplo, m, lb, a, ia, ja, x, y)
call mkl_cspblas_zbsrsymv(uplo, m, lb, a, ia, ja, x, y)
```

C:

```
mkl_cspblas_sbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_dbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_cbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_zbsrsymv(&uplo, &m, &lb, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?bsrsymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the BSR format (3-array variation) with zero-based indexing.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo — CHARACTER*1. Specifies whether the upper or low triangle of the matrix *A* is used.
If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used.
If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.

m — INTEGER. Number of block rows of the matrix *A*.

lb — INTEGER. Size of the block in the matrix *A*.

a — REAL for mkl_cspblas_sbsrsymv.
DOUBLE PRECISION for mkl_cspblas_dbsrsymv.
COMPLEX for mkl_cspblas_cbsrsymv.
DOUBLE COMPLEX for mkl_cspblas_zbsrsymv.
Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to *values* array description in BSR Format for more details.

ia — INTEGER. Array of length (*m* + 1), containing indices of block in the array *a*, such that *ia*(*i*) is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zero blocks plus one. Refer to *rowIndex* array description in BSR Format for more details.

ja — INTEGER. Array containing the column indices for each non-zero block in the matrix *A*.
Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.

x — REAL for mkl_cspblas_sbsrsymv.
DOUBLE PRECISION for mkl_cspblas_dbsrsymv.
COMPLEX for mkl_cspblas_cbsrsymv.
DOUBLE COMPLEX for mkl_cspblas_zbsrsymv.
Array, DIMENSION (*m*lb*).
On entry, the array *x* must contain the vector *x*.

Output Parameters

y — REAL for mkl_cspblas_sbsrsymv.
DOUBLE PRECISION for mkl_cspblas_dbsrsymv.
COMPLEX for mkl_cspblas_cbsrsymv.
DOUBLE COMPLEX for mkl_cspblas_zbsrsymv.
Array, DIMENSION at least (*m*lb*).
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_sbsrsymv(uplo, m, lb, a, ia, ja, x, y)

   CHARACTER*1  uplo

   INTEGER      m, lb

   INTEGER      ia(*), ja(*)

   REAL           a(*), x(*), y(*)
```

```
SUBROUTINE mkl_cspblas_dbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_cbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   COMPLEX    a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zbsrsymv(uplo, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX    a(*), x(*), y(*)
```

C:

```
void mkl_cspblas_sbsrsymv(char *uplo, int *m, int *lb,
float *a, int *ia, int *ja, float *x, float *y);
void mkl_cspblas_dbsrsymv(char *uplo, int *m, int *lb,
double *a, int *ia, int *ja, double *x, double *y);
void mkl_cspblas_cbsrsymv(char *uplo, int *m, int *lb,
MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_cspblas_zbsrsymv(char *uplo, int *m, int *lb,
MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?coosymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the coordinate format with zero-based indexing .

Syntax

Fortran:

```
call mkl_cspblas_scoosymv(uplo, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_dcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_ccoosymv(uplo, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_zcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_cspblas_scoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_dcoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
```

```
mkl_cspblas_ccoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_zcoosymv(&uplo, &m, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?coosymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and *y* are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the coordinate format with zero-based indexing.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	`CHARACTER*1`. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = `'U'` or `'u'`, then the upper triangle of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangle of the matrix *A* is used.
m	`INTEGER`. Number of rows of the matrix *A*.
val	`REAL` for `mkl_cspblas_scoosymv`. `DOUBLE PRECISION` for `mkl_cspblas_dcoosymv`. `COMPLEX` for `mkl_cspblas_ccoosymv`. `DOUBLE COMPLEX` for `mkl_cspblas_zcoosymv`. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	`INTEGER`. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	`INTEGER`. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	`INTEGER`. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	`REAL` for `mkl_cspblas_scoosymv`. `DOUBLE PRECISION` for `mkl_cspblas_dcoosymv`. `COMPLEX` for `mkl_cspblas_ccoosymv`. `DOUBLE COMPLEX` for `mkl_cspblas_zcoosymv`. Array, `DIMENSION` is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_cspblas_scoosymv. DOUBLE PRECISION for mkl_cspblas_dcoosymv. COMPLEX for mkl_cspblas_ccoosymv. DOUBLE COMPLEX for mkl_cspblas_zcoosymv. Array, DIMENSION at least *m*. On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_scoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  REAL           val(*), x(*), y(*)
SUBROUTINE mkl_cspblas_dcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION        val(*), x(*), y(*)
SUBROUTINE mkl_cspblas_ccoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX     val(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zcoosymv(uplo, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX      val(*), x(*), y(*)
```

C:

```
void mkl_cspblas_scoosymv(char *uplo, int *m, float  *val, int *rowind,
int *colind, int *nnz, float *x, float *y);
void mkl_cspblas_dcoosymv(char *uplo, int *m, double  *val, int *rowind,
int *colind, int *nnz, double *x, double *y);
void mkl_cspblas_ccoosymv(char *uplo, int *m, MKL_Complex8  *val, int *rowind,
int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_cspblas_zcoosymv(char *uplo, int *m, MKL_Complex16  *val, int *rowind,
int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?csrtrsv

Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation) with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_scsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
call mkl_cspblas_dcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
call mkl_cspblas_ccsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
call mkl_cspblas_zcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
```

C:

```
mkl_cspblas_scsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
mkl_cspblas_dcsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
mkl_cspblas_ccsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
mkl_cspblas_zcsrtrsv(&uplo, &transa, &diag, &m, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?csrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the CSR format (3-array variation) with zero-based indexing:

`A*y = x`

or

`A'*y = x`,

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	`CHARACTER*1`. Specifies whether the upper or low triangle of the matrix *A* is used. If *uplo* = `'U'` or `'u'`, then the upper triangle of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangle of the matrix *A* is used.
transa	`CHARACTER*1`. Specifies the system of linear equations. If *transa* = `'N'` or `'n'`, then *A*y = x*

If *transa* = 'T' or 't' or 'C' or 'c', then *A*'**y* = *x*,

diag CHARACTER*1. Specifies whether matrix *A* is unit triangular.
If *diag* = 'U' or 'u', then *A* is unit triangular.
If *diag* = 'N' or 'n', then *A* is not unit triangular.

m INTEGER. Number of rows of the matrix *A*.

a REAL for mkl_cspblas_scsrtrsv.
DOUBLE PRECISION for mkl_cspblas_dcsrtrsv.
COMPLEX for mkl_cspblas_ccsrtrsv.
DOUBLE COMPLEX for mkl_cspblas_zcsrtrsv.
Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia INTEGER. Array of length *m*+1, containing indices of elements in the array *a*, such that *ia*(*i*) is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia*(*m*) is equal to the number of non-zeros. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

ja INTEGER. Array containing the column indices for each non-zero element of the matrix *A*.
Its length is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

NOTE Column indices must be sorted in increasing order for each row.

x REAL for mkl_cspblas_scsrtrsv.
DOUBLE PRECISION for mkl_cspblas_dcsrtrsv.
COMPLEX for mkl_cspblas_ccsrtrsv.
DOUBLE COMPLEX for mkl_cspblas_zcsrtrsv.
Array, DIMENSION is *m*.
On entry, the array *x* must contain the vector *x*.

Output Parameters

y REAL for mkl_cspblas_scsrtrsv.
DOUBLE PRECISION for mkl_cspblas_dcsrtrsv.
COMPLEX for mkl_cspblas_ccsrtrsv.
DOUBLE COMPLEX for mkl_cspblas_zcsrtrsv.
Array, DIMENSION at least *m*.
Contains the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_scsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)

SUBROUTINE mkl_cspblas_dcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION     a(*), x(*), y(*)

SUBROUTINE mkl_cspblas_ccsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)

SUBROUTINE mkl_cspblas_zcsrtrsv(uplo, transa, diag, m, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX       a(*), x(*), y(*)
```

C:

```
void mkl_cspblas_scsrtrsv(char *uplo, char *transa, char *diag, int *m,
float *a, int *ia, int *ja, float *x, float *y);

void mkl_cspblas_dcsrtrsv(char *uplo, char *transa, char *diag, int *m,
double *a, int *ia, int *ja, double *x, double *y);

void mkl_cspblas_ccsrtrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_cspblas_zcsrtrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?bsrtrsv

Triangular solver with simplified interface for a sparse matrix stored in the BSR format (3-array variation) with zero-based indexing.

Syntax

Fortran:

```
call mkl_cspblas_sbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
call mkl_cspblas_dbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
call mkl_cspblas_cbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
call mkl_cspblas_zbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
```

C:

```
mkl_cspblas_sbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_dbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_cbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
mkl_cspblas_zbsrtrsv(&uplo, &transa, &diag, &m, &lb, a, ia, ja, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?bsrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the BSR format (3-array variation) with zero-based indexing:

```
y := A*x
```

or

```
y := A'*x,
```

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	CHARACTER*1. Specifies the upper or low triangle of the matrix *A* is used. If *uplo* = 'U' or 'u', then the upper triangle of the matrix *A* is used. If *uplo* = 'L' or 'l', then the low triangle of the matrix *A* is used.
transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-vector product is computed as *y* := *A*x* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *y* := *A'*x*.
diag	CHARACTER*1. Specifies whether matrix *A* is unit triangular or not. If *diag* = 'U' or 'u', *A* is unit triangular.

If *diag* = 'N' or 'n', *A* is not unit triangular.

m INTEGER. Number of block rows of the matrix *A*.

lb INTEGER. Size of the block in the matrix *A*.

a REAL for mkl_cspblas_sbsrtrsv.
DOUBLE PRECISION for mkl_cspblas_dbsrtrsv.
COMPLEX for mkl_cspblas_cbsrtrsv.
DOUBLE COMPLEX for mkl_cspblas_zbsrtrsv.
Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb***lb*. Refer to *values* array description in BSR Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia INTEGER. Array of length (*m* + 1), containing indices of block in the array *a*, such that *ia*(*I*) is the index in the array *a* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zero blocks. Refer to *rowIndex* array description in BSR Format for more details.

ja INTEGER. Array containing the column indices for each non-zero block in the matrix *A*.
Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.

x REAL for mkl_cspblas_sbsrtrsv.
DOUBLE PRECISION for mkl_cspblas_dbsrtrsv.
COMPLEX for mkl_cspblas_cbsrtrsv.
DOUBLE COMPLEX for mkl_cspblas_zbsrtrsv.
Array, DIMENSION (*m***lb*).
On entry, the array *x* must contain the vector *x*.

Output Parameters

y REAL for mkl_cspblas_sbsrtrsv.
DOUBLE PRECISION for mkl_cspblas_dbsrtrsv.
COMPLEX for mkl_cspblas_cbsrtrsv.
DOUBLE COMPLEX for mkl_cspblas_zbsrtrsv.
Array, DIMENSION at least (*m***lb*).
On exit, the array *y* must contain the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_sbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   REAL          a(*), x(*), y(*)
```

```
SUBROUTINE mkl_cspblas_dbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE PRECISION       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_cbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   COMPLEX       a(*), x(*), y(*)
SUBROUTINE mkl_cspblas_zbsrtrsv(uplo, transa, diag, m, lb, a, ia, ja, x, y)
   CHARACTER*1  uplo, transa, diag
   INTEGER      m, lb
   INTEGER      ia(*), ja(*)
   DOUBLE COMPLEX      a(*), x(*), y(*)
```

C:

```
void mkl_cspblas_sbsrtrsv(char *uplo, char *transa, char *diag, int *m,
int *lb, float *a, int *ia, int *ja, float *x, float *y);
void mkl_cspblas_dbsrtrsv(char *uplo, char *transa, char *diag, int *m,
int *lb, double *a, int *ia, int *ja, double *x, double *y);
void mkl_cspblas_cbsrtrsv(char *uplo, char *transa, char *diag, int *m,
int *lb, MKL_Complex8 *a, int *ia, int *ja, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_cspblas_zbsrtrsv(char *uplo, char *transa, char *diag, int *m,
int *lb, MKL_Complex16 *a, int *ia, int *ja, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_cspblas_?cootrsv

Triangular solvers with simplified interface for a sparse matrix in the coordinate format with zero-based indexing .

Syntax

Fortran:

```
call mkl_cspblas_scootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_dcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_ccootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
call mkl_cspblas_zcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_cspblas_scootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_dcootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
```

```
mkl_cspblas_ccootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
mkl_cspblas_zcootrsv(&uplo, &transa, &diag, &m, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_cspblas_?cootrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the coordinate format with zero-based indexing:

```
A*y = x
```

or

```
A'*y = x,
```

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only zero-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

uplo	`CHARACTER*1`. Specifies whether the upper or low triangle of the matrix *A* is considered. If *uplo* = `'U'` or `'u'`, then the upper triangle of the matrix *A* is used. If *uplo* = `'L'` or `'l'`, then the low triangle of the matrix *A* is used.
transa	`CHARACTER*1`. Specifies the system of linear equations. If *transa* = `'N'` or `'n'`, then *A*y = x* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *A'*y = x*,
diag	`CHARACTER*1`. Specifies whether *A* is unit triangular. If *diag* = `'U'` or `'u'`, then *A* is unit triangular. If *diag* = `'N'` or `'n'`, then *A* is not unit triangular.
m	`INTEGER`. Number of rows of the matrix *A*.
val	`REAL` for `mkl_cspblas_scootrsv`. `DOUBLE PRECISION` for `mkl_cspblas_dcootrsv`. `COMPLEX` for `mkl_cspblas_ccootrsv`. `DOUBLE COMPLEX` for `mkl_cspblas_zcootrsv`. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	`INTEGER`. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.

colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_cspblas_scootrsv. DOUBLE PRECISION for mkl_cspblas_dcootrsv. COMPLEX for mkl_cspblas_ccootrsv. DOUBLE COMPLEX for mkl_cspblas_zcootrsv. Array, DIMENSION is *m*. On entry, the array *x* must contain the vector *x*.

Output Parameters

y	REAL for mkl_cspblas_scootrsv. DOUBLE PRECISION for mkl_cspblas_dcootrsv. COMPLEX for mkl_cspblas_ccootrsv. DOUBLE COMPLEX for mkl_cspblas_zcootrsv. Array, DIMENSION at least *m*. Contains the vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_cspblas_scootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  REAL           val(*), x(*), y(*)

SUBROUTINE mkl_cspblas_dcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION      val(*), x(*), y(*)

SUBROUTINE mkl_cspblas_ccootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        val(*), x(*), y(*)

SUBROUTINE mkl_cspblas_zcootrsv(uplo, transa, diag, m, val, rowind, colind, nnz, x, y)
  CHARACTER*1   uplo, transa, diag
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX      val(*), x(*), y(*)
```

C:

```
void mkl_cspblas_scootrsv(char *uplo, char *transa, char *diag, int *m,
float *val, int *rowind, int *colind, int *nnz, float *x, float *y);

void mkl_cspblas_dcootrsv(char *uplo, char *transa, char *diag, int *m,
double *val, int *rowind, int *colind, int *nnz, double *x, double *y);

void mkl_cspblas_ccootrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex8 *val, int *rowind, int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_cspblas_zcootrsv(char *uplo, char *transa, char *diag, int *m,
MKL_Complex16 *val, int *rowind, int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?csrmv

Computes matrix - vector product of a sparse matrix stored in the CSR format.

Syntax

Fortran:

```
call mkl_scsrmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
call mkl_dcsrmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
call mkl_ccsrmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
call mkl_zcsrmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
```

C:

```
mkl_scsrmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
mkl_dcsrmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
mkl_ccsrmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
mkl_zcsrmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrmv` routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*A'*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix in the CSR format, *A'* is the transpose of *A*.

NOTE This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *y* := *alpha*A*x* + *beta*y* If *transa* = 'T' or 't' or 'C' or 'c', then *y* := *alpha*A'*x* + *beta*y*,
m	INTEGER. Number of rows of the matrix *A*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_scsrmv. DOUBLE PRECISION for mkl_dcsrmv. COMPLEX for mkl_ccsrmv. DOUBLE COMPLEX for mkl_zcsrmv. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scsrmv. DOUBLE PRECISION for mkl_dcsrmv. COMPLEX for mkl_ccsrmv. DOUBLE COMPLEX for mkl_zcsrmv. Array containing non-zero elements of the matrix *A*. For one-based indexing its length is *pntre*(m) – *pntrb*(1). For zero-based indexing its length is *pntre*(m–1) – *pntrb*(0). Refer to *values* array description in CSR Format for more details.
indx	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*.Its length is equal to length of the *val* array. Refer to *columns* array description in CSR Format for more details.
pntrb	INTEGER. Array of length *m*. For one-based indexing this array contains row indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of row *i* in the arrays *val* and *indx*. For zero-based indexing this array contains row indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of row *i* in the arrays *val* and *indx*. Refer to *pointerb* array description in CSR Format for more details.
pntre	INTEGER. Array of length *m*. For one-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of row *i* in the arrays *val* and *indx*. For zero-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(0)–1 is the last index of row *i* in the arrays *val* and *indx*.Refer to *pointerE* array description in CSR Format for more details.
x	REAL for mkl_scsrmv. DOUBLE PRECISION for mkl_dcsrmv. COMPLEX for mkl_ccsrmv. DOUBLE COMPLEX for mkl_zcsrmv.

	Array, DIMENSION at least *k* if *transa* = 'N' or 'n' and at least *m* otherwise. On entry, the array *x* must contain the vector *x*.
beta	REAL for mkl_scsrmv. DOUBLE PRECISION for mkl_dcsrmv. COMPLEX for mkl_ccsrmv. DOUBLE COMPLEX for mkl_zcsrmv. Specifies the scalar *beta*.
y	REAL for mkl_scsrmv. DOUBLE PRECISION for mkl_dcsrmv. COMPLEX for mkl_ccsrmv. DOUBLE COMPLEX for mkl_zcsrmv. Array, DIMENSION at least *m* if *transa* = 'N' or 'n' and at least *k* otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y	Overwritten by the updated vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL           alpha, beta
  REAL           val(*), x(*), y(*)

SUBROUTINE mkl_dcsrmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), x(*), y(*)

SUBROUTINE mkl_ccsrmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX      alpha, beta
  COMPLEX      val(*), x(*), y(*)
```

```
SUBROUTINE mkl_zcsrmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_scsrmv(char *transa, int *m, int *k, float *alpha, char *matdescra,
float *val, int *indx, int *pntrb, int *pntre, float *x, float *beta, float *y);

void mkl_dcsrmv(char *transa, int *m, int *k, double *alpha, char *matdescra,
double *val, int *indx, int *pntrb, int *pntre, double *x, double *beta, double *y);

void mkl_ccsrmv(char *transa, int *m, int *k, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *indx, int *pntrb, int *pntre, MKL_Complex8 *x, MKL_Complex8 *beta, double *y);

void mkl_zcsrmv(char *transa, int *m, int *k, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *indx, int *pntrb, int *pntre, MKL_Complex16 *x, MKL_Complex16 *beta,
MKL_Complex16 *y);
```

mkl_?bsrmv

Computes matrix - vector product of a sparse matrix stored in the BSR format.

Syntax

Fortran:

```
call mkl_sbsrmv(transa, m, k, lb, alpha, matdescra, val, indx, pntrb, pntre, x, beta,
y)
call mkl_dbsrmv(transa, m, k, lb, alpha, matdescra, val, indx, pntrb, pntre, x, beta,
y)
call mkl_cbsrmv(transa, m, k, lb, alpha, matdescra, val, indx, pntrb, pntre, x, beta,
y)
call mkl_zbsrmv(transa, m, k, lb, alpha, matdescra, val, indx, pntrb, pntre, x, beta,
y)
```

C:

```
mkl_sbsrmv(&transa, &m, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta,
y);
mkl_dbsrmv(&transa, &m, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta,
y);
mkl_cbsrmv(&transa, &m, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta,
y);
mkl_zbsrmv(&transa, &m, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta,
y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrmv` routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*A'*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* block sparse matrix in the BSR format, *A'* is the transpose of *A*.

NOTE This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then the matrix-vector product is computed as *y := alpha*A*x + beta*y* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then the matrix-vector product is computed as *y := alpha*A'*x + beta*y*,
m	`INTEGER`. Number of block rows of the matrix *A*.
k	`INTEGER`. Number of block columns of the matrix *A*.
lb	`INTEGER`. Size of the block in the matrix *A*.
alpha	`REAL` for `mkl_sbsrmv`. `DOUBLE PRECISION` for `mkl_dbsrmv`. `COMPLEX` for `mkl_cbsrmv`. `DOUBLE COMPLEX` for `mkl_zbsrmv`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	`REAL` for `mkl_sbsrmv`. `DOUBLE PRECISION` for `mkl_dbsrmv`. `COMPLEX` for `mkl_cbsrmv`. `DOUBLE COMPLEX` for `mkl_zbsrmv`. Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*.

	Refer to *values* array description in BSR Format for more details.
indx	INTEGER. Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A*. Refer to *columns* array description in BSR Format for more details.
pntrb	INTEGER. Array of length *m*. For one-based indexing: this array contains row indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of block row *i* in the array *indx*. For zero-based indexing: this array contains row indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of block row *i* in the array *indx* Refer to *pointerB* array description in BSR Format for more details.
pntre	INTEGER. Array of length *m*. For one-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of block row *i* in the array *indx*. For zero-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(0)–1 is the last index of block row *i* in the array *indx*. Refer to *pointerE* array description in BSR Format for more details.
x	REAL for mkl_sbsrmv. DOUBLE PRECISION for mkl_dbsrmv. COMPLEX for mkl_cbsrmv. DOUBLE COMPLEX for mkl_zbsrmv. Array, DIMENSION at least (*k***lb*) if *transa* = 'N' or 'n', and at least (*m***lb*) otherwise. On entry, the array *x* must contain the vector *x*.
beta	REAL for mkl_sbsrmv. DOUBLE PRECISION for mkl_dbsrmv. COMPLEX for mkl_cbsrmv. DOUBLE COMPLEX for mkl_zbsrmv. Specifies the scalar *beta*.
y	REAL for mkl_sbsrmv. DOUBLE PRECISION for mkl_dbsrmv. COMPLEX for mkl_cbsrmv. DOUBLE COMPLEX for mkl_zbsrmv. Array, DIMENSION at least (*m***lb*) if *transa* = 'N' or 'n', and at least (*k***lb*) otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y	Overwritten by the updated vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrmv(transa, m, k, lb, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha, beta
  REAL          val(*), x(*), y(*)

SUBROUTINE mkl_dbsrmv(transa, m, k, lb, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), x(*), y(*)

SUBROUTINE mkl_cbsrmv(transa, m, k, lb, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha, beta
  COMPLEX        val(*), x(*), y(*)

SUBROUTINE mkl_zbsrmv(transa, m, k, lb, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_sbsrmv(char *transa, int *m, int *k, int *lb,
float *alpha, char *matdescra, float *val, int *indx,
int *pntrb, int *pntre, float *x, float *beta, float *y);
```

```
void mkl_dbsrmv(char *transa, int *m, int *k, int *lb,
double *alpha, char *matdescra, double *val, int *indx,
int *pntrb, int *pntre, double *x, double *beta, double *y);
void mkl_cbsrmv(char *transa, int *m, int *k, int *lb,
MKL_Complex8 *alpha, char *matdescra, MKL_Complex8 *val, int *indx,
int *pntrb, int *pntre, MKL_Complex8 *x, MKL_Complex8 *beta, MKL_Complex8 *y);
void mkl_zbsrmv(char *transa, int *m, int *k, int *lb,
MKL_Complex16 *alpha, char *matdescra, MKL_Complex16 *val, int *indx,
int *pntrb, int *pntre, MKL_Complex16 *x, MKL_Complex16 *beta, MKL_Complex16 *y);
```

mkl_?cscmv

Computes matrix-vector product for a sparse matrix in the CSC format.

Syntax

Fortran:

```
call mkl_scscmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
call mkl_dcscmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
call mkl_ccscmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
call mkl_zcscmv(transa, m, k, alpha, matdescra, val, indx, pntrb, pntre, x, beta, y)
```

C:

```
mkl_scscmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
mkl_dcscmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
mkl_ccscmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
mkl_zcscmv(&transa, &m, &k, &alpha, matdescra, val, indx, pntrb, pntre, x, &beta, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?cscmv` routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*A'*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix in compressed sparse column (CSC) format, *A'* is the transpose of *A*.

NOTE This routine supports CSC format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *y* := *alpha*A*x* + *beta*y* If *transa* = 'T' or 't' or 'C' or 'c', then *y* := *alpha*A'*x* + *beta*y*,
m	INTEGER. Number of rows of the matrix *A*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_scscmv. DOUBLE PRECISION for mkl_dcscmv. COMPLEX for mkl_ccscmv. DOUBLE COMPLEX for mkl_zcscmv. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scscmv. DOUBLE PRECISION for mkl_dcscmv. COMPLEX for mkl_ccscmv. DOUBLE COMPLEX for mkl_zcscmv. Array containing non-zero elements of the matrix *A*. For one-based indexing its length is pntre(*k*) – *pntrb*(1). For zero-based indexing its length is *pntre*(*m*–1) – *pntrb*(0). Refer to *values* array description in CSC Format for more details.
indx	INTEGER. Array containing the row indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array. Refer to *rows* array description in CSC Format for more details.
pntrb	INTEGER. Array of length *k*. For one-based indexing this array contains column indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of column *i* in the arrays *val* and *indx*. For zero-based indexing this array contains column indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of column *i* in the arrays *val* and *indx*. Refer to *pointerb* array description in CSC Format for more details.
pntre	INTEGER. Array of length *k*. For one-based indexing this array contains column indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of column *i* in the arrays *val* and *indx*. For zero-based indexing this array contains column indices, such that *pntre*(*i*) – *pntrb*(1)–1 is the last index of column *i* in the arrays *val* and *indx*. Refer to *pointerE* array description in CSC Format for more details.

x	REAL for mkl_scscmv. DOUBLE PRECISION for mkl_dcscmv. COMPLEX for mkl_ccscmv. DOUBLE COMPLEX for mkl_zcscmv. Array, DIMENSION at least *k* if *transa* = 'N' or 'n' and at least *m* otherwise. On entry, the array *x* must contain the vector *x*.
beta	REAL for mkl_scscmv. DOUBLE PRECISION for mkl_dcscmv. COMPLEX for mkl_ccscmv. DOUBLE COMPLEX for mkl_zcscmv. Specifies the scalar *beta*.
y	REAL for mkl_scscmv. DOUBLE PRECISION for mkl_dcscmv. COMPLEX for mkl_ccscmv. DOUBLE COMPLEX for mkl_zcscmv. Array, DIMENSION at least *m* if *transa* = 'N' or 'n' and at least *k* otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y	Overwritten by the updated vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scscmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha, beta
  REAL          val(*), x(*), y(*)

SUBROUTINE mkl_dcscmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), x(*), y(*)
```

```
SUBROUTINE mkl_ccscmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha, beta
  COMPLEX        val(*), x(*), y(*)

SUBROUTINE mkl_zcscmv(transa, m, k, alpha, matdescra, val, indx,
 pntrb, pntre, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_scscmv(char *transa, int *m, int *k, float *alpha,
char *matdescra, float *val, int *indx, int *pntrb,
int *pntre, float *x, float *beta, float *y);

void mkl_dcscmv(char *transa, int *m, int *k, double *alpha,
char *matdescra, double *val, int *indx, int *pntrb,
int *pntre, double *x, double *beta, double *y);

void mkl_ccscmv(char *transa, int *m, int *k, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8 *val, int *indx, int *pntrb,
int *pntre, MKL_Complex8 *x, MKL_Complex8 *beta, MKL_Complex8 *y);

void mkl_zcscmv(char *transa, int *m, int *k, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16 *val, int *indx, int *pntrb,
int *pntre, MKL_Complex16 *x, MKL_Complex16 *beta, MKL_Complex16 *y);
```

mkl_?coomv

Computes matrix - vector product for a sparse matrix in the coordinate format.

Syntax

Fortran:

```
call mkl_scoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
call mkl_dcoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
call mkl_ccoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
```

```
call mkl_zcoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
```

C:

```
mkl_scoomv(&transa, &m, &k, &alpha, matdescra, val, rowind, colind, &nnz, x, &beta, y);
mkl_dcoomv(&transa, &m, &k, &alpha, matdescra, val, rowind, colind, &nnz, x, &beta, y);
mkl_ccoomv(&transa, &m, &k, &alpha, matdescra, val, rowind, colind, &nnz, x, &beta, y);
mkl_zcoomv(&transa, &m, &k, &alpha, matdescra, val, rowind, colind, &nnz, x, &beta, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?coomv` routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*A'*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix in compressed coordinate format, *A'* is the transpose of *A*.

NOTE This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then *y* := *alpha*A*x* + *beta*y* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *y* := *alpha*A'*x* + *beta*y*,
m	`INTEGER`. Number of rows of the matrix *A*.
k	`INTEGER`. Number of columns of the matrix *A*.
alpha	`REAL` for `mkl_scoomv`. `DOUBLE PRECISION` for `mkl_dcoomv`. `COMPLEX` for `mkl_ccoomv`. `DOUBLE COMPLEX` for `mkl_zcoomv`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)".

	Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scoomv. DOUBLE PRECISION for mkl_dcoomv. COMPLEX for mkl_ccoomv. DOUBLE COMPLEX for mkl_zcoomv. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_scoomv. DOUBLE PRECISION for mkl_dcoomv. COMPLEX for mkl_ccoomv. DOUBLE COMPLEX for mkl_zcoomv. Array, DIMENSION at least *k* if *transa* = 'N' or 'n' and at least *m* otherwise. On entry, the array *x* must contain the vector *x*.
beta	REAL for mkl_scoomv. DOUBLE PRECISION for mkl_dcoomv. COMPLEX for mkl_ccoomv. DOUBLE COMPLEX for mkl_zcoomv. Specifies the scalar *beta*.
y	REAL for mkl_scoomv. DOUBLE PRECISION for mkl_dcoomv. COMPLEX for mkl_ccoomv. DOUBLE COMPLEX for mkl_zcoomv. Array, DIMENSION at least *m* if *transa* = 'N' or 'n' and at least *k* otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y	Overwritten by the updated vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)

  CHARACTER*1   transa

  CHARACTER     matdescra(*)

  INTEGER       m, k, nnz

  INTEGER       rowind(*), colind(*)

  REAL         alpha, beta

  REAL         val(*), x(*), y(*)
```

```
SUBROUTINE mkl_dcoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), x(*), y(*)

SUBROUTINE mkl_ccoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        alpha, beta
  COMPLEX        val(*), x(*), y(*)

SUBROUTINE mkl_zcoomv(transa, m, k, alpha, matdescra, val, rowind, colind, nnz, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_scoomv(char *transa, int *m, int *k, float *alpha, char *matdescra,
float  *val, int *rowind, int *colind, int *nnz, float *x, float *beta, float *y);

void mkl_dcoomv(char *transa, int *m, int *k, double *alpha, char *matdescra,
double  *val, int *rowind, int *colind, int *nnz, double *x, double *beta, double *y);

void mkl_ccoomv(char *transa, int *m, int *k, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8  *val, int *rowind, int *colind, int *nnz, MKL_Complex8 *x, MKL_Complex8 *beta,
MKL_Complex8 *y);

void mkl_zcoomv(char *transa, int *m, int *k, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16  *val, int *rowind, int *colind, int *nnz, MKL_Complex16 *x, MKL_Complex16 *beta,
MKL_Complex16 *y);
```

mkl_?csrsv

Solves a system of linear equations for a sparse matrix in the CSR format.

Syntax

Fortran:

```
call mkl_scsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
```

```
call mkl_dcsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
call mkl_ccsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
call mkl_zcsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
```

C:

```
mkl_scsrsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_dcsrsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_ccsrsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_zcsrsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the CSR format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(A')*x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the system of linear equations. If *transa* = `'N'` or `'n'`, then *y* := *alpha*inv(*A*)*x* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *y* := *alpha*inv(*A'*)*x*,
m	`INTEGER`. Number of columns of the matrix *A*.
alpha	`REAL` for `mkl_scsrsv`. `DOUBLE PRECISION` for `mkl_dcsrsv`. `COMPLEX` for `mkl_ccsrsv`. `DOUBLE COMPLEX` for `mkl_zcsrsv`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".

val

REAL for mkl_scsrsv.
DOUBLE PRECISION for mkl_dcsrsv.
COMPLEX for mkl_ccsrsv.
DOUBLE COMPLEX for mkl_zcsrsv.
Array containing non-zero elements of the matrix *A*.
For one-based indexing its length is *pntre*(m) – *pntrb*(1).
For zero-based indexing its length is *pntre*(m-1) – *pntrb*(0).
Refer to *values* array description in CSR Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx

INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array.
Refer to *columns* array description in CSR Format for more details.

NOTE Column indices must be sorted in increasing order for each row.

pntrb

INTEGER. Array of length *m*.
For one-based indexing this array contains row indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of row *i* in the arrays *val* and *indx*.
For zero-based indexing this array contains row indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of row *i* in the arrays *val* and *indx*. Refer to *pointerb* array description in CSR Format for more details.

pntre

INTEGER. Array of length *m*.
For one-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of row *i* in the arrays *val* and *indx*.
For zero-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(0)-1 is the last index of row *i* in the arrays *val* and *indx*.Refer to *pointerE* array description in CSR Format for more details.

x

REAL for mkl_scsrsv.
DOUBLE PRECISION for mkl_dcsrsv.
COMPLEX for mkl_ccsrsv.
DOUBLE COMPLEX for mkl_zcsrsv.
Array, DIMENSION at least *m*.
On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.

y

REAL for mkl_scsrsv.
DOUBLE PRECISION for mkl_dcsrsv.
COMPLEX for mkl_ccsrsv.
DOUBLE COMPLEX for mkl_zcsrsv.
Array, DIMENSION at least *m*.
On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y

Contains solution vector *x*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha
  REAL          val(*)
  REAL          x(*), y(*)

SUBROUTINE mkl_dcsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*)
  DOUBLE PRECISION        x(*), y(*)

SUBROUTINE mkl_ccsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha
  COMPLEX        val(*)
  COMPLEX        x(*), y(*)

SUBROUTINE mkl_zcsrsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*)
  DOUBLE COMPLEX        x(*), y(*)
```

C:

```
void mkl_scsrsv(char *transa, int *m, float *alpha, char *matdescra,
float *val, int *indx, int *pntrb, int *pntre, float *x, float *y);
```

```
void mkl_dcsrsv(char *transa, int *m, double *alpha, char *matdescra,
double *val, int *indx, int *pntrb, int *pntre, double *x, double *y);

void mkl_ccsrsv(char *transa, int *m, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *indx, int *pntrb, int *pntre, MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_zcsrsv(char *transa, int *m, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *indx, int *pntrb, int *pntre, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?bsrsv

Solves a system of linear equations for a sparse matrix in the BSR format.

Syntax

Fortran:

```
call mkl_sbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
call mkl_dbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
call mkl_cbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
call mkl_zbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
```

C:

```
mkl_sbsrsv(&transa, &m, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_dbsrsv(&transa, &m, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_cbsrsv(&transa, &m, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_zbsrsv(&transa, &m, &lb, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the BSR format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(A')* x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa
CHARACTER*1. Specifies the operation.
If *transa* = 'N' or 'n', then *y* := *alpha**inv(*A*)**x*
If *transa* = 'T' or 't' or 'C' or 'c', then *y* := *alpha**inv(*A*')* *x*,

m
INTEGER. Number of block columns of the matrix *A*.

lb
INTEGER. Size of the block in the matrix *A*.

alpha
REAL for mkl_sbsrsv.
DOUBLE PRECISION for mkl_dbsrsv.
COMPLEX for mkl_cbsrsv.
DOUBLE COMPLEX for mkl_zbsrsv.
Specifies the scalar *alpha*.

matdescra
CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".

val
REAL for mkl_sbsrsv.
DOUBLE PRECISION for mkl_dbsrsv.
COMPLEX for mkl_cbsrsv.
DOUBLE COMPLEX for mkl_zbsrsv.
Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb***lb*. Refer to the *values* array description in BSR Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx
INTEGER. Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A*.
Refer to the *columns* array description in BSR Format for more details.

pntrb
INTEGER. Array of length *m*.
For one-based indexing: this array contains row indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of block row *i* in the array *indx*.
For zero-based indexing: this array contains row indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of block row *i* in the array *indx*
Refer to *pointerB* array description in BSR Format for more details.

pntre
INTEGER. Array of length *m*.
For one-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of block row *i* in the array *indx*.
For zero-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(0)–1 is the last index of block row *i* in the array *indx*.

	Refer to *pointerE* array description in BSR Format for more details.
x	REAL for mkl_sbsrsv. DOUBLE PRECISION for mkl_dbsrsv. COMPLEX for mkl_cbsrsv. DOUBLE COMPLEX for mkl_zbsrsv. Array, DIMENSION at least (*m*lb*). On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.
y	REAL for mkl_sbsrsv. DOUBLE PRECISION for mkl_dbsrsv. COMPLEX for mkl_cbsrsv. DOUBLE COMPLEX for mkl_zbsrsv. Array, DIMENSION at least (*m*lb*). On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y	Contains solution vector *x*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha
  REAL          val(*)
  REAL          x(*), y(*)

SUBROUTINE mkl_dbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*)
  DOUBLE PRECISION        x(*), y(*)
```

```
SUBROUTINE mkl_cbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha
  COMPLEX        val(*)
  COMPLEX        x(*), y(*)
SUBROUTINE mkl_zbsrsv(transa, m, lb, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lb
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*)
  DOUBLE COMPLEX        x(*), y(*)
```

C:

```
void mkl_sbsrsv(char *transa, int *m, int *lb, float *alpha, char *matdescra,
float *val, int *indx, int *pntrb, int *pntre, float *x, float *y);
void mkl_dbsrsv(char *transa, int *m, int *lb, double *alpha, char *matdescra,
double *val, int *indx, int *pntrb, int *pntre, double *x, double *y);
void mkl_cbsrsv(char *transa, int *m, int *lb, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *indx, int *pntrb, int *pntre, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zbsrsv(char *transa, int *m, int *lb, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *indx, int *pntrb, int *pntre, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?cscsv

Solves a system of linear equations for a sparse matrix in the CSC format.

Syntax

Fortran:

```
call mkl_scscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)call
mkl_dcscsv
call mkl_dcscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)call
mkl_dcscsv
call mkl_ccscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)call
mkl_dcscsv
call mkl_zcscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)call
mkl_dcscsv
```

C:

```
mkl_scscsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_dcscsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_ccscsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
mkl_zcscsv(&transa, &m, &alpha, matdescra, val, indx, pntrb, pntre, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?cscsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the CSC format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(A')* x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a CSC format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then *y* := *alpha*inv(*A*)*x* If *transa*= `'T'` or `'t'` or `'C'` or `'c'`, then *y* := *alpha*inv(*A'*)* *x*,
m	`INTEGER`. Number of columns of the matrix *A*.
alpha	`REAL` for `mkl_scscsv`. `DOUBLE PRECISION` for `mkl_dcscsv`. `COMPLEX` for `mkl_ccscsv`. `DOUBLE COMPLEX` for `mkl_zcscsv`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	`REAL` for `mkl_scscsv`. `DOUBLE PRECISION` for `mkl_dcscsv`. `COMPLEX` for `mkl_ccscsv`. `DOUBLE COMPLEX` for `mkl_zcscsv`.

Array containing non-zero elements of the matrix *A*.
For one-based indexing its length is *pntre*(*m*) – *pntrb*(1).
For zero-based indexing its length is *pntre*(*m*–1) – *pntrb*(0).
Refer to *values* array description in CSC Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx INTEGER. Array containing the row indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array.
Refer to *columns* array description in CSC Format for more details.

NOTE Row indices must be sorted in increasing order for each column.

pntrb INTEGER. Array of length *m*.
For one-based indexing this array contains column indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of column *i* in the arrays *val* and *indx*.
For zero-based indexing this array contains column indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of column *i* in the arrays *val* and *indx*.
Refer to *pointerb* array description in CSC Format for more details.

pntre INTEGER. Array of length *m*.
For one-based indexing this array contains column indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of column *i* in the arrays *val* and *indx*.
For zero-based indexing this array contains column indices, such that *pntre*(*i*) – *pntrb*(1)–1 is the last index of column *i* in the arrays *val* and *indx*.
Refer to *pointerE* array description in CSC Format for more details.

x REAL for mkl_scscsv.
DOUBLE PRECISION for mkl_dcscsv.
COMPLEX for mkl_ccscsv.
DOUBLE COMPLEX for mkl_zcscsv.
Array, DIMENSION at least *m*.
On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.

y REAL for mkl_scscsv.
DOUBLE PRECISION for mkl_dcscsv.
COMPLEX for mkl_ccscsv.
DOUBLE COMPLEX for mkl_zcscsv.
Array, DIMENSION at least *m*.
On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y Contains the solution vector *x*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha
  REAL          val(*)
  REAL          x(*), y(*)

SUBROUTINE mkl_dcscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*)
  DOUBLE PRECISION        x(*), y(*)

SUBROUTINE mkl_ccscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha
  COMPLEX        val(*)
  COMPLEX        x(*), y(*)

SUBROUTINE mkl_zcscsv(transa, m, alpha, matdescra, val, indx, pntrb, pntre, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*)
  DOUBLE COMPLEX        x(*), y(*)
```

C:

```
void mkl_scscsv(char *transa, int *m, float *alpha, char *matdescra,
float *val, int *indx, int *pntrb, int *pntre, float *x, float *y);
```

```
void mkl_dcscsv(char *transa, int *m, double *alpha, char *matdescra,
double *val, int *indx, int *pntrb, int *pntre, double *x, double *y);

void mkl_ccscsv(char *transa, int *m, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *indx, int *pntrb, int *pntre, MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_zcscsv(char *transa, int *m, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *indx, int *pntrb, int *pntre, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?coosv

Solves a system of linear equations for a sparse matrix in the coordinate format.

Syntax

Fortran:

```
call mkl_scoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
call mkl_dcoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
call mkl_ccoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
call mkl_zcoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
```

C:

```
mkl_scoosv(&transa, &m, &alpha, matdescra, val, rowind, colind, &nnz, x, y);
mkl_dcoosv(&transa, &m, &alpha, matdescra, val, rowind, colind, &nnz, x, y);
mkl_ccoosv(&transa, &m, &alpha, matdescra, val, rowind, colind, &nnz, x, y);
mkl_zcoosv(&transa, &m, &alpha, matdescra, val, rowind, colind, &nnz, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?coosv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the coordinate format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(A')*x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the system of linear equations. If *transa* = 'N' or 'n', then *y* := *alpha**inv(*A*)**x* If *transa* = 'T' or 't' or 'C' or 'c', then *y* := *alpha**inv(*A*')* *x*,
m	INTEGER. Number of rows of the matrix *A*.
alpha	REAL for mkl_scoosv. DOUBLE PRECISION for mkl_dcoosv. COMPLEX for mkl_ccoosv. DOUBLE COMPLEX for mkl_zcoosv. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scoosv. DOUBLE PRECISION for mkl_dcoosv. COMPLEX for mkl_ccoosv. DOUBLE COMPLEX for mkl_zcoosv. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
x	REAL for mkl_scoosv. DOUBLE PRECISION for mkl_dcoosv. COMPLEX for mkl_ccoosv. DOUBLE COMPLEX for mkl_zcoosv. Array, DIMENSION at least *m*. On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.
y	REAL for mkl_scoosv. DOUBLE PRECISION for mkl_dcoosv. COMPLEX for mkl_ccoosv. DOUBLE COMPLEX for mkl_zcoosv. Array, DIMENSION at least *m*. On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y	Contains solution vector *x*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  REAL          alpha
  REAL          val(*)
  REAL          x(*), y(*)

SUBROUTINE mkl_dcoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*)
  DOUBLE PRECISION        x(*), y(*)

SUBROUTINE mkl_ccoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        alpha
  COMPLEX        val(*)
  COMPLEX        x(*), y(*)

SUBROUTINE mkl_zcoosv(transa, m, alpha, matdescra, val, rowind, colind, nnz, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*)
  DOUBLE COMPLEX        x(*), y(*)
```

C:

```
void mkl_scoosv(char *transa, int *m, float *alpha, char *matdescra,
float *val, int *rowind, int *colind, int *nnz,
float *x, float *y);

void mkl_dcoosv(char *transa, int *m, double *alpha, char *matdescra,
double *val, int *rowind, int *colind, int *nnz,
double *x, double *y);

void mkl_ccoosv(char *transa, int *m, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *rowind, int *colind, int *nnz,
MKL_Complex8 *x, MKL_Complex8 *y);

void mkl_zcoosv(char *transa, int *m, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *rowind, int *colind, int *nnz,
MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?csrmm

Computes matrix - matrix product of a sparse matrix stored in the CSR format.

Syntax

Fortran:

```
call mkl_scsrmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)

call mkl_dcsrmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)

call mkl_ccsrmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)

call mkl_zcsrmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)
```

C:

```
mkl_scsrmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);

mkl_dcsrmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);

mkl_ccsrmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);

mkl_zcsrmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrmm` routine performs a matrix-matrix operation defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*A'*B + beta*C,
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in compressed sparse row (CSR) format, *A'* is the transpose of *A*.

NOTE This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *C* := *alpha*A*B + beta*C* If *transa* = 'T' or 't' or 'C' or 'c', then *C* := *alpha*A'*B + beta*C*,
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_scsrmm. DOUBLE PRECISION for mkl_dcsrmm. COMPLEX for mkl_ccsrmm. DOUBLE COMPLEX for mkl_zcsrmm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scsrmm. DOUBLE PRECISION for mkl_dcsrmm. COMPLEX for mkl_ccsrmm. DOUBLE COMPLEX for mkl_zcsrmm. Array containing non-zero elements of the matrix *A*. For one-based indexing its length is *pntre*(m) – *pntrb*(1). For zero-based indexing its length is *pntre*(–1) – *pntrb*(0). Refer to *values* array description in CSR Format for more details.
indx	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array. Refer to *columns* array description in CSR Format for more details.
pntrb	INTEGER. Array of length *m*. For one-based indexing this array contains row indices, such that *pntrb*(*I*) – *pntrb*(1)+1 is the first index of row *I* in the arrays *val* and *indx*. For zero-based indexing this array contains row indices, such that *pntrb*(*I*) – *pntrb*(0) is the first index of row *I* in the arrays *val* and *indx*. Refer to *pointerb* array description in CSR Format for more details.

pntre	INTEGER. Array of length *m*. For one-based indexing this array contains row indices, such that *pntre*(*I*) – *pntrb*(1) is the last index of row *I* in the arrays *val* and *indx*. For zero-based indexing this array contains row indices, such that *pntre*(*I*) – *pntrb*(0)–1 is the last index of row *I* in the arrays *val* and *indx*. Refer to *pointerE* array description in CSR Format for more details.
b	REAL for mkl_scsrmm. DOUBLE PRECISION for mkl_dcsrmm. COMPLEX for mkl_ccsrmm. DOUBLE COMPLEX for mkl_zcsrmm. Array, DIMENSION (*ldb*, at least *n* for non-transposed matrix *A* and at least *m* for transposed) for one-based indexing, and (at least *k* for non-transposed matrix *A* and at least *m* for transposed, *ldb*) for zero-based indexing. On entry with *transa*= 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* for one-based indexing, and the second dimension of *b* for zero-based indexing, as declared in the calling (sub)program.
beta	REAL for mkl_scsrmm. DOUBLE PRECISION for mkl_dcsrmm. COMPLEX for mkl_ccsrmm. DOUBLE COMPLEX for mkl_zcsrmm. Specifies the scalar *beta*.
c	REAL for mkl_scsrmm. DOUBLE PRECISION for mkl_dcsrmm. COMPLEX for mkl_ccsrmm. DOUBLE COMPLEX for mkl_zcsrmm. Array, DIMENSION (*ldc*, *n*) for one-based indexing, and (*m*, *ldc*) for zero-based indexing. On entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, otherwise the leading *k*-by-*n* part of the array *c* must contain the matrix *C*.
ldc	INTEGER. Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c	Overwritten by the matrix (*alpha***A***B* + *beta** *C*) or (*alpha***A*'**B* + *beta***C*).

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_dcsrmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha, beta
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcsrmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcsrmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha, beta
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcsrmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_scsrmm(char *transa, int *m, int *n, int *k, float *alpha,
char *matdescra, float  *val, int *indx, int *pntrb, int *pntre,
float *b, int *ldb, float *beta, float *c, int *ldc,);
```

```
void mkl_dcsrmm(char *transa, int *m, int *n, int *k, double *alpha,
char *matdescra, double  *val, int *indx, int *pntrb, int *pntre,
double *b, int *ldb, double *beta, double *c, int *ldc,);
void mkl_ccsrmm(char *transa, int *m, int *n, int *k, double *alpha,
char *matdescra, double  *val, int *indx, int *pntrb, int *pntre,
double *b, int *ldb, double *beta, double *c, int *ldc,);
void mkl_zcsrmm(char *transa, int *m, int *n, int *k, double *alpha,
char *matdescra, double  *val, int *indx, int *pntrb, int *pntre,
double *b, int *ldb, double *beta, double *c, int *ldc,);
```

mkl_?bsrmm

Computes matrix - matrix product of a sparse matrix stored in the BSR format.

Syntax

Fortran:

```
call mkl_sbsrmm(transa, m, n, k, lb, alpha, matdescra, val, indx, pntrb, pntre, b,
ldb, beta, c, ldc)
call mkl_dbsrmm(transa, m, n, k, lb, alpha, matdescra, val, indx, pntrb, pntre, b,
ldb, beta, c, ldc)
call mkl_cbsrmm(transa, m, n, k, lb, alpha, matdescra, val, indx, pntrb, pntre, b,
ldb, beta, c, ldc)
call mkl_zbsrmm(transa, m, n, k, lb, alpha, matdescra, val, indx, pntrb, pntre, b,
ldb, beta, c, ldc)
```

C:

```
mkl_sbsrmm(&transa, &m, &n, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b,
&ldb, &beta, c, &ldc);
mkl_dbsrmm(&transa, &m, &n, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b,
&ldb, &beta, c, &ldc);
mkl_cbsrmm(&transa, &m, &n, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b,
&ldb, &beta, c, &ldc);
mkl_zbsrmm(&transa, &m, &n, &k, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b,
&ldb, &beta, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrmm` routine performs a matrix-matrix operation defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*A'*B + beta*C,
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in block sparse row (BSR) format, *A'* is the transpose of *A*.

NOTE This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then the matrix-matrix product is computed as *C* := *alpha*A*B* + *beta*C* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *C* := *alpha*A'*B* + *beta*C*,
m	INTEGER. Number of block rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
k	INTEGER. Number of block columns of the matrix *A*.
lb	INTEGER. Size of the block in the matrix *A*.
alpha	REAL for mkl_sbsrmm. DOUBLE PRECISION for mkl_dbsrmm. COMPLEX for mkl_cbsrmm. DOUBLE COMPLEX for mkl_zbsrmm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_sbsrmm. DOUBLE PRECISION for mkl_dbsrmm. COMPLEX for mkl_cbsrmm. DOUBLE COMPLEX for mkl_zbsrmm. Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to the *values* array description in BSR Format for more details.
indx	INTEGER. Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A*. Refer to the *columns* array description in BSR Format for more details.
pntrb	INTEGER. Array of length *m*. For one-based indexing: this array contains row indices, such that *pntrb*(*I*) – *pntrb*(1)+1 is the first index of block row *I* in the array *indx*. For zero-based indexing: this array contains row indices, such that *pntrb*(*I*) – *pntrb*(0) is the first index of block row *I* in the array *indx*.

	Refer to *pointerB* array description in BSR Format for more details.
pntre	INTEGER. Array of length *m*. For one-based indexing this array contains row indices, such that *pntre*(*I*) – *pntrb*(1) is the last index of block row *I* in the array *indx*. For zero-based indexing this array contains row indices, such that *pntre*(*I*) – *pntrb*(0)–1 is the last index of block row *I* in the array *indx*. Refer to *pointerE* array description in BSR Format for more details.
b	REAL for mkl_sbsrmm. DOUBLE PRECISION for mkl_dbsrmm. COMPLEX for mkl_cbsrmm. DOUBLE COMPLEX for mkl_zbsrmm. Array, DIMENSION (*ldb*, at least *n* for non-transposed matrix *A* and at least *m* for transposed) for one-based indexing, and (at least *k* for non-transposed matrix *A* and at least *m* for transposed, *ldb*) for zero-based indexing. On entry with *transa*= 'N' or 'n', the leading *n*-by-*k* block part of the array *b* must contain the matrix *B*, otherwise the leading *m*-by-*n* block part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension (in blocks) of *b* as declared in the calling (sub)program.
beta	REAL for mkl_sbsrmm. DOUBLE PRECISION for mkl_dbsrmm. COMPLEX for mkl_cbsrmm. DOUBLE COMPLEX for mkl_zbsrmm. Specifies the scalar *beta*.
c	REAL for mkl_sbsrmm. DOUBLE PRECISION for mkl_dbsrmm. COMPLEX for mkl_cbsrmm. DOUBLE COMPLEX for mkl_zbsrmm. Array, DIMENSION (*ldc*, *n*) for one-based indexing, DIMENSION (*k*, *ldc*) for zero-based indexing. On entry, the leading *m*-by-*n* block part of the array *c* must contain the matrix *C*, otherwise the leading *n*-by-*k* block part of the array *c* must contain the matrix *C*.
ldc	INTEGER. Specifies the leading dimension (in blocks) of *c* as declared in the calling (sub)program.

Output Parameters

c	Overwritten by the matrix (*alpha***A***B* + *beta***C*) or (*alpha***A*'**B* + *beta***C*).

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrmm(transa, m, n, k, lb, alpha, matdescra, val,
 indx, pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ld, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha, beta
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dbsrmm(transa, m, n, k, lb, alpha, matdescra, val,
 indx, pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ld, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_cbsrmm(transa, m, n, k, lb, alpha, matdescra, val,
 indx, pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ld, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha, beta
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zbsrmm(transa, m, n, k, lb, alpha, matdescra, val,
 indx, pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ld, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_sbsrmm(char *transa, int *m, int *n, int *k,  int *lb,
float *alpha, char *matdescra, float  *val, int *indx, int *pntrb,
int *pntre, float *b, int *ldb, float *beta, float *c, int *ldc,);
```

```
void mkl_dbsrmm(char *transa, int *m, int *n, int *k,  int *lb,
double *alpha, char *matdescra, double  *val, int *indx, int *pntrb,
int *pntre, double *b, int *ldb, double *beta, double *c, int *ldc,);
void mkl_cbsrmm(char *transa, int *m, int *n, int *k,  int *lb,
MKL_Complex8 *alpha, char *matdescra, MKL_Complex8  *val, int *indx, int *pntrb,
int *pntre, MKL_Complex8 *b, int *ldb, MKL_Complex8 *beta, MKL_Complex8 *c, int *ldc,);
void mkl_zbsrmm(char *transa, int *m, int *n, int *k,  int *lb,
MKL_Complex16 *alpha, char *matdescra, MKL_Complex16  *val, int *indx, int *pntrb,
int *pntre, MKL_Complex16 *b, int *ldb, MKL_Complex16 *beta, MKL_Complex16 *c, int *ldc,);
```

mkl_?cscmm

Computes matrix-matrix product of a sparse matrix stored in the CSC format.

Syntax

Fortran:

```
call mkl_scscmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)
call mkl_dcscmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)
call mkl_ccscmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)
call mkl_zcscmm(transa, m, n, k, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
beta, c, ldc)
```

C:

```
mkl_scscmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);
mkl_dcscmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);
mkl_ccscmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);
mkl_zcscmm(&transa, &m, &n, &k, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb,
&beta, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?cscmm` routine performs a matrix-matrix operation defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*A'*B + beta*C,
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in compressed sparse column (CSC) format, *A'* is the transpose of *A*.

NOTE This routine supports CSC format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *C* := *alpha*A* B + beta*C* If *transa* = 'T' or 't' or 'C' or 'c', then *C* := *alpha*A'*B + beta*C*,
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_scscmm. DOUBLE PRECISION for mkl_dcscmm. COMPLEX for mkl_ccscmm. DOUBLE COMPLEX for mkl_zcscmm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scscmm. DOUBLE PRECISION for mkl_dcscmm. COMPLEX for mkl_ccscmm. DOUBLE COMPLEX for mkl_zcscmm. Array containing non-zero elements of the matrix *A*. For one-based indexing its length is pntre(*k*) – *pntrb*(1). For zero-based indexing its length is *pntre*(*m*–1) – *pntrb*(0). Refer to *values* array description in CSC Format for more details.
indx	INTEGER. Array containing the row indices for each non-zero element of the matrix *A*.Its length is equal to length of the *val* array. Refer to *rows* array description in CSC Format for more details.
pntrb	INTEGER. Array of length *k*. For one-based indexing this array contains column indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of column *i* in the arrays *val* and *indx*. For zero-based indexing this array contains column indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of column *i* in the arrays *val* and *indx*. Refer to *pointerb* array description in CSC Format for more details.
pntre	INTEGER. Array of length *k*.

	For one-based indexing this array contains column indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of column *i* in the arrays *val* and *indx*. For zero-based indexing this array contains column indices, such that *pntre*(*i*) – *pntrb*(1)-1 is the last index of column *i* in the arrays *val* and *indx*. Refer to *pointerE* array description in CSC Format for more details.
b	REAL for mkl_scscmm. DOUBLE PRECISION for mkl_dcscmm. COMPLEX for mkl_ccscmm. DOUBLE COMPLEX for mkl_zcscmm. Array, DIMENSION (*ldb*, at least *n* for non-transposed matrix *A* and at least *m* for transposed) for one-based indexing, and (at least *k* for non-transposed matrix *A* and at least *m* for transposed, *ldb*) for zero-based indexing. On entry with *transa* = 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* for one-based indexing, and the second dimension of *b* for zero-based indexing, as declared in the calling (sub)program.
beta	REAL*8. Specifies the scalar *beta*.
c	REAL for mkl_scscmm. DOUBLE PRECISION for mkl_dcscmm. COMPLEX for mkl_ccscmm. DOUBLE COMPLEX for mkl_zcscmm. Array, DIMENSION (*ldc*, *n*) for one-based indexing, and (*m*, *ldc*) for zero-based indexing. On entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, otherwise the leading *k*-by-*n* part of the array *c* must contain the matrix *C*.
ldc	INTEGER. Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c	Overwritten by the matrix (*alpha***A***B* + *beta** *C*) or (*alpha***A*'**B* + *beta***C*).

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scscmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(k), pntre(k)
  REAL          alpha, beta
  REAL          val(*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_dcscmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(k), pntre(k)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_ccscmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(k), pntre(k)
  COMPLEX        alpha, beta
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zcscmm(transa, m, n, k, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       indx(*), pntrb(k), pntre(k)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_scscmm(char *transa, int *m, int *n, int *k,
float *alpha, char *matdescra, float  *val, int *indx,
 int *pntrb, int *pntre, float *b, int *ldb,
float *beta, float *c, int *ldc);

void mkl_dcscmm(char *transa, int *m, int *n, int *k,
double *alpha, char *matdescra, double  *val, int *indx,
int *pntrb, int *pntre, double *b, int *ldb,
double *beta, double *c, int *ldc);

void mkl_ccscmm(char *transa, int *m, int *n, int *k,
MKL_Complex8 *alpha, char *matdescra, MKL_Complex8  *val, int *indx,
int *pntrb, int *pntre, MKL_Complex8 *b, int *ldb,
MKL_Complex8 *beta, MKL_Complex8 *c, int *ldc);
```

```
void mkl_zcscmm(char *transa, int *m, int *n, int *k,
MKL_Complex16 *alpha, char *matdescra, MKL_Complex16  *val, int *indx,
int *pntrb, int *pntre, MKL_Complex16 *b, int *ldb,
MKL_Complex16 *beta, MKL_Complex16 *c, int *ldc);
```

mkl_?coomm

Computes matrix-matrix product of a sparse matrix stored in the coordinate format.

Syntax

Fortran:

```
call mkl_scoomm(transa, m, n, k, alpha, matdescra, val, rowind, colind, nnz, b, ldb, beta, c, ldc)
call mkl_dcoomm(transa, m, n, k, alpha, matdescra, val, rowind, colind, nnz, b, ldb, beta, c, ldc)
call mkl_ccoomm(transa, m, n, k, alpha, matdescra, val, rowind, colind, nnz, b, ldb, beta, c, ldc)
call mkl_zcoomm(transa, m, n, k, alpha, matdescra, val, rowind, colind, nnz, b, ldb, beta, c, ldc)
```

C:

```
mkl_scoomm(&transa, &m, &n, &k, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, &beta, c, &ldc);
mkl_dcoomm(&transa, &m, &n, &k, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, &beta, c, &ldc);
mkl_ccoomm(&transa, &m, &n, &k, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, &beta, c, &ldc);
mkl_zcoomm(&transa, &m, &n, &k, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, &beta, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?coomm` routine performs a matrix-matrix operation defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*A'*B + beta*C,
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in the coordinate format, *A'* is the transpose of *A*.

NOTE This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *C* := *alpha***A***B* + *beta***C* If *transa* = 'T' or 't' or 'C' or 'c', then *C* := *alpha***A'***B* + *beta***C*,
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_scoomm. DOUBLE PRECISION for mkl_dcoomm. COMPLEX for mkl_ccoomm. DOUBLE COMPLEX for mkl_zcoomm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scoomm. DOUBLE PRECISION for mkl_dcoomm. COMPLEX for mkl_ccoomm. DOUBLE COMPLEX for mkl_zcoomm. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.
colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
b	REAL for mkl_scoomm. DOUBLE PRECISION for mkl_dcoomm. COMPLEX for mkl_ccoomm. DOUBLE COMPLEX for mkl_zcoomm. Array, DIMENSION (*ldb*, at least *n* for non-transposed matrix *A* and at least *m* for transposed) for one-based indexing, and (at least *k* for non-transposed matrix *A* and at least *m* for transposed, *ldb*) for zero-based indexing.

	On entry with *transa* = 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* for one-based indexing, and the second dimension of *b* for zero-based indexing, as declared in the calling (sub)program.
beta	REAL for mkl_scoomm. DOUBLE PRECISION for mkl_dcoomm. COMPLEX for mkl_ccoomm. DOUBLE COMPLEX for mkl_zcoomm. Specifies the scalar *beta*.
c	REAL for mkl_scoomm. DOUBLE PRECISION for mkl_dcoomm. COMPLEX for mkl_ccoomm. DOUBLE COMPLEX for mkl_zcoomm. Array, DIMENSION (*ldc*, *n*) for one-based indexing, and (*m*, *ldc*) for zero-based indexing. On entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, otherwise the leading *k*-by-*n* part of the array *c* must contain the matrix *C*.
ldc	INTEGER. Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c	Overwritten by the matrix (*alpha*A*B + beta*C*) or (*alpha*A'*B + beta*C*).

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scoomm(transa, m, n, k, alpha, matdescra, val,
 rowind, colind, nnz, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  REAL          alpha, beta
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcoomm(transa, m, n, k, alpha, matdescra, val,
 rowind, colind, nnz, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_ccoomm(transa, m, n, k, alpha, matdescra, val,
 rowind, colind, nnz, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        alpha, beta
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zcoomm(transa, m, n, k, alpha, matdescra, val,
 rowind, colind, nnz, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_scoomm(char *transa, int *m, int *n, int *k, float *alpha,
char *matdescra, float  *val, int *rowind, int *colind, int *nnz,
float *b, int *ldb, float *beta, float *c, int *ldc);

void mkl_dcoomm(char *transa, int *m, int *n, int *k, double *alpha,
char *matdescra, double  *val, int *rowind, int *colind, int *nnz,
double *b, int *ldb, double *beta, double *c, int *ldc);

void mkl_ccoomm(char *transa, int *m, int *n, int *k, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8  *val, int *rowind, int *colind, int *nnz,
MKL_Complex8 *b, int *ldb, MKL_Complex8 *beta, MKL_Complex8 *c, int *ldc);

void mkl_zcoomm(char *transa, int *m, int *n, int *k, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16  *val, int *rowind, int *colind, int *nnz,
MKL_Complex16 *b, int *ldb, MKL_Complex16 *beta, MKL_Complex16 *c, int *ldc);
```

mkl_?csrsm

Solves a system of linear matrix equations for a sparse matrix in the CSR format.

Syntax

Fortran:

```
call mkl_scsrsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c,
ldc)

call mkl_dcsrsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c,
ldc)
```

```
call mkl_ccsrsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c, ldc)
call mkl_zcsrsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c, ldc)
```

C:

```
mkl_scsrsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_dcsrsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_ccsrsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_zcsrsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrsm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the CSR format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(A')*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the system of linear equations. If *transa* = `'N'` or `'n'`, then *C* := *alpha**inv(*A*)**B* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *C* := *alpha**inv(*A'*)**B*,
m	`INTEGER`. Number of columns of the matrix *A*.
n	`INTEGER`. Number of columns of the matrix *C*.
alpha	`REAL` for `mkl_scsrsm`. `DOUBLE PRECISION` for `mkl_dcsrsm`. `COMPLEX` for `mkl_ccsrsm`. `DOUBLE COMPLEX` for `mkl_zcsrsm`. Specifies the scalar *alpha*.

matdescra CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".

val REAL for mkl_scsrsm.
DOUBLE PRECISION for mkl_dcsrsm.
COMPLEX for mkl_ccsrsm.
DOUBLE COMPLEX for mkl_zcsrsm.
Array containing non-zero elements of the matrix *A*.
For one-based indexing its length is *pntre*(m) – *pntrb*(1).
For zero-based indexing its length is *pntre*(m-1) – *pntrb*(0).
Refer to *values* array description in CSR Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array.
Refer to *columns* array description in CSR Format for more details.

NOTE Column indices must be sorted in increasing order for each row.

pntrb INTEGER. Array of length *m*.
For one-based indexing this array contains row indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of row *i* in the arrays *val* and *indx*.
For zero-based indexing this array contains row indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of row *i* in the arrays *val* and *indx*. Refer to *pointerb* array description in CSR Format for more details.

pntre INTEGER. Array of length *m*.
For one-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of row *i* in the arrays *val* and *indx*.
For zero-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(0)–1 is the last index of row *i* in the arrays *val* and *indx*.Refer to *pointerE* array description in CSR Format for more details.

b REAL for mkl_scsrsm.
DOUBLE PRECISION for mkl_dcsrsm.
COMPLEX for mkl_ccsrsm.
DOUBLE COMPLEX for mkl_zcsrsm.
Array, DIMENSION (*ldb*, *n*) for one-based indexing, and (*m*, *ldb*) for zero-based indexing.
On entry the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb INTEGER. Specifies the leading dimension of *b* for one-based indexing, and the second dimension of *b* for zero-based indexing, as declared in the calling (sub)program.

ldc	INTEGER. Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c	REAL*8. Array, DIMENSION (*ldc*, *n*) for one-based indexing, and (*m*, *ldc*) for zero-based indexing. The leading *m*-by-*n* part of the array *c* contains the output matrix *C*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcsrsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_ccsrsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha
  COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_zcsrsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_scsrsm(char *transa, int *m, int *n, float *alpha,
char *matdescra, float  *val, int *indx, int *pntrb,
int *pntre, float *b, int *ldb, float *c, int *ldc);

void mkl_dcsrsm(char *transa, int *m, int *n, double *alpha,
char *matdescra, double  *val, int *indx, int *pntrb,
int *pntre, double *b, int *ldb, double *c, int *ldc);

void mkl_ccsrsm(char *transa, int *m, int *n, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8  *val, int *indx, int *pntrb,
int *pntre, MKL_Complex8 *b, int *ldb, MKL_Complex8 *c, int *ldc);

void mkl_zcsrsm(char *transa, int *m, int *n, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16  *val, int *indx, int *pntrb,
int *pntre, MKL_Complex16 *b, int *ldb, MKL_Complex16 *c, int *ldc);
```

mkl_?cscsm

Solves a system of linear matrix equations for a sparse matrix in the CSC format.

Syntax

Fortran:

```
call mkl_scscsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c,
ldc)

call mkl_dcscsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c,
ldc)

call mkl_ccscsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c,
ldc)

call mkl_zcscsm(transa, m, n, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c,
ldc)
```

C:

```
mkl_scscsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c,
&ldc);
```

```
mkl_dcscsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_ccscsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_zcscsm(&transa, &m, &n, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?cscsm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the CSC format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(A')*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a CSC format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the system of equations. If *transa* = `'N'` or `'n'`, then *C* := *alpha**inv(*A*)**B* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *C* := *alpha**inv(*A'*)**B*,
m	`INTEGER`. Number of columns of the matrix *A*.
n	`INTEGER`. Number of columns of the matrix *C*.
alpha	`REAL` for `mkl_scscsm`. `DOUBLE PRECISION` for `mkl_dcscsm`. `COMPLEX` for `mkl_ccscsm`. `DOUBLE COMPLEX` for `mkl_zcscsm`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	`REAL` for `mkl_scscsm`. `DOUBLE PRECISION` for `mkl_dcscsm`. `COMPLEX` for `mkl_ccscsm`.

DOUBLE COMPLEX for mkl_zcscsm.
Array containing non-zero elements of the matrix *A*.
For one-based indexing its length is pntre(*k*) – *pntrb*(1).
For zero-based indexing its length is *pntre*(*m*–1) – *pntrb*(0).
Refer to *values* array description in CSC Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx INTEGER. Array containing the row indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array.
Refer to *rows* array description in CSC Format for more details.

NOTE Row indices must be sorted in increasing order for each column.

pntrb INTEGER. Array of length *m*.
For one-based indexing this array contains column indices, such that *pntrb*(*I*) – *pntrb*(1)+1 is the first index of column *I* in the arrays *val* and *indx*.
For zero-based indexing this array contains column indices, such that *pntrb*(*I*) – *pntrb*(0) is the first index of column *I* in the arrays *val* and *indx*.
Refer to *pointerb* array description in CSC Format for more details.

pntre INTEGER. Array of length *m*.
For one-based indexing this array contains column indices, such that *pntre*(*I*) – *pntrb*(1) is the last index of column *I* in the arrays *val* and *indx*.
For zero-based indexing this array contains column indices, such that *pntre*(*I*) – *pntrb*(1)–1 is the last index of column *I* in the arrays *val* and *indx*.
Refer to *pointerE* array description in CSC Format for more details.

b REAL for mkl_scscsm.
DOUBLE PRECISION for mkl_dcscsm.
COMPLEX for mkl_ccscsm.
DOUBLE COMPLEX for mkl_zcscsm.
Array, DIMENSION (*ldb*, *n*) for one-based indexing, and (*m*, *ldb*) for zero-based indexing.
On entry the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb INTEGER. Specifies the leading dimension of *b* for one-based indexing, and the second dimension of *b* for zero-based indexing, as declared in the calling (sub)program.

ldc INTEGER. Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c REAL for mkl_scscsm.

DOUBLE PRECISION for mkl_dcscsm.
COMPLEX for mkl_ccscsm.
DOUBLE COMPLEX for mkl_zcscsm.
Array, DIMENSION (*ldc*, *n*) for one-based indexing, and (*m*, *ldc*) for zero-based indexing.
The leading *m*-by-*n* part of the array *c* contains the output matrix *C*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scscsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcscsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_ccscsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zcscsm(transa, m, n, alpha, matdescra, val, indx,
 pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_scscsm(char *transa, int *m, int *n, float *alpha,
char *matdescra, float  *val, int *indx, int *pntrb,
int *pntre, float *b, int *ldb, float *c, int *ldc);

void mkl_dcscsm(char *transa, int *m, int *n, double *alpha,
char *matdescra, double  *val, int *indx, int *pntrb,
int *pntre, double *b, int *ldb, double *c, int *ldc);

void mkl_ccscsm(char *transa, int *m, int *n, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8  *val, int *indx, int *pntrb,
int *pntre, MKL_Complex8 *b, int *ldb, MKL_Complex8 *c, int *ldc);

void mkl_zcscsm(char *transa, int *m, int *n, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16  *val, int *indx, int *pntrb,
int *pntre, MKL_Complex16 *b, int *ldb, MKL_Complex16 *c, int *ldc);
```

mkl_?coosm

Solves a system of linear matrix equations for a sparse matrix in the coordinate format.

Syntax

Fortran:

```
call mkl_scoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)

call mkl_dcoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)

call mkl_ccoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)

call mkl_zcoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)
```

C:

```
mkl_scoosm(&transa, &m, &n, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, c, &ldc);

mkl_dcoosm(&transa, &m, &n, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, c, &ldc);

mkl_ccoosm(&transa, &m, &n, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, c, &ldc);

mkl_zcoosm(&transa, &m, &n, &alpha, matdescra, val, rowind, colind, &nnz, b, &ldb, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The mkl_?coosm routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the coordinate format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(A')*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the system of linear equations. If *transa* = 'N' or 'n', then the matrix-matrix product is computed as *C* := *alpha**inv(*A*)**B* If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as *C* := *alpha**inv(*A'*)**B*,
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
alpha	REAL for mkl_scoosm. DOUBLE PRECISION for mkl_dcoosm. COMPLEX for mkl_ccoosm. DOUBLE COMPLEX for mkl_zcoosm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_scoosm. DOUBLE PRECISION for mkl_dcoosm. COMPLEX for mkl_ccoosm. DOUBLE COMPLEX for mkl_zcoosm. Array of length *nnz*, contains non-zero elements of the matrix *A* in the arbitrary order. Refer to *values* array description in Coordinate Format for more details.
rowind	INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.

colind	INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.
nnz	INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.
b	REAL for mkl_scoosm. DOUBLE PRECISION for mkl_dcoosm. COMPLEX for mkl_ccoosm. DOUBLE COMPLEX for mkl_zcoosm. Array, DIMENSION (*ldb*, *n*) for one-based indexing, and (*m*, *ldb*) for zero-based indexing. Before entry the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* for one-based indexing, and the second dimension of *b* for zero-based indexing, as declared in the calling (sub)program.
ldc	INTEGER. Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c	REAL for mkl_scoosm. DOUBLE PRECISION for mkl_dcoosm. COMPLEX for mkl_ccoosm. DOUBLE COMPLEX for mkl_zcoosm. Array, DIMENSION (*ldc*, *n*) for one-based indexing, and (*m*, *ldc*) for zero-based indexing. The leading *m*-by-*n* part of the array *c* contains the output matrix *C*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  REAL          alpha
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dcoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_ccoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  COMPLEX        alpha
  COMPLEX       val(*), b(ldb,*), c(ldc,*)
SUBROUTINE mkl_zcoosm(transa, m, n, alpha, matdescra, val, rowind, colind, nnz, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, nnz
  INTEGER       rowind(*), colind(*)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_scoosm(char *transa, int *m, int *n, float *alpha, char *matdescra,
float *val, int *rowind, int *colind, int *nnz, float *b, int *ldb, float *c, int *ldc);
void mkl_dcoosm(char *transa, int *m, int *n, double *alpha, char *matdescra,
double *val, int *rowind, int *colind, int *nnz, double *b, int *ldb, double *c, int *ldc);
void mkl_ccoosm(char *transa, int *m, int *n, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *rowind, int *colind, int *nnz, MKL_Complex8 *b, int *ldb, MKL_Complex8 *c, int
*ldc);
void mkl_zcoosm(char *transa, int *m, int *n, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *rowind, int *colind, int *nnz, MKL_Complex16 *b, int *ldb, MKL_Complex16 *c,
int *ldc);
```

mkl_?bsrsm

Solves a system of linear matrix equations for a sparse matrix in the BSR format.

Syntax

Fortran:

```
call mkl_scsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
c, ldc)
call mkl_dcsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
c, ldc)
call mkl_ccsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
c, ldc)
call mkl_zcsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb,
c, ldc)
```

C:

```
mkl_scsrsm(&transa, &m, &n, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_dcsrsm(&transa, &m, &n, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_ccsrsm(&transa, &m, &n, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
mkl_zcsrsm(&transa, &m, &n, &lb, &alpha, matdescra, val, indx, pntrb, pntre, b, &ldb, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?bsrsm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the BSR format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(A')*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then the matrix-matrix product is computed as *C* := *alpha*`*inv(`*A*`)*`*B*. If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then the matrix-vector product is computed as *C* := *alpha*`*inv(`*A'*`)*`*B*.
m	`INTEGER`. Number of block columns of the matrix *A*.
n	`INTEGER`. Number of columns of the matrix *C*.
lb	`INTEGER`. Size of the block in the matrix *A*.
alpha	`REAL` for `mkl_sbsrsm`. `DOUBLE PRECISION` for `mkl_dbsrsm`. `COMPLEX` for `mkl_cbsrsm`. `DOUBLE COMPLEX` for `mkl_zbsrsm`. Specifies the scalar *alpha*.

matdescra CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".

val REAL for mkl_sbsrsm.
DOUBLE PRECISION for mkl_dbsrsm.
COMPLEX for mkl_cbsrsm.
DOUBLE COMPLEX for mkl_zbsrsm.
Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the *ABAB* number *ABAB* of non-zero blocks in the matrix *A* multiplied by *lb*lb*. Refer to the *values* array description in BSR Format for more details.

NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx INTEGER. Array containing the column indices for each non-zero block in the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A*. Refer to the *columns* array description in BSR Format for more details.

pntrb INTEGER. Array of length *m*.
For one-based indexing: this array contains row indices, such that *pntrb*(*i*) – *pntrb*(1)+1 is the first index of block row *i* in the array *indx*.
For zero-based indexing: this array contains row indices, such that *pntrb*(*i*) – *pntrb*(0) is the first index of block row *i* in the array *indx*.
Refer to *pointerB* array description in BSR Format for more details.

pntre INTEGER. Array of length *m*.
For one-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(1) is the last index of block row *i* in the array *indx*.
For zero-based indexing this array contains row indices, such that *pntre*(*i*) – *pntrb*(0)–1 is the last index of block row *i* in the array *indx*.
Refer to *pointerE* array description in BSR Format for more details.

b REAL for mkl_sbsrsm.
DOUBLE PRECISION for mkl_dbsrsm.
COMPLEX for mkl_cbsrsm.
DOUBLE COMPLEX for mkl_zbsrsm.
Array, DIMENSION (*ldb*, *n*) for one-based indexing, DIMENSION (*m*, *ldb*) for zero-based indexing.
On entry the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb INTEGER. Specifies the leading dimension (in blocks) of *b* as declared in the calling (sub)program.

ldc INTEGER. Specifies the leading dimension (in blocks) of *c* as declared in the calling (sub)program.

Output Parameters

c	REAL for mkl_sbsrsm. DOUBLE PRECISION for mkl_dbsrsm. COMPLEX for mkl_cbsrsm. DOUBLE COMPLEX for mkl_zbsrsm. Array, DIMENSION (*ldc*, *n*) for one-based indexing, DIMENSION (*m*, *ldc*) for zero-based indexing. The leading *m*-by-*n* part of the array *c* contains the output matrix *C*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sbsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1     transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, lb, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  REAL          alpha
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dbsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1     transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, lb, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_cbsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1     transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, lb, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  COMPLEX        alpha
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zbsrsm(transa, m, n, lb, alpha, matdescra, val, indx, pntrb, pntre, b, ldb, c, ldc)
  CHARACTER*1     transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, lb, ldb, ldc
  INTEGER       indx(*), pntrb(m), pntre(m)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_sbsrsm(char *transa, int *m, int *n, int *lb, float *alpha, char *matdescra,
float  *val, int *indx, int *pntrb, int *pntre, float *b, int *ldb, float *c, int *ldc);

void mkl_dbsrsm(char *transa, int *m, int *n, int *lb, double *alpha, char *matdescra,
double  *val, int *indx, int *pntrb, int *pntre, double *b, int *ldb, double *c, int *ldc);

void mkl_cbsrsm(char *transa, int *m, int *n, int *lb, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8  *val, int *indx, int *pntrb, int *pntre, MKL_Complex8 *b, int *ldb, MKL_Complex8 *c, int
*ldc);

void mkl_zbsrsm(char *transa, int *m, int *n, int *lb, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16  *val, int *indx, int *pntrb, int *pntre, MKL_Complex16 *b, int *ldb, MKL_Complex16 *c,
int *ldc);
```

mkl_?diamv

Computes matrix - vector product for a sparse matrix in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiamv(transa, m, k, alpha, matdescra, val, lval, idiag, ndiag, x, beta, y)
call mkl_ddiamv(transa, m, k, alpha, matdescra, val, lval, idiag, ndiag, x, beta, y)
call mkl_cdiamv(transa, m, k, alpha, matdescra, val, lval, idiag, ndiag, x, beta, y)
call mkl_zdiamv(transa, m, k, alpha, matdescra, val, lval, idiag, ndiag, x, beta, y)
```

C:

```
mkl_sdiamv(&transa, &m, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, x, &beta,
y);
mkl_ddiamv(&transa, &m, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, x, &beta,
y);
mkl_cdiamv(&transa, &m, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, x, &beta,
y);
mkl_zdiamv(&transa, &m, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, x, &beta,
y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?diamv` routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*A'*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix stored in the diagonal format, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *y* := *alpha*A*x* + *beta*y*, If *transa* = 'T' or 't' or 'C' or 'c', then *y* := *alpha*A'*x* + *beta*y*.
m	INTEGER. Number of rows of the matrix *A*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_sdiamv. DOUBLE PRECISION for mkl_ddiamv. COMPLEX for mkl_cdiamv. DOUBLE COMPLEX for mkl_zdiamv. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_sdiamv. DOUBLE PRECISION for mkl_ddiamv. COMPLEX for mkl_cdiamv. DOUBLE COMPLEX for mkl_zdiamv. Two-dimensional array of size *lval* by *ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.
lval	INTEGER. Leading dimension of *val*, *lval* ≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.
idiag	INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*. Refer to *distance* array description in Diagonal Storage Scheme for more details.
ndiag	INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.
x	REAL for mkl_sdiamv. DOUBLE PRECISION for mkl_ddiamv. COMPLEX for mkl_cdiamv. DOUBLE COMPLEX for mkl_zdiamv. Array, DIMENSION at least *k* if *transa* = 'N' or 'n', and at least *m* otherwise. On entry, the array *x* must contain the vector *x*.
beta	REAL for mkl_sdiamv. DOUBLE PRECISION for mkl_ddiamv. COMPLEX for mkl_cdiamv.

	DOUBLE COMPLEX for mkl_zdiamv. Specifies the scalar *beta*.
y	REAL for mkl_sdiamv. DOUBLE PRECISION for mkl_ddiamv. COMPLEX for mkl_cdiamv. DOUBLE COMPLEX for mkl_zdiamv. Array, DIMENSION at least *m* if *transa* = 'N' or 'n', and at least *k* otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y	Overwritten by the updated vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiamv(transa, m, k, alpha, matdescra, val, lval, idiag,
ndiag, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lval, ndiag
  INTEGER      idiag(*)
  REAL          alpha, beta
  REAL          val(lval,*), x(*), y(*)

SUBROUTINE mkl_ddiamv(transa, m, k, alpha, matdescra, val, lval, idiag,
ndiag, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE PRECISION          alpha, beta
  DOUBLE PRECISION          val(lval,*), x(*), y(*)

SUBROUTINE mkl_cdiamv(transa, m, k, alpha, matdescra, val, lval, idiag,
ndiag, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lval, ndiag
  INTEGER      idiag(*)
  COMPLEX          alpha, beta
  COMPLEX          val(lval,*), x(*), y(*)
```

```
SUBROUTINE mkl_zdiamv(transa, m, k, alpha, matdescra, val, lval, idiag,
ndiag, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE COMPLEX          alpha, beta
  DOUBLE COMPLEX          val(lval,*), x(*), y(*)
```

C:

```
void mkl_sdiamv(char *transa, int *m, int *k, float *alpha,
char *matdescra, float  *val, int *lval, int *idiag,
int *ndiag, float *x, float *beta, float *y);

void mkl_ddiamv(char *transa, int *m, int *k, double *alpha,
char *matdescra, double  *val, int *lval, int *idiag,
int *ndiag, double *x, double *beta, double *y);

void mkl_cdiamv(char *transa, int *m, int *k, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8  *val, int *lval, int *idiag,
int *ndiag, MKL_Complex8 *x, MKL_Complex8 *beta, MKL_Complex8 *y);

void mkl_zdiamv(char *transa, int *m, int *k, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16  *val, int *lval, int *idiag,
int *ndiag, MKL_Complex16 *x, MKL_Complex16 *beta, MKL_Complex16 *y);
```

mkl_?skymv

Computes matrix - vector product for a sparse matrix in the skyline storage format with one-based indexing.

Syntax

Fortran:

```
call mkl_sskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
call mkl_dskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
call mkl_cskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
call mkl_zskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
```

C:

```
mkl_sskymv(&transa, &m, &k, &alpha, matdescra, val, pntr, x, &beta, y);
mkl_dskymv(&transa, &m, &k, &alpha, matdescra, val, pntr, x, &beta, y);
mkl_cskymv(&transa, &m, &k, &alpha, matdescra, val, pntr, x, &beta, y);
mkl_zskymv(&transa, &m, &k, &alpha, matdescra, val, pntr, x, &beta, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`

- C: mkl_spblas.h

Description

The mkl_?skymv routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*A'*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix stored using the skyline storage scheme, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *y := alpha*A*x + beta*y* If *transa* = 'T' or 't' or 'C' or 'c', then *y := alpha*A'*x + beta*y*,
m	INTEGER. Number of rows of the matrix *A*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_sskymv. DOUBLE PRECISION for mkl_dskymv. COMPLEX for mkl_cskymv. DOUBLE COMPLEX for mkl_zskymv. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*". **NOTE** General matrices (*matdescra* (1)='G') is not supported.
val	REAL for mkl_sskymv. DOUBLE PRECISION for mkl_dskymv. COMPLEX for mkl_cskymv. DOUBLE COMPLEX for mkl_zskymv. Array containing the set of elements of the matrix *A* in the skyline profile form. If *matdescrsa*(2)= 'L', then *val* contains elements from the low triangle of the matrix *A*.

	If *matdescrsa*(2)= 'U', then *val* contains elements from the upper triangle of the matrix *A*. Refer to *values* array description in Skyline Storage Scheme for more details.
pntr	INTEGER. Array of length (*m*+*m*) for lower triangle, and (*k*+*k*) for upper triangle. It contains the indices specifying in the *val* the positions of the first element in each row (column) of the matrix *A*. Refer to *pointers* array description in Skyline Storage Scheme for more details.
x	REAL for mkl_sskymv. DOUBLE PRECISION for mkl_dskymv. COMPLEX for mkl_cskymv. DOUBLE COMPLEX for mkl_zskymv. Array, DIMENSION at least *k* if *transa* = 'N' or 'n' and at least *m* otherwise. On entry, the array *x* must contain the vector *x*.
beta	REAL for mkl_sskymv. DOUBLE PRECISION for mkl_dskymv. COMPLEX for mkl_cskymv. DOUBLE COMPLEX for mkl_zskymv. Specifies the scalar *beta*.
y	REAL for mkl_sskymv. DOUBLE PRECISION for mkl_dskymv. COMPLEX for mkl_cskymv. DOUBLE COMPLEX for mkl_zskymv. Array, DIMENSION at least *m* if *transa* = 'N' or 'n' and at least *k* otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y	Overwritten by the updated vector *y*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       pntr(*)
  REAL          alpha, beta
  REAL          val(*), x(*), y(*)

SUBROUTINE mkl_dskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       pntr(*)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), x(*), y(*)
```

```
SUBROUTINE mkl_cdskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       pntr(*)
  COMPLEX        alpha, beta
  COMPLEX        val(*), x(*), y(*)
SUBROUTINE mkl_zskymv(transa, m, k, alpha, matdescra, val, pntr, x, beta, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, k
  INTEGER       pntr(*)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_sskymv (char *transa, int *m, int *k, float *alpha, char *matdescra,
float  *val, int *pntr, float *x, float *beta, float *y);
void mkl_dskymv (char *transa, int *m, int *k, double *alpha, char *matdescra,
double  *val, int *pntr, double *x, double *beta, double *y);
void mkl_cskymv (char *transa, int *m, int *k, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8  *val, int *pntr, MKL_Complex8 *x, MKL_Complex8 *beta, MKL_Complex8 *y);
void mkl_zskymv (char *transa, int *m, int *k, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16  *val, int *pntr, MKL_Complex16 *x, MKL_Complex16 *beta, MKL_Complex16 *y);
```

mkl_?diasv

Solves a system of linear equations for a sparse matrix in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
call mkl_ddiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
call mkl_cdiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
call mkl_zdiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
```

C:

```
mkl_sdiasv(&transa, &m, &alpha, matdescra, val, &lval, idiag, &ndiag, x, y);
mkl_ddiasv(&transa, &m, &alpha, matdescra, val, &lval, idiag, &ndiag, x, y);
mkl_cdiasv(&transa, &m, &alpha, matdescra, val, &lval, idiag, &ndiag, x, y);
```

```
mkl_zdiasv(&transa, &m, &alpha, matdescra, val, &lval, idiag, &ndiag, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?diasv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the diagonal format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(A')* x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the system of linear equations. If *transa* = `'N'` or `'n'`, then *y* := *alpha**inv(*A*)**x* If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *y* := *alpha**inv(*A'*)**x*,
m	`INTEGER`. Number of rows of the matrix *A*.
alpha	`REAL` for `mkl_sdiasv`. `DOUBLE PRECISION` for `mkl_ddiasv`. `COMPLEX` for `mkl_cdiasv`. `DOUBLE COMPLEX` for `mkl_zdiasv`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	`REAL` for `mkl_sdiasv`. `DOUBLE PRECISION` for `mkl_ddiasv`. `COMPLEX` for `mkl_cdiasv`. `DOUBLE COMPLEX` for `mkl_zdiasv`. Two-dimensional array of size *lval* by *ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.
lval	`INTEGER`. Leading dimension of *val*, *lval*≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.

idiag	INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*. **NOTE** All elements of this array must be sorted in increasing order. Refer to *distance* array description in Diagonal Storage Scheme for more details.
ndiag	INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.
x	REAL for mkl_sdiasv. DOUBLE PRECISION for mkl_ddiasv. COMPLEX for mkl_cdiasv. DOUBLE COMPLEX for mkl_zdiasv. Array, DIMENSION at least *m*. On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.
y	REAL for mkl_sdiasv. DOUBLE PRECISION for mkl_ddiasv. COMPLEX for mkl_cdiasv. DOUBLE COMPLEX for mkl_zdiasv. Array, DIMENSION at least *m*. On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y	Contains solution vector *x*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  REAL        alpha
  REAL        val(lval,*), x(*), y(*)

SUBROUTINE mkl_ddiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(lval,*), x(*), y(*)
```

```
SUBROUTINE mkl_cdiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  COMPLEX        alpha
  COMPLEX        val(lval,*), x(*), y(*)
SUBROUTINE mkl_zdiasv(transa, m, alpha, matdescra, val, lval, idiag, ndiag, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, lval, ndiag
  INTEGER       indiag(*)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(lval,*), x(*), y(*)
```

C:

```
void mkl_sdiasv(char *transa, int *m, float *alpha, char *matdescra,
float *val, int *lval, int *idiag, int *ndiag, float *x, float *y);
void mkl_ddiasv(char *transa, int *m, double *alpha, char *matdescra,
double *val, int *lval, int *idiag, int *ndiag, double *x, double *y);
void mkl_cdiasv(char *transa, int *m, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *lval, int *idiag, int *ndiag, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zdiasv(char *transa, int *m, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *lval, int *idiag, int *ndiag, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?skysv

Solves a system of linear equations for a sparse matrix in the skyline format with one-based indexing.

Syntax

Fortran:

```
call mkl_sskysv(transa, m, alpha, matdescra, val, pntr, x, y)
call mkl_dskysv(transa, m, alpha, matdescra, val, pntr, x, y)
call mkl_cskysv(transa, m, alpha, matdescra, val, pntr, x, y)
call mkl_zskysv(transa, m, alpha, matdescra, val, pntr, x, y)
```

C:

```
mkl_sskysv(&transa, &m, &alpha, matdescra, val, pntr, x, y);
mkl_dskysv(&transa, &m, &alpha, matdescra, val, pntr, x, y);
mkl_cskysv(&transa, &m, &alpha, matdescra, val, pntr, x, y);
mkl_zskysv(&transa, &m, &alpha, matdescra, val, pntr, x, y);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?skysv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the skyline storage format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(A')*x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa
`CHARACTER*1`. Specifies the system of linear equations.
If *transa* = `'N'` or `'n'`, then *y* := *alpha**inv(*A*)**x*
If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *y* := *alpha**inv(*A'*)* *x*,

m
`INTEGER`. Number of rows of the matrix *A*.

alpha
`REAL` for `mkl_sskysv`.
`DOUBLE PRECISION` for `mkl_dskysv`.
`COMPLEX` for `mkl_cskysv`.
`DOUBLE COMPLEX` for `mkl_zskysv`.
Specifies the scalar *alpha*.

matdescra
`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".

NOTE General matrices (*matdescra* `(1)='G'`) is not supported.

val
`REAL` for `mkl_sskysv`.
`DOUBLE PRECISION` for `mkl_dskysv`.
`COMPLEX` for `mkl_cskysv`.
`DOUBLE COMPLEX` for `mkl_zskysv`.
Array containing the set of elements of the matrix *A* in the skyline profile form.
If *matdescrsa*`(2)= 'L'`, then *val* contains elements from the low triangle of the matrix *A*.

	If *matdescrsa*(2)= 'U', then *val* contains elements from the upper triangle of the matrix *A*. Refer to *values* array description in Skyline Storage Scheme for more details.
pntr	INTEGER. Array of length (*m*+*m*) for lower triangle, and (*k*+*k*) for upper triangle. It contains the indices specifying in the *val* the positions of the first element in each row (column) of the matrix *A*. Refer to *pointers* array description in Skyline Storage Scheme for more details.
x	REAL for mkl_sskysv. DOUBLE PRECISION for mkl_dskysv. COMPLEX for mkl_cskysv. DOUBLE COMPLEX for mkl_zskysv. Array, DIMENSION at least *m*. On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.
y	REAL for mkl_sskysv. DOUBLE PRECISION for mkl_dskysv. COMPLEX for mkl_cskysv. DOUBLE COMPLEX for mkl_zskysv. Array, DIMENSION at least *m*. On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y	Contains solution vector *x*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sskysv(transa, m, alpha, matdescra, val, pntr, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       pntr(*)
  REAL        alpha
  REAL        val(*), x(*), y(*)

SUBROUTINE mkl_dskysv(transa, m, alpha, matdescra, val, pntr, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       pntr(*)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*), x(*), y(*)
```

```
SUBROUTINE mkl_cskysv(transa, m, alpha, matdescra, val, pntr, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       pntr(*)
  COMPLEX        alpha
  COMPLEX        val(*), x(*), y(*)
SUBROUTINE mkl_zskysv(transa, m, alpha, matdescra, val, pntr, x, y)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m
  INTEGER       pntr(*)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*), x(*), y(*)
```

C:

```
void mkl_sskysv(char *transa, int *m, float *alpha, char *matdescra,
float *val, int *pntr, float *x, float *y);
void mkl_dskysv(char *transa, int *m, double *alpha, char *matdescra,
double *val, int *pntr, double *x, double *y);
void mkl_cskysv(char *transa, int *m, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *pntr, MKL_Complex8 *x, MKL_Complex8 *y);
void mkl_zskysv(char *transa, int *m, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *pntr, MKL_Complex16 *x, MKL_Complex16 *y);
```

mkl_?diamm

Computes matrix-matrix product of a sparse matrix stored in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiamm(transa, m, n, k, alpha, matdescra, val, lval, idiag, ndiag, b, ldb,
beta, c, ldc)
call mkl_ddiamm(transa, m, n, k, alpha, matdescra, val, lval, idiag, ndiag, b, ldb,
beta, c, ldc)
call mkl_cdiamm(transa, m, n, k, alpha, matdescra, val, lval, idiag, ndiag, b, ldb,
beta, c, ldc)
call mkl_zdiamm(transa, m, n, k, alpha, matdescra, val, lval, idiag, ndiag, b, ldb,
beta, c, ldc)
```

C:

```
mkl_sdiamm(&transa, &m, &n, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, &beta, c, &ldc);
mkl_ddiamm(&transa, &m, &n, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, &beta, c, &ldc);
mkl_cdiamm(&transa, &m, &n, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, &beta, c, &ldc);
mkl_zdiamm(&transa, &m, &n, &k, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, &beta, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?diamm` routine performs a matrix-matrix operation defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*A'*B + beta*C,
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in the diagonal format, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the operation. If *transa* = `'N'` or `'n'`, then *C* := *alpha*A*B + beta*C*, If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *C* := *alpha*A'*B + beta*C*.
m	`INTEGER`. Number of rows of the matrix *A*.
n	`INTEGER`. Number of columns of the matrix *C*.
k	`INTEGER`. Number of columns of the matrix *A*.
alpha	`REAL` for `mkl_sdiamm`. `DOUBLE PRECISION` for `mkl_ddiamm`. `COMPLEX` for `mkl_cdiamm`. `DOUBLE COMPLEX` for `mkl_zdiamm`. Specifies the scalar *alpha*.
matdescra	`CHARACTER`. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)".

	Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".
val	REAL for mkl_sdiamm. DOUBLE PRECISION for mkl_ddiamm. COMPLEX for mkl_cdiamm. DOUBLE COMPLEX for mkl_zdiamm. Two-dimensional array of size *lval* by *ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.
lval	INTEGER. Leading dimension of *val*, *lval*≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.
idiag	INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*. Refer to *distance* array description in Diagonal Storage Scheme for more details.
ndiag	INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.
b	REAL for mkl_sdiamm. DOUBLE PRECISION for mkl_ddiamm. COMPLEX for mkl_cdiamm. DOUBLE COMPLEX for mkl_zdiamm. Array, DIMENSION (*ldb*, *n*). On entry with *transa* = 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program.
beta	REAL for mkl_sdiamm. DOUBLE PRECISION for mkl_ddiamm. COMPLEX for mkl_cdiamm. DOUBLE COMPLEX for mkl_zdiamm. Specifies the scalar *beta*.
c	REAL for mkl_sdiamm. DOUBLE PRECISION for mkl_ddiamm. COMPLEX for mkl_cdiamm. DOUBLE COMPLEX for mkl_zdiamm. Array, DIMENSION (*ldc*, *n*). On entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, otherwise the leading *k*-by-*n* part of the array *c* must contain the matrix *C*.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program.

Output Parameters

c	Overwritten by the matrix (*alpha***A***B* + *beta***C*) or (*alpha***A*'**B* + *beta***C*).

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiamm(transa, m, n, k, alpha, matdescra, val, lval,
 idiag, ndiag, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  REAL          alpha, beta
  REAL          val(lval,*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_ddiamm(transa, m, n, k, alpha, matdescra, val, lval,
 idiag, ndiag, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(lval,*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_cdiamm(transa, m, n, k, alpha, matdescra, val, lval,
 idiag, ndiag, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  COMPLEX        alpha, beta
  COMPLEX        val(lval,*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zdiamm(transa, m, n, k, alpha, matdescra, val, lval,
 idiag, ndiag, b, ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(lval,*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_sdiamm(char *transa, int *m, int *n, int *k, float *alpha,
char *matdescra, float *val, int *lval, int *idiag, int *ndiag,
float *b, int *ldb, float *beta, float *c, int *ldc);
```

```
void mkl_ddiamm(char *transa, int *m, int *n, int *k, double *alpha,
char *matdescra, double *val, int *lval, int *idiag, int *ndiag,
double *b, int *ldb, double *beta, double *c, int *ldc);
void mkl_cdiamm(char *transa, int *m, int *n, int *k, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8 *val, int *lval, int *idiag, int *ndiag,
MKL_Complex8 *b, int *ldb, MKL_Complex8 *beta, MKL_Complex8 *c, int *ldc);
void mkl_zdiamm(char *transa, int *m, int *n, int *k, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16 *val, int *lval, int *idiag, int *ndiag,
MKL_Complex16 *b, int *ldb, MKL_Complex16 *beta, MKL_Complex16 *c, int *ldc);
```

mkl_?skymm

Computes matrix-matrix product of a sparse matrix stored using the skyline storage scheme with one-based indexing.

Syntax

Fortran:

```
call mkl_sskymm(transa, m, n, k, alpha, matdescra, val, pntr, b, ldb, beta, c, ldc)
call mkl_dskymm(transa, m, n, k, alpha, matdescra, val, pntr, b, ldb, beta, c, ldc)
call mkl_cskymm(transa, m, n, k, alpha, matdescra, val, pntr, b, ldb, beta, c, ldc)
call mkl_zskymm(transa, m, n, k, alpha, matdescra, val, pntr, b, ldb, beta, c, ldc)
```

C:

```
mkl_sskymm(&transa, &m, &n, &k, &alpha, matdescra, val, pntr, b, &ldb, &beta, c,
&ldc);
mkl_dskymm(&transa, &m, &n, &k, &alpha, matdescra, val, pntr, b, &ldb, &beta, c,
&ldc);
mkl_cskymm(&transa, &m, &n, &k, &alpha, matdescra, val, pntr, b, &ldb, &beta, c,
&ldc);
mkl_zskymm(&transa, &m, &n, &k, &alpha, matdescra, val, pntr, b, &ldb, &beta, c,
&ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?skymm` routine performs a matrix-matrix operation defined as

```
C := alpha*A*B + beta*C
```

or

```
C := alpha*A'*B + beta*C,
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in the skyline storage format, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the operation. If *transa* = 'N' or 'n', then *C* := *alpha*A*B + beta*C*, If *transa* = 'T' or 't' or 'C' or 'c', then *C* := *alpha*A'*B + beta*C*,
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
k	INTEGER. Number of columns of the matrix *A*.
alpha	REAL for mkl_sskymm. DOUBLE PRECISION for mkl_dskymm. COMPLEX for mkl_cskymm. DOUBLE COMPLEX for mkl_zskymm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*". **NOTE** General matrices (*matdescra* (1)='G') is not supported.
val	REAL for mkl_sskymm. DOUBLE PRECISION for mkl_dskymm. COMPLEX for mkl_cskymm. DOUBLE COMPLEX for mkl_zskymm. Array containing the set of elements of the matrix *A* in the skyline profile form. If *matdescrsa*(2)= 'L', then *val* contains elements from the low triangle of the matrix *A*. If *matdescrsa*(2)= 'U', then *val* contains elements from the upper triangle of the matrix *A*. Refer to *values* array description in Skyline Storage Scheme for more details.
pntr	INTEGER. Array of length (*m*+*m*) for lower triangle, and (*k*+*k*) for upper triangle. It contains the indices specifying in the *val* the positions of the first element in each row (column) of the matrix *A*. Refer to *pointers* array description in Skyline Storage Scheme for more details.
b	REAL for mkl_sskymm. DOUBLE PRECISION for mkl_dskymm.

	COMPLEX for mkl_cskymm. DOUBLE COMPLEX for mkl_zskymm. Array, DIMENSION (*ldb*, *n*). On entry with *transa* = 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program.
beta	REAL for mkl_sskymm. DOUBLE PRECISION for mkl_dskymm. COMPLEX for mkl_cskymm. DOUBLE COMPLEX for mkl_zskymm. Specifies the scalar *beta*.
c	REAL for mkl_sskymm. DOUBLE PRECISION for mkl_dskymm. COMPLEX for mkl_cskymm. DOUBLE COMPLEX for mkl_zskymm. Array, DIMENSION (*ldc*, *n*). On entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, otherwise the leading *k*-by-*n* part of the array *c* must contain the matrix *C*.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program.

Output Parameters

c	Overwritten by the matrix (*alpha***A***B* + *beta***C*) or (*alpha***A*'**B* + *beta***C*).

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sskymm(transa, m, n, k, alpha, matdescra, val, pntr, b,
 ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       pntr(*)
  REAL          alpha, beta
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dskymm(transa, m, n, k, alpha, matdescra, val, pntr, b,
 ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       pntr(*)
  DOUBLE PRECISION        alpha, beta
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_cskymm(transa, m, n, k, alpha, matdescra, val, pntr, b,
 ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       pntr(*)
  COMPLEX        alpha, beta
  COMPLEX        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zskymm(transa, m, n, k, alpha, matdescra, val, pntr, b,
 ldb, beta, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, k, ldb, ldc
  INTEGER       pntr(*)
  DOUBLE COMPLEX        alpha, beta
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_sskymm(char *transa, int *m, int *n, int *k, float *alpha,
char *matdescra, float *val, int *pntr, float *b, int *ldb,
float *beta, float *c, int *ldc);

void mkl_dskymm(char *transa, int *m, int *n, int *k, double *alpha,
char *matdescra, double *val, int *pntr, double *b, int *ldb,
double *beta, double *c, int *ldc);

void mkl_cskymm(char *transa, int *m, int *n, int *k, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8 *val, int *pntr, MKL_Complex8 *b, int *ldb,
MKL_Complex8 *beta, MKL_Complex8 *c, int *ldc);

void mkl_zskymm(char *transa, int *m, int *n, int *k, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16 *val, int *pntr, MKL_Complex16 *b, int *ldb,
MKL_Complex16 *beta, MKL_Complex16 *c, int *ldc);
```

mkl_?diasm

Solves a system of linear matrix equations for a sparse matrix in the diagonal format with one-based indexing.

Syntax

Fortran:

```
call mkl_sdiasm(transa, m, n, alpha, matdescra, val, lval, idiag, ndiag, b, ldb, c,
ldc)
```

```
call mkl_ddiasm(transa, m, n, alpha, matdescra, val, lval, idiag, ndiag, b, ldb, c, ldc)
call mkl_cdiasm(transa, m, n, alpha, matdescra, val, lval, idiag, ndiag, b, ldb, c, ldc)
call mkl_zdiasm(transa, m, n, alpha, matdescra, val, lval, idiag, ndiag, b, ldb, c, ldc)
```

C:

```
mkl_sdiasm(&transa, &m, &n, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, c, &ldc);
mkl_ddiasm(&transa, &m, &n, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, c, &ldc);
mkl_cdiasm(&transa, &m, &n, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, c, &ldc);
mkl_zdiasm(&transa, &m, &n, &alpha, matdescra, val, &lval, idiag, &ndiag, b, &ldb, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?diasm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the diagonal format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(A')*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	`CHARACTER*1`. Specifies the system of linear equations. If *transa* = `'N'` or `'n'`, then *C* := *alpha*`inv(`*A*`)*`*B*, If *transa* = `'T'` or `'t'` or `'C'` or `'c'`, then *C* := *alpha*`inv(`*A'*`)*`*B*.
m	`INTEGER`. Number of rows of the matrix *A*.
n	`INTEGER`. Number of columns of the matrix *C*.
alpha	`REAL` for `mkl_sdiasm`. `DOUBLE PRECISION` for `mkl_ddiasm`. `COMPLEX` for `mkl_cdiasm`. `DOUBLE COMPLEX` for `mkl_zdiasm`.

Specifies the scalar *alpha*.

matdescra — CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*".

val — REAL for mkl_sdiasm.
DOUBLE PRECISION for mkl_ddiasm.
COMPLEX for mkl_cdiasm.
DOUBLE COMPLEX for mkl_zdiasm.
Two-dimensional array of size *lval* by *ndiag*, contains non-zero diagonals of the matrix *A*. Refer to *values* array description in Diagonal Storage Scheme for more details.

lval — INTEGER. Leading dimension of *val*, *lval*≥*m*. Refer to *lval* description in Diagonal Storage Scheme for more details.

idiag — INTEGER. Array of length *ndiag*, contains the distances between main diagonal and each non-zero diagonals in the matrix *A*.

NOTE All elements of this array must be sorted in increasing order.

Refer to *distance* array description in Diagonal Storage Scheme for more details.

ndiag — INTEGER. Specifies the number of non-zero diagonals of the matrix *A*.

b — REAL for mkl_sdiasm.
DOUBLE PRECISION for mkl_ddiasm.
COMPLEX for mkl_cdiasm.
DOUBLE COMPLEX for mkl_zdiasm.
Array, DIMENSION (*ldb*, *n*).
On entry the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb — INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program.

ldc — INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program.

Output Parameters

c — REAL for mkl_sdiasm.
DOUBLE PRECISION for mkl_ddiasm.
COMPLEX for mkl_cdiasm.
DOUBLE COMPLEX for mkl_zdiasm.
Array, DIMENSION (*ldc*, *n*).
The leading *m*-by-*n* part of the array *c* contains the matrix *C*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdiasm(transa, m, n, alpha, matdescra, val, lval, idiag,
 ndiag, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  REAL          alpha
  REAL          val(lval,*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_ddiasm(transa, m, n, alpha, matdescra, val, lval, idiag,
 ndiag, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(lval,*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_cdiasm(transa, m, n, alpha, matdescra, val, lval, idiag,
 ndiag, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  COMPLEX        alpha
  COMPLEX        val(lval,*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_zdiasm(transa, m, n, alpha, matdescra, val, lval, idiag,
 ndiag, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc, lval, ndiag
  INTEGER      idiag(*)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(lval,*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_sdiasm(char *transa, int *m, int *n, float *alpha,
char *matdescra, float *val, int *lval, int *idiag, int *ndiag,
float *b, int *ldb, float *c, int *ldc);
```

```
void mkl_ddiasm(char *transa, int *m, int *n, double *alpha,
char *matdescra, double *val, int *lval, int *idiag, int *ndiag,
double *b, int *ldb, double *c, int *ldc);
void mkl_cdiasm(char *transa, int *m, int *n, MKL_Complex8 *alpha,
char *matdescra, MKL_Complex8 *val, int *lval, int *idiag, int *ndiag,
MKL_Complex8 *b, int *ldb, MKL_Complex8 *c, int *ldc);
void mkl_zdiasm(char *transa, int *m, int *n, MKL_Complex16 *alpha,
char *matdescra, MKL_Complex16 *val, int *lval, int *idiag, int *ndiag,
MKL_Complex16 *b, int *ldb, MKL_Complex16 *c, int *ldc);
```

mkl_?skysm

Solves a system of linear matrix equations for a sparse matrix stored using the skyline storage scheme with one-based indexing.

Syntax

Fortran:

```
call mkl_sskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
call mkl_dskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
call mkl_cskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
call mkl_zskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
```

C:

```
mkl_sskysm(&transa, &m, &n, &alpha, matdescra, val, pntr, b, &ldb, c, &ldc);
mkl_dskysm(&transa, &m, &n, &alpha, matdescra, val, pntr, b, &ldb, c, &ldc);
mkl_cskysm(&transa, &m, &n, &alpha, matdescra, val, pntr, b, &ldb, c, &ldc);
mkl_zskysm(&transa, &m, &n, &alpha, matdescra, val, pntr, b, &ldb, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?skysm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the skyline storage format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(A')*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A'* is the transpose of *A*.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

transa	CHARACTER*1. Specifies the system of linear equations. If *transa* = 'N' or 'n', then *C* := *alpha**inv(*A*)**B*, If *transa* = 'T' or 't' or 'C' or 'c', then *C* := *alpha**inv(*A*')**B*,
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *C*.
alpha	REAL for mkl_sskysm. DOUBLE PRECISION for mkl_dskysm. COMPLEX for mkl_cskysm. DOUBLE COMPLEX for mkl_zskysm. Specifies the scalar *alpha*.
matdescra	CHARACTER. Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter *matdescra* (*descra*)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter *matdescra*". **NOTE** General matrices (*matdescra* (1)='G') is not supported.
val	REAL for mkl_sskysm. DOUBLE PRECISION for mkl_dskysm. COMPLEX for mkl_cskysm. DOUBLE COMPLEX for mkl_zskysm. Array containing the set of elements of the matrix *A* in the skyline profile form. If *matdescrsa*(2)= 'L', then *val* contains elements from the low triangle of the matrix *A*. If *matdescrsa*(2)= 'U', then *val* contains elements from the upper triangle of the matrix *A*. Refer to *values* array description in Skyline Storage Scheme for more details.
pntr	INTEGER. Array of length (*m*+*m*). It contains the indices specifying in the *val* the positions of the first non-zero element of each *i*-row (column) of the matrix *A* such that *pointers*(*i*)- *pointers*(1)+1. Refer to *pointers* array description in Skyline Storage Scheme for more details.
b	REAL for mkl_sskysm. DOUBLE PRECISION for mkl_dskysm. COMPLEX for mkl_cskysm. DOUBLE COMPLEX for mkl_zskysm. Array, DIMENSION (*ldb*, *n*). On entry the leading *m*-by-*n* part of the array *b* must contain the matrix *B*.

ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program.

Output Parameters

c	REAL for mkl_sskysm. DOUBLE PRECISION for mkl_dskysm. COMPLEX for mkl_cskysm. DOUBLE COMPLEX for mkl_zskysm. Array, DIMENSION (*ldc*, *n*). The leading *m*-by-*n* part of the array *c* contains the matrix *C*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       pntr(*)
  REAL          alpha
  REAL          val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_dskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       pntr(*)
  DOUBLE PRECISION        alpha
  DOUBLE PRECISION        val(*), b(ldb,*), c(ldc,*)

SUBROUTINE mkl_cskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       pntr(*)
  COMPLEX        alpha
  COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_zskysm(transa, m, n, alpha, matdescra, val, pntr, b, ldb, c, ldc)
  CHARACTER*1   transa
  CHARACTER     matdescra(*)
  INTEGER       m, n, ldb, ldc
  INTEGER       pntr(*)
  DOUBLE COMPLEX        alpha
  DOUBLE COMPLEX        val(*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_sskysm(char *transa, int *m, int *n, float *alpha, char *matdescra,
float *val, int *pntr, float *b, int *ldb, float *c, int *ldc);

void mkl_dskysm(char *transa, int *m, int *n, double *alpha, char *matdescra,
double *val, int *pntr, double *b, int *ldb, double *c, int *ldc);

void mkl_cskysm(char *transa, int *m, int *n, MKL_Complex8 *alpha, char *matdescra,
MKL_Complex8 *val, int *pntr, MKL_Complex8 *b, int *ldb, MKL_Complex8 *c, int *ldc);

void mkl_zskysm(char *transa, int *m, int *n, MKL_Complex16 *alpha, char *matdescra,
MKL_Complex16 *val, int *pntr, MKL_Complex16 *b, int *ldb, MKL_Complex16 *c, int *ldc);
```

mkl_?dnscsr

Convert a sparse matrix in dense representation to the CSR format and vice versa.

Syntax

Fortran:

```
call mkl_sdnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
call mkl_ddnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
call mkl_cdnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
call mkl_zdnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
```

C:

```
mkl_sdnscsr(job, &m, &n, adns, &lda, acsr, ja, ia, &info);
mkl_ddnscsr(job, &m, &n, adns, &lda, acsr, ja, ia, &info);
mkl_cdnscsr(job, &m, &n, adns, &lda, acsr, ja, ia, &info);
mkl_zdnscsr(job, &m, &n, adns, &lda, acsr, ja, ia, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

This routine converts an sparse matrix stored as a rectangular m-by-n matrix A (dense representation) to the compressed sparse row (CSR) format (3-array variation) and vice versa.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

job — INTEGER
Array, contains the following conversion parameters:
job(1)
If *job*(1)=0, the rectangular matrix *A* is converted to the CSR format;
if *job*(1)=1, the rectangular matrix *A* is restored from the CSR format.
job(2)
If *job*(2)=0, zero-based indexing for the rectangular matrix *A* is used;
if *job*(2)=1, one-based indexing for the rectangular matrix *A* is used.
job(3)
If *job*(3)=0, zero-based indexing for the matrix in CSR format is used;
if *job*(3)=1, one-based indexing for the matrix in CSR format is used.
job(4)
If *job*(4)=0, *adns* is a lower triangular part of matrix *A*;
If *job*(4)=1, *adns* is an upper triangular part of matrix *A*;
If *job*(4)=2, *adns* is a whole matrix *A*.
job(5)
job(5)=*nzmax* - maximum number of the non-zero elements allowed if *job*(1)=0.
job(6) - job indicator for conversion to CSR format.
If *job*(6)=0, only array *ia* is generated for the output storage.
If *job*(6)>0, arrays *acsr*, *ia*, *ja* are generated for the output storage.

m — INTEGER. Number of rows of the matrix *A*.

n — INTEGER. Number of columns of the matrix *A*.

adns — (input/output)
REAL for mkl_sdnscsr.
DOUBLE PRECISION for mkl_ddnscsr.
COMPLEX for mkl_cdnscsr.
DOUBLE COMPLEX for mkl_zdnscsr.
Array containing non-zero elements of the matrix *A*.

lda — (input/output)INTEGER. Specifies the leading dimension of *adns* as declared in the calling (sub)program, must be at least max(1, *m*).

acsr — (input/output)
REAL for mkl_sdnscsr.
DOUBLE PRECISION for mkl_ddnscsr.
COMPLEX for mkl_cdnscsr.
DOUBLE COMPLEX for mkl_zdnscsr.
Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

ja — (input/output)INTEGER. Array containing the column indices for each non-zero element of the matrix *A*.
Its length is equal to the length of the array *acsr*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ia — (input/output)INTEGER. Array of length *m* + 1, containing indices of elements in the array *acsr*, such that *ia*(*I*) is the index in the array *acsr* of the first non-zero element from the row *I*. The value of the last element

ia(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

Output Parameters

info INTEGER. Integer info indicator only for restoring the matrix *A* from the CSR format.
If *info*=0, the execution is successful.
If *info*=*i*, the routine is interrupted processing the *i*-th row because there is no space in the arrays *adns* and *ja* according to the value *nzmax*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_sdnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
  INTEGER       job(8)
  INTEGER       m, n, lda, info
  INTEGER      ja(*), ia(m+1)
  REAL         adns(*), acsr(*)
SUBROUTINE mkl_ddnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
  INTEGER       job(8)
  INTEGER       m, n, lda, info
  INTEGER      ja(*), ia(m+1)
  DOUBLE PRECISION       adns(*), acsr(*)
SUBROUTINE mkl_cdnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
  INTEGER       job(8)
  INTEGER       m, n, lda, info
  INTEGER      ja(*), ia(m+1)
  COMPLEX      adns(*), acsr(*)
SUBROUTINE mkl_zdnscsr(job, m, n, adns, lda, acsr, ja, ia, info)
  INTEGER       job(8)
  INTEGER       m, n, lda, info
  INTEGER      ja(*), ia(m+1)
  DOUBLE COMPLEX      adns(*), acsr(*)
```

C:

```
void mkl_sdnscsr(int *job, int *m, int *n, float *adns,
int *lda, float *acsr, int *ja, int *ia, int *info);
void mkl_ddnscsr(int *job, int *m, int *n, double *adns,
int *lda, double *acsr, int *ja, int *ia, int *info);
void mkl_cdnscsr(int *job, int *m, int *n, MKL_Complex8 *adns,
int *lda, MKL_Complex8 *acsr, int *ja, int *ia, int *info);
```

```
void mkl_zdnscsr(int *job, int *m, int *n, MKL_Complex16 *adns,
int *lda, MKL_Complex16 *acsr, int *ja, int *ia, int *info);
```

mkl_?csrcoo

Converts a sparse matrix in the CSR format to the coordinate format and vice versa.

Syntax

Fortran:

```
call mkl_scsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
call mkl_dcsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
call mkl_ccsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
call mkl_zcsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
```

C:

```
mkl_scsrcoo(job, &n, acsr, ja, ia, &nnz, acoo, rowind, colind, &info);
mkl_dcsrcoo(job, &n, acsr, ja, ia, &nnz, acoo, rowind, colind, &info);
mkl_ccsrcoo(job, &n, acsr, ja, ia, &nnz, acoo, rowind, colind, &info);
mkl_zcsrcoo(job, &n, acsr, ja, ia, &nnz, acoo, rowind, colind, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

This routine converts a sparse matrix `A` stored in the compressed sparse row (CSR) format (3-array variation) to coordinate format and vice versa.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

job — INTEGER

Array, contains the following conversion parameters:

job(1)

If *job*(1)=0, the matrix in the CSR format is converted to the coordinate format;

if *job*(1)=1, the matrix in the coordinate format is converted to the CSR format.

if *job*(1)=2, the matrix in the coordinate format is converted to the CSR format, and the column indices in CSR representation are sorted in the increasing order within each row.

job(2)

If *job*(2)=0, zero-based indexing for the matrix in CSR format is used;

if *job*(2)=1, one-based indexing for the matrix in CSR format is used.

job(3)

	If *job*(3)=0, zero-based indexing for the matrix in coordinate format is used; if *job*(3)=1, one-based indexing for the matrix in coordinate format is used. *job*(5) *job*(5)=*nzmax* - maximum number of the non-zero elements allowed if *job*(1)=0. *job*(5)=*nnz* - sets number of the non-zero elements of the matrix *A* if *job*(1)=1. *job*(6) - job indicator. For conversion to the coordinate format: If *job*(6)=1, only array *rowind* is filled in for the output storage. If *job*(6)=2, arrays *rowind*, *colind* are filled in for the output storage. If *job*(6)=3, all arrays *rowind*, *colind*, *acoo* are filled in for the output storage. For conversion to the CSR format: If *job*(6)=0, all arrays *acsr*, *ja*, *ia* are filled in for the output storage. If *job*(6)=1, only array *ia* is filled in for the output storage. If *job*(6)=2, then it is assumed that the routine already has been called with the *job*(6)=1, and the user allocated the required space for storing the output arrays *acsr* and *ja*.
n	INTEGER. Dimension of the matrix *A*.
acsr	(input/output) REAL for mkl_scsrcoo. DOUBLE PRECISION for mkl_dcsrcoo. COMPLEX for mkl_ccsrcoo. DOUBLE COMPLEX for mkl_zcsrcoo. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	(input/output) INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *acsr*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia	(input/output) INTEGER. Array of length *n* + 1, containing indices of elements in the array *acsr*, such that *ia*(*I*) is the index in the array *acsr* of the first non-zero element from the row *I*. The value of the last element *ia*(*n* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
acoo	(input/output) REAL for mkl_scsrcoo. DOUBLE PRECISION for mkl_dcsrcoo. COMPLEX for mkl_ccsrcoo. DOUBLE COMPLEX for mkl_zcsrcoo. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
rowind	(input/output)INTEGER. Array of length *nnz*, contains the row indices for each non-zero element of the matrix *A*. Refer to *rows* array description in Coordinate Format for more details.

colind (input/output) INTEGER. Array of length *nnz*, contains the column indices for each non-zero element of the matrix *A*. Refer to *columns* array description in Coordinate Format for more details.

Output Parameters

nnz INTEGER. Specifies the number of non-zero element of the matrix *A*. Refer to *nnz* description in Coordinate Format for more details.

info INTEGER. Integer info indicator only for converting the matrix *A* from the CSR format.
If *info*=0, the execution is successful.
If *info*=1, the routine is interrupted because there is no space in the arrays *acoo*, *rowind*, *colind* according to the value *nzmax*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
  INTEGER      job(8)
  INTEGER      n, nnz, info
  INTEGER      ja(*), ia(n+1), rowind(*), colind(*)
  REAL         acsr(*), acoo(*)

SUBROUTINE mkl_dcsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
  INTEGER      job(8)
  INTEGER      n, nnz, info
  INTEGER      ja(*), ia(n+1), rowind(*), colind(*)
  DOUBLE PRECISION       acsr(*), acoo(*)

SUBROUTINE mkl_ccsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
  INTEGER      job(8)
  INTEGER      n, nnz, info
  INTEGER      ja(*), ia(n+1), rowind(*), colind(*)
  COMPLEX      acsr(*), acoo(*)

SUBROUTINE mkl_zcsrcoo(job, n, acsr, ja, ia, nnz, acoo, rowind, colind, info)
  INTEGER      job(8)
  INTEGER      n, nnz, info
  INTEGER      ja(*), ia(n+1), rowind(*), colind(*)
  DOUBLE COMPLEX      acsr(*), acoo(*)
```

C:

```
void mkl_scsrcoo(int *job, int *n, float *acsr, int *ja,
int *ia, int *nnz, float *acoo, int *rowind, int *colind, int *info);

void mkl_dcsrcoo(int *job, int *n, double *acsr, int *ja,
int *ia, int *nnz, double *acoo, int *rowind, int *colind, int *info);
```

```
void mkl_ccsrcoo(int *job, int *n, MKL_Complex8 *acsr, int *ja,
int *ia, int *nnz, MKL_Complex8 *acoo, int *rowind, int *colind, int *info);
void mkl_zcsrcoo(int *job, int *n, MKL_Complex16 *acsr, int *ja,
int *ia, int *nnz, MKL_Complex16 *acoo, int *rowind, int *colind, int *info);
```

mkl_?csrbsr

Converts a sparse matrix in the CSR format to the BSR format and vice versa.

Syntax

Fortran:

```
call mkl_scsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
call mkl_dcsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
call mkl_ccsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
call mkl_zcsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
```

C:

```
mkl_scsrbsr(job, &m, &mblk, &ldabsr, acsr, ja, ia, absr, jab, iab, &info);
mkl_dcsrbsr(job, &m, &mblk, &ldabsr, acsr, ja, ia, absr, jab, iab, &info);
mkl_ccsrbsr(job, &m, &mblk, &ldabsr, acsr, ja, ia, absr, jab, iab, &info);
mkl_zcsrbsr(job, &m, &mblk, &ldabsr, acsr, ja, ia, absr, jab, iab, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

This routine converts a sparse matrix `A` stored in the compressed sparse row (CSR) format (3-array variation) to the block sparse row (BSR) format and vice versa.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

job	INTEGER Array, contains the following conversion parameters: *job*(1) If *job*(1)=0, the matrix in the CSR format is converted to the BSR format; if *job*(1)=1, the matrix in the BSR format is converted to the CSR format. *job*(2) If *job*(2)=0, zero-based indexing for the matrix in CSR format is used; if *job*(2)=1, one-based indexing for the matrix in CSR format is used. *job*(3) If *job*(3)=0, zero-based indexing for the matrix in the BSR format is used; if *job*(3)=1, one-based indexing for the matrix in the BSR format is used.

	job(4) is only used for conversion to CSR format. By default, the converter saves the blocks without checking whether an element is zero or not. If *job*(4)=1, then the converter only saves non-zero elements in blocks. *job*(6) - job indicator. For conversion to the BSR format: If *job*(6)=0, only arrays *jab*, *iab* are generated for the output storage. If *job*(6)>0, all output arrays *absr*, *jab*, and *iab* are filled in for the output storage. If *job*(6)=-1, *iab*(1) returns the number of non-zero blocks. For conversion to the CSR format: If *job*(6)=0, only arrays *ja*, *ia* are generated for the output storage.
m	INTEGER. Actual row dimension of the matrix *A* for convert to the BSR format; block row dimension of the matrix *A* for convert to the CSR format.
mblk	INTEGER. Size of the block in the matrix *A*.
ldabsr	INTEGER. Leading dimension of the array *absr* as declared in the calling program. *ldabsr* must be greater than or equal to *mblk*mblk*.
acsr	(input/output) REAL for mkl_scsrbsr. DOUBLE PRECISION for mkl_dcsrbsr. COMPLEX for mkl_ccsrbsr. DOUBLE COMPLEX for mkl_zcsrbsr. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	(input/output) INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *acsr*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia	(input/output) INTEGER. Array of length *m* + 1, containing indices of elements in the array *acsr*, such that *ia*(*I*) is the index in the array *acsr* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
absr	(input/output) REAL for mkl_scsrbsr. DOUBLE PRECISION for mkl_dcsrbsr. COMPLEX for mkl_ccsrbsr. DOUBLE COMPLEX for mkl_zcsrbsr. Array containing elements of non-zero blocks of the matrix *A*. Its length is equal to the number of non-zero blocks in the matrix *A* multiplied by *mblk*mblk*. Refer to *values* array description in BSR Format for more details.
jab	(input/output) INTEGER. Array containing the column indices for each non-zero block of the matrix *A*. Its length is equal to the number of non-zero blocks of the matrix *A*. Refer to *columns* array description in BSR Format for more details.

iab	(input/output) INTEGER. Array of length (*m* + 1), containing indices of blocks in the array *absr*, such that *iab*(*i*) is the index in the array *absr* of the first non-zero element from the *i*-th row . The value of the last element *iab*(*m* + 1) is equal to the number of non-zero blocks plus one. Refer to *rowIndex* array description in BSR Format for more details.

Output Parameters

info	INTEGER. Integer info indicator only for converting the matrix *A* from the CSR format. If *info*=0, the execution is successful. If *info*=1, it means that *mblk* is equal to 0. If *info*=2, it means that *ldabsr* is less than *mblk***mblk* and there is no space for all blocks.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
  INTEGER      job(8)
  INTEGER      m, mblk, ldabsr, info
  INTEGER      ja(*), ia(m+1), jab(*), iab(*)
  REAL         acsr(*), absr(ldabsr,*)

SUBROUTINE mkl_dcsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
  INTEGER      job(8)
  INTEGER      m, mblk, ldabsr, info
  INTEGER      ja(*), ia(m+1), jab(*), iab(*)
  DOUBLE PRECISION       acsr(*), absr(ldabsr,*)

SUBROUTINE mkl_ccsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
  INTEGER      job(8)
  INTEGER      m, mblk, ldabsr, info
  INTEGER      ja(*), ia(m+1), jab(*), iab(*)
  COMPLEX       acsr(*), absr(ldabsr,*)

SUBROUTINE mkl_zcsrbsr(job, m, mblk, ldabsr, acsr, ja, ia, absr, jab, iab, info)
  INTEGER      job(8)
  INTEGER      m, mblk, ldabsr, info
  INTEGER      ja(*), ia(m+1), jab(*), iab(*)
  DOUBLE COMPLEX       acsr(*), absr(ldabsr,*)
```

C:

```
void mkl_scsrbsr(int *job, int *m, int *mblk, int *ldabsr, float *acsr, int *ja,
int *ia, float *absr, int *jab, int *iab, int *info);

void mkl_dcsrbsr(int *job, int *m, int *mblk, int *ldabsr, double *acsr, int *ja,
int *ia, double *absr, int *jab, int *iab, int *info);
```

```
void mkl_ccsrbsr(int *job, int *m, int *mblk, int *ldabsr, MKL_Complex8 *acsr, int *ja,
int *ia, MKL_Complex8 *absr, int *jab, int *iab, int *info);
void mkl_zcsrbsr(int *job, int *m, int *mblk, int *ldabsr, MKL_Complex16 *acsr, int *ja,
int *ia, MKL_Complex16 *absr, int *jab, int *iab, int *info);
```

mkl_?csrcsc

Converts a square sparse matrix in the CSR format to the CSC format and vice versa.

Syntax

Fortran:

```
call mkl_scsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
call mkl_dcsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
call mkl_ccsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
call mkl_zcsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
```

C:

```
mkl_scsrcsc(job, &m, acsr, ja, ia, acsc, ja1, ia1, &info);
mkl_dcsrcsc(job, &m, acsr, ja, ia, acsc, ja1, ia1, &info);
mkl_ccsrcsc(job, &m, acsr, ja, ia, acsc, ja1, ia1, &info);
mkl_zcsrcsc(job, &m, acsr, ja, ia, acsc, ja1, ia1, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

This routine converts a square sparse matrix A stored in the compressed sparse row (CSR) format (3-array variation) to the compressed sparse column (CSC) format and vice versa.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

job — INTEGER

Array, contains the following conversion parameters:

job(1)

If *job*(1)=0, the matrix in the CSR format is converted to the CSC format; if *job*(1)=1, the matrix in the CSC format is converted to the CSR format.

job(2)

If *job*(2)=0, zero-based indexing for the matrix in CSR format is used; if *job*(2)=1, one-based indexing for the matrix in CSR format is used.

job(3)

If *job*(3)=0, zero-based indexing for the matrix in the CSC format is used; if *job*(3)=1, one-based indexing for the matrix in the CSC format is used.

job(6) - job indicator.

	For conversion to the CSC format: If *job*(6)=0, only arrays *ja1*, *ia1* are filled in for the output storage. If *job*(6)≠0, all output arrays *acsc*, *ja1*, and *ia1* are filled in for the output storage. For conversion to the CSR format: If *job*(6)=0, only arrays *ja*, *ia* are filled in for the output storage. If *job*(6)≠0, all output arrays *acsr*, *ja*, and *ia* are filled in for the output storage.
m	INTEGER. Dimension of the square matrix *A*.
acsr	(input/output) REAL for mkl_scsrcsc. DOUBLE PRECISION for mkl_dcsrcsc. COMPLEX for mkl_ccsrcsc. DOUBLE COMPLEX for mkl_zcsrcsc. Array containing non-zero elements of the square matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	(input/output) INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *acsr*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia	(input/output) INTEGER. Array of length *m* + 1, containing indices of elements in the array *acsr*, such that *ia*(*I*) is the index in the array *acsr* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
acsc	(input/output) REAL for mkl_scsrcsc. DOUBLE PRECISION for mkl_dcsrcsc. COMPLEX for mkl_ccsrcsc. DOUBLE COMPLEX for mkl_zcsrcsc. Array containing non-zero elements of the square matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja1	(input/output) INTEGER. Array containing the row indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *acsc*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia1	(input/output) INTEGER. Array of length *m* + 1, containing indices of elements in the array *acsc*, such that *ia1*(*I*) is the index in the array *acsc* of the first non-zero element from the column *I*. The value of the last element *ia1*(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

Output Parameters

info	INTEGER. This parameter is not used now.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), ja1(*), ia1(m+1)
  REAL         acsr(*), acsc(*)

SUBROUTINE mkl_dcsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), ja1(*), ia1(m+1)
  DOUBLE PRECISION       acsr(*), acsc(*)

SUBROUTINE mkl_ccsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), ja1(*), ia1(m+1)
  COMPLEX      acsr(*), acsc(*)

SUBROUTINE mkl_zcsrcsc(job, m, acsr, ja, ia, acsc, ja1, ia1, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), ja1(*), ia1(m+1)
  DOUBLE COMPLEX        acsr(*), acsc(*)
```

C:

```
void mkl_scsrcsc(int *job, int *m, float *acsr, int *ja,
int *ia, float *acsc, int *ja1, int *ia1, int *info);

void mkl_dcsrcsc(int *job, int *m, double *acsr, int *ja,
int *ia, double *acsc, int *ja1, int *ia1, int *info);

void mkl_ccsrcsc(int *job, int *m, MKL_Complex8 *acsr, int *ja,
int *ia, MKL_Complex8 *acsc, int *ja1, int *ia1, int *info);

void mkl_zcsrcsc(int *job, int *m, MKL_Complex16 *acsr, int *ja,
int *ia, MKL_Complex16 *acsc, int *ja1, int *ia1, int *info);
```

mkl_?csrdia

Converts a sparse matrix in the CSR format to the diagonal format and vice versa.

Syntax

Fortran:

```
call mkl_scsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem, info)
call mkl_dcsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem, info)
call mkl_ccsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem, info)
call mkl_zcsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem, info)
```

C:

```
mkl_scsrdia(job, &m, acsr, ja, ia, adia, &ngiag, distance, &idiag, acsr_rem, ja_rem, ia_rem, &info);
mkl_dcsrdia(job, &m, acsr, ja, ia, adia, &ngiag, distance, &idiag, acsr_rem, ja_rem, ia_rem, &info);
mkl_ccsrdia(job, &m, acsr, ja, ia, adia, &ngiag, distance, &idiag, acsr_rem, ja_rem, ia_rem, &info);
mkl_zcsrdia(job, &m, acsr, ja, ia, adia, &ngiag, distance, &idiag, acsr_rem, ja_rem, ia_rem, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

This routine converts a sparse matrix `A` stored in the compressed sparse row (CSR) format (3-array variation) to the diagonal format and vice versa.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

job — INTEGER

Array, contains the following conversion parameters:

job(1)

If *job*(1)=0, the matrix in the CSR format is converted to the diagonal format;

if *job*(1)=1, the matrix in the diagonal format is converted to the CSR format.

job(2)

If *job*(2)=0, zero-based indexing for the matrix in CSR format is used;

if *job*(2)=1, one-based indexing for the matrix in CSR format is used.

job(3)

If *job*(3)=0, zero-based indexing for the matrix in the diagonal format is used;

if *job*(3)=1, one-based indexing for the matrix in the diagonal format is used.

	job(6) - job indicator. For conversion to the diagonal format: If *job*(6)=0, diagonals are not selected internally, and *acsr_rem*, *ja_rem*, *ia_rem* are not filled in for the output storage. If *job*(6)=1, diagonals are not selected internally, and *acsr_rem*, *ja_rem*, *ia_rem* are filled in for the output storage. If *job*(6)=10, diagonals are selected internally, and *acsr_rem*, *ja_rem*, *ia_rem* are not filled in for the output storage. If *job*(6)=11, diagonals are selected internally, and *csr_rem*, *ja_rem*, *ia_rem* are filled in for the output storage. For conversion to the CSR format: If *job*(6)=0, each entry in the array *adia* is checked whether it is zero. Zero entries are not included in the array *acsr*. If *job*(6)≠0, each entry in the array *adia* is not checked whether it is zero.
m	INTEGER. Dimension of the matrix *A*.
acsr	(input/output) REAL for mkl_scsrdia. DOUBLE PRECISION for mkl_dcsrdia. COMPLEX for mkl_ccsrdia. DOUBLE COMPLEX for mkl_zcsrdia. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	(input/output) INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *acsr*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia	(input/output) INTEGER. Array of length *m* + 1, containing indices of elements in the array *acsr*, such that *ia*(*I*) is the index in the array *acsr* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
adia	(input/output) REAL for mkl_scsrdia. DOUBLE PRECISION for mkl_dcsrdia. COMPLEX for mkl_ccsrdia. DOUBLE COMPLEX for mkl_zcsrdia. Array of size (*ndiag* x *idiag*) containing diagonals of the matrix *A*. The key point of the storage is that each element in the array *adia* retains the row number of the original matrix. To achieve this diagonals in the lower triangular part of the matrix are padded from the top, and those in the upper triangular part are padded from the bottom.
ndiag	INTEGER. Specifies the leading dimension of the array *adia* as declared in the calling (sub)program, must be at least max(1, *m*).
distance	INTEGER. Array of length *idiag*, containing the distances between the main diagonal and each non-zero diagonal to be extracted. The distance is positive if the diagonal is above the main diagonal, and negative if the diagonal is below the main diagonal. The main diagonal has a distance equal to zero.

idiag	INTEGER. Number of diagonals to be extracted. For conversion to diagonal format on return this parameter may be modified.
acsr_rem, *ja_rem*, *ia_rem*	Remainder of the matrix in the CSR format if it is needed for conversion to the diagonal format.

Output Parameters

info	INTEGER. This parameter is not used now.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem,
info)

  INTEGER      job(8)

  INTEGER      m, info, ndiag, idiag

  INTEGER      ja(*), ia(m+1), distance(*), ja_rem(*), ia_rem(*)

  REAL         acsr(*), adia(*), acsr_rem(*)

SUBROUTINE mkl_dcsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem,
info)

  INTEGER      job(8)

  INTEGER      m, info, ndiag, idiag

  INTEGER      ja(*), ia(m+1), distance(*), ja_rem(*), ia_rem(*)

  DOUBLE PRECISION       acsr(*), adia(*), acsr_rem(*)

SUBROUTINE mkl_ccsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem,
info)

  INTEGER      job(8)

  INTEGER      m, info, ndiag, idiag

  INTEGER      ja(*), ia(m+1), distance(*), ja_rem(*), ia_rem(*)

  COMPLEX      acsr(*), adia(*), acsr_rem(*)

SUBROUTINE mkl_zcsrdia(job, m, acsr, ja, ia, adia, ndiag, distance, idiag, acsr_rem, ja_rem, ia_rem,
info)

  INTEGER      job(8)

  INTEGER      m, info, ndiag, idiag

  INTEGER      ja(*), ia(m+1), distance(*), ja_rem(*), ia_rem(*)

  DOUBLE COMPLEX       acsr(*), adia(*), acsr_rem(*)
```

C:

```
void mkl_scsrdia(int *job, int *m, float *acsr, int *ja,

int *ia, float *adia, int *ndiag, int *distance, int *distance,
int *idiag, float *acsr_rem, int *ja_rem, int *ia_rem, int *info);

void mkl_dcsrdia(int *job, int *m, double *acsr, int *ja,

int *ia, double *adia, int *ndiag, int *distance, int *distance,
int *idiag, double *acsr_rem, int *ja_rem, int *ia_rem, int *info);
```

```
void mkl_ccsrdia(int *job, int *m, MKL_Complex8 *acsr, int *ja,
int *ia, MKL_Complex8 *adia, int *ndiag, int *distance, int *distance,
int *idiag, MKL_Complex8 *acsr_rem, int *ja_rem, int *ia_rem, int *info);

void mkl_zcsrdia(int *job, int *m, MKL_Complex16 *acsr, int *ja,
int *ia, MKL_Complex16 *adia, int *ndiag, int *distance, int *distance,
int *idiag, MKL_Complex16 *acsr_rem, int *ja_rem, int *ia_rem, int *info);
```

mkl_?csrsky

Converts a sparse matrix in CSR format to the skyline format and vice versa.

Syntax

Fortran:

```
call mkl_scsrsky(job, m, acsr, ja, ia, asky, pointers, info)
call mkl_dcsrsky(job, m, acsr, ja, ia, asky, pointers, info)
call mkl_ccsrsky(job, m, acsr, ja, ia, asky, pointers, info)
call mkl_zcsrsky(job, m, acsr, ja, ia, asky, pointers, info)
```

C:

```
mkl_scsrsky(job, &m, acsr, ja, ia, asky, pointers, &info);
mkl_dcsrsky(job, &m, acsr, ja, ia, asky, pointers, &info);
mkl_ccsrsky(job, &m, acsr, ja, ia, asky, pointers, &info);
mkl_zcsrsky(job, &m, acsr, ja, ia, asky, pointers, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

This routine converts a sparse matrix `A` stored in the compressed sparse row (CSR) format (3-array variation) to the skyline format and vice versa.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

job — INTEGER

Array, contains the following conversion parameters:

job(1)

If *job*(1)=0, the matrix in the CSR format is converted to the skyline format;

if *job*(1)=1, the matrix in the skyline format is converted to the CSR format.

job(2)

If *job*(2)=0, zero-based indexing for the matrix in CSR format is used;

if *job*(2)=1, one-based indexing for the matrix in CSR format is used.

	job(3) If *job*(3)=0, zero-based indexing for the matrix in the skyline format is used; if *job*(3)=1, one-based indexing for the matrix in the skyline format is used. *job*(4) For conversion to the skyline format: If *job*(4)=0, the upper part of the matrix *A* in the CSR format is converted. If *job*(4)=1, the lower part of the matrix *A* in the CSR format is converted. For conversion to the CSR format: If *job*(4)=0, the matrix is converted to the upper part of the matrix *A* in the CSR format. If *job*(4)=1, the matrix is converted to the lower part of the matrix *A* in the CSR format. *job*(5) *job*(5)=*nzmax* - maximum number od the non-zero elements of the matrix *A* if *job*(1)=0. *job*(6) - job indicator. Only for conversion to the skyline format: If *job*(6)=0, only arrays *pointers* is filled in for the output storage. If *job*(6)=1, all output arrays *asky* and *pointers* are filled in for the output storage.
m	INTEGER. Dimension of the matrix *A*.
acsr	(input/output) REAL for mkl_scsrsky. DOUBLE PRECISION for mkl_dcsrsky. COMPLEX for mkl_ccsrsky. DOUBLE COMPLEX for mkl_zcsrsky. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	(input/output) INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to the length of the array *acsr*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia	(input/output) INTEGER. Array of length *m* + 1, containing indices of elements in the array *acsr*, such that *ia*(*I*) is the index in the array *acsr* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zeros plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
asky	(input/output) REAL for mkl_scsrsky. DOUBLE PRECISION for mkl_dcsrsky. COMPLEX for mkl_ccsrsky. DOUBLE COMPLEX for mkl_zcsrsky. Array, for a lower triangular part of *A* it contains the set of elements from each row starting from the first none-zero element to and including the diagonal element. For an upper triangular matrix it contains the set of elements from each column of the matrix starting with the first non-zero

element down to and including the diagonal element. Encountered zero elements are included in the sets. Refer to *values* array description in Skyline Storage Format for more details.

pointers (input/output) INTEGER.
Array with dimension (*m*+1), where *m* is number of rows for lower triangle (columns for upper triangle), *pointers*(*I*) - *pointers*(1)+1 gives the index of element in the array *asky* that is first non-zero element in row (column)*I* . The value of *pointers*(*m* +1) is set to*nnz* + *pointers*(1), where*nnz* is the number of elements in the array *asky*. Refer to *pointers* array description in Skyline Storage Format for more details

Output Parameters

info INTEGER. Integer info indicator only for converting the matrix *A* from the CSR format.
If *info*=0, the execution is successful.
If *info*=1, the routine is interrupted because there is no space in the array *asky* according to the value *nzmax*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrsky(job, m, acsr, ja, ia, asky, pointers, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), pointers(m+1)
  REAL         acsr(*), asky(*)

SUBROUTINE mkl_dcsrsky(job, m, acsr, ja, ia, asky, pointers, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), pointers(m+1)
  DOUBLE PRECISION       acsr(*), asky(*)

SUBROUTINE mkl_ccsrsky(job, m, acsr, ja, ia, asky, pointers, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), pointers(m+1)
  COMPLEX      acsr(*), asky(*)

SUBROUTINE mkl_zcsrsky(job, m, acsr, ja, ia, asky, pointers, info)
  INTEGER      job(8)
  INTEGER      m, info
  INTEGER      ja(*), ia(m+1), pointers(m+1)
  DOUBLE COMPLEX       acsr(*), asky(*)
```

C:

```
void mkl_scsrsky(int *job, int *m, float *acsr, int *ja,
     int *ia, float *asky, int *pointers, int *info);
void mkl_dcsrsky(int *job, int *m, double *acsr, int *ja,
     int *ia, double *asky, int *pointers, int *info);
void mkl_ccsrsky(int *job, int *m, MKL_COMPLEX8 *acsr, int *ja,
     int *ia, MKL_COMPLEX8 *asky, int *pointers, int *info);
void mkl_zcsrsky(int *job, int *m, MKL_COMPLEX16 *acsr, int *ja,
     int *ia, MKL_COMPLEX16 *asky, int *pointers, int *info);
```

mkl_?csradd

Computes the sum of two matrices stored in the CSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_scsradd(trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic,
nzmax, info)
call mkl_dcsradd(trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic,
nzmax, info)
call mkl_ccsradd(trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic,
nzmax, info)
call mkl_zcsradd(trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic,
nzmax, info)
```

C:

```
mkl_scsradd(&trans, &request, &sort, &m, &n, a, ja, ia, &beta, b, jb, ib, c, jc, ic,
&nzmax, &info);
mkl_dcsradd(&trans, &request, &sort, &m, &n, a, ja, ia, &beta, b, jb, ib, c, jc, ic,
&nzmax, &info);
mkl_ccsradd(&trans, &request, &sort, &m, &n, a, ja, ia, &beta, b, jb, ib, c, jc, ic,
&nzmax, &info);
mkl_zcsradd(&trans, &request, &sort, &m, &n, a, ja, ia, &beta, b, jb, ib, c, jc, ic,
&nzmax, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csradd` routine performs a matrix-matrix operation defined as

```
C := A+beta*op(B)
```

where:

A, *B*, *C* are the sparse matrices in the CSR format (3-array variation).

op(*B*) is one of op(*B*) = *B*, or op(*B*) = *B*', or op(*A*) = conjg(*B*')

beta is a scalar.

The routine works correctly if and only if the column indices in sparse matrix representations of matrices *A* and *B* are arranged in the increasing order for each row. If not, use the parameter *sort* (see below) to reorder column indices and the corresponding elements of the input matrices.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

trans	CHARACTER*1. Specifies the operation. If *trans* = 'N' or 'n', then *C* := *A*+*beta***B* If *trans* = 'T' or 't' or 'C' or 'c', then *C* := *A*+*beta***B*'.
request	INTEGER. If *request*=0, the routine performs addition, the memory for the output arrays *ic*, *jc*, *c* must be allocated beforehand. If *request*=1, the routine computes only values of the array *ic* of length *m* + 1, the memory for this array must be allocated beforehand. On exit the value *ic*(*m*+1) – 1 is the actual number of the elements in the arrays *c* and *jc*. If *request*=2, the routine has been called previously with the parameter *request*=1, the output arrays *jc* and *c* are allocated in the calling program and they are of the length (*m*+1)–1 at least.
sort	INTEGER. Specifies the type of reordering. If this parameter is not set (default), the routine does not perform reordering. If *sort*=1, the routine arranges the column indices *ja* for each row in the increasing order and reorders the corresponding values of the matrix *A* in the array *a*. If *sort*=2, the routine arranges the column indices *jb* for each row in the increasing order and reorders the corresponding values of the matrix *B* in the array *b*. If *sort*=3, the routine performs reordering for both input matrices *A* and *B*.
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *A*.
a	REAL for mkl_scsradd. DOUBLE PRECISION for mkl_dcsradd. COMPLEX for mkl_ccsradd. DOUBLE COMPLEX for mkl_zcsradd. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. For each row the column indices must be arranged in the increasing order. The length of this array is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ia — INTEGER. Array of length *m* + 1, containing indices of elements in the array *a*, such that *ia*(*I*) is the index in the array *a* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zero elements of the matrix *B* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

beta — REAL for mkl_scsradd.
DOUBLE PRECISION for mkl_dcsradd.
COMPLEX for mkl_ccsradd.
DOUBLE COMPLEX for mkl_zcsradd.
Specifies the scalar *beta*.

b — REAL for mkl_scsradd.
DOUBLE PRECISION for mkl_dcsradd.
COMPLEX for mkl_ccsradd.
DOUBLE COMPLEX for mkl_zcsradd.
Array containing non-zero elements of the matrix *B*. Its length is equal to the number of non-zero elements in the matrix *B*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

jb — INTEGER. Array containing the column indices for each non-zero element of the matrix *B*. For each row the column indices must be arranged in the increasing order.
The length of this array is equal to the length of the array *b*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ib — INTEGER. Array of length *m* + 1 when *trans* = 'N' or 'n', or *n* + 1 otherwise.
This array contains indices of elements in the array *b*, such that *ib*(*I*) is the index in the array *b* of the first non-zero element from the row *I*. The value of the last element *ib*(*m* + 1) or *ib*(*n* + 1) is equal to the number of non-zero elements of the matrix *B* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

nzmax — INTEGER. The length of the arrays *c* and *jc*.
This parameter is used only if *request*=0. The routine stops calculation if the number of elements in the result matrix *C* exceeds the specified value of *nzmax*.

Output Parameters

c — REAL for mkl_scsradd.
DOUBLE PRECISION for mkl_dcsradd.
COMPLEX for mkl_ccsradd.
DOUBLE COMPLEX for mkl_zcsradd.
Array containing non-zero elements of the result matrix *C*. Its length is equal to the number of non-zero elements in the matrix *C*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

jc — INTEGER. Array containing the column indices for each non-zero element of the matrix *C*.
The length of this array is equal to the length of the array *c*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ic	INTEGER. Array of length *m* + 1, containing indices of elements in the array *c*, such that *ic*(*I*) is the index in the array *c* of the first non-zero element from the row *I*. The value of the last element *ic*(*m* + 1) is equal to the number of non-zero elements of the matrix *C* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
info	INTEGER. If *info*=0, the execution is successful. If *info*=*I*>0, the routine stops calculation in the *I*-th row of the matrix *C* because number of elements in *C* exceeds *nzmax*. If *info*=-1, the routine calculates only the size of the arrays *c* and *jc* and returns this value plus 1 as the last element of the array *ic*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsradd( trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER  trans

   INTEGER       request, sort, m, n, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   REAL            a(*), b(*), c(*), beta

SUBROUTINE mkl_dcsradd( trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER  trans

   INTEGER       request, sort, m, n, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   DOUBLE PRECISION         a(*), b(*), c(*), beta

SUBROUTINE mkl_ccsradd( trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER  trans

   INTEGER       request, sort, m, n, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   COMPLEX     a(*), b(*), c(*), beta

SUBROUTINE mkl_zcsradd( trans, request, sort, m, n, a, ja, ia, beta, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER  trans

   INTEGER       request, sort, m, n, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   DOUBLE COMPLEX       a(*), b(*), c(*), beta
```

C:

```
void mkl_scsradd(char *trans, int *request, int *sort, int *m, int *n, float *a, int *ja, int *ia,
float *beta, float *b, int *jb, int *ib, float *c,
int *jc, int *ic, int *nzmax, int *info);

void mkl_dcsradd(char *trans, int *request, int *sort, int *m, int *n,
double *a, int *ja, int *ia, double *beta, double *b, int *jb, int *ib, double *c,
int *jc, int *ic, int *nzmax, int *info);
```

```
void mkl_ccsradd(char *trans, int *request, int *sort, int *m, int *n,
 MKL_Complex8 *a, int *ja, int *ia,  MKL_Complex8 *beta, MKL_Complex8 *b,
int *jb, int *ib,  MKL_Complex8 *c, int *jc, int *ic, int *nzmax, int *info);
```

```
void mkl_zcsradd(char *trans, int *request, int *sort, int *m, int *n,
MKL_Complex16 *a, int *ja, int *ia, MKL_Complex16 *beta, MKL_Complex16 *b,
int *jb, int *ib, MKL_Complex16 *c, int *jc, int *ic, int *nzmax, int *info);
```

mkl_?csrmultcsr

Computes product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing.

Syntax

Fortran:

```
call mkl_scsrmultcsr(trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic,
nzmax, info)
```

```
call mkl_dcsrmultcsr(trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic,
nzmax, info)
```

```
call mkl_ccsrmultcsr(trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic,
nzmax, info)
```

```
call mkl_zcsrmultcsr(trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic,
nzmax, info)
```

C:

```
mkl_scsrmultcsr(&trans, &request, &sort, &m, &n, &k, a, ja, ia, b, jb, ib, c, jc, ic,
&nzmax, &info);
```

```
mkl_dcsrmultcsr(&trans, &request, &sort, &m, &n, &k, a, ja, ia, b, jb, ib, c, jc, ic,
&nzmax, &info);
```

```
mkl_ccsrmultcsr(&trans, &request, &sort, &m, &n, &k, a, ja, ia, b, jb, ib, c, jc, ic,
&nzmax, &info);
```

```
mkl_zcsrmultcsr(&trans, &request, &sort, &m, &n, &k, a, ja, ia, b, jb, ib, c, jc, ic,
&nzmax, &info);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrmultcsr` routine performs a matrix-matrix operation defined as

```
C := op(A)*B
```

where:

A, *B*, *C* are the sparse matrices in the CSR format (3-array variation);

`op(A)` is one of `op(A) = A`, or `op(A) =A'`, or `op(A) = conjg(A')` .

You can use the parameter *sort* to perform or not perform reordering of non-zero entries in input and output sparse matrices. The purpose of reordering is to rearrange non-zero entries in compressed sparse row matrix so that column indices in compressed sparse representation are sorted in the increasing order for each row.

The following table shows correspondence between the value of the parameter *sort* and the type of reordering performed by this routine for each sparse matrix involved:

Value of the parameter *sort*	Reordering of *A* (arrays *a*, *ja*, *ia*)	Reordering of *B* (arrays *b*, *ja*, *ib*)	Reordering of *C* (arrays *c*, *jc*, *ic*)
1	yes	no	yes
2	no	yes	yes
3	yes	yes	yes
4	yes	no	no
5	no	yes	no
6	yes	yes	no
7	no	no	no
arbitrary value not equal to 1, 2,..., 7	no	no	yes

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

trans — CHARACTER*1. Specifies the operation.
If *trans* = 'N' or 'n', then *C* := *A***B*
If *trans* = 'T' or 't' or 'C' or 'c', then *C* := *A*'**B*.

request — INTEGER.
If *request*=0, the routine performs multiplication, the memory for the output arrays *ic*, *jc*, *c* must be allocated beforehand.
If *request*=1, the routine computes only values of the array *ic* of length *m* + 1, the memory for this array must be allocated beforehand. On exit the value *ic*(*m*+1) – 1 is the actual number of the elements in the arrays *c* and *jc*.
If *request*=2, the routine has been called previously with the parameter *request*=1, the output arrays *jc* and *c* are allocated in the calling program and they are of the length *ic*(*m*+1) – 1 at least.

sort — INTEGER. Specifies whether the routine performs reordering of non-zeros entries in input and/or output sparse matrices (see table above).

m — INTEGER. Number of rows of the matrix *A*.

n — INTEGER. Number of columns of the matrix *A*.

k — INTEGER. Number of columns of the matrix *B*.

a — REAL for mkl_scsrmultcsr.
DOUBLE PRECISION for mkl_dcsrmultcsr.
COMPLEX for mkl_ccsrmultcsr.
DOUBLE COMPLEX for mkl_zcsrmultcsr.
Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

ja — INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. For each row the column indices must be arranged in the increasing order.

The length of this array is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ia — INTEGER. Array of length *m* + 1.
This array contains indices of elements in the array *a*, such that *ia*(*I*) is the index in the array *a* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) is equal to the number of non-zero elements of the matrix *A* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

b — REAL for mkl_scsrmultcsr.
DOUBLE PRECISION for mkl_dcsrmultcsr.
COMPLEX for mkl_ccsrmultcsr.
DOUBLE COMPLEX for mkl_zcsrmultcsr.
Array containing non-zero elements of the matrix *B*. Its length is equal to the number of non-zero elements in the matrix *B*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

jb — INTEGER. Array containing the column indices for each non-zero element of the matrix *B*. For each row the column indices must be arranged in the increasing order.
The length of this array is equal to the length of the array *b*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ib — INTEGER. Array of length *n* + 1 when *trans* = 'N' or 'n', or *m* + 1 otherwise.
This array contains indices of elements in the array *b*, such that *ib*(*I*) is the index in the array *b* of the first non-zero element from the row *I*. The value of the last element *ib*(*n* + 1) or *ib*(*m* + 1) is equal to the number of non-zero elements of the matrix *B* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

nzmax — INTEGER. The length of the arrays *c* and *jc*.
This parameter is used only if *request*=0. The routine stops calculation if the number of elements in the result matrix *C* exceeds the specified value of *nzmax*.

Output Parameters

c — REAL for mkl_scsrmultcsr.
DOUBLE PRECISION for mkl_dcsrmultcsr.
COMPLEX for mkl_ccsrmultcsr.
DOUBLE COMPLEX for mkl_zcsrmultcsr.
Array containing non-zero elements of the result matrix *C*. Its length is equal to the number of non-zero elements in the matrix *C*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.

jc — INTEGER. Array containing the column indices for each non-zero element of the matrix *C*.
The length of this array is equal to the length of the array *c*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.

ic — INTEGER. Array of length *m* + 1 when *trans* = 'N' or 'n', or *n* + 1 otherwise.

This array contains indices of elements in the array *c*, such that *ic*(*I*) is the index in the array *c* of the first non-zero element from the row *I*. The value of the last element *ic*(*m* + 1) or *ic*(*n* + 1) is equal to the number of non-zero elements of the matrix *C* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

info INTEGER.
If *info*=0, the execution is successful.
If *info*=*I*>0, the routine stops calculation in the *I*-th row of the matrix *C* because number of elements in *C* exceeds *nzmax*.
If *info*=-1, the routine calculates only the size of the arrays *c* and *jc* and returns this value plus 1 as the last element of the array *ic*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrmultcsr( trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER*1  trans

   INTEGER       request, sort, m, n, k, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   REAL            a(*), b(*), c(*)

SUBROUTINE mkl_dcsrmultcsr( trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER*1  trans

   INTEGER       request, sort, m, n, k, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   DOUBLE PRECISION            a(*), b(*), c(*)

SUBROUTINE mkl_ccsrmultcsr( trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER*1  trans

   INTEGER       request, sort, m, n, k, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   COMPLEX     a(*), b(*), c(*)

SUBROUTINE mkl_zcsrmultcsr( trans, request, sort, m, n, k, a, ja, ia, b, jb, ib, c, jc, ic, nzmax,
info)

   CHARACTER*1  trans

   INTEGER       request, sort, m, n, k, nzmax, info

   INTEGER       ja(*), jb(*), jc(*), ia(*), ib(*), ic(*)

   DOUBLE COMPLEX       a(*), b(*), c(*)
```

C:

```
void mkl_scsrmultcsr(char *trans, int *request, int *sort, int *m, int *n, int *k,
float *a, int *ja, int *ia, float *b, int *jb, int *ib, float *c,
int *jc, int *ic, int *nzmax, int *info);
```

```
void mkl_dcsrmultcsr(char *trans, int *request, int *sort, int *m, int *n, int *k,
double *a, int *ja, int *ia, double *b, int *jb, int *ib, double *c,
int *jc, int *ic, int *nzmax, int *info);
```

```
void mkl_ccsrmultcsr(char *trans, int *request, int *sort, int *m, int *n, int *k,
MKL_Complex8 *a, int *ja, int *ia,  MKL_Complex8 *b, int *jb, int *ib,
MKL_Complex8 *c, int *jc, int *ic, int *nzmax, int *info);
```

```
void mkl_zcsrmultcsr(char *trans, int *request, int *sort, int *m, int *n, int *k,
MKL_Complex16 *a, int *ja, int *ia, MKL_Complex16 *b, int *jb, int *ib,
MKL_Complex16 *c, int *jc, int *ic, int *nzmax, int *info);
```

mkl_?csrmultd

Computes product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing. The result is stored in the dense matrix.

Syntax

Fortran:

```
call mkl_scsrmultd(trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
call mkl_dcsrmultd(trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
call mkl_ccsrmultd(trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
call mkl_zcsrmultd(trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
```

C:

```
mkl_scsrmultd(&trans, &m, &n, &k, a, ja, ia, b, jb, ib, c, &ldc);
mkl_dcsrmultd(&trans, &m, &n, &k, a, ja, ia, b, jb, ib, c, &ldc);
mkl_ccsrmultd(&trans, &m, &n, &k, a, ja, ia, b, jb, ib, c, &ldc);
mkl_zcsrmultd(&trans, &m, &n, &k, a, ja, ia, b, jb, ib, c, &ldc);
```

Include Files

- FORTRAN 77: `mkl_spblas.fi`
- C: `mkl_spblas.h`

Description

The `mkl_?csrmultd` routine performs a matrix-matrix operation defined as

```
C := op(A)*B
```

where:

A, *B* are the sparse matrices in the CSR format (3-array variation), *C* is dense matrix;

`op(A)` is one of `op(A) = A`, or `op(A) =A'`, or `op(A) = conjg(A')` .

The routine works correctly if and only if the column indices in sparse matrix representations of matrices *A* and *B* are arranged in the increasing order for each row. If not, use the parameter *sort* (see below) to reorder column indices and the corresponding elements of the input matrices.

NOTE This routine supports only one-based indexing of the input arrays.

Input Parameters

Parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

trans	CHARACTER*1. Specifies the operation. If *trans* = 'N' or 'n', then *C* := *A***B* If *trans* = 'T' or 't' or 'C' or 'c', then *C* := *A*'**B*.
m	INTEGER. Number of rows of the matrix *A*.
n	INTEGER. Number of columns of the matrix *A*.
k	INTEGER. Number of columns of the matrix *B*.
a	REAL for mkl_scsrmultd. DOUBLE PRECISION for mkl_dcsrmultd. COMPLEX for mkl_ccsrmultd. DOUBLE COMPLEX for mkl_zcsrmultd. Array containing non-zero elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
ja	INTEGER. Array containing the column indices for each non-zero element of the matrix *A*. For each row the column indices must be arranged in the increasing order. The length of this array is equal to the length of the array *a*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ia	INTEGER. Array of length *m* + 1 when *trans* = 'N' or 'n', or *n* + 1 otherwise. This array contains indices of elements in the array *a*, such that *ia*(*I*) is the index in the array *a* of the first non-zero element from the row *I*. The value of the last element *ia*(*m* + 1) or *ia*(*n* + 1) is equal to the number of non-zero elements of the matrix *A* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.
b	REAL for mkl_scsrmultd. DOUBLE PRECISION for mkl_dcsrmultd. COMPLEX for mkl_ccsrmultd. DOUBLE COMPLEX for mkl_zcsrmultd. Array containing non-zero elements of the matrix *B*. Its length is equal to the number of non-zero elements in the matrix *B*. Refer to *values* array description in Sparse Matrix Storage Formats for more details.
jb	INTEGER. Array containing the column indices for each non-zero element of the matrix *B*. For each row the column indices must be arranged in the increasing order. The length of this array is equal to the length of the array *b*. Refer to *columns* array description in Sparse Matrix Storage Formats for more details.
ib	INTEGER. Array of length *m* + 1. This array contains indices of elements in the array *b*, such that *ib*(*I*) is the index in the array *b* of the first non-zero element from the row *I*. The value of the last element *ib*(*m* + 1) is equal to the number of non-zero elements of the matrix *B* plus one. Refer to *rowIndex* array description in Sparse Matrix Storage Formats for more details.

Output Parameters

c	REAL for mkl_scsrmultd. DOUBLE PRECISION for mkl_dcsrmultd. COMPLEX for mkl_ccsrmultd. DOUBLE COMPLEX for mkl_zcsrmultd. Array containing non-zero elements of the result matrix *c*.
ldc	INTEGER. Specifies the leading dimension of the dense matrix *c* as declared in the calling (sub)program. Must be at least max(*m*, 1) when *trans* = 'N' or 'n', or max(1, *n*) otherwise.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_scsrmultd( trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
   CHARACTER*1  trans
   INTEGER       m, n, k, ldc
   INTEGER       ja(*), jb(*), ia(*), ib(*)
   REAL            a(*), b(*), c(ldc, *)
SUBROUTINE mkl_dcsrmultd( trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
   CHARACTER*1  trans
   INTEGER       m, n, k, ldc
   INTEGER       ja(*), jb(*), ia(*), ib(*)
   DOUBLE PRECISION         a(*), b(*), c(ldc, *)
SUBROUTINE mkl_ccsrmultd( trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
   CHARACTER*1  trans
   INTEGER       m, n, k, ldc
   INTEGER       ja(*), jb(*), ia(*), ib(*)
   COMPLEX     a(*), b(*), c(ldc, *)
SUBROUTINE mkl_zcsrmultd( trans, m, n, k, a, ja, ia, b, jb, ib, c, ldc)
   CHARACTER*1  trans
   INTEGER       m, n, k, ldc
   INTEGER       ja(*), jb(*), ia(*), ib(*)
   DOUBLE COMPLEX      a(*), b(*), c(ldc, *)
```

C:

```
void mkl_scsrmultd(char *trans, int *m, int *n, int *k,
float *a, int *ja, int *ia, float *b, int *jb, int *ib, float *c, int *ldc);

void mkl_dcsrmultd(char *trans, int *m, int *n, int *k,
double *a, int *ja, int *ia, double *b, int *jb, int *ib, double *c, int *ldc);

void mkl_ccsrmultd(char *trans, int *m, int *n, int *k,
MKL_Complex8 *a, int *ja, int *ia,  MKL_Complex8 *b, int *jb, int *ib,
MKL_Complex8 *c, int *ldc);
```

```
void mkl_zcsrmultd(char *trans, int *m, int *n, int *k,
MKL_Complex16 *a, int *ja, int *ia,  MKL_Complex16 *b, int *jb, int *ib,
MKL_Complex16 *c, int *ldc);
```

BLAS-like Extensions

Intel MKL provides C and Fortran routines to extend the functionality of the BLAS routines. These include routines to compute vector products, matrix-vector products, and matrix-matrix products.

Intel MKL also provides routines to perform certain data manipulation, including matrix in-place and out-of-place transposition operations combined with simple matrix arithmetic operations. Transposition operations are Copy As Is, Conjugate transpose, Transpose, and Conjugate. Each routine adds the possibility of scaling during the transposition operation by giving some *alpha* and/or *beta* parameters. Each routine supports both row-major orderings and column-major orderings.

Table "BLAS-like Extensions" lists these routines.

The <?> symbol in the routine short names is a precision prefix that indicates the data type:

s	REAL for Fortran interface, or float for C interface
d	DOUBLE PRECISION for Fortran interface, or double for C interface.
c	COMPLEX for Fortran interface, or MKL_Complex8 for C interface.
z	DOUBLE COMPLEX for Fortran interface, or MKL_Complex16 for C interface.

BLAS-like Extensions

Routine	Data Types	Description
axpby	s, d, c, z	Scales two vectors, adds them to one another and stores result in the vector (routines)
gem2vu	s, d	Two matrix-vector products using a general matrix, real data
gem2vc	c, z	Two matrix-vector products using a general matrix, complex data
?gemm3m	c, z	Computes a scalar-matrix-matrix product using matrix multiplications and adds the result to a scalar-matrix product.
mkl_?imatcopy	s, d, c, z	Performs scaling and in-place transposition/copying of matrices.
mkl_?omatcopy	s, d, c, z	Performs scaling and out-of-place transposition/copying of matrices.
mkl_?omatcopy2	s, d, c, z	Performs two-strided scaling and out-of-place transposition/copying of matrices.
mkl_?omatadd	s, d, c, z	Performs scaling and sum of two matrices including their out-of-place transposition/copying.

?axpby

Scales two vectors, adds them to one another and stores result in the vector.

Syntax

Fortran 77:

```
call saxpby(n, a, x, incx, b, y, incy)
```

```
call daxpby(n, a, x, incx, b, y, incy)
call caxpby(n, a, x, incx, b, y, incy)
call zaxpby(n, a, x, incx, b, y, incy)
```

Fortran 95:

```
call axpby(x, y [,a] [,b])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?axpby` routines perform a vector-vector operation defined as

```
y := a*x + b*y
```

where:

a and *b* are scalars

x and *y* are vectors each with *n* elements.

Input Parameters

n	`INTEGER`. Specifies the number of elements in vectors *x* and *y*.
a	`REAL` for `saxpby` `DOUBLE PRECISION` for `daxpby` `COMPLEX` for `caxpby` `DOUBLE COMPLEX` for `zaxpby` Specifies the scalar *a*.
x	`REAL` for `saxpby` `DOUBLE PRECISION` for `daxpby` `COMPLEX` for `caxpby` `DOUBLE COMPLEX` for `zaxpby` Array, `DIMENSION` at least `(1 + (n-1)*abs(incx))`.
incx	`INTEGER`. Specifies the increment for the elements of *x*.
b	`REAL` for `saxpby` `DOUBLE PRECISION` for `daxpby` `COMPLEX` for `caxpby` `DOUBLE COMPLEX` for `zaxpby` Specifies the scalar *b*.
y	`REAL` for `saxpby` `DOUBLE PRECISION` for `daxpby` `COMPLEX` for `caxpby` `DOUBLE COMPLEX` for `zaxpby` Array, `DIMENSION` at least `(1 + (n-1)*abs(incy))`.
incy	`INTEGER`. Specifies the increment for the elements of *y*.

Output Parameters

y	Contains the updated vector *y*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `axpby` interface are the following:

x	Holds the array of size *n*.
y	Holds the array of size *n*.
a	The default value is 1.
b	The default value is 1.

?gem2vu

Computes two matrix-vector products using a general matrix (real data)

Syntax

Fortran 77:

```
call sgem2vu(m, n, alpha, a, lda, x1, incx1, x2, incx2, beta, y1, incy1, y2, incy2)
call dgem2vu(m, n, alpha, a, lda, x1, incx1, x2, incx2, beta, y1, incy1, y2, incy2)
```

Fortran 95:

```
call gem2vu(a, x1, x2, y1, y2 [,alpha][,beta] )
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?gem2vu` routines perform two matrix-vector operations defined as

y1 := *alpha***A***x1* + *beta***y1*,

and

y2 := *alpha***A*'**x2* + *beta***y2*,

where:

alpha and *beta* are scalars,

x1, *x2*, *y1*, and *y2* are vectors,

A is an *m*-by-*n* matrix.

Input Parameters

m	`INTEGER`. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	`INTEGER`. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
alpha	`REAL` for `sgem2vu` `DOUBLE PRECISION` for `dgem2vu`

	Specifies the scalar *alpha*.
a	REAL for sgem2vu DOUBLE PRECISION for dgem2vu Array, DIMENSION (*lda*, *n*). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).
x1	REAL for sgem2vu DOUBLE PRECISION for dgem2vu Array, DIMENSION at least (1+(*n*–1)*abs(*incx1*)). Before entry, the incremented array *x1* must contain the vector *x1*.
incx1	INTEGER. Specifies the increment for the elements of *x1*. The value of *incx1* must not be zero.
x2	REAL for sgem2vu DOUBLE PRECISION for dgem2vu Array, DIMENSION at least (1+(*m*–1)*abs(*incx2*)). Before entry, the incremented array *x2* must contain the vector *x2*.
incx2	INTEGER. Specifies the increment for the elements of *x2*. The value of *incx2* must not be zero.
beta	REAL for sgem2vu DOUBLE PRECISION for dgem2vu Specifies the scalar *beta*. When *beta* is set to zero, then *y1* and *y2* need not be set on input.
y1	REAL for sgem2vu DOUBLE PRECISION for dgem2vu Array, DIMENSION at least (1+(*m*–1)*abs(*incy1*)). Before entry with non-zero *beta*, the incremented array *y1* must contain the vector *y1*.
incy1	INTEGER. Specifies the increment for the elements of *y1*. The value of *incy1* must not be zero.
y	REAL for sgem2vu DOUBLE PRECISION for dgem2vu Array, DIMENSION at least (1+(*n*–1)*abs(*incy2*)). Before entry with non-zero *beta*, the incremented array *y2* must contain the vector *y2*.
incy2	INTEGER. Specifies the increment for the elements of *y2*. The value of *incy2* must not be zero.

Output Parameters

y1	Updated vector *y1*.
y2	Updated vector *y2*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gem2vu interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
x1	Holds the vector with the number of elements *rx1* where *rx1* = *n*.
x2	Holds the vector with the number of elements *rx2* where *rx2* = *m*.

y1	Holds the vector with the number of elements *ry1* where *ry1* = *m*.
y2	Holds the vector with the number of elements *ry2* where *ry2* = *n*.
alpha	The default value is 1.
beta	The default value is 0.

?gem2vc

Computes two matrix-vector products using a general matrix (complex data)

Syntax

Fortran 77:

```
call cgem2vc(m, n, alpha, a, lda, x1, incx1, x2, incx2, beta, y1, incy1, y2, incy2)
call zgem2vc(m, n, alpha, a, lda, x1, incx1, x2, incx2, beta, y1, incy1, y2, incy2)
```

Fortran 95:

```
call gem2vc(a, x1, x2, y1, y2 [,alpha][,beta] )
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?gem2vc` routines perform two matrix-vector operations defined as

y1 := *alpha***A***x1* + *beta***y1*,

and

y2 := *alpha**conjg(*A*')**x2* + *beta***y2*,

where:

alpha and *beta* are scalars,

x1, *x2*, *y1*, and *y2* are vectors,

A is an *m*-by-*n* matrix.

Input Parameters

m	`INTEGER`. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.
n	`INTEGER`. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.
alpha	`COMPLEX` for `cgem2vc` `DOUBLE COMPLEX` for `zgem2vc` Specifies the scalar *alpha*.
a	`COMPLEX` for `cgem2vc` `DOUBLE COMPLEX` for `zgem2vc` Array, `DIMENSION (lda, n)`. Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients.

lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).
x1	COMPLEX for cgem2vc DOUBLE COMPLEX for zgem2vc Array, DIMENSION at least (1+(*n*–1)*abs(*incx1*)). Before entry, the incremented array *x1* must contain the vector *x1*.
incx1	INTEGER. Specifies the increment for the elements of *x1*. The value of *incx1* must not be zero.
x2	COMPLEX for cgem2vc DOUBLE COMPLEX for zgem2vc Array, DIMENSION at least (1+(*m*–1)*abs(*incx2*)). Before entry, the incremented array *x2* must contain the vector *x2*.
incx2	INTEGER. Specifies the increment for the elements of *x2*. The value of *incx2* must not be zero.
beta	COMPLEX for cgem2vc DOUBLE COMPLEX for zgem2vc Specifies the scalar *beta*. When *beta* is set to zero, then *y1* and *y2* need not be set on input.
y1	COMPLEX for cgem2vc DOUBLE COMPLEX for zgem2vc Array, DIMENSION at least (1+(*m*–1)*abs(*incy1*)). Before entry with non-zero *beta*, the incremented array *y1* must contain the vector *y1*.
incy1	INTEGER. Specifies the increment for the elements of *y1*. The value of *incy1* must not be zero.
y2	COMPLEX for cgem2vc DOUBLE COMPLEX for zgem2vc Array, DIMENSION at least (1+(*n*–1)*abs(*incy2*)). Before entry with non-zero *beta*, the incremented array *y2* must contain the vector *y2*.
incy2	INTEGER. Specifies the increment for the elements of *y2*. The value of *incy* must not be zero. INTEGER. Specifies the increment for the elements of *y*.

Output Parameters

y1	Updated vector *y1*.
y2	Updated vector *y2*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gem2vc interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
x1	Holds the vector with the number of elements *rx1* where *rx1* = *n*.
x2	Holds the vector with the number of elements *rx2* where *rx2* = *m*.
y1	Holds the vector with the number of elements *ry1* where *ry1* = *m*.
y2	Holds the vector with the number of elements *ry2* where *ry2* = *n*.
alpha	The default value is 1.

beta	The default value is 0.

?gemm3m

Computes a scalar-matrix-matrix product using matrix multiplications and adds the result to a scalar-matrix product.

Syntax

Fortran 77:

```
call cgemm3m(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
call zgemm3m(transa, transb, m, n, k, alpha, a, lda, b, ldb, beta, c, ldc)
```

Fortran 95:

```
call gemm3m(a, b, c [,transa][,transb] [,alpha][,beta])
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- Fortran 95: `blas.f90`
- C: `mkl_blas.h`

Description

The `?gemm3m` routines perform a matrix-matrix operation with general complex matrices. These routines are similar to the `?gemm` routines, but they use matrix multiplications(see *Application Notes* below).

The operation is defined as

```
C := alpha*op(A)*op(B) + beta*C,
```

where:

`op(x)` is one of `op(x) = x`, or `op(x) = x'`, or `op(x) = conjg(x')`,

alpha and *beta* are scalars,

A, *B* and *C* are matrices:

`op(A)` is an *m*-by-*k* matrix,

`op(B)` is a *k*-by-*n* matrix,

C is an *m*-by-*n* matrix.

Input Parameters

transa	`CHARACTER*1`. Specifies the form of `op(A)` used in the matrix multiplication: if *transa* = `'N'` or `'n'`, then `op(A) = A`; if *transa* = `'T'` or `'t'`, then `op(A) = A'`; if *transa* = `'C'` or `'c'`, then `op(A) = conjg(A')`.
transb	`CHARACTER*1`. Specifies the form of `op(B)` used in the matrix multiplication: if *transb* = `'N'` or `'n'`, then `op(B) = B`; if *transb* = `'T'` or `'t'`, then `op(B) = B'`; if *transb* = `'C'` or `'c'`, then `op(B) = conjg(B')`.

m	INTEGER. Specifies the number of rows of the matrix op(*A*) and of the matrix *C*. The value of m must be at least zero.
n	INTEGER. Specifies the number of columns of the matrix op(*B*) and the number of columns of the matrix *C*. The value of *n* must be at least zero.
k	INTEGER. Specifies the number of columns of the matrix op(*A*) and the number of rows of the matrix op(*B*). The value of *k* must be at least zero.
alpha	COMPLEX for cgemm3m DOUBLE COMPLEX for zgemm3m Specifies the scalar *alpha*.
a	COMPLEX for cgemm3m DOUBLE COMPLEX for zgemm3m Array, DIMENSION (*lda*, *ka*), where *ka* is *k* when *transa*= 'N' or 'n', and is *m* otherwise. Before entry with *transa*= 'N' or 'n', the leading *m*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*m* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *transa*= 'N' or 'n', then *lda* must be at least max(1, *m*), otherwise *lda* must be at least max(1, *k*).
b	COMPLEX for cgemm3m DOUBLE COMPLEX for zgemm3m Array, DIMENSION (*ldb*, *kb*), where *kb* is *n* when *transb* = 'N' or 'n', and is *k* otherwise. Before entry with *transb* = 'N' or 'n', the leading *k*-by-*n* part of the array *b* must contain the matrix *B*, otherwise the leading *n*-by-*k* part of the array *b* must contain the matrix *B*.
ldb	INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. When *transb* = 'N' or 'n', then *ldb* must be at least max(1, *k*), otherwise *ldb* must be at least max(1, *n*).
beta	COMPLEX for cgemm3m DOUBLE COMPLEX for zgemm3m Specifies the scalar *beta*. When *beta* is equal to zero, then *c* need not be set on input.
c	COMPLEX for cgemm3m DOUBLE COMPLEX for zgemm3m Array, DIMENSION (*ldc*, *n*). Before entry, the leading *m*-by-*n* part of the array *c* must contain the matrix *C*, except when *beta* is equal to zero, in which case *c* need not be set on entry.
ldc	INTEGER. Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least max(1, *m*).

Output Parameters

c	Overwritten by the *m*-by-*n* matrix (*alpha**op(*A*)*op(*B*) + *beta***C*).

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gemm3m interface are the following:

a	Holds the matrix *A* of size (*ma*,*ka*) where *ka* = *k* if *transa*= 'N', *ka* = *m* otherwise, *ma* = *m* if *transa*= 'N', *ma* = *k* otherwise.
b	Holds the matrix *B* of size (*mb*,*kb*) where *kb* = *n* if *transb* = 'N', *kb* = *k* otherwise, *mb* = *k* if *transb* = 'N', *mb* = *n* otherwise.
c	Holds the matrix *C* of size (*m*,*n*).
transa	Must be 'N', 'C', or 'T'. The default value is 'N'.
transb	Must be 'N', 'C', or 'T'. The default value is 'N'.
alpha	The default value is 1.
beta	The default value is 1.

Application Notes

These routines perform the complex multiplication by forming the real and imaginary parts of the input matrices. It allows to use three real matrix multiplications and five real matrix additions, instead of the conventional four real matrix multiplications and two real matrix additions. The use of three real matrix multiplications only gives a 25% reduction of time in matrix operations. This can result in significant savings in computing time for large matrices.

If the errors in the floating point calculations satisfy the following conditions:

$fl(x \text{ op } y)=(x \text{ op } y)(1+\delta), |\delta|\le u, \text{ op}=\times,/, \quad fl(x\pm y)=x(1+\alpha)\pm y(1+\beta), \quad |\alpha|,|\beta|\le u$

then for n-by-n matrix $\hat{C}=fl(C_1+iC_2)= fl((A_1+iA_2)(B_1+iB_2))=\hat{C}_1+i\hat{C}_2$ the following estimations are correct

$\|\hat{C}_1-C_2\|\le 2(n+1)u\|A\|_\infty\|B\|_\infty+O(u^2)$,

$\|\hat{C}_2-C_1\|\le 4(n+4)u\|A\|_\infty\|B\|_\infty+O(u^2)$,

where $\|A\|_\infty=\max(\|A_1\|_\infty,\|A_2\|_\infty)$, and $\|B\|_\infty=\max(\|B_1\|_\infty,\|B_2\|_\infty)$.

and hence the matrix multiplications are stable.

mkl_?imatcopy

Performs scaling and in-place transposition/copying of matrices.

Syntax

Fortran:

```
call mkl_simatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda)
call mkl_dimatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda)
call mkl_cimatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda)
call mkl_zimatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda)
```

C:

```
mkl_simatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda);
mkl_dimatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda);
mkl_cimatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda);
mkl_zimatcopy(ordering, trans, rows, cols, alpha, a, src_lda, dst_lda);
```

Include Files

- FORTRAN 77: `mkl_trans.fi`
- C: `mkl_trans.h`

Description

The `mkl_?imatcopy` routine performs scaling and in-place transposition/copying of matrices. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The operation is defined as follows:

A := *alpha**op(*A*).

The routine parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

Note that different arrays should not overlap.

Input Parameters

ordering	CHARACTER*1. Ordering of the matrix storage. If *ordering* = 'R' or 'r', the ordering is row-major. If *ordering* = 'C' or 'c', the ordering is column-major.
trans	CHARACTER*1. Parameter that specifies the operation type. If *trans* = 'N' or 'n', op(*A*)=*A* and the matrix *A* is assumed unchanged on input. If *trans* = 'T' or 't', it is assumed that *A* should be transposed. If *trans* = 'C' or 'c', it is assumed that *A* should be conjugate transposed. If *trans* = 'R' or 'r', it is assumed that *A* should be only conjugated. If the data is real, then *trans* = 'R' is the same as *trans* = 'N', and *trans* = 'C' is the same as *trans* = 'T'.
rows	INTEGER. The number of matrix rows.
cols	INTEGER. The number of matrix columns.
a	REAL for mkl_simatcopy. DOUBLE PRECISION for mkl_dimatcopy. COMPLEX for mkl_cimatcopy. DOUBLE COMPLEX for mkl_zimatcopy. Array, DIMENSION *a*(*scr_lda*,*).
alpha	REAL for mkl_simatcopy. DOUBLE PRECISION for mkl_dimatcopy. COMPLEX for mkl_cimatcopy. DOUBLE COMPLEX for mkl_zimatcopy. This parameter scales the input matrix by *alpha*.

src_lda INTEGER. Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix; measured in the number of elements.
This parameter must be at least max(1,*rows*) if *ordering* = 'C' or 'c', and max(1,*cols*) otherwise.

dst_lda INTEGER. Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the destination matrix; measured in the number of elements.
To determine the minimum value of *dst_lda* on output, consider the following guideline:
If *ordering* = 'C' or 'c', then

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*rows*)
- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*cols*)

If *ordering* = 'R' or 'r', then

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*cols*)
- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*rows*)

Output Parameters

a REAL for mkl_simatcopy.
DOUBLE PRECISION for mkl_dimatcopy.
COMPLEX for mkl_cimatcopy.
DOUBLE COMPLEX for mkl_zimatcopy.
Array, DIMENSION at least *m*.
Contains the matrix *A*.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_simatcopy ( ordering, trans, rows, cols, alpha, a, src_lda, dst_lda )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_ld, dst_ld
   REAL a(*), alpha*
```

```
SUBROUTINE mkl_dimatcopy ( ordering, trans, rows, cols, alpha, a, src_lda, dst_lda )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_ld, dst_ld
   DOUBLE PRECISION a(*), alpha*
```

```
SUBROUTINE mkl_cimatcopy ( ordering, trans, rows, cols, alpha, a, src_lda, dst_lda )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_ld, dst_ld
   COMPLEX a(*), alpha*
```

```
SUBROUTINE mkl_zimatcopy ( ordering, trans, rows, cols, alpha, a, src_lda, dst_lda )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_ld, dst_ld
   DOUBLE COMPLEX a(*), alpha*
```

C:

```
void mkl_simatcopy(char ordering, char trans, size_t rows, size_t cols, float *alpha, float *a, size_t
src_lda, size_t dst_lda);
```

```
void mkl_dimatcopy(char ordering, char trans, size_t rows, size_t cols, double *alpha, float *a, size_t
src_lda, size_t dst_lda);

void mkl_cimatcopy(char ordering, char trans, size_t rows, size_t cols, MKL_Complex8 *alpha,
MKL_Complex8 *a, size_t src_lda, size_t dst_lda);

void mkl_zimatcopy(char ordering, char trans, size_t rows, size_t cols, MKL_Complex16 *alpha,
MKL_Complex16 *a, size_t src_lda, size_t dst_lda);
```

mkl_?omatcopy

Performs scaling and out-place transposition/copying of matrices.

Syntax

Fortran:

```
call mkl_somatcopy(ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld)
call mkl_domatcopy(ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld)
call mkl_comatcopy(ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld)
call mkl_zomatcopy(ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld)
```

C:

```
mkl_somatcopy(ordering, trans, rows, cols, alpha, SRC, src_stride, DST, dst_stride);
mkl_domatcopy(ordering, trans, rows, cols, alpha, SRC, src_stride, DST, dst_stride);
mkl_comatcopy(ordering, trans, rows, cols, alpha, SRC, src_stride, DST, dst_stride);
mkl_zomatcopy(ordering, trans, rows, cols, alpha, SRC, src_stride, DST, dst_stride);
```

Include Files

- FORTRAN 77: `mkl_trans.fi`
- C: `mkl_trans.h`

Description

The `mkl_?omatcopy` routine performs scaling and out-of-place transposition/copying of matrices. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The operation is defined as follows:

B := *alpha**op(*A*)

The routine parameter descriptions are common for all implemented interfaces with the exception of data types that mostly refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

Note that different arrays should not overlap.

Input Parameters

ordering	CHARACTER*1. Ordering of the matrix storage. If *ordering* = 'R' or 'r', the ordering is row-major. If *ordering* = 'C' or 'c', the ordering is column-major.
trans	CHARACTER*1. Parameter that specifies the operation type. If *trans* = 'N' or 'n', op(*A*)=*A* and the matrix *A* is assumed unchanged on input.

If *trans* = 'T' or 't', it is assumed that *A* should be transposed.
If *trans* = 'C' or 'c', it is assumed that *A* should be conjugate transposed.
If *trans* = 'R' or 'r', it is assumed that *A* should be only conjugated.
If the data is real, then *trans* = 'R' is the same as *trans* = 'N', and *trans* = 'C' is the same as *trans* = 'T'.

rows — INTEGER. The number of matrix rows.

cols — INTEGER. The number of matrix columns.

alpha — REAL for mkl_somatcopy.
DOUBLE PRECISION for mkl_domatcopy.
COMPLEX for mkl_comatcopy.
DOUBLE COMPLEX for mkl_zomatcopy.
This parameter scales the input matrix by *alpha*.

src — REAL for mkl_somatcopy.
DOUBLE PRECISION for mkl_domatcopy.
COMPLEX for mkl_comatcopy.
DOUBLE COMPLEX for mkl_zomatcopy.
Array, DIMENSION *src*(*scr_ld*,*).

src_ld — INTEGER. (Fortran interface). Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix; measured in the number of elements.
This parameter must be at least max(1,*rows*) if *ordering* = 'C' or 'c', and max(1,*cols*) otherwise.

src_stride — INTEGER. (C interface). Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix; measured in the number of elements.
This parameter must be at least max(1,*rows*) if *ordering* = 'C' or 'c', and max(1,*cols*) otherwise.

dst — REAL for mkl_somatcopy.
DOUBLE PRECISION for mkl_domatcopy.
COMPLEX for mkl_comatcopy.
DOUBLE COMPLEX for mkl_zomatcopy.
Array, DIMENSION *dst*(*dst_ld*,*).

dst_ld — INTEGER. (Fortran interface). Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the destination matrix; measured in the number of elements.
To determine the minimum value of *dst_lda* on output, consider the following guideline:
If *ordering* = 'C' or 'c', then

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*rows*)
- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*cols*)

If *ordering* = 'R' or 'r', then

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*cols*)

- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*rows*)

dst_stride INTEGER. (C interface). Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the destination matrix; measured in the number of elements.
To determine the minimum value of *dst_lda* on output, consider the following guideline:
If *ordering* = 'C' or 'c', then

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*rows*)
- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*cols*)

If *ordering* = 'R' or 'r', then

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*cols*)
- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*rows*)

Output Parameters

dst REAL for mkl_somatcopy.
DOUBLE PRECISION for mkl_domatcopy.
COMPLEX for mkl_comatcopy.
DOUBLE COMPLEX for mkl_zomatcopy.
Array, DIMENSION at least *m*.
Contains the destination matrix.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_somatcopy ( ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_ld, dst_ld
   REAL alpha, dst(dst_ld,*), src(src_ld,*)
```

```
SUBROUTINE mkl_domatcopy ( ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_lda, dst_lda
   DOUBLE PRECISION alpha, dst(dst_ld,*), src(src_ld,*)
```

```
SUBROUTINE mkl_comatcopy ( ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_lda, dst_lda
   COMPLEX alpha, dst(dst_ld,*), src(src_ld,*)
```

```
SUBROUTINE mkl_zomatcopy ( ordering, trans, rows, cols, alpha, src, src_ld, dst, dst_ld )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_lda, dst_lda
   DOUBLE COMPLEX alpha, dst(dst_ld,*), src(src_ld,*)
```

C:

```
void mkl_somatcopy(char ordering, char trans, size_t rows, size_t cols, float alpha, float *SRC, size_t
src_stride, float *DST, size_t dst_stride);
```

```
void mkl_domatcopy(char ordering, char trans, size_t rows, size_t cols, double alpha, double *SRC,
size_t src_stride, double *DST, size_t dst_stride);

void mkl_comatcopy(char ordering, char trans, size_t rows, size_t cols, MKL_Complex8 alpha,
MKL_Complex8 *SRC, size_t src_stride, MKL_Complex8 *DST, size_t dst_stride);

void mkl_zomatcopy(char ordering, char trans, size_t rows, size_t cols, MKL_Complex16 alpha,
MKL_Complex16 *SRC, size_t src_stride, MKL_Complex16 *DST, size_t dst_stride);
```

mkl_?omatcopy2

Performs two-strided scaling and out-of-place transposition/copying of matrices.

Syntax

Fortran:

```
call mkl_somatcopy2(ordering, trans, rows, cols, alpha, src, src_row, src_col, dst,
dst_row, dst_col)

call mkl_domatcopy2(ordering, trans, rows, cols, alpha, src, src_row, src_col, dst,
dst_row, dst_col)

call mkl_comatcopy2(ordering, trans, rows, cols, alpha, src, src_row, src_col, dst,
dst_row, dst_col)

call mkl_zomatcopy2(ordering, trans, rows, cols, alpha, src, src_row, src_col, dst,
dst_row, dst_col)
```

C:

```
mkl_somatcopy2(ordering, trans, rows, cols, alpha, SRC, src_row, src_col, DST,
dst_row, dst_col);

mkl_domatcopy2(ordering, trans, rows, cols, alpha, SRC, src_row, src_col, DST,
dst_row, dst_col);

mkl_comatcopy2(ordering, trans, rows, cols, alpha, SRC, src_row, src_col, DST,
dst_row, dst_col);

mkl_zomatcopy2(ordering, trans, rows, cols, alpha, SRC, src_row, src_col, DST,
dst_row, dst_col);
```

Include Files

- FORTRAN 77: `mkl_trans.fi`
- C: `mkl_trans.h`

Description

The `mkl_?omatcopy2` routine performs two-strided scaling and out-of-place transposition/copying of matrices. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The operation is defined as follows:

`B := alpha*op(A)`

Normally, matrices in the BLAS or LAPACK are specified by a single stride index. For instance, in the column-major order, `A(2,1)` is stored in memory one element away from `A(1,1)`, but `A(1,2)` is a leading dimension away. The leading dimension in this case is the single stride. If a matrix has two strides, then both `A(2,1)` and `A(1,2)` may be an arbitrary distance from `A(1,1)`.

The routine parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

Note that different arrays should not overlap.

Input Parameters

ordering	CHARACTER*1. Ordering of the matrix storage. If *ordering* = 'R' or 'r', the ordering is row-major. If *ordering* = 'C' or 'c', the ordering is column-major.
trans	CHARACTER*1. Parameter that specifies the operation type. If *trans* = 'N' or 'n', op(*A*)=*A* and the matrix *A* is assumed unchanged on input. If *trans* = 'T' or 't', it is assumed that *A* should be transposed. If *trans* = 'C' or 'c', it is assumed that *A* should be conjugate transposed. If *trans* = 'R' or 'r', it is assumed that *A* should be only conjugated. If the data is real, then *trans* = 'R' is the same as *trans* = 'N', and *trans* = 'C' is the same as *trans* = 'T'.
rows	INTEGER. The number of matrix rows.
cols	INTEGER. The number of matrix columns.
alpha	REAL for mkl_somatcopy2. DOUBLE PRECISION for mkl_domatcopy2. COMPLEX for mkl_comatcopy2. DOUBLE COMPLEX for mkl_zomatcopy2. This parameter scales the input matrix by *alpha*.
src	REAL for mkl_somatcopy2. DOUBLE PRECISION for mkl_domatcopy2. COMPLEX for mkl_comatcopy2. DOUBLE COMPLEX for mkl_zomatcopy2. Array, DIMENSION *src*(*).
src_row	INTEGER. Distance between the first elements in adjacent rows in the source matrix; measured in the number of elements. This parameter must be at least max(1,*rows*).
src_col	INTEGER. Distance between the first elements in adjacent columns in the source matrix; measured in the number of elements. This parameter must be at least max(1,*cols*).
dst	REAL for mkl_somatcopy2. DOUBLE PRECISION for mkl_domatcopy2. COMPLEX for mkl_comatcopy2. DOUBLE COMPLEX for mkl_zomatcopy2. Array, DIMENSION *dst*(*).
dst_row	INTEGER. Distance between the first elements in adjacent rows in the destination matrix; measured in the number of elements. To determine the minimum value of *dst_row* on output, consider the following guideline: • If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*cols*) • If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*rows*)

dst_col INTEGER. Distance between the first elements in adjacent columns in the destination matrix; measured in the number of elements.
To determine the minimum value of *dst_lda* on output, consider the following guideline:

- If *trans* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*rows*)
- If *trans* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*cols*)

Output Parameters

dst REAL for mkl_somatcopy2.
DOUBLE PRECISION for mkl_domatcopy2.
COMPLEX for mkl_comatcopy2.
DOUBLE COMPLEX for mkl_zomatcopy2.
Array, DIMENSION at least *m*.
Contains the destination matrix.

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_somatcopy2 ( ordering, trans, rows, cols, alpha, src, src_row, src_col, dst, dst_row,
dst_col )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_row, src_col, dst_row, dst_col
   REAL  alpha, dst(*), src(*)
```

```
SUBROUTINE mkl_domatcopy2 ( ordering, trans, rows, cols, alpha, src, src_row, src_col, dst, dst_row,
dst_col )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_row, src_col, dst_row, dst_col
   DOUBLE PRECISION alpha, dst(*), src(*)
```

```
SUBROUTINE mkl_comatcopy2 ( ordering, trans, rows, cols, alpha, src, src_row, src_col, dst, dst_row,
dst_col )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_row, src_col, dst_row, dst_col
   COMPLEX alpha, dst(*), src(*)
```

```
SUBROUTINE mkl_zomatcopy2 ( ordering, trans, rows, cols, alpha, src, src_row, src_col, dst, dst_row,
dst_col )
   CHARACTER*1 ordering, trans
   INTEGER rows, cols, src_row, src_col, dst_row, dst_col
   DOUBLE COMPLEX alpha, dst(*), src(*)
```

C:

```
void mkl_somatcopy2(char ordering, char trans, size_t rows, size_t cols, float *alpha, float *SRC,
size_t src_row, size_t src_col, float *DST, size_t dst_row, size_t dst_col);
```

```
void mkl_domatcopy2(char ordering, char trans, size_t rows, size_t cols, float *alpha, double *SRC,
size_t src_row, size_t src_col, double *DST, size_t dst_row, size_t dst_col);
```

```
void mkl_comatcopy2(char ordering, char trans, size_t rows, size_t cols, MKL_Complex8 *alpha,
MKL_Complex8 *SRC, size_t src_row, size_t src_col, MKL_Complex8 *DST, size_t dst_row, size_t dst_col);
```

```
void mkl_zomatcopy2(char ordering, char trans, size_t rows, size_t cols, MKL_Complex16 *alpha,
MKL_Complex16 *SRC, size_t src_row, size_t src_col, MKL_Complex16 *DST, size_t dst_row, size_t dst_col);
```

mkl_?omatadd

Performs scaling and sum of two matrices including their out-of-place transposition/copying.

Syntax

Fortran:

```
call mkl_somatadd(ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc)
call mkl_domatadd(ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc)
call mkl_comatadd(ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc)
call mkl_zomatadd(ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc)
```

C:

```
mkl_somatadd(ordering, transa, transb, m, n, alpha, A, lda, beta, B, ldb, C, ldc);
mkl_domatadd(ordering, transa, transb, m, n, alpha, A, lda, beta, B, ldb, C, ldc);
mkl_comatadd(ordering, transa, transb, m, n, alpha, A, lda, beta, B, ldb, C, ldc);
mkl_zomatadd(ordering, transa, transb, m, n, alpha, A, lda, beta, B, ldb, C, ldc);
```

Include Files

- FORTRAN 77: `mkl_trans.fi`
- C: `mkl_trans.h`

Description

The `mkl_?omatadd` routine scaling and sum of two matrices including their out-of-place transposition/copying. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The following out-of-place memory movement is done:

```
C := alpha*op(A) + beta*op(B)
```

`op(A)` is either transpose, conjugate-transpose, or leave alone depending on *transa*. If no transposition of the source matrices is required, *m* is the number of rows and *n* is the number of columns in the source matrices *A* and *B*. In this case, the output matrix *C* is *m*-by-*n*.

The routine parameter descriptions are common for all implemented interfaces with the exception of data types that refer here to the FORTRAN 77 standard types. Data types specific to the different interfaces are described in the section **"Interfaces"** below.

Note that different arrays should not overlap.

Input Parameters

ordering — `CHARACTER*1`. Ordering of the matrix storage.
If *ordering* = `'R'` or `'r'`, the ordering is row-major.
If *ordering* = `'C'` or `'c'`, the ordering is column-major.

transa — `CHARACTER*1`. Parameter that specifies the operation type on matrix *A*.
If *transa* = `'N'` or `'n'`, `op(A)=A` and the matrix *A* is assumed unchanged on input.
If *transa* = `'T'` or `'t'`, it is assumed that *A* should be transposed.
If *transa* = `'C'` or `'c'`, it is assumed that *A* should be conjugate transposed.
If *transa* = `'R'` or `'r'`, it is assumed that *A* should be only conjugated.

If the data is real, then *transa* = 'R' is the same as *transa* = 'N', and *transa* = 'C' is the same as *transa* = 'T'.

transb — CHARACTER*1. Parameter that specifies the operation type on matrix *B*.
If *transb* = 'N' or 'n', op(*B*)=*B* and the matrix *B* is assumed unchanged on input.
If *transb* = 'T' or 't', it is assumed that *B* should be transposed.
If *transb* = 'C' or 'c', it is assumed that *B* should be conjugate transposed.
If *transb* = 'R' or 'r', it is assumed that *B* should be only conjugated.
If the data is real, then *transb* = 'R' is the same as *transb* = 'N', and *transb* = 'C' is the same as *transb* = 'T'.

m — INTEGER. The number of matrix rows.

n — INTEGER. The number of matrix columns.

alpha — REAL for mkl_somatadd.
DOUBLE PRECISION for mkl_domatadd.
COMPLEX for mkl_comatadd.
DOUBLE COMPLEX for mkl_zomatadd.
This parameter scales the input matrix by *alpha*.

a — REAL for mkl_somatadd.
DOUBLE PRECISION for mkl_domatadd.
COMPLEX for mkl_comatadd.
DOUBLE COMPLEX for mkl_zomatadd.
Array, DIMENSION *a*(*lda*,*).

lda — INTEGER. Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix *A*; measured in the number of elements.
This parameter must be at least max(1,*rows*) if *ordering* = 'C' or 'c', and max(1,*cols*) otherwise.

beta — REAL for mkl_somatadd.
DOUBLE PRECISION for mkl_domatadd.
COMPLEX for mkl_comatadd.
DOUBLE COMPLEX for mkl_zomatadd.
This parameter scales the input matrix by *beta*.

b — REAL for mkl_somatadd.
DOUBLE PRECISION for mkl_domatadd.
COMPLEX for mkl_comatadd.
DOUBLE COMPLEX for mkl_zomatadd.
Array, DIMENSION *b*(*ldb*,*).

ldb — INTEGER. Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix *B*; measured in the number of elements.
This parameter must be at least max(1,*rows*) if *ordering* = 'C' or 'c', and max(1,*cols*) otherwise.

Output Parameters

c — REAL for mkl_somatadd.
DOUBLE PRECISION for mkl_domatadd.
COMPLEX for mkl_comatadd.
DOUBLE COMPLEX for mkl_zomatadd.
Array, DIMENSION *c*(*ldc*,*).

ldc INTEGER. Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the destination matrix *c*; measured in the number of elements. To determine the minimum value of *ldc*, consider the following guideline:
If *ordering* = 'C' or 'c', then

- If *transa* or *transb* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*rows*)
- If *transa* or *transb* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*cols*)

If *ordering* = 'R' or 'r', then

- If *transa* or *transb* = 'T' or 't' or 'C' or 'c', this parameter must be at least max(1,*cols*)
- If *transa* or *transb* = 'N' or 'n' or 'R' or 'r', this parameter must be at least max(1,*rows*)

Interfaces

FORTRAN 77:

```
SUBROUTINE mkl_somatadd ( ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc )
   CHARACTER*1 ordering, transa, transb
   INTEGER m, n, lda, ldb, ldc
   REAL alpha, beta
   REAL a(lda,*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_domatadd ( ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc )
   CHARACTER*1 ordering, transa, transb
   INTEGER m, n, lda, ldb, ldc
   DOUBLE PRECISION alpha, beta
   DOUBLE PRECISION a(lda,*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_comatadd ( ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc )
   CHARACTER*1 ordering, transa, transb
   INTEGER m, n, lda, ldb, ldc
   COMPLEX alpha, beta
   COMPLEX a(lda,*), b(ldb,*), c(ldc,*)
```

```
SUBROUTINE mkl_zomatadd ( ordering, transa, transb, m, n, alpha, a, lda, beta, b, ldb, c, ldc )
   CHARACTER*1 ordering, transa, transb
   INTEGER m, n, lda, ldb, ldc
   DOUBLE COMPLEX alpha, beta
   DOUBLE COMPLEX a(lda,*), b(ldb,*), c(ldc,*)
```

C:

```
void mkl_somatadd(char ordering, char transa, char transb, size_t m, size_t n, float *alpha, float *A,
size_t lda, float *beta, float *B, size_t ldb, float *C, size_t ldc);
```

```
void mkl_domatadd(char ordering, char transa, char transb, size_t m, size_t n, double *alpha, double
*A, size_t lda, double *beta, float *B, size_t ldb, double *C, size_t ldc);
```

```
void mkl_comatadd(char ordering, char transa, char transb, size_t m, size_t n,
 MKL_Complex8 *alpha, MKL_Complex8 *A, size_t lda, float *beta, float *B, size_t ldb, MKL_Complex8 *C,
size_t ldc);
```

```
void mkl_zomatadd(char ordering, char transa, char transb, size_t m, size_t n,
 MKL_Complex16 *alpha, MKL_Complex16 *A, size_t lda, float *beta, float *B, size_t ldb, MKL_Complex16
*C, size_t ldc);
```

LAPACK Routines: Linear Equations

3

This chapter describes the Intel® Math Kernel Library implementation of routines from the LAPACK package that are used for solving systems of linear equations and performing a number of related computational tasks. The library includes LAPACK routines for both real and complex data. Routines are supported for systems of equations with the following types of matrices:

- general
- banded
- symmetric or Hermitian positive-definite (full, packed, and rectangular full packed (RFP) storage)
- symmetric or Hermitian positive-definite banded
- symmetric or Hermitian indefinite (both full and packed storage)
- symmetric or Hermitian indefinite banded
- triangular (full, packed, and RFP storage)
- triangular banded
- tridiagonal
- diagonally dominant tridiagonal.

For each of the above matrix types, the library includes routines for performing the following computations:

- factoring the matrix (except for triangular matrices)
- equilibrating the matrix (except for RFP matrices)
- solving a system of linear equations
- estimating the condition number of a matrix (except for RFP matrices)
- refining the solution of linear equations and computing its error bounds (except for RFP matrices)
- inverting the matrix.

To solve a particular problem, you can call two or more computational routines or call a corresponding driver routine that combines several tasks in one call. For example, to solve a system of linear equations with a general matrix, call `?getrf` (*LU* factorization) and then `?getrs` (computing the solution). Then, call `?gerfs` to refine the solution and get the error bounds. Alternatively, use the driver routine `?gesvx` that performs all these tasks in one call.

WARNING LAPACK routines assume that input matrices do not contain IEEE 754 special values such as `INF` or `NaN` values. Using these special values may cause LAPACK to return unexpected results or become unstable.

Starting from release 8.0, Intel MKL along with the FORTRAN 77 interface to LAPACK computational and driver routines also supports the Fortran 95 interface that uses simplified routine calls with shorter argument lists. The syntax section of the routine description gives the calling sequence for the Fortran 95 interface, where available, immediately after the FORTRAN 77 calls.

Routine Naming Conventions

To call each routine introduced in this chapter from the FORTRAN 77 program, you can use the LAPACK name.

LAPACK names are listed in Table "Computational Routines for Systems of Equations with Real Matrices" and Table "Computational Routines for Systems of Equations with Complex Matrices", and have the structure `?yyzzz` or `?yyzz`, which is described below.

The initial symbol `?` indicates the data type:

`s`	real, single precision

c	complex, single precision
d	real, double precision
z	complex, double precision

Some routines can have combined character codes, such as ds or zc.

The second and third letters yy indicate the matrix type and storage scheme:

ge	general
gb	general band
gt	general tridiagonal
dt	diagonally dominant tridiagonal
po	symmetric or Hermitian positive-definite
pp	symmetric or Hermitian positive-definite (packed storage)
pf	symmetric or Hermitian positive-definite (RFP storage)
pb	symmetric or Hermitian positive-definite band
pt	symmetric or Hermitian positive-definite tridiagonal
sy	symmetric indefinite
sp	symmetric indefinite (packed storage)
he	Hermitian indefinite
hp	Hermitian indefinite (packed storage)
tr	triangular
tp	triangular (packed storage)
tf	triangular (RFP storage)
tb	triangular band

The last three letters zzz indicate the computation performed:

trf	perform a triangular matrix factorization
trs	solve the linear system with a factored matrix
con	estimate the matrix condition number
rfs	refine the solution and compute error bounds
rfsx	refine the solution and compute error bounds using extra-precise iterative refinement
tri	compute the inverse matrix using the factorization
equ, equb	equilibrate a matrix.

For example, the sgetrf routine performs the triangular factorization of general real matrices in single precision; the corresponding routine for complex matrices is cgetrf.

Driver routine names can end with -sv (meaning a *simple* driver), or with -svx (meaning an *expert* driver) or with -svxx (meaning an extra-precise iterative refinement *expert* driver).

The Fortran 95 interfaces to the LAPACK computational and driver routines are the same as the FORTRAN 77 names but without the first letter that indicates the data type. For example, the name of the routine that performs a triangular factorization of general real matrices in Fortran 95 is getrf. Different data types are handled through the definition of a specific internal parameter that refers to a module block with named constants for single and double precision.

C Interface Conventions

The C interfaces are implemented for most of the Intel MKL LAPACK driver and computational routines.

The arguments of the C interfaces for the Intel MKL LAPACK functions comply with the following rules:

- Scalar input arguments are passed by value.

- Array arguments are passed by reference.
- Array input arguments are declared with the `const` modifier.
- Function arguments are passed by pointer.
- An integer return value replaces the *info* output parameter. The return value equal to 0 means the function operation is completed successfully. See also special error codes below.

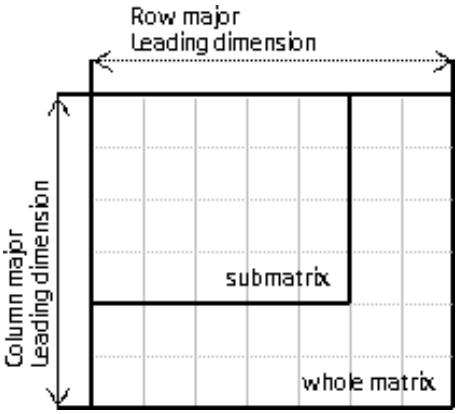
Matrix Order

Most of the LAPACK C interfaces have an additional parameter *matrix_order* of type `int` as their first argument. This parameter specifies whether the two-dimensional arrays are row-major (`LAPACK_ROW_MAJOR`) or column-major (`LAPACK_COL_MAJOR`).

In general the leading dimension *lda* is equal to the number of elements in the major dimension. It is also equal to the distance in elements between two neighboring elements in a line in the minor dimension. If there are no extra elements in a matrix with *m* rows and *n* columns, then

- For row-major ordering: the number of elements in a row is *n*, and row *i* is stored in memory right after row *i*-1. Therefore *lda* is *n*.
- For column-major ordering: the number of elements in a column is *m*, and column *i* is stored in memory right after column *i*-1. Therefore *lda* is *m*.

To refer to a submatrix with dimensions *k* by *l*, use the number of elements in the major dimension of the whole matrix (as above) as the leading dimension and *k* and *l* in the subroutine's input parameters to describe the size of the submatrix.



Workspace Arrays

The LAPACK C interface omits workspace parameters because workspace is allocated during runtime and released upon completion of the function operation.

For some functions, *work* arrays contain valuable information on exit. In such cases, the interface contains an additional argument or arguments, namely:

- `?gesvx` and `?gbsvx` contain *rpivot*
- `?gesvd` contains *superb*
- `?gejsv` and `?gesvj` contain *istat* and *stat*, respectively.

Function Types

The function types are used in non-symmetric eigenproblem functions only.

```
typedef lapack_logical (*LAPACK_S_SELECT2) (const float*, const float*);
typedef lapack_logical (*LAPACK_S_SELECT3) (const float*, const float*, const float*);
typedef lapack_logical (*LAPACK_D_SELECT2) (const double*, const double*);
typedef lapack_logical (*LAPACK_D_SELECT3) (const double*, const double*, const double*);
```

```
typedef lapack_logical (*LAPACK_C_SELECT1) (const lapack_complex_float*);
typedef lapack_logical (*LAPACK_C_SELECT2) (const lapack_complex_float*, const lapack_complex_float*);
typedef lapack_logical (*LAPACK_Z_SELECT1) (const lapack_complex_double*);
typedef lapack_logical (*LAPACK_Z_SELECT2) (const lapack_complex_double*, const lapack_complex_double*);
```

Mapping FORTRAN Data Types against C Data Types

FORTRAN Data Types vs. C Data Types

FORTRAN	C
INTEGER	lapack_int
LOGICAL	lapack_logical
REAL	float
DOUBLE PRECISION	double
COMPLEX	lapack_complex_float
COMPLEX*16/DOUBLE COMPLEX	lapack_complex_double
CHARACTER	char

C Type Definitions

```
#ifndef lapack_int
#define lapack_int MKL_INT
#endif

#ifndef lapack_logical
#define lapack_logical lapack_int
#endif
```

Complex Type Definitions

Complex type for single precision:

```
#ifndef lapack_complex_float
#define lapack_complex_float   MKL_Complex8
#endif
```

Complex type for double precision:

```
#ifndef lapack_complex_double
#define lapack_complex_double   MKL_Complex16
#endif
```

Matrix Order Definitions

```
#define LAPACK_ROW_MAJOR  101
#define LAPACK_COL_MAJOR  102
```

See Matrix Order for an explanation of row-major order and column-major order storage.

Error Code Definitions

```
#define LAPACK_WORK_MEMORY_ERROR       -1010  /* Failed to allocate memory
                                                 for a working array */
#define LAPACK_TRANSPOSE_MEMORY_ERROR  -1011  /* Failed to allocate memory
                                                 for transposed matrix */
```

If the return value is `-i`, the `-i`-th parameter has an invalid value.

Function Prototypes

Some Intel MKL functions differ in data types they support and vary in the parameters they take.

Each function type has a unique prototype defined. Use this prototype when you call the function from your application program. In most cases, Intel MKL supports four distinct floating-point precisions. Each corresponding prototype looks similar, usually differing only in the data type. To avoid listing all the prototypes in every supported precision, a generic prototype template is provided.

<?> denotes precision and is s, d, c, or z:

- s for real, single precision
- d for real, double precision
- c for complex, single precision
- z for complex, double precision

<datatype> stands for a respective data type: float, double, lapack_complex_float, or lapack_complex_double.

For example, the C prototype template for the ?pptrs function that solves a system of linear equations with a packed Cholesky-factored symmetric (Hermitian) positive-definite matrix looks as follows:

```
lapack_int LAPACKE_<?>pptrs(int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const <datatype>* ap, <datatype>* b, lapack_int ldb);
```

To obtain the function name and parameter list that corresponds to a specific precision, replace the <?> symbol with s, d, c, or z and the <datatype> field with the corresponding data type (float, double, lapack_complex_float, or lapack_complex_double respectively).

A specific example follows. To solve a system of linear equations with a packed Cholesky-factored Hermitian positive-definite matrix with complex precision, use the following:

```
lapack_int LAPACKE_cpptrs(int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* ap, lapack_complex_float* b, lapack_int ldb);
```

NOTE For the *select* parameter, the respective values of the <datatype> field for s, d, c, or z are as follows: LAPACK_S_SELECT3, LAPACK_D_SELECT3, LAPACK_C_SELECT2, and LAPACK_Z_SELECT2.

Fortran 95 Interface Conventions

Intel® MKL implements the Fortran 95 interface to LAPACK through wrappers that call respective FORTRAN 77 routines. This interface uses such Fortran 95 features as assumed-shape arrays and optional arguments to provide simplified calls to LAPACK routines with fewer arguments.

NOTE For LAPACK, Intel MKL offers two types of the Fortran 95 interfaces:

- using mkl_lapack.fi only through the include 'mkl_lapack.fi' statement. Such interfaces allow you to make use of the original LAPACK routines with all their arguments
- using lapack.f90 that includes improved interfaces. This file is used to generate the module files lapack95.mod and f95_precision.mod. The module files mkl95_lapack.mod and mkl95_precision.mod are also generated. See also the section "Fortran 95 interfaces and wrappers to LAPACK and BLAS" of the *Intel® MKL User's Guide* for details. The module files are used to process the FORTRAN use clauses referencing the LAPACK interface: use lapack95 (or an equivalent use mkl95_lapack) and use f95_precision (or an equivalent use mkl95_precision).

The main conventions for the Fortran 95 interface are as follows:

- The names of arguments used in Fortran 95 call are typically the same as for the respective generic (FORTRAN 77) interface. In rare cases, formal argument names may be different. For instance, *select* instead of *selctg*.
- Input arguments such as array dimensions are not required in Fortran 95 and are skipped from the calling sequence. Array dimensions are reconstructed from the user data that must exactly follow the required array shape.

  Another type of generic arguments that are skipped in the Fortran 95 interface are arguments that represent workspace arrays (such as *work*, *rwork*, and so on). The only exception are cases when workspace arrays return significant information on output.

  An argument can also be skipped if its value is completely defined by the presence or absence of another argument in the calling sequence, and the restored value is the only meaningful value for the skipped argument.
- Some generic arguments are declared as optional in the Fortran 95 interface and may or may not be present in the calling sequence. An argument can be declared optional if it meets one of the following conditions:
  - If an argument value is completely defined by the presence or absence of another argument in the calling sequence, it can be declared optional. The difference from the skipped argument in this case is that the optional argument can have some meaningful values that are distinct from the value reconstructed by default. For example, if some argument (like *jobz*) can take only two values and one of these values directly implies the use of another argument, then the value of *jobz* can be uniquely reconstructed from the actual presence or absence of this second argument, and *jobz* can be omitted.
  - If an input argument can take only a few possible values, it can be declared as optional. The default value of such argument is typically set as the first value in the list and all exceptions to this rule are explicitly stated in the routine description.
  - If an input argument has a natural default value, it can be declared as optional. The default value of such optional argument is set to its natural default value.
- Argument *info* is declared as optional in the Fortran 95 interface. If it is present in the calling sequence, the value assigned to *info* is interpreted as follows:
  - If this value is more than -1000, its meaning is the same as in the FORTRAN 77 routine.
  - If this value is equal to -1000, it means that there is not enough work memory.
  - If this value is equal to -1001, incompatible arguments are present in the calling sequence.
  - If this value is equal to -*i*, the *i*th parameter (counting parameters in the FORTRAN 77 interface, not the Fortran 95 interface) had an illegal value.
- Optional arguments are given in square brackets in the Fortran 95 call syntax.

The "Fortran 95 Notes" subsection at the end of the topic describing each routine details concrete rules for reconstructing the values of the omitted optional parameters.

Intel® MKL Fortran 95 Interfaces for LAPACK Routines vs. Netlib Implementation

The following list presents general digressions of the Intel MKL LAPACK95 implementation from the Netlib analog:

- The Intel MKL Fortran 95 interfaces are provided for pure procedures.
- Names of interfaces do not contain the `LA_` prefix.
- An optional array argument always has the `target` attribute.
- Functionality of the Intel MKL LAPACK95 wrapper is close to the FORTRAN 77 original implementation in the `getrf`, `gbtrf`, and `potrf` interfaces.
- If *jobz* argument value specifies presence or absence of *z* argument, then *z* is always declared as optional and *jobz* is restored depending on whether *z* is present or not. It is not always so in the Netlib version (see "Modified Netlib Interfaces" in Appendix E).
- To avoid double error checking, processing of the *info* argument is limited to checking of the allocated memory and disarranging of optional arguments.
- If an argument that is present in the list of arguments completely defines another argument, the latter is always declared as optional.

You can transform an application that uses the Netlib LAPACK interfaces to ensure its work with the Intel MKL interfaces providing that:

a. The application is correct, that is, unambiguous, compiler-independent, and contains no errors.

b. Each routine name denotes only one specific routine. If any routine name in the application coincides with a name of the original Netlib routine (for example, after removing the `LA_` prefix) but denotes a routine different from the Netlib original routine, this name should be modified through context name replacement.

You should transform your application in the following cases (see Appendix E for specific differences of individual interfaces):

- When using the Netlib routines that differ from the Intel MKL routines only by the `LA_` prefix or in the array attribute `target`. The only transformation required in this case is context name replacement. See "Interfaces Identical to Netlib" in Appendix E for details.
- When using Netlib routines that differ from the Intel MKL routines by the `LA_` prefix, the `target` array attribute, and the names of formal arguments. In the case of positional passing of arguments, no additional transformation except context name replacement is required. In the case of the keywords passing of arguments, in addition to the context name replacement the names of mismatching keywords should also be modified. See "Interfaces with Replaced Argument Names" in Appendix E for details.
- When using the Netlib routines that differ from the respective Intel MKL routines by the `LA_` prefix, the `target` array attribute, sequence of the arguments, arguments missing in Intel MKL but present in Netlib and, vice versa, present in Intel MKL but missing in Netlib. Remove the differences in the sequence and range of the arguments in process of all the transformations when you use the Netlib routines specified by this bullet and the preceding bullet. See "Modified Netlib Interfaces" in Appendix E for details.
- When using the `getrf`, `gbtrf`, and `potrf` interfaces, that is, new functionality implemented in Intel MKL but unavailable in the Netlib source. To override the differences, build the desired functionality explicitly with the Intel MKL means or create a new subroutine with the new functionality, using specific MKL interfaces corresponding to LAPACK 77 routines. You can call the LAPACK 77 routines directly but using the new Intel MKL interfaces is preferable. See "Interfaces Absent From Netlib" and "Interfaces of New Functionality" in Appendix E for details. Note that if the transformed application calls `getrf`, `gbtrf` or `potrf` without controlling arguments *rcond* and *norm*, just context name replacement is enough in modifying the calls into the Intel MKL interfaces, as described in the first bullet above. The Netlib functionality is preserved in such cases.
- When using the Netlib auxiliary routines. In this case, call a corresponding subroutine directly, using the Intel MKL LAPACK 77 interfaces.

Transform your application as follows:

1. Make sure conditions a. and b. are met.
2. Select Netlib LAPACK 95 calls. For each call, do the following:
   - Select the type of digression and do the required transformations.
   - Revise results to eliminate unneeded code or data, which may appear after several identical calls.
3. Make sure the transformations are correct and complete.

Matrix Storage Schemes

LAPACK routines use the following matrix storage schemes:

- *Full storage*: a matrix A is stored in a two-dimensional array a, with the matrix element a_{ij} stored in the array element $a(i,j)$.
- *Packed storage* scheme allows you to store symmetric, Hermitian, or triangular matrices more compactly: the upper or lower triangle of the matrix is packed by columns in a one-dimensional array.
- *Band storage*: an m-by-n band matrix with kl sub-diagonals and ku superdiagonals is stored compactly in a two-dimensional array ab with $kl+ku+1$ rows and n columns. Columns of the matrix are stored in the corresponding columns of the array, and *diagonals* of the matrix are stored in rows of the array.
- *Rectangular Full Packed (RFP) storage*: the upper or lower triangle of the matrix is packed combining the full and packed storage schemes. This combination enables using half of the full storage as packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels as the full storage.

In Chapters 4 and 5, arrays that hold matrices in packed storage have names ending in p; arrays with matrices in band storage have names ending in b; arrays with matrices in the RFP storage have names ending in fp.

For more information on matrix storage schemes, see "Matrix Arguments" in Appendix B.

Mathematical Notation

Descriptions of LAPACK routines use the following notation:

$Ax = b$	A system of linear equations with an n-by-n matrix $A = \{a_{ij}\}$, a right-hand side vector $b = \{b_i\}$, and an unknown vector $x = \{x_i\}$.
$AX = B$	A set of systems with a common matrix A and multiple right-hand sides. The columns of B are individual right-hand sides, and the columns of X are the corresponding solutions.
$\|x\|$	the vector with elements $\|x_i\|$ (absolute values of x_i).
$\|A\|$	the matrix with elements $\|a_{ij}\|$ (absolute values of a_{ij}).
$\|\|x\|\|_\infty = \max_i \|x_i\|$	The *infinity-norm* of the vector x.
$\|\|A\|\|_\infty = \max_i \Sigma_j \|a_{ij}\|$	The *infinity-norm* of the matrix A.
$\|\|A\|\|_1 = \max_j \Sigma_i \|a_{ij}\|$	The *one-norm* of the matrix A. $\|\|A\|\|_1 = \|\|A^T\|\|_\infty = \|\|A^H\|\|_\infty$
$\kappa(A) = \|\|A\|\| \ \|\|A^{-1}\|\|$	The *condition number* of the matrix A.

Error Analysis

In practice, most computations are performed with rounding errors. Besides, you often need to solve a system $Ax = b$, where the data (the elements of A and b) are not known exactly. Therefore, it is important to understand how the data errors and rounding errors can affect the solution x.

Data perturbations. If x is the exact solution of $Ax = b$, and $x + \delta x$ is the exact solution of a perturbed problem $(A + \delta A)x = (b + \delta b)$, then

where

$$\kappa(A) = \|A\| \left\|A^{-1}\right\|.$$

In other words, relative errors in A or b may be amplified in the solution vector x by a factor $\kappa(A) = \|\|A\|\| \ \|\|A^{-1}\|\|$ called the *condition number* of A.

Rounding errors have the same effect as relative perturbations $c(n)\varepsilon$ in the original data. Here ε is the *machine precision*, and $c(n)$ is a modest function of the matrix order n. The corresponding solution error is $\|\|\delta x\|\| / \|\|x\|\| \leq c(n)\kappa(A)\varepsilon$. (The value of $c(n)$ is seldom greater than $10n$.)

Thus, if your matrix A is *ill-conditioned* (that is, its condition number $\kappa(A)$ is very large), then the error in the solution x is also large; you may even encounter a complete loss of precision. LAPACK provides routines that allow you to estimate $\kappa(A)$ (see Routines for Estimating the Condition Number) and also give you a more precise estimate for the actual solution error (see Refining the Solution and Estimating Its Error).

Computational Routines

Table "Computational Routines for Systems of Equations with Real Matrices" lists the LAPACK computational routines (FORTRAN 77 and Fortran 95 interfaces) for factorizing, equilibrating, and inverting *real* matrices, estimating their condition numbers, solving systems of equations with real matrices, refining the solution, and estimating its error. Table "Computational Routines for Systems of Equations with Complex Matrices" lists similar routines for *complex* matrices. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Systems of Equations with Real Matrices

Matrix type, storage scheme	Factorize matrix	Equilibrate matrix	Solve system	Condition number	Estimate error	Invert matrix
general	?getrf	?geequ, ?geequb	?getrs	?gecon	?gerfs, ?gerfsx	?getri
general band	?gbtrf	?gbequ, ?gbequb	?gbtrs	?gbcon	?gbrfs, ?gbrfsx	
general tridiagonal	?gttrf		?gttrs	?gtcon	?gtrfs	
diagonally dominant tridiagonal	?dttrfb		?dttrsb			
symmetric positive-definite	?potrf	?poequ, ?poequb	?potrs	?pocon	?porfs, ?porfsx	?potri
symmetric positive-definite, packed storage	?pptrf	?ppequ	?pptrs	?ppcon	?pprfs	?pptri
symmetric positive-definite, RFP storage	?pftrf		?pftrs			?pftri
symmetric positive-definite, band	?pbtrf	?pbequ	?pbtrs	?pbcon	?pbrfs	
symmetric positive-definite, tridiagonal	?pttrf		?pttrs	?ptcon	?ptrfs	
symmetric indefinite	?sytrf	?syequb	?sytrs ?sytrs2	?sycon ?syconv	?syrfs, ?syrfsx	?sytri ?sytri2 ?sytri2x
symmetric indefinite, packed storage	?sptrf		?sptrs	?spcon	?sprfs	?sptri
triangular			?trtrs	?trcon	?trrfs	?trtri
triangular, packed storage			?tptrs	?tpcon	?tprfs	?tptri
triangular, RFP storage						?tftri
triangular band			?tbtrs	?tbcon	?tbrfs	

In the table above, ? denotes s (single precision) or d (double precision) for the FORTRAN 77 interface.

Computational Routines for Systems of Equations with Complex Matrices

Matrix type, storage scheme	Factorize matrix	Equilibrate matrix	Solve system	Condition number	Estimate error	Invert matrix
general	?getrf	?geequ, ?geequb	?getrs	?gecon	?gerfs, ?gerfsx	?getri
general band	?gbtrf	?gbequ, ?gbequb	?gbtrs	?gbcon	?gbrfs, ?gbrfsx	
general tridiagonal	?gttrf		?gttrs	?gtcon	?gtrfs	
Hermitian positive-definite	?potrf	?poequ, ?poequb	?potrs	?pocon	?porfs, ?porfsx	?potri
Hermitian positive-definite, packed storage	?pptrf	?ppequ	?pptrs	?ppcon	?pprfs	?pptri
Hermitian positive-definite, RFP storage	?pftrf		?pftrs			?pftri
Hermitian positive-definite, band	?pbtrf	?pbequ	?pbtrs	?pbcon	?pbrfs	
Hermitian positive-definite, tridiagonal	?pttrf		?pttrs	?ptcon	?ptrfs	
Hermitian indefinite	?hetrf	?heequb	?hetrs ?hetrs2	?hecon	?herfs, ?herfsx	?hetri ?hetri2 ?hetri2x
symmetric indefinite	?sytrf	?syequb	?sytrs ?sytrs2	?sycon ?syconv	?syrfs, ?syrfsx	?sytri ?sytri2 ?sytri2x
Hermitian indefinite, packed storage	?hptrf		?hptrs	?hpcon	?hprfs	?hptri
symmetric indefinite, packed storage	?sptrf		?sptrs	?spcon	?sprfs	?sptri
triangular			?trtrs	?trcon	?trrfs	?trtri
triangular, packed storage			?tptrs	?tpcon	?tprfs	?tptri
triangular, RFP storage						?tftri
triangular band			?tbtrs	?tbcon	?tbrfs	

In the table above, ? stands for c (single precision complex) or z (double precision complex) for FORTRAN 77 interface.

Routines for Matrix Factorization

This section describes the LAPACK routines for matrix factorization. The following factorizations are supported:

- *LU* factorization
- Cholesky factorization of real symmetric positive-definite matrices
- Cholesky factorization of real symmetric positive-definite matrices with pivoting
- Cholesky factorization of Hermitian positive-definite matrices
- Cholesky factorization of Hermitian positive-definite matrices with pivoting
- Bunch-Kaufman factorization of real and complex symmetric matrices
- Bunch-Kaufman factorization of Hermitian matrices.

You can compute:

- the *LU* factorization using full and band storage of matrices
- the Cholesky factorization using full, packed, RFP, and band storage
- the Bunch-Kaufman factorization using full and packed storage.

?getrf

Computes the LU factorization of a general m-by-n matrix.

Syntax

Fortran 77:

```
call sgetrf( m, n, a, lda, ipiv, info )
call dgetrf( m, n, a, lda, ipiv, info )
call cgetrf( m, n, a, lda, ipiv, info )
call zgetrf( m, n, a, lda, ipiv, info )
```

Fortran 95:

```
call getrf( a [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>getrf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the *LU* factorization of a general *m*-by-*n* matrix *A* as

```
A = P*L*U,
```

where *P* is a permutation matrix, *L* is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and *U* is upper triangular (upper trapezoidal if $m < n$). The routine uses partial pivoting, with row interchanges.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix *A* ($m \geq 0$).
n	INTEGER. The number of columns in *A*; $n \geq 0$.
a	REAL for sgetrf DOUBLE PRECISION for dgetrf COMPLEX for cgetrf DOUBLE COMPLEX for zgetrf. Array, DIMENSION (*lda*,*). Contains the matrix *A*. The second dimension of *a* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of array *a*.

Output Parameters

a	Overwritten by *L* and *U*. The unit diagonal elements of *L* are not stored.
ipiv	INTEGER. Array, DIMENSION at least max(1,min(*m*, *n*)). The pivot indices; for $1 \leq i \leq \min(m, n)$, row *i* was interchanged with row *ipiv*(*i*).
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, u_{ii} is 0. The factorization has been completed, but *U* is exactly singular. Division by 0 will occur if you use the factor *U* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine getrf interface are as follows:

a	Holds the matrix *A* of size (*m*, *n*).
ipiv	Holds the vector of length min(*m*, *n*).

Application Notes

The computed *L* and *U* are the exact factors of a perturbed matrix *A* + *E*, where

$$|E| \leq c(\min(m,n))\varepsilon P|L||U|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

The approximate number of floating-point operations for real flavors is

$(2/3)n^3$	If $m = n$,
$(1/3)n^2(3m-n)$	If $m > n$,
$(1/3)m^2(3n-m)$	If $m < n$.

The number of operations for complex flavors is four times greater.

After calling this routine with $m = n$, you can call the following:

?getrs	to solve $A*x = B$ or $A^T X = B$ or $A^H X = B$
?gecon	to estimate the condition number of A
?getri	to compute the inverse of A.

See Also

mkl_progress

?gbtrf

Computes the LU factorization of a general m-by-n band matrix.

Syntax

Fortran 77:

```
call sgbtrf( m, n, kl, ku, ab, ldab, ipiv, info )
call dgbtrf( m, n, kl, ku, ab, ldab, ipiv, info )
call cgbtrf( m, n, kl, ku, ab, ldab, ipiv, info )
call zgbtrf( m, n, kl, ku, ab, ldab, ipiv, info )
```

Fortran 95:

```
call gbtrf( ab [,kl] [,m] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>gbtrf( int matrix_order, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, <datatype>* ab, lapack_int ldab, lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the LU factorization of a general m-by-n band matrix A with kl non-zero subdiagonals and ku non-zero superdiagonals, that is,

```
A = P*L*U,
```

where P is a permutation matrix; L is lower triangular with unit diagonal elements and at most kl non-zero elements in each column; U is an upper triangular band matrix with $kl + ku$ superdiagonals. The routine uses partial pivoting, with row interchanges (which creates the additional kl superdiagonals in U).

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in matrix A; $m \geq 0$.

n	INTEGER. The number of columns in matrix *A*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
ab	REAL for sgbtrf DOUBLE PRECISION for dgbtrf COMPLEX for cgbtrf DOUBLE COMPLEX for zgbtrf. Array, DIMENSION (*ldab*,*). The array *ab* contains the matrix *A* in band storage, in rows *kl* + 1 to 2*kl* + *ku* + 1; rows 1 to *kl* of the array need not be set. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*kl* + *ku* + 1 + *i* – *j*, *j*) = *a*(*i*,*j*) for max(1,*j*–*ku*) ≤ *i* ≤ min(*m*,*j*+*kl*).
ldab	INTEGER. The leading dimension of the array *ab*. (*ldab* ≥ 2**kl* + *ku* + 1)

Output Parameters

ab	Overwritten by *L* and *U*. *U* is stored as an upper triangular band matrix with *kl* + *ku* superdiagonals in rows 1 to *kl* + *ku* + 1, and the multipliers used during the factorization are stored in rows *kl* + *ku* + 2 to 2**kl* + *ku* + 1. See Application Notes below for further details.
ipiv	INTEGER. Array, DIMENSION at least max(1,min(*m*, *n*)). The pivot indices; for 1 ≤ *i* ≤ min(*m*, *n*) , row *i* was interchanged with row *ipiv*(*i*). .
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = *i*, u_{ii} is 0. The factorization has been completed, but *U* is exactly singular. Division by 0 will occur if you use the factor *U* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbtrf interface are as follows:

ab	Holds the array *A* of size (2**kl*+*ku*+1,*n*).
ipiv	Holds the vector of length min(*m*,*n*).
kl	If omitted, assumed *kl* = *ku*.
ku	Restored as *ku* = *lda*–2**kl*–1.
m	If omitted, assumed *m* = *n*.

Application Notes

The computed *L* and *U* are the exact factors of a perturbed matrix *A* + *E*, where

$$|E| \leq c(kl+ku+1)\ \varepsilon\ P|L||U|$$

$c(k)$ is a modest linear function of k, and ε is the machine precision.

The total number of floating-point operations for real flavors varies between approximately $2n(ku+1)kl$ and $2n(kl+ku+1)kl$. The number of operations for complex flavors is four times greater. All these estimates assume that kl and ku are much less than $\min(m,n)$.

The band storage scheme is illustrated by the following example, when $m = n = 6$, $kl = 2$, $ku = 1$:

on entry

$$\begin{bmatrix} * & * & * & + & + & + \\ * & * & + & + & + & + \\ * & a_{12} & a_{23} & a_{34} & a_{45} & a_{56} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} & a_{66} \\ a_{21} & a_{32} & a_{43} & a_{54} & a_{65} & * \\ a_{31} & a_{42} & a_{53} & a_{64} & * & * \end{bmatrix}$$

on exit

$$\begin{bmatrix} * & * & * & u_{14} & u_{25} & u_{36} \\ * & * & u_{13} & u_{24} & u_{35} & u_{46} \\ * & u_{12} & u_{23} & u_{34} & u_{45} & u_{56} \\ u_{11} & u_{22} & u_{33} & u_{44} & u_{55} & u_{66} \\ m_{21} & m_{32} & m_{43} & m_{54} & m_{65} & * \\ m_{31} & m_{42} & m_{53} & m_{64} & * & * \end{bmatrix}$$

Array elements marked * are not used by the routine; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

After calling this routine with $m = n$, you can call the following routines:

gbtrs	to solve $A*X = B$ or $A^T*X = B$ or $A^H*X = B$
gbcon	to estimate the condition number of A.

See Also

mkl_progress

?gttrf

Computes the LU factorization of a tridiagonal matrix.

Syntax

Fortran 77:

```
call sgttrf( n, dl, d, du, du2, ipiv, info )
call dgttrf( n, dl, d, du, du2, ipiv, info )
call cgttrf( n, dl, d, du, du2, ipiv, info )
call zgttrf( n, dl, d, du, du2, ipiv, info )
```

Fortran 95:

```
call gttrf( dl, d, du, du2 [, ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>gttrf( lapack_int n, <datatype>* dl, <datatype>* d, <datatype>*
du, <datatype>* du2, lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the *LU* factorization of a real or complex tridiagonal matrix *A* in the form

```
A = P*L*U,
```

where *P* is a permutation matrix; *L* is lower bidiagonal with unit diagonal elements; and *U* is an upper triangular matrix with nonzeroes in only the main diagonal and first two superdiagonals. The routine uses elimination with partial pivoting and row interchanges.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix *A*; $n \geq 0$.
dl, *d*, *du*	REAL for sgttrf DOUBLE PRECISION for dgttrf COMPLEX for cgttrf DOUBLE COMPLEX for zgttrf. Arrays containing elements of *A*. The array *dl* of dimension (*n* – 1) contains the subdiagonal elements of *A*. The array *d* of dimension *n* contains the diagonal elements of *A*. The array *du* of dimension (*n* – 1) contains the superdiagonal elements of *A*.

Output Parameters

dl	Overwritten by the (*n*–1) multipliers that define the matrix *L* from the *LU* factorization of *A*. The matrix *L* has unit diagonal elements, and the (*n*–1) elements of *dl* form the subdiagonal. All other elements of *L* are zero.
d	Overwritten by the *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*.
du	Overwritten by the (*n*–1) elements of the first superdiagonal of *U*.
du2	REAL for sgttrf DOUBLE PRECISION for dgttrf COMPLEX for cgttrf DOUBLE COMPLEX for zgttrf. Array, dimension (*n* –2). On exit, *du2* contains (*n*–2) elements of the second superdiagonal of *U*.
ipiv	INTEGER. Array, dimension (*n*). The pivot indices: for $1 \leq i \leq n$, row *i* was interchanged with row *ipiv*(*i*). *ipiv*(*i*) is always *i* or *i*+1; *ipiv*(*i*) = *i* indicates a row interchange was not required.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = *i*, u_{ii} is 0. The factorization has been completed, but *U* is exactly singular. Division by zero will occur if you use the factor *U* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `gttrf` interface are as follows:

dl	Holds the vector of length (*n*-1).
d	Holds the vector of length *n*.
du	Holds the vector of length (*n*-1).
du2	Holds the vector of length (*n*-2).
ipiv	Holds the vector of length *n*.

Application Notes

?gbtrs	to solve $A*X = B$ or $A^T*X = B$ or $A^H*X = B$
?gbcon	to estimate the condition number of *A*.

?dttrfb

Computes the factorization of a diagonally dominant tridiagonal matrix.

Syntax

Fortran 77:

```
call sdttrfb( n, dl, d, du, info )
call ddttrfb( n, dl, d, du, info )
call cdttrfb( n, dl, d, du, info )
call zdttrfb( n, dl, d, du, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?dttrfb` routine computes the factorization of a real or complex diagonally dominant tridiagonal matrix *A* with the BABE (Burning At Both Ends) algorithm in the form

$A = L_1*U*L_2$

where

- L_1, L_2 are lower bidiagonal with unit diagonal elements corresponding to the Gaussian elimination taken from both ends of the matrix.
- *U* is an upper triangular matrix with nonzeroes in only the main diagonal and first two superdiagonals.

Input Parameters

n	`INTEGER`. The order of the matrix *A*; $n \geq 0$.
dl, *d*, *du*	`REAL` for `sdttrfb` `DOUBLE PRECISION` for `ddttrfb` `COMPLEX` for `cdttrfb` `DOUBLE COMPLEX` for `zdttrfb`.

	Arrays containing elements of *A*. The array *dl* of dimension *(n – 1)* contains the subdiagonal elements of *A*. The array *d* of dimension *n* contains the diagonal elements of *A*. The array *du* of dimension *(n – 1)* contains the superdiagonal elements of *A*.

Output Parameters

dl	Overwritten by the *(n –1)* multipliers that define the matrix *L* from the *LU* factorization of *A*.
d	Overwritten by the *n* diagonal element reciprocals of the upper triangular matrix *U* from the factorization of *A*.
info	INTEGER. If *info = 0*, the execution is successful. If *info = -i*, the *i*-th parameter had an illegal value. If *info = i*, u_{ii} is 0. The factorization has been completed, but *U* is exactly singular. Division by zero will occur if you use the factor *U* for solving a system of linear equations.

Application Notes

A diagonally dominant tridiagonal system is defined such that $|d_i| > |dl_{i-1}| + |du_i|$ for any i:

$1 < i < n$, and $|d_1| > |du_1|$, $|d_n| > |dl_{n-1}|$

The underlying BABE algorithm is designed for diagonally dominant systems. Such systems are free from the numerical stability issue unlike the canonical systems that use elimination with partial pivoting (see ?gttrf). The diagonally dominant systems are much faster than the canonical systems.

NOTE

- The current implementation of BABE has a potential accuracy issue on very small or large data close to the underflow or overflow threshold respectively. Scale the matrix before applying the solver in the case of such input data.
- Applying the ?dttrfb factorization to non-diagonally dominant systems may lead to an accuracy loss, or false singularity detected due to no pivoting.

?potrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite matrix.

Syntax

Fortran 77:

```
call spotrf( uplo, n, a, lda, info )
call dpotrf( uplo, n, a, lda, info )
call cpotrf( uplo, n, a, lda, info )
call zpotrf( uplo, n, a, lda, info )
```

Fortran 95:

```
call potrf( a [, uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>potrf( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix *A*:

A = U^T**U* for real data, *A* = U^H**U* for complex data if *uplo*='U'

A = *L**L^T for real data, *A* = *L**L^H for complex data if *uplo*='L'

where *L* is a lower triangular matrix and *U* is upper triangular.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo — CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored and how *A* is factored:
If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix *A*.
If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix *A*.

n — INTEGER. The order of matrix *A*; *n* ≥ 0.

a — REAL for spotrf
DOUBLE PRECISION for dpotrf
COMPLEX for cpotrf
DOUBLE COMPLEX for zpotrf.
Array, DIMENSION (*lda*,*). The array *a* contains either the upper or the lower triangular part of the matrix *A* (see *uplo*). The second dimension of *a* must be at least max(1, *n*).

lda — INTEGER. The leading dimension of *a*.

Output Parameters

a — The upper or lower triangular part of *a* is overwritten by the Cholesky factor *U* or *L*, as specified by *uplo*.

info — INTEGER. If *info*=0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

	If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix *A*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine potrf interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

If *uplo* = 'U', the computed factor *U* is the exact factor of a perturbed matrix *A* + *E*, where

$$|E| \le c(n)\varepsilon |U^H| |U|, |e_{ij}| \le c(n)\varepsilon \sqrt{a_{ii}a_{jj}}$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

A similar estimate holds for *uplo* = 'L'.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

?potrs	to solve *A***X* = *B*
?pocon	to estimate the condition number of *A*
?potri	to compute the inverse of *A*.

See Also

mkl_progress

?pstrf

Computes the Cholesky factorization with complete pivoting of a real symmetric (complex Hermitian) positive semidefinite matrix.

Syntax

Fortran 77:

```
call spstrf( uplo, n, a, lda, piv, rank, tol, work, info )
call dpstrf( uplo, n, a, lda, piv, rank, tol, work, info )
call cpstrf( uplo, n, a, lda, piv, rank, tol, work, info )
call zpstrf( uplo, n, a, lda, piv, rank, tol, work, info )
```

C:

```
lapack_int LAPACKE_spstrf( int matrix_order, char uplo, lapack_int n, float* a,
lapack_int lda, lapack_int* piv, lapack_int* rank, float tol );
lapack_int LAPACKE_dpstrf( int matrix_order, char uplo, lapack_int n, double* a,
lapack_int lda, lapack_int* piv, lapack_int* rank, double tol );
```

```
lapack_int LAPACKE_cpstrf( int matrix_order, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int* piv, lapack_int* rank, float
tol );

lapack_int LAPACKE_zpstrf( int matrix_order, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int* piv, lapack_int* rank, double
tol );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the Cholesky factorization with complete pivoting of a real symmetric (complex Hermitian) positive semidefinite matrix. The form of the factorization is:

$P^T * A * P = U^T * U$, if *uplo* = 'U' for real flavors,
$P^H * A * P = U^H * U$, if *uplo* = 'U' for complex flavors,
$P^T * A * P = L * L^T$, if *uplo* = 'L' for real flavors,
$P^H * A * P = L * L^H$, if *uplo* = 'L' for complex flavors,

where P is stored as vector *piv*, 'U' and 'L' are upper and lower triangular matrices respectively.

This algorithm does not attempt to check that A is positive semidefinite. This version of the algorithm calls level 3 BLAS.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix A, and the strictly lower triangular part of the matrix is not referenced. If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix A, and the strictly upper triangular part of the matrix is not referenced.
n	INTEGER. The order of matrix A; $n \geq 0$.
a, *work*	REAL for spstrf DOUBLE PRECISION for dpstrf COMPLEX for cpstrf DOUBLE COMPLEX for zpstrf. Array *a*, DIMENSION (*lda*,*). The array *a* contains either the upper or the lower triangular part of the matrix A (see *uplo*). The second dimension of *a* must be at least max(1, *n*). *work(*)* is a workspace array. The dimension of *work* is at least max(1,2*n).
tol	REAL for single precision flavors DOUBLE PRECISION for double precision flavors.

	User difined tolerance. If *tol* < 0, then n*U*max(a(k,k)) will be used. The algorithm terminates at the (k-1)-th step, if the pivot ≤ *tol*.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).

Output Parameters

a	If *info* = 0, the factor U or L from the Cholesky factorization is as described in *Description*.
piv	INTEGER. Array, DIMENSION at least max(1, *n*). The array *piv* is such that the nonzero entries are p(piv(k),k) = 1.
rank	INTEGER. The rank of *a* given by the number of steps the algorithm completed.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*k*, the *k*-th argument had an illegal value. If *info* > 0, the matrix *A* is either rank deficient with a computed rank as returned in *rank*, or is indefinite.

?pftrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite matrix using the Rectangular Full Packed (RFP) format .

Syntax

Fortran 77:

```
call spftrf( transr, uplo, n, a, info )
call dpftrf( transr, uplo, n, a, info )
call cpftrf( transr, uplo, n, a, info )
call zpftrf( transr, uplo, n, a, info )
```

C:

```
lapack_int LAPACKE_<?>pftrf( int matrix_order, char transr, char uplo, lapack_int n,
<datatype>* a );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, a Hermitian positive-definite matrix *A*:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data	if *uplo*='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data	if *uplo*='L'

where *L* is a lower triangular matrix and *U* is upper triangular.

The matrix *A* is in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see Matrix Storage Schemes.

This is the block version of the algorithm, calling Level 3 BLAS.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

transr	CHARACTER*1. Must be 'N', 'T' (for real data) or 'C' (for complex data). If *transr* = 'N', the Normal *transr* of RFP *A* is stored. If *transr* = 'T', the Transpose *transr* of RFP *A* is stored. If *transr* = 'C', the Conjugate-Transpose *transr* of RFP *A* is stored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix *A*. If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix *A*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a	REAL for spftrf DOUBLE PRECISION for dpftrf COMPLEX for cpftrf DOUBLE COMPLEX for zpftrf. Array, DIMENSION (*n**(*n*+1)/2). The array *a* contains the matrix *A* in the RFP format.

Output Parameters

a	The upper or lower triangular part of *a* is overwritten by the Cholesky factor *U* or *L*, as specified by *info*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix *A*.

?pptrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite matrix using packed storage.

Syntax

Fortran 77:

```
call spptrf( uplo, n, ap, info )
call dpptrf( uplo, n, ap, info )
call cpptrf( uplo, n, ap, info )
call zpptrf( uplo, n, ap, info )
```

Fortran 95:

```
call pptrf( ap [, uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>pptrf( int matrix_order, char uplo, lapack_int n, <datatype>*
ap );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite packed matrix A:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data if $uplo$='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data if $uplo$='L'

where L is a lower triangular matrix and U is upper triangular.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is packed in the array ap, and how A is factored: If $uplo$ = 'U', the array ap stores the upper triangular part of the matrix A, and A is factored as U^H*U. If $uplo$ = 'L', the array ap stores the lower triangular part of the matrix A; A is factored as $L*L^H$.
n	INTEGER. The order of matrix A; $n \geq 0$.
ap	REAL for spptrf DOUBLE PRECISION for dpptrf COMPLEX for cpptrf DOUBLE COMPLEX for zpptrf. Array, DIMENSION at least max(1, n(n+1)/2). The array ap contains either the upper or the lower triangular part of the matrix A (as specified by $uplo$) in packed storage (see Matrix Storage Schemes).

Output Parameters

ap	The upper or lower triangular part of A in packed storage is overwritten by the Cholesky factor U or L, as specified by $uplo$.
info	INTEGER. If $info$=0, the execution is successful. If $info$ = -i, the i-th parameter had an illegal value. If $info$ = i, the leading minor of order i (and therefore the matrix A itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix A.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `pptrf` interface are as follows:

ap	Holds the array *A* of size `(n*(n+1)/2)`.
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.

Application Notes

If *uplo* = `'U'`, the computed factor *U* is the exact factor of a perturbed matrix *A* + *E*, where

$$|E| \le c(n)\varepsilon |U^H||U|, |e_{ij}| \le c(n)\varepsilon \sqrt{a_{ii}a_{jj}}$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

A similar estimate holds for *uplo* = `'L'`.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors and $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

`?pptrs`	to solve *A***X* = *B*
`?ppcon`	to estimate the condition number of *A*
`?pptri`	to compute the inverse of *A*.

See Also

mkl_progress

?pbtrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite band matrix.

Syntax

Fortran 77:

```
call spbtrf( uplo, n, kd, ab, ldab, info )
call dpbtrf( uplo, n, kd, ab, ldab, info )
call cpbtrf( uplo, n, kd, ab, ldab, info )
call zpbtrf( uplo, n, kd, ab, ldab, info )
```

Fortran 95:

```
call pbtrf( ab [, uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>pbtrf( int matrix_order, char uplo, lapack_int n, lapack_int kd,
<datatype>* ab, lapack_int ldab );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`

- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite band matrix A:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data if $uplo$='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data if $uplo$='L'

where L is a lower triangular matrix and U is upper triangular.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored in the array *ab*, and how A is factored: If *uplo* = 'U', the upper triangle of A is stored. If *uplo* = 'L', the lower triangle of A is stored.
n	INTEGER. The order of matrix A; $n \geq 0$.
kd	INTEGER. The number of superdiagonals or subdiagonals in the matrix A; $kd \geq 0$.
ab	REAL for spbtrf DOUBLE PRECISION for dpbtrf COMPLEX for cpbtrf DOUBLE COMPLEX for zpbtrf. Array, DIMENSION (,*). The array *ab* contains either the upper or the lower triangular part of the matrix A (as specified by *uplo*) in band storage (see Matrix Storage Schemes). The second dimension of *ab* must be at least max(1, *n*).
ldab	INTEGER. The leading dimension of the array *ab*. (*ldab* ≥ *kd* + 1)

Output Parameters

ab	The upper or lower triangular part of A (in band storage) is overwritten by the Cholesky factor U or L, as specified by *uplo*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the leading minor of order *i* (and therefore the matrix A itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix A.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `pbtrf` interface are as follows:

ab	Holds the array *A* of size (*kd*+1,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

If *uplo* = 'U', the computed factor *U* is the exact factor of a perturbed matrix *A* + *E*, where

$$|E| \le c(kd+1)\varepsilon |U^H||U|, |e_{ij}| \le c(kd+1)\varepsilon\sqrt{a_{ii}a_{jj}}$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

A similar estimate holds for *uplo* = 'L'.

The total number of floating-point operations for real flavors is approximately $n(kd+1)^2$. The number of operations for complex flavors is 4 times greater. All these estimates assume that *kd* is much less than *n*.

After calling this routine, you can call the following routines:

?pbtrs	to solve *A***X* = *B*
?pbcon	to estimate the condition number of *A*.

See Also

mkl_progress

?pttrf

Computes the factorization of a symmetric (Hermitian) positive-definite tridiagonal matrix.

Syntax

Fortran 77:

```
call spttrf( n, d, e, info )
call dpttrf( n, d, e, info )
call cpttrf( n, d, e, info )
call zpttrf( n, d, e, info )
```

Fortran 95:

```
call pttrf( d, e [,info] )
```

C:

```
lapack_int LAPACKE_spttrf( lapack_int n, float* d, float* e );
lapack_int LAPACKE_dpttrf( lapack_int n, double* d, double* e );
lapack_int LAPACKE_cpttrf( lapack_int n, float* d, lapack_complex_float* e );
lapack_int LAPACKE_zpttrf( lapack_int n, double* d, lapack_complex_double* e );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite tridiagonal matrix A:

$A = L*D*L^T$ for real flavors, or

$A = L*D*L^H$ for complex flavors,

where D is diagonal and L is unit lower bidiagonal. The factorization may also be regarded as having the form $A = U^T*D*U$ for real flavors, or $A = U^H*D*U$ for complex flavors, where D is unit upper bidiagonal.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix A; $n \geq 0$.
d	REAL for sptttrf, cpttrf DOUBLE PRECISION for dpttrf, zpttrf. Array, dimension (n). Contains the diagonal elements of A.
e	REAL for sptttrf DOUBLE PRECISION for dpttrf COMPLEX for cpttrf DOUBLE COMPLEX for zpttrf. Array, dimension (n -1). Contains the subdiagonal elements of A.

Output Parameters

d	Overwritten by the n diagonal elements of the diagonal matrix D from the $L*D*L^T$ (for real flavors) or $L*D*L^H$ (for complex flavors) factorization of A.
e	Overwritten by the $(n - 1)$ off-diagonal elements of the unit bidiagonal factor L or U from the factorization of A.
$info$	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value. If $info = i$, the leading minor of order i (and therefore the matrix A itself) is not positive-definite; if $i < n$, the factorization could not be completed, while if $i = n$, the factorization was completed, but $d(n) \leq 0$.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pttrf interface are as follows:

d	Holds the vector of length n.
e	Holds the vector of length (n-1).

?sytrf

Computes the Bunch-Kaufman factorization of a symmetric matrix.

Syntax

Fortran 77:

```
call ssytrf( uplo, n, a, lda, ipiv, work, lwork, info )
call dsytrf( uplo, n, a, lda, ipiv, work, lwork, info )
call csytrf( uplo, n, a, lda, ipiv, work, lwork, info )
call zsytrf( uplo, n, a, lda, ipiv, work, lwork, info )
```

Fortran 95:

```
call sytrf( a [, uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sytrf( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the factorization of a real/complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is:

if *uplo*='U', $A = P*U*D*U^T*P^T$

if *uplo*='L', $A = P*L*D*L^T*P^T$,

where A is the input matrix, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D.

NOTE This routine supports the Progress Routine feature. See Progress Routine section for details.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If *uplo* = `'U'`, the array *a* stores the upper triangular part of the matrix A, and A is factored as $P*U*D*U^T*P^T$. If *uplo* = `'L'`, the array *a* stores the lower triangular part of the matrix A, and A is factored as $P*L*D*L^T*P^T$.
n	`INTEGER`. The order of matrix A; $n \geq 0$.
a	`REAL` for `ssytrf` `DOUBLE PRECISION` for `dsytrf` `COMPLEX` for `csytrf` `DOUBLE COMPLEX` for `zsytrf`.

	Array, DIMENSION (*lda*,*). The array *a* contains either the upper or the lower triangular part of the matrix *A* (see *uplo*). The second dimension of *a* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
work	Same type as *a*. A workspace array, dimension at least max(1,*lwork*).
lwork	INTEGER. The size of the *work* array (*lwork* ≥ *n*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See Application Notes for the suggested value of *lwork*.

Output Parameters

a	The upper or lower triangular part of *a* is overwritten by details of the block-diagonal matrix *D* and the multipliers used to obtain the factor *U* (or *L*).
work(1)	If *info*=0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*. If *ipiv*(*i*) = *k* >0, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column. If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column. If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, D_{ii} is 0. The factorization has been completed, but *D* is exactly singular. Division by 0 will occur if you use *D* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sytrf interface are as follows:

a	holds the matrix *A* of size (*n*, *n*)
ipiv	holds the vector of length *n*
uplo	must be 'U' or 'L'. The default value is 'U'.

Application Notes

For better performance, try using *lwork* = *n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The 2-by-2 unit diagonal blocks and the unit diagonal elements of *U* and *L* are not stored. The remaining elements of *U* and *L* are stored in the corresponding columns of the array *a*, but additional row interchanges are required to recover *U* or *L* explicitly (which is seldom necessary).

If *ipiv*(*i*) = *i* for all *i* =1...*n*, then all off-diagonal elements of *U* (*L*) are stored explicitly in the corresponding elements of the array *a*.

If *uplo* = 'U', the computed factors *U* and *D* are the exact factors of a perturbed matrix *A* + *E*, where

$|E| \leq c(n)\varepsilon\ P|U||D||U^T|P^T$

c(*n*) is a modest linear function of *n*, and ε is the machine precision. A similar estimate holds for the computed *L* and *D* when *uplo* = 'L'.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

?sytrs	to solve *A*X* = *B*
?sycon	to estimate the condition number of *A*
?sytri	to compute the inverse of *A*.

If *uplo* = 'U', then *A* = U*D*U', where

U = P(*n*)*U(*n*)* ... *P(*k*)*U(*k*)*...,

that is, U is a product of terms P(*k*)*U(*k*), where

- *k* decreases from *n* to 1 in steps of 1 and 2.
- D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(*k*).
- P(*k*) is a permutation matrix as defined by *ipiv*(*k*).
- U(*k*) is a unit upper triangular matrix, such that if the diagonal block D(*k*) is of order *s* (*s* = 1 or 2), then

$$U(k) = \begin{pmatrix} I & v & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \begin{matrix} k-s \\ s \\ n-k \end{matrix}$$
$$\begin{matrix} k-s & s & n-k \end{matrix}$$

If *s* = 1, D(*k*) overwrites *A*(*k*,*k*), and *v* overwrites *A*(1:*k*-1,*k*).

If s = 2, the upper triangle of D(k) overwrites A(k-1,k-1), A(k-1,k) and A(k,k), and v overwrites A(1:k-2,k-1:k).

If *uplo* = 'L', then *A* = L*D*L', where

```
L = P(1)*L(1)* ... *P(k)*L(k)*...,
```

that is, L is a product of terms P(k)*L(k), where

- k decreases from 1 to n in steps of 1 and 2.
- D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks D(k).
- P(k) is a permutation matrix as defined by *ipiv*(k).
- L(k) is a unit lower triangular matrix, such that if the diagonal block D(k) is of order s (s = 1 or 2), then

$$L(k) = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & v & I \end{pmatrix} \begin{matrix} k-1 \\ s \\ n-k-s+1 \end{matrix}$$
$$\begin{matrix} k-1 & s & n-k-s+1 \end{matrix}$$

If s = 1, D(k) overwrites A(k,k), and v overwrites A(k+1:n,k).

If s = 2, the lower triangle of D(k) overwrites A(k,k), A(k+1,k), and A(k+1,k+1), and v overwrites A(k+2:n,k:k+1).

See Also

mkl_progress

?hetrf

Computes the Bunch-Kaufman factorization of a complex Hermitian matrix.

Syntax

Fortran 77:

```
call chetrf( uplo, n, a, lda, ipiv, work, lwork, info )
call zhetrf( uplo, n, a, lda, ipiv, work, lwork, info )
```

Fortran 95:

```
call hetrf( a [, uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hetrf( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method:

if $uplo$='U', $A = P*U*D*U^H*P^T$
if $uplo$='L', $A = P*L*D*L^H*P^T$,

where A is the input matrix, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D.

NOTE This routine supports the Progress Routine feature. See Progress Routine section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If $uplo$ = 'U', the array a stores the upper triangular part of the matrix A, and A is factored as $P*U*D*U^H*P^T$. If $uplo$ = 'L', the array a stores the lower triangular part of the matrix A, and A is factored as $P*L*D*L^H*P^T$.
n	INTEGER. The order of matrix A; $n \geq 0$.
a, *work*	COMPLEX for chetrf DOUBLE COMPLEX for zhetrf. Arrays, DIMENSION $a(lda,*)$, $work(*)$. The array a contains the upper or the lower triangular part of the matrix A (see $uplo$). The second dimension of a must be at least max(1, n). $work(*)$ is a workspace array of dimension at least max(1, $lwork$).
lda	INTEGER. The leading dimension of a; at least max(1, n).
lwork	INTEGER. The size of the $work$ array ($lwork \geq n$). If $lwork$ = -1, then a workspace query is assumed; the routine only calculates the optimal size of the $work$ array, returns this value as the first entry of the $work$ array, and no error message related to $lwork$ is issued by xerbla. See Application Notes for the suggested value of $lwork$.

Output Parameters

a	The upper or lower triangular part of a is overwritten by details of the block-diagonal matrix D and the multipliers used to obtain the factor U (or L).
work(1)	If $info$ = 0, on exit $work(1)$ contains the minimum value of $lwork$ required for optimum performance. Use this $lwork$ for subsequent runs.
ipiv	INTEGER.

	Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*. If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column. If *uplo* = 'U' and *ipiv*(*i*) = *ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and the (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column. If *uplo* = 'L' and *ipiv*(*i*) = *ipiv*(*i*+1) = –*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and the (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular. Division by 0 will occur if you use *D* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hetrf interface are as follows:

a	holds the matrix *A* of size (*n*, *n*)
ipiv	holds the vector of length *n*
uplo	must be 'U' or 'L'. The default value is 'U'.

Application Notes

This routine is suitable for Hermitian matrices that are not known to be positive-definite. If *A* is in fact positive-definite, the routine does not perform interchanges, and no 2-by-2 diagonal blocks occur in *D*.

For better performance, try using *lwork* = *n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = –1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The 2-by-2 unit diagonal blocks and the unit diagonal elements of *U* and *L* are not stored. The remaining elements of *U* and *L* are stored in the corresponding columns of the array *a*, but additional row interchanges are required to recover *U* or *L* explicitly (which is seldom necessary).

If *ipiv*(*i*) = *i* for all *i* =1...*n*, then all off-diagonal elements of *U* (*L*) are stored explicitly in the corresponding elements of the array *a*.

If *uplo* = 'U', the computed factors *U* and *D* are the exact factors of a perturbed matrix *A* + *E*, where

$$|E| \leq c(n)\varepsilon\ P|U||D||U^T|P^T$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

A similar estimate holds for the computed L and D when $uplo$ = 'L'.

The total number of floating-point operations is approximately $(4/3)n^3$.

After calling this routine, you can call the following routines:

?hetrs	to solve $A*X = B$
?hecon	to estimate the condition number of A
?hetri	to compute the inverse of A.

See Also

mkl_progress

?sptrf

Computes the Bunch-Kaufman factorization of a symmetric matrix using packed storage.

Syntax

Fortran 77:

```
call ssptrf( uplo, n, ap, ipiv, info )
call dsptrf( uplo, n, ap, ipiv, info )
call csptrf( uplo, n, ap, ipiv, info )
call zsptrf( uplo, n, ap, ipiv, info )
```

Fortran 95:

```
call sptrf( ap [,uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sptrf( int matrix_order, char uplo, lapack_int n, <datatype>* ap,
lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the factorization of a real/complex symmetric matrix A stored in the packed format using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is:

if $uplo$='U', $A = P*U*D*U^T*P^T$
if $uplo$='L', $A = P*L*D*L^T*P^T$,

where P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is packed in the array *ap* and how *A* is factored: If *uplo* = 'U', the array *ap* stores the upper triangular part of the matrix *A*, and *A* is factored as $P*U*D*U^T*P^T$. If *uplo* = 'L', the array *ap* stores the lower triangular part of the matrix *A*, and *A* is factored as $P*L*D*L^T*P^T$.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
ap	REAL for ssptrf DOUBLE PRECISION for dsptrf COMPLEX for csptrf DOUBLE COMPLEX for zsptrf. Array, DIMENSION at least max(1, *n*(*n*+1)/2). The array *ap* contains the upper or the lower triangular part of the matrix *A* (as specified by *uplo*) in *packed storage* (see Matrix Storage Schemes).

Output Parameters

ap	The upper or lower triangle of *A* (as specified by *uplo*) is overwritten by details of the block-diagonal matrix *D* and the multipliers used to obtain the factor *U* (or *L*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*. If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column. If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and the (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column. If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and the (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular. Division by 0 will occur if you use *D* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sptrf interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The 2-by-2 unit diagonal blocks and the unit diagonal elements of U and L are not stored. The remaining elements of U and L overwrite elements of the corresponding columns of the matrix A, but additional row interchanges are required to recover U or L explicitly (which is seldom necessary).

If $ipiv(i) = i$ for all $i = 1...n$, then all off-diagonal elements of U (L) are stored explicitly in packed form.

If $uplo$ = 'U', the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\varepsilon\ P|U||D||U^T|P^T$$

$c(n)$ is a modest linear function of n, and ε is the machine precision. A similar estimate holds for the computed L and D when $uplo$ = 'L'.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

?sptrs	to solve $A*X = B$
?spcon	to estimate the condition number of A
?sptri	to compute the inverse of A.

See Also

mkl_progress

?hptrf

Computes the Bunch-Kaufman factorization of a complex Hermitian matrix using packed storage.

Syntax

Fortran 77:

```
call chptrf( uplo, n, ap, ipiv, info )
call zhptrf( uplo, n, ap, ipiv, info )
```

Fortran 95:

```
call hptrf( ap [,uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hptrf( int matrix_order, char uplo, lapack_int n, <datatype>* ap,
lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the factorization of a complex Hermitian packed matrix A using the Bunch-Kaufman diagonal pivoting method:

if $uplo$='U', $A = P*U*D*U^H*P^T$

if $uplo$='L', $A = P*L*D*L^H*P^T$,

where *A* is the input matrix, *P* is a permutation matrix, *U* and *L* are upper and lower triangular matrices with unit diagonal, and *D* is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. *U* and *L* have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of *D*.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo — CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is packed and how *A* is factored:
If *uplo* = 'U', the array *ap* stores the upper triangular part of the matrix *A*, and *A* is factored as $P*U*D*U^H*P^T$.
If *uplo* = 'L', the array *ap* stores the lower triangular part of the matrix *A*, and *A* is factored as $P*L*D*L^H*P^T$.

n — INTEGER. The order of matrix *A*; $n \geq 0$.

ap — COMPLEX for chptrf
DOUBLE COMPLEX for zhptrf.
Array, DIMENSION at least max(1, *n*(*n*+1)/2). The array *ap* contains the upper or the lower triangular part of the matrix *A* (as specified by *uplo*) in *packed storage* (see Matrix Storage Schemes).

Output Parameters

ap — The upper or lower triangle of *A* (as specified by *uplo*) is overwritten by details of the block-diagonal matrix *D* and the multipliers used to obtain the factor *U* (or *L*).

ipiv — INTEGER.
Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*. If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column.
If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and the (*i*–1)-th row and column of *A* was interchanged with the *m*–th row and column.
If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and the (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.

info — INTEGER. If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular. Division by 0 will occur if you use *D* for solving a system of linear equations.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hptrf interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The 2-by-2 unit diagonal blocks and the unit diagonal elements of *U* and *L* are not stored. The remaining elements of *U* and *L* are stored in the corresponding columns of the array *a*, but additional row interchanges are required to recover *U* or *L* explicitly (which is seldom necessary).

If *ipiv*(*i*) = *i* for all *i* = 1...*n*, then all off-diagonal elements of *U* (*L*) are stored explicitly in the corresponding elements of the array *a*.

If *uplo* = 'U', the computed factors *U* and *D* are the exact factors of a perturbed matrix *A* + *E*, where

$$|E| \leq c(n)\varepsilon\ P|U||D||U^T|P^T$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

A similar estimate holds for the computed *L* and *D* when *uplo* = 'L'.

The total number of floating-point operations is approximately $(4/3)n^3$.

After calling this routine, you can call the following routines:

?hptrs	to solve *A*X* = *B*
?hpcon	to estimate the condition number of *A*
?hptri	to compute the inverse of *A*.

See Also

mkl_progress

Routines for Solving Systems of Linear Equations

This section describes the LAPACK routines for solving systems of linear equations. Before calling most of these routines, you need to factorize the matrix of your system of equations (see Routines for Matrix Factorization in this chapter). However, the factorization is not necessary if your system of equations has a triangular matrix.

?getrs

Solves a system of linear equations with an LU-factored square matrix, with multiple right-hand sides.

Syntax

Fortran 77:

```
call sgetrs( trans, n, nrhs, a, lda, ipiv, b, ldb, info )
call dgetrs( trans, n, nrhs, a, lda, ipiv, b, ldb, info )
call cgetrs( trans, n, nrhs, a, lda, ipiv, b, ldb, info )
call zgetrs( trans, n, nrhs, a, lda, ipiv, b, ldb, info )
```

Fortran 95:

```
call getrs( a, ipiv, b [, trans] [,info] )
```

C:

```
lapack_int LAPACKE_<?>getrs( int matrix_order, char trans, lapack_int n, lapack_int
nrhs, const <datatype>* a, lapack_int lda, const lapack_int* ipiv, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the following systems of linear equations:

$A*X = B$	if *trans*='N',
$A^T*X = B$	if *trans*='T',
$A^H*X = B$	if *trans*='C' (for complex matrices only).

Before calling this routine, you must call ?getrf to compute the LU factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then $A*X = B$ is solved for X. If *trans* = 'T', then $A^T*X = B$ is solved for X. If *trans* = 'C', then $A^H*X = B$ is solved for X.
n	INTEGER. The order of A; the number of rows in B ($n \geq 0$).
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
a, *b*	REAL for sgetrs DOUBLE PRECISION for dgetrs COMPLEX for cgetrs DOUBLE COMPLEX for zgetrs. Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains LU factorization of matrix A resulting from the call of ?getrf . The array *b* contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of *a* must be at least max(1,*n*), the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* $\geq$ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* $\geq$ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?getrf.

Output Parameters

b	Overwritten by the solution matrix X.

info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine getrs interface are as follows:

a	Holds the matrix A of size (n, n).
b	Holds the matrix B of size (n, $nrhs$).
ipiv	Holds the vector of length n.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

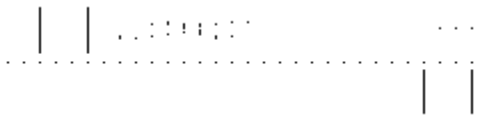
Application Notes

For each right-hand side b, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon\ P|L||U|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:



where $\text{cond}(A,x) = \| \ |A^{-1}||A| \ |x| \ \|_\infty \ / \ \|x\|_\infty \leq \||A^{-1}|\|_\infty \ \||A|\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A,x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is $2n^2$ for real flavors and $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?gecon.

To refine the solution and estimate the error, call ?gerfs.

?gbtrs

Solves a system of linear equations with an LU-factored band matrix, with multiple right-hand sides.

Syntax

Fortran 77:

```
call sgbtrs( trans, n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
call dgbtrs( trans, n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
call cgbtrs( trans, n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
call zgbtrs( trans, n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
```

Fortran 95:

```
call gbtrs( ab, b, ipiv, [, kl] [, trans] [, info] )
```

C:

```
lapack_int LAPACKE_<?>gbtrs( int matrix_order, char trans, lapack_int n, lapack_int kl,
lapack_int ku, lapack_int nrhs, const <datatype>* ab, lapack_int ldab, const
lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for *X* the following systems of linear equations:

$A*X = B$	if *trans*='N',
$A^T*X = B$	if *trans*='T',
$A^H*X = B$	if *trans*='C' (for complex matrices only).

Here *A* is an *LU*-factored general band matrix of order *n* with *kl* non-zero subdiagonals and *ku* nonzero superdiagonals. Before calling this routine, call ?gbtrf to compute the *LU* factorization of *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'.
n	INTEGER. The order of *A*; the number of rows in *B*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ab, *b*	REAL for sgbtrs DOUBLE PRECISION for dgbtrs COMPLEX for cgbtrs DOUBLE COMPLEX for zgbtrs. Arrays: *ab*(*ldab*,*), *b*(*ldb*,*). The array *ab* contains the matrix *A* in *band storage* (see Matrix Storage Schemes). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *ab* must be at least max(1, *n*), and the second dimension of *b* at least max(1,*nrhs*).
ldab	INTEGER. The leading dimension of the array *ab*; $ldab \geq 2*kl + ku + 1$.
ldb	INTEGER. The leading dimension of *b*; $ldb \geq \max(1, n)$.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?gbtrf.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbtrs interface are as follows:

ab	Holds the array *A* of size (2**kl*+*ku*+1,*n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length min(*m*, *n*).
kl	If omitted, assumed *kl* = *ku*.
ku	Restored as *lda*-2**kl*-1.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

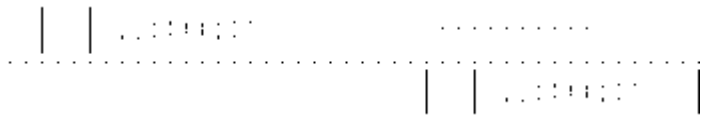
Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$$|E| \leq c(kl + ku + 1)\varepsilon\, P|L||U|$$

$c(k)$ is a modest linear function of k, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:



where $\text{cond}(A,x) = \| \, |A^{-1}||A| \, |x| \, \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A,x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector is $2n(ku + 2kl)$ for real flavors. The number of operations for complex flavors is 4 times greater. All these estimates assume that *kl* and *ku* are much less than min(*m*,*n*).

To estimate the condition number $\kappa_\infty(A)$, call ?gbcon.

To refine the solution and estimate the error, call ?gbrfs.

?gttrs

Solves a system of linear equations with a tridiagonal matrix using the LU factorization computed by ?gttrf.

Syntax

Fortran 77:

```
call sgttrs( trans, n, nrhs, dl, d, du, du2, ipiv, b, ldb, info )
call dgttrs( trans, n, nrhs, dl, d, du, du2, ipiv, b, ldb, info )
call cgttrs( trans, n, nrhs, dl, d, du, du2, ipiv, b, ldb, info )
call zgttrs( trans, n, nrhs, dl, d, du, du2, ipiv, b, ldb, info )
```

Fortran 95:

```
call gttrs( dl, d, du, du2, b, ipiv [, trans] [,info] )
```

C:

```
lapack_int LAPACKE_<?>gttrs( int matrix_order, char trans, lapack_int n, lapack_int
nrhs, const <datatype>* dl, const <datatype>* d, const <datatype>* du, const
<datatype>* du2, const lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the following systems of linear equations with multiple right hand sides:

$A*X = B$	if *trans*='N',
$A^T*X = B$	if *trans*='T',
$A^H*X = B$	if *trans*='C' (for complex matrices only).

Before calling this routine, you must call ?gttrf to compute the LU factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then $A*X = B$ is solved for X. If *trans* = 'T', then $A^T*X = B$ is solved for X. If *trans* = 'C', then $A^H*X = B$ is solved for X.
n	INTEGER. The order of A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, that is, the number of columns in B; $nrhs \geq 0$.
dl,*d*,*du*,*du2*,*b*	REAL for sgttrs DOUBLE PRECISION for dgttrs COMPLEX for cgttrs DOUBLE COMPLEX for zgttrs. Arrays: *dl*(*n* -1), *d*(*n*), *du*(*n* -1), *du2*(*n* -2), *b*(*ldb*,*nrhs*). The array *dl* contains the (*n* - 1) multipliers that define the matrix L from the LU factorization of A.

	The array *d* contains the *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*. The array *du* contains the (*n* - 1) elements of the first superdiagonal of *U*. The array *du2* contains the (*n* - 2) elements of the second superdiagonal of *U*. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION (*n*). The *ipiv* array, as returned by ?gttrf.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gttrs interface are as follows:

dl	Holds the vector of length (*n*-1).
d	Holds the vector of length *n*.
du	Holds the vector of length (*n*-1).
du2	Holds the vector of length (*n*-2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

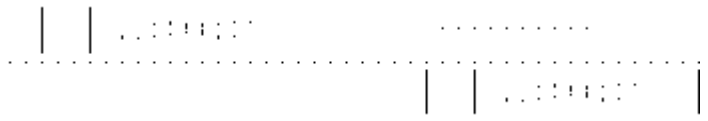
Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$$|E| \leq c(n)\varepsilon\ P|L||U|$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

If x_0 is the true solution, the computed solution *x* satisfies this error bound:



where cond(*A*, *x*) = || $|A^{-1}||A|\ |x|\ ||_\infty$ / $||x||_\infty \leq ||A^{-1}||_\infty\ ||A||_\infty = \kappa_\infty(A)$.

Note that cond(*A*, *x*) can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is $7n$ (including n divisions) for real flavors and $34n$ (including $2n$ divisions) for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?gtcon.

To refine the solution and estimate the error, call ?gtrfs.

?dttrsb

Solves a system of linear equations with a diagonally dominant tridiagonal matrix using the LU factorization computed by ?dttrfb.

Syntax

Fortran 77:

```
call sdttrsb( trans, n, nrhs, dl, d, du, b, ldb, info )
call ddttrsb( trans, n, nrhs, dl, d, du, b, ldb, info )
call cdttrsb( trans, n, nrhs, dl, d, du, b, ldb, info )
call zdttrsb( trans, n, nrhs, dl, d, du, b, ldb, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?dttrsb routine solves the following systems of linear equations with multiple right hand sides for X:

$A*X = B$	if *trans*='N',
$A^T*X = B$	if *trans*='T',
$A^H*X = B$	if *trans*='C' (for complex matrices only).

Before calling this routine, call ?dttrfb to compute the factorization of A.

Input Parameters

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations solved for X: If *trans* = 'N', then $A*X = B$. If *trans* = 'T', then $A^T*X = B$. If *trans* = 'C', then $A^H*X = B$.
n	INTEGER. The order of A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, that is, the number of columns in B; $nrhs \geq 0$.
dl, *d*, *du*, *b*	REAL for sdttrsb DOUBLE PRECISION for ddttrsb COMPLEX for cdttrsb DOUBLE COMPLEX for zdttrsb. Arrays: *dl*(*n* -1), *d*(*n*), *du*(*n* -1), *b*(*ldb*,*nrhs*). The array *dl* contains the (*n* - 1) multipliers that define the matrices L_1, L_2 from the factorization of A. The array *d* contains the *n* diagonal elements of the upper triangular matrix U from the factorization of A. The array *du* contains the (*n* - 1) elements of the superdiagonal of U.

	The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?potrs

Solves a system of linear equations with a Cholesky-factored symmetric (Hermitian) positive-definite matrix.

Syntax

Fortran 77:

```
call spotrs( uplo, n, nrhs, a, lda, b, ldb, info )
call dpotrs( uplo, n, nrhs, a, lda, b, ldb, info )
call cpotrs( uplo, n, nrhs, a, lda, b, ldb, info )
call zpotrs( uplo, n, nrhs, a, lda, b, ldb, info )
```

Fortran 95:

```
call potrs( a, b [,uplo] [, info] )
```

C:

```
lapack_int LAPACKE_<?>potrs( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for *X* the system of linear equations *A*X* = *B* with a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix *A*, given the Cholesky factorization of *A*:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data	if *uplo*='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data	if *uplo*='L'

where *L* is a lower triangular matrix and *U* is upper triangular. The system is solved with multiple right-hand sides stored in the columns of the matrix *B*.

Before calling this routine, you must call ?potrf to compute the Cholesky factorization of *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides ($nrhs \geq 0$).
a, *b*	REAL for spotrs DOUBLE PRECISION for dpotrs COMPLEX for cpotrs DOUBLE COMPLEX for zpotrs. Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the factor *U* or *L* (see *uplo*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *a* must be at least max(1,*n*), the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* $\geq$ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* $\geq$ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine potrs interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

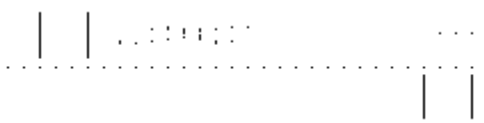
Application Notes

If *uplo* = 'U', the computed solution for each right-hand side *b* is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon\ |U^H||U|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

A similar estimate holds for *uplo* = 'L'. If x_0 is the true solution, the computed solution x satisfies this error bound:



where $\mathrm{cond}(A,x) = \| |A^{-1}||A|\ |x|\ \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that cond(A,x) can be much smaller than $\kappa_\infty(A)$. The approximate number of floating-point operations for one right-hand side vector b is $2n^2$ for real flavors and $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?pocon.

To refine the solution and estimate the error, call ?porfs.

?pftrs

Solves a system of linear equations with a Cholesky-factored symmetric (Hermitian) positive-definite matrix using the Rectangular Full Packed (RFP) format.

Syntax

Fortran 77:

```
call spftrs( transr, uplo, n, nrhs, a, b, ldb, info )
call dpftrs( transr, uplo, n, nrhs, a, b, ldb, info )
call cpftrs( transr, uplo, n, nrhs, a, b, ldb, info )
call zpftrs( transr, uplo, n, nrhs, a, b, ldb, info )
```

C:

```
lapack_int LAPACKE_<?>pftrs( int matrix_order, char transr, char uplo, lapack_int n,
lapack_int nrhs, const <datatype>* a, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves a system of linear equations A*X = B with a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A using the Cholesky factorization of A:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data	if uplo='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data	if uplo='L'

computed by ?pftrf. L stands for a lower triangular matrix and U - for an upper triangular matrix.

The matrix A is in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see Matrix Storage Schemes.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

transr	CHARACTER*1. Must be 'N', 'T' (for real data) or 'C' (for complex data). If transr = 'N', the Normal transr of RFP A is stored. If transr = 'T', the Transpose transr of RFP A is stored. If transr = 'C', the Conjugate-Transpose transr of RFP A is stored.
uplo	CHARACTER*1. Must be 'U' or 'L'.

	Indicates whether the upper or lower triangular part of the RFP matrix *A* is stored: If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix *A*. If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix *A*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, that is, the number of columns of the matrix *B*; $nrhs \geq 0$.
a, *b*	REAL for spftrs DOUBLE PRECISION for dpftrs COMPLEX for cpftrs DOUBLE COMPLEX for zpftrs. Arrays: *a*(*n**(*n*+1)/2), *b*(*ldb*,*nrhs*). The array *a* contains the matrix *A* in the RFP format. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	The solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?pptrs

Solves a system of linear equations with a packed Cholesky-factored symmetric (Hermitian) positive-definite matrix.

Syntax

Fortran 77:

```
call spptrs( uplo, n, nrhs, ap, b, ldb, info )
call dpptrs( uplo, n, nrhs, ap, b, ldb, info )
call cpptrs( uplo, n, nrhs, ap, b, ldb, info )
call zpptrs( uplo, n, nrhs, ap, b, ldb, info )
```

Fortran 95:

```
call pptrs( ap, b [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>pptrs( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* ap, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the system of linear equations $A*X = B$ with a packed symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A, given the Cholesky factorization of A:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data if $uplo$='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data if $uplo$='L'

where L is a lower triangular matrix and U is upper triangular. The system is solved with multiple right-hand sides stored in the columns of the matrix B.

Before calling this routine, you must call ?pptrf to compute the Cholesky factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo$ = 'U', the upper triangle of A is stored. If $uplo$ = 'L', the lower triangle of A is stored.
n	INTEGER. The order of matrix A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides ($nrhs \geq 0$).
ap, *b*	REAL for spptrs DOUBLE PRECISION for dpptrs COMPLEX for cpptrs DOUBLE COMPLEX for zpptrs. Arrays: $ap(*)$, $b(ldb,*)$ The dimension of ap must be at least max(1,n(n+1)/2). The array ap contains the factor U or L, as specified by $uplo$, in *packed storage* (see Matrix Storage Schemes). The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of b must be at least max(1, $nrhs$).
ldb	INTEGER. The leading dimension of b; $ldb \geq$ max(1, n).

Output Parameters

b	Overwritten by the solution matrix X.
info	INTEGER. If $info$ = 0, the execution is successful. If $info$ = -i, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pptrs interface are as follows:

ap	Holds the array A of size (n*(n+1)/2).
b	Holds the matrix B of size (n, $nrhs$).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

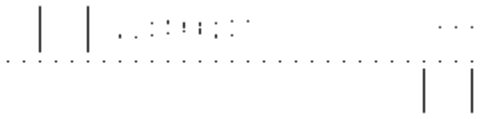
If `uplo = 'U'`, the computed solution for each right-hand side b is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon\ |U^H||U|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

A similar estimate holds for `uplo = 'L'`.

If x_0 is the true solution, the computed solution x satisfies this error bound:



where $\mathrm{cond}(A,x) = \| |A^{-1}||A|\ |x|\ \|_\infty / \|x\|_\infty \leq \||A^{-1}|\|_\infty\ \||A|\|_\infty = \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is $2n^2$ for real flavors and $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?ppcon.

To refine the solution and estimate the error, call ?pprfs.

?pbtrs

Solves a system of linear equations with a Cholesky-factored symmetric (Hermitian) positive-definite band matrix.

Syntax

Fortran 77:

```
call spbtrs( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
call dpbtrs( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
call cpbtrs( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
call zpbtrs( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
```

Fortran 95:

```
call pbtrs( ab, b [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>pbtrs( int matrix_order, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const <datatype>* ab, lapack_int ldab, <datatype>* b, lapack_int
ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for real data a system of linear equations *A*X = B* with a symmetric positive-definite or, for complex data, Hermitian positive-definite *band* matrix *A*, given the Cholesky factorization of *A*:

$A = U^T*U$ for real data, $A = U^H*U$ for complex data	if *uplo*='U'
$A = L*L^T$ for real data, $A = L*L^H$ for complex data	if *uplo*='L'

where *L* is a lower triangular matrix and *U* is upper triangular. The system is solved with multiple right-hand sides stored in the columns of the matrix *B*.

Before calling this routine, you must call ?pbtrf to compute the Cholesky factorization of *A* in the band storage form.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the upper triangular factor is stored in *ab*. If *uplo* = 'L', the lower triangular factor is stored in *ab*.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
kd	INTEGER. The number of superdiagonals or subdiagonals in the matrix *A*; $kd \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ab, *b*	REAL for spbtrs DOUBLE PRECISION for dpbtrs COMPLEX for cpbtrs DOUBLE COMPLEX for zpbtrs. Arrays: *ab*(*ldab*,*), *b*(*ldb*,*). The array *ab* contains the Cholesky factor, as returned by the factorization routine, in *band storage* form. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *ab* must be at least max(1, *n*), and the second dimension of *b* at least max(1,*nrhs*).
ldab	INTEGER. The leading dimension of the array *ab*; $ldab \geq kd + 1$.
ldb	INTEGER. The leading dimension of *b*; $ldb \geq \max(1, n)$.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pbtrs interface are as follows:

ab	Holds the array *A* of size (*kd*+1, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$|E| \le c(kd+1)\varepsilon\, P|U^H||U|$ or $|E| \le c(kd+1)\varepsilon\, P|L^H||L|$

$c(k)$ is a modest linear function of k, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \le c(kd+1)\, cond\,(A, x)\varepsilon$$

where $cond(A,x) = \| |A^{-1}||A|\,|x| \|_\infty / \||x|\|_\infty \le \||A^{-1}|\|_\infty \||A|\|_\infty = \kappa_\infty(A)$.

Note that $cond(A,x)$ can be much smaller than $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector is $4n*kd$ for real flavors and $16n*kd$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?pbcon.

To refine the solution and estimate the error, call ?pbrfs.

?pttrs

Solves a system of linear equations with a symmetric (Hermitian) positive-definite tridiagonal matrix using the factorization computed by ?pttrf.

Syntax

Fortran 77:

```
call spttrs( n, nrhs, d, e, b, ldb, info )
call dpttrs( n, nrhs, d, e, b, ldb, info )
call cpttrs( uplo, n, nrhs, d, e, b, ldb, info )
call zpttrs( uplo, n, nrhs, d, e, b, ldb, info )
```

Fortran 95:

```
call pttrs( d, e, b [,info] )
call pttrs( d, e, b [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_spttrs( int matrix_order, lapack_int n, lapack_int nrhs, const
float* d, const float* e, float* b, lapack_int ldb );
lapack_int LAPACKE_dpttrs( int matrix_order, lapack_int n, lapack_int nrhs, const
double* d, const double* e, double* b, lapack_int ldb );
```

```
lapack_int LAPACKE_cpttrs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const float* d, const lapack_complex_float* e, lapack_complex_float* b, lapack_int
ldb );

lapack_int LAPACKE_zpttrs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const double* d, const lapack_complex_double* e, lapack_complex_double* b, lapack_int
ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for *X* a system of linear equations *A*X = B* with a symmetric (Hermitian) positive-definite tridiagonal matrix *A*. Before calling this routine, call ?pttrf to compute the *L*D*L'* for real data and the *L*D*L'* or *U'*D*U* factorization of *A* for complex data.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Used for cpttrs/zpttrs only. Must be 'U' or 'L'. Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix *A* is stored and how *A* is factored: If *uplo* = 'U', the array *e* stores the superdiagonal of *A*, and *A* is factored as *U'*D*U*. If *uplo* = 'L', the array *e* stores the subdiagonal of *A*, and *A* is factored as *L*D*L'*.
n	INTEGER. The order of *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides, that is, the number of columns of the matrix *B*; *nrhs* ≥ 0.
d	REAL for spttrs, cpttrs DOUBLE PRECISION for dpttrs, zpttrs. Array, dimension (*n*). Contains the diagonal elements of the diagonal matrix *D* from the factorization computed by ?pttrf.
e, *b*	REAL for spttrs DOUBLE PRECISION for dpttrs COMPLEX for cpttrs DOUBLE COMPLEX for zpttrs. Arrays: *e*(*n* -1), *b*(*ldb*, *nrhs*). The array *e* contains the (*n* - 1) off-diagonal elements of the unit bidiagonal factor *U* or *L* from the factorization computed by ?pttrf (see *uplo*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.

info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pttrs interface are as follows:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Used in complex flavors only. Must be 'U' or 'L'. The default value is 'U'.

?sytrs

Solves a system of linear equations with a UDU- or LDL-factored symmetric matrix.

Syntax

Fortran 77:

```
call ssytrs( uplo, n, nrhs, a, lda, ipiv, b, ldb, info )
call dsytrs( uplo, n, nrhs, a, lda, ipiv, b, ldb, info )
call csytrs( uplo, n, nrhs, a, lda, ipiv, b, ldb, info )
call zsytrs( uplo, n, nrhs, a, lda, ipiv, b, ldb, info )
```

Fortran 95:

```
call sytrs( a, b, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sytrs( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* a, lapack_int lda, const lapack_int* ipiv, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for *X* the system of linear equations *A*X* = *B* with a symmetric matrix *A*, given the Bunch-Kaufman factorization of *A*:

if *uplo*='U',	$A = P*U*D*U^T*P^T$
if *uplo*='L',	$A = P*L*D*L^T*P^T$,

where *P* is a permutation matrix, *U* and *L* are upper and lower triangular matrices with unit diagonal, and *D* is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix *B*. You must supply to this routine the factor *U* (or *L*) and the array *ipiv* returned by the factorization routine ?sytrf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the upper triangular factor *U* of the factorization *A* = *P*U*D*U^T*P^T*. If *uplo* = 'L', the array *a* stores the lower triangular factor *L* of the factorization *A* = *P*L*D*L^T*P^T*.
n	INTEGER. The order of matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides; *nrhs* ≥ 0.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?sytrf.
a, *b*	REAL for ssytrs DOUBLE PRECISION for dsytrs COMPLEX for csytrs DOUBLE COMPLEX for zsytrs. Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the factor *U* or *L* (see *uplo*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the system of equations. The second dimension of *a* must be at least max(1,*n*), and the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sytrs interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$$|E| \le c(n)\varepsilon\ P|U||D||U^T|P^T \text{ or } |E| \le c(n)\varepsilon\ P|L||D||U^T|P^T$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n)\, cond\,(A, x)\varepsilon$$

where cond(A, x) = || $|A^{-1}||A|\,|x|\,||_\infty$ / $||x||_\infty \leq ||A^{-1}||_\infty\,||A||_\infty = \kappa_\infty(A)$.

Note that cond(A, x) can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $2n^2$ for real flavors or $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?sycon.

To refine the solution and estimate the error, call ?syrfs.

?hetrs

Solves a system of linear equations with a UDU- or LDL-factored Hermitian matrix.

Syntax

Fortran 77:

```
call chetrs( uplo, n, nrhs, a, lda, ipiv, b, ldb, info )
call zhetrs( uplo, n, nrhs, a, lda, ipiv, b, ldb, info )
```

Fortran 95:

```
call hetrs( a, b, ipiv [, uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hetrs( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* a, lapack_int lda, const lapack_int* ipiv, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the system of linear equations $A*X = B$ with a Hermitian matrix A, given the Bunch-Kaufman factorization of A:

if uplo = 'U' $A = P*U*D*U^H*P^T$

if uplo = 'L' $A = P*L*D*L^H*P^T$,

where P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B. You must supply to this routine the factor U (or L) and the array ipiv returned by the factorization routine ?hetrf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the upper triangular factor *U* of the factorization *A* = *P*U*D*U^H*P^T*. If *uplo* = 'L', the array *a* stores the lower triangular factor *L* of the factorization *A* = *P*L*D*L^H*P^T*.
n	INTEGER. The order of matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides; *nrhs* ≥ 0.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?hetrf.
a, *b*	COMPLEX for chetrs DOUBLE COMPLEX for zhetrs. Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the factor *U* or *L* (see *uplo*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the system of equations. The second dimension of *a* must be at least max(1,*n*), the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hetrs interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

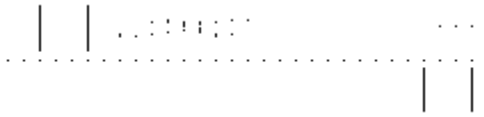
Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \le c(n)\varepsilon\ P|U||D||U^H|P^T \text{ or } |E| \le c(n)\varepsilon\ P|L||D||L^H|P^T$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:



where cond(A, x) = || $|A^{-1}||A|$ $|x|$ $||_\infty$ / $||x||_\infty \le ||A^{-1}||_\infty$ $||A||_\infty$ = $\kappa_\infty(A)$.

Note that cond(A, x) can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $8n^2$.

To estimate the condition number $\kappa_\infty(A)$, call ?hecon.

To refine the solution and estimate the error, call ?herfs.

?sytrs2

Solves a system of linear equations with a UDU- or LDL-factored symmetric matrix computed by ?sytrf *and converted by* ?syconv.

Syntax

Fortran 77:

```
call ssytrs2( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, info )
call dsytrs2( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, info )
call csytrs2( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, info )
call zsytrs2( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, info )
```

Fortran 95:

```
call sytrs2( a,b,ipiv[,uplo][,info] )
```

C:

```
lapack_int LAPACKE_<?>sytrs2( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* a, lapack_int lda, const lapack_int* ipiv, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves a system of linear equations A*X = B with a symmetric matrix A using the factorization of A:

if uplo='U', $A = U*D*U^T$

if uplo='L', $A = L*D*L^T$

where

- U and L are upper and lower triangular matrices with unit diagonal
- D is a symmetric block-diagonal matrix.

The factorization is computed by ?sytrf and converted by ?syconv.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the upper triangular factor *U* of the factorization *A* = *U*D*U*^{T}. If *uplo* = 'L', the array *a* stores the lower triangular factor *L* of the factorization *A* = *L*D*L*^{T}.
n	INTEGER. The order of matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides; *nrhs* ≥ 0.
a, *b*	REAL for ssytrs2 DOUBLE PRECISION for dsytrs2 COMPLEX for csytrs2 DOUBLE COMPLEX for zsytrs2 Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* as computed by ?sytrf. The array *b* contains the right-hand side matrix *B*. The second dimension of *a* must be at least max(1,*n*), and the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ipiv	INTEGER. Array of DIMENSION *n*. The *ipiv* array contains details of the interchanges and the block structure of *D* as determined by ?sytrf.
work	REAL for ssytrs2 DOUBLE PRECISION for dsytrs2 COMPLEX for csytrs2 DOUBLE COMPLEX for zsytrs2 Workspace array, DIMENSION *n*.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sytrs2 interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.

uplo	Indicates how the input matrix *A* has been factored. Must be 'U' or 'L'.

See Also

?sytrf
?syconv

?hetrs2

Solves a system of linear equations with a UDU- or LDL-factored Hermitian matrix computed by ?hetrf *and converted by* ?syconv.

Syntax

Fortran 77:

```
call chetrs2( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, info )
call zhetrs2( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, info )
```

Fortran 95:

```
call hetrs2( a, b, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hetrs2( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* a, lapack_int lda, const lapack_int* ipiv, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves a system of linear equations *A*X = B* with a complex Hermitian matrix *A* using the factorization of *A*:

if *uplo*='U', $A = U*D*U^H$

if *uplo*='L', $A = L*D*L^H$

where

- *U* and *L* are upper and lower triangular matrices with unit diagonal
- *D* is a Hermitian block-diagonal matrix.

The factorization is computed by ?hetrf and converted by ?syconv.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the upper triangular factor *U* of the factorization *A* = $U*D*U^H$.

	If *uplo* = 'L', the array *a* stores the lower triangular factor *L* of the factorization *A* = *L*D*L*^{H}.
n	INTEGER. The order of matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides; *nrhs* ≥ 0.
a, *b*	COMPLEX for chetrs2 DOUBLE COMPLEX for zhetrs2 Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* as computed by ?hetrf. The array *b* contains the right-hand side matrix *B*. The second dimension of *a* must be at least max(1,*n*), and the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ipiv	INTEGER. Array of DIMENSION *n*. The *ipiv* array contains details of the interchanges and the block structure of *D* as determined by ?hetrf.
work	COMPLEX for chetrs2 DOUBLE COMPLEX for zhetrs2 Workspace array, DIMENSION *n*.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hetrs2 interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

See Also

?hetrf

?syconv

?sptrs

Solves a system of linear equations with a UDU- or LDL-factored symmetric matrix using packed storage.

Syntax

Fortran 77:

```
call ssptrs( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call dsptrs( uplo, n, nrhs, ap, ipiv, b, ldb, info )
```

```
call csptrs( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call zsptrs( uplo, n, nrhs, ap, ipiv, b, ldb, info )
```

Fortran 95:

```
call sptrs( ap, b, ipiv [, uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sptrs( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* ap, const lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the system of linear equations $A*X = B$ with a symmetric matrix A, given the Bunch-Kaufman factorization of A:

if uplo='U', $A = PUDU^TP^T$

if uplo='L', $A = PLDL^TP^T$,

where P is a permutation matrix, U and L are upper and lower *packed* triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B. You must supply the factor U (or L) and the array ipiv returned by the factorization routine ?sptrf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If uplo = 'U', the array ap stores the packed factor U of the factorization $A = P*U*D*U^T*P^T$. If uplo = 'L', the array ap stores the packed factor L of the factorization $A = P*L*D*L^T*P^T$.
n	INTEGER. The order of matrix A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The ipiv array, as returned by ?sptrf.
ap, b	REAL for ssptrs DOUBLE PRECISION for dsptrs COMPLEX for csptrs DOUBLE COMPLEX for zsptrs. Arrays: ap(*), b(ldb,*). The dimension of ap must be at least max(1,n(n+1)/2). The array ap contains the factor U or L, as specified by uplo, in *packed storage* (see Matrix Storage Schemes).

	The array *b* contains the matrix *B* whose columns are the right-hand sides for the system of equations. The second dimension of *b* must be at least max(1, *nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sptrs interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$|E| \le c(n)\varepsilon\ P|U||D||U^T|P^T$ or $|E| \le c(n)\varepsilon\ P|L||D||L^T|P^T$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \le c(n)\, cond\,(A, x)\varepsilon$$

where $cond(A, x) = \| |A^{-1}||A|\,|x|\,\|_\infty / \|x\|_\infty \le \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $cond(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $2n^2$ for real flavors or $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?spcon.

To refine the solution and estimate the error, call ?sprfs.

?hptrs

Solves a system of linear equations with a UDU- or LDL-factored Hermitian matrix using packed storage.

Syntax

Fortran 77:

```
call chptrs( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call zhptrs( uplo, n, nrhs, ap, ipiv, b, ldb, info )
```

Fortran 95:

```
call hptrs( ap, b, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hptrs( int matrix_order, char uplo, lapack_int n, lapack_int
nrhs, const <datatype>* ap, const lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the system of linear equations $A*X = B$ with a Hermitian matrix A, given the Bunch-Kaufman factorization of A:

if uplo='U', $A = P*U*D*U^H*P^T$

if uplo='L', $A = P*L*D*L^H*P^T$,

where P is a permutation matrix, U and L are upper and lower *packed* triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B.

You must supply to this routine the arrays ap (containing U or L)and $ipiv$ in the form returned by the factorization routine ?hptrf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If uplo = 'U', the array ap stores the packed factor U of the factorization $A = P*U*D*U^H*P^T$. If uplo = 'L', the array ap stores the packed factor L of the factorization $A = P*L*D*L^H*P^T$.
n	INTEGER. The order of matrix A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The $ipiv$ array, as returned by ?hptrf.
ap, b	COMPLEX for chptrs DOUBLE COMPLEX for zhptrs. Arrays: ap(*), b(ldb,*). The dimension of ap must be at least max(1,n(n+1)/2). The array ap contains the factor U or L, as specified by uplo, in *packed storage* (see Matrix Storage Schemes).

	The array *b* contains the matrix *B* whose columns are the right-hand sides for the system of equations. The second dimension of *b* must be at least max(1, *nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hptrs interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$|E| \le c(n)\varepsilon\ P|U||D||U^H|P^T$ or $|E| \le c(n)\varepsilon\ P|L||D||L^H|P^T$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \le c(n)\, cond\,(A, x)\varepsilon$$

where $cond(A, x) = \| |A^{-1}||A|\,|x|\,\|_\infty / \|x\|_\infty \le \||A^{-1}|\|_\infty \||A|\|_\infty = \kappa_\infty(A)$.

Note that $cond(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?hpcon.

To refine the solution and estimate the error, call ?hprfs.

?trtrs

Solves a system of linear equations with a triangular matrix, with multiple right-hand sides.

Syntax

Fortran 77:

```
call strtrs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, info )
call dtrtrs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, info )
call ctrtrs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, info )
call ztrtrs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, info )
```

Fortran 95:

```
call trtrs( a, b [,uplo] [, trans] [,diag] [,info] )
```

C:

```
lapack_int LAPACKE_<?>trtrs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const <datatype>* a, lapack_int lda, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for X the following systems of linear equations with a triangular matrix A, with multiple right-hand sides stored in B:

$A*X = B$	if *trans*='N',
$A^T*X = B$	if *trans*='T',
$A^H*X = B$	if *trans*='C' (for complex matrices only).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If *uplo* = 'U', then A is upper triangular. If *uplo* = 'L', then A is lower triangular.
trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. If *trans* = 'N', then $A*X = B$ is solved for X. If *trans* = 'T', then $A^T*X = B$ is solved for X. If *trans* = 'C', then $A^H*X = B$ is solved for X.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then A is not a unit triangular matrix. If *diag* = 'U', then A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array *a*.
n	INTEGER. The order of A; the number of rows in B; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
a, *b*	REAL for strtrs

	DOUBLE PRECISION for dtrtrs COMPLEX for ctrtrs DOUBLE COMPLEX for ztrtrs. Arrays: a(lda,*), b(ldb,*). The array a contains the matrix A. The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of a must be at least max(1,n), the second dimension of b at least max(1,nrhs).
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).
ldb	INTEGER. The leading dimension of b; ldb ≥ max(1, n).

Output Parameters

b	Overwritten by the solution matrix X.
info	INTEGER. If info=0, the execution is successful. If info = -i, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trtrs interface are as follows:

a	Stands for argument ap in FORTRAN 77 interface. Holds the matrix A of size (n*(n+1)/2).
b	Holds the matrix B of size (n, nrhs).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

For each right-hand side b, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \le c(n)\varepsilon\,|A|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision. If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \le c(n)\,cond\,(A, x)\varepsilon \text{ provided } c(n)\,cond\,(A, x)\varepsilon < 1$$

where $cond(A,x) = \| |A^{-1}||A|\,|x|\,\|_\infty / \|x\|_\infty \le \||A^{-1}|\|_\infty \||A|\|_\infty = \kappa_\infty(A)$.

Note that $cond(A,x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is n^2 for real flavors and $4n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?trcon.

To estimate the error in the solution, call ?trrfs.

?tptrs

Solves a system of linear equations with a packed triangular matrix, with multiple right-hand sides.

Syntax

Fortran 77:

```
call stptrs( uplo, trans, diag, n, nrhs, ap, b, ldb, info )
call dtptrs( uplo, trans, diag, n, nrhs, ap, b, ldb, info )
call ctptrs( uplo, trans, diag, n, nrhs, ap, b, ldb, info )
call ztptrs( uplo, trans, diag, n, nrhs, ap, b, ldb, info )
```

Fortran 95:

```
call tptrs( ap, b [,uplo] [, trans] [,diag] [,info] )
```

C:

```
lapack_int LAPACKE_<?>tptrs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const <datatype>* ap, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the following systems of linear equations with a packed triangular matrix A, with multiple right-hand sides stored in B:

$A*X = B$	if *trans*='N',
$A^T*X = B$	if *trans*='T',
$A^H*X = B$	if *trans*='C' (for complex matrices only).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If *uplo* = 'U', then A is upper triangular. If *uplo* = 'L', then A is lower triangular.
trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. If *trans* = 'N', then $A*X = B$ is solved for X. If *trans* = 'T', then $A^T*X = B$ is solved for X. If *trans* = 'C', then $A^H*X = B$ is solved for X.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then A is not a unit triangular matrix.

	If *diag* = 'U', then *A* is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array *ap*.
n	INTEGER. The order of *A*; the number of rows in *B*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ap, *b*	REAL for stptrs DOUBLE PRECISION for dtptrs COMPLEX for ctptrs DOUBLE COMPLEX for ztptrs. Arrays: *ap*(*), *b*(*ldb*,*). The dimension of *ap* must be at least max(1,*n*(*n*+1)/2). The array *ap* contains the matrix *A* in *packed storage* (see Matrix Storage Schemes). The array *b* contains the matrix *B* whose columns are the right-hand sides for the system of equations. The second dimension of *b* must be at least max(1, *nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tptrs interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$$|E| \leq c(n)\varepsilon\ |A|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

where $\mathrm{cond}(A, x) = \| |A^{-1}||A|\ |x|\ \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that cond(A,x) can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is n^2 for real flavors and $4n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call ?tpcon.

To estimate the error in the solution, call ?tprfs.

?tbtrs

Solves a system of linear equations with a band triangular matrix, with multiple right-hand sides.

Syntax

Fortran 77:

```
call stbtrs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, info )
call dtbtrs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, info )
call ctbtrs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, info )
call ztbtrs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, info )
```

Fortran 95:

```
call tbtrs( ab, b [,uplo] [, trans] [,diag] [,info] )
```

C:

```
lapack_int LAPACKE_<?>tbtrs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const <datatype>* ab, lapack_int ldab,
<datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the following systems of linear equations with a band triangular matrix A, with multiple right-hand sides stored in B:

A*X = B	if trans='N',
A^T*X = B	if trans='T',
A^H*X = B	if trans='C' (for complex matrices only).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If uplo = 'U', then A is upper triangular. If uplo = 'L', then A is lower triangular.

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. If *trans* = 'N', then *A*X* = *B* is solved for *X*. If *trans* = 'T', then $A^T*X = B$ is solved for *X*. If *trans* = 'C', then $A^H*X = B$ is solved for *X*.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then *A* is not a unit triangular matrix. If *diag* = 'U', then *A* is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array *ab*.
n	INTEGER. The order of *A*; the number of rows in *B*; $n \geq 0$.
kd	INTEGER. The number of superdiagonals or subdiagonals in the matrix *A*; $kd \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ab, *b*	REAL for stbtrs DOUBLE PRECISION for dtbtrs COMPLEX for ctbtrs DOUBLE COMPLEX for ztbtrs. Arrays: *ab*(*ldab*,*), *b*(*ldb*,*). The array *ab* contains the matrix *A* in *band storage* form. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *ab* must be at least max(1, *n*), the second dimension of *b* at least max(1,*nrhs*).
ldab	INTEGER. The leading dimension of *ab*; $ldab \geq kd + 1$.
ldb	INTEGER. The leading dimension of *b*; $ldb \geq \max(1, n)$.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tbtrs interface are as follows:

ab	Holds the array *A* of size (*kd*+1,*n*)
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations (*A* + *E*)*x* = *b*, where

$$|E| \leq c(n)\varepsilon|A|$$

$c(n)$ is a modest linear function of n, and ε is the machine precision. If x_0 is the true solution, the computed solution x satisfies this error bound:

where cond(A, x) = || |A^-1||A| |x| ||∞ / ||x||∞ ≤ ||A^-1||∞ ||A||∞ = κ∞(A).

Note that cond(A, x) can be much smaller than κ∞(A); the condition number of A^T and A^H might or might not be equal to κ∞(A).

The approximate number of floating-point operations for one right-hand side vector *b* is *2n*kd* for real flavors and *8n*kd* for complex flavors.

To estimate the condition number κ∞(A), call ?tbcon.

To estimate the error in the solution, call ?tbrfs.

Routines for Estimating the Condition Number

This section describes the LAPACK routines for estimating the *condition number* of a matrix. The condition number is used for analyzing the errors in the solution of a system of linear equations (see Error Analysis). Since the condition number may be arbitrarily large when the matrix is nearly singular, the routines actually compute the *reciprocal* condition number.

?gecon

Estimates the reciprocal of the condition number of a general matrix in the 1-norm or the infinity-norm.

Syntax

Fortran 77:

```
call sgecon( norm, n, a, lda, anorm, rcond, work, iwork, info )
call dgecon( norm, n, a, lda, anorm, rcond, work, iwork, info )
call cgecon( norm, n, a, lda, anorm, rcond, work, rwork, info )
call zgecon( norm, n, a, lda, anorm, rcond, work, rwork, info )
```

Fortran 95:

```
call gecon( a, anorm, rcond [,norm] [,info] )
```

C:

```
lapack_int LAPACKE_sgecon( int matrix_order, char norm, lapack_int n, const float* a,
lapack_int lda, float anorm, float* rcond );
lapack_int LAPACKE_dgecon( int matrix_order, char norm, lapack_int n, const double* a,
lapack_int lda, double anorm, double* rcond );
lapack_int LAPACKE_cgecon( int matrix_order, char norm, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float anorm, float* rcond );
lapack_int LAPACKE_zgecon( int matrix_order, char norm, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double anorm, double* rcond );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a general matrix *A* in the 1-norm or infinity-norm:

$\kappa_1(A) = ||A||_1||A^{-1}||_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$

$\kappa_\infty(A) = ||A||_\infty||A^{-1}||_\infty = \kappa_1(A^T) = \kappa_1(A^H)$.

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?getrf` to compute the *LU* factorization of *A*.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

norm	`CHARACTER*1`. Must be `'1'` or `'O'` or `'I'`. If *norm* = `'1'` or `'O'`, then the routine estimates the condition number of matrix *A* in 1-norm. If *norm* = `'I'`, then the routine estimates the condition number of matrix *A* in infinity-norm.
n	`INTEGER`. The order of the matrix *A*; $n \geq 0$.
a, *work*	`REAL` for `sgecon` `DOUBLE PRECISION` for `dgecon` `COMPLEX` for `cgecon` `DOUBLE COMPLEX` for `zgecon`. Arrays: *a*(*lda*,*), *work*(*). The array *a* contains the *LU*-factored matrix *A*, as returned by `?getrf`. The second dimension of *a* must be at least `max(1,n)`. The array *work* is a workspace for the routine. The dimension of *work* must be at least `max(1, 4*n)` for real flavors and `max(1, 2*n)` for complex flavors.
anorm	`REAL` for single precision flavors. `DOUBLE PRECISION` for double precision flavors. The norm of the *original* matrix *A* (see Description).
lda	`INTEGER`. The leading dimension of *a*; *lda* ≥ `max(1, n)`.
iwork	`INTEGER`. Workspace array, `DIMENSION` at least `max(1, n)`.
rwork	`REAL` for `cgecon` `DOUBLE PRECISION` for `zgecon`. Workspace array, `DIMENSION` at least `max(1, 2*n)`.

Output Parameters

rcond	`REAL` for single precision flavors. `DOUBLE PRECISION` for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.

info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gecon interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
norm	Must be '1', 'O', or 'I'. The default value is '1'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b* or A^H**x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately 2*n^2 floating-point operations for real flavors and 8*n^2 for complex flavors.

?gbcon

Estimates the reciprocal of the condition number of a band matrix in the 1-norm or the infinity-norm.

Syntax

Fortran 77:

```
call sgbcon( norm, n, kl, ku, ab, ldab, ipiv, anorm, rcond, work, iwork, info )
call dgbcon( norm, n, kl, ku, ab, ldab, ipiv, anorm, rcond, work, iwork, info )
call cgbcon( norm, n, kl, ku, ab, ldab, ipiv, anorm, rcond, work, rwork, info )
call zgbcon( norm, n, kl, ku, ab, ldab, ipiv, anorm, rcond, work, rwork, info )
```

Fortran 95:

```
call gbcon( ab, ipiv, anorm, rcond [,kl] [,norm] [,info] )
```

C:

```
lapack_int LAPACKE_sgbcon( int matrix_order, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const float* ab, lapack_int ldab, const lapack_int* ipiv, float anorm,
float* rcond );
lapack_int LAPACKE_dgbcon( int matrix_order, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const double* ab, lapack_int ldab, const lapack_int* ipiv, double
anorm, double* rcond );
lapack_int LAPACKE_cgbcon( int matrix_order, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const lapack_complex_float* ab, lapack_int ldab, const lapack_int* ipiv,
float anorm, float* rcond );
lapack_int LAPACKE_zgbcon( int matrix_order, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const lapack_complex_double* ab, lapack_int ldab, const lapack_int*
ipiv, double anorm, double* rcond );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90

- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a general band matrix A in the 1-norm or infinity-norm:

$\kappa_1(A) = ||A||_1||A^{-1}||_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$

$\kappa_\infty(A) = ||A||_\infty||A^{-1}||_\infty = \kappa_1(A^T) = \kappa_1(A^H)$.

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?gbtrf` to compute the LU factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

norm	CHARACTER*1. Must be '1' or 'O' or 'I'. If *norm* = '1' or 'O', then the routine estimates the condition number of matrix A in 1-norm. If *norm* = 'I', then the routine estimates the condition number of matrix A in infinity-norm.
n	INTEGER. The order of the matrix A; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of A; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of A; $ku \geq 0$.
ldab	INTEGER. The leading dimension of the array *ab*. ($ldab \geq 2*kl + ku + 1$).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by `?gbtrf`.
ab, *work*	REAL for sgbcon DOUBLE PRECISION for dgbcon COMPLEX for cgbcon DOUBLE COMPLEX for zgbcon. Arrays: *ab*(*ldab*,*), *work*(*). The array *ab* contains the factored band matrix A, as returned by `?gbtrf`. The second dimension of *ab* must be at least max(1,*n*). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
anorm	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix A (see Description).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cgbcon DOUBLE PRECISION for zgbcon. Workspace array, DIMENSION at least max(1, 2*n).

Output Parameters

rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbcon interface are as follows:

ab	Holds the array *A* of size (2**kl*+*ku*+1,*n*).
ipiv	Holds the vector of length *n*.
norm	Must be '1', 'O', or 'I'. The default value is '1'.
kl	If omitted, assumed *kl* = *ku*.
ku	Restored as *ku* = *lda*-2**kl*-1.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b* or A^H**x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately 2*n*(*ku* + 2*kl*) floating-point operations for real flavors and 8*n*(*ku* + 2*kl*) for complex flavors.

?gtcon

Estimates the reciprocal of the condition number of a tridiagonal matrix using the factorization computed by ?gttrf*.*

Syntax

Fortran 77:

```
call sgtcon( norm, n, dl, d, du, du2, ipiv, anorm, rcond, work, iwork, info )
call dgtcon( norm, n, dl, d, du, du2, ipiv, anorm, rcond, work, iwork, info )
call cgtcon( norm, n, dl, d, du, du2, ipiv, anorm, rcond, work, info )
call zgtcon( norm, n, dl, d, du, du2, ipiv, anorm, rcond, work, info )
```

Fortran 95:

```
call gtcon( dl, d, du, du2, ipiv, anorm, rcond [,norm] [,info] )
```

C:

```
lapack_int LAPACKE_sgtcon( char norm, lapack_int n, const float* dl, const float* d,
const float* du, const float* du2, const lapack_int* ipiv, float anorm, float* rcond );
```

```
lapack_int LAPACKE_dgtcon( char norm, lapack_int n, const double* dl, const double* d,
const double* du, const double* du2, const lapack_int* ipiv, double anorm, double*
rcond );

lapack_int LAPACKE_cgtcon( char norm, lapack_int n, const lapack_complex_float* dl,
const lapack_complex_float* d, const lapack_complex_float* du, const
lapack_complex_float* du2, const lapack_int* ipiv, float anorm, float* rcond );

lapack_int LAPACKE_zgtcon( char norm, lapack_int n, const lapack_complex_double* dl,
const lapack_complex_double* d, const lapack_complex_double* du, const
lapack_complex_double* du2, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a real or complex tridiagonal matrix A in the 1-norm or infinity-norm:

$\kappa_1(A) = ||A||_1||A^{-1}||_1$

$\kappa_\infty(A) = ||A||_\infty||A^{-1}||_\infty$

An estimate is obtained for $||A^{-1}||$, and the reciprocal of the condition number is computed as $rcond = 1 / (||A|| \ ||A^{-1}||)$.

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?gttrf` to compute the LU factorization of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

norm	`CHARACTER*1`. Must be `'1'` or `'O'` or `'I'`. If *norm* = `'1'` or `'O'`, then the routine estimates the condition number of matrix A in 1-norm. If *norm* = `'I'`, then the routine estimates the condition number of matrix A in infinity-norm.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
dl,*d*,*du*,*du2*	`REAL` for `sgtcon` `DOUBLE PRECISION` for `dgtcon` `COMPLEX` for `cgtcon` `DOUBLE COMPLEX` for `zgtcon`. Arrays: *dl*(*n* -1), *d*(*n*), *du*(*n* -1), *du2*(*n* -2). The array *dl* contains the (*n* - 1) multipliers that define the matrix L from the LU factorization of A as computed by `?gttrf`. The array *d* contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A. The array *du* contains the (n - 1) elements of the first superdiagonal of U.

	The array *du2* contains the (*n* - 2) elements of the second superdiagonal of *U*.
ipiv	INTEGER. Array, DIMENSION (*n*). The array of pivot indices, as returned by ?gttrf.
anorm	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix *A* (see *Description*).
work	REAL for sgtcon DOUBLE PRECISION for dgtcon COMPLEX for cgtcon DOUBLE COMPLEX for zgtcon. Workspace array, DIMENSION (2**n*).
iwork	INTEGER. Workspace array, DIMENSION (*n*). Used for real flavors only.

Output Parameters

rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond*=0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gtcon interface are as follows:

dl	Holds the vector of length (*n*-1).
d	Holds the vector of length *n*.
du	Holds the vector of length (*n*-1).
du2	Holds the vector of length (*n*-2).
ipiv	Holds the vector of length *n*.
norm	Must be '1', 'O', or 'I'. The default value is '1'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A*x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

?pocon

Estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite matrix.

Syntax

Fortran 77:

```
call spocon( uplo, n, a, lda, anorm, rcond, work, iwork, info )
call dpocon( uplo, n, a, lda, anorm, rcond, work, iwork, info )
call cpocon( uplo, n, a, lda, anorm, rcond, work, rwork, info )
call zpocon( uplo, n, a, lda, anorm, rcond, work, rwork, info )
```

Fortran 95:

```
call pocon( a, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_spocon( int matrix_order, char uplo, lapack_int n, const float* a,
lapack_int lda, float anorm, float* rcond );
lapack_int LAPACKE_dpocon( int matrix_order, char uplo, lapack_int n, const double* a,
lapack_int lda, double anorm, double* rcond );
lapack_int LAPACKE_cpocon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float anorm, float* rcond );
lapack_int LAPACKE_zpocon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is symmetric or Hermitian, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?potrf` to compute the Cholesky factorization of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the upper triangle of A is stored. If *uplo* = `'L'`, the lower triangle of A is stored.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
a, *work*	`REAL` for `spocon` `DOUBLE PRECISION` for `dpocon` `COMPLEX` for `cpocon`

	DOUBLE COMPLEX for zpocon. Arrays: a(lda,*), work(*). The array a contains the factored matrix A, as returned by ?potrf. The second dimension of a must be at least max(1,n). The array work is a workspace for the routine. The dimension of work must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).
anorm	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix A (see *Description*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, n).
rwork	REAL for cpocon DOUBLE PRECISION for zpocon. Workspace array, DIMENSION at least max(1, n).

Output Parameters

rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets rcond =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime rcond is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pocon interface are as follows:

a	Holds the matrix A of size (n, n).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed rcond is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than 10r. A call to this routine involves solving a number of systems of linear equations A*x = b; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

?ppcon

Estimates the reciprocal of the condition number of a packed symmetric (Hermitian) positive-definite matrix.

Syntax

Fortran 77:

```
call sppcon( uplo, n, ap, anorm, rcond, work, iwork, info )
```

```
call dppcon( uplo, n, ap, anorm, rcond, work, iwork, info )
call cppcon( uplo, n, ap, anorm, rcond, work, rwork, info )
call zppcon( uplo, n, ap, anorm, rcond, work, rwork, info )
```

Fortran 95:

```
call ppcon( ap, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_sppcon( int matrix_order, char uplo, lapack_int n, const float* ap,
float anorm, float* rcond );
lapack_int LAPACKE_dppcon( int matrix_order, char uplo, lapack_int n, const double* ap,
double anorm, double* rcond );
lapack_int LAPACKE_cppcon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* ap, float anorm, float* rcond );
lapack_int LAPACKE_zppcon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* ap, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a packed symmetric (Hermitian) positive-definite matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is symmetric or Hermitian, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?pptrf` to compute the Cholesky factorization of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the upper triangle of A is stored. If *uplo* = `'L'`, the lower triangle of A is stored.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
ap, *work*	`REAL` for `sppcon` `DOUBLE PRECISION` for `dppcon` `COMPLEX` for `cppcon` `DOUBLE COMPLEX` for `zppcon`. Arrays: *ap*(*), *work*(*). The array *ap* contains the packed factored matrix A, as returned by `?pptrf`. The dimension of *ap* must be at least max(1,n(n+1)/2).

	The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
anorm	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix *A* (see *Description*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cppcon DOUBLE PRECISION for zppcon. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ppcon interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

?pbcon

Estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite band matrix.

Syntax

Fortran 77:

```
call spbcon( uplo, n, kd, ab, ldab, anorm, rcond, work, iwork, info )
call dpbcon( uplo, n, kd, ab, ldab, anorm, rcond, work, iwork, info )
call cpbcon( uplo, n, kd, ab, ldab, anorm, rcond, work, rwork, info )
call zpbcon( uplo, n, kd, ab, ldab, anorm, rcond, work, rwork, info )
```

Fortran 95:

```
call pbcon( ab, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_spbcon( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const float* ab, lapack_int ldab, float anorm, float* rcond );

lapack_int LAPACKE_dpbcon( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const double* ab, lapack_int ldab, double anorm, double* rcond );

lapack_int LAPACKE_cpbcon( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_float* ab, lapack_int ldab, float anorm, float* rcond );

lapack_int LAPACKE_zpbcon( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_double* ab, lapack_int ldab, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite band matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is symmetric or Hermitian, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?pbtrf` to compute the Cholesky factorization of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the upper triangular factor is stored in *ab*. If *uplo* = `'L'`, the lower triangular factor is stored in *ab*.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
kd	`INTEGER`. The number of superdiagonals or subdiagonals in the matrix A; $kd \geq 0$.
ldab	`INTEGER`. The leading dimension of the array *ab*. (*ldab* $\geq$ *kd* +1).
ab, *work*	`REAL` for `spbcon` `DOUBLE PRECISION` for `dpbcon` `COMPLEX` for `cpbcon` `DOUBLE COMPLEX` for `zpbcon`. Arrays: *ab*(*ldab*,*), *work*(*). The array *ab* contains the factored matrix A in band form, as returned by `?pbtrf`. The second dimension of *ab* must be at least `max(1, n)`.

	The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
anorm	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix *A* (see *Description*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cpbcon DOUBLE PRECISION for zpbcon. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pbcon interface are as follows:

ab	Holds the array *A* of size (*kd*+1,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately 4**n*(*kd* + 1) floating-point operations for real flavors and 16**n*(*kd* + 1) for complex flavors.

?ptcon

Estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite tridiagonal matrix.

Syntax

Fortran 77:

```
call sptcon( n, d, e, anorm, rcond, work, info )
call dptcon( n, d, e, anorm, rcond, work, info )
call cptcon( n, d, e, anorm, rcond, work, info )
call zptcon( n, d, e, anorm, rcond, work, info )
```

Fortran 95:

```
call ptcon( d, e, anorm, rcond [,info] )
```

C:

```
lapack_int LAPACKE_sptcon( lapack_int n, const float* d, const float* e, float anorm,
float* rcond );

lapack_int LAPACKE_dptcon( lapack_int n, const double* d, const double* e, double
anorm, double* rcond );

lapack_int LAPACKE_cptcon( lapack_int n, const float* d, const lapack_complex_float* e,
float anorm, float* rcond );

lapack_int LAPACKE_zptcon( lapack_int n, const double* d, const lapack_complex_double*
e, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the reciprocal of the condition number (in the 1-norm) of a real symmetric or complex Hermitian positive-definite tridiagonal matrix using the factorization $A = L*D*L^T$ for real flavors and $A = L*D*L^H$ for complex flavors or $A = U^T*D*U$ for real flavors and $A = U^H*D*U$ for complex flavors computed by `?pttrf` :

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is symmetric or Hermitian, $\kappa_\infty(A) = \kappa_1(A)$).

The norm $||A^{-1}||$ is computed by a direct method, and the reciprocal of the condition number is computed as $rcond = 1 / (||A|| \ ||A^{-1}||)$.

Before calling this routine:

- compute $anorm$ as $||A||_1 = \max_j \Sigma_i |a_{ij}|$
- call `?pttrf` to compute the factorization of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	`INTEGER`. The order of the matrix A; $n \geq 0$.
d, $work$	`REAL` for single precision flavors `DOUBLE PRECISION` for double precision flavors. Arrays, dimension (n). The array d contains the n diagonal elements of the diagonal matrix D from the factorization of A, as computed by `?pttrf` ; $work$ is a workspace array.
e	`REAL` for `sptcon` `DOUBLE PRECISION` for `dptcon` `COMPLEX` for `cptcon` `DOUBLE COMPLEX` for `zptcon`. Array, `DIMENSION (n -1)`.

	Contains off-diagonal elements of the unit bidiagonal factor *U* or *L* from the factorization computed by ?pttrf .
anorm	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The 1- norm of the *original* matrix *A* (see *Description*).

Output Parameters

rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gtcon interface are as follows:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately 4**n*(*kd* + 1) floating-point operations for real flavors and 16**n*(*kd* + 1) for complex flavors.

?sycon

Estimates the reciprocal of the condition number of a symmetric matrix.

Syntax

Fortran 77:

```
call ssycon( uplo, n, a, lda, ipiv, anorm, rcond, work, iwork, info )
call dsycon( uplo, n, a, lda, ipiv, anorm, rcond, work, iwork, info )
call csycon( uplo, n, a, lda, ipiv, anorm, rcond, work, info )
call zsycon( uplo, n, a, lda, ipiv, anorm, rcond, work, info )
```

Fortran 95:

```
call sycon( a, ipiv, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_ssycon( int matrix_order, char uplo, lapack_int n, const float* a,
lapack_int lda, const lapack_int* ipiv, float anorm, float* rcond );

lapack_int LAPACKE_dsycon( int matrix_order, char uplo, lapack_int n, const double* a,
lapack_int lda, const lapack_int* ipiv, double anorm, double* rcond );

lapack_int LAPACKE_csycon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, const lapack_int* ipiv, float anorm, float*
rcond );

lapack_int LAPACKE_zsycon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, const lapack_int* ipiv, double anorm, double*
rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a symmetric matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is symmetric, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?sytrf` to compute the factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the array *a* stores the upper triangular factor U of the factorization $A = P*U*D*U^T*P^T$. If *uplo* = `'L'`, the array *a* stores the lower triangular factor L of the factorization $A = P*L*D*L^T*P^T$.
n	`INTEGER`. The order of matrix A; $n \geq 0$.
a, *work*	`REAL` for `ssycon` `DOUBLE PRECISION` for `dsycon` `COMPLEX` for `csycon` `DOUBLE COMPLEX` for `zsycon`. Arrays: *a*(*lda*,*), *work*(*). The array *a* contains the factored matrix A, as returned by `?sytrf`. The second dimension of *a* must be at least `max(1,n)`. The array *work* is a workspace for the routine. The dimension of *work* must be at least `max(1, 2*n)`.
lda	`INTEGER`. The leading dimension of *a*; *lda* ≥ `max(1, n)`.
ipiv	`INTEGER`. Array, `DIMENSION` at least `max(1, n)`.

	The array *ipiv*, as returned by ?sytrf.
anorm	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix *A* (see *Description*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sycon interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

?syconv

Converts a symmetric matrix given by a triangular matrix factorization into two matrices and vice versa.

Syntax

Fortran 77:

```
call ssyconv( uplo, way, n, a, lda, ipiv, work, info )
call dsyconv( uplo, way, n, a, lda, ipiv, work, info )
call csyconv( uplo, way, n, a, lda, ipiv, work, info )
call zsyconv( uplo, way, n, a, lda, ipiv, work, info )
```

Fortran 95:

```
call sycon( a[,uplo][,way][,ipiv][,info] )
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine converts matrix *A*, which results from a triangular matrix factorization, into matrices *L* and *D* and vice versa. The routine gets non-diagonalized elements of *D* returned in the workspace and applies or reverses permutation done with the triangular matrix factorization.

Input Parameters

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether the details of the factorization are stored as an upper or lower triangular matrix: If *uplo* = `'U'`: the upper triangular, $A = U*D*U^T$. If *uplo* = `'L'`: the lower triangular, $A = L*D*L^T$.
way	`CHARACTER*1`. Must be `'C'` or `'R'`. Indicates whether the routine converts or reverts the matrix: *way* = `'C'` means conversion. *way* = `'R'` means reversion.
n	`INTEGER`. The order of matrix *A*; $n \geq 0$.
a	`REAL` for `ssyconv` `DOUBLE PRECISION` for `dsyconv` `COMPLEX` for `csyconv` `DOUBLE COMPLEX` for `zsyconv` Array of `DIMENSION` (*lda*,*n*). The block diagonal matrix *D* and the multipliers used to obtain the factor `U` or `L` as computed by `?sytrf`.
lda	`INTEGER`. The leading dimension of *a*; *lda* $\geq$ `max(1, n)`.
ipiv	`INTEGER`. Array, `DIMENSION` at least `max(1, n)`. Details of the interchanges and the block structure of *D*, as returned by `?sytrf`.
work	`INTEGER`. Workspace array, `DIMENSION` at least `max(1, n)`.

Output Parameters

info	`INTEGER`. If *info* = `0`, the execution is successful. If *info* < 0, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `syconv` interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
uplo	Must be `'U'` or `'L'`.
way	Must be `'C'` or `'R'`.
ipiv	Holds the vector of length *n*.

See Also

?sytrf

?hecon

Estimates the reciprocal of the condition number of a Hermitian matrix.

Syntax

Fortran 77:

```
call checon( uplo, n, a, lda, ipiv, anorm, rcond, work, info )
call zhecon( uplo, n, a, lda, ipiv, anorm, rcond, work, info )
```

Fortran 95:

```
call hecon( a, ipiv, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_checon( int matrix_order, char uplo, lapack_int n, const lapack_complex_float* a, lapack_int lda, const lapack_int* ipiv, float anorm, float* rcond );
lapack_int LAPACKE_zhecon( int matrix_order, char uplo, lapack_int n, const lapack_complex_double* a, lapack_int lda, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a Hermitian matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is Hermitian, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?hetrf` to compute the factorization of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the array *a* stores the upper triangular factor U of the factorization $A = P*U*D*U^H*P^T$. If *uplo* = `'L'`, the array *a* stores the lower triangular factor L of the factorization $A = P*L*D*L^H*P^T$.
n	`INTEGER`. The order of matrix A; $n \geq 0$.
a, *work*	`COMPLEX` for `checon` `DOUBLE COMPLEX` for `zhecon`. Arrays: *a*(*lda*,*), *work*(*).

	The array *a* contains the factored matrix *A*, as returned by ?hetrf. The second dimension of *a* must be at least max(1,*n*). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 2*n).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The array *ipiv*, as returned by ?hetrf.
anorm	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix *A* (see *Description*).

Output Parameters

rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hecon interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

?spcon

Estimates the reciprocal of the condition number of a packed symmetric matrix.

Syntax

Fortran 77:

```
call sspcon( uplo, n, ap, ipiv, anorm, rcond, work, iwork, info )
call dspcon( uplo, n, ap, ipiv, anorm, rcond, work, iwork, info )
call cspcon( uplo, n, ap, ipiv, anorm, rcond, work, info )
call zspcon( uplo, n, ap, ipiv, anorm, rcond, work, info )
```

Fortran 95:

```
call spcon( ap, ipiv, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_sspcon( int matrix_order, char uplo, lapack_int n, const float* ap,
const lapack_int* ipiv, float anorm, float* rcond );

lapack_int LAPACKE_dspcon( int matrix_order, char uplo, lapack_int n, const double* ap,
const lapack_int* ipiv, double anorm, double* rcond );

lapack_int LAPACKE_cspcon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* ap, const lapack_int* ipiv, float anorm, float* rcond );

lapack_int LAPACKE_zspcon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* ap, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a packed symmetric matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is symmetric, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call `?sptrf` to compute the factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If *uplo* = 'U', the array *ap* stores the packed upper triangular factor U of the factorization $A = P*U*D*U^T*P^T$. If *uplo* = 'L', the array *ap* stores the packed lower triangular factor L of the factorization $A = P*L*D*L^T*P^T$.
n	INTEGER. The order of matrix A; $n \geq 0$.
ap, *work*	REAL for sspcon DOUBLE PRECISION for dspcon COMPLEX for cspcon DOUBLE COMPLEX for zspcon. Arrays: *ap*(*), *work*(*). The array *ap* contains the packed factored matrix A, as returned by ?sptrf. The dimension of *ap* must be at least max(1,n(n+1)/2). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 2*n).
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The array *ipiv*, as returned by ?sptrf.

anorm	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix *A* (see *Description*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spcon interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

?hpcon

Estimates the reciprocal of the condition number of a packed Hermitian matrix.

Syntax

Fortran 77:

```
call chpcon( uplo, n, ap, ipiv, anorm, rcond, work, info )
call zhpcon( uplo, n, ap, ipiv, anorm, rcond, work, info )
```

Fortran 95:

```
call hpcon( ap, ipiv, anorm, rcond [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_chpcon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* ap, const lapack_int* ipiv, float anorm, float* rcond );
lapack_int LAPACKE_zhpcon( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* ap, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine estimates the reciprocal of the condition number of a Hermitian matrix A:

$\kappa_1(A) = ||A||_1 \ ||A^{-1}||_1$ (since A is Hermitian, $\kappa_\infty(A) = k_1(A)$).

Before calling this routine:

- compute *anorm* (either $||A||_1 = \max_j \Sigma_i |a_{ij}|$ or $||A||_\infty = \max_i \Sigma_j |a_{ij}|$)
- call ?hptrf to compute the factorization of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If *uplo* = 'U', the array *ap* stores the packed upper triangular factor U of the factorization $A = P*U*D*U^T*P^T$. If *uplo* = 'L', the array *ap* stores the packed lower triangular factor L of the factorization $A = P*L*D*L^T*P^T$.
n	INTEGER. The order of matrix A; $n \geq 0$.
ap, *work*	COMPLEX for chpcon DOUBLE COMPLEX for zhpcon. Arrays: *ap*(*), *work*(*). The array *ap* contains the packed factored matrix A, as returned by ?hptrf. The dimension of *ap* must be at least max(1,n(n+1)/2). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 2*n).
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The array *ipiv*, as returned by ?hptrf.
anorm	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. The norm of the *original* matrix A (see *Description*).

Output Parameters

rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `hbcon` interface are as follows:

ap	Holds the array A of size $(n*(n+1)/2)$.
ipiv	Holds the vector of length n.

Application Notes

The computed *rcond* is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

?trcon

Estimates the reciprocal of the condition number of a triangular matrix.

Syntax

Fortran 77:

```
call strcon( norm, uplo, diag, n, a, lda, rcond, work, iwork, info )
call dtrcon( norm, uplo, diag, n, a, lda, rcond, work, iwork, info )
call ctrcon( norm, uplo, diag, n, a, lda, rcond, work, rwork, info )
call ztrcon( norm, uplo, diag, n, a, lda, rcond, work, rwork, info )
```

Fortran 95:

```
call trcon( a, rcond [,uplo] [,diag] [,norm] [,info] )
```

C:

```
lapack_int LAPACKE_strcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const float* a, lapack_int lda, float* rcond );
lapack_int LAPACKE_dtrcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const double* a, lapack_int lda, double* rcond );
lapack_int LAPACKE_ctrcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_float* a, lapack_int lda, float* rcond );
lapack_int LAPACKE_ztrcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_double* a, lapack_int lda, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a triangular matrix A in either the 1-norm or infinity-norm:

$\kappa_1(A) = ||A||_1 \, ||A^{-1}||_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$

$\kappa_\infty(A) = ||A||_\infty \, ||A^{-1}||_\infty = k_1(A^T) = \kappa_1(A^H)$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

norm	CHARACTER*1. Must be '1' or 'O' or 'I'. If *norm* = '1' or 'O', then the routine estimates the condition number of matrix *A* in 1-norm. If *norm* = 'I', then the routine estimates the condition number of matrix *A* in infinity-norm.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether *A* is upper or lower triangular: If *uplo* = 'U', the array *a* stores the upper triangle of *A*, other array elements are not referenced. If *uplo* = 'L', the array *a* stores the lower triangle of *A*, other array elements are not referenced.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then *A* is not a unit triangular matrix. If *diag* = 'U', then *A* is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array *a*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a, *work*	REAL for strcon DOUBLE PRECISION for dtrcon COMPLEX for ctrcon DOUBLE COMPLEX for ztrcon. Arrays: *a*(*lda*,*), *work*(*). The array *a* contains the matrix *A*. The second dimension of *a* must be at least max(1,*n*). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for ctrcon DOUBLE PRECISION for ztrcon. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `trcon` interface are as follows:

a	Holds the matrix A of size (n, n).
norm	Must be '1', 'O', or 'I'. The default value is '1'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

The computed *rcond* is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors and $4n^2$ operations for complex flavors.

?tpcon

Estimates the reciprocal of the condition number of a packed triangular matrix.

Syntax

Fortran 77:

```
call stpcon( norm, uplo, diag, n, ap, rcond, work, iwork, info )
call dtpcon( norm, uplo, diag, n, ap, rcond, work, iwork, info )
call ctpcon( norm, uplo, diag, n, ap, rcond, work, rwork, info )
call ztpcon( norm, uplo, diag, n, ap, rcond, work, rwork, info )
```

Fortran 95:

```
call tpcon( ap, rcond [,uplo] [,diag] [,norm] [,info] )
```

C:

```
lapack_int LAPACKE_stpcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const float* ap, float* rcond );
lapack_int LAPACKE_dtpcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const double* ap, double* rcond );
lapack_int LAPACKE_ctpcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_float* ap, float* rcond );
lapack_int LAPACKE_ztpcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_double* ap, double* rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the reciprocal of the condition number of a packed triangular matrix A in either the 1-norm or infinity-norm:

$\kappa_1(A) = ||A||_1 \, ||A^{-1}||_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$

$\kappa_\infty(A) = ||A||_\infty \, ||A^{-1}||_\infty = \kappa_1(A^T) = \kappa_1(A^H)$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

norm	CHARACTER*1. Must be '1' or 'O' or 'I'. If *norm* = '1' or 'O', then the routine estimates the condition number of matrix *A* in 1-norm. If *norm* = 'I', then the routine estimates the condition number of matrix *A* in infinity-norm.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether *A* is upper or lower triangular: If *uplo* = 'U', the array *ap* stores the upper triangle of *A* in packed form. If *uplo* = 'L', the array *ap* stores the lower triangle of *A* in packed form.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then *A* is not a unit triangular matrix. If *diag* = 'U', then *A* is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array *ap*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
ap, *work*	REAL for stpcon DOUBLE PRECISION for dtpcon COMPLEX for ctpcon DOUBLE COMPLEX for ztpcon. Arrays: *ap*(*), *work*(*). The array *ap* contains the packed matrix *A*. The dimension of *ap* must be at least max(1,*n*(*n*+1)/2). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for ctpcon DOUBLE PRECISION for ztpcon. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `tpcon` interface are as follows:

ap	Holds the array *A* of size `(n*(n+1)/2)`.
norm	Must be `'1'`, `'O'`, or `'I'`. The default value is `'1'`.
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.
diag	Must be `'N'` or `'U'`. The default value is `'N'`.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations *A*x = b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors and $4n^2$ operations for complex flavors.

?tbcon

Estimates the reciprocal of the condition number of a triangular band matrix.

Syntax

Fortran 77:

```
call stbcon( norm, uplo, diag, n, kd, ab, ldab, rcond, work, iwork, info )
call dtbcon( norm, uplo, diag, n, kd, ab, ldab, rcond, work, iwork, info )
call ctbcon( norm, uplo, diag, n, kd, ab, ldab, rcond, work, rwork, info )
call ztbcon( norm, uplo, diag, n, kd, ab, ldab, rcond, work, rwork, info )
```

Fortran 95:

```
call tbcon( ab, rcond [,uplo] [,diag] [,norm] [,info] )
```

C:

```
lapack_int LAPACKE_stbcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const float* ab, lapack_int ldab, float* rcond );
lapack_int LAPACKE_dtbcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const double* ab, lapack_int ldab, double* rcond );
lapack_int LAPACKE_ctbcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const lapack_complex_float* ab, lapack_int ldab, float*
rcond );
lapack_int LAPACKE_ztbcon( int matrix_order, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const lapack_complex_double* ab, lapack_int ldab, double*
rcond );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`

- C: mkl_lapacke.h

Description

The routine estimates the reciprocal of the condition number of a triangular band matrix A in either the 1-norm or infinity-norm:

$\kappa_1(A) = ||A||_1 \, ||A^{-1}||_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$

$\kappa_\infty(A) = ||A||_\infty \, ||A^{-1}||_\infty = \kappa_1(A^T) = \kappa_1(A^H)$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

norm	CHARACTER*1. Must be '1' or 'O' or 'I'. If *norm* = '1' or 'O', then the routine estimates the condition number of matrix *A* in 1-norm. If *norm* = 'I', then the routine estimates the condition number of matrix *A* in infinity-norm.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether *A* is upper or lower triangular: If *uplo* = 'U', the array *ap* stores the upper triangle of *A* in packed form. If *uplo* = 'L', the array *ap* stores the lower triangle of *A* in packed form.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then *A* is not a unit triangular matrix. If *diag* = 'U', then *A* is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array *ab*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
kd	INTEGER. The number of superdiagonals or subdiagonals in the matrix *A*; $kd \geq 0$.
ab, *work*	REAL for stbcon DOUBLE PRECISION for dtbcon COMPLEX for ctbcon DOUBLE COMPLEX for ztbcon. Arrays: *ab*(*ldab*,*), *work*(*). The array *ab* contains the band matrix *A*. The second dimension of *ab* must be at least max(1,*n*). The array *work* is a workspace for the routine. The dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
ldab	INTEGER. The leading dimension of the array *ab*. ($ldab \geq kd + 1$).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for ctbcon DOUBLE PRECISION for ztbcon. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

rcond	REAL for single precision flavors.

	DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal of the condition number. The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tbcon interface are as follows:

ab	Holds the array *A* of size (*kd*+1,*n*).
norm	Must be '1', 'O', or 'I'. The default value is '1'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately 2**n*(*kd* + 1) floating-point operations for real flavors and 8**n*(*kd* + 1) operations for complex flavors.

Refining the Solution and Estimating Its Error

This section describes the LAPACK routines for refining the computed solution of a system of linear equations and estimating the solution error. You can call these routines after factorizing the matrix of the system of equations and computing the solution (see Routines for Matrix Factorization and Routines for Solving Systems of Linear Equations).

?gerfs

Refines the solution of a system of linear equations with a general matrix and estimates its error.

Syntax

Fortran 77:

```
call sgerfs( trans, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call dgerfs( trans, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call cgerfs( trans, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
call zgerfs( trans, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call gerfs( a, af, ipiv, b, x [,trans] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_sgerfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const float* a, lapack_int lda, const float* af, lapack_int ldaf, const lapack_int*
ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float*
berr );

lapack_int LAPACKE_dgerfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const double* a, lapack_int lda, const double* af, lapack_int ldaf, const lapack_int*
ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr,
double* berr );

lapack_int LAPACKE_cgerfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zgerfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a general matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?getrf`
- call the solver routine `?getrs`.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', the system has the form $A*X = B$. If *trans* = 'T', the system has the form $A^T*X = B$. If *trans* = 'C', the system has the form $A^H*X = B$.
n	INTEGER. The order of the matrix A; $n \ge 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \ge 0$.

a,*af*,*b*,*x*,*work*	REAL for sgerfs DOUBLE PRECISION for dgerfs COMPLEX for cgerfs DOUBLE COMPLEX for zgerfs. Arrays: *a*(*lda*,*) contains the original matrix *A*, as supplied to ?getrf. *af*(*ldaf*,*) contains the factored matrix *A*, as returned by ?getrf. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The second dimension of *a* and *af* must be at least max(1, *n*); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 3**n*) for real flavors and max(1, 2**n*) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?getrf.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cgerfs DOUBLE PRECISION for zgerfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gerfs interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
af	Holds the matrix *AF* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).

berr	Holds the vector of length (*nrhs*).
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations *A*x = b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

?gerfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a general matrix A and provides error bounds and backward error estimates.

Syntax

Fortran 77:

```
call sgerfsx( trans, equed, n, nrhs, a, lda, af, ldaf, ipiv, r, c, b, ldb, x, ldx,
rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork,
info )

call dgerfsx( trans, equed, n, nrhs, a, lda, af, ldaf, ipiv, r, c, b, ldb, x, ldx,
rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork,
info )

call cgerfsx( trans, equed, n, nrhs, a, lda, af, ldaf, ipiv, r, c, b, ldb, x, ldx,
rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork,
info )

call zgerfsx( trans, equed, n, nrhs, a, lda, af, ldaf, ipiv, r, c, b, ldb, x, ldx,
rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork,
info )
```

C:

```
lapack_int LAPACKE_sgerfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf,
const lapack_int* ipiv, const float* r, const float* c, const float* b, lapack_int
ldb, float* x, lapack_int ldx, float* rcond, float* berr, lapack_int n_err_bnds,
float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float* params );

lapack_int LAPACKE_dgerfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf,
const lapack_int* ipiv, const double* r, const double* c, const double* b, lapack_int
ldb, double* x, lapack_int ldx, double* rcond, double* berr, lapack_int n_err_bnds,
double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double* params );

lapack_int LAPACKE_cgerfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const float* r,
```

```
const float* c, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* berr, lapack_int n_err_bnds, float*
err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float* params );

lapack_int LAPACKE_zgerfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const double* r,
const double* c, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* rcond, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed*, *r*, and *c* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X = B* (No transpose). If *trans* = 'T', the system has the form *A**T*X = B* (Transpose). If *trans* = 'C', the system has the form *A**H*X = B* (Conjugate transpose = Transpose).
equed	CHARACTER*1. Must be 'N', 'R', 'C', or 'B'. Specifies the form of equilibration that was done to *A* before calling this routine. If *equed* = 'N', no equilibration was done. If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag*(*r*). If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag*(*c*). If *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag(r)*A*diag(c)*. The right-hand side *B* has been changed accordingly.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
a, *af*, *b*, *work*	REAL for sgerfsx DOUBLE PRECISION for dgerfsx

	COMPLEX for cgerfsx DOUBLE COMPLEX for zgerfsx. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*). The array *a* contains the original *n*-by-*n* matrix *A*. The array *af* contains the factored form of the matrix *A*, that is, the factors *L* and *U* from the factorization *A = P*L*U* as computed by ?getrf. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*). *work(*)* is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Contains the pivot indices as computed by ?getrf; for row 1 ≤ *i* ≤ *n*, row *i* of the matrix was interchanged with row *ipiv*(*i*).
r, *c*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays: *r*(*n*), *c*(*n*). The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. *equed* = 'R' or 'B', *A* is multiplied on the left by *diag*(*r*); if *equed* = 'N' or 'C', *r* is not accessed. If *equed* = 'R' or 'B', each element of *r* must be positive. If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag*(*c*); if *equed* = 'N' or 'R', *c* is not accessed. If *equed* = 'C' or 'B', each element of *c* must be positive. Each element of *r* or *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).
x	REAL for sgerfsx DOUBLE PRECISION for dgerfsx COMPLEX for cgerfsx DOUBLE COMPLEX for zgerfsx. Array, DIMENSION (*ldx*,*). The solution matrix *X* as computed by ?getrs
ldx	INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).
n_err_bnds	INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.
nparams	INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.
params	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used

for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.

params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support DOUBLE PRECISION.

(Other values are reserved for futute use.)

params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for refinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork — INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3**n*); used in complex flavors only.

Output Parameters

x — REAL for sgerfsx
DOUBLE PRECISION for dgerfsx
COMPLEX for cgerfsx
DOUBLE COMPLEX for zgerfsx.
The improved solution matrix *X*.

rcond — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left|Xtrue_{ji} - X_{ji}\right|}{\max_j \left|X_{ji}\right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Output parameter only if the input contains erroneous values, namely, in *params(1)*, *params(2)*, *params(3)*. In such a case, the corresponding elements of *params* are filled with default values on output.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = −*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested

params(3) = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm*(;,1) and *err_bnds_comp*(;,1). To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?gbrfs

Refines the solution of a system of linear equations with a general band matrix and estimates its error.

Syntax

Fortran 77:

```
call sgbrfs( trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )

call dgbrfs( trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )

call cgbrfs( trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )

call zgbrfs( trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call gbrfs( ab, afb, ipiv, b, x [,kl] [,trans] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_sgbrfs( int matrix_order, char trans, lapack_int n, lapack_int kl, lapack_int ku, lapack_int nrhs, const float* ab, lapack_int ldab, const float* afb, lapack_int ldafb, const lapack_int* ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dgbrfs( int matrix_order, char trans, lapack_int n, lapack_int kl, lapack_int ku, lapack_int nrhs, const double* ab, lapack_int ldab, const double* afb, lapack_int ldafb, const lapack_int* ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );

lapack_int LAPACKE_cgbrfs( int matrix_order, char trans, lapack_int n, lapack_int kl, lapack_int ku, lapack_int nrhs, const lapack_complex_float* ab, lapack_int ldab, const lapack_complex_float* afb, lapack_int ldafb, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zgbrfs( int matrix_order, char trans, lapack_int n, lapack_int kl, lapack_int ku, lapack_int nrhs, const lapack_complex_double* ab, lapack_int ldab, const lapack_complex_double* afb, lapack_int ldafb, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a band matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \leq \beta |a_{ij}|$, $|\delta b_i| \leq \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine ?gbtrf
- call the solver routine ?gbtrs.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', the system has the form $A*X = B$. If *trans* = 'T', the system has the form $A^T*X = B$. If *trans* = 'C', the system has the form $A^H*X = B$.
n	INTEGER. The order of the matrix A; $n \geq 0$.
kl	INTEGER. The number of sub-diagonals within the band of A; $kl \geq 0$.
ku	INTEGER. The number of super-diagonals within the band of A; $ku \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ab,*afb*,*b*,*x*,*work*	REAL for sgbrfs DOUBLE PRECISION for dgbrfs COMPLEX for cgbrfs DOUBLE COMPLEX for zgbrfs. Arrays: *ab*(*ldab*,*) contains the original band matrix A, as supplied to ?gbtrf, but stored in rows from 1 to *kl* + *ku* + 1. *afb*(*ldafb*,*) contains the factored band matrix A, as returned by ?gbtrf. *b*(*ldb*,*) contains the right-hand side matrix B. *x*(*ldx*,*) contains the solution matrix X. *work*(*) is a workspace array. The second dimension of *ab* and *afb* must be at least max(1, *n*); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
ldab	INTEGER. The leading dimension of *ab*.
ldafb	INTEGER. The leading dimension of *afb* .
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
ipiv	INTEGER.

	Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?gbtrf.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cgbrfs DOUBLE PRECISION for zgbrfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* =0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbrfs interface are as follows:

ab	Holds the array *A* of size (*kl*+*ku*+1, *n*).
afb	Holds the array *AF* of size (2**kl***ku*+1, *n*).
ipiv	Holds the vector of length *n*.
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
kl	If omitted, assumed *kl* = *ku*.
ku	Restored as *ku* = *lda*-*kl*-1.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n(kl + ku)$ floating-point operations (for real flavors) or $16n(kl + ku)$ operations (for complex flavors). In addition, each step of iterative refinement involves $2n(4kl + 3ku)$ operations (for real flavors) or $8n(4kl + 3ku)$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

?gbrfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a banded matrix A and provides error bounds and backward error estimates.

Syntax

Fortran 77:

```
call sgbrfsx( trans, equed, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, r, c, b, ldb,
x, ldx, rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
iwork, info )
```

```
call dgbrfsx( trans, equed, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, r, c, b, ldb,
x, ldx, rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
iwork, info )
```

```
call cgbrfsx( trans, equed, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, r, c, b, ldb,
x, ldx, rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
rwork, info )
```

```
call zgbrfsx( trans, equed, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, r, c, b, ldb,
x, ldx, rcond, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
rwork, info )
```

C:

```
lapack_int LAPACKE_sgbrfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const float* ab, lapack_int ldab, const
float* afb, lapack_int ldafb, const lapack_int* ipiv, const float* r, const float* c,
const float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
float* params );
```

```
lapack_int LAPACKE_dgbrfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const double* ab, lapack_int ldab,
const double* afb, lapack_int ldafb, const lapack_int* ipiv, const double* r, const
double* c, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond,
double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double* err_bnds_comp,
lapack_int nparams, double* params );
```

```
lapack_int LAPACKE_cgbrfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const lapack_complex_float* ab,
lapack_int ldab, const lapack_complex_float* afb, lapack_int ldafb, const lapack_int*
ipiv, const float* r, const float* c, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* berr, lapack_int
n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float*
params );
```

```
lapack_int LAPACKE_zgbrfsx( int matrix_order, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const lapack_complex_double* ab,
lapack_int ldab, const lapack_complex_double* afb, lapack_int ldafb, const lapack_int*
ipiv, const double* r, const double* c, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double* berr, lapack_int
n_err_bnds, double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double*
params );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed*, *r*, and *c* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose). If *trans* = 'T', the system has the form *A**T*X* = *B* (Transpose). If *trans* = 'C', the system has the form *A**H*X* = *B* (Conjugate transpose = Transpose).
equed	CHARACTER*1. Must be 'N', 'R', 'C', or 'B'. Specifies the form of equilibration that was done to *A* before calling this routine. If *equed* = 'N', no equilibration was done. If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag*(*r*). If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag*(*c*). If *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*r*)*A*diag*(*c*). The right-hand side *B* has been changed accordingly.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
ab, *afb*, *b*, *work*	REAL for sgbrfsx DOUBLE PRECISION for dgbrfsx COMPLEX for cgbrfsx DOUBLE COMPLEX for zgbrfsx. Arrays: *ab*(*ldab*,*), *afb*(*ldafb*,*), *b*(*ldb*,*), *work*(*). The array *ab* contains the original matrix *A* in band storage, in rows 1 to *kl*+*ku*+1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = *A*(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*).

The array *afb* contains details of the LU factorization of the banded matrix *A* as computed by ?gbtrf. *U* is stored as an upper triangular banded matrix with *kl* + *ku* superdiagonals in rows 1 to *kl* + *ku* + 1. The multipliers used during the factorization are stored in rows *kl* + *ku* + 2 to 2**kl* + *ku* + 1. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
work(*) is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.

ldab — INTEGER. The leading dimension of the array *ab*; *ldab* ≥ *kl*+*ku*+1.

ldafb — INTEGER. The leading dimension of the array *afb*; *ldafb* ≥ 2**kl*+*ku*+1.

ipiv — INTEGER.
Array, DIMENSION at least max(1, *n*). Contains the pivot indices as computed by ?gbtrf; for row 1 ≤ *i* ≤ *n*, row *i* of the matrix was interchanged with row *ipiv*(*i*).

r, *c* — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Arrays: *r*(*n*), *c*(*n*). The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*.
If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag*(*r*); if *equed* = 'N' or 'C', *r* is not accessed.
If *equed* = 'R' or 'B', each element of *r* must be positive.
If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag*(*c*); if *equed* = 'N' or 'R', *c* is not accessed.
If *equed* = 'C' or 'B', each element of *c* must be positive.
Each element of *r* or *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb — INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

x — REAL for sgbrfsx
DOUBLE PRECISION for dgbrfsx
COMPLEX for cgbrfsx
DOUBLE COMPLEX for zgbrfsx.
Array, DIMENSION (*ldx*,*).
The solution matrix *X* as computed by sgbtrs/dgbtrs for real flavors or cgbtrs/zgbtrs for complex flavors.

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds — INTEGER. Number of error bounds to return for each right-hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams — INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used

for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support DOUBLE PRECISION.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for refinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork — INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3**n*); used in complex flavors only.

Output Parameters

x — REAL for sgbrfsx
DOUBLE PRECISION for dgbrfsx
COMPLEX for cgbrfsx
DOUBLE COMPLEX for zgbrfsx.
The improved solution matrix *X*.

rcond — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left|Xtrue_{ji} - X_{ji}\right|}{\max_j \left|X_{ji}\right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the following three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/*z*,inf)*norm(*z*,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Output parameter only if the input contains erroneous values, namely, in *params(1)*, *params(2)*, *params(3)*. In such a case, the corresponding elements of *params* are filled with default values on output.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = −*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested

params(3) = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm*(;,1) and *err_bnds_comp*(;,1). To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?gtrfs

Refines the solution of a system of linear equations with a tridiagonal matrix and estimates its error.

Syntax

Fortran 77:

```
call sgtrfs( trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x, ldx,
ferr, berr, work, iwork, info )

call dgtrfs( trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x, ldx,
ferr, berr, work, iwork, info )

call cgtrfs( trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x, ldx,
ferr, berr, work, rwork, info )

call zgtrfs( trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x, ldx,
ferr, berr, work, rwork, info )
```

Fortran 95:

```
call gtrfs( dl, d, du, dlf, df, duf, du2, ipiv, b, x [,trans] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_sgtrfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const float* dl, const float* d, const float* du, const float* dlf, const float* df,
const float* duf, const float* du2, const lapack_int* ipiv, const float* b, lapack_int
ldb, float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dgtrfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const double* dl, const double* d, const double* du, const double* dlf, const double*
df, const double* duf, const double* du2, const lapack_int* ipiv, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );

lapack_int LAPACKE_cgtrfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const lapack_complex_float* dl, const lapack_complex_float* d, const
lapack_complex_float* du, const lapack_complex_float* dlf, const lapack_complex_float*
df, const lapack_complex_float* duf, const lapack_complex_float* du2, const lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zgtrfs( int matrix_order, char trans, lapack_int n, lapack_int nrhs,
const lapack_complex_double* dl, const lapack_complex_double* d, const
lapack_complex_double* du, const lapack_complex_double* dlf, const
lapack_complex_double* df, const lapack_complex_double* duf, const
lapack_complex_double* du2, const lapack_int* ipiv, const lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a tridiagonal matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}|/|a_{ij}| \leq \beta|a_{ij}|$, $|\delta b_i|/|b_i| \leq \beta|b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty/||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?gttrf`
- call the solver routine `?gttrs`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	`CHARACTER*1`. Must be `'N'` or `'T'` or `'C'`. Indicates the form of the equations: If *trans* = `'N'`, the system has the form $A*X = B$. If *trans* = `'T'`, the system has the form $A^T*X = B$. If *trans* = `'C'`, the system has the form $A^H*X = B$.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides, that is, the number of columns of the matrix B; $nrhs \geq 0$.
dl,*d*,*du*,*dlf*, *df*,*duf*,*du2*, *b*,*x*,*work*	`REAL` for `sgtrfs` `DOUBLE PRECISION` for `dgtrfs` `COMPLEX` for `cgtrfs` `DOUBLE COMPLEX` for `zgtrfs`. Arrays: *dl*, dimension (*n* -1), contains the subdiagonal elements of A. *d*, dimension (*n*), contains the diagonal elements of A. *du*, dimension (*n* -1), contains the superdiagonal elements of A. *dlf*, dimension (*n* -1), contains the (*n* - 1) multipliers that define the matrix L from the LU factorization of A as computed by `?gttrf`. *df*, dimension (*n*), contains the *n* diagonal elements of the upper triangular matrix U from the LU factorization of A. *duf*, dimension (*n* -1), contains the (*n* - 1) elements of the first superdiagonal of U. *du2*, dimension (*n* -2), contains the (*n* - 2) elements of the second superdiagonal of U. *b*(*ldb*, *nrhs*) contains the right-hand side matrix B. *x*(*ldx*, *nrhs*) contains the solution matrix X, as computed by `?gttrs`. *work*(*) is a workspace array; the dimension of *work* must be at least `max(1, 3*n)` for real flavors and `max(1, 2*n)` for complex flavors.
ldb	`INTEGER`. The leading dimension of *b*; *ldb* ≥ `max(1, n)`.
ldx	`INTEGER`. The leading dimension of *x*; *ldx* ≥ `max(1, n)`.

ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?gttrf.
iwork	INTEGER. Workspace array, DIMENSION (*n*). Used for real flavors only.
rwork	REAL for cgtrfs DOUBLE PRECISION for zgtrfs. Workspace array, DIMENSION (*n*). Used for complex flavors only.

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gtrfs interface are as follows:

dl	Holds the vector of length (*n*-1).
d	Holds the vector of length *n*.
du	Holds the vector of length (*n*-1).
dlf	Holds the vector of length (*n*-1).
df	Holds the vector of length *n*.
duf	Holds the vector of length (*n*-1).
du2	Holds the vector of length (*n*-2).
ipiv	Holds the vector of length *n*.
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

?porfs

Refines the solution of a system of linear equations with a symmetric (Hermitian) positive-definite matrix and estimates its error.

Syntax

Fortran 77:

```
call sporfs( uplo, n, nrhs, a, lda, af, ldaf, b, ldb, x, ldx, ferr, berr, work, iwork, info )
```

```
call dporfs( uplo, n, nrhs, a, lda, af, ldaf, b, ldb, x, ldx, ferr, berr, work, iwork, info )

call cporfs( uplo, n, nrhs, a, lda, af, ldaf, b, ldb, x, ldx, ferr, berr, work, rwork, info )

call zporfs( uplo, n, nrhs, a, lda, af, ldaf, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call porfs( a, af, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_sporfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dporfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );

lapack_int LAPACKE_cporfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af, lapack_int ldaf, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zporfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af, lapack_int ldaf, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a symmetric (Hermitian) positive definite matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \leq \beta |a_{ij}|$, $|\delta b_i| \leq \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?potrf`
- call the solver routine `?potrs`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
a,*af*,*b*,*x*,*work*	REAL for sporfs DOUBLE PRECISION for dporfs COMPLEX for cporfs DOUBLE COMPLEX for zporfs. Arrays: *a*(*lda*,*) contains the original matrix *A*, as supplied to ?potrf. *af*(*ldaf*,*) contains the factored matrix *A*, as returned by ?potrf. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The second dimension of *a* and *af* must be at least max(1, *n*); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cporfs DOUBLE PRECISION for zporfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine porfs interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
af	Holds the matrix *AF* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).

ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations *A*x = b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

?porfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a symmetric/Hermitian positive-definite matrix A and provides error bounds and backward error estimates.

Syntax

Fortran 77:

```
call sporfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, s, b, ldb, x, ldx, rcond, berr,
n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork, info )

call dporfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, s, b, ldb, x, ldx, rcond, berr,
n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork, info )

call cporfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, s, b, ldb, x, ldx, rcond, berr,
n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork, info )

call zporfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, s, b, ldb, x, ldx, rcond, berr,
n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork, info )
```

C:

```
lapack_int LAPACKE_sporfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf,
const float* s, const float* b, lapack_int ldb, float* x, lapack_int ldx, float*
rcond, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp,
lapack_int nparams, float* params );

lapack_int LAPACKE_dporfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf,
const double* s, const double* b, lapack_int ldb, double* x, lapack_int ldx, double*
rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, double* params );

lapack_int LAPACKE_cporfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const float* s, const lapack_complex_float*
b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
float* params );
```

```
lapack_int LAPACKE_zporfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const double* s, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed* and *s* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo
: CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored:
If *uplo* = 'U', the upper triangle of *A* is stored.
If *uplo* = 'L', the lower triangle of *A* is stored.

equed
: CHARACTER*1. Must be 'N' or 'Y'.
Specifies the form of equilibration that was done to *A* before calling this routine.
If *equed* = 'N', no equilibration was done.
If *equed* = 'Y', both row and column equilibration was done, that is, *A* has been replaced by *diag(s)*A*diag(s)*. The right-hand side *B* has been changed accordingly.

n
: INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.

nrhs
: INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.

a, *af*, *b*, *work*
: REAL for sporfsx
DOUBLE PRECISION for dporfsx
COMPLEX for cporfsx
DOUBLE COMPLEX for zporfsx.
Arrays: *a(lda,*)*, *af(ldaf,*)*, *b(ldb,*)*, *work(*)*.
The array *a* contains the symmetric/Hermitian matrix *A* as specified by *uplo*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A* and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower

triangular part of *a* contains the lower triangular part of the matrix *A* and the strictly upper triangular part of *a* is not referenced. The second dimension of *a* must be at least max(1,*n*).
The array *af* contains the triangular factor *L* or *U* from the Cholesky factorization *A = U**T*U* or *A = L*L**T* as computed by spotrf for real flavors or dpotrf for complex flavors.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
work(*) is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.

lda — INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).

ldaf — INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).

s — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*.
If *equed* = 'N', *s* is not accessed.
If *equed* = 'Y', each element of *s* must be positive.
Each element of *s* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb — INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

x — REAL for sporfsx
DOUBLE PRECISION for dporfsx
COMPLEX for cporfsx
DOUBLE COMPLEX for zporfsx.
Array, DIMENSION (*ldx*,*).
The solution matrix *X* as computed by ?potrs

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds — INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams — INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.

=1.0	Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support DOUBLE PRECISION.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for refinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3**n*); used in complex flavors only.

Output Parameters

x REAL for sporfsx
DOUBLE PRECISION for dporfsx
COMPLEX for cporfsx
DOUBLE COMPLEX for zporfsx.
The improved solution matrix *X*.

rcond REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left| Xtrue_{ji} - X_{ji} \right|}{\left| X_{ji} \right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.

The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Output parameter only if the input contains erroneous values. namely, in *params(1)*, *params(2)*, *params(3)*. In such a case, the corresponding elements of *params* are filled with default values on output.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = −*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm(:,1)* and *err_bnds_comp(:,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?pprfs

Refines the solution of a system of linear equations with a packed symmetric (Hermitian) positive-definite matrix and estimates its error.

Syntax

Fortran 77:

```
call spprfs( uplo, n, nrhs, ap, afp, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call dpprfs( uplo, n, nrhs, ap, afp, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call cpprfs( uplo, n, nrhs, ap, afp, b, ldb, x, ldx, ferr, berr, work, rwork, info )
call zpprfs( uplo, n, nrhs, ap, afp, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call pprfs( ap, afp, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_spprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const float* ap, const float* afp, const float* b, lapack_int ldb, float* x,
lapack_int ldx, float* ferr, float* berr );
lapack_int LAPACKE_dpprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const double* ap, const double* afp, const double* b, lapack_int ldb, double* x,
lapack_int ldx, double* ferr, double* berr );
lapack_int LAPACKE_cpprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* ap, const lapack_complex_float* afp, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* ferr, float* berr );
lapack_int LAPACKE_zpprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* ap, const lapack_complex_double* afp, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a packed symmetric (Hermitian)positive definite matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution

$||x - x_e||_\infty / ||x||_\infty$

where x_e is the exact solution.

Before calling this routine:

- call the factorization routine ?pptrf
- call the solver routine ?pptrs.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ap, *afp*, *b*, *x*, *work*	REAL for spprfs DOUBLE PRECISION for dpprfs COMPLEX for cpprfs DOUBLE COMPLEX for zpprfs. Arrays: *ap*(*) contains the original packed matrix *A*, as supplied to ?pptrf. *afp*(*) contains the factored packed matrix *A*, as returned by ?pptrf. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The dimension of arrays *ap* and *afp* must be at least max(1,*n*(*n*+1)/2); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 3**n*) for real flavors and max(1, 2**n*) for complex flavors.
ldb	INTEGER. The leading dimension of *b*; *ldb* $\geq$ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* $\geq$ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cpprfs DOUBLE PRECISION for zpprfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pprfs interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
afp	Holds the array *AF* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations *A*x = b*; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

?pbrfs

Refines the solution of a system of linear equations with a band symmetric (Hermitian) positive-definite matrix and estimates its error.

Syntax

Fortran 77:

```
call spbrfs( uplo, n, kd, nrhs, ab, ldab, afb, ldafb, b, ldb, x, ldx, ferr, berr,
work, iwork, info )
```

```
call dpbrfs( uplo, n, kd, nrhs, ab, ldab, afb, ldafb, b, ldb, x, ldx, ferr, berr,
work, iwork, info )
```

```
call cpbrfs( uplo, n, kd, nrhs, ab, ldab, afb, ldafb, b, ldb, x, ldx, ferr, berr,
work, rwork, info )
```

```
call zpbrfs( uplo, n, kd, nrhs, ab, ldab, afb, ldafb, b, ldb, x, ldx, ferr, berr,
work, rwork, info )
```

Fortran 95:

```
call pbrfs( ab, afb, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_spbrfs( int matrix_order, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const float* ab, lapack_int ldab, const float* afb, lapack_int ldafb,
const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dpbrfs( int matrix_order, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const double* ab, lapack_int ldab, const double* afb, lapack_int
ldafb, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr,
double* berr );
```

```
lapack_int LAPACKE_cpbrfs( int matrix_order, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const lapack_complex_float* ab, lapack_int ldab, const
lapack_complex_float* afb, lapack_int ldafb, const lapack_complex_float* b, lapack_int
ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zpbrfs( int matrix_order, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const lapack_complex_double* ab, lapack_int ldab, const
lapack_complex_double* afb, lapack_int ldafb, const lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a symmetric (Hermitian) positive definite band matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?pbtrf`
- call the solver routine `?pbtrs`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the upper triangle of A is stored. If *uplo* = `'L'`, the lower triangle of A is stored.
n	`INTEGER`. The order of the matrix A; $n \ge 0$.
kd	`INTEGER`. The number of superdiagonals or subdiagonals in the matrix A; $kd \ge 0$.
nrhs	`INTEGER`. The number of right-hand sides; $nrhs \ge 0$.
ab,*afb*,*b*,*x*,*work*	`REAL` for `spbrfs` `DOUBLE PRECISION` for `dpbrfs` `COMPLEX` for `cpbrfs` `DOUBLE COMPLEX` for `zpbrfs`. Arrays: *ab*(*ldab*,*) contains the original band matrix A, as supplied to `?pbtrf`. *afb*(*ldafb*,*) contains the factored band matrix A, as returned by `?pbtrf`.

	b(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The second dimension of *ab* and *afb* must be at least max(1, *n*); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 3**n*) for real flavors and max(1, 2**n*) for complex flavors.
ldab	INTEGER. The leading dimension of *ab*; *ldab* ≥ *kd* + 1.
ldafb	INTEGER. The leading dimension of *afb*; *ldafb* ≥ *kd* + 1.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cpbrfs DOUBLE PRECISION for zpbrfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pbrfs interface are as follows:

ab	Holds the array *A* of size (*kd*+1, *n*).
afb	Holds the array *AF* of size (*kd*+1, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of 8*n***kd* floating-point operations (for real flavors) or 32*n***kd* operations (for complex flavors). In addition, each step of iterative refinement involves 12*n***kd* operations (for real flavors) or 48*n***kd* operations (for complex flavors); the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations `A*x = b`; the number is usually 4 or 5 and never more than 11. Each solution requires approximately `4n*kd` floating-point operations for real flavors or `16n*kd` for complex flavors.

?ptrfs

Refines the solution of a system of linear equations with a symmetric (Hermitian) positive-definite tridiagonal matrix and estimates its error.

Syntax

Fortran 77:

```
call sptrfs( n, nrhs, d, e, df, ef, b, ldb, x, ldx, ferr, berr, work, info )
call dptrfs( n, nrhs, d, e, df, ef, b, ldb, x, ldx, ferr, berr, work, info )
call cptrfs( uplo, n, nrhs, d, e, df, ef, b, ldb, x, ldx, ferr, berr, work, rwork, info )
call zptrfs( uplo, n, nrhs, d, e, df, ef, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call ptrfs( d, df, e, ef, b, x [,ferr] [,berr] [,info] )
call ptrfs( d, df, e, ef, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_sptrfs( int matrix_order, lapack_int n, lapack_int nrhs, const float* d, const float* e, const float* df, const float* ef, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );
lapack_int LAPACKE_dptrfs( int matrix_order, lapack_int n, lapack_int nrhs, const double* d, const double* e, const double* df, const double* ef, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );
lapack_int LAPACKE_cptrfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const float* d, const lapack_complex_float* e, const float* df, const lapack_complex_float* ef, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
lapack_int LAPACKE_zptrfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const double* d, const lapack_complex_double* e, const double* df, const lapack_complex_double* ef, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations *A*X = B* with a symmetric (Hermitian) positive definite tridiagonal matrix *A*, with multiple right-hand sides. For each computed solution vector *x*, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of *A* and *b* such that *x* is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine ?pttrf
- call the solver routine ?pttrs.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Used for complex flavors only. Must be 'U' or 'L'. Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix *A* is stored and how *A* is factored: If *uplo* = 'U', the array *e* stores the superdiagonal of *A*, and *A* is factored as U^H*D*U. If *uplo* = 'L', the array *e* stores the subdiagonal of *A*, and *A* is factored as $L*D*L^H$.
n	INTEGER. The order of the matrix *A*; $n \ge 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \ge 0$.
d, *df*, *rwork*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors Arrays: *d*(*n*), *df*(*n*), *rwork*(*n*). The array *d* contains the *n* diagonal elements of the tridiagonal matrix *A*. The array *df* contains the *n* diagonal elements of the diagonal matrix *D* from the factorization of *A* as computed by ?pttrf. The array *rwork* is a workspace array used for complex flavors only.
e,*ef*,*b*,*x*,*work*	REAL for sptrfs DOUBLE PRECISION for dptrfs COMPLEX for cptrfs DOUBLE COMPLEX for zptrfs. Arrays: *e*(*n* -1), *ef*(*n* -1), *b*(*ldb*,*nrhs*), *x*(*ldx*,*nrhs*), *work*(*). The array *e* contains the (*n* - 1) off-diagonal elements of the tridiagonal matrix *A* (see *uplo*). The array *ef* contains the (*n* - 1) off-diagonal elements of the unit bidiagonal factor *U* or *L* from the factorization computed by ?pttrf (see *uplo*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The array *x* contains the solution matrix *X* as computed by ?pttrs. The array *work* is a workspace array. The dimension of *work* must be at least 2**n* for real flavors, and at least *n* for complex flavors.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ptrfs interface are as follows:

d	Holds the vector of length *n*.
df	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).
ef	Holds the vector of length (*n*-1).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Used in complex flavors only. Must be 'U' or 'L'. The default value is 'U'.

?syrfs

Refines the solution of a system of linear equations with a symmetric matrix and estimates its error.

Syntax

Fortran 77:

```
call ssyrfs( uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call dsyrfs( uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call csyrfs( uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
call zsyrfs( uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call syrfs( a, af, ipiv, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_ssyrfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const float* a, lapack_int lda, const float* af, lapack_int ldaf, const lapack_int*
ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float*
berr );

lapack_int LAPACKE_dsyrfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const double* a, lapack_int lda, const double* af, lapack_int ldaf, const lapack_int*
ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr,
double* berr );

lapack_int LAPACKE_csyrfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zsyrfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a symmetric full-storage matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?sytrf`
- call the solver routine `?sytrs`.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', the upper triangle of A is stored. If *uplo* = 'L', the lower triangle of A is stored.
n	INTEGER. The order of the matrix A; $n \ge 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \ge 0$.
a,*af*,*b*,*x*,*work*	REAL for ssyrfs DOUBLE PRECISION for dsyrfs

	COMPLEX for csyrfs DOUBLE COMPLEX for zsyrfs. Arrays: a(lda,*) contains the original matrix A, as supplied to ?sytrf. af(ldaf,*) contains the factored matrix A, as returned by ?sytrf. b(ldb,*) contains the right-hand side matrix B. x(ldx,*) contains the solution matrix X. work(*) is a workspace array. The second dimension of a and af must be at least max(1, n); the second dimension of b and x must be at least max(1, nrhs); the dimension of work must be at least max(1, 3*n) for real flavors and max(1, 2*n) for complex flavors.
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).
ldaf	INTEGER. The leading dimension of af; ldaf ≥ max(1, n).
ldb	INTEGER. The leading dimension of b; ldb ≥ max(1, n).
ldx	INTEGER. The leading dimension of x; ldx ≥ max(1, n).
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The ipiv array, as returned by ?sytrf.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, n).
rwork	REAL for csyrfs DOUBLE PRECISION for zsyrfs. Workspace array, DIMENSION at least max(1, n).

Output Parameters

x	The refined solution matrix X.
ferr, berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, nrhs). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syrfs interface are as follows:

a	Holds the matrix A of size (n, n).
af	Holds the matrix AF of size (n, n).
ipiv	Holds the vector of length n.
b	Holds the matrix B of size (n, nrhs).
x	Holds the matrix X of size (n, nrhs).
ferr	Holds the vector of length (nrhs).
berr	Holds the vector of length (nrhs).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations *A*x = b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

?syrfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a symmetric indefinite matrix A and provides error bounds and backward error estimates.

Syntax

Fortran 77:

```
call ssyrfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, ipiv, s, b, ldb, x, ldx, rcond,
berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork, info )

call dsyrfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, ipiv, s, b, ldb, x, ldx, rcond,
berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork, info )

call csyrfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, ipiv, s, b, ldb, x, ldx, rcond,
berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork, info )

call zsyrfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, ipiv, s, b, ldb, x, ldx, rcond,
berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork, info )
```

C:

```
lapack_int LAPACKE_ssyrfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf,
const lapack_int* ipiv, const float* s, const float* b, lapack_int ldb, float* x,
lapack_int ldx, float* rcond, float* berr, lapack_int n_err_bnds, float*
err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float* params );

lapack_int LAPACKE_dsyrfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf,
const lapack_int* ipiv, const double* s, const double* b, lapack_int ldb, double* x,
lapack_int ldx, double* rcond, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double* params );

lapack_int LAPACKE_csyrfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const float* s,
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float*
err_bnds_comp, lapack_int nparams, float* params );

lapack_int LAPACKE_zsyrfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const double* s,
```

```
const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int
ldx, double* rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm,
double* err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed* and *s* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo
CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored:
If *uplo* = 'U', the upper triangle of *A* is stored.
If *uplo* = 'L', the lower triangle of *A* is stored.

equed
CHARACTER*1. Must be 'N' or 'Y'.
Specifies the form of equilibration that was done to *A* before calling this routine.
If *equed* = 'N', no equilibration was done.
If *equed* = 'Y', both row and column equilibration was done, that is, *A* has been replaced by *diag(s)*A*diag(s)*. The right-hand side *B* has been changed accordingly.

n
INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.

nrhs
INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.

a, *af*, *b*, *work*
REAL for ssyrfsx
DOUBLE PRECISION for dsyrfsx
COMPLEX for csyrfsx
DOUBLE COMPLEX for zsyrfsx.
Arrays: *a(lda,*)*, *af(ldaf,*)*, *b(ldb,*)*, *work(*)*.
The array *a* contains the symmetric/Hermitian matrix *A* as specified by *uplo*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A* and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A* and the strictly upper triangular part of *a* is not referenced. The second dimension of *a* must be at least max(1,*n*).

The array *af* contains the triangular factor *L* or *U* from the Cholesky factorization *A = U**T*U* or *A = L*L**T* as computed by ssytrf for real flavors or dsytrf for complex flavors.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
work(*) is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.

lda — INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).

ldaf — INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).

ipiv — INTEGER.
Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D* as determined by ssytrf for real flavors or dsytrf for complex flavors.

s — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*.
If *equed* = 'N', *s* is not accessed.
If *equed* = 'Y', each element of *s* must be positive.
Each element of *s* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb — INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

x — REAL for ssyrfsx
DOUBLE PRECISION for dsyrfsx
COMPLEX for csyrfsx
DOUBLE COMPLEX for zsyrfsx.
Array, DIMENSION (*ldx*,*).
The solution matrix *X* as computed by ?sytrs

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds — INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams — INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.

=1.0	Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support DOUBLE PRECISION.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for refinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3**n*); used in complex flavors only.

Output Parameters

x REAL for ssyrfsx
DOUBLE PRECISION for dsyrfsx
COMPLEX for csyrfsx
DOUBLE COMPLEX for zsyrfsx.
The improved solution matrix *X*.

rcond REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left| Xtrue_{ji} - X_{ji} \right|}{\left| X_{ji} \right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.

The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

params — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Output parameter only if the input contains erroneous values, namely, in *params(1)*, *params(2)*, *params(3)*. In such a case, the corresponding elements of *params* are filled with default values on output.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = −*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm(:,1)* and *err_bnds_comp(:,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?herfs

Refines the solution of a system of linear equations with a complex Hermitian matrix and estimates its error.

Syntax

Fortran 77:

```
call cherfs( uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

```
call zherfs( uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call herfs( a, af, ipiv, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_cherfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zherfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a complex Hermitian full-storage matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine ?hetrf
- call the solver routine ?hetrs.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo CHARACTER*1. Must be 'U' or 'L'.

	If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
a,*af*,*b*,*x*,*work*	COMPLEX for cherfs DOUBLE COMPLEX for zherfs. Arrays: *a*(*lda*,*) contains the original matrix *A*, as supplied to ?hetrf. *af*(*ldaf*,*) contains the factored matrix *A*, as returned by ?hetrf. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The second dimension of *a* and *af* must be at least max(1, *n*); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 2**n*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?hetrf.
rwork	REAL for cherfs DOUBLE PRECISION for zherfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for cherfs DOUBLE PRECISION for zherfs. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine herfs interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
af	Holds the matrix *AF* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
ferr	Holds the vector of length (*nrhs*).

berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ operations. In addition, each step of iterative refinement involves $24n^2$ operations; the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations *A*x = b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

The real counterpart of this routine is ?ssyrfs/?dsyrfs

?herfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a symmetric indefinite matrix A and provides error bounds and backward error estimates.

Syntax

Fortran 77:

```
call cherfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, ipiv, s, b, ldb, x, ldx, rcond,
berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork, info )
```

```
call zherfsx( uplo, equed, n, nrhs, a, lda, af, ldaf, ipiv, s, b, ldb, x, ldx, rcond,
berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork, info )
```

C:

```
lapack_int LAPACKE_cherfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const float* s,
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float*
err_bnds_comp, lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_zherfsx( int matrix_order, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const double* s,
const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int
ldx, double* rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm,
double* err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite, and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed* and *s* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo
CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored:
If *uplo* = 'U', the upper triangle of *A* is stored.
If *uplo* = 'L', the lower triangle of *A* is stored.

equed
CHARACTER*1. Must be 'N' or 'Y'.
Specifies the form of equilibration that was done to *A* before calling this routine.
If *equed* = 'N', no equilibration was done.
If *equed* = 'Y', both row and column equilibration was done, that is, *A* has been replaced by *diag(s)*A*diag(s)*. The right-hand side *B* has been changed accordingly.

n
INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.

nrhs
INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.

a, *af*, *b*, *work*
COMPLEX for cherfsx
DOUBLE COMPLEX for zherfsx.
Arrays: *a(lda,*)*, *af(ldaf,*)*, *b(ldb,*)*, *work(*)*.
The array *a* contains the Hermitian matrix *A* as specified by *uplo*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A* and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A* and the strictly upper triangular part of *a* is not referenced. The second dimension of *a* must be at least max(1,*n*).
The factored form of the matrix *A*. The array *af* contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization *A = U*D*U**T* or *A = L*D*L**T* as computed by ssytrf for cherfsx or dsytrf for zherfsx.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
work(*) is a workspace array. The dimension of *work* must be at least max(1,2**n*).

lda
INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).

ldaf
INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).

ipiv
INTEGER.

Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D* as determined by ssytrf for real flavors or dsytrf for complex flavors.

s

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*.
If *equed* = 'N', *s* is not accessed.
If *equed* = 'Y', each element of *s* must be positive.
Each element of *s* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb

INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

x

COMPLEX for cherfsx
DOUBLE COMPLEX for zherfsx.
Array, DIMENSION (*ldx*,*).
The solution matrix *X* as computed by ?hetrs

ldx

INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds

INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams

INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for cherfsx), 1.0D+0 (for zherfsx).

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support DOUBLE PRECISION.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for refinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

rwork

REAL for cherfsx
DOUBLE PRECISION for zherfsx.
Workspace array, DIMENSION at least max(1, 3*n).

Output Parameters

x

COMPLEX for cherfsx
DOUBLE COMPLEX for zherfsx.
The improved solution matrix *X*.

rcond

REAL for cherfsx
DOUBLE PRECISION for zherfsx.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr

REAL for cherfsx
DOUBLE PRECISION for zherfsx.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm

REAL for cherfsx
DOUBLE PRECISION for zherfsx.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm*(*i*,:) corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm*(:,*err*) contains the follwoing three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for cherfsx and *sqrt*(*n*)**dlamch*(ε) for zherfsx.

err=2 "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for cherfsx and *sqrt*(*n*)**dlamch*(ε) for zherfsx. This error bound should only be trusted if the previous boolean is true.

	err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for cherfsx and *sqrt*(*n*)**dlamch*(ε) for zherfsx to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for cherfsx

DOUBLE PRECISION for zherfsx.

Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.

The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.

The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for cherfsx and *sqrt*(*n*)**dlamch*(ε) for zherfsx.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for cherfsx and *sqrt*(*n*)**dlamch*(ε) for zherfsx. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for cherfsx and *sqrt*(*n*)**dlamch*(ε) for zherfsx to determine if the error estimate is "guaranteed". These

reciprocal condition numbers are `1/(norm(1/z,inf)*norm(z,inf))` for some appropriately scaled matrix *Z*.
Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

params — `REAL` for single precision flavors
`DOUBLE PRECISION` for double precision flavors.
Output parameter only if the input contains erroneous values, namely, in *params(1)*, *params(2)*, *params(3)*. In such a case, the corresponding elements of *params* are filled with default values on output.

info — `INTEGER`. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = −*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm(;,1)* and *err_bnds_comp(;,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?sprfs

Refines the solution of a system of linear equations with a packed symmetric matrix and estimates the solution error.

Syntax

Fortran 77:

```
call ssprfs( uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call dsprfs( uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call csprfs( uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
call zsprfs( uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call sprfs( ap, afp, ipiv, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_ssprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const float* ap, const float* afp, const lapack_int* ipiv, const float* b, lapack_int
ldb, float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dsprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const double* ap, const double* afp, const lapack_int* ipiv, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );

lapack_int LAPACKE_csprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* ap, const lapack_complex_float* afp, const lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zsprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* ap, const lapack_complex_double* afp, const lapack_int*
ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x,
lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a packed symmetric matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \leq \beta |a_{ij}|$, $|\delta b_i| \leq \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?sptrf`
- call the solver routine `?sptrs`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, the upper triangle of A is stored. If *uplo* = `'L'`, the lower triangle of A is stored.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides; $nrhs \geq 0$.
ap,*afp*,*b*,*x*,*work*	`REAL` for `ssprfs` `DOUBLE PRECISION` for `dsprfs` `COMPLEX` for `csprfs` `DOUBLE COMPLEX` for `zsprfs`.

	Arrays: *ap*(*) contains the original packed matrix *A*, as supplied to ?sptrf. *afp*(*) contains the factored packed matrix *A*, as returned by ?sptrf. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The dimension of arrays *ap* and *afp* must be at least max(1, *n*(*n*+1)/2); the second dimension of *b* and *x* must be at least max(1, *nrhs*); the dimension of *work* must be at least max(1, 3**n*) for real flavors and max(1, 2**n*) for complex flavors.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?sptrf.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for csprfs DOUBLE PRECISION for zsprfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sprfs interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
afp	Holds the array *AF* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
b	Holds the matrix *B* of size (*n*,*nrhs*).
x	Holds the matrix *X* of size (*n*,*nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations $A*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

?hprfs

Refines the solution of a system of linear equations with a packed complex Hermitian matrix and estimates the solution error.

Syntax

Fortran 77:

```
call chprfs( uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

```
call zhprfs( uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call hprfs( ap, afp, ipiv, b, x [,uplo] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_chprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_float* ap, const lapack_complex_float* afp, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zhprfs( int matrix_order, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_double* ap, const lapack_complex_double* afp, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ with a packed complex Hermitian matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

Finally, the routine estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?hptrf`

- call the solver routine ?hptrs.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ap,*afp*,*b*,*x*,*work*	COMPLEX for chprfs DOUBLE COMPLEX for zhprfs. Arrays: *ap*(*) contains the original packed matrix *A*, as supplied to ?hptrf. *afp*(*) contains the factored packed matrix *A*, as returned by ?hptrf. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The dimension of arrays *ap* and *afp* must be at least max(1,*n*(*n*+1)/2); the second dimension of *b* and *x* must be at least max(1,*nrhs*); the dimension of *work* must be at least max(1, 2*n).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?hptrf.
rwork	REAL for chprfs DOUBLE PRECISION for zhprfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x	The refined solution matrix *X*.
ferr, *berr*	REAL for chprfs. DOUBLE PRECISION for zhprfs. Arrays, DIMENSION at least max(1,*nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hprfs interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).

afp	Holds the array *AF* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
b	Holds the matrix *B* of size (*n*,*nrhs*).
x	Holds the matrix *X* of size (*n*,*nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ operations. In addition, each step of iterative refinement involves $24n^2$ operations; the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations *A***x* = *b*; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

The real counterpart of this routine is ?ssprfs/?dsprfs.

?trrfs

Estimates the error in the solution of a system of linear equations with a triangular matrix.

Syntax

Fortran 77:

```
call strrfs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, x, ldx, ferr, berr, work,
iwork, info )
```

```
call dtrrfs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, x, ldx, ferr, berr, work,
iwork, info )
```

```
call ctrrfs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, x, ldx, ferr, berr, work,
rwork, info )
```

```
call ztrrfs( uplo, trans, diag, n, nrhs, a, lda, b, ldb, x, ldx, ferr, berr, work,
rwork, info )
```

Fortran 95:

```
call trrfs( a, b, x [,uplo] [,trans] [,diag] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_strrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const float* a, lapack_int lda, const float* b,
lapack_int ldb, const float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dtrrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const double* a, lapack_int lda, const double* b,
lapack_int ldb, const double* x, lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_ctrrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* b, lapack_int ldb, const lapack_complex_float* x, lapack_int ldx,
float* ferr, float* berr );
```

```
lapack_int LAPACKE_ztrrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* b, lapack_int ldb, const lapack_complex_double* x, lapack_int
ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the errors in the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a triangular matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

The routine also estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine, call the solver routine `?trtrs`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether A is upper or lower triangular: If *uplo* = `'U'`, then A is upper triangular. If *uplo* = `'L'`, then A is lower triangular.
trans	`CHARACTER*1`. Must be `'N'` or `'T'` or `'C'`. Indicates the form of the equations: If *trans* = `'N'`, the system has the form $A*X = B$. If *trans* = `'T'`, the system has the form $A^T*X = B$. If *trans* = `'C'`, the system has the form $A^H*X = B$.
diag	`CHARACTER*1`. Must be `'N'` or `'U'`. If *diag* = `'N'`, then A is not a unit triangular matrix. If *diag* = `'U'`, then A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array a.
n	`INTEGER`. The order of the matrix A; $n \ge 0$.
nrhs	`INTEGER`. The number of right-hand sides; $nrhs \ge 0$.
a, *b*, *x*, *work*	`REAL` for `strrfs` `DOUBLE PRECISION` for `dtrrfs` `COMPLEX` for `ctrrfs` `DOUBLE COMPLEX` for `ztrrfs`. Arrays: *a*(*lda*,*) contains the upper or lower triangular matrix A, as specified by *uplo*. *b*(*ldb*,*) contains the right-hand side matrix B. *x*(*ldx*,*) contains the solution matrix X.

	work(*) is a workspace array. The second dimension of *a* must be at least max(1,*n*); the second dimension of *b* and *x* must be at least max(1,*nrhs*); the dimension of *work* must be at least max(1,3**n*) for real flavors and max(1,2**n*) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for ctrrfs DOUBLE PRECISION for ztrrfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trrfs interface are as follows:

a	Holds the matrix *A* of size (*n*,*n*).
b	Holds the matrix *B* of size (*n*,*nrhs*).
x	Holds the matrix *X* of size (*n*,*nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

A call to this routine involves, for each right-hand side, solving a number of systems of linear equations *A*x* = *b*; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors or $4n^2$ for complex flavors.

?tprfs

Estimates the error in the solution of a system of linear equations with a packed triangular matrix.

Syntax

Fortran 77:

```
call stprfs( uplo, trans, diag, n, nrhs, ap, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call dtprfs( uplo, trans, diag, n, nrhs, ap, b, ldb, x, ldx, ferr, berr, work, iwork, info )
call ctprfs( uplo, trans, diag, n, nrhs, ap, b, ldb, x, ldx, ferr, berr, work, rwork, info )
call ztprfs( uplo, trans, diag, n, nrhs, ap, b, ldb, x, ldx, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call tprfs( ap, b, x [,uplo] [,trans] [,diag] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_stprfs( int matrix_order, char uplo, char trans, char diag, lapack_int n, lapack_int nrhs, const float* ap, const float* b, lapack_int ldb, const float* x, lapack_int ldx, float* ferr, float* berr );
lapack_int LAPACKE_dtprfs( int matrix_order, char uplo, char trans, char diag, lapack_int n, lapack_int nrhs, const double* ap, const double* b, lapack_int ldb, const double* x, lapack_int ldx, double* ferr, double* berr );
lapack_int LAPACKE_ctprfs( int matrix_order, char uplo, char trans, char diag, lapack_int n, lapack_int nrhs, const lapack_complex_float* ap, const lapack_complex_float* b, lapack_int ldb, const lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
lapack_int LAPACKE_ztprfs( int matrix_order, char uplo, char trans, char diag, lapack_int n, lapack_int nrhs, const lapack_complex_double* ap, const lapack_complex_double* b, lapack_int ldb, const lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the errors in the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a packed triangular matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \leq \beta |a_{ij}|$, $|\delta b_i| \leq \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

The routine also estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine, call the solver routine `?tptrs`.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether *A* is upper or lower triangular: If *uplo* = 'U', then *A* is upper triangular. If *uplo* = 'L', then *A* is lower triangular.
trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', the system has the form *A*X* = *B*. If *trans* = 'T', the system has the form $A^T*X = B$. If *trans* = 'C', the system has the form $A^H*X = B$.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', *A* is not a unit triangular matrix. If *diag* = 'U', *A* is unit triangular: diagonal elements of *A* are assumed to be 1 and not referenced in the array *ap*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
ap, *b*, *x*, *work*	REAL for stprfs DOUBLE PRECISION for dtprfs COMPLEX for ctprfs DOUBLE COMPLEX for ztprfs. Arrays: *ap*(*) contains the upper or lower triangular matrix *A*, as specified by *uplo*. *b*(*ldb*,*) contains the right-hand side matrix *B*. *x*(*ldx*,*) contains the solution matrix *X*. *work*(*) is a workspace array. The dimension of *ap* must be at least max(1,*n*(*n*+1)/2); the second dimension of *b* and *x* must be at least max(1,*nrhs*); the dimension of *work* must be at least max(1,3**n*) for real flavors and max(1,2**n*) for complex flavors.
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for ctprfs DOUBLE PRECISION for ztprfs. Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `tprfs` interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*,*nrhs*).
x	Holds the matrix *X* of size (*n*,*nrhs*).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

A call to this routine involves, for each right-hand side, solving a number of systems of linear equations *A***x* = *b*; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors or $4n^2$ for complex flavors.

?tbrfs

Estimates the error in the solution of a system of linear equations with a triangular band matrix.

Syntax

Fortran 77:

```
call stbrfs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, x, ldx, ferr, berr,
work, iwork, info )

call dtbrfs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, x, ldx, ferr, berr,
work, iwork, info )

call ctbrfs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, x, ldx, ferr, berr,
work, rwork, info )

call ztbrfs( uplo, trans, diag, n, kd, nrhs, ab, ldab, b, ldb, x, ldx, ferr, berr,
work, rwork, info )
```

Fortran 95:

```
call tbrfs( ab, b, x [,uplo] [,trans] [,diag] [,ferr] [,berr] [,info] )
```

C:

```
lapack_int LAPACKE_stbrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const float* ab, lapack_int ldab, const
float* b, lapack_int ldb, const float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dtbrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const double* ab, lapack_int ldab, const
double* b, lapack_int ldb, const double* x, lapack_int ldx, double* ferr, double*
berr );
```

```
lapack_int LAPACKE_ctbrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const lapack_complex_float* ab,
lapack_int ldab, const lapack_complex_float* b, lapack_int ldb, const
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_ztbrfs( int matrix_order, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const lapack_complex_double* ab,
lapack_int ldab, const lapack_complex_double* b, lapack_int ldb, const
lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates the errors in the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a triangular band matrix A, with multiple right-hand sides. For each computed solution vector x, the routine computes the *component-wise backward error* β. This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$|\delta a_{ij}| \le \beta |a_{ij}|$, $|\delta b_i| \le \beta |b_i|$ such that $(A + \delta A)x = (b + \delta b)$.

The routine also estimates the *component-wise forward error* in the computed solution $||x - x_e||_\infty / ||x||_\infty$ (here x_e is the exact solution).

Before calling this routine, call the solver routine `?tbtrs`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether A is upper or lower triangular: If *uplo* = `'U'`, then A is upper triangular. If *uplo* = `'L'`, then A is lower triangular.
trans	`CHARACTER*1`. Must be `'N'` or `'T'` or `'C'`. Indicates the form of the equations: If *trans* = `'N'`, the system has the form $A*X = B$. If *trans* = `'T'`, the system has the form $A^T*X = B$. If *trans* = `'C'`, the system has the form $A^H*X = B$.
diag	`CHARACTER*1`. Must be `'N'` or `'U'`. If *diag* = `'N'`, A is not a unit triangular matrix. If *diag* = `'U'`, A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array *ab*.
n	`INTEGER`. The order of the matrix A; $n \ge 0$.
kd	`INTEGER`. The number of super-diagonals or sub-diagonals in the matrix A; $kd \ge 0$.
nrhs	`INTEGER`. The number of right-hand sides; $nrhs \ge 0$.
ab, *b*, *x*, *work*	`REAL` for `stbrfs` `DOUBLE PRECISION` for `dtbrfs` `COMPLEX` for `ctbrfs`

	DOUBLE COMPLEX for ztbrfs. Arrays: ab(ldab,*) contains the upper or lower triangular matrix A, as specified by uplo, in band storage format. b(ldb,*) contains the right-hand side matrix B. x(ldx,*) contains the solution matrix X. work(*) is a workspace array. The second dimension of a must be at least max(1,n); the second dimension of b and x must be at least max(1,nrhs). The dimension of work must be at least max(1,3*n) for real flavors and max(1,2*n) for complex flavors.
ldab	INTEGER. The leading dimension of the array ab; ldab ≥ kd +1.
ldb	INTEGER. The leading dimension of b; ldb ≥ max(1, n).
ldx	INTEGER. The leading dimension of x; ldx ≥ max(1, n).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, n).
rwork	REAL for ctbrfs DOUBLE PRECISION for ztbrfs. Workspace array, DIMENSION at least max(1, n).

Output Parameters

ferr, berr	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, nrhs). Contain the component-wise forward and backward errors, respectively, for each solution vector.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tbrfs interface are as follows:

ab	Holds the array A of size (kd+1,n).
b	Holds the matrix B of size (n,nrhs).
x	Holds the matrix X of size (n,nrhs).
ferr	Holds the vector of length (nrhs).
berr	Holds the vector of length (nrhs).
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

The bounds returned in ferr are not rigorous, but in practice they almost always overestimate the actual error.

A call to this routine involves, for each right-hand side, solving a number of systems of linear equations $A*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately $2n*kd$ floating-point operations for real flavors or $8n*kd$ operations for complex flavors.

Routines for Matrix Inversion

It is seldom necessary to compute an explicit inverse of a matrix. In particular, do not attempt to solve a system of equations $Ax = b$ by first computing A^{-1} and then forming the matrix-vector product $x = A^{-1}b$. Call a solver routine instead (see Routines for Solving Systems of Linear Equations); this is more efficient and more accurate.

However, matrix inversion routines are provided for the rare occasions when an explicit inverse matrix is needed.

?getri

Computes the inverse of an LU-factored general matrix.

Syntax

Fortran 77:

```
call sgetri( n, a, lda, ipiv, work, lwork, info )
call dgetri( n, a, lda, ipiv, work, lwork, info )
call cgetri( n, a, lda, ipiv, work, lwork, info )
call zgetri( n, a, lda, ipiv, work, lwork, info )
```

Fortran 95:

```
call getri( a, ipiv [,info] )
```

C:

```
lapack_int LAPACKE_<?>getri( int matrix_order, lapack_int n, <datatype>* a, lapack_int lda, const lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a general matrix A. Before calling this routine, call `?getrf` to factorize A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix A; $n \geq 0$.
a, *work*	REAL for sgetri DOUBLE PRECISION for dgetri COMPLEX for cgetri

	DOUBLE COMPLEX for zgetri. Arrays: a(lda,*), work(*). a(lda,*) contains the factorization of the matrix A, as returned by ?getrf: A = P*L*U. The second dimension of a must be at least max(1,n). work(*) is a workspace array of dimension at least max(1,lwork).
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The ipiv array, as returned by ?getrf.
lwork	INTEGER. The size of the work array; lwork ≥ n. If lwork = -1, then a workspace query is assumed; the routine only calculates the optimal size of the work array, returns this value as the first entry of the work array, and no error message related to lwork is issued by xerbla.

See *Application Notes* below for the suggested value of lwork.

Output Parameters

a	Overwritten by the n-by-n matrix inv(A).
work(1)	If info = 0, on exit work(1) contains the minimum value of lwork required for optimum performance. Use this lwork for subsequent runs.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value. If info = i, the i-th diagonal element of the factor U is zero, U is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine getri interface are as follows:

a	Holds the matrix A of size (n, n).
ipiv	Holds the vector of length n.

Application Notes

For better performance, try using lwork = n*blocksize, where blocksize is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of lwork for the first run or set lwork = -1.

If you choose the first option and set any of admissible lwork sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array work on exit. Use this value (work(1)) for subsequent runs.

If you set lwork = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (work). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed inverse *X* satisfies the following error bound:

$$|XA - I| \leq c(n)\varepsilon|X|P|L||U|,$$

where $c(n)$ is a modest linear function of n; ε is the machine precision; I denotes the identity matrix; P, L, and U are the factors of the matrix factorization $A = P*L*U$.

The total number of floating-point operations is approximately $(4/3)n^3$ for real flavors and $(16/3)n^3$ for complex flavors.

?potri

Computes the inverse of a symmetric (Hermitian) positive-definite matrix.

Syntax

Fortran 77:

```
call spotri( uplo, n, a, lda, info )
call dpotri( uplo, n, a, lda, info )
call cpotri( uplo, n, a, lda, info )
call zpotri( uplo, n, a, lda, info )
```

Fortran 95:

```
call potri( a [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>potri( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a symmetric positive definite or, for complex flavors, Hermitian positive-definite matrix *A*. Before calling this routine, call `?potrf` to factorize *A*.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether *A* is upper or lower triangular: If *uplo* = `'U'`, then *A* is upper triangular. If *uplo* = `'L'`, then *A* is lower triangular.
n	`INTEGER`. The order of the matrix *A*; $n \geq 0$.
a	`REAL` for `spotri` `DOUBLE PRECISION` for `dpotri`

	COMPLEX for cpotri DOUBLE COMPLEX for zpotri. Array a(lda,*). Contains the factorization of the matrix A, as returned by ?potrf. The second dimension of a must be at least max(1, n).
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).

Output Parameters

a	Overwritten by the n-by-n matrix inv(A).
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value. If info = i, the i-th diagonal element of the Cholesky factor (and therefore the factor itself) is zero, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine potri interface are as follows:

a	Holds the matrix A of size (n, n).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed inverse X satisfies the following error bounds:

$$||XA - I||_2 \le c(n)\varepsilon\kappa_2(A), \quad ||AX - I||_2 \le c(n)\varepsilon\kappa_2(A),$$

where $c(n)$ is a modest linear function of n, and ε is the machine precision; I denotes the identity matrix.

The 2-norm $||A||_2$ of a matrix A is defined by $||A||_2 = max_{x \cdot x=1}(Ax \cdot Ax)^{1/2}$, and the condition number $\kappa_2(A)$ is defined by $\kappa_2(A) = ||A||_2 \, ||A^{-1}||_2$.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

?pftri

Computes the inverse of a symmetric (Hermitian) positive-definite matrix in RFP format using the Cholesky factorization.

Syntax

Fortran 77:

```
call spftri( transr, uplo, n, a, info )
call dpftri( transr, uplo, n, a, info )
call cpftri( transr, uplo, n, a, info )
call zpftri( transr, uplo, n, a, info )
```

C:

```
lapack_int LAPACKE_<?>pftri( int matrix_order, char transr, char uplo, lapack_int n,
<datatype>* a );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a symmetric positive definite or, for complex data, Hermitian positive-definite matrix *A* using the Cholesky factorization:

A = U^T**U* for real data, *A* = U^H**U* for complex data if *uplo*='U'

A = *L**L^T for real data, *A* = *L**L^H for complex data if *uplo*='L'

Before calling this routine, call ?pftrf to factorize *A*.

The matrix *A* is in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see Matrix Storage Schemes.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

transr — CHARACTER*1. Must be 'N', 'T' (for real data) or 'C' (for complex data).
If *transr* = 'N', the Normal *transr* of RFP *A* is stored.
If *transr* = 'T', the Transpose *transr* of RFP *A* is stored.
If *transr* = 'C', the Conjugate-Transpose *transr* of RFP *A* is stored.

uplo — CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of the RFP matrix *A* is stored:
If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix *A*.
If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix *A*.

n — INTEGER. The order of the matrix *A*; *n* ≥ 0.

a — REAL for spftri
DOUBLE PRECISION for dpftri
COMPLEX for cpftri
DOUBLE COMPLEX for zpftri.
Array, DIMENSION (*n**(*n*+1)/2). The array *a* contains the matrix *A* in the RFP format.

Output Parameters

a — The symmetric/Hermitian inverse of the original matrix in the same storage format.

info — INTEGER. If *info*=0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, the (*i*,*i*) element of the factor U or L is zero, and the inverse could not be computed.

?pptri

Computes the inverse of a packed symmetric (Hermitian) positive-definite matrix

Syntax

Fortran 77:

```
call spptri( uplo, n, ap, info )
call dpptri( uplo, n, ap, info )
call cpptri( uplo, n, ap, info )
call zpptri( uplo, n, ap, info )
```

Fortran 95:

```
call pptri( ap [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>pptri( int matrix_order, char uplo, lapack_int n, <datatype>*
ap );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a symmetric positive definite or, for complex flavors, Hermitian positive-definite matrix *A* in *packed* form. Before calling this routine, call ?pptrf to factorize *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular factor is stored in *ap*: If *uplo* = 'U', then the upper triangular factor is stored. If *uplo* = 'L', then the lower triangular factor is stored.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
ap	REAL for spptri DOUBLE PRECISION for dpptri COMPLEX for cpptri DOUBLE COMPLEX for zpptri. Array, DIMENSION at least max(1, $n(n+1)/2$). Contains the factorization of the packed matrix *A*, as returned by ?pptrf. The dimension *ap* must be at least max(1,$n(n+1)/2$).

Output Parameters

ap	Overwritten by the packed *n*-by-*n* matrix `inv(A)`.
info	`INTEGER`. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of the Cholesky factor (and therefore the factor itself) is zero, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `pptri` interface are as follows:

ap	Holds the array *A* of size `(n*(n+1)/2)`.
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.

Application Notes

The computed inverse *X* satisfies the following error bounds:

$||XA - I||_2 \le c(n)\varepsilon\kappa_2(A), \quad ||AX - I||_2 \le c(n)\varepsilon\kappa_2(A),$

where $c(n)$ is a modest linear function of n, and ε is the machine precision; I denotes the identity matrix.

The 2-norm $||A||_2$ of a matrix A is defined by $||A||_2 = max_{x \cdot x=1}(Ax \cdot Ax)^{1/2}$, and the condition number $\kappa_2(A)$ is defined by $\kappa_2(A) = ||A||_2 \, ||A^{-1}||_2$.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

?sytri

Computes the inverse of a symmetric matrix.

Syntax

Fortran 77:

```
call ssytri( uplo, n, a, lda, ipiv, work, info )
call dsytri( uplo, n, a, lda, ipiv, work, info )
call csytri( uplo, n, a, lda, ipiv, work, info )
call zsytri( uplo, n, a, lda, ipiv, work, info )
```

Fortran 95:

```
call sytri( a, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sytri( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, const lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`

- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a symmetric matrix *A*. Before calling this routine, call ?sytrf to factorize *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the Bunch-Kaufman factorization $A = P*U*D*U^T*P^T$. If *uplo* = 'L', the array *a* stores the Bunch-Kaufman factorization $A = P*L*D*L^T*P^T$.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a, *work*	REAL for ssytri DOUBLE PRECISION for dsytri COMPLEX for csytri DOUBLE COMPLEX for zsytri. Arrays: *a*(*lda*,*) contains the factorization of the matrix *A*, as returned by ?sytrf. The second dimension of *a* must be at least max(1,*n*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1,2*n).
lda	INTEGER. The leading dimension of *a*; *lda* $\geq$ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?sytrf.

Output Parameters

a	Overwritten by the *n*-by-*n* matrix inv(*A*).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *D* is zero, *D* is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sytri interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed inverse X satisfies the following error bounds:

$$|D*U^T*P^T*X*P*U - I| \leq c(n)\varepsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$$

for *uplo* = 'U', and

$$|D*L^T*P^T*X*P*L - I| \leq c(n)\varepsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$$

for *uplo* = 'L'. Here $c(n)$ is a modest linear function of n, and ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

?hetri

Computes the inverse of a complex Hermitian matrix.

Syntax

Fortran 77:

```
call chetri( uplo, n, a, lda, ipiv, work, info )
call zhetri( uplo, n, a, lda, ipiv, work, info )
```

Fortran 95:

```
call hetri( a, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hetri( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, const lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a complex Hermitian matrix *A*. Before calling this routine, call ?hetrf to factorize *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the Bunch-Kaufman factorization $A = P*U*D*U^H*P^T$. If *uplo* = 'L', the array *a* stores the Bunch-Kaufman factorization $A = P*L*D*L^H*P^T$.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a, *work*	COMPLEX for chetri

	DOUBLE COMPLEX for zhetri. Arrays: a(lda,*) contains the factorization of the matrix A, as returned by ?hetrf. The second dimension of a must be at least max(1,n). work(*) is a workspace array. The dimension of work must be at least max(1,n).
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).
ipiv	INTEGER. Array, DIMENSION at least max(1, n). The ipiv array, as returned by ?hetrf.

Output Parameters

a	Overwritten by the n-by-n matrix inv(A).
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value. If info = i, the i-th diagonal element of D is zero, D is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hetri interface are as follows:

a	Holds the matrix A of size (n,n).
ipiv	Holds the vector of length n.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed inverse X satisfies the following error bounds:

$$|D*U^H*P^T*X*P*U - I| \le c(n)\varepsilon(|D||U^H|P^T|X|P|U| + |D||D^{-1}|)$$

for uplo = 'U', and

$$|D*L^H*P^T*X*P*L - I| \le c(n)\varepsilon(|D||L^H|P^T|X|P|L| + |D||D^{-1}|)$$

for uplo = 'L'. Here c(n) is a modest linear function of n, and ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

The real counterpart of this routine is ?sytri.

?sytri2

Computes the inverse of a symmetric indefinite matrix through setting the leading dimension of the workspace and calling ?sytri2x.

Syntax

Fortran 77:

```
call ssytri2( uplo, n, a, lda, ipiv, work, lwork, info )
call dsytri2( uplo, n, a, lda, ipiv, work, lwork, info )
call csytri2( uplo, n, a, lda, ipiv, work, lwork, info )
call zsytri2( uplo, n, a, lda, ipiv, work, lwork, info )
```

Fortran 95:

```
call sytri2( a,ipiv[,uplo][,info] )
```

C:

```
lapack_int LAPACKE_<?>sytri2( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, const lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a symmetric indefinite matrix A using the factorization $A = U*D*U^T$ or $A = L*D*L^T$ computed by `?sytrf`.

The `?sytri2` routine sets the leading dimension of the workspace before calling `?sytri2x` that actually computes the inverse.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix A has been factored: If *uplo* = `'U'`, the array *a* stores the factorization $A = U*D*U^T$. If *uplo* = `'L'`, the array *a* stores the factorization $A = L*D*L^T$.
n	`INTEGER`. The order of the matrix A; $n \geq 0$.
a, *work*	`REAL` for `ssytri2` `DOUBLE PRECISION` for `dsytri2` `COMPLEX` for `csytri2` `DOUBLE COMPLEX` for `zsytri2` Arrays: *a*(*lda*,*) contains the block diagonal matrix D and the multipliers used to obtain the factor `U` or `L` as returned by `?sytrf`. The second dimension of *a* must be at least `max(1,n)`. *work* is a workspace array of `(n+nb+1)*(nb+3)` dimension.
lda	`INTEGER`. The leading dimension of *a*; *lda* $\geq$ `max(1, n)`.
ipiv	`INTEGER`. Array, `DIMENSION` at least `max(1, n)`. Details of the interchanges and the block structure of D as returned by `?sytrf`.

lwork	INTEGER. The dimension of the *work* array. *lwork* ≥ (*n*+*nb*+1)*(*nb*+3) where *nb* is the block size parameter as returned by sytrf. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

Output Parameters

a	If *info* = 0, the symmetric inverse of the original matrix. If *info* = 'U', the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced. If *info* = 'L', the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, *D*(*i*,*i*) = 0; *D* is singular and its inversion could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sytri2 interface are as follows:

a	Holds the matrix *A* of size (*n*,*n*).
ipiv	Holds the vector of length *n*.
uplo	Indicates how the matrix *A* has been factored. Must be 'U' or 'L'.

See Also

?sytrf
?sytri2x

?hetri2

Computes the inverse of a Hermitian indefinite matrix through setting the leading dimension of the workspace and calling ?hetri2x*.*

Syntax

Fortran 77:

```
call chetri2( uplo, n, a, lda, ipiv, work, lwork, info )
call zhetri2( uplo, n, a, lda, ipiv, work, lwork, info )
```

Fortran 95:

```
call hetri2( a,ipiv[,uplo][,info] )
```

C:

```
lapack_int LAPACKE_<?>hetri2( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, const lapack_int* ipiv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a Hermitian indefinite matrix *A* using the factorization $A = U*D*U^H$ or $A = L*D*L^H$ computed by ?hetrf.

The ?hetri2 routine sets the leading dimension of the workspace before calling ?hetri2x that actually computes the inverse.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo
CHARACTER*1. Must be 'U' or 'L'.
Indicates how the input matrix *A* has been factored:
If *uplo* = 'U', the array *a* stores the factorization $A = U*D*U^H$.
If *uplo* = 'L', the array *a* stores the factorization $A = L*D*L^H$.

n
INTEGER. The order of the matrix *A*; $n \geq 0$.

a, *work*
COMPLEX for chetri2
DOUBLE COMPLEX for zhetri2
Arrays:
a(*lda*,*) contains the block diagonal matrix *D* and the multipliers used to obtain the factor U or L as returned by ?sytrf.
The second dimension of *a* must be at least max(1,*n*).
work is a workspace array of (*n*+*nb*+1)*(*nb*+3) dimension.

lda
INTEGER. The leading dimension of *a*; *lda* $\geq$ max(1, *n*).

ipiv
INTEGER.
Array, DIMENSION at least max(1, *n*).
Details of the interchanges and the block structure of *D* as returned by ?hetrf.

lwork
INTEGER. The dimension of the *work* array.
lwork $\geq$ (*n*+*nb*+1)*(*nb*+3)
where
nb is the block size parameter as returned by hetrf.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

Output Parameters

a
If *info* = 0, the inverse of the original matrix.
If *info* = 'U', the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced.
If *info* = 'L', the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.

info
INTEGER.
If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, *D*(*i*,*i*) = 0; *D* is singular and its inversion could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `hetri2` interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
uplo	Indicates how the input matrix *A* has been factored. Must be `'U'` or `'L'`.

See Also

?hetrf
?hetri2x

?sytri2x

Computes the inverse of a symmetric indefinite matrix after `?sytri2` *sets the leading dimension of the workspace.*

Syntax

Fortran 77:

```
call ssytri2x( uplo, n, a, lda, ipiv, work, nb, info )
call dsytri2x( uplo, n, a, lda, ipiv, work, nb, info )
call csytri2x( uplo, n, a, lda, ipiv, work, nb, info )
call zsytri2x( uplo, n, a, lda, ipiv, work, nb, info )
```

Fortran 95:

```
call sytri2x( a,ipiv,nb[,uplo][,info] )
```

C:

```
lapack_int LAPACKE_<?>sytri2x( int matrix_order, char uplo, lapack_int n, <datatype>*
a, lapack_int lda, const lapack_int* ipiv, lapack_int nb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a symmetric indefinite matrix *A* using the factorization $A = U*D*U^T$ or $A = L*D*L^T$ computed by `?sytrf`.

The `?sytri2x` actually computes the inverse after the `?sytri2` routine sets the leading dimension of the workspace before calling `?sytri2x`.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the factorization $A = U*D*U^T$. If *uplo* = 'L', the array *a* stores the factorization $A = L*D*L^T$.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a, *work*	REAL for ssytri2x DOUBLE PRECISION for dsytri2x COMPLEX for csytri2x DOUBLE COMPLEX for zsytri2x Arrays: *a*(*lda*,*) contains the *nb* (block size) diagonal matrix *D* and the multipliers used to obtain the factor U or L as returned by ?sytrf. The second dimension of *a* must be at least max(1,*n*). *work* is a workspace array of the dimension (*n*+*nb*+1)*(*nb*+3) where *nb* is the block size as set by ?sytrf.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Details of the interchanges and the *nb* structure of *D* as returned by ?sytrf.
nb	INTEGER. Block size.

Output Parameters

a	If *info* = 0, the symmetric inverse of the original matrix. If *info* = 'U', the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced. If *info* = 'L', the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, D_{ii} = 0; *D* is singular and its inversion could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sytri2x interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
nb	Holds the block size.

uplo	Indicates how the input matrix *A* has been factored. Must be 'U' or 'L'.

See Also

?sytrf
?sytri2

?hetri2x

Computes the inverse of a Hermitian indefinite matrix after ?hetri2 *sets the leading dimension of the workspace.*

Syntax

Fortran 77:

```
call chetri2x( uplo, n, a, lda, ipiv, work, nb, info )
call zhetri2x( uplo, n, a, lda, ipiv, work, nb, info )
```

Fortran 95:

```
call hetri2x( a,ipiv,nb[,uplo][,info] )
```

C:

```
lapack_int LAPACKE_<?>hetri2x( int matrix_order, char uplo, lapack_int n, <datatype>*
a, lapack_int lda, const lapack_int* ipiv,lapack_int nb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a Hermitian indefinite matrix *A* using the factorization $A = U*D*U^H$ or $A = L*D*L^H$ computed by ?hetrf.

The ?hetri2x actually computes the inverse after the ?hetri2 routine sets the leading dimension of the workspace before calling ?hetri2x.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the factorization $A = U*D*U^H$. If *uplo* = 'L', the array *a* stores the factorization $A = L*D*L^H$.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a, *work*	COMPLEX for chetri2x DOUBLE COMPLEX for zhetri2x Arrays: *a*(*lda*,*) contains the *nb* (block size) diagonal matrix *D* and the multipliers used to obtain the factor U or L as returned by ?hetrf.

	The second dimension of *a* must be at least max(1,*n*). *work* is a workspace array of the dimension (*n*+*nb*+1)*(*nb*+3) where *nb* is the block size as set by ?hetrf.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Details of the interchanges and the *nb* structure of *D* as returned by ?hetrf.
nb	INTEGER. Block size.

Output Parameters

a	If *info* = 0, the symmetric inverse of the original matrix. If *info* = 'U', the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced. If *info* = 'L', the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, D_{ii} = 0; *D* is singular and its inversion could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hetri2x interface are as follows:

a	Holds the matrix *A* of size (*n*,*n*).
ipiv	Holds the vector of length *n*.
nb	Holds the block size.
uplo	Indicates how the input matrix *A* has been factored. Must be 'U' or 'L'.

See Also

?hetrf
?hetri2

?sptri

Computes the inverse of a symmetric matrix using packed storage.

Syntax

Fortran 77:

```
call ssptri( uplo, n, ap, ipiv, work, info )
call dsptri( uplo, n, ap, ipiv, work, info )
call csptri( uplo, n, ap, ipiv, work, info )
call zsptri( uplo, n, ap, ipiv, work, info )
```

Fortran 95:

```
call sptri( ap, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sptri( int matrix_order, char uplo, lapack_int n, <datatype>* ap,
const lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a packed symmetric matrix `A`. Before calling this routine, call `?sptrf` to factorize `A`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix `A` has been factored: If *uplo* = `'U'`, the array *ap* stores the Bunch-Kaufman factorization $A = P*U*D*U^T*P^T$. If *uplo* = `'L'`, the array *ap* stores the Bunch-Kaufman factorization $A = P*L*D*L^T*P^T$.
n	`INTEGER`. The order of the matrix `A`; $n \geq 0$.
ap, *work*	`REAL` for `ssptri` `DOUBLE PRECISION` for `dsptri` `COMPLEX` for `csptri` `DOUBLE COMPLEX` for `zsptri`. Arrays: *ap*`(*)` contains the factorization of the matrix `A`, as returned by `?sptrf`. The dimension of *ap* must be at least `max(1,`*n*`(`*n*`+1)/2)`. *work*`(*)` is a workspace array. The dimension of *work* must be at least `max(1,`*n*`)`.
ipiv	`INTEGER`. Array, `DIMENSION` at least `max(1,` *n*`)`. The *ipiv* array, as returned by `?sptrf`.

Output Parameters

ap	Overwritten by the *n*-by-*n* matrix `inv(A)` in packed form.
info	`INTEGER`. If *info* = `0`, the execution is successful. If *info* = `-`*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of `D` is zero, `D` is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `sptri` interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed inverse *X* satisfies the following error bounds:

$$|D*U^T*P^T*X*P*U - I| \le c(n)\varepsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$$

for *uplo* = 'U', and

$$|D*L^T*P^T*X*P*L - I| \le c(n)\varepsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$$

for *uplo* = 'L'. Here $c(n)$ is a modest linear function of n, and ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

?hptri

Computes the inverse of a complex Hermitian matrix using packed storage.

Syntax

Fortran 77:

```
call chptri( uplo, n, ap, ipiv, work, info )
call zhptri( uplo, n, ap, ipiv, work, info )
```

Fortran 95:

```
call hptri( ap, ipiv [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hptri( int matrix_order, char uplo, lapack_int n, <datatype>* ap,
const lapack_int* ipiv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a complex Hermitian matrix *A* using packed storage. Before calling this routine, call `?hptrf` to factorize *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *ap* stores the packed Bunch-Kaufman factorization $A = P*U*D*U^H*P^T$. If *uplo* = 'L', the array *ap* stores the packed Bunch-Kaufman factorization $A = P*L*D*L^H*P^T$.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
ap, *work*	COMPLEX for chptri DOUBLE COMPLEX for zhptri. Arrays: *ap*(*) contains the factorization of the matrix *A*, as returned by ?hptrf. The dimension of *ap* must be at least max(1,*n*(*n*+1)/2). *work*(*) is a workspace array. The dimension of *work* must be at least max(1,*n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The *ipiv* array, as returned by ?hptrf.

Output Parameters

ap	Overwritten by the *n*-by-*n* matrix inv(*A*).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *D* is zero, *D* is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hptri interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed inverse *X* satisfies the following error bounds:

$$|D*U^H*P^T*X*P*U - I| \leq c(n)\varepsilon(|D||U^H|P^T|X|P|U| + |D||D^{-1}|)$$

for *uplo* = 'U', and

$$|D*L^H*P^T*X*PL - I| \leq c(n)\varepsilon(|D||L^H|P^T|X|P|L| + |D||D^{-1}|)$$

for *uplo* = 'L'. Here $c(n)$ is a modest linear function of n, and ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

The real counterpart of this routine is ?sptri.

?trtri

Computes the inverse of a triangular matrix.

Syntax

Fortran 77:

```
call strtri( uplo, diag, n, a, lda, info )
call dtrtri( uplo, diag, n, a, lda, info )
call ctrtri( uplo, diag, n, a, lda, info )
call ztrtri( uplo, diag, n, a, lda, info )
```

Fortran 95:

```
call trtri( a [,uplo] [,diag] [,info] )
```

C:

```
lapack_int LAPACKE_<?>trtri( int matrix_order, char uplo, char diag, lapack_int n,
<datatype>* a, lapack_int lda );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the inverse inv(*A*) of a triangular matrix *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether *A* is upper or lower triangular: If *uplo* = 'U', then *A* is upper triangular. If *uplo* = 'L', then *A* is lower triangular.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then *A* is not a unit triangular matrix. If *diag* = 'U', *A* is unit triangular: diagonal elements of *A* are assumed to be 1 and not referenced in the array *a*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a	REAL for strtri DOUBLE PRECISION for dtrtri COMPLEX for ctrtri DOUBLE COMPLEX for ztrtri. Array: DIMENSION (,*).

	Contains the matrix A. The second dimension of a must be at least $\max(1,n)$.
lda	INTEGER. The first dimension of a; $lda \geq \max(1, n)$.

Output Parameters

a	Overwritten by the n-by-n matrix $\text{inv}(A)$.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value. If $info = i$, the i-th diagonal element of A is zero, A is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `trtri` interface are as follows:

a	Holds the matrix A of size (n,n).
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.
diag	Must be `'N'` or `'U'`. The default value is `'N'`.

Application Notes

The computed inverse X satisfies the following error bounds:

$$|XA - I| \leq c(n)\varepsilon\ |X||A|$$

$$|XA - I| \leq c(n)\varepsilon\ |A^{-1}||A||X|,$$

where $c(n)$ is a modest linear function of n; ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors and $(4/3)n^3$ for complex flavors.

?tftri

Computes the inverse of a triangular matrix stored in the Rectangular Full Packed (RFP) format.

Syntax

Fortran 77:

```
call stftri( transr, uplo, diag, n, a, info )
call dtftri( transr, uplo, diag, n, a, info )
call ctftri( transr, uplo, diag, n, a, info )
call ztftri( transr, uplo, diag, n, a, info )
```

C:

```
lapack_int LAPACKE_<?>tftri( int matrix_order, char transr, char uplo, char diag,
lapack_int n, <datatype>* a );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

Computes the inverse of a triangular matrix *A* stored in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see Matrix Storage Schemes.

This is the block version of the algorithm, calling Level 3 BLAS.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

transr	CHARACTER*1. Must be 'N', 'T' (for real data) or 'C' (for complex data). If *transr* = 'N', the Normal *transr* of RFP *A* is stored. If *transr* = 'T', the Transpose *transr* of RFP *A* is stored. If *transr* = 'C', the Conjugate-Transpose *transr* of RFP *A* is stored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of RFP *A* is stored: If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix *A*. If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix *A*.
diag	CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then *A* is not a unit triangular matrix. If *diag* = 'U', *A* is unit triangular: diagonal elements of *A* are assumed to be 1 and not referenced in the array *a*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a	REAL for stftri DOUBLE PRECISION for dtftri COMPLEX for ctftri DOUBLE COMPLEX for ztftri. Array, DIMENSION (*n**(*n*+1)/2). The array *a* contains the matrix *A* in the RFP format.

Output Parameters

a	The (triangular) inverse of the original matrix in the same storage format.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, *A*(*i*,*i*) is exactly zero. The triangular matrix is singular and its inverse cannot be computed.

?tptri

Computes the inverse of a triangular matrix using packed storage.

Syntax

Fortran 77:

```
call stptri( uplo, diag, n, ap, info )
call dtptri( uplo, diag, n, ap, info )
call ctptri( uplo, diag, n, ap, info )
call ztptri( uplo, diag, n, ap, info )
```

Fortran 95:

```
call tptri( ap [,uplo] [,diag] [,info] )
```

C:

```
lapack_int LAPACKE_<?>tptri( int matrix_order, char uplo, char diag, lapack_int n,
<datatype>* ap );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the inverse `inv(A)` of a packed triangular matrix *A*.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether *A* is upper or lower triangular: If *uplo* = `'U'`, then *A* is upper triangular. If *uplo* = `'L'`, then *A* is lower triangular.
diag	`CHARACTER*1`. Must be `'N'` or `'U'`. If *diag* = `'N'`, then *A* is not a unit triangular matrix. If *diag* = `'U'`, *A* is unit triangular: diagonal elements of *A* are assumed to be 1 and not referenced in the array *ap*.
n	`INTEGER`. The order of the matrix *A*; $n \geq 0$.
ap	`REAL` for `stptri` `DOUBLE PRECISION` for `dtptri` `COMPLEX` for `ctptri` `DOUBLE COMPLEX` for `ztptri`. Array, `DIMENSION` at least `max(1,n(n+1)/2)`. Contains the packed triangular matrix A.

Output Parameters

ap	Overwritten by the packed *n*-by-*n* matrix `inv(A)` .
info	`INTEGER`. If *info* = 0, the execution is successful. If *info* = *-i*, the *i*-th parameter had an illegal value.

	If *info* = *i*, the *i*-th diagonal element of *A* is zero, *A* is singular, and the inversion could not be completed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `tptri` interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
uplo	Must be 'U' or 'L'. The default value is 'U'.
diag	Must be 'N' or 'U'. The default value is 'N'.

Application Notes

The computed inverse *X* satisfies the following error bounds:

$$|XA - I| \leq c(n)\varepsilon\ |X||A|$$

$$|X - A^{-1}| \leq c(n)\varepsilon\ |A^{-1}||A||X|,$$

where $c(n)$ is a modest linear function of n; ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors and $(4/3)n^3$ for complex flavors.

Routines for Matrix Equilibration

Routines described in this section are used to compute scaling factors needed to equilibrate a matrix. Note that these routines do not actually scale the matrices.

?geequ

Computes row and column scaling factors intended to equilibrate a general matrix and reduce its condition number.

Syntax

Fortran 77:

```
call sgeequ( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
call dgeequ( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
call cgeequ( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
call zgeequ( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
```

Fortran 95:

```
call geequ( a, r, c [,rowcnd] [,colcnd] [,amax] [,info] )
```

C:

```
lapack_int LAPACKE_sgeequ( int matrix_order, lapack_int m, lapack_int n, const float*
a, lapack_int lda, float* r, float* c, float* rowcnd, float* colcnd, float* amax );
lapack_int LAPACKE_dgeequ( int matrix_order, lapack_int m, lapack_int n, const double*
a, lapack_int lda, double* r, double* c, double* rowcnd, double* colcnd, double*
amax );
```

```
lapack_int LAPACKE_cgeequ( int matrix_order, lapack_int m, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* r, float* c, float* rowcnd, float*
colcnd, float* amax );

lapack_int LAPACKE_zgeequ( int matrix_order, lapack_int m, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* r, double* c, double* rowcnd,
double* colcnd, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate an m-by-n matrix A and reduce its condition number. The output array r returns the row scale factors and the array c the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix B with elements $b_{ij}=r(i)*a_{ij}*c(j)$ have absolute value 1.

See `?laqge` auxiliary function that uses scaling factors computed by `?geequ`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	`INTEGER`. The number of rows of the matrix A; $m \geq 0$.
n	`INTEGER`. The number of columns of the matrix A; $n \geq 0$.
a	`REAL` for `sgeequ` `DOUBLE PRECISION` for `dgeequ` `COMPLEX` for `cgeequ` `DOUBLE COMPLEX` for `zgeequ`. Array: `DIMENSION (lda,*)`. Contains the m-by-n matrix A whose equilibration factors are to be computed. The second dimension of a must be at least `max(1,n)`.
lda	`INTEGER`. The leading dimension of a; $lda \geq \max(1, m)$.

Output Parameters

r, *c*	`REAL` for single precision flavors `DOUBLE PRECISION` for double precision flavors. Arrays: $r(m)$, $c(n)$. If $info = 0$, or $info > m$, the array r contains the row scale factors of the matrix A. If $info = 0$, the array c contains the column scale factors of the matrix A.
rowcnd	`REAL` for single precision flavors `DOUBLE PRECISION` for double precision flavors. If $info = 0$ or $info > m$, $rowcnd$ contains the ratio of the smallest $r(i)$ to the largest $r(i)$.
colcnd	`REAL` for single precision flavors

	DOUBLE PRECISION for double precision flavors. If *info* = 0, *colcnd* contains the ratio of the smallest *c*(i) to the largest *c*(i).
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix *A*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i* and *i* ≤ *m*, the *i*-th row of *A* is exactly zero; *i* > *m*, the (*i*-*m*)th column of *A* is exactly zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine geequ interface are as follows:

a	Holds the matrix *A* of size (*m*, *n*).
r	Holds the vector of length (*m*).
c	Holds the vector of length *n*.

Application Notes

All the components of *r* and *c* are restricted to be between SMLNUM = smallest safe number and BIGNUM= largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of *A* but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the ?lamch routines to compute them. For example, compute single precision (real and complex) values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

If *rowcnd* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *r*.

If *colcnd* ≥ 0.1, it is not worth scaling by *c*.

If *amax* is very close to overflow or very close to underflow, the matrix *A* should be scaled.

?geequb

Computes row and column scaling factors restricted to a power of radix to equilibrate a general matrix and reduce its condition number.

Syntax

Fortran 77:

```
call sgeequb( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
call dgeequb( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
call cgeequb( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
call zgeequb( m, n, a, lda, r, c, rowcnd, colcnd, amax, info )
```

C:

```
lapack_int LAPACKE_sgeequb( int matrix_order, lapack_int m, lapack_int n, const float*
a, lapack_int lda, float* r, float* c, float* rowcnd, float* colcnd, float* amax );

lapack_int LAPACKE_dgeequb( int matrix_order, lapack_int m, lapack_int n, const double*
a, lapack_int lda, double* r, double* c, double* rowcnd, double* colcnd, double*
amax );

lapack_int LAPACKE_cgeequb( int matrix_order, lapack_int m, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* r, float* c, float* rowcnd, float*
colcnd, float* amax );

lapack_int LAPACKE_zgeequb( int matrix_order, lapack_int m, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* r, double* c, double* rowcnd,
double* colcnd, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate an *m*-by-*n* general matrix *A* and reduce its condition number. The output array *r* returns the row scale factors and the array *c* - the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix *B* with elements *b*(*ij*)=*r*(i)**a*(*ij*)**c*(j) have an absolute value of at most the radix.

r(i) and *c*(j) are restricted to be a power of the radix between SMLNUM = smallest safe number and BIGNUM = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of *a* but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the ?lamch routines to compute them. For example, compute single precision (real and complex) values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

This routine differs from ?geequ by restricting the scaling factors to a power of the radix. Except for over- and underflow, scaling by these factors introduces no additional rounding errors. However, the scaled entries' magnitudes are no longer equal to approximately 1 but lie between sqrt(radix) and 1/sqrt(radix).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *A*; $m \geq 0$.
n	INTEGER. The number of columns of the matrix *A*; $n \geq 0$.
a	REAL for sgeequb DOUBLE PRECISION for dgeequb COMPLEX for cgeequb DOUBLE COMPLEX for zgeequb. Array: DIMENSION (*lda*,*).

	Contains the m-by-n matrix A whose equilibration factors are to be computed. The second dimension of a must be at least max(1,n).
lda	INTEGER. The leading dimension of a; $lda \geq \max(1, m)$.

Output Parameters

r, *c*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays: $r(m)$, $c(n)$. If $info = 0$, or $info > m$, the array r contains the row scale factors for the matrix A. If $info = 0$, the array c contains the column scale factors for the matrix A.
rowcnd	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If $info = 0$ or $info > m$, *rowcnd* contains the ratio of the smallest $r(i)$ to the largest $r(i)$. If $rowcnd \geq 0.1$, and *amax* is neither too large nor too small, it is not worth scaling by r.
colcnd	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If $info = 0$, *colcnd* contains the ratio of the smallest $c(i)$ to the largest $c(i)$. If $colcnd \geq 0.1$, it is not worth scaling by c.
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix A. If *amax* is very close to overflow or very close to underflow, the matrix should be scaled.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value. If $info = i$ and $i \leq m$, the i-th row of A is exactly zero; $i > m$, the (i-m)-th column of A is exactly zero.

?gbequ

Computes row and column scaling factors intended to equilibrate a banded matrix and reduce its condition number.

Syntax

Fortran 77:

```
call sgbequ( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
call dgbequ( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
call cgbequ( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
call zgbequ( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
```

Fortran 95:

```
call gbequ( ab, r, c [,kl] [,rowcnd] [,colcnd] [,amax] [,info] )
```

C:

```
lapack_int LAPACKE_sgbequ( int matrix_order, lapack_int m, lapack_int n, lapack_int kl,
lapack_int ku, const float* ab, lapack_int ldab, float* r, float* c, float* rowcnd,
float* colcnd, float* amax );

lapack_int LAPACKE_dgbequ( int matrix_order, lapack_int m, lapack_int n, lapack_int kl,
lapack_int ku, const double* ab, lapack_int ldab, double* r, double* c, double*
rowcnd, double* colcnd, double* amax );

lapack_int LAPACKE_cgbequ( int matrix_order, lapack_int m, lapack_int n, lapack_int kl,
lapack_int ku, const lapack_complex_float* ab, lapack_int ldab, float* r, float* c,
float* rowcnd, float* colcnd, float* amax );

lapack_int LAPACKE_zgbequ( int matrix_order, lapack_int m, lapack_int n, lapack_int kl,
lapack_int ku, const lapack_complex_double* ab, lapack_int ldab, double* r, double* c,
double* rowcnd, double* colcnd, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate an m-by-n band matrix A and reduce its condition number. The output array r returns the row scale factors and the array c the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix B with elements b_{ij}=r(i)*a_{ij}*c(j) have absolute value 1.

See `?laqgb` auxiliary function that uses scaling factors computed by `?gbequ`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix A; $m \geq 0$.
n	INTEGER. The number of columns of the matrix A; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of A; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of A; $ku \geq 0$.
ab	REAL for sgbequ DOUBLE PRECISION for dgbequ COMPLEX for cgbequ DOUBLE COMPLEX for zgbequ. Array, DIMENSION (*ldab*,*). Contains the original band matrix A stored in rows from 1 to kl + ku + 1. The second dimension of *ab* must be at least max(1,*n*).
ldab	INTEGER. The leading dimension of *ab*; $ldab \geq kl+ku+1$.

Output Parameters

r, *c*	REAL for single precision flavors

	DOUBLE PRECISION for double precision flavors. Arrays: r(m), c(n). If info = 0, or info > m, the array r contains the row scale factors of the matrix A. If info = 0, the array c contains the column scale factors of the matrix A.
rowcnd	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If info = 0 or info > m, rowcnd contains the ratio of the smallest r(i) to the largest r(i).
colcnd	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If info = 0, colcnd contains the ratio of the smallest c(i) to the largest c(i).
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix A.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value. If info = i and i ≤ m, the i-th row of A is exactly zero; i > m, the (i-m)th column of A is exactly zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbequ interface are as follows:

ab	Holds the array A of size (kl+ku+1, n).
r	Holds the vector of length (m).
c	Holds the vector of length n.
kl	If omitted, assumed kl = ku.
ku	Restored as ku = lda-kl-1.

Application Notes

All the components of r and c are restricted to be between SMLNUM = smallest safe number and BIGNUM= largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

SMLNUM and BIGNUM are parameters representing machine precision. You can use the ?lamch routines to compute them. For example, compute single precision (real and complex) values of SMLNUM and BIGNUM as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

If rowcnd ≥ 0.1 and amax is neither too large nor too small, it is not worth scaling by r.

If colcnd ≥ 0.1, it is not worth scaling by c.

If amax is very close to overflow or very close to underflow, the matrix A should be scaled.

?gbequb

Computes row and column scaling factors restricted to a power of radix to equilibrate a banded matrix and reduce its condition number.

Syntax

Fortran 77:

```
call sgbequb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
call dgbequb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
call cgbequb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
call zgbequb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, info )
```

C:

```
lapack_int LAPACKE_sgbequb( int matrix_order, lapack_int m, lapack_int n, lapack_int kl, lapack_int ku, const float* ab, lapack_int ldab, float* r, float* c, float* rowcnd, float* colcnd, float* amax );
lapack_int LAPACKE_dgbequb( int matrix_order, lapack_int m, lapack_int n, lapack_int kl, lapack_int ku, const double* ab, lapack_int ldab, double* r, double* c, double* rowcnd, double* colcnd, double* amax );
lapack_int LAPACKE_cgbequb( int matrix_order, lapack_int m, lapack_int n, lapack_int kl, lapack_int ku, const lapack_complex_float* ab, lapack_int ldab, float* r, float* c, float* rowcnd, float* colcnd, float* amax );
lapack_int LAPACKE_zgbequb( int matrix_order, lapack_int m, lapack_int n, lapack_int kl, lapack_int ku, const lapack_complex_double* ab, lapack_int ldab, double* r, double* c, double* rowcnd, double* colcnd, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate an *m*-by-*n* banded matrix *A* and reduce its condition number. The output array *r* returns the row scale factors and the array *c* - the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix *B* with elements *b*(*ij*)=*r*(i)**a*(*ij*)**c*(j) have an absolute value of at most the radix.

r(i) and *c*(j) are restricted to be a power of the radix between *SMLNUM* = smallest safe number and *BIGNUM* = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of *a* but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the ?lamch routines to compute them. For example, compute single precision (real and complex) values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

This routine differs from ?gbequ by restricting the scaling factors to a power of the radix. Except for over- and underflow, scaling by these factors introduces no additional rounding errors. However, the scaled entries' magnitudes are no longer equal to approximately 1 but lie between sqrt(radix) and 1/sqrt(radix).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *A*; $m \geq 0$.
n	INTEGER. The number of columns of the matrix *A*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
ab	REAL for sgbequb DOUBLE PRECISION for dgbequb COMPLEX for cgbequb DOUBLE COMPLEX for zgbequb. Array: DIMENSION (*ldab*,*). Contains the original banded matrix *A* stored in rows from 1 to *kl* + *ku* + 1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = *a*(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*). The second dimension of *ab* must be at least max(1,*n*).
ldab	INTEGER. The leading dimension of *a*; *ldab* ≥ max(1, *m*).

Output Parameters

r, *c*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays: *r*(*m*), *c*(*n*). If *info* = 0, or *info* > *m*, the array *r* contains the row scale factors for the matrix *A*. If *info* = 0, the array *c* contains the column scale factors for the matrix *A*.
rowcnd	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If *info* = 0 or *info* > *m*, *rowcnd* contains the ratio of the smallest *r*(i) to the largest *r*(i). If *rowcnd* ≥ 0.1, and *amax* is neither too large nor too small, it is not worth scaling by *r*.
colcnd	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If *info* = 0, *colcnd* contains the ratio of the smallest *c*(i) to the largest *c*(i). If *colcnd* ≥ 0.1, it is not worth scaling by *c*.
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix *A*. If *amax* is very close to overflow or underflow, the matrix should be scaled.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive. *i* ≤ *m*, the *i*-th row of *A* is exactly zero; *i* > *m*, the (*i*-*m*)-th column of *A* is exactly zero.

?poequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite matrix and reduce its condition number.

Syntax

Fortran 77:

```
call spoequ( n, a, lda, s, scond, amax, info )
call dpoequ( n, a, lda, s, scond, amax, info )
call cpoequ( n, a, lda, s, scond, amax, info )
call zpoequ( n, a, lda, s, scond, amax, info )
```

Fortran 95:

```
call poequ( a, s [,scond] [,amax] [,info] )
```

C:

```
lapack_int LAPACKE_spoequ( int matrix_order, lapack_int n, const float* a, lapack_int lda, float* s, float* scond, float* amax );
lapack_int LAPACKE_dpoequ( int matrix_order, lapack_int n, const double* a, lapack_int lda, double* s, double* scond, double* amax );
lapack_int LAPACKE_cpoequ( int matrix_order, lapack_int n, const lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );
lapack_int LAPACKE_zpoequ( int matrix_order, lapack_int n, const lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive-definite matrix A and reduce its condition number (with respect to the two-norm). The output array s returns scale factors computed as

$$s(i) = \frac{1}{\sqrt{a_{i,i}}}$$

These factors are chosen so that the scaled matrix B with elements $b_{ij}=s(i)*a_{ij}*s(j)$ has diagonal elements equal to 1.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

See `?laqsy` auxiliary function that uses scaling factors computed by `?poequ`.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a	REAL for spoequ DOUBLE PRECISION for dpoequ COMPLEX for cpoequ DOUBLE COMPLEX for zpoequ. Array: DIMENSION (*lda*,*). Contains the *n*-by-*n* symmetric or Hermitian positive definite matrix *A* whose scaling factors are to be computed. Only the diagonal elements of *A* are referenced. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of *a*; $lda \geq \max(1,n)$.

Output Parameters

s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). If *info* = 0, the array *s* contains the scale factors for *A*.
scond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If *info* = 0, *scond* contains the ratio of the smallest *s*(i) to the largest *s*(i).
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix *A*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine poequ interface are as follows:

a	Holds the matrix *A* of size (*n*,*n*).
s	Holds the vector of length *n*.

Application Notes

If $scond \geq 0.1$ and *amax* is neither too large nor too small, it is not worth scaling by *s*.

If *amax* is very close to overflow or very close to underflow, the matrix *A* should be scaled.

?poequb

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite matrix and reduce its condition number.

Syntax

Fortran 77:

```
call spoequb( n, a, lda, s, scond, amax, info )
call dpoequb( n, a, lda, s, scond, amax, info )
call cpoequb( n, a, lda, s, scond, amax, info )
call zpoequb( n, a, lda, s, scond, amax, info )
```

C:

```
lapack_int LAPACKE_spoequb( int matrix_order, lapack_int n, const float* a, lapack_int lda, float* s, float* scond, float* amax );
lapack_int LAPACKE_dpoequb( int matrix_order, lapack_int n, const double* a, lapack_int lda, double* s, double* scond, double* amax );
lapack_int LAPACKE_cpoequb( int matrix_order, lapack_int n, const lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );
lapack_int LAPACKE_zpoequb( int matrix_order, lapack_int n, const lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive-definite matrix A and reduce its condition number (with respect to the two-norm).

These factors are chosen so that the scaled matrix B with elements $b(i,j)=s(i)*a(i,j)*s(j)$ has diagonal elements equal to 1. $s(i)$ is a power of two nearest to, but not exceeding $1/\mathrm{sqrt}(A(i,i))$.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix A; $n \geq 0$.
a	REAL for spoequb DOUBLE PRECISION for dpoequb COMPLEX for cpoequb DOUBLE COMPLEX for zpoequb. Array: DIMENSION (*lda*,*).

	Contains the n-by-n symmetric or Hermitian positive definite matrix A whose scaling factors are to be computed. Only the diagonal elements of A are referenced. The second dimension of a must be at least $\max(1,n)$.
lda	INTEGER. The leading dimension of a; $lda \geq \max(1, m)$.

Output Parameters

s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (n). If $info = 0$, the array s contains the scale factors for A.
scond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If $info = 0$, *scond* contains the ratio of the smallest $s(i)$ to the largest $s(i)$. If $scond \geq 0.1$, and *amax* is neither too large nor too small, it is not worth scaling by s.
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix A. If *amax* is very close to overflow or underflow, the matrix should be scaled.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value. If $info = i$, the i-th diagonal element of A is nonpositive.

?ppequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite matrix in packed storage and reduce its condition number.

Syntax

Fortran 77:

```
call sppequ( uplo, n, ap, s, scond, amax, info )
call dppequ( uplo, n, ap, s, scond, amax, info )
call cppequ( uplo, n, ap, s, scond, amax, info )
call zppequ( uplo, n, ap, s, scond, amax, info )
```

Fortran 95:

```
call ppequ( ap, s [,scond] [,amax] [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_sppequ( int matrix_order, char uplo, lapack_int n, const float* ap,
float* s, float* scond, float* amax );
lapack_int LAPACKE_dppequ( int matrix_order, char uplo, lapack_int n, const double* ap,
double* s, double* scond, double* amax );
lapack_int LAPACKE_cppequ( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* ap, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_zppequ( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* ap, double* s, double* scond, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive definite matrix A in packed storage and reduce its condition number (with respect to the two-norm). The output array s returns scale factors computed as

These factors are chosen so that the scaled matrix B with elements $b_{ij}=s(i)*a_{ij}*s(j)$ has diagonal elements equal to 1.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

See `?laqsp` auxiliary function that uses scaling factors computed by `?ppequ`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether the upper or lower triangular part of A is packed in the array ap: If $uplo$ = `'U'`, the array ap stores the upper triangular part of the matrix A. If $uplo$ = `'L'`, the array ap stores the lower triangular part of the matrix A.
n	`INTEGER`. The order of matrix A; $n \geq 0$.
ap	`REAL` for `sppequ` `DOUBLE PRECISION` for `dppequ` `COMPLEX` for `cppequ` `DOUBLE COMPLEX` for `zppequ`. Array, `DIMENSION` at least `max(1,`n`(`n`+1)/2)`. The array ap contains the upper or the lower triangular part of the matrix A (as specified by $uplo$) in *packed storage* (see Matrix Storage Schemes).

Output Parameters

s	`REAL` for single precision flavors `DOUBLE PRECISION` for double precision flavors. Array, `DIMENSION` (n). If $info$ = 0, the array s contains the scale factors for A.

scond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If *info* = 0, *scond* contains the ratio of the smallest *s*(i) to the largest *s*(i).
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix *A*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ppequ interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
s	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

If *scond* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *s*.

If *amax* is very close to overflow or very close to underflow, the matrix *A* should be scaled.

?pbequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive-definite band matrix and reduce its condition number.

Syntax

Fortran 77:

```
call spbequ( uplo, n, kd, ab, ldab, s, scond, amax, info )
call dpbequ( uplo, n, kd, ab, ldab, s, scond, amax, info )
call cpbequ( uplo, n, kd, ab, ldab, s, scond, amax, info )
call zpbequ( uplo, n, kd, ab, ldab, s, scond, amax, info )
```

Fortran 95:

```
call pbequ( ab, s [,scond] [,amax] [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_spbequ( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const float* ab, lapack_int ldab, float* s, float* scond, float* amax );
lapack_int LAPACKE_dpbequ( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const double* ab, lapack_int ldab, double* s, double* scond, double* amax );
lapack_int LAPACKE_cpbequ( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_float* ab, lapack_int ldab, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_zpbequ( int matrix_order, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_double* ab, lapack_int ldab, double* s, double* scond, double*
amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive definite matrix *A* in packed storage and reduce its condition number (with respect to the two-norm). The output array *s* returns scale factors computed as

$$s(i) = \frac{1}{\sqrt{a_{i,i}}}$$

These factors are chosen so that the scaled matrix *B* with elements b_{ij}=s(i)*a_{ij}*s(j) has diagonal elements equal to 1. This choice of *s* puts the condition number of *B* within a factor *n* of the smallest possible condition number over all possible diagonal scalings.

See `?laqsb` auxiliary function that uses scaling factors computed by `?pbequ`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether the upper or lower triangular part of *A* is packed in the array *ab*: If *uplo* = `'U'`, the array *ab* stores the upper triangular part of the matrix *A*. If *uplo* = `'L'`, the array *ab* stores the lower triangular part of the matrix *A*.
n	`INTEGER`. The order of matrix *A*; *n* ≥ 0.
kd	`INTEGER`. The number of superdiagonals or subdiagonals in the matrix *A*; *kd* ≥ 0.
ab	`REAL` for `spbequ` `DOUBLE PRECISION` for `dpbequ` `COMPLEX` for `cpbequ` `DOUBLE COMPLEX` for `zpbequ`. Array, `DIMENSION (ldab,*)`. The array *ap* contains either the upper or the lower triangular part of the matrix *A* (as specified by *uplo*) in *band storage* (see Matrix Storage Schemes). The second dimension of *ab* must be at least `max(1, n)`.
ldab	`INTEGER`. The leading dimension of the array *ab*; *ldab* ≥ *kd* +1.

Output Parameters

s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). If *info* = 0, the array *s* contains the scale factors for *A*.
scond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. If *info* = 0, *scond* contains the ratio of the smallest *s*(i) to the largest *s*(i).
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix *A*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pbequ interface are as follows:

ab	Holds the array *A* of size (*kd*+1, *n*).
s	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

If *scond* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *s*.

If *amax* is very close to overflow or very close to underflow, the matrix *A* should be scaled.

?syequb

Computes row and column scaling factors intended to equilibrate a symmetric indefinite matrix and reduce its condition number.

Syntax

Fortran 77:

```
call ssyequb( uplo, n, a, lda, s, scond, amax, work, info )
call dsyequb( uplo, n, a, lda, s, scond, amax, work, info )
call csyequb( uplo, n, a, lda, s, scond, amax, work, info )
call zsyequb( uplo, n, a, lda, s, scond, amax, work, info )
```

C:

```
lapack_int LAPACKE_ssyequb( int matrix_order, char uplo, lapack_int n, const float* a,
lapack_int lda, float* s, float* scond, float* amax );
lapack_int LAPACKE_dsyequb( int matrix_order, char uplo, lapack_int n, const double* a,
lapack_int lda, double* s, double* scond, double* amax );
```

```
lapack_int LAPACKE_csyequb( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );

lapack_int LAPACKE_zsyequb( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric indefinite matrix A and reduce its condition number (with respect to the two-norm).

The array s contains the scale factors, $s(i) = 1/\sqrt{A(i,i)}$. These factors are chosen so that the scaled matrix B with elements $b(i,j)=s(i)*a(i,j)*s(j)$ has ones on the diagonal.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix A. If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix A.
n	INTEGER. The order of the matrix A; $n \geq 0$.
a, *work*	REAL for ssyequb DOUBLE PRECISION for dsyequb COMPLEX for csyequb DOUBLE COMPLEX for zsyequb. Array *a*: DIMENSION (*lda*,*). Contains the n-by-n symmetric indefinite matrix A whose scaling factors are to be computed. Only the diagonal elements of A are referenced. The second dimension of *a* must be at least max(1,*n*). *work(*)* is a workspace array. The dimension of *work* is at least max(1,3*n).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *m*).

Output Parameters

s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). If *info* = 0, the array *s* contains the scale factors for A.
scond	REAL for single precision flavors

	DOUBLE PRECISION for double precision flavors. If *info* = 0, *scond* contains the ratio of the smallest *s*(i) to the largest *s*(i). If *scond* ≥ 0.1, and *amax* is neither too large nor too small, it is not worth scaling by *s*.
amax	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Absolute value of the largest element of the matrix *A*. If *amax* is very close to overflow or underflow, the matrix should be scaled.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

?heequb

Computes row and column scaling factors intended to equilibrate a Hermitian indefinite matrix and reduce its condition number.

Syntax

Fortran 77:

```
call cheequb( uplo, n, a, lda, s, scond, amax, work, info )
call zheequb( uplo, n, a, lda, s, scond, amax, work, info )
```

C:

```
lapack_int LAPACKE_cheequb( int matrix_order, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );
lapack_int LAPACKE_zheequb( int matrix_order, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes row and column scalings intended to equilibrate a Hermitian indefinite matrix *A* and reduce its condition number (with respect to the two-norm).

The array *s* contains the scale factors, *s*(*i*) = 1/sqrt(A(i,i)). These factors are chosen so that the scaled matrix *B* with elements *b*(*i*,*j*)=*s*(i)**a*(*i*,*j*)**s*(j) has ones on the diagonal.

This choice of *s* puts the condition number of *B* within a factor *n* of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored:

	If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix *A*. If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix *A*.
n	INTEGER. The order of the matrix *A*; $n \geq 0$.
a, *work*	COMPLEX for cheequb DOUBLE COMPLEX for zheequb. Array *a*: DIMENSION (*lda*,*). Contains the *n*-by-*n* symmetric indefinite matrix *A* whose scaling factors are to be computed. Only the diagonal elements of *A* are referenced. The second dimension of *a* must be at least max(1,*n*). *work(*)* is a workspace array. The dimension of *work* is at least max(1,3**n*).
lda	INTEGER. The leading dimension of *a*; $lda \geq \max(1, m)$.

Output Parameters

s	REAL for cheequb DOUBLE PRECISION for zheequb. Array, DIMENSION (*n*). If *info* = 0, the array *s* contains the scale factors for *A*.
scond	REAL for cheequb DOUBLE PRECISION for zheequb. If *info* = 0, *scond* contains the ratio of the smallest *s*(i) to the largest *s*(i). If $scond \geq 0.1$, and *amax* is neither too large nor too small, it is not worth scaling by *s*.
amax	REAL for cheequb DOUBLE PRECISION for zheequb. Absolute value of the largest element of the matrix *A*. If *amax* is very close to overflow or underflow, the matrix should be scaled.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

Driver Routines

Table "Driver Routines for Solving Systems of Linear Equations" lists the LAPACK driver routines for solving systems of linear equations with real or complex matrices.

Driver Routines for Solving Systems of Linear Equations

Matrix type, storage scheme	Simple Driver	Expert Driver	Expert Driver using Extra-Precise Interative Refinement
general	?gesv	?gesvx	?gesvxx
general band	?gbsv	?gbsvx	?gbsvxx
general tridiagonal	?gtsv	?gtsvx	

Matrix type, storage scheme	Simple Driver	Expert Driver	Expert Driver using Extra-Precise Interative Refinement
diagonally dominant tridiagonal	?dtsvb		
symmetric/Hermitian positive-definite	?posv	?posvx	?posvxx
symmetric/Hermitian positive-definite, storage	?ppsv	?ppsvx	
symmetric/Hermitian positive-definite, band	?pbsv	?pbsvx	
symmetric/Hermitian positive-definite, tridiagonal	?ptsv	?ptsvx	
symmetric/Hermitian indefinite	?sysv/?hesv	?sysvx/?hesvx	?sysvxx/?hesvxx
symmetric/Hermitian indefinite, packed storage	?spsv/?hpsv	?spsvx/?hpsvx	
complex symmetric	?sysv	?sysvx	
complex symmetric, packed storage	?spsv	?spsvx	

In this table ? stands for s (single precision real), d (double precision real), c (single precision complex), or z (double precision complex). In the description of ?gesv and ?posv routines, the ? sign stands for combined character codes ds and zc for the mixed precision subroutines.

?gesv

Computes the solution to the system of linear equations with a square matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sgesv( n, nrhs, a, lda, ipiv, b, ldb, info )
call dgesv( n, nrhs, a, lda, ipiv, b, ldb, info )
call cgesv( n, nrhs, a, lda, ipiv, b, ldb, info )
call zgesv( n, nrhs, a, lda, ipiv, b, ldb, info )
call dsgesv( n, nrhs, a, lda, ipiv, b, ldb, x, ldx, work, swork, iter, info )
call zcgesv( n, nrhs, a, lda, ipiv, b, ldb, x, ldx, work, swork, rwork, iter, info )
```

Fortran 95:

```
call gesv( a, b [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>gesv( int matrix_order, lapack_int n, lapack_int nrhs,
<datatype>* a, lapack_int lda, lapack_int* ipiv, <datatype>* b, lapack_int ldb );

lapack_int LAPACKE_dsgesv( int matrix_order, lapack_int n, lapack_int nrhs, double* a,
lapack_int lda, lapack_int* ipiv, double* b, lapack_int ldb, double* x, lapack_int
ldx, lapack_int* iter );

lapack_int LAPACKE_zcgesv( int matrix_order, lapack_int n, lapack_int nrhs,
lapack_complex_double* a, lapack_int lda, lapack_int* ipiv, lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, lapack_int* iter );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for X the system of linear equations $A*X = B$, where A is an n-by-n matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The LU decomposition with partial pivoting and row interchanges is used to factor A as $A = P*L*U$, where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $A*X = B$.

The `dsgesv` and `zcgesv` are mixed precision iterative refinement subroutines for exploiting fast single precision hardware. They first attempt to factorize the matrix in single precision (`dsgesv`) or single complex precision (`zcgesv`) and use this factorization within an iterative refinement procedure to produce a solution with double precision (`dsgesv`) / double complex precision (`zcgesv`) normwise backward error quality (see below). If the approach fails, the method switches to a double precision or double complex precision factorization respectively and computes the solution.

The iterative refinement is not going to be a winning strategy if the ratio single precision performance over double precision performance is too small. A reasonable strategy should take the number of right-hand sides and the size of the matrix into account. This might be done with a call to `ilaenv` in the future. At present, iterative refinement is implemented.

The iterative refinement process is stopped if

```
iter > itermax
```

or for all the right-hand sides:

```
rnmr < sqrt(n)*xnrm*anrm*eps*bwdmax
```

where

- `iter` is the number of the current iteration in the iterativerefinement process
- `rnmr` is the infinity-norm of the residual
- `xnrm` is the infinity-norm of the solution
- `anrm` is the infinity-operator-norm of the matrix A
- `eps` is the machine epsilon returned by `dlamch` (‘Epsilon’).

The values `itermax` and `bwdmax` are fixed to 30 and 1.0d+00 respectively.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides, that is, the number of columns of the matrix *B*; *nrhs* ≥ 0.
a, *b*	REAL for sgesv DOUBLE PRECISION for dgesv and dsgesv COMPLEX for cgesv DOUBLE COMPLEX for zgesv and zcgesv. Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the *n*-by-*n* coefficient matrix *A*. The array *b* contains the *n*-by-*nrhs* matrix of right hand side matrix *B*. The second dimension of *a* must be at least max(1, *n*), the second dimension of *b* at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of the array *x*; *ldx* ≥ max(1, *n*).
work	DOUBLE PRECISION for dsgesv DOUBLE COMPLEX for zcgesv. Workspace array, DIMENSION at least max(1,*n***nrhs*). This array is used to hold the residual vectors.
swork	REAL for dsgesv COMPLEX for zcgesv. Workspace array, DIMENSION at least max(1,*n**(*n*+*nrhs*)). This array is used to use the single precision matrix and the right-hand sides or solutions in single precision.
rwork	DOUBLE PRECISION. Workspace array, DIMENSION at least max(1,*n*).

Output Parameters

a	Overwritten by the factors *L* and *U* from the factorization of *A* = *P***L***U*; the unit diagonal elements of *L* are not stored. If iterative refinement has been successfully used (*info*= 0 and *iter*≥ 0), then *A* is unchanged. If double precision factorization has been used (*info*= 0 and *iter* < 0), then the array *A* contains the factors *L* and *U* from the factorization *A* = *P***L***U*; the unit diagonal elements of *L* are not stored.
b	Overwritten by the solution matrix *X* for dgesv, sgesv,zgesv,zgesv. Unchanged for dsgesv and zcgesv.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The pivot indices that define the permutation matrix *P*; row *i* of the matrix was interchanged with row *ipiv*(i). Corresponds to the single precision factorization (if *info*= 0 and *iter* ≥ 0) or the double precision factorization (if *info*= 0 and *iter* < 0).
x	DOUBLE PRECISION for dsgesv

	DOUBLE COMPLEX for zcgesv. Array, DIMENSION (*ldx*, *nrhs*). If *info* = 0, contains the *n*-by-*nrhs* solution matrix *X*.
iter	INTEGER. If *iter* < 0: iterative refinement has failed, double precision factorization has been performed • If *iter* = -1: the routine fell back to full precision for implementation- or machine-specific reason • If *iter* = -2: narrowing the precision induced an overflow, the routine fell back to full precision • If *iter* = -3: failure of sgetrf for dsgesv, or cgetrf for zcgesv • If *iter* = -31: stop the iterative refinement after the 30th iteration. If *iter* > 0: iterative refinement has been successfully used. Returns the number of iterations.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, *U*(*i*, *i*) (computed in double precision for mixed precision subroutines) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gesv interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.

NOTE Fortran 95 Interface is so far not available for the mixed precision subroutines dsgesv/zcgesv.

See Also

ilaenv
?lamch
?getrf

?gesvx

Computes the solution to the system of linear equations with a square matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sgesvx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, ferr, berr, work, iwork, info )
```

```
call dgesvx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, ferr, berr, work, iwork, info )

call cgesvx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, ferr, berr, work, rwork, info )

call zgesvx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call gesvx( a, b, x [,af] [,ipiv] [,fact] [,trans] [,equed] [,r] [,c] [,ferr] [,berr]
[,rcond] [,rpvgrw] [,info] )
```

C:

```
lapack_int LAPACKE_sgesvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, float* r, float* c, float* b, lapack_int ldb, float* x, lapack_int
ldx, float* rcond, float* ferr, float* berr, float* rpivot );

lapack_int LAPACKE_dgesvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, double* r, double* c, double* b, lapack_int ldb, double* x,
lapack_int ldx, double* rcond, double* ferr, double* berr, double* rpivot );

lapack_int LAPACKE_cgesvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* r, float* c,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr, float* rpivot );

lapack_int LAPACKE_zgesvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* r, double* c,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr, double* rpivot );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations *A*X = B*, where *A* is an *n*-by-*n* matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?gesvx` performs the following steps:

1. If *fact* = 'E', real scaling factors *r* and *c* are computed to equilibrate the system:

trans = 'N': *diag(r)*A*diag(c)*inv(diag(c))*X = diag(r)*B*

trans = 'T': $(diag(r)*A*diag(c))^{T}$*inv(diag(r))*X = diag(c)*B*

trans = 'C': $(diag(r)*A*diag(c))^{H}$*inv(diag(r))*X = diag(c)*B*

Whether or not the system will be equilibrated depends on the scaling of the matrix *A*, but if equilibration is used, *A* is overwritten by *diag(r)*A*diag(c)* and *B* by *diag(r)*B* (if *trans*='N') or *diag(c)*B* (if *trans* = 'T' or 'C').

2. If *fact* = 'N' or 'E', the *LU* decomposition is used to factor the matrix *A* (after equilibration if *fact* = 'E') as *A* = *P*L*U*, where *P* is a permutation matrix, *L* is a unit lower triangular matrix, and *U* is upper triangular.
3. If some $U_{i,i}$= 0, so that *U* is exactly singular, then the routine returns with *info* = *i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, *info* = *n* + 1 is returned as a warning, but the routine still goes on to solve for *X* and compute error bounds as described below.
4. The system of equations is solved for *X* using the factored form of *A*.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix *X* is premultiplied by *diag(c)* (if *trans* = 'N') or *diag(r)* (if *trans* = 'T' or 'C') so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface, except for *rpivot*. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored. If *fact* = 'F': on entry, *af* and *ipiv* contain the factored form of *A*. If *equed* is not 'N', the matrix *A* has been equilibrated with scaling factors given by *r* and *c*. *a*, *af*, and *ipiv* are not modified. If *fact* = 'N', the matrix *A* will be copied to *af* and factored. If *fact* = 'E', the matrix *A* will be equilibrated if necessary, then copied to *af* and factored.
trans	CHARACTER*1. Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose). If *trans* = 'T', the system has the form A^T*X = *B* (Transpose). If *trans* = 'C', the system has the form A^H*X = *B* (Transpose for real flavors, conjugate transpose for complex flavors).
n	INTEGER. The number of linear equations; the order of the matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right hand sides; the number of columns of the matrices *B* and *X*; *nrhs* ≥ 0.
a, *af*, *b*, *work*	REAL for sgesvx DOUBLE PRECISION for dgesvx COMPLEX for cgesvx DOUBLE COMPLEX for zgesvx. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*). The array *a* contains the matrix *A*. If *fact* = 'F' and *equed* is not 'N', then *A* must have been equilibrated by the scaling factors in *r* and/or *c*. The second dimension of *a* must be at least max(1,*n*).

	The array *af* is an input argument if *fact* = 'F'. It contains the factored form of the matrix *A*, that is, the factors *L* and *U* from the factorization *A* = *P*L*U* as computed by ?getrf. If *equed* is not 'N', then *af* is the factored form of the equilibrated matrix *A*. The second dimension of *af* must be at least max(1,*n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1, *nrhs*). *work(*)* is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains the pivot indices from the factorization *A* = *P*L*U* as computed by ?getrf; row *i* of the matrix was interchanged with row *ipiv*(*i*).
equed	CHARACTER*1. Must be 'N', 'R', 'C', or 'B'. *equed* is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done: If *equed* = 'N', no equilibration was done (always true if *fact* = 'N'). If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag*(*r*). If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag*(*c*). If *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*r*)**A***diag*(*c*).
r, *c*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays: *r*(*n*), *c*(*n*). The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments. If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag*(*r*); if *equed* = 'N' or 'C', *r* is not accessed. If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive. If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag*(*c*); if *equed* = 'N' or 'R', *c* is not accessed. If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive.
ldx	INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.
rwork	REAL for single precision flavors DOUBLE PRECISION for double precision flavors.

Workspace array, DIMENSION at least max(1, 2*n); used in complex flavors only.

Output Parameters

x

REAL for sgesvx
DOUBLE PRECISION for dgesvx
COMPLEX for cgesvx
DOUBLE COMPLEX for zgesvx.
Array, DIMENSION (*ldx*,*).
If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is:
$diag(C)^{-1}*X$, if *trans* = 'N' and *equed* = 'C' or 'B';
$diag(R)^{-1}*X$, if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'. The second dimension of *x* must be at least max(1,*nrhs*).

a

Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'. If *equed* ≠ 'N', *A* is scaled on exit as follows:
equed = 'R': *A* = *diag*(*R*)*A*
equed = 'C': *A* = *A***diag*(*c*)
equed = 'B': *A* = *diag*(*R*)**A***diag*(*c*).

af

If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the factors *L* and *U* from the factorization *A* = *PLU* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.

b

Overwritten by *diag*(*r*)**B* if *trans* = 'N' and *equed* = 'R'or 'B'; overwritten by *diag*(*c*)**B* if *trans* = 'T' or 'C' and *equed* = 'C' or 'B';
not changed if *equed* = 'N'.

r, *c*

These arrays are output arguments if *fact* ≠ 'F'. See the description of *r*, *c* in *Input Arguments* section.

rcond

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.

ferr

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x* (*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) – *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.

berr

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.

	Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.
ipiv	If *fact* = 'N' or 'E', then *ipiv* is an output argument and on exit contains the pivot indices from the factorization *A* = *P***L***U* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').
equed	If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
work, *rwork*, *rpivot*	On exit, *work*(1) for real flavors, or *rwork*(1) for complex flavors (the Fortran interface) and *rpivot* (the C interface), contains the reciprocal pivot growth factor norm(*A*)/norm(*U*). The "max absolute element" norm is used. If *work*(1) for real flavors, or *rwork*(1) for complex flavors is much less than 1, then the stability of the *LU* factorization of the (equilibrated) matrix *A* could be poor. This also means that the solution *x*, condition estimator *rcond*, and forward error bound *ferr* could be unreliable. If factorization fails with 0 < *info* ≤ *n*, then *work*(1) for real flavors, or *rwork*(1) for complex flavors contains the reciprocal pivot growth factor for the leading *info* columns of *A*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*, then *U*(*i*, *i*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n*+1, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gesvx interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
af	Holds the matrix *AF* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
r	Holds the vector of length *n*. Default value for each element is *r*(*i*) = 1.0_WP.
c	Holds the vector of length *n*. Default value for each element is *c*(*i*) = 1.0_WP.

ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
fact	Must be 'N', 'E', or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.
equed	Must be 'N', 'B', 'C', or 'R'. The default value is 'N'.
rpvgrw	Real value that contains the reciprocal pivot growth factor norm(*A*)/ norm(*U*).

?gesvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a square matrix A and multiple right-hand sides

Syntax

Fortran 77:

```
call sgesvxx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params,
work, iwork, info )

call dgesvxx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params,
work, iwork, info )

call cgesvxx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params,
work, rwork, info )

call zgesvxx( fact, trans, n, nrhs, a, lda, af, ldaf, ipiv, equed, r, c, b, ldb, x,
ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params,
work, rwork, info )
```

C:

```
lapack_int LAPACKE_sgesvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, float* r, float* c, float* b, lapack_int ldb, float* x, lapack_int
ldx, float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds, float*
err_bnds_norm, float* err_bnds_comp, lapack_int nparams, const float* params );

lapack_int LAPACKE_dgesvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, double* r, double* c, double* b, lapack_int ldb, double* x,
lapack_int ldx, double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds,
double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double*
params );

lapack_int LAPACKE_cgesvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* r, float* c,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds, float* err_bnds_norm,
float* err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_zgesvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* r, double* c,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations *A*X = B*, where *A* is an *n*-by-*n* matrix, the columns of the matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error (`O(eps)`, where `eps` is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with `O(eps)` errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine `?gesvxx` performs the following steps:

1. If *fact* = `'E'`, scaling factors *r* and *c* are computed to equilibrate the system:

   trans = `'N'`: *diag(r)*A*diag(c)*inv(diag(c))*X = diag(r)*B*

   trans = `'T'`: $(diag(r)*A*diag(c))^{T}$*inv(diag(r))*X = diag(c)*B*

   trans = `'C'`: $(diag(r)*A*diag(c))^{H}$*inv(diag(r))*X = diag(c)*B*

   Whether or not the system will be equilibrated depends on the scaling of the matrix *A*, but if equilibration is used, *A* is overwritten by *diag(r)*A*diag(c)* and *B* by *diag(r)*B* (if *trans*=`'N'`) or *diag(c)*B* (if *trans* = `'T'` or `'C'`).
2. If *fact* = `'N'` or `'E'`, the *LU* decomposition is used to factor the matrix *A* (after equilibration if *fact* = `'E'`) as *A = P*L*U*, where *P* is a permutation matrix, *L* is a unit lower triangular matrix, and *U* is upper triangular.
3. If some $U_{i,i}$= 0, so that *U* is exactly singular, then the routine returns with *info = i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A* (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for *X* and compute error bounds.
4. The system of equations is solved for *X* using the factored form of *A*.
5. By default, unless `params(la_linrx_itref_i)` is set to zero, the routine applies iterative refinement to improve the computed solution matrix and calculate error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix *X* is premultiplied by *diag(c)* (if *trans* = `'N'`) or *diag(r)* (if *trans* = `'T'` or `'C'`) so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact
CHARACTER*1. Must be 'F', 'N', or 'E'.
Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored.
If *fact* = 'F', on entry, *af* and *ipiv* contain the factored form of *A*. If *equed* is not 'N', the matrix *A* has been equilibrated with scaling factors given by *r* and *c*. Parameters *a*, *af*, and *ipiv* are not modified.
If *fact* = 'N', the matrix *A* will be copied to *af* and factored.
If *fact* = 'E', the matrix *A* will be equilibrated, if necessary, copied to *af* and factored.

trans
CHARACTER*1. Must be 'N', 'T', or 'C'.
Specifies the form of the system of equations:
If *trans* = 'N', the system has the form *A*X* = *B* (No transpose).
If *trans* = 'T', the system has the form *A****T***X* = *B* (Transpose).
If *trans* = 'C', the system has the form *A****H***X* = *B* (Conjugate Transpose = Transpose for real flavors, Conjugate Transpose for complex flavors).

n
INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.

nrhs
INTEGER. The number of right hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.

a, *af*, *b*, *work*
REAL for sgesvxx
DOUBLE PRECISION for dgesvxx
COMPLEX for cgesvxx
DOUBLE COMPLEX for zgesvxx.
Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*).
The array *a* contains the matrix *A*. If *fact* = 'F' and *equed* is not 'N', then *A* must have been equilibrated by the scaling factors in *r* and/or *c*. The second dimension of *a* must be at least max(1,*n*).
The array *af* is an input argument if *fact* = 'F'. It contains the factored form of the matrix *A*, that is, the factors *L* and *U* from the factorization *A* = *P*L*U* as computed by ?getrf. If *equed* is not 'N', then *af* is the factored form of the equilibrated matrix *A*. The second dimension of *af* must be at least max(1,*n*).
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
work(*) is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.

lda
INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).

ldaf
INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).

ipiv
INTEGER.
Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains the pivot indices from the factorization *A* = *P*L*U* as computed by ?getrf; row *i* of the matrix was interchanged with row *ipiv*(*i*).

equed
CHARACTER*1. Must be 'N', 'R', 'C', or 'B'.

equed is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done:
If *equed* = 'N', no equilibration was done (always true if *fact* = 'N').
If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag*(*r*).
If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag*(*c*).
If *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*r*)**A***diag*(*c*).

r, *c*

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Arrays: *r*(*n*), *c*(*n*). The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments.
If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag*(*r*); if *equed* = 'N' or 'C', *r* is not accessed.
If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.
If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag*(*c*); if *equed* = 'N' or 'R', *c* is not accessed.
If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive.
Each element of *r* or *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb

INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

ldx

INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds

INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams

INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support DOUBLE PRECISION.

(Other values are reserved for futute use.)

params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for refinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3**n*); used in complex flavors only.

Output Parameters

x REAL for sgesvxx
DOUBLE PRECISION for dgesvxx
COMPLEX for cgesvxx
DOUBLE COMPLEX for zgesvxx.
Array, DIMENSION (*ldx*,*).
If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is:
inv(*diag*(*c*))**X*, if *trans* = 'N' and *equed* = 'C' or 'B'; or
inv(*diag*(*r*))**X*, if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'. The second dimension of *x* must be at least max(1,*nrhs*).

a Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.
If *equed* ≠ 'N', *A* is scaled on exit as follows:
equed = 'R': *A* = *diag*(*r*)**A*
equed = 'C': *A* = *A***diag*(*c*)
equed = 'B': *A* = *diag*(*r*)**A***diag*(*c*).

af If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the factors *L* and *U* from the factorization *A* = *PLU* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.

b Overwritten by *diag*(*r*)**B* if *trans* = 'N' and *equed* = 'R' or 'B'; overwritten by *trans* = 'T' or 'C' and *equed* = 'C' or 'B'; not changed if *equed* = 'N'.

r, *c* These arrays are output arguments if *fact* ≠ 'F'. Each element of these arrays is a power of the radix. See the description of *r*, *c* in *Input Arguments* section.

rcond REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.

Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Contains the reciprocal pivot growth factor *norm*(*A*)/*norm*(*U*). The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the (equlibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \le n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*. In ?gesvx, this quantity is returned in *work(1)*.

berr

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm*(*i*,:) corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm*(:,*err*) contains the follwoing three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.

err=2 "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors

and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*.
Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{|Xtrue_{ji} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1
"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.

err=2
"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3
Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*.

Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

ipiv — If *fact* = 'N' or 'E', then *ipiv* is an output argument and on exit contains the pivot indices from the factorization *A* = *P***L***U* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').

equed — If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).

params — If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm(;,1)* and *err_bnds_comp(;,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?gbsv

Computes the solution to the system of linear equations with a band matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sgbsv( n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
call dgbsv( n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
call cgbsv( n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
call zgbsv( n, kl, ku, nrhs, ab, ldab, ipiv, b, ldb, info )
```

Fortran 95:

```
call gbsv( ab, b [,kl] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>gbsv( int matrix_order, lapack_int n, lapack_int kl, lapack_int
ku, lapack_int nrhs, <datatype>* ab, lapack_int ldab, lapack_int* ipiv, <datatype>* b,
lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for *X* the real or complex system of linear equations *A*X = B*, where *A* is an *n*-by-*n* band matrix with *kl* subdiagonals and *ku* superdiagonals, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

The *LU* decomposition with partial pivoting and row interchanges is used to factor *A* as *A* = *L*U*, where *L* is a product of permutation and unit lower triangular matrices with *kl* subdiagonals, and *U* is upper triangular with *kl*+*ku* superdiagonals. The factored form of *A* is then used to solve the system of equations *A*X = B*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of *A*. The number of rows in *B*; *n* ≥ 0.
kl	INTEGER. The number of subdiagonals within the band of *A*; *kl* ≥ 0.
ku	INTEGER. The number of superdiagonals within the band of *A*; *ku* ≥ 0.
nrhs	INTEGER. The number of right-hand sides. The number of columns in *B*; *nrhs* ≥ 0.
ab, *b*	REAL for sgbsv DOUBLE PRECISION for dgbsv COMPLEX for cgbsv DOUBLE COMPLEX for zgbsv. Arrays: *ab*(*ldab*,*), *b*(*ldb*,*). The array *ab* contains the matrix *A* in band storage (see Matrix Storage Schemes). The second dimension of *ab* must be at least max(1, *n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
ldab	INTEGER. The leading dimension of the array *ab*. (*ldab* ≥ 2*kl* + *ku* +1)
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

ab	Overwritten by *L* and *U*. The diagonal and *kl* + *ku* superdiagonals of *U* are stored in the first 1 + *kl* + *ku* rows of *ab*. The multipliers used to form *L* are stored in the next *kl* rows.
b	Overwritten by the solution matrix *X*.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The pivot indices: row *i* was interchanged with row *ipiv*(*i*).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

	If *info* = *i*, *U*(*i*, *i*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbsv interface are as follows:

ab	Holds the array *A* of size (2**kl*+*ku*+1,*n*).
b	Holds the matrix *B* of size (*n*,*nrhs*).
ipiv	Holds the vector of length *n*.
kl	If omitted, assumed *kl* = *ku*.
ku	Restored as *ku* = *lda*-2**kl*-1.

?gbsvx

Computes the solution to the real or complex system of linear equations with a band matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sgbsvx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c, b,
ldb, x, ldx, rcond, ferr, berr, work, iwork, info )
```

```
call dgbsvx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c, b,
ldb, x, ldx, rcond, ferr, berr, work, iwork, info )
```

```
call cgbsvx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c, b,
ldb, x, ldx, rcond, ferr, berr, work, rwork, info )
```

```
call zgbsvx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c, b,
ldb, x, ldx, rcond, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call gbsvx( ab, b, x [,kl] [,afb] [,ipiv] [,fact] [,trans] [,equed] [,r] [,c] [,ferr]
[,berr] [,rcond] [,rpvgrw] [,info] )
```

C:

```
lapack_int LAPACKE_sgbsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, float* ab, lapack_int ldab, float* afb,
lapack_int ldafb, lapack_int* ipiv, char* equed, float* r, float* c, float* b,
lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr,
float* rpivot );
```

```
lapack_int LAPACKE_dgbsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, double* ab, lapack_int ldab, double*
afb, lapack_int ldafb, lapack_int* ipiv, char* equed, double* r, double* c, double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr,
double* rpivot );
```

```
lapack_int LAPACKE_cgbsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_float* ab, lapack_int
ldab, lapack_complex_float* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
float* r, float* c, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* ferr, float* berr, float* rpivot );

lapack_int LAPACKE_zgbsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_double* ab, lapack_int
ldab, lapack_complex_double* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
double* r, double* c, lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* rcond, double* ferr, double* berr, double* rpivot );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations $A*X = B$, $A^T*X = B$, or $A^H*X = B$, where A is a band matrix of order n with kl subdiagonals and ku superdiagonals, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?gbsvx` performs the following steps:

1. If *fact* = 'E', real scaling factors r and c are computed to equilibrate the system:

   trans = 'N': $diag(r)*A*diag(c)\ *inv(diag(c))*X = diag(r)*B$

   trans = 'T': $(diag(r)*A*diag(c))^T\ *inv(diag(r))*X = diag(c)*B$

   trans = 'C': $(diag(r)*A*diag(c))^H\ *inv(diag(r))*X = diag(c)*B$

   Whether the system will be equilibrated depends on the scaling of the matrix A, but if equilibration is used, A is overwritten by $diag(r)*A*diag(c)$ and B by $diag(r)*B$ (if *trans*='N') or $diag(c)*B$ (if *trans* = 'T' or 'C').
2. If *fact* = 'N' or 'E', the *LU* decomposition is used to factor the matrix A (after equilibration if *fact* = 'E') as $A = L*U$, where L is a product of permutation and unit lower triangular matrices with kl subdiagonals, and U is upper triangular with $kl+ku$ superdiagonals.
3. If some $U_{i,i} = 0$, so that U is exactly singular, then the routine returns with *info* = *i*. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, *info* = *n* + 1 is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $diag(c)$ (if *trans* = 'N') or $diag(r)$ (if *trans* = 'T' or 'C') so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface, except for *rpivot*. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored. If *fact* = 'F': on entry, *afb* and *ipiv* contain the factored form of *A*. If *equed* is not 'N', the matrix *A* is equilibrated with scaling factors given by *r* and *c*. *ab*, *afb*, and *ipiv* are not modified. If *fact* = 'N', the matrix *A* will be copied to *afb* and factored. If *fact* = 'E', the matrix *A* will be equilibrated if necessary, then copied to *afb* and factored.
trans	CHARACTER*1. Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose). If *trans* = 'T', the system has the form $A^T*X = B$ (Transpose). If *trans* = 'C', the system has the form $A^H*X = B$ (Transpose for real flavors, conjugate transpose for complex flavors).
n	INTEGER. The number of linear equations, the order of the matrix *A*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
nrhs	INTEGER. The number of right hand sides, the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
ab, *afb*, *b*, *work*	REAL for sgesvx DOUBLE PRECISION for dgesvx COMPLEX for cgesvx DOUBLE COMPLEX for zgesvx. Arrays: *ab*(*ldab*,*), *afb*(*ldafb*,*), *b*(*ldb*,*), *work*(*). The array *ab* contains the matrix *A* in band storage (see Matrix Storage Schemes). The second dimension of *ab* must be at least max(1, *n*). If *fact* = 'F' and *equed* is not 'N', then *A* must have been equilibrated by the scaling factors in *r* and/or *c*. The array *afb* is an input argument if *fact* = 'F'. The second dimension of *afb* must be at least max(1,*n*). It contains the factored form of the matrix *A*, that is, the factors *L* and *U* from the factorization *A* = *L*U* as computed by ?gbtrf. *U* is stored as an upper triangular band matrix with *kl* + *ku* superdiagonals in the first 1 + *kl* + *ku* rows of *afb*. The multipliers used during the factorization are stored in the next *kl* rows. If *equed* is not 'N', then *afb* is the factored form of the equilibrated matrix *A*. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1, *nrhs*). *work(*)* is a workspace array. The dimension of *work* must be at least max(1,3**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
ldab	INTEGER. The leading dimension of *ab*; $ldab \geq kl+ku+1$.
ldafb	INTEGER. The leading dimension of *afb*; $ldafb \geq 2*kl+ku+1$.
ldb	INTEGER. The leading dimension of *b*; $ldb \geq \max(1, n)$.
ipiv	INTEGER.

Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains the pivot indices from the factorization *A* = *L***U* as computed by ?gbtrf; row *i* of the matrix was interchanged with row *ipiv*(*i*).

equed

CHARACTER*1. Must be 'N', 'R', 'C', or 'B'.
equed is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done:
If *equed* = 'N', no equilibration was done (always true if *fact* = 'N').
If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag*(*r*).
If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag*(*c*).
if *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*r*)**A***diag*(*c*).

r, *c*

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Arrays: *r*(*n*), *c*(*n*).
The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments.
If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag*(*r*); if *equed* = 'N' or 'C', *r* is not accessed.
If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.
If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag*(*c*); if *equed* = 'N' or 'R', *c* is not accessed.
If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive.

ldx

INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

iwork

INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, *n*); used in complex flavors only.

Output Parameters

x

REAL for sgbsvx
DOUBLE PRECISION for dgbsvx
COMPLEX for cgbsvx
DOUBLE COMPLEX for zgbsvx.
Array, DIMENSION (*ldx*,*).
If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is: inv(*diag*(*c*))**X*, if *trans* = 'N' and *equed* = 'C' or 'B'; inv(*diag*(*r*))**X*, if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'.
The second dimension of *x* must be at least max(1,*nrhs*).

ab	Array *ab* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'. If *equed* ≠ 'N', *A* is scaled on exit as follows: *equed* = 'R': *A* = *diag*(*r*)***A* *equed* = 'C': *A* = *A***diag*(*c*) *equed* = 'B': *A* = *diag*(*r*)***A***diag*(*c*).
afb	If *fact* = 'N' or 'E', then *afb* is an output argument and on exit returns details of the *LU* factorization of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *ab* for the form of the equilibrated matrix.
b	Overwritten by *diag*(*r*)***b* if *trans* = 'N' and *equed* = 'R' or 'B'; overwritten by *diag*(*c*)***b* if *trans* = 'T' or 'C' and *equed* = 'C' or 'B'; not changed if *equed* = 'N'.
r, *c*	These arrays are output arguments if *fact* ≠ 'F'. See the description of *r*, *c* in *Input Arguments* section.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision (in particular, if *rcond* =0), the matrix is singular to working precision. This condition is indicated by a return code of *info*>0.
ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) - *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.
berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.
ipiv	If *fact* = 'N' or 'E', then *ipiv* is an output argument and on exit contains the pivot indices from the factorization *A* = *L***U* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').
equed	If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
work, *rwork*, *rpivot*	On exit, *work*(1) for real flavors, or *rwork*(1) for complex flavors (the Fortran interface) and *rpivot* (the C interface), contains the reciprocal pivot growth factor norm(*A*)/norm(*U*). The "max absolute element" norm is used. If *work*(1) for real flavors, or *rwork*(1) for

complex flavors is much less than 1, then the stability of the *LU* factorization of the (equilibrated) matrix *A* could be poor. This also means that the solution *x*, condition estimator *rcond*, and forward error bound *ferr* could be unreliable. If factorization fails with 0 < *info* ≤ *n*, then *work*(1) for real flavors, or *rwork*(1) for complex flavors contains the reciprocal pivot growth factor for the leading *info* columns of *A*.

info — INTEGER. If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, and *i* ≤ *n*, then *U*(*i*, *i*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n*+1, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbsvx interface are as follows:

ab — Holds the array *A* of size (*kl*+*ku*+1, *n*).

b — Holds the matrix *B* of size (*n*, *nrhs*).

x — Holds the matrix *X* of size (*n*, *nrhs*).

afb — Holds the array *AF* of size (2**kl*+*ku*+1, *n*).

ipiv — Holds the vector of length *n*.

r — Holds the vector of length *n*. Default value for each element is *r*(*i*) = 1.0_WP.

c — Holds the vector of length *n*. Default value for each element is *c*(*i*) = 1.0_WP.

ferr — Holds the vector of length (*nrhs*).

berr — Holds the vector of length (*nrhs*).

trans — Must be 'N', 'C', or 'T'. The default value is 'N'.

equed — Must be 'N', 'B', 'C', or 'R'. The default value is 'N'.

fact — Must be 'N', 'E', or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.

rpvgrw — Real value that contains the reciprocal pivot growth factor norm(*A*)/ norm(*U*).

kl — If omitted, assumed *kl* = *ku*.

ku — Restored as *ku* = *lda*-*kl*-1.

?gbsvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a banded matrix A and multiple right-hand sides

Syntax

Fortran 77:

```
call sgbsvxx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c,
b, ldb, x, ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp,
nparams, params, work, iwork, info )

call dgbsvxx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c,
b, ldb, x, ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp,
nparams, params, work, iwork, info )

call cgbsvxx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c,
b, ldb, x, ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp,
nparams, params, work, rwork, info )

call zgbsvxx( fact, trans, n, kl, ku, nrhs, ab, ldab, afb, ldafb, ipiv, equed, r, c,
b, ldb, x, ldx, rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp,
nparams, params, work, rwork, info )
```

C:

```
lapack_int LAPACKE_sgbsvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, float* ab, lapack_int ldab, float* afb,
lapack_int ldafb, lapack_int* ipiv, char* equed, float* r, float* c, float* b,
lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* rpvgrw, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
const float* params );

lapack_int LAPACKE_dgbsvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, double* ab, lapack_int ldab, double*
afb, lapack_int ldafb, lapack_int* ipiv, char* equed, double* r, double* c, double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* rpvgrw, double*
berr, lapack_int n_err_bnds, double* err_bnds_norm, double* err_bnds_comp, lapack_int
nparams, const double* params );

lapack_int LAPACKE_cgbsvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_float* ab, lapack_int
ldab, lapack_complex_float* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
float* r, float* c, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds,
float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams, const float* params );

lapack_int LAPACKE_zgbsvxx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_double* ab, lapack_int
ldab, lapack_complex_double* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
double* r, double* c, lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds,
double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double*
params );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations *A*X = B*, where *A* is an *n*-by-*n* banded matrix, the columns of the matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error (O(eps), where eps is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with O(eps) errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine ?gbsvxx performs the following steps:

1. If *fact* = 'E', scaling factors *r* and *c* are computed to equilibrate the system:

   trans = 'N': *diag(r)*A*diag(c)*inv(diag(c))*X = diag(r)*B*

   trans = 'T': $(diag(r)*A*diag(c))^{T}$*inv(*diag(r)*X = diag(c)*B*

   trans = 'C': $(diag(r)*A*diag(c))^{H}$*inv(*diag(r)*X = diag(c)*B*

   Whether or not the system will be equilibrated depends on the scaling of the matrix *A*, but if equilibration is used, *A* is overwritten by *diag(r)*A*diag(c)* and *B* by *diag(r)*B* (if *trans*='N') or *diag(c)*B* (if *trans* = 'T' or 'C').
2. If *fact* = 'N' or 'E', the *LU* decomposition is used to factor the matrix *A* (after equilibration if *fact* = 'E') as *A = P*L*U*, where *P* is a permutation matrix, *L* is a unit lower triangular matrix, and *U* is upper triangular.
3. If some $U_{i,i}$= 0, so that *U* is exactly singular, then the routine returns with *info = i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A* (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for *X* and compute error bounds.
4. The system of equations is solved for *X* using the factored form of *A*.
5. By default, unless params(la_linrx_itref_i) is set to zero, the routine applies iterative refinement to improve the computed solution matrix and calculate error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix *X* is premultiplied by *diag(c)* (if *trans* = 'N') or *diag(r)* (if *trans* = 'T' or 'C') so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact — CHARACTER*1. Must be 'F', 'N', or 'E'.
Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored.

	If *fact* = 'F', on entry, *afb* and *ipiv* contain the factored form of *A*. If *equed* is not 'N', the matrix *A* has been equilibrated with scaling factors given by *r* and *c*. Parameters *ab*, *afb*, and *ipiv* are not modified. If *fact* = 'N', the matrix *A* will be copied to *afb* and factored. If *fact* = 'E', the matrix *A* will be equilibrated, if necessary, copied to *afb* and factored.
trans	CHARACTER*1. Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose). If *trans* = 'T', the system has the form *A****T***X* = *B* (Transpose). If *trans* = 'C', the system has the form *A****H***X* = *B* (Conjugate Transpose = Transpose for real flavors, Conjugate Transpose for complex flavors).
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
ab, *afb*, *b*, *work*	REAL for sgbsvxx DOUBLE PRECISION for dgbsvxx COMPLEX for cgbsvxx DOUBLE COMPLEX for zgbsvxx. Arrays: *ab*(*ldab*,*), *afb*(*ldafb*,*), *b*(*ldb*,*), *work*(*). The array *ab* contains the matrix *A* in band storage, in rows 1 to *kl*+*ku*+1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = *A*(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*). If *fact* = 'F' and *equed* is not 'N', then *AB* must have been equilibrated by the scaling factors in *r* and/or *c*. The second dimension of *a* must be at least max(1,*n*). The array *afb* is an input argument if *fact* = 'F'. It contains the factored form of the banded matrix *A*, that is, the factors *L* and *U* from the factorization *A* = *P*L*U* as computed by ?gbtrf. *U* is stored as an upper triangular banded matrix with *kl* + *ku* superdiagonals in rows 1 to *kl* + *ku* + 1. The multipliers used during the factorization are stored in rows *kl* + *ku* + 2 to 2**kl* + *ku* + 1. If *equed* is not 'N', then *afb* is the factored form of the equilibrated matrix *A*. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
ldab	INTEGER. The leading dimension of the array *ab*; *ldab* ≥ *kl*+*ku*+1..
ldafb	INTEGER. The leading dimension of the array *afb*; *ldafb* ≥ 2**kl*+*ku*+1..
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains the pivot indices from the factorization *A* = *P*L*U* as computed by ?gbtrf; row *i* of the matrix was interchanged with row *ipiv*(*i*).
equed	CHARACTER*1. Must be 'N', 'R', 'C', or 'B'.

equed is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done:
If *equed* = 'N', no equilibration was done (always true if *fact* = 'N').
If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag*(*r*).
If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag*(*c*).
If *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*r*)**A***diag*(*c*).

r, *c*

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Arrays: *r*(*n*), *c*(*n*). The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments.
If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag*(*r*); if *equed* = 'N' or 'C', *r* is not accessed.
If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.
If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag*(*c*); if *equed* = 'N' or 'R', *c* is not accessed.
If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive.
Each element of *r* or *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb

INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1,*n*).

ldx

INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1,*n*).

n_err_bnds

INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams

INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the extra-precise refinement algorithm.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for fefinement.

	Default	10
	Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3**n*); used in complex flavors only.

Output Parameters

x REAL for sgbsvxx
DOUBLE PRECISION for dgbsvxx
COMPLEX for cgbsvxx
DOUBLE COMPLEX for zgbsvxx.
Array, DIMENSION (*ldx*,*).
If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is:
inv(*diag*(*c*))**X*, if *trans* = 'N' and *equed* = 'C' or 'B'; or
inv(*diag*(*r*))**X*, if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'. The second dimension of *x* must be at least max(1,*nrhs*).

ab Array *ab* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.
If *equed* ≠ 'N', *A* is scaled on exit as follows:
equed = 'R': *A* = *diag*(*r*)**A*
equed = 'C': *A* = *A***diag*(*c*)
equed = 'B': *A* = *diag*(*r*)**A***diag*(*c*).

afb If *fact* = 'N' or 'E', then *afb* is an output argument and on exit returns the factors *L* and *U* from the factorization *A* = *PLU* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').

b Overwritten by *diag*(*r*)**B* if *trans* = 'N' and *equed* = 'R' or 'B'; overwritten by *trans* = 'T' or 'C' and *equed* = 'C' or 'B'; not changed if *equed* = 'N'.

r, *c* These arrays are output arguments if *fact* ≠ 'F'. Each element of these arrays is a power of the radix. See the description of *r*, *c* in *Input Arguments* section.

rcond REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.

Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Contains the reciprocal pivot growth factor *norm*(*A*)/*norm*(*U*). The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the (equlibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \le n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*. In ?gbsvx, this quantity is returned in *work(1)*.

berr

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.

err=2 "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors

and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*.
Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.

err=2 "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*.

Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

ipiv — If *fact* = 'N' or 'E', then *ipiv* is an output argument and on exit contains the pivot indices from the factorization *A* = *P***L***U* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').

equed — If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).

params — If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm(;,1)* and *err_bnds_comp(;,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?gtsv

Computes the solution to the system of linear equations with a tridiagonal matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sgtsv( n, nrhs, dl, d, du, b, ldb, info )
call dgtsv( n, nrhs, dl, d, du, b, ldb, info )
call cgtsv( n, nrhs, dl, d, du, b, ldb, info )
call zgtsv( n, nrhs, dl, d, du, b, ldb, info )
```

Fortran 95:

```
call gtsv( dl, d, du, b [,info] )
```

C:

```
lapack_int LAPACKE_<?>gtsv( int matrix_order, lapack_int n, lapack_int nrhs,
<datatype>* dl, <datatype>* d, <datatype>* du, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the system of linear equations $A*X = B$, where A is an n-by-n tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions. The routine uses Gaussian elimination with partial pivoting.

Note that the equation $A^T*X = B$ may be solved by interchanging the order of the arguments du and dl.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of A, the number of rows in B; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in B; $nrhs \geq 0$.
dl, *d*, *du*, *b*	REAL for sgtsv DOUBLE PRECISION for dgtsv COMPLEX for cgtsv DOUBLE COMPLEX for zgtsv. Arrays: *dl*(*n* - 1), *d*(*n*), *du*(*n* - 1), *b*(*ldb*,*). The array *dl* contains the (*n* - 1) subdiagonal elements of A. The array *d* contains the diagonal elements of A. The array *du* contains the (*n* - 1) superdiagonal elements of A. The array *b* contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

dl	Overwritten by the (*n*-2) elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A. These elements are stored in *dl*(1), ..., *dl*(*n*-2).
d	Overwritten by the n diagonal elements of U.
du	Overwritten by the (*n*-1) elements of the first superdiagonal of U.
b	Overwritten by the solution matrix X.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, $U(i, i)$ is exactly zero, and the solution has not been computed. The factorization has not been completed unless *i* = *n*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gtsv interface are as follows:

dl	Holds the vector of length (*n*–1).
d	Holds the vector of length *n*.
dl	Holds the vector of length (*n*–1).
b	Holds the matrix *B* of size (*n*,*nrhs*).

?gtsvx

Computes the solution to the real or complex system of linear equations with a tridiagonal matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sgtsvx( fact, trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x,
ldx, rcond, ferr, berr, work, iwork, info )
```

```
call dgtsvx( fact, trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x,
ldx, rcond, ferr, berr, work, iwork, info )
```

```
call cgtsvx( fact, trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x,
ldx, rcond, ferr, berr, work, rwork, info )
```

```
call zgtsvx( fact, trans, n, nrhs, dl, d, du, dlf, df, duf, du2, ipiv, b, ldb, x,
ldx, rcond, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call gtsvx( dl, d, du, b, x [,dlf] [,df] [,duf] [,du2] [,ipiv] [,fact] [,trans] [,ferr]
[,berr] [,rcond] [,info] )
```

C:

```
lapack_int LAPACKE_sgtsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, const float* dl, const float* d, const float* du, float* dlf, float*
df, float* duf, float* du2, lapack_int* ipiv, const float* b, lapack_int ldb, float*
x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dgtsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, const double* dl, const double* d, const double* du, double* dlf,
double* df, double* duf, double* du2, lapack_int* ipiv, const double* b, lapack_int
ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cgtsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, const lapack_complex_float* dl, const lapack_complex_float* d, const
lapack_complex_float* du, lapack_complex_float* dlf, lapack_complex_float* df,
lapack_complex_float* duf, lapack_complex_float* du2, lapack_int* ipiv, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zgtsvx( int matrix_order, char fact, char trans, lapack_int n,
lapack_int nrhs, const lapack_complex_double* dl, const lapack_complex_double* d, const
lapack_complex_double* du, lapack_complex_double* dlf, lapack_complex_double* df,
lapack_complex_double* duf, lapack_complex_double* du2, lapack_int* ipiv, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations *A*X = B*, $A^T*X = B$, or $A^H*X = B$, where *A* is a tridiagonal matrix of order *n*, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine ?gtsvx performs the following steps:

1. If *fact = 'N'*, the *LU* decomposition is used to factor the matrix *A* as *A = L*U*, where *L* is a product of permutation and unit lower bidiagonal matrices and *U* is an upper triangular matrix with nonzeroes in only the main diagonal and first two superdiagonals.
2. If some $U_{i,i} = 0$, so that *U* is exactly singular, then the routine returns with *info = i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, *info = n + 1* is returned as a warning, but the routine still goes on to solve for *X* and compute error bounds as described below.
3. The system of equations is solved for *X* using the factored form of *A*.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F' or 'N'. Specifies whether or not the factored form of the matrix *A* has been supplied on entry. If *fact = 'F'*: on entry, *dlf*, *df*, *duf*, *du2*, and *ipiv* contain the factored form of *A*; arrays *dl*, *d*, *du*, *dlf*, *df*, *duf*, *du2*, and *ipiv* will not be modified. If *fact = 'N'*, the matrix *A* will be copied to *dlf*, *df*, and *duf* and factored.
trans	CHARACTER*1. Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If *trans = 'N'*, the system has the form *A*X = B* (No transpose). If *trans = 'T'*, the system has the form $A^T*X = B$ (Transpose). If *trans = 'C'*, the system has the form $A^H*X = B$ (Conjugate transpose).
n	INTEGER. The number of linear equations, the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right hand sides, the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
dl,*d*,*du*,*dlf*,*df*, *duf*,*du2*,*b*, *x*,*work*	REAL for sgtsvx DOUBLE PRECISION for dgtsvx COMPLEX for cgtsvx DOUBLE COMPLEX for zgtsvx.

Arrays:
dl, DIMENSION (*n* -1), contains the subdiagonal elements of *A*.
d, DIMENSION (*n*), contains the diagonal elements of *A*.
du, DIMENSION (*n* -1), contains the superdiagonal elements of *A*.
dlf, DIMENSION (*n* -1). If *fact* = 'F', then *dlf* is an input argument and on entry contains the (*n* -1) multipliers that define the matrix *L* from the *LU* factorization of *A* as computed by ?gttrf.
df, DIMENSION (*n*). If *fact* = 'F', then *df* is an input argument and on entry contains the *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*.
duf, DIMENSION (*n* -1). If *fact* = 'F', then *duf* is an input argument and on entry contains the (*n* -1) elements of the first superdiagonal of *U*.
du2, DIMENSION (*n* -2). If *fact* = 'F', then *du2* is an input argument and on entry contains the (*n*-2) elements of the second superdiagonal of *U*.
b(*ldb**) contains the right-hand side matrix *B*. The second dimension of *b* must be at least max(1, *nrhs*).
x(*ldx**) contains the solution matrix *X*. The second dimension of *x* must be at least max(1, *nrhs*).
work(*) is a workspace array.
DIMENSION of *work* must be at least max(1, 3**n*) for real flavors and max(1, 2**n*) for complex flavors.

ldb — INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

ldx — INTEGER. The leading dimension of *x*; *ldx* ≥ max(1, *n*).

ipiv — INTEGER.
Array, DIMENSION at least max(1, *n*). If *fact* = 'F', then *ipiv* is an input argument and on entry contains the pivot indices, as returned by ?gttrf.

iwork — INTEGER. Workspace array, DIMENSION (*n*). Used for real flavors only.

rwork — REAL for cgtsvx
DOUBLE PRECISION for zgtsvx.
Workspace array, DIMENSION (*n*). Used for complex flavors only.

Output Parameters

x — REAL for sgtsvx
DOUBLE PRECISION for dgtsvx
COMPLEX for cgtsvx
DOUBLE COMPLEX for zgtsvx.
Array, DIMENSION (*ldx*,*).
If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X*. The second dimension of *x* must be at least max(1, *nrhs*).

dlf — If *fact* = 'N', then *dlf* is an output argument and on exit contains the (*n*-1) multipliers that define the matrix *L* from the *LU* factorization of A.

df — If *fact* = 'N', then *df* is an output argument and on exit contains the *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*.

duf — If *fact* = 'N', then *duf* is an output argument and on exit contains the (*n*-1) elements of the first superdiagonal of *U*.

du2	If *fact* = 'N', then *du2* is an output argument and on exit contains the (*n*-2) elements of the second superdiagonal of *U*.
ipiv	The array *ipiv* is an output argument if *fact* = 'N' and, on exit, contains the pivot indices from the factorization *A* = *L*U* ; row *i* of the matrix was interchanged with row *ipiv*(*i*). The value of *ipiv*(*i*) will always be *i* or *i*+1; *ipiv*(*i*)=*i* indicates a row interchange was not required.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A*. If *rcond* is less than the machine precision (in particular, if *rcond* =0), the matrix is singular to working precision. This condition is indicated by a return code of *info*>0.
ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) - *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.
berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*, then *U*(*i*, *i*) is exactly zero. The factorization has not been completed unless *i* = *n*, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n* + 1, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gtsvx interface are as follows:

dl	Holds the vector of length (*n*-1).
d	Holds the vector of length *n*.
du	Holds the vector of length (*n*-1).
b	Holds the matrix *B* of size (*n*, *nrhs*).

x	Holds the matrix *X* of size (*n*, *nrhs*).
dlf	Holds the vector of length (*n*-1).
df	Holds the vector of length *n*.
duf	Holds the vector of length (*n*-1).
du2	Holds the vector of length (*n*-2).
ipiv	Holds the vector of length *n*.
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
fact	Must be 'N' or 'F'. The default value is 'N'. If *fact* = 'F', then the arguments *dlf*, *df*, *duf*, *du2*, and *ipiv* must be present; otherwise, an error is returned.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

?dtsvb

Computes the solution to the system of linear equations with a diagonally dominant tridiagonal matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sdtsvb( n, nrhs, dl, d, du, b, ldb, info )
call ddtsvb( n, nrhs, dl, d, du, b, ldb, info )
call cdtsvb( n, nrhs, dl, d, du, b, ldb, info )
call zdtsvb( n, nrhs, dl, d, du, b, ldb, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?dtsvb routine solves a system of linear equations *A*X* = *B* for *X*, where *A* is an *n*-by-*n* diagonally dominant tridiagonal matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions. The routine uses the BABE (Burning At Both Ends) algorithm.

Note that the equation $A^T*X = B$ may be solved by interchanging the order of the arguments *du* and *dl*.

Input Parameters

n	INTEGER. The order of *A*, the number of rows in *B*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.
dl, *d*, *du*, *b*	REAL for sdtsvb DOUBLE PRECISION for ddtsvb COMPLEX for cdtsvb DOUBLE COMPLEX for zdtsvb. Arrays: *dl*(*n* - 1), *d*(*n*), *du*(*n* - 1), *b*(*ldb*,*). The array *dl* contains the (*n* - 1) subdiagonal elements of *A*. The array *d* contains the diagonal elements of *A*.

	The array *du* contains the (*n* – 1) superdiagonal elements of *A*. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

dl	Overwritten by the (*n*-1) elements of the subdiagonal of the lower triangular matrices L_1, L_2 from the factorization of *A*.
d	Overwritten by the *n* diagonal element reciprocals of *U*.
b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, u_{ii} is exactly zero, and the solution has not been computed. The factorization has not been completed unless *i* = *n*.

Application Notes

A diagonally dominant tridiagonal system is defined such that $|d_i| > |dl_{i-1}| + |du_i|$ for any *i*:

$1 < i < n$, and $|d_1| > |du_1|$, $|d_n| > |dl_{n-1}|$

The underlying BABE algorithm is designed for diagonally dominant systems. Such systems have no numerical stability issue unlike the canonical systems that use elimination with partial pivoting (see ?gtsv). The diagonally dominant systems are much faster than the canonical systems.

NOTE

- The current implementation of BABE has a potential accuracy issue on very small or large data close to the underflow or overflow threshold respectively. Scale the matrix before applying the solver in the case of such input data.
- Applying the ?dtsvb factorization to non-diagonally dominant systems may lead to an accuracy loss, or false singularity detected due to no pivoting.

?posv

Computes the solution to the system of linear equations with a symmetric or Hermitian positive-definite matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sposv( uplo, n, nrhs, a, lda, b, ldb, info )
call dposv( uplo, n, nrhs, a, lda, b, ldb, info )
call cposv( uplo, n, nrhs, a, lda, b, ldb, info )
call zposv( uplo, n, nrhs, a, lda, b, ldb, info )
call dsposv( uplo, n, nrhs, a, lda, b, ldb, x, ldx, work, swork, iter, info )
call zcposv( uplo, n, nrhs, a, lda, b, ldb, x, ldx, work, swork, rwork, iter, info )
```

Fortran 95:

```
call posv( a, b [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>posv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
<datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb );

lapack_int LAPACKE_dsposv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
double* a, lapack_int lda, double* b, lapack_int ldb, double* x, lapack_int ldx,
lapack_int* iter );

lapack_int LAPACKE_zcposv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, lapack_int* iter );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for X the real or complex system of linear equations $A*X = B$, where A is an n-by-n symmetric/Hermitian positive-definite matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The Cholesky decomposition is used to factor A as

$A = U^T*U$ (real flavors) and $A = U^H*U$ (complex flavors), if $uplo$ = 'U'

or $A = L*L^T$ (real flavors) and $A = L*L^H$ (complex flavors), if $uplo$ = 'L',

where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $A*X = B$.

The `dsposv` and `zcposv` are mixed precision iterative refinement subroutines for exploiting fast single precision hardware. They first attempt to factorize the matrix in single precision (`dsposv`) or single complex precision (`zcposv`) and use this factorization within an iterative refinement procedure to produce a solution with double precision (`dsposv`) / double complex precision (`zcposv`) normwise backward error quality (see below). If the approach fails, the method switches to a double precision or double complex precision factorization respectively and computes the solution.

The iterative refinement is not going to be a winning strategy if the ratio single precision/COMPLEX performance over double precision/DOUBLE COMPLEX performance is too small. A reasonable strategy should take the number of right-hand sides and the size of the matrix into account. This might be done with a call to `ilaenv` in the future. At present, iterative refinement is implemented.

The iterative refinement process is stopped if
`iter > itermax`
or for all the right-hand sides:
`rnmr < sqrt(n)*xnrm*anrm*eps*bwdmax`,
where

- `iter` is the number of the current iteration in the iterative refinement process
- `rnmr` is the infinity-norm of the residual
- `xnrm` is the infinity-norm of the solution
- `anrm` is the infinity-operator-norm of the matrix A
- `eps` is the machine epsilon returned by `dlamch` ('Epsilon').

The values itermax and bwdmax are fixed to 30 and 1.0d+00 respectively.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.
a, *b*	REAL for sposv DOUBLE PRECISION for dposv and dsposv. COMPLEX for cposv DOUBLE COMPLEX for zposv and zcposv. Arrays: *a*(*lda*,*), *b*(*ldb*,*). The array *a* contains the upper or the lower triangular part of the matrix *A* (see *uplo*). The second dimension of *a* must be at least max(1, *n*). Note that in the case of zcposv the imaginary parts of the diagonal elements need not be set and are assumed to be zero. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of *a*; *lda* $\geq$ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* $\geq$ max(1, *n*).
ldx	INTEGER. The leading dimension of the array *x*; *ldx* $\geq$ max(1, *n*).
work	DOUBLE PRECISION for dsposv DOUBLE COMPLEX for zcposv. Workspace array, DIMENSION (*n***nrhs*). This array is used to hold the residual vectors.
swork	REAL for dsgesv COMPLEX for zcgesv. Workspace array, DIMENSION (*n**(*n*+*nrhs*)). This array is used to use the single precision matrix and the right-hand sides or solutions in single precision.
rwork	DOUBLE PRECISION. Workspace array, DIMENSION (*n*).

Output Parameters

a	If *info* = 0, the upper or lower triangular part of *a* is overwritten by the Cholesky factor *U* or *L*, as specified by *uplo*. If iterative refinement has been successfully used (*info*= 0 and *iter*$\geq$ 0), then *A* is unchanged. If double precision factorization has been used (*info*= 0 and *iter* < 0), then the array *A* contains the factors *L* and *U* from the Cholesky factorization; the unit diagonal elements of *L* are not stored.
b	Overwritten by the solution matrix *X*.
ipiv	INTEGER.

	Array, DIMENSION at least max(1, *n*). The pivot indices that define the permutation matrix *P*; row *i* of the matrix was interchanged with row *ipiv*(i). Corresponds to the single precision factorization (if *info*= 0 and *iter* ≥ 0) or the double precision factorization (if *info*= 0 and *iter* < 0).
x	DOUBLE PRECISION for dsposv DOUBLE COMPLEX for zcposv. Array, DIMENSION (*ldx*, *nrhs*). If *info* = 0, contains the *n*-by-*nrhs* solution matrix *X*.
iter	INTEGER. If *iter* < 0: iterative refinement has failed, double precision factorization has been performed • If *iter* = -1: the routine fell back to full precision for implementation- or machine-specific reason • If *iter* = -2: narrowing the precision induced an overflow, the routine fell back to full precision • If *iter* = -3: failure of spotrf for dsposv, or cpotrf for zcposv • If *iter* = -31: stop the iterative refinement after the 30th iteration. If *iter* > 0: iterative refinement has been successfully used. Returns the number of iterations.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive definite, so the factorization could not be completed, and the solution has not been computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine posv interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

?posvx

Uses the Cholesky factorization to compute the solution to the system of linear equations with a symmetric or Hermitian positive-definite matrix A, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sposvx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
ferr, berr, work, iwork, info )
```

```
call dposvx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
ferr, berr, work, iwork, info )
```

```
call cposvx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
ferr, berr, work, rwork, info )

call zposvx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
ferr, berr, work, rwork, info )
```

Fortran 95:

```
call posvx( a, b, x [,uplo] [,af] [,fact] [,equed] [,s] [,ferr] [,berr] [,rcond]
[,info] )
```

C:

```
lapack_int LAPACKE_sposvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, char* equed,
float* s, float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float*
ferr, float* berr );

lapack_int LAPACKE_dposvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, char* equed,
double* s, double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond,
double* ferr, double* berr );

lapack_int LAPACKE_cposvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, char* equed, float* s, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_zposvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, char* equed, double* s, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *Cholesky* factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n-by-n real symmetric/Hermitian positive definite matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?posvx` performs the following steps:

1. If `fact = 'E'`, real scaling factors s are computed to equilibrate the system:

   ```
   diag(s)*A*diag(s)*inv(diag(s))*X = diag(s)*B.
   ```

   Whether or not the system will be equilibrated depends on the scaling of the matrix A, but if equilibration is used, A is overwritten by $diag(s)*A*diag(s)$ and B by $diag(s)*B$.
2. If `fact = 'N'` or `'E'`, the Cholesky decomposition is used to factor the matrix A (after equilibration if `fact = 'E'`) as

   $A = U^T*U$ (real), $A = U^H*U$ (complex), if `uplo = 'U'`,

   or $A = L*L^T$ (real), $A = L*L^H$ (complex), if `uplo = 'L'`,

where *U* is an upper triangular matrix and *L* is a lower triangular matrix.

3. If the leading *i*-by-*i* principal minor is not positive-definite, then the routine returns with *info* = *i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, *info* = *n* + 1 is returned as a warning, but the routine still goes on to solve for *X* and compute error bounds as described below.
4. The system of equations is solved for *X* using the factored form of *A*.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix *X* is premultiplied by *diag*(*s*) so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored. If *fact* = 'F': on entry, *af* contains the factored form of *A*. If *equed* = 'Y', the matrix *A* has been equilibrated with scaling factors given by *s*. *a* and *af* will not be modified. If *fact* = 'N', the matrix *A* will be copied to *af* and factored. If *fact* = 'E', the matrix *A* will be equilibrated if necessary, then copied to *af* and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; *nrhs* ≥ 0.
a, *af*, *b*, *work*	REAL for sposvx DOUBLE PRECISION for dposvx COMPLEX for cposvx DOUBLE COMPLEX for zposvx. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*). The array *a* contains the matrix *A* as specified by *uplo*. If *fact* = 'F' and *equed* = 'Y', then *A* must have been equilibrated by the scaling factors in *s*, and *a* must contain the equilibrated matrix *diag*(*s*)*A*diag*(*s*). The second dimension of *a* must be at least max(1,*n*). The array *af* is an input argument if *fact* = 'F'. It contains the triangular factor *U* or *L* from the Cholesky factorization of *A* in the same storage format as *A*. If *equed* is not 'N', then *af* is the factored form of the equilibrated matrix *diag*(*s*)*A*diag*(*s*). The second dimension of *af* must be at least max(1,*n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1, *nrhs*).

	work(*) is a workspace array. The dimension of *work* must be at least max(1,3**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
equed	CHARACTER*1. Must be 'N' or 'Y'. *equed* is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done: if *equed* = 'N', no equilibration was done (always true if *fact* = 'N'); if *equed* = 'Y', equilibration was done, that is, *A* has been replaced by *diag(s)*A*diag(s)*.
s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*. This array is an input argument if *fact* = 'F' only; otherwise it is an output argument. If *equed* = 'N', *s* is not accessed. If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive.
ldx	INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.
rwork	REAL for cposvx DOUBLE PRECISION for zposvx. Workspace array, DIMENSION at least max(1, *n*); used in complex flavors only.

Output Parameters

x	REAL for sposvx DOUBLE PRECISION for dposvx COMPLEX for cposvx DOUBLE COMPLEX for zposvx. Array, DIMENSION (*ldx*,*). If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that if *equed* = 'Y', *A* and *B* are modified on exit, and the solution to the equilibrated system is inv(*diag(s)*)**X*. The second dimension of *x* must be at least max(1,*nrhs*).
a	Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'. If *fact* = 'E' and *equed* = 'Y', *A* is overwritten by *diag(s)*A*diag(s)*.
af	If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization *A=U**T*U* or *A=L*L**T* (real routines), *A=U**H*U* or *A=L*L**H* (complex routines) of the original matrix *A* (if *fact* = 'N'), or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.

b	Overwritten by *diag(s)*B*, if *equed* = 'Y'; not changed if *equed* = 'N'.
s	This array is an output argument if *fact* ≠ 'F'. See the description of *s* in *Input Arguments* section.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision (in particular, if *rcond* =0), the matrix is singular to working precision. This condition is indicated by a return code of *info*>0.
ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x(j)* (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x(j)*, *ferr(j)* is an estimated upper bound for the magnitude of the largest element in *(x(j) - xtrue)* divided by the magnitude of the largest element in *x(j)*. The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.
berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x(j)*, that is, the smallest relative change in any element of *A* or *B* that makes *x(j)* an exact solution.
equed	If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, so the factorization could not be completed, and the solution and error bounds could not be computed; *rcond* =0 is returned. If *info* = *i*, and *i* = *n* + 1, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine posvx interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).

af	Holds the matrix *AF* of size (*n*, *n*).
s	Holds the vector of length *n*. Default value for each element is *s(i)* = 1.0_WP.
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
fact	Must be 'N', 'E', or 'F'. The default value is 'N'. If *fact* = 'F', then *af* must be present; otherwise, an error is returned.
equed	Must be 'N' or 'Y'. The default value is 'N'.

?posvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a symmetric or Hermitian positive-definite matrix A applying the Cholesky factorization.

Syntax

Fortran 77:

```
call sposvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork,
info )
```

```
call dposvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, iwork,
info )
```

```
call cposvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork,
info )
```

```
call zposvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, equed, s, b, ldb, x, ldx, rcond,
rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work, rwork,
info )
```

C:

```
lapack_int LAPACKE_sposvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, char* equed,
float* s, float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float*
rpvgrw, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float*
err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_dposvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, char* equed,
double* s, double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond,
double* rpvgrw, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, const double* params );
```

```
lapack_int LAPACKE_cposvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, char* equed, float* s, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* rpvgrw, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
const float* params );
```

```
lapack_int LAPACKE_zposvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, char* equed, double* s, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* rpvgrw, double* berr,
lapack_int n_err_bnds, double* err_bnds_norm, double* err_bnds_comp, lapack_int
nparams, const double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *Cholesky* factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is an n-by-n real symmetric/Hermitian positive definite matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error (`O(eps)`, where `eps` is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with `O(eps)` errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine `?posvxx` performs the following steps:

1. If *fact* = `'E'`, scaling factors are computed to equilibrate the system:

   *diag(s)*A*diag(s)* *inv(*diag(s)*)**X* = *diag(s)*B*

   Whether or not the system will be equilibrated depends on the scaling of the matrix A, but if equilibration is used, A is overwritten by *diag(s)*A*diag(s)* and B by *diag(s)*B*.
2. If *fact* = `'N'` or `'E'`, the Cholesky decomposition is used to factor the matrix A (after equilibration if *fact* = `'E'`) as

   $A = U^T*U$ (real), $A = U^H*U$ (complex), if *uplo* = `'U'`,

   or $A = L*L^T$ (real), $A = L*L^H$ (complex), if *uplo* = `'L'`,

   where U is an upper triangular matrix and L is a lower triangular matrix.
3. If the leading i-by-i principal minor is not positive-definite, the routine returns with *info* = *i*. Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A.
5. By default, unless `params(la_linrx_itref_i)` is set to zero, the routine applies iterative refinement to get a small error and error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by *diag(s)* so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored. If *fact* = 'F', on entry, *af* contains the factored form of *A*. If *equed* is not 'N', the matrix *A* has been equilibrated with scaling factors given by *s*. Parameters *a* and *af* are not modified. If *fact* = 'N', the matrix *A* will be copied to *af* and factored. If *fact* = 'E', the matrix *A* will be equilibrated, if necessary, copied to *af* and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
a, *af*, *b*, *work*	REAL for sposvxx DOUBLE PRECISION for dposvxx COMPLEX for cposvxx DOUBLE COMPLEX for zposvxx. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*). The array *a* contains the matrix *A* as specified by *uplo* . If *fact* = 'F' and *equed* = 'Y', then *A* must have been equilibrated by the scaling factors in *s*, and *a* must contain the equilibrated matrix *diag*(*s*)**A*diag*(*s*). The second dimension of *a* must be at least max(1,*n*). The array *af* is an input argument if *fact* = 'F'. It contains the triangular factor *U* or *L* from the Cholesky factorization of *A* in the same storage format as *A*. If *equed* is not 'N', then *af* is the factored form of the equilibrated matrix *diag*(*s*)**A*diag*(*s*). The second dimension of *af* must be at least max(1,*n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*). If *equed* = 'N', *s* is not accessed. If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive. *work(*)* is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*; *ldaf* ≥ max(1,*n*).
equed	CHARACTER*1. Must be 'N' or 'Y'. *equed* is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done: If *equed* = 'N', no equilibration was done (always true if *fact* = 'N'). if *equed* = 'Y', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*s*)**A*diag*(*s*).

s
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*. This array is an input argument if *fact* = 'F' only; otherwise it is an output argument.
If *equed* = 'N', *s* is not accessed.
If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive. Each element of *s* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb
INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

ldx
INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds
INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in the *Output Arguments* section below.

nparams
INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the extra-precise refinement algorithm.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for fefinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork
INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork
REAL for single precision flavors

DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3*n); used in complex flavors only.

Output Parameters

x
REAL for sposvxx
DOUBLE PRECISION for dposvxx
COMPLEX for cposvxx
DOUBLE COMPLEX for zposvxx.
Array, DIMENSION (*ldx*,*).
If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the original system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the equilibrated system is: inv(*diag*(*s*))*X.

a
Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.
If *fact* = 'E' and *equed* = 'Y', *A* is overwritten by *diag*(*s*)*A*diag(*s*).

af
If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization *A*=*U****T***U* or *A*=*L***L****T* (real routines), *A*=*U****H***U* or *A*=*L***L****H* (complex routines) of the original matrix *A* (if *fact* = 'N'), or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.

b
If *equed* = 'N', *B* is not modified.
If *equed* = 'Y', *B* is overwritten by *diag*(*s*)*B.

s
This array is an output argument if *fact* ≠ 'F'. Each element of this array is a power of the radix. See the description of *s* in *Input Arguments* section.

rcond
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Contains the reciprocal pivot growth factor *norm*(*A*)/*norm*(*U*). The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the (equlibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with 0 < *info* ≤ *n*, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*.

berr
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

err_bnds_norm
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.

Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left|Xtrue_{ji} - X_{ji}\right|}{\max_j \left|X_{ji}\right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

equed — If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).

params — If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = −*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested

params(3) = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(j,1) = 0.0 or *err_bnds_comp*(j,1) = 0.0. See the definition of *err_bnds_norm*(;,1) and *err_bnds_comp*(;,1). To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?ppsv

Computes the solution to the system of linear equations with a symmetric (Hermitian) positive definite packed matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sppsv( uplo, n, nrhs, ap, b, ldb, info )
call dppsv( uplo, n, nrhs, ap, b, ldb, info )
call cppsv( uplo, n, nrhs, ap, b, ldb, info )
call zppsv( uplo, n, nrhs, ap, b, ldb, info )
```

Fortran 95:

```
call ppsv( ap, b [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>ppsv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
<datatype>* ap, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for *X* the real or complex system of linear equations *A*X* = *B*, where *A* is an *n*-by-*n* real symmetric/Hermitian positive-definite matrix stored in packed format, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

The Cholesky decomposition is used to factor *A* as

A = U^T**U* (real flavors) and *A* = U^H**U* (complex flavors), if *uplo* = 'U'

or *A* = *L***L*^{T} (real flavors) and *A* = *L***L*^{H} (complex flavors), if *uplo* = 'L',

where *U* is an upper triangular matrix and *L* is a lower triangular matrix. The factored form of *A* is then used to solve the system of equations *A*X* = *B*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.
ap, *b*	REAL for sppsv DOUBLE PRECISION for dppsv COMPLEX for cppsv DOUBLE COMPLEX for zppsv. Arrays: *ap*(*), *b*(*ldb*,*). The array *ap* contains the upper or the lower triangular part of the matrix *A* (as specified by *uplo*) in *packed storage* (see Matrix Storage Schemes). The dimension of *ap* must be at least max(1,*n*(*n*+1)/2). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* $\geq$ max(1, *n*).

Output Parameters

ap	If *info* = 0, the upper or lower triangular part of *A* in packed storage is overwritten by the Cholesky factor *U* or *L*, as specified by *uplo*.
b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ppsv interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*,*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

?ppsvx

Uses the Cholesky factorization to compute the solution to the system of linear equations with a symmetric (Hermitian) positive definite packed matrix A, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sppsvx( fact, uplo, n, nrhs, ap, afp, equed, s, b, ldb, x, ldx, rcond, ferr,
berr, work, iwork, info )

call dppsvx( fact, uplo, n, nrhs, ap, afp, equed, s, b, ldb, x, ldx, rcond, ferr,
berr, work, iwork, info )

call cppsvx( fact, uplo, n, nrhs, ap, afp, equed, s, b, ldb, x, ldx, rcond, ferr,
berr, work, rwork, info )

call zppsvx( fact, uplo, n, nrhs, ap, afp, equed, s, b, ldb, x, ldx, rcond, ferr,
berr, work, rwork, info )
```

Fortran 95:

```
call ppsvx( ap, b, x [,uplo] [,af] [,fact] [,equed] [,s] [,ferr] [,berr] [,rcond]
[,info] )
```

C:

```
lapack_int LAPACKE_sppsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* ap, float* afp, char* equed, float* s, float* b, lapack_int
ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_dppsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* ap, double* afp, char* equed, double* s, double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );

lapack_int LAPACKE_cppsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* ap, lapack_complex_float* afp, char* equed,
float* s, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int
ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_zppsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* ap, lapack_complex_double* afp, char* equed,
double* s, lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x,
lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the Cholesky factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n-by-n symmetric or Hermitian positive-definite matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?ppsvx` performs the following steps:

1. If $fact$ = `'E'`, real scaling factors s are computed to equilibrate the system:

   $diag(s)*A*diag(s)*inv(diag(s))*X = diag(s)*B$.

   Whether or not the system will be equilibrated depends on the scaling of the matrix A, but if equilibration is used, A is overwritten by $diag(s)*A*diag(s)$ and B by $diag(s)*B$.

2. If *fact* = 'N' or 'E', the Cholesky decomposition is used to factor the matrix *A* (after equilibration if *fact* = 'E') as

   $A = U^T*U$ (real), $A = U^H*U$ (complex), if *uplo* = 'U',

   or $A = L*L^T$ (real), $A = L*L^H$ (complex), if *uplo* = 'L',

   where *U* is an upper triangular matrix and *L* is a lower triangular matrix.
3. If the leading *i*-by-*i* principal minor is not positive-definite, then the routine returns with *info* = *i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, *info* = *n*+1 is returned as a warning, but the routine still goes on to solve for *X* and compute error bounds as described below.
4. The system of equations is solved for *X* using the factored form of *A*.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix *X* is premultiplied by *diag(s)* so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored. If *fact* = 'F': on entry, *afp* contains the factored form of *A*. If *equed* = 'Y', the matrix *A* has been equilibrated with scaling factors given by *s*. *ap* and *afp* will not be modified. If *fact* = 'N', the matrix *A* will be copied to *afp* and factored. If *fact* = 'E', the matrix *A* will be equilibrated if necessary, then copied to *afp* and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns in *B*; $nrhs \geq 0$.
ap, *afp*, *b*, *work*	REAL for sppsvx DOUBLE PRECISION for dppsvx COMPLEX for cppsvx DOUBLE COMPLEX for zppsvx. Arrays: *ap*(*), *afp*(*), *b*(*ldb*,*), *work*(*). The array *ap* contains the upper or lower triangle of the original symmetric/Hermitian matrix *A* in *packed storage* (see Matrix Storage Schemes). In case when *fact* = 'F' and *equed* = 'Y', *ap* must contain the equilibrated matrix *diag(s)*A*diag(s)*.

The array *afp* is an input argument if *fact* = 'F' and contains the triangular factor *U* or *L* from the Cholesky factorization of *A* in the same storage format as *A*. If *equed* is not 'N', then *afp* is the factored form of the equilibrated matrix *A*.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
work(*) is a workspace array.
The dimension of arrays *ap* and *afp* must be at least max(1, *n*(*n*+1)/2); the second dimension of *b* must be at least max(1,*nrhs*); the dimension of *work* must be at least max(1, 3**n*) for real flavors and max(1, 2**n*) for complex flavors.

ldb — INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

equed — CHARACTER*1. Must be 'N' or 'Y'.
equed is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done:
if *equed* = 'N', no equilibration was done (always true if *fact* = 'N');
if *equed* = 'Y', equilibration was done, that is, *A* has been replaced by *diag*(*s*)*A***diag*(*s*).

s — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*. This array is an input argument if *fact* = 'F' only; otherwise it is an output argument.
If *equed* = 'N', *s* is not accessed.
If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive.

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

iwork — INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork — REAL for cppsvx;
DOUBLE PRECISION for zppsvx.
Workspace array, DIMENSION at least max(1, *n*); used in complex flavors only.

Output Parameters

x — REAL for sppsvx
DOUBLE PRECISION for dppsvx
COMPLEX for cppsvx
DOUBLE COMPLEX for zppsvx.
Array, DIMENSION (*ldx*,*).
If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that if *equed* = 'Y', *A* and *B* are modified on exit, and the solution to the equilibrated system is inv(*diag*(*s*))**X*. The second dimension of *x* must be at least max(1,*nrhs*).

ap — Array *ap* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.
If *fact* = 'E' and *equed* = 'Y', *A* is overwritten by *diag*(*s*)**A***diag*(*s*).

afp	If *fact* = 'N' or 'E', then *afp* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization $A=U^T*U$ or $A=L*L^T$ (real routines), $A=U^H*U$ or $A=L*L^H$ (complex routines) of the original matrix *A* (if *fact* = 'N'), or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *ap* for the form of the equilibrated matrix.
b	Overwritten by *diag(s)*B*, if *equed* = 'Y'; not changed if *equed* = 'N'.
s	This array is an output argument if *fact* ≠ 'F'. See the description of *s* in *Input Arguments* section.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.
ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x(j)* (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x(j)*, *ferr(j)* is an estimated upper bound for the magnitude of the largest element in *(x(j) - xtrue)* divided by the magnitude of the largest element in *x(j)*. The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.
berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x(j)*, that is, the smallest relative change in any element of *A* or *B* that makes *x(j)* an exact solution.
equed	If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, so the factorization could not be completed, and the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n* + 1, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `ppsvx` interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*,*nrhs*).
x	Holds the matrix *X* of size (*n*,*nrhs*).
afp	Holds the matrix *AF* of size (*n**(*n*+1)/2).
s	Holds the vector of length *n*. Default value for each element is *s*(*i*) = 1.0_WP.
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
fact	Must be 'N', 'E', or 'F'. The default value is 'N'. If *fact* = 'F', then *af* must be present; otherwise, an error is returned.
equed	Must be 'N' or 'Y'. The default value is 'N'.

?pbsv

Computes the solution to the system of linear equations with a symmetric or Hermitian positive-definite band matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call spbsv( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
call dpbsv( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
call cpbsv( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
call zpbsv( uplo, n, kd, nrhs, ab, ldab, b, ldb, info )
```

Fortran 95:

```
call pbsv( ab, b [,uplo] [,info] )
```

C:

```
lapack_int LAPACKE_<?>pbsv( int matrix_order, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, <datatype>* ab, lapack_int ldab, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for *X* the real or complex system of linear equations *A***X* = *B*, where *A* is an *n*-by-*n* symmetric/Hermitian positive definite band matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

The Cholesky decomposition is used to factor A as

$A = U^T*U$ (real flavors) and $A = U^H*U$ (complex flavors), if $uplo$ = 'U'

or $A = L*L^T$ (real flavors) and $A = L*L^H$ (complex flavors), if $uplo$ = 'L',

where U is an upper triangular band matrix and L is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as A. The factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If $uplo$ = 'U', the upper triangle of A is stored. If $uplo$ = 'L', the lower triangle of A is stored.
n	INTEGER. The order of matrix A; $n \geq 0$.
kd	INTEGER. The number of superdiagonals of the matrix A if $uplo$ = 'U', or the number of subdiagonals if $uplo$ = 'L'; $kd \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in B; $nrhs \geq 0$.
ab, *b*	REAL for spbsv DOUBLE PRECISION for dpbsv COMPLEX for cpbsv DOUBLE COMPLEX for zpbsv. Arrays: $ab(ldab, *)$, $b(ldb,*)$. The array ab contains the upper or the lower triangular part of the matrix A (as specified by $uplo$) in *band storage* (see Matrix Storage Schemes). The second dimension of ab must be at least $\max(1, n)$. The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of b must be at least $\max(1,nrhs)$.
ldab	INTEGER. The leading dimension of the array ab; $ldab \geq kd + 1$.
ldb	INTEGER. The leading dimension of b; $ldb \geq \max(1, n)$.

Output Parameters

ab	The upper or lower triangular part of A (in band storage) is overwritten by the Cholesky factor U or L, as specified by $uplo$, in the same storage format as A.
b	Overwritten by the solution matrix X.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value. If $info = i$, the leading minor of order i (and therefore the matrix A itself) is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `pbsv` interface are as follows:

ab	Holds the array *A* of size (*kd*+1, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.

?pbsvx

Uses the Cholesky factorization to compute the solution to the system of linear equations with a symmetric (Hermitian) positive-definite band matrix A, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call spbsvx( fact, uplo, n, kd, nrhs, ab, ldab, afb, ldafb, equed, s, b, ldb, x, ldx,
rcond, ferr, berr, work, iwork, info )
```

```
call dpbsvx( fact, uplo, n, kd, nrhs, ab, ldab, afb, ldafb, equed, s, b, ldb, x, ldx,
rcond, ferr, berr, work, iwork, info )
```

```
call cpbsvx( fact, uplo, n, kd, nrhs, ab, ldab, afb, ldafb, equed, s, b, ldb, x, ldx,
rcond, ferr, berr, work, rwork, info )
```

```
call zpbsvx( fact, uplo, n, kd, nrhs, ab, ldab, afb, ldafb, equed, s, b, ldb, x, ldx,
rcond, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call pbsvx( ab, b, x [,uplo] [,afb] [,fact] [,equed] [,s] [,ferr] [,berr] [,rcond]
[,info] )
```

C:

```
lapack_int LAPACKE_spbsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, float* ab, lapack_int ldab, float* afb, lapack_int
ldafb, char* equed, float* s, float* b, lapack_int ldb, float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dpbsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, double* ab, lapack_int ldab, double* afb, lapack_int
ldafb, char* equed, double* s, double* b, lapack_int ldb, double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cpbsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* afb, lapack_int ldafb, char* equed, float* s,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zpbsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* afb, lapack_int ldafb, char* equed, double* s,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the Cholesky factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n-by-n symmetric or Hermitian positive definite band matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?pbsvx` performs the following steps:

1. If $fact = 'E'$, real scaling factors s are computed to equilibrate the system:

   $diag(s)*A*diag(s)*inv(diag(s))*X = diag(s)*B$.

   Whether or not the system will be equilibrated depends on the scaling of the matrix A, but if equilibration is used, A is overwritten by $diag(s)*A*diag(s)$ and B by $diag(s)*B$.
2. If $fact = 'N'$ or $'E'$, the Cholesky decomposition is used to factor the matrix A (after equilibration if $fact = 'E'$) as

   $A = U^T*U$ (real), $A = U^H*U$ (complex), if $uplo = 'U'$,

   or $A = L*L^T$ (real), $A = L*L^H$ (complex), if $uplo = 'L'$,

   where U is an upper triangular band matrix and L is a lower triangular band matrix.
3. If the leading i-by-i principal minor is not positive definite, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $diag(s)$ so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.

	If *fact* = 'F': on entry, *afb* contains the factored form of *A*. If *equed* = 'Y', the matrix *A* has been equilibrated with scaling factors given by *s*. *ab* and *afb* will not be modified. If *fact* = 'N', the matrix *A* will be copied to *afb* and factored. If *fact* = 'E', the matrix *A* will be equilibrated if necessary, then copied to *afb* and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
kd	INTEGER. The number of superdiagonals or subdiagonals in the matrix *A*; $kd \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.
ab, *afb*, *b*, *work*	REAL for spbsvx DOUBLE PRECISION for dpbsvx COMPLEX for cpbsvx DOUBLE COMPLEX for zpbsvx. Arrays: *ab*(*ldab*,*), afb*(*ldab*,*), b*(*ldb*,*), work*(*). The array *ab* contains the upper or lower triangle of the matrix *A* in *band storage* (see Matrix Storage Schemes). If *fact* = 'F' and *equed* = 'Y', then *ab* must contain the equilibrated matrix *diag*(*s*)***A***diag*(*s*). The second dimension of *ab* must be at least max(1, *n*). The array *afb* is an input argument if *fact* = 'F'. It contains the triangular factor *U* or *L* from the Cholesky factorization of the band matrix *A* in the same storage format as *A*. If *equed* = 'Y', then *afb* is the factored form of the equilibrated matrix *A*. The second dimension of *afb* must be at least max(1, *n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1, *nrhs*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1,3***n*) for real flavors, and at least max(1,2***n*) for complex flavors.
ldab	INTEGER. The leading dimension of *ab*; $ldab \geq kd+1$.
ldafb	INTEGER. The leading dimension of *afb*; $ldafb \geq kd+1$.
ldb	INTEGER. The leading dimension of *b*; $ldb \geq \max(1, n)$.
equed	CHARACTER*1. Must be 'N' or 'Y'. *equed* is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done: if *equed* = 'N', no equilibration was done (always true if *fact* = 'N') if *equed* = 'Y', equilibration was done, that is, *A* has been replaced by *diag*(*s*)***A***diag*(*s*).
s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors.

	Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*. This array is an input argument if *fact* = 'F' only; otherwise it is an output argument. If *equed* = 'N', *s* is not accessed. If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive.
ldx	INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.
rwork	REAL for cpbsvx DOUBLE PRECISION for zpbsvx. Workspace array, DIMENSION at least max(1, *n*); used in complex flavors only.

Output Parameters

x	REAL for spbsvx DOUBLE PRECISION for dpbsvx COMPLEX for cpbsvx DOUBLE COMPLEX for zpbsvx. Array, DIMENSION (*ldx*,*). If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that if *equed* = 'Y', *A* and *B* are modified on exit, and the solution to the equilibrated system is inv(*diag*(*s*))**X*. The second dimension of *x* must be at least max(1,*nrhs*).
ab	On exit, if *fact* = 'E' and *equed* = 'Y', *A* is overwritten by *diag*(*s*)**A***diag*(*s*).
afb	If *fact* = 'N' or 'E', then *afb* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization $A=U^T*U$ or $A=L*L^T$ (real routines), $A=U^H*U$ or $A=L*L^H$ (complex routines) of the original matrix *A* (if *fact* = 'N'), or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *ab* for the form of the equilibrated matrix.
b	Overwritten by *diag*(*s*)**B*, if *equed* = 'Y'; not changed if *equed* = 'N'.
s	This array is an output argument if *fact* ≠ 'F'. See the description of *s* in *Input Arguments* section.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.
ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) – *xtrue*) divided by the magnitude of the

largest element in $x(j)$. The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.

berr — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector $x(j)$, that is, the smallest relative change in any element of A or B that makes $x(j)$ an exact solution.

equed — If *fact* ≠'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).

info — INTEGER. If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, and $i \le n$, the leading minor of order *i* (and therefore the matrix A itself) is not positive definite, so the factorization could not be completed, and the solution and error bounds could not be computed; *rcond* =0 is returned. If *info* = *i*, and $i = n + 1$, then U is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pbsvx interface are as follows:

ab — Holds the array A of size (*kd*+1, *n*).

b — Holds the matrix B of size (*n*, *nrhs*).

x — Holds the matrix X of size (*n*, *nrhs*).

afb — Holds the array *AF* of size (*kd*+1, *n*).

s — Holds the vector with the number of elements *n*. Default value for each element is *s*(*i*) = 1.0_WP.

ferr — Holds the vector with the number of elements *nrhs*.

berr — Holds the vector with the number of elements *nrhs*.

uplo — Must be 'U' or 'L'. The default value is 'U'.

fact — Must be 'N', 'E', or 'F'. The default value is 'N'. If *fact* = 'F', then *af* must be present; otherwise, an error is returned.

equed — Must be 'N' or 'Y'. The default value is 'N'.

?ptsv

Computes the solution to the system of linear equations with a symmetric or Hermitian positive definite tridiagonal matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call sptsv( n, nrhs, d, e, b, ldb, info )
call dptsv( n, nrhs, d, e, b, ldb, info )
call cptsv( n, nrhs, d, e, b, ldb, info )
call zptsv( n, nrhs, d, e, b, ldb, info )
```

Fortran 95:

```
call ptsv( d, e, b [,info] )
```

C:

```
lapack_int LAPACKE_sptsv( int matrix_order, lapack_int n, lapack_int nrhs, float* d,
float* e, float* b, lapack_int ldb );
lapack_int LAPACKE_dptsv( int matrix_order, lapack_int n, lapack_int nrhs, double* d,
double* e, double* b, lapack_int ldb );
lapack_int LAPACKE_cptsv( int matrix_order, lapack_int n, lapack_int nrhs, float* d,
lapack_complex_float* e, lapack_complex_float* b, lapack_int ldb );
lapack_int LAPACKE_zptsv( int matrix_order, lapack_int n, lapack_int nrhs, double* d,
lapack_complex_double* e, lapack_complex_double* b, lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for X the real or complex system of linear equations $A*X = B$, where A is an n-by-n symmetric/Hermitian positive-definite tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

A is factored as $A = L*D*L^T$ (real flavors) or $A = L*D*L^H$ (complex flavors), and the factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	`INTEGER`. The order of matrix A; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides, the number of columns in B; $nrhs \geq 0$.
d	`REAL` for single precision flavors `DOUBLE PRECISION` for double precision flavors. Array, dimension at least `max(1, n)`. Contains the diagonal elements of the tridiagonal matrix A.
e, *b*	`REAL` for `sptsv` `DOUBLE PRECISION` for `dptsv` `COMPLEX` for `cptsv`

DOUBLE COMPLEX for zptsv.
Arrays: e(n - 1), b(ldb,*). The array e contains the (n - 1) subdiagonal elements of A.
The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of b must be at least max(1,nrhs).

ldb — INTEGER. The leading dimension of b; ldb ≥ max(1, n).

Output Parameters

d — Overwritten by the n diagonal elements of the diagonal matrix D from the $L*D*L^T$ (real)/ $L*D*L^H$ (complex) factorization of A.

e — Overwritten by the (n - 1) subdiagonal elements of the unit bidiagonal factor L from the factorization of A.

b — Overwritten by the solution matrix X.

info — INTEGER. If info = 0, the execution is successful.
If info = -i, the i-th parameter had an illegal value.
If info = i, the leading minor of order i (and therefore the matrix A itself) is not positive-definite, and the solution has not been computed. The factorization has not been completed unless i = n.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ptsv interface are as follows:

d — Holds the vector of length n.

e — Holds the vector of length (n-1).

b — Holds the matrix B of size (n,nrhs).

?ptsvx

Uses factorization to compute the solution to the system of linear equations with a symmetric (Hermitian) positive definite tridiagonal matrix A, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sptsvx( fact, n, nrhs, d, e, df, ef, b, ldb, x, ldx, rcond, ferr, berr, work, info )
call dptsvx( fact, n, nrhs, d, e, df, ef, b, ldb, x, ldx, rcond, ferr, berr, work, info )
call cptsvx( fact, n, nrhs, d, e, df, ef, b, ldb, x, ldx, rcond, ferr, berr, work, rwork, info )
call zptsvx( fact, n, nrhs, d, e, df, ef, b, ldb, x, ldx, rcond, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call ptsvx( d, e, b, x [,df] [,ef] [,fact] [,ferr] [,berr] [,rcond] [,info] )
```

C:

```
lapack_int LAPACKE_sptsvx( int matrix_order, char fact, lapack_int n, lapack_int nrhs,
const float* d, const float* e, float* df, float* ef, const float* b, lapack_int ldb,
float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_dptsvx( int matrix_order, char fact, lapack_int n, lapack_int nrhs,
const double* d, const double* e, double* df, double* ef, const double* b, lapack_int
ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );

lapack_int LAPACKE_cptsvx( int matrix_order, char fact, lapack_int n, lapack_int nrhs,
const float* d, const lapack_complex_float* e, float* df, lapack_complex_float* ef,
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_zptsvx( int matrix_order, char fact, lapack_int n, lapack_int nrhs,
const double* d, const lapack_complex_double* e, double* df, lapack_complex_double* ef,
const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int
ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the Cholesky factorization $A = L*D*L^T$ (real)/$A = L*D*L^H$ (complex) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n-by-n symmetric or Hermitian positive definite tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?ptsvx` performs the following steps:

1. If $fact$ = `'N'`, the matrix A is factored as $A = L*D*L^T$ (real flavors)/$A = L*D*L^H$ (complex flavors), where L is a unit lower bidiagonal matrix and D is diagonal. The factorization can also be regarded as having the form $A = U^T*D*U$ (real flavors)/$A = U^H*D*U$ (complex flavors).
2. If the leading i-by-i principal minor is not positive-definite, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be `'F'` or `'N'`.

	Specifies whether or not the factored form of the matrix *A* is supplied on entry. If *fact* = 'F': on entry, *df* and *ef* contain the factored form of *A*. Arrays *d*, *e*, *df*, and *ef* will not be modified. If *fact* = 'N', the matrix *A* will be copied to *df* and *ef*, and factored.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.
d, *df*, *rwork*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays: *d*(*n*), *df*(*n*), *rwork*(*n*). The array *d* contains the *n* diagonal elements of the tridiagonal matrix *A*. The array *df* is an input argument if *fact* = 'F' and on entry contains the *n* diagonal elements of the diagonal matrix *D* from the $L*D*L^T$ (real)/ $L*D*L^H$ (complex) factorization of *A*. The array *rwork* is a workspace array used for complex flavors only.
e,*ef*,*b*,*work*	REAL for sptsvx DOUBLE PRECISION for dptsvx COMPLEX for cptsvx DOUBLE COMPLEX for zptsvx. Arrays: *e*(*n* -1), *ef*(*n* -1), *b*(*ldb**), *work*(*). The array *e* contains the (*n* - 1) subdiagonal elements of the tridiagonal matrix *A*. The array *ef* is an input argument if *fact* = 'F' and on entry contains the (*n* - 1) subdiagonal elements of the unit bidiagonal factor *L* from the $L*D*L^T$ (real)/ $L*D*L^H$ (complex) factorization of *A*. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The array *work* is a workspace array. The dimension of *work* must be at least 2**n* for real flavors, and at least *n* for complex flavors.
ldb	INTEGER. The leading dimension of *b*; $ldb \geq \max(1, n)$.
ldx	INTEGER. The leading dimension of *x*; $ldx \geq \max(1, n)$.

Output Parameters

x	REAL for sptsvx DOUBLE PRECISION for dptsvx COMPLEX for cptsvx DOUBLE COMPLEX for zptsvx. Array, DIMENSION (*ldx*,*). If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the system of equations. The second dimension of *x* must be at least max(1,*nrhs*).
df, *ef*	These arrays are output arguments if *fact* = 'N'. See the description of *df*, *ef* in *Input Arguments* section.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.

ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) – *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.
berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, so the factorization could not be completed, and the solution and error bounds could not be computed; *rcond* =0 is returned. If *info* = *i*, and *i* = *n* + 1, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ptsvx interface are as follows:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
df	Holds the vector of length *n*.
ef	Holds the vector of length (*n*-1).
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
fact	Must be 'N' or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.

?sysv

Computes the solution to the system of linear equations with a real or complex symmetric matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call ssysv( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, lwork, info )
call dsysv( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, lwork, info )
call csysv( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, lwork, info )
call zsysv( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, lwork, info )
```

Fortran 95:

```
call sysv( a, b [,uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>sysv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
<datatype>* a, lapack_int lda, lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for X the real or complex system of linear equations $A*X = B$, where A is an n-by-n symmetric matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U*D*U^T$ or $A = L*D*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether the upper or lower triangular part of A is stored: If *uplo* = `'U'`, the upper triangle of A is stored. If *uplo* = `'L'`, the lower triangle of A is stored.
n	`INTEGER`. The order of matrix A; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides; the number of columns in B; $nrhs \geq 0$.
a, *b*, *work*	`REAL` for `ssysv` `DOUBLE PRECISION` for `dsysv`

	COMPLEX for csysv DOUBLE COMPLEX for zsysv. Arrays: a(lda,*), b(ldb,*), work(*). The array a contains the upper or the lower triangular part of the symmetric matrix A (see uplo). The second dimension of a must be at least max(1, n). The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of b must be at least max(1,nrhs). work is a workspace array, dimension at least max(1,lwork).
lda	INTEGER. The leading dimension of a; lda ≥ max(1, n).
ldb	INTEGER. The leading dimension of b; ldb ≥ max(1, n).
lwork	INTEGER. The size of the work array; lwork ≥ 1. If lwork = -1, then a workspace query is assumed; the routine only calculates the optimal size of the work array, returns this value as the first entry of the work array, and no error message related to lwork is issued by xerbla. See *Application Notes* below for details and for the suggested value of lwork.

Output Parameters

a	If info = 0, a is overwritten by the block-diagonal matrix D and the multipliers used to obtain the factor U (or L) from the factorization of A as computed by ?sytrf.
b	If info = 0, b is overwritten by the solution matrix X.
ipiv	INTEGER. Array, DIMENSION at least max(1, n). Contains details of the interchanges and the block structure of D, as determined by ?sytrf. If ipiv(i) = k >0, then d_{ii} is a 1-by-1 diagonal block, and the i-th row and column of A was interchanged with the k-th row and column. If uplo = 'U' and ipiv(i) = ipiv(i-1) = -m < 0, then D has a 2-by-2 block in rows/columns i and i–1, and (i–1)-th row and column of A was interchanged with the m–th row and column. If uplo = 'L' and ipiv(i) = ipiv(i+1) = -m < 0, then D has a 2-by-2 block in rows/columns i and i+1, and (i+1)-th row and column of A was interchanged with the m-th row and column.
work(1)	If info = 0, on exit work(1) contains the minimum value of lwork required for optimum performance. Use this lwork for subsequent runs.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value. If info = i, d_{ii} is 0. The factorization has been completed, but D is exactly singular, so the solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sysv interface are as follows:

a	Holds the matrix A of size (n, n).

b	Holds the matrix *B* of size (*n*,*nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For better performance, try using *lwork* = *n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?sysvx

Uses the diagonal pivoting factorization to compute the solution to the system of linear equations with a real or complex symmetric matrix A, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call ssysvx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, lwork, iwork, info )
```

```
call dsysvx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, lwork, iwork, info )
```

```
call csysvx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, lwork, rwork, info )
```

```
call zsysvx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, lwork, rwork, info )
```

Fortran 95:

```
call sysvx( a, b, x [,uplo] [,af] [,ipiv] [,fact] [,ferr] [,berr] [,rcond] [,info] )
```

C:

```
lapack_int LAPACKE_ssysvx( int matrix_order, char fact, char uplo, lapack_int n, lapack_int nrhs, const float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int* ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dsysvx( int matrix_order, char fact, char uplo, lapack_int n, lapack_int nrhs, const double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int* ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_csysvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, lapack_complex_float*
af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_zsysvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, lapack_complex_double*
af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine uses the diagonal pivoting factorization to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n-by-n symmetric matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine ?sysvx performs the following steps:

1. If $fact = 'N'$, the diagonal pivoting method is used to factor the matrix A. The form of the factorization is $A = U*D*U^T$ or $A = L*D*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i}$= 0, so that D is exactly singular, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F' or 'N'. Specifies whether or not the factored form of the matrix A has been supplied on entry. If $fact = 'F'$: on entry, af and $ipiv$ contain the factored form of A. Arrays a, af, and $ipiv$ will not be modified. If $fact = 'N'$, the matrix A will be copied to af and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If $uplo = 'U'$, the upper triangle of A is stored. If $uplo = 'L'$, the lower triangle of A is stored.
n	INTEGER. The order of matrix A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in B; $nrhs \geq 0$.

a, *af*, *b*, *work*
REAL for ssysvx
DOUBLE PRECISION for dsysvx
COMPLEX for csysvx
DOUBLE COMPLEX for zsysvx.
Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*).
The array *a* contains the upper or the lower triangular part of the symmetric matrix *A* (see *uplo*). The second dimension of *a* must be at least max(1,*n*).
The array *af* is an input argument if *fact* = 'F'. It contains he block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization *A* = *U*D*U*T* or*A* = *L*D*L*T* as computed by ?sytrf. The second dimension of *af* must be at least max(1,*n*).
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1, *nrhs*).
work(*) is a workspace array, dimension at least max(1,*lwork*).

lda
INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).

ldaf
INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).

ldb
INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

ipiv
INTEGER.
Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D*, as determined by ?sytrf.
If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 diagonal block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column.
If *uplo* = 'U' and *ipiv*(*i*) = *ipiv*(*i*–1) = –*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column.
If *uplo* = 'L' and *ipiv*(*i*) = *ipiv*(*i*+1) = –*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.

ldx
INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

lwork
INTEGER. The size of the *work* array.
If *lwork* = –1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* below for details and for the suggested value of *lwork*.

iwork
INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork
REAL for csysvx;
DOUBLE PRECISION for zsysvx.
Workspace array, DIMENSION at least max(1, *n*); used in complex flavors only.

Output Parameters

x
REAL for ssysvx
DOUBLE PRECISION for dsysvx

	COMPLEX for csysvx DOUBLE COMPLEX for zsysvx. Array, DIMENSION (*ldx*,*). If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the system of equations. The second dimension of *x* must be at least max(1,*nrhs*).
af, *ipiv*	These arrays are output arguments if *fact* = 'N'. See the description of *af*, *ipiv* in *Input Arguments* section.
rcond	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A*. If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.
ferr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) – *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.
berr	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.
work(1)	If *info*=0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and $i \le n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n* + 1, then *D* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sysvx interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
af	Holds the matrix *AF* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
fact	Must be 'N' or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.

Application Notes

The value of *lwork* must be at least max(1,*m***n*), where for real flavors *m* = 3 and for complex flavors *m* = 2. For better performance, try using *lwork* = max(1, *m***n*, *n***blocksize*), where *blocksize* is the optimal block size for ?sytrf.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = –1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = –1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?sysvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a symmetric indefinite matrix A applying the diagonal pivoting factorization.

Syntax

Fortran 77:

```
call ssysvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, equed, s, b, ldb, x, ldx,
rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
iwork, info )
```

```
call dsysvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, equed, s, b, ldb, x, ldx,
rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
iwork, info )
```

```
call csysvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, equed, s, b, ldb, x, ldx,
rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
rwork, info )
```

```
call zsysvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, equed, s, b, ldb, x, ldx,
rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
rwork, info )
```

C:

```
lapack_int LAPACKE_ssysvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, float* s, float* b, lapack_int ldb, float* x, lapack_int ldx,
float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds, float* err_bnds_norm,
float* err_bnds_comp, lapack_int nparams, const float* params );

lapack_int LAPACKE_dsysvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, double* s, double* b, lapack_int ldb, double* x, lapack_int ldx,
double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double* params );

lapack_int LAPACKE_csysvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* s, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* rpvgrw,
float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp,
lapack_int nparams, const float* params );

lapack_int LAPACKE_zsysvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* s, lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double*
rpvgrw, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, const double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *diagonal pivoting* factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is an n-by-n real symmetric/Hermitian matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error (`O(eps)`, where `eps` is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with `O(eps)` errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine `?sysvxx` performs the following steps:

1. If *fact* = `'E'`, scaling factors are computed to equilibrate the system:

   diag(*s*)**A*diag*(*s*) *inv(*diag*(*s*))**X = diag*(*s*)**B*

Whether or not the system will be equilibrated depends on the scaling of the matrix A, but if equilibration is used, A is overwritten by $diag(s)*A*diag(s)$ and B by $diag(s)*B$.

2. If $fact$ = 'N' or 'E', the LU decomposition is used to factor the matrix A (after equilibration if $fact$ = 'E') as

   $A = U*D*U^T$, if $uplo$ = 'U',

   or $A = L*D*L^T$, if $uplo$ = 'L',

   where U or L is a product of permutation and unit upper (lower) triangular matrices, and D is a symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
3. If some $D(i,i)=0$, so that D is exactly singular, the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the $rcond$ parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A.
5. By default, unless params(la_linrx_itref_i) is set to zero, the routine applies iterative refinement to get a small error and error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by $diag(r)$ so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

$fact$	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored. If $fact$ = 'F', on entry, af and $ipiv$ contain the factored form of A. If $equed$ is not 'N', the matrix A has been equilibrated with scaling factors given by s. Parameters a, af, and $ipiv$ are not modified. If $fact$ = 'N', the matrix A will be copied to af and factored. If $fact$ = 'E', the matrix A will be equilibrated, if necessary, copied to af and factored.
$uplo$	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If $uplo$ = 'U', the upper triangle of A is stored. If $uplo$ = 'L', the lower triangle of A is stored.
n	INTEGER. The number of linear equations; the order of the matrix A; $n \geq 0$.
$nrhs$	INTEGER. The number of right-hand sides; the number of columns of the matrices B and X; $nrhs \geq 0$.
a, af, b, $work$	REAL for ssysvxx DOUBLE PRECISION for dsysvxx COMPLEX for csysvxx DOUBLE COMPLEX for zsysvxx. Arrays: $a(lda,*)$, $af(ldaf,*)$, $b(ldb,*)$, $work(*)$. The array a contains the symmetric matrix A as specified by $uplo$. If $uplo$ = 'U', the leading n-by-n upper triangular part of a contains the upper triangular part of the matrix A and the strictly lower triangular part of a is not referenced. If $uplo$ = 'L', the leading n-by-n lower triangular part of a

	contains the lower triangular part of the matrix *A* and the strictly upper triangular part of *a* is not referenced. The second dimension of *a* must be at least max(1,*n*). The array *af* is an input argument if *fact* = 'F'. It contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* and *L* from the factorization *A* = *U**D**U****T* or *A* = *L**D**L****T* as computed by ?sytrf. The second dimension of *af* must be at least max(1,*n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*). *work(*)* is a workspace array. The dimension of *work* must be at least max(1,4**n*) for real flavors, and at least max(1,2**n*) for complex flavors.
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*; *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D* as determined by ?sytrf. If *ipiv(k)* > 0, rows and columns *k* and *ipiv(k)* were intercanaged and *D(k,k)* is a 1-by-1 diagonal block. If *ipiv* = 'U' and *ipiv(k)* = *ipiv(k–1)* < 0, rows and columns *k–1* and *–ipiv(k)* were interchanged and *D*(*k*–1:*k*, *k*–1:*k*) is a 2-by-2 diagonal block. If *ipiv* = 'L' and *ipiv(k)* = *ipiv(k+1)* < 0, rows and columns *k+1* and *–ipiv(k)* were interchanged and *D*(*k*:*k*+1, *k*:*k*+1) is a 2-by-2 diagonal block.
equed	CHARACTER*1. Must be 'N' or 'Y'. *equed* is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done: If *equed* = 'N', no equilibration was done (always true if *fact* = 'N'). if *equed* = 'Y', both row and column equilibration was done, that is, *A* has been replaced by *diag(s)*A*diag(s)*.
s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*. If *equed* = 'Y', *A* is multiplied on the left and right by diag(*s*). This array is an input argument if *fact* = 'F' only; otherwise it is an output argument. If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive. Each element of *s* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).
ldx	INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).
n_err_bnds	INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in the *Output Arguments* section below.
nparams	INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the extra-precise refinement algorithm.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for fefinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

iwork

INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION at least max(1, 3*n); used in complex flavors only.

Output Parameters

x

REAL for ssysvxx
DOUBLE PRECISION for dsysvxx
COMPLEX for csysvxx
DOUBLE COMPLEX for zsysvxx.
Array, DIMENSION (*ldx*,*nrhs*).
If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the original system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the equilibrated system is: inv(*diag*(*s*))*X.

a

If *fact* = 'E' and *equed* = 'Y', overwritten by *diag*(*s*)*A*diag(*s*).

af

If *fact* = 'N', *af* is an output argument and on exit returns the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization *A* = *U*D*U*T* or *A* = *L*D*L*T*.

b

If *equed* = 'N', *B* is not modified.

If *equed* = 'Y', *B* is overwritten by *diag(s)*B*.

s — This array is an output argument if *fact* ≠ 'F'. Each element of this array is a power of the radix. See the description of *s* in *Input Arguments* section.

rcond — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Contains the reciprocal pivot growth factor *norm(A)/norm(U)*. The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the (equlibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \le n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*.

berr — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x(j)*, that is, the smallest relative change in any element of *A* or *B* that makes *x(j)* an exact solution.

err_bnds_norm — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1 — "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt(n)*slamch(ε)* for single precision flavors and *sqrt(n)*dlamch(ε)* for double precision flavors.

err=2 — "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt(n)*slamch(ε)* for single precision flavors and

sqrt(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are `1/(norm(1/z,inf)*norm(z,inf))` for some appropriately scaled matrix *Z*.
Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp `REAL` for single precision flavors
`DOUBLE PRECISION` for double precision flavors.
Array, `DIMENSION (`*nrhs*`,`*n_err_bnds*`)`. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{|Xtrue_{ji} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.

err=2 "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold

	sqrt(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/*z*,inf)*norm(*z*,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.
ipiv	If *fact* = 'N', *ipiv* is an output argument and on exit contains details of the interchanges and the block structure *D*, as determined by ssytrf for single precision flavors and dsytrf for double precision flavors.
equed	If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
params	If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.
info	INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed. If *info* = -*i*, the *i*-th parameter had an illegal value. If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(*j*,1) = 0.0 or *err_bnds_comp*(*j*,1) = 0.0. See the definition of *err_bnds_norm(;,1)* and *err_bnds_comp(;,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?hesv

Computes the solution to the system of linear equations with a Hermitian matrix A and multiple right-hand sides.

Syntax

Fortran 77:

```
call chesv( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, lwork, info )
call zhesv( uplo, n, nrhs, a, lda, ipiv, b, ldb, work, lwork, info )
```

Fortran 95:

```
call hesv( a, b [,uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hesv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
<datatype>* a, lapack_int lda, lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves for X the complex system of linear equations $A*X = B$, where A is an n-by-n symmetric matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U*D*U^H$ or $A = L*D*L^H$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If *uplo* = `'U'`, the array *a* stores the upper triangular part of the matrix A, and A is factored as $U*D*U^H$. If *uplo* = `'L'`, the array *a* stores the lower triangular part of the matrix A, and A is factored as $L*D*L^H$.
n	`INTEGER`. The order of matrix A; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides, the number of columns in B; $nrhs \geq 0$.
a, *b*, *work*	`COMPLEX` for `chesv` `DOUBLE COMPLEX` for `zhesv`. Arrays: *a*(*lda*,*), *b*(*ldb*,*), *work*(*). The array *a* contains the upper or the lower triangular part of the Hermitian matrix A (see *uplo*). The second dimension of *a* must be at least `max(1, n)`. The array *b* contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least `max(1,nrhs)`. *work* is a workspace array, dimension at least `max(1,lwork)`.
lda	`INTEGER`. The leading dimension of *a*; *lda* $\geq$ `max(1, n)`.
ldb	`INTEGER`. The leading dimension of *b*; *ldb* $\geq$ `max(1, n)`.
lwork	`INTEGER`. The size of the *work* array (*lwork* ≥ 1).

	If *lwork* = –1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* below for details and for the suggested value of *lwork*.

Output Parameters

a	If *info* = 0, *a* is overwritten by the block-diagonal matrix *D* and the multipliers used to obtain the factor *U* (or *L*) from the factorization of *A* as computed by ?hetrf.
b	If *info* = 0, *b* is overwritten by the solution matrix *X*.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*, as determined by ?hetrf. If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 diagonal block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column. If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column. If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)–th row and column of *A* was interchanged with the *m*-th row and column.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular, so the solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hesv interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For better performance, try using *lwork* = *n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = –1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hesvx

Uses the diagonal pivoting factorization to compute the solution to the complex system of linear equations with a Hermitian matrix A, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call chesvx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, lwork, rwork, info )
```

```
call zhesvx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, lwork, rwork, info )
```

Fortran 95:

```
call hesvx( a, b, x [,uplo] [,af] [,ipiv] [,fact] [,ferr] [,berr] [,rcond] [,info] )
```

C:

```
lapack_int LAPACKE_chesvx( int matrix_order, char fact, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, lapack_complex_float* af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zhesvx( int matrix_order, char fact, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, lapack_complex_double* af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations *A*X* = *B*, where *A* is an *n*-by-*n* Hermitian matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?hesvx` performs the following steps:

1. If *fact* = 'N', the diagonal pivoting method is used to factor the matrix *A*. The form of the factorization is *A* = *U***D***U*^{H} or A = *L***D***L*^{H}, where *U* (or *L*) is a product of permutation and unit upper (lower) triangular matrices, and *D* is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i}$ = 0, so that *D* is exactly singular, then the routine returns with *info* = *i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, *info* = *n*+1 is returned as a warning, but the routine still goes on to solve for *X* and compute error bounds as described below.
3. The system of equations is solved for *X* using the factored form of *A*.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F' or 'N'. Specifies whether or not the factored form of the matrix *A* has been supplied on entry. If *fact* = 'F': on entry, *af* and *ipiv* contain the factored form of *A*. Arrays *a*, *af*, and *ipiv* are not modified. If *fact* = 'N', the matrix *A* is copied to *af* and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored and how *A* is factored: If *uplo* = 'U', the array *a* stores the upper triangular part of the Hermitian matrix *A*, and *A* is factored as *U***D***U*^{H}. If *uplo* = 'L', the array *a* stores the lower triangular part of the Hermitian matrix *A*; *A* is factored as *L***D***L*^{H}.
n	INTEGER. The order of matrix *A*; *n* ≥ 0.
nrhs	INTEGER. The number of right-hand sides, the number of columns in *B*; *nrhs* ≥ 0.
a, *af*, *b*, *work*	COMPLEX for chesvx DOUBLE COMPLEX for zhesvx. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *work*(*). The array *a* contains the upper or the lower triangular part of the Hermitian matrix *A* (see *uplo*). The second dimension of *a* must be at least max(1,*n*). The array *af* is an input argument if *fact* = 'F'. It contains he block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization *A* = *U***D***U*^{H} or *A* = *L***D***L*^{H} as computed by ?hetrf. The second dimension of *af* must be at least max(1,*n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1, *nrhs*). *work*(*) is a workspace array of dimension at least max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).
ipiv	INTEGER.

Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D*, as determined by ?hetrf.
If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 diagonal block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column.
If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column.
If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

lwork — INTEGER. The size of the *work* array.
If *lwork* = –1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* below for details and for the suggested value of *lwork*.

rwork — REAL for chesvx
DOUBLE PRECISION for zhesvx.
Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x — COMPLEX for chesvx
DOUBLE COMPLEX for zhesvx.
Array, DIMENSION (*ldx*,*).
If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the system of equations. The second dimension of *x* must be at least max(1,*nrhs*).

af, *ipiv* — These arrays are output arguments if *fact* = 'N'. See the description of *af*, *ipiv* in *Input Arguments* section.

rcond — REAL for chesvx
DOUBLE PRECISION for zhesvx.
An estimate of the reciprocal condition number of the matrix *A*. If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.

ferr — REAL for chesvx
DOUBLE PRECISION for zhesvx.
Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) – *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcon*, and is almost always a slight overestimate of the true error.

berr — REAL for chesvx
DOUBLE PRECISION for zhesvx.

	Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and $i \leq n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n* + 1, then *D* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hesvx interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
af	Holds the matrix *AF* of size (*n*, *n*).
ipiv	Holds the vector of length *n*.
ferr	Holds the vector of length (*nrhs*).
berr	Holds the vector of length (*nrhs*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
fact	Must be 'N' or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.

Application Notes

The value of *lwork* must be at least 2*n. For better performance, try using *lwork* = *n*blocksize*, where *blocksize* is the optimal block size for ?hetrf.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hesvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a Hermitian indefinite matrix A applying the diagonal pivoting factorization.

Syntax

Fortran 77:

```
call chesvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, equed, s, b, ldb, x, ldx,
rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
rwork, info )
```

```
call zhesvxx( fact, uplo, n, nrhs, a, lda, af, ldaf, ipiv, equed, s, b, ldb, x, ldx,
rcond, rpvgrw, berr, n_err_bnds, err_bnds_norm, err_bnds_comp, nparams, params, work,
rwork, info )
```

C:

```
lapack_int LAPACKE_chesvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* s, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* rpvgrw,
float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp,
lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_zhesvxx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* s, lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double*
rpvgrw, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, const double* params );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the *diagonal pivoting* factorization to compute the solution to a complex/double complex system of linear equations *A*X = B*, where *A* is an *n*-by-*n* Hermitian matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error (`O(eps)`, where `eps` is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with `O(eps)` errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine ?hesvxx performs the following steps:

1. If *fact* = 'E', scaling factors are computed to equilibrate the system:

   *diag(s)*A*diag(s) *inv(diag(s))*X = diag(s)*B*

   Whether or not the system will be equilibrated depends on the scaling of the matrix *A*, but if equilibration is used, *A* is overwritten by *diag(s)*A*diag(s)* and *B* by *diag(s)*B*.
2. If *fact* = 'N' or 'E', the LU decomposition is used to factor the matrix *A* (after equilibration if *fact* = 'E') as

   $A = U*D*U^T$, if *uplo* = 'U',

   or $A = L*D*L^T$, if *uplo* = 'L',

   where *U* or *L* is a product of permutation and unit upper (lower) triangular matrices, and *D* is a symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
3. If some *D(i,i)*=0, so that *D* is exactly singular, the routine returns with *info* = *i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A* (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for *X* and compute error bounds.
4. The system of equations is solved for *X* using the factored form of *A*.
5. By default, unless params(la_linrx_itref_i) is set to zero, the routine applies iterative refinement to get a small error and error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix *X* is premultiplied by *diag(r)* so that it solves the original system before equilibration.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact	CHARACTER*1. Must be 'F', 'N', or 'E'. Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored. If *fact* = 'F', on entry, *af* and *ipiv* contain the factored form of *A*. If *equed* is not 'N', the matrix *A* has been equilibrated with scaling factors given by *s*. Parameters *a*, *af*, and *ipiv* are not modified. If *fact* = 'N', the matrix *A* will be copied to *af* and factored. If *fact* = 'E', the matrix *A* will be equilibrated, if necessary, copied to *af* and factored.
uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of *A* is stored: If *uplo* = 'U', the upper triangle of *A* is stored. If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrices *B* and *X*; $nrhs \geq 0$.
a, *af*, *b*, *work*	COMPLEX for chesvxx DOUBLE COMPLEX for zhesvxx. Arrays: *a(lda,*)*, *af(ldaf,*)*, *b(ldb,*)*, *work(*)*.

The array *a* contains the Hermitian matrix *A* as specified by *uplo*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A* and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A* and the strictly upper triangular part of *a* is not referenced. The second dimension of *a* must be at least max(1,*n*).
The array *af* is an input argument if *fact* = 'F'. It contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* and *L* from the factorization *A* = *U**D**U****T* or *A* = *L**D**L****T* as computed by ?hetrf. The second dimension of *af* must be at least max(1,*n*).
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
work(*) is a workspace array. The dimension of *work* must be at least max(1,2**n*).

lda — INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1,*n*).

ldaf — INTEGER. The leading dimension of the array *af*; *ldaf* ≥ max(1,*n*).

ipiv — INTEGER.
Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D* as determined by ?sytrf. If *ipiv(k)* > 0, rows and columns *k* and *ipiv(k)* were intercanaged and *D(k,k)* is a 1-by-1 diagonal block.
If *ipiv* = 'U' and *ipiv(k)* = *ipiv(k–1)* < 0, rows and columns *k–1* and *–ipiv(k)* were interchanged and *D*(*k*–1:*k*, *k*–1:*k*) is a 2-by-2 diagonal block.
If *ipiv* = 'L' and *ipiv(k)* = *ipiv(k+1)* < 0, rows and columns *k+1* and *–ipiv(k)* were interchanged and *D*(*k*:*k*+1, *k*:*k*+1) is a 2-by-2 diagonal block.

equed — CHARACTER*1. Must be 'N' or 'Y'.
equed is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done:
If *equed* = 'N', no equilibration was done (always true if *fact* = 'N').
if *equed* = 'Y', both row and column equilibration was done, that is, *A* has been replaced by *diag*(*s*)**A***diag*(*s*).

s — REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Array, DIMENSION (*n*). The array *s* contains the scale factors for *A*. If *equed* = 'Y', *A* is multiplied on the left and right by diag(*s*).
This array is an input argument if *fact* = 'F' only; otherwise it is an output argument.
If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive.
Each element of *s* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb — INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

n_err_bnds INTEGER. Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in the *Output Arguments* section below.

nparams INTEGER. Specifies the number of parameters set in *params*. If ≤ 0, the *params* array is never referenced and default values are used.

params REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION *nparams*. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.
params(la_linrx_itref_i = 1) : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0	No refinement is performed and no error bounds are computed.
=1.0	Use the extra-precise refinement algorithm.

(Other values are reserved for futute use.)
params(la_linrx_ithresh_i = 2) : Maximum number of resudual computations allowed for fefinement.

Default	10
Aggressive	Set to 100 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the quarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params(la_linrx_cwise_i = 3) : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

rwork REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Workspace array, DIMENSION at least max(1, 3*n); used in complex flavors only.

Output Parameters

x COMPLEX for chesvxx
DOUBLE COMPLEX for zhesvxx.
Array, DIMENSION (*ldx*,*nrhs*).
If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the original system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the equilibrated system is: inv(*diag*(*s*))*X.

a If *fact* = 'E' and *equed* = 'Y', overwritten by *diag*(*s*)*A*diag(*s*).

af If *fact* = 'N', *af* is an output argument and on exit returns the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization *A* = *U*D*U**T* or *A* = *L*D*L**T*.

b

If *equed* = 'N', *B* is not modified.
If *equed* = 'Y', *B* is overwritten by *diag(s)*B*.

s

This array is an output argument if *fact* ≠ 'F'. Each element of this array is a power of the radix. See the description of *s* in *Input Arguments* section.

rcond

REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw

REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Contains the reciprocal pivot growth factor *norm(A)/norm(U)*. The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the (equlibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \leq n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*.

berr

REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for each solution vector *x(j)*, that is, the smallest relative change in any element of *A* or *B* that makes *x(j)* an exact solution.

err_bnds_norm

REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the follwoing three fields:

err=1 — "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt(n)*slamch(ε)* for chesvxx and *sqrt(n)*dlamch(ε)* for zhesvxx.

err=2 — "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt(n)*slamch(ε)* for chesvxx and *sqrt(n)*dlamch(ε)* for zhesvxx. This error bound should only be trusted if the previous boolean is true.

	err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for chesvxx and *sqrt*(*n*)**dlamch*(ε) for zhesvxx to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

err_bnds_comp

REAL for chesvxx
DOUBLE PRECISION for zhesvxx.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for chesvxx and *sqrt*(*n*)**dlamch*(ε) for zhesvxx.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for chesvxx and *sqrt*(*n*)**dlamch*(ε) for zhesvxx. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for chesvxx and *sqrt*(*n*)**dlamch*(ε) for zhesvxx to determine if the error estimate is "guaranteed". These

reciprocal condition numbers are `1/(norm(1/z,inf)*norm(z,inf))` for some appropriately scaled matrix *Z*.
Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1.

ipiv — If *fact* = 'N', *ipiv* is an output argument and on exit contains details of the interchanges and the block structure *D*, as determined by ssytrf for single precision flavors and dsytrf for double precision flavors.

equed — If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).

params — If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

info — INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If 0 < *info* ≤ *n*: *U*(*info*,*info*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params(3)* = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that *err_bnds_norm*(j,1) = 0.0 or *err_bnds_comp*(j,1) = 0.0. See the definition of *err_bnds_norm(;,1)* and *err_bnds_comp(;,1)*. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

?spsv

Computes the solution to the system of linear equations with a real or complex symmetric matrix A stored in packed format, and multiple right-hand sides.

Syntax

Fortran 77:

```
call sspsv( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call dspsv( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call cspsv( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call zspsv( uplo, n, nrhs, ap, ipiv, b, ldb, info )
```

Fortran 95:

```
call spsv( ap, b [,uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>spsv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
<datatype>* ap, lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the real or complex system of linear equations $A*X = B$, where A is an n-by-n symmetric matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U*D*U^T$ or $A = L*D*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If *uplo* = 'U', the upper triangle of A is stored. If *uplo* = 'L', the lower triangle of A is stored.
n	INTEGER. The order of matrix A; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides, the number of columns in B; $nrhs \geq 0$.
ap, *b*	REAL for sspsv DOUBLE PRECISION for dspsv COMPLEX for cspsv DOUBLE COMPLEX for zspsv. Arrays: *ap*(*), *b*(*ldb*,*). The dimension of *ap* must be at least max(1,*n*(*n*+1)/2). The array *ap* contains the factor U or L, as specified by *uplo*, in *packed storage* (see Matrix Storage Schemes). The array *b* contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

ap	The block-diagonal matrix D and the multipliers used to obtain the factor U (or L) from the factorization of A as computed by ?sptrf, stored as a packed triangular matrix in the same storage format as A.
b	If *info* = 0, *b* is overwritten by the solution matrix X.
ipiv	INTEGER.

	Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*, as determined by ?sptrf. If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column. If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column. If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular, so the solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spsv interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
ipiv	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

?spsvx

Uses the diagonal pivoting factorization to compute the solution to the system of linear equations with a real or complex symmetric matrix A stored in packed format, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call sspsvx( fact, uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, rcond, ferr, berr,
work, iwork, info )
call dspsvx( fact, uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, rcond, ferr, berr,
work, iwork, info )
call cspsvx( fact, uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, rcond, ferr, berr,
work, rwork, info )
call zspsvx( fact, uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, rcond, ferr, berr,
work, rwork, info )
```

Fortran 95:

```
call spsvx( ap, b, x [,uplo] [,afp] [,ipiv] [,fact] [,ferr] [,berr] [,rcond] [,info] )
```

C:

```
lapack_int LAPACKE_sspsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, const float* ap, float* afp, lapack_int* ipiv, const float* b,
lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_dspsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, const double* ap, double* afp, lapack_int* ipiv, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );

lapack_int LAPACKE_cspsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_float* ap, lapack_complex_float* afp, lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* ferr, float* berr );

lapack_int LAPACKE_zspsvx( int matrix_order, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_double* ap, lapack_complex_double* afp,
lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the diagonal pivoting factorization to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n-by-n symmetric matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?spsvx` performs the following steps:

1. If $fact = 'N'$, the diagonal pivoting method is used to factor the matrix A. The form of the factorization is $A = U*D*U^T$ or $A = L*D*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i} = 0$, so that D is exactly singular, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact

`CHARACTER*1`. Must be `'F'` or `'N'`.
Specifies whether or not the factored form of the matrix A has been supplied on entry.
If $fact = 'F'$: on entry, *afp* and *ipiv* contain the factored form of A. Arrays *ap*, *afp*, and *ipiv* are not modified.

If *fact* = 'N', the matrix *A* is copied to *afp* and factored.

uplo — CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored and how *A* is factored:
If *uplo* = 'U', the array *ap* stores the upper triangular part of the symmetric matrix *A*, and *A* is factored as $U*D*U^T$.
If *uplo* = 'L', the array *ap* stores the lower triangular part of the symmetric matrix *A*; *A* is factored as $L*D*L^T$.

n — INTEGER. The order of matrix *A*; $n \geq 0$.

nrhs — INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.

ap, *afp*, *b*, *work* — REAL for sspsvx
DOUBLE PRECISION for dspsvx
COMPLEX for cspsvx
DOUBLE COMPLEX for zspsvx.
Arrays: *ap*(*), *afp*(*), *b*(*ldb*,*), *work*(*).
The array *ap* contains the upper or lower triangle of the symmetric matrix *A* in *packed storage* (see Matrix Storage Schemes).
The array *afp* is an input argument if *fact* = 'F'. It contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization $A = U*D*U^T$ or $A = L*D*L^T$ as computed by ?sptrf, in the same storage format as *A*.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
work(*) is a workspace array.
The dimension of arrays *ap* and *afp* must be at least max(1, *n*(*n*+1)/2); the second dimension of *b* must be at least max(1,*nrhs*); the dimension of *work* must be at least max(1,3**n*) for real flavors and max(1,2**n*) for complex flavors.

ldb — INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

ipiv — INTEGER.
Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D*, as determined by ?sptrf.
If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 diagonal block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column.
If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column.
If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.

ldx — INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

iwork — INTEGER. Workspace array, DIMENSION at least max(1, *n*); used in real flavors only.

rwork — REAL for cspsvx
DOUBLE PRECISION for zspsvx.

	Workspace array, DIMENSION at least max(1, *n*); used in complex flavors only.

Output Parameters

x	REAL for sspsvx DOUBLE PRECISION for dspsvx COMPLEX for cspsvx DOUBLE COMPLEX for zspsvx. Array, DIMENSION (*ldx*,*). If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the system of equations. The second dimension of *x* must be at least max(1,*nrhs*).
afp, *ipiv*	These arrays are output arguments if *fact* = 'N'. See the description of *afp*, *ipiv* in *Input Arguments* section.
rcond	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A*. If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.
ferr, *berr*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *nrhs*). Contain the component-wise forward and relative backward errors, respectively, for each solution vector.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and $i \leq n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and *i* = *n* + 1, then *D* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spsvx interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*,*nrhs*).
x	Holds the matrix *X* of size (*n*,*nrhs*).
afp	Holds the array *AF* of size (*n**(*n*+1)/2).
ipiv	Holds the vector with the number of elements *n*.
ferr	Holds the vector with the number of elements *nrhs*.

berr	Holds the vector with the number of elements *nrhs*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
fact	Must be 'N' or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.

?hpsv

Computes the solution to the system of linear equations with a Hermitian matrix A stored in packed format, and multiple right-hand sides.

Syntax

Fortran 77:

```
call chpsv( uplo, n, nrhs, ap, ipiv, b, ldb, info )
call zhpsv( uplo, n, nrhs, ap, ipiv, b, ldb, info )
```

Fortran 95:

```
call hpsv( ap, b [,uplo] [,ipiv] [,info] )
```

C:

```
lapack_int LAPACKE_<?>hpsv( int matrix_order, char uplo, lapack_int n, lapack_int nrhs,
<datatype>* ap, lapack_int* ipiv, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves for X the system of linear equations $A*X = B$, where A is an n-by-n Hermitian matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U*D*U^H$ or $A = L*D*L^H$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If *uplo* = 'U', the upper triangle of A is stored. If *uplo* = 'L', the lower triangle of A is stored.
n	INTEGER. The order of matrix A; $n \geq 0$.

nrhs	INTEGER. The number of right-hand sides; the number of columns in *B*; *nrhs* ≥ 0.
ap, *b*	COMPLEX for chpsv DOUBLE COMPLEX for zhpsv. Arrays: *ap*(*), *b*(*ldb*,*). The dimension of *ap* must be at least max(1,*n*(*n*+1)/2). The array *ap* contains the factor *U* or *L*, as specified by *uplo*, in *packed storage* (see Matrix Storage Schemes). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*).
ldb	INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

Output Parameters

ap	The block-diagonal matrix *D* and the multipliers used to obtain the factor *U* (or *L*) from the factorization of *A* as computed by ?hptrf, stored as a packed triangular matrix in the same storage format as *A*.
b	If *info* = 0, *b* is overwritten by the solution matrix *X*.
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Contains details of the interchanges and the block structure of *D*, as determined by ?hptrf. If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column. If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column. If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*-th row and column.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular, so the solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpsv interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*,*nrhs*).
ipiv	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.

?hpsvx

Uses the diagonal pivoting factorization to compute the solution to the system of linear equations with a Hermitian matrix A stored in packed format, and provides error bounds on the solution.

Syntax

Fortran 77:

```
call chpsvx( fact, uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, rwork, info )
```

```
call zhpsvx( fact, uplo, n, nrhs, ap, afp, ipiv, b, ldb, x, ldx, rcond, ferr, berr, work, rwork, info )
```

Fortran 95:

```
call hpsvx( ap, b, x [,uplo] [,afp] [,ipiv] [,fact] [,ferr] [,berr] [,rcond] [,info] )
```

C:

```
lapack_int LAPACKE_chpsvx( int matrix_order, char fact, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_float* ap, lapack_complex_float* afp, lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zhpsvx( int matrix_order, char fact, char uplo, lapack_int n, lapack_int nrhs, const lapack_complex_double* ap, lapack_complex_double* afp, lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations $A*X = B$, where A is a n-by-n Hermitian matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?hpsvx` performs the following steps:

1. If $fact = 'N'$, the diagonal pivoting method is used to factor the matrix A. The form of the factorization is $A = U*D*U^H$ or $A = L*D*L^H$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is a Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i} = 0$, so that D is exactly singular, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A. If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

fact
CHARACTER*1. Must be 'F' or 'N'.
Specifies whether or not the factored form of the matrix *A* has been supplied on entry.
If *fact* = 'F': on entry, *afp* and *ipiv* contain the factored form of *A*. Arrays *ap*, *afp*, and *ipiv* are not modified.
If *fact* = 'N', the matrix *A* is copied to *afp* and factored.

uplo
CHARACTER*1. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored and how *A* is factored:
If *uplo* = 'U', the array *ap* stores the upper triangular part of the Hermitian matrix *A*, and *A* is factored as $U*D*U^H$.
If *uplo* = 'L', the array *ap* stores the lower triangular part of the Hermitian matrix *A*, and *A* is factored as $L*D*L^H$.

n
INTEGER. The order of matrix *A*; $n \geq 0$.

nrhs
INTEGER. The number of right-hand sides, the number of columns in *B*; $nrhs \geq 0$.

ap, *afp*, *b*, *work*
COMPLEX for chpsvx
DOUBLE COMPLEX for zhpsvx.
Arrays: *ap*(*), *afp*(*), *b*(*ldb*,*), *work*(*).
The array *ap* contains the upper or lower triangle of the Hermitian matrix *A* in *packed storage* (see Matrix Storage Schemes).
The array *afp* is an input argument if *fact* = 'F'. It contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization $A = U*D*U^H$ or $A = L*D*L^H$ as computed by ?hptrf, in the same storage format as *A*.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
work(*) is a workspace array.
The dimension of arrays *ap* and *afp* must be at least max(1,*n*(*n*+1)/2); the second dimension of *b* must be at least max(1,*nrhs*); the dimension of *work* must be at least max(1,2**n*).

ldb
INTEGER. The leading dimension of *b*; *ldb* ≥ max(1, *n*).

ipiv
INTEGER.
Array, DIMENSION at least max(1, *n*). The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D*, as determined by ?hptrf.
If *ipiv*(*i*) = *k* > 0, then d_{ii} is a 1-by-1 diagonal block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column.
If *uplo* = 'U' and *ipiv*(*i*) =*ipiv*(*i*-1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*–1, and (*i*–1)-th row and column of *A* was interchanged with the *m*-th row and column.
If *uplo* = 'L' and *ipiv*(*i*) =*ipiv*(*i*+1) = -*m* < 0, then *D* has a 2-by-2 block in rows/columns *i* and *i*+1, and (*i*+1)-th row and column of *A* was interchanged with the *m*–th row and column.

ldx INTEGER. The leading dimension of the output array *x*; *ldx* ≥ max(1, *n*).

rwork REAL for chpsvx
DOUBLE PRECISION for zhpsvx.
Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

x COMPLEX for chpsvx
DOUBLE COMPLEX for zhpsvx.
Array, DIMENSION (*ldx*,*).
If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the system of equations. The second dimension of *x* must be at least max(1,*nrhs*).

afp, *ipiv* These arrays are output arguments if *fact* = 'N'. See the description of *afp*, *ipiv* in *Input Arguments* section.

rcond REAL for chpsvx
DOUBLE PRECISION for zhpsvx.
An estimate of the reciprocal condition number of the matrix *A*. If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.

ferr REAL for chpsvx
DOUBLE PRECISION for zhpsvx.
Array, DIMENSION at least max(1, *nrhs*). Contains the estimated forward error bound for each solution vector *x*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to *x*(*j*), *ferr*(*j*) is an estimated upper bound for the magnitude of the largest element in (*x*(*j*) – *xtrue*) divided by the magnitude of the largest element in *x*(*j*). The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.

berr REAL for chpsvx
DOUBLE PRECISION for zhpsvx.
Array, DIMENSION at least max(1, *nrhs*). Contains the component-wise relative backward error for each solution vector *x*(*j*), that is, the smallest relative change in any element of *A* or *B* that makes *x*(*j*) an exact solution.

info INTEGER. If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, and *i* ≤ *n*, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.
If *info* = *i*, and *i* = *n* + 1, then *D* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `hpsvx` interface are as follows:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
b	Holds the matrix *B* of size (*n*, *nrhs*).
x	Holds the matrix *X* of size (*n*, *nrhs*).
afp	Holds the array *AF* of size (*n**(*n*+1)/2).
ipiv	Holds the vector with the number of elements *n*.
ferr	Holds the vector with the number of elements *nrhs*.
berr	Holds the vector with the number of elements *nrhs*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
fact	Must be 'N' or 'F'. The default value is 'N'. If *fact* = 'F', then both arguments *af* and *ipiv* must be present; otherwise, an error is returned.

LAPACK Routines: Least Squares and Eigenvalue Problems

4

This chapter describes the Intel® Math Kernel Library implementation of routines from the LAPACK package that are used for solving linear least squares problems, eigenvalue and singular value problems, as well as performing a number of related computational tasks.

Sections in this chapter include descriptions of LAPACK computational routines and driver routines. For full reference on LAPACK routines and related information see [LUG].

Least Squares Problems. A typical *least squares problem* is as follows: given a matrix A and a vector b, find the vector x that minimizes the sum of squares $\Sigma_i((Ax)_i - b_i)^2$ or, equivalently, find the vector x that minimizes the 2-norm $||Ax - b||_2$.

In the most usual case, A is an m-by-n matrix with $m \geq n$ and $\mathrm{rank}(A) = n$. This problem is also referred to as finding the *least squares solution* to an *overdetermined* system of linear equations (here we have more equations than unknowns). To solve this problem, you can use the QR factorization of the matrix A (see QR Factorization).

If $m < n$ and $\mathrm{rank}(A) = m$, there exist an infinite number of solutions x which exactly satisfy $Ax = b$, and thus minimize the norm $||Ax - b||_2$. In this case it is often useful to find the unique solution that minimizes $||x||_2$. This problem is referred to as finding the *minimum-norm solution* to an *underdetermined* system of linear equations (here we have more unknowns than equations). To solve this problem, you can use the LQ factorization of the matrix A (see LQ Factorization).

In the general case you may have a *rank-deficient least squares problem*, with $\mathrm{rank}(A) < \min(m, n)$: find the *minimum-norm least squares solution* that minimizes both $||x||_2$ and $||Ax - b||^2$. In this case (or when the rank of A is in doubt) you can use the QR factorization with pivoting or *singular value decomposition* (see Singular Value Decomposition).

Eigenvalue Problems. The eigenvalue problems (from German *eigen* "own") are stated as follows: given a matrix A, find the *eigenvalues* λ and the corresponding *eigenvectors* z that satisfy the equation

$Az = \lambda z$ (right eigenvectors z)

or the equation

$z^H A = \lambda z^H$ (left eigenvectors z).

If A is a real symmetric or complex Hermitian matrix, the above two equations are equivalent, and the problem is called a *symmetric* eigenvalue problem. Routines for solving this type of problems are described in the sectionSymmetric Eigenvalue Problems .

Routines for solving eigenvalue problems with nonsymmetric or non-Hermitian matrices are described in the sectionNonsymmetric Eigenvalue Problems.

The library also includes routines that handle *generalized symmetric-definite eigenvalue problems*: find the eigenvalues λ and the corresponding eigenvectors x that satisfy one of the following equations:

$Az = \lambda Bz$, $ABz = \lambda z$, or $BAz = \lambda z$,

where A is symmetric or Hermitian, and B is symmetric positive-definite or Hermitian positive-definite. Routines for reducing these problems to standard symmetric eigenvalue problems are described in the sectionGeneralized Symmetric-Definite Eigenvalue Problems.

To solve a particular problem, you usually call several computational routines. Sometimes you need to combine the routines of this chapter with other LAPACK routines described in Chapter 3 as well as with BLAS routines described in Chapter 2.

For example, to solve a set of least squares problems minimizing $||Ax - b||^2$ for all columns b of a given matrix B (where A and B are real matrices), you can call ?geqrf to form the factorization $A = QR$, then call ?ormqr to compute $C = Q^H B$ and finally call the BLAS routine ?trsm to solve for X the system of equations $RX = C$.

Another way is to call an appropriate driver routine that performs several tasks in one call. For example, to solve the least squares problem the driver routine ?gels can be used.

WARNING LAPACK routines assume that input matrices do not contain IEEE 754 special values such as INF or NaN values. Using these special values may cause LAPACK to return unexpected results or become unstable.

Starting from release 8.0, Intel MKL along with the FORTRAN 77 interface to LAPACK computational and driver routines supports also the Fortran 95 interface, which uses simplified routine calls with shorter argument lists. The syntax section of the routine description gives the calling sequence for the Fortran 95 interface, where available, immediately after the FORTRAN 77 calls.

Routine Naming Conventions

For each routine in this chapter, when calling it from the FORTRAN 77 program you can use the LAPACK name.

LAPACK names have the structure xyyzzz, which is explained below.

The initial letter x indicates the data type:

s	real, single precision
c	complex, single precision
d	real, double precision
z	complex, double precision

The second and third letters yy indicate the matrix type and storage scheme:

bb	bidiagonal-block matrix
bd	bidiagonal matrix
ge	general matrix
gb	general band matrix
hs	upper Hessenberg matrix
or	(real) orthogonal matrix
op	(real) orthogonal matrix (packed storage)
un	(complex) unitary matrix
up	(complex) unitary matrix (packed storage)
pt	symmetric or Hermitian positive-definite tridiagonal matrix
sy	symmetric matrix
sp	symmetric matrix (packed storage)
sb	(real) symmetric band matrix
st	(real) symmetric tridiagonal matrix
he	Hermitian matrix
hp	Hermitian matrix (packed storage)
hb	(complex) Hermitian band matrix
tr	triangular or quasi-triangular matrix.

The last three letters zzz indicate the computation performed, for example:

qrf	form the QR factorization
lqf	form the LQ factorization.

Thus, the routine `sgeqrf` forms the QR factorization of general real matrices in single precision; the corresponding routine for complex matrices is `cgeqrf`.

Names of the LAPACK computational and driver routines for the Fortran 95 interface in Intel MKL are the same as the FORTRAN 77 names but without the first letter that indicates the data type. For example, the name of the routine that forms the QR factorization of general real matrices in the Fortran 95 interface is `geqrf`. Handling of different data types is done through defining a specific internal parameter referring to a module block with named constants for single and double precision.

For details on the design of the Fortran 95 interface for LAPACK computational and driver routines in Intel MKL and for the general information on how the optional arguments are reconstructed, see the Fortran 95 Interface Conventions in chapter 3 .

Matrix Storage Schemes

LAPACK routines use the following matrix storage schemes:

- *Full storage*: a matrix A is stored in a two-dimensional array a, with the matrix element a_{ij} stored in the array element $a(i,j)$.
- *Packed storage* scheme allows you to store symmetric, Hermitian, or triangular matrices more compactly: the upper or lower triangle of the matrix is packed by columns in a one-dimensional array.
- *Band storage*: an m-by-n band matrix with kl sub-diagonals and ku super-diagonals is stored compactly in a two-dimensional array ab with $kl+ku+1$ rows and n columns. Columns of the matrix are stored in the corresponding columns of the array, and *diagonals* of the matrix are stored in rows of the array.

In Chapters 3 and 4 , arrays that hold matrices in the packed storage have names ending in p; arrays with matrices in the band storage have names ending in b. For more information on matrix storage schemes, see "Matrix Arguments" in Appendix B .

Mathematical Notation

In addition to the mathematical notation used in description of BLAS and LAPACK Linear Equations routines, descriptions of the routines to solve Least Squares and Eigenvalue plroblems use the following notation:

λ_i	*Eigenvalues* of the matrix A (for the definition of eigenvalues, see Eigenvalue Problems).
σ_i	*Singular values* of the matrix A. They are equal to square roots of the eigenvalues of A^HA. (For more information, see Singular Value Decomposition).
$\|\|x\|\|_2$	The *2-norm* of the vector x: $\|\|x\|\|_2 = (\Sigma_i \|x_i\|^2)^{1/2} = \|\|x\|\|_E$.
$\|\|A\|\|_2$	The *2-norm* (or *spectral norm*) of the matrix A. $\|\|A\|\|_2 = \max_i \sigma_i$, $\|\|A\|\|_2^2 = \max_{\|x\|=1} (Ax \cdot Ax)$.
$\|\|A\|\|_E$	The *Euclidean norm* of the matrix A: $\|\|A\|\|_E^2 = \Sigma_i \Sigma_j \|a_{ij}\|^2$ (for vectors, the Euclidean norm and the 2-norm are equal: $\|\|x\|\|_E = \|\|x\|\|_2$).
$q(x, y)$	The *acute angle between vectors* x and y: $\cos q(x, y) = \|x \cdot y\| / (\|\|x\|\|_2 \|\|y\|\|_2)$.

Computational Routines

In the sections that follow, the descriptions of LAPACK computational routines are given. These routines perform distinct computational tasks that can be used for:

Orthogonal Factorizations

Singular Value Decomposition

Symmetric Eigenvalue Problems

Generalized Symmetric-Definite Eigenvalue Problems

Nonsymmetric Eigenvalue Problems

Generalized Nonsymmetric Eigenvalue Problems

Generalized Singular Value Decomposition

See also the respective driver routines.

Orthogonal Factorizations

This section describes the LAPACK routines for the *QR (RQ)* and *LQ (QL)* factorization of matrices. Routines for the *RZ* factorization as well as for generalized *QR* and *RQ* factorizations are also included.

QR Factorization. Assume that *A* is an *m*-by-*n* matrix to be factored.

If $m \geq n$, the *QR* factorization is given by

where *R* is an *n*-by-*n* upper triangular matrix with real diagonal elements, and *Q* is an *m*-by-*m* orthogonal (or unitary) matrix.

You can use the *QR* factorization for solving the following least squares problem: minimize $||Ax - b||^2$ where *A* is a full-rank *m*-by-*n* matrix ($m \geq n$). After factoring the matrix, compute the solution *x* by solving $Rx = (Q_1)^T b$.

If $m < n$, the *QR* factorization is given by

$A = QR = Q(R_1 R_2)$

where *R* is trapezoidal, R_1 is upper triangular and R_2 is rectangular.

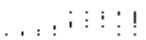
The LAPACK routines do not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of min(*m*, *n*) *elementary reflectors*. Routines are provided to work with *Q* in this representation.

LQ Factorization LQ factorization of an *m*-by-*n* matrix *A* is as follows. If $m \leq n$,

$$A = (L, 0)\,Q = (L, 0)\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = (LQ_1)$$

where *L* is an *m*-by-*m* lower triangular matrix with real diagonal elements, and *Q* is an *n*-by-*n* orthogonal (or unitary) matrix.

If $m > n$, the *LQ* factorization is



where L_1 is an *n*-by-*n* lower triangular matrix, L_2 is rectangular, and *Q* is an *n*-by-*n* orthogonal (or unitary) matrix.

You can use the *LQ* factorization to find the minimum-norm solution of an underdetermined system of linear equations $Ax = b$ where *A* is an *m*-by-*n* matrix of rank m ($m < n$). After factoring the matrix, compute the solution vector *x* as follows: solve $Ly = b$ for *y*, and then compute $x = (Q_1)^H y$.

Table "Computational Routines for Orthogonal Factorization" lists LAPACK routines (FORTRAN 77 interface) that perform orthogonal factorization of matrices. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Orthogonal Factorization

Matrix type, factorization	Factorize without pivoting	Factorize with pivoting	Generate matrix Q	Apply matrix Q
general matrices, QR factorization	geqrf	geqpf	orgqr	ormqr
	geqrfp	geqp3	ungqr	unmqr
general matrices, RQ factorization	gerqf		orgrq	ormrq
			ungrq	unmrq
general matrices, LQ factorization	gelqf		orglq	ormlq
			unglq	unmlq
general matrices, QL factorization	geqlf		orgql	ormql
			ungql	unmql
trapezoidal matrices, RZ factorization	tzrzf			ormrz
				unmrz
pair of matrices, generalized QR factorization	ggqrf			
pair of matrices, generalized RQ factorization	ggrqf			

?geqrf

Computes the QR factorization of a general m-by-n matrix.

Syntax

Fortran 77:

```
call sgeqrf(m, n, a, lda, tau, work, lwork, info)
call dgeqrf(m, n, a, lda, tau, work, lwork, info)
call cgeqrf(m, n, a, lda, tau, work, lwork, info)
call zgeqrf(m, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call geqrf(a [, tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>geqrf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the QR factorization of a general m-by-n matrix A (see Orthogonal Factorizations). No pivoting is performed.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors*. Routines are provided to work with Q in this representation.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, *work*	REAL for sgeqrf DOUBLE PRECISION for dgeqrf COMPLEX for cgeqrf DOUBLE COMPLEX for zgeqrf. Arrays: *a*(*lda*,*) contains the matrix A. The second dimension of *a* must be at least max(1, n). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, m).
lwork	INTEGER. The size of the *work* array (*lwork* $\geq n$). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See Application Notes for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the factorization data as follows: If $m \geq n$, the elements below the diagonal are overwritten by the details of the unitary matrix Q, and the upper triangle is overwritten by the corresponding elements of the upper triangular matrix R. If $m < n$, the strictly lower triangular part is overwritten by the details of the unitary matrix Q, and the remaining elements are overwritten by the corresponding elements of the m-by-n upper trapezoidal matrix R.
tau	REAL for sgeqrf DOUBLE PRECISION for dgeqrf COMPLEX for cgeqrf DOUBLE COMPLEX for zgeqrf. Array, DIMENSION at least max (1, min(m, n)). Contains additional information on the matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine *gegrf* interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length min(*m*,*n*)

Application Notes

For better performance, try using *lwork* = *n*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed factorization is the exact factorization of a matrix $A + E$, where

$||E||_2 = O(\varepsilon)||A||_2$.

The approximate number of floating-point operations for real flavors is

$(4/3)n^3$	if $m = n$,
$(2/3)n^2(3m-n)$	if $m > n$,
$(2/3)m^2(3n-m)$	if $m < n$.

The number of operations for complex flavors is 4 times greater.

To solve a set of least squares problems minimizing $||A*x - b||_2$ for all columns b of a given matrix B, you can call the following:

?geqrf (this routine)	to factorize $A = QR$;
ormqr	to compute $C = Q^T*B$ (for real matrices);
unmqr	to compute $C = Q^H*B$ (for complex matrices);
trsm (a BLAS routine)	to solve $R*X = C$.

(The columns of the computed X are the least squares solution vectors x.)

To compute the elements of Q explicitly, call

orgqr	(for real matrices)
ungqr	(for complex matrices).

See Also

mkl_progress

?geqrfp

Computes the QR factorization of a general m-by-n matrix with non-negative diagonal elements.

Syntax

Fortran 77:

```
call sgeqrfp(m, n, a, lda, tau, work, lwork, info)
call dgeqrfp(m, n, a, lda, tau, work, lwork, info)
call cgeqrfp(m, n, a, lda, tau, work, lwork, info)
call zgeqrfp(m, n, a, lda, tau, work, lwork, info)
```

C:

```
lapack_int LAPACKE_<?>geqrfp( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, <datatype>* tau );
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`
- C: `mkl_lapacke.h`

Description

The routine forms the QR factorization of a general m-by-n matrix A (see Orthogonal Factorizations). No pivoting is performed.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors*. Routines are provided to work with Q in this representation.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, *work*	REAL for sgeqrfp DOUBLE PRECISION for dgeqrfp COMPLEX for cgeqrfp DOUBLE COMPLEX for zgeqrfp. Arrays: a(*lda*,*) contains the matrix A. The second dimension of a must be at least max(1, n). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of a; at least max(1, m).
lwork	INTEGER. The size of the *work* array (*lwork* $\geq n$). If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

See Application Notes for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the factorization data as follows: If $m \geq n$, the elements below the diagonal are overwritten by the details of the unitary matrix Q, and the upper triangle is overwritten by the corresponding elements of the upper triangular matrix R. If $m < n$, the strictly lower triangular part is overwritten by the details of the unitary matrix Q, and the remaining elements are overwritten by the corresponding elements of the m-by-n upper trapezoidal matrix R. The diagonal elements of the matrix R are non-negative.
tau	REAL for sgeqrfp DOUBLE PRECISION for dgeqrfp COMPLEX for cgeqrfp DOUBLE COMPLEX for zgeqrfp. Array, DIMENSION at least max (1, min(m, n)). Contains additional information on the matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = $-i$, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine *geqrfp* interface are the following:

a	Holds the matrix A of size (m,n).
tau	Holds the vector of length min(m,n)

Application Notes

For better performance, try using *lwork* = *n*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed factorization is the exact factorization of a matrix $A + E$, where

$||E||_2 = O(\varepsilon)||A||_2$.

The approximate number of floating-point operations for real flavors is

$(4/3)n^3$	if $m = n$,
$(2/3)n^2(3m-n)$	if $m > n$,
$(2/3)m^2(3n-m)$	if $m < n$.

The number of operations for complex flavors is 4 times greater.

To solve a set of least squares problems minimizing $||A*x - b||_2$ for all columns b of a given matrix B, you can call the following:

?geqrfp (this routine)	to factorize $A = QR$;
ormqr	to compute $C = Q^T*B$ (for real matrices);
unmqr	to compute $C = Q^H*B$ (for complex matrices);
trsm (a BLAS routine)	to solve $R*X = C$.

(The columns of the computed X are the least squares solution vectors x.)

To compute the elements of Q explicitly, call

orgqr	(for real matrices)
ungqr	(for complex matrices).

See Also

mkl_progress

?geqpf

Computes the QR factorization of a general m-by-n matrix with pivoting.

Syntax

Fortran 77:

```
call sgeqpf(m, n, a, lda, jpvt, tau, work, info)
call dgeqpf(m, n, a, lda, jpvt, tau, work, info)
call cgeqpf(m, n, a, lda, jpvt, tau, work, rwork, info)
call zgeqpf(m, n, a, lda, jpvt, tau, work, rwork, info)
```

Fortran 95:

```
call geqpf(a, jpvt [,tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>geqpf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, lapack_int* jpvt, <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine is deprecated and has been replaced by routine geqp3.

The routine ?geqpf forms the QR factorization of a general m-by-n matrix A with column pivoting: $A*P = Q*R$ (see Orthogonal Factorizations). Here P denotes an n-by-n permutation matrix.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors*. Routines are provided to work with Q in this representation.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, *work*	REAL for sgeqpf DOUBLE PRECISION for dgeqpf COMPLEX for cgeqpf DOUBLE COMPLEX for zgeqpf. Arrays: a (lda,*) contains the matrix A. The second dimension of a must be at least max(1, n). $work$ ($lwork$) is a workspace array. The size of the $work$ array must be at least max(1, 3*n) for real flavors and at least max(1, n) for complex flavors.
lda	INTEGER. The leading dimension of a; at least max(1, m).
jpvt	INTEGER. Array, DIMENSION at least max(1, n). On entry, if $jpvt(i) > 0$, the i-th column of A is moved to the beginning of $A*P$ before the computation, and fixed in place during the computation. If $jpvt(i) = 0$, the ith column of A is a free column (that is, it may be interchanged during the computation with any other free column).
rwork	REAL for cgeqpf DOUBLE PRECISION for zgeqpf. A workspace array, DIMENSION at least max(1, 2*n).

Output Parameters

a	Overwritten by the factorization data as follows: If $m \geq n$, the elements below the diagonal are overwritten by the details of the unitary (orthogonal) matrix Q, and the upper triangle is overwritten by the corresponding elements of the upper triangular matrix R. If $m < n$, the strictly lower triangular part is overwritten by the details of the matrix Q, and the remaining elements are overwritten by the corresponding elements of the m-by-n upper trapezoidal matrix R.
tau	REAL for sgeqpf DOUBLE PRECISION for dgeqpf COMPLEX for cgeqpf DOUBLE COMPLEX for zgeqpf. Array, DIMENSION at least max (1, min(m, n)). Contains additional information on the matrix Q.
jpvt	Overwritten by details of the permutation matrix P in the factorization $A*P = Q*R$. More precisely, the columns of $A*P$ are the columns of A in the following order: $jpvt(1), jpvt(2), ..., jpvt(n)$.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `geqpf` interface are the following:

a	Holds the matrix A of size (m,n).
jpvt	Holds the vector of length n.
tau	Holds the vector of length min(m,n)

Application Notes

The computed factorization is the exact factorization of a matrix $A + E$, where

$||E||_2 = O(\varepsilon)||A||_2$.

The approximate number of floating-point operations for real flavors is

$(4/3)n^3$	if $m = n$,
$(2/3)n^2(3m-n)$	if $m > n$,
$(2/3)m^2(3n-m)$	if $m < n$.

The number of operations for complex flavors is 4 times greater.

To solve a set of least squares problems minimizing $||A*x - b||_2$ for all columns b of a given matrix B, you can call the following:

`?geqpf` (this routine)	to factorize $A*P = Q*R$;
ormqr	to compute $C = Q^T*B$ (for real matrices);
unmqr	to compute $C = Q^H*B$ (for complex matrices);
trsm (a BLAS routine)	to solve $R*X = C$.

(The columns of the computed X are the permuted least squares solution vectors x; the output array *jpvt* specifies the permutation order.)

To compute the elements of Q explicitly, call

orgqr	(for real matrices)
ungqr	(for complex matrices).

?geqp3

Computes the QR factorization of a general m-by-n matrix with column pivoting using level 3 BLAS.

Syntax

Fortran 77:

```
call sgeqp3(m, n, a, lda, jpvt, tau, work, lwork, info)
call dgeqp3(m, n, a, lda, jpvt, tau, work, lwork, info)
call cgeqp3(m, n, a, lda, jpvt, tau, work, lwork, rwork, info)
call zgeqp3(m, n, a, lda, jpvt, tau, work, lwork, rwork, info)
```

Fortran 95:

```
call geqp3(a, jpvt [,tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>geqp3( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, lapack_int* jpvt, <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the QR factorization of a general m-by-n matrix A with column pivoting: $A*P = Q*R$ (see Orthogonal Factorizations) using Level 3 BLAS. Here P denotes an n-by-n permutation matrix. Use this routine instead of geqpf for better performance.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors*. Routines are provided to work with Q in this representation.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	`INTEGER`. The number of rows in the matrix A ($m \geq 0$).
n	`INTEGER`. The number of columns in A ($n \geq 0$).
a, *work*	`REAL` for `sgeqp3` `DOUBLE PRECISION` for `dgeqp3` `COMPLEX` for `cgeqp3` `DOUBLE COMPLEX` for `zgeqp3`. Arrays: *a* (*lda*,*) contains the matrix A. The second dimension of *a* must be at least max(1, n). *work* is a workspace array, its dimension `max(1, lwork)`.
lda	`INTEGER`. The leading dimension of *a*; at least max(1, m).
lwork	`INTEGER`. The size of the *work* array; must be at least `max(1, 3*n+1)` for real flavors, and at least `max(1, n+1)` for complex flavors. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* below for details.
jpvt	`INTEGER`. Array, `DIMENSION` at least `max(1, n)`. On entry, if *jpvt*(*i*) ≠ 0, the *i*-th column of A is moved to the beginning of AP before the computation, and fixed in place during the computation. If *jpvt*(*i*) = 0, the *i*-th column of A is a free column (that is, it may be interchanged during the computation with any other free column).
rwork	`REAL` for `cgeqp3` `DOUBLE PRECISION` for `zgeqp3`. A workspace array, `DIMENSION` at least max(1, 2*n). Used in complex flavors only.

Output Parameters

a	Overwritten by the factorization data as follows: If $m \geq n$, the elements below the diagonal are overwritten by the details of the unitary (orthogonal) matrix Q, and the upper triangle is overwritten by the corresponding elements of the upper triangular matrix R. If $m < n$, the strictly lower triangular part is overwritten by the details of the matrix Q, and the remaining elements are overwritten by the corresponding elements of the m-by-n upper trapezoidal matrix R.
tau	`REAL` for `sgeqp3` `DOUBLE PRECISION` for `dgeqp3` `COMPLEX` for `cgeqp3` `DOUBLE COMPLEX` for `zgeqp3`. Array, `DIMENSION` at least max (1, min(m, n)). Contains scalar factors of the elementary reflectors for the matrix Q.
jpvt	Overwritten by details of the permutation matrix P in the factorization $A*P = Q*R$. More precisely, the columns of AP are the columns of A in the following order: *jpvt*(1), *jpvt*(2), ..., *jpvt*(*n*).
info	`INTEGER`. If *info* = 0, the execution is successful. If *info* = $-i$, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `geqp3` interface are the following:

a	Holds the matrix A of size (m,n).
jpvt	Holds the vector of length n.
tau	Holds the vector of length min(m,n)

Application Notes

To solve a set of least squares problems minimizing $||A*x - b||_2$ for all columns b of a given matrix B, you can call the following:

`?geqp3` (this routine)	to factorize $A*P = Q*R$;
ormqr	to compute $C = Q^T*B$ (for real matrices);
unmqr	to compute $C = Q^H*B$ (for complex matrices);
trsm (a BLAS routine)	to solve $R*X = C$.

(The columns of the computed X are the permuted least squares solution vectors x; the output array *jpvt* specifies the permutation order.)

To compute the elements of Q explicitly, call

orgqr	(for real matrices)
ungqr	(for complex matrices).

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?orgqr

Generates the real orthogonal matrix Q of the QR factorization formed by ?geqrf.

Syntax

Fortran 77:

```
call sorgqr(m, n, k, a, lda, tau, work, lwork, info)
call dorgqr(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orgqr(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>orgqr( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates the whole or part of m-by-m orthogonal matrix Q of the QR factorization formed by the routines geqrf/geqrf or geqpf/geqpf. Use this routine after a call to sgeqrf/dgeqrf or sgeqpf/dgeqpf.

Usually Q is determined from the QR factorization of an m by p matrix A with $m \geq p$. To compute the whole matrix Q, use:

```
call ?orgqr(m, m, p, a, lda, tau, work, lwork, info)
```

To compute the leading p columns of Q (which form an orthonormal basis in the space spanned by the columns of A):

```
call ?orgqr(m, p, p, a, lda, tau, work, lwork, info)
```

To compute the matrix Q^k of the QR factorization of leading k columns of the matrix A:

```
call ?orgqr(m, m, k, a, lda, tau, work, lwork, info)
```

To compute the leading k columns of Q^k (which form an orthonormal basis in the space spanned by leading k columns of the matrix A):

```
call ?orgqr(m, k, k, a, lda, tau, work, lwork, info)
```

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The order of the orthogonal matrix Q ($m \geq 0$).
n	INTEGER. The number of columns of Q to be computed ($0 \leq n \leq m$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q ($0 \leq k \leq n$).
a, *tau*, *work*	REAL for sorgqr DOUBLE PRECISION for dorgqr Arrays: *a*(*lda*,*) and *tau*(*) are the arrays returned by sgeqrf / dgeqrf or sgeqpf / dgeqpf. The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array ($lwork \geq n$). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by *n* leading columns of the *m*-by-*m* orthogonal matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orgqr interface are the following:

a	Holds the matrix A of size (m,n).
tau	Holds the vector of length (k)

Application Notes

For better performance, try using *lwork* = *n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed Q differs from an exactly orthogonal matrix by a matrix E such that

$||E||_2 = O(\varepsilon)*||A||_2$ where ε is the machine precision.

The total number of floating-point operations is approximately $4*m*n*k - 2*(m + n)*k^2 + (4/3)*k^3$.

If $n = k$, the number is approximately $(2/3)*n^2*(3m - n)$.

The complex counterpart of this routine is ungqr.

?ormqr

Multiplies a real matrix by the orthogonal matrix Q of the QR factorization formed by ?geqrf *or* ?geqpf.

Syntax

Fortran 77:

```
call sormqr(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call dormqr(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormqr(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormqr( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real matrix C by Q or Q^T, where Q is the orthogonal matrix Q of the QR factorization formed by the routines geqrf/geqrf or geqpf/geqpf.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (overwriting the result on C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* ='L', Q or Q^T is applied to C from the left. If *side* ='R', Q or Q^T is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* ='N', the routine multiplies C by Q. If *trans* ='T', the routine multiplies C by Q^T.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: $0 \leq k \leq m$ if *side* ='L'; $0 \leq k \leq n$ if *side* ='R'.
a, *tau*, *c*, *work*	REAL for sgeqrf DOUBLE PRECISION for dgeqrf. Arrays: *a*(*lda*,*) and *tau*(*) are the arrays returned by sgeqrf / dgeqrf or sgeqpf / dgeqpf. The second dimension of *a* must be at least max(1, *k*). The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the matrix C. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*. Constraints: *lda* $\geq$ max(1, *m*) if *side* = 'L'; *lda* $\geq$ max(1, *n*) if *side* = 'R'.
ldc	INTEGER. The leading dimension of *c*. Constraint: *ldc* $\geq$ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* $\geq$ max(1, *n*) if *side* = 'L'; *lwork* $\geq$ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormqr interface are the following:

a	Holds the matrix *A* of size (*r*,*k*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* (if *side* = 'L') or *lwork* = *m***blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is unmqr.

?ungqr

Generates the complex unitary matrix Q of the QR factorization formed by ?geqrf.

Syntax

Fortran 77:

```
call cungqr(m, n, k, a, lda, tau, work, lwork, info)
call zungqr(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call ungqr(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>ungqr( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates the whole or part of *m*-by-*m* unitary matrix *Q* of the *QR* factorization formed by the routines geqrf/geqrf or geqpf/geqpf. Use this routine after a call to cgeqrf/zgeqrf or cgeqpf/zgeqpf.

Usually Q is determined from the QR factorization of an m by p matrix A with $m \geq p$. To compute the whole matrix Q, use:

```
call ?ungqr(m, m, p, a, lda, tau, work, lwork, info)
```

To compute the leading p columns of Q (which form an orthonormal basis in the space spanned by the columns of A):

```
call ?ungqr(m, p, p, a, lda, tau, work, lwork, info)
```

To compute the matrix Q^k of the QR factorization of the leading k columns of the matrix A:

```
call ?ungqr(m, m, k, a, lda, tau, work, lwork, info)
```

To compute the leading k columns of Q^k (which form an orthonormal basis in the space spanned by the leading k columns of the matrix A):

```
call ?ungqr(m, k, k, a, lda, tau, work, lwork, info)
```

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The order of the unitary matrix Q ($m \geq 0$).
n	INTEGER. The number of columns of Q to be computed ($0 \leq n \leq m$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q ($0 \leq k \leq n$).
a, *tau*, *work*	COMPLEX for cungqr DOUBLE COMPLEX for zungqr Arrays: *a*(*lda*,*) and *tau*(*) are the arrays returned by cgeqrf/zgeqrf or cgeqpf/zgeqpf. The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array (*lwork* $\geq$ *n*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by *n* leading columns of the *m*-by-*m* unitary matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ungqr interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork* =*n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed *Q* differs from an exactly unitary matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)*||A||_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $16*m*n*k - 8*(m + n)*k2 + (16/3)*k^3$.

If $n = k$, the number is approximately $(8/3)*n^2*(3m - n)$.

The real counterpart of this routine is orgqr.

?unmqr

Multiplies a complex matrix by the unitary matrix Q of the QR factorization formed by ?geqrf.

Syntax

Fortran 77:

```
call cunmqr(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call zunmqr(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmqr(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmqr( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a rectangular complex matrix *C* by *Q* or Q_H, where *Q* is the unitary matrix *Q* of the *QR* factorization formed by the routines geqrf/geqrf or geqpf/geqpf.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (overwriting the result on C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q_H is applied to C from the left. If *side* = 'R', Q or Q_H is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'C'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'C', the routine multiplies C by Q^H.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: $0 \leq k \leq m$ if *side* = 'L'; $0 \leq k \leq n$ if *side* = 'R'.
a, *c*, *tau*, *work*	COMPLEX for cgeqrf DOUBLE COMPLEX for zgeqrf. Arrays: *a*(*lda*,*) and *tau*(*) are the arrays returned by cgeqrf / zgeqrf or cgeqpf / zgeqpf. The second dimension of *a* must be at least max(1, *k*). The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the matrix C. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*. Constraints: *lda* ≥ max(1, *m*) if *side* = 'L'; *lda* ≥ max(1, *n*) if *side* = 'R'.
ldc	INTEGER. The leading dimension of *c*. Constraint: *ldc* ≥ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER.

If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unmqr interface are the following:

a	Holds the matrix *A* of size (*r*,*k*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* (if *side* = 'L') or *lwork* = *m***blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is ormqr.

?gelqf

Computes the LQ factorization of a general m-by-n matrix.

Syntax

Fortran 77:

```
call sgelqf(m, n, a, lda, tau, work, lwork, info)
call dgelqf(m, n, a, lda, tau, work, lwork, info)
call cgelqf(m, n, a, lda, tau, work, lwork, info)
call zgelqf(m, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call gelqf(a [, tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>gelqf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the *LQ* factorization of a general *m*-by-*n* matrix *A* (see Orthogonal Factorizations). No pivoting is performed.

The routine does not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of min(*m*, *n*) *elementary reflectors*. Routines are provided to work with *Q* in this representation.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
a, *work*	REAL for sgelqf DOUBLE PRECISION for dgelqf COMPLEX for cgelqf DOUBLE COMPLEX for zgelqf. Arrays: *a*(*lda*,*) contains the matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; at least max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the factorization data as follows: If *m* ≤ *n*, the elements above the diagonal are overwritten by the details of the unitary (orthogonal) matrix *Q*, and the lower triangle is overwritten by the corresponding elements of the lower triangular matrix *L*. If *m* > *n*, the strictly upper triangular part is overwritten by the details of the matrix *Q*, and the remaining elements are overwritten by the corresponding elements of the *m*-by-*n* lower trapezoidal matrix *L*.

tau	REAL for sgelqf DOUBLE PRECISION for dgelqf COMPLEX for cgelqf DOUBLE COMPLEX for zgelqf. Array, DIMENSION at least max(1, min(*m*, *n*)). Contains additional information on the matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gelqf interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length min(*m*,*n*).

Application Notes

For better performance, try using *lwork* =*m***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed factorization is the exact factorization of a matrix $A + E$, where

$||E||_2 = O(\varepsilon)\ ||A||_2$.

The approximate number of floating-point operations for real flavors is

$(4/3)n^3$	if $m = n$,
$(2/3)n^2(3m-n)$	if $m > n$,
$(2/3)m^2(3n-m)$	if $m < n$.

The number of operations for complex flavors is 4 times greater.

To find the minimum-norm solution of an underdetermined least squares problem minimizing $||A*x - b||_2$ for all columns b of a given matrix B, you can call the following:

?gelqf (this routine)	to factorize $A = L*Q$;
trsm (a BLAS routine)	to solve $L*Y = B$ for Y;
ormlq	to compute $X = (Q_1)^T*Y$ (for real matrices);

unmlq	to compute $X = (Q_1)^H * Y$ (for complex matrices).

(The columns of the computed X are the minimum-norm solution vectors x. Here A is an m-by-n matrix with $m < n$; Q_1 denotes the first m columns of Q).

To compute the elements of Q explicitly, call

orglq	(for real matrices)
unglq	(for complex matrices).

See Also

mkl_progress

?orglq

Generates the real orthogonal matrix Q of the LQ factorization formed by `?gelqf`.

Syntax

Fortran 77:

```
call sorglq(m, n, k, a, lda, tau, work, lwork, info)
call dorglq(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orglq(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>orglq( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine generates the whole or part of n-by-n orthogonal matrix Q of the LQ factorization formed by the routines gelqf/gelqf. Use this routine after a call to `sgelqf`/`dgelqf`.

Usually Q is determined from the LQ factorization of an p-by-n matrix A with $n \geq p$. To compute the whole matrix Q, use:

```
call ?orglq(n, n, p, a, lda, tau, work, lwork, info)
```

To compute the leading p rows of Q, which form an orthonormal basis in the space spanned by the rows of A, use:

```
call ?orglq(p, n, p, a, lda, tau, work, lwork, info)
```

To compute the matrix Q^k of the LQ factorization of the leading k rows of A, use:

```
call ?orglq(n, n, k, a, lda, tau, work, lwork, info)
```

To compute the leading k rows of Q^k, which form an orthonormal basis in the space spanned by the leading k rows of A, use:

```
call ?orgqr(k, n, k, a, lda, tau, work, lwork, info)
```

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of Q to be computed ($0 \le m \le n$).
n	INTEGER. The order of the orthogonal matrix Q ($n \ge m$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q ($0 \le k \le m$).
a, *tau*, *work*	REAL for sorglq DOUBLE PRECISION for dorglq Arrays: *a*(*lda*,*) and *tau*(*) are the arrays returned by sgelqf/dgelqf. The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; at least max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by *m* leading rows of the *n*-by-*n* orthogonal matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orglq interface are the following:

a	Holds the matrix A of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork* =*m*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)*||A||_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $4*m*n*k - 2*(m + n)*k^2 + (4/3)*k^3$.

If $m = k$, the number is approximately $(2/3)*m^2*(3n - m)$.

The complex counterpart of this routine is unglq.

?ormlq

Multiplies a real matrix by the orthogonal matrix Q of the LQ factorization formed by ?gelqf.

Syntax

Fortran 77:

```
call sormlq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call dormlq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormlq(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormlq( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real *m*-by-*n* matrix *C* by *Q* or Q^T, where *Q* is the orthogonal matrix *Q* of the *LQ* factorization formed by the routine gelqf/gelqf.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products *Q***C*, Q^T**C*, *C***Q*, or *C**Q^T (overwriting the result on *C*).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side CHARACTER*1. Must be either 'L' or 'R'.

	If *side* = 'L', Q or Q^T is applied to C from the left. If *side* = 'R', Q or Q^T is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^T.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: $0 \leq k \leq m$ if *side* = 'L'; $0 \leq k \leq n$ if *side* = 'R'.
a, *c*, *tau*, *work*	REAL for sormlq DOUBLE PRECISION for dormlq. Arrays: *a*(*lda*,*) and *tau*(*) are arrays returned by ?gelqf. The second dimension of *a* must be: at least max(1, *m*) if *side* = 'L'; at least max(1, *n*) if *side* = 'R'. The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the matrix C. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *k*).
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormlq interface are the following:

a	Holds the matrix A of size (*k*,*m*).

tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork = n*blocksize* (if *side = 'L'*) or *lwork = m*blocksize* (if *side = 'R'*) where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork*= -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork*= -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is unmlq.

?unglq

Generates the complex unitary matrix Q of the LQ factorization formed by ?gelqf.

Syntax

Fortran 77:

```
call cunglq(m, n, k, a, lda, tau, work, lwork, info)
call zunglq(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call unglq(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>unglq( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates the whole or part of *n*-by-*n* unitary matrix *Q* of the *LQ* factorization formed by the routines gelqf/gelqf. Use this routine after a call to cgelqf/zgelqf.

Usually Q is determined from the LQ factorization of an p-by-n matrix A with $n < p$. To compute the whole matrix Q, use:

```
call ?unglq(n, n, p, a, lda, tau, work, lwork, info)
```

To compute the leading p rows of Q, which form an orthonormal basis in the space spanned by the rows of A, use:

```
call ?unglq(p, n, p, a, lda, tau, work, lwork, info)
```

To compute the matrix Q^k of the LQ factorization of the leading k rows of the matrix A, use:

```
call ?unglq(n, n, k, a, lda, tau, work, lwork, info)
```

To compute the leading k rows of Q^k, which form an orthonormal basis in the space spanned by the leading k rows of the matrix A, use:

```
call ?ungqr(k, n, k, a, lda, tau, work, lwork, info)
```

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of Q to be computed ($0 \le m \le n$).
n	INTEGER. The order of the unitary matrix Q ($n \ge m$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q ($0 \le k \le m$).
a, *tau*, *work*	COMPLEX for cunglq DOUBLE COMPLEX for zunglq Arrays: *a*(*lda*,*) and *tau*(*) are the arrays returned by sgelqf/dgelqf. The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; at least max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by *m* leading rows of the *n*-by-*n* unitary matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unglq interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork* = *m***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed *Q* differs from an exactly unitary matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)*||A||_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $16*m*n*k - 8*(m + n)*k^2 + (16/3)*k^3$.

If *m* = *k*, the number is approximately $(8/3)*m^2*(3n - m)$.

The real counterpart of this routine is orglq.

?unmlq

Multiplies a complex matrix by the unitary matrix Q of the LQ factorization formed by ?gelqf*.*

Syntax

Fortran 77:

```
call cunmlq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call zunmlq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmlq(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmlq( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real *m*-by-*n* matrix *C* by *Q* or Q^H, where *Q* is the unitary matrix *Q* of the *LQ* factorization formed by the routine gelqf/gelqf.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (overwriting the result on C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side
CHARACTER*1. Must be either 'L' or 'R'.
If *side* = 'L', Q or Q^H is applied to C from the left.
If *side* = 'R', Q or Q^H is applied to C from the right.

trans
CHARACTER*1. Must be either 'N' or 'C'.
If *trans* = 'N', the routine multiplies C by Q.
If *trans* = 'C', the routine multiplies C by Q_H.

m
INTEGER. The number of rows in the matrix C ($m \geq 0$).

n
INTEGER. The number of columns in C ($n \geq 0$).

k
INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
$0 \leq k \leq m$ if *side* = 'L';
$0 \leq k \leq n$ if *side* = 'R'.

a, *c*, *tau*, *work*
COMPLEX for cunmlq
DOUBLE COMPLEX for zunmlq.
Arrays:
a(*lda*,*) and *tau*(*) are arrays returned by ?gelqf.
The second dimension of *a* must be:
at least max(1, *m*) if *side* = 'L';
at least max(1, *n*) if *side* = 'R'.
The dimension of *tau* must be at least max(1, *k*).
c(*ldc*,*) contains the matrix C.
The second dimension of *c* must be at least max(1, *n*)
work is a workspace array, its dimension max(1, *lwork*).

lda
INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *k*).

ldc
INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).

lwork
INTEGER. The size of the *work* array. Constraints:
lwork ≥ max(1, *n*) if *side* = 'L';
lwork ≥ max(1, *m*) if *side* = 'R'.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c
Overwritten by the product $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).

work(1)
If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `unmlq` interface are the following:

a	Holds the matrix *A* of size (*k*,*m*).
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* (if *side* = 'L') or *lwork* = *m***blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is ormlq.

?geqlf

Computes the QL factorization of a general m-by-n matrix.

Syntax

Fortran 77:

```
call sgeqlf(m, n, a, lda, tau, work, lwork, info)
call dgeqlf(m, n, a, lda, tau, work, lwork, info)
call cgeqlf(m, n, a, lda, tau, work, lwork, info)
call zgeqlf(m, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call geqlf(a [, tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>geqlf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the QL factorization of a general m-by-n matrix A. No pivoting is performed.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors*. Routines are provided to work with Q in this representation.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, $work$	REAL for sgeqlf DOUBLE PRECISION for dgeqlf COMPLEX for cgeqlf DOUBLE COMPLEX for zgeqlf. Arrays: a(lda,*) contains the matrix A. The second dimension of a must be at least max(1, n). $work$ is a workspace array, its dimension max(1, $lwork$).
lda	INTEGER. The leading dimension of a; at least max(1, m).
$lwork$	INTEGER. The size of the $work$ array; at least max(1, n). If $lwork = -1$, then a workspace query is assumed; the routine only calculates the optimal size of the $work$ array, returns this value as the first entry of the $work$ array, and no error message related to $lwork$ is issued by xerbla. See *Application Notes* for the suggested value of $lwork$.

Output Parameters

a	Overwritten on exit by the factorization data as follows: if $m \geq n$, the lower triangle of the subarray a(m-n+1:m, 1:n) contains the n-by-n lower triangular matrix L; if $m \leq n$, the elements on and below the (n-m)-th superdiagonal contain the m-by-n lower trapezoidal matrix L; in both cases, the remaining elements, with the array tau, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.
tau	REAL for sgeqlf DOUBLE PRECISION for dgeqlf COMPLEX for cgeqlf DOUBLE COMPLEX for zgeqlf. Array, DIMENSION at least max(1, min(m, n)). Contains scalar factors of the elementary reflectors for the matrix Q.
$work$(1)	If $info = 0$, on exit $work$(1) contains the minimum value of $lwork$ required for optimum performance.
$info$	INTEGER.

If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine geqlf interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length min(*m*,*n*).

Application Notes

For better performance, try using *lwork* =*n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Related routines include:

orgql	to generate matrix Q (for real matrices);
ungql	to generate matrix Q (for complex matrices);
ormql	to apply matrix Q (for real matrices);
unmql	to apply matrix Q (for complex matrices).

See Also

mkl_progress

?orgql

Generates the real matrix Q of the QL factorization formed by ?geqlf.

Syntax

Fortran 77:

```
call sorgql(m, n, k, a, lda, tau, work, lwork, info)
call dorgql(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orgql(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>orgql( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates an *m*-by-*n* real matrix *Q* with orthonormal columns, which is defined as the last *n* columns of a product of *k* elementary reflectors *H*(*i*) of order *m*: *Q* = *H*(*k*) *...* *H*(2)*H*(1) as returned by the routines geqlf/geqlf. Use this routine after a call to sgeqlf/dgeqlf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *Q* (*m* ≥ 0).
n	INTEGER. The number of columns of the matrix *Q* (*m* ≥ *n* ≥ 0).
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n* ≥ *k* ≥ 0).
a, *tau*, *work*	REAL for sorgql DOUBLE PRECISION for dorgql Arrays: *a*(*lda*,*), *tau*(*). On entry, the (*n* - *k* + *i*)th column of *a* must *contain the vector which* defines the elementary reflector *H*(*i*), for *i* = 1,2,...,*k*, as returned by sgeqlf/dgeqlf in the last *k* columns of its array argument *a*; *tau*(*i*) *must contain the scalar factor of the elementary* reflector *H*(*i*), as returned by sgeqlf/dgeqlf; The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; at least max(1, *n*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the *m*-by-*n* matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orgql interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork =n*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is ungql.

?ungql

Generates the complex matrix Q of the QL factorization formed by ?geqlf*.*

Syntax

Fortran 77:

```
call cungql(m, n, k, a, lda, tau, work, lwork, info)
call zungql(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call ungql(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>ungql( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates an *m*-by-*n* complex matrix *Q* with orthonormal columns, which is defined as the last *n* columns of a product of *k* elementary reflectors *H*(*i*) of order *m*: *Q = H*(*k*) **...* H*(2)**H*(1) as returned by the routines geqlf/geqlf . Use this routine after a call to cgeqlf/zgeqlf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *Q* (*m*≥0).
n	INTEGER. The number of columns of the matrix *Q* (*m*≥*n*≥0).
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n*≥*k*≥0).
a, *tau*, *work*	COMPLEX for cungql DOUBLE COMPLEX for zungql Arrays: *a*(*lda*,*), *tau*(*), *work*(*lwork*). On entry, the (*n* - *k* + *i*)th column of *a* must contain the vector which defines the elementary reflector *H*(*i*), for i = 1,2,...,*k*, as returned by cgeqlf/zgeqlf in the last *k* columns of its array argument *a*; *tau*(i) must contain the scalar factor of the elementaryreflector *H*(*i*), as returned by cgeqlf/zgeqlf; The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; at least max(1, *n*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the *m*-by-*n* matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ungql interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork* =*n*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is orgql.

?ormql

Multiplies a real matrix by the orthogonal matrix Q of the QL factorization formed by ?geqlf.

Syntax

Fortran 77:

```
call sormql(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call dormql(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormql(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormql( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real m-by-n matrix C by Q or Q^T, where Q is the orthogonal matrix Q of the QL factorization formed by the routine geqlf/geqlf .

Depending on the parameters *side* and *trans*, the routine ormql can form one of the matrix products $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (overwriting the result over C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^T is applied to C from the left. If *side* = 'R', Q or Q^T is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^T.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).

n	INTEGER. The number of columns in *C* (*n* ≥ 0).
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. Constraints: 0 ≤ *k* ≤ *m* if *side* = 'L'; 0 ≤ *k* ≤ *n* if *side* = 'R'.
a, *tau*, *c*, *work*	REAL for sormql DOUBLE PRECISION for dormql. Arrays: *a*(*lda*,*), *tau*(*), *c*(*ldc*,*). On entry, the *i*th column of *a* must contain the vector which defines the elementary reflector H_i, for i = 1,2,...,*k*, as returned by sgeqlf/dgeqlf in the last *k* columns of its array argument *a*. The second dimension of *a* must be at least max(1, *k*). *tau*(i) must contain the scalar factor of the elementary reflector H_i, as returned by sgeqlf/dgeqlf. The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the *m*-by-*n* matrix *C*. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; if *side* = 'L', *lda* ≥ max(1, *m*); if *side* = 'R', *lda* ≥ max(1, *n*).
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = −*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormql interface are the following:

a	Holds the matrix *A* of size (*r*,*k*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*k*).

c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* (if *side* = 'L') or *lwork* = *m***blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is unmql.

?unmql

Multiplies a complex matrix by the unitary matrix Q of the QL factorization formed by ?geqlf.

Syntax

Fortran 77:

```
call cunmql(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call zunmql(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmql(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmql( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a complex *m*-by-*n* matrix *C* by *Q* or Q^H, where *Q* is the unitary matrix *Q* of the *QL* factorization formed by the routine geqlf/geqlf .

Depending on the parameters *side* and *trans*, the routine unmql can form one of the matrix products *Q***C*, Q^H**C*, *C***Q*, or *C**Q^H (overwriting the result over *C*).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side
CHARACTER*1. Must be either 'L' or 'R'.
If *side* = 'L', Q or Q^H is applied to C from the left.
If *side* = 'R', Q or Q^H is applied to C from the right.

trans
CHARACTER*1. Must be either 'N' or 'C'.
If *trans* = 'N', the routine multiplies C by Q.
If *trans* = 'C', the routine multiplies C by Q^H.

m
INTEGER. The number of rows in the matrix C ($m \geq 0$).

n
INTEGER. The number of columns in C ($n \geq 0$).

k
INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
$0 \leq k \leq m$ if *side* = 'L';
$0 \leq k \leq n$ if *side* = 'R'.

a, *tau*, *c*, *work*
COMPLEX for cunmql
DOUBLE COMPLEX for zunmql.
Arrays: *a*(*lda*,*), *tau*(*), *c*(*ldc*,*), *work*(*lwork*).
On entry, the *i*-th column of *a* must contain the vector which defines the elementary reflector *H*(*i*), for i = 1,2,...,*k*, as returned by cgeqlf/zgeqlf in the last *k* columns of its array argument *a*.
The second dimension of *a* must be at least max(1, *k*).
tau(i) must contain the scalar factor of the elementary reflector *H*(*i*), as returned by cgeqlf/zgeqlf.
The dimension of *tau* must be at least max(1, *k*).
c(*ldc*,*) contains the *m*-by-*n* matrix *C*.
The second dimension of *c* must be at least max(1, *n*)
work is a workspace array, its dimension max(1, *lwork*).

lda
INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *n*).

ldc
INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).

lwork
INTEGER. The size of the *work* array. Constraints:
lwork ≥ max(1, *n*) if *side* = 'L';
lwork ≥ max(1, *m*) if *side* = 'R'.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c
Overwritten by the product $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).

work(1)
If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `unmql` interface are the following:

a	Holds the matrix *A* of size (*r*,*k*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'L'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* (if *side* = 'L') or *lwork* = *m***blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is ormql.

?gerqf

Computes the RQ factorization of a general m-by-n matrix.

Syntax

Fortran 77:

```
call sgerqf(m, n, a, lda, tau, work, lwork, info)
call dgerqf(m, n, a, lda, tau, work, lwork, info)
call cgerqf(m, n, a, lda, tau, work, lwork, info)
call zgerqf(m, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call gerqf(a [, tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>gerqf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the *RQ* factorization of a general *m*-by-*n* matrix *A*. No pivoting is performed.

The routine does not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of min(*m*, *n*) *elementary reflectors*. Routines are provided to work with *Q* in this representation.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
a, *work*	REAL for sgerqf DOUBLE PRECISION for dgerqf COMPLEX for cgerqf DOUBLE COMPLEX for zgerqf. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; *lwork* ≥ max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See Application Notes for the suggested value of *lwork*.

Output Parameters

a	Overwritten on exit by the factorization data as follows: if *m* ≤ *n*, the upper triangle of the subarray *a*(1:*m*, *n*-*m*+1:*n*) contains the *m*-by-*m* upper triangular matrix *R*; if *m* ≥ *n*, the elements on and above the (*m*-*n*)th subdiagonal contain the *m*-by-*n* upper trapezoidal matrix *R*; in both cases, the remaining elements, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of min(*m*,*n*) elementary reflectors.
tau	REAL for sgerqf DOUBLE PRECISION for dgerqf COMPLEX for cgerqf DOUBLE COMPLEX for zgerqf.

	Array, DIMENSION at least max (1, min(*m*, *n*)). Contains scalar factors of the elementary reflectors for the matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gerqf interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length min(*m*,*n*).

Application Notes

For better performance, try using *lwork* =*m***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Related routines include:

orgrq	to generate matrix Q (for real matrices);
ungrq	to generate matrix Q (for complex matrices);
ormrq	to apply matrix Q (for real matrices);
unmrq	to apply matrix Q (for complex matrices).

See Also

mkl_progress

?orgrq

Generates the real matrix Q of the RQ factorization formed by ?gerqf.

Syntax

Fortran 77:

```
call sorgrq(m, n, k, a, lda, tau, work, lwork, info)
call dorgrq(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orgrq(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>orgrq( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates an m-by-n real matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors $H(i)$ of order n: $Q = H(1)* H(2)*...*H(k)$ as returned by the routines gerqf/gerqf. Use this routine after a call to sgerqf/dgerqf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix Q ($m \geq 0$).
n	INTEGER. The number of columns of the matrix Q ($n \geq m$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q ($m \geq k \geq 0$).
a, *tau*, *work*	REAL for sorgrq DOUBLE PRECISION for dorgrq Arrays: *a*(*lda*,*), *tau*(*). On entry, the (m - k + i)-th row of *a* must contain the vector which defines the elementary reflector $H(i)$, for i = 1,2,...,k, as returned by sgerqf/dgerqf in the last k rows of its array argument *a*; *tau*(i) must contain the scalar factor of the elementary reflector $H(i)$, as returned by sgerqf/dgerqf; The second dimension of *a* must be at least max(1, n). The dimension of *tau* must be at least max(1, k). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, m).
lwork	INTEGER. The size of the *work* array; at least max(1, m). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the m-by-n matrix Q.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orgrq interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork* =*m***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is ungrq.

?ungrq

Generates the complex matrix Q of the RQ factorization formed by ?gerqf.

Syntax

Fortran 77:

```
call cungrq(m, n, k, a, lda, tau, work, lwork, info)
call zungrq(m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call ungrq(a, tau [,info])
```

C:

```
lapack_int LAPACKE_<?>ungrq( int matrix_order, lapack_int m, lapack_int n, lapack_int
k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates an m-by-n complex matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors $H(i)$ of order n: $Q = H(1)^H * H(2)^H * \ldots * H(k)^H$ as returned by the routines gerqf/gerqf. Use this routine after a call to sgerqf/dgerqf.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix Q ($m \geq 0$).
n	INTEGER. The number of columns of the matrix Q ($n \geq m$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q ($m \geq k \geq 0$).
a, *tau*, *work*	REAL for cungrq DOUBLE PRECISION for zungrq Arrays: *a*(*lda*,*), *tau*(*), *work*(*lwork*). On entry, the (*m* - *k* + *i*)th row of *a* must contain the vector which defines the elementary reflector *H*(*i*), for i = 1,2,...,*k*, as returned by sgerqf/dgerqf in the last *k* rows of its array argument *a*; *tau*(i) must contain the scalar factor of the elementary reflector *H*(*i*), as returned by sgerqf/dgerqf; The second dimension of *a* must be at least max(1, *n*). The dimension of *tau* must be at least max(1, *k*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array; at least max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the *m*-by-*n* matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ungrq interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*k*).

Application Notes

For better performance, try using *lwork* =*m*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is orgrq.

?ormrq

Multiplies a real matrix by the orthogonal matrix Q of the RQ factorization formed by ?gerqf.

Syntax

Fortran 77:

```
call sormrq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call dormrq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormrq(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormrq( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real *m*-by-*n* matrix *C* by *Q* or Q^T, where *Q* is the real orthogonal matrix defined as a product of *k* elementary reflectors H_i : $Q = H_1 H_2 \ldots H_k$ as returned by the *RQ* factorization routine gerqf/ gerqf .

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products *Q***C*, Q^T**C*, *C***Q*, or *C**Q^T (overwriting the result over *C*).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^T is applied to C from the left. If *side* = 'R', Q or Q^T is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^T.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: $0 \leq k \leq m$, if *side* = 'L'; $0 \leq k \leq n$, if *side* = 'R'.
a, *tau*, *c*, *work*	REAL for sormrq DOUBLE PRECISION for dormrq. Arrays: *a*(*lda*,*), *tau*(*), *c*(*ldc*,*). On entry, the *i*th row of *a* must contain the vector which defines the elementary reflector H_i, for i = 1,2,...,*k*, as returned by sgerqf/dgerqf in the last *k* rows of its array argument *a*. The second dimension of *a* must be at least max(1, *m*) if *side* = 'L', and at least max(1, *n*) if *side* = 'R'. *tau*(i) must contain the scalar factor of the elementary reflector H_i, as returned by sgerqf/dgerqf. The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the *m*-by-*n* matrix *C*. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *k*).
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `ormrq` interface are the following:

a	Holds the matrix *A* of size (*k*,*m*).
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* (if *side* = 'L') or *lwork* = *m***blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is unmrq.

?unmrq

Multiplies a complex matrix by the unitary matrix Q of the RQ factorization formed by `?gerqf`.

Syntax

Fortran 77:

```
call cunmrq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call zunmrq(side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmrq(a, tau, c [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmrq( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`

- C: mkl_lapacke.h

Description

The routine multiplies a complex *m*-by-*n* matrix *C* by *Q* or Q^H, where *Q* is the complex unitary matrix defined as a product of *k* elementary reflectors *H(i)* of order *n*: *Q = H(1)H* H(2)H*...*H(k)H*as returned by the *RQ* factorization routine gerqf/gerqf .

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products *Q*C*, Q^H**C*, *C*Q*, or *C**Q^H (overwriting the result over *C*).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', *Q* or Q^H is applied to *C* from the left. If *side* = 'R', *Q* or Q^H is applied to *C* from the right.
trans	CHARACTER*1. Must be either 'N' or 'C'. If *trans* = 'N', the routine multiplies *C* by *Q*. If *trans* = 'C', the routine multiplies *C* by Q^H.
m	INTEGER. The number of rows in the matrix *C* (*m* ≥ 0).
n	INTEGER. The number of columns in *C* (*n* ≥ 0).
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. Constraints: 0 ≤ *k* ≤ *m*, if *side* = 'L'; 0 ≤ *k* ≤ *n*, if *side* = 'R'.
a, *tau*, *c*, *work*	COMPLEX for cunmrq DOUBLE COMPLEX for zunmrq. Arrays: *a*(*lda*,*), *tau*(*), *c*(*ldc*,*), *work*(*lwork*). On entry, the *i*th row of *a* must contain the vector which defines the elementary reflector *H*(*i*), for i = 1,2,...,*k*, as returned by cgerqf/zgerqf in the last *k* rows of its array argument *a*. The second dimension of *a* must be at least max(1, *m*) if *side* = 'L', and at least max(1, *n*) if *side* = 'R'. *tau*(i) must contain the scalar factor of the elementary reflector *H*(*i*), as returned by cgerqf/zgerqf. The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the *m*-by-*n* matrix *C*. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *k*) .
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product *Q*C*, Q^H**C*, *C*Q*, or *C**Q^H (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unmrq interface are the following:

a	Holds the matrix *A* of size (*k*,*m*).
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n*blocksize* (if *side* = 'L') or *lwork* = *m*blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is ormrq.

?tzrzf

Reduces the upper trapezoidal matrix A to upper triangular form.

Syntax

Fortran 77:

```
call stzrzf(m, n, a, lda, tau, work, lwork, info)
call dtzrzf(m, n, a, lda, tau, work, lwork, info)
call ctzrzf(m, n, a, lda, tau, work, lwork, info)
call ztzrzf(m, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call tzrzf(a [, tau] [,info])
```

C:

```
lapack_int LAPACKE_<?>tzrzf( int matrix_order, lapack_int m, lapack_int n, <datatype>*
a, lapack_int lda, <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces the m-by-n ($m \leq n$) real/complex upper trapezoidal matrix A to upper triangular form by means of orthogonal/unitary transformations. The upper trapezoidal matrix A is factored as

$A = (R\ 0)*Z$,

where Z is an n-by-n orthogonal/unitary matrix and R is an m-by-m upper triangular matrix.

See larz that applies an elementary reflector returned by `?tzrzf` to a general matrix.

The `?tzrzf` routine replaces the deprecated `?tzrqf` routine.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq m$).
a, *work*	REAL for `stzrzf` DOUBLE PRECISION for `dtzrzf` COMPLEX for `ctzrzf` DOUBLE COMPLEX for `ztzrzf`. Arrays: *a*(*lda*,*), *work*(*lwork*) .The leading m-by-n upper trapezoidal part of the array *a* contains the matrix A to be factorized. The second dimension of *a* must be at least max(1, n). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, m).
lwork	INTEGER. The size of the *work* array; *lwork* ≥ max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten on exit by the factorization data as follows:

	the leading *m*-by-*m* upper triangular part of *a* contains the upper triangular matrix *R*, and elements *m* +1 to *n* of the first *m* rows of *a*, with the array *tau*, represent the orthogonal matrix *Z* as a product of *m* elementary reflectors.
tau	REAL for stzrzf DOUBLE PRECISION for dtzrzf COMPLEX for ctzrzf DOUBLE COMPLEX for ztzrzf. Array, DIMENSION at least max (1, *m*). Contains scalar factors of the elementary reflectors for the matrix *Z*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tzrzf interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length (*m*).

Application Notes

The factorization is obtained by Householder's method. The *k*-th transformation matrix, *z*(*k*), which is used to introduce zeros into the (*m* - *k* + 1)-th row of *A*, is given in the form

$$Z(k)=\begin{bmatrix} I & 0 \\ 0 & T(k) \end{bmatrix}$$

where for real flavors

$$T(k) = I - tau * u(k) * u(k)^{T}, \quad u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

and for complex flavors

$$T(k) = I - tau * u(k) * u(k)^{H}, \quad u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

tau is a scalar and *z*(*k*) is an *l*-element vector. *tau* and *z*(*k*) are chosen to annihilate the elements of the *k*-th row of *X*.

The scalar *tau* is returned in the *k*-th element of *tau* and the vector *u*(*k*) in the *k*-th row of *A*, such that the elements of *z*(*k*) are in *a*(*k*, *m*+1), ..., *a*(*k*, *n*).

The elements of *r* are returned in the upper triangular part of *A*.

Z is given by

Z = *Z*(1)**Z*(2)**...**Z*(*m*).

For better performance, try using *lwork* =*m***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the blocked algorithm.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Related routines include:

ormrz	to apply matrix Q (for real matrices)
unmrz	to apply matrix Q (for complex matrices).

?ormrz

Multiplies a real matrix by the orthogonal matrix defined from the factorization formed by ?tzrzf.

Syntax

Fortran 77:

```
call sormrz(side, trans, m, n, k, l, a, lda, tau, c, ldc, work, lwork, info)
call dormrz(side, trans, m, n, k, l, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormrz(a, tau, c, l [, side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormrz( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, const <datatype>* a, lapack_int lda, const
<datatype>* tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The ?ormrz routine multiplies a real m-by-n matrix C by Q or Q^T, where Q is the real orthogonal matrix defined as a product of k elementary reflectors $H(i)$ of order n: $Q = H(1)* H(2)*...*H(k)$ as returned by the factorization routine tzrzf/tzrzf .

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (overwriting the result over C).

The matrix Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

The ?ormrz routine replaces the deprecated ?latzm routine.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^T is applied to C from the left. If *side* = 'R', Q or Q^T is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^T.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: $0 \leq k \leq m$, if *side* = 'L'; $0 \leq k \leq n$, if *side* = 'R'.
l	INTEGER. The number of columns of the matrix A containing the meaningful part of the Householder reflectors. Constraints: $0 \leq l \leq m$, if *side* = 'L'; $0 \leq l \leq n$, if *side* = 'R'.
a, *tau*, *c*, *work*	REAL for sormrz DOUBLE PRECISION for dormrz. Arrays: *a*(*lda*,*), *tau*(*), *c*(*ldc*,*). On entry, the ith row of *a* must contain the vector which defines the elementary reflector $H(i)$, for i = 1,2,...,k, as returned by stzrzf/dtzrzf in the last k rows of its array argument *a*. The second dimension of *a* must be at least max(1, m) if *side* = 'L', and at least max(1, n) if *side* = 'R'. *tau*(i) must contain the scalar factor of the elementary reflector $H(i)$, as returned by stzrzf/dtzrzf. The dimension of *tau* must be at least max(1, k). *c*(*ldc*,*) contains the m-by-n matrix C. The second dimension of *c* must be at least max(1, n) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, k) .
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, m).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, n) if *side* = 'L'; *lwork* ≥ max(1, m) if *side* = 'R'.

If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormrz interface are the following:

a	Holds the matrix A of size (*k*,*m*).
tau	Holds the vector of length (*k*).
c	Holds the matrix C of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n*blocksize* (if *side* = 'L') or *lwork* = *m*blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex counterpart of this routine is unmrz.

?unmrz

Multiplies a complex matrix by the unitary matrix defined from the factorization formed by ?tzrzf.

Syntax

Fortran 77:

```
call cunmrz(side, trans, m, n, k, l, a, lda, tau, c, ldc, work, lwork, info)
call zunmrz(side, trans, m, n, k, l, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmrz(a, tau, c, l [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmrz( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, const <datatype>* a, lapack_int lda, const
<datatype>* tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine multiplies a complex m-by-n matrix C by Q or Q^H, where Q is the unitary matrix defined as a product of k elementary reflectors $H(i)$:

$Q = H(1)^H * H(2)^H * \ldots * H(k)^H$ as returned by the factorization routine tzrzf/tzrzf .

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (overwriting the result over C).

The matrix Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^H is applied to C from the left. If *side* = 'R', Q or Q^H is applied to C from the right.
trans	CHARACTER*1. Must be either 'N' or 'C'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'C', the routine multiplies C by Q^H.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: $0 \leq k \leq m$, if *side* = 'L'; $0 \leq k \leq n$, if *side* = 'R'.
l	INTEGER. The number of columns of the matrix A containing the meaningful part of the Householder reflectors. Constraints: $0 \leq l \leq m$, if *side* = 'L'; $0 \leq l \leq n$, if *side* = 'R'.

a, *tau*, *c*, *work*	COMPLEX for cunmrz DOUBLE COMPLEX for zunmrz. Arrays: *a*(*lda*,*), *tau*(*), *c*(*ldc*,*), *work*(*lwork*). On entry, the *i*th row of *a* must contain the vector which defines the elementary reflector *H(i)*, for i = 1,2,...,*k*, as returned by ctzrzf/ztzrzf in the last *k* rows of its array argument *a*. The second dimension of *a* must be at least max(1, *m*) if *side* = 'L', and at least max(1, *n*) if *side* = 'R'. *tau*(i) must contain the scalar factor of the elementary reflector *H(i)*, as returned by ctzrzf/ztzrzf. The dimension of *tau* must be at least max(1, *k*). *c*(*ldc*,*) contains the *m*-by-*n* matrix *C*. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *k*).
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product *Q*C*, Q^H*C, *C*Q*, or $C*Q^H$ (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unmrz interface are the following:

a	Holds the matrix *A* of size (*k*,*m*).
tau	Holds the vector of length (*k*).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n*blocksize* (if *side* = 'L') or *lwork* = *m*blocksize* (if *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real counterpart of this routine is ormrz.

?ggqrf

Computes the generalized QR factorization of two matrices.

Syntax

Fortran 77:

```
call sggqrf(n, m, p, a, lda, taua, b, ldb, taub, work, lwork, info)
call dggqrf(n, m, p, a, lda, taua, b, ldb, taub, work, lwork, info)
call cggqrf(n, m, p, a, lda, taua, b, ldb, taub, work, lwork, info)
call zggqrf(n, m, p, a, lda, taua, b, ldb, taub, work, lwork, info)
```

Fortran 95:

```
call ggqrf(a, b [,taua] [,taub] [,info])
```

C:

```
lapack_int LAPACKE_<?>ggqrf( int matrix_order, lapack_int n, lapack_int m, lapack_int
p, <datatype>* a, lapack_int lda, <datatype>* taua, <datatype>* b, lapack_int ldb,
<datatype>* taub );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine forms the generalized *QR* factorization of an *n*-by-*m* matrix *A* and an *n*-by-*p* matrix *B* as *A* = *Q***R*, *B* = *Q***T***Z*, where *Q* is an *n*-by-*n* orthogonal/unitary matrix, *Z* is a *p*-by-*p* orthogonal/unitary matrix, and *R* and *T* assume one of the forms:

$$R = \begin{array}{c} \\ m \\ n-m \end{array} \begin{array}{c} \begin{array}{c} m \end{array} \\ \begin{pmatrix} R_{11} \\ 0 \end{pmatrix} \end{array}, \quad \text{if } n \geq m$$

or

$$R = \begin{array}{c} \\ n \end{array} \begin{array}{c} \begin{array}{cc} n & m-n \end{array} \\ \begin{pmatrix} R_{11} & R_{12} \end{pmatrix} \end{array}, \quad \text{if } n < m$$

where R_{11} is upper triangular, and

$$T = \begin{array}{c} \\ n \end{array} \begin{array}{c} \begin{array}{cc} p-n & n \end{array} \\ \begin{pmatrix} 0 & T_{12} \end{pmatrix} \end{array}, \text{ if } n \le p,$$

$$T = \begin{array}{c} \\ n-p \\ p \end{array} \begin{array}{c} \begin{array}{c} p \end{array} \\ \begin{pmatrix} T_{11} \\ T_{21} \end{pmatrix} \end{array}, \quad \text{if } n > p,$$

where T_{12} or T_{21} is a p-by-p upper triangular matrix.

In particular, if B is square and nonsingular, the GQR factorization of A and B implicitly gives the QR factorization of $B^{-1}A$ as:

$B^{-1}*A = Z^{T}*(T^{-1}*R)$ (for real flavors) or $B^{-1}*A = Z^{H}*(T^{-1}*R)$ (for complex flavors).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The number of rows of the matrices A and B ($n \ge 0$).
m	INTEGER. The number of columns in A ($m \ge 0$).
p	INTEGER. The number of columns in B ($p \ge 0$).
a, *b*, *work*	REAL for sggqrf DOUBLE PRECISION for dggqrf COMPLEX for cggqrf DOUBLE COMPLEX for zggqrf. Arrays: *a*(*lda*,*) contains the matrix A. The second dimension of *a* must be at least max(1, *m*). *b*(*ldb*,*) contains the matrix B. The second dimension of *b* must be at least max(1, *p*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
lwork	INTEGER. The size of the *work* array; must be at least max(1, *n*, *m*, *p*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a, *b*	Overwritten by the factorization data as follows: on exit, the elements on and above the diagonal of the array *a* contain the min(*n*,*m*)-by-*m* upper trapezoidal matrix *R* (*R* is upper triangular if $n \geq m$);the elements below the diagonal, with the array *taua*, represent the orthogonal/unitary matrix *Q* as a product of min(*n*,*m*) elementary reflectors ; if $n \leq p$, the upper triangle of the subarray *b*(1:*n*, *p*-*n*+1:*p*) contains the *n*-by-*n* upper triangular matrix *T*; if $n > p$, the elements on and above the (*n*-*p*)th subdiagonal contain the *n*-by-*p* upper trapezoidal matrix *T*; the remaining elements, with the array *taub*, represent the orthogonal/unitary matrix *Z* as a product of elementary reflectors.
taua, *taub*	`REAL` for `sggqrf` `DOUBLE PRECISION` for `dggqrf` `COMPLEX` for `cggqrf` `DOUBLE COMPLEX` for `zggqrf`. Arrays, `DIMENSION` at least max (1, min(*n*, *m*)) for *taua* and at least max (1, min(*n*, *p*)) for *taub*. The array *taua* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q*. The array *taub* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Z*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	`INTEGER`. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `ggqrf` interface are the following:

a	Holds the matrix *A* of size (*n*,*m*).
b	Holds the matrix *B* of size (*n*,*p*).
taua	Holds the vector of length min(*n*,*m*).
taub	Holds the vector of length min(*n*,*p*).

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(1)*H*(2)...*H*(*k*), where *k* = min(n,m).

Each *H*(i) has the form

$H(i) = I - taua * v * v^T$ for real flavors, or

$H(i) = I - taua * v * v^H$ for complex flavors,

where *taua* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:i-1) = 0, *v*(i) = 1.

On exit, *v*(i+1:n) is stored in *a*(i+1:*n*, i) and *taua* is stored in *taua*(i).

The matrix *Z* is represented as a product of elementary reflectors

$Z = H(1)H(2)...H(k)$, where $k = \min(n,p)$.

Each $H(i)$ has the form

$H(i) = I - taub*v*v^T$ for real flavors, or

$H(i) = I - taub*v*v^H$ for complex flavors,

where $taub$ is a real/complex scalar, and v is a real/complex vector with $v(p-k+i+1:p) = 0$, $v(p-k+i) = 1$.

On exit, $v(1:p-k+i-1)$ is stored in $b(n-k+i, 1:p-k+i-1)$ and $taub$ is stored in $taub(i)$.

For better performance, try using $lwork \geq \max(n,m, p)*\max(nb1,nb2,nb3)$, where $nb1$ is the optimal blocksize for the QR factorization of an n-by-m matrix, $nb2$ is the optimal blocksize for the RQ factorization of an n-by-p matrix, and $nb3$ is the optimal blocksize for a call of ormqr/unmqr.

If you are in doubt how much workspace to supply, use a generous value of $lwork$ for the first run or set $lwork = -1$.

If you choose the first option and set any of admissible $lwork$ sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array $work$ on exit. Use this value ($work(1)$) for subsequent runs.

If you set $lwork = -1$, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array ($work$). This operation is called a workspace query.

Note that if you set $lwork$ to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?ggrqf

Computes the generalized RQ factorization of two matrices.

Syntax

Fortran 77:

```
call sggrqf (m, p, n, a, lda, taua, b, ldb, taub, work, lwork, info)
call dggrqf (m, p, n, a, lda, taua, b, ldb, taub, work, lwork, info)
call cggrqf (m, p, n, a, lda, taua, b, ldb, taub, work, lwork, info)
call zggrqf (m, p, n, a, lda, taua, b, ldb, taub, work, lwork, info)
```

Fortran 95:

```
call ggrqf(a, b [,taua] [,taub] [,info])
```

C:

```
lapack_int LAPACKE_<?>ggrqf( int matrix_order, lapack_int m, lapack_int p, lapack_int
n, <datatype>* a, lapack_int lda, <datatype>* taua, <datatype>* b, lapack_int ldb,
<datatype>* taub );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the generalized *RQ* factorization of an *m*-by-*n* matrix *A* and an *p*-by-*n* matrix *B* as *A* = *R***Q*, *B* = *Z***T***Q*, where *Q* is an *n*-by-*n* orthogonal/unitary matrix, *Z* is a *p*-by-*p* orthogonal/unitary matrix, and *R* and *T* assume one of the forms:

$$R = \begin{array}{c} \\ m \end{array} \begin{array}{c} \begin{array}{cc} n-m & m \end{array} \\ \begin{pmatrix} 0 & R_{12} \end{pmatrix} \end{array}, \quad \text{if } m \le n,$$

or

$$R = \begin{array}{c} m-n \\ n \end{array} \begin{array}{c} n \\ \begin{pmatrix} R_{11} \\ R_{21} \end{pmatrix} \end{array}, \quad \text{if } m > n,$$

where R_{11} or R_{21} is upper triangular, and

$$T = \begin{array}{c} \\ n \\ p-n \end{array} \begin{array}{c} n \\ \begin{pmatrix} T_{11} \\ 0 \end{pmatrix} \end{array}, \quad \text{if } p \ge n,$$

or

$$T = \begin{array}{c} \\ p \end{array} \begin{array}{c} \begin{array}{cc} p & n-p \end{array} \\ \begin{pmatrix} T_{11} & T_{12} \end{pmatrix} \end{array}, \quad \text{if } p < n,$$

where T_{11} is upper triangular.

In particular, if *B* is square and nonsingular, the *GRQ* factorization of *A* and *B* implicitly gives the *RQ* factorization of $A*B^{-1}$ as:

$A*B^{-1} = (R*T^{-1})*Z^T$ (for real flavors) or $A*B^{-1} = (R*T^{-1})*Z^H$ (for complex flavors).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).
p	INTEGER. The number of rows in *B* (*p* ≥ 0).
n	INTEGER. The number of columns of the matrices *A* and *B* (*n* ≥ 0).
a, *b*, *work*	REAL for sggrqf DOUBLE PRECISION for dggrqf COMPLEX for cggrqf DOUBLE COMPLEX for zggrqf.

Arrays:
a(*lda*,*) contains the *m*-by-*n* matrix *A*.
The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the *p*-by-*n* matrix *B*.
The second dimension of *b* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

lda INTEGER. The leading dimension of *a*; at least max(1, *m*).

ldb INTEGER. The leading dimension of *b*; at least max(1, *p*).

lwork INTEGER. The size of the *work* array; must be at least max(1, *n*, *m*, *p*).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a, *b* Overwritten by the factorization data as follows:
on exit, if $m \le n$, the upper triangle of the subarray *a*(1:*m*, *n*-*m*+1:*n*) contains the *m*-by-*m* upper triangular matrix *R*;
if $m > n$, the elements on and above the (*m*-*n*)th subdiagonal contain the *m*-by-*n* upper trapezoidal matrix *R*;
the remaining elements, with the array *taua*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors; the elements on and above the diagonal of the array *b* contain the min(*p*,*n*)-by-*n* upper trapezoidal matrix *T* (*T* is upper triangular if $p \ge n$); the elements below the diagonal, with the array *taub*, represent the orthogonal/unitary matrix *Z* as a product of elementary reflectors.

taua, *taub* REAL for sggrqf
DOUBLE PRECISION for dggrqf
COMPLEX for cggrqf
DOUBLE COMPLEX for zggrqf.
Arrays, DIMENSION at least max (1, min(*m*, *n*)) for *taua* and at least max (1, min(*p*, *n*)) for *taub*.
The array *taua* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q*.
The array *taub* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Z*.

work(1) If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

info INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggrqf interface are the following:

a Holds the matrix *A* of size (*m*,*n*).

b Holds the matrix *A* of size (*p*,*n*).

taua	Holds the vector of length min(*m*,*n*).
taub	Holds the vector of length min(*p*,*n*).

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(1)*H*(2)...*H*(*k*), where *k* = min(m,n).

Each *H*(i) has the form

H(i) = *I* - *taua***v***v*^{T} for real flavors, or

H(i) = *I* - *taua***v***v*^{H} for complex flavors,

where *taua* is a real/complex scalar, and *v* is a real/complex vector with *v*(n-k+i+1:n) = 0, *v*(n-k+i) = 1.

On exit, *v*(1:n-k+i-1) is stored in *a*(m-k+i,1:n-k+i-1) and *taua* is stored in *taua*(i).

The matrix *Z* is represented as a product of elementary reflectors

Z = *H*(1)*H*(2)...*H*(*k*), where *k* = min(p,n).

Each *H*(i) has the form

H(i) = *I* - *taub***v***v*^{T} for real flavors, or

H(i) = *I* - *taub***v***v*^{H} for complex flavors,

where *taub* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:i-1) = 0, *v*(i) = 1.

On exit, *v*(i+1:p) is stored in *b*(i+1:p, i) and *taub* is stored in *taub*(i).

For better performance, try using

lwork ≥ max(*n*,*m*, *p*)*max(*nb1*,*nb2*,*nb3*),

where *nb1* is the optimal blocksize for the *RQ* factorization of an *m*-by-*n* matrix, *nb2* is the optimal blocksize for the *QR* factorization of an *p*-by-*n* matrix, and *nb3* is the optimal blocksize for a call of ?ormrq/?unmrq.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork*= -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork*= -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Singular Value Decomposition

This section describes LAPACK routines for computing the *singular value decomposition* (SVD) of a general *m*-by-*n* matrix *A*:

A = *U*Σ*V*^{H}.

In this decomposition, *U* and *V* are unitary (for complex *A*) or orthogonal (for real *A*); Σ is an *m*-by-*n* diagonal matrix with real diagonal elements σ_i:

$\sigma_1 < \sigma_2 < \ldots < \sigma_{\min(m,\ n)} < 0$.

The diagonal elements σ_i are *singular values* of A. The first $\min(m, n)$ columns of the matrices U and V are, respectively, *left* and *right singular vectors* of A. The singular values and singular vectors satisfy

$Av_i = \sigma_i u_i$ and $A^H u_i = \sigma_i v_i$

where u_i and v_i are the i-th columns of U and V, respectively.

To find the SVD of a general matrix A, call the LAPACK routine ?gebrd or ?gbbrd for reducing A to a bidiagonal matrix B by a unitary (orthogonal) transformation: $A = QBP^H$. Then call ?bdsqr, which forms the SVD of a bidiagonal matrix: $B = U_1 \Sigma V_1^H$.

Thus, the sought-for SVD of A is given by $A = U\Sigma V^H = (QU_1)\Sigma(V_1^H P^H)$.

Table "Computational Routines for Singular Value Decomposition (SVD)" lists LAPACK routines (FORTRAN 77 interface) that perform singular value decomposition of matrices. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Singular Value Decomposition (SVD)

Operation	Real matrices	Complex matrices
Reduce A to a bidiagonal matrix B: $A = QBP^H$ (full storage)	gebrd	gebrd
Reduce A to a bidiagonal matrix B: $A = QBP^H$ (band storage)	gbbrd	gbbrd
Generate the orthogonal (unitary) matrix Q or P	orgbr	ungbr
Apply the orthogonal (unitary) matrix Q or P	ormbr	unmbr
Form singular value decomposition of the bidiagonal matrix B: $B = U\Sigma V^H$	bdsqr bdsdc	bdsqr

Decision Tree: Singular Value Decomposition

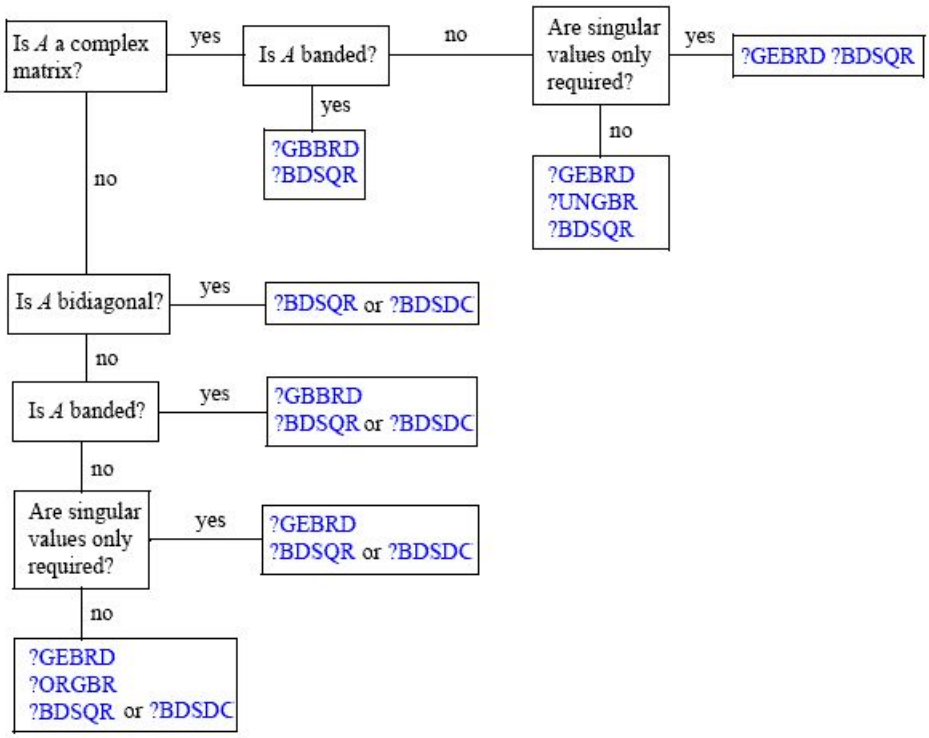


Figure "Decision Tree: Singular Value Decomposition" presents a decision tree that helps you choose the right sequence of routines for SVD, depending on whether you need singular values only or singular vectors as well, whether A is real or complex, and so on.

You can use the SVD to find a minimum-norm solution to a (possibly) rank-deficient least squares problem of minimizing $||Ax - b||^2$. The effective rank k of the matrix A can be determined as the number of singular values which exceed a suitable threshold. The minimum-norm solution is

$$x = V_k(\Sigma_k)^{-1}c$$

where Σ_k is the leading k-by-k submatrix of Σ, the matrix V_k consists of the first k columns of $V = PV_1$, and the vector c consists of the first k elements of $U^Hb = U_1^HQ^Hb$.

?gebrd

Reduces a general matrix to bidiagonal form.

Syntax

Fortran 77:

```
call sgebrd(m, n, a, lda, d, e, tauq, taup, work, lwork, info)
call dgebrd(m, n, a, lda, d, e, tauq, taup, work, lwork, info)
call cgebrd(m, n, a, lda, d, e, tauq, taup, work, lwork, info)
call zgebrd(m, n, a, lda, d, e, tauq, taup, work, lwork, info)
```

Fortran 95:

```
call gebrd(a [, d] [,e] [,tauq] [,taup] [,info])
```

C:

```
lapack_int LAPACKE_sgebrd( int matrix_order, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* d, float* e, float* tauq, float* taup );
lapack_int LAPACKE_dgebrd( int matrix_order, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* d, double* e, double* tauq, double* taup );
lapack_int LAPACKE_cgebrd( int matrix_order, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* d, float* e, lapack_complex_float*
tauq, lapack_complex_float* taup );
lapack_int LAPACKE_zgebrd( int matrix_order, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* d, double* e, lapack_complex_double*
tauq, lapack_complex_double* taup );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces a general m-by-n matrix A to a bidiagonal matrix B by an orthogonal (unitary) transformation.

If $m \geq n$, the reduction is given by

$$A = QBP^H = Q\begin{pmatrix} B_1 \\ 0 \end{pmatrix}P^H = Q_1B_1P^H,$$

where B_1 is an n-by-n upper diagonal matrix, Q and P are orthogonal or, for a complex A, unitary matrices; Q_1 consists of the first n columns of Q.

If $m < n$, the reduction is given by

$A = Q*B*P^H = Q*(B_1 0)*P^H = Q_1*B_1*P_1^H$,

where B_1 is an m-by-m lower diagonal matrix, Q and P are orthogonal or, for a complex A, unitary matrices; P_1 consists of the first m rows of P.

The routine does not form the matrices Q and P explicitly, but represents them as products of elementary reflectors. Routines are provided to work with the matrices Q and P in this representation:

If the matrix A is real,

- to compute Q and P explicitly, call orgbr.
- to multiply a general matrix by Q or P, call ormbr.

If the matrix A is complex,

- to compute Q and P explicitly, call ungbr.
- to multiply a general matrix by Q or P, call unmbr.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, *work*	REAL for sgebrd DOUBLE PRECISION for dgebrd COMPLEX for cgebrd DOUBLE COMPLEX for zgebrd. Arrays: *a*(*lda*,*) contains the matrix A. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
lwork	INTEGER. The dimension of *work*; at least max(1, *m*, *n*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	If $m \geq n$, the diagonal and first super-diagonal of *a* are overwritten by the upper bidiagonal matrix B. Elements below the diagonal are overwritten by details of Q, and the remaining elements are overwritten by details of P. If $m < n$, the diagonal and first sub-diagonal of *a* are overwritten by the lower bidiagonal matrix B. Elements above the diagonal are overwritten by details of P, and the remaining elements are overwritten by details of Q.
d	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors.

	Array, DIMENSION at least max(1, min(*m*, *n*)). Contains the diagonal elements of *B*.
e	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*) – 1). Contains the off-diagonal elements of *B*.
tauq, *taup*	REAL for sgebrd DOUBLE PRECISION for dgebrd COMPLEX for cgebrd DOUBLE COMPLEX for zgebrd. Arrays, DIMENSION at least max (1, min(*m*, *n*)). Contain further details of the matrices *Q* and *P*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gebrd interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
d	Holds the vector of length min(*m*,*n*).
e	Holds the vector of length min(*m*,*n*)-1.
tauq	Holds the vector of length min(*m*,*n*).
taup	Holds the vector of length min(*m*,*n*).

Application Notes

For better performance, try using *lwork* = (*m* + *n*)***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = –1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = –1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrices *Q*, *B*, and *P* satisfy $QBP^H = A + E$, where $||E||_2 = c(n)\varepsilon\ ||A||_2$, $c(n)$ is a modestly increasing function of *n*, and ε is the machine precision.

The approximate number of floating-point operations for real flavors is

$(4/3)*n^2*(3*m - n)$ for $m \geq n$,

$(4/3)*m^2*(3*n - m)$ for $m < n$.

The number of operations for complex flavors is four times greater.

If n is much less than m, it can be more efficient to first form the QR factorization of A by calling geqrf and then reduce the factor R to bidiagonal form. This requires approximately $2*n^2*(m + n)$ floating-point operations.

If m is much less than n, it can be more efficient to first form the LQ factorization of A by calling gelqf and then reduce the factor L to bidiagonal form. This requires approximately $2*m^2*(m + n)$ floating-point operations.

?gbbrd

Reduces a general band matrix to bidiagonal form.

Syntax

Fortran 77:

```
call sgbbrd(vect, m, n, ncc, kl, ku, ab, ldab, d, e, q, ldq, pt, ldpt, c, ldc, work, info)
call dgbbrd(vect, m, n, ncc, kl, ku, ab, ldab, d, e, q, ldq, pt, ldpt, c, ldc, work, info)
call cgbbrd(vect, m, n, ncc, kl, ku, ab, ldab, d, e, q, ldq, pt, ldpt, c, ldc, work, rwork, info)
call zgbbrd(vect, m, n, ncc, kl, ku, ab, ldab, d, e, q, ldq, pt, ldpt, c, ldc, work, rwork, info)
```

Fortran 95:

```
call gbbrd(ab [, c] [,d] [,e] [,q] [,pt] [,kl] [,m] [,info])
```

C:

```
lapack_int LAPACKE_sgbbrd( int matrix_order, char vect, lapack_int m, lapack_int n, lapack_int ncc, lapack_int kl, lapack_int ku, float* ab, lapack_int ldab, float* d, float* e, float* q, lapack_int ldq, float* pt, lapack_int ldpt, float* c, lapack_int ldc );
lapack_int LAPACKE_dgbbrd( int matrix_order, char vect, lapack_int m, lapack_int n, lapack_int ncc, lapack_int kl, lapack_int ku, double* ab, lapack_int ldab, double* d, double* e, double* q, lapack_int ldq, double* pt, lapack_int ldpt, double* c, lapack_int ldc );
lapack_int LAPACKE_cgbbrd( int matrix_order, char vect, lapack_int m, lapack_int n, lapack_int ncc, lapack_int kl, lapack_int ku, lapack_complex_float* ab, lapack_int ldab, float* d, float* e, lapack_complex_float* q, lapack_int ldq, lapack_complex_float* pt, lapack_int ldpt, lapack_complex_float* c, lapack_int ldc );
lapack_int LAPACKE_zgbbrd( int matrix_order, char vect, lapack_int m, lapack_int n, lapack_int ncc, lapack_int kl, lapack_int ku, lapack_complex_double* ab, lapack_int ldab, double* d, double* e, lapack_complex_double* q, lapack_int ldq, lapack_complex_double* pt, lapack_int ldpt, lapack_complex_double* c, lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces an m-by-n band matrix A to upper bidiagonal matrix B: $A = Q*B*P^H$. Here the matrices Q and P are orthogonal (for real A) or unitary (for complex A). They are determined as products of Givens rotation matrices, and may be formed explicitly by the routine if required. The routine can also update a matrix C as follows: $C = Q^H*C$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'N' or 'Q' or 'P' or 'B'. If *vect* = 'N', neither Q nor P^H is generated. If *vect* = 'Q', the routine generates the matrix Q. If *vect* = 'P', the routine generates the matrix P^H. If *vect* = 'B', the routine generates both Q and P^H.
m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
ncc	INTEGER. The number of columns in C ($ncc \geq 0$).
kl	INTEGER. The number of sub-diagonals within the band of A ($kl \geq 0$).
ku	INTEGER. The number of super-diagonals within the band of A ($ku \geq 0$).
ab, *c*, *work*	REAL for sgbbrd DOUBLE PRECISION for dgbbrd COMPLEX for cgbbrd DOUBLE COMPLEX for zgbbrd. Arrays: *ab*(*ldab*,*) contains the matrix A in band storage (see Matrix Storage Schemes). The second dimension of *a* must be at least max(1, n). *c*(*ldc*,*) contains an m-by-ncc matrix C. If $ncc = 0$, the array *c* is not referenced. The second dimension of *c* must be at least max(1, *ncc*). *work*(*) is a workspace array. The dimension of *work* must be at least 2*max(m, n) for real flavors, or max(m, n) for complex flavors.
ldab	INTEGER. The leading dimension of the array *ab* ($ldab \geq kl + ku + 1$).
ldq	INTEGER. The leading dimension of the output array q. $ldq \geq \max(1, m)$ if *vect* = 'Q' or 'B', $ldq \geq 1$ otherwise.
ldpt	INTEGER. The leading dimension of the output array pt. $ldpt \geq \max(1, n)$ if *vect* = 'P' or 'B', $ldpt \geq 1$ otherwise.
ldc	INTEGER. The leading dimension of the array *c*. $ldc \geq \max(1, m)$ if $ncc > 0$; $ldc \geq 1$ if $ncc = 0$.
rwork	REAL for cgbbrd DOUBLE PRECISION for zgbbrd. A workspace array, DIMENSION at least max(m, n).

Output Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

ab	Overwritten by values generated during the reduction.
d	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*)). Contains the diagonal elements of the matrix *B*.
e	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*) - 1). Contains the off-diagonal elements of *B*.
q, *pt*	REAL for sgebrd DOUBLE PRECISION for dgebrd COMPLEX for cgebrd DOUBLE COMPLEX for zgebrd. Arrays: *q*(*ldq*,*) contains the output *m*-by-*m* matrix *Q*. The second dimension of *q* must be at least max(1, *m*). *p*(*ldpt*,*) contains the output *n*-by-*n* matrix P^T. The second dimension of *pt* must be at least max(1, *n*).
c	Overwritten by the product Q^H*C. *c* is not referenced if *ncc* = 0.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gbbrd interface are the following:

ab	Holds the array *A* of size (*kl+ku+1*,*n*).
c	Holds the matrix *C* of size (*m*,*ncc*).
d	Holds the vector with the number of elements min(*m*,*n*).
e	Holds the vector with the number fo elements min(*m*,*n*)-1.
q	Holds the matrix *Q* of size (*m*,*m*).
pt	Holds the matrix *PT* of size (*n*,*n*).
m	If omitted, assumed *m* = *n*.
kl	If omitted, assumed *kl* = *ku*.
ku	Restored as *ku* = *lda-kl-1*.
vect	Restored based on the presence of arguments *q* and pt as follows: *vect* = 'B', if both *q* and *pt* are present, *vect* = 'Q', if *q* is present and *pt* omitted, *vect* = 'P', if *q* is omitted and *pt* present, *vect* = 'N', if both *q* and *pt* are omitted.

Application Notes

The computed matrices *Q*, *B*, and *P* satisfy $Q*B*P^H = A + E$, where $||E||_2 = c(n)\varepsilon ||A||_2$, $c(n)$ is a modestly increasing function of *n*, and ε is the machine precision.

If *m* = *n*, the total number of floating-point operations for real flavors is approximately the sum of:

$6*n^2*(kl + ku)$ if $vect = 'N'$ and $ncc = 0$,

$3*n^2*ncc*(kl + ku - 1)/(kl + ku)$ if C is updated, and

$3*n^3*(kl + ku - 1)/(kl + ku)$ if either Q or P^H is generated (double this if both).

To estimate the number of operations for complex flavors, use the same formulas with the coefficients 20 and 10 (instead of 6 and 3).

?orgbr

Generates the real orthogonal matrix Q or P^T determined by `?gebrd`.

Syntax

Fortran 77:

```
call sorgbr(vect, m, n, k, a, lda, tau, work, lwork, info)
call dorgbr(vect, m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orgbr(a, tau [,vect] [,info])
```

C:

```
lapack_int LAPACKE_<?>orgbr( int matrix_order, char vect, lapack_int m, lapack_int n,
lapack_int k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine generates the whole or part of the orthogonal matrices Q and P^T formed by the routines gebrd/gebrd. Use this routine after a call to `sgebrd`/`dgebrd`. All valid combinations of arguments are described in *Input parameters*. In most cases you need the following:

To compute the whole m-by-m matrix Q:

```
call ?orgbr('Q', m, m, n, a ... )
```

(note that the array a must have at least m columns).

To form the n leading columns of Q if $m > n$:

```
call ?orgbr('Q', m, n, n, a ... )
```

To compute the whole n-by-n matrix P^T:

```
call ?orgbr('P', n, n, m, a ... )
```

(note that the array a must have at least n rows).

To form the m leading rows of P^T if $m < n$:

```
call ?orgbr('P', m, n, m, a ... )
```

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'Q' or 'P'. If *vect* = 'Q', the routine generates the matrix Q. If *vect* = 'P', the routine generates the matrix P^T.
m, *n*	INTEGER. The number of rows (m) and columns (n) in the matrix Q or P^T to be returned ($m \geq 0$, $n \geq 0$). If *vect* = 'Q', $m \geq n \geq \min(m, k)$. If *vect* = 'P', $n \geq m \geq \min(n, k)$.
k	If *vect* = 'Q', the number of columns in the original m-by-k matrix reduced by gebrd. If *vect* = 'P', the number of rows in the original k-by-n matrix reduced by gebrd.
a	REAL for sorgbr DOUBLE PRECISION for dorgbr The vectors which define the elementary reflectors, as returned by gebrd.
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq \max(1, m)$.
tau	REAL for sorgbr DOUBLE PRECISION for dorgbr Array, DIMENSION min (*m*,*k*) if *vect* = 'Q', min (*n*,*k*) if *vect* = 'P'. Scalar factor of the elementary reflector $H(i)$ or $G(i)$, which determines Q and P^T as returned by gebrd in the array *tauq* or *taup*.
work	REAL for sorgbr DOUBLE PRECISION for dorgbr Workspace array, DIMENSION max(1, *lwork*).
lwork	INTEGER. Dimension of the array *work*. See *Application Notes* for the suggested value of *lwork*. If *lwork* = -1 then the routine performs a workspace query and calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

Output Parameters

a	Overwritten by the orthogonal matrix Q or P^T (or the leading rows or columns thereof) as specified by *vect*, *m*, and *n*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orgbr interface are the following:

a	Holds the matrix A of size (m,n).
tau	Holds the vector of length min(m,k) where $k = m$, if *vect* = 'P', $k = n$, if *vect* = 'Q'.
vect	Must be 'Q' or 'P'. The default value is 'Q'.

Application Notes

For better performance, try using *lwork* = min(*m*,*n*)***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the blocked algorithm.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that $||E||_2 = O(\varepsilon)$.

The approximate numbers of floating-point operations for the cases listed in *Description* are as follows:

To form the whole of Q:

$(4/3)*n*(3m^2 - 3m*n + n^2)$ if $m > n$;

$(4/3)*m^3$ if $m \le n$.

To form the n leading columns of Q when $m > n$:

$(2/3)*n^2*(3m - n^2)$ if $m > n$.

To form the whole of P^T:

$(4/3)*n^3$ if $m \ge n$;

$(4/3)*m*(3n^2 - 3m*n + m^2)$ if $m < n$.

To form the m leading columns of P^T when $m < n$:

$(2/3)*n^2*(3m - n^2)$ if $m > n$.

The complex counterpart of this routine is ungbr.

?ormbr

Multiplies an arbitrary real matrix by the real orthogonal matrix Q or P^T determined by ?gebrd.

Syntax

Fortran 77:

```
call sormbr(vect, side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call dormbr(vect, side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormbr(a, tau, c [,vect] [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormbr( int matrix_order, char vect, char side, char trans,
lapack_int m, lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const
<datatype>* tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

Given an arbitrary real matrix C, this routine forms one of the matrix products $Q*C$, Q^T*C, $C*Q$, $C*Q,^T$, $P*C$, P^T*C, $C*P$, $C*P^T$, where Q and P are orthogonal matrices computed by a call to gebrd/gebrd. The routine overwrites the product on C.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

In the descriptions below, r denotes the order of Q or P^T:

If $side$ = 'L', r = m; if $side$ = 'R', r = n.

vect	CHARACTER*1. Must be 'Q' or 'P'. If *vect* = 'Q', then Q or Q^T is applied to C. If *vect* = 'P', then P or P_T is applied to C.
side	CHARACTER*1. Must be 'L' or 'R'. If *side* = 'L', multipliers are applied to C from the left. If *side* = 'R', they are applied to C from the right.
trans	CHARACTER*1. Must be 'N' or 'T'. If *trans* = 'N', then Q or P is applied to C. If *trans* = 'T', then Q^T or P^T is applied to C.
m	INTEGER. The number of rows in C.
n	INTEGER. The number of columns in C.
k	INTEGER. One of the dimensions of A in ?gebrd: If *vect* = 'Q', the number of columns in A; If *vect* = 'P', the number of rows in A. Constraints: $m \geq 0$, $n \geq 0$, $k \geq 0$.
a, *c*, *work*	REAL for sormbr DOUBLE PRECISION for dormbr. Arrays: *a*(*lda*,*) is the array *a* as returned by ?gebrd. Its second dimension must be at least max(1, min(*r*,*k*)) for *vect* = 'Q', or max(1, *r*)) for *vect* = 'P'. *c*(*ldc*,*) holds the matrix C. Its second dimension must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*. Constraints: *lda* $\geq$ max(1, *r*) if *vect* = 'Q'; *lda* $\geq$ max(1, min(*r*,*k*)) if *vect* = 'P'.
ldc	INTEGER. The leading dimension of *c*; *ldc* $\geq$ max(1, *m*).
tau	REAL for sormbr DOUBLE PRECISION for dormbr. Array, DIMENSION at least max (1, min(*r*, *k*)).

	For *vect* = 'Q', the array *tauq* as returned by ?gebrd. For *vect* = 'P', the array *taup* as returned by ?gebrd.
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, $C*Q,^T$, $P*C$, P^T*C, $C*P$, or $C*P^T$, as specified by *vect*, *side*, and *trans*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormbr interface are the following:

a	Holds the matrix *A* of size (*r*,min(*nq*,*k*)) where *r* = *nq*, if *vect* = 'Q', *r* = min(*nq*,*k*), if *vect* = 'P', *nq* = *m*, if *side* = 'L', *nq* = *n*, if *side* = 'R', *k* = *m*, if *vect* = 'P', *k* = *n*, if *vect* = 'Q'.
tau	Holds the vector of length min(*nq*,*k*).
c	Holds the matrix *C* of size (*m*,*n*).
vect	Must be 'Q' or 'P'. The default value is 'Q'.
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using

lwork = *n***blocksize* for *side* = 'L', or

lwork = *m***blocksize* for *side* = 'R',

where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed product differs from the exact product by a matrix E such that $||E||_2 = O(\varepsilon)*||C||_2$.

The total number of floating-point operations is approximately

$2*n*k(2*m - k)$ if *side* = 'L' and $m \geq k$;

$2*m*k(2*n - k)$ if *side* = 'R' and $n \geq k$;

$2*m^2*n$ if *side* = 'L' and $m < k$;

$2*n^2*m$ if *side* = 'R' and $n < k$.

The complex counterpart of this routine is unmbr.

?ungbr

Generates the complex unitary matrix Q or P^H determined by ?gebrd.

Syntax

Fortran 77:

```
call cungbr(vect, m, n, k, a, lda, tau, work, lwork, info)
call zungbr(vect, m, n, k, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call ungbr(a, tau [,vect] [,info])
```

C:

```
lapack_int LAPACKE_<?>ungbr( int matrix_order, char vect, lapack_int m, lapack_int n,
lapack_int k, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine generates the whole or part of the unitary matrices Q and P^H formed by the routines gebrd/gebrd. Use this routine after a call to cgebrd/zgebrd. All valid combinations of arguments are described in *Input Parameters*; in most cases you need the following:

To compute the whole m-by-m matrix Q, use:

```
call ?ungbr('Q', m, m, n, a ... )
```

(note that the array a must have at least m columns).

To form the n leading columns of Q if $m > n$, use:

```
call ?ungbr('Q', m, n, n, a ... )
```

To compute the whole n-by-n matrix P^H, use:

```
call ?ungbr('P', n, n, m, a ... )
```

(note that the array a must have at least n rows).

To form the m leading rows of P^H if $m < n$, use:

```
call ?ungbr('P', m, n, m, a ... )
```

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'Q' or 'P'. If *vect* = 'Q', the routine generates the matrix Q. If *vect* = 'P', the routine generates the matrix P^H.
m	INTEGER. The number of required rows of Q or P^H.
n	INTEGER. The number of required columns of Q or P^H.
k	INTEGER. One of the dimensions of A in ?gebrd: If *vect* = 'Q', the number of columns in A; If *vect* = 'P', the number of rows in A. Constraints: $m \geq 0$, $n \geq 0$, $k \geq 0$. For *vect* = 'Q': $k \leq n \leq m$ if $m > k$, or $m = n$ if $m \leq k$. For *vect* = 'P': $k \leq m \leq n$ if $n > k$, or $m = n$ if $n \leq k$.
a, *work*	COMPLEX for cungbr DOUBLE COMPLEX for zungbr. Arrays: *a*(*lda*,*) is the array *a* as returned by ?gebrd. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
tau	COMPLEX for cungbr DOUBLE COMPLEX for zungbr. For *vect* = 'Q', the array *tauq* as returned by ?gebrd. For *vect* = 'P', the array *taup* as returned by ?gebrd. The dimension of *tau* must be at least max(1, min(*m*, *k*)) for *vect* = 'Q', or max(1, min(*m*, *k*)) for *vect* = 'P'.
lwork	INTEGER. The size of the *work* array. Constraint: *lwork* < max(1, min(*m*, *n*)). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the orthogonal matrix Q or P^T (or the leading rows or columns thereof) as specified by *vect*, *m*, and *n*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER.

If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ungbr interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
tau	Holds the vector of length min(*m*,*k*) where *k* = *m*, if *vect* = 'P', *k* = *n*, if *vect* = 'Q'.
vect	Must be 'Q' or 'P'. The default value is 'Q'.

Application Notes

For better performance, try using *lwork* = min(*m*,*n*)**blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)$.

The approximate numbers of possible floating-point operations are listed below:

To compute the whole matrix *Q*:

$(16/3)n(3m^2 - 3m*n + n^2)$ if $m > n$;

$(16/3)m^3$ if $m \leq n$.

To form the *n* leading columns of *Q* when $m > n$:

$(8/3)n^2(3m - n^2)$.

To compute the whole matrix P^H:

$(16/3)n^3$ if $m \geq n$;

$(16/3)m(3n^2 - 3m*n + m^2)$ if $m < n$.

To form the *m* leading columns of P^H when $m < n$:

$(8/3)n^2(3m - n^2)$ if $m > n$.

The real counterpart of this routine is orgbr.

?unmbr

Multiplies an arbitrary complex matrix by the unitary matrix Q or P determined by ?gebrd.

Syntax

Fortran 77:

```
call cunmbr(vect, side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
call zunmbr(vect, side, trans, m, n, k, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmbr(a, tau, c [,vect] [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmbr( int matrix_order, char vect, char side, char trans,
lapack_int m, lapack_int n, lapack_int k, const <datatype>* a, lapack_int lda, const
<datatype>* tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

Given an arbitrary complex matrix C, this routine forms one of the matrix products $Q*C$, Q^H*C, $C*Q$, $C*Q^H$, $P*C$, P^H*C, $C*P$, or $C*P^H$, where Q and P are unitary matrices computed by a call to gebrd/gebrd. The routine overwrites the product on C.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

In the descriptions below, r denotes the order of Q or P^H:

If $side$ = 'L', $r = m$; if $side$ = 'R', $r = n$.

vect	CHARACTER*1. Must be 'Q' or 'P'. If *vect* = 'Q', then Q or Q^H is applied to C. If *vect* = 'P', then P or P^H is applied to C.
side	CHARACTER*1. Must be 'L' or 'R'. If *side* = 'L', multipliers are applied to C from the left. If *side* = 'R', they are applied to C from the right.
trans	CHARACTER*1. Must be 'N' or 'C'. If *trans* = 'N', then Q or P is applied to C. If *trans* = 'C', then Q^H or P^H is applied to C.
m	INTEGER. The number of rows in C.
n	INTEGER. The number of columns in C.
k	INTEGER. One of the dimensions of A in ?gebrd: If *vect* = 'Q', the number of columns in A; If *vect* = 'P', the number of rows in A. Constraints: $m \geq 0$, $n \geq 0$, $k \geq 0$.
a, *c*, *work*	COMPLEX for cunmbr DOUBLE COMPLEX for zunmbr. Arrays:

	a(*lda*,*) is the array *a* as returned by ?gebrd. Its second dimension must be at least max(1, min(*r*,*k*)) for *vect* = 'Q', or max(1, *r*)) for *vect* = 'P'. *c*(*ldc*,*) holds the matrix *C*. Its second dimension must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*. Constraints: *lda* ≥ max(1, *r*) if *vect* = 'Q'; *lda* ≥ max(1, min(*r*,*k*)) if *vect* = 'P'.
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *m*).
tau	COMPLEX for cunmbr DOUBLE COMPLEX for zunmbr. Array, DIMENSION at least max (1, min(*r*, *k*)). For *vect* = 'Q', the array *tauq* as returned by ?gebrd. For *vect* = 'P', the array *taup* as returned by ?gebrd.
lwork	INTEGER. The size of the *work* array. *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. *lwork* ≥ 1 if *n*=0 or *m*=0. For optimum performance *lwork* ≥ max(1,*n***nb*) if *side* = 'L', and *lwork* ≥ max(1,*m***nb*) if *side* = 'R', where *nb* is the optimal blocksize. (*nb* = 0 if *m* = 0 or *n* = 0.) If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product *Q***C*, Q^H**C*, *C***Q*, *C**Q^H, *P***C*, P^H**C*, *C***P*, or *C**P^H, as specified by *vect*, *side*, and *trans*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unmbr interface are the following:

a	Holds the matrix *A* of size (*r*,min(*nq*,*k*)) where *r* = *nq*, if *vect* = 'Q', *r* = min(*nq*,*k*), if *vect* = 'P', *nq* = *m*, if *side* = 'L', *nq* = *n*, if *side* = 'R', *k* = *m*, if *vect* = 'P', *k* = *n*, if *vect* = 'Q'.

tau	Holds the vector of length min(*nq*,*k*).
c	Holds the matrix *C* of size (*m*,*n*).
vect	Must be 'Q' or 'P'. The default value is 'Q'.
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, use

lwork = *n***blocksize* for *side* = 'L', or

lwork = *m***blocksize* for *side* = 'R',

where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed product differs from the exact product by a matrix *E* such that $||E||_2 = O(\varepsilon)*||C||_2$.

The total number of floating-point operations is approximately

$8*n*k(2*m - k)$ if *side* = 'L' and $m \geq k$;

$8*m*k(2*n - k)$ if *side* = 'R' and $n \geq k$;

$8*m^2*n$ if *side* = 'L' and $m < k$;

$8*n^2*m$ if *side* = 'R' and $n < k$.

The real counterpart of this routine is ormbr.

?bdsqr

Computes the singular value decomposition of a general matrix that has been reduced to bidiagonal form.

Syntax

Fortran 77:

```
call sbdsqr(uplo, n, ncvt, nru, ncc, d, e, vt, ldvt, u, ldu, c, ldc, work, info)
call dbdsqr(uplo, n, ncvt, nru, ncc, d, e, vt, ldvt, u, ldu, c, ldc, work, info)
call cbdsqr(uplo, n, ncvt, nru, ncc, d, e, vt, ldvt, u, ldu, c, ldc, work, info)
call zbdsqr(uplo, n, ncvt, nru, ncc, d, e, vt, ldvt, u, ldu, c, ldc, work, info)
```

Fortran 95:

```
call rbdsqr(d, e [,vt] [,u] [,c] [,uplo] [,info])
call bdsqr(d, e [,vt] [,u] [,c] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_sbdsqr( int matrix_order, char uplo, lapack_int n, lapack_int ncvt,
lapack_int nru, lapack_int ncc, float* d, float* e, float* vt, lapack_int ldvt, float*
u, lapack_int ldu, float* c, lapack_int ldc );

lapack_int LAPACKE_dbdsqr( int matrix_order, char uplo, lapack_int n, lapack_int ncvt,
lapack_int nru, lapack_int ncc, double* d, double* e, double* vt, lapack_int ldvt,
double* u, lapack_int ldu, double* c, lapack_int ldc );

lapack_int LAPACKE_cbdsqr( int matrix_order, char uplo, lapack_int n, lapack_int ncvt,
lapack_int nru, lapack_int ncc, float* d, float* e, lapack_complex_float* vt,
lapack_int ldvt, lapack_complex_float* u, lapack_int ldu, lapack_complex_float* c,
lapack_int ldc );

lapack_int LAPACKE_zbdsqr( int matrix_order, char uplo, lapack_int n, lapack_int ncvt,
lapack_int nru, lapack_int ncc, double* d, double* e, lapack_complex_double* vt,
lapack_int ldvt, lapack_complex_double* u, lapack_int ldu, lapack_complex_double* c,
lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the singular values and, optionally, the right and/or left singular vectors from the Singular Value Decomposition (SVD) of a real n-by-n (upper or lower) bidiagonal matrix B using the implicit zero-shift QR algorithm. The SVD of B has the form $B = Q*S*P^H$ where S is the diagonal matrix of singular values, Q is an orthogonal matrix of left singular vectors, and P is an orthogonal matrix of right singular vectors. If left singular vectors are requested, this subroutine actually returns $U*Q$ instead of Q, and, if right singular vectors are requested, this subroutine returns P_H*VT instead of P_H, for given real/complex input matrices U and VT. When U and VT are the orthogonal/unitary matrices that reduce a general matrix A to bidiagonal form: $A = U*B*VT$, as computed by `?gebrd`, then

$A = (U*Q)*S*(P^H*VT)$

is the SVD of A. Optionally, the subroutine may also compute Q_H*C for a given real/complex input matrix C.

See also lasq1, lasq2, lasq3, lasq4, lasq5, lasq6 used by this routine.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If $uplo = 'U'$, B is an upper bidiagonal matrix. If $uplo = 'L'$, B is a lower bidiagonal matrix.
n	INTEGER. The order of the matrix B ($n \geq 0$).
ncvt	INTEGER. The number of columns of the matrix VT, that is, the number of right singular vectors ($ncvt \geq 0$). Set $ncvt = 0$ if no right singular vectors are required.
nru	INTEGER. The number of rows in U, that is, the number of left singular vectors ($nru \geq 0$).

	Set *nru* = 0 if no left singular vectors are required.
ncc	INTEGER. The number of columns in the matrix *C* used for computing the product Q^H*C (*ncc* ≥ 0). Set *ncc* = 0 if no matrix *C* is supplied.
d, *e*, *work*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays: *d*(*) contains the diagonal elements of *B*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the (*n*–1) off-diagonal elements of *B*. The dimension of *e* must be at least max(1, *n*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1, 4**n*).
vt, *u*, *c*	REAL for sbdsqr DOUBLE PRECISION for dbdsqr COMPLEX for cbdsqr DOUBLE COMPLEX for zbdsqr. Arrays: *vt*(*ldvt*,*) contains an *n*-by-*ncvt* matrix *VT*. The second dimension of *vt* must be at least max(1, *ncvt*). *vt* is not referenced if *ncvt* = 0. *u*(*ldu*,*) contains an *nru* by *n* unit matrix *U*. The second dimension of *u* must be at least max(1, *n*). *u* is not referenced if *nru* = 0. *c*(*ldc*,*) contains the matrix *C* for computing the product Q^H**C*. The second dimension of *c* must be at least max(1, *ncc*). The array is not referenced if *ncc* = 0.
ldvt	INTEGER. The leading dimension of *vt*. Constraints: *ldvt* ≥ max(1, *n*) if *ncvt* > 0; *ldvt* ≥ 1 if *ncvt* = 0.
ldu	INTEGER. The leading dimension of *u*. Constraint: *ldu* ≥ max(1, *nru*).
ldc	INTEGER. The leading dimension of *c*. Constraints: *ldc* ≥ max(1, *n*) if *ncc* > 0; *ldc* ≥ 1 otherwise.

Output Parameters

d	On exit, if *info* = 0, overwritten by the singular values in decreasing order (see *info*).
e	On exit, if *info* = 0, *e* is destroyed. See also *info* below.
c	Overwritten by the product Q^H**C*.
vt	On exit, this array is overwritten by P^H **VT*.
u	On exit, this array is overwritten by *U* **Q* .
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* > *i*, If *ncvt* = *nru* = *ncc* = 0, • *info* = 1, a split was marked by a positive value in *e* • *info* = 2, the current block of *z* not diagonalized after 30**n* iterations (in the inner while loop)

- *info* = 3, termination criterion of the outer while loop is not met (the program created more than *n* unreduced blocks).

In all other cases when *ncvt* = *nru* = *ncc* = 0, the algorithm did not converge; *d* and *e* contain the elements of a bidiagonal matrix that is orthogonally similar to the input matrix *B*; if *info* = *i*, *i* elements of *e* have not converged to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine bdsqr interface are the following:

d	Holds the vector of length (*n*).
e	Holds the vector of length (*n*).
vt	Holds the matrix *VT* of size (*n*, *ncvt*).
u	Holds the matrix *U* of size (*nru*,*n*).
c	Holds the matrix *C* of size (*n*,*ncc*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
ncvt	If argument *vt* is present, then *ncvt* is equal to the number of columns in matrix *VT*; otherwise, *ncvt* is set to zero.
nru	If argument *u* is present, then *nru* is equal to the number of rows in matrix *U*; otherwise, *nru* is set to zero.
ncc	If argument *c* is present, then *ncc* is equal to the number of columns in matrix *C*; otherwise, *ncc* is set to zero.

Note that two variants of Fortran 95 interface for bdsqr routine are needed because of an ambiguous choice between real and complex cases appear when *vt*, *u*, and *c* are omitted. Thus, the name rbdsqr is used in real cases (single or double precision), and the name bdsqr is used in complex cases (single or double precision).

Application Notes

Each singular value and singular vector is computed to high relative accuracy. However, the reduction to bidiagonal form (prior to calling the routine) may decrease the relative accuracy in the small singular values of the original matrix if its singular values vary widely in magnitude.

If s_i is an exact singular value of *B*, and s_i is the corresponding computed value, then

$$|s_i - \sigma_i| \leq p(m,n) * \varepsilon * \sigma_i$$

where $p(m, n)$ is a modestly increasing function of m and n, and ε is the machine precision.

If only singular values are computed, they are computed more accurately than when some singular vectors are also computed (that is, the function $p(m, n)$ is smaller).

If u_i is the corresponding exact left singular vector of *B*, and w_i is the corresponding computed left singular vector, then the angle $\theta(u_i, w_i)$ between them is bounded as follows:

$$\theta(u_i, w_i) \leq p(m,n) * \varepsilon / \min_{i \neq j}(|\sigma i - \sigma j| / |\sigma_i + \sigma j|).$$

Here $\min_{i \neq j}(|\sigma_i - \sigma_j| / |\sigma_i + \sigma_j|)$ is the *relative gap* between σ_i and the other singular values. A similar error bound holds for the right singular vectors.

The total number of real floating-point operations is roughly proportional to n^2 if only the singular values are computed. About $6n^2*nru$ additional operations ($12n^2*nru$ for complex flavors) are required to compute the left singular vectors and about $6n^2*ncvt$ operations ($12n^2*ncvt$ for complex flavors) to compute the right singular vectors.

?bdsdc

Computes the singular value decomposition of a real bidiagonal matrix using a divide and conquer method.

Syntax

Fortran 77:

```
call sbdsdc(uplo, compq, n, d, e, u, ldu, vt, ldvt, q, iq, work, iwork, info)
call dbdsdc(uplo, compq, n, d, e, u, ldu, vt, ldvt, q, iq, work, iwork, info)
```

Fortran 95:

```
call bdsdc(d, e [,u] [,vt] [,q] [,iq] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>bdsdc( int matrix_order, char uplo, char compq, lapack_int n,
<datatype>* d, <datatype>* e, <datatype>* u, lapack_int ldu, <datatype>* vt,
lapack_int ldvt, <datatype>* q, lapack_int* iq );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the Singular Value Decomposition (SVD) of a real n-by-n (upper or lower) bidiagonal matrix B: $B = U*\Sigma*V^T$, using a divide and conquer method, where Σ is a diagonal matrix with non-negative diagonal elements (the singular values of B), and U and V are orthogonal matrices of left and right singular vectors, respectively. ?bdsdc can be used to compute all singular values, and optionally, singular vectors or singular vectors in compact form.

This rotuine uses ?lasd0, ?lasd1, ?lasd2, ?lasd3, ?lasd4, ?lasd5, ?lasd6, ?lasd7, ?lasd8, ?lasd9, ?lasda, ?lasdq, ?lasdt.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', B is an upper bidiagonal matrix. If *uplo* = 'L', B is a lower bidiagonal matrix.
compq	CHARACTER*1. Must be 'N', 'P', or 'I'. If *compq* = 'N', compute singular values only. If *compq* = 'P', compute singular values and compute singular vectors in compact form. If *compq* = 'I', compute singular values and singular vectors.
n	INTEGER. The order of the matrix B ($n \geq 0$).

d, *e*, *work*	REAL for sbdsdc DOUBLE PRECISION for dbdsdc. Arrays: *d(*)* contains the *n* diagonal elements of the bidiagonal matrix *B*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of the bidiagonal matrix *B*. The dimension of *e* must be at least max(1, *n*). *work*(*) is a workspace array. The dimension of *work* must be at least: max(1, 4**n*), if *compq* = 'N'; max(1, 6**n*), if *compq* = 'P'; max(1, 3**n*^{2}+4**n*), if *compq* = 'I'.
ldu	INTEGER. The leading dimension of the output array *u*; *ldu* ≥ 1. If singular vectors are desired, then *ldu* ≥ max(1, *n*).
ldvt	INTEGER. The leading dimension of the output array *vt*; *ldvt* ≥ 1. If singular vectors are desired, then *ldvt* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, dimension at least max(1, 8**n*).

Output Parameters

d	If *info* = 0, overwritten by the singular values of *B*.
e	On exit, *e* is overwritten.
u, *vt*, *q*	REAL for sbdsdc DOUBLE PRECISION for dbdsdc. Arrays: *u*(*ldu*,*), *vt*(*ldvt*,*), *q*(*). If *compq* = 'I', then on exit *u* contains the left singular vectors of the bidiagonal matrix *B*, unless *info* ≠ 0 (*see* *info*). For other values of *compq*, *u* is not referenced. The second dimension of *u* must be at least max(1,*n*). if *compq* = 'I', then on exit vt^T contains the right singular vectors of the bidiagonal matrix *B*, unless *info* ≠ 0 (*see* *info*). For other values of *compq*, *vt* is not referenced. The second dimension of *vt* must be at least max(1,*n*). If *compq* = 'P', then on exit, if *info* = 0, *q* and *iq* contain the left and right singular vectors in a compact form. Specifically, *q* contains all the REAL (for sbdsdc) or DOUBLE PRECISION (for dbdsdc) data for singular vectors. For other values of *compq*, *q* is not referenced. See *Application notes* for details.
iq	INTEGER. Array: *iq*(*). If *compq* = 'P', then on exit, if *info* = 0, *q* and *iq* contain the left and right singular vectors in a compact form. Specifically, *iq* contains all the INTEGER data for singular vectors. For other values of *compq*, *iq* is not referenced. See *Application notes* for details.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the algorithm failed to compute a singular value. The update process of divide and conquer failed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `bdsdc` interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length *n*.
u	Holds the matrix *U* of size (*n*,*n*).
vt	Holds the matrix *VT* of size (*n*,*n*).
q	Holds the vector of length (*ldq*), where *ldq* ≥ *n**(11 + 2**smlsiz* + 8**int*(*log_2*(*n*/(*smlsiz* + 1)))) and *smlsiz* is returned by *ilaenv* and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25).
compq	Restored based on the presence of arguments *u*, *vt*, *q*, and *iq* as follows: *compq* = 'N', if none of *u*, *vt*, *q*, and *iq* are present, *compq* = 'I', if both *u* and *vt* are present. Arguments *u* and *vt* must either be both present or both omitted, *compq* = 'P', if both *q* and *iq* are present. Arguments *q* and *iq* must either be both present or both omitted. Note that there will be an error condition if all of *u*, *vt*, *q*, and *iq* arguments are present simultaneously.

See Also

?lasd0
?lasd1
?lasd2
?lasd3
?lasd4
?lasd5
?lasd6
?lasd7
?lasd8
?lasd9
?lasda
?lasdq
?lasdt

Symmetric Eigenvalue Problems

Symmetric eigenvalue problems are posed as follows: given an *n*-by-*n* real symmetric or complex Hermitian matrix *A*, find the *eigenvalues* λ and the corresponding *eigenvectors* *z* that satisfy the equation

$Az = \lambda z$ (or, equivalently, $z^H A = \lambda z^H$).

In such eigenvalue problems, all *n* eigenvalues are real not only for real symmetric but also for complex Hermitian matrices *A*, and there exists an orthonormal system of *n* eigenvectors. If *A* is a symmetric or Hermitian positive-definite matrix, all eigenvalues are positive.

To solve a symmetric eigenvalue problem with LAPACK, you usually need to reduce the matrix to tridiagonal form and then solve the eigenvalue problem with the tridiagonal matrix obtained. LAPACK includes routines for reducing the matrix to a tridiagonal form by an orthogonal (or unitary) similarity transformation $A = QTQ^H$ as well as for solving tridiagonal symmetric eigenvalue problems. These routines (for FORTRAN 77

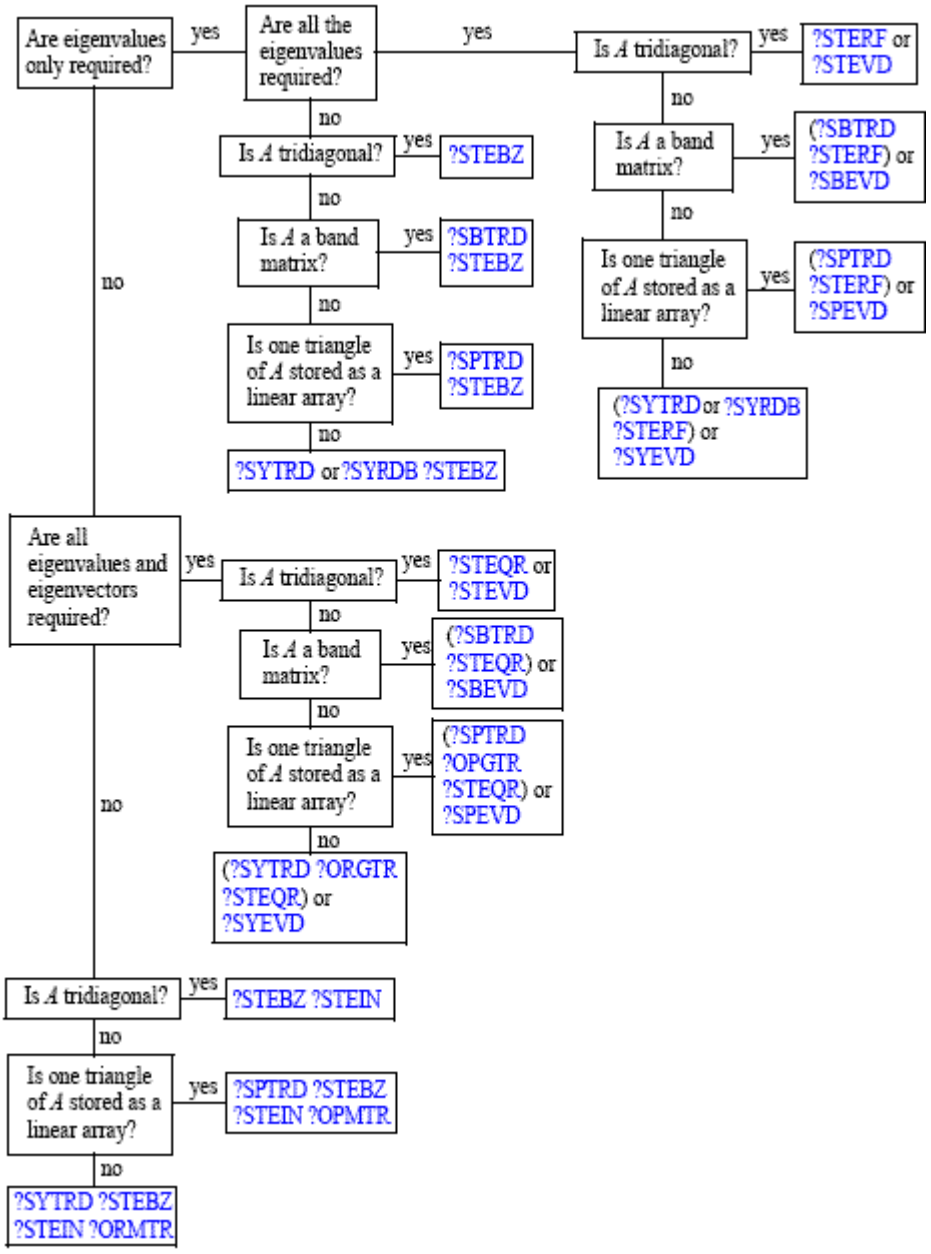
interface) are listed in Table "Computational Routines for Solving Symmetric Eigenvalue Problems". Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

There are different routines for symmetric eigenvalue problems, depending on whether you need all eigenvectors or only some of them or eigenvalues only, whether the matrix A is positive-definite or not, and so on.

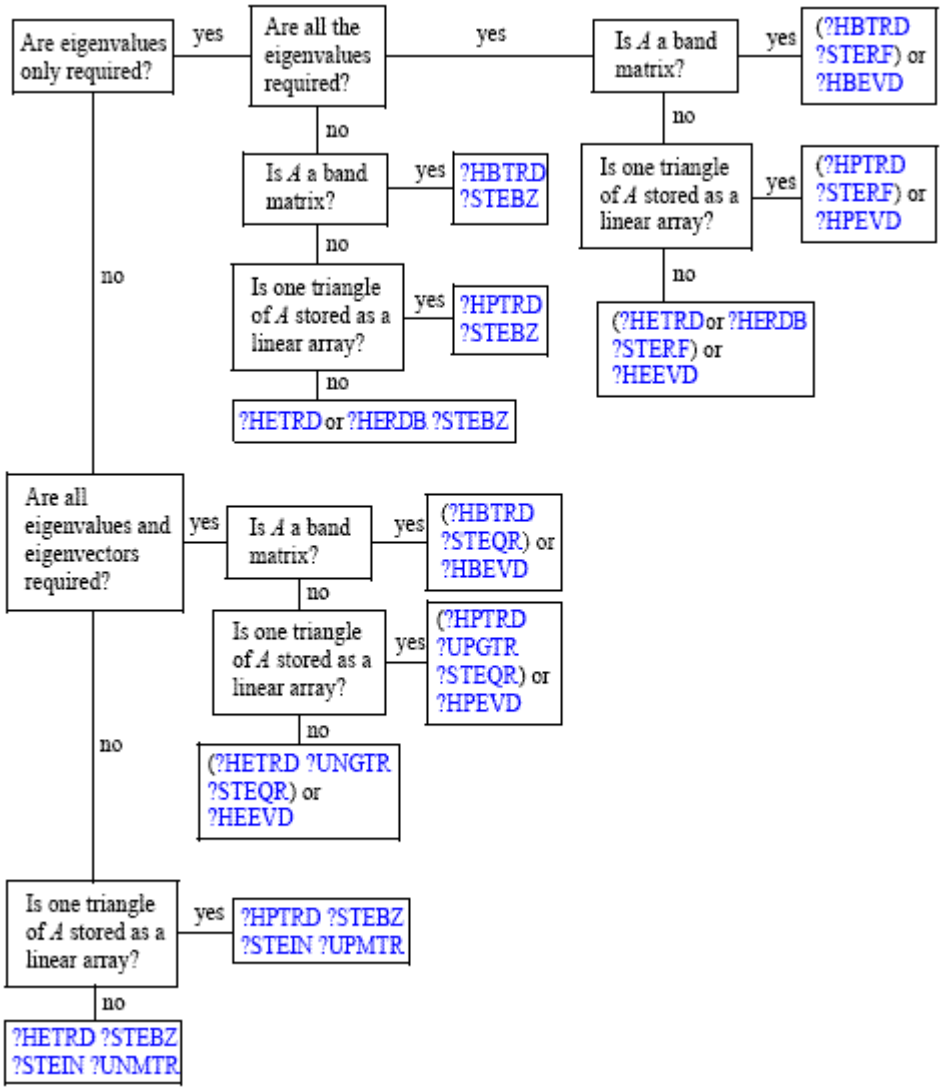
These routines are based on three primary algorithms for computing eigenvalues and eigenvectors of symmetric problems: the divide and conquer algorithm, the QR algorithm, and bisection followed by inverse iteration. The divide and conquer algorithm is generally more efficient and is recommended for computing all eigenvalues and eigenvectors. Furthermore, to solve an eigenvalue problem using the divide and conquer algorithm, you need to call only one routine. In general, more than one routine has to be called if the QR algorithm or bisection followed by inverse iteration is used.

The decision tree in Figure "Decision Tree: Real Symmetric Eigenvalue Problems" will help you choose the right routine or sequence of routines for eigenvalue problems with real symmetric matrices. Figure "Decision Tree: Complex Hermitian Eigenvalue Problems" presents a similar decision tree for complex Hermitian matrices.

Decision Tree: Real Symmetric Eigenvalue Problems



Decision Tree: Complex Hermitian Eigenvalue Problems



Computational Routines for Solving Symmetric Eigenvalue Problems

Operation	Real symmetric matrices	Complex Hermitian matrices
Reduce to tridiagonal form $A = QTQ^H$ (full storage)	sytrd syrdb	hetrd herdb
Reduce to tridiagonal form $A = QTQ^H$ (packed storage)	sptrd	hptrd
Reduce to tridiagonal form $A = QTQ^H$ (band storage).	sbtrd	hbtrd
Generate matrix Q (full storage)	orgtr	ungtr
Generate matrix Q (packed storage)	opgtr	upgtr
Apply matrix Q (full storage)	ormtr	unmtr
Apply matrix Q (packed storage)	opmtr	upmtr

Operation	Real symmetric matrices	Complex Hermitian matrices
Find all eigenvalues of a tridiagonal matrix *T*	sterf	
Find all eigenvalues and eigenvectors of a tridiagonal matrix *T*	steqr stedc	steqr stedc
Find all eigenvalues and eigenvectors of a tridiagonal positive-definite matrix *T*.	pteqr	pteqr
Find selected eigenvalues of a tridiagonal matrix *T*	stebz stegr	stegr
Find selected eigenvectors of a tridiagonal matrix *T*	stein stegr	stein stegr
Find selected eigenvalues and eigenvectors of f a real symmetric tridiagonal matrix *T*	stemr	stemr
Compute the reciprocal condition numbers for the eigenvectors	disna	disna

?sytrd

Reduces a real symmetric matrix to tridiagonal form.

Syntax

Fortran 77:

```
call ssytrd(uplo, n, a, lda, d, e, tau, work, lwork, info)
call dsytrd(uplo, n, a, lda, d, e, tau, work, lwork, info)
```

Fortran 95:

```
call sytrd(a, tau [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sytrd( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, <datatype>* d, <datatype>* e, <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces a real symmetric matrix *A* to symmetric tridiagonal form *T* by an orthogonal similarity transformation: $A = Q*T*Q^T$. The orthogonal matrix *Q* is not formed explicitly but is represented as a product of *n*-1 elementary reflectors. Routines are provided for working with *Q* in this representation (see *Application Notes* below).

This routine calls latrd to reduce a real symmetric matrix to tridiagonal form by an orthogonal similarity transformation.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* ($n \geq 0$).
a, *work*	REAL for ssytrd DOUBLE PRECISION for dsytrd. *a*(*lda*,*) is an array containing either upper or lower triangular part of the matrix *A*, as specified by *uplo*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *A* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *A* is not referenced. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
lwork	INTEGER. The size of the *work* array ($lwork \geq n$). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	On exit, if *uplo* = 'U', the diagonal and first superdiagonal of *A* are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements above the first superdiagonal, with the array *tau*, represent the orthogonal matrix *Q* as a product of elementary reflectors; if *uplo* = 'L', the diagonal and first subdiagonal of *A* are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal matrix *Q* as a product of elementary reflectors.
d, *e*, *tau*	REAL for ssytrd DOUBLE PRECISION for dsytrd. Arrays: *d*(*) contains the diagonal elements of the matrix *T*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1). *tau*(*) stores further details of the orthogonal matrix *Q* in the first *n*-1 elements. *tau*(*n*) is used as workspace. The dimension of *tau* must be at least max(1, *n*).
work(1)	If *info*=0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `sytrd` interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
tau	Holds the vector of length (*n*-1).
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.

Note that diagonal (*d*) and off-diagonal (*e*) elements of the matrix *T* are omitted because they are kept in the matrix *A* on exit.

Application Notes

For better performance, try using *lwork* =*n*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *T* is exactly similar to a matrix *A*+*E*, where $||E||_2 = c(n)*\varepsilon*||A||_2$, *c*(*n*) is a modestly increasing function of *n*, and ε is the machine precision.

The approximate number of floating-point operations is $(4/3)n^3$.

After calling this routine, you can call the following:

orgtr	to form the computed matrix *Q* explicitly
ormtr	to multiply a real matrix by *Q*.

The complex counterpart of this routine is hetrd.

?syrdb

Reduces a real symmetric matrix to tridiagonal form with Successive Bandwidth Reduction approach.

Syntax

Fortran 77:

```
call ssyrdb(jobz, uplo, n, kd, a, lda, d, e, tau, z, ldz, work, lwork, info)
call dsyrdb(jobz, uplo, n, kd, a, lda, d, e, tau, z, ldz, work, lwork, info)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = Q*T*Q^T$ and optionally multiplies matrix Z by Q, or simply forms Q.

This routine reduces a full symmetric matrix to the banded symmetric form, and then to the tridiagonal symmetric form with a Successive Bandwidth Reduction approach after Prof. C.Bischof's works (see for instance, [Bischof92]). ?syrdb is functionally close to ?sytrd routine but the tridiagonal form may differ from those obtained by ?sytrd. Unlike ?sytrd, the orthogonal matrix Q cannot be restored from the details of matrix A on exit.

Input Parameters

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only A is reduced to T. If *jobz* = 'V', then A is reduced to T and A contains Q on exit. If *jobz* = 'U', then A is reduced to T and Z contains $Z*Q$ on exit.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of A. If *uplo* = 'L', *a* stores the lower triangular part of A.
n	INTEGER. The order of the matrix A ($n \geq 0$).
kd	INTEGER. The bandwidth of the banded matrix B ($kd \geq 1$).
a,*z*, *work*	REAL for ssyrdb. DOUBLE PRECISION for dsyrdb. *a*(*lda*,*) is an array containing either upper or lower triangular part of the matrix A, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *z*(*ldz*,*), the second dimension of *z* must be at least max(1, *n*). If *jobz* = 'U', then the matrix *z* is multiplied by Q. If *jobz* = 'N' or 'V', then *z* is not referenced. *work*(*lwork*) is a workspace array.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldz	INTEGER. The leading dimension of *z*; at least max(1, *n*). Not referenced if *jobz* = 'N'
lwork	INTEGER. The size of the *work* array ($lwork \geq (2kd+1)n+kd$). If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	If *jobz* = 'V', then overwritten by Q matrix. If *jobz* = 'N' or 'U', then overwritten by the banded matrix B and details of the orthogonal matrix Q_B to reduce A to B as specified by *uplo*.
z	On exit, if *jobz* = 'U', then the matrix *z* is overwritten by $Z*Q$. If *jobz* = 'N' or 'V', then *z* is not referenced.
d, *e*, *tau*	DOUBLE PRECISION. Arrays: *d*(*) contains the diagonal elements of the matrix T.

	The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1). *tau*(*) stores further details of the orthogonal matrix *Q*. The dimension of *tau* must be at least max(1, *n*-*kd*-1).
work(1)	If *info*=0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

For better performance, try using *lwork* = *n**(3**kd*+3).

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

For better performance, try using *kd* equal to 40 if n ≤ 2000 and 64 otherwise.

Try using ?syrdb instead of ?sytrd on large matrices obtaining only eigenvalues - when no eigenvectors are needed, especially in multi-threaded environment. ?syrdb becomes faster beginning approximately with n = 1000, and much faster at larger matrices with a better scalability than ?sytrd.

Avoid applying ?syrdb for computing eigenvectors due to the two-step reduction, that is, the number of operations needed to apply orthogonal transformations to *Z* is doubled compared to the traditional one-step reduction. In that case it is better to apply ?sytrd and ?ormtr/?orgtr to obtain tridiagonal form along with the orthogonal transformation matrix *Q*.

?herdb

Reduces a complex Hermitian matrix to tridiagonal form with Successive Bandwidth Reduction approach.

Syntax

Fortran 77:

```
call cherdb(jobz, uplo, n, kd, a, lda, d, e, tau, z, ldz, work, lwork, info)
call zherdb(jobz, uplo, n, kd, a, lda, d, e, tau, z, ldz, work, lwork, info)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine reduces a complex Hermitian matrix *A* to symmetric tridiagonal form *T* by a unitary similarity transformation: $A = Q*T*Q^T$ and optionally multiplies matrix *Z* by *Q*, or simply forms *Q*.

This routine reduces a full Hermitian matrix to the banded Hermitian form, and then to the tridiagonal symmetric form with a Successive Bandwidth Reduction approach after Prof. C.Bischof's works (see for instance, [Bischof92]). ?herdb is functionally close to ?hetrd routine but the tridiagonal form may differ from those obtained by ?hetrd. Unlike ?hetrd, the orthogonal matrix Q cannot be restored from the details of matrix A on exit.

Input Parameters

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only A is reduced to T. If *jobz* = 'V', then A is reduced to T and A contains Q on exit. If *jobz* = 'U', then A is reduced to T and Z contains $Z*Q$ on exit.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of A. If *uplo* = 'L', *a* stores the lower triangular part of A.
n	INTEGER. The order of the matrix A ($n \geq 0$).
kd	INTEGER. The bandwidth of the banded matrix B ($kd \geq 1$).
a,*z*, *work*	COMPLEX for cherdb. DOUBLE COMPLEX for zherdb. *a*(*lda*,*) is an array containing either upper or lower triangular part of the matrix A, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *z*(*ldz*,*), the second dimension of *z* must be at least max(1, *n*). If *jobz* = 'U', then the matrix *z* is multiplied by Q. If *jobz* = 'N' or 'V', then *z* is not referenced. *work*(*lwork*) is a workspace array.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldz	INTEGER. The leading dimension of *z*; at least max(1, *n*). Not referenced if *jobz* = 'N'
lwork	INTEGER. The size of the *work* array (*lwork* $\geq$ (2*kd*+1)*n*+*kd*). If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	If *jobz* = 'V', then overwritten by Q matrix. If *jobz* = 'N' or 'U', then overwritten by the banded matrix B and details of the unitary matrix Q_B to reduce A to B as specified by *uplo*.
z	On exit, if *jobz* = 'U', then the matrix *z* is overwritten by $Z*Q$. If *jobz* = 'N' or 'V', then *z* is not referenced.
d, *e*	COMPLEX for cherdb. DOUBLE COMPLEX for zherdb. Arrays: *d*(*) contains the diagonal elements of the matrix T. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of T. The dimension of *e* must be at least max(1, *n*-1). *tau*(*) stores further details of the orthogonal matrix Q.

	The dimension of *tau* must be at least max(1, *n*-*kd*-1).
tau	COMPLEX for cherdb. DOUBLE COMPLEX for zherdb. Array, DIMENSION at least max(1, *n*-1) Stores further details of the unitary matrix Q_B. The dimension of *tau* must be at least max(1, *n*-*kd*-1).
work(1)	If *info*=0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

For better performance, try using *lwork* = *n**(3**kd*+3).

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

For better performance, try using *kd* equal to 40 if n ≤ 2000 and 64 otherwise.

Try using ?herdb instead of ?hetrd on large matrices obtaining only eigenvalues - when no eigenvectors are needed, especially in multi-threaded environment. ?herdb becomes faster beginning approximately with n = 1000, and much faster at larger matrices with a better scalability than ?hetrd.

Avoid applying ?herdb for computing eigenvectors due to the two-step reduction, that is, the number of operations needed to apply orthogonal transformations to *Z* is doubled compared to the traditional one-step reduction. In that case it is better to apply ?hetrd and ?unmtr/?ungtr to obtain tridiagonal form along with the unitary transformation matrix *Q*.

?orgtr

Generates the real orthogonal matrix Q determined by ?sytrd*.*

Syntax

Fortran 77:

```
call sorgtr(uplo, n, a, lda, tau, work, lwork, info)
call dorgtr(uplo, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orgtr(a, tau [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>orgtr( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine explicitly generates the n-by-n orthogonal matrix Q formed by sytrd when reducing a real symmetric matrix A to tridiagonal form: $A = Q*T*Q^T$. Use this routine after a call to ?sytrd.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?sytrd.
n	INTEGER. The order of the matrix Q ($n \geq 0$).
a, *tau*, *work*	REAL for sorgtr DOUBLE PRECISION for dorgtr. Arrays: *a*(*lda*,*) is the array *a* as returned by ?sytrd. The second dimension of *a* must be at least max(1, *n*). *tau*(*) is the array *tau* as returned by ?sytrd. The dimension of *tau* must be at least max(1, *n*-1). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
lwork	INTEGER. The size of the *work* array ($lwork \geq n$). If $lwork = -1$, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the orthogonal matrix Q.
work(1)	If $info = 0$, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orgtr interface are the following:

a	Holds the matrix A of size (n,n).
tau	Holds the vector of length (n-1).

uplo Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For better performance, try using *lwork* = (*n*–1)***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is $(4/3)n^3$.

The complex counterpart of this routine is ungtr.

?ormtr

Multiplies a real matrix by the real orthogonal matrix Q determined by ?sytrd.

Syntax

Fortran 77:

```
call sormtr(side, uplo, trans, m, n, a, lda, tau, c, ldc, work, lwork, info)
call dormtr(side, uplo, trans, m, n, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormtr(a, tau, c [,side] [,uplo] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormtr( int matrix_order, char side, char uplo, char trans,
lapack_int m, lapack_int n, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real matrix *C* by *Q* or Q^T, where *Q* is the orthogonal matrix *Q* formed by sytrd when reducing a real symmetric matrix *A* to tridiagonal form: $A = Q*T*Q^T$. Use this routine after a call to ?sytrd.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (overwriting the result on *C*).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

In the descriptions below, *r* denotes the order of *Q*:

If *side* = 'L', *r* = *m*; if *side* = 'R', *r* = *n*.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', *Q* or Q^T is applied to *C* from the left. If *side* = 'R', *Q* or Q^T is applied to *C* from the right.
uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?sytrd.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies *C* by *Q*. If *trans* = 'T', the routine multiplies *C* by Q^T.
m	INTEGER. The number of rows in the matrix *C* (*m* ≥ 0).
n	INTEGER. The number of columns in *C* (*n* ≥ 0).
a, *c*, *tau*, *work*	REAL for sormtr DOUBLE PRECISION for dormtr *a*(*lda*,*) and *tau* are the arrays returned by ?sytrd. The second dimension of *a* must be at least max(1, *r*). The dimension of *tau* must be at least max(1, *r*-1). *c*(*ldc*,*) contains the matrix *C*. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *r*).
ldc	INTEGER. The leading dimension of *c*; *ldc* ≥ max(1, *n*).
lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product *Q***C*, Q^T**C*, *C***Q*, or *C**Q^T (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormtr interface are the following:

a	Holds the matrix *A* of size (*r*,*r*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*r*-1).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n***blocksize* for *side* = 'L', or *lwork* = *m***blocksize* for *side* = 'R', where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed product differs from the exact product by a matrix E such that $||E||^2 = O(\varepsilon)*||C||^2$.

The total number of floating-point operations is approximately $2*m^2*n$, if *side* = 'L', or $2*n^2*m$, if *side* = 'R'.

The complex counterpart of this routine is unmtr.

?hetrd

Reduces a complex Hermitian matrix to tridiagonal form.

Syntax

Fortran 77:

```
call chetrd(uplo, n, a, lda, d, e, tau, work, lwork, info)
call zhetrd(uplo, n, a, lda, d, e, tau, work, lwork, info)
```

Fortran 95:

```
call hetrd(a, tau [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_chetrd( int matrix_order, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* d, float* e, lapack_complex_float*
tau );
```

```
lapack_int LAPACKE_zhetrd( int matrix_order, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* d, double* e, lapack_complex_double*
tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces a complex Hermitian matrix A to symmetric tridiagonal form T by a unitary similarity transformation: $A = Q*T*Q^H$. The unitary matrix Q is not formed explicitly but is represented as a product of n-1 elementary reflectors. Routines are provided to work with Q in this representation. (They are described later in this section .)

This routine calls latrd to reduce a complex Hermitian matrix A to Hermitian tridiagonal form by a unitary similarity transformation.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of A. If *uplo* = 'L', *a* stores the lower triangular part of A.
n	INTEGER. The order of the matrix A ($n \geq 0$).
a, *work*	COMPLEX for chetrd DOUBLE COMPLEX for zhetrd. *a*(*lda*,*) is an array containing either upper or lower triangular part of the matrix A, as specified by *uplo*. If *uplo* = 'U', the leading n-by-n upper triangular part of *a* contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If *uplo* = 'L', the leading n-by-n lower triangular part of *a* contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced. The second dimension of *a* must be at least max(1, n). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, n).
lwork	INTEGER. The size of the *work* array ($lwork \geq n$). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	On exit, if *uplo* = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array *tau*, represent the orthogonal matrix Q as a product of elementary reflectors;

	if *uplo* = 'L', the diagonal and first subdiagonal of *A* are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal matrix *Q* as a product of elementary reflectors.
d, *e*	REAL for chetrd DOUBLE PRECISION for zhetrd. Arrays: *d*(*) contains the diagonal elements of the matrix *T*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1).
tau	COMPLEX for chetrd DOUBLE COMPLEX for zhetrd. Array, DIMENSION at least max(1, *n*-1). Stores further details of the unitary matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hetrd interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
tau	Holds the vector of length (*n*-1).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Note that diagonal (*d*) and off-diagonal (*e*) elements of the matrix *T* are omitted because they are kept in the matrix *A* on exit.

Application Notes

For better performance, try using *lwork* =*n*blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *T* is exactly similar to a matrix $A + E$, where $||E||_2 = c(n)*\varepsilon*||A||_2$, $c(n)$ is a modestly increasing function of n, and ε is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

After calling this routine, you can call the following:

ungtr	to form the computed matrix *Q* explicitly
unmtr	to multiply a complex matrix by *Q*.

The real counterpart of this routine is sytrd.

?ungtr

Generates the complex unitary matrix Q determined by ?hetrd*.*

Syntax

Fortran 77:

```
call cungtr(uplo, n, a, lda, tau, work, lwork, info)
call zungtr(uplo, n, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call ungtr(a, tau [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>ungtr( int matrix_order, char uplo, lapack_int n, <datatype>* a,
lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine explicitly generates the *n*-by-*n* unitary matrix *Q* formed by hetrd when reducing a complex Hermitian matrix *A* to tridiagonal form: $A = Q*T*Q^H$. Use this routine after a call to ?hetrd.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?hetrd.
n	INTEGER. The order of the matrix *Q* ($n \geq 0$).
a, *tau*, *work*	COMPLEX for cungtr DOUBLE COMPLEX for zungtr. Arrays: *a*(*lda*,*) is the array *a* as returned by ?hetrd. The second dimension of *a* must be at least max(1, *n*). *tau*(*) is the array *tau* as returned by ?hetrd. The dimension of *tau* must be at least max(1, *n*-1). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
lwork	INTEGER. The size of the *work* array ($lwork \geq n$).

	If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the unitary matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ungtr interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
tau	Holds the vector of length (*n*-1).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For better performance, try using *lwork* = (*n*-1)**blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *Q* differs from an exactly unitary matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

The real counterpart of this routine is orgtr.

?unmtr

Multiplies a complex matrix by the complex unitary matrix Q determined by ?hetrd.

Syntax

Fortran 77:

```
call cunmtr(side, uplo, trans, m, n, a, lda, tau, c, ldc, work, lwork, info)
```

```
call zunmtr(side, uplo, trans, m, n, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmtr(a, tau, c [,side] [,uplo] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmtr( int matrix_order, char side, char uplo, char trans,
lapack_int m, lapack_int n, const <datatype>* a, lapack_int lda, const <datatype>*
tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine multiplies a complex matrix C by Q or Q^H, where Q is the unitary matrix Q formed by hetrd when reducing a complex Hermitian matrix A to tridiagonal form: $A = Q*T*Q^H$. Use this routine after a call to ?hetrd.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (overwriting the result on C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

In the descriptions below, r denotes the order of Q:

If *side* = 'L', $r = m$; if *side* = 'R', $r = n$.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^H is applied to C from the left. If *side* = 'R', Q or Q^H is applied to C from the right.
uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?hetrd.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^H.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
a, *c*, *tau*, *work*	COMPLEX for cunmtr DOUBLE COMPLEX for zunmtr. *a*(*lda*,*) and *tau* are the arrays returned by ?hetrd. The second dimension of *a* must be at least max(1, *r*). The dimension of *tau* must be at least max(1, *r*-1). *c*(*ldc*,*) contains the matrix C. The second dimension of *c* must be at least max(1, *n*) *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; *lda* $\geq$ max(1, *r*).
ldc	INTEGER. The leading dimension of *c*; *ldc* $\geq$ max(1, *n*).

lwork	INTEGER. The size of the *work* array. Constraints: *lwork* ≥ max(1, *n*) if *side* = 'L'; *lwork* ≥ max(1, *m*) if *side* = 'R'. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	Overwritten by the product *Q*C*, Q^H**C*, *C*Q*, or *C**Q^H (as specified by *side* and *trans*).
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unmtr interface are the following:

a	Holds the matrix *A* of size (*r*,*r*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*r*-1).
c	Holds the matrix *C* of size (*m*,*n*).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = *n*blocksize* (for *side* = 'L') or *lwork* = *m*blocksize* (for *side* = 'R') where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed product differs from the exact product by a matrix *E* such that $||E||_2 = O(\varepsilon)*||C||_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $8*m^2*n$ if `side = 'L'` or $8*n^2*m$ if `side = 'R'`.

The real counterpart of this routine is ormtr.

?sptrd

Reduces a real symmetric matrix to tridiagonal form using packed storage.

Syntax

Fortran 77:

```
call ssptrd(uplo, n, ap, d, e, tau, info)
call dsptrd(uplo, n, ap, d, e, tau, info)
```

Fortran 95:

```
call sptrd(ap, tau [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sptrd( int matrix_order, char uplo, lapack_int n, <datatype>* ap,
<datatype>* d, <datatype>* e, <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces a packed real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = Q*T*Q^T$. The orthogonal matrix Q is not formed explicitly but is represented as a product of n-1 elementary reflectors. Routines are provided for working with Q in this representation. See *Application Notes* below for details.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, *ap* stores the packed upper triangle of A. If *uplo* = `'L'`, *ap* stores the packed lower triangle of A.
n	`INTEGER`. The order of the matrix A ($n \geq 0$).
ap	`REAL` for `ssptrd` `DOUBLE PRECISION` for `dsptrd`. Array, `DIMENSION` at least max(1, $n(n+1)/2$). Contains either upper or lower triangle of A (as specified by *uplo*) in the packed form described in "Matrix Arguments" in Appendix B .

Output Parameters

ap	Overwritten by the tridiagonal matrix T and details of the orthogonal matrix Q, as specified by *uplo*.

d, *e*, *tau*	REAL for ssptrd DOUBLE PRECISION for dsptrd. Arrays: *d*(*) contains the diagonal elements of the matrix *T*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1). *tau*(*) stores further details of the matrix *Q*. The dimension of *tau* must be at least max(1, *n*-1).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sptrd interface are the following:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
tau	Holds the vector with the number of elements *n*-1.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Note that diagonal (*d*) and off-diagonal (*e*) elements of the matrix *T* are omitted because they are kept in the matrix *A* on exit.

Application Notes

The matrix *Q* is represented as a product of *n*-1 *elementary reflectors*, as follows :

- If *uplo* = 'U', $Q = H(n-1) \ldots H(2)H(1)$

  Each *H*(i) has the form

  $H(i) = I - tau*v*v^T$ for real flavors, or

  $H(i) = I - tau*v*v^H$ for complex flavors,

  where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(i+1:*n*) = 0 and *v*(i) = 1.

  On exit, *tau* is stored in *tau*(i), and *v*(1:i-1) is stored in *AP*, overwriting *A*(1:i-1, i+1).
- If *uplo* = 'L', $Q = H(1)H(2) \ldots H(n-1)$

  Each *H*(i) has the form

  $H(i) = I - tau*v*v^T$ for real flavors, or

  $H(i) = I - tau*v*v^H$ for complex flavors,

  where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:i) = 0 and *v*(i+1) = 1.

  On exit, *tau* is stored in *tau*(i), and *v*(i+2:n) is stored in *AP*, overwriting *A*(i+2:n, i).

The computed matrix *T* is exactly similar to a matrix *A*+*E*, where $||E||_2 = c(n)*\varepsilon*||A||_2$, *c*(*n*) is a modestly increasing function of *n*, and ε is the machine precision. The approximate number of floating-point operations is $(4/3)n^3$.

After calling this routine, you can call the following:

opgtr	to form the computed matrix *Q* explicitly
opmtr	to multiply a real matrix by *Q*.

The complex counterpart of this routine is hptrd.

?opgtr

Generates the real orthogonal matrix Q determined by ?sptrd*.*

Syntax

Fortran 77:

```
call sopgtr(uplo, n, ap, tau, q, ldq, work, info)
call dopgtr(uplo, n, ap, tau, q, ldq, work, info)
```

Fortran 95:

```
call opgtr(ap, tau, q [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>opgtr( int matrix_order, char uplo, lapack_int n, const
<datatype>* ap, const <datatype>* tau, <datatype>* q, lapack_int ldq );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine explicitly generates the n-by-n orthogonal matrix Q formed by sptrd when reducing a packed real symmetric matrix A to tridiagonal form: $A = Q*T*Q^T$. Use this routine after a call to ?sptrd.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?sptrd.
n	INTEGER. The order of the matrix Q ($n \geq 0$).
ap, *tau*	REAL for sopgtr DOUBLE PRECISION for dopgtr. Arrays *ap* and *tau*, as returned by ?sptrd. The dimension of *ap* must be at least max(1, n(n+1)/2). The dimension of *tau* must be at least max(1, n-1).
ldq	INTEGER. The leading dimension of the output array q; at least max(1, n).
work	REAL for sopgtr DOUBLE PRECISION for dopgtr. Workspace array, DIMENSION at least max(1, n-1).

Output Parameters

q	REAL for sopgtr DOUBLE PRECISION for dopgtr. Array, DIMENSION (*ldq*,*).

	Contains the computed matrix Q. The second dimension of q must be at least max(1, n).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine opgtr interface are the following:

ap	Holds the array A of size (*n*(n+1)/2*).
tau	Holds the vector with the number of elements n - 1.
q	Holds the matrix Q of size (*n,n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that $||E||_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is $(4/3)n^3$.

The complex counterpart of this routine is upgtr.

?opmtr

Multiplies a real matrix by the real orthogonal matrix Q determined by ?sptrd.

Syntax

Fortran 77:

```
call sopmtr(side, uplo, trans, m, n, ap, tau, c, ldc, work, info)
call dopmtr(side, uplo, trans, m, n, ap, tau, c, ldc, work, info)
```

Fortran 95:

```
call opmtr(ap, tau, c [,side] [,uplo] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>opmtr( int matrix_order, char side, char uplo, char trans,
lapack_int m, lapack_int n, const <datatype>* ap, const <datatype>* tau, <datatype>*
c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a real matrix C by Q or Q^T, where Q is the orthogonal matrix Q formed by sptrd when reducing a packed real symmetric matrix A to tridiagonal form: $A = Q*T*Q^T$. Use this routine after a call to ?sptrd.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (overwriting the result on C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

In the descriptions below, r denotes the order of Q:

If *side* = 'L', $r = m$; if *side* = 'R', $r = n$.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^T is applied to C from the left. If *side* = 'R', Q or Q^T is applied to C from the right.
uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?sptrd.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^T.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
ap, *tau*, *c*, *work*	REAL for sopmtr DOUBLE PRECISION for dopmtr. *ap* and *tau* are the arrays returned by ?sptrd. The dimension of *ap* must be at least max(1, $r(r+1)/2$). The dimension of *tau* must be at least max(1, r-1). c(*ldc*,*) contains the matrix C. The second dimension of c must be at least max(1, n) *work*(*) is a workspace array. The dimension of *work* must be at least max(1, n) if *side* = 'L'; max(1, m) if *side* = 'R'.
ldc	INTEGER. The leading dimension of c; *ldc* $\geq$ max(1, *n*).

Output Parameters

c	Overwritten by the product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ (as specified by *side* and *trans*).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine opmtr interface are the following:

ap	Holds the array A of size (*r*(r+1)/2*), where $r = m$ if *side* = 'L'. $r = n$ if *side* = 'R'.
tau	Holds the vector with the number of elements r - 1.

c	Holds the matrix C of size (m,n).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'. The default value is 'U'.
trans	Must be 'N', 'C', or 'T'. The default value is 'N'.

Application Notes

The computed product differs from the exact product by a matrix E such that $||E||_2 = O(\varepsilon) ||C||_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $2*m^2*n$ if *side* = 'L', or $2*n^2*m$ if *side* = 'R'.

The complex counterpart of this routine is upmtr.

?hptrd

Reduces a complex Hermitian matrix to tridiagonal form using packed storage.

Syntax

Fortran 77:

```
call chptrd(uplo, n, ap, d, e, tau, info)
call zhptrd(uplo, n, ap, d, e, tau, info)
```

Fortran 95:

```
call hptrd(ap, tau [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_chptrd( int matrix_order, char uplo, lapack_int n,
lapack_complex_float* ap, float* d, float* e, lapack_complex_float* tau );
lapack_int LAPACKE_zhptrd( int matrix_order, char uplo, lapack_int n,
lapack_complex_double* ap, double* d, double* e, lapack_complex_double* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reduces a packed complex Hermitian matrix A to symmetric tridiagonal form T by a unitary similarity transformation: $A = Q*T*Q^H$. The unitary matrix Q is not formed explicitly but is represented as a product of n-1 elementary reflectors. Routines are provided for working with Q in this representation (see *Application Notes* below).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ap* stores the packed upper triangle of A.

If *uplo* = 'L', *ap* stores the packed lower triangle of *A*.

n — INTEGER. The order of the matrix *A* (*n* ≥ 0).

ap — COMPLEX for chptrd
DOUBLE COMPLEX for zhptrd.
Array, DIMENSION at least max(1, *n*(*n*+1)/2). Contains either upper or lower triangle of *A* (as specified by *uplo*) in the packed form described in "Matrix Arguments" in Appendix B .

Output Parameters

ap — Overwritten by the tridiagonal matrix *T* and details of the orthogonal matrix *Q*, as specified by *uplo*.

d, *e* — REAL for chptrd
DOUBLE PRECISION for zhptrd.
Arrays:
d(*) contains the diagonal elements of the matrix *T*.
The dimension of *d* must be at least max(1, *n*).
e(*) contains the off-diagonal elements of *T*.
The dimension of *e* must be at least max(1, *n*-1).

tau — COMPLEX for chptrd
DOUBLE COMPLEX for zhptrd.
Arrays, DIMENSION at least max(1, *n*-1). Contains further details of the orthogonal matrix *Q*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hptrd interface are the following:

ap — Holds the array *A* of size (*n**(*n*+1)/2).

tau — Holds the vector with the number of elements *n* - 1.

uplo — Must be 'U' or 'L'. The default value is 'U'.

Note that diagonal (*d*) and off-diagonal (*e*) elements of the matrix *T* are omitted because they are kept in the matrix *A* on exit.

Application Notes

The computed matrix *T* is exactly similar to a matrix *A* + *E*, where $||E||_2 = c(n)*\varepsilon*||A||_2$, $c(n)$ is a modestly increasing function of *n*, and ε is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

After calling this routine, you can call the following:

upgtr — to form the computed matrix *Q* explicitly

upmtr — to multiply a complex matrix by *Q*.

The real counterpart of this routine is sptrd.

?upgtr

Generates the complex unitary matrix Q determined by ?hptrd*.*

Syntax

Fortran 77:

```
call cupgtr(uplo, n, ap, tau, q, ldq, work, info)
call zupgtr(uplo, n, ap, tau, q, ldq, work, info)
```

Fortran 95:

```
call upgtr(ap, tau, q [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>upgtr( int matrix_order, char uplo, lapack_int n, const
<datatype>* ap, const <datatype>* tau, <datatype>* q, lapack_int ldq );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine explicitly generates the n-by-n unitary matrix Q formed by hptrd when reducing a packed complex Hermitian matrix A to tridiagonal form: $A = Q*T*Q^H$. Use this routine after a call to ?hptrd.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?hptrd.
n	INTEGER. The order of the matrix Q ($n \geq 0$).
ap, *tau*	COMPLEX for cupgtr DOUBLE COMPLEX for zupgtr. Arrays *ap* and *tau*, as returned by ?hptrd. The dimension of *ap* must be at least max(1, $n(n+1)/2$). The dimension of *tau* must be at least max(1, $n-1$).
ldq	INTEGER. The leading dimension of the output array *q*; at least max(1, n).
work	COMPLEX for cupgtr DOUBLE COMPLEX for zupgtr. Workspace array, DIMENSION at least max(1, $n-1$).

Output Parameters

q	COMPLEX for cupgtr DOUBLE COMPLEX for zupgtr. Array, DIMENSION (*ldq*,*). Contains the computed matrix Q.

	The second dimension of *q* must be at least max(1, *n*).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine upgtr interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
tau	Holds the vector with the number of elements *n* - 1.
q	Holds the matrix *Q* of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed matrix *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is (16/3)*n3*.

The real counterpart of this routine is opgtr.

?upmtr

Multiplies a complex matrix by the unitary matrix Q determined by ?hptrd*.*

Syntax

Fortran 77:

```
call cupmtr(side, uplo, trans, m, n, ap, tau, c, ldc, work, info)
call zupmtr(side, uplo, trans, m, n, ap, tau, c, ldc, work, info)
```

Fortran 95:

```
call upmtr(ap, tau, c [,side] [,uplo] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>upmtr( int matrix_order, char side, char uplo, char trans,
lapack_int m, lapack_int n, const <datatype>* ap, const <datatype>* tau, <datatype>*
c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a complex matrix *C* by *Q* or Q^H, where *Q* is the unitary matrix formed by hptrd when reducing a packed complex Hermitian matrix *A* to tridiagonal form: $A = Q*T*Q^H$. Use this routine after a call to ?hptrd.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (overwriting the result on C).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

In the descriptions below, r denotes the order of Q:

If *side* = 'L', $r = m$; if *side* = 'R', $r = n$.

side	CHARACTER*1. Must be either 'L' or 'R'. If *side* = 'L', Q or Q^H is applied to C from the left. If *side* = 'R', Q or Q^H is applied to C from the right.
uplo	CHARACTER*1. Must be 'U' or 'L'. Use the same *uplo* as supplied to ?hptrd.
trans	CHARACTER*1. Must be either 'N' or 'T'. If *trans* = 'N', the routine multiplies C by Q. If *trans* = 'T', the routine multiplies C by Q^H.
m	INTEGER. The number of rows in the matrix C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
ap, *tau*, *c*,	COMPLEX for cupmtr DOUBLE COMPLEX for zupmtr. *ap* and *tau* are the arrays returned by ?hptrd. The dimension of *ap* must be at least max(1, $r(r+1)/2$). The dimension of *tau* must be at least max(1, r-1). c(*ldc*,*) contains the matrix C. The second dimension of c must be at least max(1, n) *work*(*) is a workspace array. The dimension of *work* must be at least max(1, n) if *side* = 'L'; max(1, m) if *side* = 'R'.
ldc	INTEGER. The leading dimension of c; *ldc* $\geq$ max(1, *n*).

Output Parameters

c	Overwritten by the product $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine upmtr interface are the following:

ap	Holds the array A of size (*r*(r+1)/2*), where $r = m$ if *side* = 'L'. $r = n$ if *side* = 'R'.
tau	Holds the vector with the number of elements n - 1.

c	Holds the matrix C of size (m,n).
side	Must be 'L' or 'R'. The default value is 'L'.
uplo	Must be 'U' or 'L'.The default value is 'U'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

The computed product differs from the exact product by a matrix E such that $||E||_2 = O(\varepsilon)*||C||_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $8*m^2*n$ if *side* = 'L' or $8*n^2*m$ if *side* = 'R'.

The real counterpart of this routine is opmtr.

?sbtrd

Reduces a real symmetric band matrix to tridiagonal form.

Syntax

Fortran 77:

```
call ssbtrd(vect, uplo, n, kd, ab, ldab, d, e, q, ldq, work, info)
call dsbtrd(vect, uplo, n, kd, ab, ldab, d, e, q, ldq, work, info)
```

Fortran 95:

```
call sbtrd(ab[, q] [,vect] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbtrd( int matrix_order, char vect, char uplo, lapack_int n,
lapack_int kd, <datatype>* ab, lapack_int ldab, <datatype>* d, <datatype>* e,
<datatype>* q, lapack_int ldq );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reduces a real symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = Q*T*Q^T$. The orthogonal matrix Q is determined as a product of Givens rotations.

If required, the routine can also form the matrix Q explicitly.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'V' or 'N'. If *vect* = 'V', the routine returns the explicit matrix Q. If *vect* = 'N', the routine does not return Q.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n*≥0).
kd	INTEGER. The number of super- or sub-diagonals in *A* (*kd*≥0).
ab, *q*, *work*	REAL for ssbtrd DOUBLE PRECISION for dsbtrd. *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the matrix *A* (as specified by *uplo*) in band storage format. The second dimension of *ab* must be at least max(1, *n*). *q* (*ldq*,*) is an array. If *vect* = 'U', the *q* array must contain an *n*-by-*n* matrix *X*. If *vect* = 'N' or 'V', the *q* parameter need not be set. The second dimension of *q* must be at least max(1, *n*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1, *n*).
ldab	INTEGER. The leading dimension of *ab*; at least *kd*+1.
ldq	INTEGER. The leading dimension of *q*. Constraints: *ldq* ≥ max(1, *n*) if *vect* = 'V'; *ldq* ≥ 1 if *vect* = 'N'.

Output Parameters

ab	On exit, the diagonal elements of the array *ab* are overwritten by the diagonal elements of the tridiagonal matrix *T*. If *kd* > 0, the elements on the first superdiagonal (if *uplo* = 'U') or the first subdiagonal (if *uplo* = 'L') are ovewritten by the off-diagonal elements of *T*. The rest of *ab* is overwritten by values generated during the reduction.
d, *e*, *q*	REAL for ssbtrd DOUBLE PRECISION for dsbtrd. Arrays: *d*(*) contains the diagonal elements of the matrix *T*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1). *q*(*ldq*,*) is not referenced if *vect* = 'N'. If *vect* = 'V', *q* contains the *n*-by-*n* matrix *Q*. The second dimension of *q* must be: at least max(1, *n*) if *vect* = 'V'; at least 1 if *vect* = 'N'.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbtrd interface are the following:

ab	Holds the array A of size (*kd*+1,*n*).
q	Holds the matrix Q of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
vect	If omitted, this argument is restored based on the presence of argument *q* as follows: *vect* = 'V', if *q* is present, *vect* = 'N', if *q* is omitted. If present, *vect* must be equal to 'V' or 'U' and the argument *q* must also be present. Note that there will be an error condition if *vect* is present and *q* omitted.

Note that diagonal (*d*) and off-diagonal (*e*) elements of the matrix T are omitted because they are kept in the matrix A on exit.

Application Notes

The computed matrix T is exactly similar to a matrix $A+E$, where $||E||_2 = c(n)*\varepsilon*||A||_2$, $c(n)$ is a modestly increasing function of n, and ε is the machine precision. The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that $||E||_2 = O(\varepsilon)$.

The total number of floating-point operations is approximately $6n^2*kd$ if *vect* = 'N', with $3n^3*(kd-1)/kd$ additional operations if *vect* = 'V'.

The complex counterpart of this routine is hbtrd.

?hbtrd

Reduces a complex Hermitian band matrix to tridiagonal form.

Syntax

Fortran 77:

```
call chbtrd(vect, uplo, n, kd, ab, ldab, d, e, q, ldq, work, info)
call zhbtrd(vect, uplo, n, kd, ab, ldab, d, e, q, ldq, work, info)
```

Fortran 95:

```
call hbtrd(ab [, q] [,vect] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_chbtrd( int matrix_order, char vect, char uplo, lapack_int n,
lapack_int kd, lapack_complex_float* ab, lapack_int ldab, float* d, float* e,
lapack_complex_float* q, lapack_int ldq );
lapack_int LAPACKE_zhbtrd( int matrix_order, char vect, char uplo, lapack_int n,
lapack_int kd, lapack_complex_double* ab, lapack_int ldab, double* d, double* e,
lapack_complex_double* q, lapack_int ldq );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reduces a complex Hermitian band matrix A to symmetric tridiagonal form T by a unitary similarity transformation: $A = Q*T*Q^H$. The unitary matrix Q is determined as a product of Givens rotations.

If required, the routine can also form the matrix Q explicitly.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'V' or 'N'. If *vect* = 'V', the routine returns the explicit matrix *Q*. If *vect* = 'N', the routine does not return *Q*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
kd	INTEGER. The number of super- or sub-diagonals in *A* (*kd* ≥ 0).
ab, *work*	COMPLEX for chbtrd DOUBLE COMPLEX for zhbtrd. *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the matrix *A* (as specified by *uplo*) in band storage format. The second dimension of *ab* must be at least max(1, *n*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1, *n*).
ldab	INTEGER. The leading dimension of *ab*; at least *kd*+1.
ldq	INTEGER. The leading dimension of *q*. Constraints: *ldq* ≥ max(1, *n*) if *vect* = 'V'; *ldq* ≥ 1 if *vect* = 'N'.

Output Parameters

ab	On exit, the diagonal elements of the array *ab* are overwritten by the diagonal elements of the tridiagonal matrix *T*. If *kd* > 0, the elements on the first superdiagonal (if *uplo* = 'U') or the first subdiagonal (if *uplo* = 'L') are ovewritten by the off-diagonal elements of *T*. The rest of *ab* is overwritten by values generated during the reduction.
d, *e*	REAL for chbtrd DOUBLE PRECISION for zhbtrd. Arrays: *d*(*) contains the diagonal elements of the matrix *T*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1).
q	COMPLEX for chbtrd DOUBLE COMPLEX for zhbtrd. Array, DIMENSION (*ldq*,*). If *vect* = 'N', *q* is not referenced. If *vect* = 'V', *q* contains the *n*-by-*n* matrix *Q*. The second dimension of *q* must be: at least max(1, *n*) if *vect* = 'V'; at least 1 if *vect* = 'N'.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `hbtrd` interface are the following:

ab	Holds the array *A* of size (*kd+1*,*n*).
q	Holds the matrix *Q* of size (*n*,*n*).
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.
vect	If omitted, this argument is restored based on the presence of argument *q* as follows: *vect* = `'V'`, if *q* is present, *vect* = `'N'`, if *q* is omitted. If present, *vect* must be equal to `'V'` or `'U'` and the argument *q* must also be present. Note that there will be an error condition if *vect* is present and *q* omitted.

Note that diagonal (*d*) and off-diagonal (*e*) elements of the matrix *T* are omitted because they are kept in the matrix *A* on exit.

Application Notes

The computed matrix *T* is exactly similar to a matrix *A* + *E*, where $||E||_2 = c(n)*\varepsilon*||A||_2$, $c(n)$ is a modestly increasing function of *n*, and ε is the machine precision. The computed matrix *Q* differs from an exactly unitary matrix by a matrix *E* such that $||E||_2 = O(\varepsilon)$.

The total number of floating-point operations is approximately $20n^2*kd$ if *vect* = `'N'`, with $10n^3*(kd-1)/kd$ additional operations if *vect* = `'V'`.

The real counterpart of this routine is sbtrd.

?sterf

Computes all eigenvalues of a real symmetric tridiagonal matrix using QR algorithm.

Syntax

Fortran 77:

```
call ssterf(n, d, e, info)
call dsterf(n, d, e, info)
```

Fortran 95:

```
call sterf(d, e [,info])
```

C:

```
lapack_int LAPACKE_<?>sterf( lapack_int n, <datatype>* d, <datatype>* e );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues of a real symmetric tridiagonal matrix *T* (which can be obtained by reducing a symmetric or Hermitian matrix to tridiagonal form). The routine uses a square-root-free variant of the *QR* algorithm.

If you need not only the eigenvalues but also the eigenvectors, call steqr.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix T ($n \geq 0$).
d, *e*	REAL for ssterf DOUBLE PRECISION for dsterf. Arrays: *d*(*) contains the diagonal elements of T. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of T. The dimension of *e* must be at least max(1, *n*-1).

Output Parameters

d	The *n* eigenvalues in ascending order, unless *info* > 0. See also *info*.
e	On exit, the array is overwritten; see *info*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, the algorithm failed to find all the eigenvalues after 30*n* iterations: *i* off-diagonal elements have not converged to zero. On exit, *d* and *e* contain, respectively, the diagonal and off-diagonal elements of a tridiagonal matrix orthogonally similar to T. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sterf interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $||E||_2 = O(\varepsilon)*||T||_2$, where ε is the machine precision.

If λ_i is an exact eigenvalue, and m_i is the corresponding computed value, then

$|\mu i - \lambda i| \leq c(n)*\varepsilon*||T||_2$

where $c(n)$ is a modestly increasing function of n.

The total number of floating-point operations depends on how rapidly the algorithm converges. Typically, it is about $14n^2$.

?steqr

Computes all eigenvalues and eigenvectors of a symmetric or Hermitian matrix reduced to tridiagonal form (QR algorithm).

Syntax

Fortran 77:

```
call ssteqr(compz, n, d, e, z, ldz, work, info)
call dsteqr(compz, n, d, e, z, ldz, work, info)
call csteqr(compz, n, d, e, z, ldz, work, info)
call zsteqr(compz, n, d, e, z, ldz, work, info)
```

Fortran 95:

```
call rsteqr(d, e [,z] [,compz] [,info])
call steqr(d, e [,z] [,compz] [,info])
```

C:

```
lapack_int LAPACKE_ssteqr( int matrix_order, char compz, lapack_int n, float* d, float*
e, float* z, lapack_int ldz );
lapack_int LAPACKE_dsteqr( int matrix_order, char compz, lapack_int n, double* d,
double* e, double* z, lapack_int ldz );
lapack_int LAPACKE_csteqr( int matrix_order, char compz, lapack_int n, float* d, float*
e, lapack_complex_float* z, lapack_int ldz );
lapack_int LAPACKE_zsteqr( int matrix_order, char compz, lapack_int n, double* d,
double* e, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues and (optionally) all the eigenvectors of a real symmetric tridiagonal matrix T. In other words, the routine can compute the spectral factorization: $T = Z*\Lambda*Z^T$. Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i; Z is an orthogonal matrix whose columns are eigenvectors. Thus,

$T*z_i = \lambda_i*z_i$ for $i = 1, 2, \ldots, n$.

The routine normalizes the eigenvectors so that $||z_i||_2 = 1$.

You can also use the routine for computing the eigenvalues and eigenvectors of an arbitrary real symmetric (or complex Hermitian) matrix A reduced to tridiagonal form T: $A = Q*T*Q^H$. In this case, the spectral factorization is as follows: $A = Q*T*Q^H = (Q*Z)*\Lambda*(Q*Z)^H$. Before calling `?steqr`, you must reduce A to tridiagonal form and generate the explicit matrix Q by calling the following routines:

	for real matrices:	for complex matrices:
full storage	`?sytrd`, `?orgtr`	`?hetrd`, `?ungtr`

	for real matrices:	for complex matrices:
packed storage	?sptrd, ?opgtr	?hptrd, ?upgtr
band storage	?sbtrd *(vect*='V')	?hbtrd *(vect*='V')

If you need eigenvalues only, it's more efficient to call sterf. If *T* is positive-definite, pteqr can compute small eigenvalues more accurately than ?steqr.

To solve the problem by a single call, use one of the divide and conquer routines stevd, syevd, spevd, or sbevd for real symmetric matrices or heevd, hpevd, or hbevd for complex Hermitian matrices.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

compz
CHARACTER*1. Must be 'N' or 'I' or 'V'.
If *compz* = 'N', the routine computes eigenvalues only.
If *compz* = 'I', the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix *T*.
If *compz* = 'V', the routine computes the eigenvalues and eigenvectors of *A* (and the array *z* must contain the matrix *Q* on entry).

n
INTEGER. The order of the matrix *T* (*n* ≥ 0).

d, *e*, *work*
REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Arrays:
d(*) contains the diagonal elements of *T*.
The dimension of *d* must be at least max(1, *n*).
e(*) contains the off-diagonal elements of *T*.
The dimension of *e* must be at least max(1, *n*-1).
work(*) is a workspace array.
The dimension of *work* must be:
at least 1 if *compz* = 'N';
at least max(1, 2**n*-2) if *compz* = 'V' or 'I'.

z
REAL for ssteqr
DOUBLE PRECISION for dsteqr
COMPLEX for csteqr
DOUBLE COMPLEX for zsteqr.
Array, DIMENSION (*ldz*, *)
If *compz* = 'N' or 'I', *z* need not be set.
If *vect* = 'V', *z* must contain the *n*-by-*n* matrix *Q*.
The second dimension of *z* must be:
at least 1 if *compz* = 'N';
at least max(1, *n*) if *compz* = 'V' or 'I'.
work (*lwork*) is a workspace array.

ldz
INTEGER. The leading dimension of *z*. Constraints:
ldz ≥ 1 if *compz* = 'N';
ldz ≥ max(1, *n*) if *compz* = 'V' or 'I'.

Output Parameters

d
The *n* eigenvalues in ascending order, unless *info* > 0.
See also *info*.

e	On exit, the array is overwritten; see *info*.
z	If *info* = 0, contains the *n* orthonormal eigenvectors, stored by columns. (The *i*-th column corresponds to the *i*th eigenvalue.)
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, the algorithm failed to find all the eigenvalues after $30n$ iterations: *i* off-diagonal elements have not converged to zero. On exit, *d* and *e* contain, respectively, the diagonal and off-diagonal elements of a tridiagonal matrix orthogonally similar to *T*. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine steqr interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).
z	Holds the matrix *Z* of size (*n*,*n*).
compz	If omitted, this argument is restored based on the presence of argument *z* as follows: *compz* = 'I', if *z* is present, *compz* = 'N', if *z* is omitted. If present, *compz* must be equal to 'I' or 'V' and the argument *z* must also be present. Note that there will be an error condition if *compz* is present and *z* omitted. Note that two variants of Fortran 95 interface for steqr routine are needed because of an ambiguous choice between real and complex cases appear when *z* is omitted. Thus, the name rsteqr is used in real cases (single or double precision), and the name steqr is used in complex cases (single or double precision).

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $||E||_2 = O(\varepsilon)*||T||_2$, where ε is the machine precision.

If λ_i is an exact eigenvalue, and μ_i is the corresponding computed value, then

$|\mu_i - \lambda_i| \leq c(n)*\varepsilon*||T||_2$

where $c(n)$ is a modestly increasing function of n.

If z_i is the corresponding exact eigenvector, and w_i is the corresponding computed vector, then the angle $\theta(z_i, w_i)$ between them is bounded as follows:

$\theta(z_i, w_i) \leq c(n)*\varepsilon*||T||_2 / \min_{i \neq j}|\lambda_i - \lambda_j|$.

The total number of floating-point operations depends on how rapidly the algorithm converges. Typically, it is about

$24n^2$ if *compz* = 'N';

$7n^3$ (for complex flavors, $14n^3$) if *compz* = 'V' or 'I'.

?stemr

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

Fortran 77:

```
call sstemr(jobz, range, n, d, e, vl, vu, il, iu, m, w, z, ldz, nzc, isuppz, tryrac,
work, lwork, iwork, liwork, info)

call dstemr(jobz, range, n, d, e, vl, vu, il, iu, m, w, z, ldz, nzc, isuppz, tryrac,
work, lwork, iwork, liwork, info)

call cstemr(jobz, range, n, d, e, vl, vu, il, iu, m, w, z, ldz, nzc, isuppz, tryrac,
work, lwork, iwork, liwork, info)

call zstemr(jobz, range, n, d, e, vl, vu, il, iu, m, w, z, ldz, nzc, isuppz, tryrac,
work, lwork, iwork, liwork, info)
```

C:

```
lapack_int LAPACKE_sstemr( int matrix_order, char jobz, char range, lapack_int n, const
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, lapack_int* m,
float* w, float* z, lapack_int ldz, lapack_int nzc, lapack_int* isuppz,
lapack_logical* tryrac );

lapack_int LAPACKE_dstemr( int matrix_order, char jobz, char range, lapack_int n, const
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, lapack_int*
m, double* w, double* z, lapack_int ldz, lapack_int nzc, lapack_int* isuppz,
lapack_logical* tryrac );

lapack_int LAPACKE_cstemr( int matrix_order, char jobz, char range, lapack_int n, const
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, lapack_int* m,
float* w, lapack_complex_float* z, lapack_int ldz, lapack_int nzc, lapack_int* isuppz,
lapack_logical* tryrac );

lapack_int LAPACKE_zstemr( int matrix_order, char jobz, char range, lapack_int n, const
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, lapack_int*
m, double* w, lapack_complex_double* z, lapack_int ldz, lapack_int nzc, lapack_int*
isuppz, lapack_logical* tryrac );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Any such unreduced matrix has a well defined set of pairwise different real eigenvalues, the corresponding real eigenvectors are pairwise orthogonal.

The spectrum may be computed either completely or partially by specifying either an interval `(vl,vu]` or a range of indices `il:iu` for the desired eigenvalues.

Depending on the number of desired eigenvalues, these are computed either by bisection or the *dqds* algorithm. Numerically orthogonal eigenvectors are computed by the use of various suitable $L*D*L^T$ factorizations near clusters of close eigenvalues (referred to as RRRs, Relatively Robust Representations). An informal sketch of the algorithm follows.

For each unreduced block (submatrix) of T,

a. Compute $T - \sigma I = L D L^T$, so that L and D define all the wanted eigenvalues to high relative accuracy. This means that small relative changes in the entries of L and D cause only small relative changes in the eigenvalues and eigenvectors. The standard (unfactored) representation of the tridiagonal matrix T does not have this property in general.

b. Compute the eigenvalues to suitable accuracy. If the eigenvectors are desired, the algorithm attains full accuracy of the computed eigenvalues only right before the corresponding vectors have to be computed, see steps c and d.

c. For each cluster of close eigenvalues, select a new shift close to the cluster, find a new factorization, and refine the shifted eigenvalues to suitable accuracy.

d. For each eigenvalue with a large enough relative separation compute the corresponding eigenvector by forming a rank revealing twisted factorization. Go back to step c for any clusters that remain.

For more details, see: [Dhillon04], [Dhillon04-02], [Dhillon97]

The routine works only on machines which follow IEEE-754 floating-point standard in their handling of infinities and NaNs (NaN stands for "not a number"). This permits the use of efficient inner loops avoiding a check for zero divisors.

LAPACK routines can be used to reduce a complex Hermitean matrix to real symmetric tridiagonal form.

(Any complex Hermitean tridiagonal matrix has real values on its diagonal and potentially complex numbers on its off-diagonals. By applying a similarity transform with an appropriate diagonal matrix $\mathrm{diag}(1, e^{i\phi_1}, \ldots, e^{i\phi_{n-1}})$, the complex Hermitean matrix can be transformed into a real symmetric matrix and complex arithmetic can be entirely avoided.) While the eigenvectors of the real symmetric tridiagonal matrix are real, the eigenvectors of original complex Hermitean matrix have complex entries in general. Since LAPACK drivers overwrite the matrix data with the eigenvectors, `zstemr` accepts complex workspace to facilitate interoperability with `zunmtr` or `zupmtr`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes all eigenvalues in the half-open interval: (*vl*, *vu*]. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
n	INTEGER. The order of the matrix T ($n \geq 0$).
d	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). Contains *n* diagonal elements of the tridiagonal matrix T.
e	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*–1). Contains (*n*–1) off-diagonal elements of the tridiagonal matrix T in elements 1 to *n*–1 of *e*. *e*(*n*) need not be set on input, but is used internally as workspace.
vl, *vu*	REAL for single precision flavors

DOUBLE PRECISION for double precision flavors.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*<*vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: 1≤*il*≤*iu*≤*n*, if *n*>0.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

ldz
INTEGER. The leading dimension of the output array *z*.
if *jobz* = 'V', then *ldz* ≥ max(1, *n*);
ldz ≥ 1 otherwise.

nzc
INTEGER. The number of eigenvectors to be held in the array *z*.
If *range* = 'A', then *nzc*≥max(1, *n*);
If *range* = 'V', then *nzc* is greater than or equal to the number of eigenvalues in the half-open interval: (*vl*, *vu*].
If *range* = 'I', then *nzc*≥*il*+*iu*+1.
This value is returned as the first entry of the array *z*, and no error message related to *nzc* is issued by the routine xerbla.

tryrac
LOGICAL.
If *tryrac* = .TRUE., it indicates that the code should check whether the tridiagonal matrix defines its eigenvalues to high relative accuracy. If so, the code uses relative-accuracy preserving algorithms that might be (a bit) slower depending on the matrix. If the matrix does not define its eigenvalues to high relative accuracy, the code can uses possibly faster algorithms.
If *tryrac* = .FALSE., the code is not required to guarantee relatively accurate eigenvalues and can use the fastest possible techniques.

work
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION (*lwork*).

lwork
INTEGER.
The dimension of the array *work*,
lwork ≥ max(1, 18**n*).
If *lwork*=-1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

iwork
INTEGER.
Workspace array, DIMENSION (*liwork*).

liwork
INTEGER.
The dimension of the array *iwork*.
lwork≥max(1, 10**n*) if the eigenvectors are desired, and *lwork*≥max(1, 8**n*) if only the eigenvalues are to be computed.
If *liwork*=-1, then a workspace query is assumed; the routine only calculates the optimal size of the *iwork* array, returns this value as the first entry of the *iwork* array, and no error message related to *liwork* is issued by xerbla.

Output Parameters

e	On exit, the array *e* is overwritten.
m	INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', then $m=n$, and if If *range* = 'I', then $m=iu-il+1$.
w	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION (*n*). The first *m* elements contain the selected eigenvalues in ascending order.
z	REAL for sstemr DOUBLE PRECISION for dstemr COMPLEX for cstemr DOUBLE COMPLEX for zstemr. Array *z*(*ldz*, *), the second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', and *info* = 0, then the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the i-th column of *z* holding the eigenvector associated with *w*(i). If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z* ; if *range* = 'V', the exact value of *m* is not known in advance and an can be computed with a workspace query by setting *nzc*=-1, see description of the parameter *nzc*.
isuppz	INTEGER. Array, DIMENSION (2*max(1, *m*)). The support of the eigenvectors in *z*, that is the indices indicating the nonzero elements in *z*. The i-th computed eigenvector is nonzero only in elements *isuppz*(2*i-1) through *isuppz*(2*i). This is relevant in the case when the matrix is split. *isuppz* is only accessed when *jobz* = 'V' and *n*>0.
tryrac	On exit, TRUE. *tryrac* is set to .FALSE. if the matrix does not define its eigenvalues to high relative accuracy.
work(1)	On exit, if *info* = 0, then *work*(1) returns the optimal (and minimal) size of *lwork*.
iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the optimal size of *liwork*.
info	INTEGER. If = 0, the execution is successful. If *info* = -i, the i-th parameter had an illegal value. If *info* = 1, internal error in ?larre occurred, if *info* = 2, internal error in ?larrv occurred.

?stedc

Computes all eigenvalues and eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method.

Syntax

Fortran 77:

```
call sstedc(compz, n, d, e, z, ldz, work, lwork, iwork, liwork, info)
call dstedc(compz, n, d, e, z, ldz, work, lwork, iwork, liwork, info)
call cstedc(compz, n, d, e, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
call zstedc(compz, n, d, e, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call rstedc(d, e [,z] [,compz] [,info])
call stedc(d, e [,z] [,compz] [,info])
```

C:

```
lapack_int LAPACKE_sstedc( int matrix_order, char compz, lapack_int n, float* d, float* e, float* z, lapack_int ldz );
lapack_int LAPACKE_dstedc( int matrix_order, char compz, lapack_int n, double* d, double* e, double* z, lapack_int ldz );
lapack_int LAPACKE_cstedc( int matrix_order, char compz, lapack_int n, float* d, float* e, lapack_complex_float* z, lapack_int ldz );
lapack_int LAPACKE_zstedc( int matrix_order, char compz, lapack_int n, double* d, double* e, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues and (optionally) all the eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band real symmetric or complex Hermitian matrix can also be found if sytrd/hetrd or sptrd/hptrd or sbtrd/hbtrd has been used to reduce this matrix to tridiagonal form.

See also laed0, laed1, laed2, laed3, laed4, laed5, laed6, laed7, laed8, laed9, and laeda used by this function.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

compz	`CHARACTER*1`. Must be `'N'` or `'I'` or `'V'`. If *compz* = `'N'`, the routine computes eigenvalues only. If *compz* = `'I'`, the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix. If *compz* = `'V'`, the routine computes the eigenvalues and eigenvectors of original symmetric/Hermitian matrix. On entry, the array *z* must contain the orthogonal/unitary matrix used to reduce the original matrix to tridiagonal form.
n	`INTEGER`. The order of the symmetric tridiagonal matrix ($n \geq 0$).

d, *e*, *rwork*

REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Arrays:
d(*) contains the diagonal elements of the tridiagonal matrix.
The dimension of *d* must be at least max(1, *n*).
e(*) contains the subdiagonal elements of the tridiagonal matrix.
The dimension of *e* must be at least max(1, *n*-1).
rwork is a workspace array, its dimension max(1, *lrwork*).

z, *work*

REAL for sstedc
DOUBLE PRECISION for dstedc
COMPLEX for cstedc
DOUBLE COMPLEX for zstedc.
Arrays: *z*(*ldz*, *), *work*(*).
If *compz* = 'V', then, on entry, *z* must contain the orthogonal/unitary matrix used to reduce the original matrix to tridiagonal form.
The second dimension of *z* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

ldz

INTEGER. The leading dimension of *z*. Constraints:
ldz ≥ 1 if *compz* = 'N';
ldz ≥ max(1, *n*) if *compz* = 'V' or 'I'.

lwork

INTEGER. The dimension of the array *work*.
For real functions sstedc and dstedc:

- If *compz* = 'N'or *n* ≤ 1, *lwork* must be at least 1.
- If *compz* = 'V' and *n* > 1, *lwork* must be at least 1 + 3*n + 2*n*$\log_2$(n) + 4*n^2, where $\log_2$(n) is the smallest integer *k* such that $2^k \geq n$.
- If *compz* = 'I' and *n* > 1 then *lwork* must be at least 1 + 4*n + n^2

  Note that for *compz* = 'I' or 'V' and if *n* is less than or equal to the minimum divide size, usually 25, then *lwork* need only be max(1, 2*(*n*-1)).

For complex functions cstedc and zstedc:

- If *compz* = 'N'or 'I', or *n* ≤ 1, *lwork* must be at least 1.
- If *compz* = 'V' and *n* > 1, *lwork* must be at least n^2.

  Note that for *compz* = 'V', and if *n* is less than or equal to the minimum divide size, usually 25, then *lwork* need only be 1.

If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for the required value of *lwork*.

lrwork

INTEGER. The dimension of the array *rwork* (used for complex flavors only).
If *compz* = 'N', or *n* ≤ 1, *lrwork* must be at least 1.
If *compz* = 'V' and *n* > 1, *lrwork* must be at least (1+3*n+2*n*lg(n) +4*n*n), where lg(n) is the smallest integer *k* such that 2**k≥n.
If *compz* = 'I' and *n* > 1, *lrwork* must be at least (1+4*n+2*n*n).

Note that for *compz* = 'V'or 'I', and if *n* is less than or equal to the minimum divide size, usually 25, then *lrwork* need only be max(1, 2*(*n*-1)).
If *lrwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla.
See *Application Notes* for the required value of *lrwork*.

iwork — INTEGER. Workspace array, its dimension max(1, *liwork*).

liwork — INTEGER. The dimension of the array *iwork*.
If *compz* = 'N', or *n* ≤ 1, *liwork* must be at least 1.
If *compz* = 'V' and *n* > 1, *liwork* must be at least (6+6*n+5*n*lg(*n*), where lg(*n*) is the smallest integer *k* such that 2**k≥n.
If *compz* = 'I' and *n* > 1, *liwork* must be at least (3+5**n*).
Note that for *compz* = 'V'or 'I', and if *n* is less than or equal to the minimum divide size, usually 25, then *liwork* need only be 1.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla.
See *Application Notes* for the required value of *liwork*.

Output Parameters

d — The *n* eigenvalues in ascending order, unless *info* ≠ 0.
See also *info*.

e — On exit, the array is overwritten; see *info*.

z — If *info* = 0, then if *compz* = 'V', *z* contains the orthonormal eigenvectors of the original symmetric/Hermitian matrix, and if *compz* = 'I', *z* contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If *compz* = 'N', *z* is not referenced.

work(1) — On exit, if *info* = 0, then *work(1)* returns the optimal *lwork*.

rwork(1) — On exit, if *info* = 0, then *rwork(1)* returns the optimal *lrwork* (for complex flavors only).

iwork(1) — On exit, if *info* = 0, then *iwork(1)* returns the optimal *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *i*/(*n*+1) through mod(*i*, *n*+1).

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stedc interface are the following:

d — Holds the vector of length *n*.

e — Holds the vector of length (*n*-1).

z — Holds the matrix *Z* of size (*n*,*n*).

compz	If omitted, this argument is restored based on the presence of argument *z* as follows: *compz* = 'I', if *z* is present, *compz* = 'N', if *z* is omitted. If present, *compz* must be equal to 'I' or 'V' and the argument *z* must also be present. Note that there will be an error condition if *compz* is present and *z* omitted.

Note that two variants of Fortran 95 interface for stedc routine are needed because of an ambiguous choice between real and complex cases appear when *z* and *work* are omitted. Thus, the name rstedc is used in real cases (single or double precision), and the name stedc is used in complex cases (single or double precision).

Application Notes

The required size of workspace arrays must be as follows.

For sstedc/dstedc:

If *compz* = 'N' or $n \le 1$ then *lwork* must be at least 1.

If *compz* = 'V' and $n > 1$ then *lwork* must be at least $(1 + 3n + 2n \cdot \lg n + 3n^2)$, where lg($n$) = smallest integer k such that $2^k \ge n$.

If *compz* = 'I' and $n > 1$ then *lwork* must be at least $(1 + 4n + n^2)$.

If *compz* = 'N' or $n \le 1$ then *liwork* must be at least 1.

If *compz* = 'V' and $n > 1$ then *liwork* must be at least $(6 + 6n + 5n \cdot \lg n)$.

If *compz* = 'I' and $n > 1$ then *liwork* must be at least $(3 + 5n)$.

For cstedc/zstedc:

If *compz* = 'N' or'I', or $n \le 1$, *lwork* must be at least 1.

If *compz* = 'V' and $n > 1$, *lwork* must be at least n^2.

If *compz* = 'N' or $n \le 1$, *lrwork* must be at least 1.

If *compz* = 'V' and $n > 1$, *lrwork* must be at least $(1 + 3n + 2n \cdot \lg n + 3n^2)$, where lg($n$) = smallest integer k such that $2^k \ge n$.

If *compz* = 'I' and $n > 1$, *lrwork* must be at least$(1 + 4n + 2n^2)$.

The required value of *liwork* for complex flavors is the same as for real flavors.

If *lwork* (or *liwork* or *lrwork*, if supplied) is equal to -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*, *lrwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?stegr

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

Fortran 77:

```
call sstegr(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, isuppz, work, lwork, iwork, liwork, info)
```

```
call dstegr(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, isuppz, work,
lwork, iwork, liwork, info)

call cstegr(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, isuppz, work,
lwork, iwork, liwork, info)

call zstegr(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, isuppz, work,
lwork, iwork, liwork, info)
```

Fortran 95:

```
call rstegr(d, e, w [,z] [,vl] [,vu] [,il] [,iu] [,m] [,isuppz] [,abstol] [,info])

call stegr(d, e, w [,z] [,vl] [,vu] [,il] [,iu] [,m] [,isuppz] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_sstegr( int matrix_order, char jobz, char range, lapack_int n,
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int* isuppz );

lapack_int LAPACKE_dstegr( int matrix_order, char jobz, char range, lapack_int n,
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, double* z, lapack_int ldz, lapack_int* isuppz );

lapack_int LAPACKE_cstegr( int matrix_order, char jobz, char range, lapack_int n,
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int* isuppz );

lapack_int LAPACKE_zstegr( int matrix_order, char jobz, char range, lapack_int n,
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, lapack_complex_double* z, lapack_int ldz,
lapack_int* isuppz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Any such unreduced matrix has a well defined set of pairwise different real eigenvalues, the corresponding real eigenvectors are pairwise orthogonal.

The spectrum may be computed either completely or partially by specifying either an interval $(vl, vu]$ or a range of indices $il:iu$ for the desired eigenvalues.

`?sregr` is a compatibility wrapper around the improved stemr routine. See its description for further details.

Note that the *abstol* parameter no longer provides any benefit and hence is no longer used.

See also auxiliary lasq2 lasq5, lasq6, used by this routine.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz — CHARACTER*1. Must be 'N' or 'V'.
If *job* = 'N', then only eigenvalues are computed.

If *job* = 'V', then eigenvalues and eigenvectors are computed.

range — CHARACTER*1. Must be 'A' or 'V' or 'I'.
If *range* = 'A', the routine computes all eigenvalues.
If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval:
vl< *lambda*(*i*) ≤ *vu*.
If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.

n — INTEGER. The order of the matrix *T* (*n* ≥ 0).

d, *e*, *work* — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Arrays:
d(*) contains the diagonal elements of *T*.
The dimension of *d* must be at least max(1, *n*).
e(*) contains the subdiagonal elements of *T* in elements 1 to *n*-1; *e*(*n*) need not be set on input, but it is used as a workspace.
The dimension of *e* must be at least max(1, *n*).
work(*lwork*) is a workspace array.

vl, *vu* — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl*< *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu* — INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol — REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Unused. Was the absolute error tolerance for the eigenvalues/ eigenvectors in previous versions.

ldz — INTEGER. The leading dimension of the output array *z*. Constraints:
ldz < 1 if *jobz* = 'N';
ldz < max(1, *n*) *jobz* = 'V', an.

lwork — INTEGER.
The dimension of the array *work*,
lwork≥max(1, 18**n*) if *jobz* = 'V', and
lwork≥max(1, 12**n*) if *jobz* = 'N'.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* below for details.

iwork — INTEGER.
Workspace array, DIMENSION (*liwork*).

liwork — INTEGER.
The dimension of the array *iwork*, *lwork* ≥ max(1, 10**n*) if the eigenvectors are desired, and *lwork* ≥ max(1, 8**n*) if only the eigenvalues are to be computed..

	If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *iwork* array, returns this value as the first entry of the *iwork* array, and no error message related to *liwork* is issued by xerbla. See *Application Notes* below for details.

Output Parameters

d, *e*	On exit, *d* and *e* are overwritten.
m	INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.
w	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *n*). The selected eigenvalues in ascending order, stored in *w*(1) to *w*(*m*).
z	REAL for sstegr DOUBLE PRECISION for dstegr COMPLEX for cstegr DOUBLE COMPLEX for zstegr. Array *z*(*ldz*, *), the second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', and if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z* ; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used. Supplying *n* columns is always safe.
isuppz	INTEGER. Array, DIMENSION at least (2*max(1, *m*)). The support of the eigenvectors in *z*, that is the indices indicating the nonzero elements in *z*. The *i*-th computed eigenvector is nonzero only in elements *isuppz*(2*i-1) through *isuppz*(2*i). This is relevant in the case when the matrix is split. *isuppz* is only accessed when *jobz* = 'V', and *n* > 0.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
iwork(1)	On exit, if *info* = 0, then *iwork(1)* returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1*x*, internal error in ?larre occurred, If *info* = 2*x*, internal error in ?larrv occurred. Here the digit *x* = abs(*iinfo*) < 10, where *iinfo* is the non-zero error code returned by ?larre or ?larrv, respectively.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `stegr` interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length *n*.
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*,*m*).
isuppz	Holds the vector of length (*2*m*).
vl	Default value for this argument is *vl* = – HUGE (*vl*) where HUGE(*a*) means the largest machine number of the same precision as argument *a*.
vu	Default value for this argument is *vu* = HUGE (*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this argument is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Note that two variants of Fortran 95 interface for `stegr` routine are needed because of an ambiguous choice between real and complex cases appear when *z* is omitted. Thus, the name `rstegr` is used in real cases (single or double precision), and the name `stegr` is used in complex cases (single or double precision).

Application Notes

Currently `?stegr` is only set up to find *all* the *n* eigenvalues and eigenvectors of *T* in O(n^2) time, that is, only *range* = 'A' is supported.

`?stegr` works only on machines which follow IEEE-754 floating-point standard in their handling of infinities and NaNs. Normal execution of `?stegr` may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not conform to the IEEE-754 standard.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run, or set *lwork* = –1 (*liwork* = –1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = –1 (*liwork* = –1), then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?pteqr

Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric positive-definite tridiagonal matrix.

Syntax

Fortran 77:

```
call spteqr(compz, n, d, e, z, ldz, work, info)
call dpteqr(compz, n, d, e, z, ldz, work, info)
call cpteqr(compz, n, d, e, z, ldz, work, info)
call zpteqr(compz, n, d, e, z, ldz, work, info)
```

Fortran 95:

```
call rpteqr(d, e [,z] [,compz] [,info])
call pteqr(d, e [,z] [,compz] [,info])
```

C:

```
lapack_int LAPACKE_spteqr( int matrix_order, char compz, lapack_int n, float* d, float* e, float* z, lapack_int ldz );
lapack_int LAPACKE_dpteqr( int matrix_order, char compz, lapack_int n, double* d, double* e, double* z, lapack_int ldz );
lapack_int LAPACKE_cpteqr( int matrix_order, char compz, lapack_int n, float* d, float* e, lapack_complex_float* z, lapack_int ldz );
lapack_int LAPACKE_zpteqr( int matrix_order, char compz, lapack_int n, double* d, double* e, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues and (optionally) all the eigenvectors of a real symmetric positive-definite tridiagonal matrix T. In other words, the routine can compute the spectral factorization: $T = Z*\Lambda*Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i; Z is an orthogonal matrix whose columns are eigenvectors. Thus,

$T*z_i = \lambda_i*z_i$ for $i = 1, 2, \ldots, n$.

(The routine normalizes the eigenvectors so that $||z_i||_2 = 1$.)

You can also use the routine for computing the eigenvalues and eigenvectors of real symmetric (or complex Hermitian) positive-definite matrices A reduced to tridiagonal form T: $A = Q*T*Q^H$. In this case, the spectral factorization is as follows: $A = Q*T*Q^H = (QZ)*\Lambda*(QZ)^H$. Before calling `?pteqr`, you must reduce A to tridiagonal form and generate the explicit matrix Q by calling the following routines:

	for real matrices:	for complex matrices:
full storage	?sytrd, ?orgtr	?hetrd, ?ungtr
packed storage	?sptrd, ?opgtr	?hptrd, ?upgtr
band storage	?sbtrd *(vect*='V')	?hbtrd *(vect*='V')

The routine first factorizes T as $L*D*L^H$ where L is a unit lower bidiagonal matrix, and D is a diagonal matrix. Then it forms the bidiagonal matrix $B = L*D^{1/2}$ and calls ?bdsqr to compute the singular values of B, which are the same as the eigenvalues of T.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

compz — CHARACTER*1. Must be 'N' or 'I' or 'V'.
If *compz* = 'N', the routine computes eigenvalues only.
If *compz* = 'I', the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix T.
If *compz* = 'V', the routine computes the eigenvalues and eigenvectors of A (and the array z must contain the matrix Q on entry).

n — INTEGER. The order of the matrix T ($n \geq 0$).

d, *e*, *work* — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Arrays:
d(*) contains the diagonal elements of T.
The dimension of d must be at least max(1, n).
e(*) contains the off-diagonal elements of T.
The dimension of e must be at least max(1, n-1).
work(*) is a workspace array.
The dimension of *work* must be:
at least 1 if *compz* = 'N';
at least max(1, 4*n-4) if *compz* = 'V' or 'I'.

z — REAL for spteqr
DOUBLE PRECISION for dpteqr
COMPLEX for cpteqr
DOUBLE COMPLEX for zpteqr.
Array, DIMENSION (*ldz*,*)
If *compz* = 'N' or 'I', z need not be set.
If *compz* = 'V', z must contain the n-by-n matrix Q.
The second dimension of z must be:
at least 1 if *compz* = 'N';
at least max(1, n) if *compz* = 'V' or 'I'.

ldz — INTEGER. The leading dimension of z. Constraints:
ldz $\geq$ 1 if *compz* = 'N';
ldz $\geq$ max(1, n) if *compz* = 'V' or 'I'.

Output Parameters

d — The n eigenvalues in descending order, unless *info* > 0.
See also *info*.

e	On exit, the array is overwritten.
z	If *info* = 0, contains the *n* orthonormal eigenvectors, stored by columns. (The *i*-th column corresponds to the *i*-th eigenvalue.)
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, the leading minor of order *i* (and hence *T* itself) is not positive-definite. If *info* = *n* + *i*, the algorithm for computing singular values failed to converge; *i* off-diagonal elements have not converged to zero. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pteqr interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).
z	Holds the matrix *Z* of size (*n*,*n*).
compz	If omitted, this argument is restored based on the presence of argument *z* as follows: *compz* = 'I', if *z* is present, *compz* = 'N', if *z* is omitted. If present, *compz* must be equal to 'I' or 'V' and the argument *z* must also be present. Note that there will be an error condition if *compz* is present and *z* omitted.

Note that two variants of Fortran 95 interface for pteqr routine are needed because of an ambiguous choice between real and complex cases appear when *z* is omitted. Thus, the name rpteqr is used in real cases (single or double precision), and the name pteqr is used in complex cases (single or double precision).

Application Notes

If λ_i is an exact eigenvalue, and μ_i is the corresponding computed value, then

$|\mu_i - \lambda_i| \le c(n)*\varepsilon*K*\lambda_i$

where $c(n)$ is a modestly increasing function of n, ε is the machine precision, and $K = ||DTD||_2 * ||(DTD)^{-1}||_2$, D is diagonal with $d_{ii} = t_{ii}^{-1/2}$.

If z_i is the corresponding exact eigenvector, and w_i is the corresponding computed vector, then the angle $\theta(z_i, w_i)$ between them is bounded as follows:

$\theta(u_i, w_i) \le c(n)\varepsilon K / \min_{i \ne j}(|\lambda_i - \lambda_j|/|\lambda_i + \lambda_j|)$.

Here $\min_{i \ne j}(|\lambda_i - \lambda_j|/|\lambda_i + \lambda_j|)$ is the *relative gap* between λ_i and the other eigenvalues.

The total number of floating-point operations depends on how rapidly the algorithm converges.

Typically, it is about

$30n^2$ if *compz* = 'N';

$6n^3$ (for complex flavors, $12n^3$) if *compz* = 'V' or 'I'.

?stebz

Computes selected eigenvalues of a real symmetric tridiagonal matrix by bisection.

Syntax

Fortran 77:

```
call sstebz (range, order, n, vl, vu, il, iu, abstol, d, e, m, nsplit, w, iblock, isplit, work, iwork, info)
call dstebz (range, order, n, vl, vu, il, iu, abstol, d, e, m, nsplit, w, iblock, isplit, work, iwork, info)
```

Fortran 95:

```
call stebz(d, e, m, nsplit, w, iblock, isplit [, order] [,vl] [,vu] [,il] [,iu] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>stebz( char range, char order, lapack_int n, <datatype> vl, <datatype> vu, lapack_int il, lapack_int iu, <datatype> abstol, const <datatype>* d, const <datatype>* e, lapack_int* m, lapack_int* nsplit, <datatype>* w, lapack_int* iblock, lapack_int* isplit );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes some (or all) of the eigenvalues of a real symmetric tridiagonal matrix T by bisection. The routine searches for zero or negligible off-diagonal elements to see if T splits into block-diagonal form $T = \mathrm{diag}(T_1, T_2, \ldots)$. Then it performs bisection on each of the blocks T_i and returns the block index of each computed eigenvalue, so that a subsequent call to stein can also take advantage of the block structure.

See also laebz.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: $vl < lambda(i) \leq vu$. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
order	CHARACTER*1. Must be 'B' or 'E'. If *order* = 'B', the eigenvalues are to be ordered from smallest to largest within each split-off block. If *order* = 'E', the eigenvalues for the entire matrix are to be ordered from smallest to largest.
n	INTEGER. The order of the matrix T ($n \geq 0$).

vl, *vu*
REAL for sstebz
DOUBLE PRECISION for dstebz.
If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval:
vl < *lambda*(*i*) ≤ *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
INTEGER. Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*.
If *range* = 'I', the routine computes eigenvalues *lambda*(*i*) such that *il*≤ *i* ≤ *iu* (assuming that the eigenvalues *lambda*(*i*) are in ascending order).
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for sstebz
DOUBLE PRECISION for dstebz.
The absolute tolerance to which each eigenvalue is required. An eigenvalue (or cluster) is considered to have converged if it lies in an interval of width *abstol*.
If *abstol* ≤ 0.0, then the tolerance is taken as eps*|*T*|, where eps is the machine precision, and |*T*| is the 1-norm of the matrix *T*.

d, *e*, *work*
REAL for sstebz
DOUBLE PRECISION for dstebz.
Arrays:
d(*) contains the diagonal elements of *T*.
The dimension of *d* must be at least max(1, *n*).
e(*) contains the off-diagonal elements of *T*.
The dimension of *e* must be at least max(1, *n*-1).
work(*) is a workspace array.
The dimension of *work* must be at least max(1, 4*n*).

iwork
INTEGER. Workspace.
Array, DIMENSION at least max(1, 3*n*).

Output Parameters

m
INTEGER. The actual number of eigenvalues found.

nsplit
INTEGER. The number of diagonal blocks detected in *T*.

w
REAL for sstebz
DOUBLE PRECISION for dstebz.
Array, DIMENSION at least max(1, *n*). The computed eigenvalues, stored in *w*(1) to *w*(*m*).

iblock, *isplit*
INTEGER.
Arrays, DIMENSION at least max(1, *n*).
A positive value *iblock*(*i*) is the block number of the eigenvalue stored in *w*(*i*) (see also *info*).
The leading *nsplit* elements of *isplit* contain points at which *T* splits into blocks T_1 as follows: the block T_1 contains rows/columns 1 to *isplit*(1); the block T_2 contains rows/columns *isplit*(1)+1 to *isplit*(2), and so on.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = 1, for *range* = 'A' or 'V', the algorithm failed to compute some of the required eigenvalues to the desired accuracy; *iblock*(*i*)<0 indicates that the eigenvalue stored in *w*(*i*) failed to converge.

If *info* = 2, for *range* = 'I', the algorithm failed to compute some of the required eigenvalues. Try calling the routine again with *range* = 'A'.
If *info* = 3:
for *range* = 'A' or 'V', same as *info* = 1;
for *range* = 'I', same as *info* = 2.
If *info* = 4, no eigenvalues have been computed. The floating-point arithmetic on the computer is not behaving as expected.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stebz interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length (*n*-1).
w	Holds the vector of length *n*.
iblock	Holds the vector of length *n*.
isplit	Holds the vector of length *n*.
order	Must be 'B' or 'E'. The default value is 'B'.
vl	Default value for this argument is *vl* = - HUGE (*vl*) where HUGE(*a*) means the largest machine number of the same precision as argument *a*.
vu	Default value for this argument is *vu* = HUGE (*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this argument is *abstol* = 0.0_WP.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

The eigenvalues of *T* are computed to high relative accuracy which means that if they vary widely in magnitude, then any small eigenvalues will be computed more accurately than, for example, with the standard *QR* method. However, the reduction to tridiagonal form (prior to calling the routine) may exclude the possibility of obtaining high relative accuracy in the small eigenvalues of the original matrix if its eigenvalues vary widely in magnitude.

?stein

Computes the eigenvectors corresponding to specified eigenvalues of a real symmetric tridiagonal matrix.

Syntax

Fortran 77:

```
call sstein(n, d, e, m, w, iblock, isplit, z, ldz, work, iwork, ifailv, info)
call dstein(n, d, e, m, w, iblock, isplit, z, ldz, work, iwork, ifailv, info)
```

```
call cstein(n, d, e, m, w, iblock, isplit, z, ldz, work, iwork, ifailv, info)
call zstein(n, d, e, m, w, iblock, isplit, z, ldz, work, iwork, ifailv, info)
```

Fortran 95:

```
call stein(d, e, w, iblock, isplit, z [,ifailv] [,info])
```

C:

```
lapack_int LAPACKE_sstein( int matrix_order, lapack_int n, const float* d, const float*
e, lapack_int m, const float* w, const lapack_int* iblock, const lapack_int* isplit,
float* z, lapack_int ldz, lapack_int* ifailv );
lapack_int LAPACKE_dstein( int matrix_order, lapack_int n, const double* d, const
double* e, lapack_int m, const double* w, const lapack_int* iblock, const lapack_int*
isplit, double* z, lapack_int ldz, lapack_int* ifailv );
lapack_int LAPACKE_cstein( int matrix_order, lapack_int n, const float* d, const float*
e, lapack_int m, const float* w, const lapack_int* iblock, const lapack_int* isplit,
lapack_complex_float* z, lapack_int ldz, lapack_int* ifailv );
lapack_int LAPACKE_zstein( int matrix_order, lapack_int n, const double* d, const
double* e, lapack_int m, const double* w, const lapack_int* iblock, const lapack_int*
isplit, lapack_complex_double* z, lapack_int ldz, lapack_int* ifailv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, by inverse iteration. It is designed to be used in particular after the specified eigenvalues have been computed by ?stebz with *order* = 'B', but may also be used when the eigenvalues have been computed by other routines.

If you use this routine after ?stebz, it can take advantage of the block structure by performing inverse iteration on each block T_i separately, which is more efficient than using the whole matrix T.

If T has been formed by reduction of a full symmetric or Hermitian matrix A to tridiagonal form, you can transform eigenvectors of T to eigenvectors of A by calling ?ormtr or ?opmtr (for real flavors) or by calling ?unmtr or ?upmtr (for complex flavors).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix T ($n \geq 0$).
m	INTEGER. The number of eigenvectors to be returned.
d, *e*, *w*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays: *d*(*) contains the diagonal elements of T. The dimension of *d* must be at least max(1, n). *e*(*) contains the sub-diagonal elements of T stored in elements 1 to n-1

The dimension of *e* must be at least max(1, *n*-1).
w(*) contains the eigenvalues of *T*, stored in *w*(1) to *w*(*m*) (as returned by stebz). Eigenvalues of T_1 must be supplied first, in non-decreasing order; then those of T_2, again in non-decreasing order, and so on. Constraint: if *iblock*(*i*) = *iblock*(*i*+1), *w*(*i*) ≤ *w*(*i*+1).
The dimension of *w* must be at least max(1, *n*).

iblock, *isplit* — INTEGER.
Arrays, DIMENSION at least max(1, *n*). The arrays *iblock* and *isplit*, as returned by ?stebz with *order* = 'B'.
If you did not call ?stebz with *order* = 'B', set all elements of *iblock* to 1, and *isplit*(1) to *n*.)

ldz — INTEGER. The leading dimension of the output array *z*; *ldz* ≥ max(1, *n*).

work — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Workspace array, DIMENSION at least max(1, 5*n*).

iwork — INTEGER.
Workspace array, DIMENSION at least max(1, *n*).

Output Parameters

z — REAL for sstein
DOUBLE PRECISION for dstein
COMPLEX for cstein
DOUBLE COMPLEX for zstein.
Array, DIMENSION (*ldz*, *).
If *info* = 0, *z* contains the *m* orthonormal eigenvectors, stored by columns. (The *i*th column corresponds to the *i*-th specified eigenvalue.)

ifailv — INTEGER.
Array, DIMENSION at least max(1, *m*).
If *info* = *i* > 0, the first *i* elements of *ifailv* contain the indices of any eigenvectors that failed to converge.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = *i*, then *i* eigenvectors (as indicated by the parameter *ifailv*) each failed to converge in 5 iterations. The current iterates are stored in the corresponding columns of the array *z*.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stein interface are the following:

d — Holds the vector of length *n*.

e — Holds the vector of length *n*.

w — Holds the vector of length *n*.

iblock — Holds the vector of length *n*.

isplit — Holds the vector of length *n*.

z — Holds the matrix *Z* of size (*n*,*m*).

ifailv Holds the vector of length (m).

Application Notes

Each computed eigenvector z_i is an exact eigenvector of a matrix $T+E_i$, where $||E_i||_2 = O(\varepsilon)*||T||_2$. However, a set of eigenvectors computed by this routine may not be orthogonal to so high a degree of accuracy as those computed by ?steqr.

?disna

Computes the reciprocal condition numbers for the eigenvectors of a symmetric/ Hermitian matrix or for the left or right singular vectors of a general matrix.

Syntax

Fortran 77:

```
call sdisna(job, m, n, d, sep, info)
call ddisna(job, m, n, d, sep, info)
```

Fortran 95:

```
call disna(d, sep [,job] [,minmn] [,info])
```

C:

```
lapack_int LAPACKE_<?>disna( char job, lapack_int m, lapack_int n, const <datatype>* d,
<datatype>* sep );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m-by-n matrix.

The reciprocal condition number is the 'gap' between the corresponding eigenvalue or singular value and the nearest other one.

The bound on the error, measured by angle in radians, in the i-th computed vector is given by

slamch('E')*(*anorm*/*sep*(*i*))

where *anorm* = $||A||_2$ = max(|*d*(j)|). *sep*(*i*) is not allowed to be smaller than slamch('E')**anorm* in order to limit the size of the error bound.

?disna may also be used to compute error bounds for eigenvectors of the generalized symmetric definite eigenproblem.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job CHARACTER*1. Must be 'E','L', or 'R'. Specifies for which problem the reciprocal condition numbers should be computed:
job = 'E': for the eigenvectors of a symmetric/Hermitian matrix;

	job = 'L': for the left singular vectors of a general matrix; *job* = 'R': for the right singular vectors of a general matrix.
m	INTEGER. The number of rows of the matrix (*m* ≥ 0).
n	INTEGER. If *job* = 'L', or 'R', the number of columns of the matrix (*n* ≥ 0). Ignored if *job* = 'E'.
d	REAL for sdisna DOUBLE PRECISION for ddisna. Array, dimension at least max(1,*m*) if *job* = 'E', and at least max(1, min(*m*,*n*)) if *job* = 'L' or 'R'. This array must contain the eigenvalues (if *job* = 'E') or singular values (if *job* = 'L' or 'R') of the matrix, in either increasing or decreasing order. If singular values, they must be non-negative.

Output Parameters

sep	REAL for sdisna DOUBLE PRECISION for ddisna. Array, dimension at least max(1,*m*) if *job* = 'E', and at least max(1, min(*m*,*n*)) if *job* = 'L' or 'R'. The reciprocal condition numbers of the vectors.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine disna interface are the following:

d	Holds the vector of length min(*m*,*n*).
sep	Holds the vector of length min(*m*,*n*).
job	Must be 'E', 'L', or 'R'. The default value is 'E'.
minmn	Indicates which of the values *m* or *n* is smaller. Must be either 'M' or 'N', the default is 'M'. If *job* = 'E', this argument is superfluous, If *job* = 'L' or 'R', this argument is used by the routine.

Generalized Symmetric-Definite Eigenvalue Problems

Generalized symmetric-definite eigenvalue problems are as follows: find the eigenvalues λ and the corresponding eigenvectors *z* that satisfy one of these equations:

$Az = \lambda Bz$, $ABz = \lambda z$, or $BAz = \lambda z$,

where *A* is an *n*-by-*n* symmetric or Hermitian matrix, and *B* is an *n*-by-*n* symmetric positive-definite or Hermitian positive-definite matrix.

In these problems, there exist *n* real eigenvectors corresponding to real eigenvalues (even for complex Hermitian matrices *A* and *B*).

Routines described in this section allow you to reduce the above generalized problems to standard symmetric eigenvalue problem $Cy = \lambda y$, which you can solve by calling LAPACK routines described earlier in this chapter (see Symmetric Eigenvalue Problems).

Different routines allow the matrices to be stored either conventionally or in packed storage. Prior to reduction, the positive-definite matrix B must first be factorized using either potrf or pptrf.

The reduction routine for the banded matrices A and B uses a split Cholesky factorization for which a specific routine pbstf is provided. This refinement halves the amount of work required to form matrix C.

Table "Computational Routines for Reducing Generalized Eigenproblems to Standard Problems" lists LAPACK routines (FORTRAN 77 interface) that can be used to solve generalized symmetric-definite eigenvalue problems. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Reducing Generalized Eigenproblems to Standard Problems

Matrix type	Reduce to standard problems (full storage)	Reduce to standard problems (packed storage)	Reduce to standard problems (band matrices)	Factorize band matrix
real symmetric matrices	sygst	spgst	sbgst	pbstf
complex Hermitian matrices	hegst	hpgst	hbgst	pbstf

?sygst

Reduces a real symmetric-definite generalized eigenvalue problem to the standard form.

Syntax

Fortran 77:

```
call ssygst(itype, uplo, n, a, lda, b, ldb, info)
call dsygst(itype, uplo, n, a, lda, b, ldb, info)
```

Fortran 95:

```
call sygst(a, b [,itype] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sygst( int matrix_order, lapack_int itype, char uplo, lapack_int
n, <datatype>* a, lapack_int lda, const <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reduces real symmetric-definite generalized eigenproblems

$A*z = \lambda*B*z$, $A*B*z = \lambda*z$, or $B*A*z = \lambda*z$

to the standard form $C*y = \lambda*y$. Here A is a real symmetric matrix, and B is a real symmetric positive-definite matrix. Before calling this routine, call ?potrf to compute the Cholesky factorization: $B = U^T*U$ or $B = L*L^T$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype — INTEGER. Must be 1 or 2 or 3.
If *itype* = 1, the generalized eigenproblem is *A*z = lambda*B*z*
for *uplo* = 'U': *C* = inv(U^T)**A**inv(*U*), *z* = inv(*U*)**y*;
for *uplo* = 'L': *C* = inv(*L*)**A**inv(L^T), *z* = inv(L^T)**y*.
If *itype* = 2, the generalized eigenproblem is *A*B*z = lambda*z*
for *uplo* = 'U': *C* = *U***A**U^T, *z* = inv(*U*)**y*;
for *uplo* = 'L': *C* = L^T**A***L*, *z* = inv(L^T)**y*.
If *itype* = 3, the generalized eigenproblem is *B*A*z = lambda*z*
for *uplo* = 'U': *C* = *U***A**U^T, *z* = U^T**y*;
for *uplo* = 'L': *C* = L^T**A***L*, *z* = *L***y*.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', the array *a* stores the upper triangle of *A*; you must supply *B* in the factored form *B* = U^T**U*.
If *uplo* = 'L', the array *a* stores the lower triangle of *A*; you must supply *B* in the factored form *B* = *L**L^T.

n — INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).

a, *b* — REAL for ssygst
DOUBLE PRECISION for dsygst.
Arrays:
a(*lda*,*) contains the upper or lower triangle of *A*.
The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the Cholesky-factored matrix *B*:
B = U^T**U* or *B* = *L**L^T (as returned by ?potrf).
The second dimension of *b* must be at least max(1, *n*).

lda — INTEGER. The leading dimension of *a*; at least max(1, *n*).

ldb — INTEGER. The leading dimension of *b*; at least max(1, *n*).

Output Parameters

a — The upper or lower triangle of *A* is overwritten by the upper or lower triangle of *C*, as specified by the arguments *itype* and *uplo*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sygst interface are the following:

a — Holds the matrix *A* of size (*n*,*n*).

b — Holds the matrix *B* of size (*n*,*n*).

itype — Must be 1, 2, or 3. The default value is 1.

uplo — Must be 'U' or 'L'. The default value is 'U'.

Application Notes

Forming the reduced matrix *C* is a stable procedure. However, it involves implicit multiplication by inv(*B*) (if *itype* = 1) or *B* (if *itype* = 2 or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if *B* is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is *n3*.

?hegst

Reduces a complex Hermitian-definite generalized eigenvalue problem to the standard form.

Syntax

Fortran 77:

```
call chegst(itype, uplo, n, a, lda, b, ldb, info)
call zhegst(itype, uplo, n, a, lda, b, ldb, info)
```

Fortran 95:

```
call hegst(a, b [,itype] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>hegst( int matrix_order, lapack_int itype, char uplo, lapack_int
n, <datatype>* a, lapack_int lda, const <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reduces complex Hermitian-definite generalized eigenvalue problems

$A*x = \lambda*B*x$, $A*B*x = \lambda*x$, or $B*A*x = \lambda*x$.

to the standard form $Cy = \lambda y$. Here the matrix *A* is complex Hermitian, and *B* is complex Hermitian positive-definite. Before calling this routine, you must call ?potrf to compute the Cholesky factorization: $B = U^H*U$ or $B = L*L^H$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype — INTEGER. Must be 1 or 2 or 3.
If *itype* = 1, the generalized eigenproblem is $A*z = lambda*B*z$
for *uplo* = 'U': $C = (U^H)^{-1}*A*U^{-1}$, z = inv(U)$*y$;
for *uplo* = 'L': $C = L^{-1}*A*(L^H)^{-1}$, $z = (L^H)^{-1}*y$.
If *itype* = 2, the generalized eigenproblem is $A*B*z = lambda*z$
for *uplo* = 'U': $C = U*A*U^H$, $z = U^{-1}*y$;
for *uplo* = 'L': $C = L^H*A*L$, $z = (L^H)^{-1}*y$.
If *itype* = 3, the generalized eigenproblem is $B*A*z = lambda*z$
for *uplo* = 'U': $C = U*A*U^H$, $z = U^H*y$;

	for *uplo* = 'L': $C = L^H*A*L$, $z = L*y$.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', the array *a* stores the upper triangle of *A*; you must supply *B* in the factored form $B = U^H*U$. If *uplo* = 'L', the array *a* stores the lower triangle of *A*; you must supply *B* in the factored form $B = L*L^H$.
n	INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).
a, *b*	COMPLEX for chegstDOUBLE COMPLEX for zhegst. Arrays: *a*(*lda*,*) contains the upper or lower triangle of *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the Cholesky-factored matrix *B*: $B = U^H*U$ or $B = L*L^H$ (as returned by ?potrf). The second dimension of *b* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).

Output Parameters

a	The upper or lower triangle of *A* is overwritten by the upper or lower triangle of *C*, as specified by the arguments *itype* and *uplo*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hegst interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
b	Holds the matrix *B* of size (*n*,*n*).
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

Forming the reduced matrix *C* is a stable procedure. However, it involves implicit multiplication by B^{-1} (if *itype* = 1) or *B* (if *itype* = 2 or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if *B* is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3.

?spgst

Reduces a real symmetric-definite generalized eigenvalue problem to the standard form using packed storage.

Syntax

Fortran 77:

```
call sspgst(itype, uplo, n, ap, bp, info)
call dspgst(itype, uplo, n, ap, bp, info)
```

Fortran 95:

```
call spgst(ap, bp [,itype] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>spgst( int matrix_order, lapack_int itype, char uplo, lapack_int
n, <datatype>* ap, const <datatype>* bp );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces real symmetric-definite generalized eigenproblems

$A*x = \lambda*B*x$, $A*B*x = \lambda*x$, or $B*A*x = \lambda*x$

to the standard form $C*y = \lambda*y$, using packed matrix storage. Here A is a real symmetric matrix, and B is a real symmetric positive-definite matrix. Before calling this routine, call `?pptrf` to compute the Cholesky factorization: $B = U^T*U$ or $B = L*L^T$.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	`INTEGER`. Must be 1 or 2 or 3. If $itype = 1$, the generalized eigenproblem is $A*z = lambda*B*z$ for $uplo$ = `'U'`: $C = inv(U^T)*A*inv(U)$, $z = inv(U)*y$; for $uplo$ = `'L'`: $C = inv(L)*A*inv(L^T)$, $z = inv(L^T)*y$. If $itype = 2$, the generalized eigenproblem is $A*B*z = lambda*z$ for $uplo$ = `'U'`: $C = U*A*U^T$, $z = inv(U)*y$; for $uplo$ = `'L'`: $C = L^T*A*L$, $z = inv(L^T)*y$. If $itype = 3$, the generalized eigenproblem is $B*A*z = lambda*z$ for $uplo$ = `'U'`: $C = U*A*U^T$, $z = U^T*y$; for $uplo$ = `'L'`: $C = L^T*A*L$, $z = L*y$.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If $uplo$ = `'U'`, ap stores the packed upper triangle of A; you must supply B in the factored form $B = U^T*U$. If $uplo$ = `'L'`, ap stores the packed lower triangle of A; you must supply B in the factored form $B = L*L^T$.
n	`INTEGER`. The order of the matrices A and B ($n \geq 0$).
ap, *bp*	`REAL` for `sspgst` `DOUBLE PRECISION` for `dspgst`. Arrays: $ap(*)$ contains the packed upper or lower triangle of A.

The dimension of *ap* must be at least max(1, *n**(*n*+1)/2).
bp(*) contains the packed Cholesky factor of *B* (as returned by ?pptrf with the same *uplo* value).
The dimension of *bp* must be at least max(1, *n**(*n*+1)/2).

Output Parameters

ap	The upper or lower triangle of *A* is overwritten by the upper or lower triangle of *C*, as specified by the arguments *itype* and *uplo*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spgst interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
bp	Holds the array *B* of size (*n*(n+1)/2*).
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

Forming the reduced matrix *C* is a stable procedure. However, it involves implicit multiplication by inv(*B*) (if *itype* = 1) or *B* (if *itype* = 2 or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if *B* is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3.

?hpgst

Reduces a complex Hermitian-definite generalized eigenvalue problem to the standard form using packed storage.

Syntax

Fortran 77:

```
call chpgst(itype, uplo, n, ap, bp, info)
call zhpgst(itype, uplo, n, ap, bp, info)
```

Fortran 95:

```
call hpgst(ap, bp [,itype] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>hpgst( int matrix_order, lapack_int itype, char uplo, lapack_int
n, <datatype>* ap, const <datatype>* bp );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90

- C: mkl_lapacke.h

Description

The routine reduces real symmetric-definite generalized eigenproblems

$A*z = \lambda*B*z$, $A*B*z = \lambda*z$, or $B*A*z = \lambda*z$.

to the standard form $C*y = \lambda*y$, using packed matrix storage. Here A is a real symmetric matrix, and B is a real symmetric positive-definite matrix. Before calling this routine, you must call ?pptrf to compute the Cholesky factorization: $B = U^H*U$ or $B = L*L^H$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. If *itype* = 1, the generalized eigenproblem is *A*z = lambda*B*z* for *uplo* = 'U': $C = \text{inv}(U^H)*A*\text{inv}(U)$, $z = \text{inv}(U)*y$; for *uplo* = 'L': $C = \text{inv}(L)*A*\text{inv}(L^H)$, $z = \text{inv}(L^H)*y$. If *itype* = 2, the generalized eigenproblem is *A*B*z = lambda*z* for *uplo* = 'U': $C = U*A*U^H$, $z = \text{inv}(U)*y$; for *uplo* = 'L': $C = L^H*A*L$, $z = \text{inv}(L^H)*y$. If *itype* = 3, the generalized eigenproblem is *B*A*z = lambda*z* for *uplo* = 'U': $C = U*A*U^H$, $z = U^H*y$; for *uplo* = 'L': $C = L^H*A*L$, $z = L*y$.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ap* stores the packed upper triangle of *A*; you must supply *B* in the factored form $B = U^H*U$. If *uplo* = 'L', *ap* stores the packed lower triangle of *A*; you must supply *B* in the factored form $B = L*L^H$.
n	INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).
ap, *bp*	COMPLEX for chpgstDOUBLE COMPLEX for zhpgst. Arrays: *ap*(*) contains the packed upper or lower triangle of *A*. The dimension of *a* must be at least max(1, *n**(*n*+1)/2). *bp*(*) contains the packed Cholesky factor of *B* (as returned by ?pptrf with the same *uplo* value). The dimension of *b* must be at least max(1, *n**(*n*+1)/2).

Output Parameters

ap	The upper or lower triangle of *A* is overwritten by the upper or lower triangle of *C*, as specified by the arguments *itype* and *uplo*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpgst interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
bp	Holds the array *B* of size (*n*(n+1)/2*).
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

Forming the reduced matrix *C* is a stable procedure. However, it involves implicit multiplication by inv(*B*) (if *itype* = 1) or *B* (if *itype* = 2 or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if *B* is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3.

?sbgst

Reduces a real symmetric-definite generalized eigenproblem for banded matrices to the standard form using the factorization performed by ?pbstf.

Syntax

Fortran 77:

```
call ssbgst(vect, uplo, n, ka, kb, ab, ldab, bb, ldbb, x, ldx, work, info)
call dsbgst(vect, uplo, n, ka, kb, ab, ldab, bb, ldbb, x, ldx, work, info)
```

Fortran 95:

```
call sbgst(ab, bb [,x] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbgst( int matrix_order, char vect, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, <datatype>* ab, lapack_int ldab, const <datatype>* bb,
lapack_int ldbb, <datatype>* x, lapack_int ldx );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

To reduce the real symmetric-definite generalized eigenproblem $A*z = \lambda*B*z$ to the standard form $C*y=\lambda*y$, where *A*, *B* and *C* are banded, this routine must be preceded by a call to pbstf/pbstf, which computes the split Cholesky factorization of the positive-definite matrix *B*: $B=S^T*S$. The split Cholesky factorization, compared with the ordinary Cholesky factorization, allows the work to be approximately halved.

This routine overwrites *A* with $C = X^T*A*X$, where $X = \text{inv}(S)*Q$ and *Q* is an orthogonal matrix chosen (implicitly) to preserve the bandwidth of *A*. The routine also has an option to allow the accumulation of *X*, and then, if *z* is an eigenvector of *C*, *X*z* is an eigenvector of the original system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'N' or 'V'. If *vect* = 'N', then matrix *X* is not returned; If *vect* = 'V', then matrix *X* is returned.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).
ka	INTEGER. The number of super- or sub-diagonals in *A* ($ka \geq 0$).
kb	INTEGER. The number of super- or sub-diagonals in *B* ($ka \geq kb \geq 0$).
ab, *bb*, *work*	REAL for ssbgst DOUBLE PRECISION for dsbgst *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format. The second dimension of the array *ab* must be at least max(1, *n*). *bb* (*ldbb*,*) is an array containing the banded split Cholesky factor of *B* as specified by *uplo*, *n* and *kb* and returned by pbstf/pbstf. The second dimension of the array *bb* must be at least max(1, *n*). *work(*)* is a workspace array, dimension at least max(1, 2**n*)
ldab	INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.
ldbb	INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.
ldx	The leading dimension of the output array x. Constraints: if *vect* = 'N', then *ldx* ≥ 1; if *vect* = 'V', then *ldx* ≥ max(1, *n*).

Output Parameters

ab	On exit, this array is overwritten by the upper or lower triangle of *C* as specified by *uplo*.
x	REAL for ssbgst DOUBLE PRECISION for dsbgst Array. If *vect* = 'V', then *x* (*ldx*,*) contains the *n*-by-*n* matrix *X* = inv(*S*)**Q*. If *vect* = 'N', then *x* is not referenced. The second dimension of x must be: at least max(1, *n*), if *vect* = 'V'; at least 1, if *vect* = 'N'.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbgst interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).

x	Holds the matrix *X* of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
vect	Restored based on the presence of the argument *x* as follows: *vect* = 'V', if *x* is present, *vect* = 'N', if *x* is omitted.

Application Notes

Forming the reduced matrix *C* involves implicit multiplication by inv(*B*). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if *B* is ill-conditioned with respect to inversion.

If *ka* and *kb* are much less than *n* then the total number of floating-point operations is approximately $6n^2*kb$, when *vect* = 'N'. Additional $(3/2)n^3*(kb/ka)$ operations are required when *vect* = 'V'.

?hbgst

Reduces a complex Hermitian-definite generalized eigenproblem for banded matrices to the standard form using the factorization performed by ?pbstf.

Syntax

Fortran 77:

```
call chbgst(vect, uplo, n, ka, kb, ab, ldab, bb, ldbb, x, ldx, work, rwork, info)
call zhbgst(vect, uplo, n, ka, kb, ab, ldab, bb, ldbb, x, ldx, work, rwork, info)
```

Fortran 95:

```
call hbgst(ab, bb [,x] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>hbgst( int matrix_order, char vect, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, <datatype>* ab, lapack_int ldab, const <datatype>* bb,
lapack_int ldbb, <datatype>* x, lapack_int ldx );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

To reduce the complex Hermitian-definite generalized eigenproblem $A*z = \lambda*B*z$ to the standard form $C*x = \lambda*y$, where *A*, *B* and *C* are banded, this routine must be preceded by a call to pbstf/pbstf, which computes the split Cholesky factorization of the positive-definite matrix *B*: $B = S^H*S$. The split Cholesky factorization, compared with the ordinary Cholesky factorization, allows the work to be approximately halved.

This routine overwrites *A* with $C = X^H*A*X$, where X = inv(*S*)*Q, and *Q* is a unitary matrix chosen (implicitly) to preserve the bandwidth of *A*. The routine also has an option to allow the accumulation of *X*, and then, if *z* is an eigenvector of *C*, *X*z* is an eigenvector of the original system.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

vect	CHARACTER*1. Must be 'N' or 'V'. If *vect* = 'N', then matrix *X* is not returned; If *vect* = 'V', then matrix *X* is returned.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).
ka	INTEGER. The number of super- or sub-diagonals in *A* ($ka \geq 0$).
kb	INTEGER. The number of super- or sub-diagonals in *B* ($ka \geq kb \geq 0$).
ab, *bb*, *work*	COMPLEX for chbgstDOUBLE COMPLEX for zhbgst *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format. The second dimension of the array *ab* must be at least max(1, *n*). *bb* (*ldbb*,*) is an array containing the banded split Cholesky factor of *B* as specified by *uplo*, *n* and *kb* and returned by pbstf/pbstf. The second dimension of the array *bb* must be at least max(1, *n*). *work(*)* is a workspace array, dimension at least max(1, *n*)
ldab	INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.
ldbb	INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.
ldx	The leading dimension of the output array x. Constraints: if *vect* = 'N', then *ldx* ≥ 1; if *vect* = 'V', then *ldx* ≥ max(1, *n*).
rwork	REAL for chbgst DOUBLE PRECISION for zhbgst Workspace array, dimension at least max(1, *n*)

Output Parameters

ab	On exit, this array is overwritten by the upper or lower triangle of *C* as specified by *uplo*.
x	COMPLEX for chbgst DOUBLE COMPLEX for zhbgst Array. If *vect* = 'V', then *x* (*ldx*,*) contains the *n*-by-*n* matrix *X* = inv(*S*)***Q*. If *vect* = 'N', then *x* is not referenced. The second dimension of x must be: at least max(1, *n*), if *vect* = 'V'; at least 1, if *vect* = 'N'.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbgst interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).
x	Holds the matrix *X* of size (*n*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
vect	Restored based on the presence of the argument *x* as follows: *vect* = 'V', if *x* is present, *vect* = 'N', if *x* is omitted.

Application Notes

Forming the reduced matrix *C* involves implicit multiplication by inv(*B*). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if *B* is ill-conditioned with respect to inversion. The total number of floating-point operations is approximately $20n^2*kb$, when *vect* = 'N'. Additional $5n^3*(kb/ka)$ operations are required when *vect* = 'V'. All these estimates assume that both *ka* and *kb* are much less than *n*.

?pbstf

Computes a split Cholesky factorization of a real symmetric or complex Hermitian positive-definite banded matrix used in ?sbgst/?hbgst *.*

Syntax

Fortran 77:

```
call spbstf(uplo, n, kb, bb, ldbb, info)
call dpbstf(uplo, n, kb, bb, ldbb, info)
call cpbstf(uplo, n, kb, bb, ldbb, info)
call zpbstf(uplo, n, kb, bb, ldbb, info)
```

Fortran 95:

```
call pbstf(bb [, uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>pbstf( int matrix_order, char uplo, lapack_int n, lapack_int kb,
<datatype>* bb, lapack_int ldbb );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes a split Cholesky factorization of a real symmetric or complex Hermitian positive-definite band matrix *B*. It is to be used in conjunction with sbgst/hbgst.

The factorization has the form $B = S^T*S$ (or $B = S^H*S$ for complex flavors), where *S* is a band matrix of the same bandwidth as *B* and the following structure: S is upper triangular in the first (*n*+*kb*)/2 rows and lower triangular in the remaining rows.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *bb* stores the upper triangular part of *B*. If *uplo* = 'L', *bb* stores the lower triangular part of *B*.
n	INTEGER. The order of the matrix *B* (*n* ≥ 0).
kb	INTEGER. The number of super- or sub-diagonals in *B* (*kb* ≥ 0).
bb	REAL for spbstf DOUBLE PRECISION for dpbstf COMPLEX for cpbstf DOUBLE COMPLEX for zpbstf. *bb* (*ldbb*,*) is an array containing either upper or lower triangular part of the matrix *B* (as specified by *uplo*) in band storage format. The second dimension of the array *bb* must be at least max(1, *n*).
ldbb	INTEGER. The leading dimension of *bb*; must be at least *kb*+1.

Output Parameters

bb	On exit, this array is overwritten by the elements of the split Cholesky factor *S*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, then the factorization could not be completed, because the updated element b_{ii} would be the square root of a negative number; hence the matrix *B* is not positive-definite. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine pbstf interface are the following:

bb	Holds the array *B* of size (*kb+1*,*n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed factor *S* is the exact factor of a perturbed matrix *B* + *E*, where

$$|E| \le c(kb+1)\varepsilon |S^H||S|, \quad |e_{ij}| \le c(kb+1)\varepsilon \sqrt{b_{ii}b_{jj}}$$

$c(n)$ is a modest linear function of n, and ε is the machine precision.

The total number of floating-point operations for real flavors is approximately $n(kb+1)^2$. The number of operations for complex flavors is 4 times greater. All these estimates assume that *kb* is much less than *n*.

After calling this routine, you can call sbgst/hbgst to solve the generalized eigenproblem $Az = \lambda Bz$, where *A* and *B* are banded and *B* is positive-definite.

Nonsymmetric Eigenvalue Problems

This section describes LAPACK routines for solving nonsymmetric eigenvalue problems, computing the Schur factorization of general matrices, as well as performing a number of related computational tasks.

A *nonsymmetric eigenvalue problem* is as follows: given a nonsymmetric (or non-Hermitian) matrix A, find the *eigenvalues* λ and the corresponding *eigenvectors* z that satisfy the equation

$Az = \lambda z$ (right eigenvectors z)

or the equation

$z^H A = \lambda z^H$ (left eigenvectors z).

Nonsymmetric eigenvalue problems have the following properties:

- The number of eigenvectors may be less than the matrix order (but is not less than the number of *distinct eigenvalues* of A).
- Eigenvalues may be complex even for a real matrix A.
- If a real nonsymmetric matrix has a complex eigenvalue $a+bi$ corresponding to an eigenvector z, then $a-bi$ is also an eigenvalue. The eigenvalue $a-bi$ corresponds to the eigenvector whose elements are complex conjugate to the elements of z.

To solve a nonsymmetric eigenvalue problem with LAPACK, you usually need to reduce the matrix to the upper Hessenberg form and then solve the eigenvalue problem with the Hessenberg matrix obtained. Table "Computational Routines for Solving Nonsymmetric Eigenvalue Problems" lists LAPACK routines (FORTRAN 77 interface) to reduce the matrix to the upper Hessenberg form by an orthogonal (or unitary) similarity transformation $A = QHQ^H$ as well as routines to solve eigenvalue problems with Hessenberg matrices, forming the Schur factorization of such matrices and computing the corresponding condition numbers. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

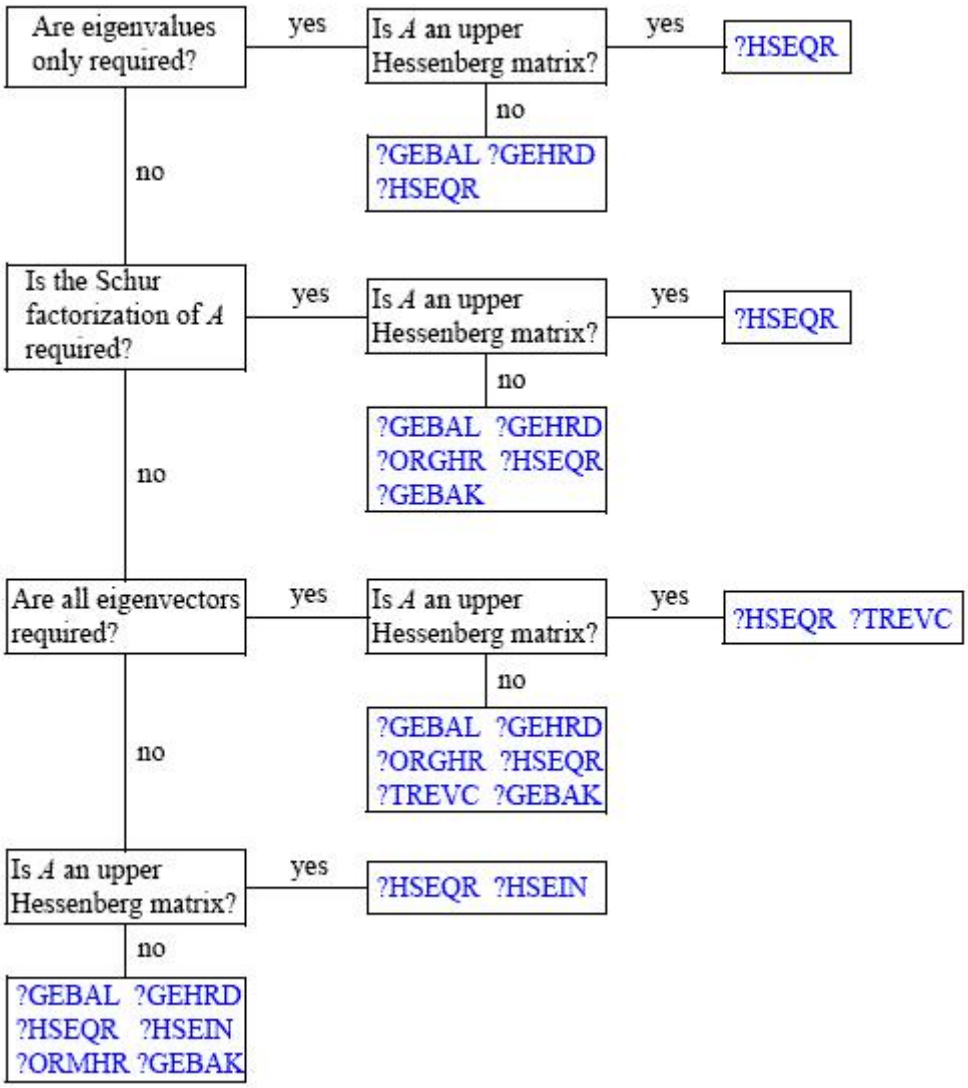
The decision tree in Figure "Decision Tree: Real Nonsymmetric Eigenvalue Problems" helps you choose the right routine or sequence of routines for an eigenvalue problem with a real nonsymmetric matrix. If you need to solve an eigenvalue problem with a complex non-Hermitian matrix, use the decision tree shown in Figure "Decision Tree: Complex Non-Hermitian Eigenvalue Problems".

Computational Routines for Solving Nonsymmetric Eigenvalue Problems

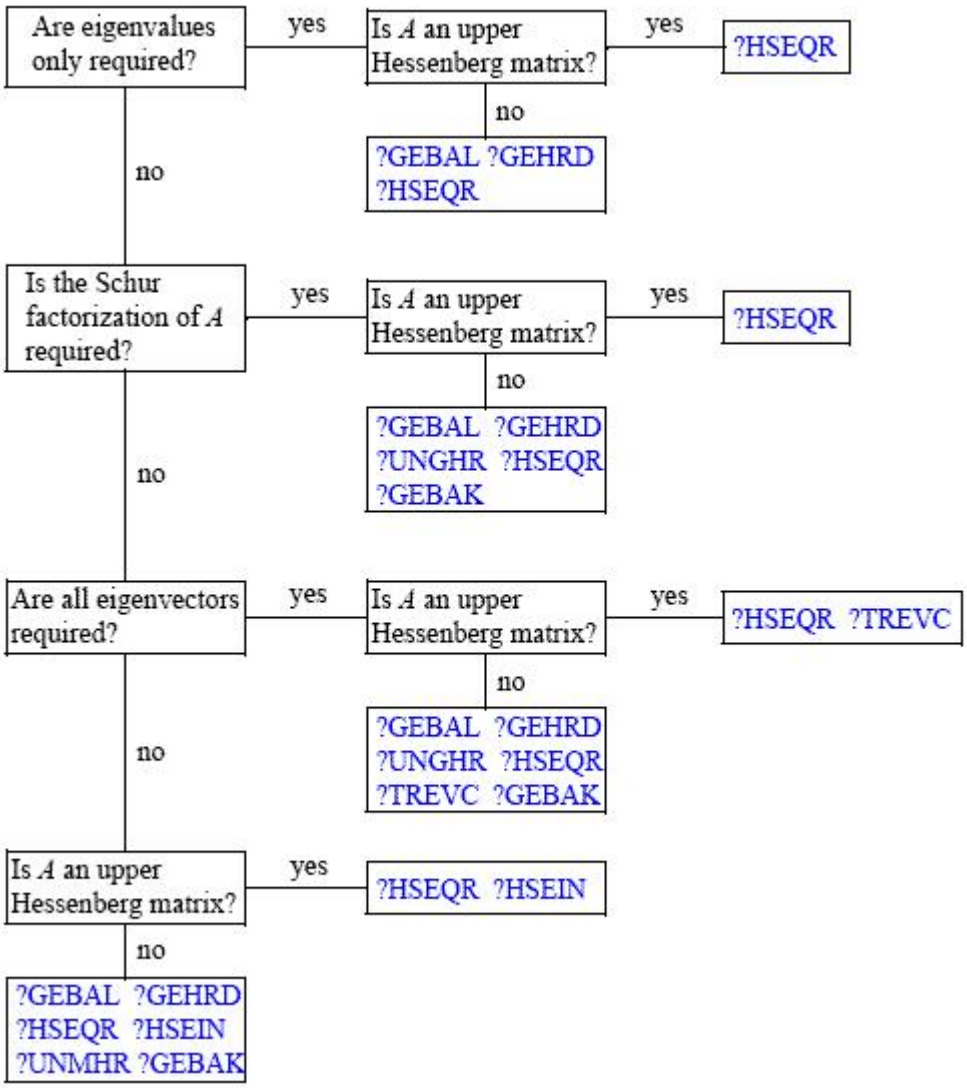
Operation performed	Routines for real matrices	Routines for complex matrices
Reduce to Hessenberg form $A = QHQ^H$	?gehrd,	?gehrd
Generate the matrix Q	?orghr	?unghr
Apply the matrix Q	?ormhr	?unmhr
Balance matrix	?gebal	?gebal
Transform eigenvectors of balanced matrix to those of the original matrix	?gebak	?gebak
Find eigenvalues and Schur factorization (QR algorithm)	?hseqr	?hseqr
Find eigenvectors from Hessenberg form (inverse iteration)	?hsein	?hsein
Find eigenvectors from Schur factorization	?trevc	?trevc

Operation performed	Routines for real matrices	Routines for complex matrices
Estimate sensitivities of eigenvalues and eigenvectors	?trsna	?trsna
Reorder Schur factorization	?trexc	?trexc
Reorder Schur factorization, find the invariant subspace and estimate sensitivities	?trsen	?trsen
Solves Sylvester's equation.	?trsyl	?trsyl

Decision Tree: Real Nonsymmetric Eigenvalue Problems



Decision Tree: Complex Non-Hermitian Eigenvalue Problems



?gehrd

Reduces a general matrix to upper Hessenberg form.

Syntax

Fortran 77:

```
call sgehrd(n, ilo, ihi, a, lda, tau, work, lwork, info)
call dgehrd(n, ilo, ihi, a, lda, tau, work, lwork, info)
call cgehrd(n, ilo, ihi, a, lda, tau, work, lwork, info)
call zgehrd(n, ilo, ihi, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call gehrd(a [, tau] [,ilo] [,ihi] [,info])
```

C:

```
lapack_int LAPACKE_<?>gehrd( int matrix_order, lapack_int n, lapack_int ilo, lapack_int ihi, <datatype>* a, lapack_int lda, <datatype>* tau );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reduces a general matrix A to upper Hessenberg form H by an orthogonal or unitary similarity transformation $A = Q*H*Q^H$. Here H has real subdiagonal elements.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of *elementary reflectors*. Routines are provided to work with Q in this representation.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix A ($n \geq 0$).
ilo, *ihi*	INTEGER. If A is an output by ?gebal, then *ilo* and *ihi* must contain the values returned by that routine. Otherwise $ilo = 1$ and $ihi = n$. (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, $ilo = 1$ and $ihi = 0$.)
a, *work*	REAL for sgehrd DOUBLE PRECISION for dgehrd COMPLEX for cgehrd DOUBLE COMPLEX for zgehrd. Arrays: *a* (*lda*,*) contains the matrix A. The second dimension of *a* must be at least max(1, *n*). *work* (*lwork*) is a workspace array.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
lwork	INTEGER. The size of the *work* array; at least max(1, *n*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the upper Hessenberg matrix H and details of the matrix Q. The subdiagonal elements of H are real.
tau	REAL for sgehrd DOUBLE PRECISION for dgehrd COMPLEX for cgehrd DOUBLE COMPLEX for zgehrd. Array, DIMENSION at least max (1, *n*-1). Contains additional information on the matrix Q.

work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gehrd interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
tau	Holds the vector of length (*n*-1).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.

Application Notes

For better performance, try using *lwork* = *n***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed Hessenberg matrix *H* is exactly similar to a nearby matrix *A* + *E*, where $||E||_2 < c(n)\varepsilon||A||_2$, $c(n)$ is a modestly increasing function of *n*, and ε is the machine precision.

The approximate number of floating-point operations for real flavors is $(2/3)*(ihi - ilo)^2(2ihi + 2ilo + 3n)$; for complex flavors it is 4 times greater.

?orghr

Generates the real orthogonal matrix Q determined by ?gehrd*.*

Syntax

Fortran 77:

```
call sorghr(n, ilo, ihi, a, lda, tau, work, lwork, info)
call dorghr(n, ilo, ihi, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call orghr(a, tau [,ilo] [,ihi] [,info])
```

C:

```
lapack_int LAPACKE_<?>orghr( int matrix_order, lapack_int n, lapack_int ilo, lapack_int ihi, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine explicitly generates the orthogonal matrix Q that has been determined by a preceding call to `sgehrd`/`dgehrd`. (The routine `?gehrd` reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation, $A = Q*H*Q^T$, and represents the matrix Q as a product of $ihi-ilo$ *elementary reflectors*. Here ilo and ihi are values determined by `sgebal`/`dgebal` when balancing the matrix; if the matrix has not been balanced, $ilo = 1$ and $ihi = n$.)

The matrix Q generated by `?orghr` has the structure:

$$Q = \begin{bmatrix} I & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & I \end{bmatrix}$$

where Q_{22} occupies rows and columns ilo to ihi.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix Q ($n \geq 0$).
ilo, *ihi*	INTEGER. These must be the same parameters ilo and ihi, respectively, as supplied to `?gehrd`. (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, $ilo = 1$ and $ihi = 0$.)
a, *tau*, *work*	REAL for `sorghr` DOUBLE PRECISION for `dorghr` Arrays: a(lda,*) contains details of the vectors which define the elementary reflectors, as returned by `?gehrd`. The second dimension of a must be at least max(1, n). tau(*) contains further details of the elementary reflectors, as returned by `?gehrd`. The dimension of tau must be at least max (1, n-1). $work$ is a workspace array, its dimension `max(1, lwork)`.
lda	INTEGER. The leading dimension of a; at least max(1, n).
lwork	INTEGER. The size of the $work$ array; $lwork \geq$ `max(1, ihi-ilo)`.

If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	Overwritten by the *n*-by-*n* orthogonal matrix *Q*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine orghr interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
tau	Holds the vector of length (*n*-1).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.

Application Notes

For better performance, try using *lwork* =(*ihi*-*ilo*)***blocksize* where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *Q* differs from the exact result by a matrix *E* such that $||E||_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is $(4/3)(ihi-ilo)^3$.

The complex counterpart of this routine is unghr.

?ormhr

Multiplies an arbitrary real matrix C by the real orthogonal matrix Q determined by ?gehrd.

Syntax

Fortran 77:

```
call sormhr(side, trans, m, n, ilo, ihi, a, lda, tau, c, ldc, work, lwork, info)
call dormhr(side, trans, m, n, ilo, ihi, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call ormhr(a, tau, c [,ilo] [,ihi] [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>ormhr( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int ilo, lapack_int ihi, const <datatype>* a, lapack_int lda,
const <datatype>* tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine multiplies a matrix C by the orthogonal matrix Q that has been determined by a preceding call to sgehrd/dgehrd. (The routine ?gehrd reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation, $A = Q*H*Q^T$, and represents the matrix Q as a product of $ihi-ilo$ *elementary reflectors*. Here ilo and ihi are values determined by sgebal/dgebal when balancing the matrix;if the matrix has not been balanced, $ilo = 1$ and $ihi = n$.)

With ?ormhr, you can form one of the matrix products $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$, overwriting the result on C (which may be any real rectangular matrix).

A common application of ?ormhr is to transform a matrix V of eigenvectors of H to the matrix QV of eigenvectors of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	CHARACTER*1. Must be 'L' or 'R'. If *side* = 'L', then the routine forms $Q*C$ or Q^T*C. If *side* = 'R', then the routine forms $C*Q$ or $C*Q^T$.
trans	CHARACTER*1. Must be 'N' or 'T'. If *trans* = 'N', then Q is applied to C. If *trans* = 'T', then Q^T is applied to C.
m	INTEGER. The number of rows in C ($m \geq 0$).
n	INTEGER. The number of columns in C ($n \geq 0$).
ilo, *ihi*	INTEGER. These must be the same parameters ilo and ihi, respectively, as supplied to ?gehrd. If $m > 0$ and *side* = 'L', then $1 \leq ilo \leq ihi \leq m$. If $m = 0$ and *side* = 'L', then $ilo = 1$ and $ihi = 0$. If $n > 0$ and *side* = 'R', then $1 \leq ilo \leq ihi \leq n$. If $n = 0$ and *side* = 'R', then $ilo = 1$ and $ihi = 0$.

a, *tau*, *c*, *work*	REAL for sormhr DOUBLE PRECISION for dormhr Arrays: *a*(*lda*,*) contains details of the vectors which define the *elementary reflectors*, as returned by ?gehrd. The second dimension of *a* must be at least max(1, *m*) if *side* = 'L' and at least max(1, *n*) if *side* = 'R'. *tau*(*) contains further details of the *elementary reflectors*, as returned by ?gehrd . The dimension of *tau* must be at least max (1, *m*-1) if *side* = 'L' and at least max (1, *n*-1) if *side* = 'R'. *c*(*ldc*,*) contains the *m* by *n* matrix *C*. The second dimension of *c* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*) if *side* = 'L' and at least max (1, *n*) if *side* = 'R'.
ldc	INTEGER. The leading dimension of *c*; at least max(1, *m*).
lwork	INTEGER. The size of the *work* array. If *side* = 'L', *lwork* ≥ max(1, *n*). If *side* = 'R', *lwork* ≥ max(1, *m*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	*C* is overwritten by product $Q*C$, Q^T*C, $C*Q$, or $C*Q^T$ as specified by *side* and *trans*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ormhr interface are the following:

a	Holds the matrix *A* of size (*r*,*r*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*r*-1).
c	Holds the matrix *C* of size (*m*,*n*).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
side	Must be 'L' or 'R'. The default value is 'L'.

trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, *lwork* should be at least *n***blocksize* if *side* = 'L' and at least *m***blocksize* if *side* = 'R', where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *Q* differs from the exact result by a matrix *E* such that $||E||_2 = O(\varepsilon)*||C||_2$, where ε is the machine precision.

The approximate number of floating-point operations is

$2n(ihi-ilo)^2$ if *side* = 'L';

$2m(ihi-ilo)^2$ if *side* = 'R'.

The complex counterpart of this routine is unmhr.

?unghr

Generates the complex unitary matrix Q determined by ?gehrd*.*

Syntax

Fortran 77:

```
call cunghr(n, ilo, ihi, a, lda, tau, work, lwork, info)
call zunghr(n, ilo, ihi, a, lda, tau, work, lwork, info)
```

Fortran 95:

```
call unghr(a, tau [,ilo] [,ihi] [,info])
```

C:

```
lapack_int LAPACKE_<?>unghr( int matrix_order, lapack_int n, lapack_int ilo, lapack_int
ihi, <datatype>* a, lapack_int lda, const <datatype>* tau );
```

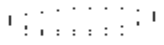
Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine is intended to be used following a call to cgehrd/zgehrd, which reduces a complex matrix A to upper Hessenberg form H by a unitary similarity transformation: $A = Q*H*Q^H$. ?gehrd represents the matrix Q as a product of $ihi-ilo$ *elementary reflectors*. Here ilo and ihi are values determined by cgebal/zgebal when balancing the matrix; if the matrix has not been balanced, $ilo = 1$ and $ihi = n$.

Use the routine unghr to generate Q explicitly as a square matrix. The matrix Q has the structure:



where Q_{22} occupies rows and columns ilo to ihi.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The order of the matrix Q ($n \geq 0$).
ilo, ihi	INTEGER. These must be the same parameters ilo and ihi, respectively, as supplied to ?gehrd . (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$. If $n = 0$, then $ilo = 1$ and $ihi = 0$.)
a, tau, $work$	COMPLEX for cunghr DOUBLE COMPLEX for zunghr. Arrays: $a(lda,*)$ contains details of the vectors which define the *elementary reflectors*, as returned by ?gehrd. The second dimension of a must be at least max(1, n). $tau(*)$ contains further details of the *elementary reflectors*, as returned by ?gehrd . The dimension of tau must be at least max (1, n-1). $work$ is a workspace array, its dimension max(1, $lwork$).
lda	INTEGER. The leading dimension of a; at least max(1, n).
$lwork$	INTEGER. The size of the $work$ array; $lwork \geq$ max(1, $ihi-ilo$). If $lwork = -1$, then a workspace query is assumed; the routine only calculates the optimal size of the $work$ array, returns this value as the first entry of the $work$ array, and no error message related to $lwork$ is issued by xerbla. See *Application Notes* for the suggested value of $lwork$.

Output Parameters

a	Overwritten by the n-by-n unitary matrix Q.
$work(1)$	If $info = 0$, on exit $work(1)$ contains the minimum value of $lwork$ required for optimum performance. Use this $lwork$ for subsequent runs.
$info$	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `unghr` interface are the following:

a	Holds the matrix A of size (n,n).
tau	Holds the vector of length (n-1).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.

Application Notes

For better performance, try using *lwork* = (*ihi*-*ilo*)**blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix Q differs from the exact result by a matrix E such that $||E||_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of real floating-point operations is $(16/3)(ihi-ilo)^3$.

The real counterpart of this routine is orghr.

?unmhr

Multiplies an arbitrary complex matrix C by the complex unitary matrix Q determined by `?gehrd`.

Syntax

Fortran 77:

```
call cunmhr(side, trans, m, n, ilo, ihi, a, lda, tau, c, ldc, work, lwork, info)
call zunmhr(side, trans, m, n, ilo, ihi, a, lda, tau, c, ldc, work, lwork, info)
```

Fortran 95:

```
call unmhr(a, tau, c [,ilo] [,ihi] [,side] [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>unmhr( int matrix_order, char side, char trans, lapack_int m,
lapack_int n, lapack_int ilo, lapack_int ihi, const <datatype>* a, lapack_int lda,
const <datatype>* tau, <datatype>* c, lapack_int ldc );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine multiplies a matrix C by the unitary matrix Q that has been determined by a preceding call to `cgehrd`/`zgehrd`. (The routine `?gehrd` reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation, $A = Q*H*Q^H$, and represents the matrix Q as a product of ihi-ilo *elementary reflectors*. Here ilo and ihi are values determined by `cgebal`/`zgebal` when balancing the matrix; if the matrix has not been balanced, $ilo = 1$ and $ihi = n$.)

With `?unmhr`, you can form one of the matrix products $Q*C$, Q^H*C, $C*Q$, or $C*Q^H$, overwriting the result on C (which may be any complex rectangular matrix). A common application of this routine is to transform a matrix V of eigenvectors of H to the matrix QV of eigenvectors of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side	`CHARACTER*1`. Must be `'L'` or `'R'`. If *side* = `'L'`, then the routine forms $Q*C$ or Q^H*C. If *side* = `'R'`, then the routine forms $C*Q$ or $C*Q^H$.
trans	`CHARACTER*1`. Must be `'N'` or `'C'`. If *trans* = `'N'`, then Q is applied to C. If *trans* = `'T'`, then Q^H is applied to C.
m	`INTEGER`. The number of rows in C ($m \geq 0$).
n	`INTEGER`. The number of columns in C ($n \geq 0$).
ilo, *ihi*	`INTEGER`. These must be the same parameters *ilo* and *ihi*, respectively, as supplied to `?gehrd` . If $m > 0$ and *side* = `'L'`, then $1 \leq ilo \leq ihi \leq m$. If $m = 0$ and *side* = `'L'`, then $ilo = 1$ and $ihi = 0$. If $n > 0$ and *side* = `'R'`, then $1 \leq ilo \leq ihi \leq n$. If $n = 0$ and *side* = `'R'`, then $ilo = 1$ and $ihi = 0$.
a, *tau*, *c*, *work*	`COMPLEX` for `cunmhr` `DOUBLE COMPLEX` for `zunmhr`. Arrays: *a* (*lda*,*) contains details of the vectors which define the elementary reflectors, as returned by `?gehrd`. The second dimension of *a* must be at least max(1, *m*) if *side* = `'L'` and at least max(1, *n*) if *side* = `'R'`. *tau*(*) contains further details of the elementary reflectors, as returned by `?gehrd`. The dimension of *tau* must be at least max (1, *m*-1) if *side* = `'L'` and at least max (1, *n*-1) if *side* = `'R'`. *c* (*ldc*,*) contains the *m*-by-*n* matrix C. The second dimension of *c* must be at least max(1, *n*). *work* is a workspace array, its dimension `max(1, lwork)`.
lda	`INTEGER`. The leading dimension of *a*; at least max(1, *m*) if *side* = `'L'` and at least max (1, *n*) if *side* = `'R'`.
ldc	`INTEGER`. The leading dimension of *c*; at least max(1, *m*).
lwork	`INTEGER`. The size of the *work* array.

If *side* = 'L', *lwork* ≥ max(1,*n*).
If *side* = 'R', *lwork* ≥ max(1,*m*).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

c	*C* is overwritten by $Q*C$, or Q^H*C, or $C*Q^H$, or $C*Q$ as specified by *side* and *trans*.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine unmhr interface are the following:

a	Holds the matrix *A* of size (*r*,*r*). *r* = *m* if *side* = 'L'. *r* = *n* if *side* = 'R'.
tau	Holds the vector of length (*r*-1).
c	Holds the matrix *C* of size (*m*,*n*).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
side	Must be 'L' or 'R'. The default value is 'L'.
trans	Must be 'N' or 'C'. The default value is 'N'.

Application Notes

For better performance, *lwork* should be at least *n***blocksize* if *side* = 'L' and at least *m***blocksize* if *side* = 'R', where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The computed matrix *Q* differs from the exact result by a matrix *E* such that $||E||_2 = O(\varepsilon)*||C||_2$, where ε is the machine precision.

The approximate number of floating-point operations is

$8n(ihi-ilo)^2$ if *side* = 'L';

$8m(ihi-ilo)^2$ if *side* = 'R'.

The real counterpart of this routine is ormhr.

?gebal

Balances a general matrix to improve the accuracy of computed eigenvalues and eigenvectors.

Syntax

Fortran 77:

```
call sgebal(job, n, a, lda, ilo, ihi, scale, info)
call dgebal(job, n, a, lda, ilo, ihi, scale, info)
call cgebal(job, n, a, lda, ilo, ihi, scale, info)
call zgebal(job, n, a, lda, ilo, ihi, scale, info)
```

Fortran 95:

```
call gebal(a [, scale] [,ilo] [,ihi] [,job] [,info])
```

C:

```
lapack_int LAPACKE_sgebal( int matrix_order, char job, lapack_int n, float* a,
lapack_int lda, lapack_int* ilo, lapack_int* ihi, float* scale );
lapack_int LAPACKE_dgebal( int matrix_order, char job, lapack_int n, double* a,
lapack_int lda, lapack_int* ilo, lapack_int* ihi, double* scale );
lapack_int LAPACKE_cgebal( int matrix_order, char job, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int* ilo, lapack_int* ihi, float*
scale );
lapack_int LAPACKE_zgebal( int matrix_order, char job, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int* ilo, lapack_int* ihi, double*
scale );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

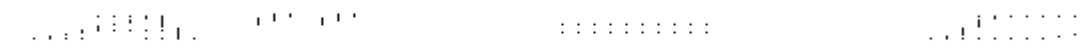
The routine *balances* a matrix *A* by performing either or both of the following two similarity transformations:

(1) The routine first attempts to permute *A* to block upper triangular form:

where P is a permutation matrix, and A'_{11} and A'_{33} are upper triangular. The diagonal elements of A'_{11} and A'_{33} are eigenvalues of A. The rest of the eigenvalues of A are the eigenvalues of the central diagonal block A'_{22}, in rows and columns ilo to ihi. Subsequent operations to compute the eigenvalues of A (or its Schur factorization) need only be applied to these rows and columns; this can save a significant amount of work if $ilo > 1$ and $ihi < n$.

If no suitable permutation exists (as is often the case), the routine sets $ilo = 1$ and $ihi = n$, and A'_{22} is the whole of A.

(2) The routine applies a diagonal similarity transformation to A', to make the rows and columns of A'_{22} as close in norm as possible:



This scaling can reduce the norm of the matrix (that is, $||A'_2{}'_2|| < ||A'_{22}||$), and hence reduce the effect of rounding errors on the accuracy of computed eigenvalues and eigenvectors.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job	CHARACTER*1. Must be 'N' or 'P' or 'S' or 'B'. If *job* = 'N', then A is neither permuted nor scaled (but *ilo*, *ihi*, and *scale* get their values). If *job* = 'P', then A is permuted but not scaled. If *job* = 'S', then A is scaled but not permuted. If *job* = 'B', then A is both scaled and permuted.
n	INTEGER. The order of the matrix A ($n \geq 0$).
a	REAL for sgebal DOUBLE PRECISION for dgebal COMPLEX for cgebal DOUBLE COMPLEX for zgebal. Arrays: *a* (*lda*,*) contains the matrix A. The second dimension of *a* must be at least max(1, *n*). *a* is not referenced if *job* = 'N'.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).

Output Parameters

a	Overwritten by the balanced matrix (*a* is not referenced if *job* = 'N').
ilo, *ihi*	INTEGER. The values *ilo* and *ihi* such that on exit *a*(*i*,*j*) is zero if $i > j$ and $1 \leq j < ilo$ or $ihi < i \leq n$. If *job* = 'N' or 'S', then *ilo* = 1 and *ihi* = *n*.
scale	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors Array, DIMENSION at least max(1, *n*).

	Contains details of the permutations and scaling factors. More precisely, if p_j is the index of the row and column interchanged with row and column j, and d_j is the scaling factor used to balance row and column j, then *scale*(*j*) = p_j for *j* = 1, 2,..., *ilo*-1, *ihi*+1,..., *n*; *scale*(*j*) = d_j for *j* = *ilo*, *ilo* + 1,..., *ihi*. The order in which the interchanges are made is *n* to *ihi*+1, then 1 to *ilo*-1.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gebal interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
scale	Holds the vector of length *n*.
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
job	Must be 'B', 'S', 'P', or 'N'. The default value is 'B'.

Application Notes

The errors are negligible, compared with those in subsequent computations.

If the matrix *A* is balanced by this routine, then any eigenvectors computed subsequently are eigenvectors of the matrix *A*'' and hence you must call gebak to transform them back to eigenvectors of *A*.

If the Schur vectors of *A* are required, do not call this routine with *job* = 'S' or 'B', because then the balancing transformation is not orthogonal (not unitary for complex flavors).

If you call this routine with *job* = 'P', then any Schur vectors computed subsequently are Schur vectors of the matrix *A*'', and you need to call gebak (with *side* = 'R') to transform them back to Schur vectors of *A*.

The total number of floating-point operations is proportional to n^2.

?gebak

Transforms eigenvectors of a balanced matrix to those of the original nonsymmetric matrix.

Syntax

Fortran 77:

```
call sgebak(job, side, n, ilo, ihi, scale, m, v, ldv, info)
call dgebak(job, side, n, ilo, ihi, scale, m, v, ldv, info)
call cgebak(job, side, n, ilo, ihi, scale, m, v, ldv, info)
call zgebak(job, side, n, ilo, ihi, scale, m, v, ldv, info)
```

Fortran 95:

```
call gebak(v, scale [,ilo] [,ihi] [,job] [,side] [,info])
```

C:

```
lapack_int LAPACKE_sgebak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* scale, lapack_int m, float* v, lapack_int
ldv );

lapack_int LAPACKE_dgebak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* scale, lapack_int m, double* v,
lapack_int ldv );

lapack_int LAPACKE_cgebak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* scale, lapack_int m, lapack_complex_float*
v, lapack_int ldv );

lapack_int LAPACKE_zgebak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* scale, lapack_int m,
lapack_complex_double* v, lapack_int ldv );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine is intended to be used after a matrix A has been balanced by a call to ?gebal, and eigenvectors of the balanced matrix A''_{22} have subsequently been computed. For a description of balancing, see gebal. The balanced matrix A'' is obtained as $A'' = D*P*A*P^T*\mathrm{inv}(D)$, where P is a permutation matrix and D is a diagonal scaling matrix. This routine transforms the eigenvectors as follows:

if x is a right eigenvector of A'', then $P^T*\mathrm{inv}(D)*x$ is a right eigenvector of A; if x is a left eigenvector of A'', then P^T*D*y is a left eigenvector of A.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job	CHARACTER*1. Must be 'N' or 'P' or 'S' or 'B'. The same parameter *job* as supplied to ?gebal.
side	CHARACTER*1. Must be 'L' or 'R'. If *side* = 'L', then left eigenvectors are transformed. If *side* = 'R', then right eigenvectors are transformed.
n	INTEGER. The number of rows of the matrix of eigenvectors ($n \geq 0$).
ilo, *ihi*	INTEGER. The values *ilo* and *ihi*, as returned by ?gebal. (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, then $ilo = 1$ and $ihi = 0$.)
scale	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors Array, DIMENSION at least max(1, n). Contains details of the permutations and/or the scaling factors used to balance the original general matrix, as returned by ?gebal.
m	INTEGER. The number of columns of the matrix of eigenvectors ($m \geq 0$).
v	REAL for sgebak

	DOUBLE PRECISION for dgebak COMPLEX for cgebak DOUBLE COMPLEX for zgebak. Arrays: *v* (*ldv*,*) contains the matrix of left or right eigenvectors to be transformed. The second dimension of *v* must be at least max(1, *m*).
ldv	INTEGER. The leading dimension of *v*; at least max(1, *n*).

Output Parameters

v	Overwritten by the transformed eigenvectors.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gebak interface are the following:

v	Holds the matrix *V* of size (*n*,*m*).
scale	Holds the vector of length *n*.
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
job	Must be 'B', 'S', 'P', or 'N'. The default value is 'B'.
side	Must be 'L' or 'R'. The default value is 'L'.

Application Notes

The errors in this routine are negligible.

The approximate number of floating-point operations is approximately proportional to *m***n*.

?hseqr

Computes all eigenvalues and (optionally) the Schur factorization of a matrix reduced to Hessenberg form.

Syntax

Fortran 77:

```
call shseqr(job, compz, n, ilo, ihi, h, ldh, wr, wi, z, ldz, work, lwork, info)
call dhseqr(job, compz, n, ilo, ihi, h, ldh, wr, wi, z, ldz, work, lwork, info)
call chseqr(job, compz, n, ilo, ihi, h, ldh, w, z, ldz, work, lwork, info)
call zhseqr(job, compz, n, ilo, ihi, h, ldh, w, z, ldz, work, lwork, info)
```

Fortran 95:

```
call hseqr(h, wr, wi [,ilo] [,ihi] [,z] [,job] [,compz] [,info])
call hseqr(h, w [,ilo] [,ihi] [,z] [,job] [,compz] [,info])
```

C:

```
lapack_int LAPACKE_shseqr( int matrix_order, char job, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, float* h, lapack_int ldh, float* wr, float* wi, float*
z, lapack_int ldz );

lapack_int LAPACKE_dhseqr( int matrix_order, char job, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, double* h, lapack_int ldh, double* wr, double* wi,
double* z, lapack_int ldz );

lapack_int LAPACKE_chseqr( int matrix_order, char job, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, lapack_complex_float* h, lapack_int ldh,
lapack_complex_float* w, lapack_complex_float* z, lapack_int ldz );

lapack_int LAPACKE_zhseqr( int matrix_order, char job, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, lapack_complex_double* h, lapack_int ldh,
lapack_complex_double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally the Schur factorization, of an upper Hessenberg matrix H: $H = Z*T*Z^H$, where T is an upper triangular (or, for real flavors, quasi-triangular) matrix (the Schur form of H), and Z is the unitary or orthogonal matrix whose columns are the Schur vectors z_i.

You can also use this routine to compute the Schur factorization of a general matrix A which has been reduced to upper Hessenberg form H:

$A = Q*H*Q^H$, where Q is unitary (orthogonal for real flavors);

$A = (QZ)*H*(QZ)^H$.

In this case, after reducing A to Hessenberg form by gehrd, call orghr to form Q explicitly and then pass Q to `?hseqr` with $compz$ = `'V'`.

You can also call gebal to balance the original matrix before reducing it to Hessenberg form by `?hseqr`, so that the Hessenberg matrix H will have the structure:

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ 0 & H_{22} & H_{23} \\ 0 & 0 & H_{33} \end{bmatrix}$$

where H_{11} and H_{33} are upper triangular.

If so, only the central diagonal block H_{22} (in rows and columns ilo to ihi) needs to be further reduced to Schur form (the blocks H_{12} and H_{23} are also affected). Therefore the values of ilo and ihi can be supplied to `?hseqr` directly. Also, after calling this routine you must call gebak to permute the Schur vectors of the balanced matrix to those of the original matrix.

If `?gebal` has not been called, however, then ilo must be set to 1 and ihi to n. Note that if the Schur factorization of A is required, `?gebal` must not be called with job = `'S'` or `'B'`, because the balancing transformation is not unitary (for real flavors, it is not orthogonal).

?hseqr uses a multishift form of the upper Hessenberg *QR* algorithm. The Schur vectors are normalized so that $||z_i||_2 = 1$, but are determined only to within a complex factor of absolute value 1 (for the real flavors, to within a factor ±1).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job
CHARACTER*1. Must be 'E' or 'S'.
If *job* = 'E', then eigenvalues only are required.
If *job* = 'S', then the Schur form *T* is required.

compz
CHARACTER*1. Must be 'N' or 'I' or 'V'.
If *compz* = 'N', then no Schur vectors are computed (and the array *z* is not referenced).
If *compz* = 'I', then the Schur vectors of *H* are computed (and the array *z* is initialized by the routine).
If *compz* = 'V', then the Schur vectors of *A* are computed (and the array *z* must contain the matrix *Q* on entry).

n
INTEGER. The order of the matrix *H* ($n \geq 0$).

ilo, *ihi*
INTEGER. If *A* has been balanced by ?gebal, then *ilo* and *ihi* must contain the values returned by ?gebal. Otherwise, *ilo* must be set to 1 and *ihi* to *n*.

h, *z*, *work*
REAL for shseqr
DOUBLE PRECISION for dhseqr
COMPLEX for chseqr
DOUBLE COMPLEX for zhseqr.
Arrays:
h(*ldh*,*) The *n*-by-*n* upper Hessenberg matrix *H*.
The second dimension of *h* must be at least max(1, *n*).
z(*ldz*,*)
If *compz* = 'V', then *z* must contain the matrix *Q* from the reduction to Hessenberg form.
If *compz* = 'I', then *z* need not be set.
If *compz* = 'N', then *z* is not referenced.
The second dimension of *z* must be
at least max(1, *n*) if *compz* = 'V' or 'I';
at least 1 if *compz* = 'N'.
work(*lwork*) is a workspace array.
The dimension of *work* must be at least max (1, *n*).

ldh
INTEGER. The leading dimension of *h*; at least max(1, *n*).

ldz
INTEGER. The leading dimension of *z*;
If *compz* = 'N', then *ldz* ≥ 1.
If *compz* = 'V' or 'I', then *ldz* ≥ max(1, *n*).

lwork
INTEGER. The dimension of the array *work*.
lwork ≥ max(1, *n*) is sufficient and delivers very good and sometimes optimal performance. However, *lwork* as large as 11*n may be required for optimal performance. A workspace query is recommended to determine the optimal workspace size.

If *lwork* = -1, then a workspace query is assumed; the routine only estimates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

w
COMPLEX for chseqr
DOUBLE COMPLEX for zhseqr.
Array, DIMENSION at least max (1, *n*). Contains the computed eigenvalues, unless *info*>0. The eigenvalues are stored in the same order as on the diagonal of the Schur form *T* (if computed).

wr, *wi*
REAL for shseqr
DOUBLE PRECISION for dhseqr
Arrays, DIMENSION at least max (1, *n*) each.
Contain the real and imaginary parts, respectively, of the computed eigenvalues, unless *info* > 0. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first. The eigenvalues are stored in the same order as on the diagonal of the Schur form *T* (if computed).

h
If *info* = 0 and *job* = 'S', *h* contains the upper triangular matrix *T* from the Schur decomposition (the Schur form).
If *info* = 0 and *job* = 'E', the contents of *h* are unspecified on exit. (The output value of *h* when *info* > 0 is given under the description of *info* below.)

z
If *compz* = 'V' and *info* = 0, then *z* contains *Q***Z*.
If *compz* = 'I' and *info* = 0, then *z* contains the unitary or orthogonal matrix *Z* of the Schur vectors of *H*.
If *compz* = 'N', then *z* is not referenced.

work(1)
On exit, if *info* = 0, then *work(1)* returns the optimal *lwork*.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, elements 1,2, ..., *ilo*-1 and *i*+1, *i*+2, ..., *n* of *wr* and *wi* contain the real and imaginary parts of those eigenvalues that have been succesively found.
If *info* > 0, and *job* = 'E', then on exit, the remaining unconverged eigenvalues are the eigenvalues of the upper Hessenberg matrix rows and columns *ilo* through *info* of the final output value of *H*.
If *info* > 0, and *job* = 'S', then on exit (initial value of *H*)**U* = *U**(final value of *H*), where *U* is a unitary matrix. The final value of *H* is upper Hessenberg and triangular in rows and columns *info*+1 through *ihi*.
If *info* > 0, and *compz* = 'V', then on exit (final value of *Z*) = (initial value of *Z*)**U*, where *U* is the unitary matrix (regardless of the value of *job*).
If *info* > 0, and *compz* = 'I', then on exit (final value of *Z*) = *U*, where *U* is the unitary matrix (regardless of the value of *job*).
If *info* > 0, and *compz* = 'N', then *Z* is not accessed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `hseqr` interface are the following:

h	Holds the matrix H of size (n,n).
wr	Holds the vector of length n. Used in real flavors only.
wi	Holds the vector of length n. Used in real flavors only.
w	Holds the vector of length n. Used in complex flavors only.
z	Holds the matrix Z of size (n,n).
job	Must be `'E'` or `'S'`. The default value is `'E'`.
compz	If omitted, this argument is restored based on the presence of argument *z* as follows: *compz* = `'I'`, if *z* is present, *compz* = `'N'`, if *z* is omitted. If present, *compz* must be equal to `'I'` or `'V'` and the argument *z* must also be present. Note that there will be an error condition if *compz* is present and *z* omitted.

Application Notes

The computed Schur factorization is the exact factorization of a nearby matrix $H + E$, where $||E||_2 < O(\varepsilon)||H||_2/s_i$, and ε is the machine precision.

If λ_i is an exact eigenvalue, and μ_i is the corresponding computed value, then $|\lambda_i - \mu_i| \leq c(n)*\varepsilon*||H||_2/s_i$, where $c(n)$ is a modestly increasing function of n, and s_i is the reciprocal condition number of λ_i. The condition numbers s_i may be computed by calling trsna.

The total number of floating-point operations depends on how rapidly the algorithm converges; typical numbers are as follows.

If only eigenvalues are computed:	$7n^3$ for real flavors $25n^3$ for complex flavors.
If the Schur form is computed:	$10n^3$ for real flavors $35n^3$ for complex flavors.
If the full Schur factorization is computed:	$20n^3$ for real flavors $70n^3$ for complex flavors.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hsein

Computes selected eigenvectors of an upper Hessenberg matrix that correspond to specified eigenvalues.

Syntax

Fortran 77:

```
call shsein(job, eigsrc, initv, select, n, h, ldh, wr, wi, vl, ldvl, vr, ldvr, mm, m,
work, ifaill, ifailr, info)
```

```
call dhsein(job, eigsrc, initv, select, n, h, ldh, wr, wi, vl, ldvl, vr, ldvr, mm, m,
work, ifaill, ifailr, info)
```

```
call chsein(job, eigsrc, initv, select, n, h, ldh, w, vl, ldvl, vr, ldvr, mm, m,
work, rwork, ifaill, ifailr, info)
```

```
call zhsein(job, eigsrc, initv, select, n, h, ldh, w, vl, ldvl, vr, ldvr, mm, m,
work, rwork, ifaill, ifailr, info)
```

Fortran 95:

```
call hsein(h, wr, wi, select [, vl] [,vr] [,ifaill] [,ifailr] [,initv] [,eigsrc] [,m]
[,info])
```

```
call hsein(h, w, select [,vl] [,vr] [,ifaill] [,ifailr] [,initv] [,eigsrc] [,m] [,info])
```

C:

```
lapack_int LAPACKE_shsein( int matrix_order, char job, char eigsrc, char initv,
lapack_logical* select, lapack_int n, const float* h, lapack_int ldh, float* wr, const
float* wi, float* vl, lapack_int ldvl, float* vr, lapack_int ldvr, lapack_int mm,
lapack_int* m, lapack_int* ifaill, lapack_int* ifailr );
```

```
lapack_int LAPACKE_dhsein( int matrix_order, char job, char eigsrc, char initv,
lapack_logical* select, lapack_int n, const double* h, lapack_int ldh, double* wr,
const double* wi, double* vl, lapack_int ldvl, double* vr, lapack_int ldvr, lapack_int
mm, lapack_int* m, lapack_int* ifaill, lapack_int* ifailr );
```

```
lapack_int LAPACKE_chsein( int matrix_order, char job, char eigsrc, char initv, const
lapack_logical* select, lapack_int n, const lapack_complex_float* h, lapack_int ldh,
lapack_complex_float* w, lapack_complex_float* vl, lapack_int ldvl,
lapack_complex_float* vr, lapack_int ldvr, lapack_int mm, lapack_int* m, lapack_int*
ifaill, lapack_int* ifailr );
```

```
lapack_int LAPACKE_zhsein( int matrix_order, char job, char eigsrc, char initv, const
lapack_logical* select, lapack_int n, const lapack_complex_double* h, lapack_int ldh,
lapack_complex_double* w, lapack_complex_double* vl, lapack_int ldvl,
lapack_complex_double* vr, lapack_int ldvr, lapack_int mm, lapack_int* m, lapack_int*
ifaill, lapack_int* ifailr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes left and/or right eigenvectors of an upper Hessenberg matrix H, corresponding to selected eigenvalues.

The right eigenvector x and the left eigenvector y, corresponding to an eigenvalue λ, are defined by: $H*x = \lambda*x$ and $y^H*H = \lambda*y^H$ (or $H^H*y = \lambda^**y$). Here λ^* denotes the conjugate of λ.

The eigenvectors are computed by inverse iteration. They are scaled so that, for a real eigenvector x, $\max|x_i| = 1$, and for a complex eigenvector, $\max(|\mathrm{Re}x_i| + |\mathrm{Im}x_i|) = 1$.

If H has been formed by reduction of a general matrix A to upper Hessenberg form, then eigenvectors of H may be transformed to eigenvectors of A by ormhr or unmhr.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job
CHARACTER*1. Must be 'R' or 'L' or 'B'.
If *job* = 'R', then only right eigenvectors are computed.
If *job* = 'L', then only left eigenvectors are computed.
If *job* = 'B', then all eigenvectors are computed.

eigsrc
CHARACTER*1. Must be 'Q' or 'N'.
If *eigsrc* = 'Q', then the eigenvalues of H were found using hseqr; thus if H has any zero sub-diagonal elements (and so is block triangular), then the j-th eigenvalue can be assumed to be an eigenvalue of the block containing the j-th row/column. This property allows the routine to perform inverse iteration on just one diagonal block. If *eigsrc* = 'N', then no such assumption is made and the routine performs inverse iteration using the whole matrix.

initv
CHARACTER*1. Must be 'N' or 'U'.
If *initv* = 'N', then no initial estimates for the selected eigenvectors are supplied.
If *initv* = 'U', then initial estimates for the selected eigenvectors are supplied in *vl* and/or *vr*.

select
LOGICAL.
Array, DIMENSION at least max (1, n). Specifies which eigenvectors are to be computed.
For real flavors:
To obtain the real eigenvector corresponding to the real eigenvalue *wr*(j), set *select*(j) to .TRUE.
To select the complex eigenvector corresponding to the complex eigenvalue (*wr*(j), *wi*(j)) with complex conjugate (*wr*(j+1), *wi*(j+1)), set *select*(j) and/or *select*(j+1) to .TRUE.; the eigenvector corresponding to the first eigenvalue in the pair is computed.
For complex flavors:
To select the eigenvector corresponding to the eigenvalue *w*(j), set *select*(j) to .TRUE.

n
INTEGER. The order of the matrix H ($n \geq 0$).

h, *vl*, *vr*,
REAL for shsein
DOUBLE PRECISION for dhsein
COMPLEX for chsein
DOUBLE COMPLEX for zhsein.
Arrays:
h(*ldh*,*) The n-by-n upper Hessenberg matrix H.
The second dimension of *h* must be at least max(1, n).
(*ldvl*,*)

If *initv* = 'V' and *job* = 'L' or 'B', then *vl* must contain starting vectors for inverse iteration for the left eigenvectors. Each starting vector must be stored in the same column or columns as will be used to store the corresponding eigenvector.
If *initv* = 'N', then *vl* need not be set.
The second dimension of *vl* must be at least max(1, *mm*) if *job* = 'L' or 'B' and at least 1 if *job* = 'R'.
The array *vl* is not referenced if *job* = 'R'.

vr(*ldvr*,*)
If *initv* = 'V' and *job* = 'R' or 'B', then *vr* must contain starting vectors for inverse iteration for the right eigenvectors. Each starting vector must be stored in the same column or columns as will be used to store the corresponding eigenvector.
If *initv* = 'N', then *vr* need not be set.
The second dimension of *vr* must be at least max(1, *mm*) if *job* = 'R' or 'B' and at least 1 if *job* = 'L'.
The array *vr* is not referenced if *job* = 'L'.

work(*) is a workspace array.
DIMENSION at least max (1, *n**(*n+2*)) for real flavors and at least max (1, *n***n*) for complex flavors.

ldh — INTEGER. The leading dimension of *h*; at least max(1, *n*).

w — COMPLEX for chsein
DOUBLE COMPLEX for zhsein.
Array, DIMENSION at least max (1, *n*).
Contains the eigenvalues of the matrix *H*.
If *eigsrc* = 'Q', the array must be exactly as returned by ?hseqr.

wr, *wi* — REAL for shsein
DOUBLE PRECISION for dhsein
Arrays, DIMENSION at least max (1, *n*) each.
Contain the real and imaginary parts, respectively, of the eigenvalues of the matrix *H*. Complex conjugate pairs of values must be stored in consecutive elements of the arrays. If *eigsrc* = 'Q', the arrays must be exactly as returned by ?hseqr.

ldvl — INTEGER. The leading dimension of *vl*.
If *job* = 'L' or 'B', *ldvl* ≥ max(1,*n*).
If *job* = 'R', *ldvl* ≥ 1.

ldvr — INTEGER. The leading dimension of *vr*.
If *job* = 'R' or 'B', *ldvr* ≥ max(1,*n*).
If *job* = 'L', *ldvr* ≥1.

mm — INTEGER. The number of columns in *vl* and/or *vr*.
Must be at least *m*, the actual number of columns required (see *Output Parameters* below).
For real flavors, *m* is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector (see *select*).
For complex flavors, *m* is the number of selected eigenvectors (see *select*).
Constraint:
0 ≤ *mm* ≤ *n*.

rwork — REAL for chsein
DOUBLE PRECISION for zhsein.
Array, DIMENSION at least max (1, *n*).

Output Parameters

select	Overwritten for real flavors only. If a complex eigenvector was selected as specified above, then *select*(*j*) is set to .TRUE. and *select*(*j*+1) to .FALSE.
w	The real parts of some elements of *w* may be modified, as close eigenvalues are perturbed slightly in searching for independent eigenvectors.
wr	Some elements of *wr* may be modified, as close eigenvalues are perturbed slightly in searching for independent eigenvectors.
vl, *vr*	If *job* = 'L' or 'B', *vl* contains the computed left eigenvectors (as specified by *select*). If *job* = 'R' or 'B', *vr* contains the computed right eigenvectors (as specified by *select*). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues. *For real flavors*: a real eigenvector corresponding to a selected real eigenvalue occupies one column; a complex eigenvector corresponding to a selected complex eigenvalue occupies two columns: the first column holds the real part and the second column holds the imaginary part.
m	INTEGER. *For real flavors*: the number of columns of *vl* and/or *vr* required to store the selected eigenvectors. *For complex flavors*: the number of selected eigenvectors.
ifaill, *ifailr*	INTEGER. Arrays, DIMENSION at least max(1, *mm*) each. *ifaill*(*i*) = 0 if the *i*th column of *vl* converged; *ifaill*(*i*) = *j* > 0 if the eigenvector stored in the *i*-th column of *vl* (corresponding to the *j*th eigenvalue) failed to converge. *ifailr*(*i*) = 0 if the *i*th column of *vr* converged; *ifailr*(*i*) = *j* > 0 if the eigenvector stored in the *i*-th column of *vr* (corresponding to the *j*th eigenvalue) failed to converge. *For real flavors*: if the *i*th and (*i*+1)th columns of *vl* contain a selected complex eigenvector, then *ifaill*(*i*) and *ifaill*(*i*+1) are set to the same value. A similar rule holds for *vr* and *ifailr*. The array *ifaill* is not referenced if *job* = 'R'. The array *ifailr* is not referenced if *job* = 'L'.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* > 0, then *i* eigenvectors (as indicated by the parameters *ifaill* and/or *ifailr* above) failed to converge. The corresponding columns of *vl* and/or *vr* contain no useful information.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hsein interface are the following:

h	Holds the matrix *H* of size (*n*,*n*).
wr	Holds the vector of length *n*. Used in real flavors only.
wi	Holds the vector of length *n*. Used in real flavors only.

w	Holds the vector of length *n*. Used in complex flavors only.
select	Holds the vector of length *n*.
vl	Holds the matrix *VL* of size (*n*,*mm*).
vr	Holds the matrix *VR* of size (*n*,*mm*).
ifaill	Holds the vector of length (*mm*). Note that there will be an error condition if *ifaill* is present and *vl* is omitted.
ifailr	Holds the vector of length (*mm*). Note that there will be an error condition if *ifailr* is present and *vr* is omitted.
initv	Must be 'N' or 'U'. The default value is 'N'.
eigsrc	Must be 'N' or 'Q'. The default value is 'N'.
job	Restored based on the presence of arguments *vl* and *vr* as follows: *job* = 'B', if both *vl* and *vr* are present, *job* = 'L', if *vl* is present and *vr* omitted, *job* = 'R', if *vl* is omitted and *vr* present, Note that there will be an error condition if both *vl* and *vr* are omitted.

Application Notes

Each computed right eigenvector x_i is the exact eigenvector of a nearby matrix $A + E_i$, such that $||E_i|| < O(\varepsilon)||A||$. Hence the residual is small:

$$||Ax_i - \lambda_i x_i|| = O(\varepsilon)||A||.$$

However, eigenvectors corresponding to close or coincident eigenvalues may not accurately span the relevant subspaces.

Similar remarks apply to computed left eigenvectors.

?trevc

Computes selected eigenvectors of an upper (quasi-) triangular matrix computed by ?hseqr.

Syntax

Fortran 77:

```
call strevc(side, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, mm, m, work, info)
call dtrevc(side, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, mm, m, work, info)
call ctrevc(side, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, mm, m, work, rwork, info)
call ztrevc(side, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, mm, m, work, rwork, info)
```

Fortran 95:

```
call trevc(t [, howmny] [,select] [,vl] [,vr] [,m] [,info])
```

C:

```
lapack_int LAPACKE_strevc( int matrix_order, char side, char howmny, lapack_logical* select, lapack_int n, const float* t, lapack_int ldt, float* vl, lapack_int ldvl, float* vr, lapack_int ldvr, lapack_int mm, lapack_int* m );
lapack_int LAPACKE_dtrevc( int matrix_order, char side, char howmny, lapack_logical* select, lapack_int n, const double* t, lapack_int ldt, double* vl, lapack_int ldvl, double* vr, lapack_int ldvr, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ctrevc( int matrix_order, char side, char howmny, const
lapack_logical* select, lapack_int n, lapack_complex_float* t, lapack_int ldt,
lapack_complex_float* vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr,
lapack_int mm, lapack_int* m );

lapack_int LAPACKE_ztrevc( int matrix_order, char side, char howmny, const
lapack_logical* select, lapack_int n, lapack_complex_double* t, lapack_int ldt,
lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int ldvr,
lapack_int mm, lapack_int* m );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes some or all of the right and/or left eigenvectors of an upper triangular matrix T (or, for real flavors, an upper quasi-triangular matrix T). Matrices of this type are produced by the Schur factorization of a general matrix: $A = Q*T*Q^H$, as computed by hseqr.

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w, are defined by:

$T*x = w*x$, $y^H*T = w*y^H$, where y^H denotes the conjugate transpose of y.

The eigenvalues are not input to this routine, but are read directly from the diagonal blocks of T.

This routine returns the matrices X and/or Y of right and left eigenvectors of T, or the products $Q*X$ and/or $Q*Y$, where Q is an input matrix.

If Q is the orthogonal/unitary factor that reduces a matrix A to Schur form T, then $Q*X$ and $Q*Y$ are the matrices of right and left eigenvectors of A.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side
`CHARACTER*1`. Must be `'R'` or `'L'` or `'B'`.
If *side* = `'R'`, then only right eigenvectors are computed.
If *side* = `'L'`, then only left eigenvectors are computed.
If *side* = `'B'`, then all eigenvectors are computed.

howmny
`CHARACTER*1`. Must be `'A'` or `'B'` or `'S'`.
If *howmny* = `'A'`, then all eigenvectors (as specified by *side*) are computed.
If *howmny* = `'B'`, then all eigenvectors (as specified by *side*) are computed and backtransformed by the matrices supplied in *vl* and *vr*.
If *howmny* = `'S'`, then selected eigenvectors (as specified by *side* and *select*) are computed.

select
`LOGICAL`.
Array, `DIMENSION` at least max (1, *n*).
If *howmny* = `'S'`, *select* specifies which eigenvectors are to be computed.
If *howmny* = `'A'` or `'B'`, *select* is not referenced.
For real flavors:
If *omega*(*j*) is a real eigenvalue, the corresponding real eigenvector is computed if *select*(*j*) is `.TRUE.`.

If *omega*(*j*) and *omega*(*j*+1) are the real and imaginary parts of a complex eigenvalue, the corresponding complex eigenvector is computed if either *select*(*j*) or *select*(*j*+1) is .TRUE., and on exit *select*(*j*) is set to .TRUE.and *select*(*j*+1) is set to .FALSE..
For complex flavors:
The eigenvector corresponding to the *j*-th eigenvalue is computed if *select*(*j*) is .TRUE..

n — INTEGER. The order of the matrix *T* (*n* ≥ 0).

t, *vl*, *vr* — REAL for strevc
DOUBLE PRECISION for dtrevc
COMPLEX for ctrevc
DOUBLE COMPLEX for ztrevc.
Arrays:
t(*ldt*,*) contains the *n*-by-*n* matrix *T* in Schur canonical form. For complex flavors ctrevc and ztrevc, contains the upper triangular matrix *T*.
The second dimension of *t* must be at least max(1, *n*).
vl(*ldvl*,*)
If *howmny* = 'B' and *side* = 'L' or 'B', then *vl* must contain an *n*-by-*n* matrix *Q* (usually the matrix of Schur vectors returned by ?hseqr).
If *howmny* = 'A' or 'S', then *vl* need not be set.
The second dimension of *vl* must be at least max(1, *mm*) if *side* = 'L' or 'B' and at least 1 if *side* = 'R'.
The array *vl* is not referenced if *side* = 'R'.
vr (*ldvr*,*)
If *howmny* = 'B' and *side* = 'R' or 'B', then *vr* must contain an *n*-by-*n* matrix *Q* (usually the matrix of Schur vectors returned by ?hseqr). .
If *howmny* = 'A' or 'S', then *vr* need not be set.
The second dimension of *vr* must be at least max(1, *mm*) if *side* = 'R' or 'B' and at least 1 if *side* = 'L'.
The array *vr* is not referenced if *side* = 'L'.
work(*) is a workspace array.
DIMENSION at least max (1, 3**n*) for real flavors and at least max (1, 2**n*) for complex flavors.

ldt — INTEGER. The leading dimension of *t*; at least max(1, *n*).

ldvl — INTEGER. The leading dimension of *vl*.
If *side* = 'L' or 'B', *ldvl* ≥ *n*.
If *side* = 'R', *ldvl* ≥ 1.

ldvr — INTEGER. The leading dimension of *vr*.
If *side* = 'R' or 'B', *ldvr* ≥ *n*.
If *side* = 'L', *ldvr* ≥ 1.

mm — INTEGER. The number of columns in the arrays *vl* and/or *vr*. Must be at least *m* (the precise number of columns required).
If *howmny* = 'A' or 'B', *m* = *n*.
If *howmny* = 'S': *for real flavors*, *m* is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector; *for complex flavors*, *m* is the number of selected eigenvectors (see *select*).
Constraint: 0 ≤ *m* ≤ *n*.

rwork — REAL for ctrevc
DOUBLE PRECISION for ztrevc.
Workspace array, DIMENSION at least max (1, *n*).

Output Parameters

select	If a complex eigenvector of a real matrix was selected as specified above, then *select*(*j*) is set to .TRUE. and *select*(*j*+1) to .FALSE.
t	COMPLEX for ctrevc DOUBLE COMPLEX for ztrevc. ctrevc/ztrevc modify the *t*(*ldt*,*) array, which is restored on exit.
vl, *vr*	If *side* = 'L' or 'B', *vl* contains the computed left eigenvectors (as specified by *howmny* and *select*). If *side* = 'R' or 'B', *vr* contains the computed right eigenvectors (as specified by *howmny* and *select*). The eigenvectors are stored consecutively in the columns of the array, in the same order as their eigenvalues. *For real flavors*: corresponding to each real eigenvalue is a real eigenvector, occupying one column;corresponding to each complex conjugate pair of eigenvalues is a complex eigenvector, occupying two columns; the first column holds the real part and the second column holds the imaginary part.
m	INTEGER. *For complex flavors*: the number of selected eigenvectors. If *howmny* = 'A' or 'B', *m* is set to *n*. *For real flavors*: the number of columns of *vl* and/or *vr* actually used to store the selected eigenvectors. If *howmny* = 'A' or 'B', *m* is set to *n*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trevc interface are the following:

t	Holds the matrix *T* of size (*n*,*n*).
select	Holds the vector of length *n*.
vl	Holds the matrix *VL* of size (*n*,*mm*).
vr	Holds the matrix *VR* of size (*n*,*mm*).
side	If omitted, this argument is restored based on the presence of arguments *vl* and *vr* as follows: *side* = 'B', if both *vl* and *vr* are present, *side* = 'L', if *vr* is omitted, *side* = 'R', if *vl* is omitted. Note that there will be an error condition if both *vl* and *vr* are omitted.
howmny	If omitted, this argument is restored based on the presence of argument *select* as follows: *howmny* = 'V', if *q* is present, *howmny* = 'N', if *q* is omitted. If present, *vect* = 'V' or 'U' and the argument *q* must also be present. Note that there will be an error condition if both *select* and *howmny* are present.

Application Notes

If x i is an exact right eigenvector and y_i is the corresponding computed eigenvector, then the angle $\theta(y_i, x_i)$ between them is bounded as follows: $\theta(y_i, x_i) \leq (c(n)\varepsilon||T||_2)/\text{sep}_i$ where sep_i is the reciprocal condition number of x_i. The condition number sep_i may be computed by calling `?trsna`.

?trsna

Estimates condition numbers for specified eigenvalues and right eigenvectors of an upper (quasi-) triangular matrix.

Syntax

Fortran 77:

```
call strsna(job, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, s, sep, mm, m, work, ldwork, iwork, info)
```

```
call dtrsna(job, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, s, sep, mm, m, work, ldwork, iwork, info)
```

```
call ctrsna(job, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, s, sep, mm, m, work, ldwork, rwork, info)
```

```
call ztrsna(job, howmny, select, n, t, ldt, vl, ldvl, vr, ldvr, s, sep, mm, m, work, ldwork, rwork, info)
```

Fortran 95:

```
call trsna(t [, s] [,sep] [,vl] [,vr] [,select] [,m] [,info])
```

C:

```
lapack_int LAPACKE_strsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const float* t, lapack_int ldt, const float* vl, lapack_int ldvl, const float* vr, lapack_int ldvr, float* s, float* sep, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_dtrsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const double* t, lapack_int ldt, const double* vl, lapack_int ldvl, const double* vr, lapack_int ldvr, double* s, double* sep, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ctrsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const lapack_complex_float* t, lapack_int ldt, const lapack_complex_float* vl, lapack_int ldvl, const lapack_complex_float* vr, lapack_int ldvr, float* s, float* sep, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ztrsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const lapack_complex_double* t, lapack_int ldt, const lapack_complex_double* vl, lapack_int ldvl, const lapack_complex_double* vr, lapack_int ldvr, double* s, double* sep, lapack_int mm, lapack_int* m );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine estimates condition numbers for specified eigenvalues and/or right eigenvectors of an upper triangular matrix T (or, for real flavors, upper quasi-triangular matrix T in canonical Schur form). These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix $A = Z*T*Z^H$ (with unitary or, for real flavors, orthogonal Z), from which T may have been derived.

The routine computes the reciprocal of the condition number of an eigenvalue *lambda*(i) as $s_i = |v^T*u| / (||u||_E||v||_E)$ for real flavors and $s_i = |v^H*u| / (||u||_E||v||_E)$ for complex flavors,

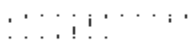
where:

- u and v are the right and left eigenvectors of T, respectively, corresponding to *lambda*(i).
- v^T/v^H denote transpose/conjugate transpose of v, respectively.

This reciprocal condition number always lies between zero (ill-conditioned) and one (well-conditioned).

An approximate error estimate for a computed eigenvalue *lambda*(i) is then given by $\varepsilon*||T||/s_i$, where ε is the *machine precision*.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to *lambda*(i), the routine first calls trexc to reorder the eigenvalues so that *lambda*(i) is in the leading position:



The reciprocal condition number of the eigenvector is then estimated as sep_i, the smallest singular value of the matrix T_{22} - *lambda*(i)*I. This number ranges from zero (ill-conditioned) to very large (well-conditioned).

An approximate error estimate for a computed right eigenvector u corresponding to *lambda*(i) is then given by $\varepsilon*||T||/sep_i$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job
CHARACTER*1. Must be 'E' or 'V' or 'B'.
If *job* = 'E', then condition numbers for eigenvalues only are computed.
If *job* = 'V', then condition numbers for eigenvectors only are computed.
If *job* = 'B', then condition numbers for both eigenvalues and eigenvectors are computed.

howmny
CHARACTER*1. Must be 'A' or 'S'.
If *howmny* = 'A', then the condition numbers for all eigenpairs are computed.
If *howmny* = 'S', then condition numbers for selected eigenpairs (as specified by *select*) are computed.

select
LOGICAL.
Array, DIMENSION at least max (1, *n*) if *howmny* = 'S' and at least 1 otherwise.
Specifies the eigenpairs for which condition numbers are to be computed if *howmny*= 'S'.
For real flavors:
To select condition numbers for the eigenpair corresponding to the real eigenvalue *lambda*(*j*), *select*(*j*) must be set .TRUE.;

	to select condition numbers for the eigenpair corresponding to a complex conjugate pair of eigenvalues *lambda*(*j*) and *lambda*(*j*+1), *select*(*j*) and/or *select*(*j*+1) must be set .TRUE. *For complex flavors* To select condition numbers for the eigenpair corresponding to the eigenvalue *lambda*(*j*), *select*(*j*) must be set .TRUE. *select* is not referenced if *howmny* = 'A'.
n	INTEGER. The order of the matrix *T* (*n* ≥ 0).
t, *vl*, *vr*, *work*	REAL for strsna DOUBLE PRECISION for dtrsna COMPLEX for ctrsna DOUBLE COMPLEX for ztrsna. Arrays: *t*(*ldt*,*) contains the *n*-by-*n* matrix *T*. The second dimension of *t* must be at least max(1, *n*). *vl*(*ldvl*,*) If *job* = 'E' or 'B', then *vl* must contain the left eigenvectors of *T* (or of any matrix $Q*T*Q^H$ with *Q* unitary or orthogonal) corresponding to the eigenpairs specified by *howmny* and *select*. The eigenvectors must be stored in consecutive columns of *vl*, as returned by trevc or hsein. The second dimension of *vl* must be at least max(1, *mm*) if *job* = 'E' or 'B' and at least 1 if *job* = 'V'. The array *vl* is not referenced if *job* = 'V'. *vr*(*ldvr*,*) If *job* = 'E' or 'B', then *vr* must contain the right eigenvectors of *T* (or of any matrix $Q*T*Q^H$ with *Q* unitary or orthogonal) corresponding to the eigenpairs specified by *howmny* and *select*. The eigenvectors must be stored in consecutive columns of *vr*, as returned by trevc or hsein. The second dimension of *vr* must be at least max(1, *mm*) if *job* = 'E' or 'B' and at least 1 if *job* = 'V'. The array *vr* is not referenced if *job* = 'V'. *work* is a workspace array, its dimension (*ldwork*, *n*+6). The array *work* is not referenced if *job* = 'E'.
ldt	INTEGER. The leading dimension of *t*; at least max(1, *n*).
ldvl	INTEGER. The leading dimension of *vl*. If *job* = 'E' or 'B', *ldvl* ≥ max(1,*n*). If *job* = 'V', *ldvl* ≥ 1.
ldvr	INTEGER. The leading dimension of *vr*. If *job* = 'E' or 'B', *ldvr* ≥ max(1,*n*). If *job* = 'R', *ldvr* ≥ 1.
mm	INTEGER. The number of elements in the arrays *s* and *sep*, and the number of columns in *vl* and *vr* (if used). Must be at least *m* (the precise number required). If *howmny* = 'A', *m* = *n*; if *howmny* = 'S', *for real flavors* *m* is obtained by counting 1 for each selected real eigenvalue and 2 for each selected complex conjugate pair of eigenvalues. *for complex flavors* *m* is the number of selected eigenpairs (see *select*). Constraint: 0 ≤ *m* ≤ *n*.
ldwork	INTEGER. The leading dimension of *work*.

If *job* = 'V' or 'B', *ldwork* ≥ max(1,*n*).
If *job* = 'E', *ldwork* ≥ 1.

rwork — REAL for ctrsna, ztrsna.
Array, DIMENSION at least max (1, *n*). The array is not referenced if *job* = 'E'.

iwork — INTEGER for strsna, dtrsna.
Array, DIMENSION at least max (1, 2*(*n* - 1)). The array is not referenced if *job* = 'E'.

Output Parameters

s — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Array, DIMENSION at least max(1, *mm*) if *job* = 'E' or 'B' and at least 1 if *job* = 'V'.
Contains the reciprocal condition numbers of the selected eigenvalues if *job* = 'E' or 'B', stored in consecutive elements of the array. Thus *s*(*j*), *sep*(*j*) and the *j*-th columns of *vl* and *vr* all correspond to the same eigenpair (but not in general the *j* th eigenpair unless all eigenpairs have been selected).
For real flavors: for a complex conjugate pair of eigenvalues, two consecutive elements of S are set to the same value. The array *s* is not referenced if *job* = 'V'.

sep — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Array, DIMENSION at least max(1, *mm*) if *job* = 'V' or 'B' and at least 1 if *job* = 'E'. Contains the estimated reciprocal condition numbers of the selected right eigenvectors if *job* = 'V' or 'B', stored in consecutive elements of the array.
For real flavors: for a complex eigenvector, two consecutive elements of *sep* are set to the same value; if the eigenvalues cannot be reordered to compute *sep*(*j*), then *sep*(*j*) is set to zero; this can only occur when the true value would be very small anyway. The array *sep* is not referenced if *job* = 'E'.

m — INTEGER.
For complex flavors: the number of selected eigenpairs.
If *howmny* = 'A', *m* is set to *n*.
For real flavors: the number of elements of *s* and/or *sep* actually used to store the estimated condition numbers.
If *howmny* = 'A', *m* is set to *n*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trsna interface are the following:

t — Holds the matrix *T* of size (*n*,*n*).

s — Holds the vector of length (*mm*).

sep	Holds the vector of length (*mm*).
vl	Holds the matrix *VL* of size (*n*,*mm*).
vr	Holds the matrix *VR* of size (*n*,*mm*).
select	Holds the vector of length *n*.
job	Restored based on the presence of arguments *s* and sep as follows: *job* = 'B', if both *s* and *sep* are present, *job* = 'E', if *s* is present and *sep* omitted, *job* = 'V', if *s* is omitted and *sep* present. Note an error condition if both *s* and *sep* are omitted.
howmny	Restored based on the presence of the argument *select* as follows: *howmny* = 'S', if *select* is present, *howmny* = 'A', if *select* is omitted.

Note that the arguments *s*, *vl*, and *vr* must either be all present or all omitted.

Otherwise, an error condition is observed.

Application Notes

The computed values sep_i may overestimate the true value, but seldom by a factor of more than 3.

?trexc

Reorders the Schur factorization of a general matrix.

Syntax

Fortran 77:

```
call strexc(compq, n, t, ldt, q, ldq, ifst, ilst, work, info)
call dtrexc(compq, n, t, ldt, q, ldq, ifst, ilst, work, info)
call ctrexc(compq, n, t, ldt, q, ldq, ifst, ilst, info)
call ztrexc(compq, n, t, ldt, q, ldq, ifst, ilst, info)
```

Fortran 95:

```
call trexc(t, ifst, ilst [,q] [,info])
```

C:

```
lapack_int LAPACKE_strexc( int matrix_order, char compq, lapack_int n, float* t,
lapack_int ldt, float* q, lapack_int ldq, lapack_int* ifst, lapack_int* ilst );
lapack_int LAPACKE_dtrexc( int matrix_order, char compq, lapack_int n, double* t,
lapack_int ldt, double* q, lapack_int ldq, lapack_int* ifst, lapack_int* ilst );
lapack_int LAPACKE_ctrexc( int matrix_order, char compq, lapack_int n,
lapack_complex_float* t, lapack_int ldt, lapack_complex_float* q, lapack_int ldq,
lapack_int ifst, lapack_int ilst );
lapack_int LAPACKE_ztrexc( int matrix_order, char compq, lapack_int n,
lapack_complex_double* t, lapack_int ldt, lapack_complex_double* q, lapack_int ldq,
lapack_int ifst, lapack_int ilst );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reorders the Schur factorization of a general matrix $A = Q*T*Q^H$, so that the diagonal element or block of T with row index $ifst$ is moved to row $ilst$.

The reordered Schur form S is computed by an unitary (or, for real flavors, orthogonal) similarity transformation: $S = Z^H*T*Z$. Optionally the updated matrix P of Schur vectors is computed as $P = Q*Z$, giving $A = P*S*P^H$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

compq	CHARACTER*1. Must be 'V' or 'N'. If *compq* = 'V', then the Schur vectors (Q) are updated. If *compq* = 'N', then no Schur vectors are updated.
n	INTEGER. The order of the matrix T ($n \geq 0$).
t, *q*	REAL for strexc DOUBLE PRECISION for dtrexc COMPLEX for ctrexc DOUBLE COMPLEX for ztrexc. Arrays: *t*(*ldt*,*) contains the n-by-n matrix T. The second dimension of *t* must be at least max(1, n). *q*(*ldq*,*) If *compq* = 'V', then *q* must contain Q (Schur vectors). If *compq* = 'N', then *q* is not referenced. The second dimension of *q* must be at least max(1, n) if *compq* = 'V' and at least 1 if *compq* = 'N'.
ldt	INTEGER. The leading dimension of *t*; at least max(1, n).
ldq	INTEGER. The leading dimension of *q*; If *compq* = 'N', then *ldq*$\geq$ 1. If *compq* = 'V', then *ldq*$\geq$ max(1, n).
ifst, *ilst*	INTEGER. $1 \leq ifst \leq n$; $1 \leq ilst \leq n$. Must specify the reordering of the diagonal elements (or blocks, which is possible for real flavors) of the matrix T. The element (or block) with row index *ifst* is moved to row *ilst* by a sequence of exchanges between adjacent elements (or blocks).
work	REAL for strexc DOUBLE PRECISION for dtrexc. Array, DIMENSION at least max (1, n).

Output Parameters

t	Overwritten by the updated matrix S.
q	If *compq* = 'V', *q* contains the updated matrix of Schur vectors.
ifst, *ilst*	Overwritten for real flavors only. If *ifst* pointed to the second row of a 2 by 2 block on entry, it is changed to point to the first row; *ilst* always points to the first row of the block in its final position (which may differ from its input value by ±1).
info	INTEGER. If *info* = 0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `trexc` interface are the following:

t	Holds the matrix *T* of size (*n*,*n*).
q	Holds the matrix *Q* of size (*n*,*n*).
compq	Restored based on the presence of the argument *q* as follows: *compq* = 'V', if *q* is present, *compq* = 'N', if *q* is omitted.

Application Notes

The computed matrix *S* is exactly similar to a matrix *T*+*E*, where $||E||_2 = O(\varepsilon)*||T||_2$, and ε is the machine precision.

Note that if a 2 by 2 diagonal block is involved in the re-ordering, its off-diagonal elements are in general changed; the diagonal elements and the eigenvalues of the block are unchanged unless the block is sufficiently ill-conditioned, in which case they may be noticeably altered. It is possible for a 2 by 2 block to break into two 1 by 1 blocks, that is, for a pair of complex eigenvalues to become purely real.

The values of eigenvalues however are never changed by the re-ordering.

The approximate number of floating-point operations is

for real flavors:	$6n(ifst-ilst)$ if *compq* = 'N'; $12n(ifst-ilst)$ if *compq* = 'V';
for complex flavors:	$20n(ifst-ilst)$ if *compq* = 'N'; $40n(ifst-ilst)$ if *compq* = 'V'.

?trsen

Reorders the Schur factorization of a matrix and (optionally) computes the reciprocal condition numbers and invariant subspace for the selected cluster of eigenvalues.

Syntax

Fortran 77:

```
call strsen(job, compq, select, n, t, ldt, q, ldq, wr, wi, m, s, sep, work, lwork, iwork, liwork, info)
call dtrsen(job, compq, select, n, t, ldt, q, ldq, wr, wi, m, s, sep, work, lwork, iwork, liwork, info)
call ctrsen(job, compq, select, n, t, ldt, q, ldq, w, m, s, sep, work, lwork, info)
call ztrsen(job, compq, select, n, t, ldt, q, ldq, w, m, s, sep, work, lwork, info)
```

Fortran 95:

```
call trsen(t, select [,wr] [,wi] [,m] [,s] [,sep] [,q] [,info])
call trsen(t, select [,w] [,m] [,s] [,sep] [,q] [,info])
```

C:

```
lapack_int LAPACKE_strsen( int matrix_order, char job, char compq, const
lapack_logical* select, lapack_int n, float* t, lapack_int ldt, float* q, lapack_int
ldq, float* wr, float* wi, lapack_int* m, float* s, float* sep );

lapack_int LAPACKE_dtrsen( int matrix_order, char job, char compq, const
lapack_logical* select, lapack_int n, double* t, lapack_int ldt, double* q, lapack_int
ldq, double* wr, double* wi, lapack_int* m, double* s, double* sep );

lapack_int LAPACKE_ctrsen( int matrix_order, char job, char compq, const
lapack_logical* select, lapack_int n, lapack_complex_float* t, lapack_int ldt,
lapack_complex_float* q, lapack_int ldq, lapack_complex_float* w, lapack_int* m, float*
s, float* sep );

lapack_int LAPACKE_ztrsen( int matrix_order, char job, char compq, const
lapack_logical* select, lapack_int n, lapack_complex_double* t, lapack_int ldt,
lapack_complex_double* q, lapack_int ldq, lapack_complex_double* w, lapack_int* m,
double* s, double* sep );
```

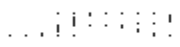
Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reorders the Schur factorization of a general matrix $A = Q*T*Q^T$ (for real flavors) or $A = Q*T*Q^H$ (for complex flavors) so that a selected cluster of eigenvalues appears in the leading diagonal elements (or, for real flavors, diagonal blocks) of the Schur form. The reordered Schur form R is computed by a unitary (orthogonal) similarity transformation: $R = Z^H*T*Z$. Optionally the updated matrix P of Schur vectors is computed as $P = Q*Z$, giving $A = P*R*P^H$.

Let



where the selected eigenvalues are precisely the eigenvalues of the leading m-by-m submatrix T_{11}. Let P be correspondingly partitioned as $(Q_1\ Q_2)$ where Q_1 consists of the first m columns of Q. Then $A*Q_1 = Q_1*T_{11}$, and so the m columns of Q_1 form an orthonormal basis for the invariant subspace corresponding to the selected cluster of eigenvalues.

Optionally the routine also computes estimates of the reciprocal condition numbers of the average of the cluster of eigenvalues and of the invariant subspace.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job — `CHARACTER*1`. Must be `'N'` or `'E'` or `'V'` or `'B'`.
If *job* = `'N'`, then no condition numbers are required.

	If *job* = 'E', then only the condition number for the cluster of eigenvalues is computed. If *job* = 'V', then only the condition number for the invariant subspace is computed. If *job* = 'B', then condition numbers for both the cluster and the invariant subspace are computed.
compq	CHARACTER*1. Must be 'V' or 'N'. If *compq* = 'V', then *Q* of the Schur vectors is updated. If *compq* = 'N', then no Schur vectors are updated.
select	LOGICAL. Array, DIMENSION at least max (1, *n*). Specifies the eigenvalues in the selected cluster. To select an eigenvalue *lambda*(*j*), *select*(*j*) must be .TRUE. *For real flavors*: to select a complex conjugate pair of eigenvalues *lambda*(*j*) and *lambda*(*j*+1) (corresponding 2 by 2 diagonal block), *select*(*j*) and/or *select*(*j*+1) must be .TRUE.; the complex conjugate *lambda*(*j*)and *lambda*(*j*+1) must be either both included in the cluster or both excluded.
n	INTEGER. The order of the matrix *T* (*n* ≥ 0).
t, *q*, *work*	REAL for strsen DOUBLE PRECISION for dtrsen COMPLEX for ctrsen DOUBLE COMPLEX for ztrsen. Arrays: *t* (*ldt*,*) The *n*-by-*n* *T*. The second dimension of *t* must be at least max(1, *n*). *q* (*ldq*,*) If *compq* = 'V', then *q* must contain *Q* of Schur vectors. If *compq* = 'N', then *q* is not referenced. The second dimension of *q* must be at least max(1, *n*) if *compq* = 'V' and at least 1 if *compq* = 'N'. *work* is a workspace array, its dimension max(1, *lwork*).
ldt	INTEGER. The leading dimension of *t*; at least max(1, *n*).
ldq	INTEGER. The leading dimension of *q*; If *compq* = 'N', then *ldq* ≥ 1. If *compq* = 'V', then *ldq* ≥ max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. If *job* = 'V' or 'B', *lwork* ≥ max(1,2*m*(*n*-*m*)). If *job* = 'E', then *lwork* ≥ max(1, *m*(*n*-*m*)) If *job* = 'N', then *lwork* ≥ 1 for complex flavors and *lwork* ≥ max(1,*n*) for real flavors. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.
iwork	INTEGER.*iwork*(*liwork*) is a workspace array. The array *iwork* is not referenced if *job* = 'N' or 'E'. The actual amount of workspace required cannot exceed $n^2/2$ if *job* = 'V' or 'B'.
liwork	INTEGER. The dimension of the array *iwork*.

If *job* = 'V' or 'B', *liwork* ≥ max(1,2*m*(*n*-*m*)).
If *job* = 'E' or 'E', *liwork* ≥ 1.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *iwork* array, returns this value as the first entry of the *iwork* array, and no error message related to *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

t — Overwritten by the updated matrix *R*.

q — If *compq* = 'V', *q* contains the updated matrix of Schur vectors; the first m columns of the *Q* form an orthogonal basis for the specified invariant subspace.

w — COMPLEX for ctrsen
DOUBLE COMPLEX for ztrsen.
Array, DIMENSION at least max(1, *n*). The recorded eigenvalues of *R*. The eigenvalues are stored in the same order as on the diagonal of *R*.

wr, *wi* — REAL for strsen
DOUBLE PRECISION for dtrsen
Arrays, DIMENSION at least max(1, *n*). Contain the real and imaginary parts, respectively, of the reordered eigenvalues of *R*. The eigenvalues are stored in the same order as on the diagonal of *R*. Note that if a complex eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

m — INTEGER.
For complex flavors: the number of the specified invariant subspaces, which is the same as the number of selected eigenvalues (see *select*).
For real flavors: the dimension of the specified invariant subspace. The value of *m* is obtained by counting 1 for each selected real eigenvalue and 2 for each selected complex conjugate pair of eigenvalues (see *select*).
Constraint: 0 ≤ *m* ≤ *n*.

s — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
If *job* = 'E' or 'B', *s* is a lower bound on the reciprocal condition number of the average of the selected cluster of eigenvalues.
If *m* = 0 or *n*, then *s* = 1.
For real flavors: if *info* = 1, then *s* is set to zero.*s* is not referenced if *job* = 'N' or 'V'.

sep — REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors.
If *job* = 'V' or 'B', *sep* is the estimated reciprocal condition number of the specified invariant subspace.
If *m* = 0 or *n*, then *sep* = |*T*|.
For real flavors: if *info* = 1, then *sep* is set to zero.
sep is not referenced if *job* = 'N' or 'E'.

work(1) — On exit, if *info* = 0, then *work*(1) returns the optimal size of *lwork*.

iwork(1) — On exit, if *info* = 0, then *iwork*(1) returns the optimal size of *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `trsen` interface are the following:

t	Holds the matrix *T* of size (*n*,*n*).
select	Holds the vector of length *n*.
wr	Holds the vector of length *n*. Used in real flavors only.
wi	Holds the vector of length *n*. Used in real flavors only.
w	Holds the vector of length *n*. Used in complex flavors only.
q	Holds the matrix *Q* of size (*n*,*n*).
compq	Restored based on the presence of the argument *q* as follows: *compq* = 'V', if *q* is present, *compq* = 'N', if *q* is omitted.
job	Restored based on the presence of arguments *s* and *sep* as follows: *job* = 'B', if both *s* and *sep* are present, *job* = 'E', if *s* is present and *sep* omitted, *job* = 'V', if *s* is omitted and *sep* present, *job* = 'N', if both *s* and *sep* are omitted.

Application Notes

The computed matrix *R* is exactly similar to a matrix $T+E$, where $||E||_2 = O(\varepsilon)*||T||_2$, and ε is the machine precision. The computed *s* cannot underestimate the true reciprocal condition number by more than a factor of $(\min(m, n-m))_{1/2}$; *sep* may differ from the true value by $(m*n-m^2)_{1/2}$. The angle between the computed invariant subspace and the true subspace is $O(\varepsilon)*||A||_2/sep$. Note that if a 2-by-2 diagonal block is involved in the re-ordering, its off-diagonal elements are in general changed; the diagonal elements and the eigenvalues of the block are unchanged unless the block is sufficiently ill-conditioned, in which case they may be noticeably altered. It is possible for a 2-by-2 block to break into two 1-by-1 blocks, that is, for a pair of complex eigenvalues to become purely real. The values of eigenvalues however are never changed by the re-ordering.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?trsyl

Solves Sylvester equation for real quasi-triangular or complex triangular matrices.

Syntax

Fortran 77:

```
call strsyl(trana, tranb, isgn, m, n, a, lda, b, ldb, c, ldc, scale, info)
```

```
call dtrsyl(trana, tranb, isgn, m, n, a, lda, b, ldb, c, ldc, scale, info)
call ctrsyl(trana, tranb, isgn, m, n, a, lda, b, ldb, c, ldc, scale, info)
call ztrsyl(trana, tranb, isgn, m, n, a, lda, b, ldb, c, ldc, scale, info)
```

Fortran 95:

```
call trsyl(a, b, c, scale [, trana] [,tranb] [,isgn] [,info])
```

C:

```
lapack_int LAPACKE_strsyl( int matrix_order, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const float* a, lapack_int lda, const float* b, lapack_int
ldb, float* c, lapack_int ldc, float* scale );
lapack_int LAPACKE_dtrsyl( int matrix_order, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const double* a, lapack_int lda, const double* b,
lapack_int ldb, double* c, lapack_int ldc, double* scale );
lapack_int LAPACKE_ctrsyl( int matrix_order, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* c, lapack_int ldc,
float* scale );
lapack_int LAPACKE_ztrsyl( int matrix_order, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* c, lapack_int ldc,
double* scale );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves the Sylvester matrix equation $\mathrm{op}(A)*X \pm X*\mathrm{op}(B) = \alpha*C$, where $\mathrm{op}(A) = A$ or A^H, and the matrices A and B are upper triangular (or, for real flavors, upper quasi-triangular in canonical Schur form); $\alpha \leq 1$ is a scale factor determined by the routine to avoid overflow in X; A is m-by-m, B is n-by-n, and C and X are both m-by-n. The matrix X is obtained by a straightforward process of back substitution.

The equation has a unique solution if and only if $\alpha_i \pm \beta_i \neq 0$, where $\{\alpha_i\}$ and $\{\beta_i\}$ are the eigenvalues of A and B, respectively, and the sign (+ or -) is the same as that used in the equation to be solved.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trana	CHARACTER*1. Must be 'N' or 'T' or 'C'. If *trana* = 'N', then $\mathrm{op}(A) = A$. If *trana* = 'T', then $\mathrm{op}(A) = A^T$ (real flavors only). If *trana* = 'C' then $\mathrm{op}(A) = A^H$.
tranb	CHARACTER*1. Must be 'N' or 'T' or 'C'. If *tranb* = 'N', then $\mathrm{op}(B) = B$. If *tranb* = 'T', then $\mathrm{op}(B) = B^T$ (real flavors only). If *tranb* = 'C', then $\mathrm{op}(B) = B^H$.

isgn	INTEGER. Indicates the form of the Sylvester equation. If *isgn* = +1, op(*A*)**X* + *X**op(*B*) = alpha**C*. If *isgn* = -1, op(*A*)**X* - *X**op(*B*) = alpha**C*.
m	INTEGER. The order of *A*, and the number of rows in *X* and *C* ($m \geq 0$).
n	INTEGER. The order of *B*, and the number of columns in *X* and *C* ($n \geq 0$).
a, *b*, *c*	REAL for strsyl DOUBLE PRECISION for dtrsyl COMPLEX for ctrsyl DOUBLE COMPLEX for ztrsyl. Arrays: *a*(*lda*,*) contains the matrix *A*. The second dimension of *a* must be at least max(1, *m*). *b*(*ldb*,*) contains the matrix *B*. The second dimension of *b* must be at least max(1, *n*). *c*(*ldc*,*) contains the matrix *C*. The second dimension of *c* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
ldc	INTEGER. The leading dimension of *c*; at least max(1, *n*).

Output Parameters

c	Overwritten by the solution matrix *X*.
scale	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. The value of the scale factor α.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1, *A* and *B* have common or close eigenvalues perturbed values were used to solve the equation.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine trsyl interface are the following:

a	Holds the matrix *A* of size (*m*,*m*).
b	Holds the matrix *B* of size (*n*,*n*).
c	Holds the matrix *C* of size (*m*,*n*).
trana	Must be 'N', 'C', or 'T'. The default value is 'N'.
tranb	Must be 'N', 'C', or 'T'. The default value is 'N'.
isgn	Must be +1 or -1. The default value is +1.

Application Notes

Let *X* be the exact, *Y* the corresponding computed solution, and *R* the residual matrix: *R* = *C* - (*AY* ± *YB*). Then the residual is always small:

$||R||_F = O(\varepsilon) * (||A||_F + ||B||_F) * ||Y||_F$.

However, Y is not necessarily the exact solution of a slightly perturbed equation; in other words, the solution is not backwards stable.

For the forward error, the following bound holds:

$||Y - X||_F \leq ||R||_F / \mathrm{sep}(A,B)$

but this may be a considerable overestimate. See [Golub96] for a definition of sep(A, B).

The approximate number of floating-point operations for real flavors is $m*n*(m + n)$. For complex flavors it is 4 times greater.

Generalized Nonsymmetric Eigenvalue Problems

This section describes LAPACK routines for solving generalized nonsymmetric eigenvalue problems, reordering the generalized Schur factorization of a pair of matrices, as well as performing a number of related computational tasks.

A *generalized nonsymmetric eigenvalue problem* is as follows: given a pair of nonsymmetric (or non-Hermitian) n-by-n matrices A and B, find the *generalized eigenvalues* λ and the corresponding *generalized eigenvectors* x and y that satisfy the equations

$Ax = \lambda Bx$ (right generalized eigenvectors x)

and

$y^H A = \lambda y^H B$ (left generalized eigenvectors y).

Table "Computational Routines for Solving Generalized Nonsymmetric Eigenvalue Problems" lists LAPACK routines (FORTRAN 77 interface) used to solve the generalized nonsymmetric eigenvalue problems and the generalized Sylvester equation. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Solving Generalized Nonsymmetric Eigenvalue Problems

Routine name	Operation performed
gghrd	Reduces a pair of matrices to generalized upper Hessenberg form using orthogonal/ unitary transformations.
ggbal	Balances a pair of general real or complex matrices.
ggbak	Forms the right or left eigenvectors of a generalized eigenvalue problem.
hgeqz	Implements the QZ method for finding the generalized eigenvalues of the matrix pair (H,T).
tgevc	Computes some or all of the right and/or left generalized eigenvectors of a pair of upper triangular matrices
tgexc	Reorders the generalized Schur decomposition of a pair of matrices (A,B) so that one diagonal block of (A,B) moves to another row index.
tgsen	Reorders the *generalized* Schur decomposition of a pair of matrices (A,B) so that a selected cluster of eigenvalues appears in the leading diagonal blocks of (A,B).
tgsyl	Solves the generalized Sylvester equation.
tgsyl	Estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a pair of matrices in generalized real Schur canonical form.

?gghrd

Reduces a pair of matrices to generalized upper Hessenberg form using orthogonal/unitary transformations.

Syntax

Fortran 77:

```
call sgghrd(compq, compz, n, ilo, ihi, a, lda, b, ldb, q, ldq, z, ldz, info)
call dgghrd(compq, compz, n, ilo, ihi, a, lda, b, ldb, q, ldq, z, ldz, info)
call cgghrd(compq, compz, n, ilo, ihi, a, lda, b, ldb, q, ldq, z, ldz, info)
call zgghrd(compq, compz, n, ilo, ihi, a, lda, b, ldb, q, ldq, z, ldz, info)
```

Fortran 95:

```
call gghrd(a, b [,ilo] [,ihi] [,q] [,z] [,compq] [,compz] [,info])
```

C:

```
lapack_int LAPACKE_<?>gghrd( int matrix_order, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, <datatype>* a, lapack_int lda, <datatype>* b,
lapack_int ldb, <datatype>* q, lapack_int ldq, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reduces a pair of real/complex matrices (A,B) to generalized upper Hessenberg form using orthogonal/unitary transformations, where A is a general matrix and B is upper triangular. The form of the generalized eigenvalue problem is $A*x = \lambda*B*x$, and B is typically made upper triangular by computing its QR factorization and moving the orthogonal matrix Q to the left side of the equation.

This routine simultaneously reduces A to a Hessenberg matrix H:

$$Q^H*A*Z = H$$

and transforms B to another upper triangular matrix T:

$$Q^H*B*Z = T$$

in order to reduce the problem to its standard form $H*y = \lambda*T*y$, where $y = Z^H*x$.

The orthogonal/unitary matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1, so that

$$Q_1*A*Z_1^H = (Q_1*Q)*H*(Z_1*Z)^H$$

$$Q_1*B*Z_1^H = (Q_1*Q)*T*(Z_1*Z)^H$$

If Q_1 is the orthogonal/unitary matrix from the QR factorization of B in the original equation $A*x = \lambda*B*x$, then the routine `?gghrd` reduces the original problem to generalized Hessenberg form.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

compq — CHARACTER*1. Must be 'N', 'I', or 'V'.
If *compq* = 'N', matrix Q is not computed.
If *compq* = 'I', Q is initialized to the unit matrix, and the orthogonal/unitary matrix Q is returned;
If *compq* = 'V', Q must contain an orthogonal/unitary matrix Q_1 on entry, and the product Q_1*Q is returned.

compz — CHARACTER*1. Must be 'N', 'I', or 'V'.
If *compz* = 'N', matrix Z is not computed.
If *compz* = 'I', Z is initialized to the unit matrix, and the orthogonal/unitary matrix Z is returned;
If *compz* = 'V', Z must contain an orthogonal/unitary matrix Z_1 on entry, and the product Z_1*Z is returned.

n — INTEGER. The order of the matrices A and B ($n \geq 0$).

ilo, *ihi* — INTEGER. *ilo* and *ihi* mark the rows and columns of A which are to be reduced. It is assumed that A is already upper triangular in rows and columns 1:*ilo*-1 and *ihi*+1:*n*. Values of *ilo* and *ihi* are normally set by a previous call to ggbal; otherwise they should be set to 1 and *n* respectively.
Constraint:
If $n > 0$, then $1 \leq ilo \leq ihi \leq n$;
if $n = 0$, then $ilo = 1$ and $ihi = 0$.

a, *b*, *q*, *z* — REAL for sgghrd
DOUBLE PRECISION for dgghrd
COMPLEX for cgghrd
DOUBLE COMPLEX for zgghrd.
Arrays:
a(*lda*,*) contains the n-by-n general matrix A. The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the n-by-n upper triangular matrix B.
The second dimension of *b* must be at least max(1, *n*).
q (*ldq*,*)
If *compq* = 'N', then *q* is not referenced.
If *compq* = 'V', then *q* must contain the orthogonal/unitary matrix Q_1, typically from the QR factorization of B.
The second dimension of *q* must be at least max(1, *n*).
z (*ldz*,*)
If *compz* = 'N', then *z* is not referenced.
If *compz* = 'V', then *z* must contain the orthogonal/unitary matrix Z_1.
The second dimension of *z* must be at least max(1, *n*).

lda — INTEGER. The leading dimension of *a*; at least max(1, *n*).

ldb — INTEGER. The leading dimension of *b*; at least max(1, *n*).

ldq — INTEGER. The leading dimension of *q*;
If *compq* = 'N', then $ldq \geq 1$.
If *compq* = 'I'or 'V', then $ldq \geq$ max(1, *n*).

ldz — INTEGER. The leading dimension of *z*;
If *compz* = 'N', then $ldz \geq 1$.
If *compz* = 'I'or 'V', then $ldz \geq$ max(1, *n*).

Output Parameters

a — On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H, and the rest is set to zero.

b	On exit, overwritten by the upper triangular matrix $T = Q^H * B * Z$. The elements below the diagonal are set to zero.
q	If *compq* = 'I', then *q* contains the orthogonal/unitary matrix Q, ; If *compq* = 'V', then *q* is overwritten by the product $Q_1 * Q$.
z	If *compz* = 'I', then *z* contains the orthogonal/unitary matrix Z; If *compz* = 'V', then *z* is overwritten by the product $Z_1 * Z$.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *-i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gghrd interface are the following:

a	Holds the matrix A of size (*n*,*n*).
b	Holds the matrix B of size (*n*,*n*).
q	Holds the matrix Q of size (*n*,*n*).
z	Holds the matrix Z of size (*n*,*n*).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
compq	If omitted, this argument is restored based on the presence of argument *q* as follows: *compq* = 'I', if *q* is present, *compq* = 'N', if *q* is omitted. If present, *compq* must be equal to 'I' or 'V' and the argument *q* must also be present. Note that there will be an error condition if *compq* is present and *q* omitted.
compz	If omitted, this argument is restored based on the presence of argument *z* as follows: *compz* = 'I', if *z* is present, *compz* = 'N', if *z* is omitted. If present, *compz* must be equal to 'I' or 'V' and the argument *z* must also be present. Note that there will be an error condition if *compz* is present and *z* omitted.

?ggbal

Balances a pair of general real or complex matrices.

Syntax

Fortran 77:

```
call sggbal(job, n, a, lda, b, ldb, ilo, ihi, lscale, rscale, work, info)
call dggbal(job, n, a, lda, b, ldb, ilo, ihi, lscale, rscale, work, info)
call cggbal(job, n, a, lda, b, ldb, ilo, ihi, lscale, rscale, work, info)
call zggbal(job, n, a, lda, b, ldb, ilo, ihi, lscale, rscale, work, info)
```

Fortran 95:

```
call ggbal(a, b [,ilo] [,ihi] [,lscale] [,rscale] [,job] [,info])
```

C:

```
lapack_int LAPACKE_sggbal( int matrix_order, char job, lapack_int n, float* a,
lapack_int lda, float* b, lapack_int ldb, lapack_int* ilo, lapack_int* ihi, float*
lscale, float* rscale );

lapack_int LAPACKE_dggbal( int matrix_order, char job, lapack_int n, double* a,
lapack_int lda, double* b, lapack_int ldb, lapack_int* ilo, lapack_int* ihi, double*
lscale, double* rscale );

lapack_int LAPACKE_cggbal( int matrix_order, char job, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
lapack_int* ilo, lapack_int* ihi, float* lscale, float* rscale );

lapack_int LAPACKE_zggbal( int matrix_order, char job, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_int* ilo, lapack_int* ihi, double* lscale, double* rscale );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine balances a pair of general real/complex matrices (*A*,*B*). This involves, first, permuting *A* and *B* by similarity transformations to isolate eigenvalues in the first 1 to *ilo*-1 and last *ihi*+1 to *n* elements on the diagonal;and second, applying a diagonal similarity transformation to rows and columns *ilo* to *ihi* to make the rows and columns as close in norm as possible. Both steps are optional. Balancing may reduce the 1-norm of the matrices, and improve the accuracy of the computed eigenvalues and/or eigenvectors in the generalized eigenvalue problem $A*x = \lambda*B*x$.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job	`CHARACTER*1`. Specifies the operations to be performed on *A* and *B*. Must be `'N'` or `'P'` or `'S'` or `'B'`. If *job* = `'N '`, then no operations are done; simply set *ilo* =1, *ihi*=*n*, *lscale*(i) =1.0 and *rscale*(i)=1.0 for i = 1,..., *n*. If *job* = `'P'`, then permute only. If *job* = `'S'`, then scale only. If *job* = `'B'`, then both permute and scale.
n	`INTEGER`. The order of the matrices *A* and *B* ($n \geq 0$).
a, *b*	`REAL` for `sggbal` `DOUBLE PRECISION` for `dggbal` `COMPLEX` for `cggbal` `DOUBLE COMPLEX` for `zggbal`. Arrays: *a*(*lda*,*) contains the matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the matrix *B*. The second dimension of *b* must be at least max(1, *n*).

	If *job* = 'N', *a* and *b* are not referenced.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
work	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Workspace array, DIMENSION at least max(1, 6*n*) when *job* = 'S'or 'B', or at least 1 when *job* = 'N'or 'P'.

Output Parameters

a, *b*	Overwritten by the balanced matrices *A* and *B*, respectively.
ilo, *ihi*	INTEGER. *ilo* and *ihi* are set to integers such that on exit *a*(*i*,*j*)=0 and *b*(*i*,*j*)=0 if *i*>*j* and *j*=1,...,*ilo*-1 or *i*=*ihi*+1,..., *n*. If *job* = 'N'or 'S', then *ilo* = 1 and *ihi* = *n*.
lscale, *rscale*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *n*). *lscale* contains details of the permutations and scaling factors applied to the left side of *A* and *B*. If P_j is the index of the row interchanged with row *j*, and D_j is the scaling factor applied to row *j*, then *lscale*(*j*) = P_j, for *j* = 1,..., *ilo*-1 = D_j, for *j* = *ilo*,...,*ihi* = P_j, for *j* = *ihi*+1,..., *n*. *rscale* contains details of the permutations and scaling factors applied to the right side of *A* and *B*. If P_j is the index of the column interchanged with column *j*, and D_j is the scaling factor applied to column *j*, then *rscale*(*j*) = P_j, for *j* = 1,..., *ilo*-1 = D_j, for *j* = *ilo*,...,*ihi* = P_j, for *j* = *ihi*+1,..., *n* The order in which the interchanges are made is *n* to *ihi*+1, then 1 to *ilo*-1.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggbal interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
b	Holds the matrix *B* of size (*n*,*n*).
lscale	Holds the vector of length (*n*).
rscale	Holds the vector of length (*n*).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
job	Must be 'B', 'S', 'P', or 'N'. The default value is 'B'.

?ggbak

Forms the right or left eigenvectors of a generalized eigenvalue problem.

Syntax

Fortran 77:

```
call sggbak(job, side, n, ilo, ihi, lscale, rscale, m, v, ldv, info)
call dggbak(job, side, n, ilo, ihi, lscale, rscale, m, v, ldv, info)
call cggbak(job, side, n, ilo, ihi, lscale, rscale, m, v, ldv, info)
call zggbak(job, side, n, ilo, ihi, lscale, rscale, m, v, ldv, info)
```

Fortran 95:

```
call ggbak(v [, ilo] [,ihi] [,lscale] [,rscale] [,job] [,info])
```

C:

```
lapack_int LAPACKE_sggbak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* lscale, const float* rscale, lapack_int
m, float* v, lapack_int ldv );
lapack_int LAPACKE_dggbak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* lscale, const double* rscale, lapack_int
m, double* v, lapack_int ldv );
lapack_int LAPACKE_cggbak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* lscale, const float* rscale, lapack_int
m, lapack_complex_float* v, lapack_int ldv );
lapack_int LAPACKE_zggbak( int matrix_order, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* lscale, const double* rscale, lapack_int
m, lapack_complex_double* v, lapack_int ldv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine forms the right or left eigenvectors of a real/complex generalized eigenvalue problem

$A*x = \lambda*B*x$

by backward transformation on the computed eigenvectors of the balanced pair of matrices output by ggbal.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job — `CHARACTER*1`. Specifies the type of backward transformation required.
Must be `'N'`, `'P'`, `'S'`, or `'B'`.
If *job* = `'N'`, then no operations are done; return.
If *job* = `'P'`, then do backward transformation for permutation only.

	If *job* = 'S', then do backward transformation for scaling only. If *job* = 'B', then do backward transformation for both permutation and scaling. This argument must be the same as the argument *job* supplied to ?ggbal.
side	CHARACTER*1. Must be 'L' or 'R'. If *side* = 'L', then *v* contains left eigenvectors. If *side* = 'R', then *v* contains right eigenvectors.
n	INTEGER. The number of rows of the matrix *V* (*n* ≥ 0).
ilo, *ihi*	INTEGER. The integers *ilo* and *ihi* determined by ?gebal. Constraint: If *n* > 0, then 1 ≤ *ilo* ≤ *ihi* ≤ *n*; if *n* = 0, then *ilo* = 1 and *ihi* = 0.
lscale, *rscale*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *n*). The array *lscale* contains details of the permutations and/or scaling factors applied to the left side of *A* and *B*, as returned by ?ggbal. The array *rscale* contains details of the permutations and/or scaling factors applied to the right side of *A* and *B*, as returned by ?ggbal.
m	INTEGER. The number of columns of the matrix *V* (*m* ≥ 0).
v	REAL for sggbak DOUBLE PRECISION for dggbak COMPLEX for cggbak DOUBLE COMPLEX for zggbak. Array *v*(*ldv*,*). Contains the matrix of right or left eigenvectors to be transformed, as returned by tgevc. The second dimension of *v* must be at least max(1, *m*).
ldv	INTEGER. The leading dimension of *v*; at least max(1, *n*).

Output Parameters

v	Overwritten by the transformed eigenvectors
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggbak interface are the following:

v	Holds the matrix *V* of size (*n*,*m*).
lscale	Holds the vector of length *n*.
rscale	Holds the vector of length *n*.
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.
job	Must be 'B', 'S', 'P', or 'N'. The default value is 'B'.
side	If omitted, this argument is restored based on the presence of arguments *lscale* and *rscale* as follows:

side = 'L', if *lscale* is present and *rscale* omitted,
side = 'R', if *lscale* is omitted and *rscale* present.
Note that there will be an error condition if both *lscale* and *rscale* are present or if they both are omitted.

?hgeqz

Implements the QZ method for finding the generalized eigenvalues of the matrix pair (H,T).

Syntax

Fortran 77:

```
call shgeqz(job, compq, compz, n, ilo, ihi, h, ldh, t, ldt, alphar, alphai, beta, q,
ldq, z, ldz, work, lwork, info)
```

```
call dhgeqz(job, compq, compz, n, ilo, ihi, h, ldh, t, ldt, alphar, alphai, beta, q,
ldq, z, ldz, work, lwork, info)
```

```
call chgeqz(job, compq, compz, n, ilo, ihi, h, ldh, t, ldt, alpha, beta, q, ldq, z,
ldz, work, lwork, rwork, info)
```

```
call zhgeqz(job, compq, compz, n, ilo, ihi, h, ldh, t, ldt, alpha, beta, q, ldq, z,
ldz, work, lwork, rwork, info)
```

Fortran 95:

```
call hgeqz(h, t [,ilo] [,ihi] [,alphar] [,alphai] [,beta] [,q] [,z] [,job] [,compq]
[,compz] [,info])
```

```
call hgeqz(h, t [,ilo] [,ihi] [,alpha] [,beta] [,q] [,z] [,job] [,compq] [, compz]
[,info])
```

C:

```
lapack_int LAPACKE_shgeqz( int matrix_order, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, float* h, lapack_int ldh, float* t,
lapack_int ldt, float* alphar, float* alphai, float* beta, float* q, lapack_int ldq,
float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_dhgeqz( int matrix_order, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, double* h, lapack_int ldh, double* t,
lapack_int ldt, double* alphar, double* alphai, double* beta, double* q, lapack_int
ldq, double* z, lapack_int ldz );
```

```
lapack_int LAPACKE_chgeqz( int matrix_order, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, lapack_complex_float* h, lapack_int ldh,
lapack_complex_float* t, lapack_int ldt, lapack_complex_float* alpha,
lapack_complex_float* beta, lapack_complex_float* q, lapack_int ldq,
lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhgeqz( int matrix_order, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, lapack_complex_double* h, lapack_int
ldh, lapack_complex_double* t, lapack_int ldt, lapack_complex_double* alpha,
lapack_complex_double* beta, lapack_complex_double* q, lapack_int ldq,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`

- C: mkl_lapacke.h

Description

The routine computes the eigenvalues of a real/complex matrix pair (*H*,*T*), where *H* is an upper Hessenberg matrix and *T* is upper triangular, using the double-shift version (for real flavors) or single-shift version (for complex flavors) of the *QZ* method. Matrix pairs of this type are produced by the reduction to generalized upper Hessenberg form of a real/complex matrix pair (*A*,*B*):

$A = Q_1 * H * Z_1^H$, $B = Q_1 * T * Z_1^H$,

as computed by ?gghrd.

For real flavors:

If *job* = 'S', then the Hessenberg-triangular pair (*H*,*T*) is also reduced to generalized Schur form,

$H = Q * S * Z^T$, $T = Q * P * Z^T$,

where *Q* and *Z* are orthogonal matrices, *P* is an upper triangular matrix, and *S* is a quasi-triangular matrix with 1-by-1 and 2-by-2 diagonal blocks. The 1-by-1 blocks correspond to real eigenvalues of the matrix pair (*H*,*T*) and the 2-by-2 blocks correspond to complex conjugate pairs of eigenvalues.

Additionally, the 2-by-2 upper triangular diagonal blocks of *P* corresponding to 2-by-2 blocks of *S* are reduced to positive diagonal form, that is, if *S*(j+1,j) is non-zero, then *P*(j+1,j) = *P*(j,j+1) = 0, *P*(j,j) > 0, and *P*(j+1,j+1) > 0.

For complex flavors:

If *job* = 'S', then the Hessenberg-triangular pair (*H*,*T*) is also reduced to generalized Schur form,

$H = Q * S * Z^H$, $T = Q * P * Z^H$,

where *Q* and *Z* are unitary matrices, and *S* and *P* are upper triangular.

For all function flavors:

Optionally, the orthogonal/unitary matrix *Q* from the generalized Schur factorization may be postmultiplied into an input matrix Q_1, and the orthogonal/unitary matrix *Z* may be postmultiplied into an input matrix Z_1.

If Q_1 and Z_1 are the orthogonal/unitary matrices from ?gghrd that reduced the matrix pair (*A*,*B*) to generalized upper Hessenberg form, then the output matrices $Q_1 Q$ and Z 1Z are the orthogonal/unitary factors from the generalized Schur factorization of (*A*,*B*):

$A = (Q_1 Q) * S * (Z_1 Z)^H$, $B = (Q_1 Q) * P * (Z_1 Z)^H$.

To avoid overflow, eigenvalues of the matrix pair (*H*,*T*) (equivalently, of (*A*,*B*)) are computed as a pair of values (*alpha*,*beta*). For chgeqz/zhgeqz, *alpha* and *beta* are complex, and for shgeqz/dhgeqz, *alpha* is complex and *beta* real. If *beta* is nonzero, $\lambda = alpha/beta$ is an eigenvalue of the generalized nonsymmetric eigenvalue problem (GNEP)

$A * x = \lambda * B * x$

and if *alpha* is nonzero, $\mu = beta/alpha$ is an eigenvalue of the alternate form of the GNEP

$\mu * A * y = B * y$.

Real eigenvalues (for real flavors) or the values of *alpha* and *beta* for the i-th eigenvalue (for complex flavors) can be read directly from the generalized Schur form:

alpha = *S*(i,i), *beta* = *P*(i,i).

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job CHARACTER*1. Specifies the operations to be performed. Must be 'E' or 'S'.
If *job* = 'E', then compute eigenvalues only;
If *job* = 'S', then compute eigenvalues and the Schur form.

compq CHARACTER*1. Must be 'N', 'I', or 'V'.
If *compq* = 'N', left Schur vectors (*q*) are not computed;
If *compq* = 'I', *q* is initialized to the unit matrix and the matrix of left Schur vectors of (*H*,*T*) is returned;
If *compq* = 'V', *q* must contain an orthogonal/unitary matrix Q_1 on entry and the product $Q_1 * Q$ is returned.

compz CHARACTER*1. Must be 'N', 'I', or 'V'.
If *compz* = 'N', right Schur vectors (*z*) are not computed;
If *compz* = 'I', *z* is initialized to the unit matrix and the matrix of right Schur vectors of (*H*,*T*) is returned;
If *compz* = 'V', *z* must contain an orthogonal/unitary matrix Z_1 on entry and the product $Z_1 * Z$ is returned.

n INTEGER. The order of the matrices *H*, *T*, *Q*, and *Z* ($n \geq 0$).

ilo, *ihi* INTEGER. *ilo* and *ihi* mark the rows and columns of *H* which are in Hessenberg form. It is assumed that *H* is already upper triangular in rows and columns 1:*ilo*-1 and *ihi*+1:*n*.
Constraint:
If $n > 0$, then $1 \leq ilo \leq ihi \leq n$;
if $n = 0$, then $ilo = 1$ and $ihi = 0$.

h, *t*, *q*, *z*, *work* REAL for shgeqz
DOUBLE PRECISION for dhgeqz
COMPLEX for chgeqz
DOUBLE COMPLEX for zhgeqz.
Arrays:
On entry, *h*(*ldh*,*) contains the *n*-by-*n* upper Hessenberg matrix *H*.
The second dimension of *h* must be at least max(1, *n*).
On entry, *t*(*ldt*,*) contains the *n*-by-*n* upper triangular matrix *T*.
The second dimension of *t* must be at least max(1, *n*).
q (*ldq*,*):
On entry, if *compq* = 'V', this array contains the orthogonal/unitary matrix Q_1 used in the reduction of (*A*,*B*) to generalized Hessenberg form.
If *compq* = 'N', then *q* is not referenced.
The second dimension of *q* must be at least max(1, *n*).
z (*ldz*,*):
On entry, if *compz* = 'V', this array contains the orthogonal/unitary matrix Z_1 used in the reduction of (*A*,*B*) to generalized Hessenberg form.
If *compz* = 'N', then *z* is not referenced.
The second dimension of *z* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

ldh INTEGER. The leading dimension of *h*; at least max(1, *n*).

ldt INTEGER. The leading dimension of *t*; at least max(1, *n*).

ldq INTEGER. The leading dimension of *q*;
If *compq* = 'N', then $ldq \geq 1$.
If *compq* = 'I'or 'V', then $ldq \geq \max(1, n)$.

ldz INTEGER. The leading dimension of *z*;

If *compq* = 'N', then *ldz* ≥ 1.
If *compq* = 'I'or 'V', then *ldz* ≥ max(1, *n*).

lwork — INTEGER. The dimension of the array *work*.
lwork ≥ max(1, *n*).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.

rwork — REAL for chgeqz
DOUBLE PRECISION for zhgeqz.
Workspace array, DIMENSION at least max(1, *n*). Used in complex flavors only.

Output Parameters

h — *For real flavors*:
If *job* = 'S', then on exit *h* contains the upper quasi-triangular matrix *S* from the generalized Schur factorization.
If *job* = 'E', then on exit the diagonal blocks of *h* match those of *S*, but the rest of *h* is unspecified.
For complex flavors:
If *job* = 'S', then, on exit, *h* contains the upper triangular matrix *S* from the generalized Schur factorization.
If *job* = 'E', then on exit the diagonal of *h* matches that of *S*, but the rest of *h* is unspecified.

t — If *job* = 'S', then, on exit, *t* contains the upper triangular matrix *P* from the generalized Schur factorization.
For real flavors:
2-by-2 diagonal blocks of *P* corresponding to 2-by-2 blocks of *S* are reduced to positive diagonal form, that is, if *h*(j+1,j) is non-zero, then *t*(j+1,j)=*t*(j,j+1)=0 and *t*(j,j) and *t*(j+1,j+1) will be positive.
If *job* = 'E', then on exit the diagonal blocks of *t* match those of *P*, but the rest of *t* is unspecified.
For complex flavors:
if *job* = 'E', then on exit the diagonal of *t* matches that of *P*, but the rest of *t* is unspecified.

alphar, *alphai* — REAL for shgeqz;
DOUBLE PRECISION for dhgeqz.
Arrays, DIMENSION at least max(1, *n*). The real and imaginary parts, respectively, of each scalar *alpha* defining an eigenvalue of GNEP.
If *alphai*(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-th eigenvalues are a complex conjugate pair, with *alphai*(j+1) = -*alphai*(j).

alpha — COMPLEX for chgeqz;
DOUBLE COMPLEX for zhgeqz.
Array, DIMENSION at least max(1, *n*).
The complex scalars *alpha* that define the eigenvalues of GNEP. *alphai*(i) = *S*(i,i) in the generalized Schur factorization.

beta — REAL for shgeqz
DOUBLE PRECISION for dhgeqz
COMPLEX for chgeqz
DOUBLE COMPLEX for zhgeqz.

	Array, DIMENSION at least max(1, *n*). *For real flavors*: The scalars *beta* that define the eigenvalues of GNEP. Together, the quantities *alpha* = (*alphar*(j), *alphai*(j)) and *beta* = *beta*(j) represent the j-th eigenvalue of the matrix pair (*A*,*B*), in one of the forms *lambda* = *alpha*/*beta* or *mu* = *beta*/*alpha*. Since either *lambda* or *mu* may overflow, they should not, in general, be computed. *For complex flavors*: The real non-negative scalars *beta* that define the eigenvalues of GNEP. *beta*(i) = *P*(i,i) in the generalized Schur factorization. Together, the quantities *alpha* = *alpha*(j) and *beta* = *beta*(j) represent the j-th eigenvalue of the matrix pair (*A*,*B*), in one of the forms *lambda* = *alpha*/*beta* or *mu* = *beta*/*alpha*. Since either *lambda* or *mu* may overflow, they should not, in general, be computed.
q	On exit, if *compq* = 'I', *q* is overwritten by the orthogonal/unitary matrix of left Schur vectors of the pair (*H*,*T*), and if *compq* = 'V', *q* is overwritten by the orthogonal/unitary matrix of left Schur vectors of (*A*,*B*).
z	On exit, if *compz* = 'I', *z* is overwritten by the orthogonal/unitary matrix of right Schur vectors of the pair (*H*,*T*), and if *compz* = 'V', *z* is overwritten by the orthogonal/unitary matrix of right Schur vectors of (*A*,*B*).
work(1)	If *info* ≥ 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1,..., *n*, the *QZ* iteration did not converge. (*H*,*T*) is not in Schur form, but *alphar*(i), *alphai*(i) (for real flavors), *alpha*(i) (for complex flavors), and *beta*(i), i=*info*+1,..., *n* should be correct. If *info* = *n*+1,...,2*n*, the shift calculation failed. (*H*,*T*) is not in Schur form, but *alphar*(i), *alphai*(i) (for real flavors), *alpha*(i) (for complex flavors), and *beta*(i), i =*info*-*n*+1,..., *n* should be correct.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hgeqz interface are the following:

h	Holds the matrix *H* of size (*n*,*n*).
t	Holds the matrix *T* of size (*n*,*n*).
alphar	Holds the vector of length *n*. Used in real flavors only.
alphai	Holds the vector of length *n*. Used in real flavors only.
alpha	Holds the vector of length *n*. Used in complex flavors only.
beta	Holds the vector of length *n*.
q	Holds the matrix *Q* of size (*n*,*n*).
z	Holds the matrix *Z* of size (*n*,*n*).
ilo	Default value for this argument is *ilo* = 1.
ihi	Default value for this argument is *ihi* = *n*.

job	Must be 'E' or 'S'. The default value is 'E'.
compq	If omitted, this argument is restored based on the presence of argument *q* as follows: *compq* = 'I', if *q* is present, *compq* = 'N', if *q* is omitted. If present, *compq* must be equal to 'I' or 'V' and the argument *q* must also be present. Note that there will be an error condition if *compq* is present and *q* omitted.
compz	If omitted, this argument is restored based on the presence of argument *z* as follows: *compz* = 'I', if *z* is present, *compz* = 'N', if *z* is omitted. If present, *compz* must be equal to 'I' or 'V' and the argument *z* must also be present. Note an error condition if *compz* is present and *z* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?tgevc

Computes some or all of the right and/or left generalized eigenvectors of a pair of upper triangular matrices.

Syntax

Fortran 77:

```
call stgevc(side, howmny, select, n, s, lds, p, ldp, vl, ldvl, vr, ldvr, mm, m, work, info)
call dtgevc(side, howmny, select, n, s, lds, p, ldp, vl, ldvl, vr, ldvr, mm, m, work, info)
call ctgevc(side, howmny, select, n, s, lds, p, ldp, vl, ldvl, vr, ldvr, mm, m, work, rwork, info)
call ztgevc(side, howmny, select, n, s, lds, p, ldp, vl, ldvl, vr, ldvr, mm, m, work, rwork, info)
```

Fortran 95:

```
call tgevc(s, p [,howmny] [,select] [,vl] [,vr] [,m] [,info])
```

C:

```
lapack_int LAPACKE_<?>tgevc( int matrix_order, char side, char howmny, const
lapack_logical* select, lapack_int n, const <datatype>* s, lapack_int lds, const
<datatype>* p, lapack_int ldp, <datatype>* vl, lapack_int ldvl, <datatype>* vr,
lapack_int ldvr, lapack_int mm, lapack_int* m );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes some or all of the right and/or left eigenvectors of a pair of real/complex matrices (*S*,*P*), where *S* is quasi-triangular (for real flavors) or upper triangular (for complex flavors) and *P* is upper triangular.

Matrix pairs of this type are produced by the generalized Schur factorization of a real/complex matrix pair (*A*,*B*):

$A = Q*S*Z^H$, $B = Q*P*Z^H$

as computed by `?gghrd` plus `?hgeqz`.

The right eigenvector *x* and the left eigenvector *y* of (*S*,*P*) corresponding to an eigenvalue *w* are defined by:

$S*x = w*P*x$, $y^H*S = w*y^H*P$

The eigenvalues are not input to this routine, but are computed directly from the diagonal blocks or diagonal elements of *S* and *P*.

This routine returns the matrices *X* and/or *Y* of right and left eigenvectors of (*S*,*P*), or the products *Z***X* and/or *Q***Y*, where *Z* and *Q* are input matrices.

If *Q* and *Z* are the orthogonal/unitary factors from the generalized Schur factorization of a matrix pair (*A*,*B*), then *Z***X* and *Q***Y* are the matrices of right and left eigenvectors of (*A*,*B*).

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

side
`CHARACTER*1`. Must be `'R'`, `'L'`, or `'B'`.
If *side* = `'R'`, compute right eigenvectors only.
If *side* = `'L'`, compute left eigenvectors only.
If *side* = `'B'`, compute both right and left eigenvectors.

howmny
`CHARACTER*1`. Must be `'A'`, `'B'`, or `'S'`.
If *howmny* = `'A'`, compute all right and/or left eigenvectors.
If *howmny* = `'B'`, compute all right and/or left eigenvectors, backtransformed by the matrices in *vr* and/or *vl*.
If *howmny* = `'S'`, compute selected right and/or left eigenvectors, specified by the logical array *select*.

select
`LOGICAL`.
Array, `DIMENSION` at least max (1, *n*).
If *howmny* = `'S'`, *select* specifies the eigenvectors to be computed.
If *howmny* = `'A'or 'B'`, *select* is not referenced.
For real flavors:

	If *omega*(*j*) is a real eigenvalue, the corresponding real eigenvector is computed if *select*(*j*) is .TRUE.. If *omega*(*j*) and *omega*(*j*+1) are the real and imaginary parts of a complex eigenvalue, the corresponding complex eigenvector is computed if either *select*(*j*) or *select*(*j*+1) is .TRUE., and on exit *select*(*j*) is set to .TRUE. and *select*(*j*+1) is set to .FALSE.. *For complex flavors:* The eigenvector corresponding to the *j*-th eigenvalue is computed if *select*(*j*) is .TRUE..
n	INTEGER. The order of the matrices *S* and *P* (*n* ≥ 0).
s, *p*, *vl*, *vr*, *work*	REAL for stgevc DOUBLE PRECISION for dtgevc COMPLEX for ctgevc DOUBLE COMPLEX for ztgevc. Arrays: *s*(*lds*,*) contains the matrix *S* from a generalized Schur factorization as computed by ?hgeqz. This matrix is upper quasi-triangular for real flavors, and upper triangular for complex flavors. The second dimension of *s* must be at least max(1, *n*). *p*(*ldp*,*) contains the upper triangular matrix *P* from a generalized Schur factorization as computed by ?hgeqz. For real flavors, 2-by-2 diagonal blocks of *P* corresponding to 2-by-2 blocks of *S* must be in positive diagonal form. For complex flavors, *P* must have real diagonal elements. The second dimension of *p* must be at least max(1, *n*). If *side* = 'L' or 'B' and *howmny* = 'B', *vl*(*ldvl*,*) must contain an *n*-by-*n* matrix *Q* (usually the orthogonal/unitary matrix *Q* of left Schur vectors returned by ?hgeqz). The second dimension of *vl* must be at least max(1, *mm*). If *side* = 'R' , *vl* is not referenced. If *side* = 'R' or 'B' and *howmny* = 'B', *vr*(*ldvr*,*) must contain an *n*-by-*n* matrix *Z* (usually the orthogonal/unitary matrix *Z* of right Schur vectors returned by ?hgeqz). The second dimension of *vr* must be at least max(1, *mm*). If *side* = 'L', *vr* is not referenced. *work*(*) is a workspace array. DIMENSION at least max (1, 6**n*) for real flavors and at least max (1, 2**n*) for complex flavors.
lds	INTEGER. The leading dimension of *s*; at least max(1, *n*).
ldp	INTEGER. The leading dimension of *p*; at least max(1, *n*).
ldvl	INTEGER. The leading dimension of *vl*; If *side* = 'L' or 'B', then *ldvl* ≥*n*. If *side* = 'R', then *ldvl* ≥ 1.
ldvr	INTEGER. The leading dimension of *vr*; If *side* = 'R' or 'B', then *ldvr* ≥*n*. If *side* = 'L', then *ldvr* ≥ 1.
mm	INTEGER. The number of columns in the arrays *vl* and/or *vr* (*mm* ≥ *m*).
rwork	REAL for ctgevc DOUBLE PRECISION for ztgevc. Workspace array, DIMENSION at least max (1, 2**n*). Used in complex flavors only.

Output Parameters

vl — On exit, if *side* = 'L' or 'B', *vl* contains:
if *howmny* = 'A', the matrix *Y* of left eigenvectors of (*S*,*P*);
if *howmny* = 'B', the matrix *Q***Y*;
if *howmny* = 'S', the left eigenvectors of (*S*,*P*) specified by *select*, stored consecutively in the columns of *vl*, in the same order as their eigenvalues.
For real flavors:
A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part.

vr — On exit, if *side* = 'R' or 'B', *vr* contains:
if *howmny* = 'A', the matrix *X* of right eigenvectors of (*S*,*P*);
if *howmny* = 'B', the matrix *Z***X*;
if *howmny* = 'S', the right eigenvectors of (*S*,*P*) specified by *select*, stored consecutively in the columns of *vr*, in the same order as their eigenvalues.
For real flavors:
A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part.

m — INTEGER. The number of columns in the arrays *vl* and/or *vr* actually used to store the eigenvectors.
If *howmny* = 'A' or 'B', *m* is set to *n*.
For real flavors:
Each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.
For complex flavors:
Each selected eigenvector occupies one column.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = –*i*, the *i*–th parameter had an illegal value.
For real flavors:
if *info* = *i*>0, the 2-by-2 block (*i*:*i*+1) does not have a complex eigenvalue.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tgevc interface are the following:

s — Holds the matrix *S* of size (*n*,*n*).

p — Holds the matrix *P* of size (*n*,*n*).

select — Holds the vector of length *n*.

vl — Holds the matrix *VL* of size (*n*,*mm*).

vr — Holds the matrix *VR* of size (*n*,*mm*).

side — Restored based on the presence of arguments *vl* and *vr* as follows:
side = 'B', if both *vl* and *vr* are present,
side = 'L', if *vl* is present and *vr* omitted,
side = 'R', if *vl* is omitted and *vr* present,
Note that there will be an error condition if both *vl* and *vr* are omitted.

howmny If omitted, this argument is restored based on the presence of argument *select* as follows:
howmny = 'S', if *select* is present,
howmny = 'A', if *select* is omitted.
If present, *howmny* must be equal to 'A' or 'B' and the argument *select* must be omitted.
Note that there will be an error condition if both *howmny* and *select* are present.

?tgexc

Reorders the generalized Schur decomposition of a pair of matrices (A,B) so that one diagonal block of (A,B) moves to another row index.

Syntax

Fortran 77:

```
call stgexc(wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, ifst, ilst, work, lwork, info)
call dtgexc(wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, ifst, ilst, work, lwork, info)
call ctgexc(wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, ifst, ilst, info)
call ztgexc(wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, ifst, ilst, info)
```

Fortran 95:

```
call tgexc(a, b [,ifst] [,ilst] [,z] [,q] [,info])
```

C:

```
lapack_int LAPACKE_<?>tgexc( int matrix_order, lapack_logical wantq, lapack_logical
wantz, lapack_int n, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb,
<datatype>* q, lapack_int ldq, <datatype>* z, lapack_int ldz, lapack_int* ifst,
lapack_int* ilst );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine reorders the generalized real-Schur/Schur decomposition of a real/complex matrix pair (*A*,*B*) using an orthogonal/unitary equivalence transformation

$(A,B) = Q*(A,B)*Z^H$,

so that the diagonal block of (*A*, *B*) with row index *ifst* is moved to row *ilst*. Matrix pair (*A*, *B*) must be in a generalized real-Schur/Schur canonical form (as returned by gges), that is, *A* is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks and *B* is upper triangular. Optionally, the matrices *Q* and *Z* of generalized Schur vectors are updated.

Q(in)*A(in)*Z(in)' = *Q*(out)*A(out)*Z(out)'

Q(in)*B(in)*Z(in)' = *Q*(out)*B(out)*Z(out)'.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

wantq, *wantz*
LOGICAL.
If *wantq* = .TRUE., update the left transformation matrix *Q*;
If *wantq* = .FALSE., do not update *Q*;
If *wantz* = .TRUE., update the right transformation matrix *Z*;
If *wantz* = .FALSE., do not update *Z*.

n
INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).

a, *b*, *q*, *z*
REAL for stgexc
DOUBLE PRECISION for dtgexc
COMPLEX for ctgexc
DOUBLE COMPLEX for ztgexc.
Arrays:
a(*lda*,*) contains the matrix *A*.
The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the matrix *B*. The second dimension of *b* must be at least max(1, *n*).
q (*ldq*,*)
If *wantq* = .FALSE., then *q* is not referenced.
If *wantq* = .TRUE., then *q* must contain the orthogonal/unitary matrix *Q*.
The second dimension of *q* must be at least max(1, *n*).
z (*ldz*,*)
If *wantz* = .FALSE., then *z* is not referenced.
If *wantz* = .TRUE., then *z* must contain the orthogonal/unitary matrix *Z*.
The second dimension of *z* must be at least max(1, *n*).

lda
INTEGER. The leading dimension of *a*; at least max(1, *n*).

ldb
INTEGER. The leading dimension of *b*; at least max(1, *n*).

ldq
INTEGER. The leading dimension of *q*;
If *wantq* = .FALSE., then $ldq \geq 1$.
If *wantq* = .TRUE., then $ldq \geq \max(1, n)$.

ldz
INTEGER. The leading dimension of *z*;
If *wantz* = .FALSE., then $ldz \geq 1$.
If *wantz* = .TRUE., then $ldz \geq \max(1, n)$.

ifst, *ilst*
INTEGER. Specify the reordering of the diagonal blocks of (*A*, *B*). The block with row index *ifst* is moved to row *ilst*, by a sequence of swapping between adjacent blocks. Constraint: $1 \leq ifst, ilst \leq n$.

work
REAL for stgexc;
DOUBLE PRECISION for dtgexc.
Workspace array, DIMENSION (*lwork*). Used in real flavors only.

lwork
INTEGER. The dimension of *work*; must be at least $4n + 16$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

a, *b*, *q*, *z*
Overwritten by the updated matrices *A*,*B*, *Q*, and *Z* respectively.

ifst, *ilst*	Overwritten for real flavors only. If *ifst* pointed to the second row of a 2 by 2 block on entry, it is changed to point to the first row; *ilst* always points to the first row of the block in its final position (which may differ from its input value by ±1).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *-i*, the *i*-th parameter had an illegal value. If *info* = 1, the transformed matrix pair (*A*, *B*) would be too far from generalized Schur form; the problem is ill-conditioned. (*A*, *B*) may have been partially reordered, and *ilst* points to the first row of the current position of the block being moved.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tgexc interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
b	Holds the matrix *B* of size (*n*,*n*).
z	Holds the matrix *Z* of size (*n*,*n*).
q	Holds the matrix *Q* of size (*n*,*n*).
wantq	Restored based on the presence of the argument *q* as follows: *wantq* = .TRUE., if *q* is present, *wantq* = .FALSE., if *q* is omitted.
wantz	Restored based on the presence of the argument *z* as follows: *wantz* = .TRUE., if *z* is present, *wantz* = .FALSE., if z is omitted.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?tgsen

Reorders the generalized Schur decomposition of a pair of matrices (A,B) so that a selected cluster of eigenvalues appears in the leading diagonal blocks of (A,B).

Syntax

Fortran 77:

```
call stgsen(ijob, wantq, wantz, select, n, a, lda, b, ldb, alphar, alphai, beta, q,
ldq, z, ldz, m, pl, pr, dif, work, lwork, iwork, liwork, info)
```

```
call dtgsen(ijob, wantq, wantz, select, n, a, lda, b, ldb, alphar, alphai, beta, q,
ldq, z, ldz, m, pl, pr, dif, work, lwork, iwork, liwork, info)
```

```
call ctgsen(ijob, wantq, wantz, select, n, a, lda, b, ldb, alpha, beta, q, ldq, z,
ldz, m, pl, pr, dif, work, lwork, iwork, liwork, info)
```

```
call ztgsen(ijob, wantq, wantz, select, n, a, lda, b, ldb, alpha, beta, q, ldq, z,
ldz, m, pl, pr, dif, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call tgsen(a, b, select [,alphar] [,alphai] [,beta] [,ijob] [,q] [,z] [,pl] [,pr] [,dif]
[,m] [,info])
```

```
call tgsen(a, b, select [,alpha] [,beta] [,ijob] [,q] [,z] [,pl] [,pr] [, dif] [,m]
[,info])
```

C:

```
lapack_int LAPACKE_stgsen( int matrix_order, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n, float* a, lapack_int
lda, float* b, lapack_int ldb, float* alphar, float* alphai, float* beta, float* q,
lapack_int ldq, float* z, lapack_int ldz, lapack_int* m, float* pl, float* pr, float*
dif );
```

```
lapack_int LAPACKE_dtgsen( int matrix_order, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n, double* a, lapack_int
lda, double* b, lapack_int ldb, double* alphar, double* alphai, double* beta, double*
q, lapack_int ldq, double* z, lapack_int ldz, lapack_int* m, double* pl, double* pr,
double* dif );
```

```
lapack_int LAPACKE_ctgsen( int matrix_order, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n, lapack_complex_float*
a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, lapack_complex_float*
alpha, lapack_complex_float* beta, lapack_complex_float* q, lapack_int ldq,
lapack_complex_float* z, lapack_int ldz, lapack_int* m, float* pl, float* pr, float*
dif );
```

```
lapack_int LAPACKE_ztgsen( int matrix_order, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* alpha, lapack_complex_double* beta, lapack_complex_double* q,
lapack_int ldq, lapack_complex_double* z, lapack_int ldz, lapack_int* m, double* pl,
double* pr, double* dif );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine reorders the generalized real-Schur/Schur decomposition of a real/complex matrix pair (A, B) (in terms of an orthogonal/unitary equivalence transformation $Q^{T} * (A, B) * Z$ for real flavors or $Q^{H} * (A, B) * Z$ for complex flavors), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the pair (A, B). The leading columns of Q and Z form orthonormal/unitary bases of the corresponding left and right eigenspaces (deflating subspaces).

(A, B) must be in generalized real-Schur/Schur canonical form (as returned by gges), that is, A and B are both upper triangular.

?tgsen also computes the generalized eigenvalues

ω_j = (*alphar*(j) + *alphai*(j)*i)/*beta*(j) (for real flavors)

ω_j = *alpha*(j)/*beta*(j) (for complex flavors)

of the reordered matrix pair (*A*, *B*).

Optionally, the routine computes the estimates of reciprocal condition numbers for eigenvalues and eigenspaces. These are Difu[(A_{11}, B_{11}), (A_{22}, B_{22})] and Difl[(A_{11}, B_{11}), (A_{22}, B_{22})], that is, the separation(s) between the matrix pairs (A_{11}, B_{11}) and (A_{22}, B_{22}) that correspond to the selected cluster and the eigenvalues outside the cluster, respectively, and norms of "projections" onto left and right eigenspaces with respect to the selected cluster in the (1,1)-block.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

ijob
INTEGER. Specifies whether condition numbers are required for the cluster of eigenvalues (*pl* and *pr*) or the deflating subspaces Difu and Difl.
If *ijob* =0, only reorder with respect to *select*;
If *ijob* =1, reciprocal of norms of "projections" onto left and right eigenspaces with respect to the selected cluster (*pl* and *pr*);
If *ijob* =2, compute upper bounds on Difu and Difl, using F-norm-based estimate (*dif* (1:2));
If *ijob* =3, compute estimate of Difu and Difl, sing 1-norm-based estimate (*dif* (1:2)). This option is about 5 times as expensive as *ijob* =2;
If *ijob* =4,>compute *pl*, *pr* and *dif* (i.e., options 0, 1 and 2 above). This is an economic version to get it all;
If *ijob* =5, compute *pl*, *pr* and *dif* (i.e., options 0, 1 and 3 above).

wantq, *wantz*
LOGICAL.
If *wantq* = .TRUE., update the left transformation matrix *Q*;
If *wantq* = .FALSE., do not update *Q*;
If *wantz* = .TRUE., update the right transformation matrix *Z*;
If *wantz* = .FALSE., do not update *Z*.

select
LOGICAL.
Array, DIMENSION at least max (1, *n*). Specifies the eigenvalues in the selected cluster.
To select an eigenvalue *omega*(*j*), *select*(*j*) must be .TRUE. *For real flavors*: to select a complex conjugate pair of eigenvalues *omega*(*j*) and *omega*(*j*+1) (corresponding 2 by 2 diagonal block), *select*(*j*) and/or *select*(*j*+1) must be set to .TRUE.; the complex conjugate *omega*(*j*) and *omega*(*j*+1) must be either both included in the cluster or both excluded.

n
INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).

a, *b*, *q*, *z*, *work*
REAL for stgsen
DOUBLE PRECISION for dtgsen
COMPLEX for ctgsen
DOUBLE COMPLEX for ztgsen.
Arrays:
a(*lda*,*) contains the matrix *A*.
For real flavors: *A* is upper quasi-triangular, with (*A*, *B*) in generalized real Schur canonical form.

For complex flavors: `A` is upper triangular, in generalized Schur canonical form.
The second dimension of `a` must be at least max(1, `n`).

`b`(`ldb`,*) contains the matrix `B`.
For real flavors: `B` is upper triangular, with (`A`, `B`) in generalized real Schur canonical form.
For complex flavors: `B` is upper triangular, in generalized Schur canonical form. The second dimension of `b` must be at least max(1, `n`).

`q` (`ldq`,*)
If `wantq` = .TRUE., then `q` is an `n`-by-`n` matrix;
If `wantq` = .FALSE., then `q` is not referenced.
The second dimension of `q` must be at least max(1, `n`).

`z` (`ldz`,*)
If `wantz` = .TRUE., then `z` is an `n`-by-`n` matrix;
If `wantz` = .FALSE., then `z` is not referenced.
The second dimension of `z` must be at least max(1, `n`).

`work` is a workspace array, its dimension max(1, `lwork`).

`lda` — INTEGER. The leading dimension of `a`; at least max(1, `n`).

`ldb` — INTEGER. The leading dimension of `b`; at least max(1, `n`).

`ldq` — INTEGER. The leading dimension of `q`; `ldq` ≥ 1.
If `wantq` = .TRUE., then `ldq` ≥ max(1, `n`).

`ldz` — INTEGER. The leading dimension of `z`; `ldz` ≥ 1.
If `wantz` = .TRUE., then `ldz` ≥ max(1, `n`).

`lwork` — INTEGER. The dimension of the array `work`.
For real flavors:
If `ijob` = 1, 2, or 4, `lwork` ≥ max(4`n`+16, 2`m`(`n`-`m`)).
If `ijob` = 3 or 5, `lwork` ≥ max(4`n`+16, 4`m`(`n`-`m`)).
For complex flavors:
If `ijob` = 1, 2, or 4, `lwork` ≥ max(1, 2`m`(`n`-`m`)).
If `ijob` = 3 or 5, `lwork` ≥ max(1, 4`m`(`n`-`m`)).
If `lwork` = -1, then a workspace query is assumed; the routine only calculates the optimal size of the `work` array, returns this value as the first entry of the `work` array, and no error message related to `lwork` is issued by xerbla. See *Application Notes* for details.

`iwork` — INTEGER. Workspace array, its dimension max(1, `liwork`).

`liwork` — INTEGER. The dimension of the array `iwork`.
For real flavors:
If `ijob` = 1, 2, or 4, `liwork` ≥ `n`+6.
If `ijob` = 3 or 5, `liwork` ≥ max(`n`+6, 2`m`(`n`-`m`)).
For complex flavors:
If `ijob` = 1, 2, or 4, `liwork` ≥ `n`+2.
If `ijob` = 3 or 5, `liwork` ≥ max(`n`+2, 2`m`(`n`-`m`)).
If `liwork` = -1, then a workspace query is assumed; the routine only calculates the optimal size of the `iwork` array, returns this value as the first entry of the `iwork` array, and no error message related to `liwork` is issued by xerbla. See *Application Notes* for details.

Output Parameters

`a`, `b` — Overwritten by the reordered matrices `A` and `B`, respectively.

`alphar`, `alphai` — REAL for stgsen;

	DOUBLE PRECISION for dtgsen. Arrays, DIMENSION at least max(1, *n*). Contain values that form generalized eigenvalues in real flavors. See *beta*.
alpha	COMPLEX for ctgsen; DOUBLE COMPLEX for ztgsen. Array, DIMENSION at least max(1, *n*). Contain values that form generalized eigenvalues in complex flavors. See *beta*.
beta	REAL for stgsen DOUBLE PRECISION for dtgsen COMPLEX for ctgsen DOUBLE COMPLEX for ztgsen. Array, DIMENSION at least max(1, *n*). *For real flavors*: On exit, (*alphar*(j) + *alphai*(j)*i)/*beta*(j), j=1,..., *n*, will be the generalized eigenvalues. *alphar*(j) + *alphai*(j)*i and *beta*(j), j=1,..., *n* are the diagonals of the complex Schur form (*S*,*T*) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (*A*,*B*) were further reduced to triangular form using complex unitary transformations. If *alphai*(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with *alphai*(j+1) negative. *For complex flavors:* The diagonal elements of *A* and *B*, respectively, when the pair (*A*,*B*) has been reduced to generalized Schur form. *alpha*(i)/*beta*(i), i=1,..., *n* are the generalized eigenvalues.
q	If *wantq* =.TRUE., then, on exit, *Q* has been postmultiplied by the left orthogonal transformation matrix which reorder (*A*, *B*). The leading *m* columns of *Q* form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces).
z	If *wantz* =.TRUE., then, on exit, *Z* has been postmultiplied by the left orthogonal transformation matrix which reorder (*A*, *B*). The leading *m* columns of *Z* form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces).
m	INTEGER. The dimension of the specified pair of left and right eigen-spaces (deflating subspaces); $0 \le m \le n$.
pl, *pr*	REAL for single precision flavors; DOUBLE PRECISION for double precision flavors. If *ijob* = 1, 4, or 5, *pl* and *pr* are lower bounds on the reciprocal of the norm of "projections" onto left and right eigenspaces with respect to the selected cluster. $0 < pl, pr \le 1$. If $m = 0$ or $m = n$, $pl = pr = 1$. If *ijob* = 0, 2 or 3, *pl* and *pr* are not referenced
dif	REAL for single precision flavors;DOUBLE PRECISION for double precision flavors. Array, DIMENSION (2). If $ijob \ge 2$, *dif*(1:2) store the estimates of Difu and Difl.

	If *ijob* = 2 or 4, *dif*(1:2) are F-norm-based upper bounds on Difu and Difl. If *ijob* = 3 or 5, *dif*(1:2) are 1-norm-based estimates of Difu and Difl. If *m* = 0 or *n*, *dif*(1:2) = F-norm([*A*, *B*]). If *ijob* = 0 or 1, *dif* is not referenced.
work(1)	If *ijob* is not 0 and *info* = 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
iwork(1)	If *ijob* is not 0 and *info* = 0, on exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance. Use this *liwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1, Reordering of (*A*, *B*) failed because the transformed matrix pair (*A*, *B*) would be too far from generalized Schur form; the problem is very ill-conditioned. (*A*, *B*) may have been partially reordered. If requested, 0 is returned in *dif*(*), *pl* and *pr*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tgsen interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
b	Holds the matrix *B* of size (*n*,*n*).
select	Holds the vector of length *n*.
alphar	Holds the vector of length *n*. Used in real flavors only.
alphai	Holds the vector of length *n*. Used in real flavors only.
alpha	Holds the vector of length *n*. Used in complex flavors only.
beta	Holds the vector of length *n*.
q	Holds the matrix *Q* of size (*n*,*n*).
z	Holds the matrix *Z* of size (*n*,*n*).
dif	Holds the vector of length (2).
ijob	Must be 0, 1, 2, 3, 4, or 5. The default value is 0.
wantq	Restored based on the presence of the argument *q* as follows: *wantq* = .TRUE., if *q* is present, *wantq* = .FALSE., if *q* is omitted.
wantz	Restored based on the presence of the argument *z* as follows: *wantz* = .TRUE., if *z* is present, *wantz* = .FALSE., if z is omitted.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?tgsyl

Solves the generalized Sylvester equation.

Syntax

Fortran 77:

```
call stgsyl(trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf, scale,
dif, work, lwork, iwork, info)
```

```
call dtgsyl(trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf, scale,
dif, work, lwork, iwork, info)
```

```
call ctgsyl(trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf, scale,
dif, work, lwork, iwork, info)
```

```
call ztgsyl(trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf, scale,
dif, work, lwork, iwork, info)
```

Fortran 95:

```
call tgsyl(a, b, c, d, e, f [,ijob] [,trans] [,scale] [,dif] [,info])
```

C:

```
lapack_int LAPACKE_stgsyl( int matrix_order, char trans, lapack_int ijob, lapack_int m,
lapack_int n, const float* a, lapack_int lda, const float* b, lapack_int ldb, float*
c, lapack_int ldc, const float* d, lapack_int ldd, const float* e, lapack_int lde,
float* f, lapack_int ldf, float* scale, float* dif );
```

```
lapack_int LAPACKE_dtgsyl( int matrix_order, char trans, lapack_int ijob, lapack_int m,
lapack_int n, const double* a, lapack_int lda, const double* b, lapack_int ldb,
double* c, lapack_int ldc, const double* d, lapack_int ldd, const double* e,
lapack_int lde, double* f, lapack_int ldf, double* scale, double* dif );
```

```
lapack_int LAPACKE_ctgsyl( int matrix_order, char trans, lapack_int ijob, lapack_int m,
lapack_int n, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* c, lapack_int ldc, const
lapack_complex_float* d, lapack_int ldd, const lapack_complex_float* e, lapack_int lde,
lapack_complex_float* f, lapack_int ldf, float* scale, float* dif );
```

```
lapack_int LAPACKE_ztgsyl( int matrix_order, char trans, lapack_int ijob, lapack_int m,
lapack_int n, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* c, lapack_int ldc,
const lapack_complex_double* d, lapack_int ldd, const lapack_complex_double* e,
lapack_int lde, lapack_complex_double* f, lapack_int ldf, double* scale, double* dif );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h

- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves the generalized Sylvester equation:

*A*R-L*B = scale*C*

*D*R-L*E = scale*F*

where *R* and *L* are unknown *m*-by-*n* matrices, (*A*, *D*), (*B*, *E*) and (*C*, *F*) are given matrix pairs of size *m*-by-*m*, *n*-by-*n* and *m*-by-*n*, respectively, with real/complex entries. (*A*, *D*) and (*B*, *E*) must be in generalized real-Schur/Schur canonical form, that is, *A*, *B* are upper quasi-triangular/triangular and *D*, *E* are upper triangular.

The solution (*R*, *L*) overwrites (*C*, *F*). The factor *scale*, $0 \le scale \le 1$, is an output scaling factor chosen to avoid overflow.

In matrix notation the above equation is equivalent to the following: solve *Z*x = scale*b*, where *Z* is defined as

$$Z = \begin{pmatrix} kron(I_n, A) - kron(B^T, I_m) \\ kron(I_n, D) - kron(E^T, I_m) \end{pmatrix}$$

Here I_k is the identity matrix of size *k* and *X*' is the transpose/conjugate-transpose of *X*. *kron(X, Y)* is the Kronecker product between the matrices *X* and *Y*.

If *trans = 'T'* (for real flavors), or *trans = 'C'* (for complex flavors), the routine `?tgsyl` solves the transposed/conjugate-transposed system *Z'*y = scale*b*, which is equivalent to solve for *R* and *L* in

*A'*R+D'*L = scale*C*

*R*B'+L*E' = scale*(-F)*

This case (*trans = 'T'* for `stgsyl`/`dtgsyl` or *trans = 'C'* for `ctgsyl`/`ztgsyl`) is used to compute an one-norm-based estimate of `Dif[(A, D), (B, E)]`, the separation between the matrix pairs (*A*,*D*) and (*B*,*E*), using lacon/lacon.

If *ijob* ≥ 1, `?tgsyl` computes a Frobenius norm-based estimate of `Dif[(A, D), (B,E)]`. That is, the reciprocal of a lower bound on the reciprocal of the smallest singular value of *Z*. This is a level 3 BLAS algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	`CHARACTER*1`. Must be `'N'`, `'T'`, or `'C'`. If *trans = 'N'*, solve the generalized Sylvester equation. If *trans = 'T'*, solve the 'transposed' system (for real flavors only). If *trans = 'C'*, solve the ' conjugate transposed' system (for complex flavors only).
ijob	`INTEGER`. Specifies what kind of functionality to be performed: If *ijob* =0, solve the generalized Sylvester equation only; If *ijob* =1, perform the functionality of *ijob* =0 and *ijob* =3; If *ijob* =2, perform the functionality of *ijob* =0 and *ijob* =4;

	If *ijob* =3, only an estimate of Dif[(*A*, *D*), (*B*, *E*)] is computed (look ahead strategy is used); If *ijob* =4, only an estimate of Dif[(*A*, *D*), (*B*,*E*)] is computed (?gecon on sub-systems is used). If *trans* = 'T' or 'C', *ijob* is not referenced.
m	INTEGER. The order of the matrices *A* and *D*, and the row dimension of the matrices *C*, *F*, *R* and *L*.
n	INTEGER. The order of the matrices *B* and *E*, and the column dimension of the matrices *C*, *F*, *R* and *L*.
a, *b*, *c*, *d*, *e*, *f*, *work*	REAL for stgsyl DOUBLE PRECISION for dtgsyl COMPLEX for ctgsyl DOUBLE COMPLEX for ztgsyl. Arrays: *a*(*lda*,*) contains the upper quasi-triangular (for real flavors) or upper triangular (for complex flavors) matrix *A*. The second dimension of *a* must be at least max(1, *m*). *b*(*ldb*,*) contains the upper quasi-triangular (for real flavors) or upper triangular (for complex flavors) matrix *B*. The second dimension of *b* must be at least max(1, *n*). *c* (*ldc*,*) contains the right-hand-side of the first matrix equation in the generalized Sylvester equation (as defined by *trans*) The second dimension of *c* must be at least max(1, *n*). *d* (*ldd*,*) contains the upper triangular matrix *D*. The second dimension of *d* must be at least max(1, *m*). *e* (*lde*,*) contains the upper triangular matrix *E*. The second dimension of *e* must be at least max(1, *n*). *f* (*ldf*,*) contains the right-hand-side of the second matrix equation in the generalized Sylvester equation (as defined by *trans*) The second dimension of *f* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
ldc	INTEGER. The leading dimension of *c*; at least max(1, *m*).
ldd	INTEGER. The leading dimension of *d*; at least max(1, *m*).
lde	INTEGER. The leading dimension of *e*; at least max(1, *n*).
ldf	INTEGER. The leading dimension of *f*; at least max(1, *m*).
lwork	INTEGER. The dimension of the array *work*. *lwork* ≥ 1. If *ijob* = 1 or 2 and *trans* = 'N', *lwork* ≥ max(1, 2*m*n). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.
iwork	INTEGER. Workspace array, DIMENSION at least (*m*+*n*+6) for real flavors, and at least (*m*+*n*+2) for complex flavors.

Output Parameters

c	If *ijob*=0, 1, or 2, overwritten by the solution *R*. If *ijob*=3 or 4 and *trans* = 'N', *c* holds *R*, the solution achieved during the computation of the Dif-estimate.

f	If *ijob*=0, 1, or 2, overwritten by the solution *L*. If *ijob*=3 or 4 and *trans* = 'N', *f* holds *L*, the solution achieved during the computation of the Dif-estimate.
dif	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. On exit, *dif* is the reciprocal of a lower bound of the reciprocal of the Dif-function, that is, *dif* is an upper bound of Dif[(*A*, *D*), (*B*, *E*)] = sigma_min(*Z*), where *Z* as in (2). If *ijob* = 0, or *trans* = 'T' (for real flavors), or *trans* = 'C' (for complex flavors), *dif* is not touched.
scale	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. On exit, *scale* is the scaling factor in the generalized Sylvester equation. If 0 < *scale* < 1, *c* and *f* hold the solutions *R* and *L*, respectively, to a slightly perturbed system but the input matrices *A*, *B*, *D* and *E* have not been changed. If *scale* = 0, *c* and *f* hold the solutions *R* and *L*, respectively, to the homogeneous system with *C* = *F* = 0. Normally, *scale* = 1.
work(1)	If *info* = 0, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* > 0, (*A*, *D*) and (*B*, *E*) have common or close eigenvalues.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tgsyl interface are the following:

a	Holds the matrix *A* of size (*m*,*m*).
b	Holds the matrix *B* of size (*n*,*n*).
c	Holds the matrix *C* of size (*m*,*n*).
d	Holds the matrix *D* of size (*m*,*m*).
e	Holds the matrix *E* of size (*n*,*n*).
f	Holds the matrix *F* of size (*m*,*n*).
ijob	Must be 0, 1, 2, 3, or 4. The default value is 0.
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?tgsna

Estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a pair of matrices in generalized real Schur canonical form.

Syntax

Fortran 77:

```
call stgsna(job, howmny, select, n, a, lda, b, ldb, vl, ldvl, vr, ldvr, s, dif, mm, m, work, lwork, iwork, info)
call dtgsna(job, howmny, select, n, a, lda, b, ldb, vl, ldvl, vr, ldvr, s, dif, mm, m, work, lwork, iwork, info)
call ctgsna(job, howmny, select, n, a, lda, b, ldb, vl, ldvl, vr, ldvr, s, dif, mm, m, work, lwork, iwork, info)
call ztgsna(job, howmny, select, n, a, lda, b, ldb, vl, ldvl, vr, ldvr, s, dif, mm, m, work, lwork, iwork, info)
```

Fortran 95:

```
call tgsna(a, b [,s] [,dif] [,vl] [,vr] [,select] [,m] [,info])
```

C:

```
lapack_int LAPACKE_stgsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const float* a, lapack_int lda, const float* b, lapack_int ldb, const float* vl, lapack_int ldvl, const float* vr, lapack_int ldvr, float* s, float* dif, lapack_int mm, lapack_int* m );
lapack_int LAPACKE_dtgsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const double* a, lapack_int lda, const double* b, lapack_int ldb, const double* vl, lapack_int ldvl, const double* vr, lapack_int ldvr, double* s, double* dif, lapack_int mm, lapack_int* m );
lapack_int LAPACKE_ctgsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* b, lapack_int ldb, const lapack_complex_float* vl, lapack_int ldvl, const lapack_complex_float* vr, lapack_int ldvr, float* s, float* dif, lapack_int mm, lapack_int* m );
lapack_int LAPACKE_ztgsna( int matrix_order, char job, char howmny, const lapack_logical* select, lapack_int n, const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* b, lapack_int ldb, const lapack_complex_double* vl, lapack_int ldvl, const lapack_complex_double* vr, lapack_int ldvr, double* s, double* dif, lapack_int mm, lapack_int* m );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The real flavors stgsna/dtgsna of this routine estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (*A*, *B*) in generalized real Schur canonical form (or of any matrix pair ($Q*A*Z^T$, $Q*B*Z^T$) with orthogonal matrices *Q* and *Z*.

(*A*, *B*) must be in generalized real Schur form (as returned by gges/gges), that is, *A* is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

The complex flavors ctgsna/ztgsna estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (*A*, *B*). (*A*, *B*) must be in generalized Schur canonical form, that is, *A* and *B* are both upper triangular.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

job — CHARACTER*1. Specifies whether condition numbers are required for eigenvalues or eigenvectors. Must be 'E' or 'V' or 'B'.
If *job* = 'E', for eigenvalues only (compute *s*).
If *job* = 'V', for eigenvectors only (compute *dif*).
If *job* = 'B', for both eigenvalues and eigenvectors (compute both *s* and *dif*).

howmny — CHARACTER*1. Must be 'A' or 'S'.
If *howmny* = 'A', compute condition numbers for all eigenpairs.
If *howmny* = 'S', compute condition numbers for selected eigenpairs specified by the logical array *select*.

select — LOGICAL.
Array, DIMENSION at least max (1, *n*).
If *howmny* = 'S', *select* specifies the eigenpairs for which condition numbers are required.
If *howmny* = 'A', *select* is not referenced.
For real flavors:
To select condition numbers for the eigenpair corresponding to a real eigenvalue omega(*j*), *select*(*j*) must be set to .TRUE.; to select condition numbers corresponding to a complex conjugate pair of eigenvalues omega(*j*) and omega(*j*+1), either *select*(*j*) or *select*(*j*+1) must be set to .TRUE.
For complex flavors:
To select condition numbers for the corresponding *j*-th eigenvalue and/or eigenvector, *select*(*j*) must be set to .TRUE..

n — INTEGER. The order of the square matrix pair (*A*, *B*)
(*n* ≥ 0).

a, *b*, *vl*, *vr*, *work* — REAL for stgsna
DOUBLE PRECISION for dtgsna
COMPLEX for ctgsna
DOUBLE COMPLEX for ztgsna.
Arrays:
a(*lda*,*) contains the upper quasi-triangular (for real flavors) or upper triangular (for complex flavors) matrix *A* in the pair (*A*, *B*).
The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the upper triangular matrix *B* in the pair (*A*, *B*). The second dimension of *b* must be at least max(1, *n*).

If *job* = 'E' or 'B', *vl*(*ldvl*,*) must contain left eigenvectors of (*A*, *B*), corresponding to the eigenpairs specified by *howmny* and *select*. The eigenvectors must be stored in consecutive columns of *vl*, as returned by ?tgevc.
If *job* = 'V', *vl* is not referenced. The second dimension of *vl* must be at least max(1, *m*).
If *job* = 'E' or 'B', *vr*(*ldvr*,*) must contain right eigenvectors of (*A*, *B*), corresponding to the eigenpairs specified by *howmny* and *select*. The eigenvectors must be stored in consecutive columns of *vr*, as returned by ?tgevc.
If *job* = 'V', *vr* is not referenced. The second dimension of *vr* must be at least max(1, *m*).
work is a workspace array, its dimension max(1, *lwork*).
If *job* = 'E', *work* is not referenced.

lda — INTEGER. The leading dimension of *a*; at least max(1, *n*).

ldb — INTEGER. The leading dimension of *b*; at least max(1, *n*).

ldvl — INTEGER. The leading dimension of *vl*; *ldvl* ≥ 1.
If *job* = 'E' or 'B', then *ldvl* ≥ max(1, *n*).

ldvr — INTEGER. The leading dimension of *vr*; *ldvr* ≥ 1.
If *job* = 'E' or 'B', then *ldvr* ≥ max(1, *n*).

mm — INTEGER. The number of elements in the arrays *s* and *dif* (*mm* ≥ *m*).

lwork — INTEGER. The dimension of the array *work*.
lwork ≥ max(1, *n*).
If *job* = 'V' or 'B', *lwork* ≥ 2**n**(*n*+2)+16 for real flavors, and *lwork* ≥ max(1, 2**n**n) for complex flavors.
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER. Workspace array, DIMENSION at least (*n*+6) for real flavors, and at least (*n*+2) for complex flavors.
If *job* = 'E', *iwork* is not referenced.

Output Parameters

s — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Array, DIMENSION (*mm*).
If *job* = 'E' or 'B', contains the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array.
If *job* = 'V', *s* is not referenced.
For real flavors:
For a complex conjugate pair of eigenvalues two consecutive elements of *s* are set to the same value. Thus, *s*(j), *dif*(j), and the j-th columns of *vl* and *vr* all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected).

dif — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Array, DIMENSION (*mm*).
If *job* = 'V' or 'B', contains the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array.

	If the eigenvalues cannot be reordered to compute *dif*(j), *dif*(j) is set to 0; this can only occur when the true value would be very small anyway. If *job* = 'E', *dif* is not referenced. *For real flavors*: For a complex eigenvector, two consecutive elements of *dif* are set to the same value. *For complex flavors*: For each eigenvalue/vector specified by *select*, *dif* stores a Frobenius norm-based estimate of Difl.
m	INTEGER. The number of elements in the arrays *s* and *dif* used to store the specified condition numbers; for each selected eigenvalue one element is used. If *howmny* = 'A', *m* is set to *n*.
work(1)	*work*(1) If *job* is not 'E' and *info* = 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tgsna interface are the following:

a	Holds the matrix *A* of size (*n*,*n*).
b	Holds the matrix *B* of size (*n*,*n*).
s	Holds the vector of length (*mm*).
dif	Holds the vector of length (*mm*).
vl	Holds the matrix *VL* of size (*n*,*mm*).
vr	Holds the matrix *VR* of size (*n*,*mm*).
select	Holds the vector of length *n*.
howmny	Restored based on the presence of the argument *select* as follows: *howmny* = 'S', if *select* is present, *howmny* = 'A', if *select* is omitted.
job	Restored based on the presence of arguments *s* and dif as follows: *job* = 'B', if both *s* and *dif* are present, *job* = 'E', if *s* is present and *dif* omitted, *job* = 'V', if *s* is omitted and *dif* present, Note that there will be an error condition if both *s* and *dif* are omitted.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Generalized Singular Value Decomposition

This section describes LAPACK computational routines used for finding the generalized singular value decomposition (GSVD) of two matrices *A* and *B* as

$U^H A Q = D_1 * (0 \ R)$,

$V^H B Q = D_2 * (0 \ R)$,

where *U*, *V*, and *Q* are orthogonal/unitary matrices, *R* is a nonsingular upper triangular matrix, and D_1, D_2 are "diagonal" matrices of the structure detailed in the routines description section.

Table "Computational Routines for Generalized Singular Value Decomposition" lists LAPACK routines (FORTRAN 77 interface) that perform generalized singular value decomposition of matrices. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Generalized Singular Value Decomposition

Routine name	Operation performed
ggsvp	Computes the preprocessing decomposition for the generalized SVD
tgsja	Computes the generalized SVD of two upper triangular or trapezoidal matrices

You can use routines listed in the above table as well as the driver routine ggsvd to find the GSVD of a pair of general rectangular matrices.

?ggsvp

Computes the preprocessing decomposition for the generalized SVD.

Syntax

Fortran 77:

```
call sggsvp(jobu, jobv, jobq, m, p, n, a, lda, b, ldb, tola, tolb, k, l, u, ldu, v,
ldv, q, ldq, iwork, tau, work, info)
call dggsvp(jobu, jobv, jobq, m, p, n, a, lda, b, ldb, tola, tolb, k, l, u, ldu, v,
ldv, q, ldq, iwork, tau, work, info)
call cggsvp(jobu, jobv, jobq, m, p, n, a, lda, b, ldb, tola, tolb, k, l, u, ldu, v,
ldv, q, ldq, iwork, rwork, tau, work, info)
call zggsvp(jobu, jobv, jobq, m, p, n, a, lda, b, ldb, tola, tolb, k, l, u, ldu, v,
ldv, q, ldq, iwork, rwork, tau, work, info)
```

Fortran 95:

```
call ggsvp(a, b, tola, tolb [, k] [,l] [,u] [,v] [,q] [,info])
```

C:

```
lapack_int LAPACKE_sggsvp( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, float* a, lapack_int lda, float* b,
lapack_int ldb, float tola, float tolb, lapack_int* k, lapack_int* l, float* u,
lapack_int ldu, float* v, lapack_int ldv, float* q, lapack_int ldq );
```

```
lapack_int LAPACKE_dggsvp( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, double* a, lapack_int lda, double* b,
lapack_int ldb, double tola, double tolb, lapack_int* k, lapack_int* l, double* u,
lapack_int ldu, double* v, lapack_int ldv, double* q, lapack_int ldq );

lapack_int LAPACKE_cggsvp( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_complex_float* a, lapack_int lda,
lapack_complex_float* b, lapack_int ldb, float tola, float tolb, lapack_int* k,
lapack_int* l, lapack_complex_float* u, lapack_int ldu, lapack_complex_float* v,
lapack_int ldv, lapack_complex_float* q, lapack_int ldq );

lapack_int LAPACKE_zggsvp( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_complex_double* b, lapack_int ldb, double tola, double tolb, lapack_int* k,
lapack_int* l, lapack_complex_double* u, lapack_int ldu, lapack_complex_double* v,
lapack_int ldv, lapack_complex_double* q, lapack_int ldq );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes orthogonal matrices U, V and Q such that

$$= \begin{array}{c} \\ k \\ m-k \end{array} \begin{array}{c} \begin{array}{ccc} n-k-l & k & l \end{array} \\ \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \end{pmatrix} \end{array}, \quad \text{if } m-k-l < 0$$

where the k-by-k matrix A_{12} and l-by-l matrix B_{13} are nonsingular upper triangular; A_{23} is l-by-l upper triangular if $m-k-l \geq 0$, otherwise A_{23} is $(m-k)$-by-l upper trapezoidal. The sum $k+l$ is equal to the effective numerical rank of the $(m+p)$-by-n matrix $(A^H, B^H)^H$.

This decomposition is the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see subroutine ggsvp.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobu	CHARACTER*1. Must be 'U' or 'N'. If *jobu* = 'U', orthogonal/unitary matrix *U* is computed. If *jobu* = 'N', *U* is not computed.
jobv	CHARACTER*1. Must be 'V' or 'N'. If *jobv* = 'V', orthogonal/unitary matrix *V* is computed. If *jobv* = 'N', *V* is not computed.
jobq	CHARACTER*1. Must be 'Q' or 'N'. If *jobq* = 'Q', orthogonal/unitary matrix *Q* is computed. If *jobq* = 'N', *Q* is not computed.
m	INTEGER. The number of rows of the matrix *A* ($m \geq 0$).
p	INTEGER. The number of rows of the matrix *B* ($p \geq 0$).
n	INTEGER. The number of columns of the matrices *A* and *B* ($n \geq 0$).
a, *b*, *tau*, *work*	REAL for sggsvp DOUBLE PRECISION for dggsvp COMPLEX for cggsvp DOUBLE COMPLEX for zggsvp. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the *p*-by-*n* matrix *B*. The second dimension of *b* must be at least max(1, *n*). *tau*(*) is a workspace array. The dimension of *tau* must be at least max(1, *n*). *work*(*) is a workspace array. The dimension of *work* must be at least max(1, 3*n*, *m*, *p*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *p*).
tola, *tolb*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. *tola* and *tolb* are the thresholds to determine the effective numerical rank of matrix *B* and a subblock of *A*. Generally, they are set to *tola* = max(*m*, *n*)*||*A*||*MACHEPS, *tolb* = max(*p*, *n*)*||*B*||*MACHEPS. The size of *tola* and *tolb* may affect the size of backward errors of the decomposition.
ldu	INTEGER. The leading dimension of the output array *u*. *ldu* ≥ max(1, *m*) if *jobu* = 'U'; *ldu* ≥ 1 otherwise.
ldv	INTEGER. The leading dimension of the output array *v*. *ldv* ≥ max(1, *p*) if *jobv* = 'V'; *ldv* ≥ 1 otherwise.
ldq	INTEGER. The leading dimension of the output array *q*. *ldq* ≥ max(1, *n*) if *jobq* = 'Q'; *ldq* ≥ 1 otherwise.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).

rwork — REAL for cggsvp
DOUBLE PRECISION for zggsvp.
Workspace array, DIMENSION at least max(1, 2*n*). Used in complex flavors only.

Output Parameters

a — Overwritten by the triangular (or trapezoidal) matrix described in the *Description* section.

b — Overwritten by the triangular matrix described in the *Description* section.

k, *l* — INTEGER. On exit, *k* and *l* specify the dimension of subblocks. The sum *k* + *l* is equal to effective numerical rank of $(A^H, B^H)^H$.

u, *v*, *q* — REAL for sggsvp
DOUBLE PRECISION for dggsvp
COMPLEX for cggsvp
DOUBLE COMPLEX for zggsvp.
Arrays:
If *jobu* = 'U', *u*(*ldu*,*) contains the orthogonal/unitary matrix *U*.
The second dimension of *u* must be at least max(1, *m*).
If *jobu* = 'N', *u* is not referenced.
If *jobv* = 'V', *v*(*ldv*,*) contains the orthogonal/unitary matrix *V*.
The second dimension of *v* must be at least max(1, *m*).
If *jobv* = 'N', *v* is not referenced.
If *jobq* = 'Q', *q*(*ldq*,*) contains the orthogonal/unitary matrix *Q*.
The second dimension of *q* must be at least max(1, *n*).
If *jobq* = 'N', *q* is not referenced.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggsvp interface are the following:

a — Holds the matrix *A* of size (*m*,*n*).

b — Holds the matrix *B* of size (*p*,*n*).

u — Holds the matrix *U* of size (*m*,*m*).

v — Holds the matrix *V* of size (*p*,*m*).

q — Holds the matrix *Q* of size (*n*,*n*).

jobu — Restored based on the presence of the argument *u* as follows:
jobu = 'U', if *u* is present,
jobu = 'N', if *u* is omitted.

jobv — Restored based on the presence of the argument *v* as follows:
jobz = 'V', if *v* is present,
jobz = 'N', if *v* is omitted.

jobq — Restored based on the presence of the argument *q* as follows:
jobz = 'Q', if *q* is present,
jobz = 'N', if *q* is omitted.

?tgsja

Computes the generalized SVD of two upper triangular or trapezoidal matrices.

Syntax

Fortran 77:

```
call stgsja(jobu, jobv, jobq, m, p, n, k, l, a, lda, b, ldb, tola, tolb, alpha, beta,
u, ldu, v, ldv, q, ldq, work, ncycle, info)

call dtgsja(jobu, jobv, jobq, m, p, n, k, l, a, lda, b, ldb, tola, tolb, alpha, beta,
u, ldu, v, ldv, q, ldq, work, ncycle, info)

call ctgsja(jobu, jobv, jobq, m, p, n, k, l, a, lda, b, ldb, tola, tolb, alpha, beta,
u, ldu, v, ldv, q, ldq, work, ncycle, info)

call ztgsja(jobu, jobv, jobq, m, p, n, k, l, a, lda, b, ldb, tola, tolb, alpha, beta,
u, ldu, v, ldv, q, ldq, work, ncycle, info)
```

Fortran 95:

```
call tgsja(a, b, tola, tolb, k, l [,u] [,v] [,q] [,jobu] [,jobv] [,jobq] [,alpha]
[,beta] [,ncycle] [,info])
```

C:

```
lapack_int LAPACKE_stgsja( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l, float* a,
lapack_int lda, float* b, lapack_int ldb, float tola, float tolb, float* alpha, float*
beta, float* u, lapack_int ldu, float* v, lapack_int ldv, float* q, lapack_int ldq,
lapack_int* ncycle );

lapack_int LAPACKE_dtgsja( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l, double* a,
lapack_int lda, double* b, lapack_int ldb, double tola, double tolb, double* alpha,
double* beta, double* u, lapack_int ldu, double* v, lapack_int ldv, double* q,
lapack_int ldq, lapack_int* ncycle );

lapack_int LAPACKE_ctgsja( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, float
tola, float tolb, float* alpha, float* beta, lapack_complex_float* u, lapack_int ldu,
lapack_complex_float* v, lapack_int ldv, lapack_complex_float* q, lapack_int ldq,
lapack_int* ncycle );

lapack_int LAPACKE_ztgsja( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
double tola, double tolb, double* alpha, double* beta, lapack_complex_double* u,
lapack_int ldu, lapack_complex_double* v, lapack_int ldv, lapack_complex_double* q,
lapack_int ldq, lapack_int* ncycle );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the generalized singular value decomposition (GSVD) of two real/complex upper triangular (or trapezoidal) matrices A and B. On entry, it is assumed that matrices A and B have the following forms, which may be obtained by the preprocessing subroutine ggsvp from a general m-by-n matrix A and p-by-n matrix B:

$$A = \begin{array}{c} \\ k \\ l \\ m-k-l \end{array} \begin{array}{c} \begin{array}{ccc} n-k-l & k & l \end{array} \\ \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & 0 \end{pmatrix} \end{array}, \quad \text{if } m-k-l \geq 0$$

[illegible]

where the k-by-k matrix A_{12} and l-by-l matrix B_{13} are nonsingular upper triangular; A_{23} is l-by-l upper triangular if $m-k-l \geq 0$, otherwise A_{23} is $(m-k)$-by-l upper trapezoidal.

On exit,

$U^H*A*Q = D_1*(0\ R)$, $V^H*B*Q = D_2*(0\ R)$,

where U, V and Q are orthogonal/unitary matrices, R is a nonsingular upper triangular matrix, and D_1 and D_2 are "diagonal" matrices, which are of the following structures:

If $m-k-l \geq 0$,

$$D_1 = \begin{array}{c} \\ k \\ l \\ m-k-l \end{array} \begin{array}{c} \begin{array}{cc} k & l \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & C \\ 0 & 0 \end{pmatrix} \end{array}$$

$$(0\ R) = \begin{array}{c} \\ k \\ l \end{array} \begin{array}{c} \begin{array}{ccc} n-k-l & k & l \end{array} \\ \begin{pmatrix} 0 & R_{11} & R_{12} \\ 0 & 0 & R_{22} \end{pmatrix} \end{array}$$

where

$C = \mathrm{diag}(alpha(k+1),...,alpha(k+l))$

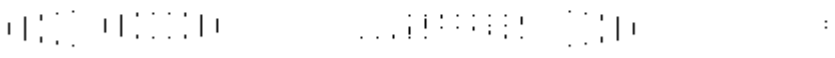
$S = \mathrm{diag}(beta(k+1),...,beta(k+l))$

$C^2 + S^2 = I$

R is stored in $a(1{:}k+l,\ n-k-l+1{:}n)$ on exit.

If $m-k-l < 0$,

$$D_1 = \begin{array}{c} \\ k \\ m-k \end{array} \begin{array}{c} \begin{array}{ccc} k & m-l & k+l-m \end{array} \\ \begin{pmatrix} I & 0 & 0 \\ 0 & C & 0 \end{pmatrix} \end{array}$$



where

C = diag($alpha(K+1)$,...,$alpha(m)$),

S = diag($beta(K+1)$,...,$beta(m)$),

$C^2 + S^2 = I$

On exit, $\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ 0 & R_{22} & R_{23} \end{pmatrix}$ is stored in $a(1{:}m, n{-}k{-}l{+}1{:}n)$ and R_{33} is stored in $b(m{-}k{+}1{:}l, n{+}m{-}k{-}l{+}1{:}n)$.

The computation of the orthogonal/unitary transformation matrices U, V or Q is optional. These matrices may either be formed explicitly, or they *may* be postmultiplied into input matrices U_1, V_1, or Q_1.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobu	CHARACTER*1. Must be 'U', 'I', or 'N'. If *jobu* = 'U', *u* must contain an orthogonal/unitary matrix U_1 on entry. If *jobu* = 'I', *u* is initialized to the unit matrix. If *jobu* = 'N', *u* is not computed.
jobv	CHARACTER*1. Must be 'V', 'I', or 'N'. If *jobv* = 'V', *v* must contain an orthogonal/unitary matrix V_1 on entry. If *jobv* = 'I', *v* is initialized to the unit matrix. If *jobv* = 'N', *v* is not computed.
jobq	CHARACTER*1. Must be 'Q', 'I', or 'N'. If *jobq* = 'Q', *q* must contain an orthogonal/unitary matrix Q_1 on entry. If *jobq* = 'I', *q* is initialized to the unit matrix. If *jobq* = 'N', *q* is not computed.
m	INTEGER. The number of rows of the matrix *A* ($m \geq 0$).
p	INTEGER. The number of rows of the matrix *B* ($p \geq 0$).
n	INTEGER. The number of columns of the matrices *A* and *B* ($n \geq 0$).
k, *l*	INTEGER. Specify the subblocks in the input matrices *A* and *B*, whose GSVD is computed.
a,*b*,*u*,*v*,*q*,*work*	REAL for stgsja DOUBLE PRECISION for dtgsja COMPLEX for ctgsja DOUBLE COMPLEX for ztgsja. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the *p*-by-*n* matrix *B*. The second dimension of *b* must be at least max(1, *n*). If *jobu* = 'U', *u*(*ldu*,*) must contain a matrix U_1 (usually the orthogonal/unitary matrix returned by ?ggsvp). The second dimension of *u* must be at least max(1, *m*). If *jobv* = 'V', *v*(*ldv*,*) must contain a matrix V_1 (usually the orthogonal/unitary matrix returned by ?ggsvp). The second dimension of *v* must be at least max(1, *p*).

If *jobq* = 'Q', *q*(*ldq*,*) must contain a matrix Q_1 (usually the orthogonal/unitary matrix returned by ?ggsvp).
The second dimension of *q* must be at least max(1, *n*).
work(*) is a workspace array.
The dimension of *work* must be at least max(1, 2*n*).

lda — INTEGER. The leading dimension of *a*; at least max(1, *m*).

ldb — INTEGER. The leading dimension of *b*; at least max(1, *p*).

ldu — INTEGER. The leading dimension of the array *u* .
ldu ≥ max(1, *m*) if *jobu* = 'U'; *ldu* ≥ 1 otherwise.

ldv — INTEGER. The leading dimension of the array *v* .
ldv ≥ max(1, *p*) if *jobv* = 'V'; *ldv* ≥ 1 otherwise.

ldq — INTEGER. The leading dimension of the array *q* .
ldq ≥ max(1, *n*) if *jobq* = 'Q'; *ldq* ≥ 1 otherwise.

tola, *tolb* — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
tola and *tolb* are the convergence criteria for the Jacobi-Kogbetliantz iteration procedure. Generally, they are the same as used in ?ggsvp:
tola = max(*m*, *n*)*|*A*|*MACHEPS,
tolb = max(*p*, *n*)*|*B*|*MACHEPS.

Output Parameters

a — On exit, *a*(*n*-*k*+1:*n*, 1:min(*k*+*l*, *m*)) contains the triangular matrix *R* or part of *R*.

b — On exit, if necessary, *b*(*m*-*k*+1: *l*, *n*+*m*-*k*-*l*+1: *n*)) contains a part of *R*.

alpha, *beta* — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Arrays, DIMENSION at least max(1, *n*). Contain the generalized singular value pairs of *A* and *B*:
alpha(1:*k*) = 1,
beta(1:*k*) = 0,
and if *m*-*k*-*l* ≥ 0,
alpha(*k*+1:*k*+*l*) = diag(*C*),
beta(*k*+1:*k*+*l*) = diag(*S*),
or if *m*-*k*-*l* < 0,
alpha(*k*+1:*m*)= *C*, *alpha*(*m*+1:*k*+*l*)=0
beta(*K*+1:*M*) = *S*,
beta(*m*+1:*k*+*l*) = 1.
Furthermore, if *k*+*l* < *n*,
alpha(*k*+*l*+1:*n*)= 0 and
beta(*k*+*l*+1:*n*) = 0.

u — If *jobu* = 'I', *u* contains the orthogonal/unitary matrix *U*.
If *jobu* = 'U', *u* contains the product U_1*U.
If *jobu* = 'N', *u* is not referenced.

v — If *jobv* = 'I', *v* contains the orthogonal/unitary matrix *U*.
If *jobv* = 'V', *v* contains the product V_1*V.
If *jobv* = 'N', *v* is not referenced.

q — If *jobq* = 'I', *q* contains the orthogonal/unitary matrix *U*.
If *jobq* = 'Q', *q* contains the product Q_1*Q.
If *jobq* = 'N', *q* is not referenced.

ncycle	INTEGER. The number of cycles required for convergence.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *-i*, the *i*-th parameter had an illegal value. If *info* = 1, the procedure does not converge after MAXIT cycles.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine tgsja interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
b	Holds the matrix *B* of size (*p*,*n*).
u	Holds the matrix *U* of size (*m*,*m*).
v	Holds the matrix *V* of size (*p*,*p*).
q	Holds the matrix *Q* of size (*n*,*n*).
alpha	Holds the vector of length *n*.
beta	Holds the vector of length *n*.
jobu	If omitted, this argument is restored based on the presence of argument *u* as follows: *jobu* = 'U', if *u* is present, *jobu* = 'N', if *u* is omitted. If present, *jobu* must be equal to 'I' or 'U' and the argument *u* must also be present. Note that there will be an error condition if *jobu* is present and *u* omitted.
jobv	If omitted, this argument is restored based on the presence of argument *v* as follows: *jobv* = 'V', if *v* is present, *jobv* = 'N', if *v* is omitted. If present, *jobv* must be equal to 'I' or 'V' and the argument *v* must also be present. Note that there will be an error condition if *jobv* is present and *v* omitted.
jobq	If omitted, this argument is restored based on the presence of argument *q* as follows: *jobq* = 'Q', if *q* is present, *jobq* = 'N', if *q* is omitted. If present, *jobq* must be equal to 'I' or 'Q' and the argument *q* must also be present. Note that there will be an error condition if *jobq* is present and *q* omitted.

Cosine-Sine Decomposition

This section describes LAPACK computational routines for computing the *cosine-sine decomposition* (CS decomposition) of a partitioned unitary/orthogonal matrix. The algorithm computes a complete 2-by-2 CS decomposition, which requires simultaneous diagonalization of all the four blocks of a unitary/orthogonal matrix partitioned into a 2-by-2 block structure.

The computation has the following phases:

1. The matrix is reduced to a bidiagonal block form.
2. The blocks are simultaneously diagonalized using techniques from the bidiagonal SVD algorithms.

Table "Computational Routines for Cosine-Sine Decomposition (CSD)" lists LAPACK routines (FORTRAN 77 interface) that perform CS decomposition of matrices. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Cosine-Sine Decomposition (CSD)

Operation	Real matrices	Complex matrices
Compute the CS decomposition of an orthogonal/unitary matrix in bidiagonal-block form	bbcsd/bbcsd	bbcsd/bbcsd
Simultaneously bidiagonalize the blocks of a partitioned orthogonal matrix	orbdb unbdb	
Simultaneously bidiagonalize the blocks of a partitioned unitary matrix		orbdb unbdb

See Also

Cosine-Sine Decomposition

?bbcsd

Computes the CS decomposition of an orthogonal/ unitary matrix in bidiagonal-block form.

Syntax

Fortran 77:

```
call sbbcsd( jobu1, jobu2, jobv1t, jobv2t, trans, m, p, q, theta, phi, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t, b11d, b11e, b12d, b12e, b21d, b21e, b21e, b22e, work, lwork, info )

call dbbcsd( jobu1, jobu2, jobv1t, jobv2t, trans, m, p, q, theta, phi, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t, b11d, b11e, b12d, b12e, b21d, b21e, b21e, b22e, work, lwork, info )

call cbbcsd( jobu1, jobu2, jobv1t, jobv2t, trans, m, p, q, theta, phi, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t, b11d, b11e, b12d, b12e, b21d, b21e, b21e, b22e, rwork, rlwork, info )

call zbbcsd( jobu1, jobu2, jobv1t, jobv2t, trans, m, p, q, theta, phi, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t, b11d, b11e, b12d, b12e, b21d, b21e, b21e, b22e, rwork, rlwork, info )
```

Fortran 95:

```
call bbcsd( theta,phi,u1,u2,v1t,v2t[,b11d][,b11e][,b12d][,b12e][,b21d][,b21e][,b22d][,b22e][,jobu1][,jobu2][,jobv1t][,jobv2t][,trans][,info] )
```

C:

```
lapack_int LAPACKE_sbbcsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, float* theta, float* phi, float* u1, lapack_int ldu1, float* u2, lapack_int ldu2, float* v1t, lapack_int ldv1t, float* v2t, lapack_int ldv2t, float* b11d, float* b11e, float* b12d, float* b12e, float* b21d, float* b21e, float* b22d, float* b22e );

lapack_int LAPACKE_dbbcsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, double* theta, double* phi, double* u1, lapack_int ldu1, double* u2, lapack_int ldu2, double* v1t, lapack_int ldv1t, double* v2t, lapack_int ldv2t, double* b11d, double* b11e, double* b12d, double* b12e, double* b21d, double* b21e, double* b22d, double* b22e );
```

```
lapack_int LAPACKE_cbbcsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char
jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, float* theta, float*
phi, lapack_complex_float* u1, lapack_int ldu1, lapack_complex_float* u2, lapack_int
ldu2, lapack_complex_float* v1t, lapack_int ldv1t, lapack_complex_float* v2t,
lapack_int ldv2t, float* b11d, float* b11e, float* b12d, float* b12e, float* b21d,
float* b21e, float* b22d, float* b22e );

lapack_int LAPACKE_zbbcsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char
jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, double* theta, double*
phi, lapack_complex_double* u1, lapack_int ldu1, lapack_complex_double* u2, lapack_int
ldu2, lapack_complex_double* v1t, lapack_int ldv1t, lapack_complex_double* v2t,
lapack_int ldv2t, double* b11d, double* b11e, double* b12d, double* b12e, double*
b21d, double* b21e, double* b22d, double* b22e );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

`mkl_lapack.fi`The routine `?bbcsd` computes the CS decomposition of an orthogonal or unitary matrix in bidiagonal-block form:

$$X=\begin{pmatrix} b_{11}|b_{12} & 0 & 0 \\ 0\ |\ 0 & -I & 0 \\ \hline b_{21}|b_{22} & 0 & 0 \\ 0\ |\ 0 & 0 & I \end{pmatrix}=\begin{pmatrix} u_1 & | & \\ & | & u_2 \end{pmatrix}\begin{pmatrix} C & |-S & 0 & 0 \\ 0 & |\ 0 & -I & 0 \\ \hline S & |\ C & 0 & 0 \\ 0 & |\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} v_1 & | & \\ & | & v_2 \end{pmatrix}^T$$

or

$$X=\begin{pmatrix} b_{11}|b_{12} & 0 & 0 \\ 0\ |\ 0 & -I & 0 \\ \hline b_{21}|b_{22} & 0 & 0 \\ 0\ |\ 0 & 0 & I \end{pmatrix}=\begin{pmatrix} u_1 & | & \\ & | & u_2 \end{pmatrix}\begin{pmatrix} C & |-S & 0 & 0 \\ 0 & |\ 0 & -I & 0 \\ \hline S & |\ C & 0 & 0 \\ 0 & |\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} v_1 & | & \\ & | & v_2 \end{pmatrix}^H$$

respectively.

x is m-by-m with the top-left block p-by-q. Note that q must not be larger than p, m-p, or m-q. If q is not the smallest index, x must be transposed and/or permuted in constant time using the *trans* option. See `?orcsd/?uncsd` for details.

The bidiagonal matrices b_{11}, b_{12}, b_{21}, and b_{22} are represented implicitly by angles *theta*`(1:`q`)` and *phi*`(1:`q`-1)`.

The orthogonal/unitary matrices u_1, u_2, ${v_1}^t$, and ${v_2}^t$ are input/output. The input matrices are pre- or post-multiplied by the appropriate singular vector matrices.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobu1 CHARACTER. If equals *Y*, then u_1 is updated. Otherwise, u_1 is not updated.

jobu2 CHARACTER. If equals *Y*, then u_2 is updated. Otherwise, u_2 is not updated.

jobv1t CHARACTER. If equals *Y*, then v_1^t is updated. Otherwise, v_1^t is not updated.

jobv2t CHARACTER. If equals *Y*, then v_2^t is updated. Otherwise, v_2^t is not updated.

trans CHARACTER

= 'T':	x, u_1, u_2, v_1^t, v_2^t are stored in row-major order.
otherwise	x, u_1, u_2, v_1^t, v_2^t are stored in column-major order.

m INTEGER. The number of rows and columns of the orthogonal/unitary matrix *X* in bidiagonal-block form.

p INTEGER. The number of rows in the top-left block of *x*. 0 ? *p* ? *m*.

q INTEGER. The number of columns in the top-left block of *x*. 0 ? *q* ? min(*p*,*m*-*p*,*m*-*q*).

theta REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*q*).
On entry, the angles *theta*(1), ..., *theta*(*q*) that, along with *phi*(1), ..., *phi*(*q*-1), define the matrix in bidiagonal-block form.

phi REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*q*-1).
The angles *phi*(1), ..., *phi*(*q*-1) that, along with *theta*(1), ..., *theta*(*q*), define the matrix in bidiagonal-block form.

u1 REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*ldu1*,*p*).
On entry, an *ldu1*-by-*p* matrix.

ldu1 INTEGER. The leading dimension of the array u_1.

u2 REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*ldu2*,*m*-*p*).
On entry, an *ldu2*-by-(*m*-*p*) matrix.

ldu2 INTEGER. The leading dimension of the array u_2.

v1t REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*ldv1t*,*q*).
On entry, an *ldv1t*-by-*q* matrix.

ldv1t INTEGER. The leading dimension of the array *v1t*.

v2t
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*ldv2t*,*m*-*q*).
On entry, an *ldv2t*-by-(*m*-*q*) matrix.

ldv2t
INTEGER. The leading dimension of the array *v2t*.

work
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Workspace array, DIMENSION (max(1,*lwork*)).

lwork
INTEGER. The size of the *work* array. *lwork* ? max(1,8*q)
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

Output Parameters

theta
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
On exit, the angles whose cosines and sines define th edaigonal blocks in the CS decomposition.

u1
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
On exit, *u1* is postmultiplied by the left singular vector matrix common to [b11 ; 0] and [b12 0 0 ; 0 -I 0 0].

u2
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
On exit, *u2* is postmultiplied by the left singular vector matrix common to [b21 ; 0] and [b22 0 0 ; 0 0 I].

v1t
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
Array, DIMENSION (*q*).
On exit, *v1t* is premultiplied by the transpose of the right singular vector matrix common to [b11 ; 0] and [b21 ; 0].

v2t
REAL for sbbcsd
DOUBLE PRECISION for dbbcsd
COMPLEX for cbbcsd
DOUBLE COMPLEX for zbbcsd
On exit, *v2t* is premultiplied by the transpose of the right singular vector matrix common to [b12 0 0 ; 0 -I 0] and [b22 0 0 ; 0 0 I].

b11d	REAL for sbbcsd DOUBLE PRECISION for dbbcsd COMPLEX for cbbcsd DOUBLE COMPLEX for zbbcsd Array, DIMENSION (*q*). When ?bbcsd converges, *b11d* contains the cosines of *theta*(1), ..., *theta*(*q*). If ?bbcsd fails to converge, *b11d* contains the diagonal of the partially reduced top left block.
b11e	REAL for sbbcsd DOUBLE PRECISION for dbbcsd COMPLEX for cbbcsd DOUBLE COMPLEX for zbbcsd Array, DIMENSION (*q*-1). When ?bbcsd converges, *b11e* contains zeros. If ?bbcsd fails to converge, *b11e* contains the superdiagonal of the partially reduced top left block.
b12d	REAL for sbbcsd DOUBLE PRECISION for dbbcsd COMPLEX for cbbcsd DOUBLE COMPLEX for zbbcsd Array, DIMENSION (*q*). When ?bbcsd converges, *b12d* contains the negative sines of *theta*(1), ..., *theta*(*q*). If ?bbcsd fails to converge, *b12d* contains the diagonal of the partially reduced top right block.
b12e	REAL for sbbcsd DOUBLE PRECISION for dbbcsd COMPLEX for cbbcsd DOUBLE COMPLEX for zbbcsd Array, DIMENSION (*q*-1). When ?bbcsd converges, *b12e* contains zeros. If ?bbcsd fails to converge, *b11e* contains the superdiagonal of the partially reduced top right block.
info	INTEGER. = 0: successful exit < 0: if *info* = -*i*, the *i*-th argument has an illegal value > 0: if ?bbcsd did not converge, *info* specifies the number of nonzero entries in *phi*, and *b11d*, *b11e*, etc. and contains the partially reduced matrix.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ?bbcsd interface are as follows:

theta	Holds the vector of length *q*.
phi	Holds the vector of length *q*-1.
u1	Holds the matrix of size (*p*,*p*).
u2	Holds the matrix of size (*m*-*p*,*m*-*p*).
v1t	Holds the matrix of size (*q*,*q*).
v2t	Holds the matrix of size (*m*-*q*,*m*-*q*).
b11d	Holds the vector of length *q*.

b11e	Holds the vector of length q-1.
b12d	Holds the vector of length q.
b12e	Holds the vector of length q-1.
b21d	Holds the vector of length q.
b21e	Holds the vector of length q-1.
b22d	Holds the vector of length q.
b22e	Holds the vector of length q-1.
jobsu1	Indicates whether u_1 is computed. Must be 'Y' or 'O'.
jobsu2	Indicates whether u_2 is computed. Must be 'Y' or 'O'.
jobv1t	Indicates whether ${v_1}^t$ is computed. Must be 'Y' or 'O'.
jobv2t	Indicates whether ${v_2}^t$ is computed. Must be 'Y' or 'O'.
trans	Must be 'N' or 'T'.

See Also

?orcsd/?uncsd
xerbla

?orbdb/?unbdb

Simultaneously bidiagonalizes the blocks of a partitioned orthogonal/unitary matrix.

Syntax

Fortran 77:

```
call sorbdb( trans, signs, m, p, q, x11, ldx11, x12, ldx12, x21, ldx21, x22, ldx22,
theta, phi, taup1, taup2, tauq1, tauq2, work, lwork, info )
```

```
call dorbdb( trans, signs, m, p, q, x11, ldx11, x12, ldx12, x21, ldx21, x22, ldx22,
theta, phi, taup1, taup2, tauq1, tauq2, work, lwork, info )
```

```
call cunbdb( trans, signs, m, p, q, x11, ldx11, x12, ldx12, x21, ldx21, x22, ldx22,
theta, phi, taup1, taup2, tauq1, tauq2, work, lwork, info )
```

```
call zunbdb( trans, signs, m, p, q, x11, ldx11, x12, ldx12, x21, ldx21, x22, ldx22,
theta, phi, taup1, taup2, tauq1, tauq2, work, lwork, info )
```

Fortran 95:

```
call orbdb( x11,x12,x21,x22,theta,phi,taup1,taup2,tauq1,tauq2[,trans][,signs][,info] )
```

```
call unbdb( x11,x12,x21,x22,theta,phi,taup1,taup2,tauq1,tauq2[,trans][,signs][,info] )
```

C:

```
lapack_int LAPACKE_sorbdb( int matrix_order, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, float* x11, lapack_int ldx11, float* x12, lapack_int
ldx12, float* x21, lapack_int ldx21, float* x22, lapack_int ldx22, float* theta,
float* phi, float* taup1, float* taup2, float* tauq1, float* tauq2 );
```

```
lapack_int LAPACKE_dorbdb( int matrix_order, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, double* x11, lapack_int ldx11, double* x12, lapack_int
ldx12, double* x21, lapack_int ldx21, double* x22, lapack_int ldx22, double* theta,
double* phi, double* taup1, double* taup2, double* tauq1, double* tauq );
```

```
lapack_int LAPACKE_cunbdb( int matrix_order, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, lapack_complex_float* x11, lapack_int ldx11,
lapack_complex_float* x12, lapack_int ldx12, lapack_complex_float* x21, lapack_int
```

```
ldx21, lapack_complex_float* x22, lapack_int ldx22, float* theta, float* phi,
lapack_complex_float* taup1, lapack_complex_float* taup2, lapack_complex_float* tauq1,
lapack_complex_float* tauq2 );

lapack_int LAPACKE_zunbdb( int matrix_order, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, lapack_complex_double* x11, lapack_int ldx11,
lapack_complex_double* x12, lapack_int ldx12, lapack_complex_double* x21, lapack_int
ldx21, lapack_complex_double* x22, lapack_int ldx22, double* theta, double* phi,
lapack_complex_double* taup1, lapack_complex_double* taup2, lapack_complex_double*
tauq1, lapack_complex_double* tauq2 );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routines `?orbdb`/`?unbdb` simultaneously bidiagonalizes the blocks of an m-by-m partitioned orthogonal matrix X:

$$X=\left(\begin{array}{c|c} x_{11} & x_{12} \\ \hline x_{21} & x_{22} \end{array}\right)=\left(\begin{array}{c|c} p_1 & \\ \hline & p_2 \end{array}\right)\left(\begin{array}{c|c|c|c} b_{11} & b_{12} & 0 & 0 \\ \hline 0 & 0 & -I & 0 \\ \hline b_{21} & b_{22} & 0 & 0 \\ \hline 0 & 0 & 0 & I \end{array}\right)\left(\begin{array}{c|c} q_1 & \\ \hline & q_2 \end{array}\right)^T$$

or unitary matrix:

$$X=\left(\begin{array}{c|c} x_{11} & x_{12} \\ \hline x_{21} & x_{22} \end{array}\right)=\left(\begin{array}{c|c} p_1 & \\ \hline & p_2 \end{array}\right)\left(\begin{array}{c|c|c|c} b_{11} & b_{12} & 0 & 0 \\ \hline 0 & 0 & -I & 0 \\ \hline b_{21} & b_{22} & 0 & 0 \\ \hline 0 & 0 & 0 & I \end{array}\right)\left(\begin{array}{c|c} q_1 & \\ \hline & q_2 \end{array}\right)^H$$

x_{11} is p-by-q. q must not be larger than p, m-p, or m-q. Otherwise, x must be transposed and/or permuted in constant time using the *trans* and *signs* options. See `?orcsd`/`?uncsd` for details.

The orthogonal/unitary matrices p_1, p_2, q_1, and q_2 are p-by-p, $(m-p)$-by-$(m-p)$, q-by-q, $(m-q)$-by-$(m-q)$, respectively. They are represented implicitly by Housholder vectors.

The bidiagonal matrices b_{11}, b_{12}, b_{21}, and b_{22} are q-by-q bidiagonal matrices represented implicitly by angles *theta*(1), ..., *theta*(q) and *phi*(1), ..., *phi*(q-1). b_{11} and b_{12} are upper bidiagonal, while b_{21} and b_{22} are lower bidiagonal. Every entry in each bidiagonal band is a product of a sine or cosine of *theta* with a sine or cosine of *phi*. See [Sutton09] or description of `?orcsd`/`?uncsd` for details.

p_1, p_2, q_1, and q_2 are represented as products of elementary reflectors. See description of `?orcsd`/`?uncsd` for details on generating p_1, p_2, q_1, and q_2 using `?orgqr` and `?orglq`.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans CHARACTER

= 'T':	x, u_1, u_2, v_1^t, v_2^t are stored in row-major order.
otherwise	x, u_1, u_2, v_1^t, v_2^t are stored in column-major order.

signs CHARACTER

= 'O':	The lower-left block is made nonpositive (the "other" convention).
otherwise	The upper-right block is made nonpositive (the "default" convention).

m INTEGER. The number of rows and columns of the matrix X.

p INTEGER. The number of rows in x_{11} and x_{12}. $0 \le p \le m$.

q INTEGER. The number of columns in x_{11} and x_{21}. $0 \le q \le \min(p, m-p, m-q)$.

x11 REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (*ldx11*,*q*).
On entry, the top-left block of the orthogonal/unitary matrix to be reduced.

ldx11 INTEGER. The leading dimension of the array X_{11}. If *trans* = 'T', *ldx11* $\ge$ *p*. Otherwise, *ldx11* $\ge$ *q*.

x12 REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (*ldx12*,*m*-*q*).
On entry, the top-right block of the orthogonal/unitary matrix to be reduced.

ldx12 INTEGER. The leading dimension of the array X_{12}. If *trans* = 'N', *ldx12* $\ge$ *p*. Otherwise, *ldx12* $\ge$ *m*-*q*.

x21 REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (*ldx21*,*q*).
On entry, the bottom-left block of the orthogonal/unitary matrix to be reduced.

ldx21 INTEGER. The leading dimension of the array X_{21}. If *trans* = 'N', *ldx21* $\ge$ *m*-*p*. Otherwise, *ldx21* $\ge$ *q*.

x22 REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (*ldx22*,*m*-*q*).
On entry, the bottom-right block of the orthogonal/unitary matrix to be reduced.

ldx22 INTEGER. The leading dimension of the array X_{21}. If *trans* = 'N', *ldx22* $\ge$ *m*-*p*. Otherwise, *ldx22* $\ge$ *m*-*q*.

work
REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Workspace array, DIMENSION (*lwork*).

lwork
INTEGER. The size of the *work* array. *lwork* ≥ *m*-*q*
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

Output Parameters

x11
On exit, the form depends on *trans*:

If *trans*='N',	the columns of tril(*x11*) specify reflectors for p_1, the rows of triu(*x11*,1) specify reflectors for q_1
otherwise *trans*='T',	the rows of triu(*x11*) specify reflectors for p_1, the columns of tril(*x11*,-1) specify reflectors for q_1

x12
On exit, the form depends on *trans*:

If *trans*='N',	the columns of triu(*x12*) specify the first *p* reflectors for q_2
otherwise *trans*='T',	the columns of tril(*x12*) specify the first *p* reflectors for q_2

x21
On exit, the form depends on *trans*:

If *trans*='N',	the columns of tril(*x21*) specify the reflectors for p_2
otherwise *trans*='T',	the columns of triu(*x21*) specify the reflectors for p_2

x22
On exit, the form depends on *trans*:

If *trans*='N',	the rows of triu(*x22*(*q*+1:*m*-*p*,*p*+1:*m*-*q*)) specify the last *m*-*p*-*q* reflectors for q_2
otherwise *trans*='T',	the columns of tril(*x22*(*p*+1:*m*-*q*,*q*+1:*m*-*p*)) specify the last *m*-*p*-*q* reflectors for p_2

theta
REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (*q*). The entries of bidiagonal blocks b_{11}, b_{12}, b_{21}, and b_{22} can be computed from the angles *theta* and *phi*. See the Description section for details.

phi
REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (*q*-1). The entries of bidiagonal blocks b_{11}, b_{12}, b_{21}, and b_{22} can be computed from the angles *theta* and *phi*. See the Description section for details.

taup1
REAL for sorbdb
DOUBLE PRECISION for dorbdb

COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (p).
Scalar factors of the elementary reflectors that define p_1.

taup2
REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (m-p).
Scalar factors of the elementary reflectors that define p_2.

tauq1
REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (q).
Scalar factors of the elementary reflectors that define q_1.

tauq2
REAL for sorbdb
DOUBLE PRECISION for dorbdb
COMPLEX for cunbdb
DOUBLE COMPLEX for zunbdb
Array, DIMENSION (m-q).
Scalar factors of the elementary reflectors that define q_2.

info
INTEGER.
= 0: successful exit
< 0: if *info* = -i, the i-th argument has an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ?orbdb/?unbdb interface are as follows:

x11 Holds the block of matrix X of size (p, q).
x12 Holds the block of matrix X of size (p, m-q).
x21 Holds the block of matrix X of size (m-p, q).
x22 Holds the block of matrix X of size (m-p, m-q).
theta Holds the vector of length q.
phi Holds the vector of length q-1.
taup1 Holds the vector of length p.
taup2 Holds the vector of length m-p.
tauq1 Holds the vector of length q.
taupq2 Holds the vector of length m-q.
trans Must be 'N' or 'T'.
signs Must be 'O' or 'D'.

See Also

?orcsd/?uncsd
?orgqr
?ungqr

?orglq
?unglq
xerbla

Driver Routines

Each of the LAPACK driver routines solves a complete problem. To arrive at the solution, driver routines typically call a sequence of appropriate computational routines.

Driver routines are described in the following sections :

Linear Least Squares (LLS) Problems

Generalized LLS Problems

Symmetric Eigenproblems

Nonsymmetric Eigenproblems

Singular Value Decomposition

Cosine-Sine Decomposition

Generalized Symmetric Definite Eigenproblems

Generalized Nonsymmetric Eigenproblems

Linear Least Squares (LLS) Problems

This section describes LAPACK driver routines used for solving linear least squares problems. Table "Driver Routines for Solving LLS Problems" lists all such routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Solving LLS Problems

Routine Name	Operation performed
gels	Uses QR or LQ factorization to solve a overdetermined or underdetermined linear system with full rank matrix.
gelsy	Computes the minimum-norm solution to a linear least squares problem using a complete orthogonal factorization of A.
gelss	Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A.
gelsd	Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A and a divide and conquer method.

?gels

Uses QR or LQ factorization to solve a overdetermined or underdetermined linear system with full rank matrix.

Syntax

Fortran 77:

```
call sgels(trans, m, n, nrhs, a, lda, b, ldb, work, lwork, info)
call dgels(trans, m, n, nrhs, a, lda, b, ldb, work, lwork, info)
call cgels(trans, m, n, nrhs, a, lda, b, ldb, work, lwork, info)
call zgels(trans, m, n, nrhs, a, lda, b, ldb, work, lwork, info)
```

Fortran 95:

```
call gels(a, b [,trans] [,info])
```

C:

```
lapack_int LAPACKE_<?>gels( int matrix_order, char trans, lapack_int m, lapack_int n,
lapack_int nrhs, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves overdetermined or underdetermined real/ complex linear systems involving an m-by-n matrix A, or its transpose/ conjugate-transpose, using a QR or LQ factorization of A. It is assumed that A has full rank.

The following options are provided:

1. If $trans$ = 'N' and $m \geq n$: find the least squares solution of an overdetermined system, that is, solve the least squares problem

$$\text{minimize } ||b - A*x||_2$$

2. If $trans$ = 'N' and $m < n$: find the minimum norm solution of an underdetermined system $A*X = B$.

3. If $trans$ = 'T' or 'C' and $m \geq n$: find the minimum norm solution of an undetermined system $A_H*X = B$.

4. If $trans$ = 'T' or 'C' and $m < n$: find the least squares solution of an overdetermined system, that is, solve the least squares problem

$$\text{minimize } ||b - A^H*x||_2$$

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m-by-$nrhs$ right hand side matrix B and the n-by-nrh solution matrix X.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

trans	CHARACTER*1. Must be 'N', 'T', or 'C'. If $trans$ = 'N', the linear system involves matrix A; If $trans$ = 'T', the linear system involves the transposed matrix A^T (for real flavors only); If $trans$ = 'C', the linear system involves the conjugate-transposed matrix A^H (for complex flavors only).
m	INTEGER. The number of rows of the matrix A ($m \geq 0$).
n	INTEGER. The number of columns of the matrix A ($n \geq 0$).
nrhs	INTEGER. The number of right-hand sides; the number of columns in B ($nrhs \geq 0$).
a, *b*, *work*	REAL for sgels DOUBLE PRECISION for dgels

	COMPLEX for cgels DOUBLE COMPLEX for zgels. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the matrix *B* of right hand side vectors, stored columnwise; *B* is *m*-by-*nrhs* if *trans* = 'N', or *n*-by-*nrhs* if *trans* = 'T' or 'C'. The second dimension of *b* must be at least max(1, *nrhs*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; must be at least max(1, *m*, *n*).
lwork	INTEGER. The size of the *work* array; must be at least min (*m*, *n*)+max(1, *m*, *n*, *nrhs*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	On exit, overwritten by the factorization data as follows: if *m* ≥ *n*, array *a* contains the details of the *QR* factorization of the matrix *A* as returned by ?geqrf; if *m* < *n*, array *a* contains the details of the *LQ* factorization of the matrix *A* as returned by ?gelqf.
b	If *info* = 0, *b* overwritten by the solution vectors, stored columnwise: if *trans* = 'N' and *m* ≥ *n*, rows 1 to *n* of *b* contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of modulus of elements *n*+1 to *m* in that column; if *trans* = 'N' and *m* < *n*, rows 1 to *n* of *b* contain the minimum norm solution vectors; if *trans* = 'T' or 'C' and *m* ≥ *n*, rows 1 to *m* of *b* contain the minimum norm solution vectors; if *trans* = 'T' or 'C' and *m* < *n*, rows 1 to *m* of *b* contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of modulus of elements *m*+1 to *n* in that column.
work(1)	If *info* = 0, on exit *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, the *i*-th diagonal element of the triangular factor of *A* is zero, so that *A* does not have full rank; the least squares solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gels interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
b	Holds the matrix of size max(*m*,*n*)-by-*nrhs*. If *trans* = 'N', then, on entry, the size of *b* is *m*-by-*nrhs*, If *trans* = 'T', then, on entry, the size of *b* is *n*-by-*nrhs*,
trans	Must be 'N' or 'T'. The default value is 'N'.

Application Notes

For better performance, try using *lwork* = min (*m*, *n*)+max(1, *m*, *n*, *nrhs*)***blocksize*, where *blocksize* is a machine-dependent value (typically, 16 to 64) required for optimum performance of the *blocked algorithm*.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?gelsy

Computes the minimum-norm solution to a linear least squares problem using a complete orthogonal factorization of A.

Syntax

Fortran 77:

```
call sgelsy(m, n, nrhs, a, lda, b, ldb, jpvt, rcond, rank, work, lwork, info)
call dgelsy(m, n, nrhs, a, lda, b, ldb, jpvt, rcond, rank, work, lwork, info)
call cgelsy(m, n, nrhs, a, lda, b, ldb, jpvt, rcond, rank, work, lwork, rwork, info)
call zgelsy(m, n, nrhs, a, lda, b, ldb, jpvt, rcond, rank, work, lwork, rwork, info)
```

Fortran 95:

```
call gelsy(a, b [,rank] [,jpvt] [,rcond] [,info])
```

C:

```
lapack_int LAPACKE_sgelsy( int matrix_order, lapack_int m, lapack_int n, lapack_int
nrhs, float* a, lapack_int lda, float* b, lapack_int ldb, lapack_int* jpvt, float
rcond, lapack_int* rank );
lapack_int LAPACKE_dgelsy( int matrix_order, lapack_int m, lapack_int n, lapack_int
nrhs, double* a, lapack_int lda, double* b, lapack_int ldb, lapack_int* jpvt, double
rcond, lapack_int* rank );
lapack_int LAPACKE_cgelsy( int matrix_order, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int
ldb, lapack_int* jpvt, float rcond, lapack_int* rank );
```

```
lapack_int LAPACKE_zgelsy( int matrix_order, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int
ldb, lapack_int* jpvt, double rcond, lapack_int* rank );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

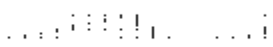
Description

The `?gelsy` routine computes the minimum-norm solution to a real/complex linear least squares problem:

`minimize ||b - A*x||`$_2$

using a complete orthogonal factorization of *A*. *A* is an *m*-by-*n* matrix which may be rank-deficient. Several right hand side vectors *b* and solution vectors *x* can be handled in a single call; they are stored as the columns of the *m*-by-*nrhs* right hand side matrix *B* and the *n*-by-*nrhs* solution matrix *X*.

The routine first computes a *QR* factorization with column pivoting:

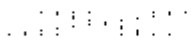


with R_{11} defined as the largest leading submatrix whose estimated condition number is less than 1/*rcond*. The order of R_{11}, *rank*, is the effective rank of *A*. Then, R_{22} is considered to be negligible, and R_{12} is annihilated by orthogonal/unitary transformations from the right, arriving at the complete orthogonal factorization:

$$AP = Q \begin{pmatrix} T_{11} & 0 \\ 0 & 0 \end{pmatrix} Z$$

The minimum-norm solution is then

$$X = PZ^T \begin{pmatrix} T_{11}^{-1} & Q_1^T b \\ 0 & 0 \end{pmatrix}$$ for real flavors and



for complex flavors,

where Q_1 consists of the first *rank* columns of *Q*.

The `?gelsy` routine is identical to the original deprecated `?gelsx` routine except for the following differences:

- The call to the subroutine `?geqpf` has been substituted by the call to the subroutine `?geqp3`, which is a BLAS-3 version of the *QR* factorization with column pivoting.
- The matrix *B* (the right hand side) is updated with BLAS-3.
- The permutation of the matrix *B* (the right hand side) is faster and more simple.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns of the matrix *A* (*n* ≥ 0).
nrhs	INTEGER. The number of right-hand sides; the number of columns in *B* (*nrhs* ≥ 0).
a, *b*, *work*	REAL for sgelsy DOUBLE PRECISION for dgelsy COMPLEX for cgelsy DOUBLE COMPLEX for zgelsy. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the *m*-by-*nrhs* right hand side matrix *B*. The second dimension of *b* must be at least max(1, *nrhs*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; must be at least max(1, *m*, *n*).
jpvt	INTEGER. Array, DIMENSION at least max(1, *n*). On entry, if *jpvt*(*i*)≠ 0, the *i*-th column of *A* is permuted to the front of *AP*, otherwise the *i*-th column of *A* is a free column.
rcond	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. *rcond* is used to determine the effective rank of *A*, which is defined as the order of the largest leading triangular submatrix R_{11} in the *QR* factorization with pivoting of *A*, whose estimated condition number < 1/*rcond*.
lwork	INTEGER. The size of the *work* array. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.
rwork	REAL for cgelsy DOUBLE PRECISION for zgelsy. Workspace array, DIMENSION at least max(1, 2*n*). Used in complex flavors only.

Output Parameters

a	On exit, overwritten by the details of the complete orthogonal factorization of *A*.
b	Overwritten by the *n*-by-*nrhs* solution matrix *X*.
jpvt	On exit, if *jpvt*(*i*)= *k*, then the *i*-th column of *AP* was the *k*-th column of *A*.
rank	INTEGER. The effective rank of *A*, that is, the order of the submatrix R_{11}. This is the same as the order of the submatrix T_{11} in the complete orthogonal factorization of *A*.
info	INTEGER. If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gelsy interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
b	Holds the matrix of size max(*m*,*n*)-by-*nrhs*. On entry, contains the *m*-by-*nrhs* right hand side matrix *B*, On exit, overwritten by the *n*-by-*nrhs* solution matrix *X*.
jpvt	Holds the vector of length *n*. Default value for this element is *jpvt*(*i*) = 0.
rcond	Default value for this element is *rcond* = 100*EPSILON(1.0_WP).

Application Notes

For real flavors:

The unblocked strategy requires that:

lwork ≥ max(*mn*+3*n*+1, 2**mn* + *nrhs*),

where *mn* = min(*m*, *n*).

The block algorithm requires that:

lwork ≥ max(*mn*+2*n*+*nb**(*n*+1), 2**mn*+*nb**nrhs*),

where *nb* is an upper bound on the blocksize returned by ilaenv for the routines sgeqp3/dgeqp3, stzrzf/dtzrzf, stzrqf/dtzrqf, sormqr/dormqr, and sormrz/dormrz.

For complex flavors:

The unblocked strategy requires that:

lwork ≥ *mn* + max(2**mn*, *n*+1, *mn* + *nrhs*),

where *mn* = min(*m*, *n*).

The block algorithm requires that:

lwork < *mn* + max(2**mn*, *nb**(*n*+1), *mn*+*mn**nb*, *mn*+ *nb**nrhs*),

where *nb* is an upper bound on the blocksize returned by ilaenv for the routines cgeqp3/zgeqp3, ctzrzf/ztzrzf, ctzrqf/ztzrqf, cunmqr/zunmqr, and cunmrz/zunmrz.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?gelss

Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A.

Syntax

Fortran 77:

```
call sgelss(m, n, nrhs, a, lda, b, ldb, s, rcond, rank, work, lwork, info)
call dgelss(m, n, nrhs, a, lda, b, ldb, s, rcond, rank, work, lwork, info)
call cgelss(m, n, nrhs, a, lda, b, ldb, s, rcond, rank, work, lwork, rwork, info)
call zgelss(m, n, nrhs, a, lda, b, ldb, s, rcond, rank, work, lwork, rwork, info)
```

Fortran 95:

```
call gelss(a, b [,rank] [,s] [,rcond] [,info])
```

C:

```
lapack_int LAPACKE_sgelss( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, float* a, lapack_int lda, float* b, lapack_int ldb, float* s, float rcond, lapack_int* rank );
lapack_int LAPACKE_dgelss( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, double* a, lapack_int lda, double* b, lapack_int ldb, double* s, double rcond, lapack_int* rank );
lapack_int LAPACKE_cgelss( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, float* s, float rcond, lapack_int* rank );
lapack_int LAPACKE_zgelss( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb, double* s, double rcond, lapack_int* rank );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the minimum norm solution to a real linear least squares problem:

$$\text{minimize } ||b - A*x||_2$$

using the singular value decomposition (SVD) of A. A is an m-by-n matrix which may be rank-deficient. Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m-by-$nrhs$ right hand side matrix B and the n-by-$nrhs$ solution matrix X. The effective rank of A is determined by treating as zero those singular values which are less than $rcond$ times the largest singular value.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns of the matrix *A* (*n* ≥ 0).
nrhs	INTEGER. The number of right-hand sides; the number of columns in *B* (*nrhs* ≥ 0).
a, *b*, *work*	REAL for sgelss DOUBLE PRECISION for dgelss COMPLEX for cgelss DOUBLE COMPLEX for zgelss. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the *m*-by-*nrhs* right hand side matrix *B*. The second dimension of *b* must be at least max(1, *nrhs*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; must be at least max(1, *m*, *n*).
rcond	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. *rcond* is used to determine the effective rank of *A*. Singular values *s*(*i*) ≤ *rcond* **s*(1) are treated as zero. If *rcond* <0, machine precision is used instead.
lwork	INTEGER. The size of the *work* array; *lwork*≥ 1. If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.
rwork	REAL for cgelss DOUBLE PRECISION for zgelss. Workspace array used in complex flavors only. DIMENSION at least max(1, 5*min(*m*, *n*)).

Output Parameters

a	On exit, the first min(*m*, *n*) rows of *A* are overwritten with its right singular vectors, stored row-wise.
b	Overwritten by the *n*-by-*nrhs* solution matrix *X*. If *m*≥*n* and *rank* = *n*, the residual sum-of-squares for the solution in the *i*-th column is given by the sum of squares of modulus of elements *n*+1:*m* in that column.
s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*)). The singular values of *A* in decreasing order. The condition number of *A* in the 2-norm is $k_2(A) = s(1) / s(\min(m, n))$.
rank	INTEGER. The effective rank of *A*, that is, the number of singular values which are greater than *rcond* **s*(1).
work(1)	If *info* = 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm for computing the SVD failed to converge; *i* indicates the number of off-diagonal elements of an intermediate bidiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gelss interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
b	Holds the matrix of size max(*m*,*n*)-by-*nrhs*. On entry, contains the *m*-by-*nrhs* right hand side matrix *B*, On exit, overwritten by the *n*-by-*nrhs* solution matrix *X*.
s	Holds the vector of length min(*m*,*n*).
rcond	Default value for this element is *rcond* = 100*EPSILON(1.0_WP).

Application Notes

For real flavors:

lwork ≥ 3*min(*m*, *n*)+ max(2*min(*m*, *n*), max(*m*, *n*), *nrhs*)

For complex flavors:

lwork ≥ 2*min(*m*, *n*)+ max(*m*, *n*, *nrhs*)

For good performance, *lwork* should generally be larger.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?gelsd

Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A and a divide and conquer method.

Syntax

Fortran 77:

call sgelsd(*m*, *n*, *nrhs*, *a*, *lda*, *b*, *ldb*, *s*, *rcond*, *rank*, *work*, *lwork*, *iwork*, *info*)

call dgelsd(*m*, *n*, *nrhs*, *a*, *lda*, *b*, *ldb*, *s*, *rcond*, *rank*, *work*, *lwork*, *iwork*, *info*)

call cgelsd(*m*, *n*, *nrhs*, *a*, *lda*, *b*, *ldb*, *s*, *rcond*, *rank*, *work*, *lwork*, *rwork*, *iwork*, *info*)

```
call zgelsd(m, n, nrhs, a, lda, b, ldb, s, rcond, rank, work, lwork, rwork, iwork, info)
```

Fortran 95:

```
call gelsd(a, b [,rank] [,s] [,rcond] [,info])
```

C:

```
lapack_int LAPACKE_sgelsd( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, float* a, lapack_int lda, float* b, lapack_int ldb, float* s, float rcond, lapack_int* rank );

lapack_int LAPACKE_dgelsd( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, double* a, lapack_int lda, double* b, lapack_int ldb, double* s, double rcond, lapack_int* rank );

lapack_int LAPACKE_cgelsd( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, float* s, float rcond, lapack_int* rank );

lapack_int LAPACKE_zgelsd( int matrix_order, lapack_int m, lapack_int n, lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb, double* s, double rcond, lapack_int* rank );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the minimum-norm solution to a real linear least squares problem:

$$\text{minimize } ||b - A*x||_2$$

using the singular value decomposition (SVD) of A. A is an m-by-n matrix which may be rank-deficient.

Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m-by-$nrhs$ right hand side matrix B and the n-by-$nrhs$ solution matrix X.

The problem is solved in three steps:

1. Reduce the coefficient matrix A to bidiagonal form with Householder transformations, reducing the original problem into a "bidiagonal least squares problem" (BLS).
2. Solve the BLS using a divide and conquer approach.
3. Apply back all the Householder transformations to solve the original least squares problem.

The effective rank of A is determined by treating as zero those singular values which are less than $rcond$ times the largest singular value.

The routine uses auxiliary routines lals0 and lalsa.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix A ($m \geq 0$).
n	INTEGER. The number of columns of the matrix A ($n \geq 0$).

nrhs	INTEGER. The number of right-hand sides; the number of columns in *B* (*nrhs* ≥ 0).
a, *b*, *work*	REAL for sgelsd DOUBLE PRECISION for dgelsd COMPLEX for cgelsd DOUBLE COMPLEX for zgelsd. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the *m*-by-*nrhs* right hand side matrix *B*. The second dimension of *b* must be at least max(1, *nrhs*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; must be at least max(1, *m*, *n*).
rcond	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. *rcond* is used to determine the effective rank of *A*. Singular values *s*(*i*) ≤ *rcond* **s*(1) are treated as zero. If *rcond* ≤ 0, machine precision is used instead.
lwork	INTEGER. The size of the *work* array; *lwork* ≥ 1. If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the array *work* and the minimum sizes of the arrays *rwork* and *iwork*, and returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.
iwork	INTEGER. Workspace array. See *Application Notes* for the suggested dimension of *iwork*.
rwork	REAL for cgelsd DOUBLE PRECISION for zgelsd. Workspace array, used in complex flavors only. See *Application Notes* for the suggested dimension of *rwork*.

Output Parameters

a	On exit, *A* has been overwritten.
b	Overwritten by the *n*-by-*nrhs* solution matrix *X*. If *m* ≥ *n* and *rank* = *n*, the residual sum-of-squares for the solution in the *i*-th column is given by the sum of squares of modulus of elements *n*+1:*m* in that column.
s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*)). The singular values of *A* in decreasing order. The condition number of *A* in the 2-norm is $k^2(A) = s(1)/s(\min(m, n))$.
rank	INTEGER. The effective rank of *A*, that is, the number of singular values which are greater than *rcond* **s*(1).
work(1)	If *info* = 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

rwork(1)	If *info* = 0, on exit, *rwork*(1) returns the minimum size of the workspace array *iwork* required for optimum performance.
iwork(1)	If *info* = 0, on exit, *iwork*(1) returns the minimum size of the workspace array *iwork* required for optimum performance.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm for computing the SVD failed to converge; *i* indicates the number of off-diagonal elements of an intermediate bidiagonal form that did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gelsd interface are the following:

a	Holds the matrix *A* of size (*m*,*n*).
b	Holds the matrix of size max(*m*,*n*)-by-*nrhs*. On entry, contains the *m*-by-*nrhs* right hand side matrix *B*, On exit, overwritten by the *n*-by-*nrhs* solution matrix *X*.
s	Holds the vector of length min(*m*,*n*).
rcond	Default value for this element is *rcond* = 100*EPSILON(1.0_WP).

Application Notes

The divide and conquer algorithm makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

The exact minimum amount of workspace needed depends on *m*, *n* and *nrhs*. The size *lwork* of the workspace array *work* must be as given below.

For real flavors:

If $m \geq n$,

$lwork \geq 12n + 2n*smlsiz + 8n*nlvl + n*nrhs + (smlsiz+1)^2$;

If $m < n$,

$lwork \geq 12m + 2m*smlsiz + 8m*nlvl + m*nrhs + (smlsiz+1)^2$;

For complex flavors:

If $m \geq n$,

$lwork < 2n + n*nrhs$;

If $m < n$,

$lwork \geq 2m + m*nrhs$;

where *smlsiz* is returned by ilaenv and is equal to the maximum size of the subproblems at the bottom of the computation tree (usually about 25), and

$nlvl = \mathrm{INT}(\log_2(\min(m, n)/(smlsiz+1))) + 1$.

For good performance, *lwork* should generally be larger.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The dimension of the workspace array *iwork* must be at least

3*min(*m*, *n*)*nlvl* + 11*min(*m*, *n*).

The dimension of the workspace array *iwork* (for complex flavors) must be at least max(1, *lrwork*).

lrwork $\geq$ 10*n* + 2*n***smlsiz* + 8*n***nlvl* + 3**smlsiz***nrhs* + (*smlsiz*+1)^2 if *m* $\geq$ *n*, and

lrwork $\geq$ 10*m* + 2*m***smlsiz* + 8*m***nlvl* + 3**smlsiz***nrhs* + (*smlsiz*+1)^2 if *m* < *n*.

Generalized LLS Problems

This section describes LAPACK driver routines used for solving generalized linear least squares problems. Table "Driver Routines for Solving Generalized LLS Problems" lists all such routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Solving Generalized LLS Problems

Routine Name	Operation performed
gglse	Solves the linear equality-constrained least squares problem using a generalized RQ factorization.
ggglm	Solves a general Gauss-Markov linear model problem using a generalized QR factorization.

?gglse

Solves the linear equality-constrained least squares problem using a generalized RQ factorization.

Syntax

Fortran 77:

```
call sgglse(m, n, p, a, lda, b, ldb, c, d, x, work, lwork, info)
call dgglse(m, n, p, a, lda, b, ldb, c, d, x, work, lwork, info)
call cgglse(m, n, p, a, lda, b, ldb, c, d, x, work, lwork, info)
call zgglse(m, n, p, a, lda, b, ldb, c, d, x, work, lwork, info)
```

Fortran 95:

```
call gglse(a, b, c, d, x [,info])
```

C:

```
lapack_int LAPACKE_<?>gglse( int matrix_order, lapack_int m, lapack_int n, lapack_int
p, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb, <datatype>* c,
<datatype>* d, <datatype>* x );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine solves the linear equality-constrained least squares (LSE) problem:

minimize ||c - A*x||^2 subject to B*x = d

where A is an m-by-n matrix, B is a p-by-n matrix, c is a given d is a given p-vector. It is assumed that $p \leq n \leq m+p$, and

These conditions ensure that the LSE problem has a unique solution, which is obtained using a generalized RQ factorization of the matrices (B, A) given by

```
B=(0 R)*Q, A=Z*T*Q
```

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

m	INTEGER. The number of rows of the matrix A ($m \geq 0$).
n	INTEGER. The number of columns of the matrices A and B ($n \geq 0$).
p	INTEGER. The number of rows of the matrix B ($0 \leq p \leq n \leq m+p$).
a, b, c, d, $work$	REAL for sgglse DOUBLE PRECISION for dgglse COMPLEX for cgglse DOUBLE COMPLEX for zgglse. Arrays: a(lda,*) contains the m-by-n matrix A. The second dimension of a must be at least max(1, n). b(ldb,*) contains the p-by-n matrix B. The second dimension of b must be at least max(1, n). c(*), dimension at least max(1, m), contains the right hand side vector for the least squares part of the LSE problem. d(*), dimension at least max(1, p), contains the right hand side vector for the constrained equation. $work$ is a workspace array, its dimension max(1, $lwork$).
lda	INTEGER. The leading dimension of a; at least max(1, m).
ldb	INTEGER. The leading dimension of b; at least max(1, p).
$lwork$	INTEGER. The size of the $work$ array; $lwork \geq$ max(1, $m+n+p$).

If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

Output Parameters

x — REAL for sgglse

b — On exit, the upper triangle of the subarray *b*(1:*p*, *n*-*p*+1:*n*) contains the *p*-by-*p* upper triangular matrix *R*.

d — On exit, *d* is destroyed.

c — On exit, the residual sum-of-squares for the solution is given by the sum of squares of elements *n*-*p*+1 to *m* of vector *c*.

work(1) — If *info* = 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance. Use this *lwork* for subsequent runs.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = 1, the upper triangular factor *R* associated with *B* in the generalized RQ factorization of the pair (*B*, *A*) is singular, so that rank(*B*) < *P*; the least squares solution could not be computed.
If *info* = 2, the (*n*-*p*)-by-(*n*-*p*) part of the upper trapezoidal factor *T* associated with *A* in the generalized RQ factorization of the pair (*B*, *A*) is singular, so that

$$\operatorname{rank}\begin{pmatrix} A \\ B \end{pmatrix} < n$$

; the least squares solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gglse interface are the following:

a — Holds the matrix *A* of size (*m*,*n*).

b — Holds the matrix *B* of size (*p*,*n*).

c — Holds the vector of length (*m*).

d — Holds the vector of length (*p*).

x — Holds the vector of length *n*.

Application Notes

For optimum performance, use

lwork ≥ *p* + min(*m*, *n*) + max(*m*, *n*)**nb*,

where *nb* is an upper bound for the optimal blocksizes for ?geqrf, ?gerqf, ?ormqr/?unmqr and ?ormrq/?unmrq.

You may set *lwork* to -1. The routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?ggglm

Solves a general Gauss-Markov linear model problem using a generalized QR factorization.

Syntax

Fortran 77:

```
call sggglm(n, m, p, a, lda, b, ldb, d, x, y, work, lwork, info)
call dggglm(n, m, p, a, lda, b, ldb, d, x, y, work, lwork, info)
call cggglm(n, m, p, a, lda, b, ldb, d, x, y, work, lwork, info)
call zggglm(n, m, p, a, lda, b, ldb, d, x, y, work, lwork, info)
```

Fortran 95:

```
call ggglm(a, b, d, x, y [,info])
```

C:

```
lapack_int LAPACKE_<?>ggglm( int matrix_order, lapack_int n, lapack_int m, lapack_int
p, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb, <datatype>* d,
<datatype>* x, <datatype>* y );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine solves a general Gauss-Markov linear model (GLM) problem:

$\text{minimize}_x \; ||y||_2$ subject to $d = A*x + B*y$

where A is an n-by-m matrix, B is an n-by-p matrix, and d is a given n-vector. It is assumed that $m \le n \le m+p$, and $\text{rank}(A) = m$ and $\text{rank}(A\ B) = n$.

Under these assumptions, the constrained equation is always consistent, and there is a unique solution x and a minimal 2-norm solution y, which is obtained using a generalized QR factorization of the matrices $(A,\ B)$ given by

$$A = Q \begin{pmatrix} R \\ 0 \end{pmatrix}; \qquad B = Q * T * Z.$$

In particular, if matrix B is square nonsingular, then the problem GLM is equivalent to the following weighted linear least squares problem

$\text{minimize}_x \; ||B^{-1}(d-A*x)||_2$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

n	INTEGER. The number of rows of the matrices *A* and *B* ($n \geq 0$).
m	INTEGER. The number of columns in *A* ($m \geq 0$).
p	INTEGER. The number of columns in *B* ($p \geq n - m$).
a, *b*, *d*, *work*	REAL for sggglm DOUBLE PRECISION for dggglm COMPLEX for cggglm DOUBLE COMPLEX for zggglm. Arrays: *a*(*lda*,*) contains the *n*-by-*m* matrix *A*. The second dimension of *a* must be at least max(1, *m*). *b*(*ldb*,*) contains the *n*-by-*p* matrix *B*. The second dimension of *b* must be at least max(1, *p*). *d*(*), dimension at least max(1, *n*), contains the left hand side of the GLM equation. *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
lwork	INTEGER. The size of the *work* array; *lwork* $\geq$ max(1, *n*+*m*+*p*). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

x, *y*	REAL for sggglm DOUBLE PRECISION for dggglm COMPLEX for cggglm DOUBLE COMPLEX for zggglm. Arrays *x*(*), *y*(*). DIMENSION at least max(1, *m*) for *x* and at least max(1, *p*) for *y*. On exit, *x* and *y* are the solutions of the GLM problem.
a	On exit, the upper triangular part of the array *a* contains the *m*-by-*m* upper triangular matrix *R*.
b	On exit, if $n \leq p$, the upper triangle of the subarray *b*(1:*n*,*p*-*n*+1:*p*) contains the *n*-by-*n* upper triangular matrix *T*; if $n > p$, the elements on and above the (*n*-*p*)-th subdiagonal contain the *n*-by-*p* upper trapezoidal matrix *T*.
d	On exit, *d* is destroyed
work(1)	If *info* = 0, on exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = 1, the upper triangular factor *R* associated with *A* in the generalized QR factorization of the pair (*A*, *B*) is singular, so that rank(*A*) < *m*; the least squares solution could not be computed.
If *info* = 2, the bottom (*n*-*m*)-by-(*n*-*m*) part of the upper trapezoidal factor *T* associated with *B* in the generalized QR factorization of the pair (*A*, *B*) is singular, so that rank(*A B*) < *n*; the least squares solution could not be computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggglm interface are the following:

a	Holds the matrix *A* of size (*n*,*m*).
b	Holds the matrix *B* of size (*n*,*p*).
d	Holds the vector of length *n*.
x	Holds the vector of length (*m*).
y	Holds the vector of length (*p*).

Application Notes

For optimum performance, use

lwork ≥ *m* + min(*n*, *p*) + max(*n*, *p*)*nb*,

where *nb* is an upper bound for the optimal blocksizes for ?geqrf, ?gerqf, ?ormqr/?unmqr and ?ormrq/?unmrq.

You may set *lwork* to -1. The routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Symmetric Eigenproblems

This section describes LAPACK driver routines used for solving symmetric eigenvalue problems. See also computational routines that can be called to solve these problems. Table "Driver Routines for Solving Symmetric Eigenproblems" lists all such driver routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Solving Symmetric Eigenproblems

Routine Name	Operation performed
syev/heev	Computes all eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix.
syevd/heevd	Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric / Hermitian matrix using divide and conquer algorithm.
syevx/heevx	Computes selected eigenvalues and, optionally, eigenvectors of a symmetric / Hermitian matrix.
syevr/heevr	Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix using the Relatively Robust Representations.

Routine Name	Operation performed
spev/hpev	Computes all eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix in packed storage.
spevd/hpevd	Uses divide and conquer algorithm to compute all eigenvalues and (optionally) all eigenvectors of a real symmetric / Hermitian matrix held in packed storage.
spevx/hpevx	Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix in packed storage.
sbev /hbev	Computes all eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian band matrix.
sbevd/hbevd	Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric / Hermitian band matrix using divide and conquer algorithm.
sbevx/hbevx	Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian band matrix.
stev	Computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix.
stevd	Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric tridiagonal matrix using divide and conquer algorithm.
stevx	Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.
stevr	Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix using the Relatively Robust Representations.

?syev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix.

Syntax

Fortran 77:

```
call ssyev(jobz, uplo, n, a, lda, w, work, lwork, info)
call dsyev(jobz, uplo, n, a, lda, w, work, lwork, info)
```

Fortran 95:

```
call syev(a, w [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>syev( int matrix_order, char jobz, char uplo, lapack_int n,
<datatype>* a, lapack_int lda, <datatype>* w );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A.

Note that for most cases of real symmetric eigenvalue problems the default choice should be syevr function as its underlying algorithm is faster and uses less workspace.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* ($n \geq 0$).
a, *work*	REAL for ssyev DOUBLE PRECISION for dsyev Arrays: *a*(*lda*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. Constraint: *lwork* $\geq$ max(1, 3*n*-1). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	On exit, if *jobz* = 'V', then if *info* = 0, array *a* contains the orthonormal eigenvectors of the matrix *A*. If *jobz* = 'N', then on exit the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.
w	REAL for ssyev DOUBLE PRECISION for dsyev Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order.
work(1)	On exit, if *lwork* > 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `syev` interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
job	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For optimum performance set *lwork* ≥ (*nb*+2)**n*, where *nb* is the blocksize for `?sytrd` returned by `ilaenv`.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

If *lwork* has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array *work*. This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?heev

Computes all eigenvalues and, optionally, eigenvectors of a Hermitian matrix.

Syntax

Fortran 77:

```
call cheev(jobz, uplo, n, a, lda, w, work, lwork, rwork, info)
call zheev(jobz, uplo, n, a, lda, w, work, lwork, rwork, info)
```

Fortran 95:

```
call heev(a, w [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_cheev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* w );

lapack_int LAPACKE_zheev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* w );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix *A*.

Note that for most cases of complex Hermitian eigenvalue problems the default choice should be heevr function as its underlying algorithm is faster and uses less workspace.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* ($n \geq 0$).
a, *work*	COMPLEX for cheev DOUBLE COMPLEX for zheev Arrays: *a*(*lda*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. C onstraint: *lwork* $\geq$ max(1, 2*n*-1). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.
rwork	REAL for cheev DOUBLE PRECISION for zheev. Workspace array, DIMENSION at least max(1, 3*n*-2).

Output Parameters

a	On exit, if *jobz* = 'V', then if *info* = 0, array *a* contains the orthonormal eigenvectors of the matrix *A*. If *jobz* = 'N', then on exit the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.
w	REAL for cheev DOUBLE PRECISION for zheev Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order.
work(1)	On exit, if *lwork* > 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine heev interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
job	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For optimum performance use

lwork ≥ (*nb*+1)*n,

where *nb* is the blocksize for ?hetrd returned by ilaenv.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?syevd

Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric matrix using divide and conquer algorithm.

Syntax

Fortran 77:

```
call ssyevd(jobz, uplo, n, a, lda, w, work, lwork, iwork, liwork, info)
call dsyevd(jobz, uplo, n, a, lda, w, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call syevd(a, w [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>syevd( int matrix_order, char jobz, char uplo, lapack_int n,
<datatype>* a, lapack_int lda, <datatype>* w );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric matrix A. In other words, it can compute the spectral factorization of A as: $A = Z*\Lambda*Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the orthogonal matrix whose columns are the eigenvectors z_i. Thus,

$A*z_i = \lambda_i*z_i$ for $i = 1, 2, ..., n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Note that for most cases of real symmetric eigenvalue problems the default choice should be syevr function as its underlying algorithm is faster and uses less workspace. `?syevd` requires more workspace but is faster in some cases, especially for large matrices.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then only eigenvalues are computed. If *jobz* = `'V'`, then eigenvalues and eigenvectors are computed.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, *a* stores the upper triangular part of A. If *uplo* = `'L'`, *a* stores the lower triangular part of A.
n	`INTEGER`. The order of the matrix A ($n \geq 0$).
a	`REAL` for `ssyevd`

DOUBLE PRECISION for dsyevd
Array, DIMENSION (*lda*, *).
a(*lda*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A*, as specified by *uplo*.
The second dimension of *a* must be at least max(1, *n*).

lda — INTEGER. The leading dimension of the array *a*.
Must be at least max(1, *n*).

work — REAL for ssyevd
DOUBLE PRECISION for dsyevd.
Workspace array, DIMENSION at least *lwork*.

lwork — INTEGER.
The dimension of the array *work*.
Constraints:
if $n \leq 1$, then $lwork \geq 1$;
if *jobz* = 'N' and $n > 1$, then $lwork \geq 2*n + 1$;
if *jobz* = 'V' and $n > 1$, then $lwork \geq 2*n^2 + 6*n + 1$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER.
Workspace array, its dimension max(1, *liwork*).

liwork — INTEGER.
The dimension of the array *iwork*.
Constraints:
if $n \leq 1$, then $liwork \geq 1$;
if *jobz* = 'N' and $n > 1$, then $liwork \geq 1$;
if *jobz* = 'V' and $n > 1$, then $liwork \geq 5*n + 3$.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

w — REAL for ssyevd
DOUBLE PRECISION for dsyevd
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order.
See also *info*.

a — If *jobz* = 'V', then on exit this array is overwritten by the orthogonal matrix *Z* which contains the eigenvectors of *A*.

work(1) — On exit, if *lwork* > 0, then *work(1)* returns the required minimal size of *lwork*.

iwork(1) — On exit, if *liwork* > 0, then *iwork(1)* returns the required minimal size of *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.

If *info* = *i*, and *jobz* = 'N', then the algorithm failed to converge; *i* indicates the number of off-diagonal elements of an intermediate tridiagonal form which did not converge to zero.
If *info* = *i*, and *jobz* = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *info*/(*n*+1) through mod(*info*,*n*+1).
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syevd interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
jobz	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $||E||_2 = O(\varepsilon)*||T||_2$, where ε is the machine precision.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run, or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex analogue of this routine is heevd

?heevd

Computes all eigenvalues and (optionally) all eigenvectors of a complex Hermitian matrix using divide and conquer algorithm.

Syntax

Fortran 77:

```
call cheevd(jobz, uplo, n, a, lda, w, work, lwork, rwork, lrwork, iwork, liwork, info)
call zheevd(jobz, uplo, n, a, lda, w, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call heevd(a, w [,job] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_cheevd( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* w );

lapack_int LAPACKE_zheevd( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* w );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a complex Hermitian matrix A. In other words, it can compute the spectral factorization of A as: $A = Z*\Lambda*Z^H$.

Here Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i. Thus,

$A*z_i = \lambda_i*z_i$ for $i = 1, 2, \ldots, n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Note that for most cases of complex Hermetian eigenvalue problems the default choice should be heevr function as its underlying algorithm is faster and uses less workspace. `?heevd` requires more workspace but is faster in some cases, especially for large matrices.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of A. If *uplo* = 'L', *a* stores the lower triangular part of A.
n	INTEGER. The order of the matrix A ($n \geq 0$).
a	COMPLEX for cheevd DOUBLE COMPLEX for zheevd Array, DIMENSION (*lda*, *). *a*(*lda*,*) is an array containing either upper or lower triangular part of the Hermitian matrix A, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
work	COMPLEX for cheevd DOUBLE COMPLEX for zheevd. Workspace array, DIMENSION max(1, *lwork*).
lwork	INTEGER. The dimension of the array *work*. Constraints:

	if $n \leq 1$, then $lwork \geq 1$; if $jobz$ = 'N' and $n > 1$, then $lwork \geq n+1$; if $jobz$ = 'V' and $n > 1$, then $lwork \geq n^2+2*n$. If $lwork$ = -1, then a workspace query is assumed; the routine only calculates the optimal size of the $work$, $rwork$ and $iwork$ arrays, returns these values as the first entries of the $work$, $rwork$ and $iwork$ arrays, and no error message related to $lwork$ or $lrwork$ or $liwork$ is issued by xerbla. See *Application Notes* for details.
rwork	REAL for cheevd DOUBLE PRECISION for zheevd Workspace array, DIMENSION at least $lrwork$.
lrwork	INTEGER. The dimension of the array $rwork$. Constraints: if $n \leq 1$, then $lrwork \geq 1$; if job = 'N' and $n > 1$, then $lrwork \geq n$; if job = 'V' and $n > 1$, then $lrwork \geq 2*n^2+ 5*n + 1$. If $lrwork$ = -1, then a workspace query is assumed; the routine only calculates the optimal size of the $work$, $rwork$ and $iwork$ arrays, returns these values as the first entries of the $work$, $rwork$ and $iwork$ arrays, and no error message related to $lwork$ or $lrwork$ or $liwork$ is issued by xerbla. See *Application Notes* for details.
iwork	INTEGER. Workspace array, its dimension max(1, $liwork$).
liwork	INTEGER. The dimension of the array $iwork$. Constraints: if $n \leq 1$, then $liwork \geq 1$; if $jobz$ = 'N' and $n > 1$, then $liwork \geq 1$; if $jobz$ = 'V' and $n > 1$, then $liwork \geq 5*n+3$. If $liwork$ = -1, then a workspace query is assumed; the routine only calculates the optimal size of the $work$, $rwork$ and $iwork$ arrays, returns these values as the first entries of the $work$, $rwork$ and $iwork$ arrays, and no error message related to $lwork$ or $lrwork$ or $liwork$ is issued by xerbla. See *Application Notes* for details.

Output Parameters

w	REAL for cheevd DOUBLE PRECISION for zheevd Array, DIMENSION at least max(1, n). If $info$ = 0, contains the eigenvalues of the matrix A in ascending order. See also $info$.
a	If $jobz$ = 'V', then on exit this array is overwritten by the unitary matrix Z which contains the eigenvectors of A.
work(1)	On exit, if $lwork > 0$, then the real part of $work(1)$ returns the required minimal size of $lwork$.
rwork(1)	On exit, if $lrwork > 0$, then $rwork(1)$ returns the required minimal size of $lrwork$.
iwork(1)	On exit, if $liwork > 0$, then $iwork(1)$ returns the required minimal size of $liwork$.
info	INTEGER. If $info$ = 0, the execution is successful.

If *info* = *i*, and *jobz* = 'N', then the algorithm failed to converge; *i* off-diagonal elements of an intermediate tridiagonal form did not converge to zero;
if *info* = *i*, and *jobz* = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *info*/(*n*+1) through mod(*info*, *n*+1).
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine heevd interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length (*n*).
jobz	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A + E$ such that $||E||_2 = O(\varepsilon)*||A||_2$, where ε is the machine precision.

If you are in doubt how much workspace to supply, use a generous value of *lwork* (*liwork* or *lrwork*) for the first run or set *lwork* = -1 (*liwork* = -1, *lrwork* = -1).

If you choose the first option and set any of admissible *lwork* (*liwork* or *lrwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*) on exit. Use this value (*work*(1), *iwork*(1), *rwork*(1)) for subsequent runs.

If you set *lwork* = -1 (*liwork* = -1, *lrwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*, *lrwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real analogue of this routine is syevd. See also hpevd for matrices held in packed storage, and hbevd for banded matrices.

?syevx

Computes selected eigenvalues and, optionally, eigenvectors of a symmetric matrix.

Syntax

Fortran 77:

```
call ssyevx(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz, work, lwork, iwork, ifail, info)
```

```
call dsyevx(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz, work, lwork, iwork, ifail, info)
```

Fortran 95:

```
call syevx(a, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>syevx( int matrix_order, char jobz, char range, char uplo,
lapack_int n, <datatype>* a, lapack_int lda, <datatype> vl, <datatype> vu, lapack_int
il, lapack_int iu, <datatype> abstol, lapack_int* m, <datatype>* w, <datatype>* z,
lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix *A*. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Note that for most cases of real symmetric eigenvalue problems the default choice should be syevr function as its underlying algorithm is faster and uses less workspace. `?syevx` is faster for a few selected eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then only eigenvalues are computed. If *jobz* = `'V'`, then eigenvalues and eigenvectors are computed.
range	`CHARACTER*1`. Must be `'A'`, `'V'`, or `'I'`. If *range* = `'A'`, all eigenvalues will be found. If *range* = `'V'`, all eigenvalues in the half-open interval (*vl*, *vu*] will be found. If *range* = `'I'`, the eigenvalues with indices *il* through *iu* will be found.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, *a* stores the upper triangular part of *A*. If *uplo* = `'L'`, *a* stores the lower triangular part of *A*.
n	`INTEGER`. The order of the matrix *A* ($n \geq 0$).
a, *work*	`REAL` for `ssyevx` `DOUBLE PRECISION` for `dsyevx`. Arrays: *a*(*lda*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension `max(1, lwork)`.
lda	`INTEGER`. The leading dimension of the array *a*. Must be at least max(1, *n*).
vl, *vu*	`REAL` for `ssyevx` `DOUBLE PRECISION` for `dsyevx`.

If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; *vl* ≤ *vu*. Not referenced if *range* = 'A'or 'I'.

il, *iu*
INTEGER.
If *range* = 'I', the indices of the smallest and largest eigenvalues to be returned.
Constraints: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0;
il = 1 and *iu* = 0, if *n* = 0.
Not referenced if *range* = 'A'or 'V'.

abstol
REAL for ssyevx
DOUBLE PRECISION for dsyevx.
The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.

ldz
INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1.
If *jobz* = 'V', then *ldz* ≥ max(1, *n*).

lwork
INTEGER.
The dimension of the array *work*.
If *n* ≤ 1 then *lwork* ≥ 1, otherwise *lwork*=8**n*.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

iwork
INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

a
On exit, the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

m
INTEGER. The total number of eigenvalues found;
0 ≤ *m* ≤ *n*.
If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+*1*.

w
REAL for ssyevx
DOUBLE PRECISION for dsyevx
Array, DIMENSION at least max(1, *n*). The first *m* elements contain the selected eigenvalues of the matrix *A* in ascending order.

z
REAL for ssyevx
DOUBLE PRECISION for dsyevx.
Array *z*(*ldz*,*) contains eigenvectors.
The second dimension of *z* must be at least max(1, *m*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the i-th column of *z* holding the eigenvector associated with *w*(i).
If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
If *jobz* = 'N', then *z* is not referenced.
Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

work(1)	On exit, if *lwork* > 0, then *work*(1) returns the required minimal size of *lwork*.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, then *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'V', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syevx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
a	Holds the matrix *A* of size (*m*, *n*).
ifail	Holds the vector of length *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

For optimum performance use *lwork* ≥ (*nb*+3)*n, where *nb* is the maximum of the blocksize for ?sytrd and ?ormtr returned by ilaenv.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If *lwork* has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array *work*. This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*|*T*| is used as tolerance, where|*T*| is the 1-norm of the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues are computed most accurately when *abstol* is set to twice the underflow threshold 2*slamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*slamch('S').

?heevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix.

Syntax

Fortran 77:

```
call cheevx(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz, work, lwork, rwork, iwork, ifail, info)
```

```
call zheevx(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz, work, lwork, rwork, iwork, ifail, info)
```

Fortran 95:

```
call heevx(a, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_cheevx( int matrix_order, char jobz, char range, char uplo, lapack_int n, lapack_complex_float* a, lapack_int lda, float vl, float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zheevx( int matrix_order, char jobz, char range, char uplo, lapack_int n, lapack_complex_double* a, lapack_int lda, double vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix *A*. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Note that for most cases of complex Hermetian eigenvalue problems the default choice should be heevr function as its underlying algorithm is faster and uses less workspace. ?heevx is faster for a few selected eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
range	CHARACTER*1. Must be 'A', 'V', or 'I'. If *range* = 'A', all eigenvalues will be found. If *range* = 'V', all eigenvalues in the half-open interval (*vl*, *vu*] will be found. If *range* = 'I', the eigenvalues with indices *il* through *iu* will be found.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
a, *work*	COMPLEX for cheevx DOUBLE COMPLEX for zheevx. Arrays: *a*(*lda*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
vl, *vu*	REAL for cheevx DOUBLE PRECISION for zheevx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; *vl* ≤ *vu*. Not referenced if *range* = 'A'or 'I'.
il, *iu*	INTEGER. If *range* = 'I', the indices of the smallest and largest eigenvalues to be returned. Constraints: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0;*il* = 1 and *iu* = 0, if *n* = 0. Not referenced if *range* = 'A'or 'V'.
abstol	REAL for cheevx DOUBLE PRECISION for zheevx. The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', then *ldz* ≥max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. *lwork* ≥ 1 if *n*≤1; otherwise at least 2*n. If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.
rwork	REAL for cheevx DOUBLE PRECISION for zheevx. Workspace array, DIMENSION at least max(1, 7*n*).

iwork	INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

a	On exit, the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.
m	INTEGER. The total number of eigenvalues found; $0 \le m \le n$. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.
w	REAL for cheevx DOUBLE PRECISION for zheevx Array, DIMENSION at least max(1, *n*). The first *m* elements contain the selected eigenvalues of the matrix *A* in ascending order.
z	COMPLEX for cheevx DOUBLE COMPLEX for zheevx. Array *z*(*ldz*,*) contains eigenvectors. The second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the i-th column of *z* holding the eigenvector associated with *w*(i). If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*. If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
work(1)	On exit, if *lwork* > 0, then *work(1)* returns the required minimal size of *lwork*.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, then *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'V', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine heevx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
ifail	Holds the vector of length *n*.

uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

For optimum performance use *lwork* ≥ (*nb*+1)*n, where *nb* is the maximum of the blocksize for ?hetrd and ?unmtr returned by ilaenv.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*|*T*| will be used in its place, where |*T*| is the 1-norm of the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*slamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*slamch('S').

?syevr

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix using the Relatively Robust Representations.

Syntax

Fortran 77:

```
call ssyevr(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz,
isuppz, work, lwork, iwork, liwork, info)
```

```
call dsyevr(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz,
isuppz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call syevr(a, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,isuppz] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>syevr( int matrix_order, char jobz, char range, char uplo,
lapack_int n, <datatype>* a, lapack_int lda, <datatype> vl, <datatype> vu, lapack_int
il, lapack_int iu, <datatype> abstol, lapack_int* m, <datatype>* w, <datatype>* z,
lapack_int ldz, lapack_int* isuppz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

The routine first reduces the matrix A to tridiagonal form T with a call to sytrd. Then, whenever possible, `?syevr` calls stemr to compute the eigenspectrum using Relatively Robust Representations. stemr computes eigenvalues by the *dqds* algorithm, while orthogonal eigenvectors are computed from various "good" $L*D*L^T$ representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the each unreduced block of T:

a. Compute $T - \sigma*I = L*D*L^T$, so that L and D define all the wanted eigenvalues to high relative accuracy. This means that small relative changes in the entries of D and L cause only small relative changes in the eigenvalues and eigenvectors. The standard (unfactored) representation of the tridiagonal matrix T does not have this property in general.

b. Compute the eigenvalues to suitable accuracy. If the eigenvectors are desired, the algorithm attains full accuracy of the computed eigenvalues only right before the corresponding vectors have to be computed, see Steps c) and d).

c. For each cluster of close eigenvalues, select a new shift close to the cluster, find a new factorization, and refine the shifted eigenvalues to suitable accuracy.

d. For each eigenvalue with a large enough relative separation, compute the corresponding eigenvector by forming a rank revealing twisted factorization. Go back to Step c) for any clusters that remain.

The desired accuracy of the output can be specified by the input parameter *abstol*.

The routine `?syevr` calls stemr when the full spectrum is requested on machines that conform to the IEEE-754 floating point standard. `?syevr` calls stebz and stein on non-IEEE machines and when partial spectrum requests are made.

Note that `?syevr` is preferable for most cases of real symmetric eigenvalue problems as its underlying algorithm is fast and uses less workspace.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz — `CHARACTER*1`. Must be `'N'` or `'V'`.
If *jobz* = `'N'`, then only eigenvalues are computed.
If *jobz* = `'V'`, then eigenvalues and eigenvectors are computed.

range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl* < *lambda*(*i*) ≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*. For *range* = 'V'or 'I' and *iu*-*il* < *n*-1, sstebz/dstebz and sstein/dstein are called.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
a, *work*	REAL for ssyevr DOUBLE PRECISION for dsyevr. Arrays: *a*(*lda*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
vl, *vu*	REAL for ssyevr DOUBLE PRECISION for dsyevr. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.
il, *iu*	INTEGER. If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0, if *n* = 0. If *range* = 'A' or 'V', *il* and *iu* are not referenced.
abstol	REAL for ssyevr DOUBLE PRECISION for dsyevr. The absolute error tolerance to which each eigenvalue/eigenvector is required. If *jobz* = 'V', the eigenvalues and eigenvectors output have residual norms bounded by *abstol*, and the dot products between different eigenvectors are bounded by *abstol*. If *abstol* < *n* *eps*|*T*|, then *n* *eps*|*T*| is used instead, where eps is the machine precision, and |*T*| is the 1-norm of the matrix *T*. The eigenvalues are computed to an accuracy of eps*|*T*| irrespective of *abstol*. If high relative accuracy is important, set *abstol* to ?lamch('S').
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: *ldz* ≥ 1 and *ldz* ≥ max(1, *n*) if *jobz* = 'V'.
lwork	INTEGER. The dimension of the array *work*.

Constraint: *lwork* ≥ max(1, 26*n*).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

iwork — INTEGER. Workspace array, its dimension max(1, *liwork*).

liwork — INTEGER.
The dimension of the array *iwork*, *lwork* ≥ max(1, 10*n*).
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *iwork* array, returns this value as the first entry of the *iwork* array, and no error message related to *liwork* is issued by xerbla.

Output Parameters

a — On exit, the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

m — INTEGER. The total number of eigenvalues found, 0 ≤ *m* ≤ *n*.
If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.

w, *z* — REAL for ssyevr
DOUBLE PRECISION for dsyevr.
Arrays:
w(*), DIMENSION at least max(1, *n*), contains the selected eigenvalues in ascending order, stored in *w*(1) to *w*(*m*);
z(*ldz*, *), the second dimension of *z* must be at least max(1, *m*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*).
If *jobz* = 'N', then *z* is not referenced. Note that you must ensure that at least max(1, *m*) columns are supplied in the array *z* ; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

isuppz — INTEGER.
Array, DIMENSION at least 2 *max(1, *m*).
The support of the eigenvectors in *z*, i.e., the indices indicating the nonzero elements in *z*. The *i*-th eigenvector is nonzero only in elements *isuppz*(2*i*-1) through *isuppz*(2*i*). Referenced only if eigenvectors are needed (*jobz* = 'V') and all eigenvalues are needed, that is, *range* = 'A' or *range* = 'I' and *il* = 1 and *iu* = *n*.

work(1) — On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.

iwork(1) — On exit, if *info* = 0, then *iwork(1)* returns the required minimal size of *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, an internal error has occurred.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `syevr` interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
isuppz	Holds the vector of length (2**m*), where the values (2**m*) are significant.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *isuppz* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

For optimum performance use *lwork* ≥ (*nb*+6)**n*, where *nb* is the maximum of the blocksize for `?sytrd` and `?ormtr` returned by `ilaenv`.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Normal execution of `?stegr` may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the IEEE standard default manner.

?heevr

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix using the Relatively Robust Representations.

Syntax

Fortran 77:

```
call cheevr(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz,
isuppz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

```
call zheevr(jobz, range, uplo, n, a, lda, vl, vu, il, iu, abstol, m, w, z, ldz,
isuppz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call heevr(a, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,isuppz] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_cheevr( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_complex_float* a, lapack_int lda, float vl, float vu, lapack_int
il, lapack_int iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z,
lapack_int ldz, lapack_int* isuppz );
```

```
lapack_int LAPACKE_zheevr( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_complex_double* a, lapack_int lda, double vl, double vu,
lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w,
lapack_complex_double* z, lapack_int ldz, lapack_int* isuppz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

The routine first reduces the matrix A to tridiagonal form T with a call to hetrd. Then, whenever possible, `?heevr` calls stegr to compute the eigenspectrum using Relatively Robust Representations. `?stegr` computes eigenvalues by the *dqds* algorithm, while orthogonal eigenvectors are computed from various "good" $L*D*L^T$ representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For each unreduced block (submatrix) of T:

a. Compute $T - \sigma*I = L*D*L^T$, so that L and D define all the wanted eigenvalues to high relative accuracy. This means that small relative changes in the entries of D and L cause only small relative changes in the eigenvalues and eigenvectors. The standard (unfactored) representation of the tridiagonal matrix T does not have this property in general.

b. Compute the eigenvalues to suitable accuracy. If the eigenvectors are desired, the algorithm attains full accuracy of the computed eigenvalues only right before the corresponding vectors have to be computed, see Steps c) and d).

c. For each cluster of close eigenvalues, select a new shift close to the cluster, find a new factorization, and refine the shifted eigenvalues to suitable accuracy.

d. For each eigenvalue with a large enough relative separation, compute the corresponding eigenvector by forming a rank revealing twisted factorization. Go back to Step c) for any clusters that remain.

The desired accuracy of the output can be specified by the input parameter *abstol*.

The routine ?heevr calls stemr when the full spectrum is requested on machines which conform to the IEEE-754 floating point standard, or stebz and stein on non-IEEE machines and when partial spectrum requests are made.

Note that the routine ?heevr is preferable for most cases of complex Hermitian eigenvalue problems as its underlying algorithm is fast and uses less workspace.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz
CHARACTER*1. Must be 'N' or 'V'.
If *job* = 'N', then only eigenvalues are computed.
If *job* = 'V', then eigenvalues and eigenvectors are computed.

range
CHARACTER*1. Must be 'A' or 'V' or 'I'.
If *range* = 'A', the routine computes all eigenvalues.
If *range* = 'V', the routine computes eigenvalues *lambda(i)* in the half-open interval: *vl*< *lambda(i)* ≤ *vu*.
If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
For *range* = 'V'or 'I', sstebz/dstebz and cstein/zstein are called.

uplo
CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', *a* stores the upper triangular part of *A*.
If *uplo* = 'L', *a* stores the lower triangular part of *A*.

n
INTEGER. The order of the matrix *A* (*n* ≥ 0).

a, *work*
COMPLEX for cheevr
DOUBLE COMPLEX for zheevr.
Arrays:
a(*lda*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A*, as specified by *uplo*.
The second dimension of *a* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

lda
INTEGER. The leading dimension of the array *a*.
Must be at least max(1, *n*).

vl, *vu*
REAL for cheevr
DOUBLE PRECISION for zheevr.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl*< *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for cheevr
DOUBLE PRECISION for zheevr.
The absolute error tolerance to which each eigenvalue/eigenvector is required.
If *jobz* = 'V', the eigenvalues and eigenvectors output have residual norms bounded by *abstol*, and the dot products between different eigenvectors are bounded by *abstol*.

If *abstol* < *n* *eps*|*T*|, then *n* *eps*|*T*| will be used in its place, where eps is the machine precision, and |*T*| is the 1-norm of the matrix *T*. The eigenvalues are computed to an accuracy of eps*|*T*| irrespective of *abstol*.
If high relative accuracy is important, set *abstol* to ?lamch('S').

ldz — INTEGER. The leading dimension of the output array *z*. Constraints:
ldz ≥ 1 if *jobz* = 'N';
ldz ≥ max(1, *n*) if *jobz* = 'V'.

lwork — INTEGER.
The dimension of the array *work*.
Constraint: *lwork* ≥ max(1, 2*n*).
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

rwork — REAL for cheevr
DOUBLE PRECISION for zheevr.
Workspace array, DIMENSION max(1, *lwork*).

lrwork — INTEGER.
The dimension of the array *rwork*;
lwork ≥ max(1, 24*n*).
If *lrwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla.

iwork — INTEGER. Workspace array, its dimension max(1, *liwork*).

liwork — INTEGER.
The dimension of the array *iwork*,
lwork ≥ max(1, 10*n*).
If *liwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla.

Output Parameters

a — On exit, the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

m — INTEGER. The total number of eigenvalues found,
0 ≤ *m* ≤ *n*.
If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.

w — REAL for cheevr
DOUBLE PRECISION for zheevr.
Array, DIMENSION at least max(1, *n*), contains the selected eigenvalues in ascending order, stored in *w*(1) to *w*(*m*).

z — COMPLEX for cheevr
DOUBLE COMPLEX for zheevr.
Array *z*(*ldz*, *); the second dimension of *z* must be at least max(1, *m*).

	If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
isuppz	INTEGER. Array, DIMENSION at least 2 *max(1, *m*). The support of the eigenvectors in *z*, i.e., the indices indicating the nonzero elements in *z*. The *i*-th eigenvector is nonzero only in elements *isuppz*(2*i*-1) through *isuppz*(2*i*). Referenced only if eigenvectors are needed (*jobz* = 'V') and all eigenvalues are needed, that is, *range* = 'A' or *range* = 'I' and *il* = 1 and *iu* = *n*.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
rwork(1)	On exit, if *info* = 0, then *rwork(1)* returns the required minimal size of *lrwork*.
iwork(1)	On exit, if *info* = 0, then *iwork(1)* returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, an internal error has occurred.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine heevr interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
isuppz	Holds the vector of length (2**n*), where the values (2**m*) are significant.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *isuppz* is present and *z* is omitted.

range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

For optimum performance use *lwork* ≥ (*nb*+1)*n, where *nb* is the maximum of the blocksize for ?hetrd and ?unmtr returned by ilaenv.

If you are in doubt how much workspace to supply, use a generous value of *lwork* (or *lrwork*, or *liwork*) for the first run or set *lwork* = -1 (*lrwork* = -1, *liwork* = -1).

If you choose the first option and set any of admissible *lwork* (or *lrwork*, *liwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *rwork*, *iwork*) on exit. Use this value (*work*(1), *rwork*(1), *iwork*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *rwork*, *iwork*). This operation is called a workspace query.

Note that if you set *lwork* (*lrwork*, *liwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Normal execution of ?stegr may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the IEEE standard default manner.

?spev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix in packed storage.

Syntax

Fortran 77:

```
call sspev(jobz, uplo, n, ap, w, z, ldz, work, info)
call dspev(jobz, uplo, n, ap, w, z, ldz, work, info)
```

Fortran 95:

```
call spev(ap, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>spev( int matrix_order, char jobz, char uplo, lapack_int n,
<datatype>* ap, <datatype>* w, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix *A* in packed storage.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *job* = 'N', then only eigenvalues are computed. If *job* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ap* stores the packed upper triangular part of *A*. If *uplo* = 'L', *ap* stores the packed lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
ap, *work*	REAL for sspev DOUBLE PRECISION for dspev Arrays: *ap(*)* contains the packed upper or lower triangle of symmetric matrix *A*, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2).
work	(*) is a workspace array, DIMENSION at least max(1, 3*n*).
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: if *jobz* = 'N', then *ldz* ≥ 1; if *jobz* = 'V', then *ldz* ≥ max(1, *n*).

Output Parameters

w, *z*	REAL for sspev DOUBLE PRECISION for dspev Arrays: *w*(*), DIMENSION at least max(1, *n*). If *info* = 0, *w* contains the eigenvalues of the matrix *A* in ascending order. *z*(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the orthonormal eigenvectors of the matrix *A*, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced.
ap	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of *A*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spev interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

?hpev

Computes all eigenvalues and, optionally, eigenvectors of a Hermitian matrix in packed storage.

Syntax

Fortran 77:

```
call chpev(jobz, uplo, n, ap, w, z, ldz, work, rwork, info)
call zhpev(jobz, uplo, n, ap, w, z, ldz, work, rwork, info)
```

Fortran 95:

```
call hpev(ap, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chpev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_complex_float* ap, float* w, lapack_complex_float* z, lapack_int ldz );
lapack_int LAPACKE_zhpev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_complex_double* ap, double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix *A* in packed storage.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *job* = 'N', then only eigenvalues are computed. If *job* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ap* stores the packed upper triangular part of *A*. If *uplo* = 'L', *ap* stores the packed lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
ap, *work*	COMPLEX for chpev DOUBLE COMPLEX for zhpev.

	Arrays: *ap(*)* contains the packed upper or lower triangle of Hermitian matrix A, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2).
work	(*) is a workspace array, DIMENSION at least max(1, 2*n*-1).
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: if *jobz* = 'N', then *ldz* ≥ 1; if *jobz* = 'V', then *ldz* ≥ max(1, *n*) .
rwork	REAL for chpev DOUBLE PRECISION for zhpev. Workspace array, DIMENSION at least max(1, 3*n*-2).

Output Parameters

w	REAL for chpev DOUBLE PRECISION for zhpev. Array, DIMENSION at least max(1, *n*). If *info* = 0, *w* contains the eigenvalues of the matrix *A* in ascending order.
z	COMPLEX for chpev DOUBLE COMPLEX for zhpev. Array z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the orthonormal eigenvectors of the matrix *A*, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced.
ap	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpev interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

?spevd

Uses divide and conquer algorithm to compute all eigenvalues and (optionally) all eigenvectors of a real symmetric matrix held in packed storage.

Syntax

Fortran 77:

```
call sspevd(jobz, uplo, n, ap, w, z, ldz, work, lwork, iwork, liwork, info)
call dspevd(jobz, uplo, n, ap, w, z, ldz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call spevd(ap, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>spevd( int matrix_order, char jobz, char uplo, lapack_int n,
<datatype>* ap, <datatype>* w, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric matrix A (held in packed storage). In other words, it can compute the spectral factorization of A as:

$A = Z*\Lambda*Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the orthogonal matrix whose columns are the eigenvectors z_i. Thus,

$A*z_i = \lambda_i*z_i$ for $i = 1, 2, \ldots, n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then only eigenvalues are computed. If *jobz* = `'V'`, then eigenvalues and eigenvectors are computed.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, *ap* stores the packed upper triangular part of A. If *uplo* = `'L'`, *ap* stores the packed lower triangular part of A.
n	`INTEGER`. The order of the matrix A ($n \geq 0$).
ap, *work*	`REAL` for `sspevd` `DOUBLE PRECISION` for `dspevd` Arrays:

	ap(*) contains the packed upper or lower triangle of symmetric matrix A, as specified by *uplo*. The dimension of *ap* must be max(1, *n**(*n*+1)/2) *work* is a workspace array, its dimension max(1, *lwork*).
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: if *jobz* = 'N', then *ldz* ≥ 1; if *jobz* = 'V', then *ldz* ≥ max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. Constraints: if *n* ≤ 1, then *lwork* ≥ 1; if *jobz* = 'N' and *n* > 1, then *lwork* ≥ 2**n*; if *jobz* = 'V' and *n* > 1, then *lwork* ≥ n^2+ 6**n* + 1. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.
iwork	INTEGER. Workspace array, its dimension max(1, *liwork*).
liwork	INTEGER. The dimension of the array *iwork*. Constraints: if *n* ≤ 1, then *liwork* ≥ 1; if *jobz* = 'N' and *n* > 1, then *liwork* ≥ 1; if *jobz* = 'V' and *n* > 1, then *liwork* ≥ 5**n*+3. If *liwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

w, *z*	REAL for sspevd DOUBLE PRECISION for dspevd Arrays: *w(*)*, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order. See also *info*. z(*ldz*,*). The second dimension of *z* must be: at least 1 if *jobz* = 'N';at least max(1, *n*) if *jobz* = 'V'. If *jobz* = 'V', then this array is overwritten by the orthogonal matrix *Z* which contains the eigenvectors of *A*. If *jobz* = 'N', then *z* is not referenced.
ap	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.
work(1)	On exit, if *info* = 0, then *work*(1) returns the required *lwork*.

iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the required *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spevd interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $||E||_2 = O(\varepsilon)*||T||_2$, where ε is the machine precision.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex analogue of this routine is hpevd.

See also syevd for matrices held in full storage, and sbevd for banded matrices.

?hpevd

Uses divide and conquer algorithm to compute all eigenvalues and (optionally) all eigenvectors of a complex Hermitian matrix held in packed storage.

Syntax

Fortran 77:

```
call chpevd(jobz, uplo, n, ap, w, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

```
call zhpevd(jobz, uplo, n, ap, w, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call hpevd(ap, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chpevd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_complex_float* ap, float* w, lapack_complex_float* z, lapack_int ldz );

lapack_int LAPACKE_zhpevd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_complex_double* ap, double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a complex Hermitian matrix A (held in packed storage). In other words, it can compute the spectral factorization of A as: $A = Z*\Lambda*Z^H$.

Here Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i. Thus,

$A*z_i = \lambda_i*z_i$ for $i = 1, 2, ..., n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then only eigenvalues are computed. If *jobz* = `'V'`, then eigenvalues and eigenvectors are computed.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, *ap* stores the packed upper triangular part of A. If *uplo* = `'L'`, *ap* stores the packed lower triangular part of A.
n	`INTEGER`. The order of the matrix A ($n \geq 0$).
ap, *work*	`COMPLEX` for `chpevd` `DOUBLE COMPLEX` for `zhpevd` Arrays: *ap(*)* contains the packed upper or lower triangle of Hermitian matrix A, as specified by *uplo*. The dimension of *ap* must be at least max(1, n*(n+1)/2). *work* is a workspace array, its dimension `max(1, lwork)`.
ldz	`INTEGER`. The leading dimension of the output array z. Constraints: if *jobz* = `'N'`, then *ldz* $\geq$ 1; if *jobz* = `'V'`, then *ldz* $\geq$ max(1, n).

lwork INTEGER.
The dimension of the array *work*.
Constraints:
if $n \leq 1$, then $lwork \geq 1$;
if *jobz* = 'N' and $n > 1$, then $lwork \geq n$;
if *jobz* = 'V' and $n > 1$, then $lwork \geq 2*n$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

rwork REAL for chpevd
DOUBLE PRECISION for zhpevd
Workspace array, its dimension max(1, *lrwork*).

lrwork INTEGER.
The dimension of the array *rwork*. Constraints:
if $n \leq 1$, then $lrwork \geq 1$;
if *jobz* = 'N' and $n > 1$, then $lrwork \geq n$;
if *jobz* = 'V' and $n > 1$, then $lrwork \geq 2*n^2 + 5*n + 1$.
If *lrwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork INTEGER. Workspace array, its dimension max(1, *liwork*).

liwork INTEGER.
The dimension of the array *iwork*.
Constraints:
if $n \leq 1$, then $liwork \geq 1$;
if *jobz* = 'N' and $n > 1$, then $liwork \geq 1$;
if *jobz* = 'V' and $n > 1$, then $liwork \geq 5*n+3$.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

w REAL for chpevd
DOUBLE PRECISION for zhpevd
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order. See also *info*.

z COMPLEX for chpevd
DOUBLE COMPLEX for zhpevd
Array, DIMENSION (*ldz*,*).
The second dimension of *z* must be:
at least 1 if *jobz* = 'N';
at least max(1, *n*) if *jobz* = 'V'.
If *jobz* = 'V', then this array is overwritten by the unitary matrix *Z* which contains the eigenvectors of *A*.

	If *jobz* = 'N', then *z* is not referenced.
ap	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A.
work(1)	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
rwork(1)	On exit, if *info* = 0, then *rwork*(1) returns the required minimal size of *lrwork*.
iwork(1)	On exit, if *info* = 0, then *iwork(1)* returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpevd interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T + E$ such that $||E||_2 = O(\varepsilon)*||T||_2$, where ε is the machine precision.

If you are in doubt how much workspace to supply, use a generous value of *lwork* (*liwork* or *lrwork*) for the first run or set *lwork* = -1 (*liwork* = -1, *lrwork* = -1).

If you choose the first option and set any of admissible *lwork* (*liwork* or *lrwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*) on exit. Use this value (*work*(1), *iwork*(1), *rwork*(1)) for subsequent runs.

If you set *lwork* = -1 (*liwork* = -1, *lrwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*, *lrwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real analogue of this routine is spevd.

See also heevd for matrices held in full storage, and hbevd for banded matrices.

?spevx

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix in packed storage.

Syntax

Fortran 77:

```
call sspevx(jobz, range, uplo, n, ap, vl, vu, il, iu, abstol, m, w, z, ldz, work, iwork, ifail, info)
```

```
call dspevx(jobz, range, uplo, n, ap, vl, vu, il, iu, abstol, m, w, z, ldz, work, iwork, ifail, info)
```

Fortran 95:

```
call spevx(ap, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>spevx( int matrix_order, char jobz, char range, char uplo, lapack_int n, <datatype>* ap, <datatype> vl, <datatype> vu, lapack_int il, lapack_int iu, <datatype> abstol, lapack_int* m, <datatype>* w, <datatype>* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *job* = 'N', then only eigenvalues are computed. If *job* = 'V', then eigenvalues and eigenvectors are computed.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: $vl < lambda(i) \leq vu$. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ap* stores the packed upper triangular part of A. If *uplo* = 'L', *ap* stores the packed lower triangular part of A.
n	INTEGER. The order of the matrix A ($n \geq 0$).

ap, *work*
REAL for sspevx
DOUBLE PRECISION for dspevx
Arrays:
ap(*) contains the packed upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*.
The dimension of *ap* must be at least max(1, *n**(*n*+1)/2).
work(*) is a workspace array, DIMENSION at least max(1, 8*n*).

vl, *vu*
REAL for sspevx
DOUBLE PRECISION for dspevx
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl*< *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for sspevx
DOUBLE PRECISION for dspevx
The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.

ldz
INTEGER. The leading dimension of the output array *z*.
Constraints:
if *jobz* = 'N', then *ldz* ≥ 1;
if *jobz* = 'V', then *ldz* ≥ max(1, *n*).

iwork
INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

ap
On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of *A*.

m
INTEGER. The total number of eigenvalues found,
0 ≤ *m* ≤ *n*. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.

w, *z*
REAL for sspevx
DOUBLE PRECISION for dspevx
Arrays:
w(*), DIMENSION at least max(1, *n*).
If *info* = 0, contains the selected eigenvalues of the matrix *A* in ascending order.
z(*ldz*,*).
The second dimension of *z* must be at least max(1, *m*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*).
If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If *jobz* = 'N', then *z* is not referenced.
Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

ifail — INTEGER.
Array, DIMENSION at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices the eigenvectors that failed to converge.
If *jobz* = 'N', then *ifail* is not referenced.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spevx interface are the following:

ap — Holds the array *A* of size (*n**(*n*+1)/2).

w — Holds the vector with the number of elements *n*.

z — Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.

ifail — Holds the vector with the number of elements *n*.

uplo — Must be 'U' or 'L'. The default value is 'U'.

vl — Default value for this element is *vl* = -HUGE(*vl*).

vu — Default value for this element is *vu* = HUGE(*vl*).

il — Default value for this argument is *il* = 1.

iu — Default value for this argument is *iu* = *n*.

abstol — Default value for this element is *abstol* = 0.0_WP.

jobz — Restored based on the presence of the argument *z* as follows:
jobz = 'V', if *z* is present,
jobz = 'N', if *z* is omitted
Note that there will be an error condition if *ifail* is present and *z* is omitted.

range — Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows:
range = 'V', if one of or both *vl* and *vu* are present,
range = 'I', if one of or both *il* and *iu* are present,
range = 'A', if none of *vl*, *vu*, *il*, *iu* is present,
Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then $\varepsilon * ||T||_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

?hpevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix in packed storage.

Syntax

Fortran 77:

```
call chpevx(jobz, range, uplo, n, ap, vl, vu, il, iu, abstol, m, w, z, ldz, work, rwork, iwork, ifail, info)
```

```
call zhpevx(jobz, range, uplo, n, ap, vl, vu, il, iu, abstol, m, w, z, ldz, work, rwork, iwork, ifail, info)
```

Fortran 95:

```
call hpevx(ap, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_chpevx( int matrix_order, char jobz, char range, char uplo, lapack_int n, lapack_complex_float* ap, float vl, float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zhpevx( int matrix_order, char jobz, char range, char uplo, lapack_int n, lapack_complex_double* ap, double vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix *A* in packed storage. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *job* = 'N', then only eigenvalues are computed. If *job* = 'V', then eigenvalues and eigenvectors are computed.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl*<*lambda*(*i*) ≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.

uplo CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', *ap* stores the packed upper triangular part of *A*.
If *uplo* = 'L', *ap* stores the packed lower triangular part of *A*.

n INTEGER. The order of the matrix *A* ($n \geq 0$).

ap, *work* COMPLEX for chpevx
DOUBLE COMPLEX for zhpevx
Arrays:
ap(*) contains the packed upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*.
The dimension of *ap* must be at least max(1, n*(n+1)/2).
work(*) is a workspace array, DIMENSION at least max(1, 2n).

vl, *vu* REAL for chpevx
DOUBLE PRECISION for zhpevx
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl*< *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu* INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; *il*=1 and *iu*=0 if $n = 0$.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol REAL for chpevx
DOUBLE PRECISION for zhpevx
The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.

ldz INTEGER. The leading dimension of the output array *z*.
Constraints:
if *jobz* = 'N', then *ldz* ≥ 1;
if *jobz* = 'V', then *ldz* $\geq$ max(1, *n*).

rwork REAL for chpevx
DOUBLE PRECISION for zhpevx
Workspace array, DIMENSION at least max(1, 7n).

iwork INTEGER. Workspace array, DIMENSION at least max(1, 5n).

Output Parameters

ap On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of *A*.

m INTEGER. The total number of eigenvalues found, $0 \leq m \leq n$.
If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu-il+1$.

w REAL for chpevx
DOUBLE PRECISION for zhpevx
Array, DIMENSION at least max(1, n).
If *info* = 0, contains the selected eigenvalues of the matrix *A* in ascending order.

z COMPLEX for chpevx
DOUBLE COMPLEX for zhpevx
Array z(*ldz*,*).

	The second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*. If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpevx interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector with the number of elements *n*.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present,

Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*T*||$_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

?sbev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric band matrix.

Syntax

Fortran 77:

```
call ssbev(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, info)
call dsbev(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, info)
```

Fortran 95:

```
call sbev(ab, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int kd, <datatype>* ab, lapack_int ldab, <datatype>* w, <datatype>* z,
lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric band matrix *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).

kd	INTEGER. The number of super- or sub-diagonals in *A* (*kd* ≥ 0).
ab, *work*	REAL for ssbev DOUBLE PRECISION for dsbev. Arrays: *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format. The second dimension of *ab* must be at least max(1, *n*). *work* (*) is a workspace array. The dimension of *work* must be at least max(1, 3*n*-2).
ldab	INTEGER. The leading dimension of *ab*; must be at least *kd* +1.
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: if *jobz* = 'N', then *ldz* ≥ 1; if *jobz* = 'V', then *ldz* ≥ max(1, *n*) .

Output Parameters

w, *z*	REAL for ssbev DOUBLE PRECISION for dsbev Arrays: *w(*)*, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the orthonormal eigenvectors of the matrix *A*, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced.
ab	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. If *uplo* = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix *T* are returned in rows *kd* and *kd*+1 of *ab*, and if *uplo* = 'L', the diagonal and first subdiagonal of *T* are returned in the first two rows of *ab*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbev interface are the following:

ab	Holds the array *A* of size (*kd+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.

jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

?hbev

Computes all eigenvalues and, optionally, eigenvectors of a Hermitian band matrix.

Syntax

Fortran 77:

```
call chbev(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, rwork, info)
call zhbev(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, rwork, info)
```

Fortran 95:

```
call hbev(ab, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chbev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int kd, lapack_complex_float* ab, lapack_int ldab, float* w,
lapack_complex_float* z, lapack_int ldz );
lapack_int LAPACKE_zhbev( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int kd, lapack_complex_double* ab, lapack_int ldab, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
kd	INTEGER. The number of super- or sub-diagonals in *A* (*kd* ≥ 0).
ab, *work*	COMPLEX for chbev DOUBLE COMPLEX for zhbev. Arrays:

	ab (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format. The second dimension of *ab* must be at least max(1, *n*). *work* (*) is a workspace array. The dimension of *work* must be at least max(1, *n*).
ldab	INTEGER. The leading dimension of *ab*; must be at least *kd* +1.
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: if *jobz* = 'N', then *ldz* ≥ 1; if *jobz* = 'V', then *ldz* ≥ max(1, *n*) .
rwork	REAL for chbev DOUBLE PRECISION for zhbev Workspace array, DIMENSION at least max(1, 3*n*-2).

Output Parameters

w	REAL for chbev DOUBLE PRECISION for zhbev Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order.
z	COMPLEX for chbev DOUBLE COMPLEX for zhbev. Array z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the orthonormal eigenvectors of the matrix *A*, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced.
ab	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. If *uplo* = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix *T* are returned in rows *kd* and *kd*+1 of *ab*, and if *uplo* = 'L', the diagonal and first subdiagonal of *T* are returned in the first two rows of *ab*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbev interface are the following:

ab	Holds the array *A* of size (*kd+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows:

jobz = 'V', if *z* is present,
jobz = 'N', if *z* is omitted.

?sbevd

Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric band matrix using divide and conquer algorithm.

Syntax

Fortran 77:

```
call ssbevd(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, lwork, iwork, liwork, info)
call dsbevd(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call sbevd(ab, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbevd( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int kd, <datatype>* ab, lapack_int ldab, <datatype>* w, <datatype>* z,
lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric band matrix *A*. In other words, it can compute the spectral factorization of *A* as:

$A = Z*\Lambda*Z^T$

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i, and *Z* is the orthogonal matrix whose columns are the eigenvectors z_i. Thus,

$A*z_i = \lambda_i*z_i$ for $i = 1, 2, \ldots, n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the *QL* or *QR* algorithm.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz — CHARACTER*1. Must be 'N' or 'V'.
If *jobz* = 'N', then only eigenvalues are computed.
If *jobz* = 'V', then eigenvalues and eigenvectors are computed.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', *ab* stores the upper triangular part of *A*.
If *uplo* = 'L', *ab* stores the lower triangular part of *A*.

n	INTEGER. The order of the matrix *A* ($n \geq 0$).
kd	INTEGER. The number of super- or sub-diagonals in *A* ($kd \geq 0$).
ab, *work*	REAL for ssbevd DOUBLE PRECISION for dsbevd. Arrays: *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format. The second dimension of *ab* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
ldab	INTEGER. The leading dimension of *ab*; must be at least *kd*+1.
ldz	INTEGER. The leading dimension of the output array *z*. Constraints: if *jobz* = 'N', then *ldz* $\geq$ 1; if *jobz* = 'V', then *ldz* $\geq$ max(1, *n*) .
lwork	INTEGER. The dimension of the array *work*. Constraints: if *n* $\leq$ 1, then *lwork* $\geq$ 1; if *jobz* = 'N' and *n* > 1, then *lwork* $\geq 2n$; if *jobz* = 'V' and *n* > 1, then *lwork* $\geq 2*n^2 + 5*n + 1$. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.
iwork	INTEGER. Workspace array, its dimension max(1, *liwork*).
liwork	INTEGER. The dimension of the array *iwork*. Constraints: if *n* $\leq$ 1, then *liwork* < 1; if *job* = 'N' and *n* > 1, then *liwork* < 1; if *job* = 'V' and *n* > 1, then *liwork* < 5*n+3. If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

w, *z*	REAL for ssbevd DOUBLE PRECISION for dsbevd Arrays: *w(*)*, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order. See also *info*. z(*ldz*,*). The second dimension of *z* must be: at least 1 if *job* = 'N'; at least max(1, *n*) if *job* = 'V'.

	If *job* = 'V', then this array is overwritten by the orthogonal matrix *Z* which contains the eigenvectors of *A*. The *i*-th column of *Z* contains the eigenvector which corresponds to the eigenvalue *w(i)*. If *job* = 'N', then *z* is not referenced.
ab	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form.
work(1)	On exit, if *lwork* > 0, then *work*(1) returns the required minimal size of *lwork*.
iwork(1)	On exit, if *liwork* > 0, then *iwork*(1) returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero. If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbevd interface are the following:

ab	Holds the array *A* of size (*kd+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $||E||_2 = O(\varepsilon) * ||T||_2$, where ε is the machine precision.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If any of admissible *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *work* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The complex analogue of this routine is hbevd.

See also syevd for matrices held in full storage, and spevd for matrices held in packed storage.

?hbevd

Computes all eigenvalues and (optionally) all eigenvectors of a complex Hermitian band matrix using divide and conquer algorithm.

Syntax

Fortran 77:

```
call chbevd(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

```
call zhbevd(jobz, uplo, n, kd, ab, ldab, w, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call hbevd(ab, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chbevd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_int kd, lapack_complex_float* ab, lapack_int ldab, float* w, lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhbevd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_int kd, lapack_complex_double* ab, lapack_int ldab, double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a complex Hermitian band matrix A. In other words, it can compute the spectral factorization of A as: $A = Z*\Lambda*Z^H$.

Here Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i. Thus,

$A*z_i = \lambda_i*z_i$ for $i = 1, 2, ..., n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of A. If *uplo* = 'L', *ab* stores the lower triangular part of A.

n INTEGER. The order of the matrix *A* ($n \geq 0$).

kd INTEGER. The number of super- or sub-diagonals in *A* ($kd \geq 0$).

ab, *work* COMPLEX for chbevd
DOUBLE COMPLEX for zhbevd.
Arrays:
ab (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format.
The second dimension of *ab* must be at least max(1, *n*).
work (*) is a workspace array, its dimension max(1, *lwork*).

ldab INTEGER. The leading dimension of *ab*; must be at least *kd*+1.

ldz INTEGER. The leading dimension of the output array *z*.
Constraints:
if *jobz* = 'N', then *ldz* ≥ 1;
if *jobz* = 'V', then *ldz* ≥ max(1, *n*) .

lwork INTEGER.
The dimension of the array *work*.
Constraints:
if $n \leq 1$, then *lwork* ≥ 1;
if *jobz* = 'N' and $n > 1$, then *lwork* ≥ *n*;
if *jobz* = 'V' and $n > 1$, then *lwork* ≥ $2*n^2$.
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

rwork REAL for chbevd
DOUBLE PRECISION for zhbevd
Workspace array, DIMENSION at least *lrwork*.

lrwork INTEGER.
The dimension of the array *rwork*.
Constraints:
if $n \leq 1$, then *lrwork* ≥ 1;
if *jobz* = 'N' and $n > 1$, then *lrwork* ≥ *n*;
if *jobz* = 'V' and $n > 1$, then *lrwork* ≥ $2*n^2 + 5*n + 1$.
If *lrwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork INTEGER. Workspace array, DIMENSION max(1, *liwork*).

liwork INTEGER.
The dimension of the array *iwork*.
Constraints:
if *jobz* = 'N' or $n \leq 1$, then *liwork* ≥ 1;
if *jobz* = 'V' and $n > 1$, then *liwork* ≥ 5*n+3.

If *liwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

w
REAL for chbevd
DOUBLE PRECISION for zhbevd
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order.
See also *info*.

z
COMPLEX for chbevd
DOUBLE COMPLEX for zhbevd
Array, DIMENSION (*ldz*,*).
The second dimension of *z* must be:
at least 1 if *jobz* = 'N';
at least max(1, *n*) if *jobz* = 'V'.
If *jobz* = 'V', then this array is overwritten by the unitary matrix *Z* which contains the eigenvectors of *A*. The *i*-th column of *Z* contains the eigenvector which corresponds to the eigenvalue *w(i)*.
If *jobz* = 'N', then *z* is not referenced.

ab
On exit, this array is overwritten by the values generated during the reduction to tridiagonal form.

work(1)
On exit, if *lwork* > 0, then the real part of *work*(1) returns the required minimal size of *lwork*.

rwork(1)
On exit, if *lrwork* > 0, then *rwork*(1) returns the required minimal size of *lrwork*.

iwork(1)
On exit, if *liwork* > 0, then *iwork*(1) returns the required minimal size of *liwork*.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.
If *info* = −*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbevd interface are the following:

ab
Holds the array *A* of size (*kd+1*,*n*).

w
Holds the vector with the number of elements *n*.

z
Holds the matrix *Z* of size (*n*, *n*).

uplo
Must be 'U' or 'L'. The default value is 'U'.

jobz
Restored based on the presence of the argument *z* as follows:
jobz = 'V', if *z* is present,
jobz = 'N', if *z* is omitted.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T + E$ such that $||E||_2 = O(\varepsilon)||T||_2$, where ε is the machine precision.

If you are in doubt how much workspace to supply, use a generous value of *lwork* (*liwork* or *lrwork*) for the first run or set *lwork* = -1 (*liwork* = -1, *lrwork* = -1).

If you choose the first option and set any of admissible *lwork* (*liwork* or *lrwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*) on exit. Use this value (*work*(1), *iwork*(1), *rwork*(1)) for subsequent runs.

If you set *lwork* = -1 (*liwork* = -1, *lrwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*, *lrwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The real analogue of this routine is sbevd.

See also heevd for matrices held in full storage, and hpevd for matrices held in packed storage.

?sbevx

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix.

Syntax

Fortran 77:

```
call ssbevx(jobz, range, uplo, n, kd, ab, ldab, q, ldq, vl, vu, il, iu, abstol, m, w, z, ldz, work, iwork, ifail, info)
```

```
call dsbevx(jobz, range, uplo, n, kd, ab, ldab, q, ldq, vl, vu, il, iu, abstol, m, w, z, ldz, work, iwork, ifail, info)
```

Fortran 95:

```
call sbevx(ab, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,q] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbevx( int matrix_order, char jobz, char range, char uplo, lapack_int n, lapack_int kd, <datatype>* ab, lapack_int ldab, <datatype>* q, lapack_int ldq, <datatype> vl, <datatype> vu, lapack_int il, lapack_int iu, <datatype> abstol, lapack_int* m, <datatype>* w, <datatype>* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix *A*. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz — CHARACTER*1. Must be 'N' or 'V'.
If *jobz* = 'N', then only eigenvalues are computed.
If *jobz* = 'V', then eigenvalues and eigenvectors are computed.

range — CHARACTER*1. Must be 'A' or 'V' or 'I'.
If *range* = 'A', the routine computes all eigenvalues.
If *range* = 'V', the routine computes eigenvalues *lambda*(i) in the half-open interval: $vl < lambda(i) \le vu$.
If *range* = 'I', the routine computes eigenvalues in range *il* to *iu*.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', *ab* stores the upper triangular part of *A*.
If *uplo* = 'L', *ab* stores the lower triangular part of *A*.

n — INTEGER. The order of the matrix *A* ($n \ge 0$).

kd — INTEGER. The number of super- or sub-diagonals in *A* ($kd \ge 0$).

ab, *work* — REAL for ssbevx
DOUBLE PRECISION for dsbevx.
Arrays:
ab (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format.
The second dimension of *ab* must be at least max(1, *n*).
work (*) is a workspace array.
The dimension of *work* must be at least max(1, 7*n*).

ldab — INTEGER. The leading dimension of *ab*; must be at least *kd* +1.

vl, *vu* — REAL for ssbevx
DOUBLE PRECISION for dsbevx.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl* < *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu* — INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: $1 \le il \le iu \le n$, if $n > 0$; *il*=1 and *iu*=0 if $n = 0$.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol — REAL for chpevx
DOUBLE PRECISION for zhpevx
The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.

ldq, *ldz* — INTEGER. The leading dimensions of the output arrays *q* and *z*, respectively.
Constraints:

	ldq ≥ 1, *ldz* ≥ 1; If *jobz* = 'V', then *ldq* ≥ max(1, *n*) and *ldz* ≥ max(1, *n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

q	REAL for ssbevx DOUBLE PRECISION for dsbevx. Array, DIMENSION (*ldz*,*n*). If *jobz* = 'V', the *n*-by-*n* orthogonal matrix is used in the reduction to tridiagonal form. If *jobz* = 'N', the array *q* is not referenced.
m	INTEGER. The total number of eigenvalues found, 0 ≤ *m* ≤ *n*. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.
w, *z*	REAL for ssbevx DOUBLE PRECISION for dsbevx Arrays: *w(*)*, DIMENSION at least max(1, *n*). The first *m* elements of *w* contain the selected eigenvalues of the matrix *A* in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*. If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
ab	On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. If *uplo* = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix *T* are returned in rows *kd* and *kd*+1 of *ab*, and if *uplo* = 'L', the diagonal and first subdiagonal of *T* are returned in the first two rows of *ab*.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbevx interface are the following:

ab	Holds the array *A* of size (*kd+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector with the number of elements *n*.
q	Holds the matrix *Q* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if either *ifail* or *q* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*T*||$_1$ is used as tolerance, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

?hbevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian band matrix.

Syntax

Fortran 77:

call chbevx(*jobz*, *range*, *uplo*, *n*, *kd*, *ab*, *ldab*, *q*, *ldq*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *w*, *z*, *ldz*, *work*, *rwork*, *iwork*, *ifail*, *info*)

call zhbevx(*jobz*, *range*, *uplo*, *n*, *kd*, *ab*, *ldab*, *q*, *ldq*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *w*, *z*, *ldz*, *work*, *rwork*, *iwork*, *ifail*, *info*)

Fortran 95:

call hbevx(*ab*, *w* [,*uplo*] [,*z*] [,*vl*] [,*vu*] [,*il*] [,*iu*] [,*m*] [,*ifail*] [,*q*] [,*abstol*] [,*info*])

C:

```
lapack_int LAPACKE_chbevx( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_int kd, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* q, lapack_int ldq, float vl, float vu, lapack_int il, lapack_int
iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz,
lapack_int* ifail );
```

```
lapack_int LAPACKE_zhbevx( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_int kd, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* q, lapack_int ldq, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z,
lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix *A*. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *job* = 'N', then only eigenvalues are computed. If *job* = 'V', then eigenvalues and eigenvectors are computed.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl*< *lambda*(*i*) ≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', *ab* stores the upper triangular part of *A*. If *uplo* = 'L', *ab* stores the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
kd	INTEGER. The number of super- or sub-diagonals in *A* (*kd* ≥ 0).
ab, *work*	COMPLEX for chbevx DOUBLE COMPLEX for zhbevx. Arrays: *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format. The second dimension of *ab* must be at least max(1, *n*). *work* (*) is a workspace array. The dimension of *work* must be at least max(1, *n*).

ldab	INTEGER. The leading dimension of *ab*; must be at least *kd* +1.
vl, *vu*	REAL for chbevx DOUBLE PRECISION for zhbevx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.
il, *iu*	INTEGER. If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0. If *range* = 'A' or 'V', *il* and *iu* are not referenced.
abstol	REAL for chbevx DOUBLE PRECISION for zhbevx. The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.
ldq, *ldz*	INTEGER. The leading dimensions of the output arrays *q* and *z*, respectively. Constraints: *ldq* ≥ 1, *ldz* ≥ 1; If *jobz* = 'V', then *ldq* ≥ max(1, *n*) and *ldz* ≥ max(1, *n*).
rwork	REAL for chbevx DOUBLE PRECISION for zhbevx Workspace array, DIMENSION at least max(1, 7*n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

q	COMPLEX for chbevx DOUBLE COMPLEX for zhbevx. Array, DIMENSION (*ldz*,*n*). If *jobz* = 'V', the *n*-by-*n* unitary matrix is used in the reduction to tridiagonal form. If *jobz* = 'N', the array *q* is not referenced.
m	INTEGER. The total number of eigenvalues found, 0 ≤ *m* ≤ *n*. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.
w	REAL for chbevx DOUBLE PRECISION for zhbevx Array, DIMENSION at least max(1, *n*). The first *m* elements contain the selected eigenvalues of the matrix *A* in ascending order.
z	COMPLEX for chbevx DOUBLE COMPLEX for zhbevx. Array z(*ldz*,*). The second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If *jobz* = 'N', then *z* is not referenced.
Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

ab — On exit, this array is overwritten by the values generated during the reduction to tridiagonal form.
If *uplo* = 'U', the first superdiagonal and the diagonal of the tridiagonal matrix *T* are returned in rows *kd* and *kd*+1 of *ab*, and if *uplo* = 'L', the diagonal and first subdiagonal of *T* are returned in the first two rows of *ab*.

ifail — INTEGER.
Array, DIMENSION at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge.
If *jobz* = 'N', then *ifail* is not referenced.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbevx interface are the following:

ab — Holds the array *A* of size (*kd+1*,*n*).

w — Holds the vector with the number of elements *n*.

z — Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.

ifail — Holds the vector with the number of elements *n*.

q — Holds the matrix *Q* of size (*n*, *n*).

uplo — Must be 'U' or 'L'. The default value is 'U'.

vl — Default value for this element is *vl* = -HUGE(*vl*).

vu — Default value for this element is *vu* = HUGE(*vl*).

il — Default value for this argument is *il* = 1.

iu — Default value for this argument is *iu* = *n*.

abstol — Default value for this element is *abstol* = 0.0_WP.

jobz — Restored based on the presence of the argument *z* as follows:
jobz = 'V', if *z* is present,
jobz = 'N', if *z* is omitted
Note that there will be an error condition if either *ifail* or *q* is present and *z* is omitted.

range — Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows:
range = 'V', if one of or both *vl* and *vu* are present,
range = 'I', if one of or both *il* and *iu* are present,
range = 'A', if none of *vl*, *vu*, *il*, *iu* is present,
Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol* + ε * max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then $\varepsilon * ||T||_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

?stev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix.

Syntax

Fortran 77:

```
call sstev(jobz, n, d, e, z, ldz, work, info)
call dstev(jobz, n, d, e, z, ldz, work, info)
```

Fortran 95:

```
call stev(d, e [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>stev( int matrix_order, char jobz, lapack_int n, <datatype>* d,
<datatype>* e, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix *A*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
n	INTEGER. The order of the matrix *A* ($n \geq 0$).
d, *e*, *work*	REAL for sstev DOUBLE PRECISION for dstev. Arrays: *d*(*) contains the *n* diagonal elements of the tridiagonal matrix *A*. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the *n*-1 subdiagonal elements of the tridiagonal matrix *A*.

	The dimension of *e* must be at least max(1, *n*-1). The *n*-th element of this array is used as workspace. *work*(*) is a workspace array. The dimension of *work* must be at least max(1, 2*n*-2). If *jobz* = 'N', *work* is not referenced.
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V' then *ldz* ≥ max(1, *n*).

Output Parameters

d	On exit, if *info* = 0, contains the eigenvalues of the matrix *A* in ascending order.
z	REAL for sstev DOUBLE PRECISION for dstev Array, DIMENSION (*ldz*, *). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the orthonormal eigenvectors of the matrix *A*, with the *i*-th column of *z* holding the eigenvector associated with the eigenvalue returned in *d*(*i*). If *job* = 'N', then *z* is not referenced.
e	On exit, this array is overwritten with intermediate results.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then the algorithm failed to converge; *i* elements of *e* did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stev interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

?stevd

Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric tridiagonal matrix using divide and conquer algorithm.

Syntax

Fortran 77:

```
call sstevd(jobz, n, d, e, z, ldz, work, lwork, iwork, liwork, info)
call dstevd(jobz, n, d, e, z, ldz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call stevd(d, e [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>stevd( int matrix_order, char jobz, lapack_int n, <datatype>* d,
<datatype>* e, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric tridiagonal matrix T. In other words, the routine can compute the spectral factorization of T as: $T = Z*\Lambda*Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i, and Z is the orthogonal matrix whose columns are the eigenvectors z_i. Thus,

$T*z_i = \lambda_i*z_i$ for $i = 1, 2, \ldots, n$.

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

There is no complex analogue of this routine.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then only eigenvalues are computed. If *jobz* = `'V'`, then eigenvalues and eigenvectors are computed.
n	`INTEGER`. The order of the matrix T ($n \geq 0$).
d, *e*, *work*	`REAL` for `sstevd` `DOUBLE PRECISION` for `dstevd`. Arrays: *d*(*) contains the n diagonal elements of the tridiagonal matrix T. The dimension of *d* must be at least max(1, n). *e*(*) contains the n-1 off-diagonal elements of T. The dimension of *e* must be at least max(1, n-1). The n-th element of this array is used as workspace. *work*(*) is a workspace array. The dimension of *work* must be at least *lwork*.
ldz	`INTEGER`. The leading dimension of the output array z. Constraints: *ldz* $\geq$ 1 if *job* = `'N'`; *ldz* < max(1, *n*) if *job* = `'V'`.
lwork	`INTEGER`. The dimension of the array *work*. Constraints: if *jobz* = `'N'` or $n \leq 1$, then *lwork* $\geq$ 1;

if *jobz* = 'V' and $n > 1$, then *lwork* $\geq n^2 + 4*n + 1$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER. Workspace array, its dimension max(1, *liwork*).

liwork — INTEGER.
The dimension of the array *iwork*.
Constraints:
if *jobz* = 'N' or $n \leq 1$, then *liwork* ≥ 1;
if *jobz* = 'V' and $n > 1$, then *liwork* $\geq 5*n+3$.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

d — On exit, if *info* = 0, contains the eigenvalues of the matrix *T* in ascending order.
See also *info*.

z — REAL for sstevd
DOUBLE PRECISION for dstevd
Array, DIMENSION (*ldz*, *).
The second dimension of *z* must be:
at least 1 if *jobz* = 'N';
at least max(1, *n*) if *jobz* = 'V'.
If *jobz* = 'V', then this array is overwritten by the orthogonal matrix *Z* which contains the eigenvectors of *T*.
If *jobz* = 'N', then *z* is not referenced.

e — On exit, this array is overwritten with intermediate results.

work(1) — On exit, if *lwork* > 0, then *work*(1) returns the required minimal size of *lwork*.

iwork(1) — On exit, if *liwork* > 0, then *iwork*(1) returns the required minimal size of *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.
If *info* = -*i*, the *i*-th parameter had an illegal value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stevd interface are the following:

d — Holds the vector of length *n*.

e	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix *T*+*E* such that $||E||_2 = O(\varepsilon)*||T||_2$, where ε is the machine precision.

If λ_i is an exact eigenvalue, and m_i is the corresponding computed value, then

$|\mu_i - \lambda_i| \le c(n)*\varepsilon*||T||_2$

where *c*(*n*) is a modestly increasing function of *n*.

If z_i is the corresponding exact eigenvector, and w_i is the corresponding computed vector, then the angle $\theta(z_i, w_i)$ between them is bounded as follows:

$\theta(z_i, w_i) \le c(n)*\varepsilon*||T||_2 / \min_{i \ne j}|\lambda_i - \lambda_j|$.

Thus the accuracy of a computed eigenvector depends on the gap between its eigenvalue and all the other eigenvalues.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run, or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?stevx

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

Fortran 77:

```
call sstevx(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, work, iwork, ifail, info)
```

```
call dstevx(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, work, iwork, ifail, info)
```

Fortran 95:

```
call stevx(d, e, w [, z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>stevx( int matrix_order, char jobz, char range, lapack_int n,
<datatype>* d, <datatype>* e, <datatype> vl, <datatype> vu, lapack_int il, lapack_int
iu, <datatype> abstol, lapack_int* m, <datatype>* w, <datatype>* z, lapack_int ldz,
lapack_int* ifail );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *job* = `'N'`, then only eigenvalues are computed. If *job* = `'V'`, then eigenvalues and eigenvectors are computed.
range	`CHARACTER*1`. Must be `'A'` or `'V'` or `'I'`. If *range* = `'A'`, the routine computes all eigenvalues. If *range* = `'V'`, the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl*<*lambda*(*i*)≤ *vu*. If *range* = `'I'`, the routine computes eigenvalues with indices *il* to *iu*.
n	`INTEGER`. The order of the matrix A ($n \geq 0$).
d, *e*, *work*	`REAL` for `sstevx` `DOUBLE PRECISION` for `dstevx`. Arrays: *d*(*) contains the n diagonal elements of the tridiagonal matrix A. The dimension of *d* must be at least max(1, n). *e*(*) contains the n-1 subdiagonal elements of A. The dimension of *e* must be at least max(1, n-1). The n-th element of this array is used as workspace. *work*(*) is a workspace array. The dimension of *work* must be at least max(1, 5n).
vl, *vu*	`REAL` for `sstevx` `DOUBLE PRECISION` for `dstevx`. If *range* = `'V'`, the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = `'A'` or `'I'`, *vl* and *vu* are not referenced.
il, *iu*	`INTEGER`. If *range* = `'I'`, the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: 1 ≤ *il* ≤ *iu* ≤ n, if n > 0; *il*=1 and *iu*=0 if n = 0.

If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol — REAL for sstevx
DOUBLE PRECISION for dstevx. The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.

ldz — INTEGER. The leading dimensions of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', then *ldz* ≥ max(1, *n*).

iwork — INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

m — INTEGER. The total number of eigenvalues found,
0 ≤ *m* ≤ *n*.
If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.

w, *z* — REAL for sstevx
DOUBLE PRECISION for dstevx.
Arrays:
w(*), DIMENSION at least max(1, *n*).
The first *m* elements of *w* contain the selected eigenvalues of the matrix *A* in ascending order.

z(*ldz*,*).
The second dimension of *z* must be at least max(1, *m*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*).
If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
If *jobz* = 'N', then *z* is not referenced.
Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

d, *e* — On exit, these arrays may be multiplied by a constant factor chosen to avoid overflow or underflow in computing the eigenvalues.

ifail — INTEGER.
Array, DIMENSION at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge.
If *jobz* = 'N', then *ifail* is not referenced.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stevx interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length *n*.
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector of length *n*.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*A*||$_1$ is used instead. Eigenvalues are computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, set *abstol* to 2*?lamch('S').

?stevr

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix using the Relatively Robust Representations.

Syntax

Fortran 77:

```
call sstevr(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, isuppz, work, lwork, iwork, liwork, info)
```

```
call dstevr(jobz, range, n, d, e, vl, vu, il, iu, abstol, m, w, z, ldz, isuppz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call stevr(d, e, w [, z] [,vl] [,vu] [,il] [,iu] [,m] [,isuppz] [,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>stevr( int matrix_order, char jobz, char range, lapack_int n,
<datatype>* d, <datatype>* e, <datatype> vl, <datatype> vu, lapack_int il, lapack_int
iu, <datatype> abstol, lapack_int* m, <datatype>* w, <datatype>* z, lapack_int ldz,
lapack_int* isuppz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, the routine calls stemr to compute the eigenspectrum using Relatively Robust Representations. stegr computes eigenvalues by the *dqds* algorithm, while orthogonal eigenvectors are computed from various "good" $L*D*L^T$ representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i-th unreduced block of T:

a. Compute $T - \sigma_i = L_i*D_i*L_i^T$, such that $L_i*D_i*L_i^T$ is a relatively robust representation.
b. Compute the eigenvalues, λ_j, of $L_i*D_i*L_i^T$ to high relative accuracy by the *dqds* algorithm.
c. If there is a cluster of close eigenvalues, "choose" σ_i close to the cluster, and go to Step (a).
d. Given the approximate eigenvalue λ_j of $L_i*D_i*L_i^T$, compute the corresponding eigenvector by forming a rank-revealing twisted factorization.

The desired accuracy of the output can be specified by the input parameter *abstol*.

The routine ?stevr calls stemr when the full spectrum is requested on machines which conform to the IEEE-754 floating point standard. ?stevr calls stebz and stein on non-IEEE machines and when partial spectrum requests are made.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda(i)* in the half-open interval: *vl*<*lambda(i)* ≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*. For *range* = 'V'or 'I' and *iu*-*il* < *n*-1, sstebz/dstebz and sstein/dstein are called.
n	INTEGER. The order of the matrix T ($n \geq 0$).
d, *e*, *work*	REAL for sstevr DOUBLE PRECISION for dstevr.

Arrays:
d(*) contains the *n* diagonal elements of the tridiagonal matrix *T*.
The dimension of *d* must be at least max(1, *n*).
e(*) contains the *n*-1 subdiagonal elements of *A*.
The dimension of *e* must be at least max(1, *n*-1). The *n*-th element of this array is used as workspace.
work is a workspace array, its dimension max(1, *lwork*).

vl, *vu*
REAL for sstevr
DOUBLE PRECISION for dstevr.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl*< *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: 1 ≤ *il* ≤ *iu*≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for sstevr
DOUBLE PRECISION for dstevr.
The absolute error tolerance to which each eigenvalue/eigenvector is required.
If *jobz* = 'V', the eigenvalues and eigenvectors output have residual norms bounded by *abstol*, and the dot products between different eigenvectors are bounded by *abstol*. If *abstol* < *n* *eps*|*T*|, then *n* *eps*|*T*| will be used in its place, where eps is the machine precision, and |*T*| is the 1-norm of the matrix *T*. The eigenvalues are computed to an accuracy of eps*|*T*| irrespective of *abstol*.
If high relative accuracy is important, set *abstol* to ?lamch('S').

ldz
INTEGER. The leading dimension of the output array *z*.
Constraints:
ldz ≥ 1 if *jobz* = 'N';
ldz ≥ max(1, *n*) if *jobz* = 'V'.

lwork
INTEGER.
The dimension of the array *work*. Constraint:
lwork ≥ max(1, 20**n*).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork
INTEGER.
Workspace array, its dimension max(1, *liwork*).

liwork
INTEGER.
The dimension of the array *iwork*,
lwork ≥ max(1, 10**n*).
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

m	INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.
w, *z*	REAL for sstevr DOUBLE PRECISION for dstevr. Arrays: *w*(*), DIMENSION at least max(1, *n*). The first *m* elements of *w* contain the selected eigenvalues of the matrix *T* in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *m*). If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). If *jobz* = 'N', then *z* is not referenced. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
d, *e*	On exit, these arrays may be multiplied by a constant factor chosen to avoid overflow or underflow in computing the eigenvalues.
isuppz	INTEGER. Array, DIMENSION at least 2 *max(1, *m*). The support of the eigenvectors in *z*, i.e., the indices indicating the nonzero elements in *z*. The *i*-th eigenvector is nonzero only in elements *isuppz*(2*i*-1) through *isuppz*(2*i*). Implemented only for *range* = 'A' or 'I' and *iu*-*il* = *n*-1.
work(1)	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, an internal error has occurred.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine stevr interface are the following:

d	Holds the vector of length *n*.
e	Holds the vector of length *n*.
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
isuppz	Holds the vector of length (2**n*), where the values (2**m*) are significant.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).

il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

Normal execution of the routine ?stegr may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the IEEE standard default manner.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run, or set *lwork* = –1 (*liwork* = –1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = –1 (*liwork* = –1), then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Nonsymmetric Eigenproblems

This section describes LAPACK driver routines used for solving nonsymmetric eigenproblems. See also computational routines that can be called to solve these problems.

Table "Driver Routines for Solving Nonsymmetric Eigenproblems" lists all such driver routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Solving Nonsymmetric Eigenproblems

Routine Name	Operation performed
gees	Computes the eigenvalues and Schur factorization of a general matrix, and orders the factorization so that selected eigenvalues are at the top left of the Schur form.
geesx	Computes the eigenvalues and Schur factorization of a general matrix, orders the factorization and computes reciprocal condition numbers.
geev	Computes the eigenvalues and left and right eigenvectors of a general matrix.

Routine Name	Operation performed
geevx	Computes the eigenvalues and left and right eigenvectors of a general matrix, with preliminary matrix balancing, and computes reciprocal condition numbers for the eigenvalues and right eigenvectors.

?gees

Computes the eigenvalues and Schur factorization of a general matrix, and orders the factorization so that selected eigenvalues are at the top left of the Schur form.

Syntax

Fortran 77:

```
call sgees(jobvs, sort, select, n, a, lda, sdim, wr, wi, vs, ldvs, work, lwork, bwork, info)
```

```
call dgees(jobvs, sort, select, n, a, lda, sdim, wr, wi, vs, ldvs, work, lwork, bwork, info)
```

```
call cgees(jobvs, sort, select, n, a, lda, sdim, w, vs, ldvs, work, lwork, rwork, bwork, info)
```

```
call zgees(jobvs, sort, select, n, a, lda, sdim, w, vs, ldvs, work, lwork, rwork, bwork, info)
```

Fortran 95:

```
call gees(a, wr, wi [,vs] [,select] [,sdim] [,info])
```

```
call gees(a, w [,vs] [,select] [,sdim] [,info])
```

C:

```
lapack_int LAPACKE_sgees( int matrix_order, char jobvs, char sort, LAPACK_S_SELECT2 select, lapack_int n, float* a, lapack_int lda, lapack_int* sdim, float* wr, float* wi, float* vs, lapack_int ldvs );
```

```
lapack_int LAPACKE_dgees( int matrix_order, char jobvs, char sort, LAPACK_D_SELECT2 select, lapack_int n, double* a, lapack_int lda, lapack_int* sdim, double* wr, double* wi, double* vs, lapack_int ldvs );
```

```
lapack_int LAPACKE_cgees( int matrix_order, char jobvs, char sort, LAPACK_C_SELECT1 select, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_int* sdim, lapack_complex_float* w, lapack_complex_float* vs, lapack_int ldvs );
```

```
lapack_int LAPACKE_zgees( int matrix_order, char jobvs, char sort, LAPACK_Z_SELECT1 select, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_int* sdim, lapack_complex_double* w, lapack_complex_double* vs, lapack_int ldvs );
```

Include Files

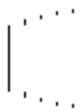
- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes for an n-by-n real/complex nonsymmetric matrix A, the eigenvalues, the real Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization $A = Z*T*Z^H$.

Optionally, it also orders the eigenvalues on the diagonal of the real-Schur/Schur form so that selected eigenvalues are at the top left. The leading columns of *Z* then form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A real matrix is in real-Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form



where *b*c* < 0. The eigenvalues of such a block are $a \pm \sqrt{bc}$

A complex matrix is in Schur form if it is upper triangular.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobvs
CHARACTER*1. Must be 'N' or 'V'.
If *jobvs* = 'N', then Schur vectors are not computed.
If *jobvs* = 'V', then Schur vectors are computed.

sort
CHARACTER*1. Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.
If *sort* = 'N', then eigenvalues are not ordered.
If *sort* = 'S', eigenvalues are ordered (see *select*).

select
LOGICAL FUNCTION of two REAL arguments for real flavors.
LOGICAL FUNCTION of one COMPLEX argument for complex flavors.
select must be declared EXTERNAL in the calling subroutine.
If *sort* = 'S', *select* is used to select eigenvalues to sort to the top left of the Schur form.
If *sort* = 'N', *select* is not referenced.
For real flavors:
An eigenvalue *wr*(j)+sqrt(-1)**wi*(j) is selected if *select*(*wr*(j), *wi*(j)) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected.
Note that a selected complex eigenvalue may no longer satisfy *select*(*wr*(j), *wi*(j))= .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case *info* may be set to *n*+2 (see *info* below).
For complex flavors:
An eigenvalue *w*(j) is selected if *select*(*w*(j)) is true.

n
INTEGER. The order of the matrix *A* (*n* ≥ 0).

a, *work*
REAL for sgees
DOUBLE PRECISION for dgees
COMPLEX for cgees
DOUBLE COMPLEX for zgees.
Arrays:
a(*lda*,*) is an array containing the *n*-by-*n* matrix *A*.
The second dimension of *a* must be at least max(1, *n*).

	work is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldvs	INTEGER. The leading dimension of the output array *vs*. Constraints: *ldvs* ≥ 1; *ldvs* ≥ max(1, *n*) if *jobvs* = 'V'.
lwork	INTEGER. The dimension of the array *work*. Constraint: *lwork* ≥ max(1, 3*n*) for real flavors; *lwork* ≥ max(1, 2*n*) for complex flavors. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
rwork	REAL for cgees DOUBLE PRECISION for zgees Workspace array, DIMENSION at least max(1, *n*). Used in complex flavors only.
bwork	LOGICAL. Workspace array, DIMENSION at least max(1, *n*). Not referenced if *sort* = 'N'.

Output Parameters

a	On exit, this array is overwritten by the real-Schur/Schur form *T*.
sdim	INTEGER. If *sort* = 'N', *sdim*= 0. If *sort* = 'S', *sdim* is equal to the number of eigenvalues (after sorting) for which *select* is true. Note that for real flavors complex conjugate pairs for which *select* is true for either eigenvalue count as 2.
wr, *wi*	REAL for sgees DOUBLE PRECISION for dgees Arrays, DIMENSION at least max (1, *n*) each. Contain the real and imaginary parts, respectively, of the computed eigenvalues, in the same order that they appear on the diagonal of the output real-Schur form *T*. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first.
w	COMPLEX for cgees DOUBLE COMPLEX for zgees. Array, DIMENSION at least max(1, *n*). Contains the computed eigenvalues. The eigenvalues are stored in the same order as they appear on the diagonal of the output Schur form *T*.
vs	REAL for sgees DOUBLE PRECISION for dgees COMPLEX for cgees DOUBLE COMPLEX for zgees. Array *vs*(*ldvs*,*);the second dimension of *vs* must be at least max(1, *n*). If *jobvs* = 'V', *vs* contains the orthogonal/unitary matrix *Z* of Schur vectors. If *jobvs* = 'N', *vs* is not referenced.

work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*: the *QR* algorithm failed to compute all the eigenvalues; elements 1:*ilo*-1 and *i*+1:*n* of *wr* and *wi* (for real flavors) or *w* (for complex flavors) contain those eigenvalues which have converged; if *jobvs* = 'V', *vs* contains the matrix which reduces *A* to its partially converged Schur form; *i* = *n*+1: the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned); *i* = *n*+2: after reordering, round-off changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy *select* = .TRUE.. This could also be caused by underflow due to scaling.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gees interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
wr	Holds the vector of length *n*. Used in real flavors only.
wi	Holds the vector of length *n*. Used in real flavors only.
w	Holds the vector of length *n*. Used in complex flavors only.
vs	Holds the matrix *VS* of size (*n*, *n*).
jobvs	Restored based on the presence of the argument *vs* as follows: *jobvs* = 'V', if *vs* is present, *jobvs* = 'N', if *vs* is omitted.
sort	Restored based on the presence of the argument *select* as follows: *sort* = 'S', if *select* is present, *sort* = 'N', if *select* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?geesx

Computes the eigenvalues and Schur factorization of a general matrix, orders the factorization and computes reciprocal condition numbers.

Syntax

Fortran 77:

```
call sgeesx(jobvs, sort, select, sense, n, a, lda, sdim, wr, wi, vs, ldvs, rconde,
rcondv, work, lwork, iwork, liwork, bwork, info)
```

```
call dgeesx(jobvs, sort, select, sense, n, a, lda, sdim, wr, wi, vs, ldvs, rconde,
rcondv, work, lwork, iwork, liwork, bwork, info)
```

```
call cgeesx(jobvs, sort, select, sense, n, a, lda, sdim, w, vs, ldvs, rconde, rcondv,
work, lwork, rwork, bwork, info)
```

```
call zgeesx(jobvs, sort, select, sense, n, a, lda, sdim, w, vs, ldvs, rconde, rcondv,
work, lwork, rwork, bwork, info)
```

Fortran 95:

```
call geesx(a, wr, wi [,vs] [,select] [,sdim] [,rconde] [,rcondev] [,info])
```

```
call geesx(a, w [,vs] [,select] [,sdim] [,rconde] [,rcondev] [,info])
```

C:

```
lapack_int LAPACKE_sgeesx( int matrix_order, char jobvs, char sort, LAPACK_S_SELECT2
select, char sense, lapack_int n, float* a, lapack_int lda, lapack_int* sdim, float*
wr, float* wi, float* vs, lapack_int ldvs, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dgeesx( int matrix_order, char jobvs, char sort, LAPACK_D_SELECT2
select, char sense, lapack_int n, double* a, lapack_int lda, lapack_int* sdim, double*
wr, double* wi, double* vs, lapack_int ldvs, double* rconde, double* rcondv );
```

```
lapack_int LAPACKE_cgeesx( int matrix_order, char jobvs, char sort, LAPACK_C_SELECT1
select, char sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_int*
sdim, lapack_complex_float* w, lapack_complex_float* vs, lapack_int ldvs, float*
rconde, float* rcondv );
```

```
lapack_int LAPACKE_zgeesx( int matrix_order, char jobvs, char sort, LAPACK_Z_SELECT1
select, char sense, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_int* sdim, lapack_complex_double* w, lapack_complex_double* vs, lapack_int ldvs,
double* rconde, double* rcondv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

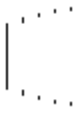
Description

The routine computes for an n-by-n real/complex nonsymmetric matrix A, the eigenvalues, the real-Schur/Schur form T, and, optionally, the matrix of Schur vectors Z. This gives the Schur factorization $A = Z*T*Z^H$.

Optionally, it also orders the eigenvalues on the diagonal of the real-Schur/Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (*rconde*); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (*rcondv*). The leading columns of *Z* form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers *rconde* and *rcondv*, see [LUG], Section 4.10 (where these quantities are called *s* and *sep* respectively).

A real matrix is in real-Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form



where *b*c* < 0. The eigenvalues of such a block are

A complex matrix is in Schur form if it is upper triangular.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobvs — CHARACTER*1. Must be 'N' or 'V'.
If *jobvs* = 'N', then Schur vectors are not computed.
If *jobvs* = 'V', then Schur vectors are computed.

sort — CHARACTER*1. Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.
If *sort* = 'N', then eigenvalues are not ordered.
If *sort* = 'S', eigenvalues are ordered (see *select*).

select — LOGICAL FUNCTION of two REAL arguments for real flavors.
LOGICAL FUNCTION of one COMPLEX argument for complex flavors.
select must be declared EXTERNAL in the calling subroutine.
If *sort* = 'S', *select* is used to select eigenvalues to sort to the top left of the Schur form.
If *sort* = 'N', *select* is not referenced.
For real flavors:
An eigenvalue *wr*(j)+sqrt(-1)**wi*(j) is selected if *select*(*wr*(j), *wi*(j)) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected.
Note that a selected complex eigenvalue may no longer satisfy *select*(*wr*(j), *wi*(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case *info* may be set to *n*+2 (see *info* below).
For complex flavors:
An eigenvalue *w*(j) is selected if *select*(*w*(j)) is true.

sense — CHARACTER*1. Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed.
If *sense* = 'N', none are computed;
If *sense* = 'E', computed for average of selected eigenvalues only;

	If *sense* = 'V', computed for selected right invariant subspace only; If *sense* = 'B', computed for both. If *sense* is 'E', 'V', or 'B', then *sort* must equal 'S'.
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
a, *work*	REAL for sgeesx DOUBLE PRECISION for dgeesx COMPLEX for cgeesx DOUBLE COMPLEX for zgeesx. Arrays: *a*(*lda*,*) is an array containing the *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldvs	INTEGER. The leading dimension of the output array *vs*. Constraints: *ldvs* ≥ 1; *ldvs* ≥ max(1, *n*) if *jobvs* = 'V'.
lwork	INTEGER. The dimension of the array *work*. Constraint: *lwork* ≥ max(1, 3*n*) for real flavors; *lwork* ≥ max(1, 2*n*) for complex flavors. Also, if *sense* = 'E', 'V', or 'B', then *lwork* ≥ *n*+2**sdim**(*n*−*sdim*) for real flavors; *lwork* ≥ 2**sdim**(*n*−*sdim*) for complex flavors; where *sdim* is the number of selected eigenvalues computed by this routine. Note that 2**sdim**(*n*−*sdim*) ≤ *n***n*/2. Note also that an error is only returned if *lwork*<max(1, 2**n*), but if *sense* = 'E', or 'V', or 'B' this may not be large enough. For good performance, *lwork* must generally be larger. If *lwork* = −1, then a workspace query is assumed; the routine only calculates upper bound on the optimal size of the array *work*, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
iwork	INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only. Not referenced if *sense* = 'N' or 'E'.
liwork	INTEGER. The dimension of the array *iwork*. Used in real flavors only. Constraint: *liwork* ≥ 1; if *sense* = 'V' or 'B', *liwork* ≥ *sdim**(*n*−*sdim*).
rwork	REAL for cgeesx DOUBLE PRECISION for zgeesx Workspace array, DIMENSION at least max(1, *n*). Used in complex flavors only.
bwork	LOGICAL. Workspace array, DIMENSION at least max(1, *n*). Not referenced if *sort* = 'N'.

Output Parameters

a	On exit, this array is overwritten by the real-Schur/Schur form *T*.

sdim
INTEGER.
If *sort* = 'N', *sdim*= 0.
If *sort* = 'S', *sdim* is equal to the number of eigenvalues (after sorting) for which *select* is true.
Note that for real flavors complex conjugate pairs for which *select* is true for either eigenvalue count as 2.

wr, *wi*
REAL for sgeesx
DOUBLE PRECISION for dgeesx
Arrays, DIMENSION at least max (1, *n*) each. Contain the real and imaginary parts, respectively, of the computed eigenvalues, in the same order that they appear on the diagonal of the output real-Schur form *T*. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first.

w
COMPLEX for cgeesx
DOUBLE COMPLEX for zgeesx.
Array, DIMENSION at least max(1, *n*). Contains the computed eigenvalues. The eigenvalues are stored in the same order as they appear on the diagonal of the output Schur form *T*.

vs
REAL for sgeesx
DOUBLE PRECISION for dgeesx
COMPLEX for cgeesx
DOUBLE COMPLEX for zgeesx.
Array *vs*(*ldvs*,*);the second dimension of *vs* must be at least max(1, *n*).
If *jobvs* = 'V', *vs* contains the orthogonal/unitary matrix *Z* of Schur vectors.
If *jobvs* = 'N', *vs* is not referenced.

rconde, *rcondv*
REAL for single precision flavors DOUBLE PRECISION for double precision flavors.
If *sense* = 'E' or 'B', *rconde* contains the reciprocal condition number for the average of the selected eigenvalues.
If *sense* = 'N' or 'V', *rconde* is not referenced.
If *sense* = 'V' or 'B', *rcondv* contains the reciprocal condition number for the selected right invariant subspace.
If *sense* = 'N' or 'E', *rcondv* is not referenced.

work(1)
On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = –*i*, the *i*th parameter had an illegal value.
If *info* = *i*, and
i ≤ *n*:
the *QR* algorithm failed to compute all the eigenvalues; elements 1:*ilo*-1 and *i*+1:*n* of *wr* and *wi* (for real flavors) or *w* (for complex flavors) contain those eigenvalues which have converged; if *jobvs* = 'V', *vs* contains the transformation which reduces *A* to its partially converged Schur form;
i = *n*+1:
the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned);
i = *n*+2:
after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy *select* = .TRUE.. This could also be caused by underflow due to scaling.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `geesx` interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
wr	Holds the vector of length (*n*). Used in real flavors only.
wi	Holds the vector of length (*n*). Used in real flavors only.
w	Holds the vector of length (*n*). Used in complex flavors only.
vs	Holds the matrix *VS* of size (*n*, *n*).
jobvs	Restored based on the presence of the argument *vs* as follows: *jobvs* = 'V', if *vs* is present, *jobvs* = 'N', if *vs* is omitted.
sort	Restored based on the presence of the argument *select* as follows: *sort* = 'S', if *select* is present, *sort* = 'N', if *select* is omitted.
sense	Restored based on the presence of arguments *rconde* and rcondv as follows: *sense* = 'B', if both *rconde* and *rcondv* are present, *sense* = 'E', if *rconde* is present and *rcondv* omitted, *sense* = 'V', if *rconde* is omitted and *rcondv* present, *sense* = 'N', if both *rconde* and *rcondv* are omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = –1 (*liwork* = –1).

If you choose the first option and set any of admissible *lwork* (or *liwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If you set *lwork* = –1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?geev

Computes the eigenvalues and left and right eigenvectors of a general matrix.

Syntax

Fortran 77:

```
call sgeev(jobvl, jobvr, n, a, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info)
call dgeev(jobvl, jobvr, n, a, lda, wr, wi, vl, ldvl, vr, ldvr, work, lwork, info)
call cgeev(jobvl, jobvr, n, a, lda, w, vl, ldvl, vr, ldvr, work, lwork, rwork, info)
call zgeev(jobvl, jobvr, n, a, lda, w, vl, ldvl, vr, ldvr, work, lwork, rwork, info)
```

Fortran 95:

```
call geev(a, wr, wi [,vl] [,vr] [,info])
```

```
call geev(a, w [,vl] [,vr] [,info])
```

C:

```
lapack_int LAPACKE_sgeev( int matrix_order, char jobvl, char jobvr, lapack_int n,
float* a, lapack_int lda, float* wr, float* wi, float* vl, lapack_int ldvl, float* vr,
lapack_int ldvr );

lapack_int LAPACKE_dgeev( int matrix_order, char jobvl, char jobvr, lapack_int n,
double* a, lapack_int lda, double* wr, double* wi, double* vl, lapack_int ldvl,
double* vr, lapack_int ldvr );

lapack_int LAPACKE_cgeev( int matrix_order, char jobvl, char jobvr, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* w, lapack_complex_float*
vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr );

lapack_int LAPACKE_zgeev( int matrix_order, char jobvl, char jobvr, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* w,
lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int
ldvr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes for an n-by-n real/complex nonsymmetric matrix A, the eigenvalues and, optionally, the left and/or right eigenvectors. The right eigenvector v(j) of A satisfies

$A*v(j)= \lambda(j)*v(j)$

where $\lambda(j)$ is its eigenvalue.

The left eigenvector u(j) of A satisfies

$u(j)^T*A = \lambda(j)*u(j)^T$

where $u(j)^T$ denotes the transpose of u(j). The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobvl	CHARACTER*1. Must be 'N' or 'V'. If *jobvl* = 'N', then left eigenvectors of A are not computed. If *jobvl* = 'V', then left eigenvectors of A are computed.
jobvr	CHARACTER*1. Must be 'N' or 'V'. If *jobvr* = 'N', then right eigenvectors of A are not computed. If *jobvr* = 'V', then right eigenvectors of A are computed.
n	INTEGER. The order of the matrix A ($n \geq 0$).
a, *work*	REAL for sgeev DOUBLE PRECISION for dgeev COMPLEX for cgeev DOUBLE COMPLEX for zgeev.

	Arrays: *a*(*lda*,*) is an array containing the *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldvl, *ldvr*	INTEGER. The leading dimensions of the output arrays *vl* and *vr*, respectively. Constraints: *ldvl* ≥ 1; *ldvr* ≥ 1. If *jobvl* = 'V', *ldvl* ≥ max(1, *n*); If *jobvr* = 'V', *ldvr* ≥ max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. Constraint: *lwork* ≥ max(1, 3*n*), and if *jobvl* = 'V' or *jobvr* = 'V', *lwork* < max(1, 4*n*) (for real flavors); *lwork* < max(1, 2*n*) (for complex flavors). For good performance, *lwork* must generally be larger. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
rwork	REAL for cgeev DOUBLE PRECISION for zgeev Workspace array, DIMENSION at least max(1, 2*n*). Used in complex flavors only.

Output Parameters

a	On exit, this array is overwritten by intermediate results.
wr, *wi*	REAL for sgeev DOUBLE PRECISION for dgeev Arrays, DIMENSION at least max (1, *n*) each. Contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first.
w	COMPLEX for cgeev DOUBLE COMPLEX for zgeev. Array, DIMENSION at least max(1, *n*). Contains the computed eigenvalues.
vl, *vr*	REAL for sgeev DOUBLE PRECISION for dgeev COMPLEX for cgeev DOUBLE COMPLEX for zgeev. Arrays: *vl*(*ldvl*,*);the second dimension of *vl* must be at least max(1, *n*). If *jobvl* = 'V', the left eigenvectors *u*(j) are stored one after another in the columns of *vl*, in the same order as their eigenvalues. If *jobvl* = 'N', *vl* is not referenced. *For real flavors*: If the j-th eigenvalue is real, then *u*(j) = *vl*(:,j), the j-th column of *vl*.

If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then `u(j) = vl(:,j) + i*vl(:,j+1)` and `u(j+1) = vl(:,j)- i*vl(:,j+1)`, where `i = sqrt(-1)`.
For complex flavors:
`u(j) = vl(:,j)`, the j-th column of `vl`.
`vr`(`ldvr`,*); the second dimension of `vr` must be at least max(1, `n`).
If `jobvr = 'V'`, the right eigenvectors `v`(j) are stored one after another in the columns of `vr`, in the same order as their eigenvalues.
If `jobvr = 'N'`, `vr` is not referenced.
For real flavors:
If the j-th eigenvalue is real, then `v(j) = vr(:,j)`, the j-th column of `vr`.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then `v(j) = vr(:,j) + i*vr(:,j+1)` and `v(j+1) = vr(:,j) - i*vr(:,j+1)`, where `i = sqrt(-1)`.
For complex flavors:
`v(j) = vr(:,j)`, the j-th column of `vr`.

`work(1)` — On exit, if `info = 0`, then `work(1)` returns the required minimal size of `lwork`.

`info` — INTEGER.
If `info = 0`, the execution is successful.
If `info = -i`, the `i`th parameter had an illegal value.
If `info = i`, the `QR` algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements `i`+1:`n` of `wr` and `wi` (for real flavors) or `w` (for complex flavors) contain those eigenvalues which have converged.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `geev` interface are the following:

`a` — Holds the matrix `A` of size (`n`, `n`).

`wr` — Holds the vector of length `n`. Used in real flavors only.

`wi` — Holds the vector of length `n`. Used in real flavors only.

`w` — Holds the vector of length `n`. Used in complex flavors only.

`vl` — Holds the matrix `VL` of size (`n`, `n`).

`vr` — Holds the matrix `VR` of size (`n`, `n`).

`jobvl` — Restored based on the presence of the argument `vl` as follows:
`jobvl = 'V'`, if `vl` is present,
`jobvl = 'N'`, if `vl` is omitted.

`jobvr` — Restored based on the presence of the argument `vr` as follows:
`jobvr = 'V'`, if `vr` is present,
`jobvr = 'N'`, if `vr` is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of `lwork` for the first run or set `lwork = -1`.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?geevx

Computes the eigenvalues and left and right eigenvectors of a general matrix, with preliminary matrix balancing, and computes reciprocal condition numbers for the eigenvalues and right eigenvectors.

Syntax

Fortran 77:

```
call sgeevx(balanc, jobvl, jobvr, sense, n, a, lda, wr, wi, vl, ldvl, vr, ldvr, ilo, ihi, scale, abnrm, rconde, rcondv, work, lwork, iwork, info)
```

```
call dgeevx(balanc, jobvl, jobvr, sense, n, a, lda, wr, wi, vl, ldvl, vr, ldvr, ilo, ihi, scale, abnrm, rconde, rcondv, work, lwork, iwork, info)
```

```
call cgeevx(balanc, jobvl, jobvr, sense, n, a, lda, w, vl, ldvl, vr, ldvr, ilo, ihi, scale, abnrm, rconde, rcondv, work, lwork, rwork, info)
```

```
call zgeevx(balanc, jobvl, jobvr, sense, n, a, lda, w, vl, ldvl, vr, ldvr, ilo, ihi, scale, abnrm, rconde, rcondv, work, lwork, rwork, info)
```

Fortran 95:

```
call geevx(a, wr, wi [,vl] [,vr] [,balanc] [,ilo] [,ihi] [,scale] [,abnrm] [, rconde] [,rcondv] [,info])
```

```
call geevx(a, w [,vl] [,vr] [,balanc] [,ilo] [,ihi] [,scale] [,abnrm] [,rconde] [, rcondv] [,info])
```

C:

```
lapack_int LAPACKE_sgeevx( int matrix_order, char balanc, char jobvl, char jobvr, char sense, lapack_int n, float* a, lapack_int lda, float* wr, float* wi, float* vl, lapack_int ldvl, float* vr, lapack_int ldvr, lapack_int* ilo, lapack_int* ihi, float* scale, float* abnrm, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dgeevx( int matrix_order, char balanc, char jobvl, char jobvr, char sense, lapack_int n, double* a, lapack_int lda, double* wr, double* wi, double* vl, lapack_int ldvl, double* vr, lapack_int ldvr, lapack_int* ilo, lapack_int* ihi, double* scale, double* abnrm, double* rconde, double* rcondv );
```

```
lapack_int LAPACKE_cgeevx( int matrix_order, char balanc, char jobvl, char jobvr, char sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* w, lapack_complex_float* vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr, lapack_int* ilo, lapack_int* ihi, float* scale, float* abnrm, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_zgeevx( int matrix_order, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double*
w, lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int
ldvr, lapack_int* ilo, lapack_int* ihi, double* scale, double* abnrm, double* rconde,
double* rcondv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes for an *n*-by-*n* real/complex nonsymmetric matrix *A*, the eigenvalues and, optionally, the left and/or right eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (*ilo*, *ihi*, *scale*, and *abnrm*), reciprocal condition numbers for the eigenvalues (*rconde*), and reciprocal condition numbers for the right eigenvectors (*rcondv*).

The right eigenvector *v*(j) of *A* satisfies

*A*v*(j) = λ(j)*v*(j)

where λ(j) is its eigenvalue.

The left eigenvector *u*(j) of *A* satisfies

$u(j)^T * A = \lambda(j) * u(j)^T$

where $u(j)^T$ denotes the transpose of *u*(j). The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Balancing a matrix means permuting the rows and columns to make it more nearly upper triangular, and applying a diagonal similarity transformation *D*A*inv(*D*), where *D* is a diagonal matrix, to make its rows and columns closer in norm and the condition numbers of its eigenvalues and eigenvectors smaller. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see [LUG], Section 4.10.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

balanc

CHARACTER*1. Must be 'N', 'P', 'S', or 'B'. Indicates how the input matrix should be diagonally scaled and/or permuted to improve the conditioning of its eigenvalues.
If *balanc* = 'N', do not diagonally scale or permute;
If *balanc* = 'P', perform permutations to make the matrix more nearly upper triangular. Do not diagonally scale;
If *balanc* = 'S', diagonally scale the matrix, i.e. replace *A* by *D*A*inv(*D*), where *D* is a diagonal matrix chosen to make the rows and columns of *A* more equal in norm. Do not permute;
If *balanc* = 'B', both diagonally scale and permute *A*.
Computed reciprocal condition numbers will be for the matrix after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.

jobvl	CHARACTER*1. Must be 'N' or 'V'. If *jobvl* = 'N', left eigenvectors of *A* are not computed; If *jobvl* = 'V', left eigenvectors of *A* are computed. If *sense* = 'E' or 'B', then *jobvl* must be 'V'.
jobvr	CHARACTER*1. Must be 'N' or 'V'. If *jobvr* = 'N', right eigenvectors of *A* are not computed; If *jobvr* = 'V', right eigenvectors of *A* are computed. If *sense* = 'E' or 'B', then *jobvr* must be 'V'.
sense	CHARACTER*1. Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed. If *sense* = 'N', none are computed; If *sense* = 'E', computed for eigenvalues only; If *sense* = 'V', computed for right eigenvectors only; If *sense* = 'B', computed for eigenvalues and right eigenvectors. If *sense* is 'E' or 'B', both left and right eigenvectors must also be computed (*jobvl* = 'V' and *jobvr* = 'V').
n	INTEGER. The order of the matrix *A* ($n \geq 0$).
a, *work*	REAL for sgeevx DOUBLE PRECISION for dgeevx COMPLEX for cgeevx DOUBLE COMPLEX for zgeevx. Arrays: *a*(*lda*,*) is an array containing the *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldvl, *ldvr*	INTEGER. The leading dimensions of the output arrays *vl* and *vr*, respectively. Constraints: *ldvl* $\geq$ 1; *ldvr* $\geq$ 1. If *jobvl* = 'V', *ldvl* $\geq$ max(1, *n*); If *jobvr* = 'V', *ldvr* $\geq$ max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. *For real flavors*: If *sense* = 'N' or 'E', *lwork* $\geq$ max(1, 2*n*), and if *jobvl* = 'V' or *jobvr* = 'V', *lwork* $\geq$ 3*n*; If *sense* = 'V' or 'B', *lwork* $\geq$ *n**(*n*+6). For good performance, *lwork* must generally be larger. *For complex flavors*: If *sense* = 'N'or 'E', *lwork* $\geq$ max(1, 2*n*); If *sense* = 'V' or 'B', *lwork* $\geq n^2+2n$. For good performance, *lwork* must generally be larger. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
rwork	REAL for cgeevx DOUBLE PRECISION for zgeevx Workspace array, DIMENSION at least max(1, 2*n*). Used in complex flavors only.

iwork INTEGER.
Workspace array, DIMENSION at least max(1, 2*n*-2). Used in real flavors only. Not referenced if *sense* = 'N' or 'E'.

Output Parameters

a On exit, this array is overwritten.
If *jobvl* = 'V' or *jobvr* = 'V', it contains the real-Schur/Schur form of the balanced version of the input matrix *A*.

wr, *wi* REAL for sgeevx
DOUBLE PRECISION for dgeevx
Arrays, DIMENSION at least max (1, *n*) each. Contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first.

w COMPLEX for cgeevx
DOUBLE COMPLEX for zgeevx.
Array, DIMENSION at least max(1, *n*). Contains the computed eigenvalues.

vl, *vr* REAL for sgeevx
DOUBLE PRECISION for dgeevx
COMPLEX for cgeevx
DOUBLE COMPLEX for zgeevx.
Arrays:
vl(*ldvl*,*); the second dimension of *vl* must be at least max(1, *n*).
If *jobvl* = 'V', the left eigenvectors *u*(j) are stored one after another in the columns of *vl*, in the same order as their eigenvalues.
If *jobvl* = 'N', *vl* is not referenced.
For real flavors:
If the j-th eigenvalue is real, then *u*(j) = *vl*(:,j), the j-th column of *vl*.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then *u*(j) = *vl*(:,j) + *i***vl*(:,j+1) and (j+1) = *vl*(:,j) - *i***vl*(:,j+1), where *i* = sqrt(-1).
For complex flavors:
u(j) = *vl*(:,j), the j-th column of *vl*.
vr(*ldvr*,*); the second dimension of *vr* must be at least max(1, *n*).
If *jobvr* = 'V', the right eigenvectors *v*(j) are stored one after another in the columns of *vr*, in the same order as their eigenvalues.
If *jobvr* = 'N', *vr* is not referenced.
For real flavors:
If the j-th eigenvalue is real, then *v*(j) = *vr*(:,j), the j-th column of *vr*.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then *v*(j) = *vr*(:,j) + *i***vr*(:,j+1) and *v*(j+1) = *vr*(:,j) - *i***vr*(:,j+1), where *i* = sqrt(-1) .
For complex flavors:
v(j) = *vr*(:,j), the j-th column of *vr*.

ilo, *ihi* INTEGER. *ilo* and *ihi* are integer values determined when *A* was balanced. The balanced *A*(i,j) = 0 if i > j and j = 1,..., *ilo*-1 or i = *ihi*+1,..., *n*.
If *balanc* = 'N' or 'S', *ilo* = 1 and *ihi* = *n*.

scale REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.

	Array, DIMENSION at least max(1, *n*). Details of the permutations and scaling factors applied when balancing *A*. If *P*(j) is the index of the row and column interchanged with row and column j, and *D*(j) is the scaling factor applied to row and column j, then *scale*(j) = *P*(j), for j = 1,...,*ilo*-1 = *D*(j), for j = *ilo*,...,*ihi* = *P*(j) for j = *ihi*+1,..., *n*. The order in which the interchanges are made is *n* to *ihi*+1, then 1 to *ilo*-1.
abnrm	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. The one-norm of the balanced matrix (the maximum of the sum of absolute values of elements of any column).
rconde, *rcondv*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *n*) each. *rconde*(j) is the reciprocal condition number of the j-th eigenvalue. *rcondv*(j) is the reciprocal condition number of the j-th right eigenvector.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value. If *info* = *i*, the *QR* algorithm failed to compute all the eigenvalues, and no eigenvectors or condition numbers have been computed; elements 1:*ilo*-1 and *i*+1:*n* of *wr* and *wi* (for real flavors) or *w* (for complex flavors) contain eigenvalues which have converged.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine geevx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
wr	Holds the vector of length *n*. Used in real flavors only.
wi	Holds the vector of length *n*. Used in real flavors only.
w	Holds the vector of length *n*. Used in complex flavors only.
vl	Holds the matrix *VL* of size (*n*, *n*).
vr	Holds the matrix *VR* of size (*n*, *n*).
scale	Holds the vector of length *n*.
rconde	Holds the vector of length *n*.
rcondv	Holds the vector of length *n*.
balanc	Must be 'N', 'B', 'P' or 'S'. The default value is 'N'.
jobvl	Restored based on the presence of the argument *vl* as follows: *jobvl* = 'V', if *vl* is present, *jobvl* = 'N', if *vl* is omitted.
jobvr	Restored based on the presence of the argument *vr* as follows: *jobvr* = 'V', if *vr* is present,

jobvr = 'N', if *vr* is omitted.

sense — Restored based on the presence of arguments *rconde* and rcondv as follows:
sense = 'B', if both *rconde* and *rcondv* are present,
sense = 'E', if *rconde* is present and *rcondv* omitted,
sense = 'V', if *rconde* is omitted and *rcondv* present,
sense = 'N', if both *rconde* and *rcondv* are omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

Singular Value Decomposition

This section describes LAPACK driver routines used for solving singular value problems. See also computational routines that can be called to solve these problems. Table "Driver Routines for Singular Value Decomposition" lists all such driver routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Singular Value Decomposition

Routine Name	Operation performed
gesvd	Computes the singular value decomposition of a general rectangular matrix.
gesdd	Computes the singular value decomposition of a general rectangular matrix using a divide and conquer method.
gejsv	Computes the singular value decomposition of a real matrix using a preconditioned Jacobi SVD method.
gesvj	Computes the singular value decomposition of a real matrix using Jacobi plane rotations.
ggsvd	Computes the generalized singular value decomposition of a pair of general rectangular matrices.

?gesvd

Computes the singular value decomposition of a general rectangular matrix.

Syntax

Fortran 77:

```
call sgesvd(jobu, jobvt, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, info)
call dgesvd(jobu, jobvt, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, info)
call cgesvd(jobu, jobvt, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, rwork, info)
```

```
call zgesvd(jobu, jobvt, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, rwork, info)
```

Fortran 95:

```
call gesvd(a, s [,u] [,vt] [,ww] [,job] [,info])
```

C:

```
lapack_int LAPACKE_sgesvd( int matrix_order, char jobu, char jobvt, lapack_int m,
lapack_int n, float* a, lapack_int lda, float* s, float* u, lapack_int ldu, float* vt,
lapack_int ldvt, float* superb );

lapack_int LAPACKE_dgesvd( int matrix_order, char jobu, char jobvt, lapack_int m,
lapack_int n, double* a, lapack_int lda, double* s, double* u, lapack_int ldu, double*
vt, lapack_int ldvt, double* superb );

lapack_int LAPACKE_cgesvd( int matrix_order, char jobu, char jobvt, lapack_int m,
lapack_int n, lapack_complex_float* a, lapack_int lda, float* s, lapack_complex_float*
u, lapack_int ldu, lapack_complex_float* vt, lapack_int ldvt, float* superb );

lapack_int LAPACKE_zgesvd( int matrix_order, char jobu, char jobvt, lapack_int m,
lapack_int n, lapack_complex_double* a, lapack_int lda, double* s,
lapack_complex_double* u, lapack_int ldu, lapack_complex_double* vt, lapack_int ldvt,
double* superb );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the singular value decomposition (SVD) of a real/complex m-by-n matrix A, optionally computing the left and/or right singular vectors. The SVD is written as

$A = U*\Sigma*V^T$ for real routines

$A = U*\Sigma*V^H$ for complex routines

where Σ is an m-by-n matrix which is zero except for its min(m,n) diagonal elements, U is an m-by-m orthogonal/unitary matrix, and V is an n-by-n orthogonal/unitary matrix. The diagonal elements of Σ are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m, n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V^T (for real flavors) or V^H (for complex flavors), not V.

Input Parameters

The data types are given for the Fortran interface, except for *superb*. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobu

`CHARACTER*1`. Must be `'A'`, `'S'`, `'O'`, or `'N'`. Specifies options for computing all or part of the matrix U.
If *jobu* = `'A'`, all m columns of U are returned in the array u;
if *jobu* = `'S'`, the first min(m, n) columns of U (the left singular vectors) are returned in the array u;
if *jobu* = `'O'`, the first min(m, n) columns of U (the left singular vectors) are overwritten on the array a;
if *jobu* = `'N'`, no columns of U (no left singular vectors) are computed.

jobvt — CHARACTER*1. Must be 'A', 'S', 'O', or 'N'. Specifies options for computing all or part of the matrix V^T/V^H.
If *jobvt* = 'A', all *n* rows of V^T/V^H are returned in the array *vt*;
if *jobvt* = 'S', the first min(*m*,*n*) rows of V^T/V^H (the right singular vectors) are returned in the array *vt*;
if *jobvt* = 'O', the first min(*m*,*n*) rows of V^T/V^H) (the right singular vectors) are overwritten on the array *a*;
if *jobvt* = 'N', no rows of V^T/V^H (no right singular vectors) are computed.
jobvt and *jobu* cannot both be 'O'.

m — INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).

n — INTEGER. The number of columns in *A* (*n* ≥ 0).

a, *work* — REAL for sgesvd
DOUBLE PRECISION for dgesvd
COMPLEX for cgesvd
DOUBLE COMPLEX for zgesvd.
Arrays:
a(*lda*,*) is an array containing the *m*-by-*n* matrix *A*.
The second dimension of *a* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

lda — INTEGER. The leading dimension of the array *a*.
Must be at least max(1, *m*).

ldu, *ldvt* — INTEGER. The leading dimensions of the output arrays *u* and *vt*, respectively.
Constraints:
ldu ≥ 1; *ldvt* ≥ 1.
If *jobu* = 'S' or 'A', *ldu* ≥ *m*;
If *jobvt* = 'A', *ldvt* ≥ *n*;
If *jobvt* = 'S', *ldvt* ≥ min(*m*, *n*).

lwork — INTEGER.
The dimension of the array *work*.
Constraints:
lwork ≥ 1
lwork ≥ max(3*min(*m*, *n*)+max(*m*, *n*), 5*min(*m*,*n*)) (for real flavors);
lwork ≥ 2*min(*m*, *n*)+max(*m*, *n*) (for complex flavors).
For good performance, *lwork* must generally be larger.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for details.

rwork — REAL for cgesvd
DOUBLE PRECISION for zgesvd
Workspace array, DIMENSION at least max(1, 5*min(*m*, *n*)). Used in complex flavors only.

Output Parameters

a — On exit,
If *jobu* = 'O', *a* is overwritten with the first min(*m*,*n*) columns of *U* (the left singular vectors stored columnwise);
If *jobvt* = 'O', *a* is overwritten with the first min(*m*, *n*) rows of V^T/V^H (the right singular vectors stored rowwise);
If *jobu*≠'O' and *jobvt*≠'O', the contents of *a* are destroyed.

s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, min(*m*,*n*)). Contains the singular values of *A* sorted so that *s*(i) ≥ *s*(i+1).
u, *vt*	REAL for sgesvd DOUBLE PRECISION for dgesvd COMPLEX for cgesvd DOUBLE COMPLEX for zgesvd. Arrays: *u*(*ldu*,*); the second dimension of *u* must be at least max(1, *m*) if *jobu* = 'A', and at least max(1, min(*m*, *n*)) if *jobu* = 'S'. If *jobu* = 'A', *u* contains the *m*-by-*m* orthogonal/unitary matrix *U*. If *jobu* = 'S', *u* contains the first min(*m*, *n*) columns of *U* (the left singular vectors stored column-wise). If *jobu* = 'N' or 'O', *u* is not referenced. *vt*(*ldvt*,*); the second dimension of *vt* must be at least max(1, *n*). If *jobvt* = 'A', *vt* contains the *n*-by-*n* orthogonal/unitary matrix V^T/V^H. If *jobvt* = 'S', *vt* contains the first min(*m*, *n*) rows of V^T/V^H (the right singular vectors stored row-wise). If *jobvt* = 'N'or 'O', *vt* is not referenced.
work	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*. *For real flavors*: If *info* > 0, *work*(2:min(*m*,*n*)) contains the unconverged superdiagonal elements of an upper bidiagonal matrix *B* whose diagonal is in *s* (not necessarily sorted). *B* satisfies *A*=*u***B***vt*, so it has the same singular values as *A*, and singular vectors related by *u* and *vt*.
rwork	On exit (for complex flavors), if *info* > 0, *rwork*(1:min(*m*,*n*)-1) contains the unconverged superdiagonal elements of an upper bidiagonal matrix *B* whose diagonal is in *s* (not necessarily sorted). *B* satisfies *A* = *u***B***vt*, so it has the same singular values as *A*, and singular vectors related by *u* and *vt*.
superb (C interface)	On exit, *superb*(0:min(*m*,*n*)-2) contains the unconverged superdiagonal elements of an upper bidiagonal matrix *B* whose diagonal is in *s* (not necessarily sorted). *B* satisfies *A* = *u***B***vt*, so it has the same singular values as *A*, and singular vectors related by *u* and *vt*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then if ?bdsqr did not converge, *i* specifies how many superdiagonals of the intermediate bidiagonal form *B* did not converge to zero.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gesvd interface are the following:

a	Holds the matrix *A* of size (*m*, *n*).
s	Holds the vector of length min(*m*, *n*).

u	If present and is a square *m*-by-*m* matrix, on exit contains the *m*-by-*m* orthogonal/unitary matrix *U*. Otherwise, if present, on exit contains the first `min(m,n)` columns of the matrix *U* (left singular vectors stored column-wise).
vt	If present and is a square *n*-by-*n* matrix, on exit contains the *n*-by-*n* orthogonal/unitary matrix V'^T/V'^H. Otherwise, if present, on exit contains the first `min(m,n)` rows of the matrix V'^T/V'^H (right singular vectors stored row-wise).
ww	Holds the vector of length min(*m*, *n*)-1. *ww* contains the unconverged superdiagonal elements of an upper bidiagonal matrix *B* whose diagonal is in *s* (not necessarily sorted). *B* satisfies *A = U*B*VT*, so it has the same singular values as *A*, and singular vectors related by *U* and *VT*.
job	Must be either `'N'`, or `'U'`, or `'V'`. The default value is `'N'`. If *job* = `'U'`, and *u* is not present, then *u* is returned in the array *a*. If *job* = `'V'`, and *vt* is not present, then *vt* is returned in the array *a*.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?gesdd

Computes the singular value decomposition of a general rectangular matrix using a divide and conquer method.

Syntax

Fortran 77:

```
call sgesdd(jobz, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, iwork, info)
call dgesdd(jobz, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, iwork, info)
call cgesdd(jobz, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, rwork, iwork, info)
call zgesdd(jobz, m, n, a, lda, s, u, ldu, vt, ldvt, work, lwork, rwork, iwork, info)
```

Fortran 95:

```
call gesdd(a, s [,u] [,vt] [,jobz] [,info])
```

C:

```
lapack_int LAPACKE_sgesdd( int matrix_order, char jobz, lapack_int m, lapack_int n,
float* a, lapack_int lda, float* s, float* u, lapack_int ldu, float* vt, lapack_int
ldvt );
```

```
lapack_int LAPACKE_dgesdd( int matrix_order, char jobz, lapack_int m, lapack_int n,
double* a, lapack_int lda, double* s, double* u, lapack_int ldu, double* vt,
lapack_int ldvt );

lapack_int LAPACKE_cgesdd( int matrix_order, char jobz, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* s, lapack_complex_float* u, lapack_int
ldu, lapack_complex_float* vt, lapack_int ldvt );

lapack_int LAPACKE_zgesdd( int matrix_order, char jobz, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* s, lapack_complex_double* u,
lapack_int ldu, lapack_complex_double* vt, lapack_int ldvt );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes the singular value decomposition (SVD) of a real/complex *m*-by-*n* matrix *A*, optionally computing the left and/or right singular vectors.

If singular vectors are desired, it uses a divide-and-conquer algorithm. The SVD is written

A = *U**Σ**V'* for real routines,

A = *U**Σ*conjg(*V'*) for complex routines,

where Σ is an *m*-by-*n* matrix which is zero except for its min(*m*,*n*) diagonal elements, *U* is an *m*-by-*m* orthogonal/unitary matrix, and *V* is an *n*-by-*n* orthogonal/unitary matrix. The diagonal elements of Σ are the singular values of *A*; they are real and non-negative, and are returned in descending order. The first min(*m*, *n*) columns of *U* and *V* are the left and right singular vectors of *A*.

Note that the routine returns *vt* = *V'* (for real flavors) or *vt* = conjg(*V'*) (for complex flavors), not *V*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'A', 'S', 'O', or 'N'. Specifies options for computing all or part of the matrix *U*. If *jobz* = 'A', all *m* columns of *U* and all *n* rows of *V'*/conjg(*V'*) are returned in the arrays *u* and *vt*; if *jobz* = 'S', the first min(*m*, *n*) columns of *U* and the first min(*m*, *n*) rows of *V'*/conjg(*V'*) are returned in the arrays *u* and *vt*; if *jobz* = 'O', then if *m* ≥ *n*, the first *n* columns of *U* are overwritten in the array *a* and all rows of *V'*/conjg(*V'*) are returned in the array *vt*; if *m* < *n*, all columns of *U* are returned in the array *u* and the first *m* rows of *V'*/conjg(*V'*) are overwritten in the array *a*; if *jobz* = 'N', no columns of *U* or rows of *V'*/conjg(*V'*) are computed.
m	INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
a, *work*	REAL for sgesdd DOUBLE PRECISION for dgesdd

	COMPLEX for cgesdd DOUBLE COMPLEX for zgesdd. Arrays: *a*(*lda*,*) is an array containing the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *m*).
ldu, *ldvt*	INTEGER. The leading dimensions of the output arrays *u* and *vt*, respectively. Constraints: *ldu* ≥ 1; *ldvt* ≥ 1. If *jobz* = 'S' or 'A', or *jobz* = 'O' and *m* < *n*, then *ldu* ≥ *m*; If *jobz* = 'A' or *jobz* = 'O' and *m* ≥ *n*, then *ldvt* ≥ *n*; If *jobz* = 'S', *ldvt* ≥ min(*m*, *n*).
lwork	INTEGER. The dimension of the array *work*; *lwork* ≥ 1. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the *work*(1), and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.
rwork	REAL for cgesdd DOUBLE PRECISION for zgesdd Workspace array, DIMENSION at least max(1, 5*min(*m*,*n*)) if *jobz* = 'N'. Otherwise, the dimension of *rwork* must be at least max(1,min(*m*,*n*)*max(5*min(*m*,*n*)+7,2*max(*m*,*n*)+2*min(*m*,*n*)+1)). This array is used in complex flavors only.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, 8 *min(*m*, *n*)).

Output Parameters

a	On exit: If *jobz* = 'O', then if *m*≥ *n*, *a* is overwritten with the first *n* columns of *U* (the left singular vectors, stored columnwise). If *m* < *n*, *a* is overwritten with the first *m* rows of V^T (the right singular vectors, stored rowwise); If *jobz*≠'O', the contents of *a* are destroyed.
s	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, min(*m*,*n*)). Contains the singular values of *A* sorted so that *s*(i) ≥ *s*(i+1).
u, *vt*	REAL for sgesdd DOUBLE PRECISION for dgesdd COMPLEX for cgesdd DOUBLE COMPLEX for zgesdd. Arrays: *u*(*ldu*,*); the second dimension of *u* must be at least max(1, *m*) if *jobz* = 'A' or *jobz* = 'O' and *m* < *n*. If *jobz* = 'S', the second dimension of *u* must be at least max(1, min(*m*, *n*)).

	If *jobz* = 'A'or *jobz* = 'O' and *m* < *n*, *u* contains the *m*-by-*m* orthogonal/unitary matrix *U*. If *jobz* = 'S', *u* contains the first min(*m*, *n*) columns of *U* (the left singular vectors, stored columnwise). If *jobz* = 'O' and *m*≥*n*, or *jobz* = 'N', *u* is not referenced.
vt(*ldvt*,*)	the second dimension of *vt* must be at least max(1, *n*). If *jobz* = 'A'or *jobz* = 'O' and *m*≥*n*, *vt* contains the *n*-by-*n* orthogonal/unitary matrix V^T. If *jobz* = 'S', *vt* contains the first min(*m*, *n*) rows of V^T (the right singular vectors, stored rowwise). If *jobz* = 'O' and *m* < *n*, or *jobz* = 'N', *vt* is not referenced.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, then ?bdsdc did not converge, updating process failed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gesdd interface are the following:

a	Holds the matrix *A* of size (*m*, *n*).
s	Holds the vector of length min(*m*, *n*).
u	Holds the matrix *U* of size • (*m*,*m*) if *jobz*='A' or *jobz*='O' and *m* < *n* • (*m*,min(*m*, *n*)) if *jobz*='S' *u* is not referenced if *jobz* is not supplied or if *jobz*='N' or *jobz*='O' and *m* ≥ *n*.
vt	Holds the matrix *VT* of size • (*n*,*n*) if *jobz*='A' or *jobz*='O' and *m* ≥ *n* • (min(*m*, *n*), *n*) if *jobz*='S' *vt* is not referenced if *jobz* is not supplied or if *jobz*='N' or *jobz*='O' and *m* < *n*.
job	Must be 'N', 'A', 'S', or 'O'. The default value is 'N'.

Application Notes

For real flavors:

If *jobz* = 'N', *lwork* ≥ 3*min(*m*, *n*) + max (max(*m*,*n*), 6*min(*m*, *n*));

If *jobz* = 'O', *lwork* ≥ 3*(min(*m*, *n*))^2 + max (max(*m*, *n*), 5*(min(*m*, *n*))^2 + 4*min(*m*, *n*));

If *jobz* = 'S' or 'A', *lwork* ≥ 3*(min(*m*, *n*))^2 + max (max(*m*, *n*), 4*(min(*m*, *n*))^2 + 4*min(*m*, *n*))

For complex flavors:

If *jobz* = 'N', *lwork* ≥ 2*min(*m*, *n*) + max(*m*, *n*);

If *jobz* = 'O', *lwork* ≥ 2*(min(*m*, *n*))^2 + max(*m*, *n*) + 2*min(*m*, *n*);

If *jobz* = 'S' or 'A', *lwork* ≥ (min(*m*, *n*))^2 + max(*m*, *n*) + 2*min(*m*, *n*);

For good performance, *lwork* should generally be larger.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?gejsv

Computes the singular value decomposition of a real matrix using a preconditioned Jacobi SVD method.

Syntax

Fortran 77:

```
call sgejsv(joba, jobu, jobv, jobr, jobt, jobp, m, n, a, lda, sva, u, ldu, v, ldv,
work, lwork, iwork, info)
```

```
call dgejsv(joba, jobu, jobv, jobr, jobt, jobp, m, n, a, lda, sva, u, ldu, v, ldv,
work, lwork, iwork, info)
```

C:

```
lapack_int LAPACKE_<?>gejsv( int matrix_order, char joba, char jobu, char jobv, char
jobr, char jobt, char jobp, lapack_int m, lapack_int n, const <datatype>* a,
lapack_int lda, <datatype>* sva, <datatype>* u, lapack_int ldu, <datatype>* v,
lapack_int ldv, <datatype>* stat, lapack_int* istat );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The routine computes the singular value decomposition (SVD) of a real *m*-by-*n* matrix *A*, where *m* ≥ *n*.

The SVD is written as

$A = U*\Sigma*V^T$,

where Σ is an *m*-by-*n* matrix which is zero except for its *n* diagonal elements, *U* is an *m*-by-*n* (or *m*-by-*m*) orthonormal matrix, and *V* is an *n*-by-*n* orthogonal matrix. The diagonal elements of Σ are the singular values of *A*; the columns of *U* and *V* are the left and right singular vectors of *A*, respectively. The matrices *U* and *V* are computed and stored in the arrays *u* and *v*, respectively. The diagonal of Σ is computed and stored in the array *sva*.

The routine implements a preconditioned Jacobi SVD algorithm. It uses ?geqp3, ?geqrf, and ?gelqf as preprocessors and preconditioners. Optionally, an additional row pivoting can be used as a preprocessor, which in some cases results in much higher accuracy. An example is matrix *A* with the structure A = D1 * C

* D2, where D1, D2 are arbitrarily ill-conditioned diagonal matrices and C is a well-conditioned matrix. In that case, complete pivoting in the first QR factorizations provides accuracy dependent on the condition number of C, and independent of D1, D2. Such higher accuracy is not completely understood theoretically, but it works well in practice.

If *A* can be written as A = B*D, with well-conditioned *B* and some diagonal *D*, then the high accuracy is guaranteed, both theoretically and in software independent of *D*. For more details see [Drmac08-1], [Drmac08-2].

The computational range for the singular values can be the full range (UNDERFLOW,OVERFLOW), provided that the machine arithmetic and the BLAS and LAPACK routines called by ?gejsv are implemented to work in that range. If that is not the case, the restriction for safe computation with the singular values in the range of normalized IEEE numbers is that the spectral condition number kappa(A)=sigma_max(A)/sigma_min(A) does not overflow. This code (?gejsv) is best used in this restricted range, meaning that singular values of magnitude below ||A||_2 / slamch('O') (for single precision) or ||A||_2 / dlamch('O') (for double precision) are returned as zeros. See *jobr* for details on this.

This implementation is slower than the one described in [Drmac08-1], [Drmac08-2] due to replacement of some non-LAPACK components, and because the choice of some tuning parameters in the iterative part (?gesvj) is left to the implementer on a particular machine.

The rank revealing QR factorization (in this code: ?geqp3) should be implemented as in [Drmac08-3].

If *m* is much larger than *n*, it is obvious that the inital QRF with column pivoting can be preprocessed by the QRF without pivoting. That well known trick is not used in ?gejsv because in some cases heavy row weighting can be treated with complete pivoting. The overhead in cases *m* much larger than *n* is then only due to pivoting, but the benefits in accuracy have prevailed. You can incorporate this extra QRF step easily and also improve data movement (matrix transpose, matrix copy, matrix transposed copy) - this implementation of ?gejsv uses only the simplest, naive data movement.

Input Parameters

The data types are given for the Fortran interface, except for *istat*. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

joba

CHARACTER*1. Must be 'C', 'E', 'F', 'G', 'A', or 'R'.
Specifies the level of accuracy:
If *joba* = 'C', high relative accuracy is achieved if *A* = *B*D* with well-conditioned *B* and arbitrary diagonal matrix *D*. The accuracy cannot be spoiled by column scaling. The accuracy of the computed output depends on the condition of *B*, and the procedure aims at the best theoretical accuracy. The relative error max_{i=1:N}|d sigma_i| / sigma_i is bounded by f(M,N)*epsilon* cond(B), independent of *D*. The input matrix is preprocessed with the QRF with column pivoting. This initial preprocessing and preconditioning by a rank revealing QR factorization is common for all values of *joba*. Additional actions are specified as follows:
If *joba* = 'E', computation as with 'C' with an additional estimate of the condition number of *B*. It provides a realistic error bound.
If *joba* = 'F', accuracy higher than in the 'C' option is achieved, if *A* = *D1*C*D2* with ill-conditioned diagonal scalings *D1*, *D2*, and a well-conditioned matrix *C*. This option is advisable, if the structure of the input matrix is not known and relative accuracy is desirable. The input matrix *A* is preprocessed with QR factorization with full (row and column) pivoting.
If *joba* = 'G', computation as with 'F' with an additional estimate of the condition number of *B*, where *A* = *B*D*. If *A* has heavily weighted rows, using this condition number gives too pessimistic error bound.

If *joba* = 'A', small singular values are the noise and the matrix is treated as numerically rank defficient. The error in the computed singular values is bounded by f(m,n)*epsilon*||A||. The computed SVD A = U*S*V**t restores *A* up to f(m,n)*epsilon*||A||. This enables the procedure to set all singular values below n*epsilon*||A|| to zero.
If *joba* = 'R', the procedure is similar to the 'A' option. Rank revealing property of the initial QR factorization is used to reveal (using triangular factor) a gap sigma_{r+1} < epsilon * sigma_r, in which case the numerical rank is declared to be *r*. The SVD is computed with absolute error bounds, but more accurately than with 'A'.

jobu CHARACTER*1. Must be 'U', 'F', 'W', or 'N'.
Specifies whether to compute the columns of the matrix *U*:
If *jobu* = 'U', *n* columns of *U* are returned in the array *u*
If *jobu* = 'F', a full set of *m* left singular vectors is returned in the array *u*.
If *jobu* = 'W', *u* may be used as workspace of length *m***n*. See the description of *u*.
If *jobu* = 'N', *u* is not computed.

jobv CHARACTER*1. Must be 'V', 'J', 'W', or 'N'.
Specifies whether to compute the matrix *V*:
If *jobv* = 'V', *n* columns of *V* are returned in the array *v*; Jacobi rotations are not explicitly accumulated.
If *jobv* = 'J', *n* columns of *V* are returned in the array *v* but they are computed as the product of Jacobi rotations. This option is allowed only if *jobu* ≠ *n*
If *jobv* = 'W', *v* may be used as workspace of length *n***n*. See the description of *v*.
If *jobv* = 'N', *v* is not computed.

jobr CHARACTER*1. Must be 'N' or 'R'.
Specifies the range for the singular values. If small positive singular values are outside the specified range, they may be set to zero. If *A* is scaled so that the largest singular value of the scaled matrix is around sqrt(big), big = ?lamch('O'), the function can remove columns of *A* whose norm in the scaled matrix is less than sqrt(?lamch('S')) (for *jobr* = 'R'), or less than small = ?lamch('S')/?lamch('E').
If *jobr* = 'N', the function does not remove small columns of the scaled matrix. This option assumes that BLAS and QR factorizations and triangular solvers are implemented to work in that range. If the condition of *A* if greater that big, use ?gesvj.
If *jobr* = 'R', restricted range for singular values of the scaled matrix *A* is [sqrt(?lamch('S'), sqrt(big)], roughly as described above. This option is recommended.
For computing the singular values in the full range [?lamch('S'),big], use ?gesvj.

jobt CHARACTER*1. Must be 'T' or 'N'.
If the matrix is square, the procedure may determine to use a transposed *A* if *A***t seems to be better with respect to convergence. If the matrix is not square, *jobt* is ignored. This is subject to changes in the future.
The decision is based on two values of entropy over the adjoint orbit of *A***t * *A*. See the descriptions of *work(6)* and *work(7)*.
If *jobt* = 'T', the function perfomrs transpositon if the entropy test indicates possibly faster convergence of the Jacobi process, if *A* is taken as input. If *A* is replaced with *A***t, the row pivoting is included automatically.

If *jobt* = 'N', the functions attempts no speculations. This option can be used to compute only the singular values, or the full SVD (*u*, *sigma*, and *v*). For only one set of singular vectors (*u* or *v*), the caller should provide both *u* and *v*, as one of the matrices is used as workspace if the matrix *A* is transposed. The implementer can easily remove this constraint and make the code more complicated. See the descriptions of *u* and *v*.

jobp

CHARACTER*1. Must be 'P' or 'N'.
Enables structured perturbations of denormalized numbers. This option should be active if the denormals are poorly implemented, causing slow computation, especially in cases of fast convergence. For details, see [Drmac08-1], [Drmac08-2] . For simplicity, such perturbations are included only when the full SVD or only the singular values are requested. You can add the perturbation for the cases of computing one set of singular vectors.
If *jobp* = 'P', the function introduces perturbation.
If *jobp* = 'N', the function introduces no perturbation.

m

INTEGER. The number of rows of the input matrix *A*; $m \geq 0$.

n

INTEGER. The number of columns in the input matrix *A*; $n \geq 0$.

a, *work*, *sva*, *u*, *v*

REAL for sgejsv
DOUBLE PRECISION for dgejsv.
Array *a*(*lda*,*) is an array containing the *m*-by-*n* matrix *A*.
The second dimension of *a* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).
sva is a workspace array, its dimension is *n*.
u is a workspace array, its dimension is (*ldu*,*); the second dimension of *u* must be at least max(1, *n*).
v is a workspace array, its dimension is (*ldv*,*); the second dimension of *u* must be at least max(1, *n*).

lda

INTEGER. The leading dimension of the array *a*. Must be at least max(1, *m*).

ldu

INTEGER. The leading dimension of the array *u*; $ldu \geq 1$.
jobu = 'U' or 'F' or 'W', $ldu \geq m$.

ldv

INTEGER. The leading dimension of the array *v*; $ldv \geq 1$.
jobv = 'V' or 'J' or 'W', $ldv \geq n$.

lwork

INTEGER.
Length of *work* to confirm proper allocation of work space. *lwork* depends on the task performed:
If only *sigma* is needed (*jobu* = 'N', *jobv* = 'N') and

- ... no scaled condition estimate is required, then *lwork* ≥ max(2*m+n, 4*n+1,7). This is the minimal requirement. For optimal performance (blocked code) the optimal value is *lwork* ≥ max(2*m+n,3*n+(n+1)*nb,7). Here *nb* is the optimal block size for ?geqp3/?geqrf.

  In general, the optimal length *lwork* is computed as

  lwork ≥ max(2*m+n,n+*lwork*(sgeqp3),n+*lwork*(sgeqrf),7) for sgejsv

  lwork ≥ max(2*m+n,n+*lwork*(dgeqp3),n+*lwork*(dgeqrf),7) for dgejsv

– ... an estimate of the scaled condition number of *A* is required (*joba* = 'E', 'G'). In this case, *lwork* is the maximum of the above and *n***n* +4**n*, that is, *lwork* ≥ max(2**m*+*n*,*n***n*+4**n*,7). For optimal performance (blocked code) the optimal value is *lwork* ≥ max(2**m*+*n*, 3**n*+(*n*+1)**nb*, *n***n*+4**n*, 7).

  In general, the optimal length *lwork* is computed as

  lwork ≥ max(2**m*+*n*,*n*+*lwork*(sgeqp3),*n*+*lwork*(sgeqrf),*n*+*n***n* +*lwork*(spocon, 7) for sgejsv

  lwork ≥ max(2**m*+*n*,*n*+*lwork*(dgeqp3),*n*+*lwork*(dgeqrf),*n*+*n***n* +*lwork*(dpocon, 7) for dgejsv

If *sigma* and the right singular vectors are needed (*jobv* = 'V'),

– the minimal requirement is *lwork* ≥ max(2**m*+*n*,4**n*+1,7).
– for optimal performance, *lwork* ≥ max(2**m*+*n*,3**n*+(*n*+1)**nb*,7), where *nb* is the optimal block size for ?geqp3, ?geqrf, ?gelqf, ?ormlq. In general, the optimal length *lwork* is computed as

  lwork ≥ max(2**m*+*n*, *n*+*lwork*(sgeqp3), *n*+*lwork*(spocon), *n* +*lwork*(sgelqf), 2**n*+*lwork*(sgeqrf), *n*+*lwork*(sormlq) for sgejsv

  lwork ≥ max(2**m*+*n*, *n*+*lwork*(dgeqp3), *n*+*lwork*(dpocon), *n* +*lwork*(dgelqf), 2**n*+*lwork*(dgeqrf), *n*+*lwork*(dormlq) for dgejsv

If *sigma* and the left singular vectors are needed

– the minimal requirement is *lwork* ≥ max(2**n*+*m*,4**n*+1,7).
– for optimal performance,

  if *jobu* = 'U' :: *lwork* ≥ max(2**m*+*n*,3**n*+(*n*+1)**nb*, 7),

  if *jobu* = 'F' :: *lwork* ≥ max(2**m*+*n*,3**n*+(*n*+1)**nb*, *n*+*m***nb*, 7),

  where *nb* is the optimal block size for ?geqp3, ?geqrf, ?ormlq . In general, the optimal length *lwork* is computed as

  lwork ≥ max(2**m*+*n*, *n*+*lwork*(sgeqp3), *n*+*lwork*(spocon), 2**n* +*lwork*(sgeqrf), *n*+*lwork*(sormlq) for sgejsv

  lwork ≥ max(2**m*+*n*, *n*+*lwork*(dgeqp3), *n*+*lwork*(dpocon), 2**n* +*lwork*(dgeqrf), *n*+*lwork*(dormlq) for dgejsv

  Here *lwork*(?ormlq) equals *n***nb* (for *jobu* = 'U') or *m***nb* (for *jobu* = 'F')

If full SVD is needed (*jobu* = 'U' or 'F') and

– if *jobv* = 'V',

  the minimal requirement is *lwork* ≥ max(2**m*+*n*, 6**n*+2**n***n*)
– if *jobv* = 'J',

  the minimal requirement is *lwork* ≥ max(2**m*+*n*, 4**n*+*n***n*, 2**n*+*n***n* +6)
– For optimal performance, *lwork* should be additionally larger than *n* +*m***nb*, where *nb* is the optimal block size for ?ormlq.

iwork INTEGER. Workspace array, DIMENSION max(3, *m*+3**n*).

Output Parameters

sva On exit:
For *work(1)/work(2)* = one: the singular values of *A*. During the computation *sva* contains Euclidean column norms of the iterated matrices in the array *a*.
For *work(1)≠work(2)*: the singular values of *A* are (*work(1)/work(2)*) * *sva*(1:*n*). This factored form is used if *sigma_max(A)* overflows or if small singular values have been saved from underflow by scaling the input matrix *A*.
jobr = 'R', some of the singular values may be returned as exact zeros obtained by 'setting to zero' because they are below the numerical rank threshold or are denormalized numbers.

u On exit:
If *jobu* = 'U', contains the *m*-by-*n* matrix of the left singular vectors.
If *jobu* = 'F', contains the *m*-by-*m* matrix of the left singular vectors, including an orthonormal basis of the orthogonal complement of the range of *A*.
If *jobu* = 'W' and *jobv* = 'V', *jobt* = 'T', and *m* = *n*, then *u* is used as workspace if the procedure replaces *A* with *A***t*. In that case, *v* is computed in *u* as left singular vectors of *A***t* and copied back to the *v* array. This 'W' option is just a reminder to the caller that in this case *u* is reserved as workspace of length *n***n*.
If *jobu* = 'N', *u* is not referenced.

v On exit:
If *jobv* = 'V' or 'J', contains the *n*-by-*n* matrix of the right singular vectors.
If *jobv* = 'W' and *jobv* = 'U', *jobt* = 'T', and *m* = *n*, then *v* is used as workspace if the procedure replaces *A* with *A***t*. In that case, *u* is computed in *v* as right singular vectors of *A***t* and copied back to the *u* array. This 'W' option is just a reminder to the caller that in this case *v* is reserved as workspace of length *n***n*.
If *jobv* = 'N', *v* is not referenced.

work On exit,
work(1) = *scale* = *work(2)*/*work(1)* is the scaling factor such that *scale***sva*(1:*n*) are the computed singular values of *A*. See the description of *sva()*.
work(2) = see the description of *work(1)*.
work(3) = *sconda* is an estimate for the condition number of column equilibrated *A*. If *joba* = 'E' or 'G', *sconda* is an estimate of sqrt(|| (R**t * R)**(-1)||_1). It is computed using ?pocon. It holds *n***(-1/4) * sconda* ≤ ||R**(-1)||_2 ≤ *n***(1/4) * sconda*, where R is the triangular factor from the QRF of *A*. However, if R is truncated and the numerical rank is determined to be strictly smaller than *n*, *sconda* is returned as -1, indicating that the smallest singular values might be lost.
If full SVD is needed, the following two condition numbers are useful for the analysis of the algorithm. They are provied for a user who is familiar with the details of the method.
work(4) = an estimate of the scaled condition number of the triangular factor in the first QR factorization.

	work(5) = an estimate of the scaled condition number of the triangular factor in the second QR factorization. The following two parameters are computed if *jobt* = 'T'. They are provided for a user who is familiar with the details of the method. *work(6)* = the entropy of *A**t*A* :: this is the Shannon entropy of diag(*A**t*A*) / Trace(*A**t*A*) taken as point in the probability simplex. *work(7)* = the entropy of *A*A**t*.
iwork (Fortran), *istat* (C)	INTEGER. On exit, *iwork(1)*/*istat[0]* = the numerical rank determined after the initial QR factorization with pivoting. See the descriptions of *joba* and *jobr*. *iwork(2)*/*istat[1]* = the number of the computed nonzero singular value. *iwork(3)*/*istat[2]* = if nonzero, a warning message. If *iwork(3)*/*istat[2]*=1, some of the column norms of *A* were denormalized floats. The requested high accuracy is not warranted by the data.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* > 0, the function did not converge in the maximal number of sweeps. The computed values may be inaccurate.

See Also

?geqp3
?geqrf
?gelqf
?gesvj
?lamch
?pocon
?ormlq

?gesvj

Computes the singular value decomposition of a real matrix using Jacobi plane rotations.

Syntax

Fortran 77:

```
call sgesvj(joba, jobu, jobv, m, n, a, lda, sva, mv, v, ldv, work, lwork, info)
call dgesvj(joba, jobu, jobv, m, n, a, lda, sva, mv, v, ldv, work, lwork, info)
```

C:

```
lapack_int LAPACKE_<?>gesvj( int matrix_order, char joba, char jobu, char jobv,
lapack_int m, lapack_int n, <datatype>* a, lapack_int lda, <datatype>* sva, lapack_int
mv, <datatype>* v, lapack_int ldv, <datatype>* stat );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The routine computes the singular value decomposition (SVD) of a real *m*-by-*n* matrix *A*, where $m \geq n$.

The SVD of A is written as

$A = U*\Sigma*V'$,

where Σ is an m-by-n diagonal matrix, U is an m-by-n orthonormal matrix, and V is an n-by-n orthogonal matrix. The diagonal elements of Σ are the singular values of A; the columns of U and V are the left and right singular vectors of A, respectively. The matrices U and V are computed and stored in the arrays u and v, respectively. The diagonal of Σ is computed and stored in the array sva.

The n-by-n orthogonal matrix V is obtained as a product of Jacobi plane rotations. The rotations are implemented as fast scaled rotations of Anda and Park [AndaPark94]. In the case of underflow of the Jacobi angle, a modified Jacobi transformation of Drmac ([Drmac08-4]) is used. Pivot strategy uses column interchanges of de Rijk ([deRijk98]). The relative accuracy of the computed singular values and the accuracy of the computed singular vectors (in angle metric) is as guaranteed by the theory of Demmel and Veselic [Demmel92]. The condition number that determines the accuracy in the full rank case is essentially

$$(\min_i d_{ii}) \cdot \kappa(A \cdot D)$$

where $\kappa(.)$ is the spectral condition number. The best performance of this Jacobi SVD procedure is achieved if used in an accelerated version of Drmac and Veselic [Drmac08-1], [Drmac08-2]. Some tunning parameters (marked with TP) are available for the implementer.

The computational range for the nonzero singular values is the machine number interval (UNDERFLOW,OVERFLOW). In extreme cases, even denormalized singular values can be computed with the corresponding gradual loss of accurate digit.

Input Parameters

The data types are given for the Fortran interface, except for *stat*. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

joba
CHARACTER*1. Must be 'L', 'U' or 'G'.
Specifies the structure of A:
If *joba* = 'L', the input matrix A is lower triangular.
If *joba* = 'U', the input matrix A is upper triangular.
If *joba* = 'G', the input matrix A is a general m-by-n, $m \geq n$.

jobu
CHARACTER*1. Must be 'U', 'C' or 'N'.
Specifies whether to compute the left singular vectors (columns of U):
If *jobu* = 'U', the left singular vectors corresponding to the nonzero singular values are computed and returned in the leading columns of A. See more details in the description of a. The default numerical orthogonality threshold is set to approximately TOL=CTOL*EPS, CTOL=sqrt(m), EPS = ?lamch('E')
If *jobu* = 'C', analogous to *jobu* = 'U', except that you can control the level of numerical orthogonality of the computed left singular vectors. TOL can be set to TOL=CTOL*EPS, where CTOL is given on input in the array *work*. No CTOL smaller than ONE is allowed. CTOL greater than 1 / EPS is meaningless. The option 'C' can be used if m*EPS is satisfactory orthogonality of the computed left singular vectors, so CTOL=m could save a few sweeps of Jacobi rotations. See the descriptions of a and *work(1)*.
If *jobu* = 'N', u is not computed. However, see the description of a.

jobv
CHARACTER*1. Must be 'V', 'A' or 'N'.
Specifies whether to compute the right singular vectors, that is, the matrix V:

If *jobv* = 'V', the matrix *V* is computed and returned in the array *v*.
If *jobv* = 'A', the Jacobi rotations are applied to the *mv*-by*n* array *v*. In other words, the right singular vector matrix *V* is not computed explicitly, instead it is applied to an *mv*-by*n* matrix initially stored in the first *mv* rows of *V*.
If *jobv* = 'N', the matrix *V* is not computed and the array *v* is not referenced.

m — INTEGER. The number of rows of the input matrix *A*.
1/slamch('E')> *m* ≥ 0 for sgesvj.
1/dlamch('E')> *m* ≥ 0 for dgesvj.

n — INTEGER. The number of columns in the input matrix *A*; *n* ≥ 0.

a, *work*, *sva*, *v* — REAL for sgesvj
DOUBLE PRECISION for dgesvj.
Array *a*(*lda*,*) is an array containing the *m*-by-*n* matrix *A*.
The second dimension of *a* is max(1, *n*).
work is a workspace array, its dimension max(4, *m*+*n*).
If *jobu* = 'C', *work(1)*=CTOL, where CTOL defines the threshold for convergence. The process stops if all columns of *A* are mutually orthogonal up to CTOL*EPS, EPS=?lamch('E'). It is required that CTOL ≥ 1, that is, it is not allowed to force the routine to obtain orthogonality below ε.
sva is a workspace array, its dimension is *n*.
v is a workspace array, its dimension is (*ldv*,*); the second dimension of *u* must be at least max(1, *n*).

lda — INTEGER. The leading dimension of the array *a*. Must be at least max(1, *m*).

mv — INTEGER.
jobv = 'A', the product of Jacobi rotations in ?gesvj is applied to the first *mv* rows of *v*. See the description of *jobv*.

ldv — INTEGER. The leading dimension of the array *v*; *ldv* ≥ 1.
jobv = 'V', *ldv* ≥ max(1, *n*).
jobv = 'A', *ldv* ≥ max(1, *mv*).

lwork — INTEGER.
Length of *work*, *work* ≥ max(6,*m*+*n*).

Output Parameters

a — On exit:
If *jobu* = 'U' or *jobu* = 'C':

- if *info* = 0, the leading columns of *A* contain left singular vectors corresponding to the computed singular values of *a* that are above the underflow threshold ?lamch('S'), that is, non-zero singular values. The number of the computed non-zero singular values is returned in *work(2)*. Also see the descriptions of *sva* and *work*. The computed columns of *u* are mutually numerically orthogonal up to approximately TOL=sqrt(*m*)*EPS (default); or TOL=CTOL*EPS *jobu* = 'C', see the description of *jobu*.
- if *info* > 0, the procedure ?gesvj did not converge in the given number of iterations (sweeps). In that case, the computed columns of *u* may not be orthogonal up to TOL. The output *u* (stored in *a*), *sigma*

(given by the computed singular values in *sva*(1:n)) and *v* is still a decomposition of the input matrix *A* in the sense that the residual ||a-scale*u*sigma*v**t||_2 / ||a||_2 is small.

If *jobu* = 'N':

- if *info* = 0, note that the left singular vectors are 'for free' in the one-sided Jacobi SVD algorithm. However, if only the singular values are needed, the level of numerical orthogonality of *u* is not an issue and iterations are stopped when the columns of the iterated matrix are numerically orthogonal up to approximately *m**EPS. Thus, on exit, *a* contains the columns of *u* scaled with the corresponding singular values.
- if *info* > 0, the procedure ?gesvj did not converge in the given number of iterations (sweeps).

sva

On exit:
If *info* = 0, depending on the value *scale* = *work(1)*, where *scale* is the scaling factor:

- if *scale* = 1, *sva*(1:n) contains the computed singular values of *a*. During the computation, *sva* contains the Euclidean column norms of the iterated matrices in the array *a*.
- if *scale* ≠ 1, the singular values of *a* are *scale***sva*(1:n), and this factored representation is due to the fact that some of the singular values of *a* might underflow or overflow.

If *info* > 0, the procedure ?gesvj did not converge in the given number of iterations (sweeps) and *scale***sva*(1:n) may not be accurate.

v

On exit:
If *jobv* = 'V', contains the *n*-by-*n* matrix of the right singular vectors.
If *jobv* = 'A', then *v* contains the product of the computed right singular vector matrix and the initial matrix in the array *v*.
If *jobv* = 'N', *v* is not referenced.

work (Fortarn), *stat* (C)

On exit,
work(1)/*stat[0]* = *scale* is the scaling factor such that *scale***sva*(1:*n*) are the computed singular values of *A*. See the description of *sva()*.
work(2)/*stat[1]* is the number of the computed nonzero singular value.
work(3)/*stat[2]* is the number of the computed singular values that are larger than the underflow threshold.
work(4)/*stat[3]* is the number of sweeps of Jacobi rotations needed for numerical convergence.
work(5)/*stat[4]* = max_{i.NE.j} |COS(A(:,i),A(:,j))| in the last sweep. This is useful information in cases when ?gesvj did not converge, as it can be used to estimate whether the output is still useful and for post festum analysis.
work(6)/*stat[5]* is the largest absolute value over all sines of the Jacobi rotation angles in the last sweep. It can be useful in a post festum analysis.

info

INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* > 0, the function did not converge in the maximal number (30) of sweeps. The output may still be useful. See the description of *work*.

See Also

?lamch

?ggsvd

Computes the generalized singular value decomposition of a pair of general rectangular matrices.

Syntax

Fortran 77:

```
call sggsvd(jobu, jobv, jobq, m, n, p, k, l, a, lda, b, ldb, alpha, beta, u, ldu, v, ldv, q, ldq, work, iwork, info)
call dggsvd(jobu, jobv, jobq, m, n, p, k, l, a, lda, b, ldb, alpha, beta, u, ldu, v, ldv, q, ldq, work, iwork, info)
call cggsvd(jobu, jobv, jobq, m, n, p, k, l, a, lda, b, ldb, alpha, beta, u, ldu, v, ldv, q, ldq, work, rwork, iwork, info)
call zggsvd(jobu, jobv, jobq, m, n, p, k, l, a, lda, b, ldb, alpha, beta, u, ldu, v, ldv, q, ldq, work, rwork, iwork, info)
```

Fortran 95:

```
call ggsvd(a, b, alpha, beta [, k] [,l] [,u] [,v] [,q] [,iwork] [,info])
```

C:

```
lapack_int LAPACKE_sggsvd( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l, float* a,
lapack_int lda, float* b, lapack_int ldb, float* alpha, float* beta, float* u,
lapack_int ldu, float* v, lapack_int ldv, float* q, lapack_int ldq, lapack_int*
iwork );
lapack_int LAPACKE_dggsvd( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l, double* a,
lapack_int lda, double* b, lapack_int ldb, double* alpha, double* beta, double* u,
lapack_int ldu, double* v, lapack_int ldv, double* q, lapack_int ldq, lapack_int*
iwork );
lapack_int LAPACKE_cggsvd( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
float* alpha, float* beta, lapack_complex_float* u, lapack_int ldu,
lapack_complex_float* v, lapack_int ldv, lapack_complex_float* q, lapack_int ldq,
lapack_int* iwork );
lapack_int LAPACKE_zggsvd( int matrix_order, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
double* alpha, double* beta, lapack_complex_double* u, lapack_int ldu,
lapack_complex_double* v, lapack_int ldv, lapack_complex_double* q, lapack_int ldq,
lapack_int* iwork );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

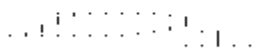
The routine computes the generalized singular value decomposition (GSVD) of an *m*-by-*n* real/complex matrix *A* and *p*-by-*n* real/complex matrix *B*:

U'**A***Q* = *D*$_1$*(0 *R*), *V*'**B***Q* = *D*$_2$*(0 *R*),

where *U*, *V* and *Q* are orthogonal/unitary matrices and *U*', *V*' mean transpose/conjugate transpose of *U* and *V* respectively.

Let *k*+*l* = the effective numerical rank of the matrix (*A*', *B*')', then *R* is a (*k*+*l*)-by-(*k*+*l*) nonsingular upper triangular matrix, *D*$_1$ and *D*$_2$ are *m*-by-(*k*+*l*) and *p*-by-(*k*+*l*) "diagonal" matrices and of the following structures, respectively:

If *m*-*k*-*l* ≥0,



$$D_2 = \begin{array}{c} \\ l \\ p-l \end{array} \begin{array}{c} \begin{array}{cc} k & l \end{array} \\ \begin{pmatrix} 0 & S \\ 0 & 0 \end{pmatrix} \end{array}$$

where

C = diag(*alpha*(*K*+1),..., *alpha*(*K*+*l*))

S = diag(*beta*(*K*+1),...,*beta*(*K*+*l*))

C^2 + *S*^{2} = I

R is stored in *a*(1:*k*+*l*, *n*-*k*-*l*+1:*n*) on exit.

If *m*-*k*-*l* < 0,

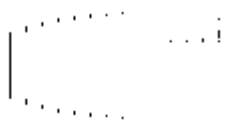
$$(0 \quad R) = \begin{array}{c} \\ k \\ m-k \\ k+l-m \end{array} \begin{array}{c} \begin{array}{cccc} n-k-l & k & m-k & k+l-m \end{array} \\ \begin{pmatrix} 0 & R_{11} & R_{12} & R_{13} \\ 0 & 0 & R_{22} & R_{23} \\ 0 & 0 & 0 & R_{33} \end{pmatrix} \end{array}$$

where

C = diag($alpha(K+1)$,..., $alpha(m)$),

S = diag($beta(K+1)$,...,$beta(m)$),

$C_2 + S_2 = I$



On exit, is stored in $a(1{:}m, n{-}k{-}l{+}1{:}n)$ and R_{33} is stored in $b(m{-}k{+}1{:}l, n{+}m{-}k{-}l{+}1{:}n)$.

The routine computes C, S, R, and optionally the orthogonal/unitary transformation matrices U, V and Q.

In particular, if B is an n-by-n nonsingular matrix, then the GSVD of A and B implicitly gives the SVD of $A*B^{-1}$:

$A*B^{-1} = U*(D_1*D_2^{-1})*V'$.

If (A', B')' has orthonormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B. Furthermore, the GSVD can be used to derive the solution of the eigenvalue problem:

$A'*A*x = \lambda*B'*B*x$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobu	CHARACTER*1. Must be 'U' or 'N'. If *jobu* = 'U', orthogonal/unitary matrix U is computed. If *jobu* = 'N', U is not computed.

jobv	CHARACTER*1. Must be 'V' or 'N'. If *jobv* = 'V', orthogonal/unitary matrix *V* is computed. If *jobv* = 'N', *V* is not computed.
jobq	CHARACTER*1. Must be 'Q' or 'N'. If *jobq* = 'Q', orthogonal/unitary matrix *Q* is computed. If *jobq* = 'N', *Q* is not computed.
m	INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns of the matrices *A* and *B* (*n* ≥ 0).
p	INTEGER. The number of rows of the matrix *B* (*p* ≥ 0).
a, *b*, *work*	REAL for sggsvd DOUBLE PRECISION for dggsvd COMPLEX for cggsvd DOUBLE COMPLEX for zggsvd. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the *p*-by-*n* matrix *B*. The second dimension of *b* must be at least max(1, *n*). *work*(*) is a workspace array. The dimension of *work* must be at least max(3*n*, *m*, *p*)+*n*.
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *p*).
ldu	INTEGER. The leading dimension of the array *u* . *ldu* ≥ max(1, *m*) if *jobu* = 'U'; *ldu* ≥ 1 otherwise.
ldv	INTEGER. The leading dimension of the array *v* . *ldv* ≥ max(1, *p*) if *jobv* = 'V'; *ldv* ≥ 1 otherwise.
ldq	INTEGER. The leading dimension of the array *q* . *ldq* ≥ max(1, *n*) if *jobq* = 'Q'; *ldq* ≥ 1 otherwise.
iwork	INTEGER. Workspace array, DIMENSION at least max(1, *n*).
rwork	REAL for cggsvd DOUBLE PRECISION for zggsvd. Workspace array, DIMENSION at least max(1, 2*n*). Used in complex flavors only.

Output Parameters

k, *l*	INTEGER. On exit, *k* and *l* specify the dimension of the subblocks. The sum *k*+*l* is equal to the effective numerical rank of (*A*', *B*')'.
a	On exit, *a* contains the triangular matrix *R* or part of *R*.
b	On exit, *b* contains part of the triangular matrix R if *m*-*k*-*l* < 0.
alpha, *beta*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays, DIMENSION at least max(1, *n*) each. Contain the generalized singular value pairs of *A* and *B*: *alpha*(1:*k*) = 1, *beta*(1:*k*) = 0, and if *m*-*k*-*l* ≥ 0, *alpha*(*k*+1:*k*+*l*) = *C*, *beta*(*k*+1:*k*+*l*) = *S*,

	or if *m*-*k*-*l* < 0, *alpha*(*k*+1:*m*)= *C*, *alpha*(*m*+1:*k*+*l*)=0 *beta*(*k*+1:*m*) = *S*, *beta*(*m*+1:*k*+*l*) = 1 and *alpha*(*k*+*l*+1:*n*) = 0 *beta*(*k*+*l*+1:*n*) = 0.
u, *v*, *q*	REAL for sggsvd DOUBLE PRECISION for dggsvd COMPLEX for cggsvd DOUBLE COMPLEX for zggsvd. Arrays: *u*(*ldu*,*); the second dimension of *u* must be at least max(1, *m*). If *jobu* = 'U', *u* contains the *m*-by-*m* orthogonal/unitary matrix *U*. If *jobu* = 'N', *u* is not referenced. *v*(*ldv*,*); the second dimension of *v* must be at least max(1, *p*). If *jobv* = 'V', *v* contains the *p*-by-*p* orthogonal/unitary matrix *V*. If *jobv* = 'N', *v* is not referenced. *q*(*ldq*,*); the second dimension of *q* must be at least max(1, *n*). If *jobq* = 'Q', *q* contains the *n*-by-*n* orthogonal/unitary matrix *Q*. If *jobq* = 'N', *q* is not referenced.
iwork	On exit, *iwork* stores the sorting information.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1, the Jacobi-type procedure failed to converge. For further details, see subroutine tgsja.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggsvd interface are the following:

a	Holds the matrix *A* of size (*m*, *n*).
b	Holds the matrix *B* of size (*p*, *n*).
alpha	Holds the vector of length *n*.
beta	Holds the vector of length *n*.
u	Holds the matrix *U* of size (*m*, *m*).
v	Holds the matrix *V* of size (*p*, *p*).
q	Holds the matrix *Q* of size (*n*, *n*).
iwork	Holds the vector of length *n*.
jobu	Restored based on the presence of the argument *u* as follows: *jobu* = 'U', if *u* is present, *jobu* = 'N', if *u* is omitted.
jobv	Restored based on the presence of the argument *v* as follows: *jobz* = 'V', if *v* is present, *jobz* = 'N', if *v* is omitted.
jobq	Restored based on the presence of the argument *q* as follows: *jobz* = 'Q', if *q* is present, *jobz* = 'N', if *q* is omitted.

Cosine-Sine Decomposition

This section describes LAPACK driver routines for computing the *cosine-sine decomposition* (CS decomposition). You can also call the corresponding computational routines to perform the same task.

The computation has the following phases:

1. The matrix is reduced to a bidiagonal block form.
2. The blocks are simultaneously diagonalized using techniques from the bidiagonal SVD algorithms.

Table "Driver Routines for Cosine-Sine Decomposition (CSD)" lists LAPACK routines (FORTRAN 77 interface) that perform CS decomposition of matrices. Respective routine names in Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Computational Routines for Cosine-Sine Decomposition (CSD)

Operation	Real matrices	Complex matrices
Compute the CS decomposition of a block-partitioned orthogonal matrix	orcsd uncsd	
Compute the CS decomposition of a block-partitioned unitary matrix		orcsd uncsd

See Also

Cosine-Sine Decomposition

?orcsd/?uncsd

Computes the CS decomposition of a block-partitioned orthogonal/unitary matrix.

Syntax

Fortran 77:

```
call sorcsd( jobu1, jobu2, jobv1t, jobv2t, trans, signs, m, p, q, x11, ldx11, x12,
ldx12, x21, ldx21, x22, ldx22, theta, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t,
work, lwork, iwork, info )
```

```
call dorcsd( jobu1, jobu2, jobv1t, jobv2t, trans, signs, m, p, q, x11, ldx11, x12,
ldx12, x21, ldx21, x22, ldx22, theta, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t,
work, lwork, iwork, info )
```

```
call cuncsd( jobu1, jobu2, jobv1t, jobv2t, trans, signs, m, p, q, x11, ldx11, x12,
ldx12, x21, ldx21, x22, ldx22, theta, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t,
work, lwork, rwork, lrwork, iwork, info )
```

```
call zuncsd( jobu1, jobu2, jobv1t, jobv2t, trans, signs, m, p, q, x11, ldx11, x12,
ldx12, x21, ldx21, x22, ldx22, theta, u1, ldu1, u2, ldu2, v1t, ldv1t, v2t, ldv2t,
work, lwork, rwork, lrwork, iwork, info )
```

Fortran 95:

```
call orcsd( x11,x12,x21,x22,theta,u1,u2,v1t,v2t[,jobu1][,jobu2][,jobv1t][,jobv2t]
[,trans][,signs][,info] )
```

```
call uncsd( x11,x12,x21,x22,theta,u1,u2,v1t,v2t[,jobu1][,jobu2][,jobv1t][,jobv2t]
[,trans][,signs][,info] )
```

C:

```
lapack_int LAPACKE_sorcsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q, float* x11,
lapack_int ldx11, float* x12, lapack_int ldx12, float* x21, lapack_int ldx21, float*
x22, lapack_int ldx22, float* theta, float* u1, lapack_int ldu1, float* u2, lapack_int
ldu2, float* v1t, lapack_int ldv1t, float* v2t, lapack_int ldv2t );

lapack_int LAPACKE_dorcsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q, double* x11,
lapack_int ldx11, double* x12, lapack_int ldx12, double* x21, lapack_int ldx21,
double* x22, lapack_int ldx22, double* theta, double* u1, lapack_int ldu1, double* u2,
lapack_int ldu2, double* v1t, lapack_int ldv1t, double* v2t, lapack_int ldv2t );

lapack_int LAPACKE_cuncsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q,
lapack_complex_float* x11, lapack_int ldx11, lapack_complex_float* x12, lapack_int
ldx12, lapack_complex_float* x21, lapack_int ldx21, lapack_complex_float* x22,
lapack_int ldx22, float* theta, lapack_complex_float* u1, lapack_int ldu1,
lapack_complex_float* u2, lapack_int ldu2, lapack_complex_float* v1t, lapack_int ldv1t,
lapack_complex_float* v2t, lapack_int ldv2t );

lapack_int LAPACKE_zuncsd( int matrix_order, char jobu1, char jobu2, char jobv1t, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q,
lapack_complex_double* x11, lapack_int ldx11, lapack_complex_double* x12, lapack_int
ldx12, lapack_complex_double* x21, lapack_int ldx21, lapack_complex_double* x22,
lapack_int ldx22, double* theta, lapack_complex_double* u1, lapack_int ldu1,
lapack_complex_double* u2, lapack_int ldu2, lapack_complex_double* v1t, lapack_int
ldv1t, lapack_complex_double* v2t, lapack_int ldv2t );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routines `?orcsd`/`?uncsd` compute the CS decomposition of an m-by-m partitioned orthogonal matrix X:

$$X=\left(\begin{array}{c|c} x_{11} & x_{12} \\ \hline x_{21} & x_{22} \end{array}\right)=\left(\begin{array}{c|c} u_1 & \\ \hline & u_2 \end{array}\right)\left(\begin{array}{ccc|ccc} I & 0 & 0 & 0 & 0 \\ 0 & C & 0 & -S & 0 \\ 0 & 0 & 0 & 0 & -I \\ \hline 0 & 0 & 0 & I & 0 & 0 \\ 0 & S & 0 & 0 & C & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{array}\right)\left(\begin{array}{c|c} v_1 & \\ \hline & v_2 \end{array}\right)^T$$

or unitary matrix:

$$X=\left(\begin{array}{c|c} x_{11} & x_{12} \\ \hline x_{21} & x_{22} \end{array}\right)=\left(\begin{array}{c|c} u_1 & \\ \hline & u_2 \end{array}\right)\left(\begin{array}{ccc|ccc} I & 0 & 0 & 0 & 0 \\ 0 & C & 0 & -S & 0 \\ 0 & 0 & 0 & 0 & -I \\ \hline 0 & 0 & I & 0 & 0 \\ 0 & S & 0 & C & 0 \\ 0 & 0 & I & 0 & 0 \end{array}\right)\left(\begin{array}{c|c} v_1 & \\ \hline & v_2 \end{array}\right)^H$$

x_{11} is p-by-q. The orthogonal/unitary matrices u_1, u_2, v_1, and v_2 are p-by-p, $(m-p)$-by-$(m-p)$, q-by-q, $(m-q)$-by-$(m-q)$, respectively. C and S are r-by-r nonnegative diagonal matrices satisfying $C^2 + S^2 = I$, in which $r = \min(p, m-p, q, m-q)$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobu1 — CHARACTER. If equals 'Y', then u_1 is computed. Otherwise, u_1 is not computed.

jobu2 — CHARACTER. If equals 'Y', then u_2 is computed. Otherwise, u_2 is not computed.

jobv1t — CHARACTER. If equals 'Y', then v_1^t is computed. Otherwise, v_1^t is not computed.

jobv2t — CHARACTER. If equals 'Y', then v_2^t is computed. Otherwise, v_2^t is not computed.

trans — CHARACTER

= 'T':	x, u_1, u_2, v_1^t, v_2^t are stored in row-major order.
otherwise	x, u_1, u_2, v_1^t, v_2^t are stored in column-major order.

signs — CHARACTER

= 'O':	The lower-left block is made nonpositive (the "other" convention).
otherwise	The upper-right block is made nonpositive (the "default" convention).

m — INTEGER. The number of rows and columns of the matrix X.

p — INTEGER. The number of rows in x_{11} and x_{12}. $0 \le p \le m$.

q — INTEGER. The number of columns in x_{11} and x_{21}. $0 \le q \le m$.

x — REAL for sorcsd
DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Array, DIMENSION (*ldx*,*m*).
On entry, the orthogonal/unitary matrix whose CSD is desired.

ldx — INTEGER. The leading dimension of the array X. $ldx \ge \max(1, m)$.

ldu1 INTEGER. The leading dimension of the array u_1. If *jobu1* = '*Y*', *ldu1* ≥ max(1,*p*).

ldu2 INTEGER. The leading dimension of the array u_2. If *jobu2* = '*Y*', *ldu2* ≥ max(1,*m*-*p*).

ldv1t INTEGER. The leading dimension of the array *v1t*. If *jobv1t* = '*Y*', *ldv1t* ≥ max(1,*q*).

ldv2t INTEGER. The leading dimension of the array *v2t*. If *jobv2t* = '*Y*', *ldv2t* ≥ max(1,*m*-*q*).

work REAL for sorcsd
DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Workspace array, DIMENSION (max(1,*lwork*)).

lwork INTEGER. The size of the *work* array. Constraints:
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

rwork REAL for cuncsd
DOUBLE PRECISION for zuncsd
Workspace array, DIMENSION (max(1,*lrwork*)).

lrwork INTEGER. The size of the *rwork* array. Constraints:
If *lrwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *rwork* array, returns this value as the first entry of the *rwork* array, and no error message related to *lrwork* is issued by xerbla.

iwork INTEGER. Workspace array, dimension *m*.

Output Parameters

theta REAL for sorcsd
DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Array, DIMENSION (*r*), in which *r* = min(*p*,*m*-*p*,*q*,*m*-*q*).
C = diag(cos(*theta*(1)), ..., cos(*theta*(r))), and
S = diag(sin(*theta*(1)), ..., sin(*theta*(r))).

u1 REAL for sorcsd
DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Array, DIMENSION (*p*).
If *jobu1* = '*Y*', *u1* contains the *p*-by-*p* orthogonal/unitary matrix u_1.

u2 REAL for sorcsd
DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Array, DIMENSION (*ldu2*,*m*-*p*).
If *jobu2* = '*Y*', *u2* contains the (*m*-*p*)-by-(*m*-*p*) orthogonal/unitary matrix u_2.

v1t REAL for sorcsd

DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Array, DIMENSION (*ldv1t*,*q*).
If *jobv1t* = '*Y*', *v1t* contains the *q*-by-*q* orthogonal matrix v_1^T or unitary matrix v_1^H.

v2t

REAL for sorcsd
DOUBLE PRECISION for dorcsd
COMPLEX for cuncsd
DOUBLE COMPLEX for zuncsd
Array, DIMENSION (*ldv2t*,*m*-*q*).
If *jobv2t* = '*Y*', *v2t* contains the (*m*-*q*)-by-(*m*-*q*) orthogonal matrix v_2^T or unitary matrix v_2^H.

work

On exit,

If *info* = 0, *work(1)* returns the optimal *lwork*.

If *info* > 0, *work(2:r)* contains the values *phi(1)*, ..., *phi(r-1)* that, together with *theta(1)*, ..., *theta(r)* define the matrix in intermediate bidiagonal-block form remaining after nonconvergence. *info* specifies the number of nonzero *phi*'s.

rwork

On exit,

If *info* = 0, *rwork(1)* returns the optimal *lrwork*.

If *info* > 0, *rwork(2:r)* contains the values *phi(1)*, ..., *phi(r-1)* that, together with *theta(1)*, ..., *theta(r)* define the matrix in intermediate bidiagonal-block form remaining after nonconvergence. *info* specifies the number of nonzero *phi*'s.

info

INTEGER.
= 0: successful exit
< 0: if *info* = -*i*, the *i*-th argument has an illegal value
> 0: ?bbcsd did not converge. See the description of *work* above for details.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ?orcsd/?uncsd interface are as follows:

x11 Holds the block of matrix *X* of size (*p*, *q*).

x12 Holds the block of matrix *X* of size (*p*, *m*-*q*).

x21 Holds the block of matrix *X* of size (*m*-*p*, *q*).

x22 Holds the block of matrix *X* of size (*m*-*p*, *m*-*q*).

theta Holds the vector of length *r* = min(*p*,*m*-*p*,*q*,*m*-*q*).

u1 Holds the matrix of size (*p*,*p*).

u2 Holds the matrix of size (*m*-*p*,*m*-*p*).

v1t	Holds the matrix of size (q,q).
v2t	Holds the matrix of size (m-q,m-q).
jobsu1	Indicates whether u_1 is computed. Must be 'Y' or 'O'.
jobsu2	Indicates whether u_2 is computed. Must be 'Y' or 'O'.
jobv1t	Indicates whether v_1^t is computed. Must be 'Y' or 'O'.
jobv2t	Indicates whether v_2^t is computed. Must be 'Y' or 'O'.
trans	Must be 'N' or 'T'.
signs	Must be 'O' or 'D'.

See Also
?bbcsd
xerbla

Generalized Symmetric Definite Eigenproblems

This section describes LAPACK driver routines used for solving generalized symmetric definite eigenproblems. See also computational routines that can be called to solve these problems. Table "Driver Routines for Solving Generalized Symmetric Definite Eigenproblems" lists all such driver routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Solving Generalized Symmetric Definite Eigenproblems

Routine Name	Operation performed
sygv/hegv	Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem.
sygvd/hegvd	Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem. If eigenvectors are desired, it uses a divide and conquer method.
sygvx/hegvx	Computes selected eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem.
spgv/hpgv	Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem with matrices in packed storage.
spgvd/hpgvd	Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem with matrices in packed storage. If eigenvectors are desired, it uses a divide and conquer method.
spgvx/hpgvx	Computes selected eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem with matrices in packed storage.
sbgv/hbgv	Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem with banded matrices.
sbgvd/hbgvd	Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem with banded matrices. If eigenvectors are desired, it uses a divide and conquer method.
sbgvx/hbgvx	Computes selected eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian definite eigenproblem with banded matrices.

?sygv

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem.

Syntax

Fortran 77:

```
call ssygv(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, info)
call dsygv(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, info)
```

Fortran 95:

```
call sygv(a, b, w [,itype] [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sygv( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb,
<datatype>* w );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$A*x = \lambda*B*x$, $A*B*x = \lambda*x$, or $B*A*x = \lambda*x$.

Here A and B are assumed to be symmetric and B is also positive definite.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	`INTEGER`. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x* = *lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then compute eigenvalues only. If *jobz* = `'V'`, then compute eigenvalues and eigenvectors.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, arrays *a* and *b* store the upper triangles of *A* and *B*; If *uplo* = `'L'`, arrays *a* and *b* store the lower triangles of *A* and *B*.
n	`INTEGER`. The order of the matrices *A* and *B* ($n \geq 0$).
a, *b*, *work*	`REAL` for `ssygv` `DOUBLE PRECISION` for `dsygv`. Arrays:

	a(*lda*,*) contains the upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the upper or lower triangle of the symmetric positive definite matrix *B*, as specified by *uplo*. The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
lwork	INTEGER. The dimension of the array *work*; *lwork* ≥ max(1, 3*n*-1). If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla. See *Application Notes* for the suggested value of *lwork*.

Output Parameters

a	On exit, if *jobz* = 'V', then if *info* = 0, *a* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows: if *itype* = 1 or 2, $Z^T*B*Z = I$; if *itype* = 3, $Z^T*\mathrm{inv}(B)*Z = I$; If *jobz* = 'N', then on exit the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is destroyed.
b	On exit, if *info* ≤ *n*, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^T*U$ or $B = L*L^T$.
w	REAL for ssygv DOUBLE PRECISION for dsygv. Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, spotrf/dpotrf and ssyev/dsyev returned an error code: If *info* = *i* ≤ *n*, ssyev/dsyev failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sygv interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).

b	Holds the matrix *B* of size (*n*, *n*).
w	Holds the vector of length *n*.
itype	Must be 1, 2, or 3. The default value is 1.
jobz	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For optimum performance use *lwork* ≥ (*nb*+2)*n, where *nb* is the blocksize for ssytrd/dsytrd returned by ilaenv.

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *work* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hegv

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem.

Syntax

Fortran 77:

```
call chegv(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, rwork, info)
call zhegv(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, rwork, info)
```

Fortran 95:

```
call hegv(a, b, w [,itype] [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_chegv( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb, float* w );
lapack_int LAPACKE_zhegv( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b,
lapack_int ldb, double* w );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be Hermitian and *B* is also positive definite.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype — INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved:
if *itype* = 1, the problem type is *A*x = lambda*B*x*;
if *itype* = 2, the problem type is *A*B*x = lambda*x*;
if *itype* = 3, the problem type is *B*A*x = lambda*x*.

jobz — CHARACTER*1. Must be 'N' or 'V'.
If *jobz* = 'N', then compute eigenvalues only.
If *jobz* = 'V', then compute eigenvalues and eigenvectors.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', arrays *a* and *b* store the upper triangles of *A* and *B*;
If *uplo* = 'L', arrays *a* and *b* store the lower triangles of *A* and *B*.

n — INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).

a, *b*, *work* — COMPLEX for chegv
DOUBLE COMPLEX for zhegv.
Arrays:
a(*lda*,*) contains the upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*.
The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the upper or lower triangle of the Hermitian positive definite matrix *B*, as specified by *uplo*.
The second dimension of *b* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

lda — INTEGER. The leading dimension of *a*; at least max(1, *n*).

ldb — INTEGER. The leading dimension of *b*; at least max(1, *n*).

lwork — INTEGER.
The dimension of the array *work*; *lwork* ≥ max(1, 2*n*-1).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

rwork — REAL for chegv
DOUBLE PRECISION for zhegv.
Workspace array, DIMENSION at least max(1, 3*n*-2).

Output Parameters

a — On exit, if *jobz* = 'V', then if *info* = 0, *a* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows:
if *itype* = 1 or 2, Z^H**B***Z* = I;
if *itype* = 3, Z^H*inv(*B*)**Z* = I;

	If *jobz* = 'N', then on exit the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is destroyed.
b	On exit, if *info* ≤ *n*, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^H*U$ or $B = L*L^H$.
w	REAL for chegv DOUBLE PRECISION for zhegv. Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument has an illegal value. If *info* > 0, cpotrf/zpotrf and cheev/zheev return an error code: If *info* = *i* ≤ *n*, cheev/zheev fails to converge, and *i* off-diagonal elements of an intermediate tridiagonal do not converge to zero; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* can not be completed and no eigenvalues or eigenvectors are computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hegv interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).
w	Holds the vector of length *n*.
itype	Must be 1, 2, or 3. The default value is 1.
jobz	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

For optimum performance use *lwork* ≥ (*nb*+1)*n, where *nb* is the blocksize for chetrd/zhetrd returned by ilaenv.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?sygvd

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

Fortran 77:

```
call ssygvd(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, iwork, liwork, info)
call dsygvd(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call sygvd(a, b, w [,itype] [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_<?>sygvd( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb,
<datatype>* w );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$A*x = \lambda*B*x$, $A*B*x = \lambda*x$, or $B*A*x = \lambda*x$.

Here A and B are assumed to be symmetric and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	`INTEGER`. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then compute eigenvalues only. If *jobz* = `'V'`, then compute eigenvalues and eigenvectors.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, arrays *a* and *b* store the upper triangles of *A* and *B*; If *uplo* = `'L'`, arrays *a* and *b* store the lower triangles of *A* and *B*.
n	`INTEGER`. The order of the matrices *A* and *B* ($n \geq 0$).
a, *b*, *work*	`REAL` for `ssygvd` `DOUBLE PRECISION` for `dsygvd`.

	Arrays: *a*(*lda*,*) contains the upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the upper or lower triangle of the symmetric positive definite matrix *B*, as specified by *uplo*. The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. Constraints: If $n \leq 1$, $lwork \geq 1$; If *jobz* = 'N' and $n>1$, $lwork < 2n+1$; If *jobz* = 'V' and $n>1$, $lwork < 2n^2+6n+1$. If *lwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.
iwork	INTEGER. Workspace array, its dimension max(1, *lwork*).
liwork	INTEGER. The dimension of the array *iwork*. Constraints: If $n \leq 1$, $liwork \geq 1$; If *jobz* = 'N' and $n>1$, $liwork \geq 1$; If *jobz* = 'V' and $n>1$, $liwork \geq 5n+3$. If *liwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

a	On exit, if *jobz* = 'V', then if *info* = 0, *a* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows: if *itype* = 1 or 2, $Z^T*B*Z = I$; if *itype* = 3, $Z^T*\text{inv}(B)*Z = I$; If *jobz* = 'N', then on exit the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is destroyed.
b	On exit, if $info \leq n$, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^T*U$ or $B = L*L^T$.
w	REAL for ssygvd DOUBLE PRECISION for dsygvd. Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order.

work(1)	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, an error code is returned as specified below. • For *info*≤*N*: • If *info* = *i*, with *i*≤*n*, and *jobz* = 'N', then the algorithm falied to converge; *i* off-diagonal elements of an intermediate tridiagonal form did not converge to zero. • If *jobz* = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *info*/(*n* +1) through mod(*info*,*n*+1). • For *info* > *N*: • If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sygvd interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).
w	Holds the vector of length *n*.
itype	Must be 1, 2, or 3. The default value is 1.
jobz	Must be 'N' or 'V'. The default value is 'N'.
uplo	Must be 'U' or 'L'. The default value is 'U'.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *work* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hegvd

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

Fortran 77:

```
call chegvd(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, rwork, lrwork, iwork, liwork, info)
```

```
call zhegvd(itype, jobz, uplo, n, a, lda, b, ldb, w, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call hegvd(a, b, w [,itype] [,jobz] [,uplo] [,info])
```

C:

```
lapack_int LAPACKE_chegvd( int matrix_order, lapack_int itype, char jobz, char uplo, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, float* w );
```

```
lapack_int LAPACKE_zhegvd( int matrix_order, lapack_int itype, char jobz, char uplo, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb, double* w );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

$A*x = \lambda*B*x$, $A*B*x = \lambda*x$, or $B*A*x = \lambda*x$.

Here A and B are assumed to be Hermitian and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
uplo	CHARACTER*1. Must be 'U' or 'L'.

If *uplo* = 'U', arrays *a* and *b* store the upper triangles of *A* and *B*;
If *uplo* = 'L', arrays *a* and *b* store the lower triangles of *A* and *B*.

n — INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).

a, *b*, *work* — COMPLEX for chegvd
DOUBLE COMPLEX for zhegvd.
Arrays:
a(*lda*,*) contains the upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*.
The second dimension of *a* must be at least max(1, *n*).
b(*ldb*,*) contains the upper or lower triangle of the Hermitian positive definite matrix *B*, as specified by *uplo*.
The second dimension of *b* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

lda — INTEGER. The leading dimension of *a*; at least max(1, *n*).

ldb — INTEGER. The leading dimension of *b*; at least max(1, *n*).

lwork — INTEGER.
The dimension of the array *work*.
Constraints:
If *n* ≤ 1, *lwork* ≥ 1;
If *jobz* = 'N' and *n*>1, *lwork* ≥ *n*+1;
If *jobz* = 'V' and *n*>1, *lwork* ≥ n^2+2n.
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

rwork — REAL for chegvd
DOUBLE PRECISION for zhegvd.
Workspace array, DIMENSION max(1, *lrwork*).

lrwork — INTEGER.
The dimension of the array *rwork*.
Constraints:
If *n* ≤ 1, *lrwork* ≥ 1;
If *jobz* = 'N' and *n*>1, *lrwork* ≥ *n*;
If *jobz* = 'V' and *n*>1, *lrwork* ≥ $2n^2+5n+1$.
If *lrwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER.
Workspace array, DIMENSION max(1, *liwork*).

liwork — INTEGER.
The dimension of the array *iwork*.
Constraints:
If *n* ≤ 1, *liwork* ≥ 1;
If *jobz* = 'N' and *n*>1, *liwork* ≥ 1;
If *jobz* = 'V' and *n*>1, *liwork* ≥ 5*n*+3.

If *liwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

a — On exit, if *jobz* = 'V', then if *info* = 0, *a* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows:
if *itype* = 1 or 2, Z^H* *B*Z* = I;
if *itype* = 3, Z^H*inv(*B*)*Z* = I;
If *jobz* = 'N', then on exit the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is destroyed.

b — On exit, if *info* ≤ *n*, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization *B* = U^H**U* or *B* = *L***L*^{H}.

w — REAL for chegvd
DOUBLE PRECISION for zhegvd.
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues in ascending order.

work(1) — On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.

rwork(1) — On exit, if *info* = 0, then *rwork*(1) returns the required minimal size of *lrwork*.

iwork(1) — On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = −*i*, the *i*-th argument had an illegal value.
If *info* = *i*, and *jobz* = 'N', then the algorithm failed to converge; *i* off-diagonal elements of an intermediate tridiagonal form did not converge to zero;
if *info* = *i*, and *jobz* = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *info*/(*n*+1) through mod(*info*, *n*+1).
If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hegvd interface are the following:

a — Holds the matrix *A* of size (*n*, *n*).

b — Holds the matrix *B* of size (*n*, *n*).

w — Holds the vector of length *n*.

itype — Must be 1, 2, or 3. The default value is 1.

jobz — Must be 'N' or 'V'. The default value is 'N'.

uplo Must be 'U' or 'L'. The default value is 'U'.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* (*liwork* or *lrwork*) for the first run or set *lwork* = -1 (*liwork* = -1, *lrwork* = -1).

If you choose the first option and set any of admissible *lwork* (*liwork* or *lrwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*) on exit. Use this value (*work*(1), *iwork*(1), *rwork*(1)) for subsequent runs.

If you set *lwork* = -1 (*liwork* = -1, *lrwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*, *lrwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?sygvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem.

Syntax

Fortran 77:

```
call ssygvx(itype, jobz, range, uplo, n, a, lda, b, ldb, vl, vu, il, iu, abstol, m,
w, z, ldz, work, lwork, iwork, ifail, info)
```

```
call dsygvx(itype, jobz, range, uplo, n, a, lda, b, ldb, vl, vu, il, iu, abstol, m,
w, z, ldz, work, lwork, iwork, ifail, info)
```

Fortran 95:

```
call sygvx(a, b, w [,itype] [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol]
[,info])
```

C:

```
lapack_int LAPACKE_<?>sygvx( int matrix_order, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, <datatype>* a, lapack_int lda, <datatype>* b, lapack_int ldb,
<datatype> vl, <datatype> vu, lapack_int il, lapack_int iu, <datatype> abstol,
lapack_int* m, <datatype>* w, <datatype>* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be symmetric and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A***B***x* = *lambda***x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(i) in the half-open interval: *vl*<*lambda*(i)≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *a* and *b* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *a* and *b* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
a, *b*, *work*	REAL for ssygvx DOUBLE PRECISION for dsygvx. Arrays: *a*(*lda*,*) contains the upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the upper or lower triangle of the symmetric positive definite matrix *B*, as specified by *uplo*. The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
vl, *vu*	REAL for ssygvx DOUBLE PRECISION for dsygvx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.
il, *iu*	INTEGER. If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0. If *range* = 'A' or 'V', *il* and *iu* are not referenced.
abstol	REAL for ssygvx

DOUBLE PRECISION for dsygvx. The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.

ldz — INTEGER. The leading dimension of the output array *z*. Constraints: *ldz* ≥ 1; if *jobz* = 'V', *ldz* ≥ max(1, *n*).

lwork — INTEGER.
The dimension of the array *work*;
lwork < max(1, 8*n*).
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

iwork — INTEGER.
Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

a — On exit, the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is overwritten.

b — On exit, if *info* ≤ *n*, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^T*U$ or $B = L*L^T$.

m — INTEGER. The total number of eigenvalues found,
0 ≤ *m* ≤ *n*. If *range* = 'A', *m* = *n*, and if *range* = 'I',
m = *iu*-*il*+1.

w, *z* — REAL for ssygvx
DOUBLE PRECISION for dsygvx.
Arrays:
w(*), DIMENSION at least max(1, *n*).
The first *m* elements of *w* contain the selected eigenvalues in ascending order.
z(*ldz*,*).
The second dimension of *z* must be at least max(1, *m*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized as follows:
if *itype* = 1 or 2, Z^T*B*Z = I;
if *itype* = 3, Z^T*inv(*B*)*Z* = I;
If *jobz* = 'N', then *z* is not referenced.
If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

work(1) — On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.

ifail — INTEGER.
Array, DIMENSION at least max(1, *n*).

	If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th argument had an illegal value. If *info* > 0, spotrf/dpotrf and ssyevx/dsyevx returned an error code: If *info* = *i* ≤ *n*, ssyevx/dsyevx failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sygvx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector of length *n*.
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted. Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then $\varepsilon * ||T||_1$ is used as tolerance, where T is the tridiagonal matrix obtained by reducing C to tridiagonal form, where C is the symmetric matrix of the standard symmetric problem to which the generalized problem is transformed. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, set *abstol* to 2*?lamch('S').

For optimum performance use *lwork* ≥ (*nb*+3)*n, where *nb* is the blocksize for ssytrd/dsytrd returned by ilaenv.

If it is not clear how much workspace to supply, use a generous value of *lwork* for the first run, or set *lwork* = -1.

In first case the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If *lwork* = -1, then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if *lwork* is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hegvx

Computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem.

Syntax

Fortran 77:

```
call chegvx(itype, jobz, range, uplo, n, a, lda, b, ldb, vl, vu, il, iu, abstol, m,
w, z, ldz, work, lwork, rwork, iwork, ifail, info)
```

```
call zhegvx(itype, jobz, range, uplo, n, a, lda, b, ldb, vl, vu, il, iu, abstol, m,
w, z, ldz, work, lwork, rwork, iwork, ifail, info)
```

Fortran 95:

```
call hegvx(a, b, w [,itype] [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,abstol]
[,info])
```

C:

```
lapack_int LAPACKE_chegvx( int matrix_order, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float*
b, lapack_int ldb, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zhegvx( int matrix_order, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_complex_double* b, lapack_int ldb, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z,
lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be Hermitian and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl*<*lambda*(*i*)≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *a* and *b* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *a* and *b* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
a, *b*, *work*	COMPLEX for chegvx DOUBLE COMPLEX for zhegvx. Arrays: *a*(*lda*,*) contains the upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*. The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) contains the upper or lower triangle of the Hermitian positive definite matrix *B*, as specified by *uplo*. The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ldb	INTEGER. The leading dimension of *b*; at least max(1, *n*).
vl, *vu*	REAL for chegvx DOUBLE PRECISION for zhegvx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.
il, *iu*	INTEGER.

If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: $1 \le il \le iu \le n$, if $n > 0$; *il*=1 and *iu*=0 if $n = 0$.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for chegvx
DOUBLE PRECISION for zhegvx.
The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.

ldz
INTEGER. The leading dimension of the output array *z*. Constraints: $ldz \ge 1$; if *jobz* = 'V', $ldz \ge \max(1, n)$.

lwork
INTEGER.
The dimension of the array *work*; $lwork \ge \max(1, 2n)$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
See *Application Notes* for the suggested value of *lwork*.

rwork
REAL for chegvx
DOUBLE PRECISION for zhegvx.
Workspace array, DIMENSION at least max(1, 7*n*).

iwork
INTEGER.
Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

a
On exit, the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is overwritten.

b
On exit, if $info \le n$, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^H*U$ or $B = L*L^H$.

m
INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu-il+1$.

w
REAL for chegvx
DOUBLE PRECISION for zhegvx.
Array, DIMENSION at least max(1, *n*).
The first *m* elements of *w* contain the selected eigenvalues in ascending order.

z
COMPLEX for chegvx
DOUBLE COMPLEX for zhegvx.
Array z(*ldz*,*). The second dimension of *z* must be at least max(1, *m*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized as follows:
if *itype* = 1 or 2, $Z^H*B*Z = I$;
if *itype* = 3, $Z^H*\text{inv}(B)*Z = I$;
If *jobz* = 'N', then *z* is not referenced.

	If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th argument had an illegal value. If *info* > 0, cpotrf/zpotrf and cheevx/zheevx returned an error code: If *info* = *i* ≤ *n*, cheevx/zheevx failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hegvx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).
w	Holds the vector of length *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector of length *n*.
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted. Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present,

range = 'A', if none of *vl*, *vu*, *il*, *iu* is present,
Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*T*||$_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *C* to tridiagonal form, where *C* is the symmetric matrix of the standard symmetric problem to which the generalized problem is transformed. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

For optimum performance use *lwork* ≥ (*nb*+1)**n*, where *nb* is the blocksize for chetrd/zhetrd returned by ilaenv.

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?spgv

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with matrices in packed storage.

Syntax

Fortran 77:

```
call sspgv(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, info)
call dspgv(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, info)
```

Fortran 95:

```
call spgv(ap, bp, w [,itype] [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>spgv( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, <datatype>* ap, <datatype>* bp, <datatype>* w, <datatype>* z, lapack_int
ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be symmetric, stored in packed format, and *B* is also positive definite.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ap* and *bp* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ap* and *bp* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
ap, *bp*, *work*	REAL for sspgv DOUBLE PRECISION for dspgv. Arrays: *ap(*)* contains the packed upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2). *bp(*)* contains the packed upper or lower triangle of the symmetric matrix *B*, as specified by *uplo*. The dimension of *bp* must be at least max(1, *n**(*n*+1)/2). *work(*)* is a workspace array, DIMENSION at least max(1, 3*n*).
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', *ldz* ≥ max(1, *n*).

Output Parameters

ap	On exit, the contents of *ap* are overwritten.
bp	On exit, contains the triangular factor *U* or *L* from the Cholesky factorization *B* = U^T**U* or *B* = *L**L^T, in the same storage format as *B*.
w, *z*	REAL for sspgv DOUBLE PRECISION for dspgv. Arrays: *w*(*), DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows: if *itype* = 1 or 2, Z^T**B***Z* = I;

	if *itype* = 3, Z^T*inv(*B*)**Z* = I; If *jobz* = 'N', then *z* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, spptrf/dpptrf and sspev/dspev returned an error code: If *info* = *i* ≤ *n*, sspev/dspev failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spgv interface are the following:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
bp	Holds the array *B* of size (*n**(*n*+1)/2).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

?hpgv

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem with matrices in packed storage.

Syntax

Fortran 77:

```
call chpgv(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, rwork, info)
call zhpgv(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, rwork, info)
```

Fortran 95:

```
call hpgv(ap, bp, w [,itype] [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chpgv( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* ap, lapack_complex_float* bp, float* w,
lapack_complex_float* z, lapack_int ldz );
lapack_int LAPACKE_zhpgv( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* ap, lapack_complex_double* bp, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be Hermitian, stored in packed format, and *B* is also positive definite.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ap* and *bp* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ap* and *bp* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
ap, *bp*, *work*	COMPLEX for chpgv DOUBLE COMPLEX for zhpgv. Arrays: *ap(*)* contains the packed upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2). *bp(*)* contains the packed upper or lower triangle of the Hermitian matrix *B*, as specified by *uplo*. The dimension of *bp* must be at least max(1, *n**(*n*+1)/2). *work(*)* is a workspace array, DIMENSION at least max(1, 2*n*-1).
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', *ldz* ≥ max(1, *n*).
rwork	REAL for chpgv DOUBLE PRECISION for zhpgv. Workspace array, DIMENSION at least max(1, 3*n*-2).

Output Parameters

ap	On exit, the contents of *ap* are overwritten.
bp	On exit, contains the triangular factor *U* or *L* from the Cholesky factorization *B* = U^H**U* or *B* = *L**L^H, in the same storage format as *B*.
w	REAL for chpgv

DOUBLE PRECISION for zhpgv.
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues in ascending order.

z — COMPLEX for chpgv
DOUBLE COMPLEX for zhpgv.
Array z(*ldz*,*).
The second dimension of *z* must be at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors.
The eigenvectors are normalized as follows:
if *itype* = 1 or 2, $Z^H*B*Z = I$;
if *itype* = 3, $Z^H*\mathrm{inv}(B)*Z = I$;
If *jobz* = 'N', then *z* is not referenced.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th argument had an illegal value.
If *info* > 0, cpptrf/zpptrf and chpev/zhpev returned an error code:
If *info* = *i* ≤ *n*, chpev/zhpev failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;
If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpgv interface are the following:

ap — Holds the array *A* of size (*n**(*n*+1)/2).

bp — Holds the array *B* of size (*n**(*n*+1)/2).

w — Holds the vector with the number of elements *n*.

z — Holds the matrix *Z* of size (*n*, *n*).

itype — Must be 1, 2, or 3. The default value is 1.

uplo — Must be 'U' or 'L'. The default value is 'U'.

jobz — Restored based on the presence of the argument *z* as follows:
jobz = 'V', if *z* is present,
jobz = 'N', if *z* is omitted.

?spgvd

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with matrices in packed storage. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

Fortran 77:

```
call sspgvd(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, lwork, iwork, liwork, info)
call dspgvd(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call spgvd(ap, bp, w [,itype] [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>spgvd( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, <datatype>* ap, <datatype>* bp, <datatype>* w, <datatype>* z, lapack_int
ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be symmetric, stored in packed format, and *B* is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x* = *lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ap* and *bp* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ap* and *bp* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
ap, *bp*, *work*	REAL for sspgvd DOUBLE PRECISION for dspgvd. Arrays: *ap(*)* contains the packed upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2). *bp(*)* contains the packed upper or lower triangle of the symmetric matrix *B*, as specified by *uplo*. The dimension of *bp* must be at least max(1, *n**(*n*+1)/2). *work* is a workspace array, its dimension max(1, *lwork*).
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', *ldz* ≥ max(1, *n*).
lwork	INTEGER.

The dimension of the array *work*.
Constraints:
If $n \leq 1$, $lwork \geq 1$;
If *jobz* = 'N' and $n>1$, $lwork \geq 2n$;
If *jobz* = 'V' and $n>1$, $lwork \geq 2n^2+6n+1$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER.
Workspace array, dimension max(1, *lwork*).

liwork — INTEGER.
The dimension of the array *iwork*.
Constraints:
If $n \leq 1$, $liwork \geq 1$;
If *jobz* = 'N' and $n>1$, $liwork \geq 1$;
If *jobz* = 'V' and $n>1$, $liwork \geq 5n+3$.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the required sizes of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

ap — On exit, the contents of *ap* are overwritten.

bp — On exit, contains the triangular factor *U* or *L* from the Cholesky factorization $B = U^T*U$ or $B = L*L^T$, in the same storage format as *B*.

w, *z* — REAL for sspgv
DOUBLE PRECISION for dspgv.
Arrays:
w(*), DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues in ascending order.
z(*ldz*,*).
The second dimension of *z* must be at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors.
The eigenvectors are normalized as follows:
if *itype* = 1 or 2, $Z^T*B*Z = I$;
if *itype* = 3, $Z^T*\mathrm{inv}(B)*Z = I$;
If *jobz* = 'N', then *z* is not referenced.

work(1) — On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.

iwork(1) — On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th argument had an illegal value.
If *info* > 0, spptrf/dpptrf and sspevd/dspevd returned an error code:
If *info* = *i* ≤ *n*, sspevd/dspevd failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;

If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spgvd interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
bp	Holds the array *B* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run, or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, then the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), then the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *lwork* (*liwork*) is less than the minimal required value and is not equal to -1, then the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hpgvd

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem with matrices in packed storage. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

Fortran 77:

```
call chpgvd(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, lwork, rwork, lrwork,
iwork, liwork, info)
```

```
call zhpgvd(itype, jobz, uplo, n, ap, bp, w, z, ldz, work, lwork, rwork, lrwork,
iwork, liwork, info)
```

Fortran 95:

```
call hpgvd(ap, bp, w [,itype] [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chpgvd( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* ap, lapack_complex_float* bp, float* w,
lapack_complex_float* z, lapack_int ldz );

lapack_int LAPACKE_zhpgvd( int matrix_order, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* ap, lapack_complex_double* bp, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

$A*x = \lambda*B*x$, $A*B*x = \lambda*x$, or $B*A*x = \lambda*x$.

Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	`INTEGER`. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then compute eigenvalues only. If *jobz* = `'V'`, then compute eigenvalues and eigenvectors.
uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, arrays *ap* and *bp* store the upper triangles of *A* and *B*; If *uplo* = `'L'`, arrays *ap* and *bp* store the lower triangles of *A* and *B*.
n	`INTEGER`. The order of the matrices *A* and *B* ($n \geq 0$).
ap, *bp*, *work*	`COMPLEX` for `chpgvd` `DOUBLE COMPLEX` for `zhpgvd`. Arrays: *ap(*)* contains the packed upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2). *bp(*)* contains the packed upper or lower triangle of the Hermitian matrix *B*, as specified by *uplo*. The dimension of *bp* must be at least max(1, *n**(*n*+1)/2). *work* is a workspace array, its dimension `max(1, lwork)`.
ldz	`INTEGER`. The leading dimension of the output array *z*; $ldz \geq 1$. If *jobz* = `'V'`, $ldz \geq \max(1, n)$.

lwork INTEGER.
The dimension of the array *work*.
Constraints:
If $n \leq 1$, $lwork \geq 1$;
If *jobz* = 'N' and $n>1$, $lwork \geq n$;
If *jobz* = 'V' and $n>1$, $lwork \geq 2n$.
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

rwork REAL for chpgvd
DOUBLE PRECISION for zhpgvd.
Workspace array, its dimension max(1, *lrwork*).

lrwork INTEGER.
The dimension of the array *rwork*.
Constraints:
If $n \leq 1$, $lrwork \geq 1$;
If *jobz* = 'N' and $n>1$, $lrwork \geq n$;
If *jobz* = 'V' and $n>1$, $lrwork \geq 2n^2+5n+1$.
If *lrwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork INTEGER.
Workspace array, its dimension max(1, *liwork*).

liwork INTEGER.
The dimension of the array *iwork*.
Constraints:
If $n \leq 1$, $liwork \geq 1$;
If *jobz* = 'N' and $n>1$, $liwork \geq 1$;
If *jobz* = 'V' and $n>1$, $liwork \geq 5n+3$.
If *liwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

ap On exit, the contents of *ap* are overwritten.

bp On exit, contains the triangular factor *U* or *L* from the Cholesky factorization $B = U^H*U$ or $B = L*L^H$, in the same storage format as *B*.

w REAL for chpgvd
DOUBLE PRECISION for zhpgvd.
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues in ascending order.

z COMPLEX for chpgvd
DOUBLE COMPLEX for zhpgvd.
Array z(*ldz*,*).

	The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows: if *itype* = 1 or 2, $Z^H*B*Z = I$; if *itype* = 3, $Z^H*\text{inv}(B)*Z = I$; If *jobz* = 'N', then *z* is not referenced.
work(1)	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
rwork(1)	On exit, if *info* = 0, then *rwork*(1) returns the required minimal size of *lrwork*.
iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, cpptrf/zpptrf and chpevd/zhpevd returned an error code: If *info* = *i* ≤ *n*, chpevd/zhpevd failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpgvd interface are the following:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
bp	Holds the array *B* of size (*n**(*n*+1)/2).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* (*liwork* or *lrwork*) for the first run or set *lwork* = -1 (*liwork* = -1, *lrwork* = -1).

If you choose the first option and set any of admissible *lwork* (*liwork* or *lrwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*) on exit. Use this value (*work*(1), *iwork*(1), *rwork*(1)) for subsequent runs.

If you set *lwork* = -1 (*liwork* = -1, *lrwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*, *lrwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?spgvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with matrices in packed storage.

Syntax

Fortran 77:

```
call sspgvx(itype, jobz, range, uplo, n, ap, bp, vl, vu, il, iu, abstol, m, w, z,
ldz, work, iwork, ifail, info)
```

```
call dspgvx(itype, jobz, range, uplo, n, ap, bp, vl, vu, il, iu, abstol, m, w, z,
ldz, work, iwork, ifail, info)
```

Fortran 95:

```
call spgvx(ap, bp, w [,itype] [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail]
[,abstol] [,info])
```

C:

```
lapack_int LAPACKE_<?>spgvx( int matrix_order, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, <datatype>* ap, <datatype>* bp, <datatype> vl, <datatype> vu,
lapack_int il, lapack_int iu, <datatype> abstol, lapack_int* m, <datatype>* w,
<datatype>* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

*A*x* = λ**B*x*, *A*B*x* = λ**x*, or *B*A*x* = λ**x*.

Here *A* and *B* are assumed to be symmetric, stored in packed format, and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype — `INTEGER`. Must be 1 or 2 or 3. Specifies the problem type to be solved:
if *itype* = 1, the problem type is *A*x = lambda*B*x*;
if *itype* = 2, the problem type is *A***B***x* = *lambda***x*;
if *itype* = 3, the problem type is *B*A*x = lambda*x*.

jobz — `CHARACTER*1`. Must be `'N'` or `'V'`.
If *jobz* = `'N'`, then compute eigenvalues only.

If *jobz* = 'V', then compute eigenvalues and eigenvectors.

range — CHARACTER*1. Must be 'A' or 'V' or 'I'.
If *range* = 'A', the routine computes all eigenvalues.
If *range* = 'V', the routine computes eigenvalues *lambda(i)* in the half-open interval:
vl<*lambda(i)*≤ *vu*.
If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', arrays *ap* and *bp* store the upper triangles of *A* and *B*;
If *uplo* = 'L', arrays *ap* and *bp* store the lower triangles of *A* and *B*.

n — INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).

ap, *bp*, *work* — REAL for sspgvx
DOUBLE PRECISION for dspgvx.
Arrays:
ap(*) contains the packed upper or lower triangle of the symmetric matrix *A*, as specified by *uplo*.
The dimension of *ap* must be at least max(1, *n**(*n*+1)/2).
bp(*) contains the packed upper or lower triangle of the symmetric matrix *B*, as specified by *uplo*.
The dimension of *bp* must be at least max(1, *n**(*n*+1)/2).
work(*) is a workspace array, DIMENSION at least max(1, 8*n*).

vl, *vu* — REAL for sspgvx
DOUBLE PRECISION for dspgvx.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
Constraint: *vl*< *vu*.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu* — INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol — REAL for sspgvx
DOUBLE PRECISION for dspgvx.
The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.

ldz — INTEGER. The leading dimension of the output array *z*. Constraints:
ldz ≥ 1; if *jobz* = 'V', *ldz* ≥ max(1, *n*).

iwork — INTEGER.
Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

ap — On exit, the contents of *ap* are overwritten.

bp — On exit, contains the triangular factor *U* or *L* from the Cholesky factorization *B* = U^T**U* or *B* = *L**L^T, in the same storage format as *B*.

m — INTEGER. The total number of eigenvalues found,
0 ≤ *m* ≤ *n*. If *range* = 'A', *m* = *n*, and if *range* = 'I', *m* = *iu*-*il*+1.

w, *z*	REAL for sspgvx DOUBLE PRECISION for dspgvx. Arrays: *w*(*), DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized as follows: if *itype* = 1 or 2, $Z^T*B*Z = I$; if *itype* = 3, $Z^T*\text{inv}(B)*Z = I$; If *jobz* = 'N', then *z* is not referenced. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*. Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, spptrf/dpptrf and sspevx/dspevx returned an error code: If *info* = *i* ≤ *n*, sspevx/dspevx failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine spgvx interface are the following:

ap	Holds the array *A* of size (*n**(*n*+1)/2).
bp	Holds the array *B* of size (*n**(*n*+1)/2).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector with the number of elements *n*.
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).

vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted. Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*T*||$_1$ is used instead, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues are computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, set *abstol* to 2*?lamch('S').

?hpgvx

Computes selected eigenvalues and, optionally, eigenvectors of a generalized Hermitian definite eigenproblem with matrices in packed storage.

Syntax

Fortran 77:

```
call chpgvx(itype, jobz, range, uplo, n, ap, bp, vl, vu, il, iu, abstol, m, w, z,
ldz, work, rwork, iwork, ifail, info)
```

```
call zhpgvx(itype, jobz, range, uplo, n, ap, bp, vl, vu, il, iu, abstol, m, w, z,
ldz, work, rwork, iwork, ifail, info)
```

Fortran 95:

```
call hpgvx(ap, bp, w [,itype] [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail]
[,abstol] [,info])
```

C:

```
lapack_int LAPACKE_chpgvx( int matrix_order, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_float* ap, lapack_complex_float* bp, float vl,
float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float* w,
lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zhpgvx( int matrix_order, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_double* ap, lapack_complex_double* bp, double
vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w,
lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

*A*x = λ*B*x*, *A*B*x = λ*x*, or *B*A*x = λ*x*.

Here *A* and *B* are assumed to be Hermitian, stored in packed format, and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

itype	INTEGER. Must be 1 or 2 or 3. Specifies the problem type to be solved: if *itype* = 1, the problem type is *A*x = lambda*B*x*; if *itype* = 2, the problem type is *A*B*x = lambda*x*; if *itype* = 3, the problem type is *B*A*x = lambda*x*.
jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda(i)* in the half-open interval: *vl< lambda(i) ≤ vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ap* and *bp* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ap* and *bp* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
ap, *bp*, *work*	COMPLEX for chpgvx DOUBLE COMPLEX for zhpgvx. Arrays: *ap(*)* contains the packed upper or lower triangle of the Hermitian matrix *A*, as specified by *uplo*. The dimension of *ap* must be at least max(1, *n**(*n*+1)/2). *bp(*)* contains the packed upper or lower triangle of the Hermitian matrix *B*, as specified by *uplo*. The dimension of *bp* must be at least max(1, *n**(*n*+1)/2). *work(*)* is a workspace array, DIMENSION at least max(1, 2*n*).
vl, *vu*	REAL for chpgvx DOUBLE PRECISION for zhpgvx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl< vu*.

If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: $1 \le il \le iu \le n$, if $n > 0$; *il*=1 and *iu*=0 if $n = 0$.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for chpgvx
DOUBLE PRECISION for zhpgvx.
The absolute error tolerance for the eigenvalues.
See *Application Notes* for more information.

ldz
INTEGER. The leading dimension of the output array *z*; $ldz \ge 1$. If *jobz* = 'V', $ldz \ge \max(1, n)$.

rwork
REAL for chpgvx
DOUBLE PRECISION for zhpgvx.
Workspace array, DIMENSION at least max(1, 7*n*).

iwork
INTEGER.
Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

ap
On exit, the contents of *ap* are overwritten.

bp
On exit, contains the triangular factor *U* or *L* from the Cholesky factorization $B = U^H*U$ or $B = L*L^H$, in the same storage format as *B*.

m
INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu-il+1$.

w
REAL for chpgvx
DOUBLE PRECISION for zhpgvx.
Array, DIMENSION at least max(1, *n*).
If *info* = 0, contains the eigenvalues in ascending order.

z
COMPLEX for chpgvx
DOUBLE COMPLEX for zhpgvx.
Array z(*ldz*,*).
The second dimension of *z* must be at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized as follows:
if *itype* = 1 or 2, $Z^H*B*Z = I$;
if *itype* = 3, $Z^H*\mathrm{inv}(B)*Z = I$;
If *jobz* = 'N', then *z* is not referenced.
If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
Note: you must ensure that at least max(1,*m*) columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

ifail
INTEGER.
Array, DIMENSION at least max(1, *n*).

	If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, cpptrf/zpptrf and chpevx/zhpevx returned an error code: If *info* = *i* ≤ *n*, chpevx/zhpevx failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*; If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hpgvx interface are the following:

ap	Holds the array *A* of size (*n*(n+1)/2*).
bp	Holds the array *B* of size (*n*(n+1)/2*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*), where the values *n* and *m* are significant.
ifail	Holds the vector with the number of elements *n*.
itype	Must be 1, 2, or 3. The default value is 1.
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted. Note that there will be an error condition if *ifail* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*T*||$_1$ is used as tolerance, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

?sbgv

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with banded matrices.

Syntax

Fortran 77:

```
call ssbgv(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, info)
call dsbgv(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, info)
```

Fortran 95:

```
call sbgv(ab, bb, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbgv( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, <datatype>* ab, lapack_int ldab, <datatype>* bb,
lapack_int ldbb, <datatype>* w, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form $A*x = \lambda*B*x$. Here A and B are assumed to be symmetric and banded, and B is also positive definite.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ab* and *bb* store the upper triangles of A and B; If *uplo* = 'L', arrays *ab* and *bb* store the lower triangles of A and B.
n	INTEGER. The order of the matrices A and B ($n \geq 0$).
ka	INTEGER. The number of super- or sub-diagonals in A ($ka \geq 0$).
kb	INTEGER. The number of super- or sub-diagonals in B ($kb \geq 0$).
ab, *bb*, *work*	REAL for ssbgv DOUBLE PRECISION for dsbgv Arrays:

	ab (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format. The second dimension of the array *ab* must be at least max(1, *n*). *bb*(*ldbb*,*) is an array containing either upper or lower triangular part of the symmetric matrix *B* (as specified by *uplo*) in band storage format. The second dimension of the array *bb* must be at least max(1, *n*). *work(*)* is a workspace array, dimension at least max(1, 3*n*)
ldab	INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.
ldbb	INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', *ldz* ≥ max(1, *n*).

Output Parameters

ab	On exit, the contents of *ab* are overwritten.
bb	On exit, contains the factor *S* from the split Cholesky factorization $B = S^T*S$, as returned by pbstf/pbstf.
w, *z*	REAL for ssbgv DOUBLE PRECISION for dsbgv Arrays: *w*(*), DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^T*B*Z = I$. If *jobz* = 'N', then *z* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th argument had an illegal value. If *info* > 0, and if *i* ≤ *n*, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; if *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then pbstf/pbstf returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbgv interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows:

jobz = 'V', if *z* is present,
jobz = 'N', if *z* is omitted.

?hbgv

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem with banded matrices.

Syntax

Fortran 77:

```
call chbgv(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, rwork, info)
call zhbgv(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, rwork, info)
```

Fortran 95:

```
call hbgv(ab, bb, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chbgv( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* bb, lapack_int ldbb, float* w, lapack_complex_float* z,
lapack_int ldz );
lapack_int LAPACKE_zhbgv( int matrix_order, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* bb, lapack_int ldbb, double* w, lapack_complex_double* z,
lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form $A*x = \lambda*B*x$. Here A and B are Hermitian and banded matrices, and matrix B is also positive definite.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ab* and *bb* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ab* and *bb* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).
ka	INTEGER. The number of super- or sub-diagonals in *A*

	(*ka* ≥ 0).
kb	INTEGER. The number of super- or sub-diagonals in *B* (*kb* ≥ 0).
ab, *bb*, *work*	COMPLEX for chbgv DOUBLE COMPLEX for zhbgv Arrays: *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format. The second dimension of the array *ab* must be at least max(1, *n*). *bb*(*ldbb*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *B* (as specified by *uplo*) in band storage format. The second dimension of the array *bb* must be at least max(1, *n*). *work(*)* is a workspace array, dimension at least max(1, *n*).
ldab	INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.
ldbb	INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', *ldz* ≥ max(1, *n*).
rwork	REAL for chbgv DOUBLE PRECISION for zhbgv. Workspace array, DIMENSION at least max(1, 3*n*).

Output Parameters

ab	On exit, the contents of *ab* are overwritten.
bb	On exit, contains the factor *S* from the split Cholesky factorization $B = S^H*S$, as returned by pbstf/pbstf.
w	REAL for chbgv DOUBLE PRECISION for zhbgv. Array, DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order.
z	COMPLEX for chbgv DOUBLE COMPLEX for zhbgv Array z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^H*B*Z = I$. If *jobz* = 'N', then *z* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, and if *i* ≤ *n*, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; if *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then pbstf/pbstf returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbgv interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

?sbgvd

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with banded matrices. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

Fortran 77:

```
call ssbgvd(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, lwork, iwork, liwork, info)
```

```
call dsbgvd(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, lwork, iwork, liwork, info)
```

Fortran 95:

```
call sbgvd(ab, bb, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_<?>sbgvd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_int ka, lapack_int kb, <datatype>* ab, lapack_int ldab, <datatype>* bb, lapack_int ldbb, <datatype>* w, <datatype>* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form *A*x* = λ*B*x*. Here *A* and *B* are assumed to be symmetric and banded, and *B* is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only.

If *jobz* = 'V', then compute eigenvalues and eigenvectors.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', arrays *ab* and *bb* store the upper triangles of *A* and *B*;
If *uplo* = 'L', arrays *ab* and *bb* store the lower triangles of *A* and *B*.

n — INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).

ka — INTEGER. The number of super- or sub-diagonals in *A* ($ka \geq 0$).

kb — INTEGER. The number of super- or sub-diagonals in *B* ($kb \geq 0$).

ab, *bb*, *work* — REAL for ssbgvd
DOUBLE PRECISION for dsbgvd
Arrays:
ab (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format.
The second dimension of the array *ab* must be at least max(1, *n*).
bb(*ldbb*,*) is an array containing either upper or lower triangular part of the symmetric matrix *B* (as specified by *uplo*) in band storage format.
The second dimension of the array *bb* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

ldab — INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.

ldbb — INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.

ldz — INTEGER. The leading dimension of the output array *z*; $ldz \geq 1$. If *jobz* = 'V', $ldz \geq \max(1, n)$.

lwork — INTEGER.
The dimension of the array *work*.
Constraints:
If $n \leq 1$, $lwork \geq 1$;
If *jobz* = 'N' and $n>1$, $lwork \geq 3n$;
If *jobz* = 'V' and $n>1$, $lwork \geq 2n^2+5n+1$.
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER.
Workspace array, its dimension max(1, *liwork*).

liwork — INTEGER.
The dimension of the array *iwork*.
Constraints:
If $n \leq 1$, $liwork \geq 1$;
If *jobz* = 'N' and $n>1$, $liwork \geq 1$;
If *jobz* = 'V' and $n>1$, $liwork \geq 5n+3$.
If *liwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* and *iwork* arrays, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

ab — On exit, the contents of *ab* are overwritten.

bb	On exit, contains the factor *S* from the split Cholesky factorization $B = S^T*S$, as returned by pbstf/pbstf.
w, *z*	REAL for ssbgvd DOUBLE PRECISION for dsbgvd Arrays: *w*(*), DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order. z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^T*B*Z = I$. If *jobz* = 'N', then *z* is not referenced.
work(1)	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, and if $i \le n$, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; if $info = n + i$, for $1 \le i \le n$, then pbstf/pbstf returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine sbgvd interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

If it is not clear how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = -1 (*liwork* = -1).

If *lwork* (or *liwork*) has any of admissible sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If *lwork* = -1 (*liwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if *work* (*liwork*) is less than the minimal required value and is not equal to -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?hbgvd

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem with banded matrices. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

Fortran 77:

```
call chbgvd(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

```
call zhbgvd(jobz, uplo, n, ka, kb, ab, ldab, bb, ldbb, w, z, ldz, work, lwork, rwork, lrwork, iwork, liwork, info)
```

Fortran 95:

```
call hbgvd(ab, bb, w [,uplo] [,z] [,info])
```

C:

```
lapack_int LAPACKE_chbgvd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab, lapack_complex_float* bb, lapack_int ldbb, float* w, lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhbgvd( int matrix_order, char jobz, char uplo, lapack_int n, lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int ldab, lapack_complex_double* bb, lapack_int ldbb, double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form *A*x* = λ*B*x*. Here *A* and *B* are assumed to be Hermitian and banded, and *B* is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz — CHARACTER*1. Must be 'N' or 'V'.
If *jobz* = 'N', then compute eigenvalues only.

If *jobz* = 'V', then compute eigenvalues and eigenvectors.

uplo — CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', arrays *ab* and *bb* store the upper triangles of *A* and *B*;
If *uplo* = 'L', arrays *ab* and *bb* store the lower triangles of *A* and *B*.

n — INTEGER. The order of the matrices *A* and *B* ($n \geq 0$).

ka — INTEGER. The number of super- or sub-diagonals in *A* ($ka \geq 0$).

kb — INTEGER. The number of super- or sub-diagonals in *B* ($kb \geq 0$).

ab, *bb*, *work* — COMPLEX for chbgvd
DOUBLE COMPLEX for zhbgvd
Arrays:
ab (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format.
The second dimension of the array *ab* must be at least max(1, *n*).
bb(*ldbb*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *B* (as specified by *uplo*) in band storage format.
The second dimension of the array *bb* must be at least max(1, *n*).
work is a workspace array, its dimension max(1, *lwork*).

ldab — INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.

ldbb — INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.

ldz — INTEGER. The leading dimension of the output array *z*; $ldz \geq 1$. If *jobz* = 'V', $ldz \geq \max(1, n)$.

lwork — INTEGER.
The dimension of the array *work*.
Constraints:
If $n \leq 1$, $lwork \geq 1$;
If *jobz* = 'N' and $n>1$, $lwork \geq n$;
If *jobz* = 'V' and $n>1$, $lwork \geq 2n^2$.
If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

rwork — REAL for chbgvd
DOUBLE PRECISION for zhbgvd.
Workspace array, DIMENSION max(1, *lrwork*).

lrwork — INTEGER.
The dimension of the array *rwork*.
Constraints:
If $n \leq 1$, $lrwork \geq 1$;
If *jobz* = 'N' and $n>1$, $lrwork \geq n$;
If *jobz* = 'V' and $n>1$, $lrwork \geq 2n^2+5n+1$.
If *lrwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

iwork — INTEGER.
Workspace array, DIMENSION max(1, *liwork*).

liwork	INTEGER. The dimension of the array *iwork*. Constraints: If $n \le 1$, $lwork \ge 1$; If *jobz* = 'N' and $n>1$, $liwork \ge 1$; If *jobz* = 'V' and $n>1$, $liwork \ge 5n+3$. If *liwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work*, *rwork* and *iwork* arrays, returns these values as the first entries of the *work*, *rwork* and *iwork* arrays, and no error message related to *lwork* or *lrwork* or *liwork* is issued by xerbla. See *Application Notes* for details.

Output Parameters

ab	On exit, the contents of *ab* are overwritten.
bb	On exit, contains the factor *S* from the split Cholesky factorization $B = S^H*S$, as returned by pbstf/pbstf.
w	REAL for chbgvd DOUBLE PRECISION for zhbgvd. Array, DIMENSION at least max(1, *n*) . If *info* = 0, contains the eigenvalues in ascending order.
z	COMPLEX for chbgvd DOUBLE COMPLEX for zhbgvd Array z(*ldz*,*) . The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^H*B*Z = I$. If *jobz* = 'N', then *z* is not referenced.
work(1)	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
rwork(1)	On exit, if *info* = 0, then *rwork*(1) returns the required minimal size of *lrwork*.
iwork(1)	On exit, if *info* = 0, then *iwork*(1) returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = −*i*, the *i*-th argument had an illegal value. If *info* > 0, and if $i \le n$, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; if $info = n + i$, for $1 \le i \le n$, then pbstf/pbstf returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbgvd interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).
w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* (*liwork* or *lrwork*) for the first run or set *lwork* = -1 (*liwork* = -1, *lrwork* = -1).

If you choose the first option and set any of admissible *lwork* (*liwork* or *lrwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*) on exit. Use this value (*work*(1), *iwork*(1), *rwork*(1)) for subsequent runs.

If you set *lwork* = -1 (*liwork* = -1, *lrwork* = -1), the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*, *rwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*, *lrwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

?sbgvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with banded matrices.

Syntax

Fortran 77:

```
call ssbgvx(jobz, range, uplo, n, ka, kb, ab, ldab, bb, ldbb, q, ldq, vl, vu, il, iu,
abstol, m, w, z, ldz, work, iwork, ifail, info)
```

```
call dsbgvx(jobz, range, uplo, n, ka, kb, ab, ldab, bb, ldbb, q, ldq, vl, vu, il, iu,
abstol, m, w, z, ldz, work, iwork, ifail, info)
```

Fortran 95:

```
call sbgvx(ab, bb, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,q] [,abstol]
[,info])
```

C:

```
lapack_int LAPACKE_<?>sbgvx( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, <datatype>* ab, lapack_int ldab,
<datatype>* bb, lapack_int ldbb, <datatype>* q, lapack_int ldq, <datatype> vl,
<datatype> vu, lapack_int il, lapack_int iu, <datatype> abstol, lapack_int* m,
<datatype>* w, <datatype>* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: mkl_lapack.fi and mkl_lapack.h

- Fortran 95: lapack.f90
- C: mkl_lapacke.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form *A*x = λ*B*x*. Here *A* and *B* are assumed to be symmetric and banded, and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	CHARACTER*1. Must be 'N' or 'V'. If *jobz* = 'N', then compute eigenvalues only. If *jobz* = 'V', then compute eigenvalues and eigenvectors.
range	CHARACTER*1. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl*<*lambda*(*i*)≤ *vu*. If *range* = 'I', the routine computes eigenvalues in range *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ab* and *bb* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ab* and *bb* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
ka	INTEGER. The number of super- or sub-diagonals in *A* (*ka* ≥ 0).
kb	INTEGER. The number of super- or sub-diagonals in *B* (*kb* ≥ 0).
ab, *bb*, *work*	REAL for ssbgvx DOUBLE PRECISION for dsbgvx Arrays: *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the symmetric matrix *A* (as specified by *uplo*) in band storage format. The second dimension of the array *ab* must be at least max(1, *n*). *bb*(*ldbb*,*) is an array containing either upper or lower triangular part of the symmetric matrix *B* (as specified by *uplo*) in band storage format. The second dimension of the array *bb* must be at least max(1, *n*). *work(*)* is a workspace array, DIMENSION (7**n*).
ldab	INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.
ldbb	INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.
vl, *vu*	REAL for ssbgvx DOUBLE PRECISION for dsbgvx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.
il, *iu*	INTEGER.

If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
Constraint: $1 \le il \le iu \le n$, if $n > 0$; *il*=1 and *iu*=0 if $n = 0$.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
REAL for ssbgvx
DOUBLE PRECISION for dsbgvx.
The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.

ldz
INTEGER. The leading dimension of the output array *z*; $ldz \ge 1$. If *jobz* = 'V', $ldz \ge \max(1, n)$.

ldq
INTEGER. The leading dimension of the output array *q*; $ldq < 1$.
If *jobz* = 'V', $ldq < \max(1, n)$.

iwork
INTEGER.
Workspace array, DIMENSION (5*n).

Output Parameters

ab
On exit, the contents of *ab* are overwritten.

bb
On exit, contains the factor *S* from the split Cholesky factorization $B = S^T*S$, as returned by pbstf/pbstf.

m
INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu-il+1$.

w, *z*, *q*
REAL for ssbgvx
DOUBLE PRECISION for dsbgvx
Arrays:
w(*), DIMENSION at least max(1, *n*) .
If *info* = 0, contains the eigenvalues in ascending order.
z(*ldz*,*) .
The second dimension of *z* must be at least max(1, *n*).
If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^T*B*Z = I$.
If *jobz* = 'N', then *z* is not referenced.
q(*ldq*,*) .
The second dimension of *q* must be at least max(1, *n*).
If *jobz* = 'V', then *q* contains the *n*-by-*n* matrix used in the reduction of *A*x* = lambda*B*x* to standard form, that is, *C*x= lambda*x* and consequently *C* to tridiagonal form.
If *jobz* = 'N', then *q* is not referenced.

ifail
INTEGER.
Array, DIMENSION (*m*).
If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge.
If *jobz* = 'N', then *ifail* is not referenced.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th argument had an illegal value.
If *info* > 0, and

	if $i \leq n$, the algorithm failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero; if $info = n + i$, for $1 \leq i \leq n$, then pbstf/pbstf returned $info = i$ and B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `sbgvx` interface are the following:

ab	Holds the array A of size (*ka+1*,*n*).
bb	Holds the array B of size (*kb+1*,*n*).
w	Holds the vector with the number of elements n.
z	Holds the matrix Z of size (n, n).
ifail	Holds the vector with the number of elements n.
q	Holds the matrix Q of size (n, n).
uplo	Must be `'U'` or `'L'`. The default value is `'U'`.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument z as follows: *jobz* = `'V'`, if z is present, *jobz* = `'N'`, if z is omitted. Note that there will be an error condition if *ifail* or q is present and z is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = `'V'`, if one of or both *vl* and *vu* are present, *range* = `'I'`, if one of or both *il* and *iu* are present, *range* = `'A'`, if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then $\varepsilon * ||T||_1$ is used as tolerance, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

?hbgvx

Computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem with banded matrices.

Syntax

Fortran 77:

```
call chbgvx(jobz, range, uplo, n, ka, kb, ab, ldab, bb, ldbb, q, ldq, vl, vu, il, iu,
abstol, m, w, z, ldz, work, rwork, iwork, ifail, info)
```

```
call zhbgvx(jobz, range, uplo, n, ka, kb, ab, ldab, bb, ldbb, q, ldq, vl, vu, il, iu,
abstol, m, w, z, ldz, work, rwork, iwork, ifail, info)
```

Fortran 95:

```
call hbgvx(ab, bb, w [,uplo] [,z] [,vl] [,vu] [,il] [,iu] [,m] [,ifail] [,q] [,abstol]
[,info])
```

C:

```
lapack_int LAPACKE_chbgvx( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* bb, lapack_int ldbb, lapack_complex_float* q, lapack_int ldq,
float vl, float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float*
w, lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zhbgvx( int matrix_order, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int
ldab, lapack_complex_double* bb, lapack_int ldbb, lapack_complex_double* q, lapack_int
ldq, double vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m,
double* w, lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form $A*x = \lambda*B*x$. Here A and B are assumed to be Hermitian and banded, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobz	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobz* = `'N'`, then compute eigenvalues only. If *jobz* = `'V'`, then compute eigenvalues and eigenvectors.
range	`CHARACTER*1`. Must be `'A'` or `'V'` or `'I'`. If *range* = `'A'`, the routine computes all eigenvalues.

	If *range* = 'V', the routine computes eigenvalues *lambda*(*i*) in the half-open interval: *vl*< *lambda*(*i*) ≤ *vu*. If *range* = 'I', the routine computes eigenvalues with indices *il* to *iu*.
uplo	CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', arrays *ab* and *bb* store the upper triangles of *A* and *B*; If *uplo* = 'L', arrays *ab* and *bb* store the lower triangles of *A* and *B*.
n	INTEGER. The order of the matrices *A* and *B* (*n* ≥ 0).
ka	INTEGER. The number of super- or sub-diagonals in *A* (*ka* ≥ 0).
kb	INTEGER. The number of super- or sub-diagonals in *B* (*kb* ≥ 0).
ab, *bb*, *work*	COMPLEX for chbgvx DOUBLE COMPLEX for zhbgvx Arrays: *ab* (*ldab*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *A* (as specified by *uplo*) in band storage format. The second dimension of the array *ab* must be at least max(1, *n*). *bb*(*ldbb*,*) is an array containing either upper or lower triangular part of the Hermitian matrix *B* (as specified by *uplo*) in band storage format. The second dimension of the array *bb* must be at least max(1, *n*). *work(*)* is a workspace array, DIMENSION at least max(1, *n*).
ldab	INTEGER. The leading dimension of the array *ab*; must be at least *ka*+1.
ldbb	INTEGER. The leading dimension of the array *bb*; must be at least *kb*+1.
vl, *vu*	REAL for chbgvx DOUBLE PRECISION for zhbgvx. If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: *vl*< *vu*. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.
il, *iu*	INTEGER. If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: 1 ≤ *il* ≤ *iu* ≤ *n*, if *n* > 0; *il*=1 and *iu*=0 if *n* = 0. If *range* = 'A' or 'V', *il* and *iu* are not referenced.
abstol	REAL for chbgvx DOUBLE PRECISION for zhbgvx. The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.
ldz	INTEGER. The leading dimension of the output array *z*; *ldz* ≥ 1. If *jobz* = 'V', *ldz* ≥ max(1, *n*).
ldq	INTEGER. The leading dimension of the output array *q*; *ldq* ≥ 1. If *jobz* = 'V', *ldq* ≥ max(1, *n*).
rwork	REAL for chbgvx DOUBLE PRECISION for zhbgvx. Workspace array, DIMENSION at least max(1, 7*n*).
iwork	INTEGER. Workspace array, DIMENSION at least max(1, 5*n*).

Output Parameters

ab	On exit, the contents of *ab* are overwritten.
bb	On exit, contains the factor *S* from the split Cholesky factorization $B = S^H*S$, as returned by pbstf/pbstf.
m	INTEGER. The total number of eigenvalues found, $0 \le m \le n$. If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu-il+1$.
w	REAL for chbgvx DOUBLE PRECISION for zhbgvx. Array *w*(*), DIMENSION at least max(1, *n*). If *info* = 0, contains the eigenvalues in ascending order.
z, *q*	COMPLEX for chbgvx DOUBLE COMPLEX for zhbgvx Arrays: z(*ldz*,*). The second dimension of *z* must be at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^H*B*Z = I$. If *jobz* = 'N', then *z* is not referenced. q(*ldq*,*). The second dimension of *q* must be at least max(1, *n*). If *jobz* = 'V', then *q* contains the *n*-by-*n* matrix used in the reduction of $Ax = \lambda Bx$ to standard form, that is, $Cx = \lambda x$ and consequently *C* to tridiagonal form. If *jobz* = 'N', then *q* is not referenced.
ifail	INTEGER. Array, DIMENSION at least max(1, *n*). If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge. If *jobz* = 'N', then *ifail* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0, and if $i \le n$, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero; if *info* = *n* + *i*, for $1 \le i \le n$, then pbstf/pbstf returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine hbgvx interface are the following:

ab	Holds the array *A* of size (*ka+1*,*n*).
bb	Holds the array *B* of size (*kb+1*,*n*).

w	Holds the vector with the number of elements *n*.
z	Holds the matrix *Z* of size (*n*, *n*).
ifail	Holds the vector with the number of elements *n*.
q	Holds the matrix *Q* of size (*n*, *n*).
uplo	Must be 'U' or 'L'. The default value is 'U'.
vl	Default value for this element is *vl* = -HUGE(*vl*).
vu	Default value for this element is *vu* = HUGE(*vl*).
il	Default value for this argument is *il* = 1.
iu	Default value for this argument is *iu* = *n*.
abstol	Default value for this element is *abstol* = 0.0_WP.
jobz	Restored based on the presence of the argument *z* as follows: *jobz* = 'V', if *z* is present, *jobz* = 'N', if *z* is omitted. Note that there will be an error condition if *ifail* or *q* is present and *z* is omitted.
range	Restored based on the presence of arguments *vl*, *vu*, *il*, *iu* as follows: *range* = 'V', if one of or both *vl* and *vu* are present, *range* = 'I', if one of or both *il* and *iu* are present, *range* = 'A', if none of *vl*, *vu*, *il*, *iu* is present, Note that there will be an error condition if one of or both *vl* and *vu* are present and at the same time one of or both *il* and *iu* are present.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to *abstol*+ε*max(|a|,|b|), where ε is the machine precision.

If *abstol* is less than or equal to zero, then ε*||*T*||$_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*?lamch('S'), not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to 2*?lamch('S').

Generalized Nonsymmetric Eigenproblems

This section describes LAPACK driver routines used for solving generalized nonsymmetric eigenproblems. See also computational routines that can be called to solve these problems. Table "Driver Routines for Solving Generalized Nonsymmetric Eigenproblems" lists all such driver routines for the FORTRAN 77 interface. Respective routine names in the Fortran 95 interface are without the first symbol (see Routine Naming Conventions).

Driver Routines for Solving Generalized Nonsymmetric Eigenproblems

Routine Name	Operation performed
gges	Computes the generalized eigenvalues, Schur form, and the left and/or right Schur vectors for a pair of nonsymmetric matrices.
ggesx	Computes the generalized eigenvalues, Schur form, and, optionally, the left and/or right matrices of Schur vectors.
ggev	Computes the generalized eigenvalues, and the left and/or right generalized eigenvectors for a pair of nonsymmetric matrices.

Routine Name	Operation performed
ggevx	Computes the generalized eigenvalues, and, optionally, the left and/or right generalized eigenvectors.

?gges

Computes the generalized eigenvalues, Schur form, and the left and/or right Schur vectors for a pair of nonsymmetric matrices.

Syntax

Fortran 77:

```
call sgges(jobvsl, jobvsr, sort, selctg, n, a, lda, b, ldb, sdim, alphar, alphai, beta, vsl, ldvsl, vsr, ldvsr, work, lwork, bwork, info)
```

```
call dgges(jobvsl, jobvsr, sort, selctg, n, a, lda, b, ldb, sdim, alphar, alphai, beta, vsl, ldvsl, vsr, ldvsr, work, lwork, bwork, info)
```

```
call cgges(jobvsl, jobvsr, sort, selctg, n, a, lda, b, ldb, sdim, alpha, beta, vsl, ldvsl, vsr, ldvsr, work, lwork, rwork, bwork, info)
```

```
call zgges(jobvsl, jobvsr, sort, selctg, n, a, lda, b, ldb, sdim, alpha, beta, vsl, ldvsl, vsr, ldvsr, work, lwork, rwork, bwork, info)
```

Fortran 95:

```
call gges(a, b, alphar, alphai, beta [,vsl] [,vsr] [,select] [,sdim] [,info])
```

```
call gges(a, b, alpha, beta [, vsl] [,vsr] [,select] [,sdim] [,info])
```

C:

```
lapack_int LAPACKE_sgges( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_S_SELECT3 select, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, lapack_int* sdim, float* alphar, float* alphai, float* beta, float* vsl, lapack_int ldvsl, float* vsr, lapack_int ldvsr );
```

```
lapack_int LAPACKE_dgges( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_D_SELECT3 select, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, lapack_int* sdim, double* alphar, double* alphai, double* beta, double* vsl, lapack_int ldvsl, double* vsr, lapack_int ldvsr );
```

```
lapack_int LAPACKE_cgges( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_C_SELECT2 select, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, lapack_int* sdim, lapack_complex_float* alpha, lapack_complex_float* beta, lapack_complex_float* vsl, lapack_int ldvsl, lapack_complex_float* vsr, lapack_int ldvsr );
```

```
lapack_int LAPACKE_zgges( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_Z_SELECT2 select, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb, lapack_int* sdim, lapack_complex_double* alpha, lapack_complex_double* beta, lapack_complex_double* vsl, lapack_int ldvsl, lapack_complex_double* vsr, lapack_int ldvsr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The ?gges routine computes the generalized eigenvalues, the generalized real/complex Schur form (*S*,*T*), optionally, the left and/or right matrices of Schur vectors (*vsl* and *vsr*) for a pair of *n*-by-*n* real/complex nonsymmetric matrices (*A*,*B*). This gives the generalized Schur factorization

*(A,B) = (vsl*S *vsr^H, vsl*T*vsr^H)*

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix *S* and the upper triangular matrix *T*. The leading columns of *vsl* and *vsr* then form an orthonormal/unitary basis for the corresponding left and right eigenspaces (deflating subspaces).

If only the generalized eigenvalues are needed, use the driver ggev instead, which is faster.

A generalized eigenvalue for a pair of matrices (*A*,*B*) is a scalar *w* or a ratio *alpha* / *beta* = *w*, such that *A* - *w*B* is singular. It is usually represented as the pair (*alpha*, *beta*), as there is a reasonable interpretation for *beta*=0 or for both being zero. A pair of matrices (*S*,*T*) is in the generalized real Schur form if *T* is upper triangular with non-negative diagonal and *S* is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of *S* are "standardized" by making the corresponding elements of *T* have the form:

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

and the pair of corresponding 2-by-2 blocks in *S* and *T* will have a complex conjugate pair of generalized eigenvalues. A pair of matrices (*S*,*T*) is in generalized complex Schur form if *S* and *T* are upper triangular and, in addition, the diagonal of *T* are non-negative real numbers.

The ?gges routine replaces the deprecated ?gegs routine.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobvsl
CHARACTER*1. Must be 'N' or 'V'.
If *jobvsl* = 'N', then the left Schur vectors are not computed.
If *jobvsl* = 'V', then the left Schur vectors are computed.

jobvsr
CHARACTER*1. Must be 'N' or 'V'.
If *jobvsr* = 'N', then the right Schur vectors are not computed.
If *jobvsr* = 'V', then the right Schur vectors are computed.

sort
CHARACTER*1. Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form.
If *sort* = 'N', then eigenvalues are not ordered.
If *sort* = 'S', eigenvalues are ordered (see *selctg*).

selctg
LOGICAL FUNCTION of three REAL arguments for real flavors.
LOGICAL FUNCTION of two COMPLEX arguments for complex flavors.
selctg must be declared EXTERNAL in the calling subroutine.
If *sort* = 'S', *selctg* is used to select eigenvalues to sort to the top left of the Schur form.
If *sort* = 'N', *selctg* is not referenced.
For real flavors:

	An eigenvalue (*alphar*(j) + *alphai*(j))/*beta*(j) is selected if *selctg*(*alphar*(j), *alphai*(j), *beta*(j)) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy *selctg*(*alphar*(j), *alphai*(j), *beta*(j)) = .TRUE. after ordering. In this case *info* is set to *n*+2 . *For complex flavors*: An eigenvalue *alpha*(j) / *beta*(j) is selected if *selctg*(*alpha*(j), *beta*(j)) is true. Note that a selected complex eigenvalue may no longer satisfy *selctg*(*alpha*(j), *beta*(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case *info* is set to *n*+2 (see *info* below).
n	INTEGER. The order of the matrices *A*, *B*, *vsl*, and *vsr* (*n* ≥ 0).
a, *b*, *work*	REAL for sgges DOUBLE PRECISION for dgges COMPLEX for cgges DOUBLE COMPLEX for zgges. Arrays: *a*(*lda*,*) is an array containing the *n*-by-*n* matrix *A* (first of the pair of matrices). The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) is an array containing the *n*-by-*n* matrix *B* (second of the pair of matrices). The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldb	INTEGER. The leading dimension of the array *b*. Must be at least max(1, *n*).
ldvsl, *ldvsr*	INTEGER. The leading dimensions of the output matrices *vsl* and *vsr*, respectively. Constraints: *ldvsl* ≥ 1. If *jobvsl* = 'V', *ldvsl* ≥ max(1, *n*). *ldvsr* ≥ 1. If *jobvsr* = 'V', *ldvsr* ≥ max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. *lwork* ≥ max(1, 8*n*+16) for real flavors; *lwork* ≥ max(1, 2*n*) for complex flavors. For good performance, *lwork* must generally be larger. If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
rwork	REAL for cgges DOUBLE PRECISION for zgges Workspace array, DIMENSION at least max(1, 8*n*). This array is used in complex flavors only.
bwork	LOGICAL. Workspace array, DIMENSION at least max(1, *n*). Not referenced if *sort* = 'N'.

Output Parameters

a	On exit, this array has been overwritten by its generalized Schur form *S*.
b	On exit, this array has been overwritten by its generalized Schur form *T*.
sdim	INTEGER. If *sort* = 'N', *sdim*= 0. If *sort* = 'S', *sdim* is equal to the number of eigenvalues (after sorting) for which *selctg* is true. Note that for real flavors complex conjugate pairs for which *selctg* is true for either eigenvalue count as 2.
alphar, *alphai*	REAL for sgges; DOUBLE PRECISION for dgges. Arrays, DIMENSION at least max(1, *n*) each. Contain values that form generalized eigenvalues in real flavors. See *beta*.
alpha	COMPLEX for cgges; DOUBLE COMPLEX for zgges. Array, DIMENSION at least max(1, *n*). Contain values that form generalized eigenvalues in complex flavors. See *beta*.
beta	REAL for sgges DOUBLE PRECISION for dgges COMPLEX for cgges DOUBLE COMPLEX for zgges. Array, DIMENSION at least max(1, *n*). *For real flavors*: On exit, (*alphar*(j) + *alphai*(j)*i)/*beta*(j), j=1,..., *n*, will be the generalized eigenvalues. *alphar*(j) + *alphai*(j)*i and *beta*(j), j=1,..., *n* are the diagonals of the complex Schur form (*S*,*T*) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (*A*,*B*) were further reduced to triangular form using complex unitary transformations. If *alphai*(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with *alphai*(j+1) negative. *For complex flavors:* On exit, *alpha*(j)/*beta*(j), j=1,..., *n*, will be the generalized eigenvalues. *alpha*(j), j=1,...,*n*, and *beta*(j), j=1,..., *n* are the diagonals of the complex Schur form (*S*,*T*) output by cgges/zgges. The *beta*(j) will be non-negative real. See also *Application Notes* below.
vsl, *vsr*	REAL for sgges DOUBLE PRECISION for dgges COMPLEX for cgges DOUBLE COMPLEX for zgges. Arrays: *vsl*(*ldvsl*,*), the second dimension of *vsl* must be at least max(1, *n*). If *jobvsl* = 'V', this array will contain the left Schur vectors. If *jobvsl* = 'N', *vsl* is not referenced. *vsr*(*ldvsr*,*), the second dimension of *vsr* must be at least max(1, *n*). If *jobvsr* = 'V', this array will contain the right Schur vectors. If *jobvsr* = 'N', *vsr* is not referenced.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.

info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*: the *QZ* iteration failed. (*A*, *B*) is not in Schur form, but *alphar*(j), *alphai*(j) (for real flavors), or *alpha*(j) (for complex flavors), and *beta*(j), j=*info* +1,..., *n* should be correct. *i* > *n*: errors that usually indicate LAPACK problems: *i* = *n*+1: other than *QZ* iteration failed in hgeqz; *i* = *n*+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the generalized Schur form no longer satisfy *selctg* = .TRUE.. This could also be caused due to scaling; *i* = *n*+3: reordering failed in tgsen.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine gges interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).
alphar	Holds the vector of length *n*. Used in real flavors only.
alphai	Holds the vector of length *n*. Used in real flavors only.
alpha	Holds the vector of length *n*. Used in complex flavors only.
beta	Holds the vector of length *n*.
vsl	Holds the matrix *VSL* of size (*n*, *n*).
vsr	Holds the matrix *VSR* of size (*n*, *n*).
jobvsl	Restored based on the presence of the argument *vsl* as follows: *jobvsl* = 'V', if *vsl* is present, *jobvsl* = 'N', if *vsl* is omitted.
jobvsr	Restored based on the presence of the argument *vsr* as follows: *jobvsr* = 'V', if *vsr* is present, *jobvsr* = 'N', if *vsr* is omitted.
sort	Restored based on the presence of the argument *select* as follows: *sort* = 'S', if *select* is present, *sort* = 'N', if *select* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The quotients *alphar*(j)/*beta*(j) and *alphai*(j)/*beta*(j) may easily over- or underflow, and *beta*(j) may even be zero. Thus, you should avoid simply computing the ratio. However, *alphar* and *alphai* will be always less than and usually comparable with norm(*A*) in magnitude, and *beta* always less than and usually comparable with norm(*B*).

?ggesx

Computes the generalized eigenvalues, Schur form, and, optionally, the left and/or right matrices of Schur vectors.

Syntax

Fortran 77:

```
call sggesx (jobvsl, jobvsr, sort, selctg, sense, n, a, lda, b, ldb, sdim, alphar, alphai, beta, vsl, ldvsl, vsr, ldvsr, rconde, rcondv, work, lwork, iwork, liwork, bwork, info)
```

```
call dggesx (jobvsl, jobvsr, sort, selctg, sense, n, a, lda, b, ldb, sdim, alphar, alphai, beta, vsl, ldvsl, vsr, ldvsr, rconde, rcondv, work, lwork, iwork, liwork, bwork, info)
```

```
call cggesx (jobvsl, jobvsr, sort, selctg, sense, n, a, lda, b, ldb, sdim, alpha, beta, vsl, ldvsl, vsr, ldvsr, rconde, rcondv, work, lwork, rwork, iwork, liwork, bwork, info)
```

```
call zggesx (jobvsl, jobvsr, sort, selctg, sense, n, a, lda, b, ldb, sdim, alpha, beta, vsl, ldvsl, vsr, ldvsr, rconde, rcondv, work, lwork, rwork, iwork, liwork, bwork, info)
```

Fortran 95:

```
call ggesx(a, b, alphar, alphai, beta [,vsl] [,vsr] [,select] [,sdim] [,rconde] [,rcondv] [,info])
```

```
call ggesx(a, b, alpha, beta [, vsl] [,vsr] [,select] [,sdim] [,rconde] [,rcondv] [,info])
```

C:

```
lapack_int LAPACKE_sggesx( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_S_SELECT3 select, char sense, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, lapack_int* sdim, float* alphar, float* alphai, float* beta, float* vsl, lapack_int ldvsl, float* vsr, lapack_int ldvsr, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dggesx( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_D_SELECT3 select, char sense, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, lapack_int* sdim, double* alphar, double* alphai, double* beta, double* vsl, lapack_int ldvsl, double* vsr, lapack_int ldvsr, double* rconde, double* rcondv );
```

```
lapack_int LAPACKE_cggesx( int matrix_order, char jobvsl, char jobvsr, char sort, LAPACK_C_SELECT2 select, char sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, lapack_int* sdim, lapack_complex_float* alpha, lapack_complex_float* beta, lapack_complex_float* vsl, lapack_int ldvsl, lapack_complex_float* vsr, lapack_int ldvsr, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_zggesx( int matrix_order, char jobvsl, char jobvsr, char sort,
LAPACK_Z_SELECT2 select, char sense, lapack_int n, lapack_complex_double* a, lapack_int
lda, lapack_complex_double* b, lapack_int ldb, lapack_int* sdim, lapack_complex_double*
alpha, lapack_complex_double* beta, lapack_complex_double* vsl, lapack_int ldvsl,
lapack_complex_double* vsr, lapack_int ldvsr, double* rconde, double* rcondv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes for a pair of *n*-by-*n* real/complex nonsymmetric matrices (*A*,*B*), the generalized eigenvalues, the generalized real/complex Schur form (*S*,*T*), optionally, the left and/or right matrices of Schur vectors (*vsl* and *vsr*). This gives the generalized Schur factorization

$(A,B) = (vsl*S*vsr^H, vsl*T*vsr^H)$

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix *S* and the upper triangular matrix *T*; computes a reciprocal condition number for the average of the selected eigenvalues (*rconde*); and computes a reciprocal condition number for the right and left deflating subspaces corresponding to the selected eigenvalues (*rcondv*). The leading columns of *vsl* and *vsr* then form an orthonormal/unitary basis for the corresponding left and right eigenspaces (deflating subspaces).

A generalized eigenvalue for a pair of matrices (*A*,*B*) is a scalar *w* or a ratio *alpha / beta = w*, such that *A - w*B* is singular. It is usually represented as the pair (*alpha*, *beta*), as there is a reasonable interpretation for *beta*=0 or for both being zero. A pair of matrices (*S*,*T*) is in generalized real Schur form if *T* is upper triangular with non-negative diagonal and *S* is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of *S* will be "standardized" by making the corresponding elements of *T* have the form:

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

and the pair of corresponding 2-by-2 blocks in *S* and *T* will have a complex conjugate pair of generalized eigenvalues. A pair of matrices (*S*,*T*) is in generalized complex Schur form if *S* and *T* are upper triangular and, in addition, the diagonal of *T* are non-negative real numbers.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobvsl	CHARACTER*1. Must be 'N' or 'V'. If *jobvsl* = 'N', then the left Schur vectors are not computed. If *jobvsl* = 'V', then the left Schur vectors are computed.
jobvsr	CHARACTER*1. Must be 'N' or 'V'. If *jobvsr* = 'N', then the right Schur vectors are not computed. If *jobvsr* = 'V', then the right Schur vectors are computed.

sort	CHARACTER*1. Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. If *sort* = 'N', then eigenvalues are not ordered. If *sort* = 'S', eigenvalues are ordered (see *selctg*).
selctg	LOGICAL FUNCTION of three REAL arguments for real flavors. LOGICAL FUNCTION of two COMPLEX arguments for complex flavors. *selctg* must be declared EXTERNAL in the calling subroutine. If *sort* = 'S', *selctg* is used to select eigenvalues to sort to the top left of the Schur form. If *sort* = 'N', *selctg* is not referenced. *For real flavors*: An eigenvalue (*alphar*(j) + *alphai*(j))/*beta*(j) is selected if *selctg*(*alphar*(j), *alphai*(j), *beta*(j)) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy *selctg*(*alphar*(j), *alphai*(j), *beta*(j)) = .TRUE. after ordering. In this case *info* is set to *n*+2. *For complex flavors*: An eigenvalue *alpha*(j) / *beta*(j) is selected if *selctg*(*alpha*(j), *beta*(j)) is true. Note that a selected complex eigenvalue may no longer satisfy *selctg*(*alpha*(j), *beta*(j)) = .TRUE. after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case *info* is set to *n*+2 (see *info* below).
sense	CHARACTER*1. Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed. If *sense* = 'N', none are computed; If *sense* = 'E', computed for average of selected eigenvalues only; If *sense* = 'V', computed for selected deflating subspaces only; If *sense* = 'B', computed for both. If *sense* is 'E', 'V', or 'B', then *sort* must equal 'S'.
n	INTEGER. The order of the matrices *A*, *B*, *vsl*, and *vsr* (*n* ≥ 0).
a, *b*, *work*	REAL for sggesx DOUBLE PRECISION for dggesx COMPLEX for cggesx DOUBLE COMPLEX for zggesx. Arrays: *a*(*lda*,*) is an array containing the *n*-by-*n* matrix *A* (first of the pair of matrices). The second dimension of *a* must be at least max(1, *n*). *b*(*ldb*,*) is an array containing the *n*-by-*n* matrix *B* (second of the pair of matrices). The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldb	INTEGER. The leading dimension of the array *b*. Must be at least max(1, *n*).
ldvsl, *ldvsr*	INTEGER. The leading dimensions of the output matrices *vsl* and *vsr*, respectively. Constraints: *ldvsl* ≥ 1. If *jobvsl* = 'V', *ldvsl* ≥ max(1, *n*).

ldvsr ≥ 1. If *jobvsr* = 'V', *ldvsr* ≥ max(1, *n*).

lwork

INTEGER.
The dimension of the array *work*.
For real flavors:
If *n*=0 then *lwork*≥1.
If *n*>0 and *sense* = 'N', then *lwork* ≥ max(8**n*, 6**n*+16).
If *n*>0 and *sense* = 'E', 'V', or 'B', then *lwork* ≥ max(8**n*, 6**n*+16, 2**sdim**(*n*-*sdim*));
For complex flavors:
If *n*=0 then *lwork*≥1.
If *n*>0 and *sense* = 'N', then *lwork* ≥ max(1, 2**n*);
If *n*>0 and *sense* = 'E', 'V', or 'B', then *lwork* ≥ max(1, 2**n*, 2**sdim**(*n*-*sdim*)).
Note that 2**sdim**(*n*-*sdim*) ≤ *n***n*/2.
An error is only returned if *lwork* < max(8**n*, 6**n*+16) for real flavors, and *lwork* < max(1, 2**n*) for complex flavors, but if *sense* = 'E', 'V', or 'B', this may not be large enough.
If *lwork*=-1, then a workspace query is assumed; the routine only calculates the bound on the optimal size of the *work* array and the minimum size of the *iwork* array, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla.

rwork

REAL for cggesx
DOUBLE PRECISION for zggesx
Workspace array, DIMENSION at least max(1, 8*n*).
This array is used in complex flavors only.

iwork

INTEGER.
Workspace array, DIMENSION max(1, *liwork*).

liwork

INTEGER.
The dimension of the array *iwork*.
If *sense* = 'N', or *n*=0, then *liwork*≥1,
otherwise *liwork* ≥ (*n*+6) for real flavors, and *liwork* ≥ (*n*+2) for complex flavors.
If *liwork*=-1, then a workspace query is assumed; the routine only calculates the bound on the optimal size of the *work* array and the minimum size of the *iwork* array, returns these values as the first entries of the *work* and *iwork* arrays, and no error message related to *lwork* or *liwork* is issued by xerbla.

bwork

LOGICAL.
Workspace array, DIMENSION at least max(1, *n*).
Not referenced if *sort* = 'N'.

Output Parameters

a

On exit, this array has been overwritten by its generalized Schur form *S*.

b

On exit, this array has been overwritten by its generalized Schur form *T*.

sdim

INTEGER.
If *sort* = 'N', *sdim*= 0.
If *sort* = 'S', *sdim* is equal to the number of eigenvalues (after sorting) for which *selctg* is true.

Note that for real flavors complex conjugate pairs for which *selctg* is true for either eigenvalue count as 2.

alphar, *alphai*
REAL for sggesx;
DOUBLE PRECISION for dggesx.
Arrays, DIMENSION at least max(1, *n*) each. Contain values that form generalized eigenvalues in real flavors.
See *beta*.

alpha
COMPLEX for cggesx;
DOUBLE COMPLEX for zggesx.
Array, DIMENSION at least max(1, *n*). Contain values that form generalized eigenvalues in complex flavors. See *beta*.

beta
REAL for sggesx
DOUBLE PRECISION for dggesx
COMPLEX for cggesx
DOUBLE COMPLEX for zggesx.
Array, DIMENSION at least max(1, *n*).
For real flavors:
On exit, (*alphar*(j) + *alphai*(j)*i)/*beta*(j), j=1,..., *n* will be the generalized eigenvalues.
alphar(j) + *alphai*(j)*i and *beta*(j), j=1,..., *n* are the diagonals of the complex Schur form (*S*,*T*) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (*A*,*B*) were further reduced to triangular form using complex unitary transformations. If *alphai*(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with *alphai*(j+1) negative.
For complex flavors:
On exit, *alpha*(j)/*beta*(j), j=1,..., *n* will be the generalized eigenvalues.
alpha(j), j=1,..., *n*, and *beta*(j), j=1,...,*n* are the diagonals of the complex Schur form (*S*,*T*) output by cggesx/zggesx. The *beta*(j) will be non-negative real.
See also *Application Notes* below.

vsl, *vsr*
REAL for sggesx
DOUBLE PRECISION for dggesx
COMPLEX for cggesx
DOUBLE COMPLEX for zggesx.
Arrays:
vsl(*ldvsl*,*), the second dimension of *vsl* must be at least max(1, *n*).
If *jobvsl* = 'V', this array will contain the left Schur vectors.
If *jobvsl* = 'N', *vsl* is not referenced.
vsr(*ldvsr*,*), the second dimension of *vsr* must be at least max(1, *n*).
If *jobvsr* = 'V', this array will contain the right Schur vectors.
If *jobvsr* = 'N', *vsr* is not referenced.

rconde, *rcondv*
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Arrays, DIMENSION (2) each
If *sense* = 'E' or 'B', *rconde*(1) and *rconde*(2) contain the reciprocal condition numbers for the average of the selected eigenvalues.
Not referenced if *sense* = 'N' or 'V'.
If *sense* = 'V' or 'B', *rcondv*(1) and *rcondv*(2) contain the reciprocal condition numbers for the selected deflating subspaces.
Not referenced if *sense* = 'N' or 'E'.

work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
iwork(1)	On exit, if *info* = 0, then *iwork(1)* returns the required minimal size of *liwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*: the *QZ* iteration failed. (*A*, *B*) is not in Schur form, but *alphar*(j), *alphai*(j) (for real flavors), or *alpha*(j) (for complex flavors), and *beta*(j), j=*info* +1,..., *n* should be correct. *i* > *n*: errors that usually indicate LAPACK problems: *i* = *n*+1: other than *QZ* iteration failed in ?hgeqz; *i* = *n*+2: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the generalized Schur form no longer satisfy *selctg* = .TRUE.. This could also be caused due to scaling; *i* = *n*+3: reordering failed in tgsen.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggesx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).
alphar	Holds the vector of length *n*. Used in real flavors only.
alphai	Holds the vector of length *n*. Used in real flavors only.
alpha	Holds the vector of length *n*. Used in complex flavors only.
beta	Holds the vector of length *n*.
vsl	Holds the matrix *VSL* of size (*n*, *n*).
vsr	Holds the matrix *VSR* of size (*n*, *n*).
rconde	Holds the vector of length (2).
rcondv	Holds the vector of length (2).
jobvsl	Restored based on the presence of the argument *vsl* as follows: *jobvsl* = 'V', if *vsl* is present, *jobvsl* = 'N', if *vsl* is omitted.
jobvsr	Restored based on the presence of the argument *vsr* as follows: *jobvsr* = 'V', if *vsr* is present, *jobvsr* = 'N', if *vsr* is omitted.
sort	Restored based on the presence of the argument *select* as follows: *sort* = 'S', if *select* is present, *sort* = 'N', if *select* is omitted.
sense	Restored based on the presence of arguments *rconde* and rcondv as follows: *sense* = 'B', if both *rconde* and *rcondv* are present, *sense* = 'E', if *rconde* is present and *rcondv* omitted, *sense* = 'V', if *rconde* is omitted and *rcondv* present, *sense* = 'N', if both *rconde* and *rcondv* are omitted.

Note that there will be an error condition if *rconde* or *rcondv* are present and *select* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* (or *liwork*) for the first run or set *lwork* = –1 (*liwork* = –1).

If you choose the first option and set any of admissible *lwork* (or *liwork*) sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*) on exit. Use this value (*work*(1), *iwork*(1)) for subsequent runs.

If you set *lwork* = –1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*, *iwork*). This operation is called a workspace query.

Note that if you set *lwork* (*liwork*) to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The quotients *alphar*(j)/*beta*(j) and *alphai*(j)/*beta*(j) may easily over- or underflow, and *beta*(j) may even be zero. Thus, you should avoid simply computing the ratio. However, *alphar* and *alphai* will be always less than and usually comparable with norm(*A*) in magnitude, and *beta* always less than and usually comparable with norm(*B*).

?ggev

Computes the generalized eigenvalues, and the left and/or right generalized eigenvectors for a pair of nonsymmetric matrices.

Syntax

Fortran 77:

```
call sggev(jobvl, jobvr, n, a, lda, b, ldb, alphar, alphai, beta, vl, ldvl, vr, ldvr, work, lwork, info)
```

```
call dggev(jobvl, jobvr, n, a, lda, b, ldb, alphar, alphai, beta, vl, ldvl, vr, ldvr, work, lwork, info)
```

```
call cggev(jobvl, jobvr, n, a, lda, b, ldb, alpha, beta, vl, ldvl, vr, ldvr, work, lwork, rwork, info)
```

```
call zggev(jobvl, jobvr, n, a, lda, b, ldb, alpha, beta, vl, ldvl, vr, ldvr, work, lwork, rwork, info)
```

Fortran 95:

```
call ggev(a, b, alphar, alphai, beta [,vl] [,vr] [,info])
```

```
call ggev(a, b, alpha, beta [, vl] [,vr] [,info])
```

C:

```
lapack_int LAPACKE_sggev( int matrix_order, char jobvl, char jobvr, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float* alphar, float* alphai, float* beta, float* vl, lapack_int ldvl, float* vr, lapack_int ldvr );
```

```
lapack_int LAPACKE_dggev( int matrix_order, char jobvl, char jobvr, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double* alphar, double* alphai, double* beta, double* vl, lapack_int ldvl, double* vr, lapack_int ldvr );
```

```
lapack_int LAPACKE_cggev( int matrix_order, char jobvl, char jobvr, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* alpha, lapack_complex_float* beta, lapack_complex_float* vl,
lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr );

lapack_int LAPACKE_zggev( int matrix_order, char jobvl, char jobvr, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* alpha, lapack_complex_double* beta, lapack_complex_double* vl,
lapack_int ldvl, lapack_complex_double* vr, lapack_int ldvr );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The `?ggev` routine computes the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors for a pair of n-by-n real/complex nonsymmetric matrices (A,B).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar λ or a ratio $alpha / beta = \lambda$, such that $A - \lambda * B$ is singular. It is usually represented as the pair ($alpha$, $beta$), as there is a reasonable interpretation for $beta = 0$ and even for both being zero.

The right generalized eigenvector $v(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies

$A * v(j) = \lambda(j) * B * v(j)$.

The left generalized eigenvector $u(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies

$u(j)^H * A = \lambda(j) * u(j)^H * B$

where $u(j)^H$ denotes the conjugate transpose of $u(j)$.

The `?ggev` routine replaces the deprecated `?gegv` routine.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

jobvl	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobvl* = `'N'`, the left generalized eigenvectors are not computed; If *jobvl* = `'V'`, the left generalized eigenvectors are computed.
jobvr	`CHARACTER*1`. Must be `'N'` or `'V'`. If *jobvr* = `'N'`, the right generalized eigenvectors are not computed; If *jobvr* = `'V'`, the right generalized eigenvectors are computed.
n	`INTEGER`. The order of the matrices *A*, *B*, *vl*, and *vr* ($n \geq 0$).
a, *b*, *work*	`REAL` for `sggev` `DOUBLE PRECISION` for `dggev` `COMPLEX` for `cggev` `DOUBLE COMPLEX` for `zggev`. Arrays: *a*(*lda*,*) is an array containing the n-by-n matrix *A* (first of the pair of matrices). The second dimension of *a* must be at least max(1, n).

	b(*ldb*,*) is an array containing the *n*-by-*n* matrix *B* (second of the pair of matrices). The second dimension of *b* must be at least max(1, *n*). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, *n*).
ldb	INTEGER. The leading dimension of the array *b*. Must be at least max(1, *n*).
ldvl, *ldvr*	INTEGER. The leading dimensions of the output matrices *vl* and *vr*, respectively. Constraints: *ldvl* ≥ 1. If *jobvl* = 'V', *ldvl* ≥ max(1, *n*). *ldvr* ≥ 1. If *jobvr* = 'V', *ldvr* ≥ max(1, *n*).
lwork	INTEGER. The dimension of the array *work*. *lwork* ≥ max(1, 8*n*+16) for real flavors; *lwork* ≥ max(1, 2*n*) for complex flavors. For good performance, *lwork* must generally be larger. If *lwork* = −1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.
rwork	REAL for cggev DOUBLE PRECISION for zggev Workspace array, DIMENSION at least max(1, 8*n*). This array is used in complex flavors only.

Output Parameters

a, *b*	On exit, these arrays have been overwritten.
alphar, *alphai*	REAL for sggev; DOUBLE PRECISION for dggev. Arrays, DIMENSION at least max(1, *n*) each. Contain values that form generalized eigenvalues in real flavors. See *beta*.
alpha	COMPLEX for cggev; DOUBLE COMPLEX for zggev. Array, DIMENSION at least max(1, *n*). Contain values that form generalized eigenvalues in complex flavors. See *beta*.
beta	REAL for sggev DOUBLE PRECISION for dggev COMPLEX for cggev DOUBLE COMPLEX for zggev. Array, DIMENSION at least max(1, *n*). *For real flavors*: On exit, (*alphar*(j)+ *alphai*(j)*i)/*beta*(j), j=1,..., *n*, are the generalized eigenvalues. If *alphai*(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with *alphai*(j+1) negative. *For complex flavors:* On exit, *alpha*(j)/*beta*(j), j=1,..., *n*, are the generalized eigenvalues. See also *Application Notes* below.

vl, *vr*

REAL for sggev
DOUBLE PRECISION for dggev
COMPLEX for cggev
DOUBLE COMPLEX for zggev.
Arrays:
vl(*ldvl*,*); the second dimension of *vl* must be at least max(1, *n*).
If *jobvl* = 'V', the left generalized eigenvectors *u*(j) are stored one after another in the columns of *vl*, in the same order as their eigenvalues. Each eigenvector is scaled so the largest component has abs(Re) + abs(Im) = 1.
If *jobvl* = 'N', *vl* is not referenced.
For real flavors:
If the j-th eigenvalue is real, then *u*(j) = *vl*(:,j), the j-th column of *vl*.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then *u*(j) = *vl*(:,j) + *i***vl*(:,j+1) and *u*(j+1) = *vl*(:,j) – *i***vl*(:,j+1), where *i* = sqrt(–1).
For complex flavors:
u(j) = *vl*(:,j), the j-th column of *vl*.
vr(*ldvr*,*); the second dimension of *vr* must be at least max(1, *n*).
If *jobvr* = 'V', the right generalized eigenvectors *v*(j) are stored one after another in the columns of *vr*, in the same order as their eigenvalues. Each eigenvector is scaled so the largest component has abs(Re) + abs(Im) = 1.
If *jobvr* = 'N', *vr* is not referenced.
For real flavors:
If the j-th eigenvalue is real, then *v*(j) = *vr*(:,j), the j-th column of *vr*.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then *v*(j) = *vr*(:,j) + *i***vr*(:,j+1) and *v*(j+1) = *vr*(:,j) – *i***vr*(:,j+1).
For complex flavors:
v(j) = *vr*(:,j), the j-th column of *vr*.

work(1)

On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.

info

INTEGER.
If *info* = 0, the execution is successful.
If *info* = –*i*, the *i*-th parameter had an illegal value.
If *info* = *i*, and
i ≤ *n*: the *QZ* iteration failed. No eigenvectors have been calculated, but *alphar*(j), *alphai*(j) (for real flavors), or *alpha*(j) (for complex flavors), and *beta*(j), j=*info*+1,..., *n* should be correct.
i > *n*: errors that usually indicate LAPACK problems:
i = *n*+1: other than *QZ* iteration failed in hgeqz;
i = *n*+2: error return from tgevc.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggev interface are the following:

a

Holds the matrix *A* of size (*n*, *n*).

b

Holds the matrix *B* of size (*n*, *n*).

alphar	Holds the vector of length *n*. Used in real flavors only.
alphai	Holds the vector of length *n*. Used in real flavors only.
alpha	Holds the vector of length *n*. Used in complex flavors only.
beta	Holds the vector of length *n*.
vl	Holds the matrix *VL* of size (*n*, *n*).
vr	Holds the matrix *VR* of size (*n*, *n*).
jobvl	Restored based on the presence of the argument *vl* as follows: *jobvl* = 'V', if *vl* is present, *jobvl* = 'N', if *vl* is omitted.
jobvr	Restored based on the presence of the argument *vr* as follows: *jobvr* = 'V', if *vr* is present, *jobvr* = 'N', if *vr* is omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The quotients *alphar*(j)/*beta*(j) and *alphai*(j)/*beta*(j) may easily over- or underflow, and *beta*(j) may even be zero. Thus, you should avoid simply computing the ratio. However, *alphar* and *alphai* (for real flavors) or *alpha* (for complex flavors) will be always less than and usually comparable with norm(*A*) in magnitude, and *beta* always less than and usually comparable with norm(*B*).

?ggevx

Computes the generalized eigenvalues, and, optionally, the left and/or right generalized eigenvectors.

Syntax

Fortran 77:

```
call sggevx(balanc, jobvl, jobvr, sense, n, a, lda, b, ldb, alphar, alphai, beta, vl,
ldvl, vr, ldvr, ilo, ihi, lscale, rscale, abnrm, bbnrm, rconde, rcondv, work, lwork,
iwork, bwork, info)
```

```
call dggevx(balanc, jobvl, jobvr, sense, n, a, lda, b, ldb, alphar, alphai, beta, vl,
ldvl, vr, ldvr, ilo, ihi, lscale, rscale, abnrm, bbnrm, rconde, rcondv, work, lwork,
iwork, bwork, info)
```

```
call cggevx(balanc, jobvl, jobvr, sense, n, a, lda, b, ldb, alpha, beta, vl, ldvl, vr,
ldvr, ilo, ihi, lscale, rscale, abnrm, bbnrm, rconde, rcondv, work, lwork, rwork,
iwork, bwork, info)
```

```
call zggevx(balanc, jobvl, jobvr, sense, n, a, lda, b, ldb, alpha, beta, vl, ldvl, vr,
ldvr, ilo, ihi, lscale, rscale, abnrm, bbnrm, rconde, rcondv, work, lwork, rwork,
iwork, bwork, info)
```

Fortran 95:

```
call ggevx(a, b, alphar, alphai, beta [,vl] [,vr] [,balanc] [,ilo] [,ihi] [, lscale]
[,rscale] [,abnrm] [,bbnrm] [,rconde] [,rcondv] [,info])

call ggevx(a, b, alpha, beta [, vl] [,vr] [,balanc] [,ilo] [,ihi] [,lscale] [, rscale]
[,abnrm] [,bbnrm] [,rconde] [,rcondv] [,info])
```

C:

```
lapack_int LAPACKE_sggevx( int matrix_order, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float*
alphar, float* alphai, float* beta, float* vl, lapack_int ldvl, float* vr, lapack_int
ldvr, lapack_int* ilo, lapack_int* ihi, float* lscale, float* rscale, float* abnrm,
float* bbnrm, float* rconde, float* rcondv );

lapack_int LAPACKE_dggevx( int matrix_order, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double*
alphar, double* alphai, double* beta, double* vl, lapack_int ldvl, double* vr,
lapack_int ldvr, lapack_int* ilo, lapack_int* ihi, double* lscale, double* rscale,
double* abnrm, double* bbnrm, double* rconde, double* rcondv );

lapack_int LAPACKE_cggevx( int matrix_order, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* alpha, lapack_complex_float* beta,
lapack_complex_float* vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr,
lapack_int* ilo, lapack_int* ihi, float* lscale, float* rscale, float* abnrm, float*
bbnrm, float* rconde, float* rcondv );

lapack_int LAPACKE_zggevx( int matrix_order, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double*
b, lapack_int ldb, lapack_complex_double* alpha, lapack_complex_double* beta,
lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int ldvr,
lapack_int* ilo, lapack_int* ihi, double* lscale, double* rscale, double* abnrm,
double* bbnrm, double* rconde, double* rcondv );
```

Include Files

- Fortran: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`
- C: `mkl_lapacke.h`

Description

The routine computes for a pair of n-by-n real/complex nonsymmetric matrices (A,B), the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (*ilo*, *ihi*, *lscale*, *rscale*, *abnrm*, and *bbnrm*), reciprocal condition numbers for the eigenvalues (*rconde*), and reciprocal condition numbers for the right eigenvectors (*rcondv*).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar λ or a ratio *alpha* / *beta* = λ, such that $A - \lambda * B$ is singular. It is usually represented as the pair (*alpha*, *beta*), as there is a reasonable interpretation for *beta*=0 and even for both being zero. The right generalized eigenvector $v(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies

$A*v(j) = \lambda(j)*B*v(j)$.

The left generalized eigenvector $u(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies

$u(j)^H*A = \lambda(j)*u(j)^H*B$

where $u(j)^H$ denotes the conjugate transpose of $u(j)$.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type definitions.

balanc	CHARACTER*1. Must be 'N', 'P', 'S', or 'B'. Specifies the balance option to be performed. If *balanc* = 'N', do not diagonally scale or permute; If *balanc* = 'P', permute only; If *balanc* = 'S', scale only; If *balanc* = 'B', both permute and scale. Computed reciprocal condition numbers will be for the matrices after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.
jobvl	CHARACTER*1. Must be 'N' or 'V'. If *jobvl* = 'N', the left generalized eigenvectors are not computed; If *jobvl* = 'V', the left generalized eigenvectors are computed.
jobvr	CHARACTER*1. Must be 'N' or 'V'. If *jobvr* = 'N', the right generalized eigenvectors are not computed; If *jobvr* = 'V', the right generalized eigenvectors are computed.
sense	CHARACTER*1. Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed. If *sense* = 'N', none are computed; If *sense* = 'E', computed for eigenvalues only; If *sense* = 'V', computed for eigenvectors only; If *sense* = 'B', computed for eigenvalues and eigenvectors.
n	INTEGER. The order of the matrices A, B, vl, and vr ($n \geq 0$).
a, *b*, *work*	REAL for sggevx DOUBLE PRECISION for dggevx COMPLEX for cggevx DOUBLE COMPLEX for zggevx. Arrays: *a*(*lda*,*) is an array containing the n-by-n matrix A (first of the pair of matrices). The second dimension of *a* must be at least max(1, n). *b*(*ldb*,*) is an array containing the n-by-n matrix B (second of the pair of matrices). The second dimension of *b* must be at least max(1, n). *work* is a workspace array, its dimension max(1, *lwork*).
lda	INTEGER. The leading dimension of the array *a*. Must be at least max(1, n).
ldb	INTEGER. The leading dimension of the array *b*. Must be at least max(1, n).
ldvl, *ldvr*	INTEGER. The leading dimensions of the output matrices vl and vr, respectively.

Constraints:
ldvl ≥ 1. If *jobvl* = 'V', *ldvl* ≥ max(1, *n*).
ldvr ≥ 1. If *jobvr* = 'V', *ldvr* ≥ max(1, *n*).

lwork — INTEGER.
The dimension of the array *work*. *lwork* ≥ max(1, 2*n);
For real flavors:
If *balanc* = 'S', or 'B', or *jobvl* = 'V', or *jobvr* = 'V', then *lwork* ≥ max(1, 6*n);
if *sense* = 'E', or 'B', then *lwork* ≥ max(1, 10*n);
if *sense* = 'V', or 'B', *lwork* ≥ $(2n^2+ 8*n+16)$.
For complex flavors:
if *sense* = 'E', *lwork* ≥ max(1, 4*n);
if *sense* = 'V', or 'B', *lwork* ≥max(1, $2*n^2+ 2*n$).
If *lwork* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *work* array, returns this value as the first entry of the *work* array, and no error message related to *lwork* is issued by xerbla.

rwork — REAL for cggevx
DOUBLE PRECISION for zggevx
Workspace array, DIMENSION at least max(1, 6*n) if *balanc* = 'S', or 'B', and at least max(1, 2*n) otherwise.
This array is used in complex flavors only.

iwork — INTEGER.
Workspace array, DIMENSION at least (*n*+6) for real flavors and at least (*n* +2) for complex flavors.
Not referenced if *sense* = 'E'.

bwork — LOGICAL. Workspace array, DIMENSION at least max(1, *n*).
Not referenced if *sense* = 'N'.

Output Parameters

a, *b* — On exit, these arrays have been overwritten.
If *jobvl* = 'V' or *jobvr* = 'V' or both, then *a* contains the first part of the real Schur form of the "balanced" versions of the input *A* and *B*, and *b* contains its second part.

alphar, *alphai* — REAL for sggevx;
DOUBLE PRECISION for dggevx.
Arrays, DIMENSION at least max(1, *n*) each. Contain values that form generalized eigenvalues in real flavors.
See *beta*.

alpha — COMPLEX for cggevx;
DOUBLE COMPLEX for zggevx.
Array, DIMENSION at least max(1, *n*). Contain values that form generalized eigenvalues in complex flavors. See *beta*.

beta — REAL for sggevx
DOUBLE PRECISION for dggevx
COMPLEX for cggevx
DOUBLE COMPLEX for zggevx.
Array, DIMENSION at least max(1, *n*).
For real flavors:
On exit, (*alphar*(j) + *alphai*(j)*i)/*beta*(j), j=1,..., *n*, will be the generalized eigenvalues.

If *alphai*(j) is zero, then the j-th eigenvalue is real; if positive, then the j-th and (j+1)-st eigenvalues are a complex conjugate pair, with *alphai*(j+1) negative.
For complex flavors:
On exit, *alpha*(j)/*beta*(j), j=1,..., *n*, will be the generalized eigenvalues.
See also *Application Notes* below.

vl, *vr*

REAL for sggevx
DOUBLE PRECISION for dggevx
COMPLEX for cggevx
DOUBLE COMPLEX for zggevx.
Arrays:
vl(*ldvl*,*); the second dimension of *vl* must be at least max(1, *n*).
If *jobvl* = 'V', the left generalized eigenvectors *u*(j) are stored one after another in the columns of *vl*, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component have abs(Re) + abs(Im) = 1.
If *jobvl* = 'N', *vl* is not referenced.
For real flavors:
If the j-th eigenvalue is real, then *u*(j) = *vl*(:,j), the j-th column of *vl*.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then *u*(j) = *vl*(:,j) + *i***vl*(:,j+1) and *u*(j+1) = *vl*(:,j) – *i***vl*(:,j+1), where *i* = sqrt(-1).
For complex flavors:
u(j) = *vl*(:,j), the j-th column of *vl*.
vr(*ldvr*,*); the second dimension of *vr* must be at least max(1, *n*).
If *jobvr* = 'V', the right generalized eigenvectors *v*(j) are stored one after another in the columns of *vr*, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component have abs(Re) + abs(Im) = 1.
If *jobvr* = 'N', *vr* is not referenced.
For real flavors:
If the j-th eigenvalue is real, then *v*(j) = *vr*(:,j), the j-th column of *vr*.
If the j-th and (j+1)-st eigenvalues form a complex conjugate pair, then *v*(j) = *vr*(:,j) + *i***vr*(:,j+1) and *v*(j+1) = *vr*(:,j) – *i***vr*(:,j+1).
For complex flavors:
v(j) = *vr*(:,j), the j-th column of *vr*.

ilo, *ihi*

INTEGER. *ilo* and *ihi* are integer values such that on exit *A*(i,j) = 0 and *B*(i,j) = 0 if i > j and j = 1,..., *ilo*-1 or i = *ihi*+1,..., *n*.
If *balanc* = 'N' or 'S', *ilo* = 1 and *ihi* = *n*.

lscale, *rscale*

REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Arrays, DIMENSION at least max(1, *n*) each.
lscale contains details of the permutations and scaling factors applied to the left side of *A* and *B*.
If *PL*(j) is the index of the row interchanged with row j, and *DL*(j) is the scaling factor applied to row j, then
lscale(j) = *PL*(j), for j = 1,..., *ilo*-1
= *DL*(j), for j = *ilo*,...,*ihi*
= *PL*(j) for j = *ihi*+1,..., *n*.
The order in which the interchanges are made is *n* to *ihi*+1, then 1 to *ilo*-1.

	rscale contains details of the permutations and scaling factors applied to the right side of *A* and *B*. If *PR*(j) is the index of the column interchanged with column j, and *DR*(j) is the scaling factor applied to column j, then *rscale*(j) = *PR*(j), for j = 1,..., *ilo*-1 = *DR*(j), for j = *ilo*,...,*ihi* = *PR*(j) for j = *ihi*+1,..., *n*. The order in which the interchanges are made is *n* to *ihi*+1, then 1 to *ilo*-1.
abnrm, *bbnrm*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. The one-norms of the balanced matrices *A* and *B*, respectively.
rconde, *rcondv*	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION at least max(1, *n*) each. If *sense* = 'E', or 'B', *rconde* contains the reciprocal condition numbers of the eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of *rconde* are set to the same value. Thus *rconde*(j), *rcondv*(j), and the j-th columns of *vl* and *vr* all correspond to the same eigenpair (but not in general the j-th eigenpair, unless all eigenpairs are selected). If *sense* = 'N', or 'V', *rconde* is not referenced. If *sense* = 'V', or 'B', *rcondv* contains the estimated reciprocal condition numbers of the eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of *rcondv* are set to the same value. If the eigenvalues cannot be reordered to compute *rcondv*(j), *rcondv*(j) is set to 0; this can only occur when the true value would be very small anyway. If *sense* = 'N', or 'E', *rcondv* is not referenced.
work(1)	On exit, if *info* = 0, then *work(1)* returns the required minimal size of *lwork*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i*, and *i* ≤ *n*: the *QZ* iteration failed. No eigenvectors have been calculated, but *alphar*(j), *alphai*(j) (for real flavors), or *alpha*(j) (for complex flavors), and *beta*(j), j=*info*+1,..., *n* should be correct. *i* > *n*: errors that usually indicate LAPACK problems: *i* = *n*+1: other than *QZ* iteration failed in hgeqz; *i* = *n*+2: error return from tgevc.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or restorable arguments, see Fortran 95 Interface Conventions.

Specific details for the routine ggevx interface are the following:

a	Holds the matrix *A* of size (*n*, *n*).
b	Holds the matrix *B* of size (*n*, *n*).

alphar	Holds the vector of length *n*. Used in real flavors only.
alphai	Holds the vector of length *n*. Used in real flavors only.
alpha	Holds the vector of length *n*. Used in complex flavors only.
beta	Holds the vector of length *n*.
vl	Holds the matrix *VL* of size (*n*, *n*).
vr	Holds the matrix *VR* of size (*n*, *n*).
lscale	Holds the vector of length *n*.
rscale	Holds the vector of length *n*.
rconde	Holds the vector of length *n*.
rcondv	Holds the vector of length *n*.
balanc	Must be 'N', 'B', or 'P'. The default value is 'N'.
jobvl	Restored based on the presence of the argument *vl* as follows: *jobvl* = 'V', if *vl* is present, *jobvl* = 'N', if *vl* is omitted.
jobvr	Restored based on the presence of the argument *vr* as follows: *jobvr* = 'V', if *vr* is present, *jobvr* = 'N', if *vr* is omitted.
sense	Restored based on the presence of arguments *rconde* and rcondv as follows: *sense* = 'B', if both *rconde* and *rcondv* are present, *sense* = 'E', if *rconde* is present and *rcondv* omitted, *sense* = 'V', if *rconde* is omitted and *rcondv* present, *sense* = 'N', if both *rconde* and *rcondv* are omitted.

Application Notes

If you are in doubt how much workspace to supply, use a generous value of *lwork* for the first run or set *lwork* = -1.

If you choose the first option and set any of admissible *lwork* sizes, which is no less than the minimal value described, the routine completes the task, though probably not so fast as with a recommended workspace, and provides the recommended workspace in the first element of the corresponding array *work* on exit. Use this value (*work*(1)) for subsequent runs.

If you set *lwork* = -1, the routine returns immediately and provides the recommended workspace in the first element of the corresponding array (*work*). This operation is called a workspace query.

Note that if you set *lwork* to less than the minimal required value and not -1, the routine returns immediately with an error exit and does not provide any information on the recommended workspace.

The quotients *alphar*(j)/*beta*(j) and *alphai*(j)/*beta*(j) may easily over- or underflow, and *beta*(j) may even be zero. Thus, you should avoid simply computing the ratio. However, *alphar* and *alphai* (for real flavors) or *alpha* (for complex flavors) will be always less than and usually comparable with norm(*A*) in magnitude, and *beta* always less than and usually comparable with norm(*B*).

LAPACK Auxiliary and Utility Routines

5

This chapter describes the Intel® Math Kernel Library implementation of LAPACK auxiliary and utility routines. The library includes auxiliary routines for both real and complex data.

Auxiliary Routines

Routine naming conventions, mathematical notation, and matrix storage schemes used for LAPACK auxiliary routines are the same as for the driver and computational routines described in previous chapters.

The table below summarizes information about the available LAPACK auxiliary routines.

LAPACK Auxiliary Routines

Routine Name	Data Types	Description
?lacgv	c, z	Conjugates a complex vector.
?lacrm	c, z	Multiplies a complex matrix by a square real matrix.
?lacrt	c, z	Performs a linear transformation of a pair of complex vectors.
?laesy	c, z	Computes the eigenvalues and eigenvectors of a 2-by-2 complex symmetric matrix.
?rot	c, z	Applies a plane rotation with real cosine and complex sine to a pair of complex vectors.
?spmv	c, z	Computes a matrix-vector product for complex vectors using a complex symmetric packed matrix
?spr	c, z	Performs the symmetrical rank-1 update of a complex symmetric packed matrix.
?symv	c, z	Computes a matrix-vector product for a complex symmetric matrix.
?syr	c, z	Performs the symmetric rank-1 update of a complex symmetric matrix.
i?max1	c, z	Finds the index of the vector element whose real part has maximum absolute value.
?sum1	sc, dz	Forms the 1-norm of the complex vector using the true absolute value.
?gbtf2	s, d, c, z	Computes the LU factorization of a general band matrix using the unblocked version of the algorithm.
?gebd2	s, d, c, z	Reduces a general matrix to bidiagonal form using an unblocked algorithm.
?gehd2	s, d, c, z	Reduces a general square matrix to upper Hessenberg form using an unblocked algorithm.
?gelq2	s, d, c, z	Computes the LQ factorization of a general rectangular matrix using an unblocked algorithm.

Routine Name	Data Types	Description
?geql2	s, d, c, z	Computes the QL factorization of a general rectangular matrix using an unblocked algorithm.
?geqr2	s, d, c, z	Computes the QR factorization of a general rectangular matrix using an unblocked algorithm.
?geqr2p	s, d, c, z	Computes the QR factorization of a general rectangular matrix with non-negative diagonal elements using an unblocked algorithm.
?gerq2	s, d, c, z	Computes the RQ factorization of a general rectangular matrix using an unblocked algorithm.
?gesc2	s, d, c, z	Solves a system of linear equations using the LU factorization with complete pivoting computed by ?getc2.
?getc2	s, d, c, z	Computes the LU factorization with complete pivoting of the general n-by-n matrix.
?getf2	s, d, c, z	Computes the LU factorization of a general *m*-by-*n* matrix using partial pivoting with row interchanges (unblocked algorithm).
?gtts2	s, d, c, z	Solves a system of linear equations with a tridiagonal matrix using the LU factorization computed by ?gttrf.
?isnan	s, d,	Tests input for NaN.
?laisnan	s, d,	Tests input for NaN by comparing itwo arguments for inequality.
?labrd	s, d, c, z	Reduces the first *nb* rows and columns of a general matrix to a bidiagonal form.
?lacn2	s, d, c, z	Estimates the 1-norm of a square matrix, using reverse communication for evaluating matrix-vector products.
?lacon	s, d, c, z	Estimates the 1-norm of a square matrix, using reverse communication for evaluating matrix-vector products.
?lacpy	s, d, c, z	Copies all or part of one two-dimensional array to another.
?ladiv	s, d, c, z	Performs complex division in real arithmetic, avoiding unnecessary overflow.
?lae2	s, d	Computes the eigenvalues of a 2-by-2 symmetric matrix.
?laebz	s, d	Computes the number of eigenvalues of a real symmetric tridiagonal matrix which are less than or equal to a given value, and performs other tasks required by the routine ?stebz.
?laed0	s, d, c, z	Used by ?stedc. Computes all eigenvalues and corresponding eigenvectors of an unreduced symmetric tridiagonal matrix using the divide and conquer method.
?laed1	s, d	Used by sstedc/dstedc. Computes the updated eigensystem of a diagonal matrix after modification by a rank-one symmetric matrix. Used when the original matrix is tridiagonal.
?laed2	s, d	Used by sstedc/dstedc. Merges eigenvalues and deflates secular equation. Used when the original matrix is tridiagonal.

Routine Name	Data Types	Description
?laed3	s, d	Used by sstedc/dstedc. Finds the roots of the secular equation and updates the eigenvectors. Used when the original matrix is tridiagonal.
?laed4	s, d	Used by sstedc/dstedc. Finds a single root of the secular equation.
?laed5	s, d	Used by sstedc/dstedc. Solves the 2-by-2 secular equation.
?laed6	s, d	Used by sstedc/dstedc. Computes one Newton step in solution of the secular equation.
?laed7	s, d, c, z	Used by ?stedc. Computes the updated eigensystem of a diagonal matrix after modification by a rank-one symmetric matrix. Used when the original matrix is dense.
?laed8	s, d, c, z	Used by ?stedc. Merges eigenvalues and deflates secular equation. Used when the original matrix is dense.
?laed9	s, d	Used by sstedc/dstedc. Finds the roots of the secular equation and updates the eigenvectors. Used when the original matrix is dense.
?laeda	s, d	Used by ?stedc. Computes the Z vector determining the rank-one modification of the diagonal matrix. Used when the original matrix is dense.
?laein	s, d, c, z	Computes a specified right or left eigenvector of an upper Hessenberg matrix by inverse iteration.
?laev2	s, d, c, z	Computes the eigenvalues and eigenvectors of a 2-by-2 symmetric/Hermitian matrix.
?laexc	s, d	Swaps adjacent diagonal blocks of a real upper quasi-triangular matrix in Schur canonical form, by an orthogonal similarity transformation.
?lag2	s, d	Computes the eigenvalues of a 2-by-2 generalized eigenvalue problem, with scaling as necessary to avoid over-/underflow.
?lags2	s, d	Computes 2-by-2 orthogonal matrices U, V, and Q, and applies them to matrices A and B such that the rows of the transformed A and B are parallel.
?lagtf	s, d	Computes an LU factorization of a matrix $T-\lambda I$, where T is a general tridiagonal matrix, and λ a scalar, using partial pivoting with row interchanges.
?lagtm	s, d, c, z	Performs a matrix-matrix product of the form $C = \alpha AB+\beta C$, where A is a tridiagonal matrix, B and C are rectangular matrices, and α and β are scalars, which may be 0, 1, or -1.
?lagts	s, d	Solves the system of equations $(T-\lambda I)x = y$ or $(T-\lambda I)^T x = y$,where T is a general tridiagonal matrix and λ a scalar, using the LU factorization computed by ?lagtf.
?lagv2	s, d	Computes the Generalized Schur factorization of a real 2-by-2 matrix pencil (A,B) where B is upper triangular.

Routine Name	Data Types	Description
?lahqr	s, d, c, z	Computes the eigenvalues and Schur factorization of an upper Hessenberg matrix, using the double-shift/single-shift QR algorithm.
?lahrd	s, d, c, z	Reduces the first *nb* columns of a general rectangular matrix *A* so that elements below the *k*-th subdiagonal are zero, and returns auxiliary matrices which are needed to apply the transformation to the unreduced part of *A*.
?lahr2	s, d, c, z	Reduces the specified number of first columns of a general rectangular matrix *A* so that elements below thespecified subdiagonal are zero, and returns auxiliary matrices which are needed to apply the transformation to the unreduced part of *A*.
?laic1	s, d, c, z	Applies one step of incremental condition estimation.
?laln2	s, d	Solves a 1-by-1 or 2-by-2 linear system of equations of the specified form.
?lals0	s, d, c, z	Applies back multiplying factors in solving the least squares problem using divide and conquer SVD approach. Used by ?gelsd.
?lalsa	s, d, c, z	Computes the SVD of the coefficient matrix in compact form. Used by ?gelsd.
?lalsd	s, d, c, z	Uses the singular value decomposition of *A* to solve the least squares problem.
?lamrg	s, d	Creates a permutation list to merge the entries of two independently sorted sets into a single set sorted in ascending order.
?laneg	s, d	Computes the Sturm count.
?langb	s, d, c, z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of general band matrix.
?lange	s, d, c, z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general rectangular matrix.
?langt	s, d, c, z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general tridiagonal matrix.
?lanhs	s, d, c, z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of an upper Hessenberg matrix.
?lansb	s, d, c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a symmetric band matrix.
?lanhb	c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a Hermitian band matrix.
?lansp	s, d, c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a symmetric matrix supplied in packed form.

Routine Name	Data Types	Description
?lanhp	c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix supplied in packed form.
?lanst/?lanht	s, d/c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric or complex Hermitian tridiagonal matrix.
?lansy	s, d, c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/ complex symmetric matrix.
?lanhe	c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix.
?lantb	s, d, c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a triangular band matrix.
?lantp	s, d, c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a triangular matrix supplied in packed form.
?lantr	s, d, c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular matrix.
?lanv2	s, d	Computes the Schur factorization of a real 2-by-2 nonsymmetric matrix in standard form.
?lapll	s, d, c, z	Measures the linear dependence of two vectors.
?lapmr	s, d, c, z	Rearranges rows of a matrix as specified by a permutation vector.
?lapmt	s, d, c, z	Performs a forward or backward permutation of the columns of a matrix.
?lapy2	s, d	Returns sqrt(x^2+y^2).
?lapy3	s, d	Returns sqrt($x^2+y^2+z^2$).
?laqgb	s, d, c, z	Scales a general band matrix, using row and column scaling factors computed by ?gbequ.
?laqge	s, d, c, z	Scales a general rectangular matrix, using row and column scaling factors computed by ?geequ.
?laqhb	c, z	Scales a Hermetian band matrix, using scaling factors computed by ?pbequ.
?laqp2	s, d, c, z	Computes a QR factorization with column pivoting of the matrix block.
?laqps	s, d, c, z	Computes a step of QR factorization with column pivoting of a real m-by-n matrix A by using BLAS level 3.
?laqr0	s, d, c, z	Computes the eigenvalues of a Hessenberg matrix, and optionally the matrices from the Schur decomposition.
?laqr1	s, d, c, z	Sets a scalar multiple of the first column of the product of 2-by-2 or 3-by-3 matrix H and specified shifts.

Routine Name	Data Types	Description
?laqr2	s, d, c, z	Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation).
?laqr3	s, d, c, z	Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation).
?laqr4	s, d, c, z	Computes the eigenvalues of a Hessenberg matrix, and optionally the matrices from the Schur decomposition.
?laqr5	s, d, c, z	Performs a single small-bulge multi-shift QR sweep.
?laqsb	s, d, c, z	Scales a symmetric/Hermitian band matrix, using scaling factors computed by ?pbequ.
?laqsp	s, d, c, z	Scales a symmetric/Hermitian matrix in packed storage, using scaling factors computed by ?ppequ.
?laqsy	s, d, c, z	Scales a symmetric/Hermitian matrix, using scaling factors computed by ?poequ.
?laqtr	s, d	Solves a real quasi-triangular system of equations, or a complex quasi-triangular system of special form, in real arithmetic.
?lar1v	s, d, c, z	Computes the (scaled) r-th column of the inverse of the submatrix in rows $b1$ through bn of the tridiagonal matrix $LDL^T - \sigma I$.
?lar2v	s, d, c, z	Applies a vector of plane rotations with real cosines and real/ complex sines from both sides to a sequence of 2-by-2 symmetric/ Hermitian matrices.
?larf	s, d, c, z	Applies an elementary reflector to a general rectangular matrix.
?larfb	s, d, c, z	Applies a block reflector or its transpose/conjugate-transpose to a general rectangular matrix.
?larfg	s, d, c, z	Generates an elementary reflector (Householder matrix).
?larfgp	s, d, c, z	Generates an elementary reflector (Householder matrix) with non-negatibe beta.
?larft	s, d, c, z	Forms the triangular factor T of a block reflector $H = I - vtv^H$
?larfx	s, d, c, z	Applies an elementary reflector to a general rectangular matrix, with loop unrolling when the reflector has order ≤ 10.
?largv	s, d, c, z	Generates a vector of plane rotations with real cosines and real/ complex sines.
?larnv	s, d, c, z	Returns a vector of random numbers from a uniform or normal distribution.
?larra	s, d	Computes the splitting points with the specified threshold.
?larrb	s, d	Provides limited bisection to locate eigenvalues for more accuracy.
?larrc	s, d	Computes the number of eigenvalues of the symmetric tridiagonal matrix.

Routine Name	Data Types	Description
?larrd	s, d	Computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.
?larre	s, d	Given the tridiagonal matrix T, sets small off-diagonal elements to zero and for each unreduced block T_i, finds base representations and eigenvalues.
?larrf	s, d	Finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.
?larrj	s, d	Performs refinement of the initial estimates of the eigenvalues of the matrix T.
?larrk	s, d	Computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy.
?larrr	s, d	Performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.
?larrv	s, d, c, z	Computes the eigenvectors of the tridiagonal matrix $T = L D L^T$ given L, D and the eigenvalues of $L D L^T$.
?lartg	s, d, c, z	Generates a plane rotation with real cosine and real/complex sine.
?lartgp	s, d	Generates a plane rotation so that the diagonal is nonnegative.
?lartgs	s, d	Generates a plane rotation designed to introduce a bulge in implicit QR iteration for the bidiagonal SVD problem.
?lartv	s, d, c, z	Applies a vector of plane rotations with real cosines and real/ complex sines to the elements of a pair of vectors.
?laruv	s, d	Returns a vector of n random real numbers from a uniform distribution.
?larz	s, d, c, z	Applies an elementary reflector (as returned by ?tzrzf) to a general matrix.
?larzb	s, d, c, z	Applies a block reflector or its transpose/conjugate-transpose to a general matrix.
?larzt	s, d, c, z	Forms the triangular factor T of a block reflector $H = I - vtv^H$.
?las2	s, d	Computes singular values of a 2-by-2 triangular matrix.
?lascl	s, d, c, z	Multiplies a general rectangular matrix by a real scalar defined as c_{to}/c_{from}.
?lasd0	s, d	Computes the singular values of a real upper bidiagonal n-by-m matrix B with diagonal d and off-diagonal e. Used by ?bdsdc.
?lasd1	s, d	Computes the SVD of an upper bidiagonal matrix B of the specified size. Used by ?bdsdc.
?lasd2	s, d	Merges the two sets of singular values together into a single sorted set. Used by ?bdsdc.
?lasd3	s, d	Finds all square roots of the roots of the secular equation, as defined by the values in D and Z, and then updates the singular vectors by matrix multiplication. Used by ?bdsdc.

Routine Name	Data Types	Description
?lasd4	s, d	Computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. Used by ?bdsdc.
?lasd5	s, d	Computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix.Used by ?bdsdc.
?lasd6	s, d	Computes the SVD of an updated upper bidiagonal matrix obtained by merging two smaller ones by appending a row. Used by ?bdsdc.
?lasd7	s, d	Merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. Used by ?bdsdc.
?lasd8	s, d	Finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by ?bdsdc.
?lasd9	s, d	Finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by ?bdsdc.
?lasda	s, d	Computes the singular value decomposition (SVD) of a real upper bidiagonal matrix with diagonal *d* and off-diagonal *e*. Used by ?bdsdc.
?lasdq	s, d	Computes the SVD of a real bidiagonal matrix with diagonal *d* and off-diagonal *e*. Used by ?bdsdc.
?lasdt	s, d	Creates a tree of subproblems for bidiagonal divide and conquer. Used by ?bdsdc.
?laset	s, d, c, z	Initializes the off-diagonal elements and the diagonal elements of a matrix to given values.
?lasq1	s, d	Computes the singular values of a real square bidiagonal matrix. Used by ?bdsqr.
?lasq2	s, d	Computes all the eigenvalues of the symmetric positive definite tridiagonal matrix associated with the *qd* Array *z* to high relative accuracy. Used by ?bdsqr and ?stegr.
?lasq3	s, d	Checks for deflation, computes a shift and calls *dqds*. Used by ?bdsqr.
?lasq4	s, d	Computes an approximation to the smallest eigenvalue using values of *d* from the previous transform. Used by ?bdsqr.
?lasq5	s, d	Computes one *dqds* transform in ping-pong form. Used by ?bdsqr and ?stegr.
?lasq6	s, d	Computes one *dqd* transform in ping-pong form. Used by ?bdsqr and ?stegr.
?lasr	s, d, c, z	Applies a sequence of plane rotations to a general rectangular matrix.
?lasrt	s, d	Sorts numbers in increasing or decreasing order.

Routine Name	Data Types	Description
?lassq	s, d, c, z	Updates a sum of squares represented in scaled form.
?lasv2	s, d	Computes the singular value decomposition of a 2-by-2 triangular matrix.
?laswp	s, d, c, z	Performs a series of row interchanges on a general rectangular matrix.
?lasy2	s, d	Solves the Sylvester matrix equation where the matrices are of order 1 or 2.
?lasyf	s, d, c, z	Computes a partial factorization of a real/complex symmetric matrix, using the diagonal pivoting method.
?lahef	c, z	Computes a partial factorization of a complex Hermitian indefinite matrix, using the diagonal pivoting method.
?latbs	s, d, c, z	Solves a triangular banded system of equations.
?latdf	s, d, c, z	Uses the LU factorization of the n-by-n matrix computed by ?getc2 and computes a contribution to the reciprocal Dif-estimate.
?latps	s, d, c, z	Solves a triangular system of equations with the matrix held in packed storage.
?latrd	s, d, c, z	Reduces the first *nb* rows and columns of a symmetric/Hermitian matrix *A* to real tridiagonal form by an orthogonal/unitary similarity transformation.
?latrs	s, d, c, z	Solves a triangular system of equations with the scale factor set to prevent overflow.
?latrz	s, d, c, z	Factors an upper trapezoidal matrix by means of orthogonal/ unitary transformations.
?lauu2	s, d, c, z	Computes the product UU^H or L^HL, where U and L are upper or lower triangular matrices (unblocked algorithm).
?lauum	s, d, c, z	Computes the product UU^H or L^HL, where U and L are upper or lower triangular matrices (blocked algorithm).
?org2l/?ung2l	s, d/c, z	Generates all or part of the orthogonal/unitary matrix Q from a QL factorization determined by ?geqlf (unblocked algorithm).
?org2r/?ung2r	s, d/c, z	Generates all or part of the orthogonal/unitary matrix Q from a QR factorization determined by ?geqrf (unblocked algorithm).
?orgl2/?ungl2	s, d/c, z	Generates all or part of the orthogonal/unitary matrix Q from an LQ factorization determined by ?gelqf (unblocked algorithm).
?orgr2/?ungr2	s, d/c, z	Generates all or part of the orthogonal/unitary matrix Q from an RQ factorization determined by ?gerqf (unblocked algorithm).
?orm2l/?unm2l	s, d/c, z	Multiplies a general matrix by the orthogonal/unitary matrix from a QL factorization determined by ?geqlf (unblocked algorithm).
?orm2r/?unm2r	s, d/c, z	Multiplies a general matrix by the orthogonal/unitary matrix from a QR factorization determined by ?geqrf (unblocked algorithm).
?orml2/?unml2	s, d/c, z	Multiplies a general matrix by the orthogonal/unitary matrix from a LQ factorization determined by ?gelqf (unblocked algorithm).

Routine Name	Data Types	Description
?ormr2/?unmr2	s, d/c, z	Multiplies a general matrix by the orthogonal/unitary matrix from a RQ factorization determined by ?gerqf (unblocked algorithm).
?ormr3/?unmr3	s, d/c, z	Multiplies a general matrix by the orthogonal/unitary matrix from a RZ factorization determined by ?tzrzf (unblocked algorithm).
?pbtf2	s, d, c, z	Computes the Cholesky factorization of a symmetric/ Hermitian positive definite band matrix (unblocked algorithm).
?potf2	s, d, c, z	Computes the Cholesky factorization of a symmetric/Hermitian positive definite matrix (unblocked algorithm).
?ptts2	s, d, c, z	Solves a tridiagonal system of the form $AX=B$ using the $L D L^H$ factorization computed by ?pttrf.
?rscl	s, d, cs, zd	Multiplies a vector by the reciprocal of a real scalar.
?syswapr	s, d, c, z	Applies an elementary permutation on the rows and columns of a symmetric matrix.
?heswapr	c, z	Applies an elementary permutation on the rows and columns of a Hermitian matrix.
?sygs2/?hegs2	s, d/c, z	Reduces a symmetric/Hermitian definite generalized eigenproblem to standard form, using the factorization results obtained from ?potrf (unblocked algorithm).
?sytd2/?hetd2	s, d/c, z	Reduces a symmetric/Hermitian matrix to real symmetric tridiagonal form by an orthogonal/unitary similarity transformation (unblocked algorithm).
?sytf2	s, d, c, z	Computes the factorization of a real/complex symmetric indefinite matrix, using the diagonal pivoting method (unblocked algorithm).
?hetf2	c, z	Computes the factorization of a complex Hermitian matrix, using the diagonal pivoting method (unblocked algorithm).
?tgex2	s, d, c, z	Swaps adjacent diagonal blocks in an upper (quasi) triangular matrix pair by an orthogonal/unitary equivalence transformation.
?tgsy2	s, d, c, z	Solves the generalized Sylvester equation (unblocked algorithm).
?trti2	s, d, c, z	Computes the inverse of a triangular matrix (unblocked algorithm).
clag2z	c → z	Converts a complex single precision matrix to a complex double precision matrix.
dlag2s	d → s	Converts a double precision matrix to a single precision matrix.
slag2d	s → d	Converts a single precision matrix to a double precision matrix.
zlag2c	z → c	Converts a complex double precision matrix to a complex single precision matrix.
?larfp	s, d, c, z	Generates a real or complex elementary reflector.
ila?lc	s, d, c, z	Scans a matrix for its last non-zero column.
ila?lr	s, d, c, z	Scans a matrix for its last non-zero row.

Routine Name	Data Types	Description
?gsvj0	s, d	Pre-processor for the routine ?gesvj.
?gsvj1	s, d	Pre-processor for the routine ?gesvj, applies Jacobi rotations targeting only particular pivots.
?sfrk	s, d	Performs a symmetric rank-k operation for matrix in RFP format.
?hfrk	c, z	Performs a Hermitian rank-k operation for matrix in RFP format.
?tfsm	s, d, c, z	Solves a matrix equation (one operand is a triangular matrix in RFP format).
?lansf	s, d	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a symmetric matrix in RFP format.
?lanhf	c, z	Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a Hermitian matrix in RFP format.
?tfttp	s, d, c, z	Copies a triangular matrix from the rectangular full packed format (TF) to the standard packed format (TP).
?tfttr	s, d, c, z	Copies a triangular matrix from the rectangular full packed format (TF) to the standard full format (TR).
?tpttf	s, d, c, z	Copies a triangular matrix from the standard packed format (TP) to the rectangular full packed format (TF).
?tpttr	s, d, c, z	Copies a triangular matrix from the standard packed format (TP) to the standard full format (TR).
?trttf	s, d, c, z	Copies a triangular matrix from the standard full format (TR) to the rectangular full packed format (TF).
?trttp	s, d, c, z	Copies a triangular matrix from the standard full format (TR) to the standard packed format (TP).
?pstf2	s, d, c, z	Computes the Cholesky factorization with complete pivoting of a real symmetric or complex Hermitian positive semi-definite matrix.
dlat2s	d → s	Converts a double-precision triangular matrix to a single-precision triangular matrix.
zlat2c	z → c	Converts a double complex triangular matrix to a complex triangular matrix.
?lacp2	c, z	Copies all or part of a real two-dimensional array to a complex array.
?la_gbamv	s, d, c, z	Performs a matrix-vector operation to calculate error bounds.
?la_gbrcond	s, d	Estimates the Skeel condition number for a general banded matrix.
?la_gbrcond_c	c, z	Computes the infinity norm condition number of op(A)*inv(diag(c)) for general banded matrices.
?la_gbrcond_x	c, z	Computes the infinity norm condition number of op(A)*diag(x) for general banded matrices.

Routine Name	Data Types	Description
?la_gbrfsx_extended	s, d, c, z	Improves the computed solution to a system of linear equations for general banded matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.
?la_gbrpvgrw	s, d, c, z	Computes the reciprocal pivot growth factor norm(A)/norm(U) for a general banded matrix.
?la_geamv	s, d, c, z	Computes a matrix-vector product using a general matrix to calculate error bounds.
?la_gercond	s, d	Estimates the Skeel condition number for a general matrix.
?la_gercond_c	c, z	Computes the infinity norm condition number of op(A)*inv(diag(c)) for general matrices.
?la_gercond_x	c, z	Computes the infinity norm condition number of op(A)*diag(x) for general matrices.
?la_gerfsx_extended	s, d	Improves the computed solution to a system of linear equations for general matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.
?la_heamv	c, z	Computes a matrix-vector product using a Hermitian indefinite matrix to calculate error bounds.
?la_hercond_c	c, z	Computes the infinity norm condition number of op(A)*inv(diag(c)) for Hermitian indefinite matrices.
?la_hercond_x	c, z	Computes the infinity norm condition number of op(A)*diag(x) for Hermitian indefinite matrices.
?la_herfsx_extended	c, z	Improves the computed solution to a system of linear equations for Hermitian indefinite matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.
?la_lin_berr	s, d, c, z	Computes a component-wise relative backward error.
?la_porcond	s, d	Estimates the Skeel condition number for a symmetric positive-definite matrix.
?la_porcond_c	c, z	Computes the infinity norm condition number of op(A)*inv(diag(c)) for Hermitian positive-definite matrices.
?la_porcond_x	c, z	Computes the infinity norm condition number of op(A)*diag(x) for Hermitian positive-definite matrices.
?la_porfsx_extended	s, d, c, z	Improves the computed solution to a system of linear equations for symmetric or Hermitian positive-definite matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.
?la_porpvgrw	s, d, c, z	Computes the reciprocal pivot growth factor norm(A)/norm(U) for a symmetric or Hermitian positive-definite matrix.
?laqhe	c, z	Scales a Hermitian matrix.
?laqhp	c, z	Scales a Hermitian matrix stored in packed form.
?larcm	c, z	Copies all or part of a real two-dimensional array to a complex array.

Routine Name	Data Types	Description
?la_rpvgrw	c, z	Multiplies a square real matrix by a complex matrix.
?larscl2	s, d, c, z	Performs reciprocal diagonal scaling on a vector.
?lascl2	s, d, c, z	Performs diagonal scaling on a vector.
?la_syamv	s, d, c, z	Computes a matrix-vector product using a symmetric indefinite matrix to calculate error bounds.
?la_syrcond	s, d	Estimates the Skeel condition number for a symmetric indefinite matrix.
?la_syrcond_c	c, z	Computes the infinity norm condition number of op(A)*inv(diag(c)) for symmetric indefinite matrices.
?la_syrcond_x	c, z	Computes the infinity norm condition number of op(A)*diag(x) for symmetric indefinite matrices.
?la_syrfsx_extended	s, d, c, z	Improves the computed solution to a system of linear equations for symmetric indefinite matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.
?la_syrpvgrw	s, d, c, z	Computes the reciprocal pivot growth factor norm(A)/norm(U) for a symmetric indefinite matrix.
?la_wwaddw	s, d, c, z	Adds a vector into a doubled-single vector.

?lacgv

Conjugates a complex vector.

Syntax

```
call clacgv( n, x, incx )
call zlacgv( n, x, incx )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine conjugates a complex vector x of length n and increment $incx$ (see "Vector Arguments in BLAS" in Appendix B).

Input Parameters

n	INTEGER. The length of the vector x ($n \geq 0$).
x	COMPLEX for clacgv DOUBLE COMPLEX for zlacgv. Array, dimension (1+(*n*-1)* |*incx*|). Contains the vector of length n to be conjugated.
incx	INTEGER. The spacing between successive elements of x.

Output Parameters

x	On exit, overwritten with conjg(*x*).

?lacrm

Multiplies a complex matrix by a square real matrix.

Syntax

```
call clacrm( m, n, a, lda, b, ldb, c, ldc, rwork )
call zlacrm( m, n, a, lda, b, ldb, c, ldc, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs a simple matrix-matrix multiplication of the form

C = *A***B*,

where *A* is *m*-by-*n* and complex, *B* is *n*-by-*n* and real, *C* is *m*-by-*n* and complex.

Input Parameters

m	INTEGER. The number of rows of the matrix *A* and of the matrix *C* (*m* ≥ 0).
n	INTEGER. The number of columns and rows of the matrix *B* and the number of columns of the matrix *C* (*n* ≥ 0).
a	COMPLEX for clacrm DOUBLE COMPLEX for zlacrm Array, DIMENSION (*lda*, *n*). Contains the *m*-by-*n* matrix *A*.
lda	INTEGER. The leading dimension of the array *a*, *lda* ≥ max(1, *m*).
b	REAL for clacrm DOUBLE PRECISION for zlacrm Array, DIMENSION (*ldb*, *n*). Contains the *n*-by-*n* matrix *B*.
ldb	INTEGER. The leading dimension of the array *b*, *ldb* ≥ max(1, *n*).
ldc	INTEGER. The leading dimension of the output array *c*, *ldc* ≥ max(1, *n*).
rwork	REAL for clacrm DOUBLE PRECISION for zlacrm Workspace array, DIMENSION (2**m***n*).

Output Parameters

c	COMPLEX for clacrm DOUBLE COMPLEX for zlacrm Array, DIMENSION (*ldc*, *n*). Contains the *m*-by-*n* matrix *C*.

?lacrt

Performs a linear transformation of a pair of complex vectors.

Syntax

```
call clacrt( n, cx, incx, cy, incy, c, s )
call zlacrt( n, cx, incx, cy, incy, c, s )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine performs the following transformation

$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix},$$

where c, s are complex scalars and x, y are complex vectors.

Input Parameters

n	INTEGER. The number of elements in the vectors *cx* and *cy* ($n \geq 0$).
cx, *cy*	COMPLEX for clacrt DOUBLE COMPLEX for zlacrt Arrays, dimension (n). Contain input vectors x and y, respectively.
incx	INTEGER. The increment between successive elements of *cx*.
incy	INTEGER. The increment between successive elements of *cy*.
c, *s*	COMPLEX for clacrt DOUBLE COMPLEX for zlacrt Complex scalars that define the transform matrix

$$\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & \\ & & & & \\ & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

Output Parameters

cx	On exit, overwritten with *c*x + s*y* .
cy	On exit, overwritten with *-s*x + c*y* .

?laesy

Computes the eigenvalues and eigenvectors of a 2-by-2 complex symmetric matrix, and checks that the norm of the matrix of eigenvectors is larger than a threshold value.

Syntax

```
call claesy( a, b, c, rt1, rt2, evscal, cs1, sn1 )
call zlaesy( a, b, c, rt1, rt2, evscal, cs1, sn1 )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine performs the eigendecomposition of a 2-by-2 symmetric matrix

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix},$$

provided the norm of the matrix of eigenvectors is larger than some threshold value.

rt1 is the eigenvalue of larger absolute value, and *rt2* of smaller absolute value. If the eigenvectors are computed, then on return (*cs1*, *sn1*) is the unit eigenvector for *rt1*, hence

$$\begin{bmatrix} cs1 & sn1 \\ -sn1 & cs1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ b & c \end{bmatrix} \cdot \begin{bmatrix} cs1 & -sn1 \\ sn1 & cs1 \end{bmatrix} = \begin{bmatrix} rt1 & 0 \\ 0 & rt2 \end{bmatrix}$$

Input Parameters

a, *b*, *c*	`COMPLEX` for `claesy` `DOUBLE COMPLEX` for `zlaesy` Elements of the input matrix.

Output Parameters

rt1, *rt2*	`COMPLEX` for `claesy` `DOUBLE COMPLEX` for `zlaesy` Eigenvalues of larger and smaller modulus, respectively.
evscal	`COMPLEX` for `claesy` `DOUBLE COMPLEX` for `zlaesy` The complex value by which the eigenvector matrix was scaled to make it orthonormal. If *evscal* is zero, the eigenvectors were not computed. This means one of two things: the 2-by-2 matrix could not be diagonalized, or the norm of the matrix of eigenvectors before scaling was larger than the threshold value `thresh` (set to 0.1E0).
cs1, *sn1*	`COMPLEX` for `claesy` `DOUBLE COMPLEX` for `zlaesy` If *evscal* is not zero, then (*cs1*, *sn1*) is the unit right eigenvector for *rt1*.

?rot

Applies a plane rotation with real cosine and complex sine to a pair of complex vectors.

Syntax

```
call crot( n, cx, incx, cy, incy, c, s )
call zrot( n, cx, incx, cy, incy, c, s )
```

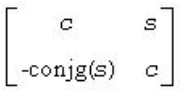
Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine applies a plane rotation, where the cosine (*c*) is real and the sine (*s*) is complex, and the vectors *cx* and *cy* are complex. This routine has its real equivalents in BLAS (see `?rot` in Chapter 2).

Input Parameters

n	`INTEGER`. The number of elements in the vectors *cx* and *cy*.
cx, *cy*	`REAL` for `srot` `DOUBLE PRECISION` for `drot` `COMPLEX` for `crot` `DOUBLE COMPLEX` for `zrot` Arrays of dimension (*n*), contain input vectors *x* and *y*, respectively.
incx	`INTEGER`. The increment between successive elements of *cx*.
incy	`INTEGER`. The increment between successive elements of *cy*.
c	`REAL` for `crot` `DOUBLE PRECISION` for `zrot`
s	`REAL` for `srot` `DOUBLE PRECISION` for `drot` `COMPLEX` for `crot` `DOUBLE COMPLEX` for `zrot` Values that define a rotation  where `c*c + s*conjg(s) = 1.0`.

Output Parameters

cx	On exit, overwritten with `c*x + s*y`.
cy	On exit, overwritten with `-conjg(s)*x + c*y`.

?spmv

Computes a matrix-vector product for complex vectors using a complex symmetric packed matrix.

Syntax

```
call cspmv( uplo, n, alpha, ap, x, incx, beta, y, incy )
call zspmv( uplo, n, alpha, ap, x, incx, beta, y, incy )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?spmv` routines perform a matrix-vector operation defined as

y := *alpha***a***x* + *beta***y*,

where:

alpha and *beta* are complex scalars,

x and *y* are *n*-element complex vectors

a is an *n*-by-*n* complex symmetric matrix, supplied in packed form.

These routines have their real equivalents in BLAS (see ?spmv in Chapter 2).

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the matrix *a* is supplied in the packed array *ap*. If *uplo* = 'U' or 'u', the upper triangular part of the matrix *a* is supplied in the array *ap*. If *uplo* = 'L' or 'l', the lower triangular part of the matrix *a* is supplied in the array *ap* .
n	INTEGER. Specifies the order of the matrix *a*. The value of *n* must be at least zero.
alpha, *beta*	COMPLEX for cspmv DOUBLE COMPLEX for zspmv Specify complex scalars *alpha* and *beta*. When *beta* is supplied as zero, then *y* need not be set on input.
ap	COMPLEX for cspmv DOUBLE COMPLEX for zspmv Array, DIMENSION at least ((*n**(*n* + 1))/2). Before entry, with *uplo* = 'U' or 'u', the array *ap* must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*(1) contains *A*(1, 1), *ap*(2) and *ap*(3) contain *A*(1, 2) and *A*(2, 2) respectively, and so on. Before entry, with *uplo* = 'L' or 'l', the array *ap* must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1, 1), *ap*(2) and *ap*(3) contain *a*(2, 1) and *a*(3, 1) respectively, and so on.
x	COMPLEX for cspmv DOUBLE COMPLEX for zspmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array x must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	COMPLEX for cspmv DOUBLE COMPLEX for zspmv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Overwritten by the updated vector *y*.

?spr

Performs the symmetrical rank-1 update of a complex symmetric packed matrix.

Syntax

```
call cspr( uplo, n, alpha, x, incx, ap )
call zspr( uplo, n, alpha, x, incx, ap )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?spr` routines perform a matrix-vector operation defined as

$a := alpha*x*x^H + a$,

where:

alpha is a complex scalar

x is an *n*-element complex vector

a is an *n*-by-*n* complex symmetric matrix, supplied in packed form.

These routines have their real equivalents in BLAS (see `?spr` in Chapter 2).

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the upper or lower triangular part of the matrix *a* is supplied in the packed array *ap*, as follows: If *uplo* = `'U'` or `'u'`, the upper triangular part of the matrix *a* is supplied in the array *ap*. If *uplo* = `'L'` or `'l'`, the lower triangular part of the matrix *a* is supplied in the array *ap* .
n	`INTEGER`. Specifies the order of the matrix *a*. The value of *n* must be at least zero.
alpha	`COMPLEX` for `cspr` `DOUBLE COMPLEX` for `zspr` Specifies the scalar *alpha*.
x	`COMPLEX` for `cspr` `DOUBLE COMPLEX` for `zspr` Array, `DIMENSION` at least `(1 + (n - 1)*abs(incx))`. Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	`INTEGER`. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
ap	`COMPLEX` for `cspr` `DOUBLE COMPLEX` for `zspr` Array, `DIMENSION` at least `((n*(n + 1))/2)`. Before entry, with *uplo* = `'U'` or `'u'`, the array *ap* must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*`(1)` contains *A*`(1,1)`, *ap*`(2)` and *ap*`(3)` contain *A*`(1, 2)` and *A*`(2,2)` respectively, and so on.

Before entry, with *uplo* = 'L' or 'l', the array *ap* must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that *ap*(1) contains *a*(1,1), *ap*(2) and *ap*(3) contain *a*(2,1) and *a*(3,1) respectively, and so on.
Note that the imaginary parts of the diagonal elements need not be set, they are assumed to be zero, and on exit they are set to zero.

Output Parameters

ap — With *uplo* = 'U' or 'u', overwritten by the upper triangular part of the updated matrix.
With *uplo* = 'L' or 'l', overwritten by the lower triangular part of the updated matrix.

?symv

Computes a matrix-vector product for a complex symmetric matrix.

Syntax

```
call csymv( uplo, n, alpha, a, lda, x, incx, beta, y, incy )
call zsymv( uplo, n, alpha, a, lda, x, incx, beta, y, incy )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs the matrix-vector operation defined as

y := *alpha***a***x* + *beta***y*,

where:

alpha and *beta* are complex scalars

x and *y* are *n*-element complex vectors

a is an *n*-by-*n* symmetric complex matrix.

These routines have their real equivalents in BLAS (see ?symv in Chapter 2).

Input Parameters

uplo — CHARACTER*1. Specifies whether the upper or lower triangular part of the array *a* is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the array *a* is used.
If *uplo* = 'L' or 'l', then the lower triangular part of the array *a* is used.

n — INTEGER. Specifies the order of the matrix *a*. The value of *n* must be at least zero.

alpha, *beta* — COMPLEX for csymv
DOUBLE COMPLEX for zsymv
Specify the scalars *alpha* and *beta*. When *beta* is supplied as zero, then *y* need not be set on input.

a — COMPLEX for csymv
DOUBLE COMPLEX for zsymv

	Array, DIMENSION (*lda*, *n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. Specifies the leading dimension of *A* as declared in the calling (sub)program. The value of *lda* must be at least max(1,*n*).
x	COMPLEX for csymv DOUBLE COMPLEX for zsymv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
y	COMPLEX for csymv DOUBLE COMPLEX for zsymv Array, DIMENSION at least (1 + (*n* – 1)*abs(*incy*)). Before entry, the incremented array *y* must contain the *n*-element vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero.

Output Parameters

y	Overwritten by the updated vector *y*.

?syr

Performs the symmetric rank-1 update of a complex symmetric matrix.

Syntax

```
call csyr( uplo, n, alpha, x, incx, a, lda )
call zsyr( uplo, n, alpha, x, incx, a, lda )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs the symmetric rank 1 operation defined as

$a := alpha*x*x^H + a$,

where:

- *alpha* is a complex scalar.
- *x* is an *n*-element complex vector.
- *a* is an *n*-by-*n* complex symmetric matrix.

These routines have their real equivalents in BLAS (see ?syr in Chapter 2).

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *a* is used: If *uplo* = 'U' or 'u', then the upper triangular part of the array *a* is used. If *uplo* = 'L' or 'l', then the lower triangular part of the array *a* is used.
n	INTEGER. Specifies the order of the matrix *a*. The value of *n* must be at least zero.
alpha	COMPLEX for csyr DOUBLE COMPLEX for zsyr Specifies the scalar *alpha*.
x	COMPLEX for csyr DOUBLE COMPLEX for zsyr Array, DIMENSION at least (1 + (*n* – 1)*abs(*incx*)). Before entry, the incremented array *x* must contain the *n*-element vector x.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.
a	COMPLEX for csyr DOUBLE COMPLEX for zsyr Array, DIMENSION (*lda*, *n*). Before entry with *uplo* = 'U' or 'u', the leading *n*-by-*n* upper triangular part of the array *a* must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of *a* is not referenced. Before entry with *uplo* = 'L' or 'l', the leading *n*-by-*n* lower triangular part of the array *a* must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1,*n*).

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix. With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix.

i?max1

Finds the index of the vector element whose real part has maximum absolute value.

Syntax

index = icmax1(*n*, *cx*, *incx*)

index = izmax1(*n*, *cx*, *incx*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given a complex vector *cx*, the i?max1 functions return the index of the vector element whose real part has maximum absolute value. These functions are based on the BLAS functions icamax/izamax, but using the absolute value of the real part. They are designed for use with clacon/zlacon.

Input Parameters

n	INTEGER. Specifies the number of elements in the vector *cx*.
cx	COMPLEX for icmax1 DOUBLE COMPLEX for izmax1 Array, DIMENSION at least (1+(*n*-1)*abs(*incx*)). Contains the input vector.
incx	INTEGER. Specifies the spacing between successive elements of *cx*.

Output Parameters

index	INTEGER. Contains the index of the vector element whose real part has maximum absolute value.

?sum1

Forms the 1-norm of the complex vector using the true absolute value.

Syntax

res = scsum1(*n*, *cx*, *incx*)

res = dzsum1(*n*, *cx*, *incx*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given a complex vector *cx*, scsum1/dzsum1 functions take the sum of the absolute values of vector elements and return a single/double precision result, respectively. These functions are based on scasum/dzasum from Level 1 BLAS, but use the true absolute value and were designed for use with clacon/zlacon.

Input Parameters

n	INTEGER. Specifies the number of elements in the vector *cx*.
cx	COMPLEX for scsum1 DOUBLE COMPLEX for dzsum1 Array, DIMENSION at least (1+(*n*-1)*abs(*incx*)). Contains the input vector whose elements will be summed.
incx	INTEGER. Specifies the spacing between successive elements of *cx* (*incx* > 0).

Output Parameters

res	REAL for scsum1 DOUBLE PRECISION for dzsum1 Contains the sum of absolute values.

?gbtf2

Computes the LU factorization of a general band matrix using the unblocked version of the algorithm.

Syntax

```
call sgbtf2( m, n, kl, ku, ab, ldab, ipiv, info )
call dgbtf2( m, n, kl, ku, ab, ldab, ipiv, info )
call cgbtf2( m, n, kl, ku, ab, ldab, ipiv, info )
call zgbtf2( m, n, kl, ku, ab, ldab, ipiv, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine forms the *LU* factorization of a general real/complex *m*-by-*n* band matrix *A* with *kl* sub-diagonals and *ku* super-diagonals. The routine uses partial pivoting with row interchanges and implements the unblocked version of the algorithm, calling Level 2 BLAS. See also `?gbtrf`.

Input Parameters

m	`INTEGER`. The number of rows of the matrix *A* ($m \geq 0$).
n	`INTEGER`. The number of columns in *A* ($n \geq 0$).
kl	`INTEGER`. The number of sub-diagonals within the band of *A* ($kl \geq 0$).
ku	`INTEGER`. The number of super-diagonals within the band of *A* ($ku \geq 0$).
ab	`REAL` for `sgbtf2` `DOUBLE PRECISION` for `dgbtf2` `COMPLEX` for `cgbtf2` `DOUBLE COMPLEX` for `zgbtf2`. Array, `DIMENSION` (*ldab*,*). The array *ab* contains the matrix *A* in band storage (see Matrix Arguments). The second dimension of *ab* must be at least `max(1, n)`.
ldab	`INTEGER`. The leading dimension of the array *ab*. ($ldab \geq 2kl + ku + 1$)

Output Parameters

ab	Overwritten by details of the factorization. The diagonal and $kl + ku$ super-diagonals of *U* are stored in the first $1 + kl + ku$ rows of *ab*. The multipliers used during the factorization are stored in the next *kl* rows.
ipiv	`INTEGER`. Array, `DIMENSION` at least max(1,min(*m*,*n*)). The pivot indices: row *i* was interchanged with row *ipiv*(*i*).
info	`INTEGER`. If $info = 0$, the execution is successful. If $info = -i$, the *i*-th parameter had an illegal value. If $info = i$, u_{ii} is 0. The factorization has been completed, but *U* is exactly singular. Division by 0 will occur if you use the factor *U* for solving a system of linear equations.

?gebd2

Reduces a general matrix to bidiagonal form using an unblocked algorithm.

Syntax

```
call sgebd2( m, n, a, lda, d, e, tauq, taup, work, info )
call dgebd2( m, n, a, lda, d, e, tauq, taup, work, info )
call cgebd2( m, n, a, lda, d, e, tauq, taup, work, info )
call zgebd2( m, n, a, lda, d, e, tauq, taup, work, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine reduces a general m-by-n matrix A to upper or lower bidiagonal form B by an orthogonal (unitary) transformation: $Q^T*A*P = B$ (for real flavors) or $Q^H*A*P = B$ (for complex flavors).

If $m \geq n$, B is upper bidiagonal; if $m < n$, B is lower bidiagonal.

The routine does not form the matrices Q and P explicitly, but represents them as products of elementary reflectors. if $m \geq n$,

$Q = H(1)*H(2)*...*H(n)$, and $P = G(1)*G(2)*...*G(n-1)$

if $m < n$,

$Q = H(1)*H(2)*...*H(m-1)$, and $P = G(1)*G(2)*...*G(m)$

Each $H(i)$ and $G(i)$ has the form

$H(i) = I - tauq*v*v^T$ and $G(i) = I - taup*u*u^T$ for real flavors, or

$H(i) = I - tauq*v*v^H$ and $G(i) = I - taup*u*u^H$ for complex flavors

where $tauq$ and $taup$ are scalars (real for `sgebd2`/`dgebd2`, complex for `cgebd2`/`zgebd2`), and v and u are vectors (real for `sgebd2`/`dgebd2`, complex for `cgebd2`/`zgebd2`).

Input Parameters

m	`INTEGER`. The number of rows in the matrix A ($m \geq 0$).
n	`INTEGER`. The number of columns in A ($n \geq 0$).
a, *work*	`REAL` for `sgebd2` `DOUBLE PRECISION` for `dgebd2` `COMPLEX` for `cgebd2` `DOUBLE COMPLEX` for `zgebd2`. Arrays: *a*(*lda*,*) contains the m-by-n general matrix A to be reduced. The second dimension of *a* must be at least max(1, n). *work*(*) is a workspace array, the dimension of *work* must be at least max(1, m, n).
lda	`INTEGER`. The leading dimension of *a*; at least `max(1, m)`.

Output Parameters

a	if *m* ≥ *n*, the diagonal and first super-diagonal of *a* are overwritten with the upper bidiagonal matrix *B*. Elements below the diagonal, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and elements above the first superdiagonal, with the array *taup*, represent the orthogonal/unitary matrix *p* as a product of elementary reflectors. if *m* < *n*, the diagonal and first sub-diagonal of *a* are overwritten by the lower bidiagonal matrix *B*. Elements below the first subdiagonal, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and elements above the diagonal, with the array *taup*, represent the orthogonal/unitary matrix *p* as a product of elementary reflectors.
d	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*)). Contains the diagonal elements of the bidiagonal matrix *B*: *d*(i) = *a*(i, i).
e	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Array, DIMENSION at least max(1, min(*m*, *n*) - 1). Contains the off-diagonal elements of the bidiagonal matrix *B*: if *m* ≥ *n*, *e*(i) = *a*(i, i+1) for i = 1,2,..., *n*-1; if *m* < *n*, *e*(i) = *a*(i+1, i) for i = 1,2,..., *m*-1.
tauq, *taup*	REAL for sgebd2 DOUBLE PRECISION for dgebd2 COMPLEX for cgebd2 DOUBLE COMPLEX for zgebd2. Arrays, DIMENSION at least max (1, min(*m*, *n*)). Contain scalar factors of the elementary reflectors which represent orthogonal/unitary matrices *Q* and *p*, respectively.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*th parameter had an illegal value.

?gehd2

Reduces a general square matrix to upper Hessenberg form using an unblocked algorithm.

Syntax

```
call sgehd2( n, ilo, ihi, a, lda, tau, work, info )
call dgehd2( n, ilo, ihi, a, lda, tau, work, info )
call cgehd2( n, ilo, ihi, a, lda, tau, work, info )
call zgehd2( n, ilo, ihi, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine reduces a real/complex general matrix A to upper Hessenberg form H by an orthogonal or unitary similarity transformation $Q^T*A*Q = H$ (for real flavors) or $Q^H*A*Q = H$ (for complex flavors).

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of *elementary reflectors*.

Input Parameters

n	INTEGER The order of the matrix A ($n \geq 0$).
ilo, *ihi*	INTEGER. It is assumed that A is already upper triangular in rows and columns 1:*ilo* -1 and *ihi*+1:*n*. If A has been output by ?gebal, then *ilo* and *ihi* must contain the values returned by that routine. Otherwise they should be set to *ilo* = 1 and *ihi* = *n*. Constraint: $1 \leq ilo \leq ihi \leq \max(1, n)$.
a, *work*	REAL for sgehd2 DOUBLE PRECISION for dgehd2 COMPLEX for cgehd2 DOUBLE COMPLEX for zgehd2. Arrays: *a* (*lda*,*) contains the *n*-by-*n* matrix A to be reduced. The second dimension of *a* must be at least max(1, *n*). *work* (*n*) is a workspace array.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).

Output Parameters

a	On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix Q as a product of elementary reflectors. See *Application Notes* below.
tau	REAL for sgehd2 DOUBLE PRECISION for dgehd2 COMPLEX for cgehd2 DOUBLE COMPLEX for zgehd2. Array, DIMENSION at least max (1, *n*–1). Contains the scalar factors of elementary reflectors. See *Application Notes* below.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The matrix Q is represented as a product of (*ihi* - *ilo*) elementary reflectors

$Q = H(ilo)*H(ilo+1)*...*H(ihi-1)$

Each H(i) has the form

$H(i) = I - tau*v*v^T$ for real flavors, or

$H(i) = I - tau*v*v^H$ for complex flavors

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:i) = 0, *v*(i+1) = 1 and *v*(*ihi*+1:*n*) = 0.

On exit, *v*(i+2:*ihi*) is stored in *a*(i+2:*ihi*, i) and *tau* in *tau*(i).

The contents of *a* are illustrated by the following example, with *n* = 7, *ilo* = 2 and *ihi* = 6:

on entry

$$\begin{bmatrix} a & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & & & & & & a \end{bmatrix}$$

on exit

$$\begin{bmatrix} a & a & h & h & h & h & a \\ & a & h & h & h & h & a \\ & h & h & h & h & h & h \\ & v_2 & h & h & h & h & h \\ & v_2 & v_3 & h & h & h & h \\ & v_2 & v_3 & v_4 & h & h & h \\ & & & & & & a \end{bmatrix}$$

where *a* denotes an element of the original matrix *A*, *h* denotes a modified element of the upper Hessenberg matrix *H*, and v_i denotes an element of the vector defining *H*(i).

?gelq2

Computes the LQ factorization of a general rectangular matrix using an unblocked algorithm.

Syntax

```
call sgelq2( m, n, a, lda, tau, work, info )
call dgelq2( m, n, a, lda, tau, work, info )
call cgelq2( m, n, a, lda, tau, work, info )
call zgelq2( m, n, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes an *LQ* factorization of a real/complex *m*-by-*n* matrix *A* as *A* = *L***Q*.

The routine does not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of min(*m*, *n*) *elementary reflectors* :

Q = *H*(k) ... *H*(2) *H*(1) (or *Q* = $H(k)^H$ ... $H(2)^H$ $H(1)^H$ for complex flavors), where *k* = min(*m*, *n*)

Each *H*(i) has the form

H(i) = *I* – *tau**v*v^T for real flavors, or

H(i) = *I* – *tau**v*v^H for complex flavors,

where *tau* is a real/complex scalar stored in *tau*(i), and *v* is a real/complex vector with *v*(1:i-1) = 0 and *v*(i) = 1.

On exit, *v*(i+1:*n*) (for real functions) and conj*g**v*(i+1:*n*) (for complex functions) are stored in *a*(i, i +1:*n*).

Input Parameters

m	INTEGER. The number of rows in the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
a, *work*	REAL for sgelq2 DOUBLE PRECISION for dgelq2 COMPLEX for cgelq2 DOUBLE COMPLEX for zgelq2. Arrays: *a*(*lda*,*) contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1, *n*). *work*(*m*) is a workspace array.
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).

Output Parameters

a	Overwritten by the factorization data as follows: on exit, the elements on and below the diagonal of the array *a* contain the *m*-by-min(*n*,*m*) lower trapezoidal matrix *L* (*L* is lower triangular if *n* ≥ *m*); the elements above the diagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of min(*n*,*m*) elementary reflectors.
tau	REAL for sgelq2 DOUBLE PRECISION for dgelq2 COMPLEX for cgelq2 DOUBLE COMPLEX for zgelq2. Array, DIMENSION at least max(1, min(*m*, *n*)). Contains scalar factors of the elementary reflectors.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?geql2

Computes the QL factorization of a general rectangular matrix using an unblocked algorithm.

Syntax

```
call sgeql2( m, n, a, lda, tau, work, info )
call dgeql2( m, n, a, lda, tau, work, info )
call cgeql2( m, n, a, lda, tau, work, info )
call zgeql2( m, n, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes a *QL* factorization of a real/complex *m*-by-*n* matrix *A* as *A* = *Q***L*.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors* :

$Q = H(k)* \ldots *H(2)*H(1)$, where $k = \min(m, n)$.

Each $H(i)$ has the form

$H(i) = I - tau*v*v^T$ for real flavors, or

$H(i) = I - tau*v*v^H$ for complex flavors

where tau is a real/complex scalar stored in $tau(i)$, and v is a real/complex vector with $v(m-k+i+1:m) = 0$ and $v(m-k+i) = 1$.

On exit, $v(1:m-k+i-1)$ is stored in $a(1:m-k+i-1, n-k+i)$.

Input Parameters

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, *work*	REAL for sgeql2 DOUBLE PRECISION for dgeql2 COMPLEX for cgeql2 DOUBLE COMPLEX for zgeql2. Arrays: $a(lda,*)$ contains the m-by-n matrix A. The second dimension of a must be at least max(1, n). $work(m)$ is a workspace array.
lda	INTEGER. The leading dimension of a; at least max(1, m).

Output Parameters

a	Overwritten by the factorization data as follows: on exit, if $m \geq n$, the lower triangle of the subarray $a(m-n+1:m, 1:n)$ contains the n-by-n lower triangular matrix L; if $m < n$, the elements on and below the (n-m)th superdiagonal contain the m-by-n lower trapezoidal matrix L; the remaining elements, with the array tau, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.
tau	REAL for sgeql2 DOUBLE PRECISION for dgeql2 COMPLEX for cgeql2 DOUBLE COMPLEX for zgeql2. Array, DIMENSION at least max(1, min(m, n)). Contains scalar factors of the elementary reflectors.
info	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value.

?geqr2

Computes the QR factorization of a general rectangular matrix using an unblocked algorithm.

Syntax

```
call sgeqr2( m, n, a, lda, tau, work, info )
call dgeqr2( m, n, a, lda, tau, work, info )
```

```
call cgeqr2( m, n, a, lda, tau, work, info )
call zgeqr2( m, n, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes a QR factorization of a real/complex m-by-n matrix A as $A = Q*R$.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors* :

$Q = H(1)*H(2)* \ldots *H(k)$, where $k = \min(m, n)$

Each H(i) has the form

$H(i) = I - tau*v*v^T$ for real flavors, or

$H(i) = I - tau*v*v^H$ for complex flavors

where tau is a real/complex scalar stored in tau(i), and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$.

On exit, $v(i+1:m)$ is stored in $a(i+1:m, i)$.

Input Parameters

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, $work$	REAL for sgeqr2 DOUBLE PRECISION for dgeqr2 COMPLEX for cgeqr2 DOUBLE COMPLEX for zgeqr2. Arrays: $a(lda,*)$ contains the m-by-n matrix A. The second dimension of a must be at least max(1, n). $work$(n) is a workspace array.
lda	INTEGER. The leading dimension of a; at least max(1, m).

Output Parameters

a	Overwritten by the factorization data as follows: on exit, the elements on and above the diagonal of the array a contain the min(n,m)-by-n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array tau, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.
tau	REAL for sgeqr2 DOUBLE PRECISION for dgeqr2 COMPLEX for cgeqr2 DOUBLE COMPLEX for zgeqr2. Array, DIMENSION at least max(1, min(m, n)). Contains scalar factors of the elementary reflectors.
$info$	INTEGER. If $info = 0$, the execution is successful. If $info = -i$, the i-th parameter had an illegal value.

?geqr2p

Computes the QR factorization of a general rectangular matrix with non-negative diagonal elements using an unblocked algorithm.

Syntax

```
call sgeqr2p( m, n, a, lda, tau, work, info )
call dgeqr2p( m, n, a, lda, tau, work, info )
call cgeqr2p( m, n, a, lda, tau, work, info )
call zgeqr2p( m, n, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes a QR factorization of a real/complex m-by-n matrix A as $A = Q*R$.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of min(m, n) *elementary reflectors* :

$Q = H(1)*H(2)* \ldots *H(k)$, where $k = \min(m, n)$

Each H(i) has the form

$H(i) = I - tau*v*v^T$ for real flavors, or

$H(i) = I - tau*v*v^H$ for complex flavors

where tau is a real/complex scalar stored in tau(i), and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$.

On exit, $v(i+1:m)$ is stored in $a(i+1:m, i)$.

Input Parameters

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a, $work$	REAL for sgeqr2p DOUBLE PRECISION for d COMPLEX for cgeqr2p DOUBLE COMPLEX for zgeqr2p. Arrays: $a(lda,*)$ contains the m-by-n matrix A. The second dimension of a must be at least $\max(1, n)$. $work(n)$ is a workspace array.
lda	INTEGER. The leading dimension of a; at least $\max(1, m)$.

Output Parameters

a	Overwritten by the factorization data as follows: on exit, the elements on and above the diagonal of the array a contain the min(n,m)-by-n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array tau, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.

	The diagonal elements of the matrix *R* are non-negative.
tau	REAL for sgeqr2p DOUBLE PRECISION for dgeqr2p COMPLEX for cgeqr2p DOUBLE COMPLEX for zgeqr2p. Array, DIMENSION at least max(1, min(*m*, *n*)). Contains scalar factors of the elementary reflectors.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?gerq2

Computes the RQ factorization of a general rectangular matrix using an unblocked algorithm.

Syntax

```
call sgerq2( m, n, a, lda, tau, work, info )
call dgerq2( m, n, a, lda, tau, work, info )
call cgerq2( m, n, a, lda, tau, work, info )
call zgerq2( m, n, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes a *RQ* factorization of a real/complex *m*-by-*n* matrix *A* as *A* = *R***Q*.

The routine does not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of min(*m*, *n*) *elementary reflectors* :

Q = *H*(1)**H*(2)* ... **H*(k) for real flavors, or

Q = $H(1)^H$**H*(2)^H* ... **H*(k)^H for complex flavors

where *k* = min(*m*, *n*).

Each *H*(i) has the form

H(i) = *I* - *tau***v**v^T for real flavors, or

H(i) = *I* - *tau***v**v^H for complex flavors

where *tau* is a real/complex scalar stored in *tau*(i), and *v* is a real/complex vector with *v*(*n*-k+i+1:*n*) = 0 and *v*(*n*-k+i) = 1.

On exit, *v*(1:*n*-k+i-1) is stored in *a*(*m*-k+i, 1:*n*-k+i-1).

Input Parameters

m	INTEGER. The number of rows in the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
a, *work*	REAL for sgerq2 DOUBLE PRECISION for dgerq2 COMPLEX for cgerq2

	DOUBLE COMPLEX for zgerq2. Arrays: a(lda,*) contains the m-by-n matrix A. The second dimension of a must be at least max(1, n). work(m) is a workspace array.
lda	INTEGER. The leading dimension of a; at least max(1, m).

Output Parameters

a	Overwritten by the factorization data as follows: on exit, if $m \leq n$, the upper triangle of the subarray a(1:m, n-m+1:n) contains the m-by-m upper triangular matrix R; if $m > n$, the elements on and above the (m-n)-th subdiagonal contain the m-by-n upper trapezoidal matrix R; the remaining elements, with the array tau, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.
tau	REAL for sgerq2 DOUBLE PRECISION for dgerq2 COMPLEX for cgerq2 DOUBLE COMPLEX for zgerq2. Array, DIMENSION at least max(1, min(m, n)). Contains scalar factors of the elementary reflectors.
info	INTEGER. If info = 0, the execution is successful. If info = -i, the i-th parameter had an illegal value.

?gesc2

Solves a system of linear equations using the LU factorization with complete pivoting computed by ?getc2.

Syntax

```
call sgesc2( n, a, lda, rhs, ipiv, jpiv, scale )
call dgesc2( n, a, lda, rhs, ipiv, jpiv, scale )
call cgesc2( n, a, lda, rhs, ipiv, jpiv, scale )
call zgesc2( n, a, lda, rhs, ipiv, jpiv, scale )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine solves a system of linear equations

*A*X = scale*RHS*

with a general n-by-n matrix A using the *LU* factorization with complete pivoting computed by ?getc2.

Input Parameters

n	INTEGER. The order of the matrix A.
a, rhs	REAL for sgesc2

	DOUBLE PRECISION for dgesc2 COMPLEX for cgesc2 DOUBLE COMPLEX for zgesc2. Arrays: *a*(*lda*,*) contains the *LU* part of the factorization of the *n*-by-*n* matrix *A* computed by ?getc2: *A = P*L*U*Q*. The second dimension of *a* must be at least max(1, *n*); *rhs*(*n*) contains on entry the right hand side vector for the system of equations.
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).
ipiv	INTEGER. Array, DIMENSION at least max(1,*n*). The pivot indices: for 1 ≤ i ≤ *n*, row i of the matrix has been interchanged with row *ipiv*(i).
jpiv	INTEGER. Array, DIMENSION at least max(1,*n*). The pivot indices: for 1 ≤ j ≤ *n*, column j of the matrix has been interchanged with column *jpiv*(j).

Output Parameters

rhs	On exit, overwritten with the solution vector *X*.
scale	REAL for sgesc2/cgesc2 DOUBLE PRECISION for dgesc2/zgesc2 Contains the scale factor. *scale* is chosen in the range 0 ≤ *scale* ≤ 1 to prevent overflow in the solution.

?getc2

Computes the LU factorization with complete pivoting of the general n-by-n matrix.

Syntax

```
call sgetc2( n, a, lda, ipiv, jpiv, info )
call dgetc2( n, a, lda, ipiv, jpiv, info )
call cgetc2( n, a, lda, ipiv, jpiv, info )
call zgetc2( n, a, lda, ipiv, jpiv, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes an *LU* factorization with complete pivoting of the *n*-by-*n* matrix *A*. The factorization has the form *A = P*L*U*Q*, where *P* and *Q* are permutation matrices, *L* is lower triangular with unit diagonal elements and *U* is upper triangular.

The LU factorization computed by this routine is used by ?latdf to compute a contribution to the reciprocal Dif-estimate.

Input Parameters

n	INTEGER. The order of the matrix *A* ($n \geq 0$).
a	REAL for sgetc2 DOUBLE PRECISION for dgetc2 COMPLEX for cgetc2 DOUBLE COMPLEX for zgetc2. Array *a*(*lda*,*) contains the *n*-by-*n* matrix *A* to be factored. The second dimension of *a* must be at least max(1, *n*);
lda	INTEGER. The leading dimension of *a*; at least max(1, *n*).

Output Parameters

a	On exit, the factors *L* and *U* from the factorization *A* = *P*L*U*Q*; the unit diagonal elements of *L* are not stored. If *U*(*k*, *k*) appears to be less than smin, *U*(*k*, *k*) is given the value of smin, that is giving a nonsingular perturbed system.
ipiv	INTEGER. Array, DIMENSION at least max(1,*n*). The pivot indices: for $1 \leq i \leq n$, row i of the matrix has been interchanged with row *ipiv*(i).
jpiv	INTEGER. Array, DIMENSION at least max(1,*n*). The pivot indices: for $1 \leq j \leq n$, column j of the matrix has been interchanged with column *jpiv*(j).
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *k* >0, *U*(*k*, *k*) is likely to produce overflow if we try to solve for *x* in *A*x* = *b*. So *U* is perturbed to avoid the overflow.

?getf2

Computes the LU factorization of a general m-by-n matrix using partial pivoting with row interchanges (unblocked algorithm).

Syntax

```
call sgetf2( m, n, a, lda, ipiv, info )
call dgetf2( m, n, a, lda, ipiv, info )
call cgetf2( m, n, a, lda, ipiv, info )
call zgetf2( m, n, a, lda, ipiv, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the *LU* factorization of a general *m*-by-*n* matrix *A* using partial pivoting with row interchanges. The factorization has the form

A = *P*L*U*

where p is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$).

Input Parameters

m	INTEGER. The number of rows in the matrix A ($m \geq 0$).
n	INTEGER. The number of columns in A ($n \geq 0$).
a	REAL for sgetf2 DOUBLE PRECISION for dgetf2 COMPLEX for cgetf2 DOUBLE COMPLEX for zgetf2. Array, DIMENSION (*lda*,*). Contains the matrix A to be factored. The second dimension of *a* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).

Output Parameters

a	Overwritten by L and U. The unit diagonal elements of L are not stored.
ipiv	INTEGER. Array, DIMENSION at least max(1,min(*m*,*n*)). The pivot indices: for $1 \leq i \leq n$, row i was interchanged with row *ipiv*(*i*).
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = *i* >0, u_{ii} is 0. The factorization has been completed, but U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations.

?gtts2

Solves a system of linear equations with a tridiagonal matrix using the LU factorization computed by ?gttrf.

Syntax

```
call sgtts2( itrans, n, nrhs, dl, d, du, du2, ipiv, b, ldb )
call dgtts2( itrans, n, nrhs, dl, d, du, du2, ipiv, b, ldb )
call cgtts2( itrans, n, nrhs, dl, d, du, du2, ipiv, b, ldb )
call zgtts2( itrans, n, nrhs, dl, d, du, du2, ipiv, b, ldb )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine solves for X one of the following systems of linear equations with multiple right hand sides:

$A*X = B$, $A^T*X = B$, or $A^H*X = B$ (for complex matrices only), with a tridiagonal matrix A using the LU factorization computed by ?gttrf.

Input Parameters

itrans	INTEGER. Must be 0, 1, or 2. Indicates the form of the equations to be solved: If *itrans* = 0, then $A*X = B$ (no transpose). If *itrans* = 1, then $A^T*X = B$ (transpose). If *itrans* = 2, then $A^H*X = B$ (conjugate transpose).
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
nrhs	INTEGER. The number of right-hand sides, i.e., the number of columns in *B* (*nrhs* ≥ 0).
dl,*d*,*du*,*du2*,*b*	REAL for sgtts2 DOUBLE PRECISION for dgtts2 COMPLEX for cgtts2 DOUBLE COMPLEX for zgtts2. Arrays: *dl*(*n* - 1), *d*(*n*), *du*(*n* - 1), *du2*(*n* - 2), *b*(*ldb*,*nrhs*). The array *dl* contains the (*n* - 1) multipliers that define the matrix *L* from the *LU* factorization of *A*. The array *d* contains the *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*. The array *du* contains the (*n* - 1) elements of the first super-diagonal of *U*. The array *du2* contains the (*n* - 2) elements of the second super-diagonal of *U*. The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
ldb	INTEGER. The leading dimension of *b*; must be *ldb* ≥ max(1, *n*).
ipiv	INTEGER. Array, DIMENSION (*n*). The pivot indices array, as returned by ?gttrf.

Output Parameters

b	Overwritten by the solution matrix *X*.

?isnan

Tests input for NaN.

Syntax

val = sisnan(*sin*)

val = disnan(*din*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This logical routine returns .TRUE. if its argument is NaN, and .FALSE. otherwise.

Input Parameters

sin	REAL for sisnan Input to test for NaN.

din	DOUBLE PRECISION for disnan Input to test for NaN.

Output Parameters

val	Logical. Result of the test.

?laisnan

Tests input for NaN.

Syntax

```
val = slaisnan( sin1, sin2 )
val = dlaisnan( din1, din2 )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This logical routine checks for NaNs (NaN stands for 'Not A Number') by comparing its two arguments for inequality. NaN is the only floating-point value where NaN ≠ NaN returns .TRUE. To check for NaNs, pass the same variable as both arguments.

This routine is not for general use. It exists solely to avoid over-optimization in ?isnan.

Input Parameters

sin1, *sin2*	REAL for sisnan Two numbers to compare for inequality.
din2, *din2*	DOUBLE PRECISION for disnan Two numbers to compare for inequality.

Output Parameters

val	Logical. Result of the comparison.

?labrd

Reduces the first nb rows and columns of a general matrix to a bidiagonal form.

Syntax

```
call slabrd( m, n, nb, a, lda, d, e, tauq, taup, x, ldx, y, ldy )
call dlabrd( m, n, nb, a, lda, d, e, tauq, taup, x, ldx, y, ldy )
call clabrd( m, n, nb, a, lda, d, e, tauq, taup, x, ldx, y, ldy )
call zlabrd( m, n, nb, a, lda, d, e, tauq, taup, x, ldx, y, ldy )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine reduces the first *nb* rows and columns of a general *m*-by-*n* matrix *A* to upper or lower bidiagonal form by an orthogonal/unitary transformation *Q'*A*P*, and returns the matrices *X* and *Y* which are needed to apply the transformation to the unreduced part of *A*.

if *m* ≥ *n*, *A* is reduced to upper bidiagonal form; if *m* < *n*, to lower bidiagonal form.

The matrices *Q* and *P* are represented as products of elementary reflectors: *Q = H(1)*(2)* ...*H(nb)*, and *P = G(1)*G(2)* ...*G(nb)*

Each *H*(i) and *G*(i) has the form

*H(i) = I - tauq*v*v'* and *G(i) = I - taup*u*u'*

where *tauq* and *taup* are scalars, and *v* and *u* are vectors.

The elements of the vectors *v* and *u* together form the *m*-by-*nb* matrix *V* and the *nb*-by-*n* matrix *U'* which are needed, with *X* and *Y*, to apply the transformation to the unreduced part of the matrix, using a block update of the form: *A := A - V*Y' - X*U'*.

This is an auxiliary routine called by ?gebrd.

Input Parameters

m	INTEGER. The number of rows in the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
nb	INTEGER. The number of leading rows and columns of *A* to be reduced.
a	REAL for slabrd DOUBLE PRECISION for dlabrd COMPLEX for clabrd DOUBLE COMPLEX for zlabrd. Array *a*(*lda*,*) contains the matrix *A* to be reduced. The second dimension of *a* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
ldx	INTEGER. The leading dimension of the output array *x*; must beat least max(1, *m*).
ldy	INTEGER. The leading dimension of the output array *y*; must beat least max(1, *n*).

Output Parameters

a	On exit, the first *nb* rows and columns of the matrix are overwritten; the rest of the array is unchanged. if *m* ≥ *n*, elements on and below the diagonal in the first *nb* columns, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors; and elements above the diagonal in the first *nb* rows, with the array *taup*, represent the orthogonal/unitary matrix *p* as a product of elementary reflectors. if *m* < *n*, elements below the diagonal in the first *nb* columns, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and elements on and above the diagonal in the first *nb* rows, with the array *taup*, represent the orthogonal/unitary matrix *p* as a product of elementary reflectors.
d, *e*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays, DIMENSION (*nb*) each. The array *d* contains the diagonal elements of the first *nb* rows and columns of the reduced matrix:

	d(i) = a(i,i). The array e contains the off-diagonal elements of the first nb rows and columns of the reduced matrix.
tauq, taup	REAL for slabrd DOUBLE PRECISION for dlabrd COMPLEX for clabrd DOUBLE COMPLEX for zlabrd. Arrays, DIMENSION (nb) each. Contain scalar factors of the elementary reflectors which represent the orthogonal/unitary matrices Q and P, respectively.
x, y	REAL for slabrd DOUBLE PRECISION for dlabrd COMPLEX for clabrd DOUBLE COMPLEX for zlabrd. Arrays, dimension x(ldx, nb), y(ldy, nb). The array x contains the m-by-nb matrix X required to update the unreduced part of A. The array y contains the n-by-nb matrix Y required to update the unreduced part of A.

Application Notes

if $m \geq n$, then for the elementary reflectors H(i) and G(i),

v(1:i-1) = 0, v(i) = 1, and v(i:m) is stored on exit in a(i:m, i); u(1:i) = 0, u(i+1) = 1, and u(i+1:n) is stored on exit in a(i, i+1:n);

tauq is stored in tauq(i) and taup in taup(i).

if $m < n$,

v(1:i) = 0, v(i+1) = 1, and v(i+1:m) is stored on exit in a(i+2:m, i); u(1:i-1) = 0, u(i) = 1, and u(i:n) is stored on exit in a(i, i+1:n); tauq is stored in tauq(i) and taup in taup(i).

The contents of a on exit are illustrated by the following examples with nb = 2:

$m = 6,\ n = 5\ (m > n)$

$$\begin{bmatrix} 1 & 1 & u_1 & u_1 & u_1 \\ v_1 & 1 & 1 & u_2 & u_2 \\ v_1 & v_2 & a & a & a \\ v_1 & v_2 & a & a & a \\ v_1 & v_2 & a & a & a \\ v_1 & v_2 & a & a & a \end{bmatrix}$$

$m = 5,\ n = 6\ (m < n)$

$$\begin{bmatrix} 1 & u_1 & u_1 & u_1 & u_1 & u_1 \\ 1 & 1 & u_2 & u_2 & u_2 & u_2 \\ v_1 & 1 & a & a & a & a \\ v_1 & v_2 & a & a & a & a \\ v_1 & v_2 & a & a & a & a \end{bmatrix}$$

where a denotes an element of the original matrix which is unchanged, v_i denotes an element of the vector defining H(i), and u_i an element of the vector defining G(i).

?lacn2

Estimates the 1-norm of a square matrix, using reverse communication for evaluating matrix-vector products.

Syntax

```
call slacn2( n, v, x, isgn, est, kase, isave )
call dlacn2( n, v, x, isgn, est, kase, isave )
call clacn2( n, v, x, est, kase, isave )
call zlacn2( n, v, x, est, kase, isave )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine estimates the 1-norm of a square, real or complex matrix A. Reverse communication is used for evaluating matrix-vector products.

Input Parameters

n	INTEGER. The order of the matrix A ($n \geq 1$).
v, *x*	REAL for slacn2 DOUBLE PRECISION for dlacn2 COMPLEX for clacn2 DOUBLE COMPLEX for zlacn2. Arrays, DIMENSION (n) each. v is a workspace array. x is used as input after an intermediate return.
isgn	INTEGER. Workspace array, DIMENSION (n), used with real flavors only.
est	REAL for slacn2/clacn2 DOUBLE PRECISION for dlacn2/zlacn2 On entry with *kase* set to 1 or 2, and *isave*(1) = 1, *est* must be unchanged from the previous call to the routine.
kase	INTEGER. On the initial call to the routine, *kase* must be set to 0.
isave	INTEGER. Array, DIMENSION (3). Contains variables from the previous call to the routine.

Output Parameters

est	An estimate (a lower bound) for norm(A).
kase	On an intermediate return, *kase* is set to 1 or 2, indicating whether x is overwritten by $A*x$ or A^T*x for real flavors and $A*x$ or A^H*x for complex flavors. On the final return, *kase* is set to 0.
v	On the final return, $v = A*w$, where *est* = norm(v)/norm(w) (w is not returned).
x	On an intermediate return, x is overwritten by

	$A*x$, if $kase = 1$, A^T*x, if $kase = 2$ (for real flavors), A^H*x, if $kase = 2$ (for complex flavors), and the routine must be re-called with all the other parameters unchanged.
isave	This parameter is used to save variables between calls to the routine.

?lacon

Estimates the 1-norm of a square matrix, using reverse communication for evaluating matrix-vector products.

Syntax

```
call slacon( n, v, x, isgn, est, kase )
call dlacon( n, v, x, isgn, est, kase )
call clacon( n, v, x, est, kase )
call zlacon( n, v, x, est, kase )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine estimates the 1-norm of a square, real/complex matrix A. Reverse communication is used for evaluating matrix-vector products.

WARNING The `?lacon` routine is not thread-safe. It is deprecated and retained for the backward compatibility only. Use the thread-safe `?lacn2` routine instead.

Input Parameters

n	INTEGER. The order of the matrix A ($n \geq 1$).
v, *x*	REAL for slacon DOUBLE PRECISION for dlacon COMPLEX for clacon DOUBLE COMPLEX for zlacon. Arrays, DIMENSION (n) each. v is a workspace array. x is used as input after an intermediate return.
isgn	INTEGER. Workspace array, DIMENSION (n), used with real flavors only.
est	REAL for slacon/clacon DOUBLE PRECISION for dlacon/zlacon An estimate that with *kase*=1 or 2 should be unchanged from the previous call to ?lacon.
kase	INTEGER. On the initial call to ?lacon, *kase* should be 0.

Output Parameters

est	REAL for slacon/clacon DOUBLE PRECISION for dlacon/zlacon An estimate (a lower bound) for norm(*A*).
kase	On an intermediate return, *kase* will be 1 or 2, indicating whether *x* should be overwritten by *A***x* or A^T**x* for real flavors and *A***x* or A^H**x* for complex flavors. On the final return from ?lacon, *kase* will again be 0.
v	On the final return, *v* = *A***w*, where *est* = norm(*v*)/norm(*w*) (*w* is not returned).
x	On an intermediate return, *x* should be overwritten by *A***x*, if *kase* = 1, A^T**x*, if *kase* = 2 (for real flavors), A^H**x*, if *kase* = 2 (for complex flavors), and ?lacon must be re-called with all the other parameters unchanged.

?lacpy

Copies all or part of one two-dimensional array to another.

Syntax

```
call slacpy( uplo, m, n, a, lda, b, ldb )
call dlacpy( uplo, m, n, a, lda, b, ldb )
call clacpy( uplo, m, n, a, lda, b, ldb )
call zlacpy( uplo, m, n, a, lda, b, ldb )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine copies all or part of a two-dimensional matrix *A* to another matrix *B*.

Input Parameters

uplo	CHARACTER*1. Specifies the part of the matrix *A* to be copied to *B*. If *uplo* = 'U', the upper triangular part of *A*; if *uplo* = 'L', the lower triangular part of *A*. Otherwise, all of the matrix *A* is copied.
m	INTEGER. The number of rows in the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A* (*n* ≥ 0).
a	REAL for slacpy DOUBLE PRECISION for dlacpy COMPLEX for clacpy DOUBLE COMPLEX for zlacpy. Array *a*(*lda*,*), contains the *m*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*).

If *uplo* = 'U', only the upper triangle or trapezoid is accessed; if *uplo* = 'L', only the lower triangle or trapezoid is accessed.

lda — INTEGER. The leading dimension of *a*; *lda* ≥ max(1, *m*).

ldb — INTEGER. The leading dimension of the output array *b*; *ldb* ≥ max(1, *m*).

Output Parameters

b — REAL for slacpy
DOUBLE PRECISION for dlacpy
COMPLEX for clacpy
DOUBLE COMPLEX for zlacpy.
Array *b*(*ldb*,*), contains the *m*-by-*n* matrix *B*.
The second dimension of *b* must be at least max(1,*n*).
On exit, *B* = *A* in the locations specified by *uplo*.

?ladiv

Performs complex division in real arithmetic, avoiding unnecessary overflow.

Syntax

```
call sladiv( a, b, c, d, p, q )
call dladiv( a, b, c, d, p, q )
res = cladiv( x, y )
res = zladiv( x, y )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routines sladiv/dladiv perform complex division in real arithmetic as

$$p + iq = \frac{a + ib}{c + id}$$

Complex functions cladiv/zladiv compute the result as

res = *x*/*y*,

where *x* and *y* are complex. The computation of *x* / *y* will not overflow on an intermediary step unless the results overflows.

Input Parameters

a, *b*, *c*, *d* — REAL for sladiv
DOUBLE PRECISION for dladiv
The scalars *a*, *b*, *c*, and *d* in the above expression (for real flavors only).

x, *y* — COMPLEX for cladiv
DOUBLE COMPLEX for zladiv
The complex scalars *x* and *y* (for complex flavors only).

Output Parameters

p, *q*	REAL for sladiv DOUBLE PRECISION for dladiv The scalars *p* and *q* in the above expression (for real flavors only).
res	COMPLEX for cladiv DOUBLE COMPLEX for zladiv Contains the result of division *x* / *y*.

?lae2

Computes the eigenvalues of a 2-by-2 symmetric matrix.

Syntax

```
call slae2( a, b, c, rt1, rt2 )
call dlae2( a, b, c, rt1, rt2 )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routines sla2/dlae2 compute the eigenvalues of a 2-by-2 symmetric matrix

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

On return, *rt1* is the eigenvalue of larger absolute value, and *rt1* is the eigenvalue of smaller absolute value.

Input Parameters

a, *b*, *c*	REAL for slae2 DOUBLE PRECISION for dlae2 The elements *a*, *b*, and *c* of the 2-by-2 matrix above.

Output Parameters

rt1, *rt2*	REAL for slae2 DOUBLE PRECISION for dlae2 The computed eigenvalues of larger and smaller absolute value, respectively.

Application Notes

rt1 is accurate to a few ulps barring over/underflow. *rt2* may be inaccurate if there is massive cancellation in the determinant *a***c*-*b***b*; higher precision or correctly rounded or correctly truncated arithmetic would be needed to compute *rt2* accurately in all cases.

Overflow is possible only if *rt1* is within a factor of 5 of overflow. Underflow is harmless if the input data is 0 or exceeds

underflow_threshold / macheps.

?laebz

Computes the number of eigenvalues of a real symmetric tridiagonal matrix which are less than or equal to a given value, and performs other tasks required by the routine `?stebz`.

Syntax

```
call slaebz( ijob, nitmax, n, mmax, minp, nbmin, abstol, reltol, pivmin, d, e, e2,
nval, ab, c, mout, nab, work, iwork, info )
```

```
call dlaebz( ijob, nitmax, n, mmax, minp, nbmin, abstol, reltol, pivmin, d, e, e2,
nval, ab, c, mout, nab, work, iwork, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?laebz` contains the iteration loops which compute and use the function $n(w)$, which is the count of eigenvalues of a symmetric tridiagonal matrix T less than or equal to its argument w. It performs a choice of two types of loops:

ijob	=1, followed by
ijob	=2: It takes as input a list of intervals and returns a list of sufficiently small intervals whose union contains the same eigenvalues as the union of the original intervals. The input intervals are $(ab(j,1),ab(j,2)]$, $j=1,...,minp$. The output interval $(ab(j,1),ab(j,2)]$ will contain eigenvalues $nab(j,1)+1,...,nab(j,2)$, where $1 \leq j \leq mout$.
ijob	=3: It performs a binary search in each input interval $(ab(j,1),ab(j,2)]$ for a point $w(j)$ such that $n(w(j))=nval(j)$, and uses $c(j)$ as the starting point of the search. If such a $w(j)$ is found, then on output $ab(j,1)=ab(j,2)=w$. If no such $w(j)$ is found, then on output $(ab(j,1),ab(j,2)]$ will be a small interval containing the point where $n(w)$ jumps through $nval(j)$, unless that point lies outside the initial interval.

Note that the intervals are in all cases half-open intervals, that is, of the form $(a,b]$, which includes b but not a .

To avoid underflow, the matrix should be scaled so that its largest element is no greater than $overflow^{1/2} * overflow^{1/4}$ in absolute value. To assure the most accurate computation of small eigenvalues, the matrix should be scaled to be not much smaller than that, either.

NOTE In general, the arguments are not checked for unreasonable values.

Input Parameters

ijob	`INTEGER`. Specifies what is to be done: = 1: Compute *nab* for the initial intervals. = 2: Perform bisection iteration to find eigenvalues of T. = 3: Perform bisection iteration to invert $n(w)$, i.e., to find a point which has a specified number of eigenvalues of T to its left. Other values will cause `?laebz` to return with *info*=-1.

nitmax — INTEGER. The maximum number of "levels" of bisection to be performed, i.e., an interval of width *W* will not be made smaller than $2^{-nitmax}*W$. If not all intervals have converged after *nitmax* iterations, then *info* is set to the number of non-converged intervals.

n — INTEGER. The dimension *n* of the tridiagonal matrix *T*. It must be at least 1.

mmax — INTEGER. The maximum number of intervals. If more than *mmax* intervals are generated, then ?laebz will quit with *info*=*mmax*+1.

minp — INTEGER. The initial number of intervals. It may not be greater than *mmax*.

nbmin — INTEGER. The smallest number of intervals that should be processed using a vector loop. If zero, then only the scalar loop will be used.

abstol — REAL for slaebz
DOUBLE PRECISION for dlaebz.
The minimum (absolute) width of an interval. When an interval is narrower than *abstol*, or than *reltol* times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, i.e., converged. This must be at least zero.

reltol — REAL for slaebz
DOUBLE PRECISION for dlaebz.
The minimum relative width of an interval. When an interval is narrower than *abstol*, or than *reltol* times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, i.e., converged. Note: this should always be at least *radix***machine epsilon*.

pivmin — REAL for slaebz
DOUBLE PRECISION for dlaebz.
The minimum absolute value of a "pivot" in the Sturm sequence loop. This value **must** be at least (max |e(j)**2|**safe_min*) and at least *safe_min*, where *safe_min* is at least the smallest number that can divide one without overflow.

d, *e*, *e2* — REAL for slaebz
DOUBLE PRECISION for dlaebz.
Arrays, dimension (*n*) each. The array *d* contains the diagonal elements of the tridiagonal matrix *T*.
The array *e* contains the off-diagonal elements of the tridiagonal matrix *T* in positions 1 through *n*-1. *e*(*n*)vis arbitrary.
The array *e2* contains the squares of the off-diagonal elements of the tridiagonal matrix *T*. *e2*(*n*) is ignored.

nval — INTEGER.
Array, dimension (*minp*).
If *ijob*=1 or 2, not referenced.
If *ijob*=3, the desired values of *n*(*w*).

ab — REAL for slaebz
DOUBLE PRECISION for dlaebz.
Array, dimension (*mmax*,2) The endpoints of the intervals. *ab*(j,1) is *a*(*j*), the left endpoint of the *j*-th interval, and *ab*(j,2) is *b*(*j*), the right endpoint of the *j*-th interval.

c — REAL for slaebz
DOUBLE PRECISION for dlaebz.
Array, dimension (*mmax*)
If *ijob*=1, ignored.
If *ijob*=2, workspace.

If *ijob*=3, then on input *c*(*j*) should be initialized to the first search point in the binary search.

nab INTEGER.
Array, dimension (*mmax*,2)
If *ijob*=2, then on input, *nab*(i,j) should be set. It must satisfy the condition:
n(*ab*(i,1)) ≤ *nab*(i,1) ≤ *nab*(i,2) ≤ *n*(*ab*(i,2)), which means that in interval i only eigenvalues *nab*(i,1)+1,...,*nab*(i,2) are considered. Usually, *nab*(i,j)=*n*(*ab*(i,j)), from a previous call to ?laebz with *ijob*=1.
If *ijob*=3, normally, *nab* should be set to some distinctive value(s) before ?laebz is called.

work REAL for slaebz
DOUBLE PRECISION for dlaebz.
Workspace array, dimension (*mmax*).

iwork INTEGER.
Workspace array, dimension (*mmax*).

Output Parameters

nval The elements of *nval* will be reordered to correspond with the intervals in *ab*. Thus, *nval*(*j*) on output will not, in general be the same as *nval*(*j*) on input, but it will correspond with the interval (*ab*(j,1),*ab*(j,2)] on output.

ab The input intervals will, in general, be modified, split, and reordered by the calculation.

mout INTEGER.
If *ijob*=1, the number of eigenvalues in the intervals.
If *ijob*=2 or 3, the number of intervals output.
If *ijob*=3, *mout* will equal *minp*.

nab If *ijob*=1, then on output *nab*(i,j) will be set to *N*(*ab*(i,j)).
If *ijob*=2, then on output, *nab*(i,j) will contain max(*na*(*k*, min(*nb*(*k*), *N*(*ab*(i,j)))), where *k* is the index of the input interval that the output interval (*ab*(j,1),*ab*(j,2)] came from, and *na*(*k*) and *nb*(*k*) are the input values of *nab*(*k*,1) and *nab*(*k*,2).
If *ijob*=3, then on output, *nab*(i,j) contains *N*(*ab*(i,j)), unless *N*(*w*) > *nval*(i) for all search points *w*, in which case *nab*(i,1) will not be modified, i.e., the output value will be the same as the input value (modulo reorderings, see *nval* and *ab*), or unless *N*(*w*) < *nval*(i) for all search points *w*, in which case *nab*(i,2) will not be modified.

info INTEGER.
If *info* = 0 - all intervals converged
If *info* = 1--*mmax* - the last *info* interval did not converge.
If info = mmax+1 - more than *mmax* intervals were generated

Application Notes

This routine is intended to be called only by other LAPACK routines, thus the interface is less user-friendly. It is intended for two purposes:

(a) finding eigenvalues. In this case, ?laebz should have one or more initial intervals set up in *ab*, and ?laebz should be called with *ijob*=1. This sets up *nab*, and also counts the eigenvalues. Intervals with no eigenvalues would usually be thrown out at this point. Also, if not all the eigenvalues in an interval i are

desired, *nab*(i,1) can be increased or *nab*(i,2) decreased. For example, set *nab*(i,1)=*nab*(i,2)-1 to get the largest eigenvalue. ?laebz is then called with *ijob*=2 and *mmax* no smaller than the value of *mout* returned by the call with *ijob*=1. After this (*ijob*=2) call, eigenvalues *nab*(i,1)+1 through *nab*(i,2) are approximately *ab*(i,1) (or *ab*(i,2)) to the tolerance specified by *abstol* and *reltol*.

(b) finding an interval (*a*',*b*'] containing eigenvalues *w*(f),...,*w*(l). In this case, start with a Gershgorin interval (*a*,*b*). Set up *ab* to contain 2 search intervals, both initially (*a*,*b*). One *nval* element should contain f-1 and the other should contain l, while *c* should contain *a* and *b*, respectively. *nab*(i,1) should be -1 and *nab*(i,2) should be *n*+1, to flag an error if the desired interval does not lie in (*a*,*b*). ?laebz is then called with *ijob*=3. On exit, if *w*(f-1) < *w*(f), then one of the intervals -- j -- will have *ab*(j,1)=*ab*(j,2) and *nab*(j,1)=*nab*(j,2)=f-1, while if, to the specified tolerance, *w*(f-k)=...=*w*(f+r), k > 0 and r ≥ 0, then the interval will have *n*(*ab*(j,1))=*nab*(j,1)=f-k and *n*(*ab*(j,2))=*nab*(j,2)=f+r. The cases *w*(l) < *w*(l+1) and *w*(l-r)=...=*w*(l+k) are handled similarly.

?laed0

Used by ?stedc. *Computes all eigenvalues and corresponding eigenvectors of an unreduced symmetric tridiagonal matrix using the divide and conquer method.*

Syntax

```
call slaed0( icompq, qsiz, n, d, e, q, ldq, qstore, ldqs, work, iwork, info )
call dlaed0( icompq, qsiz, n, d, e, q, ldq, qstore, ldqs, work, iwork, info )
call claed0( qsiz, n, d, e, q, ldq, qstore, ldqs, rwork, iwork, info )
call zlaed0( qsiz, n, d, e, q, ldq, qstore, ldqs, rwork, iwork, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Real flavors of this routine compute all eigenvalues and (optionally) corresponding eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method.

Complex flavors claed0/zlaed0 compute all eigenvalues of a symmetric tridiagonal matrix which is one diagonal block of those from reducing a dense or band Hermitian matrix and corresponding eigenvectors of the dense or band matrix.

Input Parameters

icompq	INTEGER. Used with real flavors only. If *icompq* = 0, compute eigenvalues only. If *icompq* = 1, compute eigenvectors of original dense symmetric matrix also. On entry, the array *q* must contain the orthogonal matrix used to reduce the original matrix to tridiagonal form. If *icompq* = 2, compute eigenvalues and eigenvectors of the tridiagonal matrix.
qsiz	INTEGER. The dimension of the orthogonal/unitary matrix used to reduce the full matrix to tridiagonal form; *qsiz* ≥ *n* (for real flavors, *qsiz* ≥ *n* if *icompq* = 1).

n	INTEGER. The dimension of the symmetric tridiagonal matrix ($n \geq 0$).
d, *e*, *rwork*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays: *d*(*) contains the main diagonal of the tridiagonal matrix. The dimension of *d* must be at least max(1, *n*). *e*(*) contains the off-diagonal elements of the tridiagonal matrix. The dimension of *e* must be at least max(1, *n*-1). *rwork*(*) is a workspace array used in complex flavors only. The dimension of *rwork* must be at least $(1 + 3n + 2n\lg(n) + 3n^2)$, where lg(*n*) = smallest integer k such that $2^k \geq n$.
q, *qstore*	REAL for slaed0 DOUBLE PRECISION for dlaed0 COMPLEX for claed0 DOUBLE COMPLEX for zlaed0. Arrays: *q*(*ldq*, *), *qstore*(*ldqs*, *). The second dimension of these arrays must be at least max(1, *n*). *For real flavors:* If *icompq* = 0, array *q* is not referenced. If *icompq* = 1, on entry, *q* is a subset of the columns of the orthogonal matrix used to reduce the full matrix to tridiagonal form corresponding to the subset of the full matrix which is being decomposed at this time. If *icompq* = 2, on entry, *q* will be the identity matrix. The array *qstore* is a workspace array referenced only when *icompq* = 1. Used to store parts of the eigenvector matrix when the updating matrix multiplies take place. For complex flavors: On entry, *q* must contain an *qsiz*-by-*n* matrix whose columns are unitarily orthonormal. It is a part of the unitary matrix that reduces the full dense Hermitian matrix to a (reducible) symmetric tridiagonal matrix. The array *qstore* is a workspace array used to store parts of the eigenvector matrix when the updating matrix multiplies take place.
ldq	INTEGER. The leading dimension of the array *q*; *ldq* ≥ max(1, *n*).
ldqs	INTEGER. The leading dimension of the array *qstore*; *ldqs* ≥ max(1, *n*).
work	REAL for slaed0 DOUBLE PRECISION for dlaed0. Workspace array, used in real flavors only. If *icompq* = 0 or 1, the dimension of *work* must be at least $(1 + 3n + 2n\lg(n) + 3n^2)$, where lg(*n*) = smallest integer k such that $2^k \geq n$. If *icompq* = 2, the dimension of *work* must be at least $(4n + n^2)$.
iwork	INTEGER. Workspace array. For real flavors, if *icompq* = 0 or 1, and for complex flavors, the dimension of *iwork* must be at least (6+6*n*+5*n*lg(*n*)). For real flavors, if *icompq* = 2, the dimension of *iwork* must be at least (3+5*n*).

Output Parameters

d	On exit, contains eigenvalues in ascending order.
e	On exit, the array is destroyed.
q	If *icompq* = 2, on exit, *q* contains the eigenvectors of the tridiagonal matrix.
info	INTEGER.

If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.
If *info* = *i* >0, the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *i*/(*n*+1) through mod(*i*, *n*+1).

?laed1

Used by sstedc/dstedc*. Computes the updated eigensystem of a diagonal matrix after modification by a rank-one symmetric matrix. Used when the original matrix is tridiagonal.*

Syntax

```
call slaed1( n, d, q, ldq, indxq, rho, cutpnt, work, iwork, info )
call dlaed1( n, d, q, ldq, indxq, rho, cutpnt, work, iwork, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laed1 computes the updated eigensystem of a diagonal matrix after modification by a rank-one symmetric matrix. This routine is used only for the eigenproblem which requires all eigenvalues and eigenvectors of a tridiagonal matrix. ?laed7 handles the case in which eigenvalues only or eigenvalues and eigenvectors of a full symmetric matrix (which was reduced to tridiagonal form) are desired.

$T = Q(\text{in}) * (D(\text{in}) + rho * Z * Z^T) * Q^T(\text{in}) = Q(\text{out}) * D(\text{out}) * Q^T(\text{out})$

where $Z = Q^T u$, u is a vector of length n with ones in the *cutpnt* and (*cutpnt*+1) -th elements and zeros elsewhere. The eigenvectors of the original matrix are stored in *Q*, and the eigenvalues are in *D*. The algorithm consists of three stages:

The first stage consists of deflating the size of the problem when there are multiple eigenvalues or if there is a zero in the *z* vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine ?laed2.

The second stage consists of calculating the updated eigenvalues. This is done by finding the roots of the secular equation via the routine ?laed4 (as called by ?laed3). This routine also calculates the eigenvectors of the current problem.

The final stage consists of computing the updated eigenvectors directly using the updated eigenvalues. The eigenvectors for the current problem are multiplied with the eigenvectors from the overall problem.

Input Parameters

n	INTEGER. The dimension of the symmetric tridiagonal matrix ($n \geq 0$).
d, *q*, *work*	REAL for slaed1 DOUBLE PRECISION for dlaed1. Arrays: *d*(*) contains the eigenvalues of the rank-1-perturbed matrix. The dimension of *d* must be at least max(1, *n*). *q*(*ldq*, *) contains the eigenvectors of the rank-1-perturbed matrix. The second dimension of *q* must be at least max(1, *n*). *work*(*) is a workspace array, dimension at least ($4n+n^2$).

ldq	INTEGER. The leading dimension of the array *q*; *ldq* ≥ max(1, *n*).
indxq	INTEGER. Array, dimension (*n*). On entry, the permutation which separately sorts the two subproblems in *d* into ascending order.
rho	REAL for slaed1 DOUBLE PRECISION for dlaed1. The subdiagonal entry used to create the rank-1 modification. This parameter can be modified by ?laed2, where it is input/output.
cutpnt	INTEGER. The location of the last eigenvalue in the leading sub-matrix. min(1,*n*) ≤ *cutpnt* ≤ *n*/2.
iwork	INTEGER. Workspace array, dimension (4*n*).

Output Parameters

d	On exit, contains the eigenvalues of the repaired matrix.
q	On exit, *q* contains the eigenvectors of the repaired tridiagonal matrix.
indxq	On exit, contains the permutation which will reintegrate the subproblems back into sorted order, that is, *d*(*indxq*(i = 1, *n*)) will be in ascending order.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1, an eigenvalue did not converge.

?laed2

Used by sstedc/dstedc*. Merges eigenvalues and deflates secular equation. Used when the original matrix is tridiagonal.*

Syntax

```
call slaed2( k, n, n1, d, q, ldq, indxq, rho, z, dlamda, w, q2, indx, indxc, indxp, coltyp, info )
call dlaed2( k, n, n1, d, q, ldq, indxq, rho, z, dlamda, w, q2, indx, indxc, indxp, coltyp, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laed2 merges the two sets of eigenvalues together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more eigenvalues are close together or if there is a tiny entry in the *z* vector. For each such occurrence the order of the related secular equation problem is reduced by one.

Input Parameters

k	INTEGER. The number of non-deflated eigenvalues, and the order of the related secular equation (0 ≤ *k* ≤ *n*).

n	INTEGER. The dimension of the symmetric tridiagonal matrix ($n \geq 0$).
n1	INTEGER. The location of the last eigenvalue in the leading sub-matrix; $\min(1,n) \leq n1 \leq n/2$.
d, *q*, *z*	REAL for slaed2 DOUBLE PRECISION for dlaed2. Arrays: *d*(*) contains the eigenvalues of the two submatrices to be combined. The dimension of *d* must be at least max(1, *n*). *q*(*ldq*, *) contains the eigenvectors of the two submatrices in the two square blocks with corners at (1,1), (*n1*,*n1*) and (*n1*+1,*n1*+1), (*n*,*n*). The second dimension of *q* must be at least max(1, *n*). *z*(*) contains the updating vector (the last row of the first sub-eigenvector matrix and the first row of the second sub-eigenvector matrix).
ldq	INTEGER. The leading dimension of the array *q*; $ldq \geq \max(1, n)$.
indxq	INTEGER. Array, dimension (*n*). On entry, the permutation which separately sorts the two subproblems in *d* into ascending order. Note that elements in the second half of this permutation must first have *n1* added to their values.
rho	REAL for slaed2 DOUBLE PRECISION for dlaed2. On entry, the off-diagonal element associated with the rank-1 cut which originally split the two submatrices which are now being recombined.
indx, *indxp*	INTEGER. Workspace arrays, dimension (*n*) each. Array *indx* contains the permutation used to sort the contents of *dlamda* into ascending order. Array *indxp* contains the permutation used to place deflated values of *d* at the end of the array. *indxp*(1:*k*) points to the nondeflated *d*-values and *indxp*(*k*+1:*n*) points to the deflated eigenvalues.
coltyp	INTEGER. Workspace array, dimension (*n*). During execution, a label which will indicate which of the following types a column in the *q2* matrix is: 1 : non-zero in the upper half only; 2 : dense; 3 : non-zero in the lower half only; 4 : deflated.

Output Parameters

d	On exit, *d* contains the trailing (*n*-*k*) updated eigenvalues (those which were deflated) sorted into increasing order.
q	On exit, *q* contains the trailing (*n*-*k*) updated eigenvectors (those which were deflated) in its last *n*-*k* columns.
z	On exit, *z* content is destroyed by the updating process.
indxq	Destroyed on exit.
rho	On exit, *rho* has been modified to the value required by ?laed3.
dlamda, *w*, *q2*	REAL for slaed2 DOUBLE PRECISION for dlaed2. Arrays: *dlamda*(*n*), *w*(*n*), *q2*($n1^2+(n-n1)^2$).

The array *dlamda* contains a copy of the first *k* eigenvalues which is used by ?laed3 to form the secular equation.
The array *w* contains the first *k* values of the final deflation-altered *z*-vector which is passed to ?laed3.
The array *q2* contains a copy of the first *k* eigenvectors which is used by ?laed3 in a matrix multiply (sgemm/dgemm) to solve for the new eigenvectors.

indxc — INTEGER. Array, dimension (*n*).
The permutation used to arrange the columns of the deflated *q* matrix into three groups: the first group contains non-zero elements only at and above *n1*, the second contains non-zero elements only below *n1*, and the third is dense.

coltyp — On exit, *coltyp*(i) is the number of columns of type i, for i=1 to 4 only (see the definition of types in the description of *coltyp* in *Input Parameters*).

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = -*i*, the *i*-th parameter had an illegal value.

?laed3

Used by sstedc/dstedc. *Finds the roots of the secular equation and updates the eigenvectors. Used when the original matrix is tridiagonal.*

Syntax

```
call slaed3( k, n, n1, d, q, ldq, rho, dlamda, q2, indx, ctot, w, s, info )
call dlaed3( k, n, n1, d, q, ldq, rho, dlamda, q2, indx, ctot, w, s, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laed3 finds the roots of the secular equation, as defined by the values in *d*, *w*, and *rho*, between 1 and *k*.

It makes the appropriate calls to ?laed4 and then updates the eigenvectors by multiplying the matrix of eigenvectors of the pair of eigensystems being combined by the matrix of eigenvectors of the *k*-by-*k* system which is solved here.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray X-MP, Cray Y-MP, Cray C-90, or Cray-2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but none are known.

Input Parameters

k — INTEGER. The number of terms in the rational function to be solved by ?laed4 ($k \geq 0$).

n — INTEGER. The number of rows and columns in the *q* matrix. $n \geq k$ (deflation may result in $n > k$).

n1 — INTEGER. The location of the last eigenvalue in the leading sub-matrix; $\min(1,n) \leq n1 \leq n/2$.

q	REAL for slaed3 DOUBLE PRECISION for dlaed3. Array *q*(*ldq*, *). The second dimension of *q* must be at least max(1, *n*). Initially, the first *k* columns of this array are used as workspace.
ldq	INTEGER. The leading dimension of the array *q*; *ldq* ≥ max(1, *n*).
rho	REAL for slaed3 DOUBLE PRECISION for dlaed3. The value of the parameter in the rank one update equation. *rho* ≥ 0 required.
dlamda, *q2*, *w*	REAL for slaed3 DOUBLE PRECISION for dlaed3. Arrays: *dlamda*(*k*), *q2*(*ldq2*, *), *w*(*k*). The first *k* elements of the array *dlamda* contain the old roots of the deflated updating problem. These are the poles of the secular equation. The first *k* columns of the array *q2* contain the non-deflated eigenvectors for the split problem. The second dimension of *q2* must be at least max(1, *n*). The first *k* elements of the array *w* contain the components of the deflation-adjusted updating vector.
indx	INTEGER. Array, dimension (*n*). The permutation used to arrange the columns of the deflated *q* matrix into three groups (see ?laed2). The rows of the eigenvectors found by ?laed4 must be likewise permuted before the matrix multiply can take place.
ctot	INTEGER. Array, dimension (4). A count of the total number of the various types of columns in *q*, as described in *indx*. The fourth column type is any column which has been deflated.
s	REAL for slaed3 DOUBLE PRECISION for dlaed3. Workspace array, dimension (*n1*+1)**k* . Will contain the eigenvectors of the repaired matrix which will be multiplied by the previously accumulated eigenvectors to update the system.

Output Parameters

d	REAL for slaed3 DOUBLE PRECISION for dlaed3. Array, dimension at least max(1, *n*). *d*(i) contains the updated eigenvalues for 1 ≤ i ≤ *k*.
q	On exit, the columns 1 to *k* of *q* contain the updated eigenvectors.
dlamda	May be changed on output by having lowest order bit set to zero on Cray X-MP, Cray Y-MP, Cray-2, or Cray C-90, as described above.
w	Destroyed on exit.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value. If *info* = 1, an eigenvalue did not converge.

?laed4

Used by sstedc/dstedc*. Finds a single root of the secular equation.*

Syntax

```
call slaed4( n, i, d, z, delta, rho, dlam, info )
call dlaed4( n, i, d, z, delta, rho, dlam, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine computes the *i*-th updated eigenvalue of a symmetric rank-one modification to a diagonal matrix whose elements are given in the array *d*, and that

D(i) < *D*(*j*) for *i* < *j*

and that *rho* > 0. This is arranged by the calling routine, and is no loss in generality. The rank-one modified system is thus

diag(*D*) + *rho***Z* * transpose(*Z*).

where we assume the Euclidean norm of *Z* is 1.

The method consists of approximating the rational functions in the secular equation by simpler interpolating rational functions.

Input Parameters

n	INTEGER. The length of all arrays.
i	INTEGER. The index of the eigenvalue to be computed; 1 ≤ i ≤ *n*.
d, *z*	REAL for slaed4 DOUBLE PRECISION for dlaed4 Arrays, dimension (*n*) each. The array *d* contains the original eigenvalues. It is assumed that they are in order, *d*(i) < *d*(*j*) for *i* < *j*. The array *z* contains the components of the updating vector *Z*.
rho	REAL for slaed4 DOUBLE PRECISION for dlaed4 The scalar in the symmetric updating formula.

Output Parameters

delta	REAL for slaed4 DOUBLE PRECISION for dlaed4 Array, dimension (*n*). If *n* ≠ 1, *delta* contains (*d*(*j*) – *lambda_i*) in its *j*-th component. If *n* = 1, then *delta*(1) = 1. The vector *delta* contains the information necessary to construct the eigenvectors.
dlam	REAL for slaed4 DOUBLE PRECISION for dlaed4 The computed *lambda_i*, the *i*-th updated eigenvalue.
info	INTEGER. If *info* = 0, the execution is successful.

If *info* = 1, the updating process failed.

?laed5

Used by sstedc/dstedc. *Solves the 2-by-2 secular equation.*

Syntax

```
call slaed5( i, d, z, delta, rho, dlam )
call dlaed5( i, d, z, delta, rho, dlam )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the *i*-th eigenvalue of a symmetric rank-one modification of a 2-by-2 diagonal matrix diag(*D*) + *rho***Z* * transpose(*Z*).

The diagonal elements in the array *D* are assumed to satisfy

D(i) < *D*(*j*) for *i* < *j* .

We also assume *rho* > 0 and that the Euclidean norm of the vector *Z* is one.

Input Parameters

i	INTEGER. The index of the eigenvalue to be computed; 1 ≤ i ≤ 2.
d, *z*	REAL for slaed5 DOUBLE PRECISION for dlaed5 Arrays, dimension (2) each. The array *d* contains the original eigenvalues. It is assumed that *d*(1) < *d*(2). The array *z* contains the components of the updating vector.
rho	REAL for slaed5 DOUBLE PRECISION for dlaed5 The scalar in the symmetric updating formula.

Output Parameters

delta	REAL for slaed5 DOUBLE PRECISION for dlaed5 Array, dimension (2). The vector *delta* contains the information necessary to construct the eigenvectors.
dlam	REAL for slaed5 DOUBLE PRECISION for dlaed5 The computed *lambda_i*, the *i*-th updated eigenvalue.

?laed6

Used by sstedc/dstedc. *Computes one Newton step in solution of the secular equation.*

Syntax

```
call slaed6( kniter, orgati, rho, d, z, finit, tau, info )
call dlaed6( kniter, orgati, rho, d, z, finit, tau, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes the positive or negative root (closest to the origin) of

$$f(x) = rho + \frac{z(1)}{d(1) - x} + \frac{z(2)}{d(2) - x} + \frac{z(3)}{d(3) - x}$$

It is assumed that if *orgati* = .TRUE. the root is between *d*(2) and *d*(3);otherwise it is between *d*(1) and *d*(2) This routine is called by `?laed4` when necessary. In most cases, the root sought is the smallest in magnitude, though it might not be in some extremely rare situations.

Input Parameters

kniter
INTEGER.
Refer to `?laed4` for its significance.

orgati
LOGICAL.
If *orgati* = .TRUE., the needed root is between *d*(2) and *d*(3); otherwise it is between *d*(1) and *d*(2). See `?laed4` for further details.

rho
REAL for `slaed6`
DOUBLE PRECISION for `dlaed6`
Refer to the equation for *f*(*x*) above.

d, *z*
REAL for `slaed6`
DOUBLE PRECISION for `dlaed6`
Arrays, dimension (3) each.
The array *d* satisfies *d*(1) < *d*(2) < *d*(3).
Each of the elements in the array *z* must be positive.

finit
REAL for `slaed6`
DOUBLE PRECISION for `dlaed6`
The value of *f*(*x*) at 0. It is more accurate than the one evaluated inside this routine (if someone wants to do so).

Output Parameters

tau
REAL for `slaed6`
DOUBLE PRECISION for `dlaed6`
The root of the equation for *f*(*x*).

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = 1, failure to converge.

?laed7

Used by `?stedc`*. Computes the updated eigensystem of a diagonal matrix after modification by a rank-one symmetric matrix. Used when the original matrix is dense.*

Syntax

`call slaed7( icompq, n, qsiz, tlvls, curlvl, curpbm, d, q, ldq, indxq, rho, cutpnt, qstore, qptr, prmptr, perm, givptr, givcol, givnum, work, iwork, info )`

`call dlaed7( icompq, n, qsiz, tlvls, curlvl, curpbm, d, q, ldq, indxq, rho, cutpnt, qstore, qptr, prmptr, perm, givptr, givcol, givnum, work, iwork, info )`

`call claed7( n, cutpnt, qsiz, tlvls, curlvl, curpbm, d, q, ldq, rho, indxq, qstore, qptr, prmptr, perm, givptr, givcol, givnum, work, rwork, iwork, info )`

`call zlaed7( n, cutpnt, qsiz, tlvls, curlvl, curpbm, d, q, ldq, rho, indxq, qstore, qptr, prmptr, perm, givptr, givcol, givnum, work, rwork, iwork, info )`

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?laed7` computes the updated eigensystem of a diagonal matrix after modification by a rank-one symmetric matrix. This routine is used only for the eigenproblem which requires all eigenvalues and optionally eigenvectors of a dense symmetric/Hermitian matrix that has been reduced to tridiagonal form. For real flavors, `slaed1`/`dlaed1` handles the case in which all eigenvalues and eigenvectors of a symmetric tridiagonal matrix are desired.

$T = Q(\text{in}) * (D(\text{in}) + rho * Z * Z^T) * Q^T(\text{in}) = Q(\text{out}) * D(\text{out}) * Q^T(\text{out})$ for real flavors, or

$T = Q(\text{in}) * (D(\text{in}) + rho * Z * Z^H) * Q^H(\text{in}) = Q(\text{out}) * D(\text{out}) * Q^H(\text{out})$ for complex flavors

where $Z = Q^T * u$ for real flavors and $Z = Q^H * u$ for complex flavors, u is a vector of length n with ones in the $cutpnt$ and ($cutpnt + 1$) -th elements and zeros elsewhere. The eigenvectors of the original matrix are stored in Q, and the eigenvalues are in D. The algorithm consists of three stages:

The first stage consists of deflating the size of the problem when there are multiple eigenvalues or if there is a zero in the z vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine `slaed8`/`dlaed8` (for real flavors) or by the routine `slaed2`/`dlaed2` (for complex flavors).

The second stage consists of calculating the updated eigenvalues. This is done by finding the roots of the secular equation via the routine `?laed4` (as called by `?laed9` or `?laed3`). This routine also calculates the eigenvectors of the current problem.

The final stage consists of computing the updated eigenvectors directly using the updated eigenvalues. The eigenvectors for the current problem are multiplied with the eigenvectors from the overall problem.

Input Parameters

icompq — `INTEGER`. Used with real flavors only.
If $icompq = 0$, compute eigenvalues only.
If $icompq = 1$, compute eigenvectors of original dense symmetric matrix also. On entry, the array q must contain the orthogonal matrix used to reduce the original matrix to tridiagonal form.

n	INTEGER. The dimension of the symmetric tridiagonal matrix ($n \geq 0$).
cutpnt	INTEGER. The location of the last eigenvalue in the leading sub-matrix. $\min(1,n) \leq cutpnt \leq n$.
qsiz	INTEGER. The dimension of the orthogonal/unitary matrix used to reduce the full matrix to tridiagonal form; $qsiz \geq n$ (for real flavors, $qsiz \geq n$ if $icompq = 1$).
tlvls	INTEGER. The total number of merging levels in the overall divide and conquer tree.
curlvl	INTEGER. The current level in the overall merge routine, $0 \leq curlvl \leq tlvls$.
curpbm	INTEGER. The current problem in the current level in the overall merge routine (counting from upper left to lower right).
d	REAL for slaed7/claed7 DOUBLE PRECISION for dlaed7/zlaed7. Array, dimension at least max(1, *n*). Array *d*(*) contains the eigenvalues of the rank-1-perturbed matrix.
q, *work*	REAL for slaed7 DOUBLE PRECISION for dlaed7 COMPLEX for claed7 DOUBLE COMPLEX for zlaed7. Arrays: *q*(*ldq*, *) contains the eigenvectors of the rank-1-perturbed matrix. The second dimension of *q* must be at least max(1, *n*). *work*(*) is a workspace array, dimension at least (3*n*+2**qsiz***n*) for real flavors and at least (*qsiz***n*) for complex flavors.
ldq	INTEGER. The leading dimension of the array *q*; *ldq* ≥ max(1, *n*).
indxq	INTEGER. Array, dimension (*n*). Contains the permutation that separately sorts the two sub-problems in *d* into ascending order.
rho	REAL for slaed7 /claed7 DOUBLE PRECISION for dlaed7/zlaed7. The subdiagonal element used to create the rank-1 modification.
qstore	REAL for slaed7/claed7 DOUBLE PRECISION for dlaed7/zlaed7. Array, dimension (n^2+1). Serves also as output parameter. Stores eigenvectors of submatrices encountered during divide and conquer, packed together. *qptr* points to beginning of the submatrices.
qptr	INTEGER. Array, dimension (*n*+2). Serves also as output parameter. List of indices pointing to beginning of submatrices stored in *qstore*. The submatrices are numbered starting at the bottom left of the divide and conquer tree, from left to right and bottom to top.
prmptr, *perm*, *givptr*	INTEGER. Arrays, dimension (*n* lg*n*) each. The array *prmptr*(*) contains a list of pointers which indicate where in *perm* a level's permutation is stored. *prmptr*(i+1) – *prmptr*(i) indicates the size of the permutation and also the size of the full, non-deflated problem. The array *perm*(*) contains the permutations (from deflation and sorting) to be applied to each eigenblock. This parameter can be modified by ?laed8, where it is output.

	The array *givptr*(*) contains a list of pointers which indicate where in *givcol* a level's Givens rotations are stored. *givptr*(i+1) – *givptr*(i) indicates the number of Givens rotations.
givcol	INTEGER. Array, dimension (2, $n \lg n$). Each pair of numbers indicates a pair of columns to take place in a Givens rotation.
givnum	REAL for slaed7/claed7 DOUBLE PRECISION for dlaed7/zlaed7. Array, dimension (2, $n \lg n$). Each number indicates the S value to be used in the corresponding Givens rotation.
iwork	INTEGER. Workspace array, dimension ($4n$).
rwork	REAL for claed7 DOUBLE PRECISION for zlaed7. Workspace array, dimension (3*n*+2*qsiz***n*). Used in complex flavors only.

Output Parameters

d	On exit, contains the eigenvalues of the repaired matrix.
q	On exit, *q* contains the eigenvectors of the repaired tridiagonal matrix.
indxq	INTEGER. Array, dimension (*n*). Contains the permutation that reintegrates the subproblems back into a sorted order, that is, *d*(*indxq*(i = 1, *n*)) will be in the ascending order.
rho	This parameter can be modified by ?laed8, where it is input/output.
prmptr, *perm*, *givptr*	INTEGER. Arrays, dimension ($n \lg n$) each. The array *prmptr* contains an updated list of pointers. The array *perm* contains an updated permutation. The array *givptr* contains an updated list of pointers.
givcol	This parameter can be modified by ?laed8, where it is output.
givnum	This parameter can be modified by ?laed8, where it is output.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = –*i*, the *i*-th parameter had an illegal value. If *info* = 1, an eigenvalue did not converge.

?laed8

Used by ?stedc*. Merges eigenvalues and deflates secular equation. Used when the original matrix is dense.*

Syntax

```
call slaed8( icompq, k, n, qsiz, d, q, ldq, indxq, rho, cutpnt, z, dlamda, q2, ldq2,
w, perm, givptr, givcol, givnum, indxp, indx, info )
```

```
call dlaed8( icompq, k, n, qsiz, d, q, ldq, indxq, rho, cutpnt, z, dlamda, q2, ldq2,
w, perm, givptr, givcol, givnum, indxp, indx, info )
```

```
call claed8( k, n, qsiz, q, ldq, d, rho, cutpnt, z, dlamda, q2, ldq2, w, indxp, indx,
indxq, perm, givptr, givcol, givnum, info )
```

```
call zlaed8( k, n, qsiz, q, ldq, d, rho, cutpnt, z, dlamda, q2, ldq2, w, indxp, indx,
indxq, perm, givptr, givcol, givnum, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine merges the two sets of eigenvalues together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more eigenvalues are close together or if there is a tiny element in the z vector. For each such occurrence the order of the related secular equation problem is reduced by one.

Input Parameters

icompq	INTEGER. Used with real flavors only. If $icompq = 0$, compute eigenvalues only. If $icompq = 1$, compute eigenvectors of original dense symmetric matrix also. On entry, the array q must contain the orthogonal matrix used to reduce the original matrix to tridiagonal form.
n	INTEGER. The dimension of the symmetric tridiagonal matrix ($n \geq 0$).
cutpnt	INTEGER. The location of the last eigenvalue in the leading sub-matrix. $\min(1,n) \leq cutpnt \leq n$.
qsiz	INTEGER. The dimension of the orthogonal/unitary matrix used to reduce the full matrix to tridiagonal form; $qsiz \geq n$ (for real flavors, $qsiz \geq n$ if $icompq = 1$).
d, *z*	REAL for slaed8/claed8 DOUBLE PRECISION for dlaed8/zlaed8. Arrays, dimension at least max(1, n) each. The array d(*) contains the eigenvalues of the two submatrices to be combined. On entry, z(*) contains the updating vector (the last row of the first sub-eigenvector matrix and the first row of the second sub-eigenvector matrix). The contents of z are destroyed by the updating process.
q	REAL for slaed8 DOUBLE PRECISION for dlaed8 COMPLEX for claed8 DOUBLE COMPLEX for zlaed8. Array q(ldq, *). The second dimension of q must be at least max(1, n). On entry, q contains the eigenvectors of the partially solved system which has been previously updated in matrix multiplies with other partially solved eigensystems. For real flavors, If $icompq = 0$, q is not referenced.
ldq	INTEGER. The leading dimension of the array q; $ldq \geq \max(1, n)$.
ldq2	INTEGER. The leading dimension of the output array $q2$; $ldq2 \geq \max(1, n)$.
indxq	INTEGER. Array, dimension (n). The permutation that separately sorts the two sub-problems in d into ascending order. Note that elements in the second half of this permutation must first have $cutpnt$ added to their values in order to be accurate.

rho — REAL for slaed8/claed8
DOUBLE PRECISION for dlaed8/zlaed8.
On entry, the off-diagonal element associated with the rank-1 cut which originally split the two submatrices which are now being recombined.

Output Parameters

k — INTEGER. The number of non-deflated eigenvalues, and the order of the related secular equation.

d — On exit, contains the trailing (*n*-*k*) updated eigenvalues (those which were deflated) sorted into increasing order.

z — On exit, the updating process destroys the contents of *z*.

q — On exit, *q* contains the trailing (*n*-*k*) updated eigenvectors (those which were deflated) in its last (*n*-*k*) columns.

indxq — INTEGER. Array, dimension (*n*).
The permutation of merged eigenvalues set.

rho — On exit, *rho* has been modified to the value required by ?laed3.

dlamda, *w* — REAL for slaed8/claed8
DOUBLE PRECISION for dlaed8/zlaed8.
Arrays, dimension (*n*) each. The array *dlamda*(*) contains a copy of the first *k* eigenvalues which will be used by ?laed3 to form the secular equation. The array *w*(*) will hold the first *k* values of the final deflation-altered *z*-vector and will be passed to ?laed3.

q2 — REAL for slaed8
DOUBLE PRECISION for dlaed8
COMPLEX for claed8
DOUBLE COMPLEX for zlaed8.
Array
q2(*ldq2*, *). The second dimension of *q2* must be at least max(1, *n*).
Contains a copy of the first *k* eigenvectors which will be used by slaed7/dlaed7 in a matrix multiply (sgemm/dgemm) to update the new eigenvectors. For real flavors, If *icompq* = 0, *q2* is not referenced.

indxp, *indx* — INTEGER. Workspace arrays, dimension (*n*) each.
The array *indxp*(*) will contain the permutation used to place deflated values of *d* at the end of the array. On output, *indxp*(1:*k*) points to the nondeflated *d*-values and *indxp*(*k*+1:*n*) points to the deflated eigenvalues. The array *indx*(*) will contain the permutation used to sort the contents of *d* into ascending order.

perm — INTEGER. Array, dimension (*n*).
Contains the permutations (from deflation and sorting) to be applied to each eigenblock.

givptr — INTEGER. Contains the number of Givens rotations which took place in this subproblem.

givcol — INTEGER. Array, dimension (2, *n*).
Each pair of numbers indicates a pair of columns to take place in a Givens rotation.

givnum — REAL for slaed8/claed8
DOUBLE PRECISION for dlaed8/zlaed8.
Array, dimension (2, *n*).
Each number indicates the *S* value to be used in the corresponding Givens rotation.

info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?laed9

Used by sstedc/dstedc*. Finds the roots of the secular equation and updates the eigenvectors. Used when the original matrix is dense.*

Syntax

```
call slaed9( k, kstart, kstop, n, d, q, ldq, rho, dlamda, w, s, lds, info )
call dlaed9( k, kstart, kstop, n, d, q, ldq, rho, dlamda, w, s, lds, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine finds the roots of the secular equation, as defined by the values in *d*, *z*, and *rho*, between *kstart* and *kstop*. It makes the appropriate calls to slaed4/dlaed4 and then stores the new matrix of eigenvectors for use in calculating the next level of *z* vectors.

Input Parameters

k	INTEGER. The number of terms in the rational function to be solved by slaed4/dlaed4 ($k \geq 0$).
kstart, *kstop*	INTEGER. The updated eigenvalues *lambda*(i), $kstart \leq i \leq kstop$ are to be computed. $1 \leq kstart \leq kstop \leq k$.
n	INTEGER. The number of rows and columns in the *Q* matrix. $n \geq k$ (deflation may result in $n > k$).
q	REAL for slaed9 DOUBLE PRECISION for dlaed9. Workspace array, dimension (*ldq*, *). The second dimension of *q* must be at least max(1, *n*).
ldq	INTEGER. The leading dimension of the array *q*; *ldq* ≥ max(1, *n*).
rho	REAL for slaed9 DOUBLE PRECISION for dlaed9 The value of the parameter in the rank one update equation. *rho* ≥ 0 required.
dlamda, *w*	REAL for slaed9 DOUBLE PRECISION for dlaed9 Arrays, dimension (*k*) each. The first *k* elements of the array *dlamda*(*) contain the old roots of the deflated updating problem. These are the poles of the secular equation. The first *k* elements of the array *w*(*) contain the components of the deflation-adjusted updating vector.
lds	INTEGER. The leading dimension of the output array *s*; *lds* ≥ max(1, *k*).

Output Parameters

d	REAL for slaed9 DOUBLE PRECISION for dlaed9 Array, dimension (*n*). Elements in *d(i)* are not referenced for $1 \le i < kstart$ or $kstop < i \le n$.
s	REAL for slaed9 DOUBLE PRECISION for dlaed9. Array, dimension (*lds, **) . The second dimension of *s* must be at least max(1, *k*). Will contain the eigenvectors of the repaired matrix which will be stored for subsequent *z* vector calculation and multiplied by the previously accumulated eigenvectors to update the system.
dlamda	On exit, the value is modified to make sure all *dlamda(i) – dlamda(j)* can be computed with high relative accuracy, barring overflow and underflow.
w	Destroyed on exit.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = *-i*, the *i*-th parameter had an illegal value. If *info* = 1, the eigenvalue did not converge.

?laeda

Used by ?stedc*. Computes the Z vector determining the rank-one modification of the diagonal matrix. Used when the original matrix is dense.*

Syntax

call slaeda(*n*, *tlvls*, *curlvl*, *curpbm*, *prmptr*, *perm*, *givptr*, *givcol*, *givnum*, *q*, *qptr*, *z*, *ztemp*, *info*)

call dlaeda(*n*, *tlvls*, *curlvl*, *curpbm*, *prmptr*, *perm*, *givptr*, *givcol*, *givnum*, *q*, *qptr*, *z*, *ztemp*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laeda computes the *Z* vector corresponding to the merge step in the *curlvl*-th step of the merge process with *tlvls* steps for the *curpbm*-th problem.

Input Parameters

n	INTEGER. The dimension of the symmetric tridiagonal matrix ($n \ge 0$).
tlvls	INTEGER. The total number of merging levels in the overall divide and conquer tree.
curlvl	INTEGER. The current level in the overall merge routine, $0 \le curlvl \le tlvls$.
curpbm	INTEGER. The current problem in the current level in the overall merge routine (counting from upper left to lower right).

prmptr, *perm*, *givptr* — INTEGER. Arrays, dimension ($n \lg n$) each.
The array *prmptr*(*) contains a list of pointers which indicate where in *perm* a level's permutation is stored. *prmptr*(i+1) – *prmptr*(i) indicates the size of the permutation and also the size of the full, non-deflated problem.
The array *perm*(*) contains the permutations (from deflation and sorting) to be applied to each eigenblock.
The array *givptr*(*) contains a list of pointers which indicate where in *givcol* a level's Givens rotations are stored. *givptr*(i+1) – *givptr*(i) indicates the number of Givens rotations.

givcol — INTEGER. Array, dimension (2, $n \lg n$).
Each pair of numbers indicates a pair of columns to take place in a Givens rotation.

givnum — REAL for slaeda
DOUBLE PRECISION for dlaeda.
Array, dimension (2, $n \lg n$).
Each number indicates the s value to be used in the corresponding Givens rotation.

q — REAL for slaeda
DOUBLE PRECISION for dlaeda.
Array, dimension (n^2).
Contains the square eigenblocks from previous levels, the starting positions for blocks are given by *qptr*.

qptr — INTEGER. Array, dimension (n+2). Contains a list of pointers which indicate where in *q* an eigenblock is stored. sqrt(*qptr*(i+1) – *qptr*(i)) indicates the size of the block.

ztemp — REAL for slaeda
DOUBLE PRECISION for dlaeda.
Workspace array, dimension (n).

Output Parameters

z — REAL for slaeda
DOUBLE PRECISION for dlaeda.
Array, dimension (n). Contains the updating vector (the last row of the first sub-eigenvector matrix and the first row of the second sub-eigenvector matrix).

info — INTEGER.
If *info* = 0, the execution is successful.
If *info* = –*i*, the *i*-th parameter had an illegal value.

?laein

Computes a specified right or left eigenvector of an upper Hessenberg matrix by inverse iteration.

Syntax

call slaein(*rightv*, *noinit*, *n*, *h*, *ldh*, *wr*, *wi*, *vr*, *vi*, *b*, *ldb*, *work*, *eps3*, *smlnum*, *bignum*, *info*)

call dlaein(*rightv*, *noinit*, *n*, *h*, *ldh*, *wr*, *wi*, *vr*, *vi*, *b*, *ldb*, *work*, *eps3*, *smlnum*, *bignum*, *info*)

call claein(*rightv*, *noinit*, *n*, *h*, *ldh*, *w*, *v*, *b*, *ldb*, *rwork*, *eps3*, *smlnum*, *info*)

```
call zlaein( rightv, noinit, n, h, ldh, w, v, b, ldb, rwork, eps3, smlnum, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laein uses inverse iteration to find a right or left eigenvector corresponding to the eigenvalue (*wr*,*wi*) of a real upper Hessenberg matrix *H* (for real flavors slaein/dlaein) or to the eigenvalue *w* of a complex upper Hessenberg matrix *H* (for complex flavors claein/zlaein).

Input Parameters

rightv	LOGICAL. If *rightv* = .TRUE., compute right eigenvector; if *rightv* = .FALSE., compute left eigenvector.
noinit	LOGICAL. If *noinit* = .TRUE., no initial vector is supplied in (*vr*,*vi*) or in *v* (for complex flavors); if *noinit* = .FALSE., initial vector is supplied in (*vr*,*vi*) or in *v* (for complex flavors).
n	INTEGER. The order of the matrix *H* (*n* ≥ 0).
h	REAL for slaein DOUBLE PRECISION for dlaein COMPLEX for claein DOUBLE COMPLEX for zlaein. Array *h*(*ldh*, *). The second dimension of *h* must be at least max(1, *n*). Contains the upper Hessenberg matrix *H*.
ldh	INTEGER. The leading dimension of the array *h*; *ldh* ≥ max(1, *n*).
wr, *wi*	REAL for slaein DOUBLE PRECISION for dlaein. The real and imaginary parts of the eigenvalue of *H* whose corresponding right or left eigenvector is to be computed (for real flavors of the routine).
w	COMPLEX for claein DOUBLE COMPLEX for zlaein. The eigenvalue of *H* whose corresponding right or left eigenvector is to be computed (for complex flavors of the routine).
vr, *vi*	REAL for slaein DOUBLE PRECISION for dlaein. Arrays, dimension (*n*) each. Used for real flavors only. On entry, if *noinit* = .FALSE. and *wi* = 0.0, *vr* must contain a real starting vector for inverse iteration using the real eigenvalue *wr*; if *noinit* = .FALSE. and *wi* ≠ 0.0, *vr* and *vi* must contain the real and imaginary parts of a complex starting vector for inverse iteration using the complex eigenvalue (*wr*,*wi*);otherwise *vr* and *vi* need not be set.
v	COMPLEX for claein DOUBLE COMPLEX for zlaein. Array, dimension (*n*). Used for complex flavors only. On entry, if *noinit* = .FALSE., *v* must contain a starting vector for inverse iteration; otherwise *v* need not be set.

b
REAL for slaein
DOUBLE PRECISION for dlaein
COMPLEX for claein
DOUBLE COMPLEX for zlaein.
Workspace array *b*(*ldb*, *). The second dimension of *b* must be at least max(1, *n*).

ldb
INTEGER. The leading dimension of the array *b*;
ldb ≥ *n*+1 for real flavors;
ldb ≥ max(1, *n*) for complex flavors.

work
REAL for slaein
DOUBLE PRECISION for dlaein.
Workspace array, dimension (*n*).
Used for real flavors only.

rwork
REAL for claein
DOUBLE PRECISION for zlaein.
Workspace array, dimension (*n*).
Used for complex flavors only.

eps3, *smlnum*
REAL for slaein/claein
DOUBLE PRECISION for dlaein/zlaein.
eps3 is a small machine-dependent value which is used to perturb close eigenvalues, and to replace zero pivots.
smlnum is a machine-dependent value close to underflow threshold. A suggested value for *smlnum* is slamch('s') * (*n*/slamch('p') for slaein/claein or dlamch('s') * (*n*/dlamch('p') for dlaein/zlaein. See lamch.

bignum
REAL for slaein
DOUBLE PRECISION for dlaein.
bignum is a machine-dependent value close to overflow threshold. Used for real flavors only. A suggested value for *bignum* is 1 / slamch('s') for slaein/claein or 1 / dlamch('s') for dlaein/zlaein.

Output Parameters

vr, *vi*
On exit, if *wi* = 0.0 (real eigenvalue), *vr* contains the computed real eigenvector; if *wi* ≠ 0.0 (complex eigenvalue), *vr* and *vi* contain the real and imaginary parts of the computed complex eigenvector. The eigenvector is normalized so that the component of largest magnitude has magnitude 1; here the magnitude of a complex number (*x*,*y*) is taken to be |*x*| + |*y*|. *vi* is not referenced if *wi* = 0.0.

v
On exit, *v* contains the computed eigenvector, normalized so that the component of largest magnitude has magnitude 1; here the magnitude of a complex number (*x*,*y*) is taken to be |*x*| + |*y*|.

info
INTEGER.
If *info* = 0, the execution is successful.
If *info* = 1, inverse iteration did not converge. For real flavors, *vr* is set to the last iterate, and so is *vi*, if *wi* ≠ 0.0. For complex flavors, *v* is set to the last iterate.

?laev2

Computes the eigenvalues and eigenvectors of a 2-by-2 symmetric/Hermitian matrix.

Syntax

```
call slaev2( a, b, c, rt1, rt2, cs1, sn1 )
call dlaev2( a, b, c, rt1, rt2, cs1, sn1 )
call claev2( a, b, c, rt1, rt2, cs1, sn1 )
call zlaev2( a, b, c, rt1, rt2, cs1, sn1 )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine performs the eigendecomposition of a 2-by-2 symmetric matrix

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} \text{ (for } slaev2/dlaev2\text{) or Hermitian matrix } \begin{bmatrix} a & b \\ \text{conjg}(b) & c \end{bmatrix}$$

(for `claev2`/`zlaev2`).

On return, *rt1* is the eigenvalue of larger absolute value, *rt2* of smaller absolute value, and (*cs1*, *sn1*) is the unit right eigenvector for *rt1*, giving the decomposition

$$\begin{bmatrix} cs1 & sn1 \\ -sn1 & cs1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ b & c \end{bmatrix} \cdot \begin{bmatrix} cs1 & sn1 \\ -sn1 & cs1 \end{bmatrix} = \begin{bmatrix} rt1 & 0 \\ 0 & rt2 \end{bmatrix}$$

(for `slaev2`/`dlaev2`),

or

$$\begin{bmatrix} cs1 & \text{conjg}(sn1) \\ -sn1 & cs1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ \text{conjg}(b) & c \end{bmatrix} \cdot \begin{bmatrix} cs1 & -\text{conjg}(sn1) \\ sn1 & cs1 \end{bmatrix} = \begin{bmatrix} rt1 & 0 \\ 0 & rt2 \end{bmatrix}$$

(for `claev2`/`zlaev2`).

Input Parameters

a, *b*, *c*	`REAL` for `slaev2` `DOUBLE PRECISION` for `dlaev2` `COMPLEX` for `claev2` `DOUBLE COMPLEX` for `zlaev2`. Elements of the input matrix.

Output Parameters

rt1, *rt2*	REAL for slaev2/claev2 DOUBLE PRECISION for dlaev2/zlaev2. Eigenvalues of larger and smaller absolute value, respectively.
cs1	REAL for slaev2/claev2 DOUBLE PRECISION for dlaev2/zlaev2.
sn1	REAL for slaev2 DOUBLE PRECISION for dlaev2 COMPLEX for claev2 DOUBLE COMPLEX for zlaev2. The vector (*cs1*, *sn1*) is the unit right eigenvector for rt1.

Application Notes

rt1 is accurate to a few ulps barring over/underflow. *rt2* may be inaccurate if there is massive cancellation in the determinant *a*c-b*b*; higher precision or correctly rounded or correctly truncated arithmetic would be needed to compute *rt2* accurately in all cases. *cs1* and *sn1* are accurate to a few ulps barring over/underflow. Overflow is possible only if *rt1* is within a factor of 5 of overflow. Underflow is harmless if the input data is 0 or exceeds *underflow_threshold* / macheps.

?laexc

Swaps adjacent diagonal blocks of a real upper quasi-triangular matrix in Schur canonical form, by an orthogonal similarity transformation.

Syntax

```
call slaexc( wantq, n, t, ldt, q, ldq, j1, n1, n2, work, info )
call dlaexc( wantq, n, t, ldt, q, ldq, j1, n1, n2, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine swaps adjacent diagonal blocks T_{11} and T_{22} of order 1 or 2 in an upper quasi-triangular matrix T by an orthogonal similarity transformation.

T must be in Schur canonical form, that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks; each 2-by-2 diagonal block has its diagonal elements equal and its off-diagonal elements of opposite sign.

Input Parameters

wantq	LOGICAL. If *wantq* = .TRUE., accumulate the transformation in the matrix Q; If *wantq* = .FALSE., do not accumulate the transformation.
n	INTEGER. The order of the matrix T ($n \geq 0$).
t, *q*	REAL for slaexc DOUBLE PRECISION for dlaexc Arrays: *t*(*ldt*,*) contains on entry the upper quasi-triangular matrix T, in Schur canonical form.

	The second dimension of *t* must be at least max(1, *n*). *q*(*ldq*,*) contains on entry, if *wantq* = .TRUE., the orthogonal matrix *Q*. If *wantq* = .FALSE., *q* is not referenced. The second dimension of *q* must be at least max(1, *n*).
ldt	INTEGER. The leading dimension of *t*; at least max(1, *n*).
ldq	INTEGER. The leading dimension of *q*; If *wantq* = .FALSE., then *ldq* ≥ 1. If *wantq* = .TRUE., then *ldq* ≥ max(1,*n*).
j1	INTEGER. The index of the first row of the first block T_{11}.
n1	INTEGER. The order of the first block T_{11} (*n1* = 0, 1, or 2).
n2	INTEGER. The order of the second block T_{22} (*n2* = 0, 1, or 2).
work	REAL for slaexc; DOUBLE PRECISION for dlaexc. Workspace array, DIMENSION (*n*).

Output Parameters

t	On exit, the updated matrix *T*, again in Schur canonical form.
q	On exit, if *wantq* = .TRUE., the updated matrix *Q*.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = 1, the transformed matrix *T* would be too far from Schur form; the blocks are not swapped and *T* and *Q* are unchanged.

?lag2

Computes the eigenvalues of a 2-by-2 generalized eigenvalue problem, with scaling as necessary to avoid over-/underflow.

Syntax

```
call slag2( a, lda, b, ldb, safmin, scale1, scale2, wr1, wr2, wi )
call dlag2( a, lda, b, ldb, safmin, scale1, scale2, wr1, wr2, wi )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the eigenvalues of a 2 x 2 generalized eigenvalue problem *A* - *w* **B*, with scaling as necessary to avoid over-/underflow. The scaling factor, *s*, results in a modified eigenvalue equation

*s***A* - *w***B*,

where *s* is a non-negative scaling factor chosen so that *w*, *w***B*, and *s***A* do not overflow and, if possible, do not underflow, either.

Input Parameters

a, *b*	REAL for slag2

DOUBLE PRECISION for dlag2
Arrays:
a(*lda*,2) contains, on entry, the 2 x 2 matrix *A*. It is assumed that its 1-norm is less than 1/*safmin*. Entries less than sqrt(*safmin*)*norm(*A*) are subject to being treated as zero.
b(*ldb*,2) contains, on entry, the 2 x 2 upper triangular matrix *B*. It is assumed that the one-norm of *B* is less than 1/*safmin*. The diagonals should be at least sqrt(*safmin*) times the largest element of *B* (in absolute value); if a diagonal is smaller than that, then +/- sqrt(*safmin*) will be used instead of that diagonal.

lda — INTEGER. The leading dimension of *a*; *lda* ≥ 2.

ldb — INTEGER. The leading dimension of *b*; *ldb* ≥ 2.

safmin — REAL for slag2;
DOUBLE PRECISION for dlag2.
The smallest positive number such that 1/*safmin* does not overflow. (This should always be ?lamch('S') - it is an argument in order to avoid having to call ?lamch frequently.)

Output Parameters

scale1 — REAL for slag2;
DOUBLE PRECISION for dlag2.
A scaling factor used to avoid over-/underflow in the eigenvalue equation which defines the first eigenvalue. If the eigenvalues are complex, then the eigenvalues are (*wr1* +/- *wii*)/*scale1* (which may lie outside the exponent range of the machine), *scale1*=*scale2*, and *scale1* will always be positive.
If the eigenvalues are real, then the first (real) eigenvalue is *wr1*/*scale1*, but this may overflow or underflow, and in fact, *scale1* may be zero or less than the underflow threshhold if the exact eigenvalue is sufficiently large.

scale2 — REAL for slag2;
DOUBLE PRECISION for dlag2.
A scaling factor used to avoid over-/underflow in the eigenvalue equation which defines the second eigenvalue. If the eigenvalues are complex, then *scale2*=*scale1*. If the eigenvalues are real, then the second (real) eigenvalue is *wr2*/*scale2*, but this may overflow or underflow, and in fact, *scale2* may be zero or less than the underflow threshold if the exact eigenvalue is sufficiently large.

wr1 — REAL for slag2;
DOUBLE PRECISION for dlag2.
If the eigenvalue is real, then *wr1* is *scale1* times the eigenvalue closest to the (2,2) element of *A**inv(*B*).
If the eigenvalue is complex, then *wr1*=*wr2* is *scale1* times the real part of the eigenvalues.

wr2 — REAL for slag2;
DOUBLE PRECISION for dlag2.
If the eigenvalue is real, then *wr2* is *scale2* times the other eigenvalue. If the eigenvalue is complex, then *wr1*=*wr2* is *scale1* times the real part of the eigenvalues.

wi — REAL for slag2;
DOUBLE PRECISION for dlag2.

If the eigenvalue is real, then *wi* is zero. If the eigenvalue is complex, then *wi* is *scale1* times the imaginary part of the eigenvalues. *wi* will always be non-negative.

?lags2

Computes 2-by-2 orthogonal matrices U, V, and Q, and applies them to matrices A and B such that the rows of the transformed A and B are parallel.

Syntax

```
call slags2( upper, a1, a2, a3, b1, b2, b3, csu, snu, csv, snv, csq, snq)
call dlags2( upper, a1, a2, a3, b1, b2, b3, csu, snu, csv, snv, csq, snq)
call clags2( upper, a1, a2, a3, b1, b2, b3, csu, snu, csv, snv, csq, snq)
call zlags2( upper, a1, a2, a3, b1, b2, b3, csu, snu, csv, snv, csq, snq)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

For real flavors, the routine computes 2-by-2 orthogonal matrices *U*, *V* and *Q*, such that if *upper* = .TRUE., then

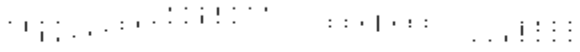
and

$$V^T*B*Q = V^T*\begin{bmatrix} B_1 & B_2 \\ 0 & B_3 \end{bmatrix}*Q = \begin{bmatrix} x & 0 \\ x & x \end{bmatrix}$$

or if *upper* = .FALSE., then

$$U^T*A*Q = U^T*\begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix}*Q = \begin{bmatrix} x & x \\ 0 & x \end{bmatrix}$$

and



The rows of the transformed A and B are parallel, where

$$U = \begin{bmatrix} csu & snu \\ -snu & csu \end{bmatrix}, V = \begin{bmatrix} csv & snv \\ -snv & csv \end{bmatrix}, Q = \begin{bmatrix} csq & snq \\ -snq & csq \end{bmatrix}$$

Here Z^T denotes the transpose of Z.

For complex flavors, the routine computes 2-by-2 unitary matrices U, V and Q, such that if *upper* = .TRUE., then

and

$$V^H * B * Q = V^H * \begin{bmatrix} B_1 & B_2 \\ 0 & B_3 \end{bmatrix} * Q = \begin{bmatrix} x & 0 \\ x & x \end{bmatrix}$$

or if *upper* = .FALSE., then

$$U^H * A * Q = U^H * \begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix} * Q = \begin{bmatrix} x & x \\ 0 & x \end{bmatrix}$$

and

$$V^H * B * Q = V^H * \begin{bmatrix} B_1 & 0 \\ B_2 & B_3 \end{bmatrix} * Q = \begin{bmatrix} x & x \\ 0 & x \end{bmatrix}$$

The rows of the transformed A and B are parallel, where

$$U = \begin{bmatrix} csu & snu \\ -snu^H & csu \end{bmatrix}, V = \begin{bmatrix} csv & snv \\ -snv^H & csv \end{bmatrix}, Q = \begin{bmatrix} csq & snq \\ -snq^H & csq \end{bmatrix}$$

Input Parameters

upper — LOGICAL.
If *upper* = .TRUE., the input matrices A and B are upper triangular; If *upper* = .FALSE., the input matrices A and B are lower triangular.

a1, *a3*	REAL for slags2 and clags2 DOUBLE PRECISION for dlags2 and zlags2
a2	REAL for slags2 DOUBLE PRECISION for dlags2 COMPLEX for clags2 COMPLEX*16 for zlags2 On entry, *a1*, *a2* and *a3* are elements of the input 2-by-2 upper (lower) triangular matrix *A*.
b1, *b3*	REAL for slags2 and clags2 DOUBLE PRECISION for dlags2 and zlags2
b2	REAL for slags2 DOUBLE PRECISION for dlags2 COMPLEX for clags2 COMPLEX*16 for zlags2 On entry, *b1*, *b2* and *b3* are elements of the input 2-by-2 upper (lower) triangular matrix *B*.

Output Parameters

csu	REAL for slags2 and clags2 DOUBLE PRECISION for dlags2 and zlags2 Element of the desired orthogonal matrix *U*.
snu	REAL for slags2 DOUBLE PRECISION for dlags2 Element of the desired orthogonal matrix *U*. COMPLEX for clags2 COMPLEX*16 for zlags2
csv	REAL for slags2 and clags2 DOUBLE PRECISION for dlags2 and zlags2 Element of the desired orthogonal matrix *V*.
snv	REAL for slags2 DOUBLE PRECISION for dlags2 COMPLEX for clags2 COMPLEX*16 for zlags2 Element of the desired orthogonal matrix *V*.
csq	REAL for slags2 and clags2 DOUBLE PRECISION for dlags2 and zlags2 Element of the desired orthogonal matrix *Q*.
snq	REAL for slags2 DOUBLE PRECISION for dlags2 Element of the desired orthogonal matrix *Q*. COMPLEX for clags2 COMPLEX*16 for zlags2

?lagtf

*Computes an LU factorization of a matrix T-λ*I, where T is a general tridiagonal matrix, and λ is a scalar, using partial pivoting with row interchanges.*

Syntax

```
call slagtf( n, a, lambda, b, c, tol, d, in, info )
```

```
call dlagtf( n, a, lambda, b, c, tol, d, in, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine factorizes the matrix ($T - lambda*I$), where T is an n-by-n tridiagonal matrix and $lambda$ is a scalar, as

$T - lambda*I = P*L*U$,

where P is a permutation matrix, L is a unit lower tridiagonal matrix with at most one non-zero sub-diagonal elements per column and U is an upper triangular matrix with at most two non-zero super-diagonal elements per column. The factorization is obtained by Gaussian elimination with partial pivoting and implicit row scaling. The parameter $lambda$ is included in the routine so that ?lagtf may be used, in conjunction with ?lagts, to obtain eigenvectors of T by inverse iteration.

Input Parameters

n	INTEGER. The order of the matrix T ($n \geq 0$).
a, *b*, *c*	REAL for slagtf DOUBLE PRECISION for dlagtf Arrays, dimension $a(n)$, $b(n-1)$, $c(n-1)$: On entry, $a(*)$ must contain the diagonal elements of the matrix T. On entry, $b(*)$ must contain the (n-1) super-diagonal elements of T. On entry, $c(*)$ must contain the (n-1) sub-diagonal elements of T.
tol	REAL for slagtf DOUBLE PRECISION for dlagtf On entry, a relative tolerance used to indicate whether or not the matrix (T - $lambda*I$) is nearly singular. tol should normally be chose as approximately the largest relative error in the elements of T. For example, if the elements of T are correct to about 4 significant figures, then tol should be set to about $5*10^{-4}$. If tol is supplied as less than eps, where eps is the relative machine precision, then the value eps is used in place of tol.

Output Parameters

a	On exit, a is overwritten by the n diagonal elements of the upper triangular matrix U of the factorization of T.
b	On exit, b is overwritten by the n-1 super-diagonal elements of the matrix U of the factorization of T.
c	On exit, c is overwritten by the n-1 sub-diagonal elements of the matrix L of the factorization of T.
d	REAL for slagtf DOUBLE PRECISION for dlagtf Array, dimension (n-2). On exit, d is overwritten by the n-2 second super-diagonal elements of the matrix U of the factorization of T.
in	INTEGER. Array, dimension (n).

	On exit, *in* contains details of the permutation matrix *p*. If an interchange occurred at the k-th step of the elimination, then *in*(k) = 1, otherwise *in*(k) = 0. The element *in*(*n*) returns the smallest positive integer j such that abs(*u*(j,j)) ≤ norm((*T* - *lambda***I*)(j))**tol*, where norm(*A*(j)) denotes the sum of the absolute values of the j-th row of the matrix *A*. If no such j exists then *in*(*n*) is returned as zero. If *in*(*n*) is returned as positive, then a diagonal element of *U* is small, indicating that (*T* - *lambda***I*) is singular or nearly singular.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*k*, the *k*-th parameter had an illegal value.

?lagtm

*Performs a matrix-matrix product of the form C = alpha*A*B+beta*C, where A is a tridiagonal matrix, B and C are rectangular matrices, and alpha and beta are scalars, which may be 0, 1, or -1.*

Syntax

```
call slagtm( trans, n, nrhs, alpha, dl, d, du, x, ldx, beta, b, ldb )
call dlagtm( trans, n, nrhs, alpha, dl, d, du, x, ldx, beta, b, ldb )
call clagtm( trans, n, nrhs, alpha, dl, d, du, x, ldx, beta, b, ldb )
call zlagtm( trans, n, nrhs, alpha, dl, d, du, x, ldx, beta, b, ldb )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs a matrix-vector product of the form:

B := *alpha***A***X* + *beta***B*

where *A* is a tridiagonal matrix of order *n*, *B* and *X* are *n*-by-*nrhs* matrices, and *alpha* and *beta* are real scalars, each of which may be 0., 1., or -1.

Input Parameters

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then *B* := *alpha***A***X* + *beta***B* (no transpose); If *trans* = 'T', then *B* := *alpha***A*^{T}**X* + *beta***B* (transpose); If *trans* = 'C', then *B* := *alpha***A*^{H}**X* + *beta***B* (conjugate transpose)
n	INTEGER. The order of the matrix *A* (*n* ≥ 0).
nrhs	INTEGER. The number of right-hand sides, i.e., the number of columns in *X* and *B* (*nrhs* ≥ 0).
alpha, *beta*	REAL for slagtm/clagtm DOUBLE PRECISION for dlagtm/zlagtm

	Specify the scalars *alpha* and *beta* respectively. *alpha* must be 0., 1., or -1.; otherwise, it is assumed to be 0. *beta* must be 0., 1., or -1.; otherwise, it is assumed to be 1.
dl, *d*, *du*	REAL for slagtm DOUBLE PRECISION for dlagtm COMPLEX for clagtm DOUBLE COMPLEX for zlagtm. Arrays: *dl*(*n* - 1), *d*(*n*), *du*(*n* - 1). The array *dl* contains the (*n* - 1) sub-diagonal elements of *T*. The array *d* contains the *n* diagonal elements of *T*. The array *du* contains the (*n* - 1) super-diagonal elements of *T*.
x, *b*	REAL for slagtm DOUBLE PRECISION for dlagtm COMPLEX for clagtm DOUBLE COMPLEX for zlagtm. Arrays: *x*(*ldx*,*) contains the *n*-by-*nrhs* matrix *X*. The second dimension of *x* must be at least max(1, *nrhs*). *b*(*ldb*,*) contains the *n*-by-*nrhs* matrix *B*. The second dimension of *b* must be at least max(1, *nrhs*).
ldx	INTEGER. The leading dimension of the array *x*; *ldx* ≥ max(1, *n*).
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

Output Parameters

b	Overwritten by the matrix expression *B := alpha*A*X + beta*B*

?lagts

*Solves the system of equations (T - lambda*I)*x = y or (T - lambda*I)^T*x = y,where T is a general tridiagonal matrix and lambda is a scalar, using the LU factorization computed by* ?lagtf*.*

Syntax

```
call slagts( job, n, a, b, c, d, in, y, tol, info )
call dlagts( job, n, a, b, c, d, in, y, tol, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine may be used to solve for *x* one of the systems of equations:

*(T - lambda*I)*x = y* or *(T - lambda*I)^T*x = y*,

where *T* is an *n*-by-*n* tridiagonal matrix, following the factorization of (*T - lambda*I*) as

*T - lambda*I = P*L*U*,

computed by the routine ?lagtf.

The choice of equation to be solved is controlled by the argument *job*, and in each case there is an option to perturb zero or very small diagonal elements of *U*, this option being intended for use in applications such as inverse iteration.

Input Parameters

job	INTEGER. Specifies the job to be performed by ?lagts as follows: = 1: The equations $(T - lambda*I)x = y$ are to be solved, but diagonal elements of *U* are not to be perturbed. = -1: The equations $(T - lambda*I)x = y$ are to be solved and, if overflow would otherwise occur, the diagonal elements of *U* are to be perturbed. See argument *tol* below. = 2: The equations $(T - lambda*I)^T x = y$ are to be solved, but diagonal elements of *U* are not to be perturbed. = -2: The equations $(T - lambda*I)^T x = y$ are to be solved and, if overflow would otherwise occur, the diagonal elements of *U* are to be perturbed. See argument *tol* below.
n	INTEGER. The order of the matrix *T* ($n \geq 0$).
a, *b*, *c*, *d*	REAL for slagts DOUBLE PRECISION for dlagts Arrays, dimension *a*(*n*), *b*(*n*-1), *c*(*n*-1), *d*(*n*-2): On entry, *a*(*) must contain the diagonal elements of *U* as returned from ?lagtf. On entry, *b*(*) must contain the first super-diagonal elements of *U* as returned from ?lagtf. On entry, *c*(*) must contain the sub-diagonal elements of *L* as returned from ?lagtf. On entry, *d*(*) must contain the second super-diagonal elements of *U* as returned from ?lagtf.
in	INTEGER. Array, dimension (*n*). On entry, *in*(*) must contain details of the matrix *p* as returned from ?lagtf.
y	REAL for slagts DOUBLE PRECISION for dlagts Array, dimension (*n*). On entry, the right hand side vector *y*.
tol	REAL for slagtf DOUBLE PRECISION for dlagtf. On entry, with *job* < 0, *tol* should be the minimum perturbation to be made to very small diagonal elements of *U*. *tol* should normally be chosen as about *eps**norm(*U*), where *eps* is the relative machine precision, but if *tol* is supplied as non-positive, then it is reset to *eps**max(abs(u(i,j))). If *job* > 0 then *tol* is not referenced.

Output Parameters

y	On exit, *y* is overwritten by the solution vector *x*.
tol	On exit, *tol* is changed as described in *Input Parameters* section above, only if *tol* is non-positive on entry. Otherwise *tol* is unchanged.
info	INTEGER. If *info* = 0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value. If *info* = *i* >0, overflow would occur when computing the *i*th element of the solution vector *x*. This can only occur when *job* is supplied as positive and either means that a diagonal element of *U* is very small, or that the elements of the right-hand side vector *y* are very large.

?lagv2

Computes the Generalized Schur factorization of a real 2-by-2 matrix pencil (A,B) where B is upper triangular.

Syntax

```
call slagv2( a, lda, b, ldb, alphar, alphai, beta, csl, snl, csr, snr )
call dlagv2( a, lda, b, ldb, alphar, alphai, beta, csl, snl, csr, snr )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes the Generalized Schur factorization of a real 2-by-2 matrix pencil (*A*,*B*) where *B* is upper triangular. The routine computes orthogonal (rotation) matrices given by *csl*, *snl* and *csr*, *snr* such that:

1) if the pencil (*A*,*B*) has two real eigenvalues (include 0/0 or 1/0 types), then

$$\begin{bmatrix} a_{11} & a_{11} \\ 0 & a_{22} \end{bmatrix} = \begin{bmatrix} csl & snl \\ -snl & csl \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} csr & -snr \\ snr & csr \end{bmatrix}$$

$$\begin{bmatrix} b_{11} & b_{11} \\ 0 & b_{22} \end{bmatrix} = \begin{bmatrix} csl & snl \\ -snl & csl \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} csr & -snr \\ snr & csr \end{bmatrix}$$

2) if the pencil (*A*,*B*) has a pair of complex conjugate eigenvalues, then

$$\begin{bmatrix} a_{11} & a_{11} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} csl & snl \\ -snl & csl \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} csr & -snr \\ snr & csr \end{bmatrix}$$

$$\begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} = \begin{bmatrix} csl & snl \\ -snl & csl \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} csr & -snr \\ snr & csr \end{bmatrix}$$

where $b_{11} \geq b_{22}>0$.

Input Parameters

a, *b*	REAL for slagv2 DOUBLE PRECISION for dlagv2 Arrays: *a*(*lda*,2) contains the 2-by-2 matrix *A*; *b*(*ldb*,2) contains the upper triangular 2-by-2 matrix *B*.
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ 2.
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ 2.

Output Parameters

a	On exit, *a* is overwritten by the "*A*-part" of the generalized Schur form.
b	On exit, *b* is overwritten by the "*B*-part" of the generalized Schur form.
alphar, *alphai*, *beta*	REAL for slagv2 DOUBLE PRECISION for dlagv2. Arrays, dimension (2) each. (*alphar*(k) + *i***alphai*(k))/*beta*(k) are the eigenvalues of the pencil (*A*,*B*), k=1,2 and *i* = sqrt(-1). Note that *beta*(k) may be zero.
csl, *snl*	REAL for slagv2 DOUBLE PRECISION for dlagv2 The cosine and sine of the left rotation matrix, respectively.
csr, *snr*	REAL for slagv2 DOUBLE PRECISION for dlagv2 The cosine and sine of the right rotation matrix, respectively.

?lahqr

Computes the eigenvalues and Schur factorization of an upper Hessenberg matrix, using the double-shift/ single-shift QR algorithm.

Syntax

```
call slahqr( wantt, wantz, n, ilo, ihi, h, ldh, wr, wi, iloz, ihiz, z, ldz, info )
call dlahqr( wantt, wantz, n, ilo, ihi, h, ldh, wr, wi, iloz, ihiz, z, ldz, info )
call clahqr( wantt, wantz, n, ilo, ihi, h, ldh, w, iloz, ihiz, z, ldz, info )
call zlahqr( wantt, wantz, n, ilo, ihi, h, ldh, w, iloz, ihiz, z, ldz, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine is an auxiliary routine called by ?hseqr to update the eigenvalues and Schur decomposition already computed by ?hseqr, by dealing with the Hessenberg submatrix in rows and columns *ilo* to *ihi*.

Input Parameters

wantt — LOGICAL.
If *wantt* = .TRUE., the full Schur form *T* is required;
If *wantt* = .FALSE., eigenvalues only are required.

wantz — LOGICAL.
If *wantz* = .TRUE., the matrix of Schur vectors *Z* is required;
If *wantz* = .FALSE., Schur vectors are not required.

n — INTEGER. The order of the matrix *H* ($n \geq 0$).

ilo, *ihi* — INTEGER.
It is assumed that *h* is already upper quasi-triangular in rows and columns *ihi*+1:*n*, and that *h*(*ilo*,*ilo*-1) = 0 (unless *ilo* = 1). The routine ?lahqr works primarily with the Hessenberg submatrix in rows and columns *ilo* to *ihi*, but applies transformations to all of *h* if *wantt* = .TRUE..
Constraints:
$1 \leq ilo \leq \max(1, ihi)$; $ihi \leq n$.

h, *z* — REAL for slahqr
DOUBLE PRECISION for dlahqr
COMPLEX for clahqr
DOUBLE COMPLEX for zlahqr.
Arrays:
h(*ldh*,*) contains the upper Hessenberg matrix *h*.
The second dimension of *h* must be at least max(1, *n*).
z(*ldz*,*)
If *wantz* = .TRUE., then, on entry, *z* must contain the current matrix *z* of transformations accumulated by ?hseqr.
If *wantz* = .FALSE., then *z* is not referenced. The second dimension of *z* must be at least max(1, *n*).

ldh — INTEGER. The leading dimension of *h*; at least max(1, *n*).

ldz — INTEGER. The leading dimension of *z*; at least max(1, *n*).

iloz, *ihiz* — INTEGER. Specify the rows of *z* to which transformations must be applied if *wantz* = .TRUE..
$1 \leq iloz \leq ilo$; $ihi \leq ihiz \leq n$.

Output Parameters

h — On exit, if *info* = 0 and *wantt* = .TRUE., then,

- for slahqr/dlahqr, *h* is upper quasi-triangular in rows and columns *ilo*:*ihi* with any 2-by-2 diagonal blocks in standard form.
- for clahqr/zlahqr, *h* is upper triangular in rows and columns *ilo*:*ihi*.

If *info* = 0 and *wantt* = .FALSE., the contents of *h* are unspecified on exit.
If *info* is positive, see description of *info* for the output state of *h*.

wr, *wi* — REAL for slahqr
DOUBLE PRECISION for dlahqr
Arrays, DIMENSION at least max(1, *n*) each. Used with real flavors only. The real and imaginary parts, respectively, of the computed eigenvalues *ilo* to *ihi* are stored in the corresponding elements of *wr* and *wi*. If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of *wr* and *wi*, say the i-th and (i+1)-th, with *wi*(i) > 0 and *wi*(i+1) < 0.

If *wantt* = .TRUE., the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *h*, with *wr*(i) = *h*(i,i), and, if *h*(i:i+1, i:i+1) is a 2-by-2 diagonal block, *wi*(i) = sqrt(*h*(i+1,i)**h*(i,i+1)) and *wi*(i+1) = -*wi*(i).

w — COMPLEX for clahqr
DOUBLE COMPLEX for zlahqr.
Array, DIMENSION at least max(1, *n*). Used with complex flavors only. The computed eigenvalues *ilo* to *ihi* are stored in the corresponding elements of *w*.
If *wantt* = .TRUE., the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *h*, with *w*(i) = *h*(i,i).

z — If *wantz* = .TRUE., then, on exit *z* has been updated; transformations are applied only to the submatrix *z*(*iloz*:*ihiz*, *ilo*:*ihi*).

info — INTEGER.
If *info* = 0, the execution is successful.
With *info* > 0,

- if *info* = *i*, ?lahqr failed to compute all the eigenvalues *ilo* to *ihi* in a total of 30 iterations per eigenvalue; elements i+1:*ihi* of *wr* and *wi* (for slahqr/dlahqr) or *w* (for clahqr/zlahqr) contain those eigenvalues which have been successfully computed.
- if *wantt* is .FALSE., then on exit the remaining unconverged eigenvalues are the eigenvalues of the upper Hessenberg matrix rows and columns *ilo* through *info* of the final output value of *h*.
- if *wantt* is .TRUE., then on exit
(initial value of *h*)**u* = *u**(final value of *h*), (*)
where *u* is an orthognal matrix. The final value of *h* is upper Hessenberg and triangular in rows and columns *info*+1 through *ihi*.
- if *wantz* is .TRUE., then on exit
(final value of *z*) = (initial value of *z*)* *u*,
where *u* is an orthognal matrix in (*) regardless of the value of *wantt*.

?lahrd

Reduces the first nb columns of a general rectangular matrix A so that elements below the k-th subdiagonal are zero, and returns auxiliary matrices which are needed to apply the transformation to the unreduced part of A.

Syntax

```
call slahrd( n, k, nb, a, lda, tau, t, ldt, y, ldy )
call dlahrd( n, k, nb, a, lda, tau, t, ldt, y, ldy )
call clahrd( n, k, nb, a, lda, tau, t, ldt, y, ldy )
call zlahrd( n, k, nb, a, lda, tau, t, ldt, y, ldy )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine reduces the first nb columns of a real/complex general n-by-(n-k+1) matrix A so that elements below the k-th subdiagonal are zero. The reduction is performed by an orthogonal/unitary similarity transformation Q^T*A*Q for real flavors, or Q^H*A*Q for complex flavors. The routine returns the matrices V and T which determine Q as a block reflector I - $V*T*V^T$ (for real flavors) or I - $V*T*V^H$ (for complex flavors), and also the matrix Y = $A*V*T$.

The matrix Q is represented as products of nb elementary reflectors:

$Q = H(1)*H(2)*... *H(nb)$

Each $H(i)$ has the form

$H(i) = I - tau*v*v^T$ for real flavors, or

$H(i) = I - tau*v*v^H$ for complex flavors, or

where tau is a real/complex scalar, and v is a real/complex vector.

This is an obsolete auxiliary routine. Please use the new routine `?lahr2` instead.

Input Parameters

n	`INTEGER`. The order of the matrix A ($n \geq 0$).
k	`INTEGER`. The offset for the reduction. Elements below the k-th subdiagonal in the first nb columns are reduced to zero.
nb	`INTEGER`. The number of columns to be reduced.
a	`REAL` for `slahrd` `DOUBLE PRECISION` for `dlahrd` `COMPLEX` for `clahrd` `DOUBLE COMPLEX` for `zlahrd`. Array a(lda, n-k+1) contains the n-by-(n-k+1) general matrix A to be reduced.
lda	`INTEGER`. The leading dimension of a; at least `max(1, n)`.
ldt	`INTEGER`. The leading dimension of the output array t; must be at least `max(1, nb)`.
ldy	`INTEGER`. The leading dimension of the output array y; must be at least `max(1, n)`.

Output Parameters

a	On exit, the elements on and above the k-th subdiagonal in the first nb columns are overwritten with the corresponding elements of the reduced matrix; the elements below the k-th subdiagonal, with the array tau, represent the matrix Q as a product of elementary reflectors. The other columns of a are unchanged. See *Application Notes* below.
tau	`REAL` for `slahrd` `DOUBLE PRECISION` for `dlahrd` `COMPLEX` for `clahrd` `DOUBLE COMPLEX` for `zlahrd`. Array, `DIMENSION` (nb). Contains scalar factors of the elementary reflectors.
t, y	`REAL` for `slahrd` `DOUBLE PRECISION` for `dlahrd` `COMPLEX` for `clahrd` `DOUBLE COMPLEX` for `zlahrd`. Arrays, dimension t(ldt, nb), y(ldy, nb).

The array *t* contains upper triangular matrix *T*.
The array *y* contains the *n*-by-*nb* matrix *Y* .

Application Notes

For the elementary reflector *H*(i),

v(1:i+*k*-1) = 0, *v*(i+*k*) = 1; *v*(i+*k*+1:*n*) is stored on exit in *a*(i+*k*+1:*n*, i) and *tau* is stored in *tau*(i).

The elements of the vectors *v* together form the (*n*-*k*+1)-by-*nb* matrix *V* which is needed, with *T* and *Y*, to apply the transformation to the unreduced part of the matrix, using an update of the form:

A := (*I* - *V***T***V*^{T}) * (*A* - *Y***V*^{T}) for real flavors, or

A := (*I* - *V***T***V*^{H}) * (*A* - *Y***V*^{H}) for complex flavors.

The contents of *A* on exit are illustrated by the following example with *n* = 7, *k* = 3 and *nb* = 2:

$$\begin{bmatrix} a & h & a & a & a \\ a & h & a & a & a \\ a & h & a & a & a \\ h & h & a & a & a \\ v_1 & h & a & a & a \\ v_1 & v_2 & a & a & a \\ v_1 & v_2 & a & a & a \end{bmatrix}$$

where *a* denotes an element of the original matrix *A*, *h* denotes a modified element of the upper Hessenberg matrix *H*, and v_i denotes an element of the vector defining *H*(i).

See Also

?lahr2

?lahr2

Reduces the specified number of first columns of a general rectangular matrix A so that elements below the specified subdiagonal are zero, and returns auxiliary matrices which are needed to apply the transformation to the unreduced part of A.

Syntax

```
call slahr2( n, k, nb, a, lda, tau, t, ldt, y, ldy )
call dlahr2( n, k, nb, a, lda, tau, t, ldt, y, ldy )
call clahr2( n, k, nb, a, lda, tau, t, ldt, y, ldy )
call zlahr2( n, k, nb, a, lda, tau, t, ldt, y, ldy )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine reduces the first *nb* columns of a real/complex general *n*-by-(*n*-*k*+1) matrix *A* so that elements below the *k*-th subdiagonal are zero. The reduction is performed by an orthogonal/unitary similarity transformation Q^T*A*Q for real flavors, or Q^H*A*Q for complex flavors. The routine returns the matrices *V* and *T* which determine *Q* as a block reflector $I - V*T*V^T$ (for real flavors) or $I - V*T*V^H$ (for real flavors), and also the matrix $Y = A*V*T$.

The matrix *Q* is represented as products of *nb* elementary reflectors:

$Q = H(1)*H(2)*... *H(nb)$

Each $H(i)$ has the form

$H(i) = I - tau*v*v^T$ for real flavors, or

$H(i) = I - tau*v*v^H$ for complex flavors

where *tau* is a real/complex scalar, and *v* is a real/complex vector.

This is an auxiliary routine called by `?gehrd`.

Input Parameters

n	`INTEGER`. The order of the matrix *A* ($n \geq 0$).
k	`INTEGER`. The offset for the reduction. Elements below the *k*-th subdiagonal in the first *nb* columns are reduced to zero ($k < n$).
nb	`INTEGER`. The number of columns to be reduced.
a	`REAL` for `slahr2` `DOUBLE PRECISION` for `dlahr2` `COMPLEX` for `clahr2` `DOUBLE COMPLEX` for `zlahr2`. Array, `DIMENSION` (*lda*, *n*-*k*+1) contains the *n*-by-(*n*-*k*+1) general matrix *A* to be reduced.
lda	`INTEGER`. The leading dimension of the array *a*; $lda \geq \max(1, n)$.
ldt	`INTEGER`. The leading dimension of the output array *t*; $ldt \geq nb$.
ldy	`INTEGER`. The leading dimension of the output array *y*; $ldy \geq n$.

Output Parameters

a	On exit, the elements on and above the *k*-th subdiagonal in the first *nb* columns are overwritten with the corresponding elements of the reduced matrix; the elements below the *k*-th subdiagonal, with the array *tau*, represent the matrix *Q* as a product of elementary reflectors. The other columns of *a* are unchanged. See *Application Notes* below.
tau	`REAL` for `slahr2` `DOUBLE PRECISION` for `dlahr2` `COMPLEX` for `clahr2` `DOUBLE COMPLEX` for `zlahr2`. Array, `DIMENSION` (*nb*). Contains scalar factors of the elementary reflectors.
t, *y*	`REAL` for `slahr2` `DOUBLE PRECISION` for `dlahr2` `COMPLEX` for `clahr2` `DOUBLE COMPLEX` for `zlahr2`. Arrays, dimension *t*(*ldt*, *nb*), *y*(*ldy*, *nb*).

The array *t* contains upper triangular matrix *T*.
The array *y* contains the *n*-by-*nb* matrix *Y* .

Application Notes

For the elementary reflector *H*(i),

v(1:i+*k*-1) = 0, *v*(i+*k*) = 1; *v*(i+*k*+1:*n*) is stored on exit in *a*(i+*k*+1:*n*, i) and *tau* is stored in *tau*(i).

The elements of the vectors *v* together form the (*n*-*k*+1)-by-*nb* matrix *V* which is needed, with *T* and *Y*, to apply the transformation to the unreduced part of the matrix, using an update of the form:

A := (*I* - *V***T***V*^{T}) * (*A* - *Y***V*^{T}) for real flavors, or

A := (*I* - *V***T***V*^{H}) * (*A* - *Y***V*^{H}) for complex flavors.

The contents of *A* on exit are illustrated by the following example with *n* = 7, *k* = 3 and *nb* = 2:

$$\begin{bmatrix} a & a & a & a & a \\ a & a & a & a & a \\ a & a & a & a & a \\ h & h & a & a & a \\ v_1 & h & a & a & a \\ v_1 & v_2 & a & a & a \\ v_1 & v_2 & a & a & a \end{bmatrix}$$

where *a* denotes an element of the original matrix *A*, *h* denotes a modified element of the upper Hessenberg matrix *H*, and v_i denotes an element of the vector defining *H*(i).

?laic1

Applies one step of incremental condition estimation.

Syntax

```
call slaic1( job, j, x, sest, w, gamma, sestpr, s, c )
call dlaic1( job, j, x, sest, w, gamma, sestpr, s, c )
call claic1( job, j, x, sest, w, gamma, sestpr, s, c )
call zlaic1( job, j, x, sest, w, gamma, sestpr, s, c )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laic1 applies one step of incremental condition estimation in its simplest version.

Let *x*, $||x||_2$ = 1 (where $||a||_2$ denotes the 2-norm of *a*), be an approximate singular vector of an *j*-by-*j* lower triangular matrix *L*, such that

$||L*x||_2$ = *sest*

Then ?laic1 computes *sestpr*, *s*, *c* such that the vector

$$xhat = \begin{bmatrix} s*x \\ c \end{bmatrix}$$

is an approximate singular vector of

$$Lhat = \begin{bmatrix} L & 0 \\ w^H & gamma \end{bmatrix}$$ (for complex flavors), or

$$Lhat = \begin{bmatrix} L & 0 \\ w^T & gamma \end{bmatrix}$$ (for real flavors), in the sense that

$||Lhat*xhat||_2 = sestpr$.

Depending on *job*, an estimate for the largest or smallest singular value is computed.

For real flavors, $[s\ c]^T$ and $sestpr^2$ is an eigenpair of the system

$$\mathrm{diag}(sest*sest, 0) + [alpha\ gamma] * \begin{bmatrix} alpha \\ gamma \end{bmatrix}$$

where $alpha = x^T*w$.

For complex flavors, $[s\ c]^H$ and $sestpr^2$ is an eigenpair of the system

$$\mathrm{diag}(sest*sest, 0) + [alpha\ gamma] * \begin{bmatrix} \mathrm{conjg}(alpha) \\ \mathrm{conjg}(gamma) \end{bmatrix}$$

where $alpha = x^H*w$.

Input Parameters

job	INTEGER. If *job* =1, an estimate for the largest singular value is computed; If *job* =2, an estimate for the smallest singular value is computed;
j	INTEGER. Length of *x* and *w*.
x, *w*	REAL for slaic1 DOUBLE PRECISION for dlaic1 COMPLEX for claic1 DOUBLE COMPLEX for zlaic1. Arrays, dimension (*j*) each. Contain vectors *x* and *w*, respectively.
sest	REAL for slaic1/claic1; DOUBLE PRECISION for dlaic1/zlaic1. Estimated singular value of *j*-by-*j* matrix *L*.
gamma	REAL for slaic1 DOUBLE PRECISION for dlaic1 COMPLEX for claic1 DOUBLE COMPLEX for zlaic1. The diagonal element *gamma*.

Output Parameters

sestpr	REAL for slaic1/claic1; DOUBLE PRECISION for dlaic1/zlaic1. Estimated singular value of (j+1)-by-(j+1) matrix *Lhat*.
s, *c*	REAL for slaic1 DOUBLE PRECISION for dlaic1 COMPLEX for claic1 DOUBLE COMPLEX for zlaic1. Sine and cosine needed in forming *xhat*.

?laln2

Solves a 1-by-1 or 2-by-2 linear system of equations of the specified form.

Syntax

```
call slaln2( ltrans, na, nw, smin, ca, a, lda, d1, d2, b, ldb, wr, wi, x, ldx, scale, xnorm, info )
```

```
call dlaln2( ltrans, na, nw, smin, ca, a, lda, d1, d2, b, ldb, wr, wi, x, ldx, scale, xnorm, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine solves a system of the form

$(ca*A - w*D)*X = s*B$, or $(ca*A^T - w*D)*X = s*B$

with possible scaling (s) and perturbation of A.

A is an na-by-na real matrix, ca is a real scalar, D is an na-by-na real diagonal matrix, w is a real or complex value, and X and B are na-by-1 matrices: real if w is real, complex if w is complex. The parameter na may be 1 or 2.

If w is complex, X and B are represented as na-by-2 matrices, the first column of each being the real part and the second being the imaginary part.

The routine computes the scaling factor s (≤ 1) so chosen that X can be computed without overflow. X is further scaled if necessary to assure that $\text{norm}(ca*A - w*D)*\text{norm}(X)$ is less than overflow.

If both singular values of $(ca*A - w*D)$ are less than $smin$, $smin*I$ (where I stands for identity) will be used instead of ($ca*A$ - $w*D$). If only one singular value is less than $smin$, one element of ($ca*A$ - $w*D$) will be perturbed enough to make the smallest singular value roughly $smin$.

If both singular values are at least $smin$, $(ca*A - w*D)$ will not be perturbed. In any case, the perturbation will be at most some small multiple of $\max(smin, ulp*\text{norm}(ca*A - w*D))$.

The singular values are computed by infinity-norm approximations, and thus will only be correct to a factor of 2 or so.

NOTE All input quantities are assumed to be smaller than overflow by a reasonable factor (see *bignum*).

Input Parameters

trans — LOGICAL.
If *trans* = .TRUE., *A*- transpose will be used.
If *trans* = .FALSE., *A* will be used (not transposed.)

na — INTEGER. The size of the matrix *A*, possible values 1 or 2.

nw — INTEGER. This parameter must be 1 if *w* is real, and 2 if *w* is complex. Possible values 1 or 2.

smin — REAL for slaln2
DOUBLE PRECISION for dlaln2.
The desired lower bound on the singular values of *A*.
This should be a safe distance away from underflow or overflow, for example, between (*underflow/machine_precision*) and (*machine_precision* * *overflow*). (See *bignum* and *ulp*).

ca — REAL for slaln2
DOUBLE PRECISION for dlaln2.
The coefficient by which *A* is multiplied.

a — REAL for slaln2
DOUBLE PRECISION for dlaln2.
Array, DIMENSION (*lda*,*na*).
The *na*-by-*na* matrix *A*.

lda — INTEGER. The leading dimension of *a*. Must be at least *na*.

d1, *d2* — REAL for slaln2
DOUBLE PRECISION for dlaln2.
The (1,1) and (2,2) elements in the diagonal matrix *D*, respectively. *d2* is not used if *nw* = 1.

b — REAL for slaln2
DOUBLE PRECISION for dlaln2.
Array, DIMENSION (*ldb*,*nw*). The *na*-by-*nw* matrix *B* (right-hand side). If *nw* =2 (*w* is complex), column 1 contains the real part of *B* and column 2 contains the imaginary part.

ldb — INTEGER. The leading dimension of *b*. Must be at least *na*.

wr, *wi* — REAL for slaln2
DOUBLE PRECISION for dlaln2.
The real and imaginary part of the scalar *w*, respectively.
wi is not used if *nw* = 1.

ldx — INTEGER. The leading dimension of the output array *x*. Must be at least *na*.

Output Parameters

x — REAL for slaln2
DOUBLE PRECISION for dlaln2.
Array, DIMENSION (*ldx*,*nw*). The *na*-by-*nw* matrix *X* (unknowns), as computed by the routine. If *nw* = 2 (*w* is complex), on exit, column 1 will contain the real part of *X* and column 2 will contain the imaginary part.

scale — REAL for slaln2
DOUBLE PRECISION for dlaln2.
The scale factor that *B* must be multiplied by to insure that overflow does not occur when computing *X*. Thus (*ca***A* - *w***D*) *X* will be *scale***B*, not *B* (ignoring perturbations of *A*.) It will be at most 1.

xnorm	REAL for slaln2 DOUBLE PRECISION for dlaln2. The infinity-norm of *X*, when *X* is regarded as an *na*-by-*nw* real matrix.
info	INTEGER. An error flag. It will be zero if no error occurs, a negative number if an argument is in error, or a positive number if (*ca*A - w*D*) had to be perturbed. The possible values are: If *info* = 0: no error occurred, and (*ca*A - w*D*) did not have to be perturbed. If *info* = 1: (*ca*A - w*D*) had to be perturbed to make its smallest (or only) singular value greater than *smin*.

NOTE For higher speed, this routine does not check the inputs for errors.

?lals0

Applies back multiplying factors in solving the least squares problem using divide and conquer SVD approach. Used by ?gelsd.

Syntax

```
call slals0( icompq, nl, nr, sqre, nrhs, b, ldb, bx, ldbx, perm, givptr, givcol,
ldgcol, givnum, ldgnum, poles, difl, difr, z, k, c, s, work, info )

call dlals0( icompq, nl, nr, sqre, nrhs, b, ldb, bx, ldbx, perm, givptr, givcol,
ldgcol, givnum, ldgnum, poles, difl, difr, z, k, c, s, work, info )

call clals0( icompq, nl, nr, sqre, nrhs, b, ldb, bx, ldbx, perm, givptr, givcol,
ldgcol, givnum, ldgnum, poles, difl, difr, z, k, c, s, rwork, info )

call zlals0( icompq, nl, nr, sqre, nrhs, b, ldb, bx, ldbx, perm, givptr, givcol,
ldgcol, givnum, ldgnum, poles, difl, difr, z, k, c, s, rwork, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine applies back the multiplying factors of either the left or right singular vector matrix of a diagonal matrix appended by a row to the right hand side matrix *B* in solving the least squares problem using the divide-and-conquer SVD approach.

For the left singular vector matrix, three types of orthogonal matrices are involved:

(1L) Givens rotations: the number of such rotations is *givptr*;the pairs of columns/rows they were applied to are stored in *givcol*;and the *c*- and *s*-values of these rotations are stored in *givnum*.

(2L) Permutation. The (*nl*+1)-st row of *B* is to be moved to the first row, and for j=2:*n*, *perm*(*j*)-th row of *B* is to be moved to the *j*-th row.

(3L) The left singular vector matrix of the remaining matrix.

For the right singular vector matrix, four types of orthogonal matrices are involved:

(1R) The right singular vector matrix of the remaining matrix.

(2R) If *sqre* = 1, one extra Givens rotation to generate the right null space.

(3R) The inverse transformation of (2L).

(4R) The inverse transformation of (1L).

Input Parameters

icompq INTEGER. Specifies whether singular vectors are to be computed in factored form:
If *icompq* = 0: Left singular vector matrix.
If *icompq* = 1: Right singular vector matrix.

nl INTEGER. The row dimension of the upper block.
nl ≥ 1.

nr INTEGER. The row dimension of the lower block.
nr ≥ 1.

sqre INTEGER.
If *sqre* = 0: the lower block is an *nr*-by-*nr* square matrix.
If *sqre* = 1: the lower block is an *nr*-by-(*nr*+1) rectangular matrix. The bidiagonal matrix has row dimension *n* = *nl* + *nr* + 1, and column dimension *m* = *n* + *sqre*.

nrhs INTEGER. The number of columns of *B* and *bx*.
Must be at least 1.

b REAL for slals0
DOUBLE PRECISION for dlals0
COMPLEX for clals0
DOUBLE COMPLEX for zlals0.
Array, DIMENSION (*ldb*, *nrhs*).
Contains the right hand sides of the least squares problem in rows 1 through *m*.

ldb INTEGER. The leading dimension of *b*.
Must be at least max(1,max(*m*, *n*)).

bx REAL for slals0
DOUBLE PRECISION for dlals0
COMPLEX for clals0
DOUBLE COMPLEX for zlals0.
Workspace array, DIMENSION (*ldbx*, *nrhs*).

ldbx INTEGER. The leading dimension of *bx*.

perm INTEGER. Array, DIMENSION (*n*).
The permutations (from deflation and sorting) applied to the two blocks.

givptr INTEGER. The number of Givens rotations which took place in this subproblem.

givcol INTEGER. Array, DIMENSION (*ldgcol*, 2). Each pair of numbers indicates a pair of rows/columns involved in a Givens rotation.

ldgcol INTEGER. The leading dimension of *givcol*, must be at least *n*.

givnum REAL for slals0/clals0
DOUBLE PRECISION for dlals0/zlals0
Array, DIMENSION (*ldgnum*, 2). Each number indicates the *c* or *s* value used in the corresponding Givens rotation.

ldgnum INTEGER. The leading dimension of arrays *difr*, *poles* and *givnum*, must be at least *k*.

poles REAL for slals0/clals0
DOUBLE PRECISION for dlals0/zlals0

	Array, DIMENSION (*ldgnum*, 2). On entry, *poles*(1:*k*, 1) contains the new singular values obtained from solving the secular equation, and *poles*(1:*k*, 2) is an array containing the poles in the secular equation.
difl	REAL for slals0/clals0 DOUBLE PRECISION for dlals0/zlals0 Array, DIMENSION (*k*). On entry, *difl*(*i*) is the distance between *i*-th updated (undeflated) singular value and the *i*-th (undeflated) old singular value.
difr	REAL for slals0/clals0 DOUBLE PRECISION for dlals0/zlals0 Array, DIMENSION (*ldgnum*, 2). On entry, *difr*(*i*, 1) contains the distances between *i*-th updated (undeflated) singular value and the *i*+*1*-th (undeflated) old singular value. And *difr*(*i*, 2) is the normalizing factor for the *i*-th right singular vector.
z	REAL for slals0/clals0 DOUBLE PRECISION for dlals0/zlals0 Array, DIMENSION (*k*). Contains the components of the deflation-adjusted updating row vector.
K	INTEGER. Contains the dimension of the non-deflated matrix. This is the order of the related secular equation. $1 \leq k \leq n$.
c	REAL for slals0/clals0 DOUBLE PRECISION for dlals0/zlals0 Contains garbage if *sqre* =0 and the *c* value of a Givens rotation related to the right null space if *sqre* = 1.
s	REAL for slals0/clals0 DOUBLE PRECISION for dlals0/zlals0 Contains garbage if *sqre* =0 and the *s* value of a Givens rotation related to the right null space if *sqre* = 1.
work	REAL for slals0 DOUBLE PRECISION for dlals0 Workspace array, DIMENSION (*k*). Used with real flavors only.
rwork	REAL for clals0 DOUBLE PRECISION for zlals0 Workspace array, DIMENSION (*k**(1+*nrhs*) + 2**nrhs*). Used with complex flavors only.

Output Parameters

b	On exit, contains the solution *X* in rows 1 through *n*.
info	INTEGER. If *info* = 0: successful exit. If *info* = -*i* < 0, the *i*-th argument had an illegal value.

?lalsa

Computes the SVD of the coefficient matrix in compact form. Used by ?gelsd.

Syntax

```
call slalsa( icompq, smlsiz, n, nrhs, b, ldb, bx, ldbx, u, ldu, vt, k, difl, difr, z,
poles, givptr, givcol, ldgcol, perm, givnum, c, s, work, iwork, info )
```

call dlalsa(*icompq*, *smlsiz*, *n*, *nrhs*, *b*, *ldb*, *bx*, *ldbx*, *u*, *ldu*, *vt*, *k*, *difl*, *difr*, *z*, *poles*, *givptr*, *givcol*, *ldgcol*, *perm*, *givnum*, *c*, *s*, *work*, *iwork*, *info*)

call clalsa(*icompq*, *smlsiz*, *n*, *nrhs*, *b*, *ldb*, *bx*, *ldbx*, *u*, *ldu*, *vt*, *k*, *difl*, *difr*, *z*, *poles*, *givptr*, *givcol*, *ldgcol*, *perm*, *givnum*, *c*, *s*, *rwork*, *iwork*, *info*)

call zlalsa(*icompq*, *smlsiz*, *n*, *nrhs*, *b*, *ldb*, *bx*, *ldbx*, *u*, *ldu*, *vt*, *k*, *difl*, *difr*, *z*, *poles*, *givptr*, *givcol*, *ldgcol*, *perm*, *givnum*, *c*, *s*, *rwork*, *iwork*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine is an intermediate step in solving the least squares problem by computing the SVD of the coefficient matrix in compact form. The singular vectors are computed as products of simple orthogonal matrices.

If *icompq* = 0, ?lalsa applies the inverse of the left singular vector matrix of an upper bidiagonal matrix to the right hand side; and if *icompq* = 1, the routine applies the right singular vector matrix to the right hand side. The singular vector matrices were generated in the compact form by ?lalsa.

Input Parameters

icompq	INTEGER. Specifies whether the left or the right singular vector matrix is involved. If *icompq* = 0: left singular vector matrix is used If *icompq* = 1: right singular vector matrix is used.
smlsiz	INTEGER. The maximum size of the subproblems at the bottom of the computation tree.
n	INTEGER. The row and column dimensions of the upper bidiagonal matrix.
nrhs	INTEGER. The number of columns of *b* and *bx*. Must be at least 1.
b	REAL for slalsa DOUBLE PRECISION for dlalsa COMPLEX for clalsa DOUBLE COMPLEX for zlalsa Array, DIMENSION (*ldb*, *nrhs*). Contains the right hand sides of the least squares problem in rows 1 through *m*.
ldb	INTEGER. The leading dimension of *b* in the calling subprogram. Must be at least max(1,max(*m*, *n*)).
ldbx	INTEGER. The leading dimension of the output array *bx*.
u	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, *smlsiz*). On entry, *u* contains the left singular vector matrices of all subproblems at the bottom level.
ldu	INTEGER, *ldu* ≥ *n*. The leading dimension of arrays *u*, *vt*, *difl*, *difr*, *poles*, *givnum*, and *z*.
vt	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, *smlsiz* +1). On entry, vt^T (for real flavors) or vt^H (for complex flavors) contains the right singular vector matrices of all subproblems at the bottom level.
k	INTEGER array, DIMENSION (*n*).
difl	REAL for slalsa/clalsa

	DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, *nlvl*), where *nlvl* = int($\log_2$(*n* /(*smlsiz* +1))) + 1.
difr	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, 2**nlvl*). On entry, *difl*(*, *i*) and *difr*(*, 2*i* -1) record distances between singular values on the *i*-th level and singular values on the (*i* -1)-th level, and *difr*(*, 2*i*) record the normalizing factors of the right singular vectors matrices of subproblems on *i*-th level.
z	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, *nlvl* . On entry, *z*(1, *i*) contains the components of the deflation- adjusted updating the row vector for subproblems on the *i*-th level.
poles	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, 2**nlvl*). On entry, *poles*(*, 2*i*-1: 2*i*) contains the new and old singular values involved in the secular equations on the *i*-th level.
givptr	INTEGER. Array, DIMENSION (*n*). On entry, *givptr*(*i*) records the number of Givens rotations performed on the *i*-th problem on the computation tree.
givcol	INTEGER. Array, DIMENSION (*ldgcol*, 2**nlvl*). On entry, for each *i*, *givcol*(*, 2*i*-1: 2*i*) records the locations of Givens rotations performed on the *i*-th level on the computation tree.
ldgcol	INTEGER, *ldgcol* ≥ *n*. The leading dimension of arrays *givcol* and *perm*.
perm	INTEGER. Array, DIMENSION (*ldgcol*, *nlvl*). On entry, *perm*(*, *i*) records permutations done on the *i*-th level of the computation tree.
givnum	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*ldu*, 2**nlvl*). On entry, *givnum*(*, 2*i*-1 : 2*i*) records the *c* and *s* values of Givens rotations performed on the *i*-th level on the computation tree.
c	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*n*). On entry, if the *i*-th subproblem is not square, *c*(*i*) contains the *c* value of a Givens rotation related to the right null space of the *i*-th subproblem.
s	REAL for slalsa/clalsa DOUBLE PRECISION for dlalsa/zlalsa Array, DIMENSION (*n*). On entry, if the *i*-th subproblem is not square, *s*(*i*) contains the *s*-value of a Givens rotation related to the right null space of the *i*-th subproblem.
work	REAL for slalsa DOUBLE PRECISION for dlalsa Workspace array, DIMENSION at least (*n*). Used with real flavors only.
rwork	REAL for clalsa DOUBLE PRECISION for zlalsa Workspace array, DIMENSION at least max(*n*, (*smlsz*+1)**nrhs**3). Used with complex flavors only.

iwork	INTEGER. Workspace array, DIMENSION at least (3*n*).

Output Parameters

b	On exit, contains the solution *X* in rows 1 through *n*.
bx	REAL for slalsa DOUBLE PRECISION for dlalsa COMPLEX for clalsa DOUBLE COMPLEX for zlalsa Array, DIMENSION (*ldbx*, *nrhs*). On exit, the result of applying the left or right singular vector matrix to *b*.
info	INTEGER. If *info* = 0: successful exit If *info* = -*i* < 0, the *i*-th argument had an illegal value.

?lalsd

Uses the singular value decomposition of A to solve the least squares problem.

Syntax

```
call slalsd( uplo, smlsiz, n, nrhs, d, e, b, ldb, rcond, rank, work, iwork, info )
call dlalsd( uplo, smlsiz, n, nrhs, d, e, b, ldb, rcond, rank, work, iwork, info )
call clalsd( uplo, smlsiz, n, nrhs, d, e, b, ldb, rcond, rank, work, rwork, iwork, info )
call zlalsd( uplo, smlsiz, n, nrhs, d, e, b, ldb, rcond, rank, work, rwork, iwork, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine uses the singular value decomposition of *A* to solve the least squares problem of finding *X* to minimize the Euclidean norm of each column of *A***X*-*B*, where *A* is *n*-by-*n* upper bidiagonal, and *X* and *B* are *n*-by-*nrhs*. The solution *X* overwrites *B*.

The singular values of *A* smaller than *rcond* times the largest singular value are treated as zero in solving the least squares problem; in this case a minimum norm solution is returned. The actual singular values are returned in *d* in ascending order.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray XMP, Cray YMP, Cray C 90, or Cray 2.

It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

Input Parameters

uplo	CHARACTER*1. If *uplo* = 'U', *d* and *e* define an upper bidiagonal matrix. If *uplo* = 'L', *d* and *e* define a lower bidiagonal matrix.

smlsiz	INTEGER. The maximum size of the subproblems at the bottom of the computation tree.
n	INTEGER. The dimension of the bidiagonal matrix. $n \geq 0$.
nrhs	INTEGER. The number of columns of *B*. Must be at least 1.
d	REAL for slalsd/clalsd DOUBLE PRECISION for dlalsd/zlalsd Array, DIMENSION (*n*). On entry, *d* contains the main diagonal of the bidiagonal matrix.
e	REAL for slalsd/clalsd DOUBLE PRECISION for dlalsd/zlalsd Array, DIMENSION (*n*-1). Contains the super-diagonal entries of the bidiagonal matrix. On exit, *e* is destroyed.
b	REAL for slalsd DOUBLE PRECISION for dlalsd COMPLEX for clalsd DOUBLE COMPLEX for zlalsd Array, DIMENSION (*ldb*,*nrhs*). On input, *b* contains the right hand sides of the least squares problem. On output, *b* contains the solution X.
ldb	INTEGER. The leading dimension of *b* in the calling subprogram. Must be at least max(1,*n*).
rcond	REAL for slalsd/clalsd DOUBLE PRECISION for dlalsd/zlalsd The singular values of *A* less than or equal to *rcond* times the largest singular value are treated as zero in solving the least squares problem. If *rcond* is negative, machine precision is used instead. For example, for the least squares problem diag(*S*)**X*=*B*, where diag(*S*) is a diagonal matrix of singular values, the solution is *X*(*i*)=*B*(*i*)/*S*(*i*) if *S*(*i*) is greater than *rcond* *max(*S*), and *X*(*i*)=0 if *S*(*i*) is less than or equal to *rcond* *max(*S*).
rank	INTEGER. The number of singular values of *A* greater than *rcond* times the largest singular value.
work	REAL for slalsd DOUBLE PRECISION for dlalsd COMPLEX for clalsd DOUBLE COMPLEX for zlalsd Workspace array. DIMENSION for real flavors at least $(9n+2n*smlsiz+8n*nlvl+n*nrhs+(smlsiz+1)^2)$, where $nlvl = \max(0, \text{int}(\log_2(n/(smlsiz+1))) + 1)$. DIMENSION for complex flavors is (*n***nrhs*).
rwork	REAL for clalsd DOUBLE PRECISION for zlalsd Workspace array, used with complex flavors only. DIMENSION at least $(9n + 2n*smlsiz + 8n*nlvl + 3*mlsiz*nrhs + (smlsiz+1)^2)$, where $nlvl = \max(0, \text{int}(\log_2(\min(m,n)/(smlsiz+1))) + 1)$.

iwork INTEGER.
Workspace array of DIMENSION (3*n**nlvl + 11*n*).

Output Parameters

d On exit, if *info* = 0, *d* contains singular values of the bidiagonal matrix.

e On exit, destroyed.

b On exit, *b* contains the solution *X*.

info INTEGER.
If *info* = 0: successful exit.
If *info* = -*i* < 0, the *i*-th argument had an illegal value.
If *info* > 0: The algorithm failed to compute a singular value while working on the submatrix lying in rows and columns *info*/(*n*+1) through mod(*info*,*n*+1).

?lamrg

Creates a permutation list to merge the entries of two independently sorted sets into a single set sorted in acsending order.

Syntax

```
call slamrg( n1, n2, a, strd1, strd2, index )
call dlamrg( n1, n2, a, strd1, strd2, index )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine creates a permutation list which will merge the elements of *a* (which is composed of two independently sorted sets) into a single set which is sorted in ascending order.

Input Parameters

n1, *n2* INTEGER. These arguments contain the respective lengths of the two sorted lists to be merged.

a REAL for slamrg
DOUBLE PRECISION for dlamrg.
Array, DIMENSION (*n1*+*n2*).
The first *n1* elements of *a* contain a list of numbers which are sorted in either ascending or descending order. Likewise for the final *n2* elements.

strd1, *strd2* INTEGER.
These are the strides to be taken through the array *a*. Allowable strides are 1 and -1. They indicate whether a subset of *a* is sorted in ascending (*strdx* = 1) or descending (*strdx* = −1) order.

Output Parameters

index INTEGER. Array, DIMENSION (*n1*+*n2*).
On exit, this array will contain a permutation such that if *b*(*i*) = *a*(*index*(*i*)) for *i*=1, *n1*+*n2*, then *b* will be sorted in ascending order.

?laneg

Computes the Sturm count, the number of negative pivots encountered while factoring tridiagonal $T - sigma*I = L*D*L^T$.

Syntax

```
value = slaneg( n, d, lld, sigma, pivmin, r )
value = dlaneg( n, d, lld, sigma, pivmin, r )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes the Sturm count, the number of negative pivots encountered while factoring tridiagonal $T - sigma*I = L*D*L^T$. This implementation works directly on the factors without forming the tridiagonal matrix T. The Sturm count is also the number of eigenvalues of T less than $sigma$. This routine is called from `?larb`. The current routine does not use the $pivmin$ parameter but rather requires IEEE-754 propagation of infinities and `NaN`s (`NaN` stands for 'Not A Number'). This routine also has no input range restrictions but does require default exception handling such that $x/0$ produces `Inf` when x is non-zero, and `Inf/Inf` produces `NaN`. (For more information see [Marques06]).

Input Parameters

n	`INTEGER`. The order of the matrix.
d	`REAL` for `slaneg` `DOUBLE PRECISION` for `dlaneg` Array, `DIMENSION` (n). Contains n diagonal elements of the matrix D.
lld	`REAL` for `slaneg` `DOUBLE PRECISION` for `dlaneg` Array, `DIMENSION` (n-1). Contains (n-1) elements $L(i)*L(i)*D(i)$.
$sigma$	`REAL` for `slaneg` `DOUBLE PRECISION` for `dlaneg` Shift amount in $T - sigma*I = L*D*L**T$.
$pivmin$	`REAL` for `slaneg` `DOUBLE PRECISION` for `dlaneg` The minimum pivot in the Sturm sequence. May be used when zero pivots are encountered on non-IEEE-754 architectures.
r	`INTEGER`. The twist index for the twisted factorization that is used for the negcount.

Output Parameters

$value$	`INTEGER`. The number of negative pivots encountered while factoring.

?langb

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of general band matrix.

Syntax

```
val = slangb( norm, n, kl, ku, ab, ldab, work )
val = dlangb( norm, n, kl, ku, ab, ldab, work )
val = clangb( norm, n, kl, ku, ab, ldab, work )
val = zlangb( norm, n, kl, ku, ab, ldab, work )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The function returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an n-by-n band matrix A, with kl sub-diagonals and ku super-diagonals.

Input Parameters

norm
CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

n
INTEGER. The order of the matrix *A*. $n \geq 0$. When $n = 0$, ?langb is set to zero.

kl
INTEGER. The number of sub-diagonals of the matrix *A*. $kl \geq 0$.

ku
INTEGER. The number of super-diagonals of the matrix *A*. $ku \geq 0$.

ab
REAL for slangb
DOUBLE PRECISION for dlangb
COMPLEX for clangb
DOUBLE COMPLEX for zlangb
Array, DIMENSION (*ldab*,*n*).
The band matrix *A*, stored in rows 1 to $kl+ku+1$. The j-th column of *A* is stored in the j-th column of the array *ab* as follows:
ab(*ku*+1+*i*-*j*,*j*) = *a*(*i*,*j*)
for $\max(1, j-ku) \leq i \leq \min(n, j+kl)$.

ldab
INTEGER. The leading dimension of the array *ab*.
$ldab \geq kl+ku+1$.

work
REAL for slangb/clangb
DOUBLE PRECISION for dlangb/zlangb
Workspace array, DIMENSION (max(1,*lwork*)), where
$lwork \geq n$ when *norm* = 'I'; otherwise, *work* is not referenced.

Output Parameters

val	REAL for slangb/clangb DOUBLE PRECISION for dlangb/zlangb Value returned by the function.

?lange

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general rectangular matrix.

Syntax

val = slange(*norm*, *m*, *n*, *a*, *lda*, *work*)

val = dlange(*norm*, *m*, *n*, *a*, *lda*, *work*)

val = clange(*norm*, *m*, *n*, *a*, *lda*, *work*)

val = zlange(*norm*, *m*, *n*, *a*, *lda*, *work*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lange returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex matrix *A*.

Input Parameters

norm	CHARACTER*1. Specifies the value to be returned by the routine: = 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*. = '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum), = 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum), = 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).
m	INTEGER. The number of rows of the matrix *A*. *m* ≥ 0. When *m* = 0, ?lange is set to zero.
n	INTEGER. The number of columns of the matrix *A*. *n* ≥ 0. When *n* = 0, ?lange is set to zero.
a	REAL for slange DOUBLE PRECISION for dlange COMPLEX for clange DOUBLE COMPLEX for zlange Array, DIMENSION (*lda*,*n*). The *m*-by-*n* matrix *A*.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(*m*,1).
work	REAL for slange and clange. DOUBLE PRECISION for dlange and zlange.

Workspace array, DIMENSION max(1,*lwork*), where *lwork* ≥ *m* when *norm* = 'I'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slange/clange
DOUBLE PRECISION for dlange/zlange
Value returned by the function.

?langt

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general tridiagonal matrix.

Syntax

```
val = slangt( norm, n, dl, d, du )
val = dlangt( norm, n, dl, d, du )
val = clangt( norm, n, dl, d, du )
val = zlangt( norm, n, dl, d, du )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex tridiagonal matrix *A*.

Input Parameters

norm — CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

n — INTEGER. The order of the matrix *A*. *n* ≥ 0. When *n* = 0, ?langt is set to zero.

dl, *d*, *du* — REAL for slangt
DOUBLE PRECISION for dlangt
COMPLEX for clangt
DOUBLE COMPLEX for zlangt
Arrays: *dl* (*n*-1), *d* (*n*), *du* (*n*-1).
The array *dl* contains the (*n*-1) sub-diagonal elements of *A*.
The array *d* contains the diagonal elements of *A*.
The array *du* contains the (*n*-1) super-diagonal elements of *A*.

Output Parameters

val — REAL for slangt/clangt

DOUBLE PRECISION for dlangt/zlangt
Value returned by the function.

?lanhs

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of an upper Hessenberg matrix.

Syntax

```
val = slanhs( norm, n, a, lda, work )
val = dlanhs( norm, n, a, lda, work )
val = clanhs( norm, n, a, lda, work )
val = zlanhs( norm, n, a, lda, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lanhs returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a Hessenberg matrix *A*.

The value *val* returned by the function is:

val = max(abs(A_{ij})), if *norm* = 'M' or 'm'

= norm1(*A*), if *norm* = '1' or 'O' or 'o'

= normI(*A*), if *norm* = 'I' or 'i'

= normF(*A*), if *norm* = 'F', 'f', 'E' or 'e'

where norm1 denotes the 1-norm of a matrix (maximum column sum), normI denotes the infinity norm of a matrix (maximum row sum) and normF denotes the Frobenius norm of a matrix (square root of sum of squares). Note that max(abs(A_{ij})) is not a consistent matrix norm.

Input Parameters

norm	CHARACTER*1. Specifies the value to be returned by the routine as described above.
n	INTEGER. The order of the matrix *A*. $n \geq 0$. When $n = 0$, ?lanhs is set to zero.
a	REAL for slanhs DOUBLE PRECISION for dlanhs COMPLEX for clanhs DOUBLE COMPLEX for zlanhs Array, DIMENSION (*lda*,*n*). The *n*-by-*n* upper Hessenberg matrix *A*; the part of *A* below the first sub-diagonal is not referenced.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(n,1)$.
work	REAL for slanhs and clanhs. DOUBLE PRECISION for dlange and zlange.

Workspace array, DIMENSION (max(1,*lwork*)), where *lwork* ≥ *n* when *norm* = 'I'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slanhs/clanhs
DOUBLE PRECISION for dlanhs/zlanhs
Value returned by the function.

?lansb

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a symmetric band matrix.

Syntax

```
val = slansb( norm, uplo, n, k, ab, ldab, work )
val = dlansb( norm, uplo, n, k, ab, ldab, work )
val = clansb( norm, uplo, n, k, ab, ldab, work )
val = zlansb( norm, uplo, n, k, ab, ldab, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lansb returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an *n*-by-*n* real/complex symmetric band matrix *A*, with *k* super-diagonals.

Input Parameters

norm — CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

uplo — CHARACTER*1.
Specifies whether the upper or lower triangular part of the band matrix *A* is supplied. If *uplo* = 'U': upper triangular part is supplied; If *uplo* = 'L': lower triangular part is supplied.

n — INTEGER. The order of the matrix *A*. *n* ≥ 0.
When *n* = 0, ?lansb is set to zero.

k — INTEGER. The number of super-diagonals or sub-diagonals of the band matrix *A*. *k* ≥ 0.

ab — REAL for slansb
DOUBLE PRECISION for dlansb
COMPLEX for clansb

DOUBLE COMPLEX for zlansb
Array, DIMENSION (*ldab*,*n*).
The upper or lower triangle of the symmetric band matrix *A*, stored in the first *k*+1 rows of *ab*. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows:
if *uplo* = 'U', *ab*(*k*+1+*i*-*j*,*j*) = *a*(*i*,*j*)
for max(1,*j*-*k*) ≤ *i*≤ *j*;
if *uplo* = 'L', *ab*(1+*i*-*j*,*j*) = *a*(*i*,*j*) for *j*≤*i*≤min(*n*,*j*+*k*).

ldab — INTEGER. The leading dimension of the array *ab*.
ldab ≥ *k*+1.

work — REAL for slansb and clansb.
DOUBLE PRECISION for dlansb and zlansb.
Workspace array, DIMENSION (max(1,*lwork*)), where
lwork ≥ *n* when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slansb/clansb
DOUBLE PRECISION for dlansb/zlansb
Value returned by the function.

?lanhb

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a Hermitian band matrix.

Syntax

val = clanhb(*norm*, *uplo*, *n*, *k*, *ab*, *ldab*, *work*)

val = zlanhb(*norm*, *uplo*, *n*, *k*, *ab*, *ldab*, *work*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an *n*-by-*n* Hermitian band matrix *A*, with *k* super-diagonals.

Input Parameters

norm — CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

uplo — CHARACTER*1.
Specifies whether the upper or lower triangular part of the band matrix *A* is supplied.

If *uplo* = 'U': upper triangular part is supplied;
If *uplo* = 'L': lower triangular part is supplied.

n — INTEGER. The order of the matrix *A*. $n \geq 0$. When $n = 0$, ?lanhb is set to zero.

k — INTEGER. The number of super-diagonals or sub-diagonals of the band matrix *A*.
$k \geq 0$.

ab — COMPLEX for clanhb.
DOUBLE COMPLEX for zlanhb.
Array, DIMENSION (*ldaB*,*n*). The upper or lower triangle of the Hermitian band matrix *A*, stored in the first $k+1$ rows of *ab*. The j-th column of *A* is stored in the j-th column of the array *ab* as follows:
if *uplo* = 'U', $ab(k+1+i-j,j) = a(i,j)$
for $\max(1,j-k) \leq i \leq j$;
if *uplo* = 'L', $ab(1+i-j,j) = a(i,j)$ for $j \leq i \leq \min(n,j+k)$.
Note that the imaginary parts of the diagonal elements need not be set and are assumed to be zero.

ldab — INTEGER. The leading dimension of the array *ab*. $ldab \geq k+1$.

work — REAL for clanhb.
DOUBLE PRECISION for zlanhb.
Workspace array, DIMENSION max(1, *lwork*), where
$lwork \geq n$ when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slanhb/clanhb
DOUBLE PRECISION for dlanhb/zlanhb
Value returned by the function.

?lansp

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a symmetric matrix supplied in packed form.

Syntax

```
val = slansp( norm, uplo, n, ap, work )
val = dlansp( norm, uplo, n, ap, work )
val = clansp( norm, uplo, n, ap, work )
val = zlansp( norm, uplo, n, ap, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lansp returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex symmetric matrix *A*, supplied in packed form.

Input Parameters

norm	CHARACTER*1. Specifies the value to be returned by the routine: = 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*. = '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum), = 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum), = 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).
uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric matrix *A* is supplied. If *uplo* = 'U': Upper triangular part of *A* is supplied If *uplo* = 'L': Lower triangular part of *A* is supplied.
n	INTEGER. The order of the matrix *A*. $n \geq 0$. When $n = 0$, ?lansp is set to zero.
ap	REAL for slansp DOUBLE PRECISION for dlansp COMPLEX for clansp DOUBLE COMPLEX for zlansp Array, DIMENSION ($n(n+1)/2$). The upper or lower triangle of the symmetric matrix *A*, packed columnwise in a linear array. The j-th column of *A* is stored in the array *ap* as follows: if *uplo* = 'U', $ap(i + (j-1)j/2) = A(i,j)$ for $1 \leq i \leq j$; if *uplo* = 'L', $ap(i + (j-1)(2n-j)/2) = A(i,j)$ for $j \leq i \leq n$.
work	REAL for slansp and clansp. DOUBLE PRECISION for dlansp and zlansp. Workspace array, DIMENSION (max(1,*lwork*)), where *lwork* $\geq n$ when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val	REAL for slansp/clansp DOUBLE PRECISION for dlansp/zlansp Value returned by the function.

?lanhp

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix supplied in packed form.

Syntax

```
val = clanhp( norm, uplo, n, ap, work )
val = zlanhp( norm, uplo, n, ap, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lanhp returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix A, supplied in packed form.

Input Parameters

norm — CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix A.
= '1' or 'O' or 'o': *val* = norm1(A), 1-norm of the matrix A (maximum column sum),
= 'I' or 'i': *val* = normI(A), infinity norm of the matrix A (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(A), Frobenius norm of the matrix A (square root of sum of squares).

uplo — CHARACTER*1.
Specifies whether the upper or lower triangular part of the Hermitian matrix A is supplied.
If *uplo* = 'U': Upper triangular part of A is supplied
If *uplo* = 'L': Lower triangular part of A is supplied.

n — INTEGER. The order of the matrix A.
$n \geq 0$. When $n = 0$, ?lanhp is set to zero.

ap — COMPLEX for clanhp.
DOUBLE COMPLEX for zlanhp.
Array, DIMENSION ($n(n+1)/2$). The upper or lower triangle of the Hermitian matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array *ap* as follows:
if *uplo* = 'U', $ap(i + (j-1)j/2) = A(i,j)$ for $1 \leq i \leq j$;
if *uplo* = 'L', $ap(i + (j-1)(2n-j)/2) = A(i,j)$ for $j \leq i \leq n$.

work — REAL for clanhp.
DOUBLE PRECISION for zlanhp.
Workspace array, DIMENSION (max(1,*lwork*)), where
lwork $\geq n$ when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for clanhp.
DOUBLE PRECISION for zlanhp.
Value returned by the function.

?lanst/?lanht

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric or complex Hermitian tridiagonal matrix.

Syntax

```
val = slanst( norm, n, d, e )
val = dlanst( norm, n, d, e )
val = clanht( norm, n, d, e )
```

```
val = zlanht( norm, n, d, e )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The functions ?lanst/?lanht return the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric or a complex Hermitian tridiagonal matrix A.

Input Parameters

norm	CHARACTER*1. Specifies the value to be returned by the routine: = 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix A. = '1' or 'O' or 'o': *val* = norm1(A), 1-norm of the matrix A (maximum column sum), = 'I' or 'i': *val* = normI(A), infinity norm of the matrix A (maximum row sum), = 'F', 'f', 'E' or 'e': *val* = normF(A), Frobenius norm of the matrix A (square root of sum of squares).
n	INTEGER. The order of the matrix A. $n \geq 0$. When $n = 0$, ?lanst/?lanht is set to zero.
d	REAL for slanst/clanht DOUBLE PRECISION for dlanst/zlanht Array, DIMENSION (n). The diagonal elements of A.
e	REAL for slanst DOUBLE PRECISION for dlanst COMPLEX for clanht DOUBLE COMPLEX for zlanht Array, DIMENSION (n-1). The (n-1) sub-diagonal or super-diagonal elements of A.

Output Parameters

val	REAL for slanst/clanht DOUBLE PRECISION for dlanst/zlanht Value returned by the function.

?lansy

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex symmetric matrix.

Syntax

```
val = slansy( norm, uplo, n, a, lda, work )
val = dlansy( norm, uplo, n, a, lda, work )
val = clansy( norm, uplo, n, a, lda, work )
val = zlansy( norm, uplo, n, a, lda, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lansy returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex symmetric matrix *A*.

Input Parameters

norm — CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

uplo — CHARACTER*1.
Specifies whether the upper or lower triangular part of the symmetric matrix *A* is to be referenced.
= 'U': Upper triangular part of *A* is referenced.
= 'L': Lower triangular part of *A* is referenced

n — INTEGER. The order of the matrix *A*. $n \geq 0$. When $n = 0$, ?lansy is set to zero.

a — REAL for slansy
DOUBLE PRECISION for dlansy
COMPLEX for clansy
DOUBLE COMPLEX for zlansy
Array, DIMENSION (*lda*,*n*). The symmetric matrix *A*.
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.

lda — INTEGER. The leading dimension of the array *a*.
lda ≥ max(*n*,1).

work — REAL for slansy and clansy.
DOUBLE PRECISION for dlansy and zlansy.
Workspace array, DIMENSION (max(1,*lwork*)), where
lwork ≥ *n* when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slansy/clansy
DOUBLE PRECISION for dlansy/zlansy
Value returned by the function.

?lanhe

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix.

Syntax

val = clanhe(*norm*, *uplo*, *n*, *a*, *lda*, *work*)

val = zlanhe(*norm*, *uplo*, *n*, *a*, *lda*, *work*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lanhe returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix *A*.

Input Parameters

norm	CHARACTER*1. Specifies the value to be returned by the routine: = 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*. = '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum), = 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum), = 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).
uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian matrix *A* is to be referenced. = 'U': Upper triangular part of *A* is referenced. = 'L': Lower triangular part of *A* is referenced
n	INTEGER. The order of the matrix *A*. $n \geq 0$. When $n = 0$, ?lanhe is set to zero.
a	COMPLEX for clanhe. DOUBLE COMPLEX for zlanhe. Array, DIMENSION (*lda*,*n*). The Hermitian matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(n,1)$.
work	REAL for clanhe. DOUBLE PRECISION for zlanhe. Workspace array, DIMENSION (max(1,*lwork*)), where $lwork \geq n$ when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val	REAL for clanhe. DOUBLE PRECISION for zlanhe. Value returned by the function.

?lantb

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a triangular band matrix.

Syntax

```
val = slantb( norm, uplo, diag, n, k, ab, ldab, work )
val = dlantb( norm, uplo, diag, n, k, ab, ldab, work )
val = clantb( norm, uplo, diag, n, k, ab, ldab, work )
val = zlantb( norm, uplo, diag, n, k, ab, ldab, work )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The function `?lantb` returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an n-by-n triangular band matrix A, with (k + 1) diagonals.

Input Parameters

norm
`CHARACTER*1`. Specifies the value to be returned by the routine:
= `'M'` or `'m'`: *val* = max(abs(A_{ij})), largest absolute value of the matrix A.
= `'1'` or `'O'` or `'o'`: *val* = norm1(A), 1-norm of the matrix A (maximum column sum),
= `'I'` or `'i'`: *val* = normI(A), infinity norm of the matrix A (maximum row sum),
= `'F'`, `'f'`, `'E'` or `'e'`: *val* = normF(A), Frobenius norm of the matrix A (square root of sum of squares).

uplo
`CHARACTER*1`.
Specifies whether the matrix A is upper or lower triangular.
= `'U'`: Upper triangular
= `'L'`: Lower triangular.

diag
`CHARACTER*1`.
Specifies whether or not the matrix A is unit triangular.
= `'N'`: Non-unit triangular
= `'U'`: Unit triangular.

n
`INTEGER`. The order of the matrix A. $n \geq 0$. When $n = 0$, `?lantb` is set to zero.

k
`INTEGER`. The number of super-diagonals of the matrix A if *uplo* = `'U'`, or the number of sub-diagonals of the matrix A if *uplo* = `'L'`. $k \geq 0$.

ab
`REAL` for `slantb`
`DOUBLE PRECISION` for `dlantb`
`COMPLEX` for `clantb`
`DOUBLE COMPLEX` for `zlantb`
Array, `DIMENSION` (*ldab*,*n*). The upper or lower triangular band matrix A, stored in the first k+1 rows of *ab*.
The j-th column of A is stored in the j-th column of the array *ab* as follows:
if *uplo* = `'U'`, *ab*(k+1+i–j,j) = *a*(i,j) for max(1,j–k) ≤ i ≤ j;

	if *uplo* = 'L', *ab*(1+*i*-*j*,*j*) = *a*(*i*,*j*) for *j*≤ *i*≤ min(*n*,*j*+*k*). Note that when *diag* = 'U', the elements of the array *ab* corresponding to the diagonal elements of the matrix *A* are not referenced, but are assumed to be one.
ldab	INTEGER. The leading dimension of the array *ab*. *ldab* ≥ *k*+1.
work	REAL for slantb and clantb. DOUBLE PRECISION for dlantb and zlantb. Workspace array, DIMENSION (max(1,*lwork*)), where *lwork* ≥ *n* when *norm* = 'I' ; otherwise, *work* is not referenced.

Output Parameters

val	REAL for slantb/clantb. DOUBLE PRECISION for dlantb/zlantb. Value returned by the function.

?lantp

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a triangular matrix supplied in packed form.

Syntax

```
val = slantp( norm, uplo, diag, n, ap, work )
val = dlantp( norm, uplo, diag, n, ap, work )
val = clantp( norm, uplo, diag, n, ap, work )
val = zlantp( norm, uplo, diag, n, ap, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lantp returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a triangular matrix *A*, supplied in packed form.

Input Parameters

norm	CHARACTER*1. Specifies the value to be returned by the routine: = 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*. = '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum), = 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum), = 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).
uplo	CHARACTER*1. Specifies whether the matrix *A* is upper or lower triangular. = 'U': Upper triangular

= 'L': Lower triangular.

diag — CHARACTER*1.
Specifies whether or not the matrix *A* is unit triangular.
= 'N': Non-unit triangular
= 'U': Unit triangular.

n — INTEGER. The order of the matrix *A*.
$n \geq 0$. When $n = 0$, ?lantp is set to zero.

ap — REAL for slantp
DOUBLE PRECISION for dlantp
COMPLEX for clantp
DOUBLE COMPLEX for zlantp
Array, DIMENSION ($n(n+1)/2$).
The upper or lower triangular matrix *A*, packed columnwise in a linear array.
The j-th column of *A* is stored in the array *ap* as follows:
if *uplo* = 'U', $AP(i + (j-1)j/2) = a(i,j)$ for $1 \leq i \leq j$;
if *uplo* = 'L', $ap(i + (j-1)(2n-j)/2) = a(i,j)$ for $j \leq i \leq n$.
Note that when *diag* = 'U', the elements of the array *ap* corresponding to the diagonal elements of the matrix *A* are not referenced, but are assumed to be one.

work — REAL for slantp and clantp.
DOUBLE PRECISION for dlantp and zlantp.
Workspace array, DIMENSION (max(1,*lwork*)), where $lwork \geq n$ when *norm* = 'I' ; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slantp/clantp.
DOUBLE PRECISION for dlantp/zlantp.
Value returned by the function.

?lantr

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular matrix.

Syntax

```
val = slantr( norm, uplo, diag, m, n, a, lda, work )
val = dlantr( norm, uplo, diag, m, n, a, lda, work )
val = clantr( norm, uplo, diag, m, n, a, lda, work )
val = zlantr( norm, uplo, diag, m, n, a, lda, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lantr returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular matrix *A*.

Input Parameters

norm CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

uplo CHARACTER*1.
Specifies whether the matrix *A* is upper or lower trapezoidal.
= 'U': Upper trapezoidal
= 'L': Lower trapezoidal.
Note that *A* is triangular instead of trapezoidal if *m* = *n*.

diag CHARACTER*1.
Specifies whether or not the matrix *A* has unit diagonal.
= 'N': Non-unit diagonal
= 'U': Unit diagonal.

m INTEGER. The number of rows of the matrix *A*. *m* ≥ 0, and if *uplo* = 'U', *m* ≤ *n*.
When *m* = 0, ?lantr is set to zero.

n INTEGER. The number of columns of the matrix *A*. *n* ≥ 0, and if *uplo* = 'L', *n* ≤ *m*.
When *n* = 0, ?lantr is set to zero.

a REAL for slantr
DOUBLE PRECISION for dlantr
COMPLEX for clantr
DOUBLE COMPLEX for zlantr
Array, DIMENSION (*lda*,*n*).
The trapezoidal matrix *A* (*A* is triangular if *m* = *n*).
If *uplo* = 'U', the leading *m*-by-*n* upper trapezoidal part of the array *a* contains the upper trapezoidal matrix, and the strictly lower triangular part of *A* is not referenced.
If *uplo* = 'L', the leading *m*-by-*n* lower trapezoidal part of the array *a* contains the lower trapezoidal matrix, and the strictly upper triangular part of *A* is not referenced. Note that when *diag* = 'U', the diagonal elements of *A* are not referenced and are assumed to be one.

lda INTEGER. The leading dimension of the array *a*.
lda ≥ max(*m*,1).

work REAL for slantr/clantrp.
DOUBLE PRECISION for dlantr/zlantr.
Workspace array, DIMENSION (max(1,*lwork*)), where
lwork ≥ *m* when *norm* = 'I' ; otherwise, *work* is not referenced.

Output Parameters

val REAL for slantr/clantrp.
DOUBLE PRECISION for dlantr/zlantr.
Value returned by the function.

?lanv2

Computes the Schur factorization of a real 2-by-2 nonsymmetric matrix in standard form.

Syntax

```
call slanv2( a, b, c, d, rt1r, rt1i, rt2r, rt2i, cs, sn )
call dlanv2( a, b, c, d, rt1r, rt1i, rt2r, rt2i, cs, sn )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes the Schur factorization of a real 2-by-2 nonsymmetric matrix in standard form:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} cs & -sn \\ sn & cs \end{bmatrix} \begin{bmatrix} aa & bb \\ cc & dd \end{bmatrix} \begin{bmatrix} cs & sn \\ -sn & cs \end{bmatrix}$$

where either

1. `cc = 0` so that `aa` and `dd` are real eigenvalues of the matrix, or
2. `aa = dd` and `bb*cc < 0`, so that `aa ± sqrt(bb*cc)` are complex conjugate eigenvalues.

The routine was adjusted to reduce the risk of cancellation errors, when computing real eigenvalues, and to ensure, if possible, that `abs(rt1r) ≥ abs(rt2r)`.

Input Parameters

a, *b*, *c*, *d*	`REAL` for `slanv2` `DOUBLE PRECISION` for `dlanv2`. On entry, elements of the input matrix.

Output Parameters

a, *b*, *c*, *d*	On exit, overwritten by the elements of the standardized Schur form.
rt1r, *rt1i*, *rt2r*, *rt2i*	`REAL` for `slanv2` `DOUBLE PRECISION` for `dlanv2`. The real and imaginary parts of the eigenvalues. If the eigenvalues are a complex conjugate pair, `rt1i > 0`.
cs, *sn*	`REAL` for `slanv2` `DOUBLE PRECISION` for `dlanv2`. Parameters of the rotation matrix.

?lapll

Measures the linear dependence of two vectors.

Syntax

```
call slapll( n, x, incx, Y, incy, ssmin )
call dlapll( n, x, incx, Y, incy, ssmin )
```

```
call clapll( n, x, incx, Y, incy, ssmin )
call zlapll( n, x, incx, Y, incy, ssmin )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given two column vectors *x* and *y* of length *n*, let

A = (*x* *y*) be the *n*-by-2 matrix.

The routine ?lapll first computes the *QR* factorization of *A* as *A* = *Q***R* and then computes the SVD of the 2-by-2 upper triangular matrix *R*. The smaller singular value of *R* is returned in *ssmin*, which is used as the measurement of the linear dependency of the vectors *x* and *y*.

Input Parameters

n	INTEGER. The length of the vectors *x* and *y*.
x	REAL for slapll DOUBLE PRECISION for dlapll COMPLEX for clapll DOUBLE COMPLEX for zlapll Array, DIMENSION (1+(*n*-1)*incx*). On entry, *x* contains the *n*-vector x.
y	REAL for slapll DOUBLE PRECISION for dlapll COMPLEX for clapll DOUBLE COMPLEX for zlapll Array, DIMENSION (1+(*n*-1)*incy*). On entry, *y* contains the *n*-vector y.
incx	INTEGER. The increment between successive elements of x; *incx* > 0.
incy	INTEGER. The increment between successive elements of y; *incy* > 0.

Output Parameters

x	On exit, *x* is overwritten.
y	On exit, *y* is overwritten.
ssmin	REAL for slapll/clapll DOUBLE PRECISION for dlapll/zlapll The smallest singular value of the *n*-by-2 matrix *A* = (*x* *y*).

?lapmr

Rearranges rows of a matrix as specified by a permutation vector.

Syntax

Fortran 77:

```
call slapmr( forwrd, m, n, x, ldx, k )
call dlapmr( forwrd, m, n, x, ldx, k )
call clapmr( forwrd, m, n, x, ldx, k )
```

```
call zlapmr( forwrd, m, n, x, ldx, k )
```

Fortran 95:

```
call lapmr( x,k[,forwrd] )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?lapmr` routine rearranges the rows of the m-by-n matrix X as specified by the permutation $k(1),k(2),...,k(m)$ of the integers $1,...,m$.

If *forwrd* = `.TRUE.`, forward permutation:

$X(k(i),*)$ is moved to $X(i,*)$ for $i = 1,2,...,m$.

If *forwrd* = `.FALSE.`, backward permutation:

$X(i,*)$ is moved to $X(k(i),*)$ for $i = 1,2,...,m$.

Input Parameters

forwrd	`LOGICAL`. If *forwrd* = `.TRUE.`, forward permutation If *forwrd* = `.FALSE.`, backward permutation
m	`INTEGER`. The number of rows of the matrix X. $m \geq 0$.
n	`INTEGER`. The number of columns of the matrix X. $n \geq 0$.
x	`REAL` for `slapmr` `DOUBLE PRECISION` for `dlapmr` `COMPLEX` for `clapmr` `DOUBLE COMPLEX` for `zlapmr` Array, `DIMENSION` (*ldx*,*n*). On entry, the m-by-n matrix X.
ldx	`INTEGER`. The leading dimension of the array X, $ldx \geq \max(1,m)$.
k	`INTEGER`. Array, `DIMENSION` (m). On entry, k contains the permutation vector and is used as internal workspace.

Output Parameters

x	On exit, x contains the permuted matrix X.
k	On exit, k is reset to its original value.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `?lapmr` interface are as follows:

x	Holds the matrix X of size (n, n).
k	Holds the vector of length m.
forwrd	Specifies the permutation. Must be `'.TRUE.'` or `'.FALSE.'`.

See Also

?lapmt

?lapmt

Performs a forward or backward permutation of the columns of a matrix.

Syntax

```
call slapmt( forwrd, m, n, x, ldx, k )
call dlapmt( forwrd, m, n, x, ldx, k )
call clapmt( forwrd, m, n, x, ldx, k )
call zlapmt( forwrd, m, n, x, ldx, k )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lapmt` rearranges the columns of the m-by-n matrix X as specified by the permutation $k(1),k(2),...,k(n)$ of the integers $1,...,n$.

If *forwrd* = `.TRUE.`, forward permutation:

$X(*,k(j))$ is moved to $X(*,j)$ for $j=1,2,...,n$.

If *forwrd* = `.FALSE.`, backward permutation:

$X(*,j)$ is moved to $X(*,k(j))$ for $j = 1,2,...,n$.

Input Parameters

forwrd	`LOGICAL`. If *forwrd* = `.TRUE.`, forward permutation If *forwrd* = `.FALSE.`, backward permutation
m	`INTEGER`. The number of rows of the matrix X. $m \geq 0$.
n	`INTEGER`. The number of columns of the matrix X. $n \geq 0$.
x	`REAL` for `slapmt` `DOUBLE PRECISION` for `dlapmt` `COMPLEX` for `clapmt` `DOUBLE COMPLEX` for `zlapmt` Array, `DIMENSION` (*ldx*,*n*). On entry, the m-by-n matrix X.
ldx	`INTEGER`. The leading dimension of the array X, $ldx \geq \max(1,m)$.
k	`INTEGER`. Array, `DIMENSION` (n). On entry, k contains the permutation vector and is used as internal workspace.

Output Parameters

x	On exit, x contains the permuted matrix X.
k	On exit, k is reset to its original value.

See Also

?lapmr

?lapy2

Returns $sqrt(x^2+y^2)$.

Syntax

val = slapy2(*x*, *y*)

val = dlapy2(*x*, *y*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lapy2 returns sqrt(x^2+y^2), avoiding unnecessary overflow or harmful underflow.

Input Parameters

x, *y*	REAL for slapy2 DOUBLE PRECISION for dlapy2 Specify the input values *x* and *y*.

Output Parameters

val	REAL for slapy2 DOUBLE PRECISION for dlapy2. Value returned by the function.

?lapy3

Returns $sqrt(x^2+y^2+z^2)$.

Syntax

val = slapy3(*x*, *y*, *z*)

val = dlapy3(*x*, *y*, *z*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lapy3 returns sqrt($x^2+y^2+z^2$), avoiding unnecessary overflow or harmful underflow.

Input Parameters

x, *y*, *z*	REAL for slapy3 DOUBLE PRECISION for dlapy3 Specify the input values *x*, *y* and *z*.

Output Parameters

val	REAL for slapy3 DOUBLE PRECISION for dlapy3. Value returned by the function.

?laqgb

Scales a general band matrix, using row and column scaling factors computed by `?gbequ`.

Syntax

```
call slaqgb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, equed )
call dlaqgb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, equed )
call claqgb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, equed )
call zlaqgb( m, n, kl, ku, ab, ldab, r, c, rowcnd, colcnd, amax, equed )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine equilibrates a general m-by-n band matrix A with kl subdiagonals and ku superdiagonals using the row and column scaling factors in the vectors r and c.

Input Parameters

m	INTEGER. The number of rows of the matrix A. $m \geq 0$.
n	INTEGER. The number of columns of the matrix A. $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of A. $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of A. $ku \geq 0$.
ab	REAL for slaqgb DOUBLE PRECISION for dlaqgb COMPLEX for claqgb DOUBLE COMPLEX for zlaqgb Array, DIMENSION (*ldab*,*n*). On entry, the matrix A in band storage, in rows 1 to $kl+ku+1$. The j-th column of A is stored in the j-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = *A*(*i*,*j*) for max(1,*j*-*ku*) ≤ i ≤ min(*m*,*j*+*kl*).
ldab	INTEGER. The leading dimension of the array *ab*. *lda* ≥ *kl*+*ku*+1.
amax	REAL for slaqgb/claqgb DOUBLE PRECISION for dlaqgb/zlaqgb Absolute value of largest matrix entry.
r, *c*	REAL for slaqgb/claqgb DOUBLE PRECISION for dlaqgb/zlaqgb Arrays r (m), c (n). Contain the row and column scale factors for A, respectively.
rowcnd	REAL for slaqgb/claqgb DOUBLE PRECISION for dlaqgb/zlaqgb Ratio of the smallest $r(i)$ to the largest $r(i)$.
colcnd	REAL for slaqgb/claqgb DOUBLE PRECISION for dlaqgb/zlaqgb Ratio of the smallest $c(i)$ to the largest $c(i)$.

Output Parameters

ab	On exit, the equilibrated matrix, in the same storage format as *A*. See *equed* for the form of the equilibrated matrix.
equed	CHARACTER*1. Specifies the form of equilibration that was done. If *equed* = 'N': No equilibration If *equed* = 'R': Row equilibration, that is, *A* has been premultiplied by diag(*r*). If *equed* = 'C': Column equilibration, that is, *A* has been postmultiplied by diag(*c*). If *equed* = 'B': Both row and column equilibration, that is, *A* has been replaced by diag(*r*)*A*diag(*c*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*, which have the following meaning. *thresh* is a threshold value used to decide if row or column scaling should be done based on the ratio of the row or column scaling factors. If *rowcnd* < *thresh*, row scaling is done, and if *colcnd* < *thresh*, column scaling is done. *large* and *small* are threshold values used to decide if row scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, row scaling is done.

?laqge

Scales a general rectangular matrix, using row and column scaling factors computed by ?geequ.

Syntax

```
call slaqge( m, n, a, lda, r, c, rowcnd, colcnd, amax, equed )
call dlaqge( m, n, a, lda, r, c, rowcnd, colcnd, amax, equed )
call claqge( m, n, a, lda, r, c, rowcnd, colcnd, amax, equed )
call zlaqge( m, n, a, lda, r, c, rowcnd, colcnd, amax, equed )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine equilibrates a general *m*-by-*n* matrix *A* using the row and column scaling factors in the vectors *r* and *c*.

Input Parameters

m	INTEGER. The number of rows of the matrix *A*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *A*. *n* ≥ 0.
a	REAL for slaqge DOUBLE PRECISION for dlaqge COMPLEX for claqge DOUBLE COMPLEX for zlaqge Array, DIMENSION (*lda*,*n*). On entry, the *m*-by-*n* matrix *A*.

lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(*m*,1).
r	REAL for slanqge/claqge DOUBLE PRECISION for dlaqge/zlaqge Array, DIMENSION (*m*). The row scale factors for A.
c	REAL for slanqge/claqge DOUBLE PRECISION for dlaqge/zlaqge Array, DIMENSION (*n*). The column scale factors for A.
rowcnd	REAL for slanqge/claqge DOUBLE PRECISION for dlaqge/zlaqge Ratio of the smallest *r*(*i*) to the largest *r*(*i*).
colcnd	REAL for slanqge/claqge DOUBLE PRECISION for dlaqge/zlaqge Ratio of the smallest *c*(*i*) to the largest *c*(*i*).
amax	REAL for slanqge/claqge DOUBLE PRECISION for dlaqge/zlaqge Absolute value of largest matrix entry.

Output Parameters

a	On exit, the equilibrated matrix. See *equed* for the form of the equilibrated matrix.
equed	CHARACTER*1. Specifies the form of equilibration that was done. If *equed* = 'N': No equilibration If *equed* = 'R': Row equilibration, that is, *A* has been premultiplied by diag(*r*). If *equed* = 'C': Column equilibration, that is, *A* has been postmultiplied by diag(*c*). If *equed* = 'B': Both row and column equilibration, that is, *A* has been replaced by diag(*r*)*A*diag(*c*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*, which have the following meaning. *thresh* is a threshold value used to decide if row or column scaling should be done based on the ratio of the row or column scaling factors. If *rowcnd* < *thresh*, row scaling is done, and if *colcnd* < *thresh*, column scaling is done. *large* and *small* are threshold values used to decide if row scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, row scaling is done.

?laqhb

Scales a Hermetian band matrix, using scaling factors computed by ?pbequ.

Syntax

```
call claqhb( uplo, n, kd, ab, ldab, s, scond, amax, equed )
call zlaqhb( uplo, n, kd, ab, ldab, s, scond, amax, equed )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine equilibrates a Hermetian band matrix A using the scaling factors in the vector s.

Input Parameters

uplo — CHARACTER*1.
Specifies whether the upper or lower triangular part of the band matrix A is stored.
If *uplo* = 'U': upper triangular.
If *uplo* = 'L': lower triangular.

n — INTEGER. The order of the matrix A.
$n \geq 0$.

kd — INTEGER. The number of super-diagonals of the matrix A if *uplo* = 'U', or the number of sub-diagonals if *uplo* = 'L'.
$kd \geq 0$.

ab — COMPLEX for claqhb
DOUBLE COMPLEX for zlaqhb
Array, DIMENSION (*ldab*,*n*). On entry, the upper or lower triangle of the band matrix A, stored in the first $kd+1$ rows of the array. The j-th column of A is stored in the j-th column of the array *ab* as follows:
if *uplo* = 'U', $ab(kd+1+i-j,j) = A(i,j)$ for $\max(1,j-kd) \leq i \leq j$;
if *uplo* = 'L', $ab(1+i-j,j) = A(i,j)$ for $j \leq i \leq \min(n,j+kd)$.

ldab — INTEGER. The leading dimension of the array *ab*.
$ldab \geq kd+1$.

scond — REAL for claqsb
DOUBLE PRECISION for zlaqsb
Ratio of the smallest $s(i)$ to the largest $s(i)$.

amax — REAL for claqsb
DOUBLE PRECISION for zlaqsb
Absolute value of largest matrix entry.

Output Parameters

ab — On exit, if *info* = 0, the triangular factor U or L from the Cholesky factorization $A = U^H*U$ or $A = L*L^H$ of the band matrix A, in the same storage format as A.

s — REAL for claqsb
DOUBLE PRECISION for zlaqsb
Array, DIMENSION (n). The scale factors for A.

equed — CHARACTER*1.
Specifies whether or not equilibration was done.
If *equed* = 'N': No equilibration.
If *equed* = 'Y': Equilibration was done, that is, A has been replaced by diag(s)*A*diag(s).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*, which have the following meaning. *thresh* is a threshold value used to decide if scaling should be based on the ratio of the scaling factors. If *scond* < *thresh*, scaling is done.

The values *large* and *small* are threshold values used to decide if scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, scaling is done.

?laqp2

Computes a QR factorization with column pivoting of the matrix block.

Syntax

```
call slaqp2( m, n, offset, a, lda, jpvt, tau, vn1, vn2, work )
call dlaqp2( m, n, offset, a, lda, jpvt, tau, vn1, vn2, work )
call claqp2( m, n, offset, a, lda, jpvt, tau, vn1, vn2, work )
call zlaqp2( m, n, offset, a, lda, jpvt, tau, vn1, vn2, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes a *QR* factorization with column pivoting of the block *A*(*offset*+1:*m*,1:*n*). The block *A*(1:*offset*,1:*n*) is accordingly pivoted, but not factorized.

Input Parameters

m	INTEGER. The number of rows of the matrix *A*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *A*. *n* ≥ 0.
offset	INTEGER. The number of rows of the matrix *A* that must be pivoted but no factorized. *offset* ≥ 0.
a	REAL for slaqp2 DOUBLE PRECISION for dlaqp2 COMPLEX for claqp2 DOUBLE COMPLEX for zlaqp2 Array, DIMENSION (*lda*,*n*). On entry, the *m*-by-*n* matrix *A*.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*m*).
jpvt	INTEGER. Array, DIMENSION (*n*). On entry, if *jpvt*(*i*) ≠ 0, the *i*-th column of *A* is permuted to the front of *A***P* (a leading column); if *jpvt*(*i*) = 0, the *i*-th column of *A* is a free column.
vn1, *vn2*	REAL for slaqp2/claqp2 DOUBLE PRECISION for dlaqp2/zlaqp2 Arrays, DIMENSION (*n*) each. Contain the vectors with the partial and exact column norms, respectively.
work	REAL for slaqp2 DOUBLE PRECISION for dlaqp2 COMPLEX for claqp2 DOUBLE COMPLEX for zlaqp2 Workspace array, DIMENSION (*n*).

Output Parameters

a	On exit, the upper triangle of block *A*(*offset*+1:*m*,1:*n*) is the triangular factor obtained; the elements in block *A*(*offset*+1:*m*,1:*n*) below the diagonal, together with the array *tau*, represent the orthogonal matrix *Q* as a product of elementary reflectors. Block *A*(1:*offset*,1:*n*) has been accordingly pivoted, but not factorized.
jpvt	On exit, if *jpvt*(*i*) = *k*, then the *i*-th column of *A***P* was the *k*-th column of *A*.
tau	REAL for slaqp2 DOUBLE PRECISION for dlaqp2 COMPLEX for claqp2 DOUBLE COMPLEX for zlaqp2 Array, DIMENSION (min(*m*,*n*)). The scalar factors of the elementary reflectors.
vn1, *vn2*	Contain the vectors with the partial and exact column norms, respectively.

?laqps

Computes a step of QR factorization with column pivoting of a real m-by-n matrix A by using BLAS level 3.

Syntax

```
call slaqps( m, n, offset, nb, kb, a, lda, jpvt, tau, vn1, vn2, auxv, f, ldf )
call dlaqps( m, n, offset, nb, kb, a, lda, jpvt, tau, vn1, vn2, auxv, f, ldf )
call claqps( m, n, offset, nb, kb, a, lda, jpvt, tau, vn1, vn2, auxv, f, ldf )
call zlaqps( m, n, offset, nb, kb, a, lda, jpvt, tau, vn1, vn2, auxv, f, ldf )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes a step of *QR* factorization with column pivoting of a real *m*-by-*n* matrix *A* by using BLAS level 3. The routine tries to factorize *NB* columns from *A* starting from the row *offset*+1, and updates all of the matrix with BLAS level 3 routine ?gemm.

In some cases, due to catastrophic cancellations, ?laqps cannot factorize *NB* columns. Hence, the actual number of factorized columns is returned in *kb*.

Block *A*(1:*offset*,1:*n*) is accordingly pivoted, but not factorized.

Input Parameters

m	INTEGER. The number of rows of the matrix *A*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *A*. *n* ≥ 0.
offset	INTEGER. The number of rows of *A* that have been factorized in previous steps.
nb	INTEGER. The number of columns to factorize.
a	REAL for slaqps

	DOUBLE PRECISION for dlaqps COMPLEX for claqps DOUBLE COMPLEX for zlaqps Array, DIMENSION (*lda*,*n*). On entry, the *m*-by-*n* matrix *A*.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*m*).
jpvt	INTEGER. Array, DIMENSION (*n*). If *jpvt*(*I*) = *k* then column *k* of the full matrix *A* has been permuted into position *i* in AP.
vn1, *vn2*	REAL for slaqps/claqps DOUBLE PRECISION for dlaqps/zlaqps Arrays, DIMENSION (*n*) each. Contain the vectors with the partial and exact column norms, respectively.
auxv	REAL for slaqps DOUBLE PRECISION for dlaqps COMPLEX for claqps DOUBLE COMPLEX for zlaqps Array, DIMENSION (*nb*). Auxiliary vector.
f	REAL for slaqps DOUBLE PRECISION for dlaqps COMPLEX for claqps DOUBLE COMPLEX for zlaqps Array, DIMENSION (*ldf*,*nb*). For real flavors, matrix F^T = *L***Y*^{T}**A*. For complex flavors, matrix F^H = *L***Y*^{H}**A*.
ldf	INTEGER. The leading dimension of the array *f*. *ldf* ≥ max(1,*n*).

Output Parameters

kb	INTEGER. The number of columns actually factorized.
a	On exit, block *A*(*offset*+1:*m*,1:*kb*) is the triangular factor obtained and block *A*(1:*offset*,1:*n*) has been accordingly pivoted, but no factorized. The rest of the matrix, block *A*(*offset*+1:*m*,*kb*+1:*n*) has been updated.
jpvt	INTEGER array, DIMENSION (*n*). If *jpvt*(*I*) = *k* then column *k* of the full matrix *A* has been permuted into position *i* in AP.
tau	REAL for slaqps DOUBLE PRECISION for dlaqps COMPLEX for claqps DOUBLE COMPLEX for zlaqps Array, DIMENSION (*kb*). The scalar factors of the elementary reflectors.
vn1, *vn2*	The vectors with the partial and exact column norms, respectively.
auxv	Auxiliary vector.
f	Matrix *F'* = *L***Y*'**A*.

?laqr0

Computes the eigenvalues of a Hessenberg matrix, and optionally the marixes from the Schur decomposition.

Syntax

```
call slaqr0( wantt, wantz, n, ilo, ihi, h, ldh, wr, wi, iloz, ihiz, z, ldz, work, lwork, info )
call dlaqr0( wantt, wantz, n, ilo, ihi, h, ldh, wr, wi, iloz, ihiz, z, ldz, work, lwork, info )
call claqr0( wantt, wantz, n, ilo, ihi, h, ldh, w, iloz, ihiz, z, ldz, work, lwork, info )
call zlaqr0( wantt, wantz, n, ilo, ihi, h, ldh, w, iloz, ihiz, z, ldz, work, lwork, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes the eigenvalues of a Hessenberg matrix H, and, optionally, the matrices T and Z from the Schur decomposition $H=Z*T*Z^H$, where T is an upper quasi-triangular/triangular matrix (the Schur form), and Z is the orthogonal/unitary matrix of Schur vectors.

Optionally Z may be postmultiplied into an input orthogonal/unitary matrix Q so that this routine can give the Schur factorization of a matrix A which has been reduced to the Hessenberg form H by the orthogonal/unitary matrix Q: $A = Q*H*Q^H = (QZ)*H*(QZ)^H$.

Input Parameters

wantt	LOGICAL. If *wantt* = .TRUE., the full Schur form T is required; If *wantt* = .FALSE., only eigenvalues are required.
wantz	LOGICAL. If *wantz* = .TRUE., the matrix of Schur vectors *Z* is required; If *wantz* = .FALSE., Schur vectors are not required.
n	INTEGER. The order of the Hessenberg matrix *H*. ($n \geq 0$).
ilo, *ihi*	INTEGER. It is assumed that *H* is already upper triangular in rows and columns 1:*ilo*-1 and *ihi*+1:*n*, and if $ilo > 1$ then $H(ilo, ilo-1) = 0$. *ilo* and *ihi* are normally set by a previous call to cgebal, and then passed to cgehrd when the matrix output by cgebal is reduced to Hessenberg form. Otherwise, *ilo* and *ihi* should be set to 1 and *n*, respectively. If $n > 0$, then $1 \leq ilo \leq ihi \leq n$. If $n=0$, then $ilo=1$ and $ihi=0$
h	REAL for slaqr0 DOUBLE PRECISION for dlaqr0 COMPLEX for claqr0 DOUBLE COMPLEX for zlaqr0. Array, DIMENSION (*ldh*, *n*), contains the upper Hessenberg matrix *H*.
ldh	INTEGER. The leading dimension of the array *h*. $ldh \geq \max(1, n)$.
iloz, *ihiz*	INTEGER. Specify the rows of *Z* to which transformations must be applied if *wantz* is .TRUE., $1 \leq iloz \leq ilo$; $ihi \leq ihiz \leq n$.
z	REAL for slaqr0

DOUBLE PRECISION for dlaqr0
COMPLEX for claqr0
DOUBLE COMPLEX for zlaqr0.
Array, DIMENSION (*ldz*, *ihi*), contains the matrix *Z* if *wantz* is .TRUE.. If *wantz* is .FALSE., *z* is not referenced.

ldz — INTEGER. The leading dimension of the array *z*.
If *wantz* is .TRUE., then *ldz* ≥ max(1, *ihiz*). Otherwise, *ldz* ≥ 1.

work — REAL for slaqr0
DOUBLE PRECISION for dlaqr0
COMPLEX for claqr0
DOUBLE COMPLEX for zlaqr0.
Workspace array with dimension *lwork*.

lwork — INTEGER. The dimension of the array *work*.
lwork ≥ max(1,*n*) is sufficient, but for the optimal performance a greater workspace may be required, typically as large as 6**n*.
It is recommended to use the workspace query to determine the optimal workspace size. If *lwork*=−1, then the routine performs a workspace query: it estimates the optimal workspace size for the given values of the input parameters *n*, *ilo*, and *ihi*. The estimate is returned in *work*(1). No error messages related to the *lwork* is issued by xerbla. Neither *H* nor *Z* are accessed.

Output Parameters

h — If *info*=0, and *wantt* is .TRUE., then *h* contains the upper quasi-triangular/triangular matrix *T* from the Schur decomposition (the Schur form).
If *info*=0, and *wantt* is .FALSE., then the contents of *h* are unspecified on exit.
(The output values of *h* when *info* > 0 are given under the description of the *info* parameter below.)
The routine may explicitly set *h*(i,j) for i>j and j=1,2,...*ilo*−1 or j=*ihi*+1, *ihi*+2,...*n*.

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

w — COMPLEX for claqr0
DOUBLE COMPLEX for zlaqr0.
Arrays, DIMENSION(*n*). The computed eigenvalues of *h*(*ilo*:*ihi*, *ilo*:*ihi*) are stored in *w*(*ilo*:*ihi*). If *wantt* is .TRUE., then the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *h*, with *w*(i) = *h*(i,i).

wr, *wi* — REAL for slaqr0
DOUBLE PRECISION for dlaqr0
Arrays, DIMENSION(*ihi*) each. The real and imaginary parts, respectively, of the computed eigenvalues of *h*(*ilo*:*ihi*, *ilo*:*ihi*) are stored in *wr*(*ilo*:*ihi*) and *wi*(*ilo*:*ihi*). If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of *wr* and *wi*, say the *i*-th and (*i*+1)-th, with *wi*(*i*)> 0 and *wi*(*i*+1) < 0. If *wantt* is .TRUE., then the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *h*, with *wr*(i) = *h*(i,i), and if *h*(i:i+1,i:i+1) is a 2-by-2 diagonal block, then *wi*(i)=sqrt(−*h*(i+1,i)**h*(i,i+1)).

z	If *wantz* is .TRUE., then *z(ilo:ihi, iloz:ihiz)* is replaced by *z(ilo:ihi, iloz:ihiz)*U*, where *U* is the orthogonal/unitary Schur factor of *h(ilo:ihi, ilo:ihi)*. If *wantz* is .FALSE., *z* is not referenced. (The output values of *z* when *info* > 0 are given under the description of the *info* parameter below.)
info	INTEGER. = 0: the execution is successful. > 0: if *info* = *i*, then the routine failed to compute all the eigenvalues. Elements 1:*ilo*-1 and i+1:*n* of *wr* and *wi* contain those eigenvalues which have been successfully computed. > 0: if *wantt* is .FALSE., then the remaining unconverged eigenvalues are the eigenvalues of the upper Hessenberg matrix rows and columns *ilo* through *info* of the final output value of *h*. > 0: if *wantt* is .TRUE., then (initial value of *h*)**U* = *U**(final value of *h*, where *U* is an orthogonal/unitary matrix. The final value of *h* is upper Hessenberg and quasi-triangular/triangular in rows and columns *info*+1 through *ihi*. > 0: if *wantz* is .TRUE., then (final value of *z*(*ilo*:*ihi*, *iloz*:*ihiz*))=(initial value of *z*(*ilo*:*ihi*, *iloz*:*ihiz*)**U*, where *U* is the orthogonal/unitary matrix in the previous expression (regardless of the value of *wantt*). > 0: if *wantz* is .FALSE., then *z* is not accessed.

?laqr1

Sets a scalar multiple of the first column of the product of 2-by-2 or 3-by-3 matrix H and specified shifts.

Syntax

```
call slaqr1( n, h, ldh, sr1, si1, sr2, si2, v )
call dlaqr1( n, h, ldh, sr1, si1, sr2, si2, v )
call claqr1( n, h, ldh, s1, s2, v )
call zlaqr1( n, h, ldh, s1, s2, v )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given a 2-by-2 or 3-by-3 matrix *H*, this routine sets *v* to a scalar multiple of the first column of the product *K* = (*H* - *s1***I*)*(*H* - *s2***I*), or *K* = (*H* - (*sr1* + i**si1*)**I*)*(*H* - (*sr2* + i**si2*)**I*)

scaling to avoid overflows and most underflows.

It is assumed that either 1) *sr1* = *sr2* and *si1* = -*si2*, or 2) *si1* = *si2* = 0.

This is useful for starting double implicit shift bulges in the QR algorithm.

Input Parameters

n	INTEGER.

	The order of the matrix H. n must be equal to 2 or 3.
sr1, *si2*, *sr2*, *si2*	REAL for slaqr1 DOUBLE PRECISION for dlaqr1 Shift values that define K in the formula above.
s1, *s2*	COMPLEX for claqr1 DOUBLE COMPLEX for zlaqr1. Shift values that define K in the formula above.
h	REAL for slaqr1 DOUBLE PRECISION for dlaqr1 COMPLEX for claqr1 DOUBLE COMPLEX for zlaqr1. Array, DIMENSION (*ldh*, *n*), contains 2-by-2 or 3-by-3 matrix H in the formula above.
ldh	INTEGER. The leading dimension of the array *h* just as declared in the calling routine. *ldh* ≥ *n*.

Output Parameters

v	REAL for slaqr1 DOUBLE PRECISION for dlaqr1 COMPLEX for claqr1 DOUBLE COMPLEX for zlaqr1. Array with dimension (*n*). A scalar multiple of the first column of the matrix K in the formula above.

?laqr2

Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation).

Syntax

```
call slaqr2( wantt, wantz, n, ktop, kbot, nw, h, ldh, iloz, ihiz, z, ldz, ns, nd, sr, si, v, ldv, nh, t, ldt, nv, wv, ldwv, work, lwork )
call dlaqr2( wantt, wantz, n, ktop, kbot, nw, h, ldh, iloz, ihiz, z, ldz, ns, nd, sr, si, v, ldv, nh, t, ldt, nv, wv, ldwv, work, lwork )
call claqr2( wantt, wantz, n, ktop, kbot, nw, h, ldh, iloz, ihiz, z, ldz, ns, nd, sh, v, ldv, nh, t, ldt, nv, wv, ldwv, work, lwork )
call zlaqr2( wantt, wantz, n, ktop, kbot, nw, h, ldh, iloz, ihiz, z, ldz, ns, nd, sh, v, ldv, nh, t, ldt, nv, wv, ldwv, work, lwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine accepts as input an upper Hessenberg matrix *H* and performs an orthogonal/unitary similarity transformation designed to detect and deflate fully converged eigenvalues from a trailing principal submatrix. On output *H* has been overwritten by a new Hessenberg matrix that is a perturbation of an orthogonal/unitary similarity transformation of *H*. It is to be hoped that the final version of *H* has many zero subdiagonal entries.

This subroutine is identical to ?laqr3 except that it avoids recursion by calling ?lahqr instead of ?laqr4.

Input Parameters

wantt — LOGICAL.
If *wantt* = .TRUE., then the Hessenberg matrix *H* is fully updated so that the quasi-triangular/triangular Schur factor may be computed (in cooperation with the calling subroutine).
If *wantt* = .FALSE., then only enough of *H* is updated to preserve the eigenvalues.

wantz — LOGICAL.
If *wantz* = .TRUE., then the orthogonal/unitary matrix *Z* is updated so that the orthogonal/unitary Schur factor may be computed (in cooperation with the calling subroutine).
If *wantz* = .FALSE., then *Z* is not referenced.

n — INTEGER. The order of the Hessenberg matrix *H* and (if *wantz* = .TRUE.) the order of the orthogonal/unitary matrix *Z*.

ktop — INTEGER.
It is assumed that either *ktop*=1 or *h*(*ktop*,*ktop*-1)=0. *ktop* and *kbot* together determine an isolated block along the diagonal of the Hessenberg matrix.

kbot — INTEGER.
It is assumed without a check that either *kbot*=*n* or *h*(*kbot*+1,*kbot*)=0. *ktop* and *kbot* together determine an isolated block along the diagonal of the Hessenberg matrix.

nw — INTEGER.
Size of the deflation window. 1 ≤ *nw* ≤ (*kbot*-*ktop*+1).

h — REAL for slaqr2
DOUBLE PRECISION for dlaqr2
COMPLEX for claqr2
DOUBLE COMPLEX for zlaqr2.
Array, DIMENSION (*ldh*, *n*), on input the initial *n*-by-*n* section of *h* stores the Hessenberg matrix *H* undergoing aggressive early deflation.

ldh — INTEGER. The leading dimension of the array *h* just as declared in the calling subroutine. *ldh*≥*n*.

iloz, *ihiz* — INTEGER. Specify the rows of *Z* to which transformations must be applied if *wantz* is .TRUE.. 1 ≤ *iloz* ≤ *ihiz* ≤ *n*.

z — REAL for slaqr2
DOUBLE PRECISION for dlaqr2
COMPLEX for claqr2
DOUBLE COMPLEX for zlaqr2.
Array, DIMENSION (*ldz*, *n*), contains the matrix *Z* if *wantz* is .TRUE.. If *wantz* is .FALSE., then *z* is not referenced.

ldz — INTEGER. The leading dimension of the array *z* just as declared in the calling subroutine. *ldz* ≥ 1.

v — REAL for slaqr2

	DOUBLE PRECISION for dlaqr2 COMPLEX for claqr2 DOUBLE COMPLEX for zlaqr2. Workspace array with dimension (*ldv*, *nw*). An *nw*-by-*nw* work array.
ldv	INTEGER. The leading dimension of the array *v* just as declared in the calling subroutine. *ldv* ≥ *nw*.
nh	INTEGER. The number of column of *t*. *nh* ≥ *nw*.
t	REAL for slaqr2 DOUBLE PRECISION for dlaqr2 COMPLEX for claqr2 DOUBLE COMPLEX for zlaqr2. Workspace array with dimension (*ldt*, *nw*).
ldt	INTEGER. The leading dimension of the array *t* just as declared in the calling subroutine. *ldt*≥*nw*.
nv	INTEGER. The number of rows of work array *wv* available for workspace. *nv*≥*nw*.
wv	REAL for slaqr2 DOUBLE PRECISION for dlaqr2 COMPLEX for claqr2 DOUBLE COMPLEX for zlaqr2. Workspace array with dimension (*ldwv*, *nw*).
ldwv	INTEGER. The leading dimension of the array *wv* just as declared in the calling subroutine. *ldwv*≥*nw*.
work	REAL for slaqr2 DOUBLE PRECISION for dlaqr2 COMPLEX for claqr2 DOUBLE COMPLEX for zlaqr2. Workspace array with dimension *lwork*.
lwork	INTEGER. The dimension of the array *work*. *lwork*=2**nw*) is sufficient, but for the optimal performance a greater workspace may be required. If *lwork*=-1, then the routine performs a workspace query: it estimates the optimal workspace size for the given values of the input parameters *n*, *nw*, *ktop*, and *kbot*. The estimate is returned in *work*(1). No error messages related to the *lwork* is issued by xerbla. Neither *H* nor *Z* are accessed.

Output Parameters

h	On output *h* has been transformed by an orthogonal/unitary similarity transformation, perturbed, and the returned to Hessenberg form that (it is to be hoped) has some zero subdiagonal entries.
work(1)	On exit *work*(1) is set to an estimate of the optimal value of *lwork* for the given values of the input parameters *n*, *nw*, *ktop*, and *kbot*.
z	If *wantz* is .TRUE., then the orthogonal/unitary similarity transformation is accumulated into *z(iloz:ihiz, ilo:ihi)* from the right. If *wantz* is .FALSE., then *z* is unreferenced.
nd	INTEGER. The number of converged eigenvalues uncovered by the routine.
ns	INTEGER. The number of unconverged, that is approximate eigenvalues returned in *sr*, *si* or in *sh* that may be used as shifts by the calling subroutine.

sh — COMPLEX for claqr2
DOUBLE COMPLEX for zlaqr2.
Arrays, DIMENSION (*kbot*).
The approximate eigenvalues that may be used for shifts are stored in the *sh*(*kbot*-*nd*-*ns*+1) through the *sh*(*kbot*-*nd*).
The converged eigenvalues are stored in the *sh*(*kbot*-*nd*+1) through the *sh*(*kbot*).

sr, *si* — REAL for slaqr2
DOUBLE PRECISION for dlaqr2
Arrays, DIMENSION (*kbot*) each.
The real and imaginary parts of the approximate eigenvalues that may be used for shifts are stored in the *sr*(*kbot*-*nd*-*ns*+1) through the *sr*(*kbot*-*nd*), and *si*(*kbot*-*nd*-*ns*+1) through the *si*(*kbot*-*nd*), respectively.
The real and imaginary parts of converged eigenvalues are stored in the *sr*(*kbot*-*nd*+1) through the *sr*(*kbot*), and *si*(*kbot*-*nd*+1) through the *si*(*kbot*), respectively.

?laqr3

Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation).

Syntax

call slaqr3(*wantt*, *wantz*, *n*, *ktop*, *kbot*, *nw*, *h*, *ldh*, *iloz*, *ihiz*, *z*, *ldz*, *ns*, *nd*, *sr*, *si*, *v*, *ldv*, *nh*, *t*, *ldt*, *nv*, *wv*, *ldwv*, *work*, *lwork*)

call dlaqr3(*wantt*, *wantz*, *n*, *ktop*, *kbot*, *nw*, *h*, *ldh*, *iloz*, *ihiz*, *z*, *ldz*, *ns*, *nd*, *sr*, *si*, *v*, *ldv*, *nh*, *t*, *ldt*, *nv*, *wv*, *ldwv*, *work*, *lwork*)

call claqr3(*wantt*, *wantz*, *n*, *ktop*, *kbot*, *nw*, *h*, *ldh*, *iloz*, *ihiz*, *z*, *ldz*, *ns*, *nd*, *sh*, *v*, *ldv*, *nh*, *t*, *ldt*, *nv*, *wv*, *ldwv*, *work*, *lwork*)

call zlaqr3(*wantt*, *wantz*, *n*, *ktop*, *kbot*, *nw*, *h*, *ldh*, *iloz*, *ihiz*, *z*, *ldz*, *ns*, *nd*, *sh*, *v*, *ldv*, *nh*, *t*, *ldt*, *nv*, *wv*, *ldwv*, *work*, *lwork*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine accepts as input an upper Hessenberg matrix *H* and performs an orthogonal/unitary similarity transformation designed to detect and deflate fully converged eigenvalues from a trailing principal submatrix. On output *H* has been overwritten by a new Hessenberg matrix that is a perturbation of an orthogonal/unitary similarity transformation of *H*. It is to be hoped that the final version of *H* has many zero subdiagonal entries.

Input Parameters

wantt — LOGICAL.
If *wantt* = .TRUE., then the Hessenberg matrix *H* is fully updated so that the quasi-triangular/triangular Schur factor may be computed (in cooperation with the calling subroutine).

	If *wantt* = .FALSE., then only enough of *H* is updated to preserve the eigenvalues.
wantz	LOGICAL. If *wantz* = .TRUE., then the orthogonal/unitary matrix *Z* is updated so that the orthogonal/unitary Schur factor may be computed (in cooperation with the calling subroutine). If *wantz* = .FALSE., then *Z* is not referenced.
n	INTEGER. The order of the Hessenberg matrix *H* and (if *wantz* = .TRUE.) the order of the orthogonal/unitary matrix *Z*.
ktop	INTEGER. It is assumed that either *ktop*=1 or *h*(*ktop*,*ktop-1*)=0. *ktop* and *kbot* together determine an isolated block along the diagonal of the Hessenberg matrix.
kbot	INTEGER. It is assumed without a check that either *kbot*=*n* or *h*(*kbot*+1,*kbot*)=0. *ktop* and *kbot* together determine an isolated block along the diagonal of the Hessenberg matrix.
nw	INTEGER. Size of the deflation window. 1≤*nw*≤(*kbot*-*ktop*+1).
h	REAL for slaqr3 DOUBLE PRECISION for dlaqr3 COMPLEX for claqr3 DOUBLE COMPLEX for zlaqr3. Array, DIMENSION (*ldh*, *n*), on input the initial *n*-by-*n* section of *h* stores the Hessenberg matrix *H* undergoing aggressive early deflation.
ldh	INTEGER. The leading dimension of the array *h* just as declared in the calling subroutine. *ldh*≥*n*.
iloz, *ihiz*	INTEGER. Specify the rows of *Z* to which transformations must be applied if *wantz* is .TRUE.. 1≤*iloz*≤*ihiz*≤*n*.
z	REAL for slaqr3 DOUBLE PRECISION for dlaqr3 COMPLEX for claqr3 DOUBLE COMPLEX for zlaqr3. Array, DIMENSION (*ldz*, *n*), contains the matrix *Z* if *wantz* is .TRUE.. If *wantz* is .FALSE., then *z* is not referenced.
ldz	INTEGER. The leading dimension of the array *z* just as declared in the calling subroutine. *ldz*≥1.
v	REAL for slaqr3 DOUBLE PRECISION for dlaqr3 COMPLEX for claqr3 DOUBLE COMPLEX for zlaqr3. Workspace array with dimension (*ldv*, *nw*). An *nw*-by-*nw* work array.
ldv	INTEGER. The leading dimension of the array *v* just as declared in the calling subroutine. *ldv*≥*nw*.
nh	INTEGER. The number of column of *t*. *nh*≥*nw*.
t	REAL for slaqr3 DOUBLE PRECISION for dlaqr3 COMPLEX for claqr3 DOUBLE COMPLEX for zlaqr3.

Workspace array with dimension (*ldt*, *nw*).

ldt — INTEGER. The leading dimension of the array *t* just as declared in the calling subroutine. *ldt*≥*nw*.

nv — INTEGER. The number of rows of work array *wv* available for workspace. *nv*≥*nw*.

wv — REAL for slaqr3
DOUBLE PRECISION for dlaqr3
COMPLEX for claqr3
DOUBLE COMPLEX for zlaqr3.
Workspace array with dimension (*ldwv*, *nw*).

ldwv — INTEGER. The leading dimension of the array *wv* just as declared in the calling subroutine. *ldwv*≥*nw*.

work — REAL for slaqr3
DOUBLE PRECISION for dlaqr3
COMPLEX for claqr3
DOUBLE COMPLEX for zlaqr3.
Workspace array with dimension *lwork*.

lwork — INTEGER. The dimension of the array *work*.
lwork=2**nw*) is sufficient, but for the optimal performance a greater workspace may be required.
If *lwork*=-1, then the routine performs a workspace query: it estimates the optimal workspace size for the given values of the input parameters *n*, *nw*, *ktop*, and *kbot*. The estimate is returned in *work*(1). No error messages related to the *lwork* is issued by xerbla. Neither *H* nor *Z* are accessed.

Output Parameters

h — On output *h* has been transformed by an orthogonal/unitary similarity transformation, perturbed, and the returned to Hessenberg form that (it is to be hoped) has some zero subdiagonal entries.

work(1) — On exit *work*(1) is set to an estimate of the optimal value of *lwork* for the given values of the input parameters *n*, *nw*, *ktop*, and *kbot*.

z — If *wantz* is .TRUE., then the orthogonal/unitary similarity transformation is accumulated into *z*(*iloz*:*ihiz*, *ilo*:*ihi*) from the right.
If *wantz* is .FALSE., then *z* is unreferenced.

nd — INTEGER. The number of converged eigenvalues uncovered by the routine.

ns — INTEGER. The number of unconverged, that is approximate eigenvalues returned in *sr*, *si* or in *sh* that may be used as shifts by the calling subroutine.

sh — COMPLEX for claqr3
DOUBLE COMPLEX for zlaqr3.
Arrays, DIMENSION (*kbot*).
The approximate eigenvalues that may be used for shifts are stored in the *sh*(*kbot*-*nd*-*ns*+1) through the *sh*(*kbot*-*nd*).
The converged eigenvalues are stored in the *sh*(*kbot*-*nd*+1) through the *sh*(*kbot*).

sr, *si* — REAL for slaqr3
DOUBLE PRECISION for dlaqr3
Arrays, DIMENSION (*kbot*) each.

The real and imaginary parts of the approximate eigenvalues that may be used for shifts are stored in the *sr*(*kbot*-*nd*-*ns*+1) through the *sr*(*kbot*-*nd*), and *si*(*kbot*-*nd*-*ns*+1) through the *si*(*kbot*-*nd*), respectively. The real and imaginary parts of converged eigenvalues are stored in the *sr*(*kbot*-*nd*+1) through the *sr*(*kbot*), and *si*(*kbot*-*nd*+1) through the *si*(*kbot*), respectively.

?laqr4

Computes the eigenvalues of a Hessenberg matrix, and optionally the matrices from the Schur decomposition.

Syntax

call slaqr4(*wantt*, *wantz*, *n*, *ilo*, *ihi*, *h*, *ldh*, *wr*, *wi*, *iloz*, *ihiz*, *z*, *ldz*, *work*, *lwork*, *info*)

call dlaqr4(*wantt*, *wantz*, *n*, *ilo*, *ihi*, *h*, *ldh*, *wr*, *wi*, *iloz*, *ihiz*, *z*, *ldz*, *work*, *lwork*, *info*)

call claqr4(*wantt*, *wantz*, *n*, *ilo*, *ihi*, *h*, *ldh*, *w*, *iloz*, *ihiz*, *z*, *ldz*, *work*, *lwork*, *info*)

call zlaqr4(*wantt*, *wantz*, *n*, *ilo*, *ihi*, *h*, *ldh*, *w*, *iloz*, *ihiz*, *z*, *ldz*, *work*, *lwork*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the eigenvalues of a Hessenberg matrix *H*, and, optionally, the matrices *T* and *Z* from the Schur decomposition $H=Z*T*Z^H$, where *T* is an upper quasi-triangular/triangular matrix (the Schur form), and *Z* is the orthogonal/unitary matrix of Schur vectors.

Optionally *Z* may be postmultiplied into an input orthogonal/unitary matrix *Q* so that this routine can give the Schur factorization of a matrix *A* which has been reduced to the Hessenberg form *H* by the orthogonal/unitary matrix *Q*: $A = Q*H*Q^H = (QZ)*H*(QZ)^H$.

This routine implements one level of recursion for ?laqr0. It is a complete implementation of the small bulge multi-shift QR algorithm. It may be called by ?laqr0 and, for large enough deflation window size, it may be called by ?laqr3. This routine is identical to ?laqr0 except that it calls ?laqr2 instead of ?laqr3.

Input Parameters

wantt — LOGICAL.
If *wantt* = .TRUE., the full Schur form T is required;
If *wantt* = .FALSE., only eigenvalues are required.

wantz — LOGICAL.
If *wantz* = .TRUE., the matrix of Schur vectors *Z* is required;
If *wantz* = .FALSE., Schur vectors are not required.

n — INTEGER. The order of the Hessenberg matrix *H*. ($n \geq 0$).

ilo, *ihi* — INTEGER.
It is assumed that *H* is already upper triangular in rows and columns 1:*ilo*-1 and *ihi*+1:*n*, and if *ilo* > 1 then *h*(*ilo*, *ilo*-1) = 0.

ilo and *ihi* are normally set by a previous call to cgebal, and then passed to cgehrd when the matrix output by cgebal is reduced to Hessenberg form. Otherwise, *ilo* and *ihi* should be set to 1 and *n*, respectively.
If $n > 0$, then $1 \le ilo \le ihi \le n$.
If $n=0$, then $ilo=1$ and $ihi=0$

h — REAL for slaqr4
DOUBLE PRECISION for dlaqr4
COMPLEX for claqr4
DOUBLE COMPLEX for zlaqr4.
Array, DIMENSION (*ldh*, *n*), contains the upper Hessenberg matrix *H*.

ldh — INTEGER. The leading dimension of the array *h*. $ldh \ge \max(1, n)$.

iloz, *ihiz* — INTEGER. Specify the rows of *Z* to which transformations must be applied if *wantz* is .TRUE., $1 \le iloz \le ilo$; $ihi \le ihiz \le n$.

z — REAL for slaqr4
DOUBLE PRECISION for dlaqr4
COMPLEX for claqr4
DOUBLE COMPLEX for zlaqr4.
Array, DIMENSION (*ldz*, *ihi*), contains the matrix *Z* if *wantz* is .TRUE.. If *wantz* is .FALSE., *z* is not referenced.

ldz — INTEGER. The leading dimension of the array *z*.
If *wantz* is .TRUE., then $ldz \ge \max(1, ihiz)$. Otherwise, $ldz \ge 1$.

work — REAL for slaqr4
DOUBLE PRECISION for dlaqr4
COMPLEX for claqr4
DOUBLE COMPLEX for zlaqr4.
Workspace array with dimension *lwork*.

lwork — INTEGER. The dimension of the array *work*.
$lwork \ge \max(1,n)$ is sufficient, but for the optimal performance a greater workspace may be required, typically as large as $6*n$.
It is recommended to use the workspace query to determine the optimal workspace size. If $lwork=-1$, then the routine performs a workspace query: it estimates the optimal workspace size for the given values of the input parameters *n*, *ilo*, and *ihi*. The estimate is returned in *work*(1). No error messages related to the *lwork* is issued by xerbla. Neither *H* nor *Z* are accessed.

Output Parameters

h — If $info=0$, and *wantt* is .TRUE., then *h* contains the upper quasi-triangular/triangular matrix *T* from the Schur decomposition (the Schur form).
If $info=0$, and *wantt* is .FALSE., then the contents of *h* are unspecified on exit.
(The output values of *h* when $info > 0$ are given under the description of the *info* parameter below.)
The routines may explicitly set *h*(i,j) for i>j and j=1,2,...*ilo*-1 or j=*ihi*+1, *ihi*+2,...*n*.

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

w — COMPLEX for claqr4

	DOUBLE COMPLEX for zlaqr4. Arrays, DIMENSION(*n*). The computed eigenvalues of *h*(*ilo*:*ihi*, *ilo*:*ihi*) are stored in *w*(*ilo*:*ihi*). If *wantt* is .TRUE., then the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *h*, with *w*(i) = *h*(i,i).
wr, *wi*	REAL for slaqr4 DOUBLE PRECISION for dlaqr4 Arrays, DIMENSION(*ihi*) each. The real and imaginary parts, respectively, of the computed eigenvalues of *h*(*ilo*:*ihi*, *ilo*:*ihi*) are stored in the *wr*(*ilo*:*ihi*) and *wi*(*ilo*:*ihi*). If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of *wr* and *wi*, say the *i*-th and (*i*+1)-th, with *wi*(*i*)> 0 and *wi*(*i*+1) < 0. If *wantt* is .TRUE., then the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *h*, with *wr*(i) = *h*(i,i), and if *h*(i:i+1,i:i+1)is a 2-by-2 diagonal block, then *wi*(i)=sqrt(-*h*(i+1,i)*h(i,i+1)).
z	If *wantz* is .TRUE., then *z(ilo:ihi, iloz:ihiz)* is replaced by *z(ilo:ihi, iloz:ihiz)*U*, where *U* is the orthogonal/unitary Schur factor of *h(ilo:ihi, ilo:ihi)*. If *wantz* is .FALSE., *z* is not referenced. (The output values of *z* when *info* > 0 are given under the description of the *info* parameter below.)
info	INTEGER. = 0: the execution is successful. > 0: if *info* = *i*, then the routine failed to compute all the eigenvalues. Elements 1:*ilo*–1 and i+1:*n* of *wr* and *wi* contain those eigenvalues which have been successfully computed. > 0: if *wantt* is .FALSE., then the remaining unconverged eigenvalues are the eigenvalues of the upper Hessenberg matrix rows and columns *ilo* through *info* of the final output value of *h*. > 0: if *wantt* is .TRUE., then (initial value of *h*)**U* = *U**(final value of *h*, where *U* is an orthogonal/unitary matrix. The final value of *h* is upper Hessenberg and quasi-triangular/triangular in rows and columns *info*+1 through *ihi*. > 0: if *wantz* is .TRUE., then (final value of *z*(*ilo*:*ihi*, *iloz*:*ihiz*))=(initial value of *z*(*ilo*:*ihi*, *iloz*:*ihiz*)**U*, where *U* is the orthogonal/unitary matrix in the previous expression (regardless of the value of *wantt*). > 0: if *wantz* is .FALSE., then *z* is not accessed.

?laqr5

Performs a single small-bulge multi-shift QR sweep.

Syntax

```
call slaqr5( wantt, wantz, kacc22, n, ktop, kbot, nshfts, sr, si, h, ldh, iloz, ihiz,
z, ldz, v, ldv, u, ldu, nv, wv, ldwv, nh, wh, ldwh )
```

```
call dlaqr5( wantt, wantz, kacc22, n, ktop, kbot, nshfts, sr, si, h, ldh, iloz, ihiz,
z, ldz, v, ldv, u, ldu, nv, wv, ldwv, nh, wh, ldwh )
```

```
call claqr5( wantt, wantz, kacc22, n, ktop, kbot, nshfts, s, h, ldh, iloz, ihiz, z,
ldz, v, ldv, u, ldu, nv, wv, ldwv, nh, wh, ldwh )
```

```
call zlaqr5( wantt, wantz, kacc22, n, ktop, kbot, nshfts, s, h, ldh, iloz, ihiz, z,
ldz, v, ldv, u, ldu, nv, wv, ldwv, nh, wh, ldwh )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

This auxiliary routine called by `?laqr0` performs a single small-bulge multi-shift QR sweep.

Input Parameters

wantt	`LOGICAL`. *wantt* = `.TRUE.` if the quasi-triangular/triangular Schur factor is computed. *wantt* is set to `.FALSE.` otherwise.
wantz	`LOGICAL`. *wantz* = `.TRUE.` if the orthogonal/unitary Schur factor is computed. *wantz* is set to `.FALSE.` otherwise.
kacc22	`INTEGER`. Possible values are 0, 1, or 2. Specifies the computation mode of far-from-diagonal orthogonal updates. = `0`: the routine does not accumulate reflections and does not use matrix-matrix multiply to update far-from-diagonal matrix entries. = `1`: the routine accumulates reflections and uses matrix-matrix multiply to update the far-from-diagonal matrix entries. = `2`: the routine accumulates reflections, uses matrix-matrix multiply to update the far-from-diagonal matrix entries, and takes advantage of 2-by-2 block structure during matrix multiplies.
n	`INTEGER`. The order of the Hessenberg matrix *H* upon which the routine operates.
ktop, *kbot*	`INTEGER`. It is assumed without a check that either *ktop*=1 or *h*(*ktop*,*ktop-1*)=0, and either *kbot*=*n* or *h*(*kbot*+1,*kbot*)=0.
nshfts	`INTEGER`. Number of simultaneous shifts, must be positive and even.
sr, *si*	`REAL` for `slaqr5` `DOUBLE PRECISION` for `dlaqr5` Arrays, `DIMENSION` (*nshfts*) each. *sr* contains the real parts and *si* contains the imaginary parts of the *nshfts* shifts of origin that define the multi-shift QR sweep.
s	`COMPLEX` for `claqr5` `DOUBLE COMPLEX` for `zlaqr5`. Arrays, `DIMENSION` (*nshfts*). *s* contains the shifts of origin that define the multi-shift QR sweep.
h	`REAL` for `slaqr5` `DOUBLE PRECISION` for `dlaqr5` `COMPLEX` for `claqr5` `DOUBLE COMPLEX` for `zlaqr5`. Array, `DIMENSION` (*ldh*, *n*), on input contains the Hessenberg matrix.
ldh	`INTEGER`. The leading dimension of the array *h* just as declared in the calling routine. *ldh* ≥ `max(1,` *n*`)`.

iloz, *ihiz*	INTEGER. Specify the rows of *Z* to which transformations must be applied if *wantz* is .TRUE.. $1 \le iloz \le ihiz \le n$.
z	REAL for slaqr5 DOUBLE PRECISION for dlaqr5 COMPLEX for claqr5 DOUBLE COMPLEX for zlaqr5. Array, DIMENSION (*ldz*, *ihi*), contains the matrix *Z* if *wantz* is .TRUE.. If *wantz* is .FALSE., then *z* is not referenced.
ldz	INTEGER. The leading dimension of the array *z* just as declared in the calling routine. $ldz \ge n$.
v	REAL for slaqr5 DOUBLE PRECISION for dlaqr5 COMPLEX for claqr5 DOUBLE COMPLEX for zlaqr5. Workspace array with dimension (*ldv*, *nshfts*/2).
ldv	INTEGER. The leading dimension of the array *v* just as declared in the calling routine. $ldv \ge 3$.
u	REAL for slaqr5 DOUBLE PRECISION for dlaqr5 COMPLEX for claqr5 DOUBLE COMPLEX for zlaqr5. Workspace array with dimension (*ldu*, 3**nshfts*-3).
ldu	INTEGER. The leading dimension of the array *u* just as declared in the calling routine. $ldu \ge 3*nshfts-3$.
nh	INTEGER. The number of column in the array *wh* available for workspace. $nh \ge 1$.
wh	REAL for slaqr5 DOUBLE PRECISION for dlaqr5 COMPLEX for claqr5 DOUBLE COMPLEX for zlaqr5. Workspace array with dimension (*ldwh*, *nh*)
ldwh	INTEGER. The leading dimension of the array *wh* just as declared in the calling routine. $ldwh \ge 3*nshfts-3$
nv	INTEGER. The number of rows of the array *wv* available for workspace. $nv \ge 1$.
wv	REAL for slaqr5 DOUBLE PRECISION for dlaqr5 COMPLEX for claqr5 DOUBLE COMPLEX for zlaqr5. Workspace array with dimension (*ldwv*, 3**nshfts*-3).
ldwv	INTEGER. The leading dimension of the array *wv* just as declared in the calling routine. $ldwv \ge nv$.

Output Parameters

sr, *si*	On output, may be reordered.
h	On output a multi-shift QR Sweep with shifts *sr*(j)+i**si*(j) or *s*(j) is applied to the isolated diagonal block in rows and columns *ktop* through *kbot* .

z	If *wantz* is .TRUE., then the QR Sweep orthogonal/unitary similarity transformation is accumulated into *z(iloz:ihiz, ilo:ihi)* from the right. If *wantz* is .FALSE., then *z* is unreferenced.

?laqsb

Scales a symmetric band matrix, using scaling factors computed by ?pbequ.

Syntax

```
call slaqsb( uplo, n, kd, ab, ldab, s, scond, amax, equed )
call dlaqsb( uplo, n, kd, ab, ldab, s, scond, amax, equed )
call claqsb( uplo, n, kd, ab, ldab, s, scond, amax, equed )
call zlaqsb( uplo, n, kd, ab, ldab, s, scond, amax, equed )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine equilibrates a symmetric band matrix *A* using the scaling factors in the vector *s*.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric matrix *A* is stored. If *uplo* = 'U': upper triangular. If *uplo* = 'L': lower triangular.
n	INTEGER. The order of the matrix *A*. *n* ≥ 0.
kd	INTEGER. The number of super-diagonals of the matrix *A* if *uplo* = 'U', or the number of sub-diagonals if *uplo* = 'L'. *kd* ≥ 0.
ab	REAL for slaqsb DOUBLE PRECISION for dlaqsb COMPLEX for claqsb DOUBLE COMPLEX for zlaqsb Array, DIMENSION (*ldab*,*n*). On entry, the upper or lower triangle of the symmetric band matrix *A*, stored in the first *kd*+1 rows of the array. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: if *uplo* = 'U', *ab*(*kd*+1+*i*-*j*,*j*) = A(*i*,*j*) for max(1,*j*-*kd*) ≤ *i* ≤ *j*; if *uplo* = 'L', *ab*(1+*i*-*j*,*j*) = A(*i*,*j*) for *j* ≤ *i* ≤ min(*n*,*j*+*kd*).
ldab	INTEGER. The leading dimension of the array *ab*. *ldab* ≥ *kd*+1.
s	REAL for slaqsb/claqsb DOUBLE PRECISION for dlaqsb/zlaqsb Array, DIMENSION (*n*). The scale factors for A.
scond	REAL for slaqsb/claqsb

DOUBLE PRECISION for dlaqsb/zlaqsb
Ratio of the smallest $s(i)$ to the largest $s(i)$.

amax — REAL for slaqsb/claqsb
DOUBLE PRECISION for dlaqsb/zlaqsb
Absolute value of largest matrix entry.

Output Parameters

ab — On exit, if *info* = 0, the triangular factor U or L from the Cholesky factorization of the band matrix A that can be $A = U^T*U$ or $A = L*L^T$ for real flavors and $A = U^H*U$ or $A = L*L^H$ for complex flavors, in the same storage format as A.

equed — CHARACTER*1.
Specifies whether or not equilibration was done.
If *equed* = 'N': No equilibration.
If *equed* = 'Y': Equilibration was done, that is, A has been replaced by diag(*s*)*A*diag(*s*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*, which have the following meaning. *thresh* is a threshold value used to decide if scaling should be based on the ratio of the scaling factors. If *scond* < *thresh*, scaling is done. *large* and *small* are threshold values used to decide if scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, scaling is done.

?laqsp

Scales a symmetric/Hermitian matrix in packed storage, using scaling factors computed by ?ppequ.

Syntax

```
call slaqsp( uplo, n, ap, s, scond, amax, equed )
call dlaqsp( uplo, n, ap, s, scond, amax, equed )
call claqsp( uplo, n, ap, s, scond, amax, equed )
call zlaqsp( uplo, n, ap, s, scond, amax, equed )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laqsp equilibrates a symmetric matrix A using the scaling factors in the vector s.

Input Parameters

uplo — CHARACTER*1.
Specifies whether the upper or lower triangular part of the symmetric matrix A is stored.
If *uplo* = 'U': upper triangular.
If *uplo* = 'L': lower triangular.

n — INTEGER. The order of the matrix A. $n \geq 0$.

ap	REAL for slaqsp DOUBLE PRECISION for dlaqsp COMPLEX for claqsp DOUBLE COMPLEX for zlaqsp Array, DIMENSION (*n*(*n*+1)/2). On entry, the upper or lower triangle of the symmetric matrix *A*, packed columnwise in a linear array. The *j*-th column of *A* is stored in the array *ap* as follows: if *uplo* = 'U', *ap*(i + (*j*-1)*j*/2) = *A*(*i*,*j*) for $1 \le i \le j$; if *uplo* = 'L', *ap*(i + (*j*-1)(2*n*-*j*)/2) = *A*(*i*,*j*) for $j \le i \le n$.
s	REAL for slaqsp/claqsp DOUBLE PRECISION for dlaqsp/zlaqsp Array, DIMENSION (*n*). The scale factors for A.
scond	REAL for slaqsp/claqsp DOUBLE PRECISION for dlaqsp/zlaqsp Ratio of the smallest *s*(*i*) to the largest *s*(*i*).
amax	REAL for slaqsp/claqsp DOUBLE PRECISION for dlaqsp/zlaqsp Absolute value of largest matrix entry.

Output Parameters

ap	On exit, the equilibrated matrix: diag(*s*)*A*diag(*s*), in the same storage format as *A*.
equed	CHARACTER*1. Specifies whether or not equilibration was done. If *equed* = 'N': No equilibration. If *equed* = 'Y': Equilibration was done, that is, *A* has been replaced by diag(*s*)*A*diag(*s*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*, which have the following meaning. *thresh* is a threshold value used to decide if scaling should be based on the ratio of the scaling factors. If *scond* < *thresh*, scaling is done. *large* and *small* are threshold values used to decide if scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, scaling is done.

?laqsy

Scales a symmetric/Hermitian matrix, using scaling factors computed by ?poequ.

Syntax

```
call slaqsy( uplo, n, a, lda, s, scond, amax, equed )
call dlaqsy( uplo, n, a, lda, s, scond, amax, equed )
call claqsy( uplo, n, a, lda, s, scond, amax, equed )
call zlaqsy( uplo, n, a, lda, s, scond, amax, equed )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine equilibrates a symmetric matrix *A* using the scaling factors in the vector *s*.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric matrix *A* is stored. If *uplo* = 'U': upper triangular. If *uplo* = 'L': lower triangular.
n	INTEGER. The order of the matrix *A*. *n* ≥ 0.
a	REAL for slaqsy DOUBLE PRECISION for dlaqsy COMPLEX for claqsy DOUBLE COMPLEX for zlaqsy Array, DIMENSION (*lda*,*n*). On entry, the symmetric matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(*n*,1).
s	REAL for slaqsy/claqsy DOUBLE PRECISION for dlaqsy/zlaqsy Array, DIMENSION (*n*). The scale factors for A.
scond	REAL for slaqsy/claqsy DOUBLE PRECISION for dlaqsy/zlaqsy Ratio of the smallest *s*(*i*) to the largest *s*(*i*).
amax	REAL for slaqsy/claqsy DOUBLE PRECISION for dlaqsy/zlaqsy Absolute value of largest matrix entry.

Output Parameters

a	On exit, if *equed* = 'Y', the equilibrated matrix: diag(*s*)*A*diag(*s*).
equed	CHARACTER*1. Specifies whether or not equilibration was done. If *equed* = 'N': No equilibration. If *equed* = 'Y': Equilibration was done, i.e., *A* has been replaced by diag(*s*)*A*diag(*s*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*, which have the following meaning. *thresh* is a threshold value used to decide if scaling should be based on the ratio of the scaling factors. If *scond* < *thresh*, scaling is done. *large* and *small* are threshold values used to decide if scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, scaling is done.

?laqtr

Solves a real quasi-triangular system of equations, or a complex quasi-triangular system of special form, in real arithmetic.

Syntax

```
call slaqtr( ltran, lreal, n, t, ldt, b, w, scale, x, work, info )
call dlaqtr( ltran, lreal, n, t, ldt, b, w, scale, x, work, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?laqtr` solves the real quasi-triangular system

`op(T) * p = scale*c`, if `lreal = .TRUE.`

or the complex quasi-triangular systems

`op(T + iB)*(p+iq) = scale*(c+id)`, if `lreal = .FALSE.`

in real arithmetic, where T is upper quasi-triangular.

If `lreal = .FALSE.`, then the first diagonal block of T must be 1-by-1, B is the specially structured matrix

`op(A) = A` or A^T, A^T denotes the transpose of matrix A.

On input,

This routine is designed for the condition number estimation in routine `?trsna`.

Input Parameters

ltran	LOGICAL. On entry, *ltran* specifies the option of conjugate transpose: = .FALSE., op(T + iB) = T + iB, = .TRUE., op(T + iB) = (T + iB)^T.

lreal	LOGICAL. On entry, *lreal* specifies the input matrix structure: = .FALSE., the input is complex = .TRUE., the input is real.
n	INTEGER. On entry, *n* specifies the order of $T + iB$. $n \geq 0$.
t	REAL for slaqtr DOUBLE PRECISION for dlaqtr Array, dimension (*ldt*,*n*). On entry, *t* contains a matrix in Schur canonical form. If *lreal* = .FALSE., then the first diagonal block of *t* must be 1-by-1.
ldt	INTEGER. The leading dimension of the matrix *T*. $ldt \geq \max(1,n)$.
b	REAL for slaqtr DOUBLE PRECISION for dlaqtr Array, dimension (*n*). On entry, *b* contains the elements to form the matrix *B* as described above. If *lreal* = .TRUE., *b* is not referenced.
w	REAL for slaqtr DOUBLE PRECISION for dlaqtr On entry, *w* is the diagonal element of the matrix *B*. If *lreal* = .TRUE., *w* is not referenced.
x	REAL for slaqtr DOUBLE PRECISION for dlaqtr Array, dimension (2*n*). On entry, *x* contains the right hand side of the system.
work	REAL for slaqtr DOUBLE PRECISION for dlaqtr Workspace array, dimension (*n*).

Output Parameters

scale	REAL for slaqtr DOUBLE PRECISION for dlaqtr On exit, *scale* is the scale factor.
x	On exit, *X* is overwritten by the solution.
info	INTEGER. If *info* = 0: successful exit. If *info* = 1: the some diagonal 1-by-1 block has been perturbed by a small number smin to keep nonsingularity. If *info* = 2: the some diagonal 2-by-2 block has been perturbed by a small number in ?laln2 to keep nonsingularity.

NOTE For higher speed, this routine does not check the inputs for errors.

?lar1v

Computes the (scaled) r-th column of the inverse of the submatrix in rows b1 through bn of tridiagonal matrix.

Syntax

```
call slar1v( n, b1, bn, lambda, d, l, ld, lld, pivmin, gaptol, z, wantnc, negcnt, ztz, mingma, r, isuppz, nrminv, resid, rqcorr, work )
call dlar1v( n, b1, bn, lambda, d, l, ld, lld, pivmin, gaptol, z, wantnc, negcnt, ztz, mingma, r, isuppz, nrminv, resid, rqcorr, work )
call clar1v( n, b1, bn, lambda, d, l, ld, lld, pivmin, gaptol, z, wantnc, negcnt, ztz, mingma, r, isuppz, nrminv, resid, rqcorr, work )
call zlar1v( n, b1, bn, lambda, d, l, ld, lld, pivmin, gaptol, z, wantnc, negcnt, ztz, mingma, r, isuppz, nrminv, resid, rqcorr, work )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lar1v` computes the (scaled) *r*-th column of the inverse of the submatrix in rows *b1* through *bn* of the tridiagonal matrix $L*D*L^T - \lambda*I$. When λ is close to an eigenvalue, the computed vector is an accurate eigenvector. Usually, *r* corresponds to the index where the eigenvector is largest in magnitude.

The following steps accomplish this computation :

- Stationary *qd* transform, $L*D*L^T - \lambda*I = L(+)*D(+)*L(+)^T$
- Progressive *qd* transform, $L*D*L^T - \lambda*I = U(-)*D(-)*U(-)^T$,
- Computation of the diagonal elements of the inverse of $L*D*L^T - \lambda*I$ by combining the above transforms, and choosing *r* as the index where the diagonal of the inverse is (one of the) largest in magnitude.
- Computation of the (scaled) *r*-th column of the inverse using the twisted factorization obtained by combining the top part of the stationary and the bottom part of the progressive transform.

Input Parameters

n — `INTEGER`. The order of the matrix $L*D*L^T$.

b1 — `INTEGER`. First index of the submatrix of $L*D*L^T$.

bn — `INTEGER`. Last index of the submatrix of $L*D*L^T$.

lambda — `REAL` for `slar1v/clar1v`
`DOUBLE PRECISION` for `dlar1v/zlar1v`
The shift. To compute an accurate eigenvector, *lambda* should be a good approximation to an eigenvalue of $L*D*L^T$.

l — `REAL` for `slar1v/clar1v`
`DOUBLE PRECISION` for `dlar1v/zlar1v`
Array, `DIMENSION` (*n*-1).
The (*n*-1) subdiagonal elements of the unit bidiagonal matrix *L*, in elements 1 to *n*-1.

d — `REAL` for `slar1v/clar1v`
`DOUBLE PRECISION` for `dlar1v/zlar1v`
Array, `DIMENSION` (*n*).
The *n* diagonal elements of the diagonal matrix *D*.

ld — `REAL` for `slar1v/clar1v`
`DOUBLE PRECISION` for `dlar1v/zlar1v`
Array, `DIMENSION` (*n*-1).
The *n*-1 elements L_i*D_i.

lld
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
Array, DIMENSION (*n*-1).
The *n*-1 elements $L_i*L_i*D_i$.

pivmin
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
The minimum pivot in the Sturm sequence.

gaptol
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
Tolerance that indicates when eigenvector entries are negligible with respect to their contribution to the residual.

z
REAL for slar1v
DOUBLE PRECISION for dlar1v
COMPLEX for clar1v
DOUBLE COMPLEX for zlar1v
Array, DIMENSION (*n*). All entries of *z* must be set to 0.

wantnc
LOGICAL.
Specifies whether *negcnt* has to be computed.

r
INTEGER.
The twist index for the twisted factorization used to compute *z*. On input, $0 \le r \le n$. If *r* is input as 0, *r* is set to the index where $(L*D*L^T - lambda*I)^{-1}$ is largest in magnitude. If $1 \le r \le n$, *r* is unchanged.

work
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
Workspace array, DIMENSION (4**n*).

Output Parameters

z
REAL for slar1v
DOUBLE PRECISION for dlar1v
COMPLEX for clar1v
DOUBLE COMPLEX for zlar1v
Array, DIMENSION (*n*). The (scaled) *r*-th column of the inverse. *z*(*r*) is returned to be 1.

negcnt
INTEGER. If *wantnc* is .TRUE. then *negcnt* = the number of pivots < *pivmin* in the matrix factorization $L*D*L^T$, and *negcnt* = -1 otherwise.

ztz
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
The square of the 2-norm of *z*.

mingma
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
The reciprocal of the largest (in magnitude) diagonal element of the inverse of $L*D*L^T - lambda*I$.

r
On output, *r* is the twist index used to compute *z*. Ideally, *r* designates the position of the maximum entry in the eigenvector.

isuppz
INTEGER. Array, DIMENSION (2). The support of the vector in *Z*, that is, the vector *z* is nonzero only in elements *isuppz*(1) through *isuppz*(2).

nrminv
REAL for slar1v/clar1v
DOUBLE PRECISION for dlar1v/zlar1v
Equals 1/sqrt(*ztz*).

resid	REAL for slar1v/clar1v DOUBLE PRECISION for dlar1v/zlar1v The residual of the FP vector. *resid* = ABS(*mingma*)/sqrt(*ztz*).
rqcorr	REAL for slar1v/clar1v DOUBLE PRECISION for dlar1v/zlar1v The Rayleigh Quotient correction to *lambda*. *rqcorr* = *mingma*/*ztz*.

?lar2v

Applies a vector of plane rotations with real cosines and real/complex sines from both sides to a sequence of 2-by-2 symmetric/Hermitian matrices.

Syntax

```
call slar2v( n, x, y, z, incx, c, s, incc )
call dlar2v( n, x, y, z, incx, c, s, incc )
call clar2v( n, x, y, z, incx, c, s, incc )
call zlar2v( n, x, y, z, incx, c, s, incc )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lar2v applies a vector of real/complex plane rotations with real cosines from both sides to a sequence of 2-by-2 real symmetric or complex Hermitian matrices, defined by the elements of the vectors x, y and z. For i = 1,2,...,*n*

$$\begin{bmatrix} x_i & z_i \\ \mathrm{conjg}(z_i) & y_i \end{bmatrix} := \begin{bmatrix} c(i) & \mathrm{conjg}(s(i)) \\ -s(i) & c(i) \end{bmatrix} \begin{bmatrix} x_i & z_i \\ \mathrm{conjg}(z_i) & y_i \end{bmatrix} \begin{bmatrix} c(i) & -\mathrm{conjg}(s(i)) \\ s(i) & c(i) \end{bmatrix}$$

Input Parameters

n	INTEGER. The number of plane rotations to be applied.
x, *y*, *z*	REAL for slar2v DOUBLE PRECISION for dlar2v COMPLEX for clar2v DOUBLE COMPLEX for zlar2v Arrays, DIMENSION (1+(*n*-1)**incx*) each. Contain the vectors x, y and z, respectively. For all flavors of ?lar2v, elements of *x* and *y* are assumed to be real.
incx	INTEGER. The increment between elements of *x*, *y*, and *z*. *incx* > 0.
c	REAL for slar2v/clar2v DOUBLE PRECISION for dlar2v/zlar2v Array, DIMENSION (1+(*n*-1)**incc*). The cosines of the plane rotations.
s	REAL for slar2v

	DOUBLE PRECISION for dlar2v COMPLEX for clar2v DOUBLE COMPLEX for zlar2v Array, DIMENSION (1+(*n*-1)**incc*). The sines of the plane rotations.
incc	INTEGER. The increment between elements of *c* and *s*. *incc* > 0.

Output Parameters

x, *y*, *z*	Vectors *x*, *y* and *z*, containing the results of transform.

?larf

Applies an elementary reflector to a general rectangular matrix.

Syntax

```
call slarf( side, m, n, v, incv, tau, c, ldc, work )
call dlarf( side, m, n, v, incv, tau, c, ldc, work )
call clarf( side, m, n, v, incv, tau, c, ldc, work )
call zlarf( side, m, n, v, incv, tau, c, ldc, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine applies a real/complex elementary reflector *H* to a real/complex *m*-by-*n* matrix *C*, from either the left or the right. *H* is represented in one of the following forms:

- $H = I - tau*v*v^T$

  where *tau* is a real scalar and v is a real vector.

  If *tau* = 0, then *H* is taken to be the unit matrix.
- $H = I - tau*v*v^H$

  where *tau* is a complex scalar and v is a complex vector.

  If *tau* = 0, then *H* is taken to be the unit matrix. For clarf/zlarf, to apply H^H (the conjugate transpose of *H*), supply conjg(*tau*) instead of *tau*.

Input Parameters

side	CHARACTER*1. If *side* = 'L': form *H***C* If *side* = 'R': form *C***H*.
m	INTEGER. The number of rows of the matrix *C*.
n	INTEGER. The number of columns of the matrix *C*.
v	REAL for slarf DOUBLE PRECISION for dlarf COMPLEX for clarf DOUBLE COMPLEX for zlarf Array, DIMENSION (1 + (*m*-1)*abs(*incv*)) if *side* = 'L' or

	(1 + (*n*-1)*abs(*incv*)) if *side* = 'R'. The vector *v* in the representation of *H*. *v* is not used if *tau* = 0.
incv	INTEGER. The increment between elements of *v*. *incv* ≠ 0.
tau	REAL for slarf DOUBLE PRECISION for dlarf COMPLEX for clarf DOUBLE COMPLEX for zlarf The value *tau* in the representation of *H*.
c	REAL for slarf DOUBLE PRECISION for dlarf COMPLEX for clarf DOUBLE COMPLEX for zlarf Array, DIMENSION (*ldc*,*n*). On entry, the *m*-by-*n* matrix *C*.
ldc	INTEGER. The leading dimension of the array *c*. *ldc* ≥ max(1,*m*).
work	REAL for slarf DOUBLE PRECISION for dlarf COMPLEX for clarf DOUBLE COMPLEX for zlarf Workspace array, DIMENSION (*n*) if *side* = 'L' or (*m*) if *side* = 'R'.

Output Parameters

c	On exit, *C* is overwritten by the matrix *H***C* if *side* = 'L', or *C***H* if *side* = 'R'.

?larfb

Applies a block reflector or its transpose/conjugate-transpose to a general rectangular matrix.

Syntax

```
call slarfb( side, trans, direct, storev, m, n, k, v, ldv, t, ldt, c, ldc, work, ldwork )
call dlarfb( side, trans, direct, storev, m, n, k, v, ldv, t, ldt, c, ldc, work, ldwork )
call clarfb( side, trans, direct, storev, m, n, k, v, ldv, t, ldt, c, ldc, work, ldwork )
call zlarfb( side, trans, direct, storev, m, n, k, v, ldv, t, ldt, c, ldc, work, ldwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The real flavors of the routine ?larfb apply a real block reflector *H* or its transpose H^T to a real *m*-by-*n* matrix *C* from either left or right.

The complex flavors of the routine ?larfb apply a complex block reflector H or its conjugate transpose H^H to a complex m-by-n matrix C from either left or right.

Input Parameters

side	CHARACTER*1. If *side* = 'L': apply H or H^T for real flavors and H or H^H for complex flavors from the left. If *side* = 'R': apply H or H^T for real flavors and H or H^H for complex flavors from the right.
trans	CHARACTER*1. If *trans* = 'N': apply H (No transpose). If *trans* = 'C': apply H^H (Conjugate transpose). If *trans* = 'T': apply H^T (Transpose).
direct	CHARACTER*1. Indicates how H is formed from a product of elementary reflectors If *direct* = 'F': $H = H(1)*H(2)*. \; . \; . \; *H(k)$ (forward) If *direct* = 'B': $H = H(k)* \; . \; . \; . \; H(2)*H(1)$ (backward)
storev	CHARACTER*1. Indicates how the vectors which define the elementary reflectors are stored: If *storev* = 'C': Column-wise If *storev* = 'R': Row-wise
m	INTEGER. The number of rows of the matrix C.
n	INTEGER. The number of columns of the matrix C.
k	INTEGER. The order of the matrix T (equal to the number of elementary reflectors whose product defines the block reflector).
v	REAL for slarfb DOUBLE PRECISION for dlarfb COMPLEX for clarfb DOUBLE COMPLEX for zlarfb Array, DIMENSION (*ldv*, *k*) if *storev* = 'C' (*ldv*, *m*) if *storev* = 'R' and *side* = 'L' (*ldv*, *n*) if *storev* = 'R' and *side* = 'R' The matrix v. See *Application Notes* below.
ldv	INTEGER. The leading dimension of the array v. If *storev* = 'C' and *side* = 'L', $ldv \geq \max(1,m)$; if *storev* = 'C' and *side* = 'R', $ldv \geq \max(1,n)$; if *storev* = 'R', $ldv \geq k$.
t	REAL for slarfb DOUBLE PRECISION for dlarfb COMPLEX for clarfb DOUBLE COMPLEX for zlarfb Array, DIMENSION (*ldt*,*k*). Contains the triangular k-by-k matrix T in the representation of the block reflector.
ldt	INTEGER. The leading dimension of the array t. $ldt \geq k$.
c	REAL for slarfb DOUBLE PRECISION for dlarfb

COMPLEX for clarfb
DOUBLE COMPLEX for zlarfb
Array, DIMENSION (*ldc*,*n*).
On entry, the *m*-by-*n* matrix *C*.

ldc — INTEGER. The leading dimension of the array *c*.
ldc ≥ max(1,*m*).

work — REAL for slarfb
DOUBLE PRECISION for dlarfb
COMPLEX for clarfb
DOUBLE COMPLEX for zlarfb
Workspace array, DIMENSION (*ldwork*, *k*).

ldwork — INTEGER. The leading dimension of the array *work*.
If *side* = 'L', *ldwork* ≥ max(1, *n*);
if *side* = 'R', *ldwork* ≥ max(1, *m*).

Output Parameters

c — On exit, *c* is overwritten by the product of the following:

- *H***C*, or H^T**C*, or *C***H*, or *C**H^T for real flavors
- *H***C*, or H^H**C*, or *C***H*, or *C**H^H for complex flavors

Application Notes

The shape of the matrix *V* and the storage of the vectors which define the *H*(*i*) is best illustrated by the following example with *n* = 5 and *k* = 3. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct = 'B' and *storev* = 'C':

$$\begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ 1 & v_2 & v_3 \\ & 1 & v_3 \\ & & 1 \end{bmatrix}$$

direct = 'B' and *storev* = 'R':

$$\begin{bmatrix} v_1 & v_1 & 1 & & \\ v_2 & v_2 & v_2 & 1 & \\ v_3 & v_3 & v_3 & v_3 & 1 \end{bmatrix}$$

?larfg

Generates an elementary reflector (Householder matrix).

Syntax

```
call slarfg( n, alpha, x, incx, tau )
call dlarfg( n, alpha, x, incx, tau )
call clarfg( n, alpha, x, incx, tau )
call zlarfg( n, alpha, x, incx, tau )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?larfg` generates a real/complex elementary reflector H of order n, such that

$$H * \begin{bmatrix} alpha \\ x \end{bmatrix} = \begin{bmatrix} beta \\ 0 \end{bmatrix}, \quad H^T * H = I,$$

for real flavors and

$$H^H * \begin{bmatrix} alpha \\ x \end{bmatrix} = \begin{bmatrix} beta \\ 0 \end{bmatrix}, \quad H^H * H = I,$$

for complex flavors,

where $alpha$ and $beta$ are scalars (with $beta$ real for all flavors), and x is an (n-1)-element real/complex vector. H is represented in the form

$$H = I - tau * \begin{bmatrix} 1 \\ v \end{bmatrix} * \begin{bmatrix} 1 & v^T \end{bmatrix}$$

for real flavors and

$$H = I - tau * \begin{bmatrix} 1 \\ v \end{bmatrix} * \begin{bmatrix} 1 & v' \end{bmatrix}$$

for complex flavors,

where tau is a real/complex scalar and v is a real/complex (n-1)-element vector, respectively. Note that for `clarfg`/`zlarfg`, H is not Hermitian.

If the elements of x are all zero (and, for complex flavors, $alpha$ is real), then $tau = 0$ and H is taken to be the unit matrix.

Otherwise, $1 \le tau \le 2$ (for real flavors), or

$1 \le \mathrm{Re}(tau) \le 2$ and $\mathrm{abs}(tau-1) \le 1$ (for complex flavors).

Input Parameters

n	`INTEGER`. The order of the elementary reflector.
$alpha$	`REAL` for `slarfg` `DOUBLE PRECISION` for `dlarfg` `COMPLEX` for `clarfg` `DOUBLE COMPLEX` for `zlarfg` On entry, the value alpha.

x	REAL for slarfg DOUBLE PRECISION for dlarfg COMPLEX for clarfg DOUBLE COMPLEX for zlarfg Array, DIMENSION (1+(*n*-2)*abs(*incx*)). On entry, the vector *x*.
incx	INTEGER. The increment between elements of *x*. *incx* > 0.

Output Parameters

alpha	On exit, it is overwritten with the value *beta*.
x	On exit, it is overwritten with the vector *v*.
tau	REAL for slarfg DOUBLE PRECISION for dlarfg COMPLEX for clarfg DOUBLE COMPLEX for zlarfg The value *tau*.

?larfgp

Generates an elementary reflector (Householder matrix) with non-negative beta .

Syntax

```
call slarfgp( n, alpha, x, incx, tau )
call dlarfgp( n, alpha, x, incx, tau )
call clarfgp( n, alpha, x, incx, tau )
call zlarfgp( n, alpha, x, incx, tau )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?larfgp generates a real/complex elementary reflector *H* of order *n*, such that

$$H * \begin{bmatrix} alpha \\ x \end{bmatrix} = \begin{bmatrix} beta \\ 0 \end{bmatrix}, \quad H^T * H = I,$$

for real flavors and

$$H^H * \begin{bmatrix} alpha \\ x \end{bmatrix} = \begin{bmatrix} beta \\ 0 \end{bmatrix}, \quad H^H * H = I,$$

for complex flavors,

where *alpha* and *beta* are scalars (with *beta* real and non-negative for all flavors), and *x* is an (*n*-1)-element real/complex vector. *H* is represented in the form

$$H = I - tau * \begin{bmatrix} 1 \\ v \end{bmatrix} * \begin{bmatrix} 1 & v^T \end{bmatrix}$$

for real flavors and

$$H = I - tau * \begin{bmatrix} 1 \\ v \end{bmatrix} * \begin{bmatrix} 1 & v' \end{bmatrix}$$

for complex flavors,

where *tau* is a real/complex scalar and v is a real/complex (*n*-1)-element vector. Note that for c/zlarfgp, *H* is not Hermitian.

If the elements of *x* are all zero (and, for complex flavors, *alpha* is real), then *tau* = 0 and *H* is taken to be the unit matrix.

Otherwise, 1 ≤ *tau* ≤ 2 (for real flavors), or

1 ≤ Re(*tau*) ≤ 2 and abs(*tau*-1) ≤ 1 (for complex flavors).

Input Parameters

n	INTEGER. The order of the elementary reflector.
alpha	REAL for slarfgp DOUBLE PRECISION for dlarfgp COMPLEX for clarfgp DOUBLE COMPLEX for zlarfgp On entry, the value alpha.
x	REAL for s DOUBLE PRECISION for dlarfgp COMPLEX for clarfgp DOUBLE COMPLEX for zlarfgp Array, DIMENSION (1+(*n*-2)*abs(*incx*)). On entry, the vector *x*.
incx	INTEGER. The increment between elements of *x*. *incx* > 0.

Output Parameters

alpha	On exit, it is overwritten with the value *beta*.
x	On exit, it is overwritten with the vector *v*.
tau	REAL for slarfgp DOUBLE PRECISION for dlarfgp COMPLEX for clarfgp DOUBLE COMPLEX for zlarfgp The value *tau*.

?larft

*Forms the triangular factor T of a block reflector H = I - V*T*V**H.*

Syntax

```
call slarft( direct, storev, n, k, v, ldv, tau, t, ldt )
call dlarft( direct, storev, n, k, v, ldv, tau, t, ldt )
call clarft( direct, storev, n, k, v, ldv, tau, t, ldt )
call zlarft( direct, storev, n, k, v, ldv, tau, t, ldt )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?larft forms the triangular factor T of a real/complex block reflector H of order n, which is defined as a product of k elementary reflectors.

If $direct$ = 'F', $H = H(1)*H(2)* \ldots *H(k)$ and T is upper triangular;

If $direct$ = 'B', $H = H(k)* \ldots *H(2)*H(1)$ and T is lower triangular.

If $storev$ = 'C', the vector which defines the elementary reflector H(i) is stored in the i-th column of the array v, and $H = I - V*T*V^T$ (for real flavors) or $H = I - V*T*V^H$ (for complex flavors) .

If $storev$ = 'R', the vector which defines the elementary reflector H(i) is stored in the i-th row of the array v, and $H = I - V^T*T*V$ (for real flavors) or $H = I - V^H*T*V$ (for complex flavors).

Input Parameters

direct	CHARACTER*1. Specifies the order in which the elementary reflectors are multiplied to form the block reflector: = 'F': $H = H(1)*H(2)* \ldots *H(k)$ (forward) = 'B': $H = H(k)* \ldots *H(2)*H(1)$ (backward)
storev	CHARACTER*1. Specifies how the vectors which define the elementary reflectors are stored (see also *Application Notes* below): = 'C': column-wise = 'R': row-wise.
n	INTEGER. The order of the block reflector H. $n \geq 0$.
k	INTEGER. The order of the triangular factor T (equal to the number of elementary reflectors). $k \geq 1$.
v	REAL for slarft DOUBLE PRECISION for dlarft COMPLEX for clarft DOUBLE COMPLEX for zlarft Array, DIMENSION (ldv, k) if $storev$ = 'C' or (ldv, n) if $storev$ = 'R'. The matrix V.
ldv	INTEGER. The leading dimension of the array v. If $storev$ = 'C', $ldv \geq \max(1,n)$; if $storev$ = 'R', $ldv \geq k$.
tau	REAL for slarft DOUBLE PRECISION for dlarft COMPLEX for clarft DOUBLE COMPLEX for zlarft Array, DIMENSION (k). tau(i) must contain the scalar factor of the elementary reflector H(i).
ldt	INTEGER. The leading dimension of the output array t. $ldt \geq k$.

Output Parameters

t	REAL for slarft DOUBLE PRECISION for dlarft COMPLEX for clarft DOUBLE COMPLEX for zlarft

Array, DIMENSION (*ldt*,*k*). The *k*-by-*k* triangular factor *T* of the block reflector. If *direct* = 'F', *T* is upper triangular; if *direct* = 'B', *T* is lower triangular. The rest of the array is not used.

v The matrix *V*.

Application Notes

The shape of the matrix *V* and the storage of the vectors which define the *H*(*i*) is best illustrated by the following example with *n* = 5 and *k* = 3. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct = 'B' and *storev* = 'C':

$$\begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ 1 & v_2 & v_3 \\ & 1 & v_3 \\ & & 1 \end{bmatrix}$$

direct = 'B' and *storev* = 'R':

$$\begin{bmatrix} v_1 & v_1 & 1 & & \\ v_2 & v_2 & v_2 & 1 & \\ v_3 & v_3 & v_3 & v_3 & 1 \end{bmatrix}$$

?larfx

Applies an elementary reflector to a general rectangular matrix, with loop unrolling when the reflector has order less than or equal to 10.

Syntax

```
call slarfx( side, m, n, v, tau, c, ldc, work )
call dlarfx( side, m, n, v, tau, c, ldc, work )
call clarfx( side, m, n, v, tau, c, ldc, work )
call zlarfx( side, m, n, v, tau, c, ldc, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?larfx applies a real/complex elementary reflector H to a real/complex m-by-n matrix C, from either the left or the right.

H is represented in the following forms:

- $H = I - tau*v*v^T$, where tau is a real scalar and v is a real vector.
- $H = I - tau*v*v^H$, where tau is a complex scalar and v is a complex vector.

If $tau = 0$, then H is taken to be the unit matrix.

Input Parameters

side	CHARACTER*1. If *side* = 'L': form $H*C$ If *side* = 'R': form $C*H$.
m	INTEGER. The number of rows of the matrix C.
n	INTEGER. The number of columns of the matrix C.
v	REAL for slarfx DOUBLE PRECISION for dlarfx COMPLEX for clarfx DOUBLE COMPLEX for zlarfx Array, DIMENSION (m) if *side* = 'L' or (n) if *side* = 'R'. The vector v in the representation of H.
tau	REAL for slarfx DOUBLE PRECISION for dlarfx COMPLEX for clarfx DOUBLE COMPLEX for zlarfx The value tau in the representation of H.
c	REAL for slarfx DOUBLE PRECISION for dlarfx COMPLEX for clarfx DOUBLE COMPLEX for zlarfx Array, DIMENSION (ldc,n). On entry, the m-by-n matrix C.
ldc	INTEGER. The leading dimension of the array c. $lda \geq (1,m)$.
work	REAL for slarfx DOUBLE PRECISION for dlarfx COMPLEX for clarfx DOUBLE COMPLEX for zlarfx Workspace array, DIMENSION (n) if *side* = 'L' or (m) if *side* = 'R'. *work* is not referenced if H has order < 11.

Output Parameters

c	On exit, C is overwritten by the matrix $H*C$ if *side* = 'L', or $C*H$ if *side* = 'R'.

?largv

Generates a vector of plane rotations with real cosines and real/complex sines.

Syntax

```
call slargv( n, x, incx, y, incy, c, incc )
call dlargv( n, x, incx, y, incy, c, incc )
call clargv( n, x, incx, y, incy, c, incc )
call zlargv( n, x, incx, y, incy, c, incc )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine generates a vector of real/complex plane rotations with real cosines, determined by elements of the real/complex vectors x and y.

For `slargv`/`dlargv`:

$$\begin{bmatrix} c(i) & s(i) \\ -s(i) & c(i) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} a_i \\ 0 \end{bmatrix}, \text{for } i = 1,2,\ldots,n$$

For `clargv`/`zlargv`:

$$\begin{bmatrix} c(i) & s(i) \\ -\text{conjg}(s(i)) & c(i) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} r_i \\ 0 \end{bmatrix}, \text{for } i = 1,2,\ldots,n$$

where $c(i)^2 + \text{abs}(s(i))^2 = 1$ and the following conventions are used (these are the same as in `clartg`/`zlartg` but differ from the BLAS Level 1 routine `crotg`/`zrotg`):

If $y_i = 0$, then $c(i) = 1$ and $s(i) = 0$;

If $x_i = 0$, then $c(i) = 0$ and $s(i)$ is chosen so that r_i is real.

Input Parameters

n	INTEGER. The number of plane rotations to be generated.
x, *y*	REAL for slargv DOUBLE PRECISION for dlargv COMPLEX for clargv DOUBLE COMPLEX for zlargv Arrays, DIMENSION (1+(*n*-1)**incx*) and (1+(*n*-1)**incy*), respectively. On entry, the vectors *x* and *y*.
incx	INTEGER. The increment between elements of *x*. *incx* > 0.
incy	INTEGER. The increment between elements of *y*.

	incy > 0.
incc	INTEGER. The increment between elements of the output array *c*. *incc* > 0.

Output Parameters

x	On exit, *x*(*i*) is overwritten by a_i (for real flavors), or by r_i (for complex flavors), for *i* = 1,...,*n*.
y	On exit, the sines *s*(*i*) of the plane rotations.
c	REAL for slargv/clargv DOUBLE PRECISION for dlargv/zlargv Array, DIMENSION (1+(*n*-1)**incc*). The cosines of the plane rotations.

?larnv

Returns a vector of random numbers from a uniform or normal distribution.

Syntax

```
call slarnv( idist, iseed, n, x )
call dlarnv( idist, iseed, n, x )
call clarnv( idist, iseed, n, x )
call zlarnv( idist, iseed, n, x )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?larnv returns a vector of *n* random real/complex numbers from a uniform or normal distribution.

This routine calls the auxiliary routine ?laruv to generate random real numbers from a uniform (0,1) distribution, in batches of up to 128 using vectorisable code. The Box-Muller method is used to transform numbers from a uniform to a normal distribution.

Input Parameters

idist	INTEGER. Specifies the distribution of the random numbers: for slarnv and dlanrv: = 1: uniform (0,1) = 2: uniform (-1,1) = 3: normal (0,1). for clarnv and zlanrv: = 1: real and imaginary parts each uniform (0,1) = 2: real and imaginary parts each uniform (-1,1) = 3: real and imaginary parts each normal (0,1) = 4: uniformly distributed on the disc abs(z) < 1 = 5: uniformly distributed on the circle abs(z) = 1
iseed	INTEGER. Array, DIMENSION (4). On entry, the seed of the random number generator; the array elements must be between 0 and 4095, and *iseed*(4) must be odd.

n	INTEGER. The number of random numbers to be generated.

Output Parameters

x	REAL for slarnv DOUBLE PRECISION for dlarnv COMPLEX for clarnv DOUBLE COMPLEX for zlarnv Array, DIMENSION (*n*). The generated random numbers.
iseed	On exit, the seed is updated.

?larra

Computes the splitting points with the specified threshold.

Syntax

```
call slarra( n, d, e, e2, spltol, tnrm, nsplit, isplit, info )
call dlarra( n, d, e, e2, spltol, tnrm, nsplit, isplit, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the splitting points with the specified threshold and sets any "small" off-diagonal elements to zero.

Input Parameters

n	INTEGER. The order of the matrix (*n* > 1).
d	REAL for slarra DOUBLE PRECISION for dlarra Array, DIMENSION (*n*). Contains *n* diagonal elements of the tridiagonal matrix *T*.
e	REAL for slarra DOUBLE PRECISION for dlarra Array, DIMENSION (*n*). First (*n*-1) entries contain the subdiagonal elements of the tridiagonal matrix *T*; *e*(*n*) need not be set.
e2	REAL for slarra DOUBLE PRECISION for dlarra Array, DIMENSION (*n*). First (*n*-1) entries contain the squares of the subdiagonal elements of the tridiagonal matrix *T*; *e2*(*n*) need not be set.
spltol	REAL for slarra DOUBLE PRECISION for dlarra The threshold for splitting. Two criteria can be used: *spltol*<0 : criterion based on absolute off-diagonal value; *spltol*>0 : criterion that preserves relative accuracy.
tnrm	REAL for slarra DOUBLE PRECISION for dlarra

The norm of the matrix.

Output Parameters

e	On exit, the entries *e*(*isplit*(i)), 1 ≤ i ≤ *nsplit*, are set to zero, the other entries of *e* are untouched.
e2	On exit, the entries *e2*(*isplit*(i)), 1 ≤ i ≤ *nsplit*, are set to zero.
nsplit	INTEGER. The number of blocks the matrix *T* splits into. 1 ≤ *nsplit* ≤ *n*
isplit	INTEGER. Array, DIMENSION (*n*). The splitting points, at which *T* breaks up into blocks. The first block consists of rows/columns 1 to *isplit*(1), the second of rows/columns *isplit*(1)+1 through *isplit*(2), and so on, and the *nsplit*-th consists of rows/columns *isplit*(*nsplit*–1)+1 through *isplit*(*nsplit*)=*n*.
info	INTEGER. = 0: successful exit.

?larrb

Provides limited bisection to locate eigenvalues for more accuracy.

Syntax

call slarrb(*n*, *d*, *lld*, *ifirst*, *ilast*, *rtol1*, *rtol2*, *offset*, *w*, *wgap*, *werr*, *work*, *iwork*, *pivmin*, *spdiam*, *twist*, *info*)

call dlarrb(*n*, *d*, *lld*, *ifirst*, *ilast*, *rtol1*, *rtol2*, *offset*, *w*, *wgap*, *werr*, *work*, *iwork*, *pivmin*, *spdiam*, *twist*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given the relatively robust representation (RRR) $L*D*L^T$, the routine does "limited" bisection to refine the eigenvalues of $L*D*L^T$, *w*(*ifirst*-*offset*) through *w*(*ilast*-*offset*), to more accuracy. Initial guesses for these eigenvalues are input in *w*. The corresponding estimate of the error in these guesses and their gaps are input in *werr* and *wgap*, respectively. During bisection, intervals [*left*, *right*] are maintained by storing their mid-points and semi-widths in the arrays *w* and *werr* respectively.

Input Parameters

n	INTEGER. The order of the matrix.
d	REAL for slarrb DOUBLE PRECISION for dlarrb Array, DIMENSION (*n*). The *n* diagonal elements of the diagonal matrix *D*.
lld	REAL for slarrb DOUBLE PRECISION for dlarrb Array, DIMENSION (*n*-1). The *n*-1 elements $L_i*L_i*D_i$.

ifirst INTEGER. The index of the first eigenvalue to be computed.

ilast INTEGER. The index of the last eigenvalue to be computed.

rtol1, *rtol2* REAL for slarrb
DOUBLE PRECISION for dlarrb
Tolerance for the convergence of the bisection intervals. An interval [*left*, *right*] has converged if RIGHT-LEFT.LT.MAX(*rtol1**gap, *rtol2**max(|left|,|right|)), where gap is the (estimated) distance to the nearest eigenvalue.

offset INTEGER. Offset for the arrays *w*, *wgap* and *werr*, that is, the *ifirst*-*offset* through *ilast*-*offset* elements of these arrays are to be used.

w REAL for slarrb
DOUBLE PRECISION for dlarrb
Array, DIMENSION (*n*). On input, *w*(*ifirst*-*offset*) through *w*(*ilast*-*offset*) are estimates of the eigenvalues of $L*D*L^T$ indexed *ifirst* through *ilast*.

wgap REAL for slarrb
DOUBLE PRECISION for dlarrb
Array, DIMENSION (*n*-1). The estimated gaps between consecutive eigenvalues of $L*D*L^T$, that is, *wgap*(*i*-*offset*) is the gap between eigenvalues *i* and *i*+1. Note that if IFIRST.EQ.ILAST then *wgap*(*ifirst*-*offset*) must be set to 0.

werr REAL for slarrb
DOUBLE PRECISION for dlarrb
Array, DIMENSION (*n*). On input, *werr*(*ifirst*-*offset*) through *werr*(*ilast*-*offset*) are the errors in the estimates of the corresponding elements in *w*.

work REAL for slarrb
DOUBLE PRECISION for dlarrb
Workspace array, DIMENSION (2**n*).

pivmin REAL for slarrb
DOUBLE PRECISION for dlarrb
The minimum pivot in the Sturm sequence.

spdiam REAL for slarrb
DOUBLE PRECISION for dlarrb
The spectral diameter of the matrix.

twist INTEGER. The twist index for the twisted factorization that is used for the negcount.

twist = *n*: Compute negcount from $L*D*L^T - lambda*i = L+* D+ *L+^T$

twist = *n*: Compute negcount from $L*D*L^T - lambda*i = U-*D-*U-^T$

twist = *n*: Compute negcount from $L*D*L^T - lambda*i = N_r*D_r*N_r$

iwork INTEGER.
Workspace array, DIMENSION (2**n*).

Output Parameters

w On output, the estimates of the eigenvalues are "refined".

wgap On output, the gaps are refined.

werr On output, "refined" errors in the estimates of *w*.

info	INTEGER. Error flag.

?larrc

Computes the number of eigenvalues of the symmetric tridiagonal matrix.

Syntax

```
call slarrc( jobt, n, vl, vu, d, e, pivmin, eigcnt, lcnt, rcnt, info )
call dlarrc( jobt, n, vl, vu, d, e, pivmin, eigcnt, lcnt, rcnt, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine finds the number of eigenvalues of the symmetric tridiagonal matrix T or of its factorization $L*D*L^T$ in the specified interval.

Input Parameters

jobt	CHARACTER*1. = 'T': computes Sturm count for matrix T. = 'L': computes Sturm count for matrix $L*D*L^T$.
n	INTEGER. The order of the matrix. ($n > 1$).
vl,*vu*	REAL for slarrc DOUBLE PRECISION for dlarrc The lower and upper bounds for the eigenvalues.
d	REAL for slarrc DOUBLE PRECISION for dlarrc Array, DIMENSION (n). If *jobt*= 'T': contains the n diagonal elements of the tridiagonal matrix T. If *jobt*= 'L': contains the n diagonal elements of the diagonal matrix D.
e	REAL for slarrc DOUBLE PRECISION for dlarrc Array, DIMENSION (n). If *jobt*= 'T': contains the $(n-1)$ offdiagonal elements of the matrix T. If *jobt*= 'L': contains the $(n-1)$ offdiagonal elements of the matrix L.
pivmin	REAL for slarrc DOUBLE PRECISION for dlarrc The minimum pivot in the Sturm sequence for the matrix T.

Output Parameters

eigcnt	INTEGER. The number of eigenvalues of the symmetric tridiagonal matrix T that are in the half-open interval $(vl, vu]$.
lcnt,*rcnt*	INTEGER. The left and right negcounts of the interval.

info	INTEGER. Now it is not used and always is set to 0.

?larrd

Computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.

Syntax

```
call slarrd( range, order, n, vl, vu, il, iu, gers, reltol, d, e, e2, pivmin, nsplit, isplit, m, w, werr, wl, wu, iblock, indexw, work, iwork, info )
```

```
call dlarrd( range, order, n, vl, vu, il, iu, gers, reltol, d, e, e2, pivmin, nsplit, isplit, m, w, werr, wl, wu, iblock, indexw, work, iwork, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes the eigenvalues of a symmetric tridiagonal matrix T to suitable accuracy. This is an auxiliary code to be called from `?stemr`. The user may ask for all eigenvalues, all eigenvalues in the half-open interval $(vl, vu]$, or the il-th through iu-th eigenvalues.

To avoid overflow, the matrix must be scaled so that its largest element is no greater than $(overflow^{1/2}*underflow^{1/4})$ in absolute value, and for greatest accuracy, it should not be much smaller than that. (For more details see [Kahan66].

Input Parameters

range	CHARACTER. = 'A': ("All") all eigenvalues will be found. = 'V': ("Value") all eigenvalues in the half-open interval $(vl, vu]$ will be found. = 'I': ("Index") the il-th through iu-th eigenvalues will be found.
order	CHARACTER. = 'B': ("By block") the eigenvalues will be grouped by split-off block (see *iblock*, *isplit* below) and ordered from smallest to largest within the block. = 'E': ("Entire matrix") the eigenvalues for the entire matrix will be ordered from smallest to largest.
n	INTEGER. The order of the tridiagonal matrix T ($n \geq 1$).
vl,*vu*	REAL for `slarrd` DOUBLE PRECISION for `dlarrd` If *range* = 'V': the lower and upper bounds of the interval to be searched for eigenvalues. Eigenvalues less than or equal to vl, or greater than vu, will not be returned. $vl < vu$. If *range* = 'A' or 'I': not referenced.
il,*iu*	INTEGER. If *range* = 'I': the indices (in ascending order) of the smallest and largest eigenvalues to be returned. $1 \leq il \leq iu \leq n$, if $n > 0$; il=1 and iu=0 if n=0. If *range* = 'A' or 'V': not referenced.

gers
REAL for slarrd
DOUBLE PRECISION for dlarrd
Array, DIMENSION (2*n).
The n Gerschgorin intervals (the i-th Gerschgorin interval is (*gers*(2*i-1), *gers*(2*i)).

reltol
REAL for slarrd
DOUBLE PRECISION for dlarrd
The minimum relative width of an interval. When an interval is narrower than *reltol* times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, that is converged. Note: this should always be at least *radix*machine epsilon*.

d
REAL for slarrd
DOUBLE PRECISION for dlarrd
Array, DIMENSION (n).
Contains n diagonal elements of the tridiagonal matrix T.

e
REAL for slarrd
DOUBLE PRECISION for dlarrd
Array, DIMENSION (n-1).
Contains (n-1) off-diagonal elements of the tridiagonal matrix T.

e2
REAL for slarrd
DOUBLE PRECISION for dlarrd
Array, DIMENSION (n-1).
Contains (n-1) squared off-diagonal elements of the tridiagonal matrix T.

pivmin
REAL for slarrd
DOUBLE PRECISION for dlarrd
The minimum pivot in the Sturm sequence for the matrix T.

nsplit
INTEGER.
The number of diagonal blocks the matrix T. $1 \le nsplit \le n$

isplit
INTEGER.
Arrays, DIMENSION (n).
The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to *isplit*(1), the second of rows/columns *isplit*(1)+1 through *isplit*(2), and so on, and the *nsplit*-th consists of rows/columns *isplit*(*nsplit*-1)+1 through *isplit*(*nsplit*)=n.
(Only the first *nsplit* elements actually is used, but since the user cannot know a priori value of *nsplit*, n words must be reserved for *isplit*.)

work
REAL for slarrd
DOUBLE PRECISION for dlarrd
Workspace array, DIMENSION (4*n).

iwork
INTEGER.
Workspace array, DIMENSION (4*n).

Output Parameters

m
INTEGER.
The actual number of eigenvalues found. $0 \le m \le n$. (See also the description of *info*=2, 3.)

w
REAL for slarrd
DOUBLE PRECISION for dlarrd
Array, DIMENSION (n).

	The first *m* elements of *w* contain the eigenvalue approximations. ?laprd computes an interval $I_j = (a_j, b_j]$ that includes eigenvalue j. The eigenvalue approximation is given as the interval midpoint *w*(j)= $(a_j+b_j)/2$. The corresponding error is bounded by *werr*(j) = abs(a_j-b_j)/2.
werr	REAL for slarrd DOUBLE PRECISION for dlarrd Array, DIMENSION (*n*). The error bound on the corresponding eigenvalue approximation in *w*.
wl, *wu*	REAL for slarrd DOUBLE PRECISION for dlarrd The interval (*wl*, *wu*] contains all the wanted eigenvalues. If *range* = 'V': then *wl*=*vl* and *wu*=*vu*. If *range* = 'A': then *wl* and *wu* are the global Gerschgorin bounds on the spectrum. If *range* = 'I': then *wl* and *wu* are computed by ?laebz from the index range specified.
iblock	INTEGER. Array, DIMENSION (*n*). At each row/column j where *e*(j) is zero or small, the matrix *T* is considered to split into a block diagonal matrix. If *info* = 0, then *iblock*(i) specifies to which block (from 1 to the number of blocks) the eigenvalue *w*(i) belongs. (The routine may use the remaining *n*–*m* elements as workspace.)
indexw	INTEGER. Array, DIMENSION (*n*). The indices of the eigenvalues within each block (submatrix); for example, *indexw*(i)= j and *iblock*(i)=k imply that the i-th eigenvalue *w*(i) is the j-th eigenvalue in block k.
info	INTEGER. = 0: successful exit. < 0: if *info* = –i, the i-th argument has an illegal value > 0: some or all of the eigenvalues fail to converge or are not computed: =1 or 3: bisection fail to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. =2 or 3: *range*='I' only: not all of the eigenvalues *il*:*iu* are found. =4: *range*='I', and the Gershgorin interval initially used is too small. No eigenvalues are computed.

?larre

Given the tridiagonal matrix T, sets small off-diagonal elements to zero and for each unreduced block T_i, finds base representations and eigenvalues.

Syntax

call slarre(*range*, *n*, *vl*, *vu*, *il*, *iu*, *d*, *e*, *e2*, *rtol1*, *rtol2*, *spltol*, *nsplit*, *isplit*, *m*, *w*, *werr*, *wgap*, *iblock*, *indexw*, *gers*, *pivmin*, *work*, *iwork*, *info*)

call dlarre(*range*, *n*, *vl*, *vu*, *il*, *iu*, *d*, *e*, *e2*, *rtol1*, *rtol2*, *spltol*, *nsplit*, *isplit*, *m*, *w*, *werr*, *wgap*, *iblock*, *indexw*, *gers*, *pivmin*, *work*, *iwork*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

To find the desired eigenvalues of a given real symmetric tridiagonal matrix T, the routine sets any "small" off-diagonal elements to zero, and for each unreduced block T_i, it finds

- a suitable shift at one end of the block spectrum
- the base representation, $T_i - \sigma_i * I = L_i * D_i * L_i^T$, and
- eigenvalues of each $L_i * D_i * L_i^T$.

The representations and eigenvalues found are then used by ?stemr to compute the eigenvectors of a symmetric tridiagonal matrix. The accuracy varies depending on whether bisection is used to find a few eigenvalues or the dqds algorithm (subroutine ?lasq2) to compute all and discard any unwanted one. As an added benefit, ?larre also outputs the n Gerschgorin intervals for the matrices $L_i * D_i * L_i^T$.

Input Parameters

range
CHARACTER.
= 'A': ("All") all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval (vl, vu] will be found.
= 'I': ("Index") the il-th through iu-th eigenvalues of the entire matrix will be found.

n
INTEGER. The order of the matrix. $n > 0$.

vl, *vu*
REAL for slarre
DOUBLE PRECISION for dlarre
If *range*='V', the lower and upper bounds for the eigenvalues. Eigenvalues less than or equal to vl, or greater than vu, are not returned. $vl < vu$.

il, *iu*
INTEGER.
If *range*='I', the indices (in ascending order) of the smallest and largest eigenvalues to be returned. $1 \le il \le iu \le n$.

d
REAL for slarre
DOUBLE PRECISION for dlarre
Array, DIMENSION (n). The n diagonal elements of the diagonal matrices T.

e
REAL for slarre
DOUBLE PRECISION for dlarre
Array, DIMENSION (n). The first (n-1) entries contain the subdiagonal elements of the tridiagonal matrix T; $e(n)$ need not be set.

e2
REAL for slarre
DOUBLE PRECISION for dlarre
Array, DIMENSION (n). The first (n-1) entries contain the squares of the subdiagonal elements of the tridiagonal matrix T; $e2(n)$ need not be set.

rtol1, *rtol2*
REAL for slarre
DOUBLE PRECISION for dlarre
Parameters for bisection. An interval [LEFT,RIGHT] has converged if RIGHT-LEFT.LT.MAX(*rtol1**gap, *rtol2**max(|LEFT|,|RIGHT|)).

spltol
REAL for slarre
DOUBLE PRECISION for dlarre

	The threshold for splitting.
work	REAL for slarre DOUBLE PRECISION for dlarre Workspace array, DIMENSION (6*n).
iwork	INTEGER. Workspace array, DIMENSION (5*n).

Output Parameters

vl, *vu*	On exit, if *range*='I' or ='A', contain the bounds on the desired part of the spectrum.
d	On exit, the n diagonal elements of the diagonal matrices D_i .
e	On exit, the subdiagonal elements of the unit bidiagonal matrices L_i . The entries *e*(*isplit*(i)), $1 \le i \le nsplit$, contain the base points $sigma_i$ on output.
e2	On exit, the entries *e2*(*isplit*(i)), $1 \le i \le nsplit$, have been set to zero.
nsplit	INTEGER. The number of blocks T splits into. $1 \le nsplit \le n$.
isplit	INTEGER. Array, DIMENSION (n). The splitting points, at which T breaks up into blocks. The first block consists of rows/columns 1 to *isplit*(1), the second of rows/columns *isplit*(1)+1 through *isplit*(2), etc., and the *nsplit*-th consists of rows/columns *isplit*(*nsplit*–1)+1 through *isplit*(*nsplit*)=n.
m	INTEGER. The total number of eigenvalues (of all the $L_i*D_i*L_i^T$) found.
w	REAL for slarre DOUBLE PRECISION for dlarre Array, DIMENSION (n). The first m elements contain the eigenvalues. The eigenvalues of each of the blocks, $L_i*D_i*L_i^T$, are sorted in ascending order. The routine may use the remaining n-m elements as workspace.
werr	REAL for slarre DOUBLE PRECISION for dlarre Array, DIMENSION (n). The error bound on the corresponding eigenvalue in w.
wgap	REAL for slarre DOUBLE PRECISION for dlarre Array, DIMENSION (n). The separation from the right neighbor eigenvalue in w. The gap is only with respect to the eigenvalues of the same block as each block has its own representation tree. Exception: at the right end of a block the left gap is stored.
iblock	INTEGER. Array, DIMENSION (n). The indices of the blocks (submatrices) associated with the corresponding eigenvalues in w; *iblock*(i)=1 if eigenvalue w(i) belongs to the first block from the top, =2 if w(i) belongs to the second block, etc.
indexw	INTEGER. Array, DIMENSION (n). The indices of the eigenvalues within each block (submatrix); for example, *indexw*(i)= 10 and *iblock*(i)=2 imply that the i-th eigenvalue w(i) is the 10-th eigenvalue in the second block.
gers	REAL for slarre DOUBLE PRECISION for dlarre

	Array, DIMENSION (2*n). The n Gerschgorin intervals (the i-th Gerschgorin interval is (gers(2*i-1), gers(2*i)).
pivmin	REAL for slarre DOUBLE PRECISION for dlarre The minimum pivot in the Sturm sequence for T.
info	INTEGER. If *info* = 0: successful exit If *info* > 0: A problem occured in ?larre. If *info* = 5, the Rayleigh Quotient Iteration failed to converge to full accuracy. If *info* < 0: One of the called subroutines signaled an internal problem. Inspection of the corresponding parameter *info* for further information is required. • If *info* = -1, there is a problem in ?larrd • If *info* = -2, no base representation could be found in maxtry iterations. Increasing maxtry and recompilation might be a remedy. • If *info* = -3, there is a problem in ?larrb when computing the refined root representation for ?lasq2. • If *info* = -4, there is a problem in ?larrb when preforming bisection on the desired part of the spectrum. • If *info* = -5, there is a problem in ?lasq2. • If *info* = -6, there is a problem in ?lasq2.

See Also

?stemr
?lasq2
?larrb
?larrd

?larrf

Finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.

Syntax

```
call slarrf( n, d, l, ld, clstrt, clend, w, wgap, werr, spdiam, clgapl, clgapr,
pivmin, sigma, dplus, lplus, work, info )
```

```
call dlarrf( n, d, l, ld, clstrt, clend, w, wgap, werr, spdiam, clgapl, clgapr,
pivmin, sigma, dplus, lplus, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given the initial representation $L*D*L^T$ and its cluster of close eigenvalues (in a relative measure), $w(clstrt)$, $w(clstrt+1)$, ... $w(clend)$, the routine ?larrf finds a new relatively robust representation

$$L*D*L^T - \sigma_i*I = L(+)*D(+)*L(+)^T$$

such that at least one of the eigenvalues of $L(+)*D*(+)*L(+)^T$ is relatively isolated.

Input Parameters

n	INTEGER. The order of the matrix (subblock, if the matrix is splitted).
d	REAL for slarrf DOUBLE PRECISION for dlarrf Array, DIMENSION (*n*). The *n* diagonal elements of the diagonal matrix *D*.
l	REAL for slarrf DOUBLE PRECISION for dlarrf Array, DIMENSION (*n*-1). The (*n*-1) subdiagonal elements of the unit bidiagonal matrix *L*.
ld	REAL for slarrf DOUBLE PRECISION for dlarrf Array, DIMENSION (*n*-1). The *n*-1 elements L_i*D_i.
clstrt	INTEGER. The index of the first eigenvalue in the cluster.
clend	INTEGER. The index of the last eigenvalue in the cluster.
w	REAL for slarrf DOUBLE PRECISION for dlarrf Array, DIMENSION ≥ (*clend* -*clstrt*+1). The eigenvalue approximations of $L*D*L^T$ in ascending order. *w*(*clstrt*) through *w*(*clend*) form the cluster of relatively close eigenvalues.
wgap	REAL for slarrf DOUBLE PRECISION for dlarrf Array, DIMENSION ≥ (*clend* -*clstrt*+1). The separation from the right neighbor eigenvalue in *w*.
werr	REAL for slarrf DOUBLE PRECISION for dlarrf Array, DIMENSION ≥ (*clend* -*clstrt*+1). On input, *werr* contains the semiwidth of the uncertainty interval of the corresponding eigenvalue approximation in *w*.
spdiam	REAL for slarrf DOUBLE PRECISION for dlarrf Estimate of the spectral diameter obtained from the Gerschgorin intervals.
clgapl, *clgapr*	REAL for slarrf DOUBLE PRECISION for dlarrf Absolute gap on each end of the cluster. Set by the calling routine to protect against shifts too close to eigenvalues outside the cluster.
pivmin	REAL for slarrf DOUBLE PRECISION for dlarrf The minimum pivot allowed in the Sturm sequence.
work	REAL for slarrf DOUBLE PRECISION for dlarrf Workspace array, DIMENSION (2**n*).

Output Parameters

wgap	On output, the gaps are refined.
sigma	REAL for slarrf DOUBLE PRECISION for dlarrf The shift used to form $L(+)*D*(+)*L(+)^T$.

dplus REAL for slarrf
DOUBLE PRECISION for dlarrf
Array, DIMENSION (*n*). The *n* diagonal elements of the diagonal matrix *D*(+).

lplus REAL for slarrf
DOUBLE PRECISION for dlarrf
Array, DIMENSION (*n*). The first (*n*-1) elements of *lplus* contain the subdiagonal elements of the unit bidiagonal matrix *L*(+).

?larrj

Performs refinement of the initial estimates of the eigenvalues of the matrix T.

Syntax

```
call slarrj( n, d, e2, ifirst, ilast, rtol, offset, w, werr, work, iwork, pivmin, spdiam, info )
call dlarrj( n, d, e2, ifirst, ilast, rtol, offset, w, werr, work, iwork, pivmin, spdiam, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Given the initial eigenvalue approximations of *T*, this routine does bisection to refine the eigenvalues of *T*, *w*(*ifirst-offset*) through *w*(*ilast-offset*), to more accuracy. Initial guesses for these eigenvalues are input in *w*, the corresponding estimate of the error in these guesses in *werr*. During bisection, intervals [a,b] are maintained by storing their mid-points and semi-widths in the arrays *w* and *werr* respectively.

Input Parameters

n INTEGER. The order of the matrix *T*.

d REAL for slarrj
DOUBLE PRECISION for dlarrj
Array, DIMENSION (*n*).
Contains *n* diagonal elements of the matrix *T*.

e2 REAL for slarrj
DOUBLE PRECISION for dlarrj
Array, DIMENSION (*n*-1).
Contains (*n*-1) squared sub-diagonal elements of the *T*.

ifirst INTEGER.
The index of the first eigenvalue to be computed.

ilast INTEGER.
The index of the last eigenvalue to be computed.

rtol REAL for slarrj
DOUBLE PRECISION for dlarrj
Tolerance for the convergence of the bisection intervals. An interval [a,b] is considered to be converged if (b-a) $\leq$ *rtol**max(|a|,|b|).

offset INTEGER.

	Offset for the arrays *w* and *werr*, that is the *ifirst-offset* through *ilast-offset* elements of these arrays are to be used.
w	REAL for slarrj DOUBLE PRECISION for dlarrj Array, DIMENSION (*n*). On input, *w(ifirst-offset)* through *w(ilast-offset)* are estimates of the eigenvalues of $L*D*L^T$ indexed *ifirst* through *ilast*.
werr	REAL for slarrj DOUBLE PRECISION for dlarrj Array, DIMENSION (*n*). On input, *werr(ifirst-offset)* through *werr(ilast-offset)* are the errors in the estimates of the corresponding elements in *w*.
work	REAL for slarrj DOUBLE PRECISION for dlarrj Workspace array, DIMENSION (2*n).
iwork	INTEGER. Workspace array, DIMENSION (2*n).
pivmin	REAL for slarrj DOUBLE PRECISION for dlarrj The minimum pivot in the Sturm sequence for the matrix *T*.
spdiam	REAL for slarrj DOUBLE PRECISION for dlarrj The spectral diameter of the matrix *T*.

Output Parameters

w	On exit, contains the refined estimates of the eigenvalues.
werr	On exit, contains the refined errors in the estimates of the corresponding elements in *w*.
info	INTEGER. Now it is not used and always is set to 0.

?larrk

Computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy.

Syntax

```
call slarrk( n, iw, gl, gu, d, e2, pivmin, reltol, w, werr, info )
call dlarrk( n, iw, gl, gu, d, e2, pivmin, reltol, w, werr, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes one eigenvalue of a symmetric tridiagonal matrix *T* to suitable accuracy. This is an auxiliary code to be called from ?stemr.

To avoid overflow, the matrix must be scaled so that its largest element is no greater than $(overflow^{1/2}*underflow^{1/4})$ in absolute value, and for greatest accuracy, it should not be much smaller than that. For more details see [Kahan66].

Input Parameters

n	INTEGER. The order of the matrix *T*. ($n \geq 1$).
iw	INTEGER. The index of the eigenvalue to be returned.
gl, *gu*	REAL for slarrk DOUBLE PRECISION for dlarrk An upper and a lower bound on the eigenvalue.
d	REAL for slarrk DOUBLE PRECISION for dlarrk Array, DIMENSION (*n*). Contains *n* diagonal elements of the matrix *T*.
e2	REAL for slarrk DOUBLE PRECISION for dlarrk Array, DIMENSION (*n*-1). Contains (*n*-1) squared off-diagonal elements of the *T*.
pivmin	REAL for slarrk DOUBLE PRECISION for dlarrk The minimum pivot in the Sturm sequence for the matrix *T*.
reltol	REAL for slarrk DOUBLE PRECISION for dlarrk The minimum relative width of an interval. When an interval is narrower than *reltol* times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, that is converged. Note: this should always be at least *radix*machine epsilon*.

Output Parameters

w	REAL for slarrk DOUBLE PRECISION for dlarrk Contains the eigenvalue approximation.
werr	REAL for slarrk DOUBLE PRECISION for dlarrk Contains the error bound on the corresponding eigenvalue approximation in *w*.
info	INTEGER. = 0: Eigenvalue converges = -1: Eigenvalue does not converge

?larrr

Performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

Syntax

```
call slarrr( n, d, e, info )
call dlarrr( n, d, e, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

Input Parameters

n	INTEGER. The order of the matrix T. ($n > 0$).
d	REAL for slarrr DOUBLE PRECISION for dlarrr Array, DIMENSION (n). Contains n diagonal elements of the matrix T.
e	REAL for slarrr DOUBLE PRECISION for dlarrr Array, DIMENSION (n). The first $(n-1)$ entries contain sub-diagonal elements of the tridiagonal matrix T; $e(n)$ is set to 0.

Output Parameters

info	INTEGER. = 0: the matrix warrants computations preserving relative accuracy (default value). = -1: the matrix warrants computations guaranteeing only absolute accuracy.

?larrv

Computes the eigenvectors of the tridiagonal matrix T = $L*D*L^T$ given L, D and the eigenvalues of $L*D*L^T$.

Syntax

```
call slarrv( n, vl, vu, d, l, pivmin, isplit, m, dol, dou, minrgp, rtol1, rtol2, w,
werr, wgap, iblock, indexw, gers, z, ldz, isuppz, work, iwork, info )

call dlarrv( n, vl, vu, d, l, pivmin, isplit, m, dol, dou, minrgp, rtol1, rtol2, w,
werr, wgap, iblock, indexw, gers, z, ldz, isuppz, work, iwork, info )

call clarrv( n, vl, vu, d, l, pivmin, isplit, m, dol, dou, minrgp, rtol1, rtol2, w,
werr, wgap, iblock, indexw, gers, z, ldz, isuppz, work, iwork, info )

call zlarrv( n, vl, vu, d, l, pivmin, isplit, m, dol, dou, minrgp, rtol1, rtol2, w,
werr, wgap, iblock, indexw, gers, z, ldz, isuppz, work, iwork, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?larrv computes the eigenvectors of the tridiagonal matrix T = $L*D*L^T$ given L, D and approximations to the eigenvalues of $L*D*L^T$.

The input eigenvalues should have been computed by slarre for real flavors (slarrv/clarrv) and by dlarre for double precision flavors (dlarre/zlarre).

Input Parameters

n	INTEGER. The order of the matrix. $n \geq 0$.
vl, *vu*	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv Lower and upper bounds respectively of the interval that contains the desired eigenvalues. $vl < vu$. Needed to compute gaps on the left or right end of the extremal eigenvalues in the desired range.
d	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv Array, DIMENSION (n). On entry, the n diagonal elements of the diagonal matrix D.
l	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv Array, DIMENSION (n). On entry, the (n-1) subdiagonal elements of the unit bidiagonal matrix L are contained in elements 1 to n-1 of L if the matrix is not splitted. At the end of each block the corresponding shift is stored as given by slarre for real flavors and by dlarre for double precision flavors.
pivmin	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv The minimum pivot allowed in the Sturm sequence.
isplit	INTEGER. Array, DIMENSION (n). The splitting points, at which T breaks up into blocks. The first block consists of rows/columns 1 to *isplit*(1), the second of rows/columns *isplit*(1)+1 through *isplit*(2), etc.
m	INTEGER. The total number of eigenvalues found. $0 \leq m \leq n$. If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu - il + 1$.
dol, *dou*	INTEGER. If you want to compute only selected eigenvectors from all the eigenvalues supplied, specify an index range *dol*:*dou*. Or else apply the setting *dol*=1, *dou*=*m*. Note that *dol* and *dou* refer to the order in which the eigenvalues are stored in w. If you want to compute only selected eigenpairs, then the columns *dol*-1 to *dou*+1 of the eigenvector space Z contain the computed eigenvectors. All other columns of Z are set to zero.
minrgp, *rtol1*, *rtol2*	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv Parameters for bisection. An interval [LEFT,RIGHT] has converged if RIGHT-LEFT.LT.MAX(*rtol1**gap, *rtol2**max(|LEFT|,|RIGHT|)).
w	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv Array, DIMENSION (n). The first m elements of w contain the approximate eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block (the output array w from ?larre is expected here). These eigenvalues are set with respect to the shift of the corresponding root representation for their block.
werr	REAL for slarrv/clarrv DOUBLE PRECISION for dlarrv/zlarrv

Array, DIMENSION (*n*). The first *m* elements contain the semiwidth of the uncertainty interval of the corresponding eigenvalue in *w*.

wgap
REAL for slarrv/clarrv
DOUBLE PRECISION for dlarrv/zlarrv
Array, DIMENSION (*n*). The separation from the right neighbor eigenvalue in *w*.

iblock
INTEGER. Array, DIMENSION (*n*).
The indices of the blocks (submatrices) associated with the corresponding eigenvalues in *w*; *iblock*(*i*)=1 if eigenvalue *w*(*i*) belongs to the first block from the top, =2 if *w*(*i*) belongs to the second block, etc.

indexw
INTEGER. Array, DIMENSION (*n*).
The indices of the eigenvalues within each block (submatrix); for example, *indexw*(*i*)= 10 and *iblock*(*i*)=2 imply that the *i*-th eigenvalue *w*(*i*) is the 10-th eigenvalue in the second block.

gers
REAL for slarrv/clarrv
DOUBLE PRECISION for dlarrv/zlarrv
Array, DIMENSION (2**n*). The *n* Gerschgorin intervals (the *i*-th Gerschgorin interval is (*gers*(2**i*-1), *gers*(2**i*)). The Gerschgorin intervals should be computed from the original unshifted matrix.

ldz
INTEGER. The leading dimension of the output array *Z*. *ldz* ≥ 1, and if *jobz* = 'V', *ldz* ≥ max(1,*n*).

work
REAL for slarrv/clarrv
DOUBLE PRECISION for dlarrv/zlarrv
Workspace array, DIMENSION (12**n*).

iwork
INTEGER.
Workspace array, DIMENSION (7**n*).

Output Parameters

d
On exit, *d* may be overwritten.

l
On exit, *l* is overwritten.

w
On exit, *w* holds the eigenvalues of the unshifted matrix.

werr
On exit, *werr* contains refined values of its input approximations.

wgap
On exit, *wgap* contains refined values of its input approximations. Very small gaps are changed.

z
REAL for slarrv
DOUBLE PRECISION for dlarrv
COMPLEX for clarrv
DOUBLE COMPLEX for zlarrv
Array, DIMENSION (*ldz*, max(1,*m*)).
If *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the input eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*).

NOTE The user must ensure that at least max(1,*m*) columns are supplied in the array *z*.

isuppz
INTEGER .

Array, DIMENSION (2*max(1,*m*)). The support of the eigenvectors in *z*, that is, the indices indicating the nonzero elements in *z*. The *i*-th eigenvector is nonzero only in elements *isuppz*(2*i*-1) through *isuppz*(2*i*).

info INTEGER.
If *info* = 0: successful exit
If *info* > 0: A problem occured in ?larrv. If *info* = 5, the Rayleigh Quotient Iteration failed to converge to full accuracy.
If *info* < 0: One of the called subroutines signaled an internal problem. Inspection of the corresponding parameter *info* for further information is required.

- If *info* = -1, there is a problem in ?larrb when refining a child eigenvalue;
- If *info* = -2, there is a problem in ?larrf when computing the relatively robust representation (RRR) of a child. When a child is inside a tight cluster, it can be difficult to find an RRR. A partial remedy from the user's point of view is to make the parameter *minrgp* smaller and recompile. However, as the orthogonality of the computed vectors is proportional to 1/*minrgp*, you should be aware that you might be trading in precision when you decrease *minrgp*.
- If *info* = -3, there is a problem in ?larrb when refining a single eigenvalue after the Rayleigh correction was rejected.

See Also

?larrb
?larre
?larrf

?lartg

Generates a plane rotation with real cosine and real/ complex sine.

Syntax

```
call slartg( f, g, cs, sn, r )
call dlartg( f, g, cs, sn, r )
call clartg( f, g, cs, sn, r )
call zlartg( f, g, cs, sn, r )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine generates a plane rotation so that

$$\begin{bmatrix} cs & sn \\ -\text{conjg}(sn) & cs \end{bmatrix} \cdot \begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

where $cs^2 + |sn|^2 = 1$

This is a slower, more accurate version of the BLAS Level 1 routine ?rotg, except for the following differences.

For slartg/dlartg:

f and *g* are unchanged on return;

If *g*=0, then *cs*=1 and *sn*=0;

If *f*=0 and *g* ≠ 0, then *cs*=0 and *sn*=1 without doing any floating point operations (saves work in ?bdsqr when there are zeros on the diagonal);

If *f* exceeds *g* in magnitude, *cs* will be positive.

For clartg/zlartg:

f and *g* are unchanged on return;

If *g*=0, then *cs*=1 and *sn*=0;

If *f*=0, then *cs*=0 and *sn* is chosen so that *r* is real.

Input Parameters

f, *g*	REAL for slartg DOUBLE PRECISION for dlartg COMPLEX for clartg DOUBLE COMPLEX for zlartg The first and second component of vector to be rotated.

Output Parameters

cs	REAL for slartg/clartg DOUBLE PRECISION for dlartg/zlartg The cosine of the rotation.
sn	REAL for slartg DOUBLE PRECISION for dlartg COMPLEX for clartg DOUBLE COMPLEX for zlartg The sine of the rotation.
r	REAL for slartg DOUBLE PRECISION for dlartg COMPLEX for clartg DOUBLE COMPLEX for zlartg The nonzero component of the rotated vector.

?lartgp

Generates a plane rotation so that the diagonal is nonnegative.

Syntax

Fortran 77:

```
call slartgp( f, g, cs, sn, r )
call dlartgp( f, g, cs, sn, r )
```

Fortran 95:

```
call lartgp( f,g,cs,sn,r )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine generates a plane rotation so that

$$\begin{bmatrix} cs & sn \\ -sn & cs \end{bmatrix} \cdot \begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

where $cs^2 + sn^2 = 1$

This is a slower, more accurate version of the BLAS Level 1 routine `?rotg`, except for the following differences:

- f and g are unchanged on return.
- If g=0, then cs=(+/-)1 and sn=0.
- If f=0 and $g \neq 0$, then cs=0 and sn=(+/-)1.

The sign is chosen so that $r \geq 0$.

Input Parameters

f, g	`REAL` for `slartgp` `DOUBLE PRECISION` for `dlartgp` The first and second component of the vector to be rotated.

Output Parameters

cs	`REAL` for `slartgp` `DOUBLE PRECISION` for `dlartgp` The cosine of the rotation.
sn	`REAL` for `slartgp` `DOUBLE PRECISION` for `dlartgp` The sine of the rotation.
r	`REAL` for `slartgp` `DOUBLE PRECISION` for `dlartgp` The nonzero component of the rotated vector.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `?lartgp` interface are as follows:

f	Holds the first component of the vector to be rotated.
g	Holds the second component of the vector to be rotated.
cs	Holds the cosine of the rotation.
sn	Holds the sine of the rotation.
r	Holds the nonzero component of the rotated vector.

See Also

?rotg

?lartg
?lartgs

?lartgs

Generates a plane rotation designed to introduce a bulge in implicit QR iteration for the bidiagonal SVD problem.

Syntax

Fortran 77:

```
call slartgs( x, y, sigma, cs, sn )
call dlartgs( x, y, sigma, cs, sn )
```

Fortran 95:

```
call lartgs( x,y,sigma,cs,sn )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine generates a plane rotation designed to introduce a bulge in Golub-Reinsch-style implicit QR iteration for the bidiagonal SVD problem. *x* and *y* are the top-row entries, and *sigma* is the shift. The computed *cs* and *sn* define a plane rotation that satisfies the following:

$$\begin{bmatrix} cs & sn \\ -sn & cs \end{bmatrix} \cdot \begin{bmatrix} x^2 - sigma \\ x * y \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

with *r* nonnegative.

If x^2 – *sigma* and *x* * *y* are 0, the rotation is by $\pi/2$

Input Parameters

x, *y*	`REAL` for `slartgs` `DOUBLE PRECISION` for `dlartgs` The (1,1) and (1,2) entries of an upper bidiagonal matrix, respectively.
sigma	`REAL` for `slartgs` `DOUBLE PRECISION` for `dlartgs` Shift

Output Parameters

cs	`REAL` for `slartgs` `DOUBLE PRECISION` for `dlartgs` The cosine of the rotation.
sn	`REAL` for `slartgs` `DOUBLE PRECISION` for `dlartgs` The sine of the rotation.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `?lartgs` interface are as follows:

x	Holds the (1,1) entry of an upper diagonal matrix.
y	Holds the (1,2) entry of an upper diagonal matrix.
sigma	Holds the shift.
cs	Holds the cosine of the rotation.
sn	Holds the sine of the rotation.

See Also

?lartg
?lartgp

?lartv

Applies a vector of plane rotations with real cosines and real/complex sines to the elements of a pair of vectors.

Syntax

```
call slartv( n, x, incx, y, incy, c, s, incc )
call dlartv( n, x, incx, y, incy, c, s, incc )
call clartv( n, x, incx, y, incy, c, s, incc )
call zlartv( n, x, incx, y, incy, c, s, incc )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine applies a vector of real/complex plane rotations with real cosines to elements of the real/complex vectors *x* and *y*. For `i = 1,2,...,n`

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} := \begin{bmatrix} c(i) & s(i) \\ -\mathrm{conjg}(s(i)) & c(i) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Input Parameters

n	`INTEGER`. The number of plane rotations to be applied.
x, *y*	`REAL` for `slartv` `DOUBLE PRECISION` for `dlartv` `COMPLEX` for `clartv` `DOUBLE COMPLEX` for `zlartv` Arrays, `DIMENSION` (1+(*n*-1)**incx*) and (1+(*n*-1)**incy*), respectively. The input vectors *x* and *y*.

incx	INTEGER. The increment between elements of *x*. *incx* > 0.
incy	INTEGER. The increment between elements of *y*. *incy* > 0.
c	REAL for slartv/clartv DOUBLE PRECISION for dlartv/zlartv Array, DIMENSION (1+(*n*-1)**incc*). The cosines of the plane rotations.
s	REAL for slartv DOUBLE PRECISION for dlartv COMPLEX for clartv DOUBLE COMPLEX for zlartv Array, DIMENSION (1+(*n*-1)**incc*). The sines of the plane rotations.
incc	INTEGER. The increment between elements of *c* and *s*. *incc* > 0.

Output Parameters

x, *y*	The rotated vectors *x* and *y*.

?laruv

Returns a vector of n random real numbers from a uniform distribution.

Syntax

```
call slaruv( iseed, n, x )
call dlaruv( iseed, n, x )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?laruv returns a vector of *n* random real numbers from a uniform (0,1) distribution (n ≤ 128).

This is an auxiliary routine called by ?larnv.

Input Parameters

iseed	INTEGER. Array, DIMENSION (4). On entry, the seed of the random number generator; the array elements must be between 0 and 4095, and *iseed*(4) must be odd.
n	INTEGER. The number of random numbers to be generated. *n* ≤ 128.

Output Parameters

x	REAL for slaruv DOUBLE PRECISION for dlaruv Array, DIMENSION (*n*). The generated random numbers.
seed	On exit, the seed is updated.

?larz

Applies an elementary reflector (as returned by ?tzrzf*) to a general matrix.*

Syntax

```
call slarz( side, m, n, l, v, incv, tau, c, ldc, work )
call dlarz( side, m, n, l, v, incv, tau, c, ldc, work )
call clarz( side, m, n, l, v, incv, tau, c, ldc, work )
call zlarz( side, m, n, l, v, incv, tau, c, ldc, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?larz applies a real/complex elementary reflector H to a real/complex m-by-n matrix C, from either the left or the right. H is represented in the forms

$H = I - tau*v*v^T$ for real flavors and $H = I - tau*v*v^H$ for complex flavors,

where tau is a real/complex scalar and v is a real/complex vector, respectively.

If $tau = 0$, then H is taken to be the unit matrix.

For complex flavors, to apply H^H (the conjugate transpose of H), supply conjg(tau) instead of tau.

H is a product of k elementary reflectors as returned by ?tzrzf.

Input Parameters

side	CHARACTER*1. If *side* = 'L': form $H*C$ If *side* = 'R': form $C*H$
m	INTEGER. The number of rows of the matrix C.
n	INTEGER. The number of columns of the matrix C.
l	INTEGER. The number of entries of the vector v containing the meaningful part of the Householder vectors. If *side* = 'L', $m \geq L \geq 0$, if *side* = 'R', $n \geq L \geq 0$.
v	REAL for slarz DOUBLE PRECISION for dlarz COMPLEX for clarz DOUBLE COMPLEX for zlarz Array, DIMENSION (1+(*l*-1)*abs(*incv*)). The vector v in the representation of H as returned by ?tzrzf. v is not used if $tau = 0$.
incv	INTEGER. The increment between elements of v. $incv \neq 0$.
tau	REAL for slarz DOUBLE PRECISION for dlarz COMPLEX for clarz DOUBLE COMPLEX for zlarz

	The value *tau* in the representation of *H*.
c	REAL for slarz DOUBLE PRECISION for dlarz COMPLEX for clarz DOUBLE COMPLEX for zlarz Array, DIMENSION (*ldc*,*n*). On entry, the *m*-by-*n* matrix *C*.
ldc	INTEGER. The leading dimension of the array *c*. *ldc* ≥ max(1,*m*).
work	REAL for slarz DOUBLE PRECISION for dlarz COMPLEX for clarz DOUBLE COMPLEX for zlarz Workspace array, DIMENSION (*n*) if *side* = 'L' or (*m*) if *side* = 'R'.

Output Parameters

c	On exit, *C* is overwritten by the matrix *H***C* if *side* = 'L', or *C***H* if *side* = 'R'.

?larzb

Applies a block reflector or its transpose/conjugate-transpose to a general matrix.

Syntax

```
call slarzb( side, trans, direct, storev, m, n, k, l, v, ldv, t, ldt, c, ldc, work, ldwork )
call dlarzb( side, trans, direct, storev, m, n, k, l, v, ldv, t, ldt, c, ldc, work, ldwork )
call clarzb( side, trans, direct, storev, m, n, k, l, v, ldv, t, ldt, c, ldc, work, ldwork )
call zlarzb( side, trans, direct, storev, m, n, k, l, v, ldv, t, ldt, c, ldc, work, ldwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine applies a real/complex block reflector *H* or its transpose H^T (or the conjugate transpose H^H for complex flavors) to a real/complex distributed *m*-by-*n* matrix *C* from the left or the right. Currently, only *storev* = 'R' and *direct* = 'B' are supported.

Input Parameters

side	CHARACTER*1. If *side* = 'L': apply *H* or H^T/H^H from the left If *side* = 'R': apply *H* or H^T/H^H from the right

trans CHARACTER*1.
If *trans* = 'N': apply H (No transpose)
If *trans*='C': apply H^H (conjugate transpose)
If *trans*='T': apply H^T (transpose transpose)

direct CHARACTER*1.
Indicates how H is formed from a product of elementary reflectors
= 'F': $H = H(1)*H(2)*...*H(k)$ (forward, not supported)
= 'B': $H = H(k)*...*H(2)*H(1)$ (backward)

storev CHARACTER*1.
Indicates how the vectors which define the elementary reflectors are stored:
= 'C': Column-wise (not supported)
= 'R': Row-wise.

m INTEGER. The number of rows of the matrix C.

n INTEGER. The number of columns of the matrix C.

k INTEGER. The order of the matrix T (equal to the number of elementary reflectors whose product defines the block reflector).

l INTEGER. The number of columns of the matrix V containing the meaningful part of the Householder reflectors.
If *side* = 'L', $m \geq l \geq 0$, if *side* = 'R', $n \geq l \geq 0$.

v REAL for slarzb
DOUBLE PRECISION for dlarzb
COMPLEX for clarzb
DOUBLE COMPLEX for zlarzb
Array, DIMENSION (*ldv*, nv).
If *storev* = 'C', nv = *k*;
if *storev* = 'R', nv = *l*.

ldv INTEGER. The leading dimension of the array v.
If *storev* = 'C', $ldv \geq l$; if *storev* = 'R', $ldv \geq k$.

t REAL for slarzb
DOUBLE PRECISION for dlarzb
COMPLEX for clarzb
DOUBLE COMPLEX for zlarzb
Array, DIMENSION (*ldt*,*k*). The triangular k-by-k matrix T in the representation of the block reflector.

ldt INTEGER. The leading dimension of the array t.
$ldt \geq k$.

c REAL for slarzb
DOUBLE PRECISION for dlarzb
COMPLEX for clarzb
DOUBLE COMPLEX for zlarzb
Array, DIMENSION (*ldc*,*n*). On entry, the m-by-n matrix C.

ldc INTEGER. The leading dimension of the array c.
$ldc \geq \max(1,m)$.

work REAL for slarzb
DOUBLE PRECISION for dlarzb
COMPLEX for clarzb
DOUBLE COMPLEX for zlarzb
Workspace array, DIMENSION (*ldwork*, *k*).

ldwork	INTEGER. The leading dimension of the array *work*. If *side* = 'L', *ldwork* ≥ max(1, *n*); if *side* = 'R', *ldwork* ≥ max(1, *m*).

Output Parameters

c	On exit, *C* is overwritten by $H*C$, or H^T/H^H*C, or $C*H$, or $C*H^T/H^H$.

?larzt

*Forms the triangular factor T of a block reflector H = I - V*T*V^H.*

Syntax

```
call slarzt( direct, storev, n, k, v, ldv, tau, t, ldt )
call dlarzt( direct, storev, n, k, v, ldv, tau, t, ldt )
call clarzt( direct, storev, n, k, v, ldv, tau, t, ldt )
call zlarzt( direct, storev, n, k, v, ldv, tau, t, ldt )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine forms the triangular factor *T* of a real/complex block reflector *H* of order > *n*, which is defined as a product of *k* elementary reflectors.

If *direct* = 'F', $H = H(1)*H(2)*...*H(k)$, and *T* is upper triangular.

If *direct* = 'B', $H = H(k)*...*H(2)*H(1)$, and *T* is lower triangular.

If *storev* = 'C', the vector which defines the elementary reflector *H*(*i*) is stored in the *i*-th column of the array *v*, and $H = I-V*T*V^T$ (for real flavors) or $H = I-V*T*V^H$ (for complex flavors).

If *storev* = 'R', the vector which defines the elementary reflector *H*(*i*) is stored in the *i*-th row of the array *v*, and $H = I-V^T*T*V$ (for real flavors) or $H = I-V^H*T*V$ (for complex flavors).

Currently, only *storev* = 'R' and *direct* = 'B' are supported.

Input Parameters

direct	CHARACTER*1. Specifies the order in which the elementary reflectors are multiplied to form the block reflector: If *direct* = 'F': $H = H(1)*H(2)*...*H(k)$ (forward, not supported) If *direct* = 'B': $H = H(k)*...*H(2)*H(1)$ (backward)
storev	CHARACTER*1. Specifies how the vectors which define the elementary reflectors are stored (see also *Application Notes* below): If *storev* = 'C': column-wise (not supported) If *storev* = 'R': row-wise
n	INTEGER. The order of the block reflector *H*. *n* ≥ 0.
k	INTEGER. The order of the triangular factor *T* (equal to the number of elementary reflectors). *k* ≥ 1.

v	REAL for slarzt DOUBLE PRECISION for dlarzt COMPLEX for clarzt DOUBLE COMPLEX for zlarzt Array, DIMENSION (*ldv*, *k*) if *storev* = 'C' (*ldv*, *n*) if *storev* = 'R' The matrix *V*.
ldv	INTEGER. The leading dimension of the array *v*. If *storev* = 'C', *ldv* ≥ max(1,*n*); if *storev* = 'R', *ldv* ≥ *k*.
tau	REAL for slarzt DOUBLE PRECISION for dlarzt COMPLEX for clarzt DOUBLE COMPLEX for zlarzt Array, DIMENSION (*k*). *tau*(*i*) must contain the scalar factor of the elementary reflector *H*(*i*).
ldt	INTEGER. The leading dimension of the output array *t*. *ldt* ≥ *k*.

Output Parameters

t	REAL for slarzt DOUBLE PRECISION for dlarzt COMPLEX for clarzt DOUBLE COMPLEX for zlarzt Array, DIMENSION (*ldt*,*k*). The *k*-by-*k* triangular factor *T* of the block reflector. If *direct* = 'F', *T* is upper triangular; if *direct* = 'B', *T* is lower triangular. The rest of the array is not used.
v	The matrix *V*. See *Application Notes* below.

Application Notes

The shape of the matrix *V* and the storage of the vectors which define the *H*(i) is best illustrated by the following example with *n* = 5 and *k* = 3. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct = 'F' and *storev* = 'C':

$$v = \begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot \\ & 1 & \cdot \\ & & 1 \end{bmatrix}$$

direct = 'F' and *storev* = 'R':

$$\underbrace{\begin{bmatrix} v_1 & v_1 & v_1 & v_1 & v_1 & \cdot & \cdot & \cdot & \cdot & 1 \\ v_2 & v_2 & v_2 & v_2 & v_2 & \cdot & \cdot & \cdot & 1 & \\ v_3 & v_3 & v_3 & v_3 & v_3 & \cdot & \cdot & 1 & & \end{bmatrix}}_{V}$$

direct = 'B' and *storev* = 'C':

$$v = \begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot \\ & 1 & \cdot \\ & & 1 \end{bmatrix}$$

direct = 'B' and *storev* = 'R':

$$\underbrace{\begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & v_1 & v_1 & v_1 & v_1 & v_1 \\ \cdot & 1 & \cdot & \cdot & \cdot & v_2 & v_2 & v_2 & v_2 & v_2 \\ \cdot & \cdot & 1 & \cdot & \cdot & v_3 & v_3 & v_3 & v_3 & v_3 \end{bmatrix}}_{V}$$

?las2

Computes singular values of a 2-by-2 triangular matrix.

Syntax

```
call slas2( f, g, h, ssmin, ssmax )
```

```
call dlas2( f, g, h, ssmin, ssmax )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?las2` computes the singular values of the 2-by-2 matrix

$$\begin{bmatrix} f & g \\ 0 & h \end{bmatrix}$$

On return, *ssmin* is the smaller singular value and *SSMAX* is the larger singular value.

Input Parameters

f, *g*, *h*	`REAL` for `slas2` `DOUBLE PRECISION` for `dlas2` The (1,1), (1,2) and (2,2) elements of the 2-by-2 matrix, respectively.

Output Parameters

ssmin, *ssmax*	`REAL` for `slas2` `DOUBLE PRECISION` for `dlas2` The smaller and the larger singular values, respectively.

Application Notes

Barring over/underflow, all output quantities are correct to within a few units in the last place (*ulps*), even in the absence of a guard digit in addition/subtraction. In ieee arithmetic, the code works correctly if one matrix element is infinite. Overflow will not occur unless the largest singular value itself overflows, or is within a few ulps of overflow. (On machines with partial overflow, like the Cray, overflow may occur if the largest singular value is within a factor of 2 of overflow.) Underflow is harmless if underflow is gradual. Otherwise, results may correspond to a matrix modified by perturbations of size near the underflow threshold.

?lascl

Multiplies a general rectangular matrix by a real scalar defined as c_{to}/c_{from}.

Syntax

```
call slascl( type, kl, ku, cfrom, cto, m, n, a, lda, info )
call dlascl( type, kl, ku, cfrom, cto, m, n, a, lda, info )
call clascl( type, kl, ku, cfrom, cto, m, n, a, lda, info )
call zlascl( type, kl, ku, cfrom, cto, m, n, a, lda, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lascl` multiplies the *m*-by-*n* real/complex matrix *A* by the real scalar c_{to}/c_{from}. The operation is performed without over/underflow as long as the final result c_{to}*A(i,j)/c_{from} does not over/underflow.

type specifies that *A* may be full, upper triangular, lower triangular, upper Hessenberg, or banded.

Input Parameters

type	CHARACTER*1. This parameter specifies the storage type of the input matrix. = 'G': *A* is a full matrix. = 'L': *A* is a lower triangular matrix. = 'U': *A* is an upper triangular matrix. = 'H': *A* is an upper Hessenberg matrix. = 'B': *A* is a symmetric band matrix with lower bandwidth *kl* and upper bandwidth *ku* and with the only the lower half stored = 'Q': *A* is a symmetric band matrix with lower bandwidth *kl* and upper bandwidth *ku* and with the only the upper half stored. = 'Z': *A* is a band matrix with lower bandwidth *kl* and upper bandwidth *ku*. See description of the ?gbtrf function for storage details.
kl	INTEGER. The lower bandwidth of *A*. Referenced only if *type* = 'B', 'Q' or 'Z'.
ku	INTEGER. The upper bandwidth of *A*. Referenced only if *type* = 'B', 'Q' or 'Z'.
cfrom, *cto*	REAL for slascl/clascl DOUBLE PRECISION for dlascl/zlascl The matrix *A* is multiplied by *cto*/*cfrom*. *A*(*i*,*j*) is computed without over/underflow if the final result *cto***A*(*i*,*j*)/*cfrom* can be represented without over/underflow. *cfrom* must be nonzero.
m	INTEGER. The number of rows of the matrix *A*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *A*. *n* ≥ 0.
a	REAL for slascl DOUBLE PRECISION for dlascl COMPLEX for clascl DOUBLE COMPLEX for zlascl Array, DIMENSION (*lda*, *n*). The matrix to be multiplied by *cto*/*cfrom*. See *type* for the storage type.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*m*).

Output Parameters

a	The multiplied matrix *A*.
info	INTEGER. If *info* = 0 - successful exit If *info* = -*i* < 0, the *i*-th argument had an illegal value.

See Also

?gbtrf

?lasd0

Computes the singular values of a real upper bidiagonal n-by-m matrix B with diagonal d and off-diagonal e. Used by ?bdsdc.

Syntax

```
call slasd0( n, sqre, d, e, u, ldu, vt, ldvt, smlsiz, iwork, work, info )
call dlasd0( n, sqre, d, e, u, ldu, vt, ldvt, smlsiz, iwork, work, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

Using a divide and conquer approach, the routine `?lasd0` computes the singular value decomposition (SVD) of a real upper bidiagonal n-by-m matrix B with diagonal d and offdiagonal e, where $m = n + sqre$.

The algorithm computes orthogonal matrices U and VT such that $B = U*S*VT$. The singular values S are overwritten on d.

The related subroutine `?lasda` computes only the singular values, and optionally, the singular vectors in compact form.

Input Parameters

n	INTEGER. On entry, the row dimension of the upper bidiagonal matrix. This is also the dimension of the main diagonal array d.
sqre	INTEGER. Specifies the column dimension of the bidiagonal matrix. If $sqre = 0$: the bidiagonal matrix has column dimension $m = n$. If $sqre = 1$: the bidiagonal matrix has column dimension $m = n+1$.
d	REAL for `slasd0` DOUBLE PRECISION for `dlasd0` Array, DIMENSION (n). On entry, d contains the main diagonal of the bidiagonal matrix.
e	REAL for `slasd0` DOUBLE PRECISION for `dlasd0` Array, DIMENSION (m-1). Contains the subdiagonal entries of the bidiagonal matrix. On exit, e is destroyed.
ldu	INTEGER. On entry, leading dimension of the output array u.
ldvt	INTEGER. On entry, leading dimension of the output array vt.
smlsiz	INTEGER. On entry, maximum size of the subproblems at the bottom of the computation tree.
iwork	INTEGER. Workspace array, `dimension` must be at least $(8n)$.
work	REAL for `slasd0` DOUBLE PRECISION for `dlasd0` Workspace array, `dimension` must be at least $(3m^2+2m)$.

Output Parameters

d	On exit d, If $info = 0$, contains singular values of the bidiagonal matrix.
u	REAL for `slasd0` DOUBLE PRECISION for `dlasd0` Array, DIMENSION at least (ldq, n). On exit, u contains the left singular vectors.
vt	REAL for `slasd0` DOUBLE PRECISION for `dlasd0`

	Array, DIMENSION at least (*ldvt*, *m*). On exit, vt^T contains the right singular vectors.
info	INTEGER. If *info* = 0: successful exit. If *info* = -*i* < 0, the *i*-th argument had an illegal value. If *info* = 1, a singular value did not converge.

?lasd1

Computes the SVD of an upper bidiagonal matrix B of the specified size. Used by ?bdsdc.

Syntax

```
call slasd1( nl, nr, sqre, d, alpha, beta, u, ldu, vt, ldvt, idxq, iwork, work, info )
call dlasd1( nl, nr, sqre, d, alpha, beta, u, ldu, vt, ldvt, idxq, iwork, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the SVD of an upper bidiagonal *n*-by-*m* matrix *B*, where *n* = *nl* + *nr* + 1 and *m* = *n* + *sqre*.

The routine ?lasd1 is called from ?lasd0.

A related subroutine ?lasd7 handles the case in which the singular values (and the singular vectors in factored form) are desired.

?lasd1 computes the SVD as follows:

$$VT = U(in)^* \begin{bmatrix} D1(in) & 0 & 0 & 0 \\ Z1^T & a & Z2^T & b \\ 0 & 0 & D2(in) & 0 \end{bmatrix} ^*VT(in)$$

= *U*(*out*)*(*D*(*out*) 0)**VT*(*out*)

where Z^T = ($Z1^T$ *a* $Z2^T$ *b*) = u^T**VT*^{T}, and *u* is a vector of dimension *m* with *alpha* and *beta* in the *nl*+1 and *nl*+2-th entries and zeros elsewhere; and the entry *b* is empty if *sqre* = 0.

The left singular vectors of the original matrix are stored in *u*, and the transpose of the right singular vectors are stored in *vt*, and the singular values are in *d*. The algorithm consists of three stages:

1. The first stage consists of deflating the size of the problem when there are multiple singular values or when there are zeros in the *z* vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine ?lasd2.
2. The second stage consists of calculating the updated singular values. This is done by finding the square roots of the roots of the secular equation via the routine ?lasd4 (as called by ?lasd3). This routine also calculates the singular vectors of the current problem.
3. The final stage consists of computing the updated singular vectors directly using the updated singular values. The singular vectors for the current problem are multiplied with the singular vectors from the overall problem.

Input Parameters

nl — INTEGER. The row dimension of the upper block.
nl ≥ 1.

nr — INTEGER. The row dimension of the lower block.
nr ≥ 1.

sqre — INTEGER.
If *sqre* = 0: the lower block is an *nr*-by-*nr* square matrix.
If *sqre* = 1: the lower block is an *nr*-by-(*nr*+1) rectangular matrix. The bidiagonal matrix has row dimension *n* = *nl* + *nr* + 1, and column dimension *m* = *n* + *sqre*.

d — REAL for slasd1
DOUBLE PRECISION for dlasd1
Array, DIMENSION (*nl*+*nr*+1). *n* = *nl*+*nr*+1. On entry *d*(1:*nl*,1:*nl*) contains the singular values of the upper block; and *d*(*nl*+2:*n*) contains the singular values of the lower block.

alpha — REAL for slasd1
DOUBLE PRECISION for dlasd1
Contains the diagonal element associated with the added row.

beta — REAL for slasd1
DOUBLE PRECISION for dlasd1
Contains the off-diagonal element associated with the added row.

u — REAL for slasd1
DOUBLE PRECISION for dlasd1
Array, DIMENSION (*ldu*, *n*). On entry *u*(1:*nl*, 1:*nl*) contains the left singular vectors of the upper block; *u*(*nl*+2:*n*, *nl*+2:*n*) contains the left singular vectors of the lower block.

ldu — INTEGER. The leading dimension of the array *U*.
ldu ≥ max(1, *n*).

vt — REAL for slasd1
DOUBLE PRECISION for dlasd1
Array, DIMENSION (*ldvt*, *m*), where *m* = *n* + *sqre*.
On entry $vt(1{:}nl+1,\ 1{:}nl+1)^T$ contains the right singular vectors of the upper block; $vt(nl+2{:}m,\ nl+2{:}m)^T$ contains the right singular vectors of the lower block.

ldvt — INTEGER. The leading dimension of the array *vt*.
ldvt ≥ max(1, *M*).

iwork — INTEGER.
Workspace array, DIMENSION (4*n*).

work — REAL for slasd1
DOUBLE PRECISION for dlasd1
Workspace array, DIMENSION ($3m_2$ + 2*m*).

Output Parameters

d — On exit *d*(1:*n*) contains the singular values of the modified matrix.

alpha — On exit, the diagonal element associated with the added row deflated by max(abs(*alpha*), abs(*beta*), abs(D(I))),I = 1,n.

beta — On exit, the off-diagonal element associated with the added row deflated by max(abs(*alpha*), abs(*beta*), abs(D(I))),I = 1,n.

u	On exit *u* contains the left singular vectors of the bidiagonal matrix.
vt	On exit vt^T contains the right singular vectors of the bidiagonal matrix.
idxq	INTEGER Array, DIMENSION (*n*). Contains the permutation which will reintegrate the subproblem just solved back into sorted order, that is, *d*(*idxq*(*i* = 1, *n*)) will be in ascending order.
info	INTEGER. If *info* = 0: successful exit. If *info* = -*i* < 0, the *i*-th argument had an illegal value. If *info* = 1, a singular value did not converge.

?lasd2

Merges the two sets of singular values together into a single sorted set. Used by ?bdsdc.

Syntax

call slasd2(*nl*, *nr*, *sqre*, *k*, *d*, *z*, *alpha*, *beta*, *u*, *ldu*, *vt*, *ldvt*, *dsigma*, *u2*, *ldu2*, *vt2*, *ldvt2*, *idxp*, *idx*, *idxp*, *idxq*, *coltyp*, *info*)

call dlasd2(*nl*, *nr*, *sqre*, *k*, *d*, *z*, *alpha*, *beta*, *u*, *ldu*, *vt*, *ldvt*, *dsigma*, *u2*, *ldu2*, *vt2*, *ldvt2*, *idxp*, *idx*, *idxp*, *idxq*, *coltyp*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasd2 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more singular values are close together or if there is a tiny entry in the *z* vector. For each such occurrence the order of the related secular equation problem is reduced by one.

The routine ?lasd2 is called from ?lasd1.

Input Parameters

nl	INTEGER. The row dimension of the upper block. *nl* ≥ 1.
nr	INTEGER. The row dimension of the lower block. *nr* ≥ 1.
sqre	INTEGER. If *sqre* = 0): the lower block is an *nr*-by-*nr* square matrix If *sqre* = 1): the lower block is an *nr*-by-(*nr*+1) rectangular matrix. The bidiagonal matrix has *n* = *nl* + *nr* + 1 rows and *m* = *n* + *sqre* ≥ *n* columns.
d	REAL for slasd2 DOUBLE PRECISION for dlasd2 Array, DIMENSION (*n*). On entry *d* contains the singular values of the two submatrices to be combined.
alpha	REAL for slasd2 DOUBLE PRECISION for dlasd2

Contains the diagonal element associated with the added row.

beta — REAL for slasd2
DOUBLE PRECISION for dlasd2
Contains the off-diagonal element associated with the added row.

u — REAL for slasd2
DOUBLE PRECISION for dlasd2
Array, DIMENSION (*ldu*, *n*). On entry *u* contains the left singular vectors of two submatrices in the two square blocks with corners at (1,1), (*nl*, *nl*), and (*nl*+2, *nl*+2), (*n*,*n*).

ldu — INTEGER. The leading dimension of the array *u*.
ldu ≥ *n*.

ldu2 — INTEGER. The leading dimension of the output array *u2*. *ldu2* ≥ *n*.

vt — REAL for slasd2
DOUBLE PRECISION for dlasd2
Array, DIMENSION (*ldvt*, *m*). On entry, vt^T contains the right singular vectors of two submatrices in the two square blocks with corners at (1,1), (*nl*+1, *nl*+1), and (*nl*+2, *nl*+2), (*m*, *m*).

ldvt — INTEGER. The leading dimension of the array *vt*. *ldvt* ≥ *m*.

ldvt2 — INTEGER. The leading dimension of the output array *vt2*. *ldvt2* ≥ *m*.

idxp — INTEGER.
Workspace array, DIMENSION (*n*). This will contain the permutation used to place deflated values of *D* at the end of the array. On output *idxp*(2:*k*) points to the nondeflated *d*-values and *idxp*(*k*+1:*n*) points to the deflated singular values.

idx — INTEGER.
Workspace array, DIMENSION (*n*). This will contain the permutation used to sort the contents of *d* into ascending order.

coltyp — INTEGER.
Workspace array, DIMENSION (*n*). As workspace, this array contains a label that indicates which of the following types a column in the *u2* matrix or a row in the *vt2* matrix is:
1 : non-zero in the upper half only
2 : non-zero in the lower half only
3 : dense
4 : deflated.

idxq — INTEGER. Array, DIMENSION (*n*). This parameter contains the permutation that separately sorts the two sub-problems in *D* in the ascending order. Note that entries in the first half of this permutation must first be moved one position backwards and entries in the second half must have *nl*+1 added to their values.

Output Parameters

k — INTEGER. Contains the dimension of the non-deflated matrix, This is the order of the related secular equation. 1 ≤ *k* ≤ *n*.

d — On exit *D* contains the trailing (*n*-*k*) updated singular values (those which were deflated) sorted into increasing order.

u — On exit *u* contains the trailing (*n*-*k*) updated left singular vectors (those which were deflated) in its last *n*-*k* columns.

z — REAL for slasd2

DOUBLE PRECISION for dlasd2
Array, DIMENSION (*n*). On exit, *z* contains the updating row vector in the secular equation.

dsigma — REAL for slasd2
DOUBLE PRECISION for dlasd2
Array, DIMENSION (*n*). Contains a copy of the diagonal elements (*k*–1 singular values and one zero) in the secular equation.

u2 — REAL for slasd2
DOUBLE PRECISION for dlasd2
Array, DIMENSION (*ldu2*, *n*). Contains a copy of the first *k*–1 left singular vectors which will be used by ?lasd3 in a matrix multiply (?gemm) to solve for the new left singular vectors. *u2* is arranged into four blocks. The first block contains a column with 1 at *nl*+1 and zero everywhere else; the second block contains non-zero entries only at and above *nl*; the third contains non-zero entries only below *nl*+1; and the fourth is dense.

vt — On exit, vt^T contains the trailing (*n*-*k*) updated right singular vectors (those which were deflated) in its last *n*-*k* columns. In case *sqre* =1, the last row of *vt* spans the right null space.

vt2 — REAL for slasd2
DOUBLE PRECISION for dlasd2
Array, DIMENSION (*ldvt2*, *n*). $vt2^T$ contains a copy of the first *k* right singular vectors which will be used by ?lasd3 in a matrix multiply (?gemm) to solve for the new right singular vectors. *vt2* is arranged into three blocks. The first block contains a row that corresponds to the special 0 diagonal element in *sigma*; the second block contains non-zeros only at and before *nl* +1; the third block contains non-zeros only at and after *nl* +2.

idxc — INTEGER. Array, DIMENSION (*n*). This will contain the permutation used to arrange the columns of the deflated *u* matrix into three groups: the first group contains non-zero entries only at and above *nl*, the second contains non-zero entries only below *nl*+2, and the third is dense.

coltyp — On exit, it is an array of dimension 4, with *coltyp*(*i*) being the dimension of the *i*-th type columns.

info — INTEGER.
If *info* = 0): successful exit
If *info* = -*i* < 0, the *i*-th argument had an illegal value.

?lasd3

Finds all square roots of the roots of the secular equation, as defined by the values in D and Z, and then updates the singular vectors by matrix multiplication. Used by ?bdsdc.

Syntax

```
call slasd3( nl, nr, sqre, k, d, q, ldq, dsigma, u, ldu, u2, ldu2, vt, ldvt, vt2,
ldvt2, idxc, ctot, z, info )
```

```
call dlasd3( nl, nr, sqre, k, d, q, ldq, dsigma, u, ldu, u2, ldu2, vt, ldvt, vt2,
ldvt2, idxc, ctot, z, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasd3 finds all the square roots of the roots of the secular equation, as defined by the values in *D* and *Z*.

It makes the appropriate calls to ?lasd4 and then updates the singular vectors by matrix multiplication.

The routine ?lasd3 is called from ?lasd1.

Input Parameters

nl INTEGER. The row dimension of the upper block.
nl ≥ 1.

nr INTEGER. The row dimension of the lower block.
nr ≥ 1.

sqre INTEGER.
If *sqre* = 0): the lower block is an *nr*-by-*nr* square matrix.
If *sqre* = 1): the lower block is an *nr*-by-(*nr*+1) rectangular matrix. The bidiagonal matrix has *n* = *nl* + *nr* + 1 rows and *m* = *n* + *sqre* ≥ *n* columns.

k INTEGER.The size of the secular equation, 1 ≤ *k* ≤ *n*.

q REAL for slasd3
DOUBLE PRECISION for dlasd3
Workspace array, DIMENSION at least (*ldq*, *k*).

ldq INTEGER. The leading dimension of the array *Q*.
ldq ≥ *k*.

dsigma REAL for slasd3
DOUBLE PRECISION for dlasd3
Array, DIMENSION (*k*). The first *k* elements of this array contain the old roots of the deflated updating problem. These are the poles of the secular equation.

ldu INTEGER. The leading dimension of the array *u*.
ldu ≥ *n*.

u2 REAL for slasd3
DOUBLE PRECISION for dlasd3
Array, DIMENSION (*ldu2*, *n*).
The first *k* columns of this matrix contain the non-deflated left singular vectors for the split problem.

ldu2 INTEGER. The leading dimension of the array *u2*.
ldu2 ≥ *n*.

ldvt INTEGER. The leading dimension of the array *vt*.
ldvt ≥ *n*.

vt2 REAL for slasd3
DOUBLE PRECISION for dlasd3
Array, DIMENSION (*ldvt2*, *n*).
The first *k* columns of *vt2*' contain the non-deflated right singular vectors for the split problem.

ldvt2 INTEGER. The leading dimension of the array *vt2*.
ldvt2 ≥ *n*.

idxc INTEGER. Array, DIMENSION (*n*).

	The permutation used to arrange the columns of u (and rows of *vt*) into three groups: the first group contains non-zero entries only at and above (or before) *nl* +1; the second contains non-zero entries only at and below (or after) *nl*+2; and the third is dense. The first column of *u* and the row of *vt* are treated separately, however. The rows of the singular vectors found by ?lasd4 must be likewise permuted before the matrix multiplies can take place.
ctot	INTEGER. Array, DIMENSION (4). A count of the total number of the various types of columns in *u* (or rows in *vt*), as described in *idxc*. The fourth column type is any column which has been deflated.
z	REAL for slasd3 DOUBLE PRECISION for dlasd3 Array, DIMENSION (*k*). The first *k* elements of this array contain the components of the deflation-adjusted updating row vector.

Output Parameters

d	REAL for slasd3 DOUBLE PRECISION for dlasd3 Array, DIMENSION (*k*). On exit the square roots of the roots of the secular equation, in ascending order.
u	REAL for slasd3 DOUBLE PRECISION for dlasd3 Array, DIMENSION (*ldu*, *n*). The last *n* - *k* columns of this matrix contain the deflated left singular vectors.
vt	REAL for slasd3 DOUBLE PRECISION for dlasd3 Array, DIMENSION (*ldvt*, *m*). The last *m* – *k* columns of *vt*' contain the deflated right singular vectors.
vt2	Destroyed on exit.
z	Destroyed on exit.
info	INTEGER. If *info* = 0): successful exit. If *info* = -*i* < 0, the *i*-th argument had an illegal value. If *info* = 1, an singular value did not converge.

Application Notes

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray XMP, Cray YMP, Cray C 90, or Cray 2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

?lasd4

Computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. Used by ?bdsdc*.*

Syntax

```
call slasd4( n, i, d, z, delta, rho, sigma, work, info)
```

```
call dlasd4( n, i, d, z, delta, rho, sigma, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix whose entries are given as the squares of the corresponding entries in the array d, and that $0 \le d(i) < d(j)$ for $i < j$ and that $rho > 0$. This is arranged by the calling routine, and is no loss in generality. The rank-one modified system is thus

$diag(d)*diag(d) + rho*Z*Z^T$,

where the Euclidean norm of Z is equal to 1.The method consists of approximating the rational functions in the secular equation by simpler interpolating rational functions.

Input Parameters

n	INTEGER. The length of all arrays.
i	INTEGER. The index of the eigenvalue to be computed. $1 \le i \le n$.
d	REAL for slasd4 DOUBLE PRECISION for dlasd4 Array, DIMENSION (*n*). The original eigenvalues. They must be in order, $0 \le d(i) < d(j)$ for $i < j$.
z	REAL for slasd4 DOUBLE PRECISION for dlasd4 Array, DIMENSION (*n*). The components of the updating vector.
rho	REAL for slasd4 DOUBLE PRECISION for dlasd4 The scalar in the symmetric updating formula.
work	REAL for slasd4 DOUBLE PRECISION for dlasd4 Workspace array, DIMENSION (*n*). If $n \ne 1$, *work* contains ($d(j) + sigma\_i$) in its j-th component. If $n = 1$, then $work(1) = 1$.

Output Parameters

delta	REAL for slasd4 DOUBLE PRECISION for dlasd4 Array, DIMENSION (*n*). If $n \ne 1$, *delta* contains ($d(j) - sigma\_i$) in its j-th component. If $n = 1$, then $delta (1) = 1$. The vector *delta* contains the information necessary to construct the (singular) eigenvectors.
sigma	REAL for slasd4 DOUBLE PRECISION for dlasd4 The computed $sigma\_i$, the i-th updated eigenvalue.
info	INTEGER. = 0: successful exit > 0: If $info = 1$, the updating process failed.

?lasd5

Computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix.Used by ?bdsdc.

Syntax

```
call slasd5( i, d, z, delta, rho, dsigma, work )
call dlasd5( i, d, z, delta, rho, dsigma, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix $diag(d)*diag(d)+rho*Z*Z^T$

The diagonal entries in the array d must satisfy $0 \le d(i) < d(j)$ for i<i, rho mustbe greater than 0, and that the Euclidean norm of the vector Z is equal to 1.

Input Parameters

i	*INTEGER*.The index of the eigenvalue to be computed. $i = 1$ or $i = 2$.
d	REAL for slasd5 DOUBLE PRECISION for dlasd5 Array, dimension (2). The original eigenvalues, $0 \le d(1) < d(2)$.
z	REAL for slasd5 DOUBLE PRECISION for dlasd5 Array, dimension (2). The components of the updating vector.
rho	REAL for slasd5 DOUBLE PRECISION for dlasd5 The scalar in the symmetric updating formula.
work	REAL for slasd5 DOUBLE PRECISION for dlasd5. Workspace array, dimension (2). Contains ($d(j) + sigma\_i$) in its j-th component.

Output Parameters

delta	REAL for slasd5 DOUBLE PRECISION for dlasd5. Array, dimension (2). Contains ($d(j) - sigma\_i$) in its j-th component. The vector *delta* contains the information necessary to construct the eigenvectors.
dsigma	REAL for slasd5 DOUBLE PRECISION for dlasd5. The computed $sigma\_i$, the i-th updated eigenvalue.

?lasd6

Computes the SVD of an updated upper bidiagonal matrix obtained by merging two smaller ones by appending a row. Used by ?bdsdc.

Syntax

call slasd6(*icompq, nl, nr, sqre, d, vf, vl, alpha, beta, idxq, perm, givptr, givcol, ldgcol, givnum, ldgnum, poles, difl, difr, z, k, c, s, work, iwork, info*)

call dlasd6(*icompq, nl, nr, sqre, d, vf, vl, alpha, beta, idxq, perm, givptr, givcol, ldgcol, givnum, ldgnum, poles, difl, difr, z, k, c, s, work, iwork, info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasd6 computes the *SVD* of an updated upper bidiagonal matrix *B* obtained by merging two smaller ones by appending a row. This routine is used only for the problem which requires all singular values and optionally singular vector matrices in factored form. *B* is an *n*-by-*m* matrix with *n = nl + nr + 1* and *m = n + sqre*. A related subroutine, ?lasd1, handles the case in which all singular values and singular vectors of the bidiagonal matrix are desired. ?lasd6 computes the *SVD* as follows:

$$B = U(in)*\begin{bmatrix} D1(in) & 0 & 0 & 0 \\ Z1' & a & Z2' & b \\ 0 & 0 & D2(in) & 0 \end{bmatrix}*VT(in)$$

$= U(\text{out})*(D(\text{out})*VT(out)$

where *Z' = (Z1' a Z2' b) = u'*VT'*, and *u* is a vector of dimension *m* with *alpha* and *beta* in the *nl*+1 and *nl*+2-th entries and zeros elsewhere; and the entry b is empty if *sqre = 0*.

The singular values of *B* can be computed using *D1*, *D2*, the first components of all the right singular vectors of the lower block, and the last components of all the right singular vectors of the upper block. These components are stored and updated in *vf* and *vl*, respectively, in ?lasd6. Hence *U* and *VT* are not explicitly referenced.

The singular values are stored in *D*. The algorithm consists of two stages:

1. The first stage consists of deflating the size of the problem when there are multiple singular values or if there is a zero in the *Z* vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine ?lasd7.
2. The second stage consists of calculating the updated singular values. This is done by finding the roots of the secular equation via the routine ?lasd4 (as called by ?lasd8). This routine also updates *vf* and *vl* and computes the distances between the updated singular values and the old singular values. ?lasd6 is called from ?lasda.

Input Parameters

icompq — INTEGER. Specifies whether singular vectors are to be computed in factored form:
= 0: Compute singular values only
= 1: Compute singular vectors in factored form as well.

nl	INTEGER. The row dimension of the upper block. *nl* ≥ 1.
nr	INTEGER. The row dimension of the lower block. *nr* ≥ 1.
sqre	INTEGER . = 0: the lower block is an *nr*-by-*nr* square matrix. = 1: the lower block is an *nr*-by-(*nr*+1) rectangular matrix. The bidiagonal matrix has row dimension *n*=*nl*+*nr*+1, and column dimension *m* = *n* + *sqre*.
d	REAL for slasd6 DOUBLE PRECISION for dlasd6 Array, dimension (*nl*+*nr*+1). On entry *d*(1:*nl*,1:*nl*) contains the singular values of the upper block, and *d*(*nl*+2:*n*) contains the singular values of the lower block.
vf	REAL for slasd6 DOUBLE PRECISION for dlasd6 Array, dimension (*m*). On entry, *vf*(1:*nl*+1) contains the first components of all right singular vectors of the upper block; and *vf*(*nl*+2:*m*) contains the first components of all right singular vectors of the lower block.
vl	REAL for slasd6 DOUBLE PRECISION for dlasd6 Array, dimension (*m*). On entry, *vl*(1:*nl*+1) contains the last components of all right singular vectors of the upper block; and *vl*(*nl*+2:*m*) contains the last components of all right singular vectors of the lower block.
alpha	REAL for slasd6 DOUBLE PRECISION for dlasd6 Contains the diagonal element associated with the added row.
beta	REAL for slasd6 DOUBLE PRECISION for dlasd6 Contains the off-diagonal element associated with the added row.
ldgcol	INTEGER.The leading dimension *of* the output array *givcol*, must be at least *n*.
ldgnum	INTEGER. The leading dimension of the output arrays *givnum* and *poles*, must be at least *n*.
work	REAL for slasd6 DOUBLE PRECISION for dlasd6 Workspace array, dimension (4*m*).
iwork	INTEGER. Workspace array, dimension (3*n*).

Output Parameters

d	On exit *d*(1:*n*) contains the singular values of the modified matrix.
vf	On exit, *vf* contains the first components of all right singular vectors of the bidiagonal matrix.
vl	On exit, *vl* contains the last components of all right singular vectors of the bidiagonal matrix.

alpha — On exit, the diagonal element associated with the added row deflated by max(abs(*alpha*), abs(*beta*), abs(D(I))), I = 1,n.

beta — On exit, the off-diagonal element associated with the added row deflated by max(abs(*alpha*), abs(*beta*), abs(D(I))), I = 1,n.

idxq — INTEGER.
Array, dimension (*n*). This contains the permutation which will reintegrate the subproblem just solved back into sorted order, that is, *d*(*idxq*(*i* = 1, *n*)) will be in ascending order.

perm — INTEGER.
Array, dimension (*n*). The permutations (from deflation and sorting) to be applied to each block. Not referenced if *icompq* = 0.

givptr — INTEGER. The number of Givens rotations which took place in this subproblem. Not referenced if *icompq* = 0.

givcol — INTEGER.
Array, dimension (*ldgcol*, 2). Each pair of numbers indicates a pair of columns to take place in a Givens rotation. Not referenced if *icompq* = 0.

givnum — REAL for slasd6
DOUBLE PRECISION for dlasd6
Array, dimension (*ldgnum*, 2). Each number indicates the *C* or *S* value to be used in the corresponding Givens rotation. Not referenced if *icompq* = 0.

poles — REAL for slasd6
DOUBLE PRECISION for dlasd6
Array, dimension (*ldgnum*, 2). On exit, *poles*(1,*) is an array containing the new singular values obtained from solving the secular equation, and *poles*(2,*) is an array containing the poles in the secular equation. Not referenced if *icompq* = 0.

difl — REAL for slasd6
DOUBLE PRECISION for dlasd6
Array, dimension (*n*). On exit, *difl*(*i*) is the distance between *i*-th updated (undeflated) singular value and the *i*-th (undeflated) old singular value.

difr — REAL for slasd6
DOUBLE PRECISION for dlasd6
Array, dimension (*ldgnum*, 2) if *icompq* = 1 and dimension (*n*) if *icompq* = 0.
On exit, *difr*(*i*, 1) is the distance between *i*-th updated (undeflated) singular value and the *i*+1-th (undeflated) old singular value. If *icompq* = 1, *difr*(1: *k*, 2) is an array containing the normalizing factors for the right singular vector matrix.
See ?lasd8 for details on *difl* and *difr*.

z — REAL for slasd6
DOUBLE PRECISION for dlasd6
Array, dimension (*m*).
The first elements of this array contain the components of the deflation-adjusted updating row vector.

k — INTEGER. Contains the dimension of the non-deflated matrix. This is the order of the related secular equation. $1 \le k \le n$.

c — REAL for slasd6

	DOUBLE PRECISION for dlasd6 *c* contains garbage if *sqre* =0 and the *C*-value of a Givens rotation related to the right null space if *sqre* = 1.
s	REAL for slasd6 DOUBLE PRECISION for dlasd6 *s* contains garbage if *sqre* =0 and the *S*-value of a Givens rotation related to the right null space if *sqre* = 1.
info	INTEGER. = 0: successful exit. < 0: if *info* = −*i*, the *i*-th argument had an illegal value. > 0: if *info* = 1, an singular value did not converge

?lasd7

Merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. Used by ?bdsdc.

Syntax

call slasd7(*icompq, nl, nr, sqre, k, d, z, zw, vf, vfw, vl, vlw, alpha, beta, dsigma, idx, idxp, idxq, perm, givptr, givcol, ldgcol, givnum, ldgnum, c, s, info*)

call dlasd7(*icompq, nl, nr, sqre, k, d, z, zw, vf, vfw, vl, vlw, alpha, beta, dsigma, idx, idxp, idxq, perm, givptr, givcol, ldgcol, givnum, ldgnum, c, s, info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasd7 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more singular values are close together or if there is a tiny entry in the *z* vector. For each such occurrence the order of the related secular equation problem is reduced by one. ?lasd7 is called from ?lasd6.

Input Parameters

icompq	INTEGER. Specifies whether singular vectors are to be computed in compact form, as follows: = 0: Compute singular values only. = 1: Compute singular vectors of upper bidiagonal matrix in compact form.
nl	INTEGER. The row dimension of the upper block. *nl* ≥ 1.
nr	INTEGER. The row dimension of the lower block. *nr* ≥ 1.
sqre	INTEGER. = 0: the lower block is an *nr*-by-*nr* square matrix. = 1: the lower block is an *nr*-by-(*nr*+1) rectangular matrix. The bidiagonal matrix has *n* = *nl* + *nr* + 1 rows and *m* = *n* + *sqre* ≥ *n* columns.
d	REAL for slasd7 DOUBLE PRECISION for dlasd7

Array, DIMENSION (n). On entry d contains the singular values of the two submatrices to be combined.

zw
REAL for slasd7
DOUBLE PRECISION for dlasd7
Array, DIMENSION (m).
Workspace for z.

vf
REAL for slasd7
DOUBLE PRECISION for dlasd7
Array, DIMENSION (m). On entry, $vf(1:nl+1)$ contains the first components of all right singular vectors of the upper block; and $vf(nl+2:m)$ contains the first components of all right singular vectors of the lower block.

vfw
REAL for slasd7
DOUBLE PRECISION for dlasd7
Array, DIMENSION (m).
Workspace for vf.

vl
REAL for slasd7
DOUBLE PRECISION for dlasd7
Array, DIMENSION (m).
On entry, $vl(1:nl+1)$ contains the last components of all right singular vectors of the upper block; and $vl(nl+2:m)$ contains the last components of all right singular vectors of the lower block.

VLW
REAL for slasd7
DOUBLE PRECISION for dlasd7
Array, DIMENSION (m).
Workspace for VL.

alpha
REAL for slasd7
DOUBLE PRECISION for dlasd7.
Contains the diagonal element associated with the added row.

beta
REAL for slasd7
DOUBLE PRECISION for dlasd7
Contains the off-diagonal element associated with the added row.

idx
INTEGER.
Workspace array, DIMENSION (n). This will contain the permutation used to sort the contents of d into ascending order.

idxp
INTEGER.
Workspace array, DIMENSION (n). This will contain the permutation used to place deflated values of d at the end of the array.

idxq
INTEGER.
Array, DIMENSION (n).
This contains the permutation which separately sorts the two sub-problems in d into ascending order. Note that entries in the first half of this permutation must first be moved one position backward; and entries in the second half must first have $nl+1$ added to their values.

ldgcol
INTEGER.The leading dimension of the output array $givcol$, must be at least n.

ldgnum
INTEGER. The leading dimension of the output array $givnum$, must be at least n.

Output Parameters

k	INTEGER. Contains the dimension of the non-deflated matrix, this is the order of the related secular equation. $1 \leq k \leq n$.
d	On exit, *d* contains the trailing (*n*-*k*) updated singular values (those which were deflated) sorted into increasing order.
z	REAL for slasd7 DOUBLE PRECISION for dlasd7. Array, DIMENSION (*m*). On exit, *Z* contains the updating row vector in the secular equation.
vf	On exit, *vf* contains the first components of all right singular vectors of the bidiagonal matrix.
vl	On exit, *vl* contains the last components of all right singular vectors of the bidiagonal matrix.
dsigma	REAL for slasd7 DOUBLE PRECISION for dlasd7. Array, DIMENSION (*n*). Contains a copy of the diagonal elements (*k*-1 singular values and one zero) in the secular equation.
idxp	On output, *idxp*(2: *k*) points to the nondeflated *d*-values and *idxp*(*k*+1:*n*) points to the deflated singular values.
perm	INTEGER. Array, DIMENSION (*n*). The permutations (from deflation and sorting) to be applied to each singular block. Not referenced if *icompq* = 0.
givptr	INTEGER. The number of Givens rotations which took place in this subproblem. Not referenced if *icompq* = 0.
givcol	INTEGER. Array, DIMENSION (*ldgcol*, 2). Each pair of numbers indicates a pair of columns to take place in a Givens rotation. Not referenced if *icompq* = 0.
givnum	REAL for slasd7 DOUBLE PRECISION for dlasd7. Array, DIMENSION (*ldgnum*, 2). Each number indicates the *C* or *S* value to be used in the corresponding Givens rotation. Not referenced if *icompq* = 0.
c	REAL for slasd7. DOUBLE PRECISION for dlasd7. If *sqre* =0, then *c* contains garbage, and if *sqre* = 1, then *c* contains *C*-value of a Givens rotation related to the right null space.
S	REAL for slasd7. DOUBLE PRECISION for dlasd7. If *sqre* =0, then *s* contains garbage, and if *sqre* = 1, then *s* contains *S*-value of a Givens rotation related to the right null space.
info	INTEGER. = 0: successful exit. < 0: if *info* = -*i*, the *i*-th argument had an illegal value.

?lasd8

Finds the square roots of the roots of the secular equation, and stores, for each element in `D`*, the distance to its two nearest poles. Used by* `?bdsdc`*.*

Syntax

```
call slasd8( icompq, k, d, z, vf, vl, difl, difr, lddifr, dsigma, work, info )
call dlasd8( icompq, k, d, z, vf, vl, difl, difr, lddifr, dsigma, work, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lasd8` finds the square roots of the roots of the secular equation, as defined by the values in *dsigma* and *z*. It makes the appropriate calls to `?lasd4`, and stores, for each element in *d*, the distance to its two nearest poles (elements in *dsigma*). It also updates the arrays *vf* and *vl*, the first and last components of all the right singular vectors of the original bidiagonal matrix. `?lasd8` is called from `?lasd6`.

Input Parameters

icompq
`INTEGER`. Specifies whether singular vectors are to be computed in factored form in the calling routine:
= 0: Compute singular values only.
= 1: Compute singular vectors in factored form as well.

k
`INTEGER`. The number of terms in the rational function to be solved by `?lasd4`. $k \geq 1$.

z
`REAL` for `slasd8`
`DOUBLE PRECISION` for `dlasd8`.
Array, `DIMENSION` (*k*).
The first *k* elements of this array contain the components of the deflation-adjusted updating row vector.

vf
`REAL` for `slasd8`
`DOUBLE PRECISION` for `dlasd8`.
Array, `DIMENSION` (*k*).
On entry, *vf* contains information passed through `dbede8`.

vl
`REAL` for `slasd8`
`DOUBLE PRECISION` for `dlasd8`.
Array, `DIMENSION` (*k*). On entry, *vl* contains information passed through `dbede8`.

lddifr
`INTEGER`. The leading dimension of the output array *difr*, must be at least *k*.

dsigma
`REAL` for `slasd8`
`DOUBLE PRECISION` for `dlasd8`.
Array, `DIMENSION` (*k*).
The first *k* elements of this array contain the old roots of the deflated updating problem. These are the poles of the secular equation.

work
`REAL` for `slasd8`
`DOUBLE PRECISION` for `dlasd8`.
Workspace array, `DIMENSION` at least (3*k*).

Output Parameters

d	REAL for slasd8 DOUBLE PRECISION for dlasd8. Array, DIMENSION (*k*). On output, *D* contains the updated singular values.
z	Updated on exit.
vf	On exit, *vf* contains the first *k* components of the first components of all right singular vectors of the bidiagonal matrix.
vl	On exit, *vl* contains the first *k* components of the last components of all right singular vectors of the bidiagonal matrix.
difl	REAL for slasd8 DOUBLE PRECISION for dlasd8. Array, DIMENSION (*k*). On exit, *difl*(*i*) = *d*(*i*) - *dsigma*(*i*).
difr	REAL for slasd8 DOUBLE PRECISION for dlasd8. Array, DIMENSION (*lddifr*, 2) if *icompq* = 1 and DIMENSION (*k*) if *icompq* = 0. On exit, *difr*(i,1) = *d*(*i*) - *dsigma*(i+1), *difr*(*k*,1) is not defined and will not be referenced. If *icompq* = 1, *difr*(1:*k*,2) is an array containing the normalizing factors for the right singular vector matrix.
dsigma	The elements of this array may be very slightly altered in value.
info	INTEGER. = 0: successful exit. < 0: if *info* = -*i*, the *i*-th argument had an illegal value. > 0: If *info* = 1, an singular value did not converge.

?lasd9

Finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by ?bdsdc.

Syntax

```
call slasd9( icompq, ldu, k, d, z, vf, vl, difl, difr, dsigma, work, info )
call dlasd9( icompq, ldu, k, d, z, vf, vl, difl, difr, dsigma, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasd9 finds the square roots of the roots of the secular equation, as defined by the values in *dsigma* and *z*. It makes the appropriate calls to ?lasd4, and stores, for each element in *d*, the distance to its two nearest poles (elements in *dsigma*). It also updates the arrays *vf* and *vl*, the first and last components of all the right singular vectors of the original bidiagonal matrix. ?lasd9 is called from ?lasd7.

Input Parameters

icompq INTEGER. Specifies whether singular vectors are to be computed in factored form in the calling routine:
If *icompq* = 0, compute singular values only;
If *icompq* = 1, compute singular vector matrices in factored form also.

k INTEGER. The number of terms in the rational function to be solved by slasd4. $k \geq 1$.

dsigma REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array, DIMENSION(*k*).
The first *k* elements of this array contain the old roots of the deflated updating problem. These are the poles of the secular equation.

z REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array, DIMENSION (*k*). The first *k* elements of this array contain the components of the deflation-adjusted updating row vector.

vf REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array, DIMENSION(*k*). On entry, *vf* contains information passed through sbede8.

vl REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array, DIMENSION(*k*). On entry, *vl* contains information passed through sbede8.

work REAL for slasd9
DOUBLE PRECISION for dlasd9.
Workspace array, DIMENSION at least (3*k*).

Output Parameters

d REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array, DIMENSION(*k*). *d*(*i*) contains the updated singular values.

vf On exit, *vf* contains the first *k* components of the first components of all right singular vectors of the bidiagonal matrix.

vl On exit, *vl* contains the first *k* components of the last components of all right singular vectors of the bidiagonal matrix.

difl REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array, DIMENSION (*k*).
On exit, *difl*(*i*) = *d*(*i*) - *dsigma*(*i*).

difr REAL for slasd9
DOUBLE PRECISION for dlasd9.
Array,
DIMENSION (*ldu*, 2) if *icompq* =1 and
DIMENSION (*k*) if *icompq* = 0.
On exit, *difr*(*i*, 1) = *d*(*i*) - *dsigma*(*i*+1), *difr*(*k*, 1) is not defined and will not be referenced.
If *icompq* = 1, *difr*(1:*k*, 2) is an array containing the normalizing factors for the right singular vector matrix.

info	INTEGER. = 0: successful exit. < 0: if *info* = -*i*, the *i*-th argument had an illegal value. > 0: If *info* = 1, an singular value did not converge

?lasda

Computes the singular value decomposition (SVD) of a real upper bidiagonal matrix with diagonal d and off-diagonal e. Used by ?bdsdc.

Syntax

```
call slasda( icompq, smlsiz, n, sqre, d, e, u, ldu, vt, k, difl, difr, z, poles,
givptr, givcol, ldgcol, perm, givnum, c, s, work, iwork, info )
```

```
call dlasda( icompq, smlsiz, n, sqre, d, e, u, ldu, vt, k, difl, difr, z, poles,
givptr, givcol, ldgcol, perm, givnum, c, s, work, iwork, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

Using a divide and conquer approach, ?lasda computes the singular value decomposition (*SVD*) of a real upper bidiagonal *n*-by-*m* matrix *B* with diagonal *d* and off-diagonal *e*, where *m* = *n* + *sqre*.

The algorithm computes the singular values in the *SVD B* = *U*S*VT*. The orthogonal matrices *U* and *VT* are optionally computed in compact form. A related subroutine ?lasd0 computes the singular values and the singular vectors in explicit form.

Input Parameters

icompq	INTEGER. Specifies whether singular vectors are to be computed in compact form, as follows: = 0: Compute singular values only. = 1: Compute singular vectors of upper bidiagonal matrix in compact form.
smlsiz	INTEGER. The maximum size of the subproblems at the bottom of the computation tree.
n	INTEGER. The row dimension of the upper bidiagonal matrix. This is also the dimension of the main diagonal array *d*.
sqre	INTEGER. Specifies the column dimension of the bidiagonal matrix. If *sqre* = 0: the bidiagonal matrix has column dimension *m* = *n* If *sqre* = 1: the bidiagonal matrix has column dimension *m* = *n* + 1.
d	REAL for slasda DOUBLE PRECISION for dlasda. Array, DIMENSION (*n*). On entry, *d* contains the main diagonal of the bidiagonal matrix.
e	REAL for slasda DOUBLE PRECISION for dlasda. Array, DIMENSION (*m* - 1). Contains the subdiagonal entries of the bidiagonal matrix. On exit, *e* is destroyed.

ldu INTEGER. The leading dimension of arrays *u*, *vt*, *difl*, *difr*, *poles*, *givnum*, and *z*. *ldu* ≥ *n*.

ldgcol INTEGER. The leading dimension of arrays *givcol* and *perm*. *ldgcol* ≥ *n*.

work REAL for slasda
DOUBLE PRECISION for dlasda.
Workspace array, DIMENSION $(6n+(smlsiz+1)^2)$.

iwork INTEGER.
Workspace array, *Dimension* must be at least $(7n)$.

Output Parameters

d On exit *d*, if *info* = 0, contains the singular values of the bidiagonal matrix.

u REAL for slasda
DOUBLE PRECISION for dlasda.
Array, DIMENSION (*ldu*, *smlsiz*) if *icompq* =1.
Not referenced if *icompq* = 0.
If *icompq* = 1, on exit, *u* contains the left singular vector matrices of all subproblems at the bottom level.

vt REAL for slasda
DOUBLE PRECISION for dlasda.
Array, DIMENSION (*ldu*, *smlsiz*+1) if *icompq* = 1, and not referenced if *icompq* = 0. If *icompq* = 1, on exit, *vt*' contains the right singular vector matrices of all subproblems at the bottom level.

k INTEGER.
Array, DIMENSION (*n*) if *icompq* = 1 and
DIMENSION (1) if *icompq* = 0.
If *icompq* = 1, on exit, *k*(*i*) is the dimension of the *i*-th secular equation on the computation tree.

difl REAL for slasda
DOUBLE PRECISION for dlasda.
Array, DIMENSION (*ldu*, *nlvl*),
where *nlvl* = floor($\log_2(n/smlsiz)$).

difr REAL for slasda
DOUBLE PRECISION for dlasda.
Array,
DIMENSION (*ldu*, 2 *nlvl*) if *icompq* = 1 and
DIMENSION (*n*) if *icompq* = 0.
If *icompq* = 1, on exit, *difl*(1:*n*, *i*) and *difr*(1:*n*,2*i* -1) record distances between singular values on the *i*-th level and singular values on the (*i* -1)-th level, and *difr*(1:*n*, 2i) contains the normalizing factors for the right singular vector matrix. See ?lasd8 for details.

z REAL for slasda
DOUBLE PRECISION for dlasda.
Array,
DIMENSION (*ldu*, *nlvl*) if *icompq* = 1 and
DIMENSION (*n*) if *icompq* = 0. The first *k* elements of *z*(1, *i*) contain the components of the deflation-adjusted updating row vector for subproblems on the *i*-th level.

poles REAL for slasda
DOUBLE PRECISION for dlasda

	Array, DIMENSION (*ldu*, 2**nlvl*) if *icompq* = 1, and not referenced if *icompq* = 0. If *icompq* = 1, on exit, *poles*(1, 2*i* - 1) and *poles*(1, 2*i*) contain the new and old singular values involved in the secular equations on the *i*-th level.
givptr	INTEGER. Array, DIMENSION (*n*) if *icompq* = 1, and not referenced if *icompq* = 0. If *icompq* = 1, on exit, *givptr*(*i*) records the number of Givens rotations performed on the *i*-th problem on the computation tree.
givcol	INTEGER . Array, DIMENSION (*ldgcol*, 2**nlvl*) if *icompq* = 1, and not referenced if *icompq* = 0. If *icompq* = 1, on exit, for each *i*, *givcol*(1, 2 *i* - 1) and *givcol*(1, 2 *i*) record the locations of Givens rotations performed on the *i*-th level on the computation tree.
perm	INTEGER . Array, DIMENSION (*ldgcol*, *nlvl*) if *icompq* = 1, and not referenced if *icompq* = 0. If *icompq* = 1, on exit, *perm* (1, *i*) records permutations done on the *i*-th level of the computation tree.
givnum	REAL for slasda DOUBLE PRECISION for dlasda. Array DIMENSION (*ldu*, 2**nlvl*) if *icompq* = 1, and not referenced if *icompq* = 0. If *icompq* = 1, on exit, for each *i*, *givnum*(1, 2 *i* - 1) and *givnum*(1, 2 *i*) record the *c*- and *s*-values of Givens rotations performed on the *i*-th level on the computation tree.
c	REAL for slasda DOUBLE PRECISION for dlasda. Array, DIMENSION (*n*) if *icompq* = 1, and DIMENSION (1) if *icompq* = 0. If *icompq* = 1 and the *i*-th subproblem is not square, on exit, *c*(*i*) contains the *c*-value of a Givens rotation related to the right null space of the *i*-th subproblem.
s	REAL for slasda DOUBLE PRECISION for dlasda. Array, DIMENSION (*n*) *icompq* = 1, and DIMENSION (1) if *icompq* = 0. If *icompq* = 1 and the *i*-th subproblem is not square, on exit, *s*(*i*) contains the *s*-value of a Givens rotation related to the right null space of the *i*-th subproblem.
info	INTEGER. = 0: successful exit. < 0: if *info* = -*i*, the *i*-th argument had an illegal value > 0: If *info* = 1, an singular value did not converge

?lasdq

Computes the SVD of a real bidiagonal matrix with diagonal d and off-diagonal e. Used by ?bdsdc.

Syntax

```
call slasdq( uplo, sqre, n, ncvt, nru, ncc, d, e, vt, ldvt, u, ldu, c, ldc, work,
info )
```

```
call dlasdq( uplo, sqre, n, ncvt, nru, ncc, d, e, vt, ldvt, u, ldu, c, ldc, work,
info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lasdq` computes the singular value decomposition (*SVD*) of a real (upper or lower) bidiagonal matrix with diagonal d and off-diagonal e, accumulating the transformations if desired. If B is the input bidiagonal matrix, the algorithm computes orthogonal matrices Q and P such that $B = Q*S*P^T$. The singular values S are overwritten on d.

The input matrix U is changed to $U*Q$ if desired.

The input matrix VT is changed to P^T*VT if desired.

The input matrix C is changed to Q^T*C if desired.

Input Parameters

uplo — `CHARACTER*1`. On entry, *uplo* specifies whether the input bidiagonal matrix is upper or lower bidiagonal.
If *uplo* = `'U'` or `'u'`, B is upper bidiagonal;
If *uplo* = `'L'` or `'l'`, B is lower bidiagonal.

sqre — `INTEGER`.
= 0: then the input matrix is n-by-n.
= 1: then the input matrix is n-by-(n+1) if *uplu* = `'U'` and (n+1)-by-n if *uplu*
= `'L'`. The bidiagonal matrix has $n = nl + nr + 1$ rows and $m = n + sqre \geq n$ columns.

n — `INTEGER`. On entry, n specifies the number of rows and columns in the matrix. n must be at least 0.

ncvt — `INTEGER`. On entry, *ncvt* specifies the number of columns of the matrix VT. *ncvt* must be at least 0.

nru — `INTEGER`. On entry, *nru* specifies the number of rows of the matrix U. *nru* must be at least 0.

ncc — `INTEGER`. On entry, *ncc* specifies the number of columns of the matrix C. *ncc* must be at least 0.

d — `REAL` for *slasdq*
`DOUBLE PRECISION` for *dlasdq*.
Array, `DIMENSION` (n). On entry, d contains the diagonal entries of the bidiagonal matrix.

e — `REAL` for *slasdq*
`DOUBLE PRECISION` for *dlasdq*.
Array, `DIMENSION` is (n-1) if *sqre* = 0 and n if *sqre* = 1. On entry, the entries of e contain the off-diagonal entries of the bidiagonal matrix.

vt — `REAL` for `slasdq`
`DOUBLE PRECISION` for `dlasdq`.
Array, `DIMENSION` (*ldvt*, *ncvt*). On entry, contains a matrix which on exit has been premultiplied by P^T, dimension n-by-*ncvt* if *sqre* = 0 and (n+1)-by-*ncvt* if *sqre* = 1 (not referenced if *ncvt*=0).

ldvt	INTEGER. On entry, *ldvt* specifies the leading dimension of *vt* as declared in the calling (sub) program. *ldvt* must be at least 1. If *ncvt* is nonzero, *ldvt* must also be at least *n*.
u	REAL for *slasdq* DOUBLE PRECISION for *dlasdq*. Array, DIMENSION (*ldu*, *n*). On entry, contains a matrix which on exit has been postmultiplied by *Q*, dimension *nru*-by-*n* if *sqre* = 0 and *nru*-by-(*n* +1) if *sqre* = 1 (not referenced if *nru*=0).
ldu	INTEGER. On entry, *ldu* specifies the leading dimension of *u* as declared in the calling (sub) program. *ldu* must be at least max(1, *nru*) .
c	REAL for *slasdq* DOUBLE PRECISION for *dlasdq*. Array, DIMENSION (*ldc*, *ncc*). On entry, contains an *n*-by-*ncc* matrix which on exit has been premultiplied by *Q'*, dimension *n*-by-*ncc* if *sqre* = 0 and (*n*+1)-by-*ncc* if *sqre* = 1 (not referenced if *ncc*=0).
ldc	INTEGER. On entry, *ldc* specifies the leading dimension of *C* as declared in the calling (sub) program. *ldc* must be at least 1. If *ncc* is non-zero, *ldc* must also be at least *n*.
work	REAL for slasdq DOUBLE PRECISION for dlasdq. Array, DIMENSION (4*n*). This is a workspace array. Only referenced if one of *ncvt*, *nru*, or *ncc* is nonzero, and if *n* is at least 2.

Output Parameters

d	On normal exit, *d* contains the singular values in ascending order.
e	On normal exit, *e* will contain 0. If the algorithm does not converge, *d* and *e* will contain the diagonal and superdiagonal entries of a bidiagonal matrix orthogonally equivalent to the one given as input.
vt	On exit, the matrix has been premultiplied by *P*'.
u	On exit, the matrix has been postmultiplied by *Q*.
c	On exit, the matrix has been premultiplied by *Q*'.
info	INTEGER. On exit, a value of 0 indicates a successful exit. If *info* < 0, argument number -*info* is illegal. If *info* > 0, the algorithm did not converge, and *info* specifies how many superdiagonals did not converge.

?lasdt

Creates a tree of subproblems for bidiagonal divide and conquer. Used by ?bdsdc.

Syntax

```
call slasdt( n, lvl, nd, inode, ndiml, ndimr, msub )
call dlasdt( n, lvl, nd, inode, ndiml, ndimr, msub )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine creates a tree of subproblems for bidiagonal divide and conquer.

Input Parameters

n	INTEGER. On entry, the number of diagonal elements of the bidiagonal matrix.
msub	INTEGER. On entry, the maximum row dimension each subproblem at the bottom of the tree can be of.

Output Parameters

lvl	INTEGER. On exit, the number of levels on the computation tree.
nd	INTEGER. On exit, the number of nodes on the tree.
inode	INTEGER. Array, DIMENSION (*n*). On exit, centers of subproblems.
ndiml	INTEGER . Array, DIMENSION (*n*). On exit, row dimensions of left children.
ndimr	INTEGER . Array, DIMENSION (*n*). On exit, row dimensions of right children.

?laset

Initializes the off-diagonal elements and the diagonal elements of a matrix to given values.

Syntax

```
call slaset( uplo, m, n, alpha, beta, a, lda )
call dlaset( uplo, m, n, alpha, beta, a, lda )
call claset( uplo, m, n, alpha, beta, a, lda )
call zlaset( uplo, m, n, alpha, beta, a, lda )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine initializes an *m*-by-*n* matrix *A* to *beta* on the diagonal and *alpha* on the off-diagonals.

Input Parameters

uplo	CHARACTER*1. Specifies the part of the matrix *A* to be set. If *uplo* = 'U', upper triangular part is set; the strictly lower triangular part of *A* is not changed. If *uplo* = 'L': lower triangular part is set; the strictly upper triangular part of *A* is not changed. Otherwise: All of the matrix *A* is set.
m	INTEGER. The number of rows of the matrix *A*. $m \geq 0$.
n	INTEGER. The number of columns of the matrix *A*. $n \geq 0$.
alpha, *beta*	REAL for slaset DOUBLE PRECISION for dlaset COMPLEX for claset

	DOUBLE COMPLEX for zlaset. The constants to which the off-diagonal and diagonal elements are to be set, respectively.
a	REAL for slaset DOUBLE PRECISION for dlaset COMPLEX for claset DOUBLE COMPLEX for zlaset. Array, DIMENSION (*lda*, *n*). On entry, the *m*-by-*n* matrix *A*.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*m*).

Output Parameters

a	On exit, the leading *m*-by-*n* submatrix of *A* is set as follows: if *uplo* = 'U', *A*(*i*,*j*) = *alpha*, 1≤*i*≤*j*-1, 1≤*j*≤*n*, if *uplo* = 'L', *A*(*i*,*j*) = *alpha*, *j*+1≤*i*≤*m*, 1≤*j*≤*n*, otherwise, *A*(*i*,*j*) = *alpha*, 1≤*i*≤*m*, 1≤*j*≤*n*, *i* ≠ *j*, and, for all *uplo*, *A*(*i*,*i*) = *beta*, 1≤*i*≤min(*m*, *n*).

?lasq1

Computes the singular values of a real square bidiagonal matrix. Used by ?bdsqr.

Syntax

```
call slasq1( n, d, e, work, info )
call dlasq1( n, d, e, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasq1 computes the singular values of a real *n*-by-*n* bidiagonal matrix with diagonal *d* and off-diagonal *e*. The singular values are computed to high relative accuracy, in the absence of denormalization, underflow and overflow.

Input Parameters

n	INTEGER.The number of rows and columns in the matrix. *n* ≥ 0.
d	REAL for slasq1 DOUBLE PRECISION for dlasq1. Array, DIMENSION (*n*). On entry, *d* contains the diagonal elements of the bidiagonal matrix whose *SVD* is desired.
e	REAL for slasq1 DOUBLE PRECISION for dlasq1. Array, DIMENSION (*n*). On entry, elements *e*(1:*n*-1) contain the off-diagonal elements of the bidiagonal matrix whose *SVD* is desired.
work	REAL for slasq1

DOUBLE PRECISION for dlasq1.
Workspace array, DIMENSION (4*n*).

Output Parameters

d — On normal exit, *d* contains the singular values in decreasing order.

e — On exit, *e* is overwritten.

info — INTEGER.
= 0: successful exit;
< 0: if *info* = -*i*, the *i*-th argument had an illegal value;
> 0: the algorithm failed:
= 1, a split was marked by a positive value in *e*;
= 2, current block of *z* not diagonalized after 30*n* iterations (in inner while loop);
= 3, termination criterion of outer while loop not met (program created more than *n* unreduced blocks.

?lasq2

Computes all the eigenvalues of the symmetric positive definite tridiagonal matrix associated with the qd array z to high relative accuracy. Used by ?bdsqr and ?stegr.

Syntax

```
call slasq2( n, z, info )
call dlasq2( n, z, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasq2 computes all the eigenvalues of the symmetric positive definite tridiagonal matrix associated with the *qd* array *z* to high relative accuracy, in the absence of denormalization, underflow and overflow.

To see the relation of *z* to the tridiagonal matrix, let *L* be a unit lower bidiagonal matrix with subdiagonals *z*(2,4,6,,..) and let *U* be an upper bidiagonal matrix with 1's above and diagonal *z*(1,3,5,,..). The tridiagonal is *LU* or, if you prefer, the symmetric tridiagonal to which it is similar.

Input Parameters

n — INTEGER. The number of rows and columns in the matrix. $n \geq 0$.

z — REAL for slasq2
DOUBLE PRECISION for dlasq2.
Array, DIMENSION (4 * *n*).
On entry, *z* holds the *qd* array.

Output Parameters

z	On exit, entries 1 to *n* hold the eigenvalues in decreasing order, *z*(2*n*+1) holds the trace, and *z*(2*n*+2) holds the sum of the eigenvalues. If *n* > 2, then *z*(2*n*+3) holds the iteration count, *z*(2*n*+4) holds ndivs/nin^2, and *z*(2*n*+5) holds the percentage of shifts that failed.
info	INTEGER. = 0: successful exit; < 0: if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*, if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+ *j*); > 0: the algorithm failed: = 1, a split was marked by a positive value in *e*; = 2, current block of *z* not diagonalized after 30**n* iterations (in inner while loop); = 3, termination criterion of outer while loop not met (program created more than *n* unreduced blocks).

Application Notes

The routine ?lasq2 defines a logical variable, *ieee*, which is .TRUE. on machines which follow ieee-754 floating-point standard in their handling of infinities and NaNs, and .FALSE. otherwise. This variable is passed to ?lasq3.

?lasq3

Checks for deflation, computes a shift and calls dqds.
Used by ?bdsqr.

Syntax

```
call slasq3( i0, n0, z, pp, dmin, sigma, desig, qmax, nfail, iter, ndiv, ieee, ttype,
dmin1, dmin2, dn, dn1, dn2, g, tau )
```

```
call dlasq3( i0, n0, z, pp, dmin, sigma, desig, qmax, nfail, iter, ndiv, ieee, ttype,
dmin1, dmin2, dn, dn1, dn2, g, tau )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasq3 checks for deflation, computes a shift *tau*, and calls *dqds*. In case of failure, it changes shifts, and tries again until output is positive.

Input Parameters

i0	INTEGER. First index.
n0	INTEGER. Last index.
z	REAL for slasq3 DOUBLE PRECISION for dlasq3. Array, DIMENSION (4*n*). *z* holds the *qd* array.
pp	INTEGER. *pp*=0 for ping, *pp*=1 for pong. *pp*=2 indicates that flipping was applied to the *z* array and that the initial tests for deflation should not be performed.

desig	REAL for slasq3 DOUBLE PRECISION for dlasq3. Lower order part of *sigma*.
qmax	REAL for slasq3 DOUBLE PRECISION for dlasq3. Maximum value of *q*.
ieee	LOGICAL. Flag for ieee or non-ieee arithmetic (passed to ?lasq5).
ttype	INTEGER. Shift type.
dmin1, *dmin2*, *dn*, *dn1*, *dn2*, *g*, *tau*	REAL for slasq3 DOUBLE PRECISION for dlasq3. These scalars are passed as arguments in order to save their values between calls to ?lasq3.

Output Parameters

dmin	REAL for slasq3 DOUBLE PRECISION for dlasq3. Minimum value of *d*.
pp	INTEGER. *pp*=0 for ping, *pp*=1 for pong. *pp*=2 indicates that flipping was applied to the *Z* array and that the initial tests for deflation should not be performed.
sigma	REAL for slasq3 DOUBLE PRECISION for dlasq3. Sum of shifts used in the current segment.
desig	Lower order part of *sigma*.
nfail	INTEGER. Number of times shift was too big.
iter	INTEGER. Number of iterations.
ndiv	INTEGER. Number of divisions.
ttype	INTEGER. Shift type.
dmin1, *dmin2*, *dn*, *dn1*, *dn2*, *g*, *tau*	REAL for slasq3 DOUBLE PRECISION for dlasq3. These scalars are passed as arguments in order to save their values between calls to ?lasq3.

?lasq4

Computes an approximation to the smallest eigenvalue using values of d from the previous transform. Used by ?bdsqr.

Syntax

```
call slasq4( i0, n0, z, pp, n0in, dmin, dmin1, dmin2, dn, dn1, dn2, tau, ttype, g )
call dlasq4( i0, n0, z, pp, n0in, dmin, dmin1, dmin2, dn, dn1, dn2, tau, ttype, g )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes an approximation *tau* to the smallest eigenvalue using values of d from the previous transform.

Input Parameters

i0	INTEGER. First index.
n0	INTEGER. Last index.
z	REAL for slasq4 DOUBLE PRECISION for dlasq4. Array, DIMENSION (4*n*). *z* holds the *qd* array.
pp	INTEGER. *pp*=0 for ping, *pp*=1 for pong.
n0in	INTEGER. The value of *n0* at start of eigtest.
dmin	REAL for slasq4 DOUBLE PRECISION for dlasq4. Minimum value of *d*.
dmin1	REAL for slasq4 DOUBLE PRECISION for dlasq4. Minimum value of *d*, excluding *d*(*n0*).
dmin2	REAL for slasq4 DOUBLE PRECISION for dlasq4. Minimum value of *d*, excluding *d*(*n0*) and *d*(*n0*-1).
dn	REAL for slasq4 DOUBLE PRECISION for dlasq4. Contains *d*(*n*).
dn1	REAL for slasq4 DOUBLE PRECISION for dlasq4. Contains *d*(*n*-1).
dn2	REAL for slasq4 DOUBLE PRECISION for dlasq4. Contains *d*(*n*-2).
g	REAL for slasq4 DOUBLE PRECISION for dlasq4. A scalar passed as an argument in order to save its value between calls to ?lasq4.

Output Parameters

tau	REAL for slasq4 DOUBLE PRECISION for dlasq4. Shift.
ttype	INTEGER. Shift type.
g	REAL for slasq4 DOUBLE PRECISION for dlasq4. A scalar passed as an argument in order to save its value between calls to ?lasq4.

?lasq5

Computes one dqds transform in ping-pong form.
Used by ?bdsqr and ?stegr.

Syntax

```
call slasq5( i0, n0, z, pp, tau, dmin, dmin1, dmin2, dn, dnm1, dnm2, ieee )
call dlasq5( i0, n0, z, pp, tau, dmin, dmin1, dmin2, dn, dnm1, dnm2, ieee )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine computes one dqds transform in ping-pong form: one version for ieee machines, another for non-ieee machines.

Input Parameters

i0	`INTEGER`. First index.
n0	`INTEGER`. Last index.
z	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Array, `DIMENSION` (4*n*). *z* holds the qd array. *emin* is stored in *z*(4**n0*) to avoid an extra argument.
pp	`INTEGER`. *pp*=0 for ping, *pp*=1 for pong.
tau	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. This is the shift.
ieee	`LOGICAL`. Flag for IEEE or non-IEEE arithmetic.

Output Parameters

dmin	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Minimum value of *d*.
dmin1	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Minimum value of *d*, excluding *d*(*n0*).
dmin2	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Minimum value of *d*, excluding *d*(*n0*) and *d*(*n0*-1).
dn	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Contains *d*(*n0*), the last value of *d*.
dnm1	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Contains *d*(*n0*-1).
dnm2	`REAL` for `slasq5` `DOUBLE PRECISION` for `dlasq5`. Contains *d*(*n0*-2).

?lasq6

Computes one dqd transform in ping-pong form. Used by `?bdsqr` *and* `?stegr`.

Syntax

```
call slasq6( i0, n0, z, pp, dmin, dmin1, dmin2, dn, dnm1, dnm2 )
```

```
call dlasq6( i0, n0, z, pp, dmin, dmin1, dmin2, dn, dnm1, dnm2 )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasq6 computes one *dqd* (shift equal to zero) transform in ping-pong form, with protection against underflow and overflow.

Input Parameters

i0	INTEGER. First index.
n0	INTEGER. Last index.
z	REAL for slasq6 DOUBLE PRECISION for dlasq6. Array, DIMENSION (4*n*). *z* holds the qd array. *emin* is stored in *z*(4**n0*) to avoid an extra argument.
pp	INTEGER. *pp*=0 for ping, *pp*=1 for pong.

Output Parameters

dmin	REAL for slasq6 DOUBLE PRECISION for dlasq6. Minimum value of *d*.
dmin1	REAL for slasq6 DOUBLE PRECISION for dlasq6. Minimum value of *d*, excluding *d*(*n0*).
dmin2	REAL for slasq6 DOUBLE PRECISION for dlasq6. Minimum value of *d*, excluding *d*(*n0*) and *d*(*n0*-1).
dn	REAL for slasq6 DOUBLE PRECISION for dlasq6. Contains *d*(*n0*), the last value of *d*.
dnm1	REAL for slasq6 DOUBLE PRECISION for dlasq6. Contains *d*(*n0*-1).
dnm2	REAL for slasq6 DOUBLE PRECISION for dlasq6. Contains *d*(*n0*-2).

?lasr

Applies a sequence of plane rotations to a general rectangular matrix.

Syntax

```
call slasr( side, pivot, direct, m, n, c, s, a, lda )
call dlasr( side, pivot, direct, m, n, c, s, a, lda )
call clasr( side, pivot, direct, m, n, c, s, a, lda )
call zlasr( side, pivot, direct, m, n, c, s, a, lda )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine applies a sequence of plane rotations to a real/complex matrix `A`, from the left or the right.

`A := P*A`, when `side = 'L'` (Left-hand side)

`A := A*P'`, when `side = 'R'` (Right-hand side)

where `P` is an orthogonal matrix consisting of a sequence of plane rotations with `z` = `m` when `side = 'L'` and `z` = `n` when `side = 'R'`.

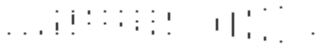
When `direct = 'F'` (Forward sequence), then

`P = P(z-1)*...P(2)*P(1)`,

and when `direct = 'B'` (Backward sequence), then

`P = P(1)*P(2)*...*P(z-1)`,

where `P( k )` is a plane rotation matrix defined by the 2-by-2 plane rotation:



When `pivot = 'V'` (Variable pivot), the rotation is performed for the plane (`k`, `k` + 1), that is, `P`(`k`) has the form

$$P(k)=\begin{bmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & 1 & & & & \\ & & & c(k) & s(k) & & \\ & & & -s(k) & c(k) & & \\ & & & & & 1 & \\ & & & & & & \ddots \\ & & & & & & & 1 \end{bmatrix}$$

where `R`(`k`) appears as a rank-2 modification to the identity matrix in rows and columns `k` and `k`+1.

When `pivot = 'T'` (Top pivot), the rotation is performed for the plane `(1,k+1)`, so `P`(`k`) has the form

$$P(k)=\begin{bmatrix} c(k) & & & & s(k) & & \\ & 1 & & & & & \\ & & \cdots & & & & \\ & & & 1 & & & \\ -s(k) & & & & c(k) & & \\ & & & & & 1 & \\ & & & & & & \cdots \\ & & & & & & & 1 \end{bmatrix}$$

where $R(k)$ appears in rows and columns k and k+1.

Similarly, when *pivot* = 'B' (Bottom pivot), the rotation is performed for the plane (k, z), giving $P(k)$ the form

$$P(k)=\begin{bmatrix} 1 & & & & & & \\ & \cdots & & & & & \\ & & 1 & & & & \\ & & & c(k) & & s(k) & \\ & & & & 1 & & \\ & & & & & \cdots & \\ & & & & & & 1 \\ & & & -s(k) & & c(k) & \end{bmatrix}$$

where $R(k)$ appears in rows and columns k and z. The rotations are performed without ever forming $P(k)$ explicitly.

Input Parameters

side — CHARACTER*1. Specifies whether the plane rotation matrix P is applied to A on the left or the right.
= 'L': left, compute $A := P*A$
= 'R': right, compute $A := A*P$'

direct — CHARACTER*1. Specifies whether P is a forward or backward sequence of plane rotations.
= 'F': forward, $P = P(z-1)*...*P(2)*P(1)$
= 'B': backward, $P = P(1)*P(2)*...*P(z-1)$

pivot — CHARACTER*1. Specifies the plane for which $P(k)$ is a plane rotation matrix.
= 'V': Variable pivot, the plane (k, k+1)
= 'T': Top pivot, the plane (1, k+1)

= 'B': Bottom pivot, the plane (k, z)

m — INTEGER. The number of rows of the matrix A.
If $m \leq 1$, an immediate return is effected.

n — INTEGER. The number of columns of the matrix A.
If $n \leq 1$, an immediate return is effected.

c, *s* — REAL for slasr/clasr
DOUBLE PRECISION for dlasr/zlasr.
Arrays, DIMENSION
(m-1) if *side* = 'L',
(n-1) if *side* = 'R' .
$c(k)$ and $s(k)$ contain the cosine and sine of the plane rotations respectively that define the 2-by-2 plane rotation part ($R(k)$) of the $P(k)$ matrix as described above in *Description*.

a — REAL for slasr
DOUBLE PRECISION for dlasr
COMPLEX for clasr
DOUBLE COMPLEX for zlasr.
Array, DIMENSION (*lda*, *n*).
The m-by-n matrix A.

lda — INTEGER. The leading dimension of the array *a*.
lda $\geq$ max(1,m).

Output Parameters

a — On exit, A is overwritten by $P*A$ if *side* = 'R', or by $A*P$' if *side* = 'L'.

?lasrt

Sorts numbers in increasing or decreasing order.

Syntax

```
call slasrt( id, n, d, info )
call dlasrt( id, n, d, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasrt sorts the numbers in d in increasing order (if *id* = 'I') or in decreasing order (if *id* = 'D'). It uses Quick Sort, reverting to Insertion Sort on arrays of size $\leq$ 20. Dimension of stack limits n to about 2^{32}.

Input Parameters

id — CHARACTER*1.
= 'I': sort d in increasing order;
= 'D': sort d in decreasing order.

n — INTEGER. The length of the array d.

d — REAL for slasrt
DOUBLE PRECISION for dlasrt.

	On entry, the array to be sorted.

Output Parameters

d	On exit, *d* has been sorted into increasing order (*d*(1) ≤ ... ≤ *d*(*n*)) or into decreasing order (*d*(1) ≥ ... ≥ *d*(*n*)), depending on *id*.
info	INTEGER. = 0: successful exit < 0: if *info* = -i, the *i*-th argument had an illegal value.

?lassq

Updates a sum of squares represented in scaled form.

Syntax

```
call slassq( n, x, incx, scale, sumsq )
call dlassq( n, x, incx, scale, sumsq )
call classq( n, x, incx, scale, sumsq )
call zlassq( n, x, incx, scale, sumsq )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The real routines slassq/dlassq return the values *scl* and *smsq* such that

$scl^2 * smsq = x(1)^2 + ... + x(n)^2 + scale^2 * sumsq$,

where $x(i) = x(1 + (i - 1) incx)$.

The value of *sumsq* is assumed to be non-negative and *scl* returns the value

scl = max(*scale*, abs(*x*(*i*))).

Values *scale* and *sumsq* must be supplied in *scale* and *sumsq*, and *scl* and *smsq* are overwritten on *scale* and *sumsq*, respectively.

The complex routines classq/zlassq return the values *scl* and *ssq* such that

$scl^2 * ssq = x(1)^2 + ... + x(n)^2 + scale^2 * sumsq$,

where *x*(*i*) = abs(*x*(1 +(*i* - 1)*incx)).

The value of *sumsq* is assumed to be at least unity and the value of *ssq* will then satisfy 1.0 ≤ *ssq* ≤ *sumsq* + 2*n*

scale is assumed to be non-negative and *scl* returns the value

scl = max(*scale*, abs(real(*x*(*i*))), abs(aimag(*x*(*i*)))).

Values *scale* and *sumsq* must be supplied in *scale* and *sumsq*, and *scl* and *ssq* are overwritten on *scale* and *sumsq*, respectively.

All routines ?lassq make only one pass through the vector *x*.

Input Parameters

n	INTEGER. The number of elements to be used from the vector *x*.
x	REAL for slassq DOUBLE PRECISION for dlassq COMPLEX for classq DOUBLE COMPLEX for zlassq. The vector for which a scaled sum of squares is computed: *x*(*i*) = *x*(1+(*i*-1)**incx*), 1 ≤ *i* ≤ *n*.
incx	INTEGER. The increment between successive values of the vector *x*. *incx* > 0.
scale	REAL for slassq/classq DOUBLE PRECISION for dlassq/zlassq. On entry, the value *scale* in the equation above.
sumsq	REAL for slassq/classq DOUBLE PRECISION for dlassq/zlassq. On entry, the value *sumsq* in the equation above.

Output Parameters

scale	On exit, *scale* is overwritten with *scl*, the scaling factor for the sum of squares.
sumsq	For real flavors: On exit, *sumsq* is overwritten with the value *smsq* in the equation above. For complex flavors: On exit, *sumsq* is overwritten with the value *ssq* in the equation above.

?lasv2

Computes the singular value decomposition of a 2-by-2 triangular matrix.

Syntax

```
call slasv2( f, g, h, ssmin, ssmax, snr, csr, snl, csl )
call dlasv2( f, g, h, ssmin, ssmax, snr, csr, snl, csl )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lasv2 computes the singular value decomposition of a 2-by-2 triangular matrix

$$\begin{bmatrix} f & g \\ 0 & h \end{bmatrix}$$

On return, abs(*ssmax*) is the larger singular value, abs(*ssmin*) is the smaller singular value, and (*csl*,*snl*) and (*csr*,*snr*) are the left and right singular vectors for abs(*ssmax*), giving the decomposition

$$\begin{bmatrix} csl & snl \\ -snl & csl \end{bmatrix} \begin{bmatrix} f & g \\ 0 & h \end{bmatrix} \begin{bmatrix} csr & -snr \\ snr & csr \end{bmatrix} = \begin{bmatrix} ssmax & 0 \\ 0 & ssmin \end{bmatrix}$$

Input Parameters

f, *g*, *h*	REAL for slasv2 DOUBLE PRECISION for dlasv2. The (1,1), (1,2) and (2,2) elements of the 2-by-2 matrix, respectively.

Output Parameters

ssmin, *ssmax*	REAL for slasv2 DOUBLE PRECISION for dlasv2. abs(*ssmin*) and abs(*ssmax*) is the smaller and the larger singular value, respectively.
snl, *csl*	REAL for slasv2 DOUBLE PRECISION for dlasv2. The vector (*csl*, *snl*) is a unit left singular vector for the singular value abs(*ssmax*).
snr, *csr*	REAL for slasv2 DOUBLE PRECISION for dlasv2. The vector (*csr*, *snr*) is a unit right singular vector for the singular value abs(*ssmax*).

Application Notes

Any input parameter may be aliased with any output parameter.

Barring over/underflow and assuming a guard digit in subtraction, all output quantities are correct to within a few units in the last place (ulps).

In ieee arithmetic, the code works correctly if one matrix element is infinite. Overflow will not occur unless the largest singular value itself overflows or is within a few ulps of overflow. (On machines with partial overflow, like the Cray, overflow may occur if the largest singular value is within a factor of 2 of overflow.) Underflow is harmless if underflow is gradual. Otherwise, results may correspond to a matrix modified by perturbations of size near the underflow threshold.

?laswp

Performs a series of row interchanges on a general rectangular matrix.

Syntax

```
call slaswp( n, a, lda, k1, k2, ipiv, incx )
call dlaswp( n, a, lda, k1, k2, ipiv, incx )
call claswp( n, a, lda, k1, k2, ipiv, incx )
call zlaswp( n, a, lda, k1, k2, ipiv, incx )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs a series of row interchanges on the matrix *A*. One row interchange is initiated for each of rows *k1* through *k2* of *A*.

Input Parameters

n	INTEGER. The number of columns of the matrix *A*.
a	REAL for slaswp DOUBLE PRECISION for dlaswp COMPLEX for claswp DOUBLE COMPLEX for zlaswp. Array, DIMENSION (*lda*, *n*). On entry, the matrix of column dimension *n* to which the row interchanges will be applied.
lda	INTEGER. The leading dimension of the array *a*.
k1	INTEGER. The first element of *ipiv* for which a row interchange will be done.
k2	INTEGER. The last element of *ipiv* for which a row interchange will be done.
ipiv	INTEGER. Array, DIMENSION (*k2*|incx|*). The vector of pivot indices. Only the elements in positions *k1* through *k2* of *ipiv* are accessed. *ipiv(k) = l* implies rows *k* and *l* are to be interchanged.
incx	INTEGER. The increment between successive values of *ipiv*. If *ipiv* is negative, the pivots are applied in reverse order.

Output Parameters

a	On exit, the permuted matrix.

?lasy2

Solves the Sylvester matrix equation where the matrices are of order 1 or 2.

Syntax

```
call slasy2( ltranl, ltranr, isgn, n1, n2, tl, ldtl, tr, ldtr, b, ldb, scale, x, ldx, xnorm, info )
```

```
call dlasy2( ltranl, ltranr, isgn, n1, n2, tl, ldtl, tr, ldtr, b, ldb, scale, x, ldx, xnorm, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine solves for the *n1*-by-*n2* matrix *X*, $1 \le n1, n2 \le 2$, in

op(*TL*)**X* + *isgn***X**op(*TR*) = *scale***B*,

where

TL is *n1*-by-*n1*,

TR is *n2*-by-*n2*,

B is *n1*-by-*n2*,

and *isgn* = 1 or -1. op(*T*) = *T* or T^T, where T^T denotes the transpose of *T*.

Input Parameters

ltranl	LOGICAL. On entry, *ltranl* specifies the op(*TL*): = .FALSE., op(*TL*) = *TL*, = .TRUE., op(*TL*) = $(TL)^T$.
ltranr	LOGICAL. On entry, *ltranr* specifies the op(*TR*): = .FALSE., op(*TR*) = *TR*, = .TRUE., op(*TR*) = $(TR)^T$.
isgn	INTEGER. On entry, *isgn* specifies the sign of the equation as described before. *isgn* may only be 1 or -1.
n1	INTEGER. On entry, *n1* specifies the order of matrix *TL*. *n1* may only be 0, 1 or 2.
n2	INTEGER. On entry, *n2* specifies the order of matrix *TR*. *n2* may only be 0, 1 or 2.
tl	REAL for slasy2 DOUBLE PRECISION for dlasy2. Array, DIMENSION (*ldtl*,2). On entry, *tl* contains an *n1*-by-*n1* matrix *TL*.
ldtl	INTEGER.The leading dimension of the matrix *TL*. *ldtl* ≥ max(1,*n1*).
tr	REAL for slasy2 DOUBLE PRECISION for dlasy2. Array, DIMENSION (*ldtr*,2). On entry, *tr* contains an *n2*-by-*n2* matrix *TR*.
ldtr	INTEGER. The leading dimension of the matrix *TR*. *ldtr* ≥ max(1,*n2*).
b	REAL for slasy2 DOUBLE PRECISION for dlasy2. Array, DIMENSION (*ldb*,2). On entry, the *n1*-by-*n2* matrix *B* contains the right-hand side of the equation.
ldb	INTEGER. The leading dimension of the matrix *B*. *ldb* ≥ max(1,*n1*).
ldx	INTEGER. The leading dimension of the output matrix *X*. *ldx* ≥ max(1,*n1*).

Output Parameters

scale	REAL for slasy2 DOUBLE PRECISION for dlasy2. On exit, *scale* contains the scale factor. *scale* is chosen less than or equal to 1 to prevent the solution overflowing.
x	REAL for slasy2 DOUBLE PRECISION for dlasy2. Array, DIMENSION (*ldx*,2). On exit, *x* contains the *n1*-by-*n2* solution.
xnorm	REAL for slasy2

DOUBLE PRECISION for dlasy2.
On exit, *xnorm* is the infinity-norm of the solution.

info INTEGER. On exit, *info* is set to 0: successful exit. 1: *TL* and *TR* have too close eigenvalues, so *TL* or *TR* is perturbed to get a nonsingular equation.

NOTE For higher speed, this routine does not check the inputs for errors.

?lasyf

Computes a partial factorization of a real/complex symmetric matrix, using the diagonal pivoting method.

Syntax

```
call slasyf( uplo, n, nb, kb, a, lda, ipiv, w, ldw, info )
call dlasyf( uplo, n, nb, kb, a, lda, ipiv, w, ldw, info )
call clasyf( uplo, n, nb, kb, a, lda, ipiv, w, ldw, info )
call zlasyf( uplo, n, nb, kb, a, lda, ipiv, w, ldw, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?*lasyf* computes a partial factorization of a real/complex symmetric matrix *A* using the Bunch-Kaufman diagonal pivoting method. The partial factorization has the form:

$$A = \begin{bmatrix} I & U_{12} \\ 0 & U_{12} \end{bmatrix} \begin{bmatrix} A_{11} & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} I & 0 \\ U_{12}' & U_{12}' \end{bmatrix} \text{ if } uplo = \text{'U'}, \text{ or}$$

$$A = \begin{bmatrix} L_{11} & 0 \\ L_{21} & I \end{bmatrix} \begin{bmatrix} D & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} L_{12}' & L_{21}' \\ 0 & I \end{bmatrix} \text{ if } uplo = \text{'L'}$$

where the order of *D* is at most *nb*.

The actual order is returned in the argument *kb*, and is either *nb* or *nb*-1, or *n* if $n \leq nb$.

This is an auxiliary routine called by ?sytrf. It uses blocked code (calling Level 3 BLAS) to update the submatrix A_{11} (if *uplo* = 'U') or A_{22} (if *uplo* = 'L').

Input Parameters

uplo CHARACTER*1.
Specifies whether the upper or lower triangular part of the symmetric matrix *A* is stored:
= 'U': Upper triangular
= 'L': Lower triangular

n	INTEGER. The order of the matrix *A*. $n \geq 0$.
nb	INTEGER. The maximum number of columns of the matrix *A* that should be factored. *nb* should be at least 2 to allow for 2-by-2 pivot blocks.
a	REAL for slasyf DOUBLE PRECISION for dlasyf COMPLEX for clasyf DOUBLE COMPLEX for zlasyf. Array, DIMENSION (*lda*, *n*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(1,n)$.
w	REAL for slasyf DOUBLE PRECISION for dlasyf COMPLEX for clasyf DOUBLE COMPLEX for zlasyf. Workspace array, DIMENSION (*ldw*, *nb*).
ldw	INTEGER. The leading dimension of the array *w*. $ldw \geq \max(1,n)$.

Output Parameters

kb	INTEGER. The number of columns of *A* that were actually factored *kb* is either *nb*-1 or *nb*, or *n* if $n \leq nb$.
a	On exit, *a* contains details of the partial factorization.
ipiv	INTEGER. Array, DIMENSION (*n*). Details of the interchanges and the block structure of *D*. If *uplo* = 'U', only the last *kb* elements of *ipiv* are set; if *uplo* = 'L', only the first *kb* elements are set. If *ipiv*(*k*) > 0, then rows and columns *k* and *ipiv*(*k*) were interchanged and *D*(*k*, *k*) is a 1-by-1 diagonal block. If *uplo* = 'U' and *ipiv*(*k*) = *ipiv*(*k*–1) < 0, then rows and columns *k*-1 and -*ipiv*(*k*) were interchanged and *D*(*k*–1:*k*, *k*–1:*k*) is a 2-by-2 diagonal block. If *uplo* = 'L' and *ipiv*(*k*) = *ipiv*(*k*+1) < 0, then rows and columns *k* +1 and –*ipiv*(*k*) were interchanged and *D*(*k*:*k*+1, *k*:*k*+1) is a 2-by-2 diagonal block.
info	INTEGER. = 0: successful exit > 0: if *info* = *k*, *D*(*k*, *k*) is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular.

?lahef

Computes a partial factorization of a complex Hermitian indefinite matrix, using the diagonal pivoting method.

Syntax

```
call clahef( uplo, n, nb, kb, a, lda, ipiv, w, ldw, info )
call zlahef( uplo, n, nb, kb, a, lda, ipiv, w, ldw, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lahef` computes a partial factorization of a complex Hermitian matrix A, using the Bunch-Kaufman diagonal pivoting method. The partial factorization has the form:

$$A = \begin{bmatrix} I & U_{12} \\ 0 & U_{22} \end{bmatrix} \begin{bmatrix} A_{11} & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} I & 0 \\ U_{12}^H & U_{22}^H \end{bmatrix} \text{ if } uplo = \text{'U'}, \text{ or}$$

$$A = \begin{bmatrix} L_{11} & 0 \\ L_{21} & I \end{bmatrix} \begin{bmatrix} D & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} L_{12}' & L_{21}' \\ 0 & I \end{bmatrix} \text{ if } uplo = \text{'L'}$$

where the order of D is at most nb.

The actual order is returned in the argument kb, and is either nb or nb-1, or n if $n \leq nb$.

Note that U^H denotes the conjugate transpose of U.

This is an auxiliary routine called by `?hetrf`. It uses blocked code (calling Level 3 BLAS) to update the submatrix A_{11} (if $uplo$ = 'U') or A_{22} (if $uplo$ = 'L').

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored: = 'U': upper triangular = 'L': lower triangular
n	INTEGER. The order of the matrix A. $n \geq 0$.
nb	INTEGER. The maximum number of columns of the matrix A that should be factored. nb should be at least 2 to allow for 2-by-2 pivot blocks.
a	COMPLEX for `clahef` DOUBLE COMPLEX for `zlahef`. Array, DIMENSION (lda, n). On entry, the Hermitian matrix A. If $uplo$ = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If $uplo$ = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.
lda	INTEGER. The leading dimension of the array a. $lda \geq \max(1,n)$.
w	COMPLEX for `clahef` DOUBLE COMPLEX for `zlahef`. Workspace array, DIMENSION (ldw, nb).
ldw	INTEGER. The leading dimension of the array w. $ldw \geq \max(1,n)$.

Output Parameters

kb	INTEGER. The number of columns of *A* that were actually factored *kb* is either *nb*-1 or *nb*, or *n* if $n \leq nb$.
a	On exit, *A* contains details of the partial factorization.
ipiv	INTEGER. Array, DIMENSION (*n*). Details of the interchanges and the block structure of *D*. If *uplo* = 'U', only the last *kb* elements of *ipiv* are set; if *uplo* = 'L', only the first *kb* elements are set. If *ipiv*(*k*) > 0, then rows and columns *k* and *ipiv*(*k*) are interchanged and *D*(*k*, *k*) is a 1-by-1 diagonal block. If *uplo* = 'U' and *ipiv*(*k*) = *ipiv*(*k*–1) < 0, then rows and columns *k*-1 and -*ipiv*(*k*) are interchanged and *D*(*k*–1:*k*, *k*–1:*k*) is a 2-by-2 diagonal block. If *uplo* = 'L' and *ipiv*(*k*) = *ipiv*(*k*+1) < 0, then rows and columns *k* +1 and -*ipiv*(*k*) are interchanged and *D*(*k*:*k*+1, *k*:*k*+1) is a 2-by-2 diagonal block.
info	INTEGER. = 0: successful exit > 0: if *info* = *k*, *D*(*k*, *k*) is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular.

?latbs

Solves a triangular banded system of equations.

Syntax

```
call slatbs( uplo, trans, diag, normin, n, kd, ab, ldab, x, scale, cnorm, info )
call dlatbs( uplo, trans, diag, normin, n, kd, ab, ldab, x, scale, cnorm, info )
call clatbs( uplo, trans, diag, normin, n, kd, ab, ldab, x, scale, cnorm, info )
call zlatbs( uplo, trans, diag, normin, n, kd, ab, ldab, x, scale, cnorm, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine solves one of the triangular systems

*A*x = s*b*, or A^T*x = s*b*, or A^H*x = s*b* (for complex flavors)

with scaling to prevent overflow, where *A* is an upper or lower triangular band matrix. Here A^T denotes the transpose of *A*, A^H denotes the conjugate transpose of *A*, *x* and *b* are *n*-element vectors, and *s* is a scaling factor, usually less than or equal to 1, chosen so that the components of *x* will be less than the overflow threshold. If the unscaled problem will not cause overflow, the Level 2 BLAS routine ?tbsv is called. If the matrix *A* is singular (*A*(*j*, *j*)=0 for some *j*), then *s* is set to 0 and a non-trivial solution to *A*x = 0 is returned.

Input Parameters

uplo	CHARACTER*1.

Specifies whether the matrix *A* is upper or lower triangular.
= 'U': upper triangular
= 'L': lower triangular

trans CHARACTER*1.
Specifies the operation applied to *A*.
= 'N': solve $A*x = s*b$ (no transpose)
= 'T': solve $A^T*x = s*b$ (transpose)
= 'C': solve $A^H*x = s*b$ (conjugate transpose)

diag CHARACTER*1.
Specifies whether the matrix *A* is unit triangular
= 'N': non-unit triangular
= 'U': unit triangular

normin CHARACTER*1.
Specifies whether *cnorm* is set.
= 'Y': *cnorm* contains the column norms on entry;
= 'N': *cnorm* is not set on entry. On exit, the norms is computed and stored in *cnorm*.

n INTEGER. The order of the matrix *A*. $n \geq 0$.

kd INTEGER. The number of subdiagonals or superdiagonals in the triangular matrix *A*. $kb \geq 0$.

ab REAL for slatbs
DOUBLE PRECISION for dlatbs
COMPLEX for clatbs
DOUBLE COMPLEX for zlatbs.
Array, DIMENSION (*ldab*, *n*).
The upper or lower triangular band matrix *A*, stored in the first *kb*+1 rows of the array. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows:
if *uplo* = 'U', $ab(kd+1+i-j,j) = A(i,j)$ for $\max(1, j-kd) \leq i \leq j$;
if *uplo* = 'L', $ab(1+i-j,j) = A(i,j)$ for $j \leq i \leq \min(n, j+kd)$.

ldab INTEGER. The leading dimension of the array *ab*. $ldab \geq kb+1$.

x REAL for slatbs
DOUBLE PRECISION for dlatbs
COMPLEX for clatbs
DOUBLE COMPLEX for zlatbs.
Array, DIMENSION (*n*).
On entry, the right hand side *b* of the triangular system.

cnorm REAL for slatbs/clatbs
DOUBLE PRECISION for dlatbs/zlatbs.
Array, DIMENSION (*n*).
If *NORMIN* = 'Y', *cnorm* is an input argument and *cnorm*(*j*) contains the norm of the off-diagonal part of the *j*-th column of *A*.
If *trans* = 'N', *cnorm*(*j*) must be greater than or equal to the infinity-norm, and if *trans* = 'T' or 'C', *cnorm*(*j*) must be greater than or equal to the 1-norm.

Output Parameters

scale REAL for slatbs/clatbs
DOUBLE PRECISION for dlatbs/zlatbs.

	The scaling factor s for the triangular system as described above. If *scale* = 0, the matrix *A* is singular or badly scaled, and the vector *x* is an exact or approximate solution to *Ax* = 0.
cnorm	If *normin* = 'N', *cnorm* is an output argument and *cnorm*(*j*) returns the 1-norm of the off-diagonal part of the *j*-th column of *A*.
info	INTEGER. = 0: successful exit < 0: if *info* = -*k*, the *k*-th argument had an illegal value

?latdf

Uses the LU factorization of the n-by-n matrix computed by ?getc2 and computes a contribution to the reciprocal Dif-estimate.

Syntax

```
call slatdf( ijob, n, z, ldz, rhs, rdsum, rdscal, ipiv, jpiv )
call dlatdf( ijob, n, z, ldz, rhs, rdsum, rdscal, ipiv, jpiv )
call clatdf( ijob, n, z, ldz, rhs, rdsum, rdscal, ipiv, jpiv )
call zlatdf( ijob, n, z, ldz, rhs, rdsum, rdscal, ipiv, jpiv )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?latdf uses the *LU* factorization of the *n*-by-*n* matrix *Z* computed by ?getc2 and computes a contribution to the reciprocal Dif-estimate by solving *Z*x* = *b* for *x*, and choosing the right-hand side *b* such that the norm of *x* is as large as possible. On entry *rhs* = *b* holds the contribution from earlier solved sub-systems, and on return *rhs* = *x*.

The factorization of *Z* returned by ?getc2 has the form *Z* = *P*L*U*Q*, where *P* and *Q* are permutation matrices. *L* is lower triangular with unit diagonal elements and *U* is upper triangular.

Input Parameters

ijob	INTEGER. *ijob* = 2: First compute an approximative null-vector *e* of *Z* using ?gecon, *e* is normalized, and solve for *Z*x* = ±*e-f* with the sign giving the greater value of 2-norm(*x*). This option is about 5 times as expensive as default. *ijob* ≠ 2 (default): Local look ahead strategy where all entries of the right-hand side *b* is chosen as either +1 or -1 .
n	INTEGER. The number of columns of the matrix *Z*.
z	REAL for slatdf/clatdf DOUBLE PRECISION for dlatdf/zlatdf. Array, DIMENSION (*ldz*, *n*) On entry, the *LU* part of the factorization of the *n*-by-*n* matrix *Z* computed by ?getc2: *Z* = *P*L*U*Q*.
ldz	INTEGER. The leading dimension of the array *Z*. *lda* ≥ max(1, *n*).
rhs	REAL for slatdf/clatdf DOUBLE PRECISION for dlatdf/zlatdf.

	Array, DIMENSION (*n*). On entry, *rhs* contains contributions from other subsystems.
rdsum	REAL for slatdf/clatdf DOUBLE PRECISION for dlatdf/zlatdf. On entry, the sum of squares of computed contributions to the *D*if-estimate under computation by ?tgsyL, where the scaling factor *rdscal* has been factored out. If *trans* = 'T', *rdsum* is not touched. Note that *rdsum* only makes sense when ?tgsy2 is called by ?tgsyL.
rdscal	REAL for slatdf/clatdf DOUBLE PRECISION for dlatdf/zlatdf. On entry, scaling factor used to prevent overflow in *rdsum*. If *trans* = *T*', *rdscal* is not touched. Note that *rdscal* only makes sense when ?tgsy2 is called by ?tgsyL.
ipiv	INTEGER. Array, DIMENSION (*n*). The pivot indices; for 1 ≤ i ≤ *n*, row *i* of the matrix has been interchanged with row *ipiv*(*i*).
jpiv	INTEGER. Array, DIMENSION (*n*). The pivot indices; for 1 ≤j≤ *n*, column *j* of the matrix has been interchanged with column *jpiv*(*j*).

Output Parameters

rhs	On exit, *rhs* contains the solution of the subsystem with entries according to the value of *ijob*.
rdsum	On exit, the corresponding sum of squares updated with the contributions from the current sub-system. If *trans* = 'T', *rdsum* is not touched.
rdscal	On exit, *rdscal* is updated with respect to the current contributions in *rdsum*. If *trans* = 'T', *rdscal* is not touched.

?latps

Solves a triangular system of equations with the matrix held in packed storage.

Syntax

```
call slatps( uplo, trans, diag, normin, n, ap, x, scale, cnorm, info )
call dlatps( uplo, trans, diag, normin, n, ap, x, scale, cnorm, info )
call clatps( uplo, trans, diag, normin, n, ap, x, scale, cnorm, info )
call zlatps( uplo, trans, diag, normin, n, ap, x, scale, cnorm, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?latps solves one of the triangular systems

*A*x = s*b*, or A^T**x = s*b*, or A^H**x = s*b* (for complex flavors)

with scaling to prevent overflow, where A is an upper or lower triangular matrix stored in packed form. Here A^T denotes the transpose of A, A^H denotes the conjugate transpose of A, x and b are n-element vectors, and s is a scaling factor, usually less than or equal to 1, chosen so that the components of x will be less than the overflow threshold. If the unscaled problem does not cause overflow, the Level 2 BLAS routine `?tpsv` is called. If the matrix A is singular ($A(j, j) = 0$ for some j), then s is set to 0 and a non-trivial solution to $A*x = 0$ is returned.

Input Parameters

uplo
CHARACTER*1.
Specifies whether the matrix A is upper or lower triangular.
= 'U': upper triangular
= 'L': uower triangular

trans
CHARACTER*1.
Specifies the operation applied to A.
= 'N': solve $A*x = s*b$ (no transpose)
= 'T': solve $A^T*x = s*b$ (transpose)
= 'C': solve $A^H*x = s*b$ (conjugate transpose)

diag
CHARACTER*1.
Specifies whether the matrix A is unit triangular.
= 'N': non-unit triangular
= 'U': unit triangular

normin
CHARACTER*1.
Specifies whether *cnorm* is set.
= 'Y': *cnorm* contains the column norms on entry;
= 'N': *cnorm* is not set on entry. On exit, the norms will be computed and stored in cnorm.

n
INTEGER. The order of the matrix A. $n \geq 0$.

ap
REAL for slatps
DOUBLE PRECISION for dlatps
COMPLEX for clatps
DOUBLE COMPLEX for zlatps.
Array, DIMENSION (n(n+1)/2).
The upper or lower triangular matrix A, packed columnwise in a linear array.
The j-th column of A is stored in the array *ap* as follows:
if *uplo* = 'U', $ap(i + (j-1)j/2) = A(i,j)$ for $1 \leq i \leq j$;
if *uplo* = 'L', $ap(i + (j-1)(2n-j)/2) = A(i, j)$ for $j \leq i \leq n$.

x
REAL for slatps DOUBLE PRECISION for dlatps
COMPLEX for clatps
DOUBLE COMPLEX for zlatps.
Array, DIMENSION (n)
On entry, the right hand side b of the triangular system.

cnorm
REAL for slatps/clatps
DOUBLE PRECISION for dlatps/zlatps.
Array, DIMENSION (n).
If *normin* = 'Y', *cnorm* is an input argument and *cnorm*(j) contains the norm of the off-diagonal part of the j-th column of A.
If *trans* = 'N', *cnorm*(j) must be greater than or equal to the infinity-norm, and if *trans* = 'T' or 'C', *cnorm*(j) must be greater than or equal to the 1-norm.

Output Parameters

x	On exit, *x* is overwritten by the solution vector *x*.
scale	REAL for slatps/clatps DOUBLE PRECISION for dlatps/zlatps. The scaling factor *s* for the triangular system as described above. If *scale* = 0, the matrix *A* is singular or badly scaled, and the vector *x* is an exact or approximate solution to *A*x* = 0.
cnorm	If *normin* = 'N', *cnorm* is an output argument and *cnorm*(*j*) returns the 1-norm of the off-diagonal part of the *j*-th column of *A*.
info	INTEGER. = 0: successful exit < 0: if *info* = -*k*, the *k*-th argument had an illegal value

?latrd

Reduces the first nb rows and columns of a symmetric/Hermitian matrix A to real tridiagonal form by an orthogonal/unitary similarity transformation.

Syntax

```
call slatrd( uplo, n, nb, a, lda, e, tau, w, ldw )
call dlatrd( uplo, n, nb, a, lda, e, tau, w, ldw )
call clatrd( uplo, n, nb, a, lda, e, tau, w, ldw )
call zlatrd( uplo, n, nb, a, lda, e, tau, w, ldw )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?latrd reduces *nb* rows and columns of a real symmetric or complex Hermitian matrix *A* to symmetric/Hermitian tridiagonal form by an orthogonal/unitary similarity transformation Q^T*A*Q for real flavors, Q^H*A*Q for complex flavors, and returns the matrices *V* and *W* which are needed to apply the transformation to the unreduced part of *A*.

If *uplo* = 'U', ?latrd reduces the last *nb* rows and columns of a matrix, of which the upper triangle is supplied;

if *uplo* = 'L', ?latrd reduces the first *nb* rows and columns of a matrix, of which the lower triangle is supplied.

This is an auxiliary routine called by ?sytrd/?hetrd.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix *A* is stored: = 'U': upper triangular = 'L': lower triangular
n	INTEGER. The order of the matrix *A*.
nb	INTEGER. The number of rows and columns to be reduced.

a	REAL for slatrd DOUBLE PRECISION for dlatrd COMPLEX for clatrd DOUBLE COMPLEX for zlatrd. Array, DIMENSION (*lda*, *n*). On entry, the symmetric/Hermitian matrix *A* If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ (1,*n*).
ldw	INTEGER. The leading dimension of the output array *w*. *ldw* ≥ max(1,*n*).

Output Parameters

a	On exit, if *uplo* = 'U', the last *nb* columns have been reduced to tridiagonal form, with the diagonal elements overwriting the diagonal elements of *a*; the elements above the diagonal with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors; if *uplo* = 'L', the first *nb* columns have been reduced to tridiagonal form, with the diagonal elements overwriting the diagonal elements of *a*; the elements below the diagonal with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors.
e	REAL for slatrd/clatrd DOUBLE PRECISION for dlatrd/zlatrd. If *uplo* = 'U', *e*(*n*-*nb*:*n*-1) contains the superdiagonal elements of the last *nb* columns of the reduced matrix; if *uplo* = 'L', *e*(1:*nb*) contains the subdiagonal elements of the first *nb* columns of the reduced matrix.
tau	REAL for slatrd DOUBLE PRECISION for dlatrd COMPLEX for clatrd DOUBLE COMPLEX for zlatrd. Array, DIMENSION (*lda*, *n*). The scalar factors of the elementary reflectors, stored in *tau*(*n*-*nb*:*n*-1) if *uplo* = 'U', and in *tau*(1:*nb*) if *uplo* = 'L'.
w	REAL for slatrd DOUBLE PRECISION for dlatrd COMPLEX for clatrd DOUBLE COMPLEX for zlatrd. Array, DIMENSION (*ldw*, *n*). The *n*-by-*nb* matrix *W* required to update the unreduced part of *A*.

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*n*)**H*(*n*-1)*...**H*(*n*-*nb*+1)

Each *H*(*i*) has the form

*H(i) = I - tau*v*v'*

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v(i:n) = 0* and *v(i-1) = 1*; *v(1: i-1)* is stored on exit in *a(1: i-1, i)*, and *tau* in *tau(i-1)*.

If *uplo = 'L'*, the matrix *Q* is represented as a product of elementary reflectors

*Q = H(1)*H(2)*...*H(nb)*

Each *H*(*i*) has the form *H(i) = I - tau*v*v'*

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v(1: i) = 0* and *v(i+1) = 1*; *v(i+1:n)* is stored on exit in *a(i+1:n, i)*, and *tau* in *tau*(*i*).

The elements of the vectors *v* together form the *n*-by-*nb* matrix *V* which is needed, with *W*, to apply the transformation to the unreduced part of the matrix, using a symmetric/Hermitian rank-2k update of the form:

A := A - VW' - WV'.

The contents of *a* on exit are illustrated by the following examples with *n = 5* and *nb = 2*:

if *uplo* = 'U':

$$\begin{bmatrix} a & a & a & v_4 & v_4 \\ & a & a & v_4 & v_5 \\ & & a & 1 & v_5 \\ & & & d & 1 \\ & & & & d \end{bmatrix}$$

if *uplo* = 'L'

$$\begin{bmatrix} d & & & & \\ 1 & d & & & \\ v_1 & 1 & a & & \\ v_1 & v_2 & a & a & \\ v_1 & v_2 & a & a & a \end{bmatrix}$$

where *d* denotes a diagonal element of the reduced matrix, *a* denotes an element of the original matrix that is unchanged, and v_i denotes an element of the vector defining *H*(*i*).

?latrs

Solves a triangular system of equations with the scale factor set to prevent overflow.

Syntax

```
call slatrs( uplo, trans, diag, normin, n, a, lda, x, scale, cnorm, info )
call dlatrs( uplo, trans, diag, normin, n, a, lda, x, scale, cnorm, info )
call clatrs( uplo, trans, diag, normin, n, a, lda, x, scale, cnorm, info )
call zlatrs( uplo, trans, diag, normin, n, a, lda, x, scale, cnorm, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine solves one of the triangular systems

*A*x = s*b*, or A^T**x = s*b*, or A^H**x = s*b* (for complex flavors)

with scaling to prevent overflow. Here A is an upper or lower triangular matrix, A^T denotes the transpose of A, A^H denotes the conjugate transpose of A, x and b are n-element vectors, and s is a scaling factor, usually less than or equal to 1, chosen so that the components of x will be less than the overflow threshold. If the unscaled problem will not cause overflow, the Level 2 BLAS routine ?trsv is called. If the matrix A is singular ($A(j,j) = 0$ for some j), then s is set to 0 and a non-trivial solution to $A*x = 0$ is returned.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix A is upper or lower triangular. = 'U': Upper triangular = 'L': Lower triangular
trans	CHARACTER*1. Specifies the operation applied to A. = 'N': solve $A*x = s*b$ (no transpose) = 'T': solve $A^T*x = s*b$ (transpose) = 'C': solve $A^H*x = s*b$ (conjugate transpose)
diag	CHARACTER*1. Specifies whether or not the matrix A is unit triangular. = 'N': non-unit triangular = 'N': non-unit triangular
normin	CHARACTER*1. Specifies whether *cnorm* has been set or not. = 'Y': *cnorm* contains the column norms on entry; = 'N': *cnorm* is not set on entry. O n exit, the norms will be computed and stored in *cnorm*.
n	INTEGER. The order of the matrix A. $n \geq 0$
a	REAL for slatrs DOUBLE PRECISION for dlatrs COMPLEX for clatrs DOUBLE COMPLEX for zlatrs. Array, DIMENSION (*lda*, *n*). Contains the triangular matrix A. If *uplo* = 'U', the leading n-by-n upper triangular part of the array *a* contains the upper triangular matrix, and the strictly lower triangular part of A is not referenced. If *uplo* = 'L', the leading n-by-n lower triangular part of the array *a* contains the lower triangular matrix, and the strictly upper triangular part of A is not referenced. If *diag* = 'U', the diagonal elements of A are also not referenced and are assumed to be 1.
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq$ max(1, *n*).
x	REAL for slatrs DOUBLE PRECISION for dlatrs COMPLEX for clatrs DOUBLE COMPLEX for zlatrs. Array, DIMENSION (*n*). On entry, the right hand side b of the triangular system.
cnorm	REAL for slatrs/clatrs DOUBLE PRECISION for dlatrs/zlatrs. Array, DIMENSION (*n*).

If *normin* = 'Y', *cnorm* is an input argument and *cnorm* (*j*) contains the norm of the off-diagonal part of the *j*-th column of *A*.
If *trans* = 'N', *cnorm* (*j*) must be greater than or equal to the infinity-norm, and if *trans* = 'T' or 'C', *cnorm*(*j*) must be greater than or equal to the 1-norm.

Output Parameters

x	On exit, *x* is overwritten by the solution vector *x*.
scale	REAL for slatrs/clatrs DOUBLE PRECISION for dlatrs/zlatrs. Array, DIMENSION (*lda*, *n*). The scaling factor *s* for the triangular system as described above. If *scale* = 0, the matrix *A* is singular or badly scaled, and the vector *x* is an exact or approximate solution to *A***x* = 0.
cnorm	If *normin* = 'N', *cnorm* is an output argument and *cnorm*(*j*) returns the 1-norm of the off-diagonal part of the *j*-th column of *A*.
info	INTEGER. = 0: successful exit < 0: if *info* = -*k*, the *k*-th argument had an illegal value

Application Notes

A rough bound on *x* is computed; if that is less than overflow, ?trsv is called, otherwise, specific code is used which checks for possible overflow or divide-by-zero at every operation.

A columnwise scheme is used for solving *Ax* = *b*. The basic algorithm if *A* is lower triangular is

```
x[1:n] := b[1:n]
for j = 1, ..., n
x(j) := x(j) / A(j,j)
x[j+1:n] := x[j+1:n] - x(j)*a[j+1:n,j]
end
```

Define bounds on the components of *x* after *j* iterations of the loop:

M(*j*) = bound on *x*[1:*j*]

G(*j*) = bound on *x*[*j*+1:*n*]

Initially, let *M*(0) = 0 and *G*(0) = max{*x*(*i*), *i*=1,...,*n*}.

Then for iteration *j*+1 we have

M(*j*+1) ≤ *G*(*j*) / | *a*(*j*+1,*j*+1)|

G(*j*+1) ≤ *G*(*j*) + *M*(*j*+1)*| *a*[*j*+2:*n*,*j*+1]|

≤ *G*(*j*)(1 + *cnorm*(*j*+1)/ | *a*(*j*+1,*j*+1)|,

where *cnorm*(*j*+1) is greater than or equal to the infinity-norm of column *j*+1 of *a*, not counting the diagonal. Hence

$$G(j) \leq G(0) \prod_{1 \leq i \leq j} (1 + cnorm(i)/|A(i,i)|)$$

and

$$|x(j)| \leq (G(0)/|A(j,j)|) \prod_{1 \leq i \leq j} (1 + cnorm(i)/|A(i,i)|)$$

Since $|x(j)| \leq M(j)$, we use the Level 2 BLAS routine ?trsv if the reciprocal of the largest $M(j)$, $j=1,..,n$, is larger than $\max(underflow, 1/overflow)$.

The bound on $x(j)$ is also used to determine when a step in the columnwise method can be performed without fear of overflow. If the computed bound is greater than a large constant, x is scaled to prevent overflow, but if the bound overflows, x is set to 0, $x(j)$ to 1, and scale to 0, and a non-trivial solution to $Ax = 0$ is found.

Similarly, a row-wise scheme is used to solve $A^T x = b$ or $A^H x = b$. The basic algorithm for A upper triangular is

```
for j = 1, ..., n
x(j) := ( b(j) - A[1:j-1,j]' x[1:j-1]) / A(j,j)
end
```

We simultaneously compute two bounds

$G(j)$ = bound on $(b(i) - A[1:i-1,i]'*x[1:i-1])$, $1 \leq i \leq j$

$M(j)$ = bound on $x(i)$, $1 \leq i \leq j$

The initial values are $G(0) = 0$, $M(0) = \max\{ b(i), i=1,..,n\}$, and we add the constraint $G(j) \geq G(j-1)$ and $M(j) \geq M(j-1)$ for $j \geq 1$.

Then the bound on $x(j)$ is

$$M(j) \leq M(j-1) *(1 + cnorm(j)) / |A(j,j)|$$

$$\leq M(0) \prod_{1 \leq i \leq j} (1 + cnorm(i)/|A(i,i)|)$$

and we can safely call ?trsv if $1/M(n)$ and $1/G(n)$ are both greater than $\max(underflow, 1/overflow)$.

?latrz

Factors an upper trapezoidal matrix by means of orthogonal/unitary transformations.

Syntax

```
call slatrz( m, n, l, a, lda, tau, work )
call dlatrz( m, n, l, a, lda, tau, work )
call clatrz( m, n, l, a, lda, tau, work )
call zlatrz( m, n, l, a, lda, tau, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?latrz factors the *m*-by-(*m*+*l*) real/complex upper trapezoidal matrix

[*A*1 *A*2] = [*A*(1:*m*,1:*m*) *A*(1: *m*, *n*-*l*+1:*n*)]

as (*R* 0)* *Z*, by means of orthogonal/unitary transformations. *Z* is an (*m*+*l*)-by-(*m*+*l*) orthogonal/unitary matrix and *R* and *A*1 are *m*-by -*m* upper triangular matrices.

Input Parameters

m	INTEGER. The number of rows of the matrix *A*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *A*. *n* ≥ 0.
l	INTEGER. The number of columns of the matrix *A* containing the meaningful part of the Householder vectors. *n*-*m* ≥ *l* ≥ 0.
a	REAL for slatrz DOUBLE PRECISION for dlatrz COMPLEX for clatrz DOUBLE COMPLEX for zlatrz. Array, DIMENSION (*lda*, *n*). On entry, the leading *m*-by-*n* upper trapezoidal part of the array *a* must contain the matrix to be factorized.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*m*).
work	REAL for slatrz DOUBLE PRECISION for dlatrz COMPLEX for clatrz DOUBLE COMPLEX for zlatrz. Workspace array, DIMENSION (*m*).

Output Parameters

a	On exit, the leading *m*-by-*m* upper triangular part of *a* contains the upper triangular matrix *R*, and elements *n*-*l*+1 to *n* of the first *m* rows of *a*, with the array *tau*, represent the orthogonal/unitary matrix *Z* as a product of *m* elementary reflectors.
tau	REAL for slatrz DOUBLE PRECISION for dlatrz COMPLEX for clatrz DOUBLE COMPLEX for zlatrz. Array, DIMENSION (*m*). The scalar factors of the elementary reflectors.

Application Notes

The factorization is obtained by Householder's method. The *k*-th transformation matrix, *z*(*k*), which is used to introduce zeros into the (*m* - *k* + 1)-th row of *A*, is given in the form

$$Z(k)=\begin{bmatrix} I & 0 \\ 0 & T(k) \end{bmatrix}$$

where for real flavors

$$T(k) = I - tau^* u(k)^* T(k)^T, \quad u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

and for complex flavors

$$T(k) = I - tau^* u(k)^* T(k)^H, \quad u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

tau is a scalar and *z*(*k*) is an *l*-element vector. *tau* and *z*(*k*) are chosen to annihilate the elements of the *k*-th row of *A*2.

The scalar *tau* is returned in the *k*-th element of *tau* and the vector *u*(*k*) in the *k*-th row of *A*2, such that the elements of *z*(*k*) are in *a*(*k*, *l*+1), ..., *a*(*k*, *n*).

The elements of *r* are returned in the upper triangular part of *A*1.

Z is given by

Z = *Z*(1)**Z*(2)*...**Z*(*m*).

?lauu2

Computes the product $U*U^T$ ($U*U^H$) or L^T*L (L^H*L), where U and L are upper or lower triangular matrices (unblocked algorithm).

Syntax

```
call slauu2( uplo, n, a, lda, info )
call dlauu2( uplo, n, a, lda, info )
call clauu2( uplo, n, a, lda, info )
call zlauu2( uplo, n, a, lda, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lauu2` computes the product $U*U^T$ or L^T*L for real flavors, and $U*U^H$ or L^H*L for complex flavors. Here the triangular factor U or L is stored in the upper or lower triangular part of the array *a*.

If *uplo* = 'U' or 'u', then the upper triangle of the result is stored, overwriting the factor *U* in *A*.

If *uplo* = 'L' or 'l', then the lower triangle of the result is stored, overwriting the factor *L* in *A*.

This is the unblocked form of the algorithm, calling BLAS Level 2 Routines.

Input Parameters

uplo	CHARACTER*1. Specifies whether the triangular factor stored in the array *a* is upper or lower triangular: = 'U': Upper triangular = 'L': Lower triangular
n	INTEGER. The order of the triangular factor *U* or *L*. $n \geq 0$.
a	REAL for slauu2 DOUBLE PRECISION for dlauu2 COMPLEX for clauu2 DOUBLE COMPLEX for zlauu2. Array, DIMENSION (*lda*, *n*). On entry, the triangular factor *U* or *L*.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(1,n)$.

Output Parameters

a	On exit, if *uplo* = 'U', then the upper triangle of *a* is overwritten with the upper triangle of the product $U*U^T$ ($U*U^H$); if *uplo* = 'L', then the lower triangle of *a* is overwritten with the lower triangle of the product L^T*L (L^H*L).
info	INTEGER. = 0: successful exit < 0: if *info* = $-k$, the k-th argument had an illegal value

?lauum

Computes the product $U*U^T$($U*U^H$) or L^T*L (L^H*L), where U and L are upper or lower triangular matrices (blocked algorithm).

Syntax

```
call slauum( uplo, n, a, lda, info )
call dlauum( uplo, n, a, lda, info )
call clauum( uplo, n, a, lda, info )
call zlauum( uplo, n, a, lda, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lauum computes the product $U*U^T$ or L^T*L for real flavors, and $U*U^H$ or L^H*L for complex flavors. Here the triangular factor U or L is stored in the upper or lower triangular part of the array *a*.

If *uplo* = 'U' or 'u', then the upper triangle of the result is stored, overwriting the factor U in A.

If *uplo* = 'L' or 'l', then the lower triangle of the result is stored, overwriting the factor L in A.

This is the blocked form of the algorithm, calling BLAS Level 3 Routines.

Input Parameters

uplo	CHARACTER*1. Specifies whether the triangular factor stored in the array *a* is upper or lower triangular: = 'U': Upper triangular = 'L': Lower triangular
n	INTEGER. The order of the triangular factor *U* or *L*. *n* ≥ 0.
a	REAL for slauum DOUBLE PRECISION for dlauum COMPLEX for clauum DOUBLE COMPLEX for zlauum . Array, DIMENSION (*lda*, *n*). On entry, the triangular factor *U* or *L*.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*n*).

Output Parameters

a	On exit, if *uplo* = 'U', then the upper triangle of *a* is overwritten with the upper triangle of the product $U*U^T$($U*U^H$); if *uplo* = 'L', then the lower triangle of *a* is overwritten with the lower triangle of the product L^T*L (L^H*L).
info	INTEGER. = 0: successful exit < 0: if *info* = -*k*, the *k*-th argument had an illegal value

?org2l/?ung2l

Generates all or part of the orthogonal/unitary matrix Q from a QL factorization determined by ?geqlf *(unblocked algorithm).*

Syntax

```
call sorg2l( m, n, k, a, lda, tau, work, info )
call dorg2l( m, n, k, a, lda, tau, work, info )
call cung2l( m, n, k, a, lda, tau, work, info )
call zung2l( m, n, k, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?org2l/?ung2l generates an *m*-by-*n* real/complex matrix *Q* with orthonormal columns, which is defined as the last *n* columns of a product of *k* elementary reflectors of order *m*:

Q = *H*(*k*)*...*H*(2)*H*(1) as returned by ?geqlf.

Input Parameters

m	INTEGER. The number of rows of the matrix *Q*. *m* ≥ 0.

n	INTEGER. The number of columns of the matrix *Q*. $m \geq n \geq 0$.
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. $n \geq k \geq 0$.
a	REAL for sorg2l DOUBLE PRECISION for dorg2l COMPLEX for cung2l DOUBLE COMPLEX for zung2l. Array, DIMENSION (*lda*,*n*). On entry, the ($n-k+i$)-th column must contain the vector which defines the elementary reflector $H(i)$, for $i = 1,2,\ldots, k$, as returned by ?geqlf in the last k columns of its array argument *A*.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(1,m)$.
tau	REAL for sorg2l DOUBLE PRECISION for dorg2l COMPLEX for cung2l DOUBLE COMPLEX for zung2l. Array, DIMENSION (*k*). *tau*(i) must contain the scalar factor of the elementary reflector $H(i)$, as returned by ?geqlf.
work	REAL for sorg2l DOUBLE PRECISION for dorg2l COMPLEX for cung2l DOUBLE COMPLEX for zung2l. Workspace array, DIMENSION (*n*).

Output Parameters

a	On exit, the m-by-n matrix *Q*.
info	INTEGER. = 0: successful exit < 0: if $info = -i$, the i-th argument has an illegal value

?org2r/?ung2r

Generates all or part of the orthogonal/unitary matrix Q from a QR factorization determined by ?geqrf *(unblocked algorithm).*

Syntax

```
call sorg2r( m, n, k, a, lda, tau, work, info )
call dorg2r( m, n, k, a, lda, tau, work, info )
call cung2r( m, n, k, a, lda, tau, work, info )
call zung2r( m, n, k, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?org2r/?ung2r generates an m-by-n real/complex matrix *Q* with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m

$Q = H(1)*H(2)*...*H(k)$

as returned by ?geqrf.

Input Parameters

m	INTEGER. The number of rows of the matrix Q. $m \geq 0$.
n	INTEGER. The number of columns of the matrix Q. $m \geq n \geq 0$.
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. $n \geq k \geq 0$.
a	REAL for sorg2r DOUBLE PRECISION for dorg2r COMPLEX for cung2r DOUBLE COMPLEX for zung2r. Array, DIMENSION (*lda*, *n*). On entry, the i-th column must contain the vector which defines the elementary reflector $H(i)$, for $i = 1,2,...,k$, as returned by ?geqrf in the first k columns of its array argument *a*.
lda	INTEGER. The first DIMENSION of the array *a*. $lda \geq \max(1,m)$.
tau	REAL for sorg2r DOUBLE PRECISION for dorg2r COMPLEX for cung2r DOUBLE COMPLEX for zung2r. Array, DIMENSION (*k*). *tau*(i) must contain the scalar factor of the elementary reflector $H(i)$, as returned by ?geqrf.
work	REAL for sorg2r DOUBLE PRECISION for dorg2r COMPLEX for cung2r DOUBLE COMPLEX for zung2r. Workspace array, DIMENSION (*n*).

Output Parameters

a	On exit, the m-by-n matrix Q.
info	INTEGER. = 0: successful exit < 0: if $info = -i$, the i-th argument has an illegal value

?orgl2/?ungl2

Generates all or part of the orthogonal/unitary matrix Q from an LQ factorization determined by ?gelqf *(unblocked algorithm).*

Syntax

```
call sorgl2( m, n, k, a, lda, tau, work, info )
call dorgl2( m, n, k, a, lda, tau, work, info )
call cungl2( m, n, k, a, lda, tau, work, info )
call zungl2( m, n, k, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?orgl2/?ungl2 generates a m-by-n real/complex matrix Q with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

$Q = H(k)*...*H(2)*H(1)$ for real flavors, or $Q = (H(k))^H*...*(H(2))^H*(H(1))^H$ for complex flavors as returned by ?gelqf.

Input Parameters

m	INTEGER. The number of rows of the matrix Q. $m \geq 0$.
n	INTEGER. The number of columns of the matrix Q. $n \geq m$.
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. $m \geq k \geq 0$.
a	REAL for sorgl2 DOUBLE PRECISION for dorgl2 COMPLEX for cungl2 DOUBLE COMPLEX for zungl2. Array, DIMENSION (*lda*, *n*). On entry, the i-th row must contain the vector which defines the elementary reflector $H(i)$, for $i = 1,2,...,k$, as returned by ?gelqf in the first k rows of its array argument a.
lda	INTEGER. The leading dimension of the array a. $lda \geq \max(1,m)$.
tau	REAL for sorgl2 DOUBLE PRECISION for dorgl2 COMPLEX for cungl2 DOUBLE COMPLEX for zungl2. Array, DIMENSION (k). $tau(i)$ must contain the scalar factor of the elementary reflector $H(i)$, as returned by ?gelqf.
work	REAL for sorgl2 DOUBLE PRECISION for dorgl2 COMPLEX for cungl2 DOUBLE COMPLEX for zungl2. Workspace array, DIMENSION (m).

Output Parameters

a	On exit, the m-by-n matrix Q.
info	INTEGER. = 0: successful exit < 0: if $info = -i$, the i-th argument has an illegal value.

?orgr2/?ungr2

Generates all or part of the orthogonal/unitary matrix Q from an RQ factorization determined by ?gerqf *(unblocked algorithm).*

Syntax

```
call sorgr2( m, n, k, a, lda, tau, work, info )
```

```
call dorgr2( m, n, k, a, lda, tau, work, info )
call cungr2( m, n, k, a, lda, tau, work, info )
call zungr2( m, n, k, a, lda, tau, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?orgr2/?ungr2 generates an m-by-n real matrix Q with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

$Q = H(1)*H(2)*...*H(k)$ for real flavors, or $Q = (H(1))^H*(H(2))^H*...*(H(k))^H$ for complex flavors as returned by ?gerqf.

Input Parameters

m	INTEGER. The number of rows of the matrix Q. $m \geq 0$.
n	INTEGER. The number of columns of the matrix Q. $n \geq m$
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. $m \geq k \geq 0$.
a	REAL for sorgr2 DOUBLE PRECISION for dorgr2 COMPLEX for cungr2 DOUBLE COMPLEX for zungr2. Array, DIMENSION (*lda*, *n*). On entry, the (m- k+i)-th row must contain the vector which defines the elementary reflector $H(i)$, for $i = 1,2,...,\ k$, as returned by ?gerqf in the last k rows of its array argument a.
lda	INTEGER. The leading dimension of the array a. $lda \geq \max(1,m)$.
tau	REAL for sorgr2 DOUBLE PRECISION for dorgr2 COMPLEX for cungr2 DOUBLE COMPLEX for zungr2. Array, DIMENSION (k).$tau(i)$ must contain the scalar factor of the elementary reflector $H(i)$, as returned by ?gerqf.
work	REAL for sorgr2 DOUBLE PRECISION for dorgr2 COMPLEX for cungr2 DOUBLE COMPLEX for zungr2. Workspace array, DIMENSION (m).

Output Parameters

a	On exit, the m-by-n matrix Q.
info	INTEGER. = 0: successful exit < 0: if $info = -i$, the i-th argument has an illegal value

?orm2l/?unm2l

Multiplies a general matrix by the orthogonal/unitary matrix from a QL factorization determined by `?geqlf` *(unblocked algorithm).*

Syntax

```
call sorm2l( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call dorm2l( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call cunm2l( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call zunm2l( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?orm2l`/`?unm2l` overwrites the general real/complex m-by-n matrix C with

$Q*C$ if *side* = 'L' and *trans* = 'N', or

Q^T*C / Q^H*C if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

$C*Q$ if *side* = 'R' and *trans* = 'N', or

$C*Q^T$ / $C*Q^H$ if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

Here Q is a real orthogonal or complex unitary matrix defined as the product of k elementary reflectors

$Q = H(k)*...*H(2)*H(1)$ as returned by `?geqlf`.

Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side	CHARACTER*1. = 'L': apply Q or Q^T / Q^H from the left = 'R': apply Q or Q^T / Q^H from the right
trans	CHARACTER*1. = 'N': apply Q (no transpose) = 'T': apply Q^T (transpose, for real flavors) = 'C': apply Q^H (conjugate transpose, for complex flavors)
m	INTEGER. The number of rows of the matrix C. $m \geq 0$.
n	INTEGER. The number of columns of the matrix C. $n \geq 0$.
k	INTEGER. The number of elementary reflectors whose product defines the matrix Q. If *side* = 'L', $m \geq k \geq 0$; if *side* = 'R', $n \geq k \geq 0$.
a	REAL for `sorm2l` DOUBLE PRECISION for `dorm2l` COMPLEX for `cunm2l` DOUBLE COMPLEX for `zunm2l`. Array, DIMENSION (*lda*,*k*).

	The i-th column must contain the vector which defines the elementary reflector $H(i)$, for $i = 1,2,\ldots, k$, as returned by ?geqlf in the last k columns of its array argument a. The array a is modified by the routine but restored on exit.
lda	INTEGER. The leading dimension of the array a. If $side = 'L'$, $lda \geq \max(1, m)$ if $side = 'R'$, $lda \geq \max(1, n)$.
tau	REAL for sorm2l DOUBLE PRECISION for dorm2l COMPLEX for cunm2l DOUBLE COMPLEX for zunm2l. Array, DIMENSION (k). $tau(i)$ must contain the scalar factor of the elementary reflector $H(i)$, as returned by ?geqlf.
c	REAL for sorm2l DOUBLE PRECISION for dorm2l COMPLEX for cunm2l DOUBLE COMPLEX for zunm2l. Array, DIMENSION (ldc, n). On entry, the m-by-n matrix C.
ldc	INTEGER. The leading dimension of the array C. $ldc \geq \max(1,m)$.
work	REAL for sorm2l DOUBLE PRECISION for dorm2l COMPLEX for cunm2l DOUBLE COMPLEX for zunm2l. Workspace array, DIMENSION: (n) if $side = 'L'$, (m) if $side = 'R'$.

Output Parameters

c	On exit, c is overwritten by $Q*C$ or Q^T*C / Q^H*C, or $C*Q$, or $C*Q^T$ / $C*Q^H$.
info	INTEGER. = 0: successful exit < 0: if $info = -i$, the i-th argument had an illegal value

?orm2r/?unm2r

Multiplies a general matrix by the orthogonal/unitary matrix from a QR factorization determined by ?geqrf *(unblocked algorithm).*

Syntax

```
call sorm2r( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call dorm2r( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call cunm2r( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call zunm2r( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?orm2r/?unm2r overwrites the general real/complex *m*-by-*n* matrix *C* with

*Q***C* if *side* = 'L' and *trans* = 'N', or

Q^T**C* / Q^H**C* if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

*C***Q* if *side* = 'R' and *trans* = 'N', or

*C**Q^T / *C**Q^H if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

Here *Q* is a real orthogonal or complex unitary matrix defined as the product of *k* elementary reflectors

Q = *H*(1)**H*(2)*...**H*(*k*) as returned by ?geqrf.

Q is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side — CHARACTER*1.
= 'L': apply *Q* or Q^T / Q^H from the left
= 'R': apply *Q* or Q^T / Q^H from the right

trans — CHARACTER*1.
= 'N': apply *Q* (no transpose)
= 'T': apply Q^T (transpose, for real flavors)
= 'C': apply Q^H (conjugate transpose, for complex flavors)

m — INTEGER. The number of rows of the matrix *C*. *m* ≥ 0.

n — INTEGER. The number of columns of the matrix *C*. *n* ≥ 0.

k — INTEGER. The number of elementary reflectors whose product defines the matrix *Q*.
If *side* = 'L', *m* ≥ *k* ≥ 0;
if *side* = 'R', *n* ≥ *k* ≥ 0.

a — REAL for sorm2r
DOUBLE PRECISION for dorm2r
COMPLEX for cunm2r
DOUBLE COMPLEX for zunm2r.
Array, DIMENSION (*lda*,*k*).
The *i*-th column must contain the vector which defines the elementary reflector *H*(*i*), for *i* = 1,2,..., *k*, as returned by ?geqrf in the first *k* columns of its array argument *a*. The array *a* is modified by the routine but restored on exit.

lda — INTEGER. The leading dimension of the array *a*.
If *side* = 'L', *lda* ≥ max(1, *m*);
if *side* = 'R', *lda* ≥ max(1, *n*).

tau — REAL for sorm2r
DOUBLE PRECISION for dorm2r
COMPLEX for cunm2r
DOUBLE COMPLEX for zunm2r.
Array, DIMENSION (*k*).
tau(*i*) must contain the scalar factor of the elementary reflector *H*(*i*), as returned by ?geqrf.

c — REAL for sorm2r
DOUBLE PRECISION for dorm2r
COMPLEX for cunm2r

	DOUBLE COMPLEX for zunm2r. Array, DIMENSION (*ldc*, *n*). On entry, the *m*-by-*n* matrix *C*.
ldc	INTEGER. The leading dimension of the array *c*. *ldc* ≥ max(1,*m*).
work	REAL for sorm2r DOUBLE PRECISION for dorm2r COMPLEX for cunm2r DOUBLE COMPLEX for zunm2r. Workspace array, DIMENSION (*n*) if *side* = 'L', (*m*) if *side* = 'R'.

Output Parameters

c	On exit, *c* is overwritten by $Q*C$ or Q^T*C / Q^H*C, or $C*Q$, or $C*Q^T$ / $C*Q^H$.
info	INTEGER. = 0: successful exit < 0: if *info* = -i, the *i*-th argument had an illegal value

?orml2/?unml2

Multiplies a general matrix by the orthogonal/unitary matrix from a LQ factorization determined by ?gelqf *(unblocked algorithm).*

Syntax

```
call sorml2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call dorml2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call cunml2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call zunml2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?orml2/?unml2 overwrites the general real/complex *m*-by-*n* matrix *C* with

$Q*C$ if *side* = 'L' and *trans* = 'N', or

Q^T*C / Q^H*C if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

$C*Q$ if *side* = 'R' and *trans* = 'N', or

$C*Q^T$ / $C*Q^H$ if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

Here Q is a real orthogonal or complex unitary matrix defined as the product of *k* elementary reflectors

$Q = H(k)*...*H(2)*H(1)$ for real flavors, or $Q = (H(k))^H*...*(H(2))^H*(H(1))^H$ for complex flavors as returned by ?gelqf.

Q is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	CHARACTER*1.

= 'L': apply Q or Q^T / Q^H from the left
= 'R': apply Q or Q^T / Q^H from the right

trans CHARACTER*1.
= 'N': apply Q (no transpose)
= 'T': apply Q^T (transpose, for real flavors)
= 'C': apply Q^H (conjugate transpose, for complex flavors)

m INTEGER. The number of rows of the matrix C. $m \geq 0$.

n INTEGER. The number of columns of the matrix C. $n \geq 0$.

k INTEGER. The number of elementary reflectors whose product defines the matrix Q.
If $side = 'L'$, $m \geq k \geq 0$;
if $side = 'R'$, $n \geq k \geq 0$.

a REAL for sorml2
DOUBLE PRECISION for dorml2
COMPLEX for cunml2
DOUBLE COMPLEX for zunml2.
Array, DIMENSION
(lda, m) if $side = 'L'$,
(lda, n) if $side = 'R'$
The i-th row must contain the vector which defines the elementary reflector $H(i)$, for $i = 1,2,\ldots, k$, as returned by ?gelqf in the first k rows of its array argument a. The array a is modified by the routine but restored on exit.

lda INTEGER. The leading dimension of the array a. $lda \geq \max(1,k)$.

tau REAL for sorml2
DOUBLE PRECISION for dorml2
COMPLEX for cunml2
DOUBLE COMPLEX for zunml2.
Array, DIMENSION (k).
$tau(i)$ must contain the scalar factor of the elementary reflector $H(i)$, as returned by ?gelqf.

c REAL for sorml2
DOUBLE PRECISION for dorml2
COMPLEX for cunml2
DOUBLE COMPLEX for zunml2.
Array, DIMENSION (ldc, n) On entry, the m-by-n matrix C.

ldc INTEGER. The leading dimension of the array c. $ldc \geq \max(1,m)$.

work REAL for sorml2
DOUBLE PRECISION for dorml2
COMPLEX for cunml2
DOUBLE COMPLEX for zunml2.
Workspace array, DIMENSION
(n) if $side = 'L'$,
(m) if $side = 'R'$

Output Parameters

c On exit, c is overwritten by $Q*C$ or Q^T*C / Q^H*C, or $C*Q$, or $C*Q^T$ / $C*Q^H$.

info INTEGER.
= 0: successful exit

< 0: if *info* = -*i*, the *i*-th argument had an illegal value

?ormr2/?unmr2

Multiplies a general matrix by the orthogonal/unitary matrix from a RQ factorization determined by ?gerqf *(unblocked algorithm).*

Syntax

```
call sormr2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call dormr2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call cunmr2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
call zunmr2( side, trans, m, n, k, a, lda, tau, c, ldc, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?ormr2/?unmr2 overwrites the general real/complex *m*-by-*n* matrix *C* with

*Q***C* if *side* = 'L' and *trans* = 'N', or

Q^T*C / Q^H*C if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

*C***Q* if *side* = 'R' and *trans* = 'N', or

$C*Q^T$ / $C*Q^H$ if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

Here *Q* is a real orthogonal or complex unitary matrix defined as the product of *k* elementary reflectors

Q = *H*(1)**H*(2)*...**H*(*k*) for real flavors, or $Q = (H(1))^H*(H(2))^H*...*(H(k))^H$ as returned by ?gerqf.

Q is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	CHARACTER*1. = 'L': apply *Q* or Q^T / Q^H from the left = 'R': apply *Q* or Q^T / Q^H from the right
trans	CHARACTER*1. = 'N': apply *Q* (no transpose) = 'T': apply Q^T (transpose, for real flavors) = 'C': apply Q^H (conjugate transpose, for complex flavors)
m	INTEGER. The number of rows of the matrix *C*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *C*. *n* ≥ 0.
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. If *side* = 'L', *m* ≥ *k* ≥ 0; if *side* = 'R', *n* ≥ *k* ≥ 0.
a	REAL for sormr2 DOUBLE PRECISION for dormr2 COMPLEX for cunmr2 DOUBLE COMPLEX for zunmr2. Array, DIMENSION

(*lda*, *m*) if *side* = 'L',
(*lda*, *n*) if *side* = 'R'
The *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), for *i* = 1,2,...,*k*, as returned by ?gerqf in the last *k* rows of its array argument *a*. The array *a* is modified by the routine but restored on exit.

lda — INTEGER.
The leading dimension of the array *a*. *lda* ≥ max(1,*k*).

tau — REAL for sormr2
DOUBLE PRECISION for dormr2
COMPLEX for cunmr2
DOUBLE COMPLEX for zunmr2.
Array, DIMENSION (*k*).
tau(*i*) must contain the scalar factor of the elementary reflector *H*(*i*), as returned by ?gerqf.

c — REAL for sormr2
DOUBLE PRECISION for dormr2
COMPLEX for cunmr2
DOUBLE COMPLEX for zunmr2.
Array, DIMENSION (*ldc*, *n*).
On entry, the *m*-by-*n* matrix *C*.

ldc — INTEGER. The leading dimension of the array *c*. *ldc* ≥ max(1,*m*).

work — REAL for sormr2
DOUBLE PRECISION for dormr2
COMPLEX for cunmr2
DOUBLE COMPLEX for zunmr2.
Workspace array, DIMENSION
(*n*) if *side* = 'L',
(*m*) if *side* = 'R'

Output Parameters

c — On exit, *c* is overwritten by $Q*C$ or Q^T*C / Q^H*C, or $C*Q$, or $C*Q^T$ / $C*Q^H$.

info — INTEGER.
= 0: successful exit
< 0: if *info* = -*i*, the *i*-th argument had an illegal value

?ormr3/?unmr3

Multiplies a general matrix by the orthogonal/unitary matrix from a RZ factorization determined by ?tzrzf *(unblocked algorithm).*

Syntax

```
call sormr3( side, trans, m, n, k, l, a, lda, tau, c, ldc, work, info )
call dormr3( side, trans, m, n, k, l, a, lda, tau, c, ldc, work, info )
call cunmr3( side, trans, m, n, k, l, a, lda, tau, c, ldc, work, info )
call zunmr3( side, trans, m, n, k, l, a, lda, tau, c, ldc, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?ormr3/?unmr3 overwrites the general real/complex *m*-by-*n* matrix *C* with

*Q***C* if *side* = 'L' and *trans* = 'N', or

Q^T**C* / Q^H**C* if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

*C***Q* if *side* = 'R' and *trans* = 'N', or

*C**Q^T / *C**Q^H if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

Here *Q* is a real orthogonal or complex unitary matrix defined as the product of *k* elementary reflectors

Q = *H*(1)**H*(2)*...**H*(*k*) as returned by ?tzrzf.

Q is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	CHARACTER*1. = 'L': apply *Q* or Q^T / Q^H from the left = 'R': apply *Q* or Q^T / Q^H from the right
trans	CHARACTER*1. = 'N': apply *Q* (no transpose) = 'T': apply Q^T (transpose, for real flavors) = 'C': apply Q^H (conjugate transpose, for complex flavors)
m	INTEGER. The number of rows of the matrix *C*. *m* ≥ 0.
n	INTEGER. The number of columns of the matrix *C*. *n* ≥ 0.
k	INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. If *side* = 'L', *m* ≥ *k* ≥ 0; if *side* = 'R', *n* ≥ *k* ≥ 0.
l	INTEGER. The number of columns of the matrix *A* containing the meaningful part of the Householder reflectors. If *side* = 'L', *m* ≥ *l* ≥ 0, if *side* = 'R', *n* ≥ *l* ≥ 0.
a	REAL for sormr3 DOUBLE PRECISION for dormr3 COMPLEX for cunmr3 DOUBLE COMPLEX for zunmr3. Array, DIMENSION (*lda*, *m*) if *side* = 'L', (*lda*, *n*) if *side* = 'R' The *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), for *i* = 1,2,...,*k*, as returned by ?tzrzf in the last *k* rows of its array argument *a*. The array *a* is modified by the routine but restored on exit.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*k*).
tau	REAL for sormr3 DOUBLE PRECISION for dormr3 COMPLEX for cunmr3 DOUBLE COMPLEX for zunmr3. Array, DIMENSION (*k*).

	tau(*i*) must contain the scalar factor of the elementary reflector H(*i*), as returned by ?tzrzf.
c	REAL for sormr3 DOUBLE PRECISION for dormr3 COMPLEX for cunmr3 DOUBLE COMPLEX for zunmr3. Array, DIMENSION (*ldc*, *n*). On entry, the *m*-by-*n* matrix *C*.
ldc	INTEGER. The leading dimension of the array *c*. *ldc* ≥ max(1,*m*).
work	REAL for sormr3 DOUBLE PRECISION for dormr3 COMPLEX for cunmr3 DOUBLE COMPLEX for zunmr3. Workspace array, DIMENSION (*n*) if *side* = 'L', (*m*) if *side* = 'R'.

Output Parameters

c	On exit, *c* is overwritten by $Q*C$ or Q^T*C / Q^H*C, or $C*Q$, or $C*Q^T$ / $C*Q^H$.
info	INTEGER. = 0: successful exit < 0: if *info* = -*i*, the *i*-th argument had an illegal value

?pbtf2

Computes the Cholesky factorization of a symmetric/ Hermitian positive-definite band matrix (unblocked algorithm).

Syntax

```
call spbtf2( uplo, n, kd, ab, ldab, info )
call dpbtf2( uplo, n, kd, ab, ldab, info )
call cpbtf2( uplo, n, kd, ab, ldab, info )
call zpbtf2( uplo, n, kd, ab, ldab, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the Cholesky factorization of a real symmetric or complex Hermitian positive definite band matrix *A*.

The factorization has the form

$A = U^T*U$ for real flavors, $A = U^H*U$ for complex flavors if *uplo* = 'U', or

$A = L*L^T$ for real flavors, $A = L*L^H$ for complex flavors if *uplo* = 'L',

where *U* is an upper triangular matrix, and *L* is lower triangular. This is the unblocked version of the algorithm, calling BLAS Level 2 Routines.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric/ Hermitian matrix *A* is stored: = 'U': upper triangular = 'L': lower triangular
n	INTEGER. The order of the matrix *A*. $n \geq 0$.
kd	INTEGER. The number of super-diagonals of the matrix *A* if *uplo* = 'U', or the number of sub-diagonals if *uplo* = 'L'. $kd \geq 0$.
ab	REAL for spbtf2 DOUBLE PRECISION for dpbtf2 COMPLEX for cpbtf2 DOUBLE COMPLEX for zpbtf2. Array, DIMENSION (*ldab*, *n*). On entry, the upper or lower triangle of the symmetric/ Hermitian band matrix *A*, stored in the first *kd*+1 rows of the array. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: if *uplo* = 'U', *ab*(*kd*+1+*i* –*j*,*j*) = *A*(*i*, *j* for max(1, *j*–*kd*) $\leq i \leq j$; if *uplo* = 'L', *ab*(1+*i* –*j*,*j*) = *A*(*i*, *j* for $j \leq i \leq$ min(*n*, *j*+*kd*).
ldab	INTEGER. The leading dimension of the array *ab*. *ldab* $\geq kd+1$.

Output Parameters

ab	On exit, If *info* = 0, the triangular factor *U* or *L* from the Cholesky factorization $A=U^T*U$ ($A=U^H*U$), or $A= L*L^T$ ($A = L*L^H$) of the band matrix *A*, in the same storage format as *A*.
info	INTEGER. = 0: successful exit < 0: if *info* = –*k*, the *k*–th argument had an illegal value > 0: if *info* = *k*, the leading minor of order *k* is not positive definite, and the factorization could not be completed.

?potf2

Computes the Cholesky factorization of a symmetric/ Hermitian positive-definite matrix (unblocked algorithm).

Syntax

```
call spotf2( uplo, n, a, lda, info )
call dpotf2( uplo, n, a, lda, info )
call cpotf2( uplo, n, a, lda, info )
call zpotf2( uplo, n, a, lda, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?potf2 computes the Cholesky factorization of a real symmetric or complex Hermitian positive definite matrix A. The factorization has the form

$A = U^T*U$ for real flavors, $A = U^H*U$ for complex flavors if $uplo$ = 'U', or

$A = L*L^T$ for real flavors, $A = L*L^H$ for complex flavors if $uplo$ = 'L',

where U is an upper triangular matrix, and L is lower triangular.

This is the unblocked version of the algorithm, calling BLAS Level 2 Routines

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric/ Hermitian matrix A is stored. = 'U': upper triangular = 'L': lower triangular
n	INTEGER. The order of the matrix A. $n \geq 0$.
a	REAL for spotf2 DOUBLE PRECISION or dpotf2 COMPLEX for cpotf2 DOUBLE COMPLEX for zpotf2. Array, DIMENSION (*lda*, *n*). On entry, the symmetric/Hermitian matrix A. If $uplo$ = 'U', the leading n-by-n upper triangular part of a contains the upper triangular part of the matrix A, and the strictly lower triangular part of a is not referenced. If $uplo$ = 'L', the leading n-by-n lower triangular part of a contains the lower triangular part of the matrix A, and the strictly upper triangular part of a is not referenced.
lda	INTEGER. The leading dimension of the array a. $lda \geq \max(1,n)$.

Output Parameters

a	On exit, If $info$ = 0, the factor U or L from the Cholesky factorization $A=U^T*U$ ($A=U^H*U$), or $A= L*L^T$ ($A = L*L^H$).
info	INTEGER. = 0: successful exit < 0: if $info = -k$, the k-th argument had an illegal value > 0: if $info = k$, the leading minor of order k is not positive definite, and the factorization could not be completed.

?ptts2

Solves a tridiagonal system of the form $A*X=B$ using the $L*D*L^H/L*D*L^H$ factorization computed by ?pttrf.

Syntax

```
call sptts2( n, nrhs, d, e, b, ldb )
call dptts2( n, nrhs, d, e, b, ldb )
call cptts2( iuplo, n, nrhs, d, e, b, ldb )
```

```
call zptts2( iuplo, n, nrhs, d, e, b, ldb )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?ptts2 solves a tridiagonal system of the form

$A*X = B$

Real flavors sptts2/dptts2 use the $L*D*L^T$ factorization of A computed by spttrf/dpttrf, and complex flavors cptts2/zptts2 use the U^H*D*U or $L*D*L^H$ factorization of A computed by cpttrf/zpttrf.

D is a diagonal matrix specified in the vector d, U (or L) is a unit bidiagonal matrix whose superdiagonal (subdiagonal) is specified in the vector e, and X and B are n-by-$nrhs$ matrices.

Input Parameters

iuplo	INTEGER. Used with complex flavors only. Specifies the form of the factorization, and whether the vector e is the superdiagonal of the upper bidiagonal factor U or the subdiagonal of the lower bidiagonal factor L. = 1: $A = U^H*D*U$, e is the superdiagonal of U; = 0: $A = L*D*L^H$, e is the subdiagonal of L
n	INTEGER. The order of the tridiagonal matrix A. $n \geq 0$.
nrhs	INTEGER. The number of right hand sides, that is, the number of columns of the matrix B. nrhs ≥ 0.
d	REAL for sptts2/cptts2 DOUBLE PRECISION for dptts2/zptts2. Array, DIMENSION (n). The n diagonal elements of the diagonal matrix D from the factorization of A.
e	REAL for sptts2 DOUBLE PRECISION for dptts2 COMPLEX for cptts2 DOUBLE COMPLEX for zptts2. Array, DIMENSION (n-1). Contains the (n-1) subdiagonal elements of the unit bidiagonal factor L from the $L*D*L^T$ (for real flavors) or $L*D*L^H$ (for complex flavors when $iuplo = 0$) factorization of A. For complex flavors when $iuplo = 1$, e contains the (n-1) superdiagonal elements of the unit bidiagonal factor U from the factorization $A = U^H*D*U$.
B	REAL for sptts2/cptts2 DOUBLE PRECISION for dptts2/zptts2. Array, DIMENSION (ldb, $nrhs$). On entry, the right hand side vectors B for the system of linear equations.
ldb	INTEGER. The leading dimension of the array B. $ldb \geq \max(1,n)$.

Output Parameters

b	On exit, the solution vectors, X.

?rscl

Multiplies a vector by the reciprocal of a real scalar.

Syntax

```
call srscl( n, sa, sx, incx )
call drscl( n, sa, sx, incx )
call csrscl( n, sa, sx, incx )
call zdrscl( n, sa, sx, incx )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?rscl` multiplies an n-element real/complex vector x by the real scalar $1/a$. This is done without overflow or underflow as long as the final result x/a does not overflow or underflow.

Input Parameters

n	`INTEGER`. The number of components of the vector x.
sa	`REAL` for `srscl`/`csrscl` `DOUBLE PRECISION` for `drscl`/`zdrscl`. The scalar a which is used to divide each component of the vector x. `sa` must be ≥ 0, or the subroutine will divide by zero.
sx	`REAL` for `srscl` `DOUBLE PRECISION` for `drscl` `COMPLEX` for `csrscl` `DOUBLE COMPLEX` for `zdrscl`. Array, `DIMENSION (1+(`n`-1)* |`$incx$`|)`. The n-element vector x.
incx	`INTEGER`. The increment between successive values of the vector sx. If $incx > 0$, $sx(1)=x(1)$, and $sx(1+(i-1)*incx)=x(i)$, $1<i\leq n$.

Output Parameters

sx	On exit, the result x/a.

?syswapr

Applies an elementary permutation on the rows and columns of a symmetric matrix.

Syntax

Fortran 77:

```
call ssyswapr( uplo, n, a, i1, i2 )
call dsyswapr( uplo, n, a, i1, i2 )
call csyswapr( uplo, n, a, i1, i2 )
call zsyswapr( uplo, n, a, i1, i2 )
```

Fortran 95:

```
call syswapr( a,i1,i2[,uplo] )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`

Description

The routine applies an elementary permutation on the rows and columns of a symmetric matrix.

Input Parameters

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix *A* has been factored: If *uplo* = `'U'`, the array *a* stores the upper triangular factor *U* of the factorization $A = U*D*U^T$. If *uplo* = `'L'`, the array *a* stores the lower triangular factor *L* of the factorization $A = L*D*L^T$.
n	`INTEGER`. The order of matrix *A*; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides; $nrhs \geq 0$.
a	`REAL` for `ssyswapr` `DOUBLE PRECISION` for `dsyswapr` `COMPLEX` for `csyswapr` `DOUBLE COMPLEX` for `zsyswapr` Array of dimension `(lda,n)`. The array *a* contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* as computed by `?sytrf`.
i1	`INTEGER`. Index of the first row to swap.
i2	`INTEGER`. Index of the second row to swap.

Output Parameters

a	If *info* = `0`, the symmetric inverse of the original matrix. If *info* = `'U'`, the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced. If *info* = `'L'`, the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `syswapr` interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
i1	Holds the index for swap.
i2	Holds the index for swap.
uplo	Indicates how the matrix *A* has been factored. Must be `'U'` or `'L'`.

See Also

?sytrf

?heswapr

Applies an elementary permutation on the rows and columns of a Hermitian matrix.

Syntax

Fortran 77:

```
call cheswapr( uplo, n, a, i1, i2 )
call zheswapr( uplo, n, a, i1, i2 )
```

Fortran 95:

```
call heswapr( a, i1, i2 [,uplo] )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`
- Fortran 95: `lapack.f90`

Description

The routine applies an elementary permutation on the rows and columns of a Hermitian matrix.

Input Parameters

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Indicates how the input matrix *A* has been factored: If *uplo* = `'U'`, the array *a* stores the upper triangular factor *U* of the factorization $A = U*D*U^H$. If *uplo* = `'L'`, the array *a* stores the lower triangular factor *L* of the factorization $A = L*D*L^H$.
n	`INTEGER`. The order of matrix *A*; $n \geq 0$.
nrhs	`INTEGER`. The number of right-hand sides; $nrhs \geq 0$.
a	`COMPLEX` for `cheswapr` `DOUBLE COMPLEX` for `zheswapr` Array of dimension `(lda,n)`. The array *a* contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* as computed by `?hetrf`.
i1	`INTEGER`. Index of the first row to swap.
i2	`INTEGER`. Index of the second row to swap.

Output Parameters

a	If *info* = `0`, the inverse of the original matrix. If *info* = `'U'`, the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced. If *info* = `'L'`, the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine `heswapr` interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
i1	Holds the index for swap.
i2	Holds the index for swap.
uplo	Must be 'U' or 'L'.

See Also

?hetrf
?syswapr1

?syswapr1

Applies an elementary permutation on the rows and columns of a symmetric matrix.

Syntax

Fortran 77:

```
call ssyswapr1( uplo, n, a, i1, i2 )
call dsyswapr1( uplo, n, a, i1, i2 )
call csyswapr1( uplo, n, a, i1, i2 )
call zsyswapr1( uplo, n, a, i1, i2 )
```

Fortran 95:

```
call syswapr1( a,i1,i2[,uplo] )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- Fortran 95: lapack.f90

Description

The routine applies an elementary permutation on the rows and columns of a symmetric matrix.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Indicates how the input matrix *A* has been factored: If *uplo* = 'U', the array *a* stores the upper triangular factor *U* of the factorization $A = U*D*U^T$. If *uplo* = 'L', the array *a* stores the lower triangular factor *L* of the factorization $A = L*D*L^T$.
n	INTEGER. The order of matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; $nrhs \geq 0$.
a	REAL for ssyswapr1 DOUBLE PRECISION for dsyswapr1 COMPLEX for csyswapr1 DOUBLE COMPLEX for zsyswapr1 Array of dimension (*lda*,*n*). The array *a* contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* as computed by ?sytrf.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq max(1,n)$.

i1	INTEGER. Index of the first row to swap.
i2	INTEGER. Index of the second row to swap.

Output Parameters

a	If *info* = 0, the symmetric inverse of the original matrix. If *info* = 'U', the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced. If *info* = 'L', the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.

Fortran 95 Interface Notes

Routines in Fortran 95 interface have fewer arguments in the calling sequence than their FORTRAN 77 counterparts. For general conventions applied to skip redundant or reconstructible arguments, see Fortran 95 Interface Conventions.

Specific details for the routine syswapr1 interface are as follows:

a	Holds the matrix *A* of size (*n*, *n*).
i1	Holds the index for swap.
i2	Holds the index for swap.
uplo	Indicates how the matrix *A* has been factored. Must be 'U' or 'L'.

See Also

?sytrf

?sygs2/?hegs2

Reduces a symmetric/Hermitian definite generalized eigenproblem to standard form, using the factorization results obtained from ?potrf (unblocked algorithm).

Syntax

```
call ssygs2( itype, uplo, n, a, lda, b, ldb, info )
call dsygs2( itype, uplo, n, a, lda, b, ldb, info )
call chegs2( itype, uplo, n, a, lda, b, ldb, info )
call zhegs2( itype, uplo, n, a, lda, b, ldb, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?sygs2/?hegs2 reduces a real symmetric-definite or a complex Hermitian-definite generalized eigenproblem to standard form.

If *itype* = 1, the problem is

$A*x = \lambda*B*x$

and *A* is overwritten by $\mathrm{inv}(U^H)*A*\mathrm{inv}(U)$ or $\mathrm{inv}(L)*A*\mathrm{inv}(L^H)$ for complex flavors and by $\mathrm{inv}(U^T)*A*\mathrm{inv}(U)$ or $\mathrm{inv}(L)*A*\mathrm{inv}(L^T)$ for real flavors.

If *itype* = 2 or 3, the problem is

$A*B*x = \lambda*x$, or $B*A*x = \lambda*x$,

and A is overwritten by $U*A*U^H$ or L^H*A*L for complex flavors and by $U*A*U^T$ or L^T*A*L for real flavors. Here U^T and L^T are the transpose while U^H and L^H are conjugate transpose of U and L.

B must be previously factorized by ?potrf as follows:

- U^H*U or $L*L^H$ for complex flavors
- U^T*U or $L*L^T$ for real flavors

Input Parameters

itype	INTEGER. For complex flavors: = 1: compute inv(U^H)*A*inv(U) or inv(L)*A*inv(L^H); = 2 or 3: compute $U*A*U^H$ or L^H*A*L. For real flavors: = 1: compute inv(U^T)*A*inv(U) or inv(L)*A*inv(L^T); = 2 or 3: compute $U*A*U^T$ or L^T*A*L.
uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix A is stored, and how B has been factorized. = 'U': upper triangular = 'L': lower triangular
n	INTEGER. The order of the matrices A and B. $n \geq 0$.
a	REAL for ssygs2 DOUBLE PRECISION for dsygs2 COMPLEX for chegs2 DOUBLE COMPLEX for zhegs2. Array, DIMENSION (lda, n). On entry, the symmetric/Hermitian matrix A. If $uplo$ = 'U', the leading n-by-n upper triangular part of a contains the upper triangular part of the matrix A, and the strictly lower triangular part of a is not referenced. If $uplo$ = 'L', the leading n-by-n lower triangular part of a contains the lower triangular part of the matrix A, and the strictly upper triangular part of a is not referenced.
lda	INTEGER. The leading dimension of the array a. $lda \geq \max(1,n)$.
b	REAL for ssygs2 DOUBLE PRECISION for dsygs2 COMPLEX for chegs2 DOUBLE COMPLEX for zhegs2. Array, DIMENSION (ldb, n). The triangular factor from the Cholesky factorization of B as returned by ?potrf.
ldb	INTEGER. The leading dimension of the array b. $ldb \geq \max(1,n)$.

Output Parameters

a	On exit, If $info$ = 0, the transformed matrix, stored in the same format as A.
info	INTEGER. = 0: successful exit. < 0: if $info$ = -i, the i-th argument had an illegal value.

?sytd2/?hetd2

Reduces a symmetric/Hermitian matrix to real symmetric tridiagonal form by an orthogonal/unitary similarity transformation(unblocked algorithm).

Syntax

```
call ssytd2( uplo, n, a, lda, d, e, tau, info )
call dsytd2( uplo, n, a, lda, d, e, tau, info )
call chetd2( uplo, n, a, lda, d, e, tau, info )
call zhetd2( uplo, n, a, lda, d, e, tau, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?sytd2/?hetd2` reduces a real symmetric/complex Hermitian matrix A to real symmetric tridiagonal form T by an orthogonal/unitary similarity transformation: $Q^T*A*Q = T$ ($Q^H*A*Q = T$).

Input Parameters

uplo
`CHARACTER*1`.
Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix A is stored:
= `'U'`: upper triangular
= `'L'`: lower triangular

n
`INTEGER`. The order of the matrix A. $n \geq 0$.

a
`REAL` for `ssytd2`
`DOUBLE PRECISION` for `dsytd2`
`COMPLEX` for `chetd2`
`DOUBLE COMPLEX` for `zhetd2`.
Array, `DIMENSION` (*lda*, *n*).
On entry, the symmetric/Hermitian matrix A.
If *uplo* = `'U'`, the leading n-by-n upper triangular part of *a* contains the upper triangular part of the matrix A, and the strictly lower triangular part of *a* is not referenced.
If *uplo* = `'L'`, the leading n-by-n lower triangular part of *a* contains the lower triangular part of the matrix A, and the strictly upper triangular part of *a* is not referenced.

lda
`INTEGER`. The leading dimension of the array *a*. $lda \geq \max(1,n)$.

Output Parameters

a
On exit, if *uplo* = `'U'`, the diagonal and first superdiagonal of *a* are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements above the first superdiagonal, with the array *tau*, represent the orthogonal/unitary matrix Q as a product of elementary reflectors;
if *uplo* = `'L'`, the diagonal and first subdiagonal of *a* are overwritten by the corresponding elements of the tridiagonal matrix T, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.

d	REAL for ssytd2/chetd2 DOUBLE PRECISION for dsytd2/zhetd2. Array, DIMENSION (*n*). The diagonal elements of the tridiagonal matrix *T*: *d*(*i*) = *a*(*i*,*i*).
e	REAL for ssytd2/chetd2 DOUBLE PRECISION for dsytd2/zhetd2. Array, DIMENSION (*n*-1). The off-diagonal elements of the tridiagonal matrix *T*: *e*(*i*) = *a*(*i*,*i*+1) if *uplo* = 'U', *e*(*i*) = *a*(*i*+1,*i*) if *uplo* = 'L'.
tau	REAL for ssytd2 DOUBLE PRECISION for dsytd2 COMPLEX for chetd2 DOUBLE COMPLEX for zhetd2. Array, DIMENSION (*n*). The first *n*–1 elements contain scalar factors of the elementary reflectors. *tau*(*n*) is used as workspace.
info	INTEGER. = 0: successful exit < 0: if *info* = -*i*, the *i*-th argument had an illegal value.

?sytf2

Computes the factorization of a real/complex symmetric indefinite matrix, using the diagonal pivoting method (unblocked algorithm).

Syntax

```
call ssytf2( uplo, n, a, lda, ipiv, info )
call dsytf2( uplo, n, a, lda, ipiv, info )
call csytf2( uplo, n, a, lda, ipiv, info )
call zsytf2( uplo, n, a, lda, ipiv, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?sytf2 computes the factorization of a real/complex symmetric matrix *A* using the Bunch-Kaufman diagonal pivoting method:

$A = U*D*U^T$, or $A = L*D*L^T$,

where *U* (or *L*) is a product of permutation and unit upper (lower) triangular matrices, and *D* is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling BLAS Level 2 Routines.

Input Parameters

uplo	CHARACTER*1.

	Specifies whether the upper or lower triangular part of the symmetric matrix *A* is stored = 'U': upper triangular = 'L': lower triangular
n	INTEGER. The order of the matrix *A*. $n \geq 0$.
a	REAL for ssytf2 DOUBLE PRECISION for dsytf2 COMPLEX for csytf2 DOUBLE COMPLEX for zsytf2. Array, DIMENSION (*lda*, *n*). On entry, the symmetric matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq$ max(1,*n*).

Output Parameters

a	On exit, the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L*.
ipiv	INTEGER. Array, DIMENSION (*n*). Details of the interchanges and the block structure of *D* If *ipiv*(*k*) > 0, then rows and columns *k* and *ipiv*(*k*) are interchanged and *D*(*k*,*k*) is a 1-by-1 diagonal block. If *uplo* = 'U' and *ipiv*(*k*) = *ipiv*(*k*–1) < 0, then rows and columns *k*-1 and -*ipiv*(*k*) are interchanged and *D*(*k*,*k*) is a 2-by-2 diagonal block. If *uplo* = 'L' and *ipiv*(*k*) = *ipiv*(*k*+1)< 0, then rows and columns *k*+1 and -*ipiv*(*k*) were interchanged and *D*(*k*:*k*+1,*k*:*k*+1) is a 2-by-2 diagonal block.
info	INTEGER. = 0: successful exit < 0: if *info* = -*k*, the *k*–th argument has an illegal value > 0: if *info* = *k*, *D*(*k*,*k*) is exactly zero. The factorization are completed, but the block diagonal matrix *D* is exactly singular, and division by zero will occur if it is used to solve a system of equations.

?hetf2

Computes the factorization of a complex Hermitian matrix, using the diagonal pivoting method (unblocked algorithm).

Syntax

```
call chetf2( uplo, n, a, lda, ipiv, info )
call zhetf2( uplo, n, a, lda, ipiv, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method:

$A = U*D*U^H$ or $A = L*D*L^H$

where U (or L) is a product of permutation and unit upper (lower) triangular matrices, U^H is the conjugate transpose of U, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

This is the unblocked version of the algorithm, calling BLAS Level 2 Routines.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored: = 'U': Upper triangular = 'L': Lower triangular
n	INTEGER. The order of the matrix A. $n \geq 0$.
A	COMPLEX for chetf2 DOUBLE COMPLEX for zhetf2. Array, DIMENSION (*lda*, *n*). On entry, the Hermitian matrix A. If *uplo* = 'U', the leading n-by-n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced. If *uplo* = 'L', the leading n-by-n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq$ max(1,*n*).

Output Parameters

a	On exit, the block diagonal matrix D and the multipliers used to obtain the factor U or L.
ipiv	INTEGER. Array, DIMENSION (*n*). Details of the interchanges and the block structure of D If *ipiv*(*k*) > 0, then rows and columns k and *ipiv*(k) were interchanged and $D(k,k)$ is a 1-by-1 diagonal block. If *uplo* = 'U' and *ipiv*(*k*) = *ipiv*(*k*–1) < 0, then rows and columns k-1 and -*ipiv*(k) were interchanged and $D(k-1:k,k-1:k)$ is a 2-by-2 diagonal block. If *uplo* = 'L' and *ipiv*(*k*) = *ipiv*(*k*+1) < 0, then rows and columns k+1 and -*ipiv*(k) were interchanged and $D(k:k+1, k:k+1)$ is a 2-by-2 diagonal block.
info	INTEGER. = 0: successful exit < 0: if *info* = –*k*, the k-th argument had an illegal value

> 0: if *info* = *k*, *D*(*k*,*k*) is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, and division by zero will occur if it is used to solve a system of equations.

?tgex2

Swaps adjacent diagonal blocks in an upper (quasi) triangular matrix pair by an orthogonal/unitary equivalence transformation.

Syntax

```
call stgex2( wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, j1, n1, n2, work, lwork, info )
call dtgex2( wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, j1, n1, n2, work, lwork, info )
call ctgex2( wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, j1, info )
call ztgex2( wantq, wantz, n, a, lda, b, ldb, q, ldq, z, ldz, j1, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The real routines stgex2/dtgex2 swap adjacent diagonal blocks (*A*11, *B*11) and (*A*22, *B*22) of size 1-by-1 or 2-by-2 in an upper (quasi) triangular matrix pair (*A*, *B*) by an orthogonal equivalence transformation. (*A*, *B*) must be in generalized real Schur canonical form (as returned by sgges/dgges), that is, *A* is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. *B* is upper triangular.

The complex routines ctgex2/ztgex2 swap adjacent diagonal 1-by-1 blocks (*A*11, *B*11) and (*A*22, *B*22) in an upper triangular matrix pair (*A*, *B*) by an unitary equivalence transformation.

(*A*, *B*) must be in generalized Schur canonical form, that is, *A* and *B* are both upper triangular.

All routines optionally update the matrices *Q* and *Z* of generalized Schur vectors:

For real flavors,

$Q(in)*A(in)*Z(in)^T = Q(out)*A(out)*Z(out)^T$

$Q(in)*B(in)*Z(in)^T = Q(out)*B(out)*Z(out)^T$.

For complex flavors,

$Q(in)*A(in)*Z(in)^H = Q(out)*A(out)*Z(out)^H$

$Q(in)*B(in)*Z(in)^H = Q(out)*B(out)*Z(out)^H$.

Input Parameters

wantq	LOGICAL. If *wantq* = .TRUE. : update the left transformation matrix *Q*; If *wantq* = .FALSE. : do not update *Q*.
wantz	LOGICAL. If *wantz* = .TRUE. : update the right transformation matrix *Z*; If *wantz* = .FALSE.: do not update *Z*.
n	INTEGER. The order of the matrices *A* and *B*. $n \geq 0$.
a, *b*	REAL for stgex2 DOUBLE PRECISION for dtgex2

	COMPLEX for ctgex2 DOUBLE COMPLEX for ztgex2. Arrays, DIMENSION (*lda*, *n*) and (*ldb*, *n*), respectively. On entry, the matrices *A* and *B* in the pair (*A*, *B*).
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*n*).
ldb	INTEGER. The leading dimension of the array *b*. *ldb* ≥ max(1,*n*).
q, *z*	REAL for stgex2 DOUBLE PRECISION for dtgex2 COMPLEX for ctgex2 DOUBLE COMPLEX for ztgex2. Arrays, DIMENSION (*ldq*, *n*) and (*ldz*, *n*), respectively. On entry, if *wantq* = .TRUE., *q* contains the orthogonal/unitary matrix *Q*, and if *wantz* = .TRUE., *z* contains the orthogonal/unitary matrix *Z*.
ldq	INTEGER. The leading dimension of the array *q*. *ldq* ≥ 1. If *wantq* = .TRUE., *ldq* ≥ *n*.
ldz	INTEGER. The leading dimension of the array *z*. *ldz* ≥ 1. If *wantz* = .TRUE., *ldz* ≥ *n*.
j1	INTEGER. The index to the first block (*A*11, *B*11). 1 ≤ *j1* ≤ *n*.
n1	INTEGER. Used with real flavors only. The order of the first block (*A*11, *B*11). *n1* = 0, 1 or 2.
n2	INTEGER. Used with real flavors only. The order of the second block (*A*22, *B*22). *n2* = 0, 1 or 2.
work	REAL for stgex2 DOUBLE PRECISION for dtgex2. Workspace array, DIMENSION (max(1,*lwork*)). Used with real flavors only.
lwork	INTEGER. The dimension of the array *work*. *lwork*≥max(*n**(*n2*+*n1*), 2*(*n2*+*n1*)^2)

Output Parameters

a	On exit, the updated matrix *A*.
B	On exit, the updated matrix *B*.
Q	On exit, the updated matrix *Q*. Not referenced if *wantq* = .FALSE..
z	On exit, the updated matrix *Z*. Not referenced if *wantz* = .FALSE..
info	INTEGER. =0: Successful exit For stgex2/dtgex2: If *info* = 1, the transformed matrix (*A*, *B*) would be too far from generalized Schur form; the blocks are not swapped and (*A*, *B*) and (*Q*, *Z*) are unchanged. The problem of swapping is too ill-conditioned. If *info* = -16: *lwork* is too small. Appropriate value for *lwork* is returned in *work*(1). For ctgex2/ztgex2: If *info* = 1, the transformed matrix pair (*A*, *B*) would be too far from generalized Schur form; the problem is ill-conditioned.

?tgsy2

Solves the generalized Sylvester equation (unblocked algorithm).

Syntax

```
call stgsy2( trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf,
scale, rdsum, rdscal, iwork, pq, info )
call dtgsy2( trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf,
scale, rdsum, rdscal, iwork, pq, info )
call ctgsy2( trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf,
scale, rdsum, rdscal, iwork, pq, info )
call ztgsy2( trans, ijob, m, n, a, lda, b, ldb, c, ldc, d, ldd, e, lde, f, ldf,
scale, rdsum, rdscal, iwork, pq, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?tgsy2` solves the generalized Sylvester equation:

```
A*R-L*B=scale*C                    (1)
D*R-L*E=scale*F
```

using Level 1 and 2 BLAS, where R and L are unknown m-by-n matrices, (A, D), (B, E) and (C, F) are given matrix pairs of size m-by -m, n-by-n and m-by-n, respectively. For `stgsy2`/`dtgsy2`, pairs (A, D) and (B, E) must be in generalized Schur canonical form, that is, A, B are upper quasi triangular and D, E are upper triangular. For `ctgsy2`/`ztgsy2`, matrices A, B, D and E are upper triangular (that is, (A, D) and (B, E) in generalized Schur form).

The solution (R, L) overwrites (C, F).

$0 \leq scale \leq 1$ is an output scaling factor chosen to avoid overflow.

In matrix notation, solving equation (1) corresponds to solve

$Z*x = scale*b$

where Z is defined for real flavors as

$$Z = \begin{bmatrix} \text{kron}(I_n, A) & -\text{kron}(B^T, I_m) \\ \text{kron}(I_n, D) & -\text{kron}(E^T, I_m) \end{bmatrix} \quad (2)$$

and for complex flavors as

$$Z = \begin{bmatrix} \text{kron}(I_n, A) & -\text{kron}(B^H, I_m) \\ \text{kron}(I_n, D) & -\text{kron}(E^H, I_m) \end{bmatrix} \quad (3)$$

Here I_k is the identity matrix of size k and X^T (X^H) is the transpose (conjugate transpose) of X. `kron`(X, Y) denotes the Kronecker product between the matrices X and Y.

For real flavors, if *trans* = 'T', solve the transposed system

$Z^T*y = scale*b$

for y, which is equivalent to solving for R and L in

$A^T*R+D^T*L=scale*C$ (4)

$R*B^T+L*E^T=scale*(-F)$

For complex flavors, if *trans* = 'C', solve the conjugate transposed system

$Z^H*y = scale*b$

for y, which is equivalent to solving for R and L in

$A^H*R+D^H*L=scale*C$ (5)

$R*B^H+L*E^H=scale*(-F)$

These cases are used to compute an estimate of Dif[(*A*, *D*), (*B*, *E*)] = *sigma_min*(*Z*) using reverse communication with ?lacon.

?tgsy2 also (for *ijob* ≥ 1) contributes to the computation in ?tgsyl of an upper bound on the separation between two matrix pairs. Then the input (*A*, *D*), (*B*, *E*) are sub-pencils of the matrix pair (two matrix pairs) in ?tgsyl. See ?tgsyl for details.

Input Parameters

trans	CHARACTER*1. If *trans* = 'N', solve the generalized Sylvester equation (1); If *trans* = 'T': solve the transposed system (4). If *trans* = 'C': solve the conjugate transposed system (5).
ijob	INTEGER. Specifies what kind of functionality is to be performed. If *ijob* = 0: solve (1) only. If *ijob* = 1: a contribution from this subsystem to a Frobenius norm-based estimate of the separation between two matrix pairs is computed (look ahead strategy is used); If *ijob* = 2: a contribution from this subsystem to a Frobenius norm-based estimate of the separation between two matrix pairs is computed (?gecon on sub-systems is used). Not referenced if *trans* = 'T'.
m	INTEGER. On entry, *m* specifies the order of *A* and *D*, and the row dimension of *C*, *F*, *R* and *L*.
n	INTEGER. On entry, *n* specifies the order of *B* and *E*, and the column dimension of *C*, *F*, *R* and *L*.
a, *b*	REAL for stgsy2 DOUBLE PRECISION for dtgsy2 COMPLEX for ctgsy2 DOUBLE COMPLEX for ztgsy2. Arrays, DIMENSION (*lda*, *m*) and (*ldb*, *n*), respectively. On entry, *a* contains an upper (quasi) triangular matrix *A*, and *b* contains an upper (quasi) triangular matrix *B*.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1, *m*).
ldb	INTEGER. The leading dimension of the array *b*. *ldb* ≥ max(1, *n*).
c, *f*	REAL for stgsy2 DOUBLE PRECISION for dtgsy2 COMPLEX for ctgsy2

DOUBLE COMPLEX for ztgsy2.
Arrays, DIMENSION (*ldc*, *n*) and (*ldf*, *n*), respectively. On entry, *c* contains the right-hand-side of the first matrix equation in (1), and *f* contains the right-hand-side of the second matrix equation in (1).

ldc — INTEGER. The leading dimension of the array *c*. *ldc* ≥ max(1, *m*).

d, *e* — REAL for stgsy2
DOUBLE PRECISION for dtgsy2
COMPLEX for ctgsy2
DOUBLE COMPLEX for ztgsy2.
Arrays, DIMENSION (*ldd*, *m*) and (*lde*, *n*), respectively. On entry, *d* contains an upper triangular matrix *D*, and *e* contains an upper triangular matrix *E*.

ldd — INTEGER. The leading dimension of the array *d*. *ldd* ≥ max(1, *m*).

lde — INTEGER. The leading dimension of the array *e*. *lde* ≥ max(1, *n*).

ldf — INTEGER. The leading dimension of the array *f*. *ldf* ≥ max(1, *m*).

rdsum — REAL for stgsy2/ctgsy2
DOUBLE PRECISION for dtgsy2/ztgsy2.
On entry, the sum of squares of computed contributions to the Dif-estimate under computation by ?tgsyL, where the scaling factor *rdscal* has been factored out.

rdscal — REAL for stgsy2/ctgsy2
DOUBLE PRECISION for dtgsy2/ztgsy2.
On entry, scaling factor used to prevent overflow in *rdsum*.

iwork — INTEGER. Used with real flavors only.
Workspace array, DIMENSION (*m*+*n*+2).

Output Parameters

c — On exit, if *ijob* = 0, *c* is overwritten by the solution *R*.

f — On exit, if *ijob* = 0, *f* is overwritten by the solution *L*.

scale — REAL for stgsy2/ctgsy2
DOUBLE PRECISION for dtgsy2/ztgsy2.
On exit, 0 ≤ *scale* ≤ 1. If 0 < *scale* < 1, the solutions *R* and *L* (*C* and *F* on entry) hold the solutions to a slightly perturbed system, but the input matrices *A*, *B*, *D* and *E* are not changed. If *scale* = 0, *R* and *L* hold the solutions to the homogeneous system with *C* = *F* = 0. Normally *scale* = 1.

rdsum — On exit, the corresponding sum of squares updated with the contributions from the current sub-system.
If *trans* = 'T', *rdsum* is not touched.
Note that *rdsum* only makes sense when ?tgsy2 is called by ?tgsyl.

rdscal — On exit, *rdscal* is updated with respect to the current contributions in *rdsum*.
If *trans* = 'T', *rdscal* is not touched.
Note that *rdscal* only makes sense when ?tgsy2 is called by ?tgsyl.

pq — INTEGER. Used with real flavors only.
On exit, the number of subsystems (of size 2-by-2, 4-by-4 and 8-by-8) solved by the routine stgsy2/dtgsy2.

info — INTEGER. On exit, if *info* is set to
= 0: Successful exit
< 0: If *info* = -*i*, the *i*-th argument has an illegal value.

> 0: The matrix pairs (*A*, *D*) and (*B*, *E*) have common or very close eigenvalues.

?trti2

Computes the inverse of a triangular matrix (unblocked algorithm).

Syntax

```
call strti2( uplo, diag, n, a, lda, info )
call dtrti2( uplo, diag, n, a, lda, info )
call ctrti2( uplo, diag, n, a, lda, info )
call ztrti2( uplo, diag, n, a, lda, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?trti2` computes the inverse of a real/complex upper or lower triangular matrix.

This is the *Level 2 BLAS* version of the algorithm.

Input Parameters

uplo	`CHARACTER*1`. Specifies whether the matrix *A* is upper or lower triangular. = `'U'`: upper triangular = `'L'`: lower triangular
diag	`CHARACTER*1`. Specifies whether or not the matrix *A* is unit triangular. = `'N'`: non-unit triangular = `'N'`: non-unit triangular
n	`INTEGER`. The order of the matrix *A*. $n \geq 0$.
a	`REAL` for `strti2` `DOUBLE PRECISION` for `dtrti2` `COMPLEX` for `ctrti2` `DOUBLE COMPLEX` for `ztrti2`. Array, `DIMENSION` (*lda*, *n*). On entry, the triangular matrix *A*. If *uplo* = `'U'`, the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular matrix, and the strictly lower triangular part of *a* is not referenced. If *uplo* = `'L'`, the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular matrix, and the strictly upper triangular part of *a* is not referenced. If *diag* = `'U'`, the diagonal elements of *a* are also not referenced and are assumed to be 1.
lda	`INTEGER`. The leading dimension of the array *a*. *lda* $\geq$ max(1,*n*).

Output Parameters

a	On exit, the (triangular) inverse of the original matrix, in the same storage format.
info	INTEGER. = 0: successful exit < 0: if *info* = -*k*, the *k*-th argument had an illegal value

clag2z

Converts a complex single precision matrix to a complex double precision matrix.

Syntax

```
call clag2z( m, n, sa, ldsa, a, lda, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine converts a complex single precision matrix *SA* to a complex double precision matrix *A*.

Note that while it is possible to overflow while converting from double to single, it is not possible to overflow when converting from single to double.

This is an auxiliary routine so there is no argument checking.

Input Parameters

m	INTEGER. The number of lines of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in the matrix *A* (*n* ≥ 0).
ldsa	INTEGER. The leading dimension of the array *sa*; *ldsa* ≥ max(1, *m*).
a	DOUBLE PRECISION array, DIMENSION (*lda*, *n*). On entry, contains the *m*-by-*n* coefficient matrix *A*.
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1, *m*).

Output Parameters

sa	REAL array, DIMENSION (*ldsa*, *n*). On exit, contains the *m*-by-*n* coefficient matrix *SA*.
info	INTEGER. If *info* = 0, the execution is successful.

dlag2s

Converts a double precision matrix to a single precision matrix.

Syntax

```
call dlag2s( m, n, a, lda, sa, ldsa, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine converts a double precision matrix *SA* to a single precision matrix *A*.

RMAX is the overflow for the single precision arithmetic. dlag2s checks that all the entries of *A* are between –RMAX and RMAX. If not, the convertion is aborted and a flag is raised.

This is an auxiliary routine so there is no argument checking.

Input Parameters

m	INTEGER. The number of lines of the matrix *A* ($m \geq 0$).
n	INTEGER. The number of columns in the matrix *A* ($n \geq 0$).
a	DOUBLE PRECISION array, DIMENSION (*lda*, *n*). On entry, contains the *m*-by-*n* coefficient matrix *A*.
lda	INTEGER. The leading dimension of the array *a*; $lda \geq \max(1, m)$.
ldsa	INTEGER. The leading dimension of the array *sa*; $ldsa \geq \max(1, m)$.

Output Parameters

sa	REAL array, DIMENSION (*ldsa*, *n*). On exit, if *info* = 0, contains the *m*-by-*n* coefficient matrix *SA*; if *info* > 0, the content of *sa* is unspecified.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = 1, an entry of the matrix *A* is greater than the single precision overflow threshold; in this case, the content of *sa* on exit is unspecified.

slag2d

Converts a single precision matrix to a double precision matrix.

Syntax

```
call slag2d( m, n, sa, ldsa, a, lda, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine converts a single precision matrix *SA* to a double precision matrix *A*.

Note that while it is possible to overflow while converting from double to single, it is not possible to overflow when converting from single to double.

This is an auxiliary routine so there is no argument checking.

Input Parameters

m	INTEGER. The number of lines of the matrix *A* ($m \geq 0$).
n	INTEGER. The number of columns in the matrix *A* ($n \geq 0$).
sa	REAL array, DIMENSION (*ldsa*, *n*). On entry, contains the *m*-by-*n* coefficient matrix *SA*.
ldsa	INTEGER. The leading dimension of the array *sa*; $ldsa \geq \max(1, m)$.

lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1, *m*).

Output Parameters

a	DOUBLE PRECISION array, DIMENSION (*lda*, *n*). On exit, contains the *m*-by-*n* coefficient matrix *A*.
info	INTEGER. If *info* = 0, the execution is successful.

zlag2c

Converts a complex double precision matrix to a complex single precision matrix.

Syntax

```
call zlag2c( m, n, a, lda, sa, ldsa, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine converts a double precision complex matrix *SA* to a single precision complex matrix *A*.

RMAX is the overflow for the single precision arithmetic. zlag2c checks that all the entries of *A* are between –RMAX and RMAX. If not, the convertion is aborted and a flag is raised.

This is an auxiliary routine so there is no argument checking.

Input Parameters

m	INTEGER. The number of lines of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in the matrix *A* (*n* ≥ 0).
a	DOUBLE COMPLEX array, DIMENSION (*lda*, *n*). On entry, contains the *m*-by-*n* coefficient matrix *A*.
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1, *m*).
ldsa	INTEGER. The leading dimension of the array *sa*; *ldsa* ≥ max(1, *m*).

Output Parameters

sa	COMPLEX array, DIMENSION (*ldsa*, *n*). On exit, if *info* = 0, contains the *m*-by-*n* coefficient matrix *SA*; if *info* > 0, the content of *sa* is unspecified.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = 1, an entry of the matrix *A* is greater than the single precision overflow threshold; in this case, the content of *sa* on exit is unspecified.

?larfp

Generates a real or complex elementary reflector.

Syntax

Fortran 77:

```
call slarfp(n, alpha, x, incx, tau)
call dlarfp(n, alpha, x, incx, tau)
call clarfp(n, alpha, x, incx, tau)
call zlarfp(n, alpha, x, incx, tau)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?larfp` routines generate a real or complex elementary reflector *H* of order *n*, such that

```
H * (alpha) = (beta),
    (  x  )   ( 0  )
```

and *H*'**H* = *I* for real flavors, `conjg(`*H*`)'`**H* = *I* for complex flavors.

Here

alpha and *beta* are scalars, *beta* is real and non-negative,

x is (*n*-1)-element vector.

H is represented in the form

```
H = I - tau*( 1 )* (1  v'),
            ( v )
```

where *tau* is scalar, and *v* is (*n*-1)-element vector .

For real flavors if the elements of *x* are all zero, then *tau* = 0 and *H* is taken to be the unit matrix. Otherwise $1 \le tau \le 2$.

For complex flavors if the elements of *x* are all zero and *alpha* is real, then *tau* = 0 and *H* is taken to be the unit matrix. Otherwise $1 \le \mathrm{real}(tau) \le 2$, and $\mathrm{abs}\ (tau-1 \le 1$.

Input Parameters

n	`INTEGER`. Specifies the order of the elementary reflector.
alpha	`REAL` for `slarfp` `DOUBLE PRECISION` for `dlarfp` `COMPLEX` for `clarfp` `DOUBLE COMPLEX` for `zlarfp` Specifies the scalar *alpha*.
x	`REAL` for `slarfp` `DOUBLE PRECISION` for `dlarfp` `COMPLEX` for `clarfp` `DOUBLE COMPLEX` for `zlarfp` Array, `DIMENSION` at least `(1 + (`*n* `- 1)*abs(`*incx*`))`. It contains the vector *x*.
incx	`INTEGER`. Specifies the increment for the elements of *x*. The value of *incx* must not be zero.

Output Parameters

alpha	Overwritten by the value *beta*.
y	Overwritten by the vector *v*.
tau	REAL for slarfp DOUBLE PRECISION for dlarfp COMPLEX for clarfp DOUBLE COMPLEX for zlarfp Contains the scalar *tau*.

ila?lc

Scans a matrix for its last non-zero column.

Syntax

Fortran 77:

```
value = ilaslc(m, n, a, lda)
value = iladlc(m, n, a, lda)
value = ilaclc(m, n, a, lda)
value = ilazlc(m, n, a, lda)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ila?lc routines scan a matrix *A* for its last non-zero column.

Input Parameters

m	INTEGER. Specifies number of rows in the matrix *A*.
n	INTEGER. Specifies number of columns in the matrix *A*.
a	REAL for ilaslc DOUBLE PRECISION for iladlc COMPLEX for ilaclc DOUBLE COMPLEX for ilazlc Array, DIMENSION (*lda*, *). The second dimension of *a* must be at least max(1, *n*). Before entry the leading *n*-by-*n* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).

Output Parameters

value	INTEGER Number of the last non-zero column.

ila?lr

Scans a matrix for its last non-zero row.

Syntax

Fortran 77:

```
value = ilaslr(m, n, a, lda)
value = iladlr(m, n, a, lda)
value = ilaclr(m, n, a, lda)
value = ilazlr(m, n, a, lda)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `ila?lr` routines scan a matrix *A* for its last non-zero row.

Input Parameters

m	`INTEGER`. Specifies number of rows in the matrix *A*.
n	`INTEGER`. Specifies number of columns in the matrix *A*.
a	`REAL` for `ilaslr` `DOUBLE PRECISION` for `iladlr` `COMPLEX` for `ilaclr` `DOUBLE COMPLEX` for `idazlr` Array, `DIMENSION (lda, *)`. The second dimension of *a* must be at least `max(1, n)`. Before entry the leading *n*-by-*n* part of the array *a* must contain the matrix *A*.
lda	`INTEGER`. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least `max(1, m)`.

Output Parameters

value	`INTEGER` Number of the last non-zero row.

?gsvj0

Pre-processor for the routine `?gesvj`.

Syntax

Fortran 77:

```
call sgsvj0(jobv, m, n, a, lda, d, sva, mv, v, ldv, eps, sfmin, tol, nsweep, work, lwork, info)
call dgsvj0(jobv, m, n, a, lda, d, sva, mv, v, ldv, eps, sfmin, tol, nsweep, work, lwork, info)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine is called from ?gesvj as a pre-processor and that is its main purpose. It applies Jacobi rotations in the same way as ?gesvj does, but it does not check convergence (stopping criterion).

The routine ?gsvj0 enables ?gesvj to use a simplified version of itself to work on a submatrix of the original matrix.

Input Parameters

jobv	CHARACTER*1. Must be 'V', 'A', or 'N'. Specifies whether the output from this routine is used to compute the matrix *V*. If *jobv* = 'V', the product of the Jacobi rotations is accumulated by post-multiplying the *n*-by-*n* array *v*. If *jobv* = 'A', the product of the Jacobi rotations is accumulated by post-multiplying the *mv*-by-*n* array *v*. If *jobv* = 'N', the Jacobi rotations are not accumulated.
m	INTEGER. The number of rows of the input matrix *A* ($m \geq 0$).
n	INTEGER. The number of columns of the input matrix *B* ($m \geq n \geq 0$).
a	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. Arrays, DIMENSION (*lda*, *). Contains the *m*-by-*n* matrix *A*, such that *A**diag(*D*) represents the input matrix. The second dimension of *a* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of *a*; at least max(1, *m*).
d	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. Arrays, DIMENSION (*n*). Contains the diagonal matrix *D* that accumulates the scaling factors from the fast scaled Jacobi rotations. On entry *A**diag(*D*) represents the input matrix.
sva	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. Arrays, DIMENSION (*n*). Contains the Euclidean norms of the columns of the matrix *A**diag(*D*).
mv	INTEGER. The leading dimension of *b*; at least max(1, *p*). If *jobv* = 'A', then *mv* rows of *v* are post-multiplied by a sequence of Jacobi rotations. If *jobv* = 'N', then *mv* is not referenced .
v	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. Array, DIMENSION (*ldv*, *). The second dimension of *a* must be at least max(1, *n*). If *jobv* = 'V', then *n* rows of *v* are post-multiplied by a sequence of Jacobi rotations. If *jobv* = 'A', then *mv* rows of *v* are post-multiplied by a sequence of Jacobi rotations. If *jobv* = 'N', then *v* is not referenced.
ldv	INTEGER. The leading dimension of the array *v*; $ldv \geq 1$

	ldv ≥*n* if *jobv* = 'V'; *ldv* ≥*mv* if *jobv* = 'A'.
eps	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. The relative machine precision (epsilon) returned by the routine ?lamch.
sfmin	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. Value of safe minimum returned by the routine ?lamch.
tol	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. The threshold for Jacobi rotations. For a pair *A*(:,p), *A*(:,q) of pivot columns, the Jacobi rotation is applied only if abs(cos(angle(*A*(:,p),*A*(:,q))))> *tol*.
nsweep	INTEGER. The number of sweeps of Jacobi rotations to be performed.
work	REAL for sgsvj0 DOUBLE PRECISION for dgsvj0. Workspace array, DIMENSION (*lwork*).
lwork	INTEGER. The size of the array *work*; at least max(1, *m*).

Output Parameters

a	On exit, *A**diag(*D*) represents the input matrix post-multiplied by a sequence of Jacobi rotations, where the rotation threshold and the total number of sweeps are given in *tol* and *nsweep*, respectively
d	On exit, *A**diag(*D*) represents the input matrix post-multiplied by a sequence of Jacobi rotations, where the rotation threshold and the total number of sweeps are given in *tol* and *nsweep*, respectively.
sva	On exit, contains the Euclidean norms of the columns of the output matrix *A**diag(*D*).
v	If *jobv* = 'V', then *n* rows of *v* are post-multiplied by a sequence of Jacobi rotations. If *jobv* = 'A', then *mv* rows of *v* are post-multiplied by a sequence of Jacobi rotations. If *jobv* = 'N', then *v* is not referenced.
info	INTEGER. If *info* = 0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?gsvj1

Pre-processor for the routine ?gesvj*, applies Jacobi rotations targeting only particular pivots.*

Syntax

Fortran 77:

```
call sgsvj1(jobv, m, n, n1, a, lda, d, sva, mv, v, ldv, eps, sfmin, tol, nsweep,
work, lwork, info)
```

```
call dgsvj1(jobv, m, n, n1, a, lda, d, sva, mv, v, ldv, eps, sfmin, tol, nsweep,
work, lwork, info)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

T

This routine is called from ?gesvj as a pre-processor and that is its main purpose. It applies Jacobi rotations in the same way as ?gesvj does, but it targets only particular pivots and it does not check convergence (stopping criterion).

The routine ?gsvj1 applies few sweeps of Jacobi rotations in the column space of the input *m*-by-*n* matrix *A*. The pivot pairs are taken from the (1,2) off-diagonal block in the corresponding *n*-by-*n* Gram matrix *A*'**A*. The block-entries (*tiles*) of the (1,2) off-diagonal block are marked by the [x]'s in the following scheme:

```
|  *   *   * [x] [x] [x] |
|  *   *   * [x] [x] [x] |
|  *   *   * [x] [x] [x] |
| [x] [x] [x] *   *   *   |
| [x] [x] [x] *   *   *   |
| [x] [x] [x] *   *   *   |
```

row-cycling in the *nblr*-by-*nblc* [x] blocks, row-cyclic pivoting inside each [x] block

In terms of the columns of the matrix *A*, the first *n1* columns are rotated 'against' the remaining *n*-*n1* columns, trying to increase the angle between the corresponding subspaces. The off-diagonal block is *n1*-by-(*n*-*n1*) and it is tiled using quadratic tiles. The number of sweeps is specified by *nsweep*, and the orthogonality threshold is set by *tol*.

Input Parameters

jobv
: CHARACTER*1. Must be 'V', 'A', or 'N'.
Specifies whether the output from this routine is used to compute the matrix *V*.
If *jobv* = 'V', the product of the Jacobi rotations is accumulated by post-multiplying the *n*-by-*n* array *v*.
If *jobv* = 'A', the product of the Jacobi rotations is accumulated by post-multiplying the *mv*-by-*n* array *v*.
If *jobv* = 'N', the Jacobi rotations are not accumulated.

m
: INTEGER. The number of rows of the input matrix *A* ($m \geq 0$).

n
: INTEGER. The number of columns of the input matrix *B* ($m \geq n \geq 0$).

n1
: INTEGER. Specifies the 2-by-2 block partition. The first *n1* columns are rotated 'against' the remaining *n*–*n1* columns of the matrix *A*.

a
: REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
Arrays, DIMENSION (*lda*, *). Contains the *m*-by-*n* matrix *A*, such that *A*diag(*D*) represents the input matrix. The second dimension of *a* must be at least max(1, *n*).

lda
: INTEGER. The leading dimension of *a*; at least max(1, *m*).

d
: REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
Arrays, DIMENSION (*n*). Contains the diagonal matrix *D* that accumulates the scaling factors from the fast scaled Jacobi rotations. On entry *A*diag(*D*) represents the input matrix.

sva
: REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.

Arrays, DIMENSION (*n*). Contains the Euclidean norms of the columns of the matrix *A**diag(*D*).

mv — INTEGER. The leading dimension of *b*; at least max(1, *p*).
If *jobv* = 'A', then *mv* rows of *v* are post-multiplied by a sequence of Jacobi rotations.
If *jobv* = 'N', then *mv* is not referenced .

v — REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
Array, DIMENSION (*ldv*, *). The second dimension of *a* must be at least max(1, *n*).
If *jobv* = 'V', then *n* rows of *v* are post-multiplied by a sequence of Jacobi rotations.
If *jobv* = 'A', then *mv* rows of *v* are post-multiplied by a sequence of Jacobi rotations.
If *jobv* = 'N', then *v* is not referenced.

ldv — INTEGER. The leading dimension of the array *v*; *ldv* ≥ 1
ldv ≥ *n* if *jobv* = 'V';
ldv ≥ *mv* if *jobv* = 'A'.

eps — REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
The relative machine precision (epsilon) returned by the routine ?lamch.

sfmin — REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
Value of safe minimum returned by the routine ?lamch.

tol — REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
The threshold for Jacobi rotations. For a pair *A*(:,p), *A*(:,q) of pivot columns, the Jacobi rotation is applied only if abs(cos(angle(*A*(:,p),*A*(:,q))))> *tol*.

nsweep — INTEGER.
The number of sweeps of Jacobi rotations to be performed.

work — REAL for sgsvj1
DOUBLE PRECISION for dgsvj1.
Workspace array, DIMENSION (*lwork*).

lwork — INTEGER. The size of the array *work*; at least max(1, *m*).

Output Parameters

a — On exit, *A**diag(*D*) represents the input matrix post-multiplied by a sequence of Jacobi rotations, where the rotation threshold and the total number of sweeps are given in *tol* and *nsweep*, respectively

d — On exit, *A**diag(*D*) represents the input matrix post-multiplied by a sequence of Jacobi rotations, where the rotation threshold and the total number of sweeps are given in *tol* and *nsweep*, respectively.

sva — On exit, contains the Euclidean norms of the columns of the output matrix *A**diag(*D*).

v — If *jobv* = 'V', then *n* rows of *v* are post-multiplied by a sequence of Jacobi rotations.
If *jobv* = 'A', then *mv* rows of *v* are post-multiplied by a sequence of Jacobi rotations.
If *jobv* = 'N', then *v* is not referenced.

info	INTEGER. If *info* = 0, the execution is successful. If *info* = *-i*, the *i*-th parameter had an illegal value.

?sfrk

Performs a symmetric rank-k operation for matrix in RFP format.

Syntax

Fortran 77:

```
call ssfrk(transr, uplo, trans, n, k, alpha, a, lda, beta, c)
call dsfrk(transr, uplo, trans, n, k, alpha, a, lda, beta, c)
```

C:

```
lapack_int LAPACKE_<?>sfrk( int matrix_order, char transr, char uplo, char trans,
lapack_int n, lapack_int k, <datatype> alpha, const <datatype>* a, lapack_int lda,
<datatype> beta, <datatype>* c );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The ?sfrk routines perform a matrix-matrix operation using symmetric matrices. The operation is defined as

$C := alpha*A*A^T + beta*C,$

or

$C := alpha*A^T*A + beta*C,$

where:

alpha and *beta* are scalars,

C is an *n*-by-*n* symmetric matrix in rectangular full packed (RFP) format,

A is an *n*-by-*k* matrix in the first case and a *k*-by-*n* matrix in the second case.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr	CHARACTER*1. if *transr* = 'N' or 'n', the normal form of RFP *C* is stored; if *transr*= 'T' or 't', the transpose form of RFP *C* is stored.
uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array *c* is used. If *uplo* = 'U' or 'u', then the upper triangular part of the array *c* is used. If *uplo* = 'L' or 'l', then the low triangular part of the array *c* is used.
trans	CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then $C := alpha*A*A^T + beta*C$;

	if *trans* = 'T' or 't', then *C* := *alpha***A*^{T}**A* + *beta***C*;
n	INTEGER. Specifies the order of the matrix *C*. The value of *n* must be at least zero.
k	INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the matrix *A*, and on entry with *trans* = 'T' or 't', *k* specifies the number of rows of the matrix *A*. The value of *k* must be at least zero.
alpha	REAL for ssfrk DOUBLE PRECISION for dsfrk Specifies the scalar *alpha*.
a	REAL for ssfrk DOUBLE PRECISION for dsfrk Array, DIMENSION (*lda*,*ka*), where *ka* is *k* when *trans* = 'N' or 'n', and is *n* otherwise. Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = 'N' or 'n', then *lda* must be at least max(1,*n*), otherwise *lda* must be at least max(1, *k*).
beta	REAL for ssfrk DOUBLE PRECISION for dsfrk Specifies the scalar *beta*.
c	REAL for ssfrk DOUBLE PRECISION for dsfrk Array, DIMENSION (*n**(*n*+1)/2). Before entry contains the symmetric matrix *C* in RFP format.

Output Parameters

c	If *trans* = 'N' or 'n', then *c* contains *C* := *alpha***A***A*' + *beta***C*; if *trans* = 'T' or 't', then *c* contains *C* := *alpha***A*'**A* + *beta***C*;

?hfrk

Performs a Hermitian rank-k operation for matrix in RFP format.

Syntax

Fortran 77:

```
call chfrk(transr, uplo, trans, n, k, alpha, a, lda, beta, c)
```

```
call zhfrk(transr, uplo, trans, n, k, alpha, a, lda, beta, c)
```

C:

```
lapack_int LAPACKE_chfrk( int matrix_order, char transr, char uplo, char trans,
lapack_int n, lapack_int k, float alpha, const lapack_complex_float* a, lapack_int
lda, float beta, lapack_complex_float* c );
```

```
lapack_int LAPACKE_zhfrk( int matrix_order, char transr, char uplo, char trans,
lapack_int n, lapack_int k, double alpha, const lapack_complex_double* a, lapack_int
lda, double beta, lapack_complex_double* c );
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`
- C: `mkl_lapacke.h`

Description

The `?hfrk` routines perform a matrix-matrix operation using Hermitian matrices. The operation is defined as

$C := alpha*A*A^H + beta*C$,

or

$C := alpha*A^H*A + beta*C$,

where:

alpha and *beta* are real scalars,

C is an *n*-by-*n* Hermitian matrix in RFP format,

A is an *n*-by-*k* matrix in the first case and a *k*-by-*n* matrix in the second case.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr
`CHARACTER*1`.
if *transr* = `'N'` or `'n'`, the normal form of RFP *C* is stored;
if *transr* = `'C'` or `'c'`, the conjugate-transpose form of RFP *C* is stored.

uplo
`CHARACTER*1`. Specifies whether the upper or lower triangular part of the array *c* is used.
If *uplo* = `'U'` or `'u'`, then the upper triangular part of the array *c* is used.
If *uplo* = `'L'` or `'l'`, then the low triangular part of the array *c* is used.

trans
`CHARACTER*1`. Specifies the operation:
if *trans* = `'N'` or `'n'`, then $C := alpha*A*A^H + beta*C$;
if *trans* = `'C'` or `'c'`, then $C := alpha*A^H*A + beta*C$.

n
`INTEGER`. Specifies the order of the matrix *C*. The value of *n* must be at least zero.

k
`INTEGER`. On entry with *trans* = `'N'` or `'n'`, *k* specifies the number of columns of the matrix *a*, and on entry with *trans* = `'T'` or `'t'` or `'C'` or `'c'`, *k* specifies the number of rows of the matrix *a*.
The value of *k* must be at least zero.

alpha
`COMPLEX` for `chfrk`
`DOUBLE COMPLEX` for `zhfrk`
Specifies the scalar *alpha*.

a
`COMPLEX` for `chfrk`
`DOUBLE COMPLEX` for `zhfrk`
Array, `DIMENSION (lda,ka)`, where *ka* is *k* when *trans* = `'N'` or `'n'`, and is *n* otherwise. Before entry with *trans* = `'N'` or `'n'`, the leading *n*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.

lda
`INTEGER`. Specifies the leading dimension of *a* as declared in the calling (sub)program. When *trans* = `'N'` or `'n'`, then *lda* must be at least `max(1,n)`, otherwise *lda* must be at least `max(1, k)`.

beta	COMPLEX for chfrk DOUBLE COMPLEX for zhfrk Specifies the scalar *beta*.
c	COMPLEX for chfrk DOUBLE COMPLEX for zhfrk Array, DIMENSION (*n**(*n*+1)/2). Before entry contains the Hermitian matrix *C* in in RFP format.

Output Parameters

c	If *trans* = 'N' or 'n', then *c* contains *C* := *alpha***A***A*^{H} + *beta***C*; if *trans* = 'C' or 'c', then *c* contains *C* := *alpha***A*^{H}**A* + *beta***C* ;

?tfsm

Solves a matrix equation (one operand is a triangular matrix in RFP format).

Syntax

Fortran 77:

```
call stfsm(transr, side, uplo, trans, diag, m, n, alpha, a, b, ldb)
call dtfsm(transr, side, uplo, trans, diag, m, n, alpha, a, b, ldb)
call ctfsm(transr, side, uplo, trans, diag, m, n, alpha, a, b, ldb)
call ztfsm(transr, side, uplo, trans, diag, m, n, alpha, a, b, ldb)
```

C:

```
lapack_int LAPACKE_<?>tfsm( int matrix_order, char transr, char side, char uplo, char
trans, char diag, lapack_int m, lapack_int n, <datatype> alpha, const <datatype>* a,
<datatype>* b, lapack_int ldb );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The ?tfsm routines solve one of the following matrix equations:

op(*A*)**X* = *alpha***B*,

or

*X**op(*A*) = *alpha***B*,

where:

alpha is a scalar,

X and *B* are *m*-by-*n* matrices,

A is a unit, or non-unit, upper or lower triangular matrix in rectangular full packed (RFP) format.

op(*A*) can be one of the following:

- op(*A*) = *A* or op(*A*) = *A*^{T} for real flavors
- op(*A*) = *A* or op(*A*) = *A*^{H} for complex flavors

The matrix *B* is overwritten by the solution matrix *X*.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr — CHARACTER*1.
if *transr* = 'N' or 'n', the normal form of RFP *A* is stored;
if *transr* = 'T' or 't', the transpose form of RFP *A* is stored;
if *transr* = 'C' or 'c', the conjugate-transpose form of RFP *A* is stored.

side — CHARACTER*1. Specifies whether op(*A*) appears on the left or right of *X* in the equation:
if *side* = 'L' or 'l', then op(*A*)***X* = *alpha***B*;
if *side* = 'R' or 'r', then *X**op(*A*) = *alpha***B*.

uplo — CHARACTER*1. Specifies whether the RFP matrix *A* is upper or lower triangular:
if *uplo* = 'U' or 'u', then the matrix is upper triangular;
if *uplo* = 'L' or 'l', then the matrix is low triangular.

trans — CHARACTER*1. Specifies the form of op(*A*) used in the matrix multiplication:
if *trans* = 'N' or 'n', then op(*A*) = *A*;
if *trans* = 'T' or 't', then op(*A*) = *A*';
if *trans* = 'C' or 'c', then op(*A*) = conjg(*A*').

diag — CHARACTER*1. Specifies whether the RFP matrix *A* is unit triangular:
if *diag* = 'U' or 'u' then the matrix is unit triangular;
if *diag* = 'N' or 'n', then the matrix is not unit triangular.

m — INTEGER. Specifies the number of rows of *B*. The value of *m* must be at least zero.

n — INTEGER. Specifies the number of columns of *B*. The value of *n* must be at least zero.

alpha — REAL for stfsm
DOUBLE PRECISION for dtfsm
COMPLEX for ctfsm
DOUBLE COMPLEX for ztfsm
Specifies the scalar *alpha*.
When *alpha* is zero, then *a* is not referenced and *b* need not be set before entry.

a — REAL for stfsm
DOUBLE PRECISION for dtfsm
COMPLEX for ctfsm
DOUBLE COMPLEX for ztfsm
Array, DIMENSION (*n**(*n*+1)/2). Contains the matrix *A* in RFP format.

b — REAL for stfsm
DOUBLE PRECISION for dtfsm
COMPLEX for ctfsm
DOUBLE COMPLEX for ztfsm
Array, DIMENSION (*ldb*,*n*). Before entry, the leading *m*-by-*n* part of the array *b* must contain the right-hand side matrix *B*.

ldb — INTEGER. Specifies the leading dimension of *b* as declared in the calling (sub)program. The value of *ldb* must be at least max(1, +*m*).

Output Parameters

b	Overwritten by the solution matrix *X*.

?lansf

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a symmetric matrix in RFP format.

Syntax

```
val = slansf(norm, transr, uplo, n, a, work)
val = dlansf(norm, transr, uplo, n, a, work)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

T

The function `?lansf` returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an *n*-by-*n* real symmetric matrix *A* in the rectangular full packed (RFP) format .

Input Parameters

norm	`CHARACTER*1`. Specifies the value to be returned by the routine: = `'M'` or `'m'`: *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*. = `'1'` or `'O'` or `'o'`: *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum), = `'I'` or `'i'`: *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum), = `'F'`, `'f'`, `'E'` or `'e'`: *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).
transr	`CHARACTER*1`. Specifies whether the RFP format of matrix *A* is normal or transposed format. If *transr* = `'N'`: RFP format is normal; if *transr* = `'T'`: RFP format is transposed.
uplo	`CHARACTER*1`. Specifies whether the RFP matrix *A* came from upper or lower triangular matrix. If *uplo* = `'U'`: RFP matrix *A* came from an upper triangular matrix; if *uplo* = `'L'`: RFP matrix *A* came from a lower triangular matrix.
n	`INTEGER`. The order of the matrix *A*. $n \geq 0$. When *n* = 0, `?lansf` is set to zero.
a	`REAL` for `slansf` `DOUBLE PRECISION` for `dlansf` Array, `DIMENSION` (*n**(*n*+1)/2). The upper (if *uplo* = `'U'`) or lower (if *uplo* = `'L'`) part of the symetric matrix *A* stored in RFP format.
work	`REAL` for `slansf`.

DOUBLE PRECISION for dlansf.
Workspace array, DIMENSION (max(1,*lwork*)), where *lwork* ≥ *n* when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val — REAL for slansf
DOUBLE PRECISION for dlansf
Value returned by the function.

?lanhf

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a Hermitian matrix in RFP format.

Syntax

```
val = clanhf(norm, transr, uplo, n, a, work)
val = zlanhf(norm, transr, uplo, n, a, work)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lanhf returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an *n*-by-*n* complex Hermitian matrix *A* in the rectangular full packed (RFP) format.

Input Parameters

norm — CHARACTER*1.
Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

transr — CHARACTER*1.
Specifies whether the RFP format of matrix *A* is normal or conjugate-transposed format.
If *transr* = 'N': RFP format is normal;
if *transr* = 'C': RFP format is conjugate-transposed.

uplo — CHARACTER*1.
Specifies whether the RFP matrix *A* came from upper or lower triangular matrix.
If *uplo* = 'U': RFP matrix *A* came from an upper triangular matrix;
if *uplo* = 'L': RFP matrix *A* came from a lower triangular matrix.

n — INTEGER. The order of the matrix *A*. *n* ≥ 0.
When *n* = 0, ?lanhf is set to zero.

a	COMPLEX for clanhf DOUBLE COMPLEX for zlanhf Array, DIMENSION (*n**(*n*+1)/2). The upper (if *uplo* = 'U') or lower (if *uplo* = 'L') part of the Hermitian matrix *A* stored in RFP format.
work	COMPLEX for clanhf. DOUBLE COMPLEX for zlanhf. Workspace array, DIMENSION (max(1,*lwork*)), where *lwork* ≥ *n* when *norm* = 'I' or '1' or 'O'; otherwise, *work* is not referenced.

Output Parameters

val	COMPLEX for clanhf DOUBLE COMPLEX for zlanhf Value returned by the function.

?tfttp

Copies a triangular matrix from the rectangular full packed format (TF) to the standard packed format (TP) .

Syntax

Fortran 77:

```
call stfttp( transr, uplo, n, arf, ap, info )
call dtfttp( transr, uplo, n, arf, ap, info )
call ctfttp( transr, uplo, n, arf, ap, info )
call ztfttp( transr, uplo, n, arf, ap, info )
```

C:

```
lapack_int LAPACKE_<?>tfttp( int matrix_order, char transr, char uplo, lapack_int n,
const <datatype>* arf, <datatype>* ap );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The routine copies a triangular matrix *A* from the Rectangular Full Packed (RFP) format to the standard packed format. For the description of the RFP format, see Matrix Storage Schemes.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr	CHARACTER*1. = 'N': *arf* is in the Normal format, = 'T': *arf* is in the Transpose format (for stfttp and dtfttp),

= 'C': *arf* is in the Conjugate-transpose format (for ctfttp and ztfttp).

uplo — CHARACTER*1.
Specifies whether *A* is upper or lower triangular:
= 'U': A is upper triangular,
= 'L': A is lower triangular.

n — INTEGER. The order of the matrix *A*. $n \geq 0$.

arf — REAL for stfttp,
DOUBLE PRECISION for dtfttp,
COMPLEX for ctfttp,
DOUBLE COMPLEX for ztfttp.
Array, DIMENSION at least *max* (1, *n**(*n*+1)/2).
On entry, the upper or lower triangular matrix *A* stored in the RFP format.

Output Parameters

ap — REAL for stfttp,
DOUBLE PRECISION for dtfttp,
COMPLEX for ctfttp,
DOUBLE COMPLEX for ztfttp.
Array, DIMENSION at least *max* (1, *n**(*n*+1)/2).
On exit, the upper or lower triangular matrix *A*, packed columnwise in a linear array. The *j*-th column of *A* is stored in the array *ap* as follows:
if *uplo* = 'U', *ap(i + (j-1)*j/2)* = *A(i,j)* for $1 \leq i \leq j$,
if *uplo* = 'L', *ap(i + (j-1)*(2n-j)/2)* = *A(i,j)* for $j \leq i \leq n$.

info — INTEGER.
=0: successful exit,
< 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

?tfttr

Copies a triangular matrix from the rectangular full packed format (TF) to the standard full format (TR) .

Syntax

Fortran 77:

```
call stfttr( transr, uplo, n, arf, a, lda, info )
call dtfttr( transr, uplo, n, arf, a, lda, info )
call ctfttr( transr, uplo, n, arf, a, lda, info )
call ztfttr( transr, uplo, n, arf, a, lda, info )
```

C:

```
lapack_int LAPACKE_<?>tfttr( int matrix_order, char transr, char uplo, lapack_int n,
const <datatype>* arf, <datatype>* a, lapack_int lda );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The routine copies a triangular matrix *A* from the Rectangular Full Packed (RFP) format to the standard full format. For the description of the RFP format, see Matrix Storage Schemes.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr	CHARACTER*1. = 'N': *arf* is in the Normal format, = 'T': *arf* is in the Transpose format (for stfttr and dtfttr), = 'C': *arf* is in the Conjugate-transpose format (for ctfttr and ztfttr).
uplo	CHARACTER*1. Specifies whether *A* is upper or lower triangular: = 'U': A is upper triangular, = 'L': A is lower triangular.
n	INTEGER. The order of the matrices *arf* and *a*. $n \geq 0$.
arf	REAL for stfttr, DOUBLE PRECISION for dtfttr, COMPLEX for ctfttr, DOUBLE COMPLEX for ztfttr. Array, DIMENSION at least *max* (1, *n**(*n*+1)/2). On entry, the upper or lower triangular matrix *A* stored in the RFP format.
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq$ *max*(1,*n*).

Output Parameters

a	REAL for stfttr, DOUBLE PRECISION for dtfttr, COMPLEX for ctfttr, DOUBLE COMPLEX for ztfttr. Array, DIMENSION (*lda, **). On exit, the triangular matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular matrix, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular matrix, and the strictly upper triangular part of *a* is not referenced.
info	INTEGER. =0: successful exit, < 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

?tpttf

Copies a triangular matrix from the standard packed format (TP) to the rectangular full packed format (TF).

Syntax

Fortran 77:

```
call stpttf( transr, uplo, n, ap, arf, info )
call dtpttf( transr, uplo, n, ap, arf, info )
call ctpttf( transr, uplo, n, ap, arf, info )
```

```
call ztpttf( transr, uplo, n, ap, arf, info )
```

C:

```
lapack_int LAPACKE_<?>tpttf( int matrix_order, char transr, char uplo, lapack_int n,
const <datatype>* ap, <datatype>* arf );
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`
- C: `mkl_lapacke.h`

Description

The routine copies a triangular matrix *A* from the standard packed format to the Rectangular Full Packed (RFP) format. For the description of the RFP format, see Matrix Storage Schemes.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr — CHARACTER*1.
= 'N': *arf* must be in the Normal format,
= 'T': *arf* must be in the Transpose format (for stpttf and dtpttf),
= 'C': *arf* must be in the Conjugate-transpose format (for ctpttf and ztpttf).

uplo — CHARACTER*1.
Specifies whether *A* is upper or lower triangular:
= 'U': A is upper triangular,
= 'L': A is lower triangular.

n — INTEGER. The order of the matrix *A*. $n \geq 0$.

ap — REAL for stpttf,
DOUBLE PRECISION for dtpttf,
COMPLEX for ctpttf,
DOUBLE COMPLEX for ztpttf.
Array, DIMENSION at least *max* (1, *n**(*n*+1)/2).
On entry, the upper or lower triangular matrix *A*, packed columnwise in a linear array. The *j*-th column of *A* is stored in the array *ap* as follows:
if *uplo* = 'U', *ap(i + (j-1)*j/2)* = *A(i,j)* for $1 \leq i \leq j$,
if *uplo* = 'L', *ap(i + (j-1)*(2n-j)/2)* = *A(i,j)* for $j \leq i \leq n$.

Output Parameters

arf — REAL for stpttf,
DOUBLE PRECISION for dtpttf,
COMPLEX for ctfttp,
DOUBLE COMPLEX for ztpttf.
Array, DIMENSION at least *max* (1, *n**(*n*+1)/2).
On exit, the upper or lower triangular matrix *A* stored in the RFP format.

info — INTEGER.
=0: successful exit,
< 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

?tpttr

Copies a triangular matrix from the standard packed format (TP) to the standard full format (TR) .

Syntax

Fortran 77:

```
call stpttr( uplo, n, ap, a, lda, info )
call dtpttr( uplo, n, ap, a, lda, info )
call ctpttr( uplo, n, ap, a, lda, info )
call ztpttr( uplo, n, ap, a, lda, info )
```

C:

```
lapack_int LAPACKE_<?>tpttr( int matrix_order, char uplo, lapack_int n, const
<datatype>* ap, <datatype>* a, lapack_int lda );
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`
- C: `mkl_lapacke.h`

Description

The routine copies a triangular matrix A from the standard packed format to the standard full format.

Input Parameters

The data types are given for the Fortran interface. A `<datatype>` placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

uplo	`CHARACTER*1`. Specifies whether A is upper or lower triangular: = 'U': A is upper triangular, = 'L': A is lower triangular.
n	`INTEGER`. The order of the matrices *ap* and *a*. $n \geq 0$.
ap	`REAL` for `stpttr`, `DOUBLE PRECISION` for `dtpttr`, `COMPLEX` for `ctpttr`, `DOUBLE COMPLEX` for `ztpttr`. Array, `DIMENSION` at least *max* (1, n*(n+1)/2). On entry, the upper or lower triangular matrix A, packed columnwise in a linear array. The j-th column of A is stored in the array *ap* as follows: if *uplo* = 'U', *ap(i + (j-1)*j/2)* = *A(i,j)* for $1 \leq i \leq j$, if *uplo* = 'L', *ap(i + (j-1)*(2n-j)/2)* = *A(i,j)* for $j \leq i \leq n$.
lda	`INTEGER`. The leading dimension of the array *a*. *lda* $\geq$ *max*(1,n).

Output Parameters

a	`REAL` for `stpttr`, `DOUBLE PRECISION` for `dtpttr`, `COMPLEX` for `ctpttr`, `DOUBLE COMPLEX` for `ztpttr`.

Array, DIMENSION (*lda, **).
On exit, the triangular matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.

info — INTEGER.
=0: successful exit,
< 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

?trttf

Copies a triangular matrix from the standard full format (TR) to the rectangular full packed format (TF).

Syntax

Fortran 77:

```
call strttf( transr, uplo, n, a, lda, arf, info )
call dtrttf( transr, uplo, n, a, lda, arf, info )
call ctrttf( transr, uplo, n, a, lda, arf, info )
call ztrttf( transr, uplo, n, a, lda, arf, info )
```

C:

```
lapack_int LAPACKE_<?>trttf( int matrix_order, char transr, char uplo, lapack_int n,
const <datatype>* a, lapack_int lda, <datatype>* arf );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The routine copies a triangular matrix *A* from the standard full format to the Rectangular Full Packed (RFP) format. For the description of the RFP format, see Matrix Storage Schemes.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

transr — CHARACTER*1.
= 'N': *arf* must be in the Normal format,
= 'T': *arf* must be in the Transpose format (for strttf and dtrttf),
= 'C': *arf* must be in the Conjugate-transpose format (for ctrttf and ztrttf).

uplo — CHARACTER*1.
Specifies whether *A* is upper or lower triangular:
= 'U': A is upper triangular,
= 'L': A is lower triangular.

n	INTEGER. The order of the matrix *A*. $n \geq 0$.
a	REAL for strttf, DOUBLE PRECISION for dtrttf, COMPLEX for ctrttf, DOUBLE COMPLEX for ztrttf. Array, DIMENSION (*lda, **). On entry, the triangular matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular matrix, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular matrix, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq$ *max*(1,*n*).

Output Parameters

arf	REAL for strttf, DOUBLE PRECISION for dtrttf, COMPLEX for ctrttf, DOUBLE COMPLEX for ztrttf. Array, DIMENSION at least *max* (1, *n**(*n*+1)/2). On exit, the upper or lower triangular matrix *A* stored in the RFP format.
info	INTEGER. =0: successful exit, < 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

?trttp

Copies a triangular matrix from the standard full format (TR) to the standard packed format (TP) .

Syntax

Fortran 77:

```
call strttp( uplo, n, a, lda, ap, info )
call dtrttp( uplo, n, a, lda, ap, info )
call ctrttp( uplo, n, a, lda, ap, info )
call ztrttp( uplo, n, a, lda, ap, info )
```

C:

```
lapack_int LAPACKE_<?>trttp( int matrix_order, char uplo, lapack_int n, const
<datatype>* a, lapack_int lda, <datatype>* ap );
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h
- C: mkl_lapacke.h

Description

The routine copies a triangular matrix *A* from the standard full format to the standard packed format.

Input Parameters

The data types are given for the Fortran interface. A <datatype> placeholder, if present, is used for the C interface data types in the C interface section above. See the C Interface Conventions section for the C interface principal conventions and type defintions.

uplo	CHARACTER*1. Specifies whether *A* is upper or lower triangular: = 'U': A is upper triangular, = 'L': A is lower triangular.
n	INTEGER. The order of the matrices *a* and *ap*. $n \geq 0$.
a	REAL for strttp, DOUBLE PRECISION for dtrttp, COMPLEX for ctrttp, DOUBLE COMPLEX for ztrttp. Array, DIMENSION (*lda, n*). On entry, the triangular matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular matrix, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular matrix, and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq max(1,n)$.

Output Parameters

ap	REAL for strttp, DOUBLE PRECISION for dtrttp, COMPLEX for ctrttp, DOUBLE COMPLEX for ztrttp. Array, DIMENSION at least *max* (1, *n**(*n*+1)/2). On exit, the upper or lower triangular matrix *A*, packed columnwise in a linear array. The *j*-th column of *A* is stored in the array *ap* as follows: if *uplo* = 'U', $ap(i + (j-1)*j/2) = A(i,j)$ for $1 \leq i \leq j$, if *uplo* = 'L', $ap(i + (j-1)*(2n-j)/2) = A(i,j)$ for $j \leq i \leq n$.
info	INTEGER. =0: successful exit, < 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

?pstf2

Computes the Cholesky factorization with complete pivoting of a real symmetric or complex Hermitian positive semi-definite matrix.

Syntax

```
call spstf2( uplo, n, a, lda, piv, rank, tol, work, info )
call dpstf2( uplo, n, a, lda, piv, rank, tol, work, info )
call cpstf2( uplo, n, a, lda, piv, rank, tol, work, info )
call zpstf2( uplo, n, a, lda, piv, rank, tol, work, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The real flavors spstf2 and dpstf2 compute the Cholesky factorization with complete pivoting of a real symmetric positive semi-definite matrix *A*. The complex flavors cpstf2 and zpstf2 compute the Cholesky factorization with complete pivoting of a complex Hermitian positive semi-definite matrix *A*. The factorization has the form:

$P^T * A * P = U^T * U$, if *uplo* = 'U' for real flavors,

$P^T * A * P = U^H * U$, if *uplo* = 'U' for complex flavors,

$P^T * A * P = L * L^T$, if *uplo* = 'L' for real flavors,

$P^T * A * P = L * L^H$, if *uplo* = 'L' for complex flavors,

where *U* is an upper triangular matrix and *L* is lower triangular, and *P* is stored as vector *piv*.

This algorithm does not check that *A* is positive semi-definite. This version of the algorithm calls level 2 BLAS.

Input Parameters

uplo
CHARACTER*1.
Specifies whether the upper or lower triangular part of the symmetric or Hermitian matrix *A* is stored:
= 'U': Upper triangular,
= 'L': Lower triangular.

n
INTEGER. The order of the matrix *A*. $n \geq 0$.

a
REAL for spstf2,
DOUBLE PRECISION for dpstf2,
COMPLEX for cpstf2,
DOUBLE COMPLEX for zpstf2.
Array, DIMENSION (*lda, **).
On entry, the symmetric matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *a* is not referenced.

tol
REAL for spstf2 and cpstf2,
DOUBLE PRECISION for dpstf2 and zpstf2.
A user-defined tolerance.
If *tol* < 0, *n*ulp*max(A(k,k))* will be used (*ulp* is the Unit in the Last Place, or Unit of Least Precision). The algorithm terminates at the (*k* - 1)-st step if the pivot is not greater than *tol*.

lda
INTEGER. The leading dimension of the matrix *A*. *lda* $\geq max(1,n)$.

work
REAL for spstf2 and cpstf2,
DOUBLE PRECISION for dpstf2 and zpstf2.
Workspace array, DIMENSION at least *max* (1, *2*n)*.

Output Parameters

piv
INTEGER. Array. DIMENSION at least *max (1,n)*.
piv is such that the non-zero entries are *P* (*piv* (*k*), *k*) = 1.

a
On exit, if *info* = 0, the factor U or L from the Cholesky factorization stored the same way as the matrix *A* is stored on entry.

rank	INTEGER. The rank of *A*, determined by the number of steps the algorithm completed.
info	INTEGER. < 0: if *info* = -*k*, the *k*-th parameter had an illegal value, =0: the algorithm completed successfully, > 0: the matrix *A* is rank-deficient with the computed rank, returned in *rank*, or indefinite.

dlat2s

Converts a double-precision triangular matrix to a single-precision triangular matrix.

Syntax

```
call dlat2s( uplo, n, a, lda, sa, ldsa, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine converts a double-precision triangular matrix *A* to a single-precision triangular matrix *SA*. dlat2s checks that all the elements of *A* are between *-RMAX* and *RMAX*, where *RMAX* is the overflow for the single-precision arithmetic. If this condition is not met, the conversion is aborted and a flag is raised. The routine does no parameter checking.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix *A* is upper or lower triangular: = 'U': *A* is upper triangular, = 'L': *A* is lower triangular.
n	INTEGER. The number of rows and columns of the matrix *A*. $n \geq 0$.
a	DOUBLE PRECISION. Array, DIMENSION (*lda, **). On entry, the *n*-by-*n* triangular matrix *A*.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq max(1,n)$.
ldsa	INTEGER. The leading dimension of the array *sa*. $ldsa \geq max(1,n)$.

Output Parameters

sa	REAL. Array, DIMENSION (*ldsa, **). Only the part of *sa* determined by *uplo* is referenced. On exit, • if *info* = 0, the *n*-by-*n* triangular matrix *SA*, • if *info* > 0, the content of the part of *sa* determined by *uplo* is unspecified.
info	INTEGER. =0: successful exit,

> 0: an element of the matrix *A* is greater than the single-precision overflow threshold; in this case, the content of the part of *sa* determined by *uplo* is unspecified on exit.

zlat2c

Converts a double complex triangular matrix to a complex triangular matrix.

Syntax

```
call zlat2c( uplo, n, a, lda, sa, ldsa, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine is declared in mkl_lapack.fi for FORTRAN 77 interface and in mkl_lapack.h for C interface.

The routine converts a DOUBLE COMPLEX triangular matrix *A* to a COMPLEX triangular matrix *SA*. zlat2c checks that the real and complex parts of all the elements of *A* are between *-RMAX* and *RMAX*, where *RMAX* is the overflow for the single-precision arithmetic. If this condition is not met, the conversion is aborted and a flag is raised. The routine does no parameter checking.

Input Parameters

uplo	CHARACTER*1. Specifies whether the matrix *A* is upper or lower triangular: = 'U': *A* is upper triangular, = 'L': *A* is lower triangular.
n	INTEGER. The number of rows and columns in the matrix *A*. $n \geq 0$.
a	DOUBLE COMPLEX. Array, DIMENSION (*lda, **). On entry, the *n*-by-*n* triangular matrix *A*.
lda	INTEGER. The leading dimension of the array *a*. $lda \geq max(1,n)$.
ldsa	INTEGER. The leading dimension of the array *sa*. $ldsa \geq max(1,n)$.

Output Parameters

sa	COMPLEX. Array, DIMENSION (*ldsa, **). Only the part of *sa* determined by *uplo* is referenced. On exit, • if *info* = 0, the *n*-by-*n* triangular matrix *sa*, • if *info* > 0, the content of the part of *sa* determined by *uplo* is unspecified.
info	INTEGER. =0: successful exit, > 0: the real or complex part of an element of the matrix *A* is greater than the single-precision overflow threshold; in this case, the content of the part of *sa* determined by *uplo* is unspecified on exit.

?lacp2

Copies all or part of a real two-dimensional array to a complex array.

Syntax

```
call clacp2( uplo, m, n, a, lda, b, ldb )
call zlacp2( uplo, m, n, a, lda, b, ldb )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine copies all or part of a two-dimensional matrix A to another matrix B.

Input Parameters

uplo	`CHARACTER*1`. Specifies the part of the matrix A to be copied to B. If *uplo* = `'U'`, the upper triangular part of A; if *uplo* = `'L'`, the lower triangular part of A. Otherwise, all of the matrix A is copied.
m	`INTEGER`. The number of rows in the matrix A ($m \geq 0$).
n	`INTEGER`. The number of columns in A ($n \geq 0$).
a	`REAL` for `clacp2` `DOUBLE PRECISION` for `zlacp2` Array *a*(*lda*,*n*), contains the m-by-n matrix A. If *uplo* = `'U'`, only the upper triangle or trapezoid is accessed; if *uplo* = `'L'`, only the lower triangle or trapezoid is accessed.
lda	`INTEGER`. The leading dimension of *a*; *lda* $\geq$ max(1, *m*).
ldb	`INTEGER`. The leading dimension of the output array *b*; *ldb* $\geq$ max(1, *m*).

Output Parameters

b	`COMPLEX` for `clacp2` `DOUBLE COMPLEX` for `zlacp2`. Array *b*(*ldb*,*m*), contains the m-by-n matrix B. On exit, $B = A$ in the locations specified by *uplo*.

?la_gbamv

Performs a matrix-vector operation to calculate error bounds.

Syntax

Fortran 77:

```
call sla_gbamv(trans, m, n, kl, ku, alpha, ab, ldab, x, incx, beta, y, incy)
call dla_gbamv(trans, m, n, kl, ku, alpha, ab, ldab, x, incx, beta, y, incy)
call cla_gbamv(trans, m, n, kl, ku, alpha, ab, ldab, x, incx, beta, y, incy)
```

call zla_gbamv(*trans*, *m*, *n*, *kl*, *ku*, *alpha*, *ab*, *ldab*, *x*, *incx*, *beta*, *y*, *incy*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_gbamv function performs one of the matrix-vector operations defined as

y := *alpha**abs(*A*)*abs(*x*) + *beta**abs(*y*),

or

y := *alpha**abs(*A*)^T*abs(*x*) + *beta**abs(*y*),

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*n* matrix, with *kl* sub-diagonals and *ku* super-diagonals.

This function is primarily used in calculating error bounds. To protect against underflow during evaluation, the function perturbs components in the resulting vector away from zero by (*n* + 1) times the underflow threshold. To prevent unnecessarily large errors for block structure embedded in general matrices, the function does not perturb *symbolically* zero components. A zero entry is considered *symbolic* if all multiplications involved in computing that entry have at least one zero multiplicand.

Input Parameters

trans	INTEGER. Specifies the operation to be performed: If *trans* = 'BLAS_NO_TRANS', then *y* := *alpha**abs(*A*)*abs(*x*) + *beta**abs(*y*) If *trans* = 'BLAS_TRANS', then *y* := *alpha**abs(A^T)*abs(*x*) + *beta**abs(*y*) If *trans* = 'BLAS_CONJ_TRANS', then *y* := *alpha**abs(A^T)*abs(*x*) + *beta**abs(*y*) The parameter is unchanged on exit.
m	INTEGER. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero. Unchanged on exit.
n	INTEGER. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero. Unchanged on exit.
kl	INTEGER. Specifies the number of sub-diagonals within the band of *A*. *kl* ≥ 0.
ku	INTEGER. Specifies the number of super-diagonals within the band of *A*. *ku* ≥ 0.
alpha	REAL for sla_gbamv and cla_gbamv DOUBLE PRECISION for dla_gbamv and zla_gbamv Specifies the scalar *alpha*. Unchanges on exit.
ab	REAL for sla_gbamv DOUBLE PRECISION for dla_gbamv COMPLEX for cla_gbamv DOUBLE COMPLEX for zla_gbamv Array, DIMENSION (*ldab*, *).

	Before entry, the leading *m*-by-*n* part of the array *ab* must contain the matrix of coefficients. The second dimension of *ab* must be at least max(1,*n*). Unchanged on exit.
ldab	INTEGER. Specifies the leading dimension of *ab* as declared in the calling (sub)program. The value of *ldab* must be at least max(1, *m*). Unchanged on exit.
x	REAL for sla_gbamv DOUBLE PRECISION for dla_gbamv COMPLEX for cla_gbamv DOUBLE COMPLEX for zla_gbamv Array, DIMENSION (1 + (*n* - 1)*abs(*incx*)) when *trans* = 'N' or 'n' and at least (1 + (*m* - 1)*abs(*incx*)) otherwise. Before entry, the incremented array *x* must contain the vector *x*.
incx	INTEGER. Specifies the increment for the elements of *x*. *incx* must not be zero.
beta	REAL for sla_gbamv and cla_gbamv DOUBLE PRECISION for dla_gbamv and zla_gbamv Specifies the scalar *beta*. When *beta* is zero, you do not need to set *y* on input.
y	REAL for sla_gbamv and cla_gbamv DOUBLE PRECISION for dla_gbamv and zla_gbamv Array, DIMENSION at least (1 +(*m* - 1)*abs(*incy*)) when *trans* = 'N' or 'n' and at least (1 +(*n* - 1)*abs(*incy*)) otherwise. Before entry with *beta* non-zero, the incremented array *y* must contain the vector *y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must not be zero. Unchanged on exit.

Output Parameters

y	Updated vector *y*.

?la_gbrcond

Estimates the Skeel condition number for a general banded matrix.

Syntax

Fortran 77:

```
call sla_gbrcond( trans, n, kl, ku, ab, ldab, afb, ldafb, ipiv, cmode, c, info, work, iwork )
call dla_gbrcond( trans, n, kl, ku, ab, ldab, afb, ldafb, ipiv, cmode, c, info, work, iwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function estimates the Skeel condition number of

op(*A*) * op2(*C*)

where

the *cmode* parameter determines op2 as follows:

cmode **Value**	**op2(C)**
1	*C*
0	*I*
-1	inv(*C*)

The Skeel condition number

cond(*A*) = norminf(|inv(*A*)||*A*|)

is computed by computing scaling factors *R* such that

diag(*R*)*A*op2(*C*)

is row equilibrated and by computing the standard infinity-norm condition number.

Input Parameters

trans — CHARACTER*1. Must be 'N' or 'T' or 'C'.
Specifies the form of the system of equations:
If *trans* = 'N', the system has the form *A***X* = *B*.
If *trans* = 'T', the system has the form A^T**X* = *B*.
If *trans* = 'C', the system has the form A^H**X* = *B*.

n — INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.

kl — INTEGER. The number of subdiagonals within the band of *A*; *kl* ≥ 0.

ku — INTEGER. The number of superdiagonals within the band of *A*; *ku* ≥ 0.

ab, *afb*, *c*, *work* — REAL for sla_gbrcond
DOUBLE PRECISION for dla_gbrcond
Arrays:
ab(*ldab*,*) contains the original band matrix *A* stored in rows from 1 to *kl* + *ku* + 1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows:
ab(*ku*+1+*i*-*j*,*j*) = A(*i*,*j*)
for
max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*)
afb(*ldafb*,*) contains details of the LU factorization of the band matrix *A*, as returned by ?gbtrf. *U* is stored as an upper triangular band matrix with *kl*+*ku* superdiagonals in rows 1 to *kl*+*ku*+1, and the multipliers used during the factorization are stored in rows *kl*+*ku*+2 to 2**kl*+*ku*+1.
c, DIMENSION *n*. The vector *C* in the formula op(*A*) * op2(*C*).
work is a workspace array of DIMENSION (5**n*).
The second dimension of *ab* and *afb* must be at least max(1, *n*).

ldab — INTEGER. The leading dimension of the array *ab*. *ldab* ≥ *kl*+*ku*+1.

ldafb — INTEGER. The leading dimension of *afb*. *ldafb* ≥ 2**kl*+*ku*+1.

ipiv — INTEGER.

	Array with DIMENSION *n*. The pivot indices from the factorization *A = P*L*U* as computed by ?gbtrf. Row *i* of the matrix was interchanged with row *ipiv(i)*.
cmode	INTEGER. Determines op2(*C*) in the formula op(*A*) * op2(*C*) as follows: If *cmode* = 1, op2(*C*) = *C*. If *cmode* = 0, op2(*C*) = *I*. If *cmode* = -1, op2(*C*) = inv(*C*).
iwork	INTEGER. Workspace array with DIMENSION *n*.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?gbtrf

?la_gbrcond_c

*Computes the infinity norm condition number of op(A)*inv(diag(c)) for general banded matrices.*

Syntax

Fortran 77:

```
call cla_gbrcond_c( trans, n, kl, ku, ab, ldab, afb, ldafb, ipiv, c, capply, info, work, rwork )
call zla_gbrcond_c( trans, n, kl, ku, ab, ldab, afb, ldafb, ipiv, c, capply, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * inv(diag(c))

where the *c* is a REAL vector for cla_gbrcond_c and a DOUBLE PRECISION vector for zla_gbrcond_c.

Input Parameters

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose) If *trans* = 'T', the system has the form $A^T*X = B$ (Transpose) If *trans* = 'C', the system has the form $A^H*X = B$ (Conjugate Transpose = Transpose)
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
kl	INTEGER. The number of subdiagonals within the band of *A*; $kl \geq 0$.
ku	INTEGER. The number of superdiagonals within the band of *A*; $ku \geq 0$.
ab, *afb*, *work*	COMPLEX for cla_gbrcond_c

	DOUBLE COMPLEX for zla_gbrcond_c Arrays: *ab*(*ldab*,*) contains the original band matrix *A* stored in rows from 1 to *kl* + *ku* + 1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = A(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*) *afb*(*ldafb*,*) contains details of the LU factorization of the band matrix *A*, as returned by ?gbtrf. *U* is stored as an upper triangular band matrix with *kl*+*ku* superdiagonals in rows 1 to *kl*+*ku*+1, and the multipliers used during the factorization are stored in rows *kl*+*ku*+2 to 2**kl*+*ku*+1. *work* is a workspace array of DIMENSION (5**n*). The second dimension of *ab* and *afb* must be at least max(1, *n*).
ldab	INTEGER. The leading dimension of the array *ab*. *ldab* ≥ *kl*+*ku*+1.
ldafb	INTEGER. The leading dimension of *afb*. *ldafb* ≥ 2**kl*+*ku*+1.
ipiv	INTEGER. Array with DIMENSION *n*. The pivot indices from the factorization *A* = *P*L*U* as computed by ?gbtrf. Row *i* of the matrix was interchanged with row *ipiv*(*i*).
c, *rwork*	REAL for cla_gbrcond_c DOUBLE PRECISION for zla_gbrcond_c Array *c* with DIMENSION *n*. The vector *c* in the formula op(*A*) * inv(diag(*c*)). Array *rwork* with DIMENSION *n* is a workspace.
capply	LOGICAL. If .TRUE., then the function uses the vector *c* from the formula op(*A*) * inv(diag(*c*)).

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?gbtrf

?la_gbrcond_x

*Computes the infinity norm condition number of op(A)*diag(x) for general banded matrices.*

Syntax

Fortran 77:

```
call cla_gbrcond_x( trans, n, kl, ku, ab, ldab, afb, ldafb, ipiv, x, info, work,
rwork )
call zla_gbrcond_x( trans, n, kl, ku, ab, ldab, afb, ldafb, ipiv, x, info, work,
rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * diag(*x*)

where the *x* is a COMPLEX vector for cla_gbrcond_x and a DOUBLE COMPLEX vector for zla_gbrcond_x.

Input Parameters

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose) If *trans* = 'T', the system has the form A^T*X = *B* (Transpose) If *trans* = 'C', the system has the form A^H*X = *B* (Conjugate Transpose = Transpose)
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.
kl	INTEGER. The number of subdiagonals within the band of *A*; *kl* ≥ 0.
ku	INTEGER. The number of superdiagonals within the band of *A*; *ku* ≥ 0.
ab, *afb*, *x*, *work*	COMPLEX for cla_gbrcond_x DOUBLE COMPLEX for zla_gbrcond_x Arrays: *ab*(*ldab*,*) contains the original band matrix *A* stored in rows from 1 to *kl* + *ku* + 1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = A(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*) *afb*(*ldafb*,*) contains details of the LU factorization of the band matrix *A*, as returned by ?gbtrf. *U* is stored as an upper triangular band matrix with *kl*+*ku* superdiagonals in rows 1 to *kl*+*ku*+1, and the multipliers used during the factorization are stored in rows *kl*+*ku*+2 to 2**kl*+*ku*+1. *x*, DIMENSION *n*. The vector *x* in the formula op(*A*) * diag(*x*). *work* is a workspace array of DIMENSION (2**n*). The second dimension of *ab* and *afb* must be at least max(1, *n*).
ldab	INTEGER. The leading dimension of the array *ab*. *ldab* ≥ *kl*+*ku*+1.
ldafb	INTEGER. The leading dimension of *afb*. *ldafb* ≥ 2**kl*+*ku*+1.
ipiv	INTEGER. Array with DIMENSION *n*. The pivot indices from the factorization *A* = *P*L*U* as computed by ?gbtrf. Row *i* of the matrix was interchanged with row *ipiv*(*i*).
rwork	REAL for cla_gbrcond_x DOUBLE PRECISION for zla_gbrcond_x Array *rwork* with DIMENSION *n* is a workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?gbtrf

?la_gbrfsx_extended

Improves the computed solution to a system of linear equations for general banded matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.

Syntax

Fortran 77:

```
call sla_gbrfsx_extended( prec_type, trans_type, n, kl, ku, nrhs, ab, ldab, afb,
ldafb, ipiv, colequ, c, b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm,
err_bnds_comp, res, ayb, dy, y_tail, rcond, ithresh, rthresh, dz_ub, ignore_cwise,
info )

call dla_gbrfsx_extended( prec_type, trans_type, n, kl, ku, nrhs, ab, ldab, afb,
ldafb, ipiv, colequ, c, b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm,
err_bnds_comp, res, ayb, dy, y_tail, rcond, ithresh, rthresh, dz_ub, ignore_cwise,
info )

call cla_gbrfsx_extended( prec_type, trans_type, n, kl, ku, nrhs, ab, ldab, afb,
ldafb, ipiv, colequ, c, b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm,
err_bnds_comp, res, ayb, dy, y_tail, rcond, ithresh, rthresh, dz_ub, ignore_cwise,
info )

call zla_gbrfsx_extended( prec_type, trans_type, n, kl, ku, nrhs, ab, ldab, afb,
ldafb, ipiv, colequ, c, b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm,
err_bnds_comp, res, ayb, dy, y_tail, rcond, ithresh, rthresh, dz_ub, ignore_cwise,
info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?la_gbrfsx_extended` subroutine improves the computed solution to a system of linear equations by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. The `?gbrfsx` routine calls `?la_gbrfsx_extended` to perform iterative refinement.

In addition to normwise error bound, the code provides maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

Use `?la_gbrfsx_extended` to set only the second fields of *err_bnds_norm* and *err_bnds_comp*.

Input Parameters

prec_type — `INTEGER`.
Specifies the intermediate precision to be used in refinement. The value is defined by `ilaprec(p)`, where `p` is a `CHARACTER` and:
If *p* = `'S'`: Single.
If *p* = `'D'`: Double.
If *p* = `'I'`: Indigenous.
If *p* = `'X'`, `'E'`: Extra.

trans_type — `INTEGER`.
Specifies the transposition operation on `A`. The value is defined by `ilatrans(t)`, where `t` is a `CHARACTER` and:
If *t* = `'N'`: No transpose.

	If *t* = 'T': Transpose. If *t* = 'C': Conjugate Transpose.
n	INTEGER. The number of linear equations; the order of the matrix *A*; *n* ≥ 0.
kl	INTEGER. The number of subdiagonals within the band of *A*; *kl* ≥ 0.
ku	INTEGER. The number of superdiagonals within the band of *A*; *ku* ≥ 0.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrix *B*.
ab, *afb*, *b*, *y*	REAL for sla_gbrfsx_extended DOUBLE PRECISION for dla_gbrfsx_extended COMPLEX for cla_gbrfsx_extended DOUBLE COMPLEX for zla_gbrfsx_extended. Arrays: *ab*(*ldab*,*), *afb*(*ldafb*,*), *b*(*ldb*,*), *y*(*ldy*,*). The array *ab* contains the original *n*-by-*n* matrix *A*. The second dimension of *ab* must be at least max(1,*n*). The array *afb* contains the factors *L* and *U* from the factorization *A = P*L*U*) as computed by ?gbtrf. The second dimension of *afb* must be at least max(1,*n*). The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least max(1,*nrhs*). The array *y* on entry contains the solution matrix *X* as computed by ?gbtrs. The second dimension of *y* must be at least max(1,*nrhs*).
ldab	INTEGER. The leading dimension of the array *ab*; *ldab* ≥ max(1,*n*).
ldafb	INTEGER. The leading dimension of the array *afb*; *ldafb* ≥ max(1,*n*).
ipiv	INTEGER. Array, DIMENSION at least max(1, *n*). Contains the pivot indices from the factorization *A = P*L*U*) as computed by ?gbtrf; row *i* of the matrix was interchanged with row *ipiv*(*i*).
colequ	LOGICAL. If *colequ* = .TRUE., column equilibration was done to *A* before calling this routine. This is needed to compute the solution and error bounds correctly.
c	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. *c* contains the column scale factors for *A*. If *colequ* = .FALSE., *c* is not accessed. If *c* is input, each element of *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by power of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).
ldy	INTEGER. The leading dimension of the array *y*; *ldy* ≥ max(1, *n*).
n_norms	INTEGER. Determines which error bounds to return. See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below. If *n_norms* ≥ 1, returns normwise error bounds. If *n_norms* ≥ 2, returns componentwise error bounds.
err_bnds_norm	REAL for single precision flavors DOUBLE PRECISION for double precision flavors.

Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the following three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.

err_bnds_comp

REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.

res, *dy*, *y_tail*	REAL for sla_gbrfsx_extended DOUBLE PRECISION for dla_gbrfsx_extended COMPLEX for cla_gbrfsx_extended DOUBLE COMPLEX for zla_gbrfsx_extended. Workspace arrays of DIMENSION *n*. *res* holds the intermediate residual. *dy* holds the intermediate solution. *y_tail* holds the trailing bits of the intermediate solution.
ayb	REAL for single precision flavors

DOUBLE PRECISION for double precision flavors.
Workspace array, DIMENSION *n*.

rcond
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

ithresh
INTEGER. The maximum number of residual computations allowed for refinement. The default is 10. For 'aggressive', set to 100 to permit convergence using approximate factorizations or factorizations other than LU. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

rthresh
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Determines when to stop refinement if the error estimate stops decreasing. Refinement stops when the next solution no longer satisfies
norm(dx_{*i*+1}) < *rthresh* * norm(dx_*i*)
where norm(*z*) is the infinity norm of *Z*.
rthresh satisfies
0 < *rthresh* ≤ 1.
The default value is 0.5. For 'aggressive' set to 0.9 to permit convergence on extremely ill-conditioned matrices.

dz_ub
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Determines when to start considering componentwise convergence. Componentwise *dz_ub* convergence is only considered after each component of the solution *y* is stable, that is, the relative change in each component is less than *dz_ub*. The default value is 0.25, requiring the first bit to be stable.

ignore_cwise
LOGICAL
If .TRUE., the function ignores componentwise convergence. Default value is .FALSE.

Output Parameters

y
REAL for sla_gbrfsx_extended
DOUBLE PRECISION for dla_gbrfsx_extended
COMPLEX for cla_gbrfsx_extended
DOUBLE COMPLEX for zla_gbrfsx_extended.
The improved solution matrix *Y*.

berr_out
REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for right-hand-side *j* from the formula
max(*i*) (abs(*res*(*i*)) / (abs(op(*A*))*abs(*y*) + abs(*B*))(*i*))
where abs(*z*) is the componentwise absolute value of the matrix or vector *Z*. This is computed by ?la_lin_berr.

err_bnds_norm, *err_bnds_comp*	Values of the corresponding input parameters improved after iterative refinement and stored in the second column of the array `( 1:nrhs, 2 )`. The other elements are kept unchanged.
info	`INTEGER`. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed. If *info* = -*i*, the *i*-th parameter had an illegal value.

See Also

?gbrfsx
?gbtrf
?gbtrs
?lamch
ilaprec
ilatrans
?la_lin_berr

?la_gbrpvgrw

Computes the reciprocal pivot growth factor `norm(A)/ norm(U)` *for a general band matrix.*

Syntax

Fortran 77:

```
call sla_gbrpvgrw( n, kl, ku, ncols, ab, ldab, afb, ldafb )
call dla_gbrpvgrw( n, kl, ku, ncols, ab, ldab, afb, ldafb )
call cla_gbrpvgrw( n, kl, ku, ncols, ab, ldab, afb, ldafb )
call zla_gbrpvgrw( n, kl, ku, ncols, ab, ldab, afb, ldafb )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?la_gbrpvgrw` routine computes the reciprocal pivot growth factor `norm(A)/norm(U)`. The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the equilibrated matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable.

Input Parameters

n	`INTEGER`. The number of linear equations, the order of the matrix *A*; *n* ≥ 0.
kl	`INTEGER`. The number of subdiagonals within the band of *A*; *kl* ≥ 0.
ku	`INTEGER`. The number of superdiagonals within the band of *A*; *ku* ≥ 0.
ncols	`INTEGER`. The number of columns of the matrix *A*; *ncols* ≥ 0.
ab, *afb*	`REAL` for `sla_gbrpvgrw` `DOUBLE PRECISION` for `dla_gbrpvgrw` `COMPLEX` for `cla_gbrpvgrw` `DOUBLE COMPLEX` for `zla_gbrpvgrw`. Arrays: *ab*(*ldab*,*), *afb*(*ldafb*,*).

	ab contains the original band matrix *A* (see Matrix Storage Schemes) stored in rows from 1 to *kl* + *ku* + 1. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*ku*+1+*i*-*j*,*j*) = A(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*n*,*j*+*kl*) *afb* contains details of the LU factorization of the band matrix *A*, as returned by ?gbtrf. *U* is stored as an upper triangular band matrix with *kl*+*ku* superdiagonals in rows 1 to *kl*+*ku*+1, and the multipliers used during the factorization are stored in rows *kl*+*ku*+2 to 2**kl*+*ku*+1.
ldab	INTEGER. The leading dimension of *ab*; *ldab* ≥ *kl*+*ku*+1.
ldafb	INTEGER. The leading dimension of *afb*; *ldafb* ≥ 2**kl*+*ku*+1.

See Also

?gbtrf

?la_geamv

Computes a matrix-vector product using a general matrix to calculate error bounds.

Syntax

Fortran 77:

```
call sla_geamv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call dla_geamv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call cla_geamv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
call zla_geamv(trans, m, n, alpha, a, lda, x, incx, beta, y, incy)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_geamv routines perform a matrix-vector operation defined as

y := *alpha**abs(*A*)*(*x*) + *beta**abs(*y*),

or

y := *alpha**abs(A^T)*abs(*x*) + *beta**abs(*y*),

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*n* matrix.

This function is primarily used in calculating error bounds. To protect against underflow during evaluation, the function perturbs components in the resulting vector away from zero by (*n* + 1) times the underflow threshold. To prevent unnecessarily large errors for block structure embedded in general matrices, the function does not perturb *symbolically* zero components. A zero entry is considered *symbolic* if all multiplications involved in computing that entry have at least one zero multiplicand.

Input Parameters

trans — CHARACTER*1. Specifies the operation:
if *trans* = BLAS_NO_TRANS , then *y* := *alpha**abs(*A*)*abs(*x*) + *beta**abs(*y*)
if *trans* = BLAS_TRANS, then *y* := *alpha**abs(A^T)*abs(*x*) + *beta**abs(*y*)
if *trans* = 'BLAS_CONJ_TRANS, then *y* := *alpha**abs(A^T)*abs(*x*) + *beta**abs(*y*).

m — INTEGER. Specifies the number of rows of the matrix *A*. The value of *m* must be at least zero.

n — INTEGER. Specifies the number of columns of the matrix *A*. The value of *n* must be at least zero.

alpha — REAL for sla_geamv and for cla_geamv
DOUBLE PRECISION for dla_geamv and zla_geamv
Specifies the scalar *alpha*.

a — REAL for sla_geamv
DOUBLE PRECISION for dla_geamv
COMPLEX for cla_geamv
DOUBLE COMPLEX for zla_geamv
Array, DIMENSION (*lda*, *). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients. The second dimension of *a* must be at least max(1,*n*).

lda — INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *m*).

x — REAL for sla_geamv
DOUBLE PRECISION for dla_geamv
COMPLEX for cla_geamv
DOUBLE COMPLEX for zla_geamv
Array, DIMENSION at least (1+(*n*–1)*abs(*incx*)) when *trans* = 'N' or 'n' and at least (1+(*m* – 1)*abs(*incx*)) otherwise. Before entry, the incremented array *x* must contain the vector *X*.

incx — INTEGER. Specifies the increment for the elements of *x*.
The value of *incx* must be non-zero.

beta — REAL for sla_geamv and for cla_geamv
DOUBLE PRECISION for dla_geamv and zla_geamv
Specifies the scalar *beta*. When *beta* is zero, you do not need to set *y* on input.

y — REAL for sla_geamv and for cla_geamv
DOUBLE PRECISION for dla_geamv and zla_geamv
Array, DIMENSION at least (1 +(*m* – 1)*abs(*incy*)) when *trans* = 'N' or 'n' and at least (1 +(*n* – 1)*abs(*incy*)) otherwise. Before entry with non-zero *beta*, the incremented array *y* must contain the vector *Y*.

incy — INTEGER. Specifies the increment for the elements of *y*.
The value of *incy* must be non-zero.

Output Parameters

y — Updated vector *Y*.

?la_gercond

Estimates the Skeel condition number for a general matrix.

Syntax

Fortran 77:

```
call sla_gercond( trans, n, a, lda, af, ldaf, ipiv, cmode, c, info, work, iwork )
call dla_gercond( trans, n, a, lda, af, ldaf, ipiv, cmode, c, info, work, iwork )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The function estimates the Skeel condition number of

`op(A) * op2(C)`

where

the *cmode* parameter determines `op2` as follows:

cmode **Value**	**op2(C)**
1	*C*
0	*I*
-1	`inv(C)`

The Skeel condition number

`cond(A) = norminf(|inv(A)||A|`

is computed by computing scaling factors *R* such that

`diag(R)*A*op2(C)`

is row equilibrated and by computing the standard infinity-norm condition number.

Input Parameters

trans	`CHARACTER*1`. Must be `'N'` or `'T'` or `'C'`. Specifies the form of the system of equations: If *trans* = `'N'`, the system has the form *A*X* = *B* (No transpose). If *trans* = `'T'`, the system has the form $A^T*X = B$ (Transpose). If *trans* = `'C'`, the system has the form $A^H*X = B$ (Conjugate Transpose = Transpose).
n	`INTEGER`. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
a, *af*, *c*, *work*	`REAL` for `sla_gercond` `DOUBLE PRECISION` for `dla_gercond` Arrays: *a*(*lda*,*) contains the original general *n*-by-*n* matrix *A*. *af*(*ldaf*,*) contains factors *L* and *U* from the factorization of the general matrix *A*=*P***L***U*, as returned by `?getrf`. *c*, `DIMENSION` *n*. The vector *C* in the formula `op(A) * op2(C)`.

	work is a workspace array of DIMENSION (3**n*). The second dimension of *a* and *af* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of *af*. *ldaf* ≥ max(1, *n*).
ipiv	INTEGER. Array with DIMENSION *n*. The pivot indices from the factorization *A = P*L*U* as computed by ?getrf. Row *i* of the matrix was interchanged with row *ipiv*(*i*).
cmode	INTEGER. Determines op2(*C*) in the formula op(*A*) * op2(*C*) as follows: If *cmode* = 1, op2(*C*) = *C*. If *cmode* = 0, op2(*C*) = *I*. If *cmode* = -1, op2(*C*) = inv(*C*).
iwork	INTEGER. Workspace array with DIMENSION *n*.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?getrf

?la_gercond_c

*Computes the infinity norm condition number of op(A)*inv(diag(c)) for general matrices.*

Syntax

Fortran 77:

```
call cla_gercond_c( trans, n, a, lda, af, ldaf, ipiv, c, capply, info, work, rwork )
call zla_gercond_c( trans, n, a, lda, af, ldaf, ipiv, c, capply, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * inv(diag(c))

where the *c* is a REAL vector for cla_gercond_c and a DOUBLE PRECISION vector for zla_gercond_c.

Input Parameters

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose) If *trans* = 'T', the system has the form A^T*X = *B* (Transpose) If *trans* = 'C', the system has the form A^H*X = *B* (Conjugate Transpose = Transpose)
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.

a, *af*, *work*	COMPLEX for cla_gercond_c DOUBLE COMPLEX for zla_gercond_c Arrays: *a*(*lda*,*) contains the original general *n*-by-*n* matrix *A*. *af*(*ldaf*,*) contains the factors *L* and *U* from the factorization *A*=*P***L***U* as returned by ?getrf. *work* is a workspace array of DIMENSION (2**n*). The second dimension of *a* and *af* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of *af*. *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array with DIMENSION *n*. The pivot indices from the factorization *A* = *P***L***U* as computed by ?getrf. Row *i* of the matrix was interchanged with row *ipiv*(*i*).
c, *rwork*	REAL for cla_gercond_c DOUBLE PRECISION for zla_gercond_c Array *c* with DIMENSION *n*. The vector *c* in the formula op(*A*) * inv(diag(*c*)). Array *rwork* with DIMENSION *n* is a workspace.
capply	LOGICAL. If *capply*=.TRUE., then the function uses the vector *c* from the formula op(*A*) * inv(diag(*c*)).

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?getrf

?la_gercond_x

*Computes the infinity norm condition number of op(A)*diag(x) for general matrices.*

Syntax

Fortran 77:

```
call cla_gercond_x( trans, n, a, lda, af, ldaf, ipiv, x, info, work, rwork )
call zla_gercond_x( trans, n, a, lda, af, ldaf, ipiv, x, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * diag(*x*)

where the *x* is a COMPLEX vector for cla_gercond_x and a DOUBLE COMPLEX vector for zla_gercond_x.

Input Parameters

trans	CHARACTER*1. Must be 'N' or 'T' or 'C'. Specifies the form of the system of equations: If *trans* = 'N', the system has the form *A*X* = *B* (No transpose) If *trans* = 'T', the system has the form A^T*X = *B* (Transpose) If *trans* = 'C', the system has the form A^H*X = *B* (Conjugate Transpose = Transpose)
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
a, *af*, *x*, *work*	COMPLEX for cla_gercond_x DOUBLE COMPLEX for zla_gercond_x Arrays: *a*(*lda*,*) contains the original general *n*-by-*n* matrix *A*. *af*(*ldaf*,*) contains the factors *L* and *U* from the factorization *A=P*L*U* as returned by ?getrf. *x*, DIMENSION *n*. The vector *x* in the formula op(*A*) * diag(*x*). *work* is a workspace array of DIMENSION (2**n*). The second dimension of *a* and *af* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(1,n)$.
ldaf	INTEGER. The leading dimension of *af*. $ldaf \geq \max(1,n)$.
ipiv	INTEGER. Array with DIMENSION *n*. The pivot indices from the factorization *A = P*L*U* as computed by ?getrf. Row *i* of the matrix was interchanged with row *ipiv*(*i*).
rwork	REAL for cla_gercond_x DOUBLE PRECISION for zla_gercond_x Array *rwork* with DIMENSION *n* is a workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?getrf

?la_gerfsx_extended

Improves the computed solution to a system of linear equations for general matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.

Syntax

Fortran 77:

```
call sla_gerfsx_extended( prec_type, trans_type, n, nrhs, a, lda, af, ldaf, ipiv,
colequ, c, b, ldb, y, ldy, berr_out, n_norms, errs_n, errs_c, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )
```

```
call dla_gerfsx_extended( prec_type, trans_type, n, nrhs, a, lda, af, ldaf, ipiv,
colequ, c, b, ldb, y, ldy, berr_out, n_norms, errs_n, errs_c, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )
```

```
call cla_gerfsx_extended( prec_type, trans_type, n, nrhs, a, lda, af, ldaf, ipiv,
colequ, c, b, ldb, y, ldy, berr_out, n_norms, errs_n, errs_c, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call zla_gerfsx_extended( prec_type, trans_type, n, nrhs, a, lda, af, ldaf, ipiv,
colequ, c, b, ldb, y, ldy, berr_out, n_norms, errs_n, errs_c, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?la_gerfsx_extended` subroutine improves the computed solution to a system of linear equations for general matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. The `?gerfsx` routine calls `?la_gerfsx_extended` to perform iterative refinement.

In addition to normwise error bound, the code provides maximum componentwise error bound, if possible. See comments for *errs_n* and *errs_c* for details of the error bounds.

Use `?la_gerfsx_extended` to set only the second fields of *errs_n* and *errs_c*.

Input Parameters

prec_type — `INTEGER`.
Specifies the intermediate precision to be used in refinement. The value is defined by `ilaprec(p)`, where `p` is a `CHARACTER` and:
If *p* = `'S'`: Single.
If *p* = `'D'`: Double.
If *p* = `'I'`: Indigenous.
If *p* = `'X'`, `'E'`: Extra.

trans_type — `INTEGER`.
Specifies the transposition operation on *A*. The value is defined by `ilatrans(t)`, where `t` is a `CHARACTER` and:
If *t* = `'N'`: No transpose.
If *t* = `'T'`: Transpose.
If *t* = `'C'`: Conjugate Transpose.

n — `INTEGER`. The number of linear equations; the order of the matrix *A*; $n \geq 0$.

nrhs — `INTEGER`. The number of right-hand sides; the number of columns of the matrix *B*.

a, *af*, *b*, *y* — `REAL` for `sla_gerfsx_extended`
`DOUBLE PRECISION` for `dla_gerfsx_extended`
`COMPLEX` for `cla_gerfsx_extended`
`DOUBLE COMPLEX` for `zla_gerfsx_extended`.
Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *y*(*ldy*,*).
The array *a* contains the original matrix *n*-by-*n* matrix *A*. The second dimension of *a* must be at least `max(1,n)`.
The array *af* contains the factors *L* and *U* from the factorization *A* = *P*L*U*) as computed by `?getrf`. The second dimension of *af* must be at least `max(1,n)`.
The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations. The second dimension of *b* must be at least `max(1,nrhs)`.

The array *y* on entry contains the solution matrix *X* as computed by ?getrs. The second dimension of *y* must be at least max(1,*nrhs*).

lda INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1,*n*).

ldaf INTEGER. The leading dimension of the array *af*; *ldaf* ≥ max(1,*n*).

ipiv INTEGER.
Array, DIMENSION at least max(1, *n*). Contains the pivot indices from the factorization *A = P*L*U*) as computed by ?getrf; row *i* of the matrix was interchanged with row *ipiv*(*i*).

colequ LOGICAL. If *colequ* = .TRUE., column equilibration was done to *A* before calling this routine. This is needed to compute the solution and error bounds correctly.

c REAL for single precision flavors (sla_gerfsx_extended, cla_gerfsx_extended)
DOUBLE PRECISION for double precision flavors (dla_gerfsx_extended, zla_gerfsx_extended).
c contains the column scale factors for *A*. If *colequ* = .FALSE., *c* is not used.
If *c* is input, each element of *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by power of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

ldy INTEGER. The leading dimension of the array *y*; *ldy* ≥ max(1, *n*).

n_norms INTEGER. Determines which error bounds to return. See *errs_n* and *errs_c* descriptions in *Output Arguments* section below.
If *n_norms* ≥ 1, returns normwise error bounds.
If *n_norms* ≥ 2, returns componentwise error bounds.

errs_n REAL for single precision flavors
DOUBLE PRECISION for double precision flavors.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *errs_n*(*i*,:) corresponds to the *i*-th right-hand side.
The second index in *errs_n*(:,*err*) contains the following three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.

	err=2	"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
	err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are `1/(norm(1/z,inf)*norm(z,inf))` for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.
errs_c		`REAL` for single precision flavors `DOUBLE PRECISION` for double precision flavors. Array, `DIMENSION (`*nrhs*`,`*n_err_bnds*`)`. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows: Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *errs_c* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *errs_c(i,:)* corresponds to the *i*-th right-hand side.
The second index in *errs_c(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and

		sqrt(*n*)**dlamch*(ε) for double precision flavors. This error bound should only be trusted if the previous boolean is true.
	err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for single precision flavors and *sqrt*(*n*)**dlamch*(ε) for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.
res, *dy*, *y_tail*		REAL for sla_gerfsx_extended DOUBLE PRECISION for dla_gerfsx_extended COMPLEX for cla_gerfsx_extended DOUBLE COMPLEX for zla_gerfsx_extended. Workspace arrays of DIMENSION *n*. *res* holds the intermediate residual. *dy* holds the intermediate solution. *y_tail* holds the trailing bits of the intermediate solution.
ayb		REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Workspace array, DIMENSION *n*.
rcond		REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.
ithresh		INTEGER. The maximum number of residual computations allowed for refinement. The default is 10. For 'aggressive', set to 100 to permit convergence using approximate factorizations or factorizations other than LU. If the factorization uses a technique other than Gaussian elimination, the guarantees in *errs_n* and *errs_c* may no longer be trustworthy.
rthresh		REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Determines when to stop refinement if the error estimate stops decreasing. Refinement stops when the next solution no longer satisfies norm(dx_{*i*+1}) < *rthresh* * norm(dx_*i*) where norm(*z*) is the infinity norm of *Z*. *rthresh* satisfies 0 < *rthresh* ≤ 1.

	The default value is 0.5. For 'aggressive' set to 0.9 to permit convergence on extremely ill-conditioned matrices.
dz_ub	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Determines when to start considering componentwise convergence. Componentwise *dz_ub* convergence is only considered after each component of the solution *y* is stable, that is, the relative change in each component is less than *dz_ub*. The default value is 0.25, requiring the first bit to be stable.
ignore_cwise	LOGICAL If .TRUE., the function ignores componentwise convergence. Default value is .FALSE.

Output Parameters

y	REAL for sla_gerfsx_extended DOUBLE PRECISION for dla_gerfsx_extended COMPLEX for cla_gerfsx_extended DOUBLE COMPLEX for zla_gerfsx_extended. The improved solution matrix *Y*.
berr_out	REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Array, DIMENSION at least max(1, *nrhs*). Contains the componentwise relative backward error for right-hand-side *j* from the formula max(*i*) (abs(*res*(*i*)) / (abs(op(*A*))*abs(*y*) + abs(*B*))(*i*)) where abs(*z*) is the componentwise absolute value of the matrix or vector *Z*. This is computed by ?la_lin_berr.
errs_n, *errs_c*	Values of the corresponding input parameters improved after iterative refinement and stored in the second column of the array (1:*nrhs*, 2). The other elements are kept unchanged.
info	INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed. If *info* = -*i*, the *i*-th parameter had an illegal value.

See Also

?gerfsx
?getrf
?getrs
?lamch
ilaprec
ilatrans
?la_lin_berr

?la_heamv

Computes a matrix-vector product using a Hermitian indefinite matrix to calculate error bounds.

Syntax

Fortran 77:

```
call cla_heamv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
call zla_heamv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_heamv routines perform a matrix-vector operation defined as

y := *alpha**abs(*A*)*abs(*x*) + *beta**abs(*y*),

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *n*-by-*n* Hermitian matrix.

This function is primarily used in calculating error bounds. To protect against underflow during evaluation, the function perturbs components in the resulting vector away from zero by (*n* + 1) times the underflow threshold. To prevent unnecessarily large errors for block structure embedded in general matrices, the function does not perturb *symbolically* zero components. A zero entry is considered *symbolic* if all multiplications involved in computing that entry have at least one zero multiplicand.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array A is to be referenced: If *uplo* = 'BLAS_UPPER', only the upper triangular part of A is to be referenced, If *uplo* = 'BLAS_LOWER', only the lower triangular part of A is to be referenced.
n	INTEGER. Specifies the number of rows and columns of the matrix *A*. The value of *n* must be at least zero.
alpha	REAL for cla_heamv DOUBLE PRECISION for zla_heamv Specifies the scalar *alpha*.
a	COMPLEX for cla_heamv DOUBLE COMPLEX for zla_heamv Array, DIMENSION (*lda*, *). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).
x	COMPLEX for cla_heamv DOUBLE COMPLEX for zla_heamv Array, DIMENSION at least (1+(*n*–1)*abs(*incx*)). Before entry, the incremented array *x* must contain the vector *X*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must be non-zero.
beta	REAL for cla_heamv DOUBLE PRECISION for zla_heamv Specifies the scalar *beta*. When *beta* is zero, you do not need to set *y* on input.
y	REAL for cla_heamv

	DOUBLE PRECISION for zla_heamv Array, DIMENSION at least (1 +(*n* - 1)*abs(*incy*)) otherwise. Before entry with non-zero *beta*, the incremented array *y* must contain the vector *Y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must be non-zero.

Output Parameters

y	Updated vector *Y*.

?la_hercond_c

*Computes the infinity norm condition number of op(A)*inv(diag(c)) for Hermitian indefinite matrices.*

Syntax

Fortran 77:

```
call cla_hercond_c( uplo, n, a, lda, af, ldaf, ipiv, c, capply, info, work, rwork )
call zla_hercond_c( uplo, n, a, lda, af, ldaf, ipiv, c, capply, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * inv(diag(c))

where the *c* is a REAL vector for cla_hercond_c and a DOUBLE PRECISION vector for zla_hercond_c.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
a	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array, DIMENSION (*lda*, *). On entry, the *n*-by-*n* matrix A. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of the array *a*. *lda* $\geq$ max(1,*n*).
af	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array, DIMENSION (*ldaf*, *). The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?hetrf. The second dimension of *af* must be at least max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*. *ldaf* $\geq$ max(1,*n*).
ipiv	INTEGER.

	Array with DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?hetrf.
c	REAL for cla_hercond_c DOUBLE PRECISION for zla_hercond_c Array *c* with DIMENSION *n*. The vector *c* in the formula op(*A*) * inv(diag(*c*)).
capply	LOGICAL. If .TRUE., then the function uses the vector *c* from the formula op(*A*) * inv(diag(*c*)).
work	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array DIMENSION 2**n*. Workspace.
rwork	REAL for cla_hercond_c DOUBLE PRECISION for zla_hercond_c Array DIMENSION *n*. Workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?hetrf

?la_hercond_x

*Computes the infinity norm condition number of op(A)*diag(x) for Hermitian indefinite matrices.*

Syntax

Fortran 77:

```
call cla_hercond_x( uplo, n, a, lda, af, ldaf, ipiv, x, info, work, rwork )
call zla_hercond_x( uplo, n, a, lda, af, ldaf, ipiv, x, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * diag(*x*)

where the *x* is a COMPLEX vector for cla_hercond_x and a DOUBLE COMPLEX vector for zla_hercond_x.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.

a	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array, DIMENSION (*lda*, *). On entry, the *n*-by-*n* matrix A. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*n*).
af	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array, DIMENSION (*ldaf*, *). The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?hetrf. The second dimension of *af* must be at least max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*. *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array with DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?hetrf.
x	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array *x* with DIMENSION *n*. The vector *x* in the formula op(*A*) * inv(diag(*x*)).
work	COMPLEX for cla_hercond_c DOUBLE COMPLEX for zla_hercond_c Array DIMENSION 2**n*. Workspace.
rwork	REAL for cla_hercond_c DOUBLE PRECISION for zla_hercond_c Array DIMENSION *n*. Workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?hetrf

?la_herfsx_extended

Improves the computed solution to a system of linear equations for Hermitian indefinite matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.

Syntax

Fortran 77:

call cla_herfsx_extended(*prec_type*, *uplo*, *n*, *nrhs*, *a*, *lda*, *af*, *ldaf*, *ipiv*, *colequ*, *c*, *b*, *ldb*, *y*, *ldy*, *berr_out*, *n_norms*, *err_bnds_norm*, *err_bnds_comp*, *res*, *ayb*, *dy*, *y_tail*, *rcond*, *ithresh*, *rthresh*, *dz_ub*, *ignore_cwise*, *info*)

call zla_herfsx_extended(*prec_type*, *uplo*, *n*, *nrhs*, *a*, *lda*, *af*, *ldaf*, *ipiv*, *colequ*, *c*, *b*, *ldb*, *y*, *ldy*, *berr_out*, *n_norms*, *err_bnds_norm*, *err_bnds_comp*, *res*, *ayb*, *dy*, *y_tail*, *rcond*, *ithresh*, *rthresh*, *dz_ub*, *ignore_cwise*, *info*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_herfsx_extended subroutine improves the computed solution to a system of linear equations by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. The ?herfsx routine calls ?la_herfsx_extended to perform iterative refinement.

In addition to normwise error bound, the code provides maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

Use ?la_herfsx_extended to set only the second fields of *err_bnds_norm* and *err_bnds_comp*.

Input Parameters

prec_type	INTEGER. Specifies the intermediate precision to be used in refinement. The value is defined by ilaprec(p), where *p* is a CHARACTER and: If *p* = 'S': Single. If *p* = 'D': Double. If *p* = 'I': Indigenous. If *p* = 'X', 'E': Extra.
uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrix *B*.
a, af, b, y	COMPLEX for cla_herfsx_extended DOUBLE COMPLEX for zla_herfsx_extended. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *y*(*ldy*,*). The array *a* contains the original *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*). The array *af* contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?hetrf. The second dimension of *af* must be at least max(1,*n*). The array *b* contains the right-hand-side of the matrix *B*. The second dimension of *b* must be at least max(1,*nrhs*). The array *y* on entry contains the solution matrix *X* as computed by ?hetrs. The second dimension of *y* must be at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of the array *a*; *lda* $\geq$ max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*; *ldaf* $\geq$ max(1,*n*).
ipiv	INTEGER. Array, DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?hetrf.
colequ	LOGICAL. If *colequ* = .TRUE., column equilibration was done to *A* before calling this routine. This is needed to compute the solution and error bounds correctly.
c	REAL for cla_herfsx_extended DOUBLE PRECISION for zla_herfsx_extended.

c contains the column scale factors for *A*. If *colequ* = .FALSE., *c* is not used.
If *c* is input, each element of *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by power of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb — INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).

ldy — INTEGER. The leading dimension of the array *y*; *ldy* ≥ max(1, *n*).

n_norms — INTEGER. Determines which error bounds to return. See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.
If *n_norms* ≥ 1, returns normwise error bounds.
If *n_norms* ≥ 2, returns componentwise error bounds.

err_bnds_norm — REAL for cla_herfsx_extended
DOUBLE PRECISION for zla_herfsx_extended.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error.
Normwise relative error in the *i*-th solution vector is defined as follows:

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the following three fields:

err=1 — "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for cla_herfsx_extended and *sqrt*(*n*)**dlamch*(ε) for zla_herfsx_extended.

err=2 — "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for cla_herfsx_extended and *sqrt*(*n*)**dlamch*(ε) for zla_herfsx_extended. This error bound should only be trusted if the previous boolean is true.

err=3 — Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for cla_herfsx_extended and *sqrt*(*n*)**dlamch*(ε) for

zla_herfsx_extended to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*.
Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.
Use this subroutine to set only the second field above.

err_bnds_comp

REAL for cla_herfsx_extended
DOUBLE PRECISION for zla_herfsx_extended.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1 — "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for cla_herfsx_extended and *sqrt*(*n*)**dlamch*(ε) for zla_herfsx_extended.

err=2 — "Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for cla_herfsx_extended and *sqrt*(*n*)**dlamch*(ε) for zla_herfsx_extended. This error bound should only be trusted if the previous boolean is true.

err=3 — Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for cla_herfsx_extended and *sqrt*(*n*)**dlamch*(ε) for

	zla_herfsx_extended to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.
res, *dy*, *y_tail*	COMPLEX for cla_herfsx_extended DOUBLE COMPLEX for zla_herfsx_extended. Workspace arrays of DIMENSION *n*. *res* holds the intermediate residual. *dy* holds the intermediate solution. *y_tail* holds the trailing bits of the intermediate solution.
ayb	REAL for cla_herfsx_extended DOUBLE PRECISION for zla_herfsx_extended. Workspace array, DIMENSION *n*.
rcond	REAL for cla_herfsx_extended DOUBLE PRECISION for zla_herfsx_extended. Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.
ithresh	INTEGER. The maximum number of residual computations allowed for refinement. The default is 10. For 'aggressive', set to 100 to permit convergence using approximate factorizations or factorizations other than LU. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.
rthresh	REAL for cla_herfsx_extended DOUBLE PRECISION for zla_herfsx_extended. Determines when to stop refinement if the error estimate stops decreasing. Refinement stops when the next solution no longer satisfies norm(dx_{*i*+1}) < *rthresh* * norm(dx_*i*) where norm(*z*) is the infinity norm of *Z*. *rthresh* satisfies 0 < *rthresh* ≤ 1. The default value is 0.5. For 'aggressive' set to 0.9 to permit convergence on extremely ill-conditioned matrices.
dz_ub	REAL for cla_herfsx_extended DOUBLE PRECISION for zla_herfsx_extended. Determines when to start considering componentwise convergence. Componentwise *dz_ub* convergence is only considered after each component of the solution *y* is stable, that is, the relative change in each component is less than *dz_ub*. The default value is 0.25, requiring the first bit to be stable.
ignore_cwise	LOGICAL

If .TRUE., the function ignores componentwise convergence. Default value is .FALSE.

Output Parameters

y	COMPLEX for cla_herfsx_extended DOUBLE COMPLEX for zla_herfsx_extended. The improved solution matrix *Y*.
berr_out	REAL for cla_herfsx_extended DOUBLE PRECISION for zla_herfsx_extended. Array, DIMENSION *nrhs*. *berr_out(j)* contains the componentwise relative backward error for right-hand-side *j* from the formula max(*i*) (abs(*res*(*i*)) / (abs(op(*A*))*abs(*y*) + abs(*B*))(*i*)) where abs(*z*) is the componentwise absolute value of the matrix or vector *Z*. This is computed by ?la_lin_berr.
err_bnds_norm, *err_bnds_comp*	Values of the corresponding input parameters improved after iterative refinement and stored in the second column of the array (1:*nrhs*, 2). The other elements are kept unchanged.
info	INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed. If *info* = -*i*, the *i*-th parameter had an illegal value.

See Also

?herfsx
?hetrf
?hetrs
?lamch
ilaprec
ilatrans
?la_lin_berr

?la_herpvgrw

Computes the reciprocal pivot growth factor norm(A)/ norm(U) for a Hermitian indefinite matrix.

Syntax

Fortran 77:

```
call cla_herpvgrw( uplo, n, info, a, lda, af, ldaf, ipiv, work )
call zla_herpvgrw( uplo, n, info, a, lda, af, ldaf, ipiv, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_herpvgrw routine computes the reciprocal pivot growth factor norm(*A*)/norm(*U*). The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the equilibrated matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, the order of the matrix *A*; *n* ≥ 0.
info	INTEGER. The value of INFO returned from ?hetrf, that is, the pivot in column *info* is exactly 0.
a, *af*	COMPLEX for cla_herpvgrw DOUBLE COMPLEX for zla_herpvgrw. Arrays: *a*(*lda*,*), *af*(*ldaf*,*). *a* contains the *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*). *af* contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?hetrf. The second dimension of *af* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of array *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of array *af*; *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array, DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?hetrf.
work	REAL for cla_herpvgrw DOUBLE PRECISION for zla_herpvgrw. Array, DIMENSION 2**n*. Workspace.

See Also

?hetrf

?la_lin_berr

Computes component-wise relative backward error.

Syntax

Fortran 77:

```
call sla_lin_berr(n, nz, nrhs, res, ayb, berr )
call dla_lin_berr(n, nz, nrhs, res, ayb, berr )
call cla_lin_berr(n, nz, nrhs, res, ayb, berr )
call zla_lin_berr(n, nz, nrhs, res, ayb, berr )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_lin_berr computes a component-wise relative backward error from the formula:

```
max(i) ( abs(R(i))/( abs(op(A_s))*abs(Y) + abs(B_s) )(i) )
```

where abs(*Z*) is the component-wise value of the matrix or vector *Z*.

Input Parameters

n	INTEGER. The number of linear equations, the order of the matrix *A*; $n \geq 0$.
nz	INTEGER. The parameter for guarding against spuriously zero residuals. (*nz*+1)*slamch('Safe minimum') is added to R(*i*) in the numerator of the relative backward error formula. The default value is *n*.
nrhs	INTEGER. Number of right-hand sides, the number of columns in the matrices *AYB*, *RES*, and *BERR*; $nrhs \geq 0$.
res, *ayb*	REAL for sla_lin_berr, cla_lin_berr DOUBLE PRECISION for dla_lin_berr, zla_lin_berr Arrays, DIMENSION (*n*,*nrhs*). *res* is the residual matrix, that is, the matrix *R* in the relative backward error formula. *ayb* is the denominator of that formula, that is, the matrix abs(op(*A_s*))*abs(*Y*) + abs(*B_s*). The matrices *A*, *Y*, and *B* are from iterative refinement. See description of ?la_gerfsx_extended.

Output Parameters

berr	REAL for sla_lin_berr DOUBLE PRECISION for dla_lin_berr COMPLEX for cla_lin_berr DOUBLE COMPLEX for zla_lin_berr The component-wise relative backward error.

See Also

?lamch
?la_gerfsx_extended

?la_porcond

Estimates the Skeel condition number for a symmetric positive-definite matrix.

Syntax

Fortran 77:

```
call sla_porcond( uplo, n, a, lda, af, ldaf, cmode, c, info, work, iwork )
call dla_porcond( uplo, n, a, lda, af, ldaf, cmode, c, info, work, iwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function estimates the Skeel condition number of

op(*A*) * op2(*C*)

where

the *cmode* parameter determines op2 as follows:

cmode **Value**	**op2(C)**
1	*C*

cmode **Value**	**op2(C)**
0	*I*
-1	inv(*C*)

The Skeel condition number

cond(*A*) = norminf(|inv(*A*)||*A*|)

is computed by computing scaling factors *R* such that

diag(*R*)*A*op2(*C*)

is row equilibrated and by computing the standard infinity-norm condition number.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.
a, *af*, *c*, *work*	REAL for sla_porcond DOUBLE PRECISION for dla_porcond Arrays: *a* (*lda*,*) contains the *n*-by-*n* matrix *A*. *af* (*ldaf*,*) contains the triangular factor L or U from the Cholesky factorization A = U^T*U or A = L*L^T, as computed by ?potrf. *c*, DIMENSION *n*. The vector *C* in the formula op(*A*) * op2(*C*). *work* is a workspace array of DIMENSION (3**n*). The second dimension of *a* and *af* must be at least max(1, *n*).
lda	INTEGER. The leading dimension of the array *ab*. *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of *af*. *ldaf* ≥ max(1,*n*).
cmode	INTEGER. Determines op2(*C*) in the formula op(*A*) * op2(*C*) as follows: If *cmode* = 1, op2(*C*) = *C*. If *cmode* = 0, op2(*C*) = *I*. If *cmode* = -1, op2(*C*) = inv(*C*).
iwork	INTEGER. Workspace array with DIMENSION *n*.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?potrf

?la_porcond_c

*Computes the infinity norm condition number of op(A)*inv(diag(c)) for Hermitian positive-definite matrices.*

Syntax

Fortran 77:

```
call cla_porcond_c( uplo, n, a, lda, af, ldaf, c, capply, info, work, rwork )
call zla_porcond_c( uplo, n, a, lda, af, ldaf, c, capply, info, work, rwork )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The function computes the infinity norm condition number of

`op(A) * inv(diag(c))`

where the *c* is a `REAL` vector for `cla_porcond_c` and a `DOUBLE PRECISION` vector for `zla_porcond_c`.

Input Parameters

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Specifies the triangle of A to store: If *uplo* = `'U'`, the upper triangle of *A* is stored, If *uplo* = `'L'`, the lower triangle of *A* is stored.
n	`INTEGER`. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.
a	`COMPLEX` for `cla_porcond_c` `DOUBLE COMPLEX` for `zla_porcond_c` Array, `DIMENSION (lda, *)`. On entry, the *n*-by-*n* matrix A. The second dimension of *a* must be at least `max(1,n)`.
lda	`INTEGER`. The leading dimension of the array *a*. *lda* ≥ `max(1,n)`.
af	`COMPLEX` for `cla_porcond_c` `DOUBLE COMPLEX` for `zla_porcond_c` Array, `DIMENSION (ldaf, *)`. The triangular factor `L` or `U` from the Cholesky factorization $A = U^H*U$ or $A = L*L^H$, as computed by `?potrf`. The second dimension of *af* must be at least `max(1,n)`.
ldaf	`INTEGER`. The leading dimension of the array *af*. *ldaf* ≥ `max(1,n)`.
c	`REAL` for `cla_porcond_c` `DOUBLE PRECISION` for `zla_porcond_c` Array *c* with `DIMENSION` *n*. The vector *c* in the formula `op(A) * inv(diag(c))`.
capply	`LOGICAL`. If `.TRUE.`, then the function uses the vector *c* from the formula `op(A) * inv(diag(c))`.
work	`COMPLEX` for `cla_porcond_c` `DOUBLE COMPLEX` for `zla_porcond_c` Array `DIMENSION` 2**n*. Workspace.
rwork	`REAL` for `cla_porcond_c` `DOUBLE PRECISION` for `zla_porcond_c` Array `DIMENSION` *n*. Workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?potrf

?la_porcond_x

*Computes the infinity norm condition number of op(A)*diag(x) for Hermitian positive-definite matrices.*

Syntax

Fortran 77:

```
call cla_porcond_x( uplo, n, a, lda, af, ldaf, x, info, work, rwork )
call zla_porcond_x( uplo, n, a, lda, af, ldaf, x, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

op(*A*) * diag(*x*)

where the *x* is a COMPLEX vector for cla_porcond_x and a DOUBLE COMPLEX vector for zla_porcond_x.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; *n* ≥ 0.
a	COMPLEX for cla_porcond_c DOUBLE COMPLEX for zla_porcond_c Array, DIMENSION (*lda*, *). On entry, the *n*-by-*n* matrix A. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(1,*n*).
af	COMPLEX for cla_porcond_c DOUBLE COMPLEX for zla_porcond_c Array, DIMENSION (*ldaf*, *). The triangular factor L or U from the Cholesky factorization $A = U^H*U$ or $A = L*L^H$, as computed by ?potrf. The second dimension of *af* must be at least max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*. *ldaf* ≥ max(1,*n*).
x	COMPLEX for cla_porcond_c DOUBLE COMPLEX for zla_porcond_c

	Array *x* with DIMENSION *n*. The vector *x* in the formula op(*A*) * inv(diag(*x*)).
work	COMPLEX for cla_porcond_c DOUBLE COMPLEX for zla_porcond_c Array DIMENSION 2**n*. Workspace.
rwork	REAL for cla_porcond_c DOUBLE PRECISION for zla_porcond_c Array DIMENSION *n*. Workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?potrf

?la_porfsx_extended

Improves the computed solution to a system of linear equations for symmetric or Hermitian positive-definite matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.

Syntax

Fortran 77:

```
call sla_porfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, colequ, c, b,
ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call dla_porfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, colequ, c, b,
ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call cla_porfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, colequ, c, b,
ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call zla_porfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, colequ, c, b,
ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_porfsx_extended subroutine improves the computed solution to a system of linear equations by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. The ?herfsx routine calls ?la_porfsx_extended to perform iterative refinement.

In addition to normwise error bound, the code provides maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

Use ?la_porfsx_extended to set only the second fields of *err_bnds_norm* and *err_bnds_comp*.

Input Parameters

prec_type	INTEGER. Specifies the intermediate precision to be used in refinement. The value is defined by ilaprec(p), where *p* is a CHARACTER and: If *p* = 'S': Single. If *p* = 'D': Double. If *p* = 'I': Indigenous. If *p* = 'X', 'E': Extra.
uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrix *B*.
a, af, b, y	REAL for sla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended COMPLEX for cla_porfsx_extended DOUBLE COMPLEX for zla_porfsx_extended. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *y*(*ldy*,*). The array *a* contains the original *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*). The array *af* contains the triangular factor L or U from the Cholesky factorization as computed by ?potrf: $A = U^T*U$ or $A = L*L^T$ for real flavors, $A = U^H*U$ or $A = L*L^H$ for complex flavors. The second dimension of *af* must be at least max(1,*n*). The array *b* contains the right-hand-side of the matrix *B*. The second dimension of *b* must be at least max(1,*nrhs*). The array *y* on entry contains the solution matrix *X* as computed by ?potrs. The second dimension of *y* must be at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*; *ldaf* ≥ max(1,*n*).
colequ	LOGICAL. If *colequ* = .TRUE., column equilibration was done to *A* before calling this routine. This is needed to compute the solution and error bounds correctly.
c	REAL for sla_porfsx_extended and cla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended. *c* contains the column scale factors for *A*. If *colequ* = .FALSE., *c* is not used. If *c* is input, each element of *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by power of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).
ldy	INTEGER. The leading dimension of the array *y*; *ldy* ≥ max(1, *n*).

n_norms INTEGER. Determines which error bounds to return. See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.
If *n_norms* ≥ 1, returns normwise error bounds.
If *n_norms* ≥ 2, returns componentwise error bounds.

err_bnds_norm REAL for sla_porfsx_extended and cla_porfsx_extended
DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended.
Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error.
Normwise relative error in the *i*-th solution vector is defined as follows:

$$\frac{\max_j \left| Xtrue_{ji} - X_{ji} \right|}{\max_j \left| X_{ji} \right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the following three fields:

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for sla_porfsx_extended/ cla_porfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_porfsx_extended/ zla_porfsx_extended.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for sla_porfsx_extended/ cla_porfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_porfsx_extended/ zla_porfsx_extended. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for sla_porfsx_extended/ cla_porfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_porfsx_extended/ zla_porfsx_extended to determine if the error estimate is "guaranteed". These reciprocal

condition numbers are `1/(norm(1/z,inf)*norm(z,inf))` for some appropriately scaled matrix *Z*.
Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.
Use this subroutine to set only the second field above.

err_bnds_comp

`REAL` for `sla_porfsx_extended` and `cla_porfsx_extended`
`DOUBLE PRECISION` for `dla_porfsx_extended` and `zla_porfsx_extended`.
Array, `DIMENSION (`*nrhs*`,`*n_err_bnds*`)`. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1
"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for `sla_porfsx_extended`/`cla_porfsx_extended` and *sqrt*(*n*)**dlamch*(ε) for `dla_porfsx_extended`/`zla_porfsx_extended`.

err=2
"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for `sla_porfsx_extended`/`cla_porfsx_extended` and *sqrt*(*n*)**dlamch*(ε) for `dla_porfsx_extended`/`zla_porfsx_extended`. This error bound should only be trusted if the previous boolean is true.

err=3
Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold

	sqrt(*n*)**slamch*(ε) for sla_porfsx_extended/ cla_porfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_porfsx_extended/ zla_porfsx_extended to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.
res, *dy*, *y_tail*	REAL for sla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended COMPLEX for cla_porfsx_extended DOUBLE COMPLEX for zla_porfsx_extended. Workspace arrays of DIMENSION *n*. *res* holds the intermediate residual. *dy* holds the intermediate solution. *y_tail* holds the trailing bits of the intermediate solution.
ayb	REAL for sla_porfsx_extended and cla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended. Workspace array, DIMENSION *n*.
rcond	REAL for sla_porfsx_extended and cla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended. Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.
ithresh	INTEGER. The maximum number of residual computations allowed for refinement. The default is 10. For 'aggressive', set to 100 to permit convergence using approximate factorizations or factorizations other than LU. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.
rthresh	REAL for sla_porfsx_extended and cla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended. Determines when to stop refinement if the error estimate stops decreasing. Refinement stops when the next solution no longer satisfies norm(dx_{*i*+1}) < *rthresh* * norm(dx_*i*) where norm(*z*) is the infinity norm of *Z*. *rthresh* satisfies 0 < *rthresh* ≤ 1.

	The default value is 0.5. For 'aggressive' set to 0.9 to permit convergence on extremely ill-conditioned matrices.
dz_ub	REAL for sla_porfsx_extended and cla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended. Determines when to start considering componentwise convergence. Componentwise *dz_ub* convergence is only considered after each component of the solution *y* is stable, that is, the relative change in each component is less than *dz_ub*. The default value is 0.25, requiring the first bit to be stable.
ignore_cwise	LOGICAL If .TRUE., the function ignores componentwise convergence. Default value is .FALSE.

Output Parameters

y	REAL for sla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended COMPLEX for cla_porfsx_extended DOUBLE COMPLEX for zla_porfsx_extended. The improved solution matrix *Y*.
berr_out	REAL for sla_porfsx_extended and cla_porfsx_extended DOUBLE PRECISION for dla_porfsx_extended and zla_porfsx_extended. Array, DIMENSION *nrhs*. *berr_out(j)* contains the componentwise relative backward error for right-hand-side *j* from the formula max(*i*) (abs(*res*(*i*)) / (abs(op(*A*))*abs(*y*) + abs(*B*))(*i*)) where abs(*z*) is the componentwise absolute value of the matrix or vector *Z*. This is computed by ?la_lin_berr.
err_bnds_norm, *err_bnds_comp*	Values of the corresponding input parameters improved after iterative refinement and stored in the second column of the array (1:*nrhs*, 2). The other elements are kept unchanged.
info	INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed. If *info* = -*i*, the *i*-th parameter had an illegal value.

See Also

?porfsx
?potrf
?potrs
?lamch
ilaprec
ilatrans
?la_lin_berr

?la_porpvgrw

Computes the reciprocal pivot growth factor norm(A)/ norm(U) for a symmetric or Hermitian positive-definite matrix.

Syntax

Fortran 77:

```
call sla_porpvgrw( uplo, ncols, a, lda, af, ldaf, work )
call dla_porpvgrw( uplo, ncols, a, lda, af, ldaf, work )
call cla_porpvgrw( uplo, ncols, a, lda, af, ldaf, work )
call zla_porpvgrw( uplo, ncols, a, lda, af, ldaf, work )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?la_porpvgrw` routine computes the reciprocal pivot growth factor `norm(A)/norm(U)`. The *max absolute element* norm is used. If this is much less than 1, the stability of the LU factorization of the equilibrated matrix A could be poor. This also means that the solution X, estimated condition numbers, and error bounds could be unreliable.

Input Parameters

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Specifies the triangle of A to store: If *uplo* = `'U'`, the upper triangle of A is stored, If *uplo* = `'L'`, the lower triangle of A is stored.
ncols	`INTEGER`. The number of columns of the matrix A; $ncols \geq 0$.
a, *af*	`REAL` for `sla_porpvgrw` `DOUBLE PRECISION` for `dla_porpvgrw` `COMPLEX` for `cla_porpvgrw` `DOUBLE COMPLEX` for `zla_porpvgrw`. Arrays: *a*(*lda*,*), *af*(*ldaf*,*). The array *a* contains the input n-by-n matrix A. The second dimension of *a* must be at least `max(1,n)`. The array *af* contains the triangular factor `L` or `U` from the Cholesky factorization as computed by `?potrf`: `A = U`^{T}`*U` or `A = L*L`^{T} for real flavors, `A = U`^{H}`*U` or `A = L*L`^{H} for complex flavors. The second dimension of *af* must be at least `max(1,n)`.
lda	`INTEGER`. The leading dimension of *a*; *lda* $\geq$ `max(1,n)`.
ldaf	`INTEGER`. The leading dimension of *af*; *ldaf* $\geq$ `max(1,n)`.
work	`REAL` for `sla_porpvgrw` and `cla_porpvgrw` `DOUBLE PRECISION` for `dla_porpvgrw` and `zla_porpvgrw`. Workspace array, dimension 2*n.

See Also

?potrf

?laqhe

Scales a Hermitian matrix.

Syntax

```
call claqhe( uplo, n, a, lda, s, scond, amax, equed )
```

```
call zlaqhe( uplo, n, a, lda, s, scond, amax, equed )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine equilibrates a Hermitian matrix *A* using the scaling factors in the vector *s*.

Input Parameters

uplo	CHARACTER*1. Specifies whether to store the upper or lower part of the Hermitian matrix *A*. If *uplo* = 'U', the upper triangular part of *A*; if *uplo* = 'L', the lower triangular part of *A*.
n	INTEGER. The order of the matrix *A*. *n* ≥ 0.
a	COMPLEX for claqhe DOUBLE COMPLEX for zlaqhe Array, DIMENSION (*lda*,*n*). On entry, the Hermitian matrix *A*. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *a* contains the upper triangular part of matrix *A* and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *a* contains the lower triangular part of matrix *A* and the strictly upper triangular part of *a* is not referenced.
lda	INTEGER. The leading dimension of the array *a*. *lda* ≥ max(*n*,1).
s	REAL for claqhe DOUBLE PRECISION for zlaqhe Array, DIMENSION (*n*). The scale factors for *A*.
scond	REAL for claqhe DOUBLE PRECISION for zlaqhe Ratio of the smallest *s*(*i*) to the largest *s*(*i*).
amax	REAL for claqhe DOUBLE PRECISION for zlaqhe Absolute value of largest matrix entry.

Output Parameters

a	If *equed* = 'Y', *a* contains the equilibrated matrix diag(*s*)*A*diag(*s*).
equed	CHARACTER*1. Specifies whether or not equilibration was done. If *equed* = 'N': No equilibration. If *equed* = 'Y': Equilibration was done, that is, *A* has been replaced by diag(*s*)*A*diag(*s*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*. The parameter *thresh* is a threshold value used to decide if scaling should be done based on the ratio of the scaling factors. If *scond* < *thresh*, scaling is done.

The *large* and *small* parameters are threshold values used to decide if scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, scaling is done.

?laqhp

Scales a Hermitian matrix stored in packed form.

Syntax

```
call claqhp( uplo, n, ap, s, scond, amax, equed )
call zlaqhp( uplo, n, ap, s, scond, amax, equed )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine equilibrates a Hermitian matrix *A* using the scaling factors in the vector *s*.

Input Parameters

uplo — `CHARACTER*1`.
Specifies whether to store the upper or lower part of the Hermitian matrix *A*.
If *uplo* = `'U'`, the upper triangular part of *A*;
if *uplo* = `'L'`, the lower triangular part of *A*.

n — `INTEGER`. The order of the matrix *A*.
$n \geq 0$.

ap — `COMPLEX` for `claqhp`
`DOUBLE COMPLEX` for `zlaqhp`
Array, `DIMENSION` (*n**(*n*+1)/2). The Hermitian matrix *A*.

- If *uplo* = `'U'`, the upper triangular part of the Hermitian matrix *A* is stored in the packed array *ap* as follows:

  ap(*i*+(*j*-1)**j*/2) = *A*(*i*,*j*) for $1 \leq i \leq j$.
- If *uplo* = `'L'`, the lower triangular part of Hermitian matrix *A* is stored in the packed array *ap* as follows:

  ap(*i*+(*j*-1)*(2*n*-j)/2) = *A*(*i*,*j*) for $j \leq i \leq n$.

s — `REAL` for `claqhp`
`DOUBLE PRECISION` for `zlaqhp`
Array, `DIMENSION` (*n*). The scale factors for *A*.

scond — `REAL` for `claqhp`
`DOUBLE PRECISION` for `zlaqhp`
Ratio of the smallest *s*(*i*) to the largest *s*(*i*).

amax — `REAL` for `claqhp`
`DOUBLE PRECISION` for `zlaqhp`
Absolute value of largest matrix entry.

Output Parameters

a — If *equed* = `'Y'`, *a* contains the equilibrated matrix `diag(s)*A*diag(s)` in the same storage format as on input.

equed	CHARACTER*1. Specifies whether or not equilibration was done. If *equed* = 'N': No equilibration. If *equed* = 'Y': Equilibration was done, that is, *A* has been replaced by diag(*s*)*A*diag(*s*).

Application Notes

The routine uses internal parameters *thresh*, *large*, and *small*. The parameter *thresh* is a threshold value used to decide if scaling should be done based on the ratio of the scaling factors. If *scond* < *thresh*, scaling is done.

The *large* and *small* parameters are threshold values used to decide if scaling should be done based on the absolute size of the largest matrix element. If *amax* > *large* or *amax* < *small*, scaling is done.

?larcm

Multiplies a square real matrix by a complex matrix.

Syntax

```
call clarcm( m, n, a, lda, b, ldb, c, ldc, rwork )
call zlarcm( m, n, a, lda, b, ldb, c, ldc, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine performs a simple matrix-matrix multiplication of the form

C = *A***B*,

where *A* is *m*-by-*m* and real, *B* is *m*-by-*n* and complex, *C* is *m*-by-*n* and complex.

Input Parameters

m	INTEGER. The number of rows and columns of the matrix *A* and of the number of rows of the matrix *C* (*m* ≥ 0).
n	INTEGER. The number of columns of the matrix *B* and the number of columns of the matrix *C* (*n* ≥ 0).
a	REAL for clarcm DOUBLE PRECISION for zlarcm Array, DIMENSION (*lda*, *m*). Contains the *m*-by-*m* matrix *A*.
lda	INTEGER. The leading dimension of the array *a*, *lda* ≥ max(1, *m*).
b	COMPLEX for clarcm DOUBLE COMPLEX for zlarcm Array, DIMENSION (*ldb*, *n*). Contains the *m*-by-*n* matrix *B*.
ldb	INTEGER. The leading dimension of the array *b*, *ldb* ≥ max(1, *n*).
ldc	INTEGER. The leading dimension of the output array *c*, *ldc* ≥ max(1, *m*).
rwork	REAL for clarcm DOUBLE PRECISION for zlarcm Workspace array, DIMENSION (2**m***n*).

Output Parameters

c	COMPLEX for clarcm DOUBLE COMPLEX for zlarcm Array, DIMENSION (*ldc*, *n*). Contains the *m*-by-*n* matrix *C*.

?la_rpvgrw

Computes the reciprocal pivot growth factor norm(A)/ norm(U) for a general matrix.

Syntax

Fortran 77:

```
call sla_rpvgrw( n, ncols, a, lda, af, ldaf )
call dla_rpvgrw( n, ncols, a, lda, af, ldaf )
call cla_rpvgrw( n, ncols, a, lda, af, ldaf )
call zla_rpvgrw( n, ncols, a, lda, af, ldaf )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_rpvgrw routine computes the reciprocal pivot growth factor norm(*A*)/norm(*U*). The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the equilibrated matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable.

Input Parameters

n	INTEGER. The number of linear equations, the order of the matrix *A*; *n* ≥ 0.
ncols	INTEGER. The number of columns of the matrix *A*; *ncols* ≥ 0.
a, *af*	REAL for sla_rpvgrw DOUBLE PRECISION for dla_rpvgrw COMPLEX for cla_rpvgrw DOUBLE COMPLEX for zla_rpvgrw. Arrays: *a*(*lda*,*), *af*(*ldaf*,*). The array *a* contains the input *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*). The array *af* contains the factors L and U from the factorization triangular factor L or U from the Cholesky factorization A = P*L*U as computed by ?getrf. The second dimension of *af* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1,*n*).

See Also

?getrf

?larscl2

Performs reciprocal diagonal scaling on a vector.

Syntax

Fortran 77:

```
call slarscl2(m, n, d, x, ldx)
```

```
call dlarscl2(m, n, d, x, ldx) call clarscl2(m, n, d, x, ldx) call zlarscl2(m, n, d, x, ldx)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?larscl2` routines perform reciprocal diagonal scaling on a vector

$x := D^{-1}*x$,

where:

x is a vector, and

D is a diagonal matrix.

Input Parameters

m	`INTEGER`. Specifies the number of rows of the matrix D and the number of elements of the vector x. The value of m must be at least zero.
n	`INTEGER`. The number of columns of D and x. The value of n must be at least zero.
d	`REAL` for `slarscl2` and `clarscl2`. `DOUBLE PRECISION` for `dlarscl2` and `zlarscl2`. Array, `DIMENSION` m. Diagonal matrix D stored as a vector of length m.
x	`REAL` for `slarscl2`. `DOUBLE PRECISION` for `dlarscl2`. `COMPLEX` for `clarscl2`. `DOUBLE COMPLEX` for `zlarscl2`. Array, `DIMENSION (ldx, n)`. The vector x to scale by D.
ldx	`INTEGER`. The leading dimension of the vector x. The value of ldx must be at least zero.

Output Parameters

x	Scaled vector x.

?lascl2

Performs diagonal scaling on a vector.

Syntax

Fortran 77:

```
call slascl2(m, n, d, x, ldx)
```

```
call dlascl2(m, n, d, x, ldx)
call clascl2(m, n, d, x, ldx)
call zlascl2(m, n, d, x, ldx)
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?lascl2` routines perform diagonal scaling on a vector

x := *D***x*,

where:

x is a vector, and

D is a diagonal matrix.

Input Parameters

m	`INTEGER`. Specifies the number of rows of the matrix *D* and the number of elements of the vector *x*. The value of *m* must be at least zero.
n	`INTEGER`. The number of columns of *D* and *x*. The value of *n* must be at least zero.
d	`REAL` for `slascl2` and `clascl2`. `DOUBLE PRECISION` for `dlascl2` and `zlascl2`. Array, `DIMENSION` *m*. Diagonal matrix *D* stored as a vector of length *m*.
x	`REAL` for `slascl2`. `DOUBLE PRECISION` for `dlascl2`. `COMPLEX` for `clascl2`. `DOUBLE COMPLEX` for `zlascl2`. Array, `DIMENSION (`*ldx*`,`*n*`)`. The vector *x* to scale by *D*.
ldx	`INTEGER`. The leading dimension of the vector *x*. The value of *ldx* must be at least zero.

Output Parameters

x	Scaled vector *x*.

?la_syamv

Computes a matrix-vector product using a symmetric indefinite matrix to calculate error bounds.

Syntax

Fortran 77:

```
call sla_syamv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
call dla_syamv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
call cla_syamv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
call zla_syamv(uplo, n, alpha, a, lda, x, incx, beta, y, incy)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_syamv routines perform a matrix-vector operation defined as

y := *alpha**abs(*A*)*abs(*x*) + *beta**abs(*y*),

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *n*-by-*n* Hermitian matrix.

This function is primarily used in calculating error bounds. To protect against underflow during evaluation, the function perturbs components in the resulting vector away from zero by (*n* + 1) times the underflow threshold. To prevent unnecessarily large errors for block structure embedded in general matrices, the function does not perturb *symbolically* zero components. A zero entry is considered *symbolic* if all multiplications involved in computing that entry have at least one zero multiplicand.

Input Parameters

uplo	CHARACTER*1. Specifies whether the upper or lower triangular part of the array A is to be referenced: If *uplo* = 'BLAS_UPPER', only the upper triangular part of A is to be referenced, If *uplo* = 'BLAS_LOWER', only the lower triangular part of A is to be referenced.
n	INTEGER. Specifies the number of rows and columns of the matrix *A*. The value of *n* must be at least zero.
alpha	REAL for sla_syamv and cla_syamv DOUBLE PRECISION for dla_syamv and zla_syamv. Specifies the scalar *alpha*.
a	REAL for sla_syamv DOUBLE PRECISION for dla_syamv COMPLEX for cla_syamv DOUBLE COMPLEX for zla_syamv. Array, DIMENSION (*lda*, *). Before entry, the leading *m*-by-*n* part of the array *a* must contain the matrix of coefficients. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least max(1, *n*).
x	REAL for sla_syamv DOUBLE PRECISION for dla_syamv COMPLEX for cla_syamv DOUBLE COMPLEX for zla_syamv. Array, DIMENSION at least (1+(*n*–1)*abs(*incx*)). Before entry, the incremented array *x* must contain the vector *X*.
incx	INTEGER. Specifies the increment for the elements of *x*. The value of *incx* must be non-zero.
beta	REAL for sla_syamv and cla_syamv DOUBLE PRECISION for dla_syamv and zla_syamv

	Specifies the scalar *beta*. When *beta* is zero, you do not need to set *y* on input.
y	REAL for sla_syamv and cla_syamv DOUBLE PRECISION for dla_syamv and zla_syamv Array, DIMENSION at least (1 +(*n* – 1)*abs(*incy*)) otherwise. Before entry with non-zero *beta*, the incremented array *y* must contain the vector *Y*.
incy	INTEGER. Specifies the increment for the elements of *y*. The value of *incy* must be non-zero.

Output Parameters

y	Updated vector *Y*.

?la_syrcond

Estimates the Skeel condition number for a symmetric indefinite matrix.

Syntax

Fortran 77:

```
call sla_syrcond( uplo, n, a, lda, af, ldaf, ipiv, cmode, c, info, work, iwork )
call dla_syrcond( uplo, n, a, lda, af, ldaf, ipiv, cmode, c, info, work, iwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function estimates the Skeel condition number of

op(*A*) * op2(*C*)

where

the *cmode* parameter determines op2 as follows:

cmode **Value**	**op2(C)**
1	*C*
0	*I*
-1	inv(*C*)

The Skeel condition number

cond(*A*) = norminf(|inv(*A*)||*A*|)

is computed by computing scaling factors *R* such that

diag(*R*)*A*op2(*C*)

is row equilibrated and by computing the standard infinity-norm condition number.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'.

	Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
a, *af*, *c*, *work*	REAL for sla_syrcond DOUBLE PRECISION for dla_syrcond Arrays: *ab* (*lda*,*) contains the *n*-by-*n* matrix *A*. *af* (*ldaf*,*) contains the The block diagonal matrix D and the multipliers used to obtain the factor L or U as computed by ?sytrf. The second dimension of *a* and *af* must be at least max(1, *n*). *c*, DIMENSION *n*. The vector *C* in the formula op(*A*) * op2(*C*). *work* is a workspace array of DIMENSION (3**n*).
lda	INTEGER. The leading dimension of the array *ab*. *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of *af*. *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array with DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?sytrf.
cmode	INTEGER. Determines op2(*C*) in the formula op(*A*) * op2(*C*) as follows: If *cmode* = 1, op2(*C*) = *C*. If *cmode* = 0, op2(*C*) = *I*. If *cmode* = -1, op2(*C*) = inv(*C*).
iwork	INTEGER. Workspace array with DIMENSION *n*.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?sytrf

?la_syrcond_c

*Computes the infinity norm condition number of op(A)*inv(diag(c)) for symmetric indefinite matrices.*

Syntax

Fortran 77:

```
call cla_syrcond_c( uplo, n, a, lda, af, ldaf, ipiv, c, capply, info, work, rwork )
call zla_syrcond_c( uplo, n, a, lda, af, ldaf, ipiv, c, capply, info, work, rwork )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function computes the infinity norm condition number of

```
op(A) * inv(diag(c))
```

where the *c* is a REAL vector for cla_syrcond_c and a DOUBLE PRECISION vector for zla_syrcond_c.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
a	COMPLEX for cla_syrcond_c DOUBLE COMPLEX for zla_syrcond_c Array, DIMENSION (*lda*, *). On entry, the *n*-by-*n* matrix A. The second dimension of *a* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of the array *a*. $lda \geq \max(1,n)$.
af	COMPLEX for cla_syrcond_c DOUBLE COMPLEX for zla_syrcond_c Array, DIMENSION (*ldaf*, *). The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?sytrf. The second dimension of *af* must be at least max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*. $ldaf \geq \max(1,n)$.
ipiv	INTEGER. Array with DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?sytrf.
c	REAL for cla_syrcond_c DOUBLE PRECISION for zla_syrcond_c Array *c* with DIMENSION *n*. The vector *c* in the formula op(*A*) * inv(diag(*c*)).
capply	LOGICAL. If .TRUE., then the function uses the vector *c* from the formula op(*A*) * inv(diag(*c*)).
work	COMPLEX for cla_syrcond_c DOUBLE COMPLEX for zla_syrcond_c Array DIMENSION 2**n*. Workspace.
rwork	REAL for cla_syrcond_c DOUBLE PRECISION for zla_syrcond_c Array DIMENSION *n*. Workspace.

Output Parameters

info	INTEGER. If *info* = 0, the execution is successful. If *i* > 0, the *i*-th parameter is invalid.

See Also

?sytrf

?la_syrcond_x

*Computes the infinity norm condition number of op(A)*diag(x) for symmetric indefinite matrices.*

Syntax

Fortran 77:

```
call cla_syrcond_x( uplo, n, a, lda, af, ldaf, ipiv, x, info, work, rwork )
call zla_syrcond_x( uplo, n, a, lda, af, ldaf, ipiv, x, info, work, rwork )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The function computes the infinity norm condition number of

`op(A) * diag(x)`

where the *x* is a `COMPLEX` vector for `cla_syrcond_x` and a `DOUBLE COMPLEX` vector for `zla_syrcond_x`.

Input Parameters

uplo	`CHARACTER*1`. Must be `'U'` or `'L'`. Specifies the triangle of A to store: If *uplo* = `'U'`, the upper triangle of *A* is stored, If *uplo* = `'L'`, the lower triangle of *A* is stored.
n	`INTEGER`. The number of linear equations, that is, the order of the matrix *A*; $n \geq 0$.
a	`COMPLEX` for `cla_syrcond_c` `DOUBLE COMPLEX` for `zla_syrcond_c` Array, `DIMENSION (lda, *)`. On entry, the *n*-by-*n* matrix A. The second dimension of *a* must be at least `max(1,n)`.
lda	`INTEGER`. The leading dimension of the array *a*. $lda \geq \max(1,n)$.
af	`COMPLEX` for `cla_syrcond_c` `DOUBLE COMPLEX` for `zla_syrcond_c` Array, `DIMENSION (ldaf, *)`. The block diagonal matrix D and the multipliers used to obtain the factor `U` or `L` as computed by `?sytrf`. The second dimension of *af* must be at least `max(1,n)`.
ldaf	`INTEGER`. The leading dimension of the array *af*. $ldaf \geq \max(1,n)$.
ipiv	`INTEGER`. Array with `DIMENSION` *n*. Details of the interchanges and the block structure of D as determined by `?sytrf`.
x	`COMPLEX` for `cla_syrcond_c` `DOUBLE COMPLEX` for `zla_syrcond_c` Array *x* with `DIMENSION` *n*. The vector *x* in the formula `op(A) * inv(diag(x))`.
work	`COMPLEX` for `cla_syrcond_c` `DOUBLE COMPLEX` for `zla_syrcond_c` Array `DIMENSION` 2**n*. Workspace.
rwork	`REAL` for `cla_syrcond_c` `DOUBLE PRECISION` for `zla_syrcond_c` Array `DIMENSION` *n*. Workspace.

Output Parameters

info	`INTEGER`.

If *info* = 0, the execution is successful.
If *i* > 0, the *i*-th parameter is invalid.

See Also

?sytrf

?la_syrfsx_extended

Improves the computed solution to a system of linear equations for symmetric indefinite matrices by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution.

Syntax

Fortran 77:

```
call sla_syrfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, ipiv, colequ, c,
b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call dla_syrfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, ipiv, colequ, c,
b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call cla_syrfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, ipiv, colequ, c,
b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )

call zla_syrfsx_extended( prec_type, uplo, n, nrhs, a, lda, af, ldaf, ipiv, colequ, c,
b, ldb, y, ldy, berr_out, n_norms, err_bnds_norm, err_bnds_comp, res, ayb, dy, y_tail,
rcond, ithresh, rthresh, dz_ub, ignore_cwise, info )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `?la_syrfsx_extended` subroutine improves the computed solution to a system of linear equations by performing extra-precise iterative refinement and provides error bounds and backward error estimates for the solution. The `?syrfsx` routine calls `?la_syrfsx_extended` to perform iterative refinement.

In addition to normwise error bound, the code provides maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

Use `?la_syrfsx_extended` to set only the second fields of *err_bnds_norm* and *err_bnds_comp*.

Input Parameters

prec_type — `INTEGER`.
Specifies the intermediate precision to be used in refinement. The value is defined by `ilaprec(p)`, where *p* is a `CHARACTER` and:
If *p* = `'S'`: Single.
If *p* = `'D'`: Double.
If *p* = `'I'`: Indigenous.
If *p* = `'X'`, `'E'`: Extra.

uplo — `CHARACTER*1`. Must be `'U'` or `'L'`.

	Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations; the order of the matrix *A*; $n \geq 0$.
nrhs	INTEGER. The number of right-hand sides; the number of columns of the matrix *B*.
a, *af*, *b*, *y*	REAL for sla_syrfsx_extended DOUBLE PRECISION for dla_syrfsx_extended COMPLEX for cla_syrfsx_extended DOUBLE COMPLEX for zla_syrfsx_extended. Arrays: *a*(*lda*,*), *af*(*ldaf*,*), *b*(*ldb*,*), *y*(*ldy*,*). The array *a* contains the original *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*). The array *af* contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?sytrf. The second dimension of *af* must be at least max(1,*n*). The array *b* contains the right-hand-side of the matrix *B*. The second dimension of *b* must be at least max(1,*nrhs*). The array *y* on entry contains the solution matrix *X* as computed by ?sytrs. The second dimension of *y* must be at least max(1,*nrhs*).
lda	INTEGER. The leading dimension of the array *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of the array *af*; *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array with DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?sytrf.
colequ	LOGICAL. If *colequ* = .TRUE., column equilibration was done to *A* before calling this routine. This is needed to compute the solution and error bounds correctly.
c	REAL for sla_syrfsx_extended and cla_syrfsx_extended DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended. *c* contains the column scale factors for *A*. If *colequ* = .FALSE., *c* is not used. If *c* is input, each element of *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by power of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.
ldb	INTEGER. The leading dimension of the array *b*; *ldb* ≥ max(1, *n*).
ldy	INTEGER. The leading dimension of the array *y*; *ldy* ≥ max(1, *n*).
n_norms	INTEGER. Determines which error bounds to return. See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below. If *n_norms* ≥ 1, returns normwise error bounds. If *n_norms* ≥ 2, returns componentwise error bounds.
err_bnds_norm	REAL for sla_syrfsx_extended and cla_syrfsx_extended DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended. Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error.

Normwise relative error in the *i*-th solution vector is defined as follows:

$$\frac{\max_j \left|Xtrue_{ji} - X_{ji}\right|}{\max_j \left|X_{ji}\right|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.
The first index in *err_bnds_norm(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_norm(:,err)* contains the following three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for sla_syrfsx_extended/ cla_syrfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_syrfsx_extended/ zla_syrfsx_extended.
err=2	"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for sla_syrfsx_extended/ cla_syrfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_syrfsx_extended/ zla_syrfsx_extended. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for sla_syrfsx_extended/ cla_syrfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_syrfsx_extended/ zla_syrfsx_extended to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*. Let *z*=*s***a*, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.

err_bnds_comp	REAL for sla_syrfsx_extended and cla_syrfsx_extended DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended.

Array, DIMENSION (*nrhs*,*n_err_bnds*). For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:
Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{\left|Xtrue_{ji} - X_{ji}\right|}{\left|X_{ji}\right|}$$

The array is indexed by the right-hand side *i*, on which the componentwise relative error depends, and by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is nit requested (*params(3)* = 0.0), then *err_bnds_comp* is not accessed. If *n_err_bnds* < 3, then at most the first *(:,n_err_bnds)* entries are returned.
The first index in *err_bnds_comp(i,:)* corresponds to the *i*-th right-hand side.
The second index in *err_bnds_comp(:,err)* contains the follwoing three fields:

err=1	"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold *sqrt*(*n*)**slamch*(ε) for sla_syrfsx_extended/ cla_syrfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_syrfsx_extended/ zla_syrfsx_extended.
err=2	"Guaranteed" error bpound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold *sqrt*(*n*)**slamch*(ε) for sla_syrfsx_extended/ cla_syrfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_syrfsx_extended/ zla_syrfsx_extended. This error bound should only be trusted if the previous boolean is true.
err=3	Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold *sqrt*(*n*)**slamch*(ε) for sla_syrfsx_extended/ cla_syrfsx_extended and *sqrt*(*n*)**dlamch*(ε) for dla_syrfsx_extended/ zla_syrfsx_extended to determine if the error estimate is "guaranteed". These reciprocal condition numbers are 1/(norm(1/ z,inf)*norm(z,inf)) for some appropriately scaled matrix *Z*.

Let *z*=*s**(*a**diag(*x*)), where *x* is the solution for the current right-hand side and *s* scales each row of *a**diag(*x*) by a power of the radix so all absolute row sums of *z* are approximately 1. Use this subroutine to set only the second field above.

res, *dy*, *y_tail*
REAL for sla_syrfsx_extended
DOUBLE PRECISION for dla_syrfsx_extended
COMPLEX for cla_syrfsx_extended
DOUBLE COMPLEX for zla_syrfsx_extended.
Workspace arrays of DIMENSION *n*.
res holds the intermediate residual.
dy holds the intermediate solution.
y_tail holds the trailing bits of the intermediate solution.

ayb
REAL for sla_syrfsx_extended and cla_syrfsx_extended
DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended.
Workspace array, DIMENSION *n*.

rcond
REAL for sla_syrfsx_extended and cla_syrfsx_extended
DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended.
Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

ithresh
INTEGER. The maximum number of residual computations allowed for refinement. The default is 10. For 'aggressive', set to 100 to permit convergence using approximate factorizations or factorizations other than LU. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

rthresh
REAL for sla_syrfsx_extended and cla_syrfsx_extended
DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended.
Determines when to stop refinement if the error estimate stops decreasing. Refinement stops when the next solution no longer satisfies
norm(dx_{*i*+1}) < *rthresh* * norm(dx_*i*)
where norm(*z*) is the infinity norm of *Z*.
rthresh satisfies
0 < *rthresh* ≤ 1.
The default value is 0.5. For 'aggressive' set to 0.9 to permit convergence on extremely ill-conditioned matrices.

dz_ub
REAL for sla_syrfsx_extended and cla_syrfsx_extended
DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended.
Determines when to start considering componentwise convergence. Componentwise *dz_ub* convergence is only considered after each component of the solution *y* is stable, that is, the relative change in each component is less than *dz_ub*. The default value is 0.25, requiring the first bit to be stable.

ignore_cwise	LOGICAL If .TRUE., the function ignores componentwise convergence. Default value is .FALSE.

Output Parameters

y	REAL for sla_syrfsx_extended DOUBLE PRECISION for dla_syrfsx_extended COMPLEX for cla_syrfsx_extended DOUBLE COMPLEX for zla_syrfsx_extended. The improved solution matrix *Y*.
berr_out	REAL for sla_syrfsx_extended and cla_syrfsx_extended DOUBLE PRECISION for dla_syrfsx_extended and zla_syrfsx_extended. Array, DIMENSION *nrhs*. *berr_out(j)* contains the componentwise relative backward error for right-hand-side *j* from the formula max(*i*) (abs(*res*(*i*)) / (abs(op(*A*))*abs(*y*) + abs(*B*))(*i*)) where abs(*z*) is the componentwise absolute value of the matrix or vector *Z*. This is computed by ?la_lin_berr.
err_bnds_norm, *err_bnds_comp*	Values of the corresponding input parameters improved after iterative refinement and stored in the second column of the array (1:*nrhs*, 2). The other elements are kept unchanged.
info	INTEGER. If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed. If *info* = -*i*, the *i*-th parameter had an illegal value.

See Also

?syrfsx
?sytrf
?sytrs
?lamch
ilaprec
ilatrans
?la_lin_berr

?la_syrpvgrw

Computes the reciprocal pivot growth factor norm(A)/ norm(U) for a symmetric indefinite matrix.

Syntax

Fortran 77:

```
call sla_syrpvgrw( uplo, n, info, a, lda, af, ldaf, ipiv, work )
call dla_syrpvgrw( uplo, n, info, a, lda, af, ldaf, ipiv, work )
call cla_syrpvgrw( uplo, n, info, a, lda, af, ldaf, ipiv, work )
call zla_syrpvgrw( uplo, n, info, a, lda, af, ldaf, ipiv, work )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_syrpvgrw routine computes the reciprocal pivot growth factor norm(*A*)/norm(*U*). The *max absolute element* norm is used. If this is much less than 1, the stability of the *LU* factorization of the equilibrated matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable.

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies the triangle of A to store: If *uplo* = 'U', the upper triangle of *A* is stored, If *uplo* = 'L', the lower triangle of *A* is stored.
n	INTEGER. The number of linear equations, the order of the matrix *A*; *n* ≥ 0.
info	INTEGER. The value of INFO returned from ?sytrf, that is, the pivot in column *info* is exactly 0.
a, *af*	REAL for sla_syrpvgrw DOUBLE PRECISION for dla_syrpvgrw COMPLEX for cla_syrpvgrw DOUBLE COMPLEX for zla_syrpvgrw. Arrays: *a*(*lda*,*), *af*(*ldaf*,*). The array *a* contains the input *n*-by-*n* matrix *A*. The second dimension of *a* must be at least max(1,*n*). The array *af* contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?sytrf. The second dimension of *af* must be at least max(1,*n*).
lda	INTEGER. The leading dimension of *a*; *lda* ≥ max(1,*n*).
ldaf	INTEGER. The leading dimension of *af*; *ldaf* ≥ max(1,*n*).
ipiv	INTEGER. Array, DIMENSION *n*. Details of the interchanges and the block structure of D as determined by ?sytrf.
work	REAL for sla_syrpvgrw and cla_syrpvgrw DOUBLE PRECISION for dla_syrpvgrw and zla_syrpvgrw. Workspace array, dimension 2**n*.

See Also

?sytrf

?la_wwaddw

Adds a vector into a doubled-single vector.

Syntax

Fortran 77:

```
call sla_wwaddw( n, x, y, w )
call dla_wwaddw( n, x, y, w )
call cla_wwaddw( n, x, y, w )
call zla_wwaddw( n, x, y, w )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ?la_wwaddw routine adds a vector *W* into a doubled-single vector (*X*, *Y*). This works for all existing IBM hex and binary floating-point arithmetics, but not for decimal.

Input Parameters

n	INTEGER. The length of vectors *X*, *Y*, and *W*.
x, *y*, *w*	REAL for sla_wwaddw DOUBLE PRECISION for dla_wwaddw COMPLEX for cla_wwaddw DOUBLE COMPLEX for zla_wwaddw. Arrays DIMENSION *n*. *x* and *y* contain the first and second parts of the doubled-single accumulation vector, respectively. *w* contains the vector *W* to be added.

Output Parameters

x, *y*	Contain the first and second parts of the doubled-single accumulation vector, respectively, after adding the vector *W*.

Utility Functions and Routines

This section describes LAPACK utility functions and routines. Summary information about these routines is given in the following table:

LAPACK Utility Routines

Routine Name	Data Types	Description
ilaver		Returns the version of the Lapack library.
ilaenv		Environmental enquiry function which returns values for tuning algorithmic performance.
iparmq		Environmental enquiry function which returns values for tuning algorithmic performance.
ieeeck		Checks if the infinity and NaN arithmetic is safe. Called by ilaenv.
lsame		Tests two characters for equality regardless of case.
lsamen		Tests two character strings for equality regardless of case.
?labad	s, d	Returns the square root of the underflow and overflow thresholds if the exponent-range is very large.
?lamch	s, d	Determines machine parameters for floating-point arithmetic.
?lamc1	s, d	Called from ?lamc2. Determines machine parameters given by *beta*, *t*, *rnd*, *ieee1*.
?lamc2	s, d	Used by ?lamch. Determines machine parameters specified in its arguments list.

Routine Name	Data Types	Description
?lamc3	s, d	Called from ?lamc1-?lamc5. Intended to force *a* and *b* to be stored prior to doing the addition of *a* and *b*.
?lamc4	s, d	This is a service routine for ?lamc2.
?lamc5	s, d	Called from ?lamc2. Attempts to compute the largest machine floating-point number, without overflow.
second/dsecnd		Return user time for a process.
chla_transtype		Translates a BLAST-specified integer constant to the character string specifying a transposition operation.
iladiag		Translates a character string specifying whether a matrix has a unit diagonal or not to the relevant BLAST-specified integer constant.
ilaprec		Translates a character string specifying an intermediate precision to the relevant BLAST-specified integer constant.
ilatrans		Translates a character string specifying a transposition operation to the BLAST-specified integer constant.
ilauplo		Translates a character string specifying an upper- or lower-triangular matrix to the relevant BLAST-specified integer constant.
xerbla		Error handling routine called by LAPACK routines.
xerbla_array		Assists other languages in calling the xerbla function.

ilaver

Returns the version of the LAPACK library.

Syntax

```
call ilaver( vers_major, vers_minor, vers_patch )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This routine returns the version of the LAPACK library.

Output Parameters

vers_major INTEGER.
Returns the major version of the LAPACK library.

vers_minor INTEGER.
Returns the minor version from the major version of the LAPACK library.

vers_patch INTEGER.
Returns the patch version from the minor version of the LAPACK library.

ilaenv

Environmental enquiry function that returns values for tuning algorithmic performance.

Syntax

```
value = ilaenv( ispec, name, opts, n1, n2, n3, n4 )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The enquiry function `ilaenv` is called from the LAPACK routines to choose problem-dependent parameters for the local environment. See *ispec* below for a description of the parameters.

This version provides a set of parameters that should give good, but not optimal, performance on many of the currently available computers.

This routine will not function correctly if it is converted to all lower case. Converting it to all upper case is allowed.

Input Parameters

ispec

INTEGER.
Specifies the parameter to be returned as the value of `ilaenv`:
= 1: the optimal blocksize; if this value is 1, an unblocked algorithm will give the best performance.
= 2: the minimum block size for which the block routine should be used; if the usable block size is less than this value, an unblocked routine should be used.
= 3: the crossover point (in a block routine, for *n* less than this value, an unblocked routine should be used)
= 4: the number of shifts, used in the nonsymmetric eigenvalue routines (deprecated)
= 5: the minimum column dimension for blocking to be used; rectangular blocks must have dimension at least *k*-by-*m*, where `k` is given by `ilaenv(2,...)` and `m` by `ilaenv(5,...)`
= 6: the crossover point for the SVD (when reducing an *m*-by-*n* matrix to bidiagonal form, if `max(m,n)/min(m,n)` exceeds this value, a *QR* factorization is used first to reduce the matrix to a triangular form.)
= 7: the number of processors
= 8: the crossover point for the multishift *QR* and *QZ* methods for nonsymmetric eigenvalue problems (deprecated).

	= 9: maximum size of the subproblems at the bottom of the computation tree in the divide-and-conquer algorithm (used by ?gelsd and ?gesdd) =10: ieee NaN arithmetic can be trusted not to trap =11: infinity arithmetic can be trusted not to trap 12 ≤ *ispec* ≤ 16: ?hseqr or one of its subroutines, see iparmq for detailed explanation.
name	CHARACTER*(*). The name of the calling subroutine, in either upper case or lower case.
opts	CHARACTER*(*). The character options to the subroutine *name*, concatenated into a single character string. For example, *uplo* = 'U', *trans* = 'T', and *diag* = 'N' for a triangular routine would be specified as *opts* = 'UTN'.
n1, *n2*, *n3*, *n4*	INTEGER. Problem dimensions for the subroutine *name*; these may not all be required.

Output Parameters

value	INTEGER. If *value* ≥ 0: the value of the parameter specified by *ispec*; If *value* = -k < 0: the k-th argument had an illegal value.

Application Notes

The following conventions have been used when calling ilaenv from the LAPACK routines:

1. *opts* is a concatenation of all of the character options to subroutine *name*, in the same order that they appear in the argument list for *name*, even if they are not used in determining the value of the parameter specified by *ispec*.
2. The problem dimensions *n1*, *n2*, *n3*, *n4* are specified in the order that they appear in the argument list for *name*. *n1* is used first, *n2* second, and so on, and unused problem dimensions are passed a value of -1.
3. The parameter value returned by ilaenv is checked for validity in the calling subroutine. For example, ilaenv is used to retrieve the optimal blocksize for strtri as follows:

```
nb = ilaenv( 1, 'strtri', uplo // diag, n, -1, -1, -1> )
if( nb.le.1 ) nb = max( 1, n )
```

Below is an example of ilaenv usage in C language:

```
#include <stdio.h>
#include "mkl.h"

int main(void)
{
   int size = 1000;
   int ispec = 1;
   int dummy = -1;
   int blockSize1 = ilaenv(&ispec, "dsytrd", "U", &size, &dummy, &dummy, &dummy);
   int blockSize2 = ilaenv(&ispec, "dormtr", "LUN", &size, &size, &dummy, &dummy);
   printf("DSYTRD blocksize = %d\n",
blockSize1);
   printf("DORMTR blocksize = %d\n", blockSize2);
   return 0;
}
```

See Also

?hseqr
iparmq

iparmq

Environmental enquiry function which returns values for tuning algorithmic performance.

Syntax

value = iparmq(*ispec*, *name*, *opts*, *n*, *ilo*, *ihi*, *lwork*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function sets problem and machine dependent parameters useful for ?hseqr and its subroutines. It is called whenever ilaenv is called with 12≤*ispec*≤16.

Input Parameters

ispec — INTEGER.
Specifies the parameter to be returned as the value of iparmq:
= 12: (*inmin*) Matrices of order *nmin* or less are sent directly to ?lahqr, the implicit double shift QR algorithm. *nmin* must be at least 11.
= 13: (*inwin*) Size of the deflation window. This is best set greater than or equal to the number of simultaneous shifts *ns*. Larger matrices benefit from larger deflation windows.
= 14: (*inibl*) Determines when to stop nibbling and invest in an (expensive) multi-shift QR sweep. If the aggressive early deflation subroutine finds *ld* converged eigenvalues from an order *nw* deflation window and *ld*>(*nw***nibble*)/100, then the next QR sweep is skipped and early deflation is applied immediately to the remaining active diagonal block. Setting iparmq(*ispec*=14)=0 causes TTQRE to skip a multi-shift QR sweep whenever early deflation finds a converged eigenvalue. Setting iparmq(*ispec*=14) greater than or equal to 100 prevents TTQRE from skipping a multi-shift QR sweep.
= 15: (*nshfts*) The number of simultaneous shifts in a multi-shift QR iteration.
= 16: (*iacc22*) iparmq is set to 0, 1 or 2 with the following meanings.
0: During the multi-shift QR sweep, ?laqr5 does not accumulate reflections and does not use matrix-matrix multiply to update the far-from-diagonal matrix entries.
1: During the multi-shift QR sweep, ?laqr5 and/or ?laqr3 accumulates reflections and uses matrix-matrix multiply to update the far-from-diagonal matrix entries.
2: During the multi-shift QR sweep, ?laqr5 accumulates reflections and takes advantage of 2-by-2 block structure during matrix-matrix multiplies. (If ?trrm is slower than ?gemm, then iparmq(*ispec*=16)=1 may be more efficient than iparmq(*ispec*=16)=2 despite the greater level of arithmetic work implied by the latter choice.)

name — CHARACTER*(*). The name of the calling subroutine.

opts — CHARACTER*(*). This is a concatenation of the string arguments to TTQRE.

n	INTEGER. *n* is the order of the Hessenberg matrix *H*.
ilo, *ihi*	INTEGER. It is assumed that *H* is already upper triangular in rows and columns 1:*ilo*-1 and *ihi*+1:*n*.
lwork	INTEGER. The amount of workspace available.

Output Parameters

value	INTEGER. If *value* ≥ 0: the value of the parameter specified by *iparmq*; If *value* = -k < 0: the k-th argument had an illegal value.

Application Notes

The following conventions have been used when calling ilaenv from the LAPACK routines:

1. *opts* is a concatenation of all of the character options to subroutine *name*, in the same order that they appear in the argument list for *name*, even if they are not used in determining the value of the parameter specified by *ispec*.
2. The problem dimensions *n1*, *n2*, *n3*, *n4* are specified in the order that they appear in the argument list for *name*. *n1* is used first, *n2* second, and so on, and unused problem dimensions are passed a value of -1.
3. The parameter value returned by ilaenv is checked for validity in the calling subroutine. For example, ilaenv is used to retrieve the optimal blocksize for strtri as follows:

```
nb = ilaenv( 1, 'strtri', uplo // diag, n, -1, -1, -1> )
if( nb.le.1 ) nb = max( 1, n )
```

ieeeck

Checks if the infinity and NaN arithmetic is safe.
Called by ilaenv.

Syntax

```
ival = ieeeck( ispec, zero, one )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ieeeck is called from ilaenv to verify that infinity and possibly NaN arithmetic is safe, that is, will not trap.

Input Parameters

ispec	INTEGER. Specifies whether to test just for infinity arithmetic or both for infinity and NaN arithmetic: If *ispec* = 0: Verify infinity arithmetic only. If *ispec* = 1: Verify infinity and NaN arithmetic.
zero	REAL. Must contain the value 0.0 This is passed to prevent the compiler from optimizing away this code.
one	REAL. Must contain the value 1.0

This is passed to prevent the compiler from optimizing away this code.

Output Parameters

ival	INTEGER. If *ival* = 0: Arithmetic failed to produce the correct answers. If *ival* = 1: Arithmetic produced the correct answers.

lsamen

Tests two character strings for equality regardless of case.

Syntax

val = lsamen(*n*, *ca*, *cb*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This logical function tests if the first *n* letters of the string *ca* are the same as the first *n* letters of *cb*, regardless of case. The function lsamen returns .TRUE. if *ca* and *cb* are equivalent except for case and .FALSE. otherwise. lsamen also returns .FALSE. if len(*ca*) or len(*cb*) is less than *n*.

Input Parameters

n	INTEGER. The number of characters in *ca* and *cb* to be compared.
ca, *cb*	CHARACTER*(*). Specify two character strings of length at least *n* to be compared. Only the first *n* characters of each string will be accessed.

Output Parameters

val	LOGICAL. Result of the comparison.

?labad

Returns the square root of the underflow and overflow thresholds if the exponent-range is very large.

Syntax

call slabad(*small*, *large*)

call dlabad(*small*, *large*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine takes as input the values computed by slamch/dlamch for underflow and overflow, and returns the square root of each of these values if the log of *large* is sufficiently large. This subroutine is intended to identify machines with a large exponent range, such as the Crays, and redefine the underflow and overflow limits to be the square roots of the values computed by ?lamch. This subroutine is needed because ?lamch does not compensate for poor arithmetic in the upper half of the exponent range, as is found on a Cray.

Input Parameters

small	REAL for slabad DOUBLE PRECISION for dlabad. The underflow threshold as computed by ?lamch.
large	REAL for slabad DOUBLE PRECISION for dlabad. The overflow threshold as computed by ?lamch.

Output Parameters

small	On exit, if log10(*large*) is sufficiently large, the square root of *small*, otherwise unchanged.
large	On exit, if log10(*large*) is sufficiently large, the square root of *large*, otherwise unchanged.

?lamch

Determines machine parameters for floating-point arithmetic.

Syntax

val = slamch(*cmach*)

val = dlamch(*cmach*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The function ?lamch determines single precision and double precision machine parameters.

Input Parameters

cmach	CHARACTER*1. Specifies the value to be returned by ?lamch: = 'E' or 'e', *val* = *eps* = 'S' or 's', *val* = *sfmin* = 'B' or 'b', *val* = *base* = 'P' or 'p', *val* = *eps***base* = 'N' or 'n', *val* = *t* = 'R' or 'r', *val* = *rnd* = 'M' or 'm', *val* = *emin* = 'U' or 'u', *val* = *rmin* = 'L' or 'l', *val* = *emax* = 'O' or 'o', *val* = *rmax* where *eps* = relative machine precision; *sfmin* = safe minimum, such that 1/*sfmin* does not overflow; *base* = base of the machine; *prec* = *eps***base*; *t* = number of (base) digits in the mantissa; *rnd* = 1.0 when rounding occurs in addition, 0.0 otherwise; *emin* = minimum exponent before (gradual) underflow;

rmin = *underflow_threshold* - *base***(*emin*-1);
emax = largest exponent before overflow;
rmax = *overflow_threshold* - (*base****emax*)*(1-*eps*).

NOTE You can use a character string for *cmach* instead of a single character in order to make your code more readable. The first character of the string determines the value to be returned. For example, 'Precision' is interpreted as 'p'.

Output Parameters

val	REAL for slamch DOUBLE PRECISION for dlamch Value returned by the function.

?lamc1

Called from ?lamc2*. Determines machine parameters given by* *beta, t, rnd, ieee1.*

Syntax

```
call slamc1( beta, t, rnd, ieee1 )
call dlamc1( beta, t, rnd, ieee1 )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lamc1 determines machine parameters given by *beta, t, rnd, ieee1*.

Output Parameters

beta	INTEGER. The base of the machine.
t	INTEGER. The number of (*beta*) digits in the mantissa.
rnd	LOGICAL. Specifies whether proper rounding (*rnd* = .TRUE.) or chopping (*rnd* = .FALSE.) occurs in addition. This may not be a reliable guide to the way in which the machine performs its arithmetic.
ieee1	LOGICAL. Specifies whether rounding appears to be done in the *ieee* 'round to nearest' style.

?lamc2

Used by ?lamch*. Determines machine parameters specified in its arguments list.*

Syntax

```
call slamc2( beta, t, rnd, eps, emin, rmin, emax, rmax )
call dlamc2( beta, t, rnd, eps, emin, rmin, emax, rmax )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine `?lamc2` determines machine parameters specified in its arguments list.

Output Parameters

beta	`INTEGER`. The base of the machine.
t	`INTEGER`. The number of (*beta*) digits in the mantissa.
rnd	`LOGICAL`. Specifies whether proper rounding (*rnd* = `.TRUE.`) or chopping (*rnd* = `.FALSE.`) occurs in addition. This may not be a reliable guide to the way in which the machine performs its arithmetic.
eps	`REAL` for `slamc2` `DOUBLE PRECISION` for `dlamc2` The smallest positive number such that *fl*(1.0 - *eps*) < 1.0, where *fl* denotes the computed value.
emin	`INTEGER`. The minimum exponent before (gradual) underflow occurs.
rmin	`REAL` for `slamc2` `DOUBLE PRECISION` for `dlamc2` The smallest normalized number for the machine, given by $base^{emin-1}$, where *base* is the floating point value of *beta*.
emax	`INTEGER`.The maximum exponent before overflow occurs.
rmax	`REAL` for `slamc2` `DOUBLE PRECISION` for `dlamc2` The largest positive number for the machine, given by $base^{emax(1 - eps)}$, where *base* is the floating point value of *beta*.

?lamc3

Called from `?lamc1`-`?lamc5`. Intended to force `a` and `b` to be stored prior to doing the addition of `a` and `b`.

Syntax

```
val = slamc3( a, b )
val = dlamc3( a, b )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The routine is intended to force *A* and *B* to be stored prior to doing the addition of *A* and *B*, for use in situations where optimizers might hold one of these in a register.

Input Parameters

a, *b*	REAL for slamc3 DOUBLE PRECISION for dlamc3 The values *a* and *b*.

Output Parameters

val	REAL for slamc3 DOUBLE PRECISION for dlamc3 The result of adding values *a* and *b*.

?lamc4

This is a service routine for ?lamc2.

Syntax

```
call slamc4( emin, start, base )
call dlamc4( emin, start, base )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

This is a service routine for ?lamc2.

Input Parameters

start	REAL for slamc4 DOUBLE PRECISION for dlamc4 The starting point for determining *emin*.
base	INTEGER. The base of the machine.

Output Parameters

emin	INTEGER. The minimum exponent before (gradual) underflow, computed by setting *a = start* and dividing by *base* until the previous *a* can not be recovered.

?lamc5

Called from ?lamc2. *Attempts to compute the largest machine floating-point number, without overflow.*

Syntax

```
call slamc5( beta, p, emin, ieee, emax, rmax)
call dlamc5( beta, p, emin, ieee, emax, rmax)
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine ?lamc5 attempts to compute *rmax*, the largest machine floating-point number, without overflow. It assumes that *emax* + abs(*emin*) sum approximately to a power of 2. It will fail on machines where this assumption does not hold, for example, the Cyber 205 (*emin* = -28625, *emax* = 28718). It will also fail if the value supplied for *emin* is too large (that is, too close to zero), probably with overflow.

Input Parameters

beta	INTEGER. The base of floating-point arithmetic.
p	INTEGER. The number of base *beta* digits in the mantissa of a floating-point value.
emin	INTEGER. The minimum exponent before (gradual) underflow.
ieee	LOGICAL. A logical flag specifying whether or not the arithmetic system is thought to comply with the IEEE standard.

Output Parameters

emax	INTEGER. The largest exponent before overflow.
rmax	REAL for slamc5 DOUBLE PRECISION for dlamc5 The largest machine floating-point number.

second/dsecnd

Return user time for a process.

Syntax

```
val = second()
val = dsecnd()
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The functions second/dsecnd return the user time for a process in seconds. These versions get the time from the system function etime. The difference is that dsecnd returns the result with double precision.

Output Parameters

val	REAL for second DOUBLE PRECISION for dsecnd User time for a process.

chla_transtype

Translates a BLAST-specified integer constant to the character string specifying a transposition operation.

Syntax

```
val = chla_transtype( trans )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The chla_transtype function translates a BLAST-specified integer constant to the character string specifying a transposition operation.

The function returns a CHARACTER*1. If the input is not an integer indicating a transposition operator, then *val* is 'X'. Otherwise, the function returns the constant value corresponding to *trans*.

Input Parameters

trans	INTEGER. Specifies the form of the system of equations: If *trans* = BLAS_NO_TRANS = 111: No transpose. If *trans* = BLAS_TRANS = 112: Transpose. If *trans* = BLAS_CONJ_TRANS = 113: Conjugate Transpose.

Output Parameters

val	CHARACTER*1 Character that specifies a transposition operation.

iladiag

Translates a character string specifying whether a matrix has a unit diagonal to the relevant BLAST-specified integer constant.

Syntax

val = iladiag(*diag*)

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The iladiag function translates a character string specifying whether a matrix has a unit diagonal or not to the relevant BLAST-specified integer constant.

The function returns an INTEGER. If *val* < 0, the input is not a character indicating a unit or non-unit diagonal. Otherwise, the function returns the constant value corresponding to *diag*.

Input Parameters

diag	CHARACTER*1. Specifies the form of the system of equations: If *diag* = 'N': *A* is non-unit triangular. If *diag* = 'U': *A* is unit triangular.

Output Parameters

val	INTEGER Value returned by the function.

ilaprec

Translates a character string specifying an intermediate precision to the relevant BLAST-specified integer constant.

Syntax

```
val = ilaprec( prec )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `ilaprec` function translates a character string specifying an intermediate precision to the relevant BLAST-specified integer constant.

The function returns an `INTEGER`. If *val* < 0, the input is not a character indicating a supported intermediate precision. Otherwise, the function returns the constant value corresponding to *prec*.

Input Parameters

prec — `CHARACTER*1`.
Specifies the form of the system of equations:
If *prec* = `'S'`: Single.
If *prec* = `'D'`: Double.
If *prec* = `'I'`: Indigenous.
If *prec* = `'X'`, `'E'`: Extra.

Output Parameters

val — `INTEGER`
Value returned by the function.

ilatrans

Translates a character string specifying a transposition operation to the BLAST-specified integer constant.

Syntax

```
val = ilatrans( trans )
```

Include Files

- FORTRAN 77: `mkl_lapack.fi` and `mkl_lapack.h`

Description

The `ilatrans` function translates a character string specifying a transposition operation to the BLAST-specified integer constant.

The function returns a `INTEGER`. If *val* < 0, the input is not a character indicating a transposition operator. Otherwise, the function returns the constant value corresponding to *trans*.

Input Parameters

trans — `CHARACTER*1`.

Specifies the form of the system of equations:
If *trans* = 'N': No transpose.
If *trans* = 'T': Transpose.
If *trans* = 'C': Conjugate Transpose.

Output Parameters

val INTEGER
Character that specifies a transposition operation.

ilauplo

Translates a character string specifying an upper- or lower-triangular matrix to the relevant BLAST-specified integer constant.

Syntax

```
val = ilauplo( uplo )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The ilauplo function translates a character string specifying an upper- or lower-triangular matrix to the relevant BLAST-specified integer constant.

The function returns an INTEGER. If *val* < 0, the input is not a character indicating an upper- or lower-triangular matrix. Otherwise, the function returns the constant value corresponding to *uplo*.

Input Parameters

diag CHARACTER.
Specifies the form of the system of equations:
If *diag* = 'U': *A* is upper triangular.
If *diag* = 'L': *A* is lower triangular.

Output Parameters

val INTEGER
Value returned by the function.

xerbla_array

Assists other languages in calling the xerbla *function.*

Syntax

```
call xerbla_array( srname_array, srname_len, info )
```

Include Files

- FORTRAN 77: mkl_lapack.fi and mkl_lapack.h

Description

The routine assists other languages in calling the error handling `xerbla` function. Rather than taking a Fortran string argument as the function name, `xerbla_array` takes an array of single characters along with the array length. The routine then copies up to 32 characters of that array into a Fortran string and passes that to `xerbla`. If called with a non-positive *`srname_len`*, the routine will call `xerbla` with a string of all blank characters.

If some macro or other device makes `xerbla_array` available to `C99` by a name `lapack_xerbla` and with a common Fortran calling convention, a `C99` program could invoke `xerbla` via:

```
{
   int flen = strlen(__func__);
   lapack_xerbla(__func__, &flen, &info);
}
```

Providing `xerbla_array` is not necessary for intercepting LAPACK errors. `xerbla_array` calls `xerbla`.

Output Parameters

`srname_array`
`CHARACTER(1)`.
Array, dimension (*`srname_len`*). The name of the routine that called `xerbla_array`.

`srname_len`
`INTEGER`.
The length of the name in *`srname_array`*.

`info`
`INTEGER`.
Position of the invalid parameter in the parameter list of the calling routine.

ScaLAPACK Routines

6

This chapter describes the Intel® Math Kernel Library implementation of routines from the ScaLAPACK package for distributed-memory architectures. Routines are supported for both real and complex dense and band matrices to perform the tasks of solving systems of linear equations, solving linear least-squares problems, eigenvalue and singular value problems, as well as performing a number of related computational tasks.

Intel MKL ScaLAPACK routines are written in FORTRAN 77 with exception of a few utility routines written in C to exploit the IEEE arithmetic. All routines are available in all precision types: single precision, double precision, complexm, and double complex precision. See the `mkl_scalapack.h` header file for C declarations of ScaLAPACK routines.

NOTE ScaLAPACK routines are provided only with Intel® MKL versions for Linux* and Windows* OSs.

Sections in this chapter include descriptions of ScaLAPACK computational routines that perform distinct computational tasks, as well as driver routines for solving standard types of problems in one call.

Generally, ScaLAPACK runs on a network of computers using MPI as a message-passing layer and a set of prebuilt communication subprograms (BLACS), as well as a set of BLAS optimized for the target architecture. Intel MKL version of ScaLAPACK is optimized for Intel® processors. For the detailed system and environment requirements, see *Intel® MKL Release Notes* and *Intel® MKL User's Guide*.

For full reference on ScaLAPACK routines and related information, see [SLUG].

Overview

The model of the computing environment for ScaLAPACK is represented as a one-dimensional array of processes (for operations on band or tridiagonal matrices) or also a two-dimensional process grid (for operations on dense matrices). To use ScaLAPACK, all global matrices or vectors should be distributed on this array or grid prior to calling the ScaLAPACK routines.

ScaLAPACK uses the two-dimensional block-cyclic data distribution as a layout for dense matrix computations. This distribution provides good work balance between available processors, as well as gives the opportunity to use BLAS Level 3 routines for optimal local computations. Information about the data distribution that is required to establish the mapping between each global array and its corresponding process and memory location is contained in the so called *array descriptor* associated with each global array. An example of an array descriptor structure is given in Table "Content of the array descriptor for dense matrices".

Content of the array descriptor for dense matrices

Array Element #	Name	Definition
1	*dtype*	Descriptor type (=1 for dense matrices)
2	*ctxt*	BLACS context handle for the process grid

Array Element #	Name	Definition
3	*m*	Number of rows in the global array
4	*n*	Number of columns in the global array
5	*mb*	Row blocking factor
6	*nb*	Column blocking factor
7	*rsrc*	Process row over which the first row of the global array is distributed
8	*csrc*	Process column over which the first column of the global array is distributed
9	*lld*	Leading dimension of the local array

The number of rows and columns of a global dense matrix that a particular process in a grid receives after data distributing is denoted by $LOC_r()$ and $LOC_c()$, respectively. To compute these numbers, you can use the ScaLAPACK tool routine `numroc`.

After the block-cyclic distribution of global data is done, you may choose to perform an operation on a submatrix of the global matrix *A*, which is contained in the global subarray sub(*A*), defined by the following 6 values (for dense matrices):

m	The number of rows of sub(*A*)
n	The number of columns of sub(*A*)
a	A pointer to the local array containing the entire global array A
ia	The row index of sub(*A*) in the global array
ja	The column index of sub(*A*) in the global array
desca	The array descriptor for the global array

Routine Naming Conventions

For each routine introduced in this chapter, you can use the ScaLAPACK name. The naming convention for ScaLAPACK routines is similar to that used for LAPACK routines (see Routine Naming Conventions in Chapter 4). A general rule is that each routine name in ScaLAPACK, which has an LAPACK equivalent, is simply the LAPACK name prefixed by initial letter `p`.

ScaLAPACK names have the structure `p?yyzzz` or `p?yyzz`, which is described below.

The initial letter `p` is a distinctive prefix of ScaLAPACK routines and is present in each such routine.

The second symbol `?` indicates the data type:

`s`	real, single precision
`d`	real, double precision
`c`	complex, single precision
`z`	complex, double precision

The second and third letters `yy` indicate the matrix type as:

`ge`	general
`gb`	general band
`gg`	a pair of general matrices (for a generalized problem)

dt	general tridiagonal (diagonally dominant-like)
db	general band (diagonally dominant-like)
po	symmetric or Hermitian positive-definite
pb	symmetric or Hermitian positive-definite band
pt	symmetric or Hermitian positive-definite tridiagonal
sy	symmetric
st	symmetric tridiagonal (real)
he	Hermitian
or	orthogonal
tr	triangular (or quasi-triangular)
tz	trapezoidal
un	unitary

For computational routines, the last three letters **zzz** indicate the computation performed and have the same meaning as for LAPACK routines.

For driver routines, the last two letters **zz** or three letters **zzz** have the following meaning:

sv	a *simple* driver for solving a linear system
svx	an *expert* driver for solving a linear system
ls	a driver for solving a linear least squares problem
ev	a simple driver for solving a symmetric eigenvalue problem
evd	a simple driver for solving an eigenvalue problem using a divide and conquer algorithm
evx	an expert driver for solving a symmetric eigenvalue problem
svd	a driver for computing a singular value decomposition
gvx	an expert driver for solving a generalized symmetric definite eigenvalue problem

Simple driver here means that the driver just solves the general problem, whereas an *expert* driver is more versatile and can also optionally perform some related computations (such, for example, as refining the solution and computing error bounds after the linear system is solved).

Computational Routines

In the sections that follow, the descriptions of ScaLAPACK computational routines are given. These routines perform distinct computational tasks that can be used for:

- Solving Systems of Linear Equations
- Orthogonal Factorizations and LLS Problems
- Symmetric Eigenproblems
- Nonsymmetric Eigenproblems
- Singular Value Decomposition
- Generalized Symmetric-Definite Eigenproblems

See also the respective driver routines.

Linear Equations

ScaLAPACK supports routines for the systems of equations with the following types of matrices:

- general
- general banded
- general diagonally dominant-like banded (including general tridiagonal)
- symmetric or Hermitian positive-definite

- symmetric or Hermitian positive-definite banded
- symmetric or Hermitian positive-definite tridiagonal

A *diagonally dominant-like* matrix is defined as a matrix for which it is known in advance that pivoting is not required in the *LU* factorization of this matrix.

For the above matrix types, the library includes routines for performing the following computations: *factoring* the matrix; *equilibrating* the matrix; *solving* a system of linear equations; *estimating the condition number* of a matrix; *refining* the solution of linear equations and computing its error bounds; *inverting* the matrix. Note that for some of the listed matrix types only part of the computational routines are provided (for example, routines that refine the solution are not provided for band or tridiagonal matrices). See Table "Computational Routines for Systems of Linear Equations" for full list of available routines.

To solve a particular problem, you can either call two or more computational routines or call a corresponding driver routine that combines several tasks in one call. Thus, to solve a system of linear equations with a general matrix, you can first call p?getrf(*LU* factorization) and then p?getrs(computing the solution). Then, you might wish to call p?gerfs to refine the solution and get the error bounds. Alternatively, you can just use the driver routine p?gesvx which performs all these tasks in one call.

Table "Computational Routines for Systems of Linear Equations" lists the ScaLAPACK computational routines for factorizing, equilibrating, and inverting matrices, estimating their condition numbers, solving systems of equations with real matrices, refining the solution, and estimating its error.

Computational Routines for Systems of Linear Equations

Matrix type, storage scheme	Factorize matrix	Equilibrate matrix	Solve system	Condition number	Estimate error	Invert matrix
general (partial pivoting)	p?getrf	p?geequ	p?getrs	p?gecon	p?gerfs	p?getri
general band (partial pivoting)	p?gbtrf		p?gbtrs			
general band (no pivoting)	p?dbtrf		p?dbtrs			
general tridiagonal (no pivoting)	p?dttrf		p?dttrs			
symmetric/Hermitian positive-definite	p?potrf	p?poequ	p?potrs	p?pocon	p?porfs	p?potri
symmetric/Hermitian positive-definite, band	p?pbtrf		p?pbtrs			
symmetric/Hermitian positive-definite, tridiagonal	p?pttrf		p?pttrs			
triangular			p?trtrs	p?trcon	p?trrfs	p?trtri

In this table ? stands for s (single precision real), d (double precision real), c (single precision complex), or z (double precision complex).

Routines for Matrix Factorization

This section describes the ScaLAPACK routines for matrix factorization. The following factorizations are supported:

- LU factorization of general matrices
- LU factorization of diagonally dominant-like matrices
- Cholesky factorization of real symmetric or complex Hermitian positive-definite matrices

You can compute the factorizations using full and band storage of matrices.

p?getrf

Computes the LU factorization of a general m-by-n distributed matrix.

Syntax

```
call psgetrf(m, n, a, ia, ja, desca, ipiv, info)
call pdgetrf(m, n, a, ia, ja, desca, ipiv, info)
call pcgetrf(m, n, a, ia, ja, desca, ipiv, info)
call pzgetrf(m, n, a, ia, ja, desca, ipiv, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?getrf routine forms the *LU* factorization of a general *m*-by-*n* distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) as

A = *P***L***U*

where *P* is a permutation matrix, *L* is lower triangular with unit diagonal elements (lower trapezoidal if *m* > *n*) and *U* is upper triangular (upper trapezoidal if *m* < *n*). *L* and *U* are stored in sub(*A*).

The routine uses partial pivoting, with row interchanges.

Input Parameters

m	(global) INTEGER. The number of rows in the distributed submatrix sub(*A*); *m*≥0.
n	(global) INTEGER. The number of columns in the distributed submatrix sub(*A*); *n*≥0.
a	(local) REAL for psgetrf DOUBLE PRECISION for pdgetrf COMPLEX for pcgetrf DOUBLE COMPLEX for pzgetrf. Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). Contains the local pieces of the distributed matrix sub(*A*) to be factored.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix *A*(*ia:ia+n-1*, *ja:ja+n-1*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a	Overwritten by local pieces of the factors *L* and *U* from the factorization *A* = *P***L***U*. The unit diagonal elements of *L* are not stored.
ipiv	(local) INTEGER array. The dimension of *ipiv* is (*LOCr*(*m_a*)+ *mb_a*). This array contains the pivoting information: local row *i* was interchanged with global row *ipiv*(*i*). This array is tied to the distributed matrix *A*.
info	(global) INTEGER. If *info*=0, the execution is successful.

info < 0: if the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
If *info* = *i*, u_{ii} is 0. The factorization has been completed, but the factor *U* is exactly singular. Division by zero will occur if you use the factor *U* for solving a system of linear equations.

p?gbtrf

Computes the LU factorization of a general n-by-n banded distributed matrix.

Syntax

```
call psgbtrf(n, bwl, bwu, a, ja, desca, ipiv, af, laf, work, lwork, info)
call pdgbtrf(n, bwl, bwu, a, ja, desca, ipiv, af, laf, work, lwork, info)
call pcgbtrf(n, bwl, bwu, a, ja, desca, ipiv, af, laf, work, lwork, info)
call pzgbtrf(n, bwl, bwu, a, ja, desca, ipiv, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gbtrf routine computes the *LU* factorization of a general *n*-by-*n* real/complex banded distributed matrix *A*(*1:n, ja:ja+n-1*) using partial pivoting with row interchanges.

The resulting factorization is not the same factorization as returned from the LAPACK routine ?gbtrf. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form

A(*1:n, ja:ja+n-1*) = *P*L*U*Q*

where *P* and *Q* are permutation matrices, and *L* and *U* are banded lower and upper triangular matrices, respectively. The matrix *Q* represents reordering of columns for the sake of parallelism, while *P* represents reordering of rows for numerical stability using classic partial pivoting.

Input Parameters

n — (global) INTEGER. The number of rows and columns in the distributed submatrix *A*(*1:n, ja:ja+n-1*); *n* ≥ 0.

bwl — (global) INTEGER. The number of sub-diagonals within the band of *A* (0 ≤ *bwl* ≤ *n-1*).

bwu — (global) INTEGER. The number of super-diagonals within the band of *A* (0 ≤ *bwu* ≤ *n-1*).

a — (local)
REAL for psgbtrf
DOUBLE PRECISION for pdgbtrf
COMPLEX for pcgbtrf
DOUBLE COMPLEX for pzgbtrf.
Pointer into the local memory to an array of local dimension (*lld_a*, LOC_c(*ja+n*-1) where
lld_a ≥ 2**bwl* + 2**bwu* +1.

Contains the local pieces of the n-by-n distributed banded matrix $A(1:n, ja:ja+n-1)$ to be factored.

ja (global) INTEGER. The index in the global array A that points to the start of the matrix to be operated on (which may be either all of A or a submatrix of A).

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix A.
If *desca*(*dtype_*) = 501, then *dlen_* ≥ 7;
else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.

laf (local) INTEGER. The dimension of the array *af*.
Must be *laf* ≥ (NB+*bwu*)*(*bwl*+*bwu*)+6*(*bwl*+*bwu*)*(*bwl*+2*bwu*).
If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

work (local) Same type as *a*. Workspace array of dimension *lwork* .

lwork (local or global) INTEGER. The size of the *work* array (*lwork* ≥ 1). If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned.

Output Parameters

a On exit, this array contains details of the factorization. Note that additional permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK.

ipiv (local) INTEGER array.
The dimension of *ipiv* must be ≥ *desca*(NB).
Contains pivot indices for local factorizations. Note that you *should not alter* the contents of this array between factorization and solve.

af (local)
REAL for psgbtrf
DOUBLE PRECISION for pdgbtrf
COMPLEX for pcgbtrf
DOUBLE COMPLEX for pzgbtrf.
Array, dimension (*laf*).
Auxiliary Fillin space. Fillin is created during the factorization routine p?gbtrf and this is stored in *af*.
Note that if a linear system is to be solved using p?gbtrs after the factorization routine, *af* must not be altered after the factorization.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
If *info*=0, the execution is successful.
info < 0:
If the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = k ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not nonsingular, and the factorization was not completed. If *info* = k > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

p?dbtrf

Computes the LU factorization of a n-by-n diagonally dominant-like banded distributed matrix.

Syntax

```
call psdbtrf(n, bwl, bwu, a, ja, desca, af, laf, work, lwork, info)
call pddbtrf(n, bwl, bwu, a, ja, desca, af, laf, work, lwork, info)
call pcdbtrf(n, bwl, bwu, a, ja, desca, af, laf, work, lwork, info)
call pzdbtrf(n, bwl, bwu, a, ja, desca, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dbtrf routine computes the LU factorization of a *n*-by-*n* real/complex diagonally dominant-like banded distributed matrix *A*(1:*n*, *ja*:*ja*+*n*-1) without pivoting.

Note that the resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

Input Parameters

n — (global) INTEGER. The number of rows and columns in the distributed submatrix *A*(1:*n*, *ja*:*ja*+*n*-1); $n \geq 0$.

bwl — (global) INTEGER. The number of sub-diagonals within the band of *A* ($0 \leq bwl \leq n-1$).

bwu — (global) INTEGER. The number of super-diagonals within the band of *A* ($0 \leq bwu \leq n-1$).

a — (local)
REAL for psdbtrf
DOUBLE PRECISION for pddbtrf
COMPLEX for pcdbtrf
DOUBLE COMPLEX for pzdbtrf.
Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)).
Contains the local pieces of the *n*-by-*n* distributed banded matrix *A*(*1:n*, *ja:ja+n-1*) to be factored.

ja — (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
If *desca*(*dtype_*) = 501, then *dlen_* ≥ 7;
else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.

laf — (local) INTEGER. The dimension of the array *af*.
Must be *laf* ≥ NB*(*bwl*+*bwu*)+6*(max(*bwl*,*bwu*))^2 .
If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

work — (local) Same type as *a*. Workspace array of dimension *lwork*.

lwork (local or global) INTEGER. The size of the *work* array, must be *lwork* ≥ (max(*bwl*,*bwu*))^2. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned.

Output Parameters

a On exit, this array contains details of the factorization. Note that additional permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK.

af (local)
REAL for psdbtrf
DOUBLE PRECISION for pddbtrf
COMPLEX for pcdbtrf
DOUBLE COMPLEX for pzdbtrf.
Array, dimension (*laf*).
Auxiliary Fillin space. Fillin is created during the factorization routine p?dbtrf and this is stored in *af*.
Note that if a linear system is to be solved using p?dbtrs after the factorization routine, *af* must not be altered after the factorization.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
If *info*=0, the execution is successful.
info < 0:
If the *i*th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. *info* > 0:
If *info* = k ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not diagonally dominant-like, and the factorization was not completed. If *info* = k > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

p?dttrf

Computes the LU factorization of a diagonally dominant-like tridiagonal distributed matrix.

Syntax

```
call psdttrf(n, dl, d, du, ja, desca, af, laf, work, lwork, info)
call pddttrf(n, dl, d, du, ja, desca, af, laf, work, lwork, info)
call pcdttrf(n, dl, d, du, ja, desca, af, laf, work, lwork, info)
call pzdttrf(n, dl, d, du, ja, desca, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dttrf routine computes the *LU* factorization of an *n*-by-*n* real/complex diagonally dominant-like tridiagonal distributed matrix *A*(*1:n*, *ja:ja+n-1*) without pivoting for stability.

The resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form:

$A(1:n, ja:ja+n-1) = P*L*U*P^T$,

where P is a permutation matrix, and L and U are banded lower and upper triangular matrices, respectively.

Input Parameters

n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix $A(1:n, ja:ja+n-1)$ ($n \geq 0$).
dl, *d*, *du*	(local) REAL for pspttrf DOUBLE PRECISON for pdpttrf COMPLEX for pcpttrf DOUBLE COMPLEX for pzpttrf. Pointers to the local arrays of dimension (*desca*(*nb_*)) each. On entry, the array *dl* contains the local part of the global vector storing the subdiagonal elements of the matrix. Globally, *dl*(1) is not referenced, and *dl* must be aligned with *d*. On entry, the array *d* contains the local part of the global vector storing the diagonal elements of the matrix. On entry, the array *du* contains the local part of the global vector storing the super-diagonal elements of the matrix. *du*(*n*) is not referenced, and *du* must be aligned with *d*.
ja	(global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*dtype_*) = 501, then *dlen_* $\geq$ 7; else if *desca*(*dtype_*) = 1, then *dlen_* $\geq$ 9.
laf	(local) INTEGER. The dimension of the array *af*. Must be *laf* $\geq$ 2*(NB+2) . If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).
work	(local) Same type as *d*. Workspace array of dimension *lwork*.
lwork	(local or global) INTEGER. The size of the *work* array, must be at least *lwork* $\geq$ 8*NPCOL.

Output Parameters

dl, *d*, *du*	On exit, overwritten by the information containing the factors of the matrix.
af	(local) REAL for psdttrf DOUBLE PRECISION for pddttrf COMPLEX for pcdttrf DOUBLE COMPLEX for pzdttrf. Array, dimension (*laf*). Auxiliary Fillin space. Fillin is created during the factorization routine p?dttrf and this is stored in *af*. Note that if a linear system is to be solved using p?dttrs after the factorization routine, *af* must not be altered.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = k ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not diagonally dominant-like, and the factorization was not completed. If *info* = k > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

p?potrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite distributed matrix.

Syntax

```
call pspotrf(uplo, n, a, ia, ja, desca, info)
call pdpotrf(uplo, n, a, ia, ja, desca, info)
call pcpotrf(uplo, n, a, ia, ja, desca, info)
call pzpotrf(uplo, n, a, ia, ja, desca, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?potrf routine computes the Cholesky factorization of a real symmetric or complex Hermitian positive-definite distributed *n*-by-*n* matrix *A*(*ia:ia+n-1, ja:ja+n-1*), denoted below as sub(*A*).

The factorization has the form

sub(*A*) = U^H*U if *uplo*='U', or

sub(*A*) = $L*L^H$ if *uplo*='L'

where *L* is a lower triangular matrix and *U* is upper triangular.

Input Parameters

uplo (global) CHARACTER*1.
Indicates whether the upper or lower triangular part of sub(*A*) is stored. Must be 'U' or 'L'.
If *uplo* = 'U', the array *a* stores the upper triangular part of the matrix sub(*A*) that is factored as U^H*U.
If *uplo* = 'L', the array *a* stores the lower triangular part of the matrix sub(*A*) that is factored as $L*L^H$.

n (global) INTEGER. The order of the distributed submatrix sub(*A*) (*n*≥0).

a (local)
REAL for pspotrf
DOUBLE PRECISON for pdpotrf
COMPLEX for pcpotrf

	DOUBLE COMPLEX for pzpotrf. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* symmetric/ Hermitian distributed matrix sub(*A*) to be factored. Depending on *uplo*, the array *a* contains either the upper or the lower triangular part of the matrix sub(*A*) (see *uplo*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a	The upper or lower triangular part of *a* is overwritten by the Cholesky factor *U* or *L*, as specified by *uplo*.
info	(global) INTEGER. If *info*=0, the execution is successful; *info* < 0: if the *i*-th argument is an array, and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. If *info* = *k* >0, the leading minor of order *k*, *A*(*ia:ia+k-1*, *ja:ja+k-1*), is not positive-definite, and the factorization could not be completed.

p?pbtrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite banded distributed matrix.

Syntax

```
call pspbtrf(uplo, n, bw, a, ja, desca, af, laf, work, lwork, info)
call pdpbtrf(uplo, n, bw, a, ja, desca, af, laf, work, lwork, info)
call pcpbtrf(uplo, n, bw, a, ja, desca, af, laf, work, lwork, info)
call pzpbtrf(uplo, n, bw, a, ja, desca, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?pbtrf routine computes the Cholesky factorization of an *n*-by-*n* real symmetric or complex Hermitian positive-definite banded distributed matrix *A*(1:*n*, *ja*:*ja*+*n*-1).

The resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form:

A(1:*n*, *ja*:*ja*+*n*-1) = *P***U*^{H}**U***P*^{T}, if *uplo*='U', or

A(1:*n*, *ja*:*ja*+*n*-1) = *P***L***L*^{H}**P*^{T}, if *uplo*='L',

where *P* is a permutation matrix and *U* and *L* are banded upper and lower triangular matrices, respectively.

Input Parameters

uplo (global) CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', upper triangle of *A(1:n, ja:ja+n-1)* is stored;
If *uplo* = 'L', lower triangle of *A(1:n, ja:ja+n-1)* is stored.

n (global) INTEGER. The order of the distributed submatrix *A(1:n, ja:ja+n-1)*.
(n≥0).

bw (global) INTEGER.
The number of superdiagonals of the distributed matrix if *uplo* = 'U', or the number of subdiagonals if *uplo* = 'L' *(bw≥0)*.

a (local)
REAL for pspbtrf
DOUBLE PRECISON for pdpbtrf
COMPLEX for pcpbtrf
DOUBLE COMPLEX for pzpbtrf.
Pointer into the local memory to an array of dimension *(lld_a,LOCc(ja+n-1))*.
On entry, this array contains the local pieces of the upper or lower triangle of the symmetric/Hermitian band distributed matrix *A(1:n, ja:ja+n-1)* to be factored.

ja (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
If *desca(dtype_) = 501*, then *dlen_ ≥ 7*;
else if *desca(dtype_) = 1*, then *dlen_ ≥ 9*.

laf (local) INTEGER. The dimension of the array *af*.
Must be *laf ≥ (NB+2*bw)*bw*.

If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

work (local) Same type as *a*. Workspace array of dimension *lwork* .

lwork (local or global) INTEGER. The size of the *work* array, must be *lwork ≥ bw*^{2}.

Output Parameters

a On exit, if *info*=0, contains the permuted triangular factor *U* or *L* from the Cholesky factorization of the band matrix *A(1:n, ja:ja+n-1)*, as specified by *uplo*.

af (local)
REAL for pspbtrf
DOUBLE PRECISON for pdpbtrf
COMPLEX for pcpbtrf
DOUBLE COMPLEX for pzpbtrf.
Array, dimension (*laf*). Auxiliary Fillin space. Fillin is created during the factorization routine p?pbtrf and this is stored in *af*. Note that if a linear system is to be solved using p?pbtrs after the factorization routine, *af* must not be altered.

work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*th argument is a scalar and had an illegal value, then *info* = -*i*. *info*>0: If *info* = k ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not positive definite, and the factorization was not completed. If *info* = k > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

p?pttrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite tridiagonal distributed matrix.

Syntax

```
call pspttrf(n, d, e, ja, desca, af, laf, work, lwork, info)
call pdpttrf(n, d, e, ja, desca, af, laf, work, lwork, info)
call pcpttrf(n, d, e, ja, desca, af, laf, work, lwork, info)
call pzpttrf(n, d, e, ja, desca, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?pttrf routine computes the Cholesky factorization of an *n*-by-*n* real symmetric or complex hermitian positive-definite tridiagonal distributed matrix *A*(*1:n, ja:ja+n-1*).

The resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form:

$A(1:n, ja:ja+n-1) = P*L*D*L^H*P^T$, or

$A(1:n, ja:ja+n-1) = P*U^H*D*U*P^T$,

where *P* is a permutation matrix, and *U* and *L* are tridiagonal upper and lower triangular matrices, respectively.

Input Parameters

n	(global) INTEGER. The order of the distributed submatrix *A*(*1:n, ja:ja+n-1*) (*n* ≥ 0).
d, *e*	(local) REAL for pspttrf DOUBLE PRECISON for pdpttrf

COMPLEX for pcpttrf
DOUBLE COMPLEX for pzpttrf.
Pointers into the local memory to arrays of dimension (*desca*(*nb_*)) each.
On entry, the array *d* contains the local part of the global vector storing the main diagonal of the distributed matrix *A*.
On entry, the array *e* contains the local part of the global vector storing the upper diagonal of the distributed matrix *A*.

ja (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
If *desca*(*dtype_*) = 501, then *dlen_* ≥ 7;
else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.

laf (local) INTEGER. The dimension of the array *af*.
Must be *laf* ≥ NB+2.
If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

work (local) Same type as *d* and *e*. Workspace array of dimension *lwork* .

lwork (local or global) INTEGER. The size of the *work* array, must be at least *lwork* ≥ 8*NPCOL.

Output Parameters

d, *e* On exit, overwritten by the details of the factorization.

af (local)
REAL for pspttrf
DOUBLE PRECISION for pdpttrf
COMPLEX for pcpttrf
DOUBLE COMPLEX for pzpttrf.
Array, dimension (*laf*).
Auxiliary Fillin space. Fillin is created during the factorization routine p?pttrf and this is stored in *af*.
Note that if a linear system is to be solved using p?pttrs after the factorization routine, *af* must not be altered.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = k ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not positive definite, and the factorization was not completed.
If *info* = k > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

Routines for Solving Systems of Linear Equations

This section describes the ScaLAPACK routines for solving systems of linear equations. Before calling most of these routines, you need to factorize the matrix of your system of equations (see Routines for Matrix Factorization in this chapter). However, the factorization is not necessary if your system of equations has a triangular matrix.

p?getrs

Solves a system of distributed linear equations with a general square matrix, using the `LU` *factorization computed by* `p?getrf`.

Syntax

```
call psgetrs(trans, n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
call pdgetrs(trans, n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
call pcgetrs(trans, n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
call pzgetrs(trans, n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?getrs` routine solves a system of distributed linear equations with a general *n*-by-*n* distributed matrix `sub(A) = A(ia:ia+n-1, ja:ja+n-1)` using the *LU* factorization computed by `p?getrf`.

The system has one of the following forms specified by *trans*:

`sub(A)*X = sub(B)` (no transpose),

$\text{sub}(A)^T * X = \text{sub}(B)$ (transpose),

$\text{sub}(A)^H * X = \text{sub}(B)$ (conjugate transpose),

where `sub(B) = B(ib:ib+n-1, jb:jb+nrhs-1)`.

Before calling this routine, you must call `p?getrf` to compute the *LU* factorization of sub(*A*).

Input Parameters

trans	(global) `CHARACTER*1`. Must be `'N'` or `'T'` or `'C'`. Indicates the form of the equations: If *trans* = `'N'`, then `sub(A)*X = sub(B)` is solved for *X*. If *trans* = `'T'`, then $\text{sub}(A)^T * X = \text{sub}(B)$ is solved for *X*. If *trans* = `'C'`, then $\text{sub}(A)^H * X = \text{sub}(B)$ is solved for *X*.
n	(global) `INTEGER`. The number of linear equations; the order of the submatrix `sub(A)` ($n \geq 0$).
nrhs	(global) `INTEGER`. The number of right hand sides; the number of columns of the distributed submatrix `sub(B)` ($nrhs \geq 0$).
a, *b*	(global) `REAL` for `psgetrs` `DOUBLE PRECISION` for `pdgetrs` `COMPLEX` for `pcgetrs` `DOUBLE COMPLEX` for `pzgetrs`.

	Pointers into the local memory to arrays of local dimension *a*(*lld_a*, *LOCc*(*ja*+*n*-1)) and *b*(*lld_b*, *LOCc*(*jb*+*nrhs*-1)), respectively. On entry, the array *a* contains the local pieces of the factors *L* and *U* from the factorization sub(*A*) = *P***L***U*; the unit diagonal elements of *L* are not stored. On entry, the array *b* contains the right hand sides sub(*B*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
ipiv	(local) INTEGER array. The dimension of *ipiv* is (*LOCr*(*m_a*) + *mb_a*). This array contains the pivoting information: local row i of the matrix was interchanged with the global row *ipiv*(i). This array is tied to the distributed matrix *A*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

Output Parameters

b	On exit, overwritten by the solution distributed matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?gbtrs

Solves a system of distributed linear equations with a general band matrix, using the *LU* *factorization computed by* p?gbtrf.

Syntax

```
call psgbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, af, laf,
work, lwork, info)
```

```
call pdgbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, af, laf,
work, lwork, info)
```

```
call pcgbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, af, laf,
work, lwork, info)
```

```
call pzgbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, af, laf,
work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gbtrs routine solves a system of distributed linear equations with a general band distributed matrix sub(*A*) = *A*(*1:n*, *ja:ja+n-1*) using the *LU* factorization computed by p?gbtrf.

The system has one of the following forms specified by *trans*:

sub(*A*)**X* = sub(*B*) (no transpose),

sub(*A*)^T**X* = sub(*B*) (transpose),

sub(*A*)^H**X* = sub(*B*) (conjugate transpose),

where sub(*B*) = *B*(*ib:ib+n-1*, *1:nrhs*) .

Before calling this routine, you must call p?gbtrf to compute the *LU* factorization of sub(*A*).

Input Parameters

trans	(global) CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then sub(*A*)**X* = sub(*B*) is solved for *X*. If *trans* = 'T', then sub(*A*)^T**X* = sub(*B*) is solved for *X*. If *trans* = 'C', then sub(*A*)^H **X* = sub(*B*) is solved for *X*.
n	(global) INTEGER. The number of linear equations; the order of the distributed submatrix sub(*A*) (*n* ≥ 0).
bwl	(global) INTEGER. The number of sub-diagonals within the band of *A* (0 ≤ *bwl* ≤ *n-1*).
bwu	(global) INTEGER. The number of super-diagonals within the band of *A* (0 ≤ *bwu* ≤ *n-1*).
nrhs	(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs* ≥ 0).
a, *b*	(global) REAL for psgbtrs DOUBLE PRECISION for pdgbtrs COMPLEX for pcgbtrs DOUBLE COMPLEX for pzgbtrs. Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOC*$_c$(*ja+n-1*)) and *b*(*lld_b*,*LOC*$_c$(*nrhs*)), respectively. The array *a* contains details of the *LU* factorization of the distributed band matrix *A*. On entry, the array *b* contains the local pieces of the right hand sides *B*(*ib:ib+n-1*, *1:nrhs*).
ja	(global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*dtype_*) = 501, then *dlen_* ≥ 7; else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.
ib	(global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*dtype_*) = 502 , then *dlen_* ≥ 7; else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.
laf	(local) INTEGER. The dimension of the array *af*. Must be *laf* ≥ NB*(*bwl*+*bwu*)+6*(*bwl*+*bwu*)*(*bwl*+2**bwu*).

	If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).
work	(local) Same type as *a*. Workspace array of dimension *lwork*.
lwork	(local or global) INTEGER. The size of the *work* array, must be at least *lwork* ≥ *nrhs**(NB+2**bwl*+4**bwu*).

Output Parameters

ipiv	(local) INTEGER array. The dimension of *ipiv* must be ≥ *desca*(NB). Contains pivot indices for local factorizations. Note that you should not alter the contents of this array between factorization and solve.
b	On exit, overwritten by the local pieces of the solution distributed matrix *X*.
af	(local) REAL for psgbtrs DOUBLE PRECISION for pdgbtrs COMPLEX for pcgbtrs DOUBLE COMPLEX for pzgbtrs. Array, dimension (*laf*). Auxiliary Fillin space. Fillin is created during the factorization routine p?gbtrf and this is stored in *af*. Note that if a linear system is to be solved using p?gbtrs after the factorization routine, *af* must not be altered after the factorization.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?dbtrs

Solves a system of linear equations with a diagonally dominant-like banded distributed matrix using the factorization computed by p?dbtrf.

Syntax

```
call psdbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pddbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pcdbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pzdbtrs(trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dbtrs routine solves for X one of the systems of equations:

sub(A)*X = sub(B),

(sub(A))^T*X = sub(B), or

(sub(A))^H*X = sub(B),

where sub(A) = $A(1:n, ja:ja+n-1)$ is a diagonally dominant-like banded distributed matrix, and sub(B) denotes the distributed matrix $B(ib:ib+n-1, 1:nrhs)$.

This routine uses the LU factorization computed by p?dbtrf.

Input Parameters

trans	(global) CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then sub(A)*X = sub(B) is solved for X. If *trans* = 'T', then (sub(A))^T*X = sub(B) is solved for X. If *trans* = 'C', then (sub(A))^H*X = sub(B) is solved for X.
n	(global) INTEGER. The order of the distributed submatrix sub(A) ($n \geq 0$).
bwl	(global) INTEGER. The number of subdiagonals within the band of A ($0 \leq bwl \leq n-1$).
bwu	(global) INTEGER. The number of superdiagonals within the band of A ($0 \leq bwu \leq n-1$).
nrhs	(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(B) ($nrhs \geq 0$).
a, *b*	(local) REAL for psdbtrs DOUBLE PRECISON for pddbtrs COMPLEX for pcdbtrs DOUBLE COMPLEX for pzdbtrs. Pointers into the local memory to arrays of local dimension $a(lld\_a, LOCc(ja+n-1))$ and $b(lld\_b, LOCc(nrhs))$, respectively. On entry, the array a contains details of the LU factorization of the band matrix A, as computed by p?dbtrf. On entry, the array b contains the local pieces of the right hand side distributed matrix sub(B).
ja	(global) INTEGER. The index in the global array A that points to the start of the matrix to be operated on (which may be either all of A or a submatrix of A).
desca	(global and local) INTEGER array, dimension ($dlen\_$). The array descriptor for the distributed matrix A. If $desca(dtype\_) = 501$, then $dlen\_ \geq 7$; else if $desca(dtype\_) = 1$, then $dlen\_ \geq 9$.
ib	(global) INTEGER. The row index in the global array B that points to the first row of the matrix to be operated on (which may be either all of B or a submatrix of B).
descb	(global and local) INTEGER array, dimension ($dlen\_$). The array descriptor for the distributed matrix B. If $descb(dtype\_) = 502$, then $dlen\_ \geq 7$; else if $descb(dtype\_) = 1$, then $dlen\_ \geq 9$.

af, *work*	(local) REAL for psdbtrs DOUBLE PRECISION for pddbtrs COMPLEX for pcdbtrs DOUBLE COMPLEX for pzdbtrs. Arrays of dimension (*laf*) and (*lwork*), respectively The array *af* contains auxiliary Fillin space. Fillin is created during the factorization routine p?dbtrf and this is stored in *af*. The array *work* is a workspace array.
laf	(local) INTEGER. The dimension of the array *af*. Must be *laf* ≥ NB*(*bwl*+*bwu*)+6*(max(*bwl*,*bwu*))^2 . If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).
lwork	(local or global) INTEGER. The size of the array *work*, must be at least *lwork* ≥ (max(*bwl*,*bwu*))^2.

Output Parameters

b	On exit, this array contains the local pieces of the solution distributed matrix *X*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: if the *i*th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?dttrs

Solves a system of linear equations with a diagonally dominant-like tridiagonal distributed matrix using the factorization computed by p?dttrf.

Syntax

```
call psdttrs(trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pddttrs(trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pcdttrs(trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pzdttrs(trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dttrs routine solves for *X* one of the systems of equations:

sub(*A*)**X* = sub(*B*),

(sub(*A*))^T**X* = sub(*B*), or

(sub(*A*))^{H}**X* = sub(*B*),

where sub(*A*) = (1:*n*, *ja*:*ja*+*n*-1); is a diagonally dominant-like tridiagonal distributed matrix, and sub(*B*) denotes the distributed matrix *B*(*ib*:*ib*+*n*-1, 1:*nrhs*).

This routine uses the *LU* factorization computed by p?dttrf.

Input Parameters

trans	(global) CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then sub(*A*)**X* = sub(*B*) is solved for *X*. If *trans* = 'T', then (sub(*A*))^{T}**X* = sub(*B*) is solved for *X*. If *trans* = 'C', then (sub(*A*))^{H}**X* = sub(*B*) is solved for *X*.
n	(global) INTEGER. The order of the distributed submatrix sub(*A*) (*n* ≥ 0).
nrhs	(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs* ≥ 0).
dl, *d*, *du*	(local) REAL for psdttrs DOUBLE PRECISON for pddttrs COMPLEX for pcdttrs DOUBLE COMPLEX for pzdttrs. Pointers to the local arrays of dimension (*desca*(*nb_*)) each. On entry, these arrays contain details of the factorization. Globally, *dl*(1) and *du*(*n*) are not referenced; *dl* and *du* must be aligned with *d*.
ja	(global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*dtype_*) = 501 or 502, then *dlen_* ≥ 7; else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.
b	(local) Same type as *d*. Pointer into the local memory to an array of local dimension *b*(*lld_b*,*LOCc*(*nrhs*)). On entry, the array *b* contains the local pieces of the *n*-by-*nrhs* right hand side distributed matrix sub(*B*).
ib	(global) INTEGER. The row index in the global array *B* that points to the first row of the matrix to be operated on (which may be either all of *B* or a submatrix of *B*).
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*. If *descb*(*dtype_*) = 502, then *dlen_* ≥ 7; else if *descb*(*dtype_*) = 1, then *dlen_* ≥ 9.
af, *work*	(local) REAL for psdttrs DOUBLE PRECISION for pddttrs COMPLEX for pcdttrs DOUBLE COMPLEX for pzdttrs. Arrays of dimension (*laf*) and (*lwork*), respectively.

The array *af* contains auxiliary Fillin space. Fillin is created during the factorization routine p?dttrf and this is stored in *af*. If a linear system is to be solved using p?dttrsafter the factorization routine, *af* must not be altered.
The array *work* is a workspace array.

laf (local) INTEGER. The dimension of the array *af*.
Must be *laf* ≥ NB*(*bwl*+*bwu*)+6*(*bwl*+*bwu*)*(*bwl*+2*bwu*).
If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

lwork (local or global) INTEGER. The size of the array *work*, must be at least *lwork* ≥ 10*NPCOL+4**nrhs*.

Output Parameters

b On exit, this array contains the local pieces of the solution distributed matrix *X*.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info INTEGER. If *info*=0, the execution is successful. *info* < 0:
if the *i*th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?potrs

Solves a system of linear equations with a Cholesky-factored symmetric/Hermitian distributed positive-definite matrix.

Syntax

```
call pspotrs(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pdpotrs(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pcpotrs(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pzpotrs(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?potrs routine solves for *X* a system of distributed linear equations in the form:

sub(*A*)**X* = sub(*B*) ,

where sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*) is an *n*-by-*n* real symmetric or complex Hermitian positive definite distributed matrix, and sub(*B*) denotes the distributed matrix *B*(*ib:ib+n-1*, *jb:jb+nrhs-1*).

This routine uses Cholesky factorization

sub(*A*) = U^H**U*, or sub(*A*) = *L**L^H

computed by p?potrf.

Input Parameters

uplo	(global) CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', upper triangle of sub(*A*) is stored; If *uplo* = 'L', lower triangle of sub(*A*) is stored.
n	(global) INTEGER. The order of the distributed submatrix sub(*A*) (*n*≥0).
nrhs	(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs*≥0).
a, *b*	(local) REAL for pspotrs DOUBLE PRECISION for pdpotrs COMPLEX for pcpotrs DOUBLE COMPLEX for pzpotrs. Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)) and *b*(*lld_b*,*LOCc*(*jb*+*nrhs*-1)), respectively. The array *a* contains the factors *L* or *U* from the Cholesky factorization sub(*A*) = *L***L*^{H} or sub(*A*) = *U*^{H}**U*, as computed by p?potrf. On entry, the array *b* contains the local pieces of the right hand sides sub(*B*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

Output Parameters

b	Overwritten by the local pieces of the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: if the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?pbtrs

Solves a system of linear equations with a Cholesky-factored symmetric/Hermitian positive-definite band matrix.

Syntax

```
call pspbtrs(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

```
call pdpbtrs(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

```
call pcpbtrs(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

call pzpbtrs(*uplo*, *n*, *bw*, *nrhs*, *a*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?pbtrs routine solves for *X* a system of distributed linear equations in the form:

sub(*A*)***X* = sub(*B*) ,

where sub(*A*) = *A*(*1:n*, *ja:ja+n-1*) is an *n*-by-*n* real symmetric or complex Hermitian positive definite distributed band matrix, and sub(*B*) denotes the distributed matrix *B*(*ib:ib+n-1*, *1:nrhs*).

This routine uses Cholesky factorization

sub(*A*) = $P*U^H*U*P^T$, or sub(*A*) = $P*L*L^H*P^T$

computed by p?pbtrf.

Input Parameters

uplo — (global) CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', upper triangle of sub(*A*) is stored;
If *uplo* = 'L', lower triangle of sub(*A*) is stored.

n — (global) INTEGER. The order of the distributed submatrix sub(*A*) (*n*≥0).

bw — (global) INTEGER. The number of superdiagonals of the distributed matrix if *uplo* = 'U', or the number of subdiagonals if *uplo* = 'L' (*bw*≥0).

nrhs — (global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs*≥0).

a, *b* — (local)
REAL for pspbtrs
DOUBLE PRECISION for pdpbtrs
COMPLEX for pcpbtrs
DOUBLE COMPLEX for pzpbtrs.
Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja+n*-1)) and *b*(*lld_b*,*LOCc*(*nrhs-1*)), respectively.
The array *a* contains the permuted triangular factor *U* or *L* from the Cholesky factorization sub(*A*) = $P*U^H*U*P^T$, or sub(*A*) = $P*L*L^H*P^T$ of the band matrix *A*, as returned by p?pbtrf.
On entry, the array *b* contains the local pieces of the *n*-by-*nrhs* right hand side distributed matrix sub(*B*).

ja — (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
If *desca*(*dtype_*) = 501, then *dlen_* ≥ 7;
else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.

ib — (global) INTEGER. The row index in the global array *B* indicating the first row of the submatrix sub(*B*).

descb — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.
If *descb*(*dtype_*) = 502, then *dlen_* ≥ 7;

	else if *descb*(*dtype_*) = 1, then *dlen_* ≥ 9.
af, *work*	(local) Arrays, same type as *a*. The array *af* is of dimension (*laf*). It contains auxiliary Fillin space. Fillin is created during the factorization routine p?dbtrf and this is stored in *af*. The array *work* is a workspace array of dimension *lwork*.
laf	(local) INTEGER. The dimension of the array *af*. Must be *laf* ≥ *nrhs***bw*. If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).
lwork	(local or global) INTEGER. The size of the array *work*, must be at least *lwork* ≥ bw^2.

Output Parameters

b	On exit, if *info*=0, this array contains the local pieces of the *n*-by-*nrhs* solution distributed matrix *X*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?pttrs

Solves a system of linear equations with a symmetric (Hermitian) positive-definite tridiagonal distributed matrix using the factorization computed by p?pttrf.

Syntax

```
call pspttrs(n, nrhs, d, e, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pdpttrs(n, nrhs, d, e, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pcpttrs(uplo, n, nrhs, d, e, ja, desca, b, ib, descb, af, laf, work, lwork, info)
call pzpttrs(uplo, n, nrhs, d, e, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?pttrs routine solves for *X* a system of distributed linear equations in the form:

sub(*A*)**X* = sub(*B*) ,

where sub(*A*) = *A*(*1:n*, *ja:ja+n-1*) is an *n*-by-*n* real symmetric or complex Hermitian positive definite tridiagonal distributed matrix, and sub(*B*) denotes the distributed matrix *B*(*ib:ib+n-1*, *1:nrhs*).

This routine uses the factorization

sub(*A*) = $P*L*D*L^H*P^T$, or sub(*A*) = $P*U^H*D*U*P^T$

computed by p?pttrf.

Input Parameters

uplo	(global, used in complex flavors only) CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', upper triangle of sub(*A*) is stored; If *uplo* = 'L', lower triangle of sub(*A*) is stored.
n	(global) INTEGER. The order of the distributed submatrix sub(*A*) (*n*≥0).
nrhs	(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs*≥0).
d, *e*	(local) REAL for pspttrs DOUBLE PRECISON for pdpttrs COMPLEX for pcpttrs DOUBLE COMPLEX for pzpttrs. Pointers into the local memory to arrays of dimension (*desca*(*nb_*)) each. These arrays contain details of the factorization as returned by p?pttrf
ja	(global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*dtype_*) = 501 or 502, then *dlen_* ≥ 7; else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.
b	(local) Same type as *d*, *e*. Pointer into the local memory to an array of local dimension *b*(*lld_b*, *LOCc*(*nrhs*)). On entry, the array *b* contains the local pieces of the *n*-by-*nrhs* right hand side distributed matrix sub(*B*).
ib	(global) INTEGER. The row index in the global array *B* that points to the first row of the matrix to be operated on (which may be either all of *B* or a submatrix of *B*).
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*. If *descb*(*dtype_*) = 502, then *dlen_* ≥ 7; else if *descb*(*dtype_*) = 1, then *dlen_* ≥ 9.
af, *work*	(local) REAL for pspttrs DOUBLE PRECISION for pdpttrs COMPLEX for pcpttrs DOUBLE COMPLEX for pzpttrs. Arrays of dimension (*laf*) and (*lwork*), respectively The array *af* contains auxiliary Fillin space. Fillin is created during the factorization routine p?pttrf and this is stored in *af*. The array *work* is a workspace array.
laf	(local) INTEGER. The dimension of the array *af*. Must be *laf* ≥ NB+2. If *laf* is not large enough, an error code is returned and the minimum acceptable size will be returned in *af*(1).
lwork	(local or global) INTEGER. The size of the array *work*, must be at least *lwork* ≥ (10+2*min(100,*nrhs*))*NPCOL+4**nrhs*.

Output Parameters

b	On exit, this array contains the local pieces of the solution distributed matrix *X*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: if the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?trtrs

Solves a system of linear equations with a triangular distributed matrix.

Syntax

```
call pstrtrs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pdtrtrs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pctrtrs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pztrtrs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?trtrs routine solves for *X* one of the following systems of linear equations:

sub(*A*)**X* = sub(*B*),

(sub(*A*))^{T}**X* = sub(*B*), or

(sub(*A*))^{H}**X* = sub(*B*),

where sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*) is a triangular distributed matrix of order *n*, and sub(*B*) denotes the distributed matrix *B*(*ib:ib+n-1*, *jb:jb+nrhs-1*).

A check is made to verify that sub(*A*) is nonsingular.

Input Parameters

uplo	(global) CHARACTER*1. Must be 'U' or 'L'. Indicates whether sub(*A*) is upper or lower triangular: If *uplo* = 'U', then sub(*A*) is upper triangular. If *uplo* = 'L', then sub(*A*) is lower triangular.
trans	(global) CHARACTER*1. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then sub(*A*)**X* = sub(*B*) is solved for *X*. If *trans* = 'T', then sub(*A*)^{T}**X* = sub(*B*) is solved for *X*. If *trans* = 'C', then sub(*A*)^{H}**X* = sub(*B*) is solved for *X*.
diag	(global) CHARACTER*1. Must be 'N' or 'U'. If *diag* = 'N', then sub(*A*) is not a unit triangular matrix. If *diag* = 'U', then sub(*A*) is unit triangular.

n	(global) INTEGER. The order of the distributed submatrix sub(*A*) (*n*≥0).
nrhs	(global) INTEGER. The number of right-hand sides; i.e., the number of columns of the distributed matrix sub(*B*) (*nrhs*≥0).
a, *b*	(local) REAL for pstrtrs DOUBLE PRECISION for pdtrtrs COMPLEX for pctrtrs DOUBLE COMPLEX for pztrtrs. Pointers into the local memory to arrays of local dimension *a*(*lld_a*, *LOCc*(*ja*+*n*-1)) and *b*(*lld_b*, *LOCc*(*jb*+*nrhs*-1)), respectively. The array *a* contains the local pieces of the distributed triangular matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular matrix, and the strictly lower triangular part of sub(*A*) is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular matrix, and the strictly upper triangular part of sub(*A*) is not referenced. If *diag* = 'U', the diagonal elements of sub(*A*) are also not referenced and are assumed to be 1. On entry, the array *b* contains the local pieces of the right hand side distributed matrix sub(*B*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

Output Parameters

b	On exit, if *info*=0, sub(*B*) is overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: if the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*th argument is a scalar and had an illegal value, then *info* = -*i*; *info* > 0: if *info* = *i*, the *i*-th diagonal element of sub(*A*) is zero, indicating that the submatrix is singular and the solutions *X* have not been computed.

Routines for Estimating the Condition Number

This section describes the ScaLAPACK routines for estimating the condition number of a matrix. The condition number is used for analyzing the errors in the solution of a system of linear equations. Since the condition number may be arbitrarily large when the matrix is nearly singular, the routines actually compute the *reciprocal* condition number.

p?gecon

Estimates the reciprocal of the condition number of a general distributed matrix in either the 1-norm or the infinity-norm.

Syntax

```
call psgecon(norm, n, a, ia, ja, desca, anorm, rcond, work, lwork, iwork, liwork, info)
call pdgecon(norm, n, a, ia, ja, desca, anorm, rcond, work, lwork, iwork, liwork, info)
call pcgecon(norm, n, a, ia, ja, desca, anorm, rcond, work, lwork, rwork, lrwork, info)
call pzgecon(norm, n, a, ia, ja, desca, anorm, rcond, work, lwork, rwork, lrwork, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?gecon` routine estimates the reciprocal of the condition number of a general distributed real/complex matrix `sub(A) = A(ia:ia+n-1, ja:ja+n-1)` in either the 1-norm or infinity-norm, using the *LU* factorization computed by `p?getrf`.

An estimate is obtained for $||(\mathrm{sub}(A))^{-1}||$, and the reciprocal of the condition number is computed as

Input Parameters

norm	(global) `CHARACTER*1`. Must be `'1'` or `'O'` or `'I'`. Specifies whether the 1-norm condition number or the infinity-norm condition number is required. If *norm* = `'1'` or `'O'`, then the 1-norm is used; If *norm* = `'I'`, then the infinity-norm is used.
n	(global) `INTEGER`. The order of the distributed submatrix sub(*A*) ($n \geq 0$).
a	(local) `REAL` for `psgecon` `DOUBLE PRECISION` for `pdgecon` `COMPLEX` for `pcgecon` `DOUBLE COMPLEX` for `pzgecon`. Pointer into the local memory to an array of dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)). The array *a* contains the local pieces of the factors *L* and *U* from the factorization sub(*A*) = *P***L***U*; the unit diagonal elements of *L* are not stored.

ia, *ja* (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

anorm (global) REAL for single precision flavors, DOUBLE PRECISION for double precision flavors.
If *norm* = '1' or 'O', the 1-norm of the original distributed matrix sub(*A*);
If *norm* = 'I', the infinity-norm of the original distributed matrix sub(*A*).

work (local)
REAL for psgecon
DOUBLE PRECISION for pdgecon
COMPLEX for pcgecon
DOUBLE COMPLEX for pzgecon.
The array *work* of dimension (*lwork*) is a workspace array.

lwork (local or global) INTEGER. The dimension of the array *work*.
For real flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))+ 2**LOCc*(*n*+mod(*ja*-1,*nb_a*))+ max(2, max(*nb_a**max(1, iceil(NPROW-1, NPCOL)),*LOCc*(*n* +mod(*ja*-1,*nb_a*)) + *nb_a**max(1, iceil(NPCOL-1, NPROW)))).
For complex flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))+ max(2, max(*nb_a**iceil(NPROW-1, NPCOL),*LOCc*(*n*+mod(*ja*-1,*nb_a*))+ *nb_a**iceil(NPCOL-1, NPROW))).
LOCr and *LOCc* values can be computed using the ScaLAPACK tool function numroc; NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.

iwork (local) INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only.

liwork (local or global) INTEGER. The dimension of the array *iwork*; used in real flavors only. Must be at least
liwork ≥ *LOCr*(*n*+mod(*ia*-1,*mb_a*)).

rwork (local) REAL for pcgecon
DOUBLE PRECISION for pzgecon
Workspace array, DIMENSION (*lrwork*). Used in complex flavors only.

lrwork (local or global) INTEGER. The dimension of the array *rwork*; used in complex flavors only. Must be at least
lrwork ≥ max(1, 2**LOCc*(*n*+mod(*ja*-1,*nb_a*))).

Output Parameters

rcond (global) REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
The reciprocal of the condition number of the distributed matrix sub(*A*). See Description.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

iwork(1) On exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance (for real flavors).

rwork(1)	On exit, *rwork*(1) contains the minimum value of *lrwork* required for optimum performance (for complex flavors).
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?pocon

Estimates the reciprocal of the condition number (in the 1 - norm) of a symmetric / Hermitian positive-definite distributed matrix.

Syntax

```
call pspocon(uplo, n, a, ia, ja, desca, anorm, rcond, work, lwork, iwork, liwork, info)
call pdpocon(uplo, n, a, ia, ja, desca, anorm, rcond, work, lwork, iwork, liwork, info)
call pcpocon(uplo, n, a, ia, ja, desca, anorm, rcond, work, lwork, rwork, lrwork, info)
call pzpocon(uplo, n, a, ia, ja, desca, anorm, rcond, work, lwork, rwork, lrwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?pocon routine estimates the reciprocal of the condition number (in the 1 - norm) of a real symmetric or complex Hermitian positive definite distributed matrix sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1), using the Cholesky factorization sub(*A*) = U^H*U or sub(*A*) = $L*L^H$ computed by p?potrf.

An estimate is obtained for $||(sub(A))^{-1}||$, and the reciprocal of the condition number is computed as

Input Parameters

uplo	(global) CHARACTER*1. Must be 'U' or 'L'. Specifies whether the factor stored in sub(*A*) is upper or lower triangular. If *uplo* = 'U', sub(*A*) stores the upper triangular factor *U* of the Cholesky factorization sub(*A*) = U^H*U. If *uplo* = 'L', sub(*A*) stores the lower triangular factor *L* of the Cholesky factorization sub(*A*) = $L*L^H$.
n	(global) INTEGER. The order of the distributed submatrix sub(*A*) (*n*≥0).
a	(local) REAL for pspocon

DOUBLE PRECISION for pdpocon
COMPLEX for pcpocon
DOUBLE COMPLEX for pzpocon.
Pointer into the local memory to an array of dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)).
The array *a* contains the local pieces of the factors *L* or *U* from the Cholesky factorization sub(*A*) = U^H*U, or sub(*A*) = $L*L^H$, as computed by p?potrf.

ia, *ja* — (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

anorm — (global) REAL for single precision flavors,
DOUBLE PRECISION for double precision flavors.
The 1-norm of the symmetric/Hermitian distributed matrix sub(*A*).

work — (local)
REAL for pspocon
DOUBLE PRECISION for pdpocon
COMPLEX for pcpocon
DOUBLE COMPLEX for pzpocon.
The array *work* of dimension (*lwork*) is a workspace array.

lwork — (local or global) INTEGER. The dimension of the array *work*.
For real flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))+ 2**LOCc*(*n*+mod(*ja*-1,*nb_a*))+ max(2, max(*nb_a**iceil(NPROW-1, NPCOL),*LOCc*(*n*+mod(*ja*-1,*nb_a*)) + *nb_a**iceil(NPCOL-1, NPROW))).
For complex flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))+ max(2, max(*nb_a**max(1,iceil(NPROW-1, NPCOL)),*LOCc*(*n*+mod(*ja*-1,*nb_a*)) + *nb_a**max(1,iceil(NPCOL-1, NPROW)))).

iwork — (local) INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only.

liwork — (local or global) INTEGER. The dimension of the array *iwork*; used in real flavors only. Must be at least *liwork* ≥ *LOCr*(*n*+mod(*ia*-1,*mb_a*)).

rwork — (local) REAL for pcpocon
DOUBLE PRECISION for pzpocon
Workspace array, DIMENSION (*lrwork*). Used in complex flavors only.

lrwork — (local or global) INTEGER. The dimension of the array *rwork*; used in complex flavors only. Must be at least *lrwork* ≥ 2**LOCc*(*n*+mod(*ja*-1,*nb_a*)).

Output Parameters

rcond — (global) REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
The reciprocal of the condition number of the distributed matrix sub(*A*).

work(1) — On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

iwork(1)	On exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance (for real flavors).
rwork(1)	On exit, *rwork*(1) contains the minimum value of *lrwork* required for optimum performance (for complex flavors).
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*th argument is a scalar and had an illegal value, then *info* = -*i*.

p?trcon

Estimates the reciprocal of the condition number of a triangular distributed matrix in either 1-norm or infinity-norm.

Syntax

call pstrcon(*norm, uplo, diag, n, a, ia, ja, desca, rcond, work, lwork, iwork, liwork, info*)

call pdtrcon(*norm, uplo, diag, n, a, ia, ja, desca, rcond, work, lwork, iwork, liwork, info*)

call pctrcon(*norm, uplo, diag, n, a, ia, ja, desca, rcond, work, lwork, rwork, lrwork, info*)

call pztrcon(*norm, uplo, diag, n, a, ia, ja, desca, rcond, work, lwork, rwork, lrwork, info*)

Include Files

- C: mkl_scalapack.h

Description

The p?trcon routine estimates the reciprocal of the condition number of a triangular distributed matrix sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*), in either the 1-norm or the infinity-norm.

The norm of sub(*A*) is computed and an estimate is obtained for $||(\text{sub}(A))^{-1}||$, then the reciprocal of the condition number is computed as

$$rcond = \frac{1}{\|\text{sub}(A)\| \times \|(\text{sub}(A))^{-1}\|}$$

Input Parameters

norm	(global) CHARACTER*1. Must be '1' or 'O' or 'I'. Specifies whether the 1-norm condition number or the infinity-norm condition number is required. If *norm* = '1' or 'O', then the 1-norm is used; If *norm* = 'I', then the infinity-norm is used.
uplo	(global) CHARACTER*1. Must be 'U' or 'L'. If *uplo* = 'U', sub(*A*) is upper triangular. If *uplo* = 'L', sub(*A*) is lower triangular.
diag	(global) CHARACTER*1. Must be 'N' or 'U'.

If *diag* = 'N', sub(*A*) is non-unit triangular. If *diag* = 'U', sub(*A*) is unit triangular.

n (global) INTEGER. The order of the distributed submatrix sub(*A*), (*n*≥0).

a (local)
REAL for pstrcon
DOUBLE PRECISION for pdtrcon
COMPLEX for pctrcon
DOUBLE COMPLEX for pztrcon.
Pointer into the local memory to an array of dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)).
The array *a* contains the local pieces of the triangular distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of this distributed matrix contains the upper triangular matrix, and its strictly lower triangular part is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of this distributed matrix contains the lower triangular matrix, and its strictly upper triangular part is not referenced.
If *diag* = 'U', the diagonal elements of sub(*A*) are also not referenced and are assumed to be 1.

ia, *ja* (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local)
REAL for pstrcon
DOUBLE PRECISION for pdtrcon
COMPLEX for pctrcon
DOUBLE COMPLEX for pztrcon.
The array *work* of dimension (*lwork*) is a workspace array.

lwork (local or global) INTEGER. The dimension of the array *work*.
For real flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))+ *LOCc*(*n*+mod(*ja*-1,*nb_a*))+
max(2, max(*nb_a**max(1,iceil(NPROW-1, NPCOL)),
LOCc(*n*+mod(*ja*-1,*nb_a*))+ *nb_a**max(1,iceil(NPCOL-1, NPROW)))).
For complex flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))+ max(2,
max(*nb_a**iceil(NPROW-1, NPCOL),
LOCc(*n*+mod(*ja*-1,*nb_a*))+ *nb_a**iceil(NPCOL-1, NPROW))).

iwork (local) INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only.

liwork (local or global) INTEGER. The dimension of the array *iwork*; used in real flavors only. Must be at least
liwork ≥ *LOCr*(*n*+mod(*ia*-1,*mb_a*)).

rwork (local) REAL for pcpocon
DOUBLE PRECISION for pzpocon
Workspace array, DIMENSION (*lrwork*). Used in complex flavors only.

lrwork	(local or global) INTEGER. The dimension of the array *rwork*; used in complex flavors only. Must be at least *lrwork* ≥ *LOCc*(*n*+mod(*ja*-1,*nb_a*)).

Output Parameters

rcond	(global) REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. The reciprocal of the condition number of the distributed matrix sub(*A*).
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
iwork(1)	On exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance (for real flavors).
rwork(1)	On exit, *rwork*(1) contains the minimum value of *lrwork* required for optimum performance (for complex flavors).
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Refining the Solution and Estimating Its Error

This section describes the ScaLAPACK routines for refining the computed solution of a system of linear equations and estimating the solution error. You can call these routines after factorizing the matrix of the system of equations and computing the solution (see Routines for Matrix Factorization and Solving Systems of Linear Equations).

p?gerfs

Improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution.

Syntax

call psgerfs(*trans*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *ipiv*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *ferr*, *berr*, *work*, *lwork*, *iwork*, *liwork*, *info*)

call pdgerfs(*trans*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *ipiv*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *ferr*, *berr*, *work*, *lwork*, *iwork*, *liwork*, *info*)

call pcgerfs(*trans*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *ipiv*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *ferr*, *berr*, *work*, *lwork*, *rwork*, *lrwork*, *info*)

call pzgerfs(*trans*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *ipiv*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *ferr*, *berr*, *work*, *lwork*, *rwork*, *lrwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?gerfs routine improves the computed solution to one of the systems of linear equations

sub(*A*)*sub(*X*) = sub(*B*),

sub(*A*)^T*sub(*X*) = sub(*B*), or

sub(*A*)^H*sub(*X*) = sub(*B*) and provides error bounds and backward error estimates for the solution.

Here sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*), sub(*B*) = *B*(*ib:ib+n-1*, *jb:jb+nrhs-1*), and sub(*X*) = *X*(*ix:ix+n-1*, *jx:jx+nrhs-1*).

Input Parameters

trans — (global) CHARACTER*1. Must be 'N' or 'T' or 'C'.
Specifies the form of the system of equations:
If *trans* = 'N', the system has the form sub(*A*)*sub(*X*) = sub(*B*) (No transpose);
If *trans* = 'T', the system has the form sub(*A*)^T*sub(*X*) = sub(*B*) (Transpose);
If *trans* = 'C', the system has the form sub(*A*)^H*sub(*X*) = sub(*B*) (Conjugate transpose).

n — (global) INTEGER. The order of the distributed submatrix sub(*A*) (*n* ≥ 0).

nrhs — (global) INTEGER. The number of right-hand sides, i.e., the number of columns of the matrices sub(*B*) and sub(*X*) (*nrhs* ≥ 0).

a, *af*, *b*, *x* — (local)
REAL for psgerfs
DOUBLE PRECISION for pdgerfs
COMPLEX for pcgerfs
DOUBLE COMPLEX for pzgerfs.
Pointers into the local memory to arrays of local dimension *a*(*lld_a*, *LOCc*(*ja+n-1*)), *af*(*lld_af*, *LOCc*(*jaf+n-1*)), *b*(*lld_b*, *LOCc*(*jb+nrhs-1*)), and *x*(*lld_x*, *LOCc*(*jx+nrhs-1*)), respectively.
The array *a* contains the local pieces of the distributed matrix sub(*A*).
The array *af* contains the local pieces of the distributed factors of the matrix sub(*A*) = *P*L*U* as computed by p?getrf.
The array *b* contains the local pieces of the distributed matrix of right hand sides sub(*B*).
On entry, the array *x* contains the local pieces of the distributed solution matrix sub(*X*).

ia, *ja* — (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

iaf, *jaf* — (global) INTEGER. The row and column indices in the global array *AF* indicating the first row and the first column of the submatrix sub(*AF*), respectively.

descaf — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *AF*.

ib, *jb* — (global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

ix, *jx* — (global) INTEGER. The row and column indices in the global array *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.

descx (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *X*.

ipiv (local) INTEGER.
Array, dimension *LOCr*(*m_af* + *mb_af*.
This array contains pivoting information as computed by p?getrf. If *ipiv*(i)=j, then the local row i was swapped with the global row j.
This array is tied to the distributed matrix *A*.

work (local)
REAL for psgerfs
DOUBLE PRECISION for pdgerfs
COMPLEX for pcgerfs
DOUBLE COMPLEX for pzgerfs.
The array *work* of dimension (*lwork*) is a workspace array.

lwork (local or global) INTEGER. The dimension of the array *work*.
For real flavors:
lwork must be at least
lwork ≥ 3**LOCr*(*n*+mod(*ia*-1,*mb_a*))
For complex flavors:
lwork must be at least
lwork ≥ 2**LOCr*(*n*+mod(*ia*-1,*mb_a*))

iwork (local) INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only.

liwork (local or global) INTEGER. The dimension of the array *iwork*; used in real flavors only. Must be at least
liwork ≥ *LOCr*(*n*+mod(*ib*-1,*mb_b*)).

rwork (local) REAL for pcgerfs
DOUBLE PRECISION for pzgerfs
Workspace array, DIMENSION (*lrwork*). Used in complex flavors only.

lrwork (local or global) INTEGER. The dimension of the array *rwork*; used in complex flavors only. Must be at least *lrwork* ≥ *LOCr*(*n* +mod(*ib*-1,*mb_b*))).

Output Parameters

x On exit, contains the improved solution vectors.

ferr, *berr* REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
Arrays, dimension *LOCc*(*jb+nrhs-1*) each.
The array *ferr* contains the estimated forward error bound for each solution vector of sub(*X*).
If XTRUE is the true solution corresponding to sub(*X*), *ferr* is an estimated upper bound for the magnitude of the largest element in (sub(*X*) – XTRUE) divided by the magnitude of the largest element in sub(*X*). The estimate is as reliable as the estimate for rcond, and is almost always a slight overestimate of the true error.
This array is tied to the distributed matrix *X*.
The array *berr* contains the component-wise relative backward error of each solution vector (that is, the smallest relative change in any entry of sub(*A*) or sub(*B*) that makes sub(*X*) an exact solution). This array is tied to the distributed matrix *X*.

work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
iwork(1)	On exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance (for real flavors).
rwork(1)	On exit, *rwork*(1) contains the minimum value of *lrwork* required for optimum performance (for complex flavors).
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?porfs

Improves the computed solution to a system of linear equations with symmetric/Hermitian positive definite distributed matrix and provides error bounds and backward error estimates for the solution.

Syntax

call psporfs(*uplo, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, iwork, liwork, info*)

call pdporfs(*uplo, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, iwork, liwork, info*)

call pcporfs(*uplo, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, rwork, lrwork, info*)

call pzporfs(*uplo, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, rwork, lrwork, info*)

Include Files

- C: mkl_scalapack.h

Description

The p?porfs routine improves the computed solution to the system of linear equations

sub(*A*)*sub(*X*) = sub(*B*),

where sub(*A*) = *A*(*ia:ia+n-1, ja:ja+n-1*) is a real symmetric or complex Hermitian positive definite distributed matrix and

sub(*B*) = *B*(*ib:ib+n-1, jb:jb+nrhs-1*),

sub(*X*) = *X*(*ix:ix+n-1, jx:jx+nrhs-1*)

are right-hand side and solution submatrices, respectively. This routine also provides error bounds and backward error estimates for the solution.

Input Parameters

uplo	(global) CHARACTER*1. Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix sub(*A*) is stored. If *uplo* = 'U', sub(*A*) is upper triangular. If *uplo* = 'L', sub(*A*) is lower triangular.

n	(global) INTEGER. The order of the distributed matrix sub(*A*) (*n*≥0).
nrhs	(global) INTEGER. The number of right-hand sides, i.e., the number of columns of the matrices sub(*B*) and sub(*X*) (*nrhs*≥0).
a, *af*, *b*, *x*	(local) REAL for psporfs DOUBLE PRECISION for pdporfs COMPLEX for pcporfs DOUBLE COMPLEX for pzporfs. Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)), *af*(*lld_af*,*LOCc*(*ja*+*n*-1)), *b*(*lld_b*,*LOCc*(*jb*+*nrhs*-1)), and *x*(*lld_x*,*LOCc*(*jx*+*nrhs*-1)), respectively. The array *a* contains the local pieces of the *n*-by-*n* symmetric/Hermitian distributed matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the distributed matrix, and its strictly upper triangular part is not referenced. The array *af* contains the factors *L* or *U* from the Cholesky factorization sub(*A*) = $L*L^H$ or sub(*A*) = U^H*U, as computed by p?potrf. On entry, the array *b* contains the local pieces of the distributed matrix of right hand sides sub(*B*). On entry, the array *x* contains the local pieces of the solution vectors sub(*X*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
iaf, *jaf*	(global) INTEGER. The row and column indices in the global array *AF* indicating the first row and the first column of the submatrix sub(*AF*), respectively.
descaf	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *AF*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.
ix, *jx*	(global) INTEGER. The row and column indices in the global array *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *X*.
work	(local) REAL for psporfs DOUBLE PRECISION for pdporfs COMPLEX for pcporfs

DOUBLE COMPLEX for pzporfs.
The array *work* of dimension (*lwork*) is a workspace array.

lwork (local) INTEGER. The dimension of the array *work*.
For real flavors:
lwork must be at least
lwork ≥ 3*LOCr(*n*+mod(*ia*-1,*mb_a*))
For complex flavors:
lwork must be at least
lwork ≥ 2*LOCr(*n*+mod(*ia*-1,*mb_a*))

iwork (local) INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only.

liwork (local or global) INTEGER. The dimension of the array *iwork*; used in real flavors only. Must be at least
liwork ≥ *LOCr*(*n*+mod(*ib*-1,*mb_b*)).

rwork (local) REAL for pcporfs
DOUBLE PRECISION for pzporfs
Workspace array, DIMENSION (*lrwork*). Used in complex flavors only.

lrwork (local or global) INTEGER. The dimension of the array *rwork*; used in complex flavors only. Must be at least *lrwork* ≥ *LOCr*(*n* +mod(*ib*-1,*mb_b*))).

Output Parameters

x On exit, contains the improved solution vectors.

ferr, *berr* REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
Arrays, dimension *LOCc*(*jb+nrhs-1*) each.
The array *ferr* contains the estimated forward error bound for each solution vector of sub(*X*).
If XTRUE is the true solution corresponding to sub(*X*), *ferr* is an estimated upper bound for the magnitude of the largest element in (sub(*X*) – XTRUE) divided by the magnitude of the largest element in sub(*X*). The estimate is as reliable as the estimate for rcond, and is almost always a slight overestimate of the true error.
This array is tied to the distributed matrix *X*.
The array *berr* contains the component-wise relative backward error of each solution vector (that is, the smallest relative change in any entry of sub(*A*) or sub(*B*) that makes sub(*X*) an exact solution). This array is tied to the distributed matrix *X*.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

iwork(1) On exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance (for real flavors).

rwork(1) On exit, *rwork*(1) contains the minimum value of *lrwork* required for optimum performance (for complex flavors).

info (global) INTEGER. If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?trrfs

Provides error bounds and backward error estimates for the solution to a system of linear equations with a distributed triangular coefficient matrix.

Syntax

```
call pstrrfs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, iwork, liwork, info)
```

```
call pdtrrfs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, iwork, liwork, info)
```

```
call pctrrfs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, rwork, lrwork, info)
```

```
call pztrrfs(uplo, trans, diag, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, x, ix, jx, descx, ferr, berr, work, lwork, rwork, lrwork, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?trrfs` routine provides error bounds and backward error estimates for the solution to one of the systems of linear equations

`sub(A)*sub(X) = sub(B)`,

$sub(A)^T*sub(X) = sub(B)$, or

$sub(A)^H*sub(X) = sub(B)$,

where $sub(A) = A(ia:ia+n-1, ja:ja+n-1)$ is a triangular matrix,

$sub(B) = B(ib:ib+n-1, jb:jb+nrhs-1)$, and

$sub(X) = X(ix:ix+n-1, jx:jx+nrhs-1)$.

The solution matrix X must be computed by `p?trtrs` or some other means before entering this routine. The routine `p?trrfs` does not do iterative refinement because doing so cannot improve the backward error.

Input Parameters

uplo	(global) `CHARACTER*1`. Must be `'U'` or `'L'`. If *uplo* = `'U'`, sub(*A*) is upper triangular. If *uplo* = `'L'`, sub(*A*) is lower triangular.
trans	(global) `CHARACTER*1`. Must be `'N'` or `'T'` or `'C'`. Specifies the form of the system of equations: If *trans* = `'N'`, the system has the form $sub(A)*sub(X) = sub(B)$ (No transpose); If *trans* = `'T'`, the system has the form $sub(A)^T*sub(X) = sub(B)$ (Transpose); If *trans* = `'C'`, the system has the form $sub(A)^H*sub(X) = sub(B)$ (Conjugate transpose).
diag	`CHARACTER*1`. Must be `'N'` or `'U'`. If *diag* = `'N'`, then sub(*A*) is non-unit triangular. If *diag* = `'U'`, then sub(*A*) is unit triangular.
n	(global) `INTEGER`. The order of the distributed matrix sub(*A*) ($n \geq 0$).

nrhs	(global) INTEGER. The number of right-hand sides, that is, the number of columns of the matrices sub(*B*) and sub(*X*) (*nrhs*≥0).
a, *b*, *x*	(local) REAL for pstrrfs DOUBLE PRECISION for pdtrrfs COMPLEX for pctrrfs DOUBLE COMPLEX for pztrrfs. Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)), *b*(*lld_b*,*LOCc*(*jb*+*nrhs*-1)), and *x*(*lld_x*,*LOCc*(*jx*+*nrhs*-1)), respectively. The array *a* contains the local pieces of the original triangular distributed matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the distributed matrix, and its strictly upper triangular part is not referenced. If *diag* = 'U', the diagonal elements of sub(*A*) are also not referenced and are assumed to be 1. On entry, the array *b* contains the local pieces of the distributed matrix of right hand sides sub(*B*). On entry, the array *x* contains the local pieces of the solution vectors sub(*X*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.
ix, *jx*	(global) INTEGER. The row and column indices in the global array *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *X*.
work	(local) REAL for pstrrfs DOUBLE PRECISION for pdtrrfs COMPLEX for pctrrfs DOUBLE COMPLEX for pztrrfs. The array *work* of dimension (*lwork*) is a workspace array.
lwork	(local) INTEGER. The dimension of the array *work*. *For real flavors:* *lwork* must be at least *lwork* ≥ 3*LOCr(*n*+mod(*ia*-1,*mb_a*)) *For complex flavors:* *lwork* must be at least *lwork* ≥ 2*LOCr(*n*+mod(*ia*-1,*mb_a*))

iwork	(local) INTEGER. Workspace array, DIMENSION (*liwork*). Used in real flavors only.
liwork	(local or global) INTEGER. The dimension of the array *iwork*; used in real flavors only. Must be at least *liwork* ≥ *LOCr*(*n*+mod(*ib*-1,*mb_b*)).
rwork	(local) REAL for pctrrfs DOUBLE PRECISION for pztrrfs Workspace array, DIMENSION (*lrwork*). Used in complex flavors only.
lrwork	(local or global) INTEGER. The dimension of the array *rwork*; used in complex flavors only. Must be at least *lrwork* ≥ *LOCr*(*n* +mod(*ib*-1,*mb_b*))).

Output Parameters

ferr, *berr*	REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. Arrays, dimension *LOCc*(*jb+nrhs-1*) each. The array *ferr* contains the estimated forward error bound for each solution vector of sub(*X*). If XTRUE is the true solution corresponding to sub(*X*), *ferr* is an estimated upper bound for the magnitude of the largest element in (sub(*X*) - XTRUE) divided by the magnitude of the largest element in sub(*X*). The estimate is as reliable as the estimate for rcond, and is almost always a slight overestimate of the true error. This array is tied to the distributed matrix *X*. The array *berr* contains the component-wise relative backward error of each solution vector (that is, the smallest relative change in any entry of sub(*A*) or sub(*B*) that makes sub(*X*) an exact solution). This array is tied to the distributed matrix *X*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
iwork(1)	On exit, *iwork*(1) contains the minimum value of *liwork* required for optimum performance (for real flavors).
rwork(1)	On exit, *rwork*(1) contains the minimum value of *lrwork* required for optimum performance (for complex flavors).
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Routines for Matrix Inversion

This sections describes ScaLAPACK routines that compute the inverse of a matrix based on the previously obtained factorization. Note that it is not recommended to solve a system of equations $Ax = b$ by first computing A^{-1} and then forming the matrix-vector product $x = A^{-1}b$. Call a solver routine instead (see Solving Systems of Linear Equations); this is more efficient and more accurate.

p?getri

Computes the inverse of a LU-factored distributed matrix.

Syntax

```
call psgetri(n, a, ia, ja, desca, ipiv, work, lwork, iwork, liwork, info)
call pdgetri(n, a, ia, ja, desca, ipiv, work, lwork, iwork, liwork, info)
call pcgetri(n, a, ia, ja, desca, ipiv, work, lwork, iwork, liwork, info)
call pzgetri(n, a, ia, ja, desca, ipiv, work, lwork, iwork, liwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?getri routine computes the inverse of a general distributed matrix sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*) using the *LU* factorization computed by p?getrf. This method inverts *U* and then computes the inverse of sub(*A*) by solving the system

inv(sub(*A*))**L* = inv(*U*)

for inv(sub(*A*)).

Input Parameters

n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*) (*n*≥0).
a	(local) REAL for psgetri DOUBLE PRECISION for pdgetri COMPLEX for pcgetri DOUBLE COMPLEX for pzgetri. Pointer into the local memory to an array of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)). On entry, the array *a* contains the local pieces of the *L* and *U* obtained by the factorization sub(*A*) = *P***L***U* computed by p?getrf.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
work	(local) REAL for psgetri DOUBLE PRECISION for pdgetri COMPLEX for pcgetri DOUBLE COMPLEX for pzgetri. The array *work* of dimension (*lwork*) is a workspace array.
lwork	(local) INTEGER. The dimension of the array *work*. *lwork* must be at least *lwork*≥*LOCr*(*n*+mod(*ia*-1,*mb_a*))**nb_a*. The array *work* is used to keep at most an entire column block of sub(*A*).
iwork	(local) INTEGER. Workspace array used for physically transposing the pivots, DIMENSION (*liwork*).
liwork	(local or global) INTEGER. The dimension of the array *iwork*.

The minimal value *liwork* of is determined by the following code:

```
if NPROW == NPCOL then
 liwork = LOCc(n_a + mod(ja-1,nb_a))+ nb_a
else
 liwork = LOCc(n_a + mod(ja-1,nb_a)) +
max(ceil(ceil(LOCr(m_a)/mb_a)/(lcm/NPROW)),nb_a)
end if
```

where `lcm` is the least common multiple of process rows and columns (`NPROW` and `NPCOL`).

Output Parameters

ipiv (local) `INTEGER`.
Array, dimension `(LOCr(m_a)+ mb_a)`.
This array contains the pivoting information.
If `ipiv(i)=j`, then the local row `i` was swapped with the global row `j`.
This array is tied to the distributed matrix `A`.

work(1) On exit, `work(1)` contains the minimum value of *lwork* required for optimum performance.

iwork(1) On exit, `iwork(1)` contains the minimum value of *liwork* required for optimum performance.

info (global) `INTEGER`. If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = *i*, *U*(*i*,*i*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, and division by zero will occur if it is used to solve a system of equations.

p?potri

Computes the inverse of a symmetric/Hermitian positive definite distributed matrix.

Syntax

```
call pspotri(uplo, n, a, ia, ja, desca, info)
call pdpotri(uplo, n, a, ia, ja, desca, info)
call pcpotri(uplo, n, a, ia, ja, desca, info)
call pzpotri(uplo, n, a, ia, ja, desca, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?potri` routine computes the inverse of a real symmetric or complex Hermitian positive definite distributed matrix `sub(A) = A(ia:ia+n-1, ja:ja+n-1)` using the Cholesky factorization `sub(A) = U^H*U` or `sub(A) = L*L^H` computed by `p?potrf`.

Input Parameters

uplo	(global) CHARACTER*1. Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix sub(*A*) is stored. If *uplo* = 'U', upper triangle of sub(*A*) is stored. If *uplo* = 'L', lower triangle of sub(*A*) is stored.
n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*) (*n*≥0).
a	(local) REAL for pspotri DOUBLE PRECISION for pdpotri COMPLEX for pcpotri DOUBLE COMPLEX for pzpotri. Pointer into the local memory to an array of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)). On entry, the array *a* contains the local pieces of the triangular factor *U* or *L* from the Cholesky factorization sub(*A*) = U^H*U, or sub(*A*) = $L*L^H$, as computed by p?potrf.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a	On exit, overwritten by the local pieces of the upper or lower triangle of the (symmetric/Hermitian) inverse of sub(*A*).
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. *info* > 0: If *info* = *i*, the (*i*, *i*) element of the factor *U* or *L* is zero, and the inverse could not be computed.

p?trtri

Computes the inverse of a triangular distributed matrix.

Syntax

```
call pstrtri(uplo, diag, n, a, ia, ja, desca, info)
call pdtrtri(uplo, diag, n, a, ia, ja, desca, info)
call pctrtri(uplo, diag, n, a, ia, ja, desca, info)
call pztrtri(uplo, diag, n, a, ia, ja, desca, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?trtri routine computes the inverse of a real or complex upper or lower triangular distributed matrix sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*).

Input Parameters

uplo (global) CHARACTER*1. Must be 'U' or 'L'.
Specifies whether the distributed matrix sub(*A*) is upper or lower triangular.
If *uplo* = 'U', sub(*A*) is upper triangular.
If *uplo* = 'L', sub(*A*) is lower triangular.

diag CHARACTER*1. Must be 'N' or 'U'.
Specifies whether or not the distributed matrix sub(*A*) is unit triangular.
If *diag* = 'N', then sub(*A*) is non-unit triangular.
If *diag* = 'U', then sub(*A*) is unit triangular.

n (global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*) (*n*≥0).

a (local)
REAL for pstrtri
DOUBLE PRECISION for pdtrtri
COMPLEX for pctrtri
DOUBLE COMPLEX for pztrtri.
Pointer into the local memory to an array of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)).
The array *a* contains the local pieces of the triangular distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular matrix to be inverted, and the strictly lower triangular part of sub(*A*) is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular matrix, and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a On exit, overwritten by the (triangular) inverse of the original matrix.

info (global) INTEGER. If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = *k*, *A*(*ia+k-1*, *ja+k-1*) is exactly zero. The triangular matrix sub(*A*) is singular and its inverse can not be computed.

Routines for Matrix Equilibration

ScaLAPACK routines described in this section are used to compute scaling factors needed to equilibrate a matrix. Note that these routines do not actually scale the matrices.

p?geequ

Computes row and column scaling factors intended to equilibrate a general rectangular distributed matrix and reduce its condition number.

Syntax

```
call psgeequ(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, info)
call pdgeequ(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, info)
call pcgeequ(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, info)
call pzgeequ(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?geequ` routine computes row and column scalings intended to equilibrate an *m*-by-*n* distributed matrix `sub(A) = A(ia:ia+m-1, ja:ja+n-1)` and reduce its condition number. The output array *r* returns the row scale factors and the array *c* the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix *B* with elements b_{ij}=`r(i)*`a_{ij}`*c(j)` have absolute value 1.

r(i) and *c*(j) are restricted to be between *SMLNUM* = smallest safe number and *BIGNUM* = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of sub(*A*) but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the `?lamch` routines to compute them. For example, compute single precision (real and complex) values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

The auxiliary function p?laqge uses scaling factors computed by `p?geequ` to scale a general rectangular matrix.

Input Parameters

m	(global) `INTEGER`. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*) (*m* ≥ 0).
n	(global) `INTEGER`. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*) (*n* ≥ 0).
a	(local) `REAL` for `psgeequ` `DOUBLE PRECISION` for `pdgeequ` `COMPLEX` for `pcgeequ` `DOUBLE COMPLEX` for `pzgeequ` . Pointer into the local memory to an array of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)).

	The array *a* contains the local pieces of the *m*-by-*n* distributed matrix whose equilibration factors are to be computed.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

r, *c*	(local) REAL for single precision flavors; DOUBLE PRECISION for double precision flavors. Arrays, dimension *LOCr(m_a)* and *LOCc(n_a)*, respectively. If *info* = 0, or *info* > *ia+m-1*, the array *r (ia:ia+m-1)* contains the row scale factors for sub(*A*). *r* is aligned with the distributed matrix *A*, and replicated across every process column. *r* is tied to the distributed matrix *A*. If *info* = 0, the array *c* (*ja:ja+n-1*) contains the column scale factors for sub(*A*). *c* is aligned with the distributed matrix *A*, and replicated down every process row. *c* is tied to the distributed matrix *A*.
rowcnd, *colcnd*	(global) REAL for single precision flavors; DOUBLE PRECISION for double precision flavors. If *info* = 0 or *info* > *ia+m-1*, *rowcnd* contains the ratio of the smallest *r*(i) to the largest *r*(i) (*ia* ≤ i ≤ *ia+m-1*). If *rowcnd* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *r (ia:ia +m-1)*. If *info* = 0, *colcnd* contains the ratio of the smallest *c*(j) to the largest *c*(j) (*ja* ≤ j ≤ *ja+n-1*). If *colcnd* ≥ 0.1, it is not worth scaling by *c(ja:ja+n-1)*.
amax	(global) REAL for single precision flavors; DOUBLE PRECISION for double precision flavors. Absolute value of the largest matrix element. If *amax* is very close to overflow or very close to underflow, the matrix should be scaled.
info	(global) INTEGER. If *info*=0, the execution is successful. *info* < 0: If the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. *info* > 0: If *info* = *i* and *i* ≤ *m*, the *i*th row of the distributed matrix sub(*A*) is exactly zero; *i* > *m*, the (*i*-*m*)th column of the distributed matrix sub(*A*) is exactly zero.

p?poequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite distributed matrix and reduce its condition number.

Syntax

```
call pspoequ(n, a, ia, ja, desca, sr, sc, scond, amax, info)
call pdpoequ(n, a, ia, ja, desca, sr, sc, scond, amax, info)
```

```
call pcpoequ(n, a, ia, ja, desca, sr, sc, scond, amax, info)
call pzpoequ(n, a, ia, ja, desca, sr, sc, scond, amax, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?poequ routine computes row and column scalings intended to equilibrate a real symmetric or complex Hermitian positive definite distributed matrix sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*) and reduce its condition number (with respect to the two-norm). The output arrays *sr* and *sc* return the row and column scale factors

These factors are chosen so that the scaled distributed matrix *B* with elements b_{ij}=*s*(i)**a*$_{ij}$**s*(j) has ones on the diagonal.

This choice of *sr* and *sc* puts the condition number of *B* within a factor *n* of the smallest possible condition number over all possible diagonal scalings.

The auxiliary function p?laqsy uses scaling factors computed by p?geequ to scale a general rectangular matrix.

Input Parameters

n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*) (*n*≥0).
a	(local) REAL for pspoequ DOUBLE PRECISION for pdpoequ COMPLEX for pcpoequ DOUBLE COMPLEX for pzpoequ. Pointer into the local memory to an array of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)). The array *a* contains the *n*-by-*n* symmetric/Hermitian positive definite distributed matrix sub(*A*) whose scaling factors are to be computed. Only the diagonal elements of sub(*A*) are referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

sr, *sc*	(local) REAL for single precision flavors; DOUBLE PRECISION for double precision flavors. Arrays, dimension *LOCr*(*m_a*) and *LOCc*(*n_a*), respectively.

If *info* = 0, the array *sr*(*ia:ia+n-1*) contains the row scale factors for sub(*A*). *sr* is aligned with the distributed matrix *A*, and replicated across every process column. *sr* is tied to the distributed matrix *A*.
If *info* = 0, the array *sc* (*ja:ja+n-1*) contains the column scale factors for sub(*A*). *sc* is aligned with the distributed matrix *A*, and replicated down every process row. *sc* is tied to the distributed matrix *A*.

scond (global)
REAL for single precision flavors;
DOUBLE PRECISION for double precision flavors.
If *info* = 0, *scond* contains the ratio of the smallest *sr*(i) (or *sc*(j)) to the largest *sr*(i) (or *sc*(j)), with
ia≤i≤*ia+n-1* and *ja*≤j≤*ja+n-1*.
If *scond* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *sr* (or *sc*).

amax (global)
REAL for single precision flavors;
DOUBLE PRECISION for double precision flavors.
Absolute value of the largest matrix element. If *amax* is very close to overflow or very close to underflow, the matrix should be scaled.

info (global) INTEGER.
If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = *k*, the *k*-th diagonal entry of sub(*A*) is nonpositive.

Orthogonal Factorizations

This section describes the ScaLAPACK routines for the *QR (RQ)* and *LQ (QL)* factorization of matrices. Routines for the *RZ* factorization as well as for generalized *QR* and *RQ* factorizations are also included. For the mathematical definition of the factorizations, see the respective LAPACK sections or refer to [SLUG].

Table "Computational Routines for Orthogonal Factorizations" lists ScaLAPACK routines that perform orthogonal factorization of matrices.

Computational Routines for Orthogonal Factorizations

Matrix type, factorization	Factorize without pivoting	Factorize with pivoting	Generate matrix Q	Apply matrix Q
general matrices, QR factorization	p?geqrf	p?geqpf	p?orgqr p?ungqr	p?ormqr p?unmqr
general matrices, RQ factorization	p?gerqf		p?orgrq p?ungrq	p?ormrq p?unmrq
general matrices, LQ factorization	p?gelqf		p?orglq p?unglq	p?ormlq p?unmlq

Matrix type, factorization	Factorize without pivoting	Factorize with pivoting	Generate matrix Q	Apply matrix Q
general matrices, QL factorization	p?geqlf		p?orgql p?ungql	p?ormql p?unmql
trapezoidal matrices, RZ factorization	p?tzrzf			p?ormrz p?unmrz
pair of matrices, generalized QR factorization	p?ggqrf			
pair of matrices, generalized RQ factorization	p?ggrqf			

p?geqrf

Computes the QR factorization of a general m-by-n matrix.

Syntax

```
call psgeqrf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgeqrf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgeqrf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgeqrf(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?geqrf routine forms the *QR* factorization of a general *m*-by-*n* distributed matrix sub(*A*)= *A*(*ia:ia*+*m*-1,*ja:ja*+*n*-*1*) as

A=*Q***R*

Input Parameters

m — (global) INTEGER. The number of rows in the distributed submatrix sub(*A*); (*m* ≥ 0).

n — (global) INTEGER. The number of columns in the distributed submatrix sub(*A*); (*n* ≥ 0).

a — (local)
REAL for psgeqrf
DOUBLE PRECISION for pdgeqrf
COMPLEX for pcgeqrf
DOUBLE COMPLEX for pzgeqrf.
Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*A*) to be factored.

ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia:ia+m-1*,*ja:ja+n-1*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*
work	(local). REAL for psgeqrf DOUBLE PRECISION for pdgeqrf. COMPLEX for pcgeqrf. DOUBLE COMPLEX for pzgeqrf Workspace array of dimension *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *nb_a* * (*mp*0+*nq*0+*nb_a*), where *iroff* = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mp*0 = numroc(*m*+*iroff*, *mb_a*, MYROW, *iarow*, NPROW), *nq*0 = numroc(*n*+*icoff*, *nb_a*, MYCOL, *iacol*, NPCOL), and numroc, indxg2p are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	The elements on and above the diagonal of sub(*A*) contain the min(*m*,*n*)-by-*n* upper trapezoidal matrix *R* (*R* is upper triangular if *m* ≥ *n*); the elements below the diagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).
tau	(local) REAL for psgeqrf DOUBLE PRECISION for pdgeqrf COMPLEX for pcgeqrf DOUBLE COMPLEX for pzgeqrf. Array, DIMENSION *LOCc*(*ja*+min(*m*,*n*)-1). Contains the scalar factor *tau* of elementary reflectors. *tau* is tied to the distributed matrix *A*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0, the execution is successful. < 0, if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ja*)**H*(*ja*+1)*...**H*(*ja*+*k*-1),

where $k = \min(m,n)$.

Each $H(i)$ has the form

$H(i) = I - tau*v*v'$

where tau is a real/complex scalar, and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:m)$ is stored on exit in $A(ia+i:ia+m-1, ja+i-1)$, and tau in $tau(ja+i-1)$.

p?geqpf

Computes the QR factorization of a general m-by-n matrix with pivoting.

Syntax

```
call psgeqpf(m, n, a, ia, ja, desca, ipiv, tau, work, lwork, info)
call pdgeqpf(m, n, a, ia, ja, desca, ipiv, tau, work, lwork, info)
call pcgeqpf(m, n, a, ia, ja, desca, ipiv, tau, work, lwork, info)
call pzgeqpf(m, n, a, ia, ja, desca, ipiv, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?geqpf routine forms the QR factorization with column pivoting of a general m-by-n distributed matrix $sub(A) = A(ia:ia+m-1, ja:ja+n-1)$ as

$sub(A)*P=Q*R$

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(A) ($m \geq 0$).
n	(global) INTEGER. The number of columns in the submatrix sub(A) ($n \geq 0$).
a	(local) REAL for psgeqpf DOUBLE PRECISION for pdgeqpf COMPLEX for pcgeqpf DOUBLE COMPLEX for pzgeqpf. Pointer into the local memory to an array of local dimension (lld_a, $LOCc(ja+n-1)$). Contains the local pieces of the distributed matrix sub(A) to be factored.
ia, ja	(global) INTEGER. The row and column indices in the global array a indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively.
$desca$	(global and local) INTEGER array, dimension ($dlen\_$). The array descriptor for the distributed matrix A.
$work$	(local). REAL for psgeqpf DOUBLE PRECISION for pdgeqpf. COMPLEX for pcgeqpf. DOUBLE COMPLEX for pzgeqpf Workspace array of dimension $lwork$.

lwork (local or global) INTEGER, dimension of *work*, must be at least
For real flavors:
lwork ≥ max(3,*mp*0+*nq*0) + *LOCc* (*ja*+*n*-*1*) + *nq*0.
For complex flavors:
lwork ≥ max(3,*mp*0+*nq*0) .
Here
iroff = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, MYROW, *iarow*, NPROW),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, MYCOL, *iacol*, NPCOL),
LOCc (*ja*+*n*-*1*) = numroc(*ja*+*n*-*1*, *nb_a*, MYCOL,*csrc_a*, NPCOL),
and numroc, indxg2p are ScaLAPACK tool functions.
You can determine MYROW, MYCOL, NPROW and NPCOL by calling the blacs_gridinfo subroutine.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a The elements on and above the diagonal of sub(*A*)contain the min(*m*, *n*)-by-*n* upper trapezoidal matrix *R* (*R* is upper triangular if *m* ≥ *n*); the elements below the diagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

ipiv (local) INTEGER. Array, DIMENSION *LOCc*(*ja*+*n*-1).
ipiv(*i*) = *k*, the local *i*-th column of sub(*A*)*P was the global *k*-th column of sub(*A*). *ipiv* is tied to the distributed matrix *A*.

tau (local)
REAL for psgeqpf
DOUBLE PRECISION for pdgeqpf
COMPLEX for pcgeqpf
DOUBLE COMPLEX for pzgeqpf.
Array, DIMENSION *LOCc*(*ja*+min(*m*, *n*)-1).
Contains the scalar factor *tau* of elementary reflectors. *tau* is tied to the distributed matrix *A*.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0, the execution is successful.
< 0, if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(1)**H*(2)*...**H*(*k*)

where *k* = min(*m*,*n*).

Each *H*(*i*) has the form

*H = I - tau*v*v'*

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*-1) = 0 and *v*(*i*) = 1; *v*(*i*+1:*m*) is stored on exit in *A*(*ia*+*i*:*ia*+*m*-1,*ja*+*i*-1).

The matrix *P* is represented in *ipiv* as follows: if *ipiv*(*j*)= *i* then the *j*-th column of *P* is the *i*-th canonical unit vector.

p?orgqr

Generates the orthogonal matrix Q of the QR factorization formed by p?geqrf.

Syntax

```
call psorgqr(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorgqr(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?orgqr routine generates the whole or part of *m*-by-*n* real distributed matrix *Q* denoting *A*(*ia:ia+m-1*, *ja:ja+n-1*) with orthonormal columns, which is defined as the first *n* columns of a product of *k* elementary reflectors of order *m*

Q= H(1)**H*(2)**...**H*(*k*)

as returned by p?geqrf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*) (*m* ≥ 0).
n	(global) INTEGER. The number of columns in the submatrix sub(*Q*) (*m* ≥ *n* ≥ 0).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n* ≥ *k* ≥ 0).
a	(local) REAL for psorgqr DOUBLE PRECISION for pdorgqr Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). The *j*-th column must contain the vector which defines the elementary reflector *H*(*j*), *ja*≤*j*≤*ja* +*k*-1, as returned by p?geqrf in the *k* columns of its distributed matrix argument *A*(*ia*:*, *ja*:*ja*+*k*-1).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia:ia*+*m*-1,*ja:ja*+*n*-1), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) REAL for psorgqr DOUBLE PRECISION for pdorgqr Array, DIMENSION *LOCc*(*ja*+*k*-1).

	Contains the scalar factor *tau* (*j*) of elementary reflectors *H*(*j*) as returned by p?geqrf. *tau* is tied to the distributed matrix *A*.
work	(local) REAL for psorgqr DOUBLE PRECISION for pdorgqr Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*. Must be at least *lwork* ≥ *nb_a**(*nqa*0 + *mpa*0 + *nb_a*), where *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW), *nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL); indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	Contains the local pieces of the *m*-by-*n* distributed matrix Q.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?ungqr

Generates the complex unitary matrix Q of the QR factorization formed by p?geqrf.

Syntax

```
call pcungqr(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzungqr(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

This routine generates the whole or part of *m*-by-*n* complex distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal columns, which is defined as the first *n* columns of a product of *k* elementary reflectors of order m

Q = *H*(1)**H*(2)*...**H*(*k*)

as returned by p?geqrf.

Input Parameters

m (global) INTEGER. The number of rows in the submatrix sub(*Q*); (*m*≥0).

n (global) INTEGER. The number of columns in the submatrix sub(*Q*) (*m*≥*n*≥0).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n*≥*k*≥0).

a (local)
COMPLEX for pcungqr
DOUBLE COMPLEX for pzungqr
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)).The *j*-th column must contain the vector which defines the elementary reflector *H*(*j*), *ja*≤ *j*≤ *ja* +*k*-1, as returned by p?geqrf in the *k* columns of its distributed matrix argument *A*(*ia*:*, *ja*:*ja*+*k*-1).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcungqr
DOUBLE COMPLEX for pzungqr
Array, DIMENSION *LOCc*(*ja*+*k*-1).
Contains the scalar factor *tau* (*j*) of elementary reflectors *H*(*j*) as returned by p?geqrf. *tau* is tied to the distributed matrix *A*.

work (local)
COMPLEX for pcungqr
DOUBLE COMPLEX for pzungqr
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *nb_a**(*nqa*0 + *mpa*0 + *nb_a*), where
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
*nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL)
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a Contains the local pieces of the *m*-by-*n* distributed matrix *Q*.

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.

< 0: if the i-th argument is an array and the j-entry had an illegal value, then $info = -(i*100+j)$, if the i-th argument is a scalar and had an illegal value, then $info = -i$.

p?ormqr

Multiplies a general matrix by the orthogonal matrix Q of the QR factorization formed by p?geqrf.

Syntax

```
call psormqr(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

```
call pdormqr(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?ormqr routine overwrites the general real m-by-n distributed matrix sub(C) = $C(ic:ic+m-1, jc:jc+n-1)$ with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'T':	Q^T*sub(C)	sub(C)*Q^T

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$Q = H(1)\ H(2)\ldots\ H(k)$

as returned by p?geqrf. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

side
(global) CHARACTER
='L': Q or Q^T is applied from the left.
='R': Q or Q^T is applied from the right.

trans
(global) CHARACTER
='N', no transpose, Q is applied.
='T', transpose, Q^T is applied.

m
(global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n
(global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

k
(global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If $side = 'L'$, $m \geq k \geq 0$
If $side = 'R'$, $n \geq k \geq 0$.

a
(local)
REAL for psormqr
DOUBLE PRECISION for pdormqr.

Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*k*-1)). The *j*-th column must contain the vector which defines the elementary reflector *H*(*j*), *ja*≤*j*≤*ja*+*k*-1, as returned by p?geqrf in the *k* columns of its distributed matrix argument *A*(*ia*:*, *ja*:*ja*+*k*-1). *A*(*ia*:*, *ja*:*ja*+*k*-1) is modified by the routine but restored on exit.
If *side* = 'L', *lld_a* ≥ max(1, *LOCr*(*ia*+*m*-1))
If *side* = 'R', *lld_a* ≥ max(1, *LOCr*(*ia*+*n*-1))

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
REAL for psormqr
DOUBLE PRECISION for pdormqr
Array, DIMENSION *LOCc*(*ja*+*k*-1).
Contains the scalar factor *tau* (*j*) of elementary reflectors *H*(*j*) as returned by p?geqrf. *tau* is tied to the distributed matrix *A*.

c (local)
REAL for psormqr
DOUBLE PRECISION for pdormqr
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psormqr
DOUBLE PRECISION for pdormqr.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
if *side* = 'L',
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0+*mpc*0)**nb_a*) + *nb_a***nb_a*
else if *side* = 'R',
lwork ≥ max((*nb_a**(*nb_a*-1))/2,
(*nqc*0+max(*npa*0+numroc(numroc(*n*+*icoffc*, *nb_a*, 0, 0, NPCOL),
nb_a, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where
lcmq = *lcm*/NPCOL with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0= numroc(*n*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0= numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0= numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),

	ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	Overwritten by the product Q*sub(C), or Q^T*sub(C), or sub(C)*Q^T, or sub(C)*Q.
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the i-th argument is an array and the j-entry had an illegal value, then *info* = - (i* 100+j), if the i-th argument is a scalar and had an illegal value, then *info* = -i.

p?unmqr

Multiplies a complex matrix by the unitary matrix Q of the QR factorization formed by p?geqrf.

Syntax

call pcunmqr(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmqr(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general complex m-by-n distributed matrix sub (C) = C(ic:ic+m-1,jc:jc+n-1) with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'T':	Q^H*sub(C)	sub(C)*Q^H

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(1)\ H(2) \ldots\ H(k)$ as returned by p?geqrf. Q is of order m if *side* = 'L' and of order n if *side* ='R'.

Input Parameters

side	(global) CHARACTER ='L': Q or Q^H is applied from the left. ='R': Q or Q^H is applied from the right.
trans	(global) CHARACTER

='N', no transpose, Q is applied.
='C', conjugate transpose, Q^H is applied.

m (global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n (global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

a (local)
COMPLEX for pcunmqr
DOUBLE COMPLEX for pzunmqr.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*k*-1)). The *j*-th column must contain the vector which defines the elementary reflector *H*(*j*), *ja*≤*j*≤*ja* +*k*-1, as returned by p?geqrf in the *k* columns of its distributed matrix argument *A*(*ia*:*, *ja*:*ja*+*k*-1). *A*(*ia*:*, *ja*:*ja*+*k*-1) is modified by the routine but restored on exit.
If *side* = 'L', *lld_a* ≥ max(1, *LOCr*(*ia*+*m*-1))
If *side* = 'R', *lld_a* ≥ max(1, *LOCr*(*ia*+*n*-1))

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcunmqr
DOUBLE COMPLEX for pzunmqr
Array, DIMENSION *LOCc*(*ja*+*k*-1).
Contains the scalar factor *tau* (*j*) of elementary reflectors *H*(*j*) as returned by p?geqrf. *tau* is tied to the distributed matrix *A*.

c (local)
COMPLEX for pcunmqr
DOUBLE COMPLEX for pzunmqr.
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(C) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix C, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix C.

work (local)
COMPLEX for pcunmqr
DOUBLE COMPLEX for pzunmqr.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L',
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + *mpc*0)**nb_a*) + *nb_a***nb_a*
else if *side* = 'R',

lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + max(*npa*0 + numroc(numroc(*n*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where
lcmq = *lcm*/NPCOL with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0 = numroc(*n*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c — Overwritten by the product *Q**sub(*C*), or Q^H*sub(*C*), or sub(*C*)*Q^H, or sub(*C*)**Q*.

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info — (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?gelqf

Computes the LQ factorization of a general rectangular matrix.

Syntax

```
call psgelqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgelqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgelqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgelqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gelqf routine computes the *LQ* factorization of a real/complex distributed *m*-by-*n* matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ia*:*ia*+*n*-1) = *L***Q*.

Input Parameters

m (global) INTEGER. The number of rows in the submatrix sub(*Q*) (*m* ≥ 0).

n (global) INTEGER. The number of columns in the submatrix sub(*Q*) (*n* ≥ 0).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n* ≥ *k* ≥ 0).

a (local)
REAL for psgelqf
DOUBLE PRECISION for pdgelqf
COMPLEX for pcgelqf
DOUBLE COMPLEX for pzgelqf
Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*A*) to be factored.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia*:*ia*+*m*-1, *ia*:*ia*+*n*-1), respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local)
REAL for psgelqf
DOUBLE PRECISION for pdgelqf
COMPLEX for pcgelqf
DOUBLE COMPLEX for pzgelqf
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *mb_a**(*mp*0 + *nq*0 + *mb_a*), where
iroff = mod(*ia*-1, *mb_a*),
icoff = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, MYROW, *iarow*, NPROW),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, MYCOL, *iacol*, NPCOL)
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a The elements on and below the diagonal of sub(*A*) contain the *m* by min(*m*,*n*) lower trapezoidal matrix *L* (*L* is lower trapezoidal if *m* ≤ *n*); the elements above the diagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau	(local) REAL for psgelqf DOUBLE PRECISION for pdgelqf COMPLEX for pcgelqf DOUBLE COMPLEX for pzgelqf Array, DIMENSION *LOCr*(*ia*+min(*m*, *n*)-1). Contains the scalar factors of elementary reflectors. *tau* is tied to the distributed matrix *A*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ia*+*k*-1)**H*(*ia*+*k*-2)*...**H*(*ia*),

where *k* = min(*m*,*n*)

Each *H*(*i*) has the form

H(*i*) = *I* - *tau***v***v*'

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*-1) = 0 and *v*(*i*) = 1; *v*(*i*+1:*n*) is stored on exit in *A*(*ia*+*i*-1, *ja*+*i*:*ja*+*n*-1), and *tau* in *tau* (*ia*+*i*-1).

p?orglq

Generates the real orthogonal matrix Q of the LQ factorization formed by p?gelqf.

Syntax

call psorglq(*m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *work*, *lwork*, *info*)

call pdorglq(*m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?orglq routine generates the whole or part of *m*-by-*n* real distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal rows, which is defined as the first *m* rows of a product of *k* elementary reflectors of order *n*

Q = *H*(*k*)*...* *H*(2)* *H*(1)

as returned by p?gelqf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*); (*m*≥0).

n (global) INTEGER. The number of columns in the submatrix sub(*Q*) (*n*≥*m*≥0).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*m*≥*k*≥0).

a (local)
REAL for psorglq
DOUBLE PRECISION for pdorglq
Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), *ia*≤*i*≤*ia*+*k*-1, as returned by p?gelqf in the *k* rows of its distributed matrix argument *A*(*ia*:*ia*+*k* -1, *ja*:*).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1), respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local)
REAL for psorglq
DOUBLE PRECISION for pdorglq
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *mb_a**(*mpa*0+*nqa*0+*mb_a*), where
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
*nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL)
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a Contains the local pieces of the *m*-by-*n* distributed matrix *Q* to be factored.

tau (local)
REAL for psorglq
DOUBLE PRECISION for pdorglq
Array, DIMENSION *LOCr*(*ia*+*k* -1).
Contains the scalar factors *tau* of elementary reflectors *H*(*i*). *tau* is tied to the distributed matrix *A*.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?unglq

Generates the unitary matrix Q of the LQ factorization formed by p?gelqf.

Syntax

```
call pcunglq(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzunglq(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

This routine generates the whole or part of *m*-by-*n* complex distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal rows, which is defined as the first *m* rows of a product of *k* elementary reflectors of order *n*

Q = (*H*(*k*))^H...*(*H*(2))^H*(*H*(1))^H as returned by p?gelqf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*) (*m*≥0).
n	(global) INTEGER. The number of columns in the submatrix sub(*Q*) (*n*≥*m*≥0).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*m*≥*k*≥0).
a	(local) COMPLEX for pcunglq DOUBLE COMPLEX for pzunglq Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), *ia*≤*i*≤*ia*+*k*-1, as returned by p?gelqf in the *k* rows of its distributed matrix argument *A*(*ia*:*ia*+*k*-1, *ja*:*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia*:*ia*+*m*-1,*ja*:*ja*+*n*-1), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) COMPLEX for pcunglq DOUBLE COMPLEX for pzunglq Array, DIMENSION *LOCr*(*ia*+*k*-1). Contains the scalar factors *tau* of elementary reflectors *H*(*i*). *tau* is tied to the distributed matrix *A*.
work	(local) COMPLEX for pcunglq DOUBLE COMPLEX for pzunglq Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *mb_a**(*mpa*0+*nqa*0+*mb_a*), where

iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
mpa0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
nqa0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL)
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	Contains the local pieces of the *m*-by-*n* distributed matrix *Q* to be factored.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?ormlq

Multiplies a general matrix by the orthogonal matrix Q of the LQ factorization formed by p?gelqf.

Syntax

```
call psormlq(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, work, lwork, info)
```

```
call pdormlq(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?ormlq routine overwrites the general real *m*-by-*n* distributed matrix sub(*C*) = *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) with

	side = 'L'	*side* = 'R'
trans = 'N':	*Q**sub(*C*)	sub(*C*)**Q*
trans = 'T':	Q^T*sub(*C*)	sub(*C*)*Q^T

where *Q* is a real orthogonal distributed matrix defined as the product of *k* elementary reflectors

Q = *H*(*k*)...*H*(2) *H*(1)

as returned by p?gelqf. *Q* is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	(global) CHARACTER ='L': Q or Q^T is applied from the left. ='R': Q or Q^T is applied from the right.
trans	(global) CHARACTER ='N', no transpose, Q is applied. ='T', transpose, Q^T is applied.
m	(global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).
n	(global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints: If *side* = 'L', $m \geq k \geq 0$ If *side* = 'R', $n \geq k \geq 0$.
a	(local) REAL for psormlq DOUBLE PRECISION for pdormlq. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)), if *side* = 'L' and (*lld_a*, *LOCc*(*ja*+*n*-1)), if *side* = 'R'.The i-th row must contain the vector which defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?gelqf in the k rows of its distributed matrix argument *A*(*ia*:*ia*+*k*-1, *ja*:*). *A*(*ia*:*ia*+*k*-1, *ja*:*) is modified by the routine but restored on exit.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix A, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix A.
tau	(local) REAL for psormlq DOUBLE PRECISION for pdormlq Array, DIMENSION *LOCc*(*ja*+*k*-1). Contains the scalar factor *tau* (i) of elementary reflectors $H(i)$ as returned by p?gelqf. *tau* is tied to the distributed matrix A.
c	(local) REAL for psormlq DOUBLE PRECISION for pdormlq Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)). Contains the local pieces of the distributed matrix sub(C) to be factored.
ic, *jc*	(global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix C, respectively.
descc	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix C.
work	(local) REAL for psormlq DOUBLE PRECISION for pdormlq. Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of the array *work*; must be at least:

If *side* = 'L',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0+max *mqa*0)+
numroc(numroc(*m* + *iroffc*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0,
lcmp), *nqc*0))* *mb_a*) + *mb_a***mb_a*
else if *side* = 'R',
lwork ≥ max((*mb_a** (*mb_a*-1))/2, (*mpc*0+nqc0)**mb_a* + *mb_a***mb_a*
end if
where
lcmp = *lcm*/NPROW with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = *numroc*(*m*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c — Overwritten by the product *Q**sub(*C*), or *Q*' *sub (*C*), or sub(*C*)**Q*', or sub(*C*)**Q*

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info — (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?unmlq

Multiplies a general matrix by the unitary matrix Q of the LQ factorization formed by p?gelqf.

Syntax

call pcunmlq(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmlq(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general complex m-by-n distributed matrix sub $(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'T':	Q^H*sub(C)	sub(C)*Q^H

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(k)' \ldots H(2)' H(1)'$

as returned by p?gelqf. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side — (global) CHARACTER
='L': Q or Q^H is applied from the left.
='R': Q or Q^H is applied from the right.

trans — (global) CHARACTER
='N', no transpose, Q is applied.
='C', conjugate transpose, Q^H is applied.

m — (global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n — (global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

k — (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

a — (local)
COMPLEX for pcunmlq
DOUBLE COMPLEX for pzunmlq.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)), if *side* = 'L', and (*lld_a*, *LOCc*(*ja*+*n*-1)), if *side* = 'R', where *lld_a* ≥ max(1, *LOCr* (*ia*+*k*-1)). The i-th column must contain the vector which defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?gelqf in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the routine but restored on exit.

ia, *ja* — (global) INTEGER. The row and column indices in the global array a indicating the first row and the first column of the submatrix A, respectively.

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix A.

tau — (local)
COMPLEX for pcunmlq
DOUBLE COMPLEX for pzunmlq
Array, DIMENSION *LOCc*(*ia*+*k*-1).
Contains the scalar factor *tau* (i) of elementary reflectors $H(i)$ as returned by p?gelqf. *tau* is tied to the distributed matrix A.

c — (local)
COMPLEX for pcunmlq

DOUBLE COMPLEX for pzunmlq.
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
COMPLEX for pcunmlq
DOUBLE COMPLEX for pzunmlq.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of the array *work*; must be at least:
If *side* = 'L',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + max *mqa*0)+ numroc(numroc(*m* + *iroffc*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a*
else if *side* = 'R',
lwork ≥ max((*mb_a** (*mb_a*-1))/2, (*mpc*0 + nqc0)**mb_a* + *mb_a***mb_a*
end if
where
lcmp = *lcm*/NPROW with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = *numroc*(m + *icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c Overwritten by the product *Q**sub(*C*), or *Q*'*sub (*C*), or sub(*C*)**Q*', or sub(*C*)**Q*

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?geqlf

Computes the QL factorization of a general matrix.

Syntax

```
call psgeqlf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgeqlf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgeqlf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgeqlf(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?geqlf routine forms the QL factorization of a real/complex distributed *m*-by-*n* matrix sub(A) = A(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) = *Q***L*.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*); (*m* ≥ 0).
n	(global) INTEGER. The number of columns in the submatrix sub(*Q*) (*n* ≥ 0).
a	(local) REAL for psgeqlf DOUBLE PRECISION for pdgeqlf COMPLEX for pcgeqlf DOUBLE COMPLEX for pzgeqlf Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). Contains the local pieces of the distributed matrix sub(*A*) to be factored.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia*:*ia*+*m*-1, *ia*:*ia*+*n*-1), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
work	(local) REAL for psgeqlf DOUBLE PRECISION for pdgeqlf COMPLEX for pcgeqlf DOUBLE COMPLEX for pzgeqlf Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *nb_a**(*mp*0 + *nq*0 + *nb_a*), where *iroff* = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mp*0 = numroc(*m*+*iroff*, *mb_a*, MYROW, *iarow*, NPROW), *nq*0 = numroc(*n*+*icoff*, *nb_a*, MYCOL, *iacol*, NPCOL)

numroc and indxg2p are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a — On exit, if $m \geq n$, the lower triangle of the distributed submatrix *A*(*ia*+*m*-*n*:*ia*+*m*-1, *ja*:*ja*+*n*-1) contains the n-by-n lower triangular matrix *L*; if $m \leq n$, the elements on and below the (n-m)-th superdiagonal contain the m-by-n lower trapezoidal matrix *L*; the remaining elements, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau — (local)
REAL for psgeqlf
DOUBLE PRECISION for pdgeqlf
COMPLEX for pcgeqlf
DOUBLE COMPLEX for pzgeqlf
Array, DIMENSION *LOCc*(*ja*+*n*-1).
Contains the scalar factors of elementary reflectors. *tau* is tied to the distributed matrix *A*.

work(1) — On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info — (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ja*+*k*-1)*...**H*(*ja*+1)**H*(*ja*)

where *k* = min(*m*,*n*)

Each *H*(*i*) has the form

H(*i*) = *I* - *tau***v***v*'

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(*m*-*k*+*i*+1:*m*) = 0 and *v*(*m*-*k*+*i*) = 1; *v*(1:*m*-*k*+*i*-1) is stored on exit in *A*(*ia*:*ia*+*m*-*k*+*i*-2, *ja*+*n*-*k*+*i*-1), and *tau* in *tau* (*ja*+*n*-*k*+*i*-1).

p?orgql

Generates the orthogonal matrix Q of the QL factorization formed by p?geqlf.

Syntax

```
call psorgql(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorgql(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?orgql routine generates the whole or part of *m*-by-*n* real distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal rows, which is defined as the first *m* rows of a product of *k* elementary reflectors of order *n*

Q = *H*(*k*)*...*H*(2)*H*(1)

as returned by p?geqlf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*), (*m*≥0).
n	(global) INTEGER. The number of columns in the submatrix sub(*Q*), (*m*≥*n*≥0).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n*≥*k*≥0).
a	(local) REAL for psorgql DOUBLE PRECISION for pdorgql Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the *j*-th column must contain the vector which defines the elementary reflector *H*(*j*), *ja*+*n*-*k*≤*j*≤*ja*+*n*-1, as returned by p?geqlf in the *k* columns of its distributed matrix argument *A*(*ia*:*,*ja*+*n*-*k*:*ja*+*n*-1).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) REAL for psorgql DOUBLE PRECISION for pdorgql Array, DIMENSION *LOCc*(*ja*+*n*-1). Contains the scalar factors *tau*(*j*) of elementary reflectors *H*(*j*). *tau* is tied to the distributed matrix *A*.
work	(local) REAL for psorgql DOUBLE PRECISION for pdorgql Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *nb_a**(*nqa*0+*mpa*0+*nb_a*), where *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW), *nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL) indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	Contains the local pieces of the *m*-by-*n* distributed matrix *Q* to be factored.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?ungql

Generates the unitary matrix Q of the QL factorization formed by p?geqlf.

Syntax

```
call pcungql(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzungql(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

This routine generates the whole or part of *m*-by-*n* complex distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal rows, which is defined as the first *n* columns of a product of *k* elementary reflectors of order *m*

Q = (*H*(*k*))^H...*(*H*(2))^H*(*H*(1))^H as returned by p?geqlf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*) (*m*≥0).
n	(global) INTEGER. The number of columns in the submatrix sub(*Q*) (*m*≥*n*≥0).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*n*≥*k*≥0).
a	(local) COMPLEX for pcungql DOUBLE COMPLEX for pzungql Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the *j*-th column must contain the vector which defines the elementary reflector *H*(*j*), *ja*+*n*-*k*≤ *j*≤ *ja*+*n*-1, as returned by p?geqlf in the *k* columns of its distributed matrix argument *A*(*ia*:*, *ja*+*n*-*k*: *ja*+*n*-1).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia*:*ia*+*m*-1,*ja*:*ja*+*n*-1), respectively.

desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) COMPLEX for pcungql DOUBLE COMPLEX for pzungql Array, DIMENSION *LOCr*(*ia*+*n*-1). Contains the scalar factors *tau* (*j*) of elementary reflectors *H*(*j*). *tau* is tied to the distributed matrix *A*.
work	(local) COMPLEX for pcungql DOUBLE COMPLEX for pzungql Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *nb_a**(*nqa*0 + *mpa*0 + *nb_a*), where *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW), *nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL) indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	Contains the local pieces of the *m*-by-*n* distributed matrix *Q* to be factored.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?ormql

Multiplies a general matrix by the orthogonal matrix Q of the QL factorization formed by p?geqlf.

Syntax

```
call psormql(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

```
call pdormql(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?ormql routine overwrites the general real m-by-n distributed matrix sub(C) = C (ic:ic+m-1,jc:jc+n-1) with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'T':	Q^T*sub(C)	sub(C)*Q^T

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$Q = H(k)' \ldots H(2)' H(1)'$

as returned by p?geqlf. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side (global) CHARACTER
='L': Q or Q^T is applied from the left.
='R': Q or Q^T is applied from the right.

trans (global) CHARACTER
='N', no transpose, Q is applied.
='T', transpose, Q^T is applied.

m (global) INTEGER. The number of rows in the distributed matrix sub(C), ($m \geq 0$).

n (global) INTEGER. The number of columns in the distributed matrix sub(C), ($n \geq 0$).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

a (local)
REAL for psormql
DOUBLE PRECISION for pdormql.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+k-1)). The j-th column must contain the vector which defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by p?gelqf in the k columns of its distributed matrix argument A(*ia*:*, *ja*:*ja*+k-1).A(*ia*:*, *ja*:*ja*+k-1) is modified by the routine but restored on exit.
If *side* = 'L', *lld_a* ≥ max(1, *LOCr*(*ia*+m-1)),
If *side* = 'R', *lld_a* ≥ max(1, *LOCr*(*ia*+n-1)).

ia, *ja* (global) INTEGER. The row and column indices in the global array a indicating the first row and the first column of the submatrix A, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix A.

tau (local)
REAL for psormql
DOUBLE PRECISION for pdormql.
Array, DIMENSION *LOCc*(*ja*+n-1).
Contains the scalar factor *tau* (j) of elementary reflectors $H(j)$ as returned by p?geqlf. *tau* is tied to the distributed matrix A.

c (local)
REAL for psormql

DOUBLE PRECISION for pdormql.
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*) to be factored.

ic, *jc* — (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work — (local)
REAL for psormql
DOUBLE PRECISION for pdormql.
Workspace array of dimension of *lwork*.

lwork — (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L',
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0+mpc0)**nb_a* + *nb_a***nb_a*
else if *side* ='R',
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0+max *npa*0)+
numroc(numroc(*n*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where
lcmp = *lcm*/NPCOL with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0= *numroc*(*n* + *iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c — Overwritten by the product *Q** sub(*C*), or *Q*'*sub (*C*), or sub(*C*)* *Q*', or sub(*C*)* *Q*

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info — (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?unmql

Multiplies a general matrix by the unitary matrix Q of the QL factorization formed by `p?geqlf`.

Syntax

`call pcunmql(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)`

`call pzunmql(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)`

Include Files

- C: `mkl_scalapack.h`

Description

This routine overwrites the general complex m-by-n distributed matrix `sub(C) = C (ic:ic+m-1,jc:jc+n-1)` with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'C':	Q^H*sub(C)	sub(C)*Q^H

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(k)' \ldots H(2)' H(1)'$

as returned by `p?geqlf`. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side — (global) CHARACTER
='L': Q or Q^H is applied from the left.
='R': Q or Q^H is applied from the right.

trans — (global) CHARACTER
='N', no transpose, Q is applied.
='C', conjugate transpose, Q^H is applied.

m — (global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n — (global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

k — (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

a — (local)
COMPLEX for `pcunmql`
DOUBLE COMPLEX for `pzunmql`.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*k*-1)). The j-th column must contain the vector which defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by `p?geqlf` in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. $A(ia:*, ja:ja+k-1)$ is modified by the routine but restored on exit.

If *side* = 'L',*lld_a* ≥ max(1, *LOCr*(*ia*+*m*-1)),
If *side* = 'R', *lld_a* ≥ max(1, *LOCr*(*ia*+*n*-1)).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcunmql
DOUBLE COMPLEX for pzunmql
Array, DIMENSION *LOCc*(*ia*+*n*-1).
Contains the scalar factor *tau* (*j*) of elementary reflectors *H*(*j*) as returned by p?geqlf. *tau* is tied to the distributed matrix *A*.

c (local)
COMPLEX for pcunmql
DOUBLE COMPLEX for pzunmql.
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
COMPLEX for pcunmql
DOUBLE COMPLEX for pzunmql.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L',
lwork ≥ max((*nb_a** (*nb_a*-1))/2, (*nqc*0+mpc0)**nb_a* + *nb_a***nb_a*
else if *side* ='R',
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0+max *npa*0)+
numroc(numroc(*n*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0,
lcmq), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where
lcmp = *lcm*/NPCOL with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0 = *numroc* (*n* + *iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	Overwritten by the product *Q** sub(*C*), or *Q*' sub (*C*), or sub(*C*)* *Q*', or sub(*C*)* *Q*
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?gerqf

Computes the RQ factorization of a general rectangular matrix.

Syntax

```
call psgerqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgerqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgerqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgerqf(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gerqf routine forms the *QR* factorization of a general *m*-by-*n* distributed matrix sub(*A*)= *A*(*ia:ia*+*m*-1,*ja:ja*+*n*-*1*) as

A= *R***Q*

Input Parameters

m	(global) INTEGER. The number of rows in the distributed submatrix sub(*A*); (*m*≥0).
n	(global) INTEGER. The number of columns in the distributed submatrix sub(*A*); (*n*≥0).
a	(local) REAL for psgeqrf DOUBLE PRECISION for pdgeqrf COMPLEX for pcgeqrf DOUBLE COMPLEX for pzgeqrf. Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). Contains the local pieces of the distributed matrix sub(*A*) to be factored.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia:ia*+*m-1*,*ja:ja*+*n-1*), respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*

work (local).
REAL for psgeqrf
DOUBLE PRECISION for pdgeqrf.
COMPLEX for pcgeqrf.
DOUBLE COMPLEX for pzgeqrf
Workspace array of dimension *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *mb_a**(*mp*0+*nq*0+*mb_a*), where
iroff = mod(*ia*-1, *mb_a*),
icoff = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, MYROW, *iarow*, NPROW),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, MYCOL, *iacol*, NPCOL) and numroc, indxg2p are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, if *m*≤*n*, the upper triangle of *A*(*ia:ia*+*m*-*1*, *ja:ja*+*n*-*1*) contains the *m*-by-*m* upper triangular matrix *R*; if *m*≥*n*, the elements on and above the (*m* - *n*)-th subdiagonal contain the *m*-by-*n* upper trapezoidal matrix *R*; the remaining elements, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau (local)
REAL for psgeqrf
DOUBLE PRECISION for pdgeqrf
COMPLEX for pcgeqrf
DOUBLE COMPLEX for pzgeqrf.
Array, DIMENSION *LOCr*(*ia*+*m*-1).
Contains the scalar factor *tau* of elementary reflectors. *tau* is tied to the distributed matrix *A*.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0, the execution is successful.
< 0, if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ia*)**H*(*ia*+*1*)*...**H*(*ia*+*k*-1),

where *k* = min(*m*,*n*).

Each *H*(*i*) has the form

H(*i*) = *I* - *tau***v***v*'

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(*n*-*k*+*i*+1:*n*) = 0 and *v*(*n*-*k*+*i*) = 1; *v*(1:*n*-*k*+*i*-1) is stored on exit in *A*(*ia*+*m*-*k*+*i*-1,*ja*:*ja*+*n*-*k*+*i*-2), and *tau* in *tau*(*ia*+*m*-*k*+*i*-1).

p?orgrq

Generates the orthogonal matrix Q of the RQ factorization formed by p?gerqf.

Syntax

```
call psorgrq(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorgrq(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?orgrq routine generates the whole or part of *m*-by-*n* real distributed matrix *Q* denoting *A*(*ia:ia*+*m*-*1*, *ja:ja*+*n*-*1*) with orthonormal columns, which is defined as the last *m* rows of a product of *k* elementary reflectors of order *m*

Q= *H*(1)**H*(2)*...**H*(*k*)

as returned by p?gerqf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*), (*m*≥0).
n	(global) INTEGER. The number of columns in the submatrix sub(*Q*), (*n*≥*m*≥0).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*m*≥*k*≥0).
a	(local) REAL for psorgrq DOUBLE PRECISION for pdorgrq Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). The *i*-th column must contain the vector which defines the elementary reflector *H*(*i*), *ja*≤*j*≤*ja*+*k*-1, as returned by p?gerqf in the *k* columns of its distributed matrix argument *A*(*ia*:*, *ja*:*ja*+*k*-1).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*(*ia:ia*+*m*-*1*,*ja:ja*+*n*-*1*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) REAL for psorgrq DOUBLE PRECISION for pdorgrq Array, DIMENSION *LOCc*(*ja*+*k*-1).

	Contains the scalar factor *tau* (*i*) of elementary reflectors *H*(*i*) as returned by p?gerqf. *tau* is tied to the distributed matrix *A*.
work	(local) REAL for psorgrq DOUBLE PRECISION for pdorgrq Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork*≥*mb_a**(*mpa*0 + *nqa*0 + *mb_a*), where *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW), *nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL) indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = –1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	Contains the local pieces of the *m*-by-*n* distributed matrix Q.
work(1)	On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = –(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = –*i*.

p?ungrq

Generates the unitary matrix Q of the RQ factorization formed by p?gerqf.

Syntax

```
call pcungrq(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzungrq(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

This routine generates the *m*-by-*n* complex distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*–1,*ja*:*ja*+*n*–1) with orthonormal rows, which is defined as the last *m* rows of a product of *k* elementary reflectors of order n

Q = (*H*(1))^H*(*H*(2))^H*...*(*H*(*k*))^H as returned by p?gerqf.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*Q*); (*m*≥0).

n (global) INTEGER. The number of columns in the submatrix sub(*Q*) (*n*≥*m*≥0).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q* (*m*≥*k*≥0).

a (local)
COMPLEX for pcungrq
DOUBLE COMPLEX for pzungrqc
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). The *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), *ia*+*m*-*k*≤*i*≤*ia*+*m*-1, as returned by p?gerqf in the *k* rows of its distributed matrix argument *A*(*ia*+*m*-*k*:*ia*+*m*-1, *ja*:*).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcungrq
DOUBLE COMPLEX for pzungrq
Array, DIMENSION *LOCr*(*ia*+*m*-1).
Contains the scalar factor *tau* (*i*) of elementary reflectors *H*(*i*) as returned by p?gerqf. *tau* is tied to the distributed matrix *A*.

work (local)
COMPLEX for pcungrq
DOUBLE COMPLEX for pzungrq
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *mb_a**(*mpa*0 +*nqa*0+*mb_a*), where
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
*nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL)
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a Contains the local pieces of the *m*-by-*n* distributed matrix *Q*.

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?ormrq

Multiplies a general matrix by the orthogonal matrix Q of the RQ factorization formed by p?gerqf.

Syntax

```
call psormrq(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

```
call pdormrq(side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?ormrq routine overwrites the general real m-by-n distributed matrix sub(C) = C(ic:ic+m-1,jc:jc+n-1) with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'T':	Q^T*sub(C)	sub(C)*Q^T

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$Q = H(1)\ H(2) \ldots\ H(k)$

as returned by p?gerqf. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side — (global) CHARACTER
='L': Q or Q^T is applied from the left.
='R': Q or Q^T is applied from the right.

trans — (global) CHARACTER
='N', no transpose, Q is applied.
='T', transpose, Q^T is applied.

m — (global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n — (global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

k — (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

a — (local)
REAL for psormqr
DOUBLE PRECISION for pdormqr.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)) if *side* = 'L', and (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R'.

The *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), *ia*≤*i*≤*ia*+*k*-1, as returned by p?gerqf in the *k* rows of its distributed matrix argument *A*(*ia*:*ia*+*k*-1, *ja*:*).*A*(*ia*:*ia*+*k*-1, *ja*:*) is modified by the routine but restored on exit.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
REAL for psormqr
DOUBLE PRECISION for pdormqr
Array, DIMENSION *LOCc*(*ja*+*k*-1).
Contains the scalar factor *tau* (*i*) of elementary reflectors *H*(*i*) as returned by p?gerqf. *tau* is tied to the distributed matrix *A*.

c (local)
REAL for psormrq
DOUBLE PRECISION for pdormrq
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psormrq
DOUBLE PRECISION for pdormrq.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + max(*mqa*0 + numroc(numroc(*n*+*iroffc*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a*
else if *side* ='R',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + *nqc*0)**mb_a*) + *mb_a***mb_a*
end if
where
lcmp = *lcm*/NPROW with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),

	ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	Overwritten by the product *Q** sub(*C*), or *Q*'*sub (*C*), or sub(*C*)* *Q*', or sub(*C*)* *Q*
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?unmrq

Multiplies a general matrix by the unitary matrix Q of the RQ factorization formed by p?gerqf.

Syntax

call pcunmrq(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmrq(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general complex *m*-by-*n* distributed matrix sub (*C*) = *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) with

	side ='L'	*side* ='R'
trans = 'N':	*Q**sub(*C*)	sub(*C*)**Q*
trans = 'C':	Q^H*sub(*C*)	sub(*C*)*Q^H

where *Q* is a complex unitary distributed matrix defined as the product of *k* elementary reflectors

Q = *H*(1)' *H*(2)'... *H*(*k*)'

as returned by p?gerqf. *Q* is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	(global) CHARACTER ='L': *Q* or Q^H is applied from the left. ='R': *Q* or Q^H is applied from the right.
trans	(global) CHARACTER

='N', no transpose, Q is applied.
='C', conjugate transpose, Q^H is applied.

m (global) INTEGER. The number of rows in the distributed matrix sub(C) , ($m \geq 0$).

n (global) INTEGER. The number of columns in the distributed matrix sub(C), ($n \geq 0$).

k (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

a (local)
COMPLEX for pcunmrq
DOUBLE COMPLEX for pzunmrq.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)) if *side* = 'L', and (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R'. The *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), $ia \leq i \leq ia+k-1$, as returned by p?gerqf in the *k* rows of its distributed matrix argument *A*(*ia*:*ia* +*k*-1, *ja**).*A*(*ia*:*ia* +*k*-1, *ja**) is modified by the routine but restored on exit.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcunmrq
DOUBLE COMPLEX for pzunmrq
Array, DIMENSION *LOCc*(*ja*+*k*-1).
Contains the scalar factor *tau* (*i*) of elementary reflectors *H*(*i*) as returned by p?gerqf. *tau* is tied to the distributed matrix *A*.

c (local)
COMPLEX for pcunmrq
DOUBLE COMPLEX for pzunmrq.
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(C) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
COMPLEX for pcunmrq
DOUBLE COMPLEX for pzunmrq.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + max(*mqa*0+numroc(numroc(*n*+*iroffc*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a*
else if *side* = 'R',

lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + *nqc*0)**mb_a*) + *mb_a***mb_a*
end if
where
lcmp = *lcm*/NPROW with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = numroc(*m*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c — Overwritten by the product *Q** sub(*C*) or *Q*'*sub (*C*), or sub(*C*)* *Q*', or sub(*C*)* *Q*

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info — (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?tzrzf

Reduces the upper trapezoidal matrix A to upper triangular form.

Syntax

```
call pstzrzf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdtzrzf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pctzrzf(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pztzrzf(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?tzrzf routine reduces the m-by-n ($m \le n$) real/complex upper trapezoidal matrix sub(A)=(ia:ia+m-1,ja:ja+n-1) to upper triangular form by means of orthogonal/unitary transformations. The upper trapezoidal matrix A is factored as

$A = (R\ 0)*Z$,

where Z is an n-by-n orthogonal/unitary matrix and R is an m-by-m upper triangular matrix.

Input Parameters

m	(global) INTEGER. The number of rows in the submatrix sub(*A*); (*m*≥0).
n	(global) INTEGER. The number of columns in the submatrix sub(*A*) (*n*≥0).
a	(local) REAL for pstzrzf DOUBLE PRECISION for pdtzrzf. COMPLEX for pctzrzf. DOUBLE COMPLEX for pztzrzf. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). Contains the local pieces of the *m*-by-*n* distributed matrix sub (*A*) to be factored.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
work	(local) REAL for pstzrzf DOUBLE PRECISION for pdtzrzf. COMPLEX for pctzrzf. DOUBLE COMPLEX for pztzrzf. Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least *lwork* ≥ *mb_a**(*mp*0+*nq*0+*mb_a*), where *iroff* = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mp*0 = numroc (*m*+*iroff*, *mb_a*, MYROW, *iarow*, NPROW), *nq*0 = numroc (*n*+*icoff*, *nb_a*, MYCOL, *iacol*, NPCOL) indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	On exit, the leading *m*-by-m upper triangular part of sub(*A*) contains the upper triangular matrix *R*, and elements *m*+1 to *n* of the first *m* rows of sub (*A*), with the array *tau*, represent the orthogonal/unitary matrix *Z* as a product of *m* elementary reflectors.

work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
tau	(local) REAL for pstzrzf DOUBLE PRECISION for pdtzrzf. COMPLEX for pctzrzf. DOUBLE COMPLEX for pztzrzf. Array, DIMENSION *LOCr*(*ia*+*m*-1). Contains the scalar factor of elementary reflectors. *tau* is tied to the distributed matrix *A*.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The factorization is obtained by the Householder's method. The *k*-th transformation matrix, *Z*(*k*), which is or whose conjugate transpose is used to introduce zeros into the (*m* - *k* +1)-th row of sub(*A*), is given in the form

where

T(*k*) = *i* - *tau***u*(*k*)**u*(*k*)',

tau is a scalar and *Z*(*k*) is an (*n* - *m*) element vector. *tau* and *Z*(*k*) are chosen to annihilate the elements of the *k*-th row of sub(*A*). The scalar *tau* is returned in the *k*-th element of *tau* and the vector *u*(*k*) in the *k*-th row of sub(*A*), such that the elements of *Z*(*k*) are in *a*(*k*, *m* + 1),..., *a*(*k*, *n*). The elements of *R* are returned in the upper triangular part of sub(*A*). *Z* is given by

Z = *Z*(1) * *Z*(2) *... * *Z*(*m*).

p?ormrz

Multiplies a general matrix by the orthogonal matrix from a reduction to upper triangular form formed by p?tzrzf.

Syntax

call psormrz(*side*, *trans*, *m*, *n*, *k*, *l*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdormrz(*side*, *trans*, *m*, *n*, *k*, *l*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general real m-by-n distributed matrix sub(C) = $C(ic:ic+m-1, jc:jc+n-1)$ with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'T':	Q^T*sub(C)	sub(C)*Q^T

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$Q = H(1)\ H(2) \ldots\ H(k)$

as returned by p?tzrzf. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side — (global) CHARACTER
='L': Q or Q^T is applied from the left.
='R': Q or Q^T is applied from the right.

trans — (global) CHARACTER
='N', no transpose, Q is applied.
='T', transpose, Q^T is applied.

m — (global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n — (global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

k — (global) INTEGER. The number of elementary reflectors whose product defines the matrix Q. Constraints:
If *side* = 'L', $m \geq k \geq 0$
If *side* = 'R', $n \geq k \geq 0$.

l — (global)
The columns of the distributed submatrix sub(A) containing the meaningful part of the Householder reflectors.
If *side* = 'L', $m \geq l \geq 0$
If *side* = 'R', $n \geq l \geq 0$.

a — (local)
REAL for psormrz
DOUBLE PRECISION for pdormrz.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)) if *side* = 'L', and (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R', where $lld\_a \geq \max(1, LOCr(ia+k-1))$.
The i-th row must contain the vector which defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?tzrzf in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the routine but restored on exit.

ia, *ja* — (global) INTEGER. The row and column indices in the global array a indicating the first row and the first column of the submatrix A, respectively.

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix A.

tau (local)
REAL for psormrz
DOUBLE PRECISION for pdormrz
Array, DIMENSION *LOCc*(*ia*+*k*-1).
Contains the scalar factor *tau* (*i*) of elementary reflectors *H*(*i*) as returned by p?tzrzf. *tau* is tied to the distributed matrix *A*.

c (local)
REAL for psormrz
DOUBLE PRECISION for pdormrz
Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*) to be factored.

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psormrz
DOUBLE PRECISION for pdormrz.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + max(*mqa*0 + numroc(numroc(*n*+*iroffc*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a*
else if *side* ='R',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + *nqc*0)**mb_a*) + *mb_a***mb_a*
end if
where
lcmp = *lcm*/NPROW with *lcm* = ilcm (NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c Overwritten by the product *Q**sub(*C*), or *Q*'*sub (*C*), or sub(*C*)**Q*', or sub(*C*)**Q*

work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?unmrz

Multiplies a general matrix by the unitary transformation matrix from a reduction to upper triangular form determined by p?tzrzf.

Syntax

call pcunmrz(*side*, *trans*, *m*, *n*, *k*, *l*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmrz(*side*, *trans*, *m*, *n*, *k*, *l*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general complex *m*-by-*n* distributed matrix sub(*C*) = *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) with

	side ='L'	*side* ='R'
trans = 'N':	*Q**sub(*C*)	sub(*C*)**Q*
trans = 'C':	Q^H*sub(*C*)	sub(*C*)*Q^H

where *Q* is a complex unitary distributed matrix defined as the product of *k* elementary reflectors

Q = *H*(1)' *H*(2)'... *H*(*k*)'

as returned by pctzrzf/pztzrzf. *Q* is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	(global) CHARACTER ='L': *Q* or Q^H is applied from the left. ='R': *Q* or Q^H is applied from the right.
trans	(global) CHARACTER ='N', no transpose, *Q* is applied. ='C', conjugate transpose, Q^H is applied.
m	(global) INTEGER. The number of rows in the distributed matrix sub(*C*), (*m*≥0).
n	(global) INTEGER. The number of columns in the distributed matrix sub(*C*), (*n*≥0).
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. Constraints: If *side* = 'L', *m*≥*k*≥0 If *side* = 'R', *n*≥*k*≥0.

l	(global) INTEGER. The columns of the distributed submatrix sub(*A*) containing the meaningful part of the Householder reflectors. If *side* = 'L', *m*≥*l*≥0 If *side* = 'R', *n*≥*l*≥0.
a	(local) COMPLEX for pcunmrz DOUBLE COMPLEX for pzunmrz. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)) if *side* = 'L', and (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R', where *lld_a* ≥ max(1, *LOCr*(*ja*+*k*-1)). The *i*-th row must contain the vector which defines the elementary reflector *H*(*i*), *ia*≤*i*≤*ia*+*k*-1, as returned by p?gerqf in the *k* rows of its distributed matrix argument *A*(*ia*:*ia* +*k*-1, *ja**). *A*(*ia*:*ia* +*k*-1, *ja**) is modified by the routine but restored on exit.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) COMPLEX for pcunmrz DOUBLE COMPLEX for pzunmrz Array, DIMENSION *LOCc*(*ia*+*k*-1). Contains the scalar factor *tau* (*i*) of elementary reflectors *H*(*i*) as returned by p?gerqf. *tau* is tied to the distributed matrix *A*.
c	(local) COMPLEX for pcunmrz DOUBLE COMPLEX for pzunmrz. Pointer into the local memory to an array of local dimension (*lld_c*, *LOCc*(*jc*+*n*-1)). Contains the local pieces of the distributed matrix sub(*C*) to be factored.
ic, *jc*	(global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.
descc	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.
work	(local) COMPLEX for pcunmrz DOUBLE COMPLEX for pzunmrz. Workspace array of dimension *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least: If *side* = 'L', *lwork* ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0+max(*mqa*0+numroc(numroc(*n*+*iroffc*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a* else if *side* ='R', *lwork* ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0+*nqc*0)**mb_a*) + *mb_a***mb_a* end if where *lcmp* = *lcm*/NPROW with *lcm* = ilcm(NPROW, NPCOL), *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*),

iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = numroc(*m*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	Overwritten by the product *Q** sub(*C*), or *Q*'*sub (*C*), or sub(*C*)**Q*', or sub(*C*)**Q*
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?ggqrf

Computes the generalized QR factorization.

Syntax

call psggqrf(*n*, *m*, *p*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

call pdggqrf(*n*, *m*, *p*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

call pcggqrf(*n*, *m*, *p*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

call pzggqrf(*n*, *m*, *p*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?ggqrf routine forms the generalized *QR* factorization of an *n*-by-*m* matrix

sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*m*-1)

and an *n*-by-*p* matrix

sub(*B*) = *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*p*-1):

as

sub(A) = Q*R, sub(B) = Q*T*Z,

where Q is an n-by-n orthogonal/unitary matrix, Z is a p-by-p orthogonal/unitary matrix, and R and T assume one of the forms:

If $n \geq m$

$$R = \begin{pmatrix} R_{11} \\ 0 \end{pmatrix} \begin{matrix} m \\ n-m \end{matrix}$$
$$m$$

or if $n < m$

$$R = \begin{matrix} (R_{11} & R_{12}) \\ n & m-n \end{matrix} n$$

where R_{11} is upper triangular, and

$$T = \begin{matrix} (0 & T_{12}) \\ p-n & n \end{matrix} n, \text{ if } n \leq p,$$

$$\text{or } T = \begin{pmatrix} T_{11} \\ T_{21} \end{pmatrix} \begin{matrix} n-p \\ p \end{matrix}, \text{ if } n > p,$$
$$p$$

where T_{12} or T_{21} is an upper triangular matrix.

In particular, if sub(B) is square and nonsingular, the GQR factorization of sub(A) and sub(B) implicitly gives the QR factorization of inv (sub(B))* sub (A):

inv(sub(B))*sub(A) = Z^H*(inv(T)*R)

Input Parameters

n	(global) INTEGER. The number of rows in the distributed matrices sub (A) and sub(B) ($n \geq 0$).
m	(global) INTEGER. The number of columns in the distributed matrix sub(A) ($m \geq 0$).
p	INTEGER. The number of columns in the distributed matrix sub(B) ($p \geq 0$).
a	(local) REAL for psggqrf DOUBLE PRECISION for pdggqrf COMPLEX for pcggqrf DOUBLE COMPLEX for pzggqrf. Pointer into the local memory to an array of dimension (lld_a, $LOCc(ja+m-1)$). Contains the local pieces of the n-by-m matrix sub(A) to be factored.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

b (local)
REAL for psggqrf
DOUBLE PRECISION for pdggqrf
COMPLEX for pcggqrf
DOUBLE COMPLEX for pzggqrf.
Pointer into the local memory to an array of dimension (*lld_b*, *LOCc*(*jb*+*p*-1)). Contains the local pieces of the *n*-by-*p* matrix sub(*B*) to be factored.

ib, *jb* (global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.

descb (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix B.

work (local)
REAL for psggqrf
DOUBLE PRECISION for pdggqrf
COMPLEX for pcggqrf
DOUBLE COMPLEX for pzggqrf.
Workspace array of dimension of *lwork*.

lwork (local or global) INTEGER. Dimension of *work*, must be at least
lwork ≥ max(*nb_a**(*npa*0+*mqa*0+*nb_a*), max((*nb_a**(*nb_a*-1))/2, (*pqb*0+*npb*0)**nb_a*)+*nb_a***nb_a*, *mb_b**(*npb*0+*pqb*0+*mb_b*)),
where
iroffa = mod(*ia-1*, *mb_A*),
icoffa = mod(*ja-1*, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
npa0 = numroc (*n*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
mqa0 = numroc (*m*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL)
iroffb = mod(*ib-1*, *mb_b*),
icoffb = mod(*jb-1*, *nb_b*),
ibrow = indxg2p(*ib*, *mb_b*, MYROW, *rsrc_b*, NPROW),
ibcol = indxg2p(*jb*, *nb_b*, MYCOL, *csrc_b*, NPCOL),
npb0 = numroc (*n*+*iroffa*, *mb_b*, MYROW, *Ibrow*, NPROW),
pqb0 = numroc(*m*+*icoffb*, *nb_b*, MYCOL, *ibcol*, NPCOL)
and numroc, indxg2p are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, the elements on and above the diagonal of sub (*A*) contain the min(*n*, *m*)-by-*m* upper trapezoidal matrix *R* (*R* is upper triangular if *n* ≥ *m*); the elements below the diagonal, with the array *taua*, represent the orthogonal/unitary matrix *Q* as a product of min(*n*,*m*) elementary reflectors. (See Application Notes below).

taua, *taub*	(local) REAL for psggqrf DOUBLE PRECISION for pdggqrf COMPLEX for pcggqrf DOUBLE COMPLEX for pzggqrf. Arrays, DIMENSION *LOCc*(*ja*+min(*n*,*m*)-1) for *taua* and *LOCr*(*ib*+*n*-1) for *taub*. The array *taua* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q*. *taua* is tied to the distributed matrix *A*. (See Application Notes below). The array *taub* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Z*. *taub* is tied to the distributed matrix *B*. (See Application Notes below).
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ja*)**H*(*ja*+1)*...**H*(*ja*+*k*-1),

where *k* = min(*n*,*m*).

Each *H*(*i*) has the form

H(*i*) = *i* - *taua***v***v*'

where *taua* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*-1) = 0 and *v*(*i*) = 1; *v*(*i*+1:*n*) is stored on exit in *A*(*ia*+*i*:*ia*+*n*-1,*ja*+*i*-1), and *taua* in *taua*(*ja*+*i*-1).To form *Q* explicitly, use ScaLAPACK subroutine p?orgqr/p?ungqr. To use *Q* to update another matrix, use ScaLAPACK subroutine p?ormqr/p?unmqr.

The matrix *Z* is represented as a product of elementary reflectors

Z = *H*(*ib*)**H*(*ib*+1)*...**H*(*ib*+*k*-1), where *k* = min(*n*,*p*).

Each *H*(*i*) has the form

H(*i*) = *i* - *taub***v***v*'

where *taub* is a real/complex scalar, and *v* is a real/complex vector with *v*(*p*-*k*+*i*+1:*p*) = 0 and *v*(*p*-*k*+*i*) = 1; *v*(1:*p*-*k*+*i*-1) is stored on exit in *B*(*ib*+*n*-*k*+*i*-1,*jb*:*jb*+*p*-*k*+*i*-2), and *taub* in *taub*(*ib*+*n*-*k*+*i*-1). To form *Z* explicitly, use ScaLAPACK subroutine p?orgrq/p?ungrq. To use *Z* to update another matrix, use ScaLAPACK subroutine p?ormrq/p?unmrq.

p?ggrqf

Computes the generalized RQ factorization.

Syntax

call psggrqf(*m*, *p*, *n*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

call pdggrqf(*m*, *p*, *n*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

call pcggrqf(*m*, *p*, *n*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

call pzggrqf(*m*, *p*, *n*, *a*, *ia*, *ja*, *desca*, *taua*, *b*, *ib*, *jb*, *descb*, *taub*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?ggrqf routine forms the generalized *RQ* factorization of an *m*-by-*n* matrix sub(*A*)=(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) and a *p*-by-*n* matrix sub(*B*)=(*ib*:*ib*+*p*-1, *ja*:*ja*+*n*-1):

sub(*A*) = *R***Q*, sub(*B*) = *Z***T***Q*,

where *Q* is an *n*-by-*n* orthogonal/unitary matrix, *Z* is a *p*-by-*p* orthogonal/unitary matrix, and *R* and *T* assume one of the forms:

$$R = \begin{matrix} \\ m \\ n-m \end{matrix} \begin{pmatrix} 0 & R_{12} \\ & \end{pmatrix} \begin{matrix} \\ \\ m \end{matrix}, \text{ if } m \le n,$$

or

$$R = \begin{matrix} \\ m-n \\ n \end{matrix} \begin{pmatrix} R_{11} \\ R_{12} \end{pmatrix} \begin{matrix} \\ n \end{matrix}, \text{ if } m > n$$

where R_{11} or R_{21} is upper triangular, and

$$T = \begin{matrix} \\ p-n \\ n \end{matrix} \begin{pmatrix} T_{11} \\ 0 \end{pmatrix} \begin{matrix} n \\ \end{matrix}, \text{ if } p \ge n$$

or

where T^{11} is upper triangular.

In particular, if sub(*B*) is square and nonsingular, the *GRQ* factorization of sub(*A*) and sub(*B*) implicitly gives the *RQ* factorization of sub (*A*)*inv(sub(*B*)):

sub(*A*)*inv(sub(*B*))= (*R**inv(*T*))**Z*'

where inv(sub(*B*)) denotes the inverse of the matrix sub(*B*), and *Z*' denotes the transpose (conjugate transpose) of matrix *Z*.

Input Parameters

m	(global) INTEGER. The number of rows in the distributed matrices sub (*A*) (*m*≥0).
p	INTEGER. The number of rows in the distributed matrix sub(*B*) (*p*≥0).
n	(global) INTEGER. The number of columns in the distributed matrices sub(*A*) and sub(*B*) (*n*≥0).
a	(local) REAL for psggrqf DOUBLE PRECISION for pdggrqf COMPLEX for pcggrqf DOUBLE COMPLEX for pzggrqf. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). Contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*) to be factored.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
b	(local) REAL for psggrqf DOUBLE PRECISION for pdggrqf COMPLEX for pcggrqf DOUBLE COMPLEX for pzggrqf. Pointer into the local memory to an array of dimension (*lld_b*, *LOCc*(*jb*+*n*-1)). Contains the local pieces of the *p*-by-*n* matrix sub(*B*) to be factored.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.
work	(local) REAL for psggrqf DOUBLE PRECISION for pdggrqf COMPLEX for pcggrqf DOUBLE COMPLEX for pzggrqf. Workspace array of dimension of *lwork*.
lwork	(local or global) INTEGER. Dimension of *work*, must be at least *lwork* ≥ max(*mb_a**(*mpa*0+*nqa*0+*mb_a*), max((*mb_a**(*mb_a*-1))/2, (*ppb*0+*nqb*0)**mb_a*) + *mb_a***mb_a*, *nb_b**(*ppb*0+*nqb*0+*nb_b*)), where *iroffa* = mod(*ia*-1, *mb_A*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW), *iacol* = indxg2p(*ja*, *nb_a*, MYCOL, *csrc_a*, NPCOL), *mpa0* = *numroc* (*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW), *nqa0* = *numroc* (*m*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL) *iroffb* = mod(*ib-1*, *mb_b*), *icoffb* = mod(*jb-1*, *nb_b*), *ibrow* = indxg2p(*ib*, *mb_b*, MYROW, *rsrc_b*, NPROW),

ibcol = indxg2p(*jb*, *nb_b*, MYCOL, *csrc_b*, NPCOL),
ppb0 = numroc (*p*+*iroffb*, *mb_b*, MYROW, *ibrow*, NPROW),
nqb0 = numroc (*n*+*icoffb*, *nb_b*, MYCOL, *ibcol*, NPCOL)
and numroc, indxg2p are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a
On exit, if *m*≤*n*, the upper triangle of *A*(*ia*:*ia*+*m*-1, *ja*+*n*-*m*:*ja*+*n*-1) contains the *m*-by-*m* upper triangular matrix *R*; if *m*≥*n*, the elements on and above the (*m*-*n*)-th subdiagonal contain the *m*-by-*n* upper trapezoidal matrix *R*; the remaining elements, with the array *taua*, represent the orthogonal/unitary matrix *Q* as a product of min(*n*, *m*) elementary reflectors (see *Application Notes* below).

taua, *taub*
(local)
REAL for psggqrf
DOUBLE PRECISION for pdggqrf
COMPLEX for pcggqrf
DOUBLE COMPLEX for pzggqrf.
Arrays, DIMENSION *LOCr*(*ia*+*m*-1)for *taua* and *LOCc*(*jb*+min(p,*n*)-1) for *taub*.
The array *taua* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q*. *taua* is tied to the distributed matrix *A*.(See *Application Notes* below).
The array *taub* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Z*. *taub* is tied to the distributed matrix *B*. (See *Application Notes* below).

work(1)
On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info
(global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ia*)**H*(*ia*+1)*...**H*(*ia*+*k*-1),

where *k* = min(*m*,*n*).

Each *H*(*i*) has the form

H(*i*) = *i* - *taua***v***v*'

where *taua* is a real/complex scalar, and *v* is a real/complex vector with *v*(*n*-*k*+*i*+1:*n*) = 0 and *v*(*n*-*k*+*i*) = 1; v(1:*n*-*k*+*i*-1) is stored on exit in *A*(*ia*+*m*-*k*+*i*-1, *ja*:*ja*+*n*-*k*+*i*-2), and *taua* in *taua*(*ia*+*m*-*k*+*i*-1). To form *Q* explicitly, use ScaLAPACK subroutine p?orgrq/p?ungrq. To use *Q* to update another matrix, use ScaLAPACK subroutine p?ormrq/p?unmrq.

The matrix *Z* is represented as a product of elementary reflectors

Z = *H*(*jb*)**H*(*jb*+1)*...**H*(*jb*+*k*-1), where *k* = min(*p*,*n*).

Each *H*(*i*) has the form

H(*i*) = *i* - *taub***v***v*'

where *taub* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*-1) = 0 and *v*(*i*)= 1; *v*(*i*+1:*p*) is stored on exit in *B*(*ib*+*i*:*ib*+*p*-1,*jb*+*i*-1), and *taub* in *taub*(*jb*+*i*-1). To form *Z* explicitly, use ScaLAPACK subroutine p?orgqr/p?ungqr. To use *Z* to update another matrix, use ScaLAPACK subroutine p?ormqr/p?unmqr.

Symmetric Eigenproblems

To solve a symmetric eigenproblem with ScaLAPACK, you usually need to reduce the matrix to real tridiagonal form *T* and then find the eigenvalues and eigenvectors of the tridiagonal matrix *T*. ScaLAPACK includes routines for reducing the matrix to a tridiagonal form by an orthogonal (or unitary) similarity transformation $A = QTQ^H$ as well as for solving tridiagonal symmetric eigenvalue problems. These routines are listed in Table "Computational Routines for Solving Symmetric Eigenproblems".

There are different routines for symmetric eigenproblems, depending on whether you need eigenvalues only or eigenvectors as well, and on the algorithm used (either the *QTQ* algorithm, or bisection followed by inverse iteration).

Computational Routines for Solving Symmetric Eigenproblems

Operation	Dense symmetric/ Hermitian matrix	Orthogonal/unitary matrix	Symmetric tridiagonal matrix
Reduce to tridiagonal form $A = QTQ^H$	p?sytrd/p?hetrd		
Multiply matrix after reduction		p?ormtr/p?unmtr	
Find all eigenvalues and eigenvectors of a tridiagonal matrix *T* by a *QTQ* method			steqr2*)
Find selected eigenvalues of a tridiagonal matrix *T* via bisection			p?stebz
Find selected eigenvectors of a tridiagonal matrix *T* by inverse iteration			p?stein

*) This routine is described as part of auxiliary ScaLAPACK routines.

p?sytrd

Reduces a symmetric matrix to real symmetric tridiagonal form by an orthogonal similarity transformation.

Syntax

call pssytrd(*uplo*, *n*, *a*, *ia*, *ja*, *desca*, *d*, *e*, *tau*, *work*, *lwork*, *info*)

call pdsytrd(*uplo*, *n*, *a*, *ia*, *ja*, *desca*, *d*, *e*, *tau*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?sytrd routine reduces a real symmetric matrix sub(*A*) to symmetric tridiagonal form *T* by an orthogonal similarity transformation:

Q'*sub(*A*)**Q* = *T*,

where sub(*A*) = *A*(*ia*:*ia*+*n*-1,*ja*:*ja*+*n*-1).

Input Parameters

uplo (global) CHARACTER.
Specifies whether the upper or lower triangular part of the symmetric matrix sub(*A*) is stored:
If *uplo* = 'U', upper triangular
If *uplo* = 'L', lower triangular

n (global) INTEGER. The order of the distributed matrix sub(*A*) (*n*≥0).

a (local)
REAL for pssytrd
DOUBLE PRECISION for pdsytrd.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the symmetric distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. See *Application Notes* below.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local)
REAL for pssytrd
DOUBLE PRECISION for pdsytrd.
Workspace array of dimension *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
lwork ≥ max(NB*(*np* +1), 3*NB),
where NB = *mb_a* = *nb_a*,
np = numroc(*n*, NB, MYROW, *iarow*, NPROW),
iarow = indxg2p(*ia*, NB, MYROW, *rsrc_a*, NPROW).
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, if *uplo* = 'U', the diagonal and first superdiagonal of sub(*A*) are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements above the first superdiagonal, with the array *tau*, represent the orthogonal matrix *Q* as a product of elementary reflectors; if *uplo* = 'L', the diagonal and first subdiagonal of sub(*A*) are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal matrix *Q* as a product of elementary reflectors. See *Application Notes* below.

d (local)

	REAL for pssytrd DOUBLE PRECISION for pdsytrd. Arrays, DIMENSION *LOCc*(*ja*+*n*-1) .The diagonal elements of the tridiagonal matrix *T*: *d*(*i*)= *A*(*i*,*i*). *d* is tied to the distributed matrix *A*.
e	(local) REAL for pssytrd DOUBLE PRECISION for pdsytrd. Arrays, DIMENSION *LOCc*(*ja*+*n*-1) if *uplo* = 'U', *LOCc*(*ja*+*n*-2) otherwise. The off-diagonal elements of the tridiagonal matrix *T*: *e*(*i*)= *A*(*i*,*i*+1) if *uplo* = 'U', *e*(*i*) = *A*(*i*+1,*i*) if *uplo* = 'L'. *e* is tied to the distributed matrix *A*.
tau	(local) REAL for pssytrd DOUBLE PRECISION for pdsytrd. Arrays, DIMENSION *LOCc*(*ja*+*n*-1). This array contains the scalar factors *tau* of the elementary reflectors. *tau* is tied to the distributed matrix *A*.
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*n*-1)... *H*(2) *H*(1).

Each *H*(*i*) has the form

H(*i*) = *i* - *tau* * *v* * *v*',

where *tau* is a real scalar, and *v* is a real vector with *v*(*i*+1:*n*) = 0 and *v*(*i*) = 1; *v*(1:*i*-1) is stored on exit in *A*(*ia*:*ia*+*i*-2, *ja*+*i*), and *tau* in *tau* (*ja*+*i*-1).

If *uplo* = 'L', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(1) *H*(2)... *H*(*n*-1).

Each *H*(*i*) has the form

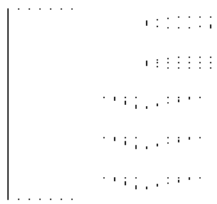
H(*i*) = *i* - *tau* * *v* * *v*',

where *tau* is a real scalar, and *v* is a real vector with *v*(1:*i*) = 0 and *v*(*i*+1) = 1; *v*(*i*+2:*n*) is stored on exit in *A*(*ia*+*i*+1:*ia*+*n*-1,*ja*+*i*-1), and *tau* in *tau*(*ja*+*i*-1).

The contents of sub(*A*) on exit are illustrated by the following examples with *n* = 5:

If *uplo* = 'U':

If *uplo* = 'L':



where *d* and *e* denote diagonal and off-diagonal elements of *T*, and *v i* denotes an element of the vector defining *H*(*i*).

p?ormtr

Multiplies a general matrix by the orthogonal transformation matrix from a reduction to tridiagonal form determined by p?sytrd.

Syntax

call psormtr(*side*, *uplo*, *trans*, *m*, *n*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdormtr(*side*, *uplo*, *trans*, *m*, *n*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general real distributed *m*-by-*n* matrix sub(*C*) = *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) with

	side = 'L'	*side* = 'R'
trans = 'N':	*Q**sub(*C*)	sub(*C*)**Q*
trans = 'T':	Q^T*sub(*C*)	sub(*C*)*Q^T

where *Q* is a real orthogonal distributed matrix of order *nq*, with *nq* = *m* if *side* = 'L' and *nq* = *n* if *side* = 'R'.

Q is defined as the product of *nq* elementary reflectors, as returned by p?sytrd.

If *uplo* = 'U', *Q* = *H*(*nq*-1)... *H*(2) *H*(1);

If *uplo* = 'L', *Q* = *H*(1) *H*(2)... *H*(*nq*-1).

Input Parameters

side (global) CHARACTER

='L': Q or Q^T is applied from the left.
='R': Q or Q^T is applied from the right.

trans (global) CHARACTER
='N', no transpose, Q is applied.
='T', transpose, Q^T is applied.

uplo (global) CHARACTER.
= 'U': Upper triangle of $A(ia{:}*, ja{:}*)$ contains elementary reflectors from p?sytrd;
= 'L': Lower triangle of $A(ia{:}*, ja{:}*)$ contains elementary reflectors from p?sytrd

m (global) INTEGER. The number of rows in the distributed matrix sub(C) ($m \geq 0$).

n (global) INTEGER. The number of columns in the distributed matrix sub(C) ($n \geq 0$).

a (local)
REAL for psormtr
DOUBLE PRECISION for pdormtr.
Pointer into the local memory to an array of dimension (lld_a, $LOCc(ja+m-1)$) if $side$='L', or (lld_a, $LOCc(ja+n-1)$) if $side$ = 'R'.
Contains the vectors which define the elementary reflectors, as returned by p?sytrd.
If $side$='L', $lld\_a \geq \max(1, LOCr(ia+m-1))$;
If $side$ ='R', $lld\_a \geq \max(1, LOCr(ia+n-1))$.

ia, *ja* (global) INTEGER. The row and column indices in the global array a indicating the first row and the first column of the submatrix A, respectively.

desca (global and local) INTEGER array, dimension ($dlen\_$). The array descriptor for the distributed matrix A.

tau (local)
REAL for psormtr
DOUBLE PRECISION for pdormtr.
Array, DIMENSION of $ltau$ where
if $side$ = 'L' and $uplo$ = 'U', $ltau$ = $LOCc(m\_a)$,
if $side$ = 'L' and $uplo$ = 'L', $ltau$ = $LOCc(ja+m-2)$,
if $side$ = 'R' and $uplo$ = 'U', $ltau$ = $LOCc(n\_a)$,
if $side$ = 'R' and $uplo$ = 'L', $ltau$ = $LOCc(ja+n-2)$.
$tau(i)$ must contain the scalar factor of the elementary reflector $H(i)$, as returned by p?sytrd. tau is tied to the distributed matrix A.

c (local) REAL for psormtr
DOUBLE PRECISION for pdormtr.
Pointer into the local memory to an array of dimension (lld_a, $LOCc\ (ja+n-1)$). Contains the local pieces of the distributed matrix sub (C).

ic, *jc* (global) INTEGER. The row and column indices in the global array c indicating the first row and the first column of the submatrix C, respectively.

descc (global and local) INTEGER array, dimension ($dlen\_$). The array descriptor for the distributed matrix C.

work (local)
REAL for psormtr
DOUBLE PRECISION for pdormtr.
Workspace array of dimension $lwork$.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
if *uplo* = 'U',
iaa= *ia*; *jaa*= *ja*+1, *icc*= *ic*; *jcc*= *jc*;
else *uplo* = 'L',
iaa= *ia*+1, *jaa*= *ja*;
If *side* = 'L',
icc= *ic*+1; *jcc*= *jc*;
else *icc*= *ic*; *jcc*= *jc*+1;
end if
end if
If *side* = 'L',
mi= *m*-1; *ni*= *n*
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + *mpc*0)**nb_a*) + *nb_a***nb_a*
else
If *side* = 'R',
mi= *m*; *mi* = *n*-1;
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + max(*npa*0+numroc(numroc(*ni*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*)+ *nb_a***nb_a*
end if
where *lcmq* = *lcm*/NPCOL with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*iaa*-1, *mb_a*),
icoffa = mod(*jaa*-1, *nb_a*),
iarow = indxg2p(*iaa*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0 = numroc(*ni*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*icc*-1, *mb_c*),
icoffc = mod(*jcc*-1, *nb_c*),
icrow = indxg2p(*icc*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jcc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*mi*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*ni*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c Overwritten by the product *Q**sub(*C*), or *Q'**sub(*C*), or sub(*C*)**Q'*, or sub(*C*)**Q*.

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?hetrd

Reduces a Hermitian matrix to Hermitian tridiagonal form by a unitary similarity transformation.

Syntax

```
call pchetrd(uplo, n, a, ia, ja, desca, d, e, tau, work, lwork, info)
call pzhetrd(uplo, n, a, ia, ja, desca, d, e, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?hetrd routine reduces a complex Hermitian matrix sub(*A*) to Hermitian tridiagonal form *T* by a unitary similarity transformation:

Q'*sub(*A*)**Q* = *T*

where sub(*A*) = *A*(*ia*:*ia*+*n*-1,*ja*:*ja*+*n*-1).

Input Parameters

uplo	(global) CHARACTER. Specifies whether the upper or lower triangular part of the Hermitian matrix sub(*A*) is stored: If *uplo* = 'U', upper triangular If *uplo* = 'L', lower triangular
n	(global) INTEGER. The order of the distributed matrix sub(*A*) (*n*≥0).
a	(local) COMPLEX for pchetrd DOUBLE COMPLEX for pzhetrd. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the Hermitian distributed matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. (see *Application Notes* below).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
work	(local) COMPLEX for pchetrd DOUBLE COMPLEX for pzhetrd. Workspace array of dimension *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least: *lwork*≥max(NB*(*np* +1), 3*NB) where NB = *mb_a* = *nb_a*, *np* = numroc(*n*, NB, MYROW, *iarow*, NPROW), *iarow* = indxg2p(*ia*, NB, MYROW, *rsrc_a*, NPROW).

indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a
On exit,
If *uplo* = 'U', the diagonal and first superdiagonal of sub(*A*) are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements above the first superdiagonal, with the array *tau*, represent the unitary matrix *Q* as a product of elementary reflectors;if *uplo* = 'L', the diagonal and first subdiagonal of sub(*A*) are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements below the first subdiagonal, with the array *tau*, represent the unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

d
(local)
REAL for pchetrd
DOUBLE PRECISION for pzhetrd.
Arrays, DIMENSION *LOCc*(*ja*+*n*-1). The diagonal elements of the tridiagonal matrix *T*:
d(*i*)= *A*(*i*,*i*).
d is tied to the distributed matrix *A*.

e
(local)
REAL for pchetrd
DOUBLE PRECISION for pzhetrd.
Arrays, DIMENSION *LOCc*(*ja*+*n*-1) if *uplo* = 'U'; *LOCc*(*ja*+*n*-2) - otherwise.
The off-diagonal elements of the tridiagonal matrix *T*:
e(*i*)= *A*(*i*,*i*+1) if *uplo* = 'U',
e(*i*)= *A*(*i*+1,*i*) if *uplo* = 'L'.
e is tied to the distributed matrix *A*.

tau
(local) COMPLEX for pchetrd
DOUBLE COMPLEX for pzhetrd.
Arrays, DIMENSION *LOCc*(*ja*+*n*-1). This array contains the scalar factors *tau* of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

work(1)
On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info
(global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*n*-1)*...**H*(2)**H*(1).

Each *H*(*i*) has the form

H(*i*) = *i* - *tau***v***v*',

where *tau* is a complex scalar, and *v* is a complex vector with *v*(*i*+1:*n*) = 0 and *v*(*i*) = 1; *v*(1:*i*-1) is stored on exit in *A*(*ia*:*ia*+*i*-2, *ja*+*i*), and *tau* in *tau* (*ja*+*i*-1).

If *uplo* = 'L', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(1)**H*(2)*...**H*(*n*-1).

Each *H*(*i*) has the form

H(*i*) = *i* - *tau***v***v*',

where *tau* is a complex scalar, and *v* is a complex vector with *v*(1:*i*) = 0 and *v*(*i*+1) = 1; *v*(*i*+2:*n*) is stored on exit in *A*(*ia*+*i*+1:*ia*+*n*-1,*ja*+*i*-1), and *tau* in *tau*(*ja*+*i*-1).

The contents of sub(*A*) on exit are illustrated by the following examples with *n* = 5:

If *uplo* = 'U':

$$\begin{bmatrix} d & e & v2 & v3 & v4 \\ & d & e & v3 & v4 \\ & & d & e & v4 \\ & & & d & e \\ & & & & d \end{bmatrix}$$

If *uplo* = 'L':

$$\begin{bmatrix} d & & & & \\ e & d & & & \\ v1 & e & d & & \\ v1 & v2 & e & d & \\ v1 & v2 & v3 & e & d \end{bmatrix}$$

where *d* and *e* denote diagonal and off-diagonal elements of *T*, and *v i* denotes an element of the vector defining *H*(*i*).

p?unmtr

Multiplies a general matrix by the unitary transformation matrix from a reduction to tridiagonal form determined by p?hetrd.

Syntax

call pcunmtr(*side*, *uplo*, *trans*, *m*, *n*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmtr(*side*, *uplo*, *trans*, *m*, *n*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general complex distributed *m*-by-*n* matrix sub(*C*) = *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'C':	Q^H*sub(C)	sub(C)*Q^H

where Q is a complex unitary distributed matrix of order *nq*, with *nq* = *m* if *side* = 'L' and *nq* = *n* if *side* = 'R'.

Q is defined as the product of *nq*-1 elementary reflectors, as returned by p?hetrd.

If *uplo* = 'U', Q = $H(nq-1)\ldots H(2)\ H(1)$;

If *uplo* = 'L', Q = $H(1)\ H(2)\ldots H(nq-1)$.

Input Parameters

side (global) CHARACTER
='L': Q or Q^H is applied from the left.
='R': Q or Q^H is applied from the right.

trans (global) CHARACTER
='N', no transpose, Q is applied.
='C', conjugate transpose, Q^H is applied.

uplo (global) CHARACTER.
= 'U': Upper triangle of *A*(*ia*:*, *ja*:*) contains elementary reflectors from p?hetrd;
= 'L': Lower triangle of *A*(*ia*:*,*ja*:*) contains elementary reflectors from p?hetrd

m (global) INTEGER. The number of rows in the distributed matrix sub(C) (*m*≥0).

n (global) INTEGER. The number of columns in the distributed matrix sub(C) (*n*≥0).

a (local)
REAL for pcunmtr
DOUBLE PRECISION for pzunmtr.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)) if *side*='L', or (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R'.
Contains the vectors which define the elementary reflectors, as returned by p?hetrd.
If *side*='L', *lld_a* ≥ max(1,*LOCr*(*ia*+*m*-1));
If *side* ='R', *lld_a* ≥ max(1, *LOCr*(*ia*+*n*-1)).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcunmtr
DOUBLE COMPLEX for pzunmtr.
Array, DIMENSION of *ltau* where
If *side* = 'L' and *uplo* = 'U', *ltau* = *LOCc*(*m_a*),
if *side* = 'L' and *uplo* = 'L', *ltau* = *LOCc*(*ja*+*m*-2),
if *side* = 'R' and *uplo* = 'U', *ltau* = *LOCc*(*n_a*),
if *side* = 'R' and *uplo* = 'L', *ltau* = *LOCc*(*ja*+*n*-2).
tau(*i*) must contain the scalar factor of the elementary reflector *H*(*i*), as returned by p?hetrd. *tau* is tied to the distributed matrix *A*.

c (local) COMPLEX for pcunmtr
DOUBLE COMPLEX for pzunmtr.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc* (*ja*+*n*-1)). Contains the local pieces of the distributed matrix sub (*C*).

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
COMPLEX for pcunmtr
DOUBLE COMPLEX for pzunmtr.
Workspace array of dimension *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *uplo* = 'U',
iaa= *ia*; *jaa*= *ja*+1, *icc*= *ic*; *jcc*= *jc*;
else *uplo* = 'L',
iaa= *ia*+1, *jaa*= *ja*;
If *side* = 'L',
icc= *ic*+1; *jcc*= *jc*;
else *icc*= *ic*; *jcc*= *jc*+1;
end if
end if
If *side* = 'L',
mi= *m*-1; *ni*= *n*
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + *mpc*0)**nb_a*) +
*nb_a***nb_a*
else
If *side* = 'R',
mi= *m*; *mi* = *n*-1;
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 +
max(*npa*0+numroc(numroc(*ni*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*,
0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where *lcmq* = *lcm*/NPCOL with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*iaa*-1, *mb_a*),
icoffa = mod(*jaa*-1, *nb_a*),
iarow = indxg2p(*iaa*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0 = numroc(*ni*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*icc*-1, *mb_c*),
icoffc = mod(*jcc*-1, *nb_c*),
icrow = indxg2p(*icc*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jcc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*mi*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*ni*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	Overwritten by the product *Q**sub(*C*), or *Q*'*sub(*C*), or sub(*C*)**Q*', or sub(*C*)**Q*.
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = – *i*

p?stebz

Computes the eigenvalues of a symmetric tridiagonal matrix by bisection.

Syntax

call psstebz(*ictxt*, *range*, *order*, *n*, *vl*, *vu*, *il*, *iu*, *abstol*, *d*, *e*, *m*, *nsplit*, *w*, *iblock*, *isplit*, *work*, *iwork*, *liwork*, *info*)

call pdstebz(*ictxt*, *range*, *order*, *n*, *vl*, *vu*, *il*, *iu*, *abstol*, *d*, *e*, *m*, *nsplit*, *w*, *iblock*, *isplit*, *work*, *iwork*, *liwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?stebz routine computes the eigenvalues of a symmetric tridiagonal matrix in parallel. These may be all eigenvalues, all eigenvalues in the interval [*v1* *vu*], or the eigenvalues indexed *il* through *iu*. A static partitioning of work is done at the beginning of p?stebz which results in all processes finding an (almost) equal number of eigenvalues.

Input Parameters

ictxt	(global) INTEGER. The BLACS context handle.
range	(global) CHARACTER. Must be 'A' or 'V' or 'I'. If *range* = 'A', the routine computes all eigenvalues. If *range* = 'V', the routine computes eigenvalues in the interval [*v1*, *vu*]. If *range* ='I', the routine computes eigenvalues with indices *il* to *iu*.
order	(global) CHARACTER. Must be 'B' or 'E'. If *order* = 'B', the eigenvalues are to be ordered from smallest to largest within each split-off block. If *order* = 'E', the eigenvalues for the entire matrix are to be ordered from smallest to largest.
n	(global) INTEGER. The order of the tridiagonal matrix *T* (*n*≥0).
vl, *vu*	(global) REAL for psstebz DOUBLE PRECISION for pdstebz. If *range* = 'V', the routine computes the lower and the upper bounds for the eigenvalues on the interval [*1*, *vu*]. If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu*
(global)
INTEGER. Constraint: $1 \le il \le iu \le n$.
If *range* = 'I', the index of the smallest eigenvalue is returned for *il* and of the largest eigenvalue for *iu* (assuming that the eigenvalues are in ascending order) must be returned. *il* must be at least 1. *iu* must be at least *il* and no greater than *n*.
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol
(global)
REAL for psstebz
DOUBLE PRECISION for pdstebz.
The absolute tolerance to which each eigenvalue is required. An eigenvalue (or cluster) is considered to have converged if it lies in an interval of width *abstol*. If $abstol \le 0$, then the tolerance is taken as $ulp||T||$, where *ulp* is the machine precision, and $||T||$ means the 1-norm of *T*
Eigenvalues will be computed most accurately when *abstol* is set to the underflow threshold slamch('U'), not 0. Note that if eigenvectors are desired later by inverse iteration (p?stein), *abstol* should be set to 2*p?lamch('S').

d
(global)
REAL for psstebz
DOUBLE PRECISION for pdstebz.
Array, DIMENSION (*n*).
Contains *n* diagonal elements of the tridiagonal matrix *T*. To avoid overflow, the matrix must be scaled so that its largest entry is no greater than the $overflow^{(1/2)} * underflow^{(1/4)}$ in absolute value, and for greatest accuracy, it should not be much smaller than that.

e
(global)
REAL for psstebz
DOUBLE PRECISION for pdstebz.
Array, DIMENSION (*n* - 1).
Contains (*n*–1) off-diagonal elements of the tridiagonal matrix *T*. To avoid overflow, the matrix must be scaled so that its largest entry is no greater than $overflow^{(1/2)} * underflow^{(1/4)}$ in absolute value, and for greatest accuracy, it should not be much smaller than that.

work
(local)
REAL for psstebz
DOUBLE PRECISION for pdstebz.
Array, DIMENSION max(5*n*, 7). This is a workspace array.

lwork
(local) INTEGER. The size of the *work* array must be ≥ max(5*n*, 7).
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

iwork
(local) INTEGER. Array, DIMENSION max(4*n*, 14). This is a workspace array.

liwork
(local) INTEGER. the size of the *iwork* array must ≥ max(4*n*, 14, NPROCS).
If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

m (global) INTEGER. The actual number of eigenvalues found. $0 \le m \le n$

nsplit (global) INTEGER. The number of diagonal blocks detected in *T*. $1 \le nsplit \le n$

w (global)
REAL for psstebz
DOUBLE PRECISION for pdstebz.
Array, DIMENSION (*n*). On exit, the first *m* elements of *w* contain the eigenvalues on all processes.

iblock (global) INTEGER.
Array, DIMENSION (*n*). At each row/column *j* where *e*(*j*) is zero or small, the matrix *T* is considered to split into a block diagonal matrix. On exit *iblock*(*i*) specifies which block (from 1 to the number of blocks) the eigenvalue *w*(*i*) belongs to.

NOTE In the (theoretically impossible) event that bisection does not converge for some or all eigenvalues, *info* is set to 1 and the ones for which it did not are identified by a negative block number.

isplit (global) INTEGER.
Array, DIMENSION (*n*).
Contains the splitting points, at which *T* breaks up into submatrices. The first submatrix consists of rows/columns 1 to *isplit*(1), the second of rows/columns *isplit*(1)+1 through *isplit*(2), etc., and the *nsplit*-th consists of rows/columns *isplit*(*nsplit*-1)+1 through *isplit*(*nsplit*)=*n*. (Only the first *nsplit* elements are used, but since the *nsplit* values are not known, *n* words must be reserved for *isplit*.)

info (global) INTEGER.
If *info* = 0, the execution is successful.
If *info* < 0, if *info* = -*i*, the *i*-th argument has an illegal value.
If *info* > 0, some or all of the eigenvalues fail to converge or not computed.
If *info* = 1, bisection fails to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances.
If *info* = 2, mismatch between the number of eigenvalues output and the number desired.
If *info* = 3: *range*='*i*', and the Gershgorin interval initially used is incorrect. No eigenvalues are computed. Probable cause: the machine has a sloppy floating point arithmetic. Increase the *fudge* parameter, recompile, and try again.

p?stein

Computes the eigenvectors of a tridiagonal matrix using inverse iteration.

Syntax

```
call psstein(n, d, e, m, w, iblock, isplit, orfac, z, iz, jz, descz, work, lwork,
iwork, liwork, ifail, iclustr, gap, info)
```

```
call pdstein(n, d, e, m, w, iblock, isplit, orfac, z, iz, jz, descz, work, lwork,
iwork, liwork, ifail, iclustr, gap, info)

call pcstein(n, d, e, m, w, iblock, isplit, orfac, z, iz, jz, descz, work, lwork,
iwork, liwork, ifail, iclustr, gap, info)

call pzstein(n, d, e, m, w, iblock, isplit, orfac, z, iz, jz, descz, work, lwork,
iwork, liwork, ifail, iclustr, gap, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?stein routine computes the eigenvectors of a symmetric tridiagonal matrix *T* corresponding to specified eigenvalues, by inverse iteration. p?stein does not orthogonalize vectors that are on different processes. The extent of orthogonalization is controlled by the input parameter *lwork*. Eigenvectors that are to be orthogonalized are computed by the same process. p?stein decides on the allocation of work among the processes and then calls ?stein2 (modified LAPACK routine) on each individual process. If insufficient workspace is allocated, the expected orthogonalization may not be done.

NOTE If the eigenvectors obtained are not orthogonal, increase *lwork* and run the code again.

p = NPROW*NPCOL is the total number of processes.

Input Parameters

n	(global) INTEGER. The order of the matrix *T* (*n* ≥ 0).
m	(global) INTEGER. The number of eigenvectors to be returned.
d, *e*, *w*	(global) REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays: *d*(*) contains the diagonal elements of *T*. DIMENSION (*n*). *e*(*) contains the off-diagonal elements of *T*. DIMENSION (*n*-1). *w*(*) contains all the eigenvalues grouped by split-off block.The eigenvalues are supplied from smallest to largest within the block. (Here the output array *w* from p?stebz with order = 'B' is expected. The array should be replicated in all processes.) DIMENSION(*m*)
iblock	(global) INTEGER. Array, DIMENSION (*n*). The submatrix indices associated with the corresponding eigenvalues in *w*--1 for eigenvalues belonging to the first submatrix from the top, 2 for those belonging to the second submatrix, etc. (The output array *iblock* from p?stebz is expected here).
isplit	(global) INTEGER. Array, DIMENSION (*n*). The splitting points, at which *T* breaks up into submatrices. The first submatrix consists of rows/columns 1 to *isplit*(1), the second of rows/columns *isplit*(1)+1 through *isplit*(2), etc., and the *nsplit*-th consists of rows/columns *isplit*(*nsplit*-1)+1 through *isplit*(*nsplit*)=*n* . (The output array *isplit* from p?stebz is expected here.)

orfac (global)
REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
orfac specifies which eigenvectors should be orthogonalized. Eigenvectors that correspond to eigenvalues within *orfac**||T|| of each other are to be orthogonalized. However, if the workspace is insufficient (see *lwork*), this tolerance may be decreased until all eigenvectors can be stored in one process. No orthogonalization is done if *orfac* is equal to zero. A default value of 1000 is used if *orfac* is negative. *orfac* should be identical on all processes

iz, *jz* (global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*.

work (local). REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Workspace array, DIMENSION (*lwork*).

lwork (local) INTEGER.
lwork controls the extent of orthogonalization which can be done. The number of eigenvectors for which storage is allocated on each process is *nvec* = floor((*lwork*-max(5**n*,*np*00**mq*00))/*n*). Eigenvectors corresponding to eigenvalue clusters of size *nvec* - ceil(*m*/*p*) + 1 are guaranteed to be orthogonal (the orthogonality is similar to that obtained from ?stein2).

NOTE *lwork* must be no smaller than max(5**n*,*np*00**mq*00) + ceil(*m*/*p*)**n* and should have the same input value on all processes.

It is the minimum value of *lwork* input on different processes that is significant.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

iwork (local) INTEGER.
Workspace array, DIMENSION (3*n*+*p*+1).

liwork (local) INTEGER. The size of the array *iwork*. It must be greater than (3**n* +*p*+1).
If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

z (local)
REAL for psstein
DOUBLE PRECISION for pdstein
COMPLEX for pcstein
DOUBLE COMPLEX for pzstein.

	Array, DIMENSION (*descz*(*dlen_*), *n*/NPCOL + NB). *z* contains the computed eigenvectors associated with the specified eigenvalues. Any vector which fails to converge is set to its current iterate after MAXIT iterations (See ?stein2). On output, *z* is distributed across the *p* processes in block cyclic format.
work(1)	On exit, *work*(1) gives a lower bound on the workspace (*lwork*) that guarantees the user desired orthogonalization (see *orfac*). Note that this may overestimate the minimum workspace needed.
iwork	On exit, *iwork*(1) contains the amount of integer workspace required. On exit, the *iwork*(2) through *iwork*(*p*+2) indicate the eigenvectors computed by each process. Process *i* computes eigenvectors indexed *iwork*(*i*+2)+1 through *iwork*(*i*+3).
ifail	(global). INTEGER. Array, DIMENSION (*m*). On normal exit, all elements of *ifail* are zero. If one or more eigenvectors fail to converge after MAXIT iterations (as in ?stein), then *info* > 0 is returned. If mod(*info*, *m*+1)>0, then for *i*=1 to mod(*info*,*m*+1), the eigenvector corresponding to the eigenvalue *w*(*ifail*(*i*)) failed to converge (*w* refers to the array of eigenvalues on output).
iclustr	(global) INTEGER. Array, DIMENSION (2*p) This output array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be orthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed *iclustr*(2*I-1) to *iclustr*(2*I), *i* = 1 to *info*/(*m*+1), could not be orthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. *iclustr* is a zero terminated array ---(*iclustr*(2*k).*ne*. 0.*and*.*iclustr*(2*k+1).*eq*.*0*) if and only if *k* is the number of clusters.
gap	(global) REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. This output array contains the gap between eigenvalues whose eigenvectors could not be orthogonalized. The *info*/*m* output values in this array correspond to the *info*/(*m*+1) clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i-th* cluster may be as high as (*O*(*n*)*macheps)/*gap*(*i*).
info	(global) INTEGER. If *info* = 0, the execution is successful. If *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i*100+*j*), If the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. If *info* < 0: if *info* = -*i*, the *i*-th argument had an illegal value. If *info* > 0: if mod(*info*, *m*+1) = *i*, then *i* eigenvectors failed to converge in MAXIT iterations. Their indices are stored in the array *ifail*. If *info*/(*m*+1) = *i*, then eigenvectors corresponding to *i* clusters of eigenvalues could not be orthogonalized due to insufficient workspace. The indices of the clusters are stored in the array *iclustr*.

Nonsymmetric Eigenvalue Problems

This section describes ScaLAPACK routines for solving nonsymmetric eigenvalue problems, computing the Schur factorization of general matrices, as well as performing a number of related computational tasks.

To solve a nonsymmetric eigenvalue problem with ScaLAPACK, you usually need to reduce the matrix to the upper Hessenberg form and then solve the eigenvalue problem with the Hessenberg matrix obtained.

Table "Computational Routines for Solving Nonsymmetric Eigenproblems" lists ScaLAPACK routines for reducing the matrix to the upper Hessenberg form by an orthogonal (or unitary) similarity transformation $A = QHQ^H$, as well as routines for solving eigenproblems with Hessenberg matrices, and multiplying the matrix after reduction.

Computational Routines for Solving Nonsymmetric Eigenproblems

Operation performed	General matrix	Orthogonal/Unitary matrix	Hessenberg matrix
Reduce to Hessenberg form $A = QHQ^H$	p?gehrd		
Multiply the matrix after reduction		p?ormhr/p?unmhr	
Find eigenvalues and Schur factorization			p?lahqr

p?gehrd

Reduces a general matrix to upper Hessenberg form.

Syntax

```
call psgehrd(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
call pdgehrd(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
call pcgehrd(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
call pzgehrd(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gehrd routine reduces a real/complex general distributed matrix sub(*A*) to upper Hessenberg form *H* by an orthogonal or unitary similarity transformation

Q'*sub(*A*)**Q* = *H*,

where sub(*A*) = *A*(*ia*+*n*-1:*ia*+*n*-1, *ja*+*n*-1:*ja*+*n*-1).

Input Parameters

n	(global) INTEGER. The order of the distributed matrix sub(*A*) (*n*≥0).
ilo, *ihi*	(global) INTEGER. It is assumed that sub(*A*) is already upper triangular in rows *ia*:*ia*+*ilo*-2 and *ia*+*ihi*:*ia*+*n*-1 and columns *ja*:*ja*+*ilo*-2 and *ja*+*ihi*:*ja*+*n*-1. (See *Application Notes* below). If *n* > 0, 1≤*ilo*≤*ihi*≤*n*; otherwise set *ilo* = 1, *ihi* = *n*.
a	(local) REAL for psgehrd DOUBLE PRECISION for pdgehrd COMPLEX for pcgehrd DOUBLE COMPLEX for pzgehrd. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* general distributed matrix sub(*A*) to be reduced.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local)
REAL for psgehrd
DOUBLE PRECISION for pdgehrd
COMPLEX for pcgehrd
DOUBLE COMPLEX for pzgehrd.
Workspace array of dimension *lwork*.

lwork (local or global) INTEGER, dimension of the array *work*. *lwork* is local input and must be at least
lwork≥NB*NB + NB*max(*ihip*+1, *ihlp*+*inlq*)
where NB = *mb_a* = *nb_a*,
iroffa = mod(*ia*-1, NB),
icoffa = mod(*ja*-1, NB),
ioff = mod(*ia*+*ilo*-2, NB), *iarow* = indxg2p(*ia*, NB, MYROW, *rsrc_a*, NPROW), *ihip* = numroc(*ihi*+*iroffa*, NB, MYROW, *iarow*, NPROW),
ilrow = indxg2p(*ia*+*ilo*-1, NB, MYROW, *rsrc_a*, NPROW),
ihlp = numroc(*ihi*-*ilo*+*ioff*+1, NB, MYROW, *ilrow*, NPROW),
ilcol = indxg2p(*ja*+*ilo*-1, NB, MYCOL, *csrc_a*, NPCOL),
inlq = numroc(*n*-*ilo*+*ioff*+1, NB, MYCOL, *ilcol*, NPCOL),
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, the upper triangle and the first subdiagonal of sub(*A*)are overwritten with the upper Hessenberg matrix *H*, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau (local). REAL for psgehrd
DOUBLE PRECISION for pdgehrd
COMPLEX for pcgehrd
DOUBLE COMPLEX for pzgehrd.
Array, DIMENSION at least max(*ja*+*n*-2).
The scalar factors of the elementary reflectors (see *Application Notes* below). Elements *ja*:*ja*+*ilo*-2 and *ja*+*ihi*:*ja*+*n*-2 of *tau* are set to zero. *tau* is tied to the distributed matrix *A*.

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of (*ihi*-*ilo*) elementary reflectors

Q = *H*(*ilo*)**H*(*ilo*+1)*...**H*(*ihi*-1).

Each *H*(*i*) has the form

H(*i*)= *i* - *tau***v***v*'

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*)= 0, *v*(*i*+1)= 1 and *v*(*ihi*+1:*n*)= 0; *v*(*i*+2:*ihi*) is stored on exit in *a*(*ia*+*ilo*+*i*:*ia*+*ihi*-1,*ja*+*ilo*+*i*-2), and *tau* in *tau*(*ja*+*ilo*+*i*-2). The contents of *a*(*ia*:*ia*+*n*-1,*ja*:*ja*+*n*-1) are illustrated by the following example, with *n* = 7, *ilo* = 2 and *ihi* = 6:

on entry

$$\begin{bmatrix} a & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & & & & & & a \end{bmatrix}$$

on exit

$$\begin{bmatrix} a & a & a & h & h & h & a \\ & a & h & h & h & h & a \\ & h & h & h & h & h & h \\ & v2 & h & h & h & h & h \\ & v2 & v3 & h & h & h & h \\ & v2 & v3 & v4 & h & h & h \\ & & & & & & a \end{bmatrix}$$

where *a* denotes an element of the original matrix sub(*A*), *H* denotes a modified element of the upper Hessenberg matrix *H*, and *vi* denotes an element of the vector defining *H*(*ja*+*ilo*+*i*-2).

p?ormhr

Multiplies a general matrix by the orthogonal transformation matrix from a reduction to Hessenberg form determined by p?gehrd*.*

Syntax

call psormhr(*side*, *trans*, *m*, *n*, *ilo*, *ihi*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdormhr(*side*, *trans*, *m*, *n*, *ilo*, *ihi*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?ormhr routine overwrites the general real distributed *m*-by-*n* matrix sub(*C*)= *C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) with

	side ='L'	*side* ='R'
trans = 'N':	*Q**sub(*C*)	sub(*C*)**Q*
trans = 'T':	Q^T*sub(*C*)	sub(*C*)*Q^T

where *Q* is a real orthogonal distributed matrix of order *nq*, with *nq* = *m* if *side* = 'L' and *nq* = *n* if *side* = 'R'.

Q is defined as the product of *ihi*-*ilo* elementary reflectors, as returned by p?gehrd.

Q = *H*(*ilo*) *H*(*ilo*+1)... *H*(*ihi*-1).

Input Parameters

side (global) CHARACTER
='L': *Q* or Q^T is applied from the left.
='R': *Q* or Q^T is applied from the right.

trans (global) CHARACTER
='N', no transpose, *Q* is applied.
='T', transpose, Q^T is applied.

m (global) INTEGER. The number of rows in the distributed matrix sub (*C*) (*m*≥0).

n (global) INTEGER. The number of columns in he distributed matrix sub (*C*) (*n*≥0).

ilo, *ihi* (global) INTEGER.
ilo and *ihi* must have the same values as in the previous call of p?gehrd.
Q is equal to the unit matrix except for the distributed submatrix *Q*(*ia*+*ilo*:*ia*+*ihi*-1,*ia*+*ilo*:*ja*+*ihi*-1).
If *side* = 'L', 1≤*ilo*≤*ihi*≤max(1,*m*);
If *side* = 'R', 1≤*ilo*≤*ihi*≤max(1,*n*);
ilo and *ihi* are relative indexes.

a (local)
REAL for psormhr
DOUBLE PRECISION for pdormhr
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*m*-1)) if *side*='L', and (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R'.
Contains the vectors which define the elementary reflectors, as returned by p?gehrd.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
REAL for psormhr
DOUBLE PRECISION for pdormhr
Array, DIMENSION *LOCc*(*ja*+*m*-2), if *side* = 'L', and *LOCc*(*ja*+*n*-2) if *side* = 'R'.
This array contains the scalar factors *tau*(*j*) of the elementary reflectors *H*(*j*) as returned by p?gehrd. *tau* is tied to the distributed matrix *A*.

c (local)
REAL for psormhr
DOUBLE PRECISION for pdormhr
Pointer into the local memory to an array of dimension (*lld_c*,*LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*).

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psormhr
DOUBLE PRECISION for pdormhr
Workspace array with dimension *lwork*.

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork must be at least *iaa = ia + ilo; jaa = ja+ilo*-1;
If *side* = 'L',
mi = ihi-ilo; *ni = n*; *icc = ic + ilo*; *jcc = jc*; *lwork* ≥
max((*nb_a**(*nb_a*-1))/2, (*nqc*0+*mpc*0)**nb_a*) + *nb_a***nb_a*
else if *side* = 'R',
mi = m; *ni = ihi-ilo*; *icc = ic*; *jcc = jc + ilo*; *lwork* ≥
max((*nb_a**(*nb_a*-1))/2, (*nqc*0+max(*npa*0+numroc(numroc(*ni*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where *lcmq* = *lcm*/NPCOL with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*iaa*-1, *mb_a*),
icoffa = mod(*jaa*-1, *nb_a*),
iarow = indxg2p(*iaa*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0 = numroc(*ni*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*icc*-1, *mb_c*), *icoffc* = mod(*jcc*-1, *nb_c*),
icrow = indxg2p(*icc*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jcc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*mi*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*ni*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c sub(*C*) is overwritten by *Q**sub(*C*), or *Q'**sub(*C*), or sub(*C*)**Q'*, or sub(*C*)**Q*.

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info (global) INTEGER.
= 0: the execution is successful.

< 0: if the i-th argument is an array and the j-entry had an illegal value, then $info = -(i*100+j)$, if the i-th argument is a scalar and had an illegal value, then $info = -i$.

p?unmhr

Multiplies a general matrix by the unitary transformation matrix from a reduction to Hessenberg form determined by p?gehrd.

Syntax

call pcunmhr(*side*, *trans*, *m*, *n*, *ilo*, *ihi*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmhr(*side*, *trans*, *m*, *n*, *ilo*, *ihi*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This routine overwrites the general complex distributed m-by-n matrix sub(C) = $C(ic:ic+m-1, jc:jc+n-1)$ with

	side ='L'	*side* ='R'
trans = 'N':	Q*sub(C)	sub(C)*Q
trans = 'H':	Q^H*sub(C)	sub(C)*Q^H

where Q is a complex unitary distributed matrix of order nq, with $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$.

Q is defined as the product of $ihi-ilo$ elementary reflectors, as returned by p?gehrd.

$Q = H(ilo)\ H(ilo+1)\ldots\ H(ihi-1)$.

Input Parameters

side — (global) CHARACTER
='L': Q or Q^H is applied from the left.
='R': Q or Q^H is applied from the right.

trans — (global) CHARACTER
='N', no transpose, Q is applied.
='C', conjugate transpose, Q^H is applied.

m — (global) INTEGER. The number of rows in the distributed submatrix sub (C) ($m \geq 0$).

n — (global) INTEGER. The number of columns in the distributed submatrix sub (C) ($n \geq 0$).

ilo, *ihi* — (global) INTEGER
These must be the same parameters ilo and ihi, respectively, as supplied to p?gehrd. Q is equal to the unit matrix except in the distributed submatrix $Q(ia+ilo:ia+ihi-1, ia+ilo:ja+ihi-1)$.
If $side = 'L'$, then $1 \leq ilo \leq ihi \leq \max(1, m)$.
If $side = 'R'$, then $1 \leq ilo \leq ihi \leq \max(1, n)$
ilo and ihi are relative indexes.

a (local)
COMPLEX for pcunmhr
DOUBLE COMPLEX for pzunmhr.
Pointer into the local memory to an array of dimension (*lld_a*, *LOC c*(*ja*+*m*-1)) if *side*='L', and (*lld_a*, *LOCc*(*ja*+*n*-1)) if *side* = 'R'.
Contains the vectors which define the elementary reflectors, as returned by p?gehrd.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
COMPLEX for pcunmhr
DOUBLE COMPLEX for pzunmhr.
Array, DIMENSION *LOCc*(*ja*+*m*-2), if *side* = 'L', and *LOCc*(*ja*+*n*-2) if *side* = 'R'.
This array contains the scalar factors *tau*(*j*) of the elementary reflectors *H*(*j*) as returned by p?gehrd. *tau* is tied to the distributed matrix *A*.

c (local)
COMPLEX for pcunmhr
DOUBLE COMPLEX for pzunmhr.
Pointer into the local memory to an array of dimension (*lld_c*, *LOCc*(*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub(*C*).

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
COMPLEX for pcunmhr
DOUBLE COMPLEX for pzunmhr.
Workspace array with dimension *lwork*.

lwork (local or global)
The dimension of the array *work*.
lwork must be at least *iaa* = *ia* + *ilo*; *jaa* = *ja*+*ilo*-1;
If *side* = 'L', *mi* = *ihi*-*ilo*; *ni* = *n*; *icc* = *ic* + *ilo*; *jcc* = *jc*;
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0+*mpc*0)**nb_a*) + *nb_a***nb_a*
else if *side* = 'R',
mi = *m*; *ni* = *ihi*-*ilo*; *icc* = *ic*; *jcc* = *jc* + *ilo*; *lwork* ≥
max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + max(*npa*0+numroc(numroc(*ni*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
where *lcmq* = *lcm*/NPCOL with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*iaa*-1, *mb_a*),
icoffa = mod(*jaa*-1, *nb_a*),
iarow = indxg2p(*iaa*, *mb_a*, MYROW, *rsrc_a*, NPROW),
*npa*0 = numroc(*ni*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*icc*-1, *mb_c*),
icoffc = mod(*jcc*-1, *nb_c*),
icrow = indxg2p(*icc*, *mb_c*, MYROW, *rsrc_c*, NPROW),

iccol = indxg2p(*jcc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*mi*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*ni*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	*C* is overwritten by *Q** sub(*C*) or *Q*'*sub(*C*) or sub(*C*)**Q*' or sub(*C*)**Q*.
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?lahqr

Computes the Schur decomposition and/or eigenvalues of a matrix already in Hessenberg form.

Syntax

call pslahqr(*wantt*, *wantz*, *n*, *ilo*, *ihi*, *a*, *desca*, *wr*, *wi*, *iloz*, *ihiz*, *z*, *descz*, *work*, *lwork*, *iwork*, *ilwork*, *info*)

call pdlahqr(*wantt*, *wantz*, *n*, *ilo*, *ihi*, *a*, *desca*, *wr*, *wi*, *iloz*, *ihiz*, *z*, *descz*, *work*, *lwork*, *iwork*, *ilwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

This is an auxiliary routine used to find the Schur decomposition and/or eigenvalues of a matrix already in Hessenberg form from columns *ilo* to *ihi*.

Input Parameters

wantt	(global) LOGICAL If *wantt* = .TRUE., the full Schur form T is required; If *wantt* = .FALSE., only eigenvalues are required.
wantz	(global) LOGICAL. If *wantz* = .TRUE., the matrix of Schur vectors *z* is required; If *wantz* = .FALSE., Schur vectors are not required.
n	(global) INTEGER. The order of the Hessenberg matrix *A* (and *z* if *wantz*). (*n*≥0).
ilo, *ihi*	(global) INTEGER.

It is assumed that *A* is already upper quasi-triangular in rows and columns *ihi*+1:*n*, and that *A*(*ilo*, *ilo*-1) = 0 (unless *ilo* = 1). p?lahqr works primarily with the Hessenberg submatrix in rows and columns *ilo* to *ihi*, but applies transformations to all of *h* if *wantt* is .TRUE.. 1≤*ilo*≤max(1,*ihi*); *ihi* ≤ *n*.

a (global)
REAL for pslahqr
DOUBLE PRECISION for pdlahqr
Array, DIMENSION (*desca*(*lld_*),*). On entry, the upper Hessenberg matrix *A*.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

iloz, *ihiz* (global) INTEGER. Specify the rows of *z* to which transformations must be applied if *wantz* is .TRUE.. 1≤*iloz*≤*ilo*; *ihi*≤*ihiz*≤*n*.

z (global) REAL for pslahqr
DOUBLE PRECISION for pdlahqr
Array. If *wantz* is .TRUE., on entry *z* must contain the current matrix *Z* of transformations accumulated by pdhseqr. If *wantz* is .FALSE., *z* is not referenced.

descz (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix Z.

work (local)
REAL for pslahqr
DOUBLE PRECISION for pdlahqr
Workspace array with dimension *lwork*.

lwork (local) INTEGER. The dimension of *work*. *lwork* is assumed big enough so that *lwork*≥3**n* + max(2*max(*descz*(*lld_*),*desca*(*lld_*)) + 2**LOCq*(*n*), 7*ceil(*n*/*hbl*)/*lcm*(NPROW,NPCOL))).
If *lwork* = –1, then *work*(1) gets set to the above number and the code returns immediately.

iwork (global and local) INTEGER array of size *ilwork*.

ilwork (local) INTEGER This holds some of the *iblk* integer arrays.

Output Parameters

a On exit, if *wantt* is .TRUE., *A* is upper quasi-triangular in rows and columns *ilo*:*ihi*, with any 2-by-2 or larger diagonal blocks not yet in standard form. If *wantt* is .FALSE., the contents of *A* are unspecified on exit.

work(1) On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

wr, *wi* (global replicated output)
REAL for pslahqr
DOUBLE PRECISION for pdlahqr
Arrays, DIMENSION(*n*) each. The real and imaginary parts, respectively, of the computed eigenvalues *ilo* to *ihi* are stored in the corresponding elements of *wr* and *wi*. If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of *wr* and *wi*, say the *i*-th and (*i*+1)-th, with *wi*(*i*)> 0 and *wi*(*i*+1) < 0. If *wantt* is .TRUE. , the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *A*. *A* may be returned with larger diagonal blocks until the next release.

z	On exit *z* has been updated; transformations are applied only to the submatrix *z*(*iloz*:*ihiz*, *ilo*:*ihi*).
info	(global) INTEGER. = 0: the execution is successful. < 0: parameter number -*info* incorrect or inconsistent > 0: p?lahqr failed to compute all the eigenvalues *ilo* to *ihi* in a total of 30*(*ihi*-*ilo*+1) iterations; if *info* = *i*, elements *i*+1:*ihi* of *wr* and *wi* contain those eigenvalues which have been successfully computed.

Singular Value Decomposition

This section describes ScaLAPACK routines for computing the singular value decomposition (SVD) of a general *m*-by-*n* matrix *A* (see "Singular Value Decomposition" in LAPACK chapter).

To find the SVD of a general matrix *A*, this matrix is first reduced to a bidiagonal matrix *B* by a unitary (orthogonal) transformation, and then SVD of the bidiagonal matrix is computed. Note that the SVD of *B* is computed using the LAPACK routine ?bdsqr .

Table "Computational Routines for Singular Value Decomposition (SVD)" lists ScaLAPACK computational routines for performing this decomposition.

Computational Routines for Singular Value Decomposition (SVD)

Operation	General matrix	Orthogonal/unitary matrix
Reduce *A* to a bidiagonal matrix	p?gebrd	
Multiply matrix after reduction		p?ormbr/p?unmbr

p?gebrd

Reduces a general matrix to bidiagonal form.

Syntax

```
call psgebrd(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
call pdgebrd(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
call pcgebrd(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
call pzgebrd(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gebrd routine reduces a real/complex general *m*-by-*n* distributed matrix sub(*A*)= *A*(*ia*:*ia*+*m*-1,*ja*:*ja*+*n*-1) to upper or lower bidiagonal form *B* by an orthogonal/unitary transformation:

Q'*sub(*A*)**P* = *B*.

If *m*≥ *n*, *B* is upper bidiagonal; if *m* < *n*, *B* is lower bidiagonal.

Input Parameters

m	(global) INTEGER. The number of rows in the distributed matrix sub(*A*) (*m*≥0).
n	(global) INTEGER. The number of columns in the distributed matrix sub(*A*) (*n*≥0).
a	(local)

REAL for psgebrd
DOUBLE PRECISION for pdgebrd
COMPLEX for pcgebrd
DOUBLE COMPLEX for pzgebrd.
Real pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the distributed matrix sub (*A*).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local)
REAL for psgebrd
DOUBLE PRECISION for pdgebrd
COMPLEX for pcgebrd
DOUBLE COMPLEX for pzgebrd. Workspace array of dimension *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
lwork ≥ *nb**(*mpa*0 + nqa0+1)+ *nqa*0
where *nb* = *mb_a* = *nb_a*,
iroffa = mod(*ia*-1, *nb*),
icoffa = mod(*ja*-1, nb),
iarow = indxg2p(*ia*, *nb*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p (*ja*, nb, MYCOL, *csrc_a*, NPCOL),
*mpa*0 = numroc(*m* +*iroffa*, nb, MYROW, iarow, NPROW),
*nqa*0 = numroc(*n* +*icoffa*, nb, MYCOL, *iacol*, NPCOL),
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, if *m*≥*n*, the diagonal and the first superdiagonal of sub(*A*) are overwritten with the upper bidiagonal matrix *B*; the elements below the diagonal, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and the elements above the first superdiagonal, with the array *taup*, represent the orthogonal matrix *P* as a product of elementary reflectors. If *m* < *n*, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix *B*; the elements below the first subdiagonal, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and the elements above the diagonal, with the array *taup*, represent the orthogonal matrix *P* as a product of elementary reflectors. See *Application Notes* below.

d (local)
REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors. Array, DIMENSION *LOCc*(*ja*+min(*m*,*n*)-1) if *m*≥*n*; *LOCr*(*ia*+min(*m*,*n*)-1) otherwise. The distributed diagonal elements of the bidiagonal matrix *B*: *d*(*i*) = *a*(*i*,*i*). *d* is tied to the distributed matrix *A*.

e (local)

	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Array, DIMENSION *LOCr*(*ia*+min(*m*,*n*)-1) if *m*≥*n*; *LOCc*(*ja*+min(*m*,*n*)-2) otherwise. The distributed off-diagonal elements of the bidiagonal distributed matrix *B*: If *m*≥*n*, *e*(*i*) = *a*(*i*,*i*+1) for *i* = 1,2,..., *n*-1; if *m* < *n*, *e*(*i*) = *a*(*i*+1, *i*) for *i* = 1,2,...,*m*-1. *e* is tied to the distributed matrix *A*.
tauq, *taup*	(local) REAL for psgebrd DOUBLE PRECISION for pdgebrd COMPLEX for pcgebrd DOUBLE COMPLEX for pzgebrd. Arrays, DIMENSION *LOCc*(*ja*+min(*m*,*n*)-1) for *tauq* and *LOCr*(*ia*+min(*m*,*n*)-1) for *taup*. Contain the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrices *Q* and *P*, respectively. *tauq* and *taup* are tied to the distributed matrix *A*. See *Application Notes* below.
work(1)	On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.
info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrices *Q* and *P* are represented as products of elementary reflectors:

If *m* ≥ *n*,

Q = *H*(1)**H*(2)*...**H*(*n*), and *P* = *G*(1)**G*(2)*...**G*(*n*-1).

Each *H*(*i*) and *G*(*i*) has the form:

H(*i*)= *i* - *tauq* * *v* * *v*' and *G*(*i*) = *i* - *taup***u***u*'

where *tauq* and *taup* are real/complex scalars, and *v* and *u* are real/complex vectors;

v(1:*i*-1) = 0, *v*(*i*) = 1, and *v*(*i*+1:*m*) is stored on exit in *A*(*ia*+*i*:*ia*+*m*-1,*ja*+*i*-1);

u(1:*i*) = 0, *u*(*i*+1) = 1, and *u*(*i*+2:*n*) is stored on exit in *A* (*ia*+*i*-1,*ja*+*i*+1:*ja*+*n*-1);

tauq is stored in *tauq*(*ja*+*i*-1) and *taup* in *taup*(*ia*+*i*-1).

If *m* < *n*,

Q = *H*(1)**H*(2)*...**H*(*m*-1), and *P* = *G*(1)* *G*(2)*...* *G*(*m*)

Each *H* (*i*) and *G*(*i*) has the form:

H(*i*)= *i*-*tauq***v***v*' and *G*(*i*)= *i*-*taup***u***u*'

here *tauq* and *taup* are real/complex scalars, and *v* and *u* are real/complex vectors;

v(1:*i*) = 0, *v*(*i*+1) = 1, and *v*(*i*+2:*m*) is stored on exit in *A* (*ia*+*i*:*ia*+*m*-1,*ja*+*i*-1); *u*(1:*i*-1) = 0, *u*(*i*) = 1, and *u*(*i*+1:*n*) is stored on exit in *A*(*ia*+*i*-1,*ja*+*i*+1:*ja*+*n*-1);

tauq is stored in *tauq*(*ja*+*i*-1) and *taup* in *taup*(*ia*+*i*-1).

The contents of sub(*A*) on exit are illustrated by the following examples:

m = 6 and *n* = 5 (*m* > *n*):

$$\begin{bmatrix} d & e & u1 & u1 & u1 \\ v1 & d & e & u2 & u2 \\ v1 & v2 & d & e & u3 \\ v1 & v2 & v3 & d & e \\ v1 & v2 & v3 & v4 & d \\ v1 & v2 & v3 & v4 & v5 \end{bmatrix}$$

$m = 5$ and $n = 6$ ($m < n$):

$$\begin{bmatrix} d & u1 & u1 & u1 & u1 & u1 \\ e & d & u2 & u2 & u2 & u2 \\ v1 & e & d & u3 & u3 & u3 \\ v1 & v2 & e & d & u4 & u4 \\ v1 & v2 & v3 & e & d & u5 \end{bmatrix}$$

where d and e denote diagonal and off-diagonal elements of B, vi denotes an element of the vector defining $H(i)$, and ui an element of the vector defining $G(i)$.

p?ormbr

Multiplies a general matrix by one of the orthogonal matrices from a reduction to bidiagonal form determined by `p?gebrd`.

Syntax

```
call psormbr(vect, side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

```
call pdormbr(vect, side, trans, m, n, k, a, ia, ja, desca, tau, c, ic, jc, descc, work, lwork, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

If *vect* = '*Q*', the `p?ormbr` routine overwrites the general real distributed m-by-n matrix sub(C) = $C(c:ic+m-1, jc:jc+n-1)$ with

	side = 'L'	*side* = 'R'
trans = 'N':	Q sub(C)	sub(C) Q
trans = 'T':	Q^T sub(C)	sub(C) Q^T

If *vect* = 'P', the routine overwrites sub(C) with

	side = 'L'	*side* = 'R'
trans = 'N':	P sub(C)	sub(C) P
trans = 'T':	P^T sub(C)	sub(C) P^T

Here Q and P^T are the orthogonal distributed matrices determined by p?gebrd when reducing a real distributed matrix $A(ia:*, ja:*)$ to bidiagonal form: $A(ia:*,ja:*) = Q*B*P^T$. Q and P^T are defined as products of elementary reflectors $H(i)$ and $G(i)$ respectively.

Let $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$. Thus nq is the order of the orthogonal matrix Q or P^T that is applied.

If $vect = 'Q'$, $A(ia:*, ja:*)$ is assumed to have been an nq-by-k matrix:

If $nq \geq k$, $Q = H(1)\ H(2)...H(k)$;

If $nq < k$, $Q = H(1)\ H(2)...H(nq-1)$.

If $vect = 'P'$, $A(ia:*, ja:*)$ is assumed to have been a k-by-nq matrix:

If $k < nq$, $P = G(1)\ G(2)...G(k)$;

If $k \geq nq$, $P = G(1)\ G(2)...G(nq-1)$.

Input Parameters

vect	(global) CHARACTER. If *vect* ='Q', then Q or Q^T is applied. If *vect* ='P', then P or P^T is applied.
side	(global) CHARACTER. If *side* ='L', then Q or Q^T, P or P^T is applied from the left. If *side* ='R', then Q or Q^T, P or P^T is applied from the right.
trans	(global) CHARACTER. If *trans* = 'N', no transpose, Q or P is applied. If *trans* = 'T', then Q^T or P^T is applied.
m	(global) INTEGER. The number of rows in the distributed matrix sub (C).
n	(global) INTEGER. The number of columns in the distributed matrix sub (C).
k	(global) INTEGER. If *vect* = 'Q', the number of columns in the original distributed matrix reduced by p?gebrd; If *vect* = 'P', the number of rows in the original distributed matrix reduced by p?gebrd. Constraints: $k \geq 0$.
a	(local) REAL for psormbr DOUBLE PRECISION for pdormbr. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+min(*nq*, *k*)-1)) If *vect*='Q', and (*lld_a*, *LOCc*(*ja*+*nq*-1)) If *vect* = 'P'. *nq* = *m* if *side* = 'L', and *nq* = *n* otherwise. The vectors which define the elementary reflectors $H(i)$ and $G(i)$, whose products determine the matrices Q and P, as returned by p?gebrd. If *vect* = 'Q', *lld_a*≥max(1, *LOCr*(*ia*+*nq*-1)); If *vect* = 'P', *lld_a*≥max(1, *LOCr*(*ia*+min(*nq*, *k*)-1)).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix A, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix A.
tau	(local)

REAL for psormbr
DOUBLE PRECISION for pdormbr.
Array, DIMENSION *LOCc*(*ja*+min(*nq*, *k*)-1), if *vect* = 'Q', and *LOCr*(*ia*+min(*nq*, *k*)-1), if *vect* = 'P'.
tau(*i*) must contain the scalar factor of the elementary reflector *H*(*i*) or *G*(*i*), which determines *Q* or *P*, as returned by pdgebrd in its array argument *tauq* or *taup*. *tau* is tied to the distributed matrix *A*.

c (local) REAL for psormbr
DOUBLE PRECISION for pdormbr
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc* (*jc*+*n*-1)).
Contains the local pieces of the distributed matrix sub (*C*).

ic, *jc* (global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.

descc (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psormbr
DOUBLE PRECISION for pdormbr.
Workspace array of dimension *lwork*.

lwork (local or global) INTEGER, dimension of *work*, must be at least:
If *side* = 'L'
nq = *m*;
if ((*vect* = 'Q' and *nq*≥*k*) or (*vect* is not equal to 'Q' and *nq*>*k*)),
iaa=*ia*; *jaa*=*ja*; *mi*=*m*; *ni*=*n*; *icc*=*ic*; *jcc*=*jc*;
else
iaa= *ia*+1; *jaa*=*ja*; *mi*=*m*-1; *ni*=*n*; *icc*=*ic*+1; *jcc*= *jc*;
end if
else
If *side* = 'R', *nq* = *n*;
if((*vect* = 'Q' and *nq*≥*k*) or (*vect* is not equal to 'Q' and *nq*>*k*)),
iaa=*ia*; *jaa*=*ja*; *mi*=*m*; *ni*=*n*; *icc*=*ic*; *jcc*=*jc*;
else
iaa= *ia*; *jaa*= *ja*+1; *mi*= *m*; *ni*= *n*-1; *icc*= *ic*; *jcc*= *jc*+1;
end if
end if
If *vect* = 'Q',
If *side* = 'L', *lwork*≥max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + *mpc*0)**nb_a*) + *nb_a* * *nb_a*
else if *side* = 'R',
lwork≥max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + max(*npa*0 + numroc(numroc(*ni*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
else if *vect* is not equal to 'Q', if *side* = 'L',
lwork≥max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + max(*mqa*0 + numroc(numroc(*mi*+iroffc, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a*
else if *side* = 'R',
lwork≥max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + *nqc*0)**mb_a*) + *mb_a***mb_a*

end if
end if
where *lcmp* = *lcm*/NPROW, *lcmq* = *lcm*/NPCOL, with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*iaa*-1, *mb_a*),
icoffa = mod(*jaa*-1, *nb_a*),
iarow = indxg2p(*iaa*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*jaa*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = numroc(*mi*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
*npa*0 = numroc(*ni*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*icc*-1, *mb_c*),
icoffc = mod(*jcc*-1, *nb_c*),
icrow = indxg2p(*icc*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jcc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*mi*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*ni*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c — On exit, if *vect*='Q', sub(*C*) is overwritten by *Q**sub(*C*), or *Q'**sub(*C*), or sub(*C*)**Q'*, or sub(*C*)**Q*; if *vect*='P', sub(*C*) is overwritten by *P**sub(*C*), or *P'**sub(*C*), or sub(*C*)**P*, or sub(*C*)**P'*.

work(1) — On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info — (global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?unmbr

Multiplies a general matrix by one of the unitary transformation matrices from a reduction to bidiagonal form determined by p?gebrd.

Syntax

call pcunmbr(*vect*, *side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmbr(*vect*, *side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

If *vect* = 'Q', the p?unmbr routine overwrites the general complex distributed *m*-by-*n* matrix sub(*C*) = *C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) with

	side ='L'	*side* ='R'
trans = 'N':	*Q**sub(*C*)	sub(*C*)**Q*
trans = 'C':	Q^H*sub(*C*)	sub(*C*)*Q^H

If *vect* = 'P', the routine overwrites sub(*C*) with

	side ='L'	*side* ='R'
trans = 'N':	*P**sub(*C*)	sub(*C*)**P*
trans = 'C':	P^H*sub(*C*)	sub(*C*)*P^H

Here *Q* and P^H are the unitary distributed matrices determined by p?gebrd when reducing a complex distributed matrix *A*(*ia*:*, *ja*:*) to bidiagonal form: *A*(*ia*:*,*ja*:*) = *Q***B***P*^{H}.

Q and P^H are defined as products of elementary reflectors *H*(*i*) and *G*(*i*) respectively.

Let *nq* = *m* if *side* = 'L' and *nq* = *n* if *side* = 'R'. Thus *nq* is the order of the unitary matrix *Q* or P^H that is applied.

If *vect* = 'Q', *A*(*ia*:*, *ja*:*) is assumed to have been an *nq*-by-*k* matrix:

If *nq* ≥ *k*, *Q* = *H*(1) *H*(2)... *H*(*k*);

If *nq* < *k*, *Q* = *H*(1) *H*(2)... *H*(*nq*-1).

If *vect* = 'P', *A*(*ia*:*, *ja*:*) is assumed to have been a *k*-by-*nq* matrix:

If *k* < *nq*, *P* = *G*(1) *G*(2)... *G*(*k*);

If *k* ≥ *nq*, *P* = *G*(1) *G*(2)... *G*(*nq*-1).

Input Parameters

vect — (global) CHARACTER.
If *vect* ='Q', then *Q* or Q^H is applied.
If *vect* ='P', then *P* or P^H is applied.

side — (global) CHARACTER.
If *side* ='L', then *Q* or Q^H, *P* or P^H is applied from the left.
If *side* ='R', then *Q* or Q^H, *P* or P^H is applied from the right.

trans — (global) CHARACTER.
If *trans* = 'N', no transpose, *Q* or *P* is applied.
If *trans* = 'C', conjugate transpose, Q^H or P^H is applied.

m — (global) INTEGER. The number of rows in the distributed matrix sub (*C*) *m*≥0.

n — (global) INTEGER. The number of columns in the distributed matrix sub (*C*) *n*≥0.

k — (global) INTEGER.
If *vect* = 'Q', the number of columns in the original distributed matrix reduced by p?gebrd;
If *vect* = 'P', the number of rows in the original distributed matrix reduced by p?gebrd.
Constraints: *k* ≥ 0.

a — (local)

	COMPLEX for psormbr DOUBLE COMPLEX for pdormbr. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+min(*nq*,*k*)-1)) if *vect*='Q', and (*lld_a*, *LOCc*(*ja*+*nq*-1)) if *vect* = 'P'. *nq* = *m* if *side* = 'L', and *nq* = *n* otherwise. The vectors which define the elementary reflectors *H*(*i*) and *G*(*i*), whose products determine the matrices *Q* and *P*, as returned by p?gebrd. If *vect* = 'Q', *lld_a* ≥ max(1, *LOCr*(*ia*+*nq*-1)); If *vect* = 'P', *lld_a* ≥ max(1, *LOCr*(*ia*+min(*nq*, *k*)-1)).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) COMPLEX for pcunmbr DOUBLE COMPLEX for pzunmbr. Array, DIMENSION *LOCc*(*ja*+min(*nq*, *k*)-1), if *vect* = 'Q', and *LOCr*(*ia*+min(*nq*, *k*)-1), if *vect* = 'P'. *tau*(*i*) must contain the scalar factor of the elementary reflector *H*(*i*) or *G*(*i*), which determines *Q* or *P*, as returned by p?gebrd in its array argument *tauq* or *taup*. *tau* is tied to the distributed matrix *A*.
c	(local) COMPLEX for pcunmbr DOUBLE COMPLEX for pzunmbr Pointer into the local memory to an array of dimension (*lld_a*, *LOCc* (*jc*+*n*-1)). Contains the local pieces of the distributed matrix sub (*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix *C*, respectively.
descc	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *C*.
work	(local) COMPLEX for pcunmbr DOUBLE COMPLEX for pzunmbr. Workspace array of dimension *lwork*.
lwork	(local or global) INTEGER, dimension of *work*, must be at least: If *side* = 'L' *nq* = *m*; if ((*vect* = 'Q' and *nq* ≥ *k*) or (*vect* is not equal to 'Q' and *nq* > *k*)), *iaa*= *ia*; *jaa*= *ja*; *mi*= *m*; *ni*= *n*; *icc*= *ic*; *jcc*= *jc*; else *iaa*= *ia*+1; *jaa*= *ja*; *mi*= *m*-1; *ni*= *n*; *icc*= *ic*+1; *jcc*= *jc*; end if else If *side* = 'R', *nq* = *n*; if ((*vect* = 'Q' and *nq* ≥ *k*) or (*vect* is not equal to 'Q' and *nq* ≥ *k*)), *iaa*= *ia*; *jaa*= *ja*; *mi*= *m*; *ni*= *n*; *icc*= *ic*; *jcc*= *jc*; else *iaa*= *ia*; *jaa*= *ja*+1; *mi*= *m*; *ni*= *n*-1; *icc*= *ic*; *jcc*= *jc*+1; end if

end if
If *vect* = 'Q',
If *side* = 'L', *lwork* ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0+*mpc*0)**nb_a*) + *nb_a***nb_a*
else if *side* = 'R',
lwork ≥ max((*nb_a**(*nb_a*-1))/2, (*nqc*0 + max(*npa*0+numroc(numroc(*ni*+*icoffc*, *nb_a*, 0, 0, NPCOL), *nb_a*, 0, 0, *lcmq*), *mpc*0))**nb_a*) + *nb_a***nb_a*
end if
else if *vect* is not equal to 'Q',
if *side* = 'L',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + max(*mqa*0+numroc(numroc(*mi*+iroffc, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0, *lcmp*), *nqc*0))**mb_a*) + *mb_a***mb_a*
else if *side* = 'R',
lwork ≥ max((*mb_a**(*mb_a*-1))/2, (*mpc*0 + *nqc*0)**mb_a*) + *mb_a***mb_a*
end if
end if
where *lcmp* = *lcm*/NPROW, *lcmq* = *lcm*/NPCOL, with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*iaa*-1, *mb_a*),
icoffa = mod(*jaa*-1, *nb_a*),
iarow = indxg2p(*iaa*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol = indxg2p(*jaa*, *nb_a*, MYCOL, *csrc_a*, NPCOL),
*mqa*0 = numroc(*mi*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
*npa*0 = numroc(*ni*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
iroffc = mod(*icc*-1, *mb_c*),
icoffc = mod(*jcc*-1, *nb_c*),
icrow = indxg2p(*icc*, *mb_c*, MYROW, *rsrc_c*, NPROW),
iccol = indxg2p(*jcc*, *nb_c*, MYCOL, *csrc_c*, NPCOL),
*mpc*0 = numroc(*mi*+*iroffc*, *mb_c*, MYROW, *icrow*, NPROW),
*nqc*0 = numroc(*ni*+*icoffc*, *nb_c*, MYCOL, *iccol*, NPCOL),
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c
On exit, if *vect*='Q', sub(*C*) is overwritten by *Q**sub(*C*), or *Q*'*sub(*C*), or sub(*C*)**Q*', or sub(*C*)**Q*; if *vect*='P', sub(*C*) is overwritten by *P**sub(*C*), or *P*'*sub(*C*), or sub(*C*)**P*, or sub(*C*)**P*'.

work(1)
On exit *work*(1) contains the minimum value of *lwork* required for optimum performance.

info
(global) INTEGER.
= 0: the execution is successful.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Generalized Symmetric-Definite Eigen Problems

This section describes ScaLAPACK routines that allow you to reduce the *generalized symmetric-definite eigenvalue problems* (see Generalized Symmetric-Definite Eigenvalue Problems in LAPACK chapters) to standard symmetric eigenvalue problem $Cy = \lambda y$, which you can solve by calling ScaLAPACK routines described earlier in this chapter (see Symmetric Eigenproblems).

Table "Computational Routines for Reducing Generalized Eigenproblems to Standard Problems" lists these routines.

Computational Routines for Reducing Generalized Eigenproblems to Standard Problems

Operation	Real symmetric matrices	Complex Hermitian matrices
Reduce to standard problems	p?sygst	p?hegst

p?sygst

Reduces a real symmetric-definite generalized eigenvalue problem to the standard form.

Syntax

```
call pssygst(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, scale, info)
call pdsygst(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, scale, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?sygst routine reduces real symmetric-definite generalized eigenproblems to the standard form.

In the following sub(*A*) denotes *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) and sub(*B*) denotes *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*n*-1).

If *ibtype* = 1, the problem is

sub(*A*)**x* = λ*sub(*B*)**x*,

and sub(*A*) is overwritten by inv(U^T)*sub(*A*)*inv(*U*), or inv(*L*)*sub(*A*)*inv(L^T).

If *ibtype* = 2 or 3, the problem is

sub(*A*)*sub(*B*)**x* = λ**x*, or sub(*B*)*sub(*A*)**x* = λ**x*,

and sub(*A*) is overwritten by *U**sub(*A*)*U^T, or L^T*sub(*A*)**L*.

sub(*B*) must have been previously factorized as U^T**U* or *L**L^T by p?potrf.

Input Parameters

ibtype — (global) INTEGER. Must be 1 or 2 or 3.
If *itype* = 1, compute inv(U^T)*sub(*A*)*inv(*U*), or inv(*L*)*sub(*A*)*inv(L^T);
If *itype* = 2 or 3, compute *U**sub(*A*)*U^T, or L^T*sub(*A*)**L*.

uplo — (global) CHARACTER. Must be 'U' or 'L'.
If *uplo* = 'U', the upper triangle of sub(*A*) is stored and sub (*B*) is factored as U^T**U*.
If *uplo* = 'L', the lower triangle of sub(*A*) is stored and sub (*B*) is factored as *L**L^T.

n	(global) INTEGER. The order of the matrices sub (*A*) and sub (*B*) (*n* ≥ 0).
a	(local) REAL for pssygst DOUBLE PRECISION for pdsygst. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the array contains the local pieces of the *n*-by-*n* symmetric distributed matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
b	(local) REAL for pssygst DOUBLE PRECISION for pdsygst. Pointer into the local memory to an array of dimension (*lld_b*, *LOCc*(*jb*+*n*-1)). On entry, the array contains the local pieces of the triangular factor from the Cholesky factorization of sub (*B*) as returned by p?potrf.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix B.

Output Parameters

a	On exit, if *info* = 0, the transformed matrix, stored in the same format as sub(*A*).
scale	(global) REAL for pssygst DOUBLE PRECISION for pdsygst. Amount by which the eigenvalues should be scaled to compensate for the scaling performed in this routine. At present, *scale* is always returned as 1.0, it is returned here to allow for future enhancement.
info	(global) INTEGER. If *info* = 0, the execution is successful. If *info* < 0, if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?hegst

Reduces a Hermitian-definite generalized eigenvalue problem to the standard form.

Syntax

```
call pchegst(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, scale, info)
call pzhegst(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, scale, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?hegst routine reduces complex Hermitian-definite generalized eigenproblems to the standard form.

In the following sub(*A*) denotes *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) and sub(*B*) denotes *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*n*-1).

If *ibtype* = 1, the problem is

sub(*A*)**x* = λ*sub(*B*)**x*,

and sub(*A*) is overwritten by inv(U^H)*sub(*A*)*inv(*U*), or inv(*L*)*sub(*A*)*inv(L^H).

If *ibtype* = 2 or 3, the problem is

sub(*A*)*sub(*B*)**x* = λ**x*, or sub(*B*)*sub(*A*)**x* = λ**x*,

and sub(*A*) is overwritten by *U**sub(*A*)*U^H, or L^H*sub(*A*)**L*.

sub(*B*) must have been previously factorized as U^H**U* or *L**L^H by p?potrf.

Input Parameters

ibtype	(global) INTEGER. Must be 1 or 2 or 3. If *itype* = 1, compute inv(U^H)*sub(*A*)*inv(*U*), or inv(*L*)*sub(*A*)*inv(L^H); If *itype* = 2 or 3, compute *U**sub(*A*)*U^H, or L^H*sub(*A*)**L*.
uplo	(global) CHARACTER. Must be 'U' or 'L'. If *uplo* = 'U', the upper triangle of sub(*A*) is stored and sub (*B*) is factored as U^H**U*. If *uplo* = 'L', the lower triangle of sub(*A*) is stored and sub (*B*) is factored as *L**L^H.
n	(global) INTEGER. The order of the matrices sub (*A*) and sub (*B*) (*n*≥0).
a	(local) COMPLEX for pchegst DOUBLE COMPLEX for pzhegst. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the array contains the local pieces of the *n*-by-*n* Hermitian distributed matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
b	(local) COMPLEX for pchegst DOUBLE COMPLEX for pzhegst. Pointer into the local memory to an array of dimension (*lld_b*, *LOCc*(*jb*+*n*-1)). On entry, the array contains the local pieces of the triangular factor from the Cholesky factorization of sub (*B*) as returned by p?potrf.

ib, *jb* (global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.

descb (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix B.

Output Parameters

a On exit, if *info* = 0, the transformed matrix, stored in the same format as sub(*A*).

scale (global)
REAL for pchegst
DOUBLE PRECISION for pzhegst.
Amount by which the eigenvalues should be scaled to compensate for the scaling performed in this routine. At present, *scale* is always returned as 1.0, it is returned here to allow for future enhancement.

info (global) INTEGER.
If *info* = 0, the execution is successful. If *info* <0, if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i*100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Driver Routines

Table "ScaLAPACK Driver Routines" lists ScaLAPACK driver routines available for solving systems of linear equations, linear least-squares problems, standard eigenvalue and singular value problems, and generalized symmetric definite eigenproblems.

ScaLAPACK Driver Routines

Type of Problem	Matrix type, storage scheme	Driver
Linear equations	general (partial pivoting)	p?gesv (simple driver)p?gesvx (expert driver)
	general band (partial pivoting)	p?gbsv (simple driver)
	general band (no pivoting)	p?dbsv (simple driver)
	general tridiagonal (no pivoting)	p?dtsv (simple driver)
	symmetric/Hermitian positive-definite	p?posv (simple driver)p?posvx (expert driver)
	symmetric/Hermitian positive-definite, band	p?pbsv (simple driver)
	symmetric/Hermitian positive-definite, tridiagonal	p?ptsv (simple driver)
Linear least squares problem	general *m*-by-*n*	p?gels
Symmetric eigenvalue problem	symmetric/Hermitian	p?syev / p?heev (simple driver); p?syevd / p?heevd (simple driver with a divide and conquer algorithm); p?syevx / p?heevx (expert driver)
Singular value decomposition	general *m*-by-*n*	p?gesvd
Generalized symmetric definite eigenvalue problem	symmetric/Hermitian, one matrix also positive-definite	p?sygvx / p?hegvx (expert driver)

p?gesv

Computes the solution to the system of linear equations with a square distributed matrix and multiple right-hand sides.

Syntax

```
call psgesv(n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
call pdgesv(n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
call pcgesv(n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
call pzgesv(n, nrhs, a, ia, ja, desca, ipiv, b, ib, jb, descb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gesv routine computes the solution to a real or complex system of linear equations sub(*A*)***X* = sub(*B*), where sub(*A*) = *A*(*ia:ia+n-1*, *ja:ja+n-1*) is an *n*-by-*n* distributed matrix and *X* and sub(*B*) = *B*(*ib:ib+n-1*, *jb:jb+nrhs-1*) are *n*-by-*nrhs* distributed matrices.

The *LU* decomposition with partial pivoting and row interchanges is used to factor sub(*A*) as sub(*A*) = *P*L*U*, where *P* is a permutation matrix, *L* is unit lower triangular, and *U* is upper triangular. *L* and *U* are stored in sub(*A*). The factored form of sub(*A*) is then used to solve the system of equations sub(*A*)***X* = sub(*B*).

Input Parameters

n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*) (*n* ≥ 0).
nrhs	(global) INTEGER. The number of right hand sides, that is, the number of columns of the distributed submatrices *B* and *X* (*nrhs* ≥ 0).
a, *b*	(local) REAL for psgesv DOUBLE PRECISION for pdgesv COMPLEX for pcgesv DOUBLE COMPLEX for pzgesv. Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)) and *b*(*lld_b*,*LOCc*(*jb*+*nrhs*-1)), respectively. On entry, the array *a* contains the local pieces of the *n*-by-*n* distributed matrix sub(*A*) to be factored. On entry, the array *b* contains the right hand side distributed matrix sub(*B*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of sub(*A*), respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of sub(*B*), respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

Output Parameters

a	Overwritten by the factors *L* and *U* from the factorization sub(*A*) = *P*L*U*; the unit diagonal elements of *L* are not stored .
b	Overwritten by the solution distributed matrix *X*.
ipiv	(local) INTEGER array.

The dimension of *ipiv* is (*LOCr(m_a)+mb_a*). This array contains the pivoting information. The (local) row *i* of the matrix was interchanged with the (global) row *ipiv*(*i*).
This array is tied to the distributed matrix *A*.

info (global) INTEGER. If *info*=0, the execution is successful.
info < 0:
If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = *k*, *U*(*ia+k-1*,*ja+k-1*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution could not be computed.

p?gesvx

Uses the LU factorization to compute the solution to the system of linear equations with a square matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

```
call psgesvx(fact, trans, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, ipiv,
equed, r, c, b, ib, jb, descb, x, ix, jx, descx, rcond, ferr, berr, work, lwork,
iwork, liwork, info)
```

```
call pdgesvx(fact, trans, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, ipiv,
equed, r, c, b, ib, jb, descb, x, ix, jx, descx, rcond, ferr, berr, work, lwork,
iwork, liwork, info)
```

```
call pcgesvx(fact, trans, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, ipiv,
equed, r, c, b, ib, jb, descb, x, ix, jx, descx, rcond, ferr, berr, work, lwork,
rwork, lrwork, info)
```

```
call pzgesvx(fact, trans, n, nrhs, a, ia, ja, desca, af, iaf, jaf, descaf, ipiv,
equed, r, c, b, ib, jb, descb, x, ix, jx, descx, rcond, ferr, berr, work, lwork,
rwork, lrwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gesvx routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations *AX = B*, where *A* denotes the *n*-by-*n* submatrix *A(ia:ia+n-1, ja:ja+n-1)*, *B* denotes the *n*-by-*nrhs* submatrix *B(ib:ib+n-1, jb:jb+nrhs-1)* and *X* denotes the *n*-by-*nrhs* submatrix *X(ix:ix+n-1, jx:jx+nrhs-1)*.

Error bounds on the solution and a condition estimate are also provided.

In the following description, *af* stands for the subarray *af(iaf:iaf+n-1, jaf:jaf+n-1)*.

The routine p?gesvx performs the following steps:

1. If *fact* = 'E', real scaling factors *R* and *C* are computed to equilibrate the system:

trans = 'N': diag(*R*)**A**diag(*C*) *diag(*C*)-1**X* = diag(*R*)*B

trans = 'T': (diag(*R*)***A**diag(*C*))*T* *diag(*R*)-1**X* = diag(*C*)*B

trans = 'C': (diag(*R*)***A**diag(*C*))*H* *diag(*R*)-1**X* = diag(*C*)*B

Whether or not the system will be equilibrated depends on the scaling of the matrix *A*, but if equilibration is used, *A* is overwritten by diag(*R*)**A**diag(*C*) and *B* by diag(*R*)**B* (if *trans*='N') or diag(*c*)**B* (if *trans* = 'T' or 'C').

2. If *fact* = 'N' or 'E', the *LU* decomposition is used to factor the matrix *A* (after equilibration if *fact* = 'E') as *A* = *P L U*, where *P* is a permutation matrix, *L* is a unit lower triangular matrix, and *U* is upper triangular.
3. The factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than relative machine precision, steps 4 - 6 are skipped.
4. The system of equations is solved for *X* using the factored form of *A*.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix *X* is premultiplied by diag(*C*) (if *trans* = 'N') or diag(*R*) (if *trans* = 'T' or 'C') so that it solves the original system before equilibration.

Input Parameters

fact
(global) CHARACTER*1. Must be 'F', 'N', or 'E'.
Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored.
If *fact* = 'F' then, on entry, *af* and *ipiv* contain the factored form of *A*. If *equed* is not 'N', the matrix *A* has been equilibrated with scaling factors given by *r* and *c*. Arrays *a*, *af*, and *ipiv* are not modified.
If *fact* = 'N', the matrix *A* is copied to *af* and factored.
If *fact* = 'E', the matrix *A* is equilibrated if necessary, then copied to *af* and factored.

trans
(global) CHARACTER*1. Must be 'N', 'T', or 'C'.
Specifies the form of the system of equations:
If *trans* = 'N', the system has the form *A*X* = *B* (No transpose);
If *trans* = 'T', the system has the form A^T**X* = *B* (Transpose);
If *trans* = 'C', the system has the form A^H**X* = *B* (Conjugate transpose);

n
(global) INTEGER. The number of linear equations; the order of the submatrix *A* (*n* ≥ 0).

nrhs
(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrices *B* and *X* (*nrhs* ≥ 0).

a, *af*, *b*, *work*
(local)
REAL for psgesvx
DOUBLE PRECISION for pdgesvx
COMPLEX for pcgesvx
DOUBLE COMPLEX for pzgesvx.
Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)), *af*(*lld_af*,*LOCc*(*ja*+*n*-1)), *b*(*lld_b*,*LOCc*(*jb*+*nrhs*-1)), *work*(*lwork*), respectively.
The array *a* contains the matrix *A*. If *fact* = 'F' and *equed* is not 'N', then *A* must have been equilibrated by the scaling factors in *r* and/or *c*.
The array *af* is an input argument if *fact* = 'F'. In this case it contains on entry the factored form of the matrix *A*, that is, the factors *L* and *U* from the factorization *A* = *P*L*U* as computed by p?getrf. If *equed* is not 'N', then *af* is the factored form of the equilibrated matrix *A*.

The array *b* contains on entry the matrix *B* whose columns are the right-hand sides for the systems of equations.
work(*) is a workspace array. The dimension of *work* is (*lwork*).

ia, *ja* — (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix *A(ia:ia+n-1, ja:ja+n-1)*, respectively.

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

iaf, *jaf* — (global) INTEGER. The row and column indices in the global array *af* indicating the first row and the first column of the subarray *af(iaf:iaf +n-1, jaf:jaf+n-1)*, respectively.

descaf — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *AF*.

ib, *jb* — (global) INTEGER. The row and column indices in the global array B indicating the first row and the first column of the submatrix *B(ib:ib+n-1, jb:jb+nrhs-1)*, respectively.

descb — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

ipiv — (local) INTEGER array.
The dimension of *ipiv* is (*LOCr*(m_a)+*mb_a*).
The array *ipiv* is an input argument if *fact* = 'F' .
On entry, it contains the pivot indices from the factorization *A = P*L*U* as computed by p?getrf; (local) row i of the matrix was interchanged with the (global) row *ipiv*(i).
This array must be aligned with *A(ia:ia+n-1, *)*.

equed — (global) CHARACTER*1. Must be 'N', 'R', 'C', or 'B'. *equed* is an input argument if *fact* = 'F' . It specifies the form of equilibration that was done:
If *equed* = 'N', no equilibration was done (always true if *fact* = 'N');
If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by diag(*r*);
If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by diag(*c*);
If *equed* = 'B', both row and column equilibration was done; *A* has been replaced by diag(*r*)**A**diag(*c*).

r, *c* — (local) REAL for single precision flavors;
DOUBLE PRECISION for double precision flavors.
Arrays, dimension *LOCr*(m_a) and *LOCc*(n_a), respectively.
The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments. If *equed* = 'R' or 'B', *A* is multiplied on the left by diag(*r*); if *equed* = 'N' or 'C', *r* is not accessed.
If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.
If *equed* = 'C' or 'B', *A* is multiplied on the right by diag(*c*); if *equed* = 'N' or 'R', *c* is not accessed.

	If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive. Array *r* is replicated in every process column, and is aligned with the distributed matrix *A*. Array *c* is replicated in every process row, and is aligned with the distributed matrix *A*.
ix, *jx*	(global) INTEGER. The row and column indices in the global array *X* indicating the first row and the first column of the submatrix *X(ix:ix+n-1, jx:jx+nrhs-1)*, respectively.
descx	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *X*.
lwork	(local or global) INTEGER. The dimension of the array *work* ; must be at least max(p?gecon(*lwork*), p?gerfs(*lwork*))+*LOCr*(n_a) .
iwork	(local, psgesvx/pdgesvx only) INTEGER. Workspace array. The dimension of *iwork* is (*liwork*).
liwork	(local, psgesvx/pdgesvx only) INTEGER. The dimension of the array *iwork* , must be at least *LOCr*(n_a) .
rwork	(local) REAL for pcgesvx DOUBLE PRECISION for pzgesvx. Workspace array, used in complex flavors only. The dimension of *rwork* is (*lrwork*).
lrwork	(local or global, pcgesvx/pzgesvx only) INTEGER. The dimension of the array *rwork*;must be at least 2**LOCc*(n_a) .

Output Parameters

x	(local) REAL for psgesvx DOUBLE PRECISION for pdgesvx COMPLEX for pcgesvx DOUBLE COMPLEX for pzgesvx. Pointer into the local memory to an array of local dimension *x*(lld_x,*LOCc*(*jx+nrhs-1*)). If *info* = 0, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is: diag(*C*)-1**X*, if *trans* = 'N' and *equed* = 'C' or 'B'; and diag(*R*)-1**X*, if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'.
a	Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'. If *equed* ≠ 'N', *A* is scaled on exit as follows: *equed* = 'R': *A* = diag(*R*)**A* *equed* = 'C': *A* = *A**diag(*c*) *equed* = 'B': *A* = diag(*R*)**A**diag(*c*)
af	If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the factors *L* and *U* from the factorization *A* = *P*L*U* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.
b	Overwritten by diag(*R*)**B* if *trans* = 'N' and *equed* = 'R' or 'B'; overwritten by diag(*c*)**B* if *trans* = 'T' and *equed* = 'C' or 'B'; not changed if *equed* = 'N'.
r, *c*	These arrays are output arguments if *fact* ≠ 'F'.

	See the description of *r*, *c* in *Input Arguments* section.
rcond	(global) REAL for single precision flavors. DOUBLE PRECISION for double precision flavors. An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). The routine sets *rcond* =0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.
ferr, *berr*	(local) REAL for single precision flavors DOUBLE PRECISION for double precision flavors. Arrays, DIMENSION *LOCc*(n_b) each. Contain the component-wise forward and relative backward errors, respectively, for each solution vector. Arrays *ferr* and *berr* are both replicated in every process row, and are aligned with the matrices *B* and *X*.
ipiv	If *fact* = 'N' or 'E', then *ipiv* is an output argument and on exit contains the pivot indices from the factorization *A* = *P*L*U* of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').
equed	If *fact* ≠ 'F' , then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
work(1)	If *info*=0, on exit *work*(1) returns the minimum value of *lwork* required for optimum performance.
iwork(1)	If *info*=0, on exit *iwork*(1) returns the minimum value of *liwork* required for optimum performance.
rwork(1)	If *info*=0, on exit *rwork*(1) returns the minimum value of *lrwork* required for optimum performance.
info	INTEGER. If *info*=0, the execution is successful. *info* < 0: if the *i*th argument is an array and the *j*th entry had an illegal value, then *info* = -(*i**100+*j*); if the *i*th argument is a scalar and had an illegal value, then *info* = -*i*. If *info* = *i*, and *i* ≤ *n*, then *U*(*i*,*i*) is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed. If *info* = *i*, and *i* = *n* +1, then *U* is nonsingular, but *rcond* is less than machine precision. The factorization has been completed, but the matrix is singular to working precision and the solution and error bounds have not been computed.

p?gbsv

Computes the solution to the system of linear equations with a general banded distributed matrix and multiple right-hand sides.

Syntax

```
call psgbsv(n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, work, lwork, info)
call pdgbsv(n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, work, lwork, info)
call pcgbsv(n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, work, lwork, info)
call pzgbsv(n, bwl, bwu, nrhs, a, ja, desca, ipiv, b, ib, descb, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gbsv routine computes the solution to a real or complex system of linear equations

sub(*A*)***X* = sub(*B*),

where sub(*A*) = *A*(*1:n*, *ja:ja+n-1*) is an *n*-by-*n* real/complex general banded distributed matrix with *bwl* subdiagonals and *bwu* superdiagonals, and *X* and sub(*B*)= *B*(*ib:ib+n-1*, *1:rhs*) are *n*-by-*nrhs* distributed matrices.

The *LU* decomposition with partial pivoting and row interchanges is used to factor sub(*A*) as sub(*A*) = *P*L*U*Q*, where *P* and *Q* are permutation matrices, and *L* and *U* are banded lower and upper triangular matrices, respectively. The matrix *Q* represents reordering of columns for the sake of parallelism, while *P* represents reordering of rows for numerical stability using classic partial pivoting.

Input Parameters

n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*) (*n* ≥ 0).
bwl	(global) INTEGER. The number of subdiagonals within the band of *A* (0≤ *bwl* ≤ *n-1*).
bwu	(global) INTEGER. The number of superdiagonals within the band of *A* (0≤ *bwu* ≤ *n-1*).
nrhs	(global) INTEGER. The number of right hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs* ≥ 0).
a, *b*	(local) REAL for psgbsv DOUBLE PRECISON for pdgbsv COMPLEX for pcgbsv DOUBLE COMPLEX for pzgbsv. Pointers into the local memory to arrays of local dimension *a*(*lld_a*,*LOCc*(*ja*+*n*-1)) and *b*(*lld_b*,*LOCc*(*nrhs*)), respectively. On entry, the array *a* contains the local pieces of the global array *A*. On entry, the array *b* contains the right hand side distributed matrix sub(*B*).
ja	(global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*dtype_*) = 501, then *dlen_* ≥ 7; else if *desca*(*dtype_*) = 1, then *dlen_* ≥ 9.
ib	(global) INTEGER. The row index in the global array *B* that points to the first row of the matrix to be operated on (which may be either all of *B* or a submatrix of *B*).
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*. If *descb*(*dtype_*) = 502, then *dlen_* ≥ 7; else if *descb*(*dtype_*) = 1, then *dlen_* ≥ 9.
work	(local) REAL for psgbsv

DOUBLE PRECISON for pdgbsv
COMPLEX for pcgbsv
DOUBLE COMPLEX for pzgbsv.
Workspace array of dimension (*lwork*).

lwork (local or global) INTEGER. The size of the array *work*, must be at least
lwork ≥ (NB+*bwu*)*(*bwl*+*bwu*)+6*(*bwl*+*bwu*)*(*bwl*+2*bwu*) +
+ max(*nrhs* *(NB+2*bwl*+4*bwu*), 1).

Output Parameters

a On exit, contains details of the factorization. Note that the resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

b On exit, this array contains the local pieces of the solution distributed matrix *X*.

ipiv (local) INTEGER array.
The dimension of *ipiv* must be at least *desca*(NB). This array contains pivot indices for local factorizations. You should not alter the contents between factorization and solve.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info INTEGER. If *info*=0, the execution is successful. *info* < 0:
If the *i*th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i*100+*j*); if the *i*th argument is a scalar and had an illegal value, then *info* = -*i*.
info > 0:
If *info* = k ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not nonsingular, and the factorization was not completed. If *info* = k > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

p?dbsv

Solves a general band system of linear equations.

Syntax

```
call psdbsv(n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
call pddbsv(n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
call pcdbsv(n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
call pzdbsv(n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dbsv routine solves the following system of linear equations:

A(1:*n*, *ja*:*ja*+*n*-1)* *X* = *B*(*ib*:*ib*+*n*-1, 1:*nrhs*),

where $A(1:n, ja:ja+n-1)$ is an n-by-n real/complex banded diagonally dominant-like distributed matrix with bandwidth bwl, bwu.

Gaussian elimination without pivoting is used to factor a reordering of the matrix into LU.

Input Parameters

n	(global) INTEGER. The order of the distributed submatrix A, $(n \geq 0)$.
bwl	(global) INTEGER. Number of subdiagonals. $0 \leq bwl \leq n-1$.
bwu	(global) INTEGER. Number of subdiagonals. $0 \leq bwu \leq n-1$.
nrhs	(global) INTEGER. The number of right-hand sides; the number of columns of the distributed submatrix B, $(nrhs \geq 0)$.
a	(local). REAL for psdbsv DOUBLE PRECISION for pddbsv COMPLEX for pcdbsv DOUBLE COMPLEX for pzdbsv. Pointer into the local memory to an array with leading dimension $lld\_a \geq (bwl+bwu+1)$ (stored in $desca$). On entry, this array contains the local pieces of the distributed matrix.
ja	(global) INTEGER. The index in the global array a that points to the start of the matrix to be operated on (which may be either all of A or a submatrix of A).
desca	(global and local) INTEGER array of dimension $dlen$. If $1d$ type $(dtype\_a=501$ or $502)$, $dlen \geq 7$; If $2d$ type $(dtype\_a=1)$, $dlen \geq 9$. The array descriptor for the distributed matrix A. Contains information of mapping of A to memory.
b	(local) REAL for psdbsv DOUBLE PRECISON for pddbsv COMPLEX for pcdbsv DOUBLE COMPLEX for pzdbsv. Pointer into the local memory to an array of local lead dimension $lld\_b \geq nb$. On entry, this array contains the local pieces of the right hand sides $B(ib:ib+n-1, 1:nrhs)$.
ib	(global) INTEGER. The row index in the global array b that points to the first row of the matrix to be operated on (which may be either all of b or a submatrix of B).
descb	(global and local) INTEGER array of dimension $dlen$. If $1d$ type $(dtype\_b=502)$, $dlen \geq 7$; If $2d$ type $(dtype\_b=1)$, $dlen \geq 9$. The array descriptor for the distributed matrix B. Contains information of mapping of B to memory.
work	(local). REAL for psdbsv DOUBLE PRECISON for pddbsv COMPLEX for pcdbsv DOUBLE COMPLEX for pzdbsv. Temporary workspace. This space may be overwritten in between calls to routines. $work$ must be the size given in $lwork$.

lwork (local or global) INTEGER. Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned.
lwork ≥ *nb*(*bwl*+*bwu*)+6max(*bwl*,*bwu*)*max(*bwl*,*bwu*)
+max((max(*bwl*,*bwu*)*nrhs*), max(*bwl*,*bwu*)*max(*bwl*,*bwu*))

Output Parameters

a On exit, this array contains information containing details of the factorization.
Note that permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK.

b On exit, this contains the local piece of the solutions distributed matrix *X*.

work On exit, *work*(1) contains the minimal *lwork*.

info (local) INTEGER. If *info*=0, the execution is successful.
< 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
> 0: If *info* = *k* < NPROCS, the submatrix stored on processor *info* and factored locally was not positive definite, and the factorization was not completed.
If *info* = *k* > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not positive definite, and the factorization was not completed.

p?dtsv

Solves a general tridiagonal system of linear equations.

Syntax

```
call psdtsv(n, nrhs, dl, d, du, ja, desca, b, ib, descb, work, lwork, info)
call pddtsv(n, nrhs, dl, d, du, ja, desca, b, ib, descb, work, lwork, info)
call pcdtsv(n, nrhs, dl, d, du, ja, desca, b, ib, descb, work, lwork, info)
call pzdtsv(n, nrhs, dl, d, du, ja, desca, b, ib, descb, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The routine solves a system of linear equations

A(1:*n*, *ja*:*ja*+*n*-1) * *X* = *B*(*ib*:*ib*+*n*-1, 1:*nrhs*),

where *A*(1:*n*, *ja*:*ja*+*n*-1) is an *n*-by-*n* complex tridiagonal diagonally dominant-like distributed matrix.

Gaussian elimination without pivoting is used to factor a reordering of the matrix into *L U*.

Input Parameters

n (global) INTEGER. The order of the distributed submatrix *A* (*n* ≥ 0).

nrhs INTEGER. The number of right hand sides; the number of columns of the distributed matrix *B* (*nrhs* ≥ 0).

dl (local). REAL for psdtsv
DOUBLE PRECISION for pddtsv
COMPLEX for pcdtsv
DOUBLE COMPLEX for pzdtsv.
Pointer to local part of global vector storing the lower diagonal of the matrix. Globally, *dl*(1)is not referenced, and *dl* must be aligned with *d*.
Must be of size > *desca*(*nb_*).

d (local). REAL for psdtsv
DOUBLE PRECISION for pddtsv
COMPLEX for pcdtsv
DOUBLE COMPLEX for pzdtsv.
Pointer to local part of global vector storing the main diagonal of the matrix.

du (local). REAL for psdtsv
DOUBLE PRECISION for pddtsv
COMPLEX for pcdtsv
DOUBLE COMPLEX for pzdtsv.
Pointer to local part of global vector storing the upper diagonal of the matrix. Globally, *du*(*n*) is not referenced, and *du* must be aligned with *d*.

ja (global) INTEGER. The index in the global array *a* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array of dimension *dlen*.
If 1*d* type (*dtype_a*=501 or 502), *dlen* ≥ 7;
If 2*d* type (*dtype_a*=1), *dlen* ≥ 9.
The array descriptor for the distributed matrix *A*.
Contains information of mapping of *A* to memory.

b (local)
REAL for psdtsv
DOUBLE PRECISONfor pddtsv
COMPLEX for pcdtsv
DOUBLE COMPLEX for pzdtsv.
Pointer into the local memory to an array of local lead dimension *lld_b* > *nb*. On entry, this array contains the local pieces of the right hand sides *B*(*ib*:*ib*+*n*-1, 1:*nrhs*).

ib (global) INTEGER. The row index in the global array *b* that points to the first row of the matrix to be operated on (which may be either all of *b* or a submatrix of *B*).

descb (global and local) INTEGER array of dimension *dlen*.
If 1*d* type (*dtype_b* =502), *dlen* ≥ 7;
If 2*d* type (*dtype_b* =1), *dlen* ≥ 9.
The array descriptor for the distributed matrix *B*.
Contains information of mapping of *B* to memory.

work (local).
REAL for psdtsv
DOUBLE PRECISON for pddtsv
COMPLEX for pcdtsv
DOUBLE COMPLEX for pzdtsv. Temporary workspace. This space may be overwritten in between calls to routines. *work* must be the size given in *lwork*.

lwork	(local or global) INTEGER. Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned. *lwork* > (12*NPCOL+3**nb*)+max((10+2*min(100, *nrhs*))*NPCOL+4**nrhs*, 8*NPCOL)

Output Parameters

dl	On exit, this array contains information containing the * factors of the matrix.
d	On exit, this array contains information containing the * factors of the matrix. Must be of size > *desca*(*nb_*).
du	On exit, this array contains information containing the * factors of the matrix. Must be of size > *desca*(*nb_*).
b	On exit, this contains the local piece of the solutions distributed matrix *X*.
work	On exit, *work*(1) contains the minimal *lwork*.
info	(local) INTEGER. If *info*=0, the execution is successful. < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. > 0: If *info* = *k* < NPROCS, the submatrix stored on processor *info* and factored locally was not positive definite, and the factorization was not completed. If *info* = *k* > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not positive definite, and the factorization was not completed.

p?posv

Solves a symmetric positive definite system of linear equations.

Syntax

```
call psposv(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pdposv(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pcposv(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
call pzposv(uplo, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?posv routine computes the solution to a real/complex system of linear equations

sub(*A*)**X* = sub(*B*),

where sub(*A*) denotes *A*(*ia*:*ia*+*n*-1,*ja*:*ja*+*n*-1) and is an *n*-by-*n* symmetric/Hermitian distributed positive definite matrix and *X* and sub(*B*) denoting *B*(*ib*:*ib*+*n*-1,*jb*:*jb*+*nrhs*-1) are *n*-by-*nrhs* distributed matrices. The Cholesky decomposition is used to factor sub(*A*) as

sub(*A*) = U^T**U*, if *uplo* = 'U', or

sub(*A*) = *L**L^T, if *uplo* = 'L',

where *U* is an upper triangular matrix and *L* is a lower triangular matrix. The factored form of sub(*A*) is then used to solve the system of equations.

Input Parameters

uplo (global). CHARACTER. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of sub(*A*) is stored.

n (global) INTEGER. The order of the distributed submatrix sub(*A*) (*n* ≥ 0).

nrhs INTEGER. The number of right-hand sides; the number of columns of the distributed submatrix sub(*B*) (*nrhs* ≥ 0).

a (local)
REAL for psposv
DOUBLE PRECISION for pdposv
COMPLEX for pcposv
COMPLEX*16 for pzposv.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* symmetric distributed matrix sub(*A*) to be factored.
If *uplo* = '*U*', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced.
If *uplo* = '*L*', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the distributed matrix, and its strictly upper triangular part is not referenced.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

b (local)
REAL for psposv
DOUBLE PRECISON for pdposv
COMPLEX for pcposv
COMPLEX*16 for pzposv.
Pointer into the local memory to an array of dimension (*lld_b*, *LOC*(*jb*+*nrhs*-1)). On entry, the local pieces of the right hand sides distributed matrix sub(*B*).

ib, *jb* (global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.

descb (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.

Output Parameters

a On exit, if *info* = 0, this array contains the local pieces of the factor *U* or *L* from the Cholesky factorization sub(*A*) = U^H*U, or $L*L^H$.

b On exit, if *info* = 0, sub(*B*) is overwritten by the solution distributed matrix *X*.

info (global) INTEGER.
If *info* =0, the execution is successful.
If *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

If *info* > 0: If *info* = *k*, the leading minor of order *k*, *A*(*ia*:*ia*+*k*-1, *ja*:*ja*+*k*-1) is not positive definite, and the factorization could not be completed, and the solution has not been computed.

p?posvx

Solves a symmetric or Hermitian positive definite system of linear equations.

Syntax

call psposvx(*fact*, *uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *equed*, *sr*, *sc*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *rcond*, *ferr*, *berr*, *work*, *lwork*, *iwork*, *liwork*, *info*)

call pdposvx(*fact*, *uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *equed*, *sr*, *sc*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *rcond*, *ferr*, *berr*, *work*, *lwork*, *iwork*, *liwork*, *info*)

call pcposvx(*fact*, *uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *equed*, *sr*, *sc*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *rcond*, *ferr*, *berr*, *work*, *lwork*, *iwork*, *liwork*, *info*)

call pzposvx(*fact*, *uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desca*, *af*, *iaf*, *jaf*, *descaf*, *equed*, *sr*, *sc*, *b*, *ib*, *jb*, *descb*, *x*, *ix*, *jx*, *descx*, *rcond*, *ferr*, *berr*, *work*, *lwork*, *iwork*, *liwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?posvx routine uses the Cholesky factorization $A=U^T*U$ or $A=L*L^T$ to compute the solution to a real or complex system of linear equations

A(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1)**X* = *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*nrhs*-1),

where *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) is a *n*-by-*n* matrix and *X* and *B*(*ib*:*ib*+*n*-1,*jb*:*jb*+*nrhs*-1) are *n*-by-*nrhs* matrices.

Error bounds on the solution and a condition estimate are also provided.

In the following comments *y* denotes *Y*(*iy*:*iy*+*m*-1, *jy*:*jy*+*k*-1) a *m*-by-*k* matrix where *y* can be *a*, *af*, *b* and *x*.

The routine p?posvx performs the following steps:

1. If *fact* = 'E', real scaling factors *s* are computed to equilibrate the system:

   diag(*sr*)***A**diag(*sc*)**inv(diag(*sc*))**X* = diag(*sr*)**B

   Whether or not the system will be equilibrated depends on the scaling of the matrix *A*, but if equilibration is used, *A* is overwritten by diag(*sr*)***A**diag(*sc*) and *B* by diag(*sr*)***B .
2. If *fact* = 'N' or 'E', the Cholesky decomposition is used to factor the matrix *A* (after equilibration if *fact* = 'E') as

   A = U^T*U, if *uplo* = 'U', or

   A = $L*L^T$, if *uplo* = 'L',

   where *U* is an upper triangular matrix and *L* is a lower triangular matrix.

3. The factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, steps 4-6 are skipped
4. The system of equations is solved for *X* using the factored form of *A*.
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix *X* is premultiplied by diag(*sr*) so that it solves the original system before equilibration.

Input Parameters

fact
(global) CHARACTER. Must be 'F', 'N', or 'E'.
Specifies whether or not the factored form of the matrix *A* is supplied on entry, and if not, whether the matrix *A* should be equilibrated before it is factored.
If *fact* = 'F': on entry, *af* contains the factored form of *A*. If *equed* = 'Y', the matrix *A* has been equilibrated with scaling factors given by *s*. *a* and *af* will not be modified.
If *fact* = 'N', the matrix *A* will be copied to *af* and factored.
If *fact* = 'E', the matrix *A* will be equilibrated if necessary, then copied to *af* and factored.

uplo
(global) CHARACTER. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular part of *A* is stored.

n
(global) INTEGER. The order of the distributed submatrix sub(*A*) ($n \geq 0$).

nrhs
(global) INTEGER. The number of right-hand sides; the number of columns of the distributed submatrices *B* and *X*. ($nrhs \geq 0$).

a
(local)
REAL for psposvx
DOUBLE PRECISION for pdposvx
COMPLEX for pcposvx
DOUBLE COMPLEX for pzposvx.
Pointer into the local memory to an array of local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the symmetric/Hermitian matrix *A*, except if *fact* = 'F' and *equed* = 'Y', then *A* must contain the equilibrated matrix diag(*sr*)*A*diag(*sc*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of *A* contains the upper triangular part of the matrix *A*, and the strictly lower triangular part of *A* is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of *A* contains the lower triangular part of the matrix *A*, and the strictly upper triangular part of *A* is not referenced. *A* is not modified if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N' on exit.

ia, *ja*
(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca
(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

af
(local)
REAL for psposvx
DOUBLE PRECISION for pdposvx
COMPLEX for pcposvx
DOUBLE COMPLEX for pzposvx.

	Pointer into the local memory to an array of local dimension (*lld_af*, *LOCc*(*ja*+*n*-1)). If *fact* = 'F', then *af* is an input argument and on entry contains the triangular factor *U* or *L* from the Cholesky factorization $A = U^T*U$ or $A = L*L^T$, in the same storage format as *A*. If *equed* ≠ 'N', then *af* is the factored form of the equilibrated matrix diag(*sr*)*A*diag(*sc*).
iaf, *jaf*	(global) INTEGER. The row and column indices in the global array *af* indicating the first row and the first column of the submatrix *AF*, respectively.
descaf	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *AF*.
equed	(global). CHARACTER. Must be 'N' or 'Y'. *equed* is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done: If *equed* = 'N', no equilibration was done (always true if *fact* = 'N'); If *equed* = 'Y', equilibration was done and *A* has been replaced by diag(*sr*)*A*diag(*sc*).
sr	(local) REAL for psposvx DOUBLE PRECISION for pdposvx COMPLEX for pcposvx DOUBLE COMPLEX for pzposvx. Array, DIMENSION (*lld_a*). The array *s* contains the scale factors for *A*. This array is an input argument if *fact* = 'F' only; otherwise it is an output argument. If *equed* = 'N', *s* is not accessed. If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive.
b	(local) REAL for psposvx DOUBLE PRECISION for pdposvx COMPLEX for pcposvx DOUBLE COMPLEX for pzposvx. Pointer into the local memory to an array of local dimension (*lld_b*, *LOCc*(*jb*+*nrhs*-1)). On entry, the *n*-by-*nrhs* right-hand side matrix *B*.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.
descb	(global and local) INTEGER. Array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.
x	(local) REAL for psposvx DOUBLE PRECISION for pdposvx COMPLEX for pcposvx DOUBLE COMPLEX for pzposvx. Pointer into the local memory to an array of local dimension (*lld_x*, *LOCc*(*jx*+*nrhs*-1)).
ix, *jx*	(global) INTEGER. The row and column indices in the global array *x* indicating the first row and the first column of the submatrix *X*, respectively.
descx	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *X*.
work	(local)

	REAL for psposvx DOUBLE PRECISION for pdposvx COMPLEX for pcposvx DOUBLE COMPLEX for pzposvx. Workspace array, DIMENSION (*lwork*).
lwork	(local or global) INTEGER. The dimension of the array *work*. *lwork* is local input and must be at least *lwork* = max(p?pocon(*lwork*), p?porfs(*lwork*)) + *LOCr*(*n_a*). *lwork* = 3**desca*(*lld_*). If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.
iwork	(local) INTEGER. Workspace array, dimension (*liwork*).
liwork	(local or global) INTEGER. The dimension of the array *iwork*. *liwork* is local input and must be at least *liwork* = *desca*(*lld_*) *liwork* = *LOCr*(*n_a*). If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	On exit, if *fact* = 'E' and *equed* = 'Y', *a* is overwritten by diag(*sr*)*a*diag(*sc*).
af	If *fact* = 'N', then *af* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization $A = U^T*U$ or $A = L*L^T$ of the original matrix *A*. If *fact* = 'E', then *af* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization $A = U^T*U$ or $A = L*L^T$ of the equilibrated matrix *A* (see the description of *A* for the form of the equilibrated matrix).
equed	If *fact* ≠ 'F' , then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).
sr	This array is an output argument if *fact* ≠ 'F'. See the description of *sr* in *Input Arguments* section.
sc	This array is an output argument if *fact* ≠ 'F'. See the description of *sc* in *Input Arguments* section.
b	On exit, if *equed* = 'N', *b* is not modified; if *trans* = 'N' and *equed* = 'R' or 'B', *b* is overwritten by diag(*r*)*b; if *trans* = 'T' or 'C' and *equed* = 'C' or 'B', *b* is overwritten by diag(*c*)*b.
x	(local) REAL for psposvx DOUBLE PRECISION for pdposvx COMPLEX for pcposvx DOUBLE COMPLEX for pzposvx. If *info* = 0 the *n*-by-*nrhs* solution matrix *X* to the original system of equations.

Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the equilibrated system is
inv(diag(*sc*))**X* if *trans* = 'N' and *equed* = 'C' or 'B', or
inv(diag(*sr*))**X* if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'.

rcond (global)
REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
An estimate of the reciprocal condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision (in particular, if *rcond*=0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.

ferr REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
Arrays, DIMENSION at least max(*LOC*, *n_b*). The estimated forward error bounds for each solution vector *X*(*j*) (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution, *ferr*(*j*) bounds the magnitude of the largest entry in (*X*(*j*) - *xtrue*) divided by the magnitude of the largest entry in *X*(*j*). The quality of the error bound depends on the quality of the estimate of norm(inv(*A*)) computed in the code; if the estimate of norm(inv(*A*)) is accurate, the error bound is guaranteed.

berr (local)
REAL for single precision flavors.
DOUBLE PRECISION for double precision flavors.
Arrays, DIMENSION at least max(*LOC*, *n_b*). The componentwise relative backward error of each solution vector *X*(*j*) (the smallest relative change in any entry of *A* or *B* that makes *X*(*j*) an exact solution).

work(1) (local) On exit, *work*(1) returns the minimal and optimal *liwork*.

info (global) INTEGER.
If *info*=0, the execution is successful.
< 0: if *info* = -*i*, the *i*-th argument had an illegal value
> 0: if *info* = *i*, and *i* is ≤ *n*: if *info* = *i*, the leading minor of order *i* of *a* is not positive definite, so the factorization could not be completed, and the solution and error bounds could not be computed.
= *n*+1: *rcond* is less than machine precision. The factorization has been completed, but the matrix is singular to working precision, and the solution and error bounds have not been computed.

p?pbsv

Solves a symmetric/Hermitian positive definite banded system of linear equations.

Syntax

```
call pspbsv(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
call pdpbsv(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
call pcpbsv(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
call pzpbsv(uplo, n, bw, nrhs, a, ja, desca, b, ib, descb, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?pbsv routine solves a system of linear equations

A(1:*n*, *ja*:*ja*+*n*-1)***X* = *B*(*ib*:*ib*+*n*-1, 1:*nrhs*),

where *A*(1:*n*, *ja*:*ja*+*n*-1) is an *n*-by-*n* real/complex banded symmetric positive definite distributed matrix with bandwidth *bw*.

Cholesky factorization is used to factor a reordering of the matrix into *L*L'*.

Input Parameters

uplo (global) CHARACTER. Must be 'U' or 'L'.
Indicates whether the upper or lower triangular of *A* is stored.
If *uplo* = 'U', the upper triangular *A* is stored
If *uplo* = 'L', the lower triangular of *A* is stored.

n (global) INTEGER. The order of the distributed matrix *A* (*n* ≥ 0).

bw (global) INTEGER. The number of subdiagonals in *L* or *U*. 0 ≤ *bw* ≤ *n*-1.

nrhs (global) INTEGER. The number of right-hand sides; the number of columns in *B* (*nrhs* ≥ 0).

a (local). REAL for pspbsv
DOUBLE PRECISON for pdpbsv
COMPLEX for pcpbsv
DOUBLE COMPLEX for pzpbsv.
Pointer into the local memory to an array with leading dimension *lld_a* ≥ (*bw*+1) (stored in *desca*). On entry, this array contains the local pieces of the distributed matrix sub(*A*) to be factored.

ja (global) INTEGER. The index in the global array *a* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

b (local)
REAL for pspbsv
DOUBLE PRECISON for pdpbsv
COMPLEX for pcpbsv
DOUBLE COMPLEX for pzpbsv.
Pointer into the local memory to an array of local lead dimension *lld_b* ≥ *nb*. On entry, this array contains the local pieces of the right hand sides *B*(*ib*:*ib*+*n*-1, 1:*nrhs*).

ib (global) INTEGER. The row index in the global array *b* that points to the first row of the matrix to be operated on (which may be either all of *b* or a submatrix of *B*).

descb (global and local) INTEGER array of dimension *dlen*.
If 1D type (*dtype_b* =502), *dlen* ≥ 7;
If 2D type (*dtype_b* =1), *dlen* ≥ 9.
The array descriptor for the distributed matrix *B*.
Contains information of mapping of *B* to memory.

work (local).
REAL for pspbsv
DOUBLE PRECISON for pdpbsv
COMPLEX for pcpbsv

DOUBLE COMPLEX for pzpbsv.
Temporary workspace. This space may be overwritten in between calls to routines. *work* must be the size given in *lwork*.

lwork — (local or global) INTEGER. Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned. *lwork* ≥ (*nb*+2***bw*)***bw* +max((*bw***nrhs*), *bw***bw*)

Output Parameters

a — On exit, this array contains information containing details of the factorization. Note that permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK.

b — On exit, contains the local piece of the solutions distributed matrix *X*.

work — On exit, *work*(1) contains the minimal *lwork*.

info — (global). INTEGER. If *info*=0, the execution is successful.
< 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
> 0: If *info* = *k* ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not positive definite, and the factorization was not completed.
If *info* = *k* > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not positive definite, and the factorization was not completed.

p?ptsv

Syntax

Solves a symmetric or Hermitian positive definite tridiagonal system of linear equations.

```
call psptsv(n, nrhs, d, e, ja, desca, b, ib, descb, work, lwork, info)
call pdptsv(n, nrhs, d, e, ja, desca, b, ib, descb, work, lwork, info)
call pcptsv(n, nrhs, d, e, ja, desca, b, ib, descb, work, lwork, info)
call pzptsv(n, nrhs, d, e, ja, desca, b, ib, descb, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?ptsv routine solves a system of linear equations

A(1:*n*, *ja*:*ja*+*n*-1)**X* = *B*(*ib*:*ib*+*n*-1, 1:*nrhs*),

where *A*(1:*n*, *ja*:*ja*+*n*-1) is an *n*-by-*n* real tridiagonal symmetric positive definite distributed matrix.

Cholesky factorization is used to factor a reordering of the matrix into *L***L*'.

Input Parameters

n — (global) INTEGER. The order of matrix *A* (*n* ≥ 0).

nrhs (global) INTEGER. The number of right-hand sides; the number of columns of the distributed submatrix *B* (*nrhs* ≥ 0).

d (local)
REAL for psptsv
DOUBLE PRECISON for pdptsv
COMPLEX for pcptsv
DOUBLE COMPLEX for pzptsv.
Pointer to local part of global vector storing the main diagonal of the matrix.

e (local)
REAL for psptsv
DOUBLE PRECISON for pdptsv
COMPLEX for pcptsv
DOUBLE COMPLEX for pzptsv.
Pointer to local part of global vector storing the upper diagonal of the matrix. Globally, *du*(n) is not referenced, and *du* must be aligned with *d*.

ja (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array of dimension *dlen*.
If 1*d* type (*dtype_a*=501 or 502), *dlen* ≥ 7;
If 2*d* type (*dtype_a*=1), *dlen* ≥ 9.
The array descriptor for the distributed matrix *A*.
Contains information of mapping of *A* to memory.

b (local)
REAL for psptsv
DOUBLE PRECISON for pdptsv
COMPLEX for pcptsv
DOUBLE COMPLEX for pzptsv.
Pointer into the local memory to an array of local lead dimension *lld_b* ≥ *nb*.
On entry, this array contains the local pieces of the right hand sides *B*(*ib*:*ib*+*n*-1, 1:*nrhs*).

ib (global) INTEGER. The row index in the global array *b* that points to the first row of the matrix to be operated on (which may be either all of *b* or a submatrix of *B*).

descb (global and local) INTEGER array of dimension *dlen*.
If 1*d* type (*dtype_b* = 502), *dlen* ≥ 7;
If 2*d* type (*dtype_b* = 1), *dlen* ≥ 9.
The array descriptor for the distributed matrix *B*.
Contains information of mapping of *B* to memory.

work (local).
REAL for psptsv
DOUBLE PRECISON for pdptsv
COMPLEX for pcptsv
DOUBLE COMPLEX for pzptsv.
Temporary workspace. This space may be overwritten in between calls to routines. *work* must be the size given in *lwork*.

lwork	(local or global) INTEGER. Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned. *lwork* > (12*NPCOL+3**nb*)+max((10+2*min(100, *nrhs*))*NPCOL+4**nrhs*, 8*NPCOL).

Output Parameters

d	On exit, this array contains information containing the factors of the matrix. Must be of size greater than or equal to *desca*(*nb_*).
e	On exit, this array contains information containing the factors of the matrix. Must be of size greater than or equal to *desca*(*nb_*).
b	On exit, this contains the local piece of the solutions distributed matrix *X*.
work	On exit, *work*(1) contains the minimal *lwork*.
info	(local) INTEGER. If *info*=0, the execution is successful. < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. > 0: If *info* = *k* ≤ NPROCS, the submatrix stored on processor *info* and factored locally was not positive definite, and the factorization was not completed. If *info* = *k* > NPROCS, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not positive definite, and the factorization was not completed.

p?gels

Solves overdetermined or underdetermined linear systems involving a matrix of full rank.

Syntax

```
call psgels(trans, m, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, work, lwork, info)
call pdgels(trans, m, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, work, lwork, info)
call pcgels(trans, m, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, work, lwork, info)
call pzgels(trans, m, n, nrhs, a, ia, ja, desca, b, ib, jb, descb, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gels routine solves overdetermined or underdetermined real/ complex linear systems involving an *m*-by-*n* matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1,*ja*:*ja*+*n*-1), or its transpose/ conjugate-transpose, using a *QTQ* or *LQ* factorization of sub(*A*). It is assumed that sub(*A*) has full rank.

The following options are provided:

1. If *trans* = 'N' and *m* ≥ *n*: find the least squares solution of an overdetermined system, that is, solve the least squares problem

   minimize ||sub(*B*) - sub(*A*)**X*||
2. If *trans* = 'N' and *m* < *n*: find the minimum norm solution of an underdetermined system sub(*A*)**X* = sub(*B*).
3. If *trans* = 'T' and *m* ≥ *n*: find the minimum norm solution of an undetermined system sub(*A*)^T**X* = sub(*B*).

4. If *trans* = 'T' and *m* < *n*: find the least squares solution of an overdetermined system, that is, solve the least squares problem

minimize ||sub(*B*) - sub(*A*)^T*X||,

where sub(*B*) denotes *B*(*ib*:*ib*+*m*-1, *jb*:*jb*+*nrhs*-1) when *trans* = 'N' and *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*nrhs*-1) otherwise. Several right hand side vectors *b* and solution vectors *x* can be handled in a single call; when *trans* = 'N', the solution vectors are stored as the columns of the *n*-by-*nrhs* right hand side matrix sub(*B*) and the *m*-by-*nrhs* right hand side matrix sub(*B*) otherwise.

Input Parameters

trans	(global) CHARACTER. Must be 'N', or 'T'. If *trans* = 'N', the linear system involves matrix sub(*A*); If *trans* = 'T', the linear system involves the transposed matrix A^T (for real flavors only).
m	(global) INTEGER. The number of rows in the distributed submatrix sub (*A*) (*m* ≥ 0).
n	(global) INTEGER. The number of columns in the distributed submatrix sub (*A*) (*n* ≥ 0).
nrhs	(global) INTEGER. The number of right-hand sides; the number of columns in the distributed submatrices sub(*B*) and *X*. (*nrhs* ≥ 0).
a	(local) REAL for psgels DOUBLE PRECISION for pdgels COMPLEX for pcgels DOUBLE COMPLEX for pzgels. Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, contains the *m*-by-*n* matrix *A*.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.
b	(local) REAL for psgels DOUBLE PRECISION for pdgels COMPLEX for pcgels DOUBLE COMPLEX for pzgels. Pointer into the local memory to an array of local dimension (*lld_b*, *LOCc*(*jb*+*nrhs*-1)). On entry, this array contains the local pieces of the distributed matrix *B* of right-hand side vectors, stored columnwise; sub(*B*) is *m*-by-*nrhs* if *trans*='N', and *n*-by-*nrhs* otherwise.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.
descb	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*.
work	(local) REAL for psgels DOUBLE PRECISION for pdgels COMPLEX for pcgels DOUBLE COMPLEX for pzgels. Workspace array with dimension *lwork*.

lwork (local or global) INTEGER.
The dimension of the array *work* *lwork* is local input and must be at least
lwork ≥ *ltau* + max(*lwf*, *lws*), where if *m* > *n*, then
ltau = numroc(*ja*+min(*m*,*n*)-1, *nb_a*, MYCOL, *csrc_a*, NPCOL),
lwf = *nb_a**(*mpa*0 + *nqa*0 + *nb_a*)
lws = max((*nb_a**(*nb_a*-1))/2, (*nrhsqb*0 + *mpb*0)**nb_a*) +
*nb_a***nb_a*
else
ltau = numroc(*ia*+min(*m*,*n*)-1, *mb_a*, MYROW, *rsrc_a*, NPROW),
lwf = *mb_a* * (*mpa*0 + *nqa*0 + *mb_a*)
lws = max((*mb_a**(*mb_a*-1))/2, (*npb*0 + max(*nqa*0 +
numroc(numroc(*n*+*iroffb*, *mb_a*, 0, 0, NPROW), *mb_a*, 0, 0,
lcmp), *nrhsqb*0))**mb_a*) + *mb_a***mb_a*
end if,
where *lcmp* = *lcm*/NPROW with *lcm* = ilcm(NPROW, NPCOL),
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, MYROW, *rsrc_a*, NPROW),
iacol= indxg2p(*ja*, *nb_a*, MYROW, *rsrc_a*, NPROW)
*mpa*0 = numroc(*m*+*iroffa*, *mb_a*, MYROW, *iarow*, NPROW),
*nqa*0 = numroc(*n*+*icoffa*, *nb_a*, MYCOL, *iacol*, NPCOL),
iroffb = mod(*ib*-1, *mb_b*),
icoffb = mod(*jb*-1, *nb_b*),
ibrow = indxg2p(*ib*, *mb_b*, MYROW, *rsrc_b*, NPROW),
ibcol = indxg2p(*jb*, *nb_b*, MYCOL, *csrc_b*, NPCOL),
*mpb*0 = numroc(*m*+*iroffb*, *mb_b*, MYROW, *icrow*, NPROW),
*nqb*0 = numroc(*n*+*icoffb*, *nb_b*, MYCOL, *ibcol*, NPCOL),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW, and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, If *m* ≥ *n*, sub(*A*) is overwritten by the details of its *QR* factorization as returned by p?geqrf; if *m* < *n*, sub(*A*) is overwritten by details of its *LQ* factorization as returned by p?gelqf.

b On exit, sub(*B*) is overwritten by the solution vectors, stored columnwise: if *trans* = 'N' and *m* ≥ *n*, rows 1 to *n* of sub(*B*) contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of elements *n*+1 to *m* in that column;
If *trans* = 'N' and *m* < *n*, rows 1 to *n* of sub(*B*) contain the minimum norm solution vectors;
If *trans* = 'T' and *m* ≥ *n*, rows 1 to *m* of sub(*B*) contain the minimum norm solution vectors; if *trans* = 'T' and *m* < *n*, rows 1 to *m* of sub(*B*) contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of elements *m*+1 to *n* in that column.

work(1) On exit, *work*(1) contains the minimum value of *lwork* required for optimum performance.

info	(global) INTEGER. = 0: the execution is successful. < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i** 100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?syev

Computes selected eigenvalues and eigenvectors of a symmetric matrix.

Syntax

```
call pssyev(jobz, uplo, n, a, ia, ja, desca, w, z, iz, jz, descz, work, lwork, info)
call pdsyev(jobz, uplo, n, a, ia, ja, desca, w, z, iz, jz, descz, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?syev routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix *A* by calling the recommended sequence of ScaLAPACK routines.

In its present form, the routine assumes a homogeneous system and makes no checks for consistency of the eigenvalues or eigenvectors across the different processes. Because of this, it is possible that a heterogeneous system may return incorrect results without any error messages.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

jobz	(global). CHARACTER. Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If *jobz* ='N', then only eigenvalues are computed. If *jobz* ='V', then eigenvalues and eigenvectors are computed.
uplo	(global). CHARACTER. Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the symmetric matrix *A* is stored: If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	(global) INTEGER. The number of rows and columns of the matrix *A* (*n* ≥ 0).
a	(local) REAL for pssyev. DOUBLE PRECISION for pdsyev. Block cyclic array of global dimension (*n*, *n*) and local dimension (*lld_a*, *LOC c*(*ja*+*n*-1)). On entry, the symmetric matrix *A*. If *uplo* = 'U', only the upper triangular part of *A* is used to define the elements of the symmetric matrix. If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the symmetric matrix.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

iz, *jz* (global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*.

work (local)
REAL for pssyev.
DOUBLE PRECISION for pdsyev.
Array, DIMENSION (*lwork*).

lwork (local) INTEGER. See below for definitions of variables used to define *lwork*.
If no eigenvectors are requested (*jobz* = 'N'), then *lwork* ≥ 5*n* + *sizesytrd* + 1,
where *sizesytrd* is the workspace for p?sytrd and is max(NB*(*np* +1), 3*NB).
If eigenvectors are requested (*jobz* = 'V') then the amount of workspace required to guarantee that all eigenvectors are computed is:
qrmem = 2*n*-2
lwmin = 5**n* + *n***ldc* + max(*sizemqrleft*, *qrmem*) + 1
Variable definitions:
nb = *desca*(*mb_*) = *desca*(*nb_*) = *descz*(*mb_*) = *descz*(*nb_*);
nn = max(*n*, *nb*, 2);
desca(*rsrc_*) = *desca*(*rsrc_*) = *descz*(*rsrc_*) = *descz*(*csrc_*) = 0
np = numroc(*nn*, *nb*, 0, 0, NPROW)
nq = numroc(max(*n*, *nb*, 2), *nb*, 0, 0, NPCOL)
nrc = numroc(*n*, *nb*, *myprowc*, 0, NPROCS)
ldc = max(1, *nrc*)
sizemqrleft is the workspace for p?ormtr when its *side* argument is 'L'.
myprowc is defined when a new context is created as follows:
call blacs_get(*desca*(*ctxt_*), 0, *contextc*)
call blacs_gridinit(*contextc*, 'R', NPROCS, 1)
call blacs_gridinfo(*contextc*, *nprowc*, *npcolc*, *myprowc*, *mypcolc*)
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, the lower triangle (if *uplo*='L') or the upper triangle (if *uplo*='U') of *A*, including the diagonal, is destroyed.

w (global). REAL for pssyev
DOUBLE PRECISION for pdsyev
Array, DIMENSION (*n*).
On normal exit, the first *m* entries contain the selected eigenvalues in ascending order.

z (local). REAL for pssyev
DOUBLE PRECISION for pdsyev

	Array, global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)). If *jobz* = 'V', then on normal exit the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If *jobz* = 'N', then *z* is not referenced.
work(1)	On output, *work*(1) returns the workspace needed to guarantee completion. If the input parameters are incorrect, *work*(1) may also be incorrect. If *jobz* = 'N' *work*(1) = minimal (optimal) amount of workspace If *jobz* = 'V' *work*(1) = minimal workspace required to generate all the eigenvectors.
info	(global) INTEGER. If *info* = 0, the execution is successful. If *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. If *info* > 0: If *info*= 1 through *n*, the *i*-th eigenvalue did not converge in ?steqr2 after a total of 30*n* iterations. If *info*= *n*+1, then p?syev has detected heterogeneity by finding that eigenvalues were not identical across the process grid. In this case, the accuracy of the results from p?syev cannot be guaranteed.

p?syevd

Computes all eigenvalues and eigenvectors of a real symmetric matrix by using a divide and conquer algorithm.

Syntax

```
call pssyevd(jobz, uplo, n, a, ia, ja, desca, w, z, iz, jz, descz, work, lwork,
iwork, liwork, info)
```

```
call pdsyevd(jobz, uplo, n, a, ia, ja, desca, w, z, iz, jz, descz, work, lwork,
iwork, liwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?syevd routine computes all eigenvalues and eigenvectors of a real symmetric matrix *A* by using a divide and conquer algorithm.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

jobz	(global). CHARACTER*1. Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	(global). CHARACTER*1. Must be 'U' or 'L'.

Specifies whether the upper or lower triangular part of the Hermitian matrix *A* is stored:
If *uplo* = 'U', *a* stores the upper triangular part of *A*.
If *uplo* = 'L', *a* stores the lower triangular part of *A*.

n — (global) INTEGER. The number of rows and columns of the matrix *A* (*n* ≥ 0).

a — (local).
REAL for pssyevd
DOUBLE PRECISION for pdsyevd.
Block cyclic array of global dimension (*n*, *n*) and local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the symmetric matrix *A*.
If *uplo* = 'U', only the upper triangular part of *A* is used to define the elements of the symmetric matrix.
If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the symmetric matrix.

ia, *ja* — (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca — (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*ctxt_*) is incorrect, p?syevd cannot guarantee correct error reporting.

iz, *jz* — (global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz — (global and local) INTEGER array, dimension *dlen_*. The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).

work — (local).
REAL for pssyevd
DOUBLE PRECISION for pdsyevd.
Array, DIMENSION *lwork*.

lwork — (local). INTEGER. The dimension of the array *work*.
If eigenvalues are requested:
lwork ≥ max(1+6*n + 2*np*nq, *trilwmin*) + 2*n
with *trilwmin* = 3*n + max(*nb*(*np* + 1), 3**nb*)
np = numroc(*n*, *nb*, *myrow*, *iarow*, NPROW)
nq = numroc(*n*, *nb*, *mycol*, *iacol*, NPCOL)
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. The required workspace is returned as the first element of the corresponding work arrays, and no error message is issued by pxerbla.

iwork — (local) INTEGER. Workspace array, dimension *liwork*.

liwork — (local) INTEGER, dimension of *iwork*.
liwork = 7*n + 8*npcol + 2.

Output Parameters

a — On exit, the lower triangle (if *uplo* = 'L'), or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

w — (global).
REAL for pssyevd
DOUBLE PRECISION for pdsyevd.

	Array, DIMENSION *n*. If *info* = 0, *w* contains the eigenvalues in the ascending order.
z	(local). REAL for pssyevd DOUBLE PRECISION for pdsyevd. Array, global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)). The *z* parameter contains the orthonormal eigenvectors of the matrix *A*.
work(1)	On exit, returns adequate workspace to allow optimal performance.
iwork(1)	(local). On exit, if *liwork* > 0, *iwork*(1) returns the optimal *liwork*.
info	(global) INTEGER. If *info* = 0, the execution is successful. If *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*). If the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. If *info* > 0: The algorithm failed to compute the *info*/(*n*+1)-th eigenvalue while working on the submatrix lying in global rows and columns mod(*info*,*n*+1).

p?syevx

Computes selected eigenvalues and, optionally, eigenvectors of a symmetric matrix.

Syntax

call pssyevx(*jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)

call pdsyevx(*jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?syevx routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix *A* by calling the recommended sequence of ScaLAPACK routines. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

jobz	(global). CHARACTER*1. Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If *jobz* ='N', then only eigenvalues are computed. If *jobz* ='V', then eigenvalues and eigenvectors are computed.
range	(global). CHARACTER*1. Must be 'A', 'V', or 'I'. If *range* = 'A', all eigenvalues will be found.

If *range* = 'V', all eigenvalues in the half-open interval [*vl*, *vu*] will be found.
If *range* = 'I', the eigenvalues with indices *il* through *iu* will be found.

uplo (global). CHARACTER*1. Must be 'U' or 'L'.
Specifies whether the upper or lower triangular part of the symmetric matrix *A* is stored:
If *uplo* = 'U', *a* stores the upper triangular part of *A*.
If *uplo* = 'L', *a* stores the lower triangular part of *A*.

n (global) INTEGER. The number of rows and columns of the matrix *A* (*n* ≥ 0).

a (local). REAL for pssyevx
DOUBLE PRECISION for pdsyevx.
Block cyclic array of global dimension (*n*, *n*) and local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the symmetric matrix *A*.
If *uplo* = 'U', only the upper triangular part of *A* is used to define the elements of the symmetric matrix.
If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the symmetric matrix.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

vl, *vu* (global)
REAL for pssyevx
DOUBLE PRECISION for pdsyevx.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; *vl* ≤ *vu*. Not referenced if *range* = 'A' or 'I'.

il, *iu* (global) INTEGER.
If *range* ='I', the indices of the smallest and largest eigenvalues to be returned.
Constraints: *il* ≥ 1
min(*il*,*n*) ≤ *iu* ≤ *n*
Not referenced if *range* = 'A' or 'V'.

abstol (global). REAL for pssyevx
DOUBLE PRECISION for pdsyevx.
If *jobz*='V', setting *abstol* to p?lamch(*context*, 'U') yields the most orthogonal eigenvectors.
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [*a*, *b*] of width less than or equal to
abstol + *eps* * max(|*a*|,|*b*|),
where *eps* is the machine precision. If *abstol* is less than or equal to zero, then *eps**norm(T) will be used in its place, where norm(T) is the 1-norm of the tridiagonal matrix obtained by reducing *A* to tridiagonal form.
Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*p?lamch('S') not zero. If this routine returns with ((mod(*info*,2).*ne*.0).*or*. * (mod(*info*/8,2).*ne*.0)), indicating that some eigenvalues or eigenvectors did not converge, try setting *abstol* to 2*p?lamch('S').

orfac (global). REAL for pssyevx
DOUBLE PRECISION for pdsyevx.

Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within *tol*=*orfac**norm(*A*) of each other are to be reorthogonalized. However, if the workspace is insufficient (see *lwork*), *tol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if *orfac* equals zero. A default value of 1.0e-3 is used if *orfac* is negative. *orfac* should be identical on all processes.

iz, *jz*

(global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz

(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).

work

(local)
REAL for pssyevx.
DOUBLE PRECISION for pdsyevx.
Array, DIMENSION (*lwork*).

lwork

(local) INTEGER. The dimension of the array *work*.
See below for definitions of variables used to define *lwork*.
If no eigenvectors are requested (*jobz* = 'N'), then *lwork* ≥ 5**n* + max(5**nn*, NB*(*np*0 + 1)).
If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is:
lwork ≥ 5**n* + max(5**nn*, *np*0**mq*0 + 2*NB*NB) + iceil(*neig*, NPROW*NPCOL)**nn*
The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and *orfac* is too small. If you want to guarantee orthogonality (at the cost of potentially poor performance) you should add the following to *lwork*:
(*clustersize*-1)**n*,
where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:
{*w*(*k*),..., *w*(*k*+*clustersize*-1)| *w*(*j*+1) ≤ *w*(*j*)) + *orfac**2*norm(*A*)},
where
neig = number of eigenvectors requested
nb = *desca*(*mb_*) = *desca*(*nb_*) = *descz*(*mb_*) = *descz*(*nb_*);
nn = max(*n*, *nb*, 2);
desca(*rsrc_*) = *desca*(*nb_*) = *descz*(*rsrc_*) = *descz*(*csrc_*) = 0;
*np*0 = numroc(*nn*, *nb*, 0, 0, NPROW);
*mq*0 = numroc(max(*neig*, *nb*, 2), *nb*, 0, 0, NPCOL)
iceil(*x*, *y*) is a ScaLAPACK function returning ceiling(*x*/*y*)
If *lwork* is too small to guarantee orthogonality, p?syevx attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.
If *lwork* is too small to compute all the eigenvectors requested, no computation is performed and *info*= -23 is returned.
Note that when *range*='V', number of requested eigenvectors are not known until the eigenvalues are computed. In this case and if *lwork* is large enough to compute the eigenvalues, p?sygvx computes the eigenvalues and as many eigenvectors as possible.
Relationship between workspace, orthogonality & performance:

Greater performance can be achieved if adequate workspace is provided. In some situations, performance can decrease as the provided workspace increases above the workspace amount shown below:
lwork ≥ max(*lwork*, 5**n* + *nsytrd_lwopt*),
where *lwork*, as defined previously, depends upon the number of eigenvectors requested, and
nsytrd_lwopt = *n* + 2*(*anb*+1)*(4**nps*+2) + (*nps* + 3)**nps;*
anb = pjlaenv(*desca*(*ctxt_*), 3, 'p?syttrd', 'L', 0, 0, 0, 0);
sqnpc = int(*sqrt*(*dble*(NPROW * NPCOL)));
nps = max(numroc(*n*, 1, 0, 0, *sqnpc*), 2**anb*);
numroc is a ScaLAPACK tool functions;
pjlaenv is a ScaLAPACK environmental inquiry function
MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
For large n, no extra workspace is needed, however the biggest boost in performance comes for small n, so it is wise to provide the extra workspace (typically less than a megabyte per process).
If *clustersize* > *n*/*sqrt*(NPROW*NPCOL), then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. At the limit (that is, *clustersize* = *n*-1) p?stein will perform no better than ?stein on single processor.
For *clustersize* = *n*/*sqrt*(NPROW*NPCOL) reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.
For *clustersize* > *n*/*sqrt*(NPROW*NPCOL) execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by pxerbla.

iwork (local) INTEGER. Workspace array.

liwork (local) INTEGER, dimension of *iwork*. *liwork* ≥ 6**nnp*
Where: *nnp* = max(*n*, NPROW*NPCOL + 1, 4)
If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U')of *A*, including the diagonal, is overwritten.

m (global) INTEGER. The total number of eigenvalues found; 0 ≤ *m* ≤ *n*.

nz (global) INTEGER. Total number of eigenvectors computed. 0 ≤ *nz* ≤ *m*.
The number of columns of *z* that are filled.
If *jobz* ≠ 'V', *nz* is not referenced.
If *jobz* = 'V', *nz* = *m* unless the user supplies insufficient space and p?syevx is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold

the eigenvectors in *z* (*m*.*le*.*descz*(*n_*)) and sufficient workspace to compute them. (See *lwork*). p?syevx is always able to detect insufficient space without computation unless *range*.*eq*.'V'.

w (global). REAL for pssyevx
DOUBLE PRECISION for pdsyevx.
Array, DIMENSION (*n*). The first *m* elements contain the selected eigenvalues in ascending order.

z (local). REAL for pssyevx
DOUBLE PRECISION for pdsyevx.
Array, global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)).
If *jobz* = 'V', then on normal exit the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
If *jobz* = 'N', then *z* is not referenced.

work(1) On exit, returns workspace adequate workspace to allow optimal performance.

iwork(1) On return, *iwork*(1) contains the amount of integer workspace required.

ifail (global) INTEGER.
Array, DIMENSION (*n*).
If *jobz* = 'V', then on normal exit, the first *m* elements of *ifail* are zero.
If (mod(*info*,2). *ne*.0) on exit, then *ifail* contains the indices of the eigenvectors that failed to converge.
If *jobz* = 'N', then *ifail* is not referenced.

iclustr (global) INTEGER. Array, DIMENSION (2*NPROW*NPCOL)
This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed *iclustr*(2*i*-1) to *iclustr*(2*i*), could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. *iclustr*() is a zero terminated array. (*iclustr*(2*k*).*ne*.0. *and*. *iclustr*(2*k*+1).*eq*.0) if and only if *k* is the number of clusters.
iclustr is not referenced if *jobz* = 'N'.

gap (global)
REAL for pssyevx
DOUBLE PRECISION for pdsyevx.
Array, DIMENSION (NPROW*NPCOL)
This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*th cluster may be as high as (*C**n*)/*gap*(*i*) where *C* is a small constant.

info (global) INTEGER.
If *info* = 0, the execution is successful.
If *info* < 0:
If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

If *info* > 0: if (mod(*info*,2).*ne*.0), then one or more eigenvectors failed to converge. Their indices are stored in *ifail*. Ensure *abstol*=2.0*p?lamch('U').
If (mod(*info*/2,2).*ne*.0), then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace.The indices of the clusters are stored in the array *iclustr*.
If (mod(*info*/4,2).*ne*.0), then space limit prevented p?syevxf rom computing all of the eigenvectors between *vl* and *vu*. The number of eigenvectors computed is returned in *nz*.
If (mod(*info*/8,2).*ne*.0), then p?stebz failed to compute eigenvalues. Ensure *abstol*=2.0*p?lamch('U').

p?heev

Computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix.

Syntax

call pcheev(*jobz*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *w*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *rwork*, *lrwork*, *info*)

call pzheev(*jobz*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *w*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *rwork*, *lrwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?heev routine computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix *A* by calling the recommended sequence of ScaLAPACK routines. The routine assumes a homogeneous system and makes spot checks of the consistency of the eigenvalues across the different processes. A heterogeneous system may return incorrect results without any error messages.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

jobz (global). CHARACTER*1. Must be 'N' or 'V'.
Specifies if it is necessary to compute the eigenvectors:
If *jobz* = 'N', then only eigenvalues are computed.
If *jobz* = 'V', then eigenvalues and eigenvectors are computed.

uplo (global). CHARACTER*1. Must be 'U' or 'L'.
Specifies whether the upper or lower triangular part of the Hermitian matrix *A* is stored:
If *uplo* = 'U', *a* stores the upper triangular part of *A*.
If *uplo* = 'L', *a* stores the lower triangular part of *A*.

n (global) INTEGER. The number of rows and columns of the matrix *A* (*n* ≥ 0).

a (local).
COMPLEX for pcheev
DOUBLE COMPLEX for pzheev.

	Block cyclic array of global dimension (*n*, *n*) and local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the Hermitian matrix *A*. If *uplo* = 'U', only the upper triangular part of *A* is used to define the elements of the Hermitian matrix. If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the Hermitian matrix.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*ctxt_*) is incorrect, p?heev cannot guarantee correct error reporting.
iz, *jz*	(global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.
descz	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).
work	(local). COMPLEX for pcheev DOUBLE COMPLEX for pzheev. Array, DIMENSION *lwork*.
lwork	(local). INTEGER. The dimension of the array *work*. If only eigenvalues are requested (*jobz* = 'N'): *lwork* ≥ max(*nb**(*np*0 + 1), 3) + 3**n* If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required: *lwork* ≥ (*np*0+*nq*0+*nb*)**nb* + 3**n* + n^2 with *nb* = *desca*(*mb_*) = *desca*(*nb_*) = *nb* = *descz*(*mb_*) = *descz*(*nb_*) *np*0 = numroc(*nn*, *nb*, 0, 0, NPROW). *nq*0 = numroc(max(*n*, *nb*, 2), *nb*, 0, 0, NPCOL). If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. The required workspace is returned as the first element of the corresponding work arrays, and no error message is issued by pxerbla.
rwork	(local). REAL for pcheev DOUBLE PRECISION for pzheev. Workspace array, DIMENSION *lrwork*.
lrwork	(local) INTEGER. The dimension of the array *rwork*. See below for definitions of variables used to define *lrwork*. If no eigenvectors are requested (*jobz* = 'N'), then *lrwork* ≥ 2**n*. If eigenvectors are requested (*jobz* = 'V'), then *lrwork* ≥ 2**n* + 2**n*-2. If *lrwork* = -1, then *lrwork* is global input and a workspace query is assumed; the routine only calculates the minimum size required for the *rwork* array. The required workspace is returned as the first element of *rwork*, and no error message is issued by pxerbla.

Output Parameters

a	On exit, the lower triangle (if *uplo* = 'L'), or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.
w	(global). REAL for pcheev DOUBLE PRECISION for pzheev. Array, DIMENSION *n*. The first *m* elements contain the selected eigenvalues in ascending order.
z	(local). COMPLEX for pcheev DOUBLE COMPLEX for pzheev. Array, global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)). If *jobz* ='V', then on normal exit the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*. If *jobz* = 'N', then *z* is not referenced.
work(1)	On exit, returns adequate workspace to allow optimal performance. If *jobz* ='N', then *work*(1) = minimal workspace only for eigenvalues. If *jobz* ='V', then *work*(1) = minimal workspace required to generate all the eigenvectors.
rwork(1)	(local) COMPLEX for pcheev DOUBLE COMPLEX for pzheev. On output, *rwork*(1) returns workspace required to guarantee completion.
info	(global) INTEGER. If *info* = 0, the execution is successful. If *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*). If the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. If *info* > 0: If *info* = 1 through *n*, the *i*-th eigenvalue did not converge in ?steqr2 after a total of 30**n* iterations. If *info* = *n*+1, then p?heev detected heterogeneity, and the accuracy of the results cannot be guaranteed.

p?heevd

Computes all eigenvalues and eigenvectors of a complex Hermitian matrix by using a divide and conquer algorithm.

Syntax

```
call pcheevd(jobz, uplo, n, a, ia, ja, desca, w, z, iz, jz, descz, work, lwork,
rwork, lrwork, iwork, liwork, info)
```

```
call pzheevd(jobz, uplo, n, a, ia, ja, desca, w, z, iz, jz, descz, work, lwork,
rwork, lrwork, iwork, liwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?heevd routine computes all eigenvalues and eigenvectors of a complex Hermitian matrix *A* by using a divide and conquer algorithm.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

jobz	(global). CHARACTER*1. Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If *jobz* = 'N', then only eigenvalues are computed. If *jobz* = 'V', then eigenvalues and eigenvectors are computed.
uplo	(global). CHARACTER*1. Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the Hermitian matrix *A* is stored: If *uplo* = 'U', *a* stores the upper triangular part of *A*. If *uplo* = 'L', *a* stores the lower triangular part of *A*.
n	(global) INTEGER. The number of rows and columns of the matrix *A* (*n* ≥ 0).
a	(local). COMPLEX for pcheevd DOUBLE COMPLEX for pzheevd. Block cyclic array of global dimension (*n*, *n*) and local dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the Hermitian matrix *A*. If *uplo* = 'U', only the upper triangular part of *A* is used to define the elements of the Hermitian matrix. If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the Hermitian matrix.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*ctxt_*) is incorrect, p?heevd cannot guarantee correct error reporting.
iz, *jz*	(global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.
descz	(global and local) INTEGER array, dimension *dlen_*. The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).
work	(local). COMPLEX for pcheevd DOUBLE COMPLEX for pzheevd. Array, DIMENSION *lwork*.
lwork	(local). INTEGER. The dimension of the array *work*. If eigenvalues are requested: *lwork* = *n* + (*nb*0 + *mq*0 + *nb*)**nb* with *np*0 = numroc(max(*n*, *nb*, 2), *nb*, 0, 0, NPROW) *mq*0 = numroc(max(*n*, *nb*, 2), *nb*, 0, 0, NPCOL)

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. The required workspace is returned as the first element of the corresponding work arrays, and no error message is issued by pxerbla.

rwork (local).
REAL for pcheevd
DOUBLE PRECISION for pzheevd.
Workspace array, DIMENSION *lrwork*.

lrwork (local) INTEGER. The dimension of the array *rwork*.
lrwork ≥ 1 + 9**n* + 3**np***nq*,
with *np* = numroc(*n*, *nb*, *myrow*, *iarow*, NPROW)
nq = numroc(*n*, *nb*, *mycol*, *iacol*, NPCOL)

iwork (local) INTEGER. Workspace array, dimension *liwork*.

liwork (local) INTEGER, dimension of *iwork*.
liwork = 7**n* + 8**npcol* + 2.

Output Parameters

a On exit, the lower triangle (if *uplo* = 'L'), or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

w (global).
REAL for pcheevd
DOUBLE PRECISION for pzheevd.
Array, DIMENSION *n*. If *info* = 0, *w* contains the eigenvalues in the ascending order.

z (local).
COMPLEX for pcheevd
DOUBLE COMPLEX for pzheevd.
Array, global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)).
The *z* parameter contains the orthonormal eigenvectors of the matrix *A*.

work(1) On exit, returns adequate workspace to allow optimal performance.

rwork(1) (local)
COMPLEX for pcheevd
DOUBLE COMPLEX for pzheevd.
On output, *rwork*(1) returns workspace required to guarantee completion.

iwork(1) (local).
On return, *iwork*(1) contains the amount of integer workspace required.

info (global) INTEGER.
If *info* = 0, the execution is successful.
If *info* < 0:
If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*). If the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
If *info* > 0:
If *info* = 1 through *n*, the *i*-th eigenvalue did not converge.

p?heevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix.

Syntax

call pcheevx(*jobz, range, uplo, n, a, ia, ja, desca, vl, vu, il, iu, abstol, m, nz, w, orfac, z, iz, jz, descz, work, lwork, rwork, lrwork, iwork, liwork, ifail, iclustr, gap, info*)

call pzheevx(*jobz, range, uplo, n, a, ia, ja, desca, vl, vu, il, iu, abstol, m, nz, w, orfac, z, iz, jz, descz, work, lwork, rwork, lrwork, iwork, liwork, ifail, iclustr, gap, info*)

Include Files

- C: mkl_scalapack.h

Description

The p?heevx routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix *A* by calling the recommended sequence of ScaLAPACK routines. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

jobz (global). CHARACTER*1. Must be 'N' or 'V'.
Specifies if it is necessary to compute the eigenvectors:
If *jobz* = 'N', then only eigenvalues are computed.
If *jobz* = 'V', then eigenvalues and eigenvectors are computed.

range (global). CHARACTER*1. Must be 'A', 'V', or 'I'.
If *range* = 'A', all eigenvalues will be found.
If *range* = 'V', all eigenvalues in the half-open interval [*vl*, *vu*] will be found.
If *range* = 'I', the eigenvalues with indices *il* through *iu* will be found.

uplo (global). CHARACTER*1. Must be 'U' or 'L'.
Specifies whether the upper or lower triangular part of the Hermitian matrix *A* is stored:
If *uplo* = 'U', *a* stores the upper triangular part of *A*.
If *uplo* = 'L', *a* stores the lower triangular part of *A*.

n (global) INTEGER. The number of rows and columns of the matrix *A* ($n \geq 0$).

a (local).
COMPLEX for pcheevx
DOUBLE COMPLEX for pzheevx.
Block cyclic array of global dimension (*n*, *n*) and local dimension (*lld_a*, *LOC c*(*ja*+*n*-1)). On entry, the Hermitian matrix *A*.
If *uplo* = 'U', only the upper triangular part of *A* is used to define the elements of the Hermitian matrix.
If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the Hermitian matrix.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*ctxt_*) is incorrect, p?heevx cannot guarantee correct error reporting.

vl, *vu*
(global)
REAL for pcheevx
DOUBLE PRECISION for pzheevx.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; not referenced if *range* = 'A' or 'I'.

il, *iu*
(global)
INTEGER. If *range* ='I', the indices of the smallest and largest eigenvalues to be returned.
Constraints:
il ≥ 1; min(*il*,*n*) ≤ *iu* ≤ *n*.
Not referenced if *range* = 'A' or 'V'.

abstol
(global).
REAL for pcheevx
DOUBLE PRECISION for pzheevx.
If *jobz*='V', setting *abstol* to p?lamch(*context*, 'U') yields the most orthogonal eigenvectors.
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [*a*, *b*] of width less than or equal to *abstol*+*eps**max(|*a*|,|*b*|), where *eps* is the machine precision. If *abstol* is less than or equal to zero, then *eps**norm(T) will be used in its place, where norm(T) is the 1-norm of the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues are computed most accurately when *abstol* is set to twice the underflow threshold 2*p?lamch('S'), not zero. If this routine returns with ((mod(*info*,2).*ne*.0).*or*.(mod(*info*/8,2).*ne*.0)), indicating that some eigenvalues or eigenvectors did not converge, try setting *abstol* to 2*p?lamch('S').

orfac
(global). REAL for pcheevx
DOUBLE PRECISION for pzheevx.
Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within *tol*=*orfac**norm(*A*) of each other are to be reorthogonalized. However, if the workspace is insufficient (see *lwork*), *tol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if *orfac* equals zero. A default value of 1.0e-3 is used if *orfac* is negative.
orfac should be identical on all processes.

iz, *jz*
(global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz
(global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).

work
(local).
COMPLEX for pcheevx
DOUBLE COMPLEX for pzheevx.
Array, DIMENSION *lwork*.

lwork
(local). INTEGER. The dimension of the array *work*.
If only eigenvalues are requested:
lwork ≥ *n* + max(*nb**(*np*0 + 1), 3)
If eigenvectors are requested:
lwork ≥ *n* + (*np*0+*mq*0+*nb*)**nb*
with *nq*0 = numroc(*nn*, *nb*, 0, 0, NPCOL).

lwork ≥ 5*n + max(5*nn, np0*mq0+2*nb*nb) + iceil(neig, NPROW*NPCOL)*nn
For optimal performance, greater workspace is needed, that is
lwork ≥ max(*lwork*, *nhetrd_lwork*)
where *lwork* is as defined above, and *nhetrd_lwork* = *n* + 2*(*anb* +1)*(4**nps*+2) + (*nps*+1)**nps*
ictxt = *desca*(*ctxt_*)
anb = pjlaenv(*ictxt*, 3, 'pchettrd', 'L', 0, 0, 0, 0)
sqnpc = *sqrt*(*dble*(NPROW * NPCOL))
nps = max(numroc(*n*, 1, 0, 0, *sqnpc*), 2**anb*)
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by pxerbla.

rwork (local)
REAL for pcheevx
DOUBLE PRECISION for pzheevx.
Workspace array, DIMENSION *lrwork*.

lrwork (local) INTEGER. The dimension of the array *work*.
See below for definitions of variables used to define *lwork*.
If no eigenvectors are requested (*jobz* = 'N'), then *lrwork* ≥ 5**nn* +4**n*.
If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is:
lrwork ≥ 4**n* + max(5**nn*, *np*0**mq*0+2**nb***nb*) + iceil(*neig*, NPROW*NPCOL)**nn*
The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and *orfac* is too small. If you want to guarantee orthogonality (at the cost of potentially poor performance) you should add the following values to *lrwork*:
(*clustersize*-1)**n*,
where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:
{*w*(*k*),..., *w*(*k*+*clustersize*-1)|*w*(*j*+1) ≤ *w*(*j*) +*orfac**2*norm*(*A*)}.
Variable definitions:
neig = number of eigenvectors requested;
nb = *desca*(*mb_*) = *desca*(*nb_*) = *descz*(*mb_*) = *descz*(*nb_*);
nn = max(*n*, NB, 2);
desca(*rsrc_*) = *desca*(*nb_*) = *descz*(*rsrc_*) = *descz*(*csrc_*) = 0;
*np*0 = numroc(*nn*, *nb*, 0, 0, NPROW);
*mq*0 = numroc(max(*neig*, *nb*, 2), *nb*, 0, 0, NPCOL);
iceil(*x*, *y*) is a ScaLAPACK function returning ceiling(*x*/*y*)
When *lrwork* is too small:
If *lwork* is too small to guarantee orthogonality, p?heevx attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues. If *lwork* is too small to compute all the eigenvectors requested, no computation is performed and *info*= -23 is returned. Note that when *range*='V', p?heevx does not know how many eigenvectors are requested until the eigenvalues are computed. Therefore, when *range*='V'

and as long as *lwork* is large enough to allow p?heevx to compute the eigenvalues, p?heevx will compute the eigenvalues and as many eigenvectors as it can.
Relationship between workspace, orthogonality and performance:
If *clustersize* ≥ *n*/*sqrt*(NPROW*NPCOL), then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. In the limit (that is, *clustersize* = *n*-1) p?stein will perform no better than ?stein on 1 processor.
For *clustersize* = *n*/*sqrt*(NPROW*NPCOL) reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.
For *clustersize* > *n*/*sqrt*(NPROW*NPCOL) execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by pxerbla.

iwork (local) INTEGER. Workspace array.

liwork (local) INTEGER, dimension of *iwork*.
liwork ≥ 6**nnp*
Where: *nnp* = max(*n*, NPROW*NPCOL+1, 4)
If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, the lower triangle (if *uplo* = 'L'), or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

m (global) INTEGER. The total number of eigenvalues found; 0 ≤ *m* ≤ *n*.

nz (global) INTEGER. Total number of eigenvectors computed. 0 ≤ *nz* ≤ *m*.
The number of columns of *z* that are filled.
If *jobz* ≠ 'V', *nz* is not referenced.
If *jobz* = 'V', *nz* = *m* unless the user supplies insufficient space and p?heevx is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in *z* (*m*.*le*.*descz*(*n*_)) and sufficient workspace to compute them. (See *lwork*). p?heevx is always able to detect insufficient space without computation unless *range*.*eq*.'V'.

w (global).
REAL for pcheevx
DOUBLE PRECISION for pzheevx.
Array, DIMENSION (*n*). The first *m* elements contain the selected eigenvalues in ascending order.

z (local).
COMPLEX for pcheevx
DOUBLE COMPLEX for pzheevx.
Array, global dimension (*n*, *n*), local dimension (*lld*_*z*, *LOCc*(*jz*+*n*-1)).

If *jobz* ='V', then on normal exit the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
If *jobz* = 'N', then *z* is not referenced.

work(1) — On exit, returns adequate workspace to allow optimal performance.

rwork — (local).
REAL for pcheevx
DOUBLE PRECISION for pzheevx.
Array, DIMENSION (*lrwork*). On return, *rwork(1)* contains the optimal amount of workspace required for efficient execution.
If *jobz*='N' *rwork(1)* = optimal amount of workspace required to compute eigenvalues efficiently.
If *jobz*='V' *rwork(1)* = optimal amount of workspace required to compute eigenvalues and eigenvectors efficiently with no guarantee on orthogonality.
If *range*='V', it is assumed that all eigenvectors may be required.

iwork(1) — (local)
On return, *iwork*(1) contains the amount of integer workspace required.

ifail — (global) INTEGER.
Array, DIMENSION (*n*).
If *jobz* ='V', then on normal exit, the first *m* elements of *ifail* are zero.
If (mod(*info*,2).*ne*.0) on exit, then *ifail* contains the indices of the eigenvectors that failed to converge.
If *jobz* = 'N', then *ifail* is not referenced.

iclustr — (global) INTEGER.
Array, DIMENSION (2*NPROW*NPCOL).
This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed *iclustr*(2*i*-1) to *iclustr*(2*i*), could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. *iclustr*() is a zero terminated array. (*iclustr*(2**k*).*ne*.0. *and*. *iclustr*(2**k* +1).*eq*.0) if and only if *k* is the number of clusters. *iclustr* is not referenced if *jobz* = 'N'.

gap — (global)
REAL for pcheevx
DOUBLE PRECISION for pzheevx.
Array, DIMENSION (NPROW*NPCOL)
This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*-th cluster may be as high as (*C**n*)/*gap*(*i*) where *C* is a small constant.

info — (global) INTEGER.
If *info* = 0, the execution is successful.
If *info* < 0:
If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*). If the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

If *info* > 0:
If (mod(*info*,2).*ne*.0), then one or more eigenvectors failed to converge. Their indices are stored in *ifail*. Ensure *abstol*=2.0*p?lamch('U')
If (mod(*info*/2,2).*ne*.0), then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace.The indices of the clusters are stored in the array *iclustr*.
If (mod(*info*/4,2).*ne*.0), then space limit prevented p?syevx from computing all of the eigenvectors between *vl* and *vu*. The number of eigenvectors computed is returned in *nz*.
If (mod(*info*/8,2).*ne*.0), then p?stebz failed to compute eigenvalues. Ensure *abstol*=2.0*p?lamch('U').

p?gesvd

Computes the singular value decomposition of a general matrix, optionally computing the left and/or right singular vectors.

Syntax

```
call psgesvd(jobu, jobvt, m, n, a, ia, ja, desca, s, u, iu, ju, descu, vt, ivt, jvt, descvt, work, lwork, info)
```

```
call pdgesvd(jobu, jobvt, m, n, a, ia, ja, desca, s, u, iu, ju, descu, vt, ivt, jvt, descvt, work, lwork, info)
```

```
call pcgesvd(jobu, jobvt, m, n, a, ia, ja, desca, s, u, iu, ju, descu, vt, ivt, jvt, descvt, work, lwork, rwork, info)
```

```
call pzgesvd(jobu, jobvt, m, n, a, ia, ja, desca, s, u, iu, ju, descu, vt, ivt, jvt, descvt, work, lwork, rwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gesvd routine computes the singular value decomposition (SVD) of an *m*-by-*n* matrix *A*, optionally computing the left and/or right singular vectors. The SVD is written

$A = U*\Sigma*V^T$,

where Σ is an *m*-by-*n* matrix that is zero except for its min(*m*, *n*) diagonal elements, *U* is an *m*-by-*m* orthogonal matrix, and *V* is an *n*-by-*n* orthogonal matrix. The diagonal elements of Σ are the singular values of *A* and the columns of *U* and *V* are the corresponding right and left singular vectors, respectively. The singular values are returned in array *s* in decreasing order and only the first min(*m*,*n*) columns of *U* and rows of *vt* = V^T are computed.

Input Parameters

mp = number of local rows in *A* and *U*

nq = number of local columns in *A* and *VT*

size = min(*m*, *n*)

sizeq = number of local columns in *U*

sizep = number of local rows in *VT*

jobu (global). CHARACTER*1. Specifies options for computing all or part of the matrix *U*.
If *jobu* = 'V', the first *size* columns of *U* (the left singular vectors) are returned in the array *u*;
If *jobu* ='N', no columns of *U* (no left singular vectors)are computed.

jobvt (global) CHARACTER*1.
Specifies options for computing all or part of the matrix V^T.
If *jobvt* = 'V', the first *size* rows of V^T (the right singular vectors) are returned in the array *vt*;
If *jobvt* = 'N', no rows of V^T(no right singular vectors) are computed.

m (global) INTEGER. The number of rows of the matrix *A* ($m \geq 0$).

n (global) INTEGER. The number of columns in *A* ($n \geq 0$).

a (local). REAL for psgesvd
DOUBLE PRECISION for pdgesvd
COMPLEX for pcgesvd
COMPLEX*16 for pzgesvd
Block cyclic array, global dimension (*m*, *n*), local dimension (*mp*, *nq*).
work(*lwork*) is a workspace array.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*.

iu, *ju* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *U*, respectively.

descu (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *U*.

ivt, *jvt* (global) INTEGER. The row and column indices in the global array *vt* indicating the first row and the first column of the submatrix *VT*, respectively.

descvt (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *VT*.

work (local). REAL for psgesvd
DOUBLE PRECISION for pdgesvd
COMPLEX for pcgesvd
COMPLEX*16 for pzgesvd
Workspace array, dimension (*lwork*)

lwork (local) INTEGER. The dimension of the array *work*;
lwork > 2 + 6**sizeb* + max(*watobd*, *wbdtosvd*),
where *sizeb* = max(*m*, *n*), and *watobd* and *wbdtosvd* refer, respectively, to the workspace required to bidiagonalize the matrix *A* and to go from the bidiagonal matrix to the singular value decomposition *U S VT*.
For *watobd*, the following holds:
watobd = max(max(*wp?lange*,*wp?gebrd*), max(*wp?lared2d*, *wp?lared1d*)),
where *wp?lange*, *wp?lared1d*, *wp?lared2d*, *wp?gebrd* are the workspaces required respectively for the subprograms p?lange, p?lared1d, p?lared2d, p?gebrd. Using the standard notation
mp = numroc(*m*, *mb*, MYROW, *desca*(*ctxt_*), NPROW),
nq = numroc(*n*, *nb*, MYCOL, *desca*(*lld_*), NPCOL),
the workspaces required for the above subprograms are

wp?lange = *mp*,
wp?lared1d = *nq*0,
wp?lared2d = *mp*0,
wp?gebrd = *nb**(*mp* + *nq* + 1) + *nq*,
where *nq*0 and *mp*0 refer, respectively, to the values obtained at MYCOL = 0 and MYROW = 0. In general, the upper limit for the workspace is given by a workspace required on processor (0,0):
watobd ≤ *nb**(*mp*0 + *nq*0 + 1) + *nq*0.
In case of a homogeneous process grid this upper limit can be used as an estimate of the minimum workspace for every processor.
For *wbdtosvd*, the following holds:
wbdtosvd = *size**(*wantu***nru* + *wantvt***ncvt*) + max(*w?bdsqr*, max(*wantu***wp?ormbrqln*, *wantvt***wp?ormbrprt*)),
where
wantu(*wantvt*) = 1, if left/right singular vectors are wanted, and *wantu*(*wantvt*) = 0, otherwise. *w?bdsqr*, *wp?ormbrqln*, and *wp?ormbrprt* refer respectively to the workspace required for the subprograms ?bdsqr, p?ormbr(*qln*), and p?ormbr(*prt*), where *qln* and *prt* are the values of the arguments *vect*, *side*, and *trans* in the call to p?ormbr. *nru* is equal to the local number of rows of the matrix *U* when distributed 1-dimensional "column" of processes. Analogously, *ncvt* is equal to the local number of columns of the matrix *VT* when distributed across 1-dimensional "row" of processes. Calling the LAPACK procedure ?bdsqr requires
w?bdsqr = max(1, 2**size* + (2**size* - 4)* max(*wantu*, *wantvt*))
on every processor. Finally,
wp?ormbrqln = max((*nb**(*nb*-1))/2, (*sizeq*+*mp*)**nb*)+*nb***nb*,
wp?ormbrprt = max((*mb**(*mb*-1))/2, (*sizep*+*nq*)**mb*)+*mb***mb*,
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum size for the work array. The required workspace is returned as the first element of *work* and no error message is issued by pxerbla.

rwork
REAL for psgesvd
DOUBLE PRECISION for pdgesvd
COMPLEX for pcgesvd
COMPLEX*16 for pzgesvd
Workspace array, dimension (1 + 4**sizeb*)

Output Parameters

a
On exit, the contents of *a* are destroyed.

s
(global). REAL for psgesvd
DOUBLE PRECISION for pdgesvd
COMPLEX for pcgesvd
COMPLEX*16 for pzgesvd
Array, DIMENSION (*size*).
Contains the singular values of *A* sorted so that *s*(i) ≥ *s*(i+1).

u
(local). REAL for psgesvd
DOUBLE PRECISION for pdgesvd
COMPLEX for pcgesvd
COMPLEX*16 for pzgesvd
local dimension (*mp*, *sizeq*), global dimension (*m*, *size*)
If *jobu* = 'V', *u* contains the first min(*m*, *n*) columns of *U*.

	If *jobu* = 'N' or 'O', *u* is not referenced.
vt	(local). REAL for psgesvd DOUBLE PRECISION for pdgesvd COMPLEX for pcgesvd COMPLEX*16 for pzgesvd local dimension (*sizep*, *nq*), global dimension (*size*, *n*) If *jobvt* = 'V', *vt* contains the first *size* rows of V^Tif *jobu* = 'N', *vt* is not referenced.
work	On exit, if *info* = 0, then *work*(1) returns the required minimal size of *lwork*.
rwork	On exit, if *info* = 0, then *rwork*(1) returns the required size of *rwork*.
info	(global) INTEGER. If *info* = 0, the execution is successful. If *info* < 0, If *info* = -*i*, the *i*th parameter had an illegal value. If *info* > 0 *i*, then if ?bdsqr did not converge, If *info* = min(*m*,*n*) + 1, then p?gesvd has detected heterogeneity by finding that eigenvalues were not identical across the process grid. In this case, the accuracy of the results from p?gesvd cannot be guaranteed.

See Also

?bdsqr
p?ormbr
pxerbla

p?sygvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem.

Syntax

call pssygvx(*ibtype*, *jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)

call pdsygvx(*ibtype*, *jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?sygvx routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

sub(*A*)**x* = λ*sub(*B*)**x*, sub(*A*) sub(*B*)**x* = λ**x*, or sub(*B*)*sub(*A*)**x* = λ**x*.

Here *x* denotes eigen vectors, λ (*lambda*) denotes eigenvalues, sub(*A*) denoting A(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) is assumed to symmetric, and sub(*B*) denoting *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*n*-1) is also positive definite.

Input Parameters

ibtype	(global) INTEGER. Must be 1 or 2 or 3.

Specifies the problem type to be solved:
If *ibtype* = 1, the problem type is sub(*A*)**x* = *lambda**sub(*B*)**x*;
If *ibtype* = 2, the problem type is sub(*A*)*sub(*B*)**x* = *lambda***x*;
If *ibtype* = 3, the problem type is sub(*B*)*sub(*A*)**x* = *lambda***x*.

jobz (global). CHARACTER*1. Must be 'N' or 'V'.
If *jobz* ='N', then compute eigenvalues only.
If *jobz* ='V', then compute eigenvalues and eigenvectors.

range (global). CHARACTER*1. Must be 'A' or 'V' or 'I'.
If *range* = 'A', the routine computes all eigenvalues.
If *range* = 'V', the routine computes eigenvalues in the interval: [*vl*, *vu*]
If *range* = 'I', the routine computes eigenvalues with indices *il* through *iu*.

uplo (global). CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', arrays *a* and *b* store the upper triangles of sub(*A*) and sub (*B*);
If *uplo* = 'L', arrays *a* and *b* store the lower triangles of sub(*A*) and sub (*B*).

n (global). INTEGER. The order of the matrices sub(*A*) and sub (*B*), $n \geq 0$.

a (local)
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* symmetric distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *A*. If *desca*(*ctxt_*) is incorrect, p?sygvx cannot guarantee correct error reporting.

b (local). REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
Pointer into the local memory to an array of dimension (*lld_b*, *LOCc*(*jb*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* symmetric distributed matrix sub(*B*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*B*) contains the upper triangular part of the matrix.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix.

ib, *jb* (global) INTEGER. The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.

descb (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *B*. *descb*(*ctxt_*) must be equal to *desca*(*ctxt_*).

vl, *vu* (global)
REAL for pssygvx

DOUBLE PRECISION for pdsygvx.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu* (global)
INTEGER.
If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \le il \le \min(il, n) \le iu \le n$
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol (global)
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
If *jobz*='V', setting *abstol* to p?lamch(*context*, 'U') yields the most orthogonal eigenvectors.
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [*a*,*b*] of width less than or equal to
abstol + *eps**max(|*a*|,|*b*|),
where *eps* is the machine precision. If *abstol* is less than or equal to zero, then *eps**norm(T) will be used in its place, where norm(T) is the 1-norm of the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*p?lamch('S') not zero. If this routine returns with ((mod(*info*,2).*ne*.0).*or*.*(*mod*(*info*/8,2).*ne*.0)), indicating that some eigenvalues or eigenvectors did not converge, try setting *abstol* to 2*p?lamch('S').

orfac (global).
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within *tol*=*orfac**norm(*A*) of each other are to be reorthogonalized. However, if the workspace is insufficient (see *lwork*), *tol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if *orfac* equals zero. A default value of 1.0e-3 is used if *orfac* is negative. *orfac* should be identical on all processes.

iz, *jz* (global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).

work (local)
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
Workspace array, dimension (*lwork*)

lwork (local) INTEGER.
Dimension of the array *work*. See below for definitions of variables used to define *lwork*.
If no eigenvectors are requested (*jobz* = 'N'), then $lwork \ge 5*n + \max(5*nn, NB*(np0 + 1))$.
If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is:

lwork ≥ 5*n + max(5*nn, np0*mq0 + 2*nb*nb) + iceil(neig, NPROW*NPCOL)*nn.

The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and *orfac* is too small. If you want to guarantee orthogonality at the cost of potentially poor performance you should add the following to *lwork*:

(*clustersize*-1)*n,

where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:

{*w*(*k*),..., *w*(*k*+*clustersize*-1)|*w*(*j*+1) ≤ *w*(*j*) + *orfac**2*norm(*A*)}

Variable definitions:

neig = number of eigenvectors requested,

nb = *desca*(*mb_*) = *desca*(*nb_*) = *descz*(*mb_*) = *descz*(*nb_*),

nn = max(*n*, *nb*, 2),

desca(*rsrc_*) = *desca*(*nb_*) = *descz*(*rsrc_*) = *descz*(*csrc_*) = 0,

*np*0 = numroc(*nn*, *nb*, 0, 0, NPROW),

*mq*0 = numroc(max(*neig*, *nb*, 2), *nb*, 0, 0, NPCOL)

iceil(*x*, *y*) is a ScaLAPACK function returning ceiling(*x*/*y*)

If *lwork* is too small to guarantee orthogonality, p?syevx attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.

If *lwork* is too small to compute all the eigenvectors requested, no computation is performed and *info*= -23 is returned.

Note that when *range*='V', number of requested eigenvectors are not known until the eigenvalues are computed. In this case and if *lwork* is large enough to compute the eigenvalues, p?sygvx computes the eigenvalues and as many eigenvectors as possible.

Greater performance can be achieved if adequate workspace is provided. In some situations, performance can decrease as the provided workspace increases above the workspace amount shown below:

lwork ≥ max(*lwork*, 5*n + *nsytrd_lwopt*, *nsygst_lwopt*), where *lwork*, as defined previously, depends upon the number of eigenvectors requested, and

nsytrd_lwopt = *n* + 2*(*anb*+1)*(4**nps*+2) + (*nps*+3)**nps*

nsygst_lwopt = 2**np*0**nb* + *nq*0**nb* + *nb***nb*

anb = pjlaenv(*desca*(*ctxt_*), 3, p?syttrd ', 'L', 0, 0, 0, 0)

sqnpc = int(*sqrt*(*dble*(NPROW * NPCOL)))

nps = max(numroc(*n*, 1, 0, 0, *sqnpc*), 2**anb*)

NB = *desca*(*mb_*)

*np*0 = numroc(*n*, *nb*, 0, 0, NPROW)

*nq*0 = numroc(*n*, *nb*, 0, 0, NPCOL)

numroc is a ScaLAPACK tool functions;

pjlaenv is a ScaLAPACK environmental inquiry function

MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.

For large n, no extra workspace is needed, however the biggest boost in performance comes for small n, so it is wise to provide the extra workspace (typically less than a Megabyte per process).

If *clustersize* ≥ *n*/*sqrt*(NPROW*NPCOL), then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. At the limit (that is, *clustersize* = *n*-1) p?stein will perform no better than ?stein on a single processor.

For *clustersize* = *n*/*sqrt*(NPROW*NPCOL) reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more. For *clustersize* > *n*/*sqrt*(NPROW*NPCOL) execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.
If *lwork* = –1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by pxerbla.

iwork (local) INTEGER. Workspace array.

liwork (local) INTEGER, dimension of *iwork*.
liwork ≥ 6**nnp*
Where:
nnp = max(*n*, NPROW*NPCOL + 1, 4)
If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit,
If *jobz* = 'V', and if *info* = 0, sub(*A*) contains the distributed matrix *Z* of eigenvectors. The eigenvectors are normalized as follows:
for *ibtype* = 1 or 2, Z^T*sub(*B*)**Z* = *I*;
for *ibtype* = 3, Z^T*inv(sub(*B*))**Z* = *I*.
If *jobz* = 'N', then on exit the upper triangle (if *uplo*='U') or the lower triangle (if *uplo*='L') of sub(*A*), including the diagonal, is destroyed.

b On exit, if *info* ≤ *n*, the part of sub(*B*) containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization sub(*B*) = U^T**U* or sub(*B*) = *L***L*^{T}.

m (global) INTEGER. The total number of eigenvalues found, 0 ≤ *m* ≤ *n*.

nz (global) INTEGER.
Total number of eigenvectors computed. 0 ≤ *nz* ≤ *m*. The number of columns of *z* that are filled.
If *jobz* ≠ 'V', *nz* is not referenced.
If *jobz* = 'V', *nz* = *m* unless the user supplies insufficient space and p?sygvx is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in *z* (*m*.*le*.*descz*(*n*_)) and sufficient workspace to compute them. (See *lwork* below.) p?sygvx is always able to detect insufficient space without computation unless *range*.*eq*.'V'.

w (global)
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
Array, DIMENSION (*n*). On normal exit, the first *m* entries contain the selected eigenvalues in ascending order.

z (local).
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.

global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)).
If *jobz* = 'V', then on normal exit the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
If *jobz* = 'N', then *z* is not referenced.

work
If *jobz*='N' *work*(1) = optimal amount of workspace required to compute eigenvalues efficiently
If *jobz* = 'V' *work*(1) = optimal amount of workspace required to compute eigenvalues and eigenvectors efficiently with no guarantee on orthogonality.
If *range*='V', it is assumed that all eigenvectors may be required.

ifail
(global) INTEGER.
Array, DIMENSION (*n*).
ifail provides additional information when *info*.*ne*.0
If (*mod*(*info*/16,2).*ne*.0) then *ifail*(1) indicates the order of the smallest minor which is not positive definite. If (*mod*(*info*,2).*ne*.0) on exit, then *ifail* contains the indices of the eigenvectors that failed to converge.
If neither of the above error conditions hold and *jobz* = 'V', then the first *m* elements of *ifail* are set to zero.

iclustr
(global) INTEGER.
Array, DIMENSION (2*NPROW*NPCOL). This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed *iclustr*(2*i*-1) to *iclustr*(2*i*), could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. *iclustr*() is a zero terminated array.
(*iclustr*(2*k*).*ne*.0.*and*. *iclustr*(2*k*+1).*eq*.0) if and only if *k* is the number of clusters *iclustr* is not referenced if *jobz* = 'N'.

gap
(global)
REAL for pssygvx
DOUBLE PRECISION for pdsygvx.
Array, DIMENSION (NPROW*NPCOL). This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*-th cluster may be as high as (*C**n*)/*gap*(*i*), where *C* is a small constant.

info
(global) INTEGER.
If *info* = 0, the execution is successful.
If *info* <0: the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
If *info* > 0:
If (mod(*info*,2).*ne*.0), then one or more eigenvectors failed to converge. Their indices are stored in *ifail*.

If (mod(*info*,2,2).*ne*.0), then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in the array *iclustr*.
If (mod(*info*/4,2).*ne*.0), then space limit prevented p?sygvx from computing all of the eigenvectors between *vl* and *vu*. The number of eigenvectors computed is returned in *nz*.
If (mod(*info*/8,2).*ne*.0), then p?stebz failed to compute eigenvalues.
If (mod(*info*/16,2).*ne*.0), then *B* was not positive definite. *ifail*(*1*) indicates the order of the smallest minor which is not positive definite.

p?hegvx

Computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian definite eigenproblem.

Syntax

call pchegvx(*ibtype*, *jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *rwork*, *lrwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)

call pzhegvx(*ibtype*, *jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *descz*, *work*, *lwork*, *rwork*, *lrwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?hegvx routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian-definite eigenproblem, of the form

sub(*A*)*x = λ*sub(*B*)*x, sub(*A*)*sub(*B*)*x = λ*x, or sub(*B*)*sub(*A*)*x = λ*x.

Here sub (*A*) denoting *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) and sub(*B*) denoting *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*n*-1) is also positive definite.

Input Parameters

ibtype — (global) INTEGER. Must be 1 or 2 or 3.
Specifies the problem type to be solved:
If *ibtype* = 1, the problem type is sub(*A*)*x = *lambda**sub(*B*)*x;
If *ibtype* = 2, the problem type is sub(*A*)*sub(*B*)*x = *lambda***x*;
If *ibtype* = 3, the problem type is sub(*B*)*sub(*A*)*x = *lambda***x*.

jobz — (global). CHARACTER*1. Must be 'N' or 'V'.
If *jobz* ='N', then compute eigenvalues only.
If *jobz* ='V', then compute eigenvalues and eigenvectors.

range — (global). CHARACTER*1. Must be 'A' or 'V' or 'I'.
If *range* = 'A', the routine computes all eigenvalues.

If *range* = 'V', the routine computes eigenvalues in the interval: [*vl*, *vu*]
If *range* = 'I', the routine computes eigenvalues with indices *il* through *iu*.

uplo (global). CHARACTER*1. Must be 'U' or 'L'.
If *uplo* = 'U', arrays *a* and *b* store the upper triangles of sub(*A*) and sub (*B*);
If *uplo* = 'L', arrays *a* and *b* store the lower triangles of sub(*A*) and sub (*B*).

n (global). INTEGER.
The order of the matrices sub(*A*) and sub (*B*) ($n \geq 0$).

a (local)
COMPLEX for pchegvx
DOUBLE COMPLEX for pzhegvx.
Pointer into the local memory to an array of dimension (*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* Hermitian distributed matrix sub(*A*). If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix.

ia, *ja* (global) INTEGER.
The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, dimension (*dlen_*).
The array descriptor for the distributed matrix *A*. If *desca*(*ctxt_*) is incorrect, p?hegvx cannot guarantee correct error reporting.

b (local).
COMPLEX for pchegvx
DOUBLE COMPLEX for pzhegvx.
Pointer into the local memory to an array of dimension (*lld_b*, *LOCc*(*jb*+*n*-1)). On entry, this array contains the local pieces of the *n*-by-*n* Hermitian distributed matrix sub(*B*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*B*) contains the upper triangular part of the matrix.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*B*) contains the lower triangular part of the matrix.

ib, *jb* (global) INTEGER.
The row and column indices in the global array *b* indicating the first row and the first column of the submatrix *B*, respectively.

descb (global and local) INTEGER array, dimension (*dlen_*).
The array descriptor for the distributed matrix *B*. *descb*(*ctxt_*) must be equal to *desca*(*ctxt_*).

vl, *vu* (global)
REAL for pchegvx
DOUBLE PRECISION for pzhegvx.
If *range* = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.
If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, *iu* (global)
INTEGER.

If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: *il* ≥ 1, min(*il*, *n*) ≤ *iu* ≤ *n*
If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol (global)
REAL for pchegvx
DOUBLE PRECISION for pzhegvx.
If *jobz*='V', setting *abstol* to p?lamch(*context*, 'U') yields the most orthogonal eigenvectors.
The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [*a*,*b*] of width less than or equal to
abstol + *eps**max(|*a*|,|*b*|),
where *eps* is the machine precision. If *abstol* is less than or equal to zero, then *eps**norm(T) will be used in its place, where norm(T) is the 1-norm of the tridiagonal matrix obtained by reducing *A* to tridiagonal form.
Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold 2*p?lamch('S') not zero. If this routine returns with ((mod(*info*,2).*ne*.0).*or*. * (*mod*(*info*/8,2).*ne*.0)), indicating that some eigenvalues or eigenvectors did not converge, try setting *abstol* to 2*p?lamch('S').

orfac (global).
REAL for pchegvx
DOUBLE PRECISION for pzhegvx.
Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within *tol*=*orfac**norm(*A*) of each other are to be reorthogonalized. However, if the workspace is insufficient (see *lwork*), *tol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if *orfac* equals zero. A default value of 1.0E-3 is used if *orfac* is negative. *orfac* should be identical on all processes.

iz, *jz* (global) INTEGER. The row and column indices in the global array *z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz (global and local) INTEGER array, dimension (*dlen_*). The array descriptor for the distributed matrix *Z*. *descz*(*ctxt_*) must equal *desca*(*ctxt_*).

work (local)
COMPLEX for pchegvx
DOUBLE COMPLEX for pzhegvx.
Workspace array, dimension (*lwork*)

lwork (local).
INTEGER. The dimension of the array *work*.
If only eigenvalues are requested:
lwork ≥ *n*+ max(NB*(*np*0 + 1), 3)
If eigenvectors are requested:
lwork ≥ *n* + (*np*0+ *mq*0 + NB)*NB
with *nq*0 = numroc(*nn*, NB, 0, 0, NPCOL).
For optimal performance, greater workspace is needed, that is
lwork ≥ max(*lwork*, *n*, *nhetrd_lwopt*, *nhegst_lwopt*)
where *lwork* is as defined above, and
nhetrd_lwork = 2*(*anb*+1)*(4**nps*+2) + (*nps* + 1)**nps*;
nhegst_lwopt = 2**np*0**nb* + *nq*0**nb* + *nb***nb*
nb = *desca*(*mb_*)

*np*0 = numroc(*n*, *nb*, 0, 0, NPROW)
*nq*0 = numroc(*n*, *nb*, 0, 0, NPCOL)
ictxt = *desca*(*ctxt_*)
anb = pjlaenv(*ictxt*, 3, 'p?hettrd', 'L', 0, 0, 0, 0)
sqnpc = *sqrt*(*dble*(NPROW * NPCOL))
nps = max(numroc(*n*, 1, 0, 0, *sqnpc*), 2**anb*)
numroc is a ScaLAPACK tool functions;
pjlaenv is a ScaLAPACK environmental inquiry function MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by pxerbla.

rwork (local)
REAL for pchegvx
DOUBLE PRECISION for pzhegvx.
Workspace array, DIMENSION (*lrwork*).

lrwork (local) INTEGER. The dimension of the array *rwork*.
See below for definitions of variables used to define *lrwork*.
If no eigenvectors are requested (*jobz* = 'N'), then *lrwork* ≥ 5**nn*+4**n*
If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is:
lrwork ≥ 4**n* + max(5**nn*, *np*0**mq*0)+ iceil(*neig*, NPROW*NPCOL)**nn*
The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and *orfac* is too small. If you want to guarantee orthogonality (at the cost of potentially poor performance) you should add the following value to *lrwork*:
(*clustersize*-1)**n*,
where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:
{*w*(*k*),..., *w*(*k*+*clustersize*-1)| *w*(*j*+1) ≤ *w*(*j*)+*orfac**2*norm*(*A*)}
Variable definitions:
neig = number of eigenvectors requested;
nb = *desca*(*mb_*) = *desca*(*nb_*) = *descz*(*mb_*) = *descz*(*nb_*);
nn = max(*n*, *nb*, 2);
desca(*rsrc_*) = *desca*(*nb_*) = *descz*(*rsrc_*) = *descz*(*csrc_*) = 0 ;
*np*0 = numroc(*nn*, *nb*, 0, 0, NPROW);
*mq*0 = numroc(max(*neig*, *nb*, 2), *nb*, 0, 0, NPCOL);
iceil(*x*, *y*) is a ScaLAPACK function returning ceiling(*x*/*y*).
When *lrwork* is too small:
If *lwork* is too small to guarantee orthogonality, p?hegvx attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.
If *lwork* is too small to compute all the eigenvectors requested, no computation is performed and *info*= -25 is returned. Note that when *range*='V', p?hegvx does not know how many eigenvectors are requested until the eigenvalues are computed. Therefore, when *range*='V' and as

long as *lwork* is large enough to allow p?hegvx to compute the eigenvalues, p?hegvx will compute the eigenvalues and as many eigenvectors as it can.
Relationship between workspace, orthogonality & performance:
If *clustersize* > *n*/*sqrt*(NPROW*NPCOL), then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. In the limit (that is, *clustersize* = *n*-1) p?stein will perform no better than ?stein on 1 processor.
For *clustersize* = *n*/*sqrt*(NPROW*NPCOL) reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.
For *clustersize* > *n*/*sqrt*(NPROW*NPCOL) execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.
If *lwork* = -1, then *lrwork* is global input and a workspace query is assumed; the routine only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by pxerbla.

iwork (local) INTEGER. Workspace array.

liwork (local) INTEGER, dimension of *iwork*.
liwork ≥ 6**nnp*
Where: *nnp* = max(*n*, NPROW*NPCOL + 1, 4)
If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, if *jobz* = 'V', then if *info* = 0, sub(*A*) contains the distributed matrix *Z* of eigenvectors.
The eigenvectors are normalized as follows:
If *ibtype* = 1 or 2, then Z^H*sub(*B*)**Z* = *i*;
If *ibtype* = 3, then Z^H*inv(sub(*B*))**Z* = *i*.
If *jobz* = 'N', then on exit the upper triangle (if *uplo*='U') or the lower triangle (if *uplo*='L') of sub(*A*), including the diagonal, is destroyed.

b On exit, if *info* ≤ *n*, the part of sub(*B*) containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization sub(*B*) = U^H**U*, or sub(*B*) = *L***L*^{H}.

m (global) INTEGER. The total number of eigenvalues found, 0 ≤ *m* ≤ *n*.

nz (global) INTEGER. Total number of eigenvectors computed. 0 < *nz* < *m*.
The number of columns of *z* that are filled.
If *jobz* ≠ 'V', *nz* is not referenced.
If *jobz* = 'V', *nz* = *m* unless the user supplies insufficient space and p?hegvx is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in *z* (*m*. *le*. *descz*(*n*_)) and sufficient workspace to compute them. (See *lwork* below.) The routine p?hegvx is always able to detect insufficient space without computation unless *range* = 'V'.

w (global)
REAL for pchegvx

DOUBLE PRECISION for pzhegvx.
Array, DIMENSION (*n*). On normal exit, the first *m* entries contain the selected eigenvalues in ascending order.

z (local).
COMPLEX for pchegvx
DOUBLE COMPLEX for pzhegvx.
global dimension (*n*, *n*), local dimension (*lld_z*, *LOCc*(*jz*+*n*-1)).
If *jobz* = 'V', then on normal exit the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
If *jobz* = 'N', then *z* is not referenced.

work On exit, *work*(1) returns the optimal amount of workspace.

rwork On exit, *rwork*(1) contains the amount of workspace required for optimal efficiency
If *jobz*='N' *rwork*(1) = optimal amount of workspace required to compute eigenvalues efficiently
If *jobz*='V' *rwork*(1) = optimal amount of workspace required to compute eigenvalues and eigenvectors efficiently with no guarantee on orthogonality.
If *range*='V', it is assumed that all eigenvectors may be required when computing optimal workspace.

ifail (global) INTEGER.
Array, DIMENSION (*n*).
ifail provides additional information when *info*.*ne*.0
If (*mod*(*info*/16,2).*ne*.0), then *ifail*(1) indicates the order of the smallest minor which is not positive definite.
If (*mod*(*info*,2).*ne*.0) on exit, then *ifail*(1) contains the indices of the eigenvectors that failed to converge.
If neither of the above error conditions are held, and *jobz* = 'V', then the first *m* elements of *ifail* are set to zero.

iclustr (global) INTEGER.
Array, DIMENSION (2*NPROW*NPCOL). This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed *iclustr*(2*i*-1) to *iclustr*(2*i*), could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal.
iclustr() is a zero terminated array. (*iclustr*(2*k*).*ne*.0.*and*.*clustr*(2*k*+1).*eq*.0) if and only if *k* is the number of clusters.
iclustr is not referenced if *jobz* = 'N'.

gap (global)
REAL for pchegvx
DOUBLE PRECISION for pzhegvx.
Array, DIMENSION (NPROW*NPCOL).
This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*-th cluster may be as high as (*C**n*)/*gap*(*i*), where *C* is a small constant.

info (global) INTEGER.
If *info* = 0, the execution is successful.
If *info* <0: the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
If *info* > 0:
If (mod(*info*,2).*ne*.0), then one or more eigenvectors failed to converge. Their indices are stored in *ifail*.
If (mod(*info*,2,2).*ne*.0), then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in the array *iclustr*.
If (mod(*info*/4,2).*ne*.0), then space limit prevented p?sygvx from computing all of the eigenvectors between *vl* and *vu*. The number of eigenvectors computed is returned in *nz*.
If (mod(*info*/8,2).*ne*.0), then p?stebz failed to compute eigenvalues.
If (mod(*info*/16,2).*ne*.0), then *B* was not positive definite. *ifail(1*) indicates the order of the smallest minor which is not positive definite.

ScaLAPACK Auxiliary and Utility Routines

7

This chapter describes the Intel® Math Kernel Library implementation of ScaLAPACK Auxiliary Routines and Utility Functions and Routines. The library includes routines for both real and complex data.

NOTE ScaLAPACK routines are provided only with Intel® MKL versions for Linux* and Windows* OSs.

Routine naming conventions, mathematical notation, and matrix storage schemes used for ScaLAPACK auxiliary and utility routines are the same as described in previous chapters. Some routines and functions may have combined character codes, such as `sc` or `dz`. For example, the routine `pscsum1` uses a complex input array and returns a real value.

Auxiliary Routines

ScaLAPACK Auxiliary Routines

Routine Name	Data Types	Description
p?lacgv	c,z	Conjugates a complex vector.
p?max1	c,z	Finds the index of the element whose real part has maximum absolute value (similar to the Level 1 PBLAS p?amax, but using the absolute value to the real part).
?combamax1	c,z	Finds the element with maximum real part absolute value and its corresponding global index.
p?sum1	sc,dz	Forms the 1-norm of a complex vector similar to Level 1 PBLAS p?asum, but using the true absolute value.
p?dbtrsv	s,d,c,z	Computes an *LU* factorization of a general tridiagonal matrix with no pivoting. The routine is called by p?dbtrs.
p?dttrsv	s,d,c,z	Computes an *LU* factorization of a general band matrix, using partial pivoting with row interchanges. The routine is called by p?dttrs.
p?gebd2	s,d,c,z	Reduces a general rectangular matrix to real bidiagonal form by an orthogonal/unitary transformation (unblocked algorithm).
p?gehd2	s,d,c,z	Reduces a general matrix to upper Hessenberg form by an orthogonal/unitary similarity transformation (unblocked algorithm).
p?gelq2	s,d,c,z	Computes an *LQ* factorization of a general rectangular matrix (unblocked algorithm).
p?geql2	s,d,c,z	Computes a *QL* factorization of a general rectangular matrix (unblocked algorithm).
p?geqr2	s,d,c,z	Computes a *QR* factorization of a general rectangular matrix (unblocked algorithm).
p?gerq2	s,d,c,z	Computes an *RQ* factorization of a general rectangular matrix (unblocked algorithm).

Routine Name	Data Types	Description
p?getf2	s,d,c,z	Computes an *LU* factorization of a general matrix, using partial pivoting with row interchanges (local blocked algorithm).
p?labrd	s,d,c,z	Reduces the first *nb* rows and columns of a general rectangular matrix A to real bidiagonal form by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A.
p?lacon	s,d,c,z	Estimates the 1-norm of a square matrix, using the reverse communication for evaluating matrix-vector products.
p?laconsb	s,d	Looks for two consecutive small subdiagonal elements.
p?lacp2	s,d,c,z	Copies all or part of a distributed matrix to another distributed matrix.
p?lacp3	s,d	Copies from a global parallel array into a local replicated array or vice versa.
p?lacpy	s,d,c,z	Copies all or part of one two-dimensional array to another.
p?laevswp	s,d,c,z	Moves the eigenvectors from where they are computed to ScaLAPACK standard block cyclic array.
p?lahrd	s,d,c,z	Reduces the first *nb* columns of a general rectangular matrix A so that elements below the k^{th} subdiagonal are zero, by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A.
p?laiect	s,d,c,z	Exploits IEEE arithmetic to accelerate the computations of eigenvalues. (C interface function).
p?lange	s,d,c,z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a general rectangular matrix.
p?lanhs	s,d,c,z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of an upper Hessenberg matrix.
p?lansy, p?lanhe	s,d,c,z/ c,z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a real symmetric or complex Hermitian matrix.
p?lantr	s,d,c,z	Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a triangular matrix.
p?lapiv	s,d,c,z	Applies a permutation matrix to a general distributed matrix, resulting in row or column pivoting.
p?laqge	s,d,c,z	Scales a general rectangular matrix, using row and column scaling factors computed by p?geequ.
p?laqsy	s,d,c,z	Scales a symmetric/Hermitian matrix, using scaling factors computed by p?poequ.
p?lared1d	s,d	Redistributes an array assuming that the input array *bycol* is distributed across rows and that all process columns contain the same copy of *bycol*.

Routine Name	Data Types	Description
p?lared2d	s,d	Redistributes an array assuming that the input array *byrow* is distributed across columns and that all process rows contain the same copy of *byrow* .
p?larf	s,d,c,z	Applies an elementary reflector to a general rectangular matrix.
p?larfb	s,d,c,z	Applies a block reflector or its transpose/conjugate-transpose to a general rectangular matrix.
p?larfc	c,z	Applies the conjugate transpose of an elementary reflector to a general matrix.
p?larfg	s,d,c,z	Generates an elementary reflector (Householder matrix).
p?larft	s,d,c,z	Forms the triangular vector *T* of a block reflector $H=I-VTV^H$
p?larz	s,d,c,z	Applies an elementary reflector as returned by p?tzrzf to a general matrix.
p?larzb	s,d,c,z	Applies a block reflector or its transpose/conjugate-transpose as returned by p?tzrzf to a general matrix.
p?larzc	c,z	Applies (multiplies by) the conjugate transpose of an elementary reflector as returned by p?tzrzf to a general matrix.
p?larzt	s,d,c,z	Forms the triangular factor *T* of a block reflector $H=I-VTV^H$ as returned by p?tzrzf.
p?lascl	s,d,c,z	Multiplies a general rectangular matrix by a real scalar defined as C_{to}/C_{from}.
p?laset	s,d,c,z	Initializes the off-diagonal elements of a matrix to α and the diagonal elements to β.
p?lasmsub	s,d	Looks for a small subdiagonal element from the bottom of the matrix that it can safely set to zero.
p?lassq	s,d,c,z	Updates a sum of squares represented in scaled form.
p?laswp	s,d,c,z	Performs a series of row interchanges on a general rectangular matrix.
p?latra	s,d,c,z	Computes the trace of a general square distributed matrix.
p?latrd	s,d,c,z	Reduces the first *nb* rows and columns of a symmetric/Hermitian matrix A to real tridiagonal form by an orthogonal/unitary similarity transformation.
p?latrz	s,d,c,z	Reduces an upper trapezoidal matrix to upper triangular form by means of orthogonal/unitary transformations.
p?lauu2	s,d,c,z	Computes the product UU^H or L^HL, where *U* and *L* are upper or lower triangular matrices (local unblocked algorithm).
p?lauum	s,d,c,z	Computes the product UU^H or L^HL, where *U* and *L* are upper or lower triangular matrices.
p?lawil	s,d	Forms the Wilkinson transform.
p?org2l/p?ung2l	s,d,c,z	Generates all or part of the orthogonal/unitary matrix *Q* from a *QL* factorization determined by p?geqlf (unblocked algorithm).

Routine Name	Data Types	Description
p?org2r/p?ung2r	s,d,c,z	Generates all or part of the orthogonal/unitary matrix *Q* from a *QR* factorization determined by p?geqrf (unblocked algorithm).
p?orgl2/p?ungl2	s,d,c,z	Generates all or part of the orthogonal/unitary matrix *Q* from an *LQ* factorization determined by p?gelqf (unblocked algorithm).
p?orgr2/p?ungr2	s,d,c,z	Generates all or part of the orthogonal/unitary matrix *Q* from an *RQ* factorization determined by p?gerqf (unblocked algorithm).
p?orm2l/p?unm2l	s,d,c,z	Multiplies a general matrix by the orthogonal/unitary matrix from a *QL* factorization determined by p?geqlf (unblocked algorithm).
p?orm2r/p?unm2r	s,d,c,z	Multiplies a general matrix by the orthogonal/unitary matrix from a *QR* factorization determined by p?geqrf (unblocked algorithm).
p?orml2/p?unml2	s,d,c,z	Multiplies a general matrix by the orthogonal/unitary matrix from an *LQ* factorization determined by p?gelqf (unblocked algorithm).
p?ormr2/p?unmr2	s,d,c,z	Multiplies a general matrix by the orthogonal/unitary matrix from an *RQ* factorization determined by p?gerqf (unblocked algorithm).
p?pbtrsv	s,d,c,z	Solves a single triangular linear system via frontsolve or backsolve where the triangular matrix is a factor of a banded matrix computed by p?pbtrf.
p?pttrsv	s,d,c,z	Solves a single triangular linear system via frontsolve or backsolve where the triangular matrix is a factor of a tridiagonal matrix computed by p?pttrf.
p?potf2	s,d,c,z	Computes the Cholesky factorization of a symmetric/Hermitian positive definite matrix (local unblocked algorithm).
p?rscl	s,d,cs,zd	Multiplies a vector by the reciprocal of a real scalar.
p?sygs2/p?hegs2	s,d,c,z	Reduces a symmetric/Hermitian definite generalized eigenproblem to standard form, using the factorization results obtained from p?potrf (local unblocked algorithm).
p?sytd2/p?hetd2	s,d,c,z	Reduces a symmetric/Hermitian matrix to real symmetric tridiagonal form by an orthogonal/unitary similarity transformation (local unblocked algorithm).
p?trti2	s,d,c,z	Computes the inverse of a triangular matrix (local unblocked algorithm).
?lamsh	s,d	Sends multiple shifts through a small (single node) matrix to maximize the number of bulges that can be sent through.
?laref	s,d	Applies Householder reflectors to matrices on either their rows or columns.
?lasorte	s,d	Sorts eigenpairs by real and complex data types.
?lasrt2	s,d	Sorts numbers in increasing or decreasing order.
?stein2	s,d	Computes the eigenvectors corresponding to specified eigenvalues of a real symmetric tridiagonal matrix, using inverse iteration.
?dbtf2	s,d,c,z	Computes an *LU* factorization of a general band matrix with no pivoting (local unblocked algorithm).

Routine Name	Data Types	Description
?dbtrf	s,d,c,z	Computes an *LU* factorization of a general band matrix with no pivoting (local blocked algorithm).
?dttrf	s,d,c,z	Computes an *LU* factorization of a general tridiagonal matrix with no pivoting (local blocked algorithm).
?dttrsv	s,d,c,z	Solves a general tridiagonal system of linear equations using the *LU* factorization computed by ?dttrf.
?pttrsv	s,d,c,z	Solves a symmetric (Hermitian) positive-definite tridiagonal system of linear equations, using the LDL^H factorization computed by ?pttrf.
?steqr2	s,d	Computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit *QL* or *QR* method.

p?lacgv

Conjugates a complex vector.

Syntax

```
call pclacgv(n, x, ix, jx, descx, incx)
call pzlacgv(n, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lacgv routine conjugates a complex vector sub(*x*) of length *n*, where sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*:*ix*+*n*-1, *jx*) if *incx* = 1.

Input Parameters

n	(global) INTEGER. The length of the distributed vector sub(*x*).
x	(local). COMPLEX for pclacgv COMPLEX*16 for pzlacgv.Pointer into the local memory to an array of DIMENSION (*lld_x*,*). On entry the vector to be conjugated *x*(i) = *X*(*ix*+(*jx*-1)*m_x+(*i*-1)**incx*), 1 ≤ *i* ≤ *n*.
ix	(global) INTEGER.The row index in the global array *x* indicating the first row of sub(*x*).
jx	(global) INTEGER. The column index in the global array *x* indicating the first column of sub(*x*).
descx	(global and local) INTEGER. Array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *X*.
incx	(global) INTEGER.The global increment for the elements of *X*. Only two values of *incx* are supported in this version, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

x	(local). On exit, the conjugated vector.

p?max1

Finds the index of the element whose real part has maximum absolute value (similar to the Level 1 PBLAS p?amax*, but using the absolute value to the real part).*

Syntax

```
call pcmax1(n, amax, indx, x, ix, jx, descx, incx)
call pzmax1(n, amax, indx, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_scalapack.h

Description

The p?max1 routine computes the global index of the maximum element in absolute value of a distributed vector sub(*x*). The global index is returned in *indx* and the value is returned in *amax*, where sub(*x*) denotes *X*(*ix*:*ix*+*n*-1, *jx*) if *incx* = 1, *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*.

Input Parameters

n	(global) pointer to INTEGER. The number of components of the distributed vector sub(*x*). $n \geq 0$.
x	(local) COMPLEX for pcmax1. COMPLEX*16 for pzmax1 Array containing the local pieces of a distributed matrix of dimension of at least ((*jx*-1)*m_x+*ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub (*x*).
ix	(global) INTEGER.The row index in the global array *X* indicating the first row of sub(*x*).
jx	(global) INTEGER. The column index in the global array *X* indicating the first column of sub(*x*)
descx	(global and local) INTEGER. Array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *X*.
incx	(global) INTEGER.The global increment for the elements of *X*. Only two values of *incx* are supported in this version, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

amax	(global output) pointer to REAL.The absolute value of the largest entry of the distributed vector sub(*x*) only in the scope of sub(*x*).
indx	(global output) pointer to INTEGER.The global index of the element of the distributed vector sub(*x*) whose real part has maximum absolute value.

?combamax1

Finds the element with maximum real part absolute value and its corresponding global index.

Syntax

```
call ccombamax1(v1, v2)
call zcombamax1(v1, v2)
```

Include Files

- C: mkl_scalapack.h

Description

The ?combamax1 routine finds the element having maximum real part absolute value as well as its corresponding global index.

Input Parameters

v1	(local) COMPLEX for ccombamax1 COMPLEX*16 for zcombamax1 Array, DIMENSION 2. The first maximum absolute value element and its global index. *v1*(1)=amax, *v1*(2)=indx.
v2	(local) COMPLEX for ccombamax1 COMPLEX*16 for zcombamax1 Array, DIMENSION 2. The second maximum absolute value element and its global index. *v2*(1)=amax, *v2*(2)=indx.

Output Parameters

v1	(local). The first maximum absolute value element and its global index. *v1*(1)=amax, *v1*(2)=indx.

p?sum1

Forms the 1-norm of a complex vector similar to Level 1 PBLAS p?asum*, but using the true absolute value.*

Syntax

```
call pscsum1(n, asum, x, ix, jx, descx, incx)
call pdzsum1(n, asum, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_scalapack.h

Description

The p?sum1 routine returns the sum of absolute values of a complex distributed vector sub(*x*) in *asum*, where sub(*x*) denotes *X*(*ix*:*ix*+*n*-1, *jx*:*jx*), if *incx* = 1, *X*(*ix*:*ix*, *jx*:*jx*+*n*-1), if *incx* = *m_x*.

Based on p?asum from the Level 1 PBLAS. The change is to use the 'genuine' absolute value.

Input Parameters

n	(global) pointer to INTEGER. The number of components of the distributed vector sub(*x*). *n* ≥ 0.
x	(local) COMPLEX for pscsum1 COMPLEX*16 for pdzsum1. Array containing the local pieces of a distributed matrix of dimension of at least ((*jx*-1)*m_x+*ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub (*x*).
ix	(global) INTEGER.The row index in the global array *X* indicating the first row of sub(*x*).
jx	(global) INTEGER. The column index in the global array *X* indicating the first column of sub(*x*)
descx	(global and local) INTEGER. Array, DIMENSION 8. The array descriptor for the distributed matrix *X*.
incx	(global) INTEGER.The global increment for the elements of *X*. Only two values of *incx* are supported in this version, namely 1 and *m_x*.

Output Parameters

asum	(local) Pointer to REAL. The sum of absolute values of the distributed vector sub(*x*) only in its scope.

p?dbtrsv

Computes an LU factorization of a general triangular matrix with no pivoting. The routine is called by p?dbtrs.

Syntax

```
call psdbtrsv(uplo, trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

```
call pddbtrsv(uplo, trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

```
call pcdbtrsv(uplo, trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

```
call pzdbtrsv(uplo, trans, n, bwl, bwu, nrhs, a, ja, desca, b, ib, descb, af, laf, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dbtrsv routine solves a banded triangular system of linear equations

A(1 :*n*, *ja*:*ja*+*n*-1) * *X* = *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*) or

A(1 :*n*, *ja*:*ja*+*n*-1)^T * *X* = *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*) (for real flavors); *A*(1 :*n*, *ja*:*ja*+*n*-1)^H * *X* = *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*) (for complex flavors),

where *A*(1 :*n*, *ja*:*ja*+*n*−1) is a banded triangular matrix factor produced by the Gaussian elimination code PD@(dom_pre)BTRF and is stored in *A*(1 :*n*, *ja*:*ja*+*n*−1) and *af*. The matrix stored in *A*(1 :*n*, *ja*:*ja*+*n*−1) is either upper or lower triangular according to *uplo*, and the choice of solving *A*(1 :*n*, *ja*:*ja*+*n*−1) or *A*(1 :*n*, *ja*:*ja*+*n*−1)^T is dictated by the user by the parameter *trans*.

Routine p?dbtrf must be called first.

Input Parameters

uplo (global) CHARACTER.
If *uplo*='U', the upper triangle of *A*(1:*n*, *ja*:*ja*+*n*−1) is stored,
if *uplo* = 'L', the lower triangle of *A*(1:*n*, *ja*:*ja*+*n*−1) is stored.

trans (global) CHARACTER.
If *trans* = 'N', solve with *A*(1:*n*, *ja*:*ja*+*n*−1),
if *trans* = 'C', solve with conjugate transpose *A*(1:*n*, *ja*:*ja*+*n*−1).

n (global) INTEGER. The order of the distributed submatrix *A*;(*n*≥ 0).

bwl (global) INTEGER. Number of subdiagonals. 0 ≤ *bwl* ≤ *n*-1.

bwu (global) INTEGER. Number of subdiagonals. 0 ≤ *bwu* ≤ *n*-1.

nrhs (global) INTEGER. The number of right-hand sides; the number of columns of the distributed submatrix *B* (*nrhs*≥ 0).

a (local).
REAL for psdbtrsv
DOUBLE PRECISION for pddbtrsv
COMPLEX for pcdbtrsv
COMPLEX*16 for pzdbtrsv.
Pointer into the local memory to an array with first DIMENSION *lld_a*≥(*bwl*+*bwu*+1)(stored in *desca*). On entry, this array contains the local pieces of the *n*-by-*n* unsymmetric banded distributed Cholesky factor *L* or L^T*A(1 :*n*, *ja*:*ja*+*n*−1). This local portion is stored in the packed banded format used in LAPACK. See the *Application Notes* below and the ScaLAPACK manual for more detail on the format of distributed matrices.

ja (global) INTEGER. The index in the global array *a* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array of DIMENSION (*dlen_*).
if 1*d* type (*dtype_a* = 501 or 502), *dlen*≥ 7;
if 2*d* type (*dtype_a* = 1), *dlen*≥ 9. The array descriptor for the distributed matrix *A*. Contains information of mapping of *A* to memory.

b (local)
REAL for psdbtrsv
DOUBLE PRECISION for pddbtrsv
COMPLEX for pcdbtrsv
COMPLEX*16 for pzdbtrsv.
Pointer into the local memory to an array of local lead DIMENSION *lld_b*≥*nb*. On entry, this array contains the local pieces of the right-hand sides *B*(*ib*:*ib*+*n*−1, 1:*nrhs*).

ib (global) INTEGER. The row index in the global array *b* that points to the first row of the matrix to be operated on (which may be either all of *b* or a submatrix of *B*).

descb (global and local) INTEGER array of DIMENSION (*dlen_*).
if 1*d* type (*dtype_b* =502), *dlen*≥7;

	if 2*d* type (*dtype_b* =1), *dlen*≥9. The array descriptor for the distributed matrix *B*. Contains information of mapping *B* to memory.
laf	(local) INTEGER. Size of user-input Auxiliary Filling space *af*. *laf* must be ≥*nb**(*bwl*+*bwu*)+6*max(*bwl*, *bwu*)*max(*bwl*, *bwu*). If *laf* is not large enough, an error code is returned and the minimum acceptable size will be returned in *af*(1).
work	(local). REAL for psdbtrsv DOUBLE PRECISION for pddbtrsv COMPLEX for pcdbtrsv COMPLEX*16 for pzdbtrsv. Temporary workspace. This space may be overwritten in between calls to routines. *work* must be the size given in *lwork*.
lwork	(local or global) INTEGER. Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned. *lwork*≥ max(*bwl*, *bwu*)**nrhs*.

Output Parameters

a	(local). This local portion is stored in the packed banded format used in LAPACK. Please see the ScaLAPACK manual for more detail on the format of distributed matrices.
b	On exit, this contains the local piece of the solutions distributed matrix *X*.
af	(local). REAL for psdbtrsv DOUBLE PRECISION for pddbtrsv COMPLEX for pcdbtrsv COMPLEX*16 for pzdbtrsv. Auxiliary Filling Space. Filling is created during the factorization routine p?dbtrf and this is stored in *af*. If a linear system is to be solved using p?dbtrf after the factorization routine, *af* must not be altered after the factorization.
work	On exit, *work*(1) contains the minimal *lwork*.
info	(local). INTEGER. If *info* = 0, the execution is successful. < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?dttrsv

Computes an LU factorization of a general band matrix, using partial pivoting with row interchanges. The routine is called by p?dttrs.

Syntax

```
call psdttrsv(uplo, trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf,
work, lwork, info)
```

```
call pddttrsv(uplo, trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf,
work, lwork, info)
```

```
call pcdttrsv(uplo, trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf,
work, lwork, info)
```

```
call pzdttrsv(uplo, trans, n, nrhs, dl, d, du, ja, desca, b, ib, descb, af, laf,
work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?dttrsv routine solves a tridiagonal triangular system of linear equations

A(1 :*n*, *ja*:*ja*+*n*-1)***X* = *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*) or

A(1 :*n*, *ja*:*ja*+*n*-1)^T * *X* = *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*) for real flavors; *A*(1 :*n*, *ja*:*ja*+*n*-1)^H * *X* = *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*) for complex flavors,

where *A*(1 :*n*, *ja*:*ja*+*n*-1) is a tridiagonal matrix factor produced by the Gaussian elimination code PS@(dom_pre)TTRF and is stored in *A*(1 :*n*, *ja*:*ja*+*n*-1) and *af*.

The matrix stored in *A*(1 :*n*, *ja*:*ja*+*n*-1) is either upper or lower triangular according to *uplo*, and the choice of solving *A*(1 :*n*, *ja*:*ja*+*n*-1) or *A*(1 :*n*, *ja*:*ja*+*n*-1)^T is dictated by the user by the parameter *trans*.

Routine p?dttrf must be called first.

Input Parameters

uplo	(global) CHARACTER. If *uplo*='U', the upper triangle of *A*(1:*n*, *ja*:*ja*+*n*-1) is stored, if *uplo* = 'L', the lower triangle of *A*(1:*n*, *ja*:*ja*+*n*-1) is stored.
trans	(global) CHARACTER. If *trans* = 'N', solve with *A*(1:*n*, *ja*:*ja*+*n*-1), if *trans* = 'C', solve with conjugate transpose *A*(1:*n*, *ja*:*ja*+*n*-1).
n	(global) INTEGER. The order of the distributed submatrix *A*; (*n*≥ 0).
nrhs	(global) INTEGER. The number of right-hand sides; the number of columns of the distributed submatrix *B*(*ib*:*ib*+*n*-1, 1:*nrhs*). (*nrhs*≥ 0).
dl	(local). REAL for psdttrsv DOUBLE PRECISION for pddttrsv COMPLEX for pcdttrsv COMPLEX*16 for pzdttrsv. Pointer to local part of global vector storing the lower diagonal of the matrix. Globally, *dl*(1) is not referenced, and *dl* must be aligned with *d*. Must be of size ≥*desca*(*nb_*).
d	(local). REAL for psdttrsv DOUBLE PRECISION for pddttrsv COMPLEX for pcdttrsv COMPLEX*16 for pzdttrsv. Pointer to local part of global vector storing the main diagonal of the matrix.

du (local).
REAL for psdttrsv
DOUBLE PRECISION for pddttrsv
COMPLEX for pcdttrsv
COMPLEX*16 for pzdttrsv.
Pointer to local part of global vector storing the upper diagonal of the matrix.
Globally, *du*(*n*) is not referenced, and *du* must be aligned with *d*.

ja (global) INTEGER. The index in the global array *a* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local). INTEGER array of DIMENSION (*dlen_*).
if 1*d* type (*dtype_a* = 501 or 502), *dlen*≥ 7;
if 2*d* type (*dtype_a* = 1), *dlen*≥ 9.
The array descriptor for the distributed matrix *A*. Contains information of mapping of *A* to memory.

b (local)
REAL for psdttrsv
DOUBLE PRECISION for pddttrsv
COMPLEX for pcdttrsv
COMPLEX*16 for pzdttrsv.
Pointer into the local memory to an array of local lead DIMENSION *lld_b*≥*nb*. On entry, this array contains the local pieces of the right-hand sides *B*(*ib*:*ib*+*n*-1, 1 :*nrhs*).

ib (global). INTEGER. The row index in the global array *b* that points to the first row of the matrix to be operated on (which may be either all of *b* or a submatrix of *B*).

descb (global and local).INTEGER array of DIMENSION (*dlen_*).
if 1*d* type (*dtype_b* = 502), *dlen*≥7;
if 2*d* type (*dtype_b* = 1), *dlen*≥ 9.
The array descriptor for the distributed matrix *B*. Contains information of mapping *B* to memory.

laf (local).
INTEGER.
Size of user-input Auxiliary Filling space *af*.
laf must be ≥ 2*(*nb*+2). If *laf* is not large enough, an error code is returned and the minimum acceptable size will be returned in *af*(1).

work (local).
REAL for psdttrsv
DOUBLE PRECISION for pddttrsv
COMPLEX for pcdttrsv
COMPLEX*16 for pzdttrsv.
Temporary workspace. This space may be overwritten in between calls to routines.
work must be the size given in *lwork*.

lwork (local or global).INTEGER.
Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned.
lwork≥ 10**npcol*+4**nrhs*.

Output Parameters

dl	(local). On exit, this array contains information containing the factors of the matrix.
d	On exit, this array contains information containing the factors of the matrix. Must be of size ≥*desca* (*nb_*).
b	On exit, this contains the local piece of the solutions distributed matrix X.
af	(local). REAL for psdttrsv DOUBLE PRECISION for pddttrsv COMPLEX for pcdttrsv COMPLEX*16 for pzdttrsv. Auxiliary Filling Space. Filling is created during the factorization routine p?dttrf and this is stored in *af*. If a linear system is to be solved using p?dttrs after the factorization routine, *af* must not be altered after the factorization.
work	On exit, *work*(1) contains the minimal *lwork*.
info	(local). INTEGER. If *info*=0, the execution is successful. if *info*< 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?gebd2

Reduces a general rectangular matrix to real bidiagonal form by an orthogonal/unitary transformation (unblocked algorithm).

Syntax

```
call psgebd2(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
call pdgebd2(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
call pcgebd2(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
call pzgebd2(m, n, a, ia, ja, desca, d, e, tauq, taup, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gebd2 routine reduces a real/complex general *m*-by-*n* distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) to upper or lower bidiagonal form *B* by an orthogonal/unitary transformation:

Q'*sub(*A*)**P* = *B*.

If *m* ≥ *n*, *B* is the upper bidiagonal; if *m*<*n*, *B* is the lower bidiagonal.

Input Parameters

m	(global) INTEGER. The number of rows of the distributed submatrix sub(*A*). (*m*≥0).
n	(global) INTEGER. The order of the distributed submatrix sub(*A*). (*n*≥0).

a	(local). REAL for psgebd2 DOUBLE PRECISION for pdgebd2 COMPLEX for pcgebd2 COMPLEX*16 for pzgebd2. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the general distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.
work	(local). REAL for psgebd2 DOUBLE PRECISION for pdgebd2 COMPLEX for pcgebd2 COMPLEX*16 for pzgebd2. This is a workspace array of DIMENSION (*lwork*).
lwork	(local or global) INTEGER. The dimension of the array *work*. *lwork* is local input and must be at least *lwork* ≥ max(*mpa*0, *nqa*0), where *nb* = *mb_a* = *nb_a*, *iroffa* = mod(*ia*-1, *nb*), *iarow* = indxg2p(*ia*, *nb*, *myrow*, *rsrc_a*, *nprow*), *iacol* = indxg2p(*ja*, *nb*, *mycol*, *csrc_a*, *npcol*), *mpa*0 = numroc(*m*+*iroffa*, *nb*, *myrow*, *iarow*, *nprow*), *nqa*0 = numroc(*n*+*icoffa*, *nb*, *mycol*, *iacol*, *npcol*). indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	(local). On exit, if *m* ≥ *n*, the diagonal and the first superdiagonal of sub(*A*) are overwritten with the upper bidiagonal matrix *B*; the elements below the diagonal, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and the elements above the first superdiagonal, with the array *taup*, represent the orthogonal matrix *P* as a product of elementary reflectors. If *m* < *n*, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix *B*; the elements below the first subdiagonal, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and the elements above the diagonal, with the array *taup*, represent the orthogonal matrix *P* as a product of elementary reflectors. See *Applications Notes* below.
d	(local) REAL for psgebd2 DOUBLE PRECISION for pdgebd2

	COMPLEX for pcgebd2 COMPLEX*16 for pzgebd2. Array, DIMENSION *LOCc*(*ja*+min(*m*,*n*)-1) if *m* ≥ *n*; *LOCr*(*ia*+min(*m*,*n*)-1) otherwise. The distributed diagonal elements of the bidiagonal matrix *B*: *d*(*i*) = *a*(*i*,*i*). *d* is tied to the distributed matrix *A*.
e	(local) REAL for psgebd2 DOUBLE PRECISION for pdgebd2 COMPLEX for pcgebd2 COMPLEX*16 for pzgebd2. Array, DIMENSION *LOCc*(*ja*+min(*m*,*n*)-1) if *m* ≥ *n*; *LOCr*(*ia*+min(*m*,*n*)-2) otherwise. The distributed diagonal elements of the bidiagonal matrix *B*: if *m* ≥ *n*, *e*(*i*) = *a*(*i*, *i*+1) for *i* = 1, 2, ... , *n*-1; if *m* < *n*, *e*(*i*) = *a*(*i*+1, *i*) for *i* = 1, 2, ..., *m*-1. *e* is tied to the distributed matrix *A*.
tauq	(local). REAL for psgebd2 DOUBLE PRECISION for pdgebd2 COMPLEX for pcgebd2 COMPLEX*16 for pzgebd2. Array, DIMENSION*LOCc*(*ja*+min(*m*,*n*)-1). The scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q*. *tauq* is tied to the distributed matrix *A*.
taup	(local). REAL for psgebd2 DOUBLE PRECISION for pdgebd2 COMPLEX for pcgebd2 COMPLEX*16 for pzgebd2. Array, DIMENSION *LOCr*(*ia*+min(*m*,*n*)-1). The scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *P*. *taup* is tied to the distributed matrix *A*.
work	On exit, *work*(1) returns the minimal and optimal *lwork*.
info	(local) INTEGER. If *info* = 0, the execution is successful. if *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrices *Q* and *P* are represented as products of elementary reflectors:

If *m*≥*n*,

Q = *H*(1)**H*(2)*...**H*(*n*), and *P* = *G*(1)**G*(2)*...**G*(*n*-1)

Each *H*(i) and *G*(i) has the form:

H(i) = *I* - *tauq***v***v'*, and *G*(i) = *I* - *taup***u***u'*,

where *tauq* and *taup* are real/complex scalars, and *v* and *u* are real/complex vectors. *v*(1: *i*-1) = 0, *v*(*i*) = 1, and *v*(*i*+*i*:*m*) is stored on exit in

A(*ia*+*i*-*ia*+*m*-1, *a*+*i*-1);

u(1:i) = 0, u(i+1) = 1, and u(i+2:n) is stored on exit in A(ia+i-1, ja+i+1:ja+n-1);

tauq is stored in TAUQ(ja+i-1) and taup in TAUP(ia+i-1).

If m < n,

v(1: i) = 0, v(i+1) = 1, and v(i+2:m) is stored on exit in A(ia+i+1: ia+m-1, ja+i-1);

u(1: i-1) = 0, u(i) = 1, and u(i+1 :n) is stored on exit in A(ia+i-1,ja+i:ja+n-1);

tauq is stored in TAUQ(ja+i-1) and taup in TAUP(ia+i-1).

The contents of sub(A) on exit are illustrated by the following examples:

$$m = 6 \text{ and } n = 5 (m > n):$$

$$\begin{bmatrix} d & e & u1 & u1 & u1 \\ v1 & d & e & u2 & u2 \\ v1 & v2 & d & e & u3 \\ v1 & v2 & v3 & d & e \\ v1 & v2 & v3 & v4 & d \\ v1 & v2 & v3 & v4 & v5 \end{bmatrix}$$

$$m = 5 \text{ and } n = 6 (m < n):$$

$$\begin{bmatrix} d & u1 & u1 & u1 & u1 & u1 \\ e & d & u2 & u2 & u2 & u2 \\ v1 & e & d & u3 & u3 & u3 \\ v1 & v2 & e & d & u4 & u4 \\ v1 & v2 & v3 & e & d & u5 \end{bmatrix}$$

where d and e denote diagonal and off-diagonal elements of B, vi denotes an element of the vector defining H(i), and ui an element of the vector defining G(i).

p?gehd2

Reduces a general matrix to upper Hessenberg form by an orthogonal/unitary similarity transformation (unblocked algorithm).

Syntax

```
call psgehd2(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
call pdgehd2(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
call pcgehd2(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
call pzgehd2(n, ilo, ihi, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gehd2 routine reduces a real/complex general distributed matrix sub(A) to upper Hessenberg form H by an orthogonal/unitary similarity transformation: Q'*sub(A)*Q = H, where sub(A) = A(ia+n-1 :ia +n-1, ja+n-1 :ja+n-1).

Input Parameters

n	(global) INTEGER. The order of the distributed submatrix A. (n≥ 0).
ilo, ihi	(global) INTEGER. It is assumed that sub(A) is already upper triangular in rows ia:ia+ilo-2 and ia+ihi:ia+n-1 and columns ja:ja+jlo-2 and ja +jhi:ja+n-1. See *Application Notes* for further information. If n≥ 0, 1 ≤ ilo ≤ ihi ≤ n; otherwise set ilo = 1, ihi = n.
a	(local). REAL for psgehd2

DOUBLE PRECISION for pdgehd2
COMPLEX for pcgehd2
COMPLEX*16 for pzgehd2.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the *n*-by-*n* general distributed matrix sub(*A*) to be reduced.

ia, *ja* (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local).
REAL for psgehd2
DOUBLE PRECISION for pdgehd2
COMPLEX for pcgehd2
COMPLEX*16 for pzgehd2.
This is a workspace array of DIMENSION (*lwork*).

lwork (local or global). INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least *lwork*≥*nb* + max(*npa*0, *nb*), where *nb* = *mb_a* = *nb_a*, *iroffa* = mod(*ia*-1, *nb*), *iarow* = indxg2p (*ia*, *nb*, *myrow*, *rsrc_a*, *nprow*),*npa*0 = numroc(*ihi* +*iroffa*, *nb*, *myrow*, *iarow*, *nprow*).
indxg2p and numroc are ScaLAPACK tool functions;*myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a (local). On exit, the upper triangle and the first subdiagonal of sub(*A*) are overwritten with the upper Hessenberg matrix *H*, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors. (see *Application Notes* below).

tau (local).
REAL for psgehd2
DOUBLE PRECISION for pdgehd2
COMPLEX for pcgehd2
COMPLEX*16 for pzgehd2.
Array, DIMENSION *LOCc*(*ja*+*n*-2) The scalar factors of the elementary reflectors (see *Application Notes* below). Elements *ja*:*ja*+*ilo*-2 and *ja* +*ihi*:*ja*+*n*-2 of *tau* are set to zero. *tau* is tied to the distributed matrix *A*.

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local).INTEGER.
If *info* = 0, the execution is successful.
if *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of (*ihi-ilo*) elementary reflectors

```
Q = H(ilo)*H(ilo+1)*...*H(ihi-1).
```

Each *H(i)* has the form

```
H(i) = I - tau*v*v',
```

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1: *i*)=0, *v*(*i*+1)=1 and *v*(*ihi* +1:*n*)=0; *v*(*i*+2:*ihi*) is stored on exit in *A*(*ia*+*ilo*+*i*:*ia*+*ihi*-1, *ia*+*ilo*+*i*-2), and *tau* in *tau*(*ja*+*ilo* +*i*-2).

The contents of *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) are illustrated by the following example, with *n* = 7, *ilo* = 2 and *ihi* = 6:

on entry

$$\begin{bmatrix} a & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & & & & & & a \end{bmatrix}$$

on exit

$$\begin{bmatrix} a & a & h & h & h & h & a \\ & a & h & h & h & h & a \\ & h & h & h & h & h & h \\ & v2 & h & h & h & h & h \\ & v2 & v3 & h & h & h & h \\ & v2 & v3 & v4 & h & h & h \\ & & & & & & a \end{bmatrix}$$

where *a* denotes an element of the original matrix sub(*A*), *h* denotes a modified element of the upper Hessenberg matrix *H*, and *vi* denotes an element of the vector defining *H*(*ja*+*ilo*+*i*-2).

p?gelq2

Computes an LQ factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
call psgelq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgelq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgelq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgelq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gelq2 routine computes an *LQ* factorization of a real/complex distributed *m*-by-*n* matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) = *L***Q*.

Input Parameters

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥0).

a (local).
REAL for psgelq2
DOUBLE PRECISION for pdgelq2
COMPLEX for pcgelq2
COMPLEX*16 for pzgelq2.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*) which is to be factored.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

work (local).
REAL for psgelq2
DOUBLE PRECISION for pdgelq2
COMPLEX for pcgelq2
COMPLEX*16 for pzgelq2.
This is a workspace array of DIMENSION (*lwork*).

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least *lwork*≥*nq*0 + max(1, *mp*0),
where *iroff* = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, *mycol*, *iacol*, *npcol*),
indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a (local).
On exit, the elements on and below the diagonal of sub(*A*) contain the *m* by min(*m*,*n*) lower trapezoidal matrix *L* (*L* is lower triangular if *m* ≤ *n*); the elements above the diagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau (local).
REAL for psgelq2
DOUBLE PRECISION for pdgelq2
COMPLEX for pcgelq2
COMPLEX*16 for pzgelq2.
Array, DIMENSION *LOCr*(*ia*+min(*m*, *n*)-1). This array contains the scalar factors of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info	(local).INTEGER. If *info* = 0, the execution is successful. if *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q =*H*(*ia*+*k*-1)**H*(*ia*+*k*-2)*. . . **H*(*ia*) for real flavors, *Q* =(*H*(*ia*+*k*-1))^H*(*H*(*ia*+*k*-2))^H...*(*H*(*ia*))^H for complex flavors,

where *k* = min(*m*,*n*).

Each *H*(*i*) has the form

H(*i*) = *I* - *tau***v***v*'

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1: *i*-1) = 0 and *v*(*i*) = 1; *v*(*i*+1: *n*) (for real flavors) or conjg(*v*(*i*+1: *n*)) (for complex flavors) is stored on exit in *A*(*ia*+*i*-1,*ja*+*i*:*ja*+*n*-1), and *tau* in *TAU*(*ia*+*i*-1).

p?geql2

Computes a QL factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
call psgeql2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgeql2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgeql2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgeql2(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?geql2 routine computes a *QL* factorization of a real/complex distributed *m*-by-*n* matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1)= *Q* **L*.

Input Parameters

m	(global) INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥ 0).
n	(global) INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥ 0).
a	(local). REAL for psgeql2 DOUBLE PRECISION for pdgeql2 COMPLEX for pcgeql2 COMPLEX*16 for pzgeql2.

Pointer into the local memory to an array of DIMENSION (*lld_a*,*LOCc* (*ja* +*n*-1)).
On entry, this array contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*) which is to be factored.

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

work (local).
REAL for psgeql2
DOUBLE PRECISION for pdgeql2
COMPLEX for pcgeql2
COMPLEX*16 for pzgeql2.
This is a workspace array of DIMENSION (*lwork*).

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least *lwork*≥*mp*0 + max(1, *nq*0),
where *iroff* = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, *mycol*, *iacol*, *npcol*),
indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a (local).
On exit,
if *m* ≥ *n*, the lower triangle of the distributed submatrix *A*(*ia*+*m*-*n*:*ia*+*m*-1, *ja*:*ja*+*n*-1) contains the *n*-by-*n* lower triangular matrix *L*;
if *m* ≤ *n*, the elements on and below the (*n*-*m*)-th superdiagonal contain the *m*-by-*n* lower trapezoidal matrix *L*; the remaining elements, with the array *tau*, represent the orthogonal/ unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau (local).
REAL for psgeql2
DOUBLE PRECISION for pdgeql2
COMPLEX for pcgeql2
COMPLEX*16 for pzgeql2.
Array, DIMENSION *LOCc*(*ja*+*n*-1). This array contains the scalar factors of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local). INTEGER.
If *info* = 0, the execution is successful. if *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = H(ja+k-1)...*H(ja+1)*H(ja)*, where *k = min(m,n)*.

Each *H(i)* has the form

*H(i) = I- tau *v*v'*

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(*m-k+i*+1: *m*) = 0 and *v*(*m-k+i*) = 1; *v*(1: *m-k+i*-1) is stored on exit in *A*(*ia*:*ia*+*m-k*+*i*-2, *ja*+*n-k*+*i*-1), and *tau* in *TAU*(*ja*+*n-k*+*i*-1).

p?geqr2

Computes a QR factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
call psgeqr2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgeqr2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgeqr2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgeqr2(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?geqr2 routine computes a *QR* factorization of a real/complex distributed *m*-by-*n* matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1)= *Q*R*.

Input Parameters

m	(global). INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).
n	(global).INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥0).
a	(local). REAL for psgeqr2 DOUBLE PRECISION for pdgeqr2 COMPLEX for pcgeqr2 COMPLEX*16 for pzgeqr2. Pointer into the local memory to an array of DIMENSION (*lld_a*, *LOCc* (*ja*+*n*-1)). On entry, this array contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*) which is to be factored.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.
work	(local). REAL for psgeqr2

DOUBLE PRECISION for pdgeqr2
COMPLEX for pcgeqr2
COMPLEX*16 for pzgeqr2.
This is a workspace array of DIMENSION (*lwork*).

lwork (local or global). INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least *lwork*≥*mp*0+max(1, *nq*0),
where *iroff* = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, *mycol*, *iacol*, *npcol*).
indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a (local).
On exit, the elements on and above the diagonal of sub(*A*) contain the min(*m*,*n*) by *n* upper trapezoidal matrix *R* (*R* is upper triangular if *m*≥*n*); the elements below the diagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau (local).
REAL for psgeqr2
DOUBLE PRECISION for pdgeqr2
COMPLEX for pcgeqr2
COMPLEX*16 for pzgeqr2.
Array, DIMENSION *LOCc*(*ja*+min(*m*,*n*)-1). This array contains the scalar factors of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local). INTEGER.
If *info* = 0, the execution is successful. if *info* < 0:
If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*),
if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ja*)**H*(*ja*+1)*. . .* *H*(*ja*+*k*-1), where *k* = min(*m*,*n*).

Each *H*(*i*) has the form

H(*j*)= *I* - *tau***v***v*',

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1: *i*-1) = 0 and *v*(*i*) = 1; *v*(*i*+1: *m*) is stored on exit in *A*(*ia*+*i*:*ia*+*m*-1, *ja*+*i*-1), and *tau* in *TAU*(*ja*+*i*-1).

p?gerq2

Computes an RQ factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
call psgerq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pdgerq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pcgerq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
call pzgerq2(m, n, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?gerq2 routine computes an *RQ* factorization of a real/complex distributed *m*-by-*n* matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) = *R***Q*.

Input Parameters

m — (global). INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).

n — (global).INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥0).

a — (local).
REAL for psgerq2
DOUBLE PRECISION for pdgerq2
COMPLEX for pcgerq2
COMPLEX*16 for pzgerq2.
Pointer into the local memory to an array of DIMENSION (*lld_a*,*LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*) which is to be factored.

ia, *ja* — (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.

desca — (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

work — (local).
REAL for psgerq2
DOUBLE PRECISION for pdgerq2
COMPLEX for pcgerq2
COMPLEX*16 for pzgerq2.
This is a workspace array of DIMENSION (*lwork*).

lwork — (local or global). INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least *lwork*≥*nq*0 + max(1, *mp*0), where
iroff = mod(*ia*-1, *mb_a*), *icoff* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),

iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*), *mp*0 = numroc(*m*+*iroff*, *mb_a*, *myrow*, *iarow*, *nprow*), *nq*0 = numroc(*n*+*icoff*, *nb_a*, *mycol*, *iacol*, *npcol*), indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a (local).
On exit,
if *m* ≤ *n*, the upper triangle of *A*(*ia*+*m*-*n*:*ia*+*m*-1, *ja*:*ja*+*n*-1) contains the *m*-by-*m* upper triangular matrix *R*;
if *m* ≥ *n*, the elements on and above the (*m*-*n*)-th subdiagonal contain the *m*-by-*n* upper trapezoidal matrix *R*; the remaining elements, with the array *tau*, represent the orthogonal/ unitary matrix *Q* as a product of elementary reflectors (see *Application Notes* below).

tau (local).
REAL for psgerq2
DOUBLE PRECISION for pdgerq2
COMPLEX for pcgerq2
COMPLEX*16 for pzgerq2.
Array, DIMENSION *LOCr*(*ia*+*m* -1). This array contains the scalar factors of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local). INTEGER.
If *info* = 0, the execution is successful.
if *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*ia*)**H*(*ia*+1)*...**H*(*ia*+*k*-1) for real flavors,

Q = (*H*(*ia*))^H*(*H*(*ia*+1))^H*...*(*H*(*ia*+*k*-1))^H for complex flavors,

where *k* = min(*m*, *n*).

Each *H*(*i*) has the form

H(*i*) = *I* - *tau***v***v*',

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(*n*-*k*+*i*+1:*n*) = 0 and *v*(*n*-*k*+*i*) = 1; *v*(1:*n*-*k*+*i*-1) for real flavors or conjg(*v*(1:*n*-*k*+*i*-1)) for complex flavors is stored on exit in *A*(*ia*+*m*-*k*+*i*-1, *ja*:*ja*+*n*-*k*+*i*-2), and *tau* in *TAU*(*ia*+*m*-*k*+*i*-1).

p?getf2

Computes an LU factorization of a general matrix, using partial pivoting with row interchanges (local blocked algorithm).

Syntax

```
call psgetf2(m, n, a, ia, ja, desca, ipiv, info)
call pdgetf2(m, n, a, ia, ja, desca, ipiv, info)
call pcgetf2(m, n, a, ia, ja, desca, ipiv, info)
call pzgetf2(m, n, a, ia, ja, desca, ipiv, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?getf2 routine computes an *LU* factorization of a general *m*-by-*n* distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) using partial pivoting with row interchanges.

The factorization has the form sub(*A*) = *P* * *L** *U*, where *P* is a permutation matrix, *L* is lower triangular with unit diagonal elements (lower trapezoidal if *m*>*n*), and *U* is upper triangular (upper trapezoidal if *m* < *n*). This is the right-looking Parallel Level 2 BLAS version of the algorithm.

Input Parameters

m	(global). INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).
n	(global).INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*nb_a* - mod(*ja*-1, *nb_a*)≥*n*≥0).
a	(local). REAL for psgetf2 DOUBLE PRECISION for pdgetf2 COMPLEX for pcgetf2 COMPLEX*16 for pzgetf2. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

Output Parameters

ipiv	(local). INTEGER. Array, DIMENSION(*LOCr*(*m_a*) + *mb_a*). This array contains the pivoting information. *ipiv*(*i*) -> The global row that local row *i* was swapped with. This array is tied to the distributed matrix *A*.
info	(local). INTEGER. If *info* = 0: successful exit. If *info* < 0: • if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*),

- if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

If *info* > 0: If *info* = *k*, *u*(*ia*+*k*-1, *ja*+*k*-1) is exactly zero. The factorization has been completed, but the factor *u* is exactly singular, and division by zero will occur if it is used to solve a system of equations.

p?labrd

Reduces the first nb rows and columns of a general rectangular matrix A to real bidiagonal form by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A.

Syntax

call pslabrd(*m*, *n*, *nb*, *a*, *ia*, *ja*, *desca*, *d*, *e*, *tauq*, *taup*, *x*, *ix*, *jx*, *descx*, *y*, *iy*, *jy*, *descy*, *work*)

call pdlabrd(*m*, *n*, *nb*, *a*, *ia*, *ja*, *desca*, *d*, *e*, *tauq*, *taup*, *x*, *ix*, *jx*, *descx*, *y*, *iy*, *jy*, *descy*, *work*)

call pclabrd(*m*, *n*, *nb*, *a*, *ia*, *ja*, *desca*, *d*, *e*, *tauq*, *taup*, *x*, *ix*, *jx*, *descx*, *y*, *iy*, *jy*, *descy*, *work*)

call pzlabrd(*m*, *n*, *nb*, *a*, *ia*, *ja*, *desca*, *d*, *e*, *tauq*, *taup*, *x*, *ix*, *jx*, *descx*, *y*, *iy*, *jy*, *descy*, *work*)

Include Files

- C: mkl_scalapack.h

Description

The p?labrd routine reduces the first *nb* rows and columns of a real/complex general *m*-by-*n* distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) to upper or lower bidiagonal form by an orthogonal/unitary transformation *Q'* * *A* * *P*, and returns the matrices *X* and *Y* necessary to apply the transformation to the unreduced part of sub(*A*).

If *m* ≥*n*, sub(*A*) is reduced to upper bidiagonal form; if *m* < *n*, sub(*A*) is reduced to lower bidiagonal form.

This is an auxiliary routine called by p?gebrd.

Input Parameters

m (global). INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m* ≥ 0).

n (global).INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n* ≥ 0).

nb (global) INTEGER.
The number of leading rows and columns of sub(*A*) to be reduced.

a (local).
REAL for pslabrd
DOUBLE PRECISION for pdlabrd
COMPLEX for pclabrd
COMPLEX*16 for pzlabrd.

Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the general distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

ix, *jx* (global) INTEGER. The row and column indices in the global array *x* indicating the first row and the first column of the submatrix sub(*X*), respectively.

descx (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix X.

iy, *jy* (global) INTEGER. The row and column indices in the global array *y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.

descy (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix Y.

work (local).
REAL for pslabrd
DOUBLE PRECISION for pdlabrd
COMPLEX for pclabrd
COMPLEX*16 for pzlabrd
Workspace array, DIMENSION(*lwork*)
lwork ≥ *nb_a* + *nq*,
with *nq* = numroc(*n*+mod(*ia*-1, *nb_y*), *nb_y*, *mycol*, *iacol*, *npcol*)
iacol = indxg2p (*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*)
indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

a (local)
On exit, the first *nb* rows and columns of the matrix are overwritten; the rest of the distributed matrix sub(*A*) is unchanged.
If *m* ≥ *n*, elements on and below the diagonal in the first *nb* columns, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors;and elements above the diagonal in the first *nb* rows, with the array *taup*, represent the orthogonal/unitary matrix *P* as a product of elementary reflectors.
If *m* < *n*, elements below the diagonal in the first *nb* columns, with the array *tauq*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors, and elements on and above the diagonal in the first *nb* rows, with the array *taup*, represent the orthogonal/unitary matrix *P* as a product of elementary reflectors. See *Application Notes* below.

d (local).
REAL for pslabrd
DOUBLE PRECISION for pdlabrd
COMPLEX for pclabrd
COMPLEX*16 for pzlabrd

	Array, DIMENSION *LOCr*(*ia*+min(*m*,*n*)-1) if *m* ≥ *n*; *LOCc*(*ja*+min(*m*,*n*)-1) otherwise. The distributed diagonal elements of the bidiagonal distributed matrix *B*: *d*(*i*) = *A*(*ia*+*i*-1, *ja*+*i*-1). *d* is tied to the distributed matrix *A*.
e	(local). REAL for pslabrd DOUBLE PRECISION for pdlabrd COMPLEX for pclabrd COMPLEX*16 for pzlabrd Array, DIMENSION *LOCr*(*ia*+min(*m*,*n*)-1) if *m* ≥ *n*; *LOCc*(*ja*+min(*m*,*n*)-2) otherwise. The distributed off-diagonal elements of the bidiagonal distributed matrix *B*: if *m* ≥ *n*, *E*(*i*) = *A*(*ia*+*i*-1, *ja*+*i*) for *i* = 1, 2, ..., *n*-1; if *m*<*n*, *E*(*i*) = *A*(*ia*+*i*, *ja*+*i*-1) for *i* = 1, 2, ..., *m*-1. *e* is tied to the distributed matrix *A*.
tauq, *taup*	(local). REAL for pslabrd DOUBLE PRECISION for pdlabrd COMPLEX for pclabrd COMPLEX*16 for pzlabrd Array DIMENSION *LOCc*(*ja*+min(*m*, *n*)-1) for *tauq*, DIMENSION *LOCr*(*ia*+min(*m*, *n*)-1) for *taup*. The scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q* for *tauq*, *P* for *taup*. *tauq* and *taup* are tied to the distributed matrix *A*. See *Application Notes* below.
x	(local) REAL for pslabrd DOUBLE PRECISION for pdlabrd COMPLEX for pclabrd COMPLEX*16 for pzlabrd Pointer into the local memory to an array of DIMENSION (*lld_x*, *nb*). On exit, the local pieces of the distributed *m*-by-*nb* matrix *X*(*ix*:*ix*+*m*-1, *jx*:*jx*+*nb*-1) required to update the unreduced part of sub(*A*).
y	(local). REAL for pslabrd DOUBLE PRECISION for pdlabrd COMPLEX for pclabrd COMPLEX*16 for pzlabrd Pointer into the local memory to an array of DIMENSION (*lld_y*, *nb*). On exit, the local pieces of the distributed *n*-by-*nb* matrix *Y*(*iy*:*iy*+*n*-1, *jy*:*jy*+*nb*-1) required to update the unreduced part of sub(*A*).

Application Notes

The matrices *Q* and *P* are represented as products of elementary reflectors:

Q = *H*(1)**H*(2)*...**H*(*nb*), and *P* = *G*(1)**G*(2)*...**G*(*nb*)

Each *H*(i) and *G*(i) has the form:

H(i) = *I* - *tauq***v***v*', and *G*(i) = *I* - *taup***u***u*',

where *tauq* and *taup* are real/complex scalars, and *v* and *u* are real/complex vectors.

If *m* ≥ *n*, *v*(1: *i*-1) = 0, *v*(*i*) = 1, and *v*(*i*:*m*) is stored on exit in

A(ia+i-1:ia+m-1, ja+i-1); u(1:i) = 0, u(i+1) = 1, and u(i+1:n) is stored on exit in A(ia+i-1, ja+i:ja+n-1); tauq is stored in TAUQ(ja+i-1) and taup in TAUP(ia+i-1).

If m < n, v(1: i) = 0, v(i+1) = 1, and v(i+1:m) is stored on exit in

A(ia+i+1:ia+m-1, ja+i-1); u(1:i-1) = 0, u(i) = 1, and u(i:n) is stored on exit in A(ia+i-1, ja+i:ja+n-1); tauq is stored in TAUQ(ja+i-1) and taup in TAUP(ia+i-1). The elements of the vectors v and u together form the m-by-nb matrix V and the nb-by-n matrix U' which are necessary, with X and Y, to apply the transformation to the unreduced part of the matrix, using a block update of the form: sub(A) := sub(A) - V*Y' - X*U'. The contents of sub(A) on exit are illustrated by the following examples with nb = 2:

m = 6 and n = 5(m > n):

$$\begin{bmatrix} 1 & 1 & u1 & u1 & u1 \\ v1 & 1 & 1 & u2 & u2 \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \end{bmatrix}$$

m = 5 and n = 6(m < n) :

$$\begin{bmatrix} 1 & u1 & u1 & u1 & u1 & u1 \\ 1 & 1 & u2 & u2 & u2 & u2 \\ v1 & 1 & a & a & a & a \\ v1 & v2 & a & a & a & a \\ v1 & v2 & a & a & a & a \end{bmatrix}$$

where a denotes an element of the original matrix which is unchanged, vi denotes an element of the vector defining H(i), and ui an element of the vector defining G(i).

p?lacon

Estimates the 1-norm of a square matrix, using the reverse communication for evaluating matrix-vector products.

Syntax

```
call pslacon(n, v, iv, jv, descv, x, ix, jx, descx, isgn, est, kase)
call pdlacon(n, v, iv, jv, descv, x, ix, jx, descx, isgn, est, kase)
call pclacon(n, v, iv, jv, descv, x, ix, jx, descx, isgn, est, kase)
call pzlacon(n, v, iv, jv, descv, x, ix, jx, descx, isgn, est, kase)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lacon routine estimates the 1-norm of a square, real/unitary distributed matrix A. Reverse communication is used for evaluating matrix-vector products. x and v are aligned with the distributed matrix A, this information is implicitly contained within iv, ix, descv, and descx.

Input Parameters

n	(global).INTEGER. The length of the distributed vectors v and x. n ≥ 0.
v	(local). REAL for pslacon DOUBLE PRECISION for pdlacon COMPLEX for pclacon COMPLEX*16 for pzlacon.

	Pointer into the local memory to an array of DIMENSION *LOCr*(*n*+mod(*iv*-1, *mb_v*)). On the final return, *v* = *a*w*, where *est* = norm(*v*)/norm(*w*) (*w* is not returned).
iv, *jv*	(global) INTEGER. The row and column indices in the global array *v* indicating the first row and the first column of the submatrix *V*, respectively.
descv	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix V.
x	(local). REAL for pslacon DOUBLE PRECISION for pdlacon COMPLEX for pclacon COMPLEX*16 for pzlacon. Pointer into the local memory to an array of DIMENSION *LOCr*(*n*+mod(*ix*-1, *mb_x*)).
ix, *jx*	(global) INTEGER. The row and column indices in the global array *x* indicating the first row and the first column of the submatrix *X*, respectively.
descx	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix X.
isgn	(local). INTEGER. Array, DIMENSION *LOCr*(*n*+mod(*ix*-1, *mb_x*)). *isgn* is aligned with *x* and *v*.
kase	(local). INTEGER. On the initial call to p?lacon, *kase* should be 0.

Output Parameters

x	(local). On an intermediate return, X should be overwritten by *A*X*, if *kase*=1, *A'*X*, if *kase*=2, p?lacon must be re-called with all the other parameters unchanged.
est	(global). REAL for single precision flavors DOUBLE PRECISION for double precision flavors
kase	(local) INTEGER. On an intermediate return, *kase* is 1 or 2, indicating whether *X* should be overwritten by *A*X,* or *A'*X*. On the final return from p?lacon, *kase* is again 0.

p?laconsb

Looks for two consecutive small subdiagonal elements.

Syntax

```
call pslaconsb(a, desca, i, l, m, h44, h33, h43h34, buf, lwork)
call pdlaconsb(a, desca, i, l, m, h44, h33, h43h34, buf, lwork)
```

Include Files

- C: mkl_scalapack.h

Description

The p?laconsb routine looks for two consecutive small subdiagonal elements by analyzing the effect of starting a double shift *QR* iteration given by *h44*, *h33*, and *h43h34* to see if this process makes a subdiagonal negligible.

Input Parameters

a	(global). REAL for pslaconsb DOUBLE PRECISION for pdlaconsb Array, DIMENSION (*desca* (*lld_*),*). On entry, the Hessenberg matrix whose tridiagonal part is being scanned. Unchanged on exit.
desca	(global and local) INTEGER. Array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
i	(global) INTEGER. The global location of the bottom of the unreduced submatrix of *A*. Unchanged on exit.
l	(global) INTEGER. The global location of the top of the unreduced submatrix of *A*. Unchanged on exit.
h44, *h33*, *h43h34*	(global). REAL for pslaconsb DOUBLE PRECISION for pdlaconsb These three values are for the double shift QR iteration.
lwork	(global) INTEGER. This must be at least 7*ceil(ceil((*i*-l)/*hbl*)/lcm(*nprow*, *npcol*)). Here lcm is least common multiple and *nprow***npcol* is the logical grid size.

Output Parameters

m	(global). On exit, this yields the starting location of the *QR* double shift. This will satisfy: *l* ≤ *m* ≤ *i*-2.
buf	(local). REAL for pslaconsb DOUBLE PRECISION for pdlaconsb Array of size *lwork*.
lwork	(global). On exit, *lwork* is the size of the work buffer.

p?lacp2

Copies all or part of a distributed matrix to another distributed matrix.

Syntax

```
call pslacp2(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
call pdlacp2(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
call pclacp2(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
call pzlacp2(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lacp2 routine copies all or part of a distributed matrix *A* to another distributed matrix *B*. No communication is performed, p?lacp2 performs a local copy sub(*A*):= sub(*B*), where sub(*A*) denotes *A*(*ia*:*ia*+*m*-1, *a*:*ja*+*n*-1) and sub(*B*) denotes *B*(*ib*:*ib*+*m*-1, *jb*:*jb*+*n*-1).

p?lacp2 requires that only dimension of the matrix operands is distributed.

Input Parameters

uplo (global) CHARACTER. Specifies the part of the distributed matrix sub(*A*) to be copied:
= 'U': Upper triangular part is copied; the strictly lower triangular part of sub(*A*) is not referenced;
= 'L': Lower triangular part is copied; the strictly upper triangular part of sub(*A*) is not referenced.
Otherwise: all of the matrix sub(*A*) is copied.

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m* ≥ 0).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n* ≥ 0).

a (local).
REAL for pslacp2
DOUBLE PRECISION for pdlacp2
COMPLEX for pclacp2
COMPLEX*16 for pzlacp2.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the *m*-by-*n* distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

ib, *jb* (global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of sub(*B*), respectively.

descb (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix B.

Output Parameters

b (local).
REAL for pslacp2
DOUBLE PRECISION for pdlacp2
COMPLEX for pclacp2
COMPLEX*16 for pzlacp2.
Pointer into the local memory to an array of DIMENSION (*lld_b*, *LOCc*(*jb*+*n*-1)). This array contains on exit the local pieces of the distributed matrix sub(B) set as follows:
if *uplo* = 'U', *B*(*ib*+*i*-1, *jb*+*j*-1) = *A*(*ia*+*i*-1, *ja*+*j*-1), 1 ≤ *i* ≤ *j*, 1 ≤ *j* ≤ *n*;

if *uplo* = 'L', *B*(*ib*+*i*-1, *jb*+*j*-1) = *A*(*ia*+*i*-1, *ja*+*j*-1), *j* ≤ *i* ≤ *m*, 1≤ *j* ≤ *n*;
otherwise, *B*(*ib*+*i*-1, *jb*+*j*-1) = *A*(*ia*+*i*-1, *ja*+*j*-1), 1 ≤ *i* ≤ *m*, 1 ≤ *j* ≤ *n*.

p?lacp3

Copies from a global parallel array into a local replicated array or vice versa.

Syntax

```
call pslacp3(m, i, a, desca, b, ldb, ii, jj, rev)
call pdlacp3(m, i, a, desca, b, ldb, ii, jj, rev)
```

Include Files

- C: mkl_scalapack.h

Description

This is an auxiliary routine that copies from a global parallel array into a local replicated array or vise versa. Note that the entire submatrix that is copied gets placed on one node or more. The receiving node can be specified precisely, or all nodes can receive, or just one row or column of nodes.

Input Parameters

m	(global) INTEGER. *m* is the order of the square submatrix that is copied. *m* ≥ 0. Unchanged on exit.
i	(global) INTEGER. *A*(*i*, *i*) is the global location that the copying starts from. Unchanged on exit.
a	(global). REAL for pslacp3 DOUBLE PRECISION for pdlacp3 Array, DIMENSION (*desca*(*lld_*),*). On entry, the parallel matrix to be copied into or from.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.
b	(local). REAL for pslacp3 DOUBLE PRECISION for pdlacp3 Array, DIMENSION (*ldb*, *m*). If *rev* = 0, this is the global portion of the array *A*(*i*:*i*+*m*-1, *i*:*i*+*m*-1). If *rev* = 1, this is the unchanged on exit.
ldb	(local) INTEGER. The leading dimension of *B*.
ii	(global) INTEGER. By using *rev* 0 and 1, data can be sent out and returned again. If *rev* = 0, then *ii* is destination row index for the node(s) receiving the replicated *B*. If *ii* ≥ 0, *jj* ≥ 0, then node (*ii*, *jj*) receives the data. If *ii* = -1, *jj* ≥ 0, then all rows in column *jj* receive the data. If *ii* ≥ 0, *jj* = -1, then all cols in row *ii* receive the data. f *ii* = -1, *jj* = -1, then all nodes receive the data. If *rev* !=0, then *ii* is the source row index for the node(s) sending the replicated *B*.
jj	(global) INTEGER. Similar description as *ii* above.

rev (global) INTEGER. Use *rev* = 0 to send global *A* into locally replicated *B* (on node (*ii*, *jj*)). Use *rev* != 0 to send locally replicated *B* from node (*ii*, *jj*) to its owner (which changes depending on its location in *A*) into the global *A*.

Output Parameters

a (global). On exit, if *rev* = 1, the copied data. Unchanged on exit if *rev* = 0.

b (local). If *rev* = 1, this is unchanged on exit.

p?lacpy

Copies all or part of one two-dimensional array to another.

Syntax

```
call pslacpy(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
call pdlacpy(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
call pclacpy(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
call pzlacpy(uplo, m, n, a, ia, ja, desca, b, ib, jb, descb)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lacpy routine copies all or part of a distributed matrix *A* to another distributed matrix *B*. No communication is performed, p?lacpy performs a local copy sub(*B*):= sub(*A*), where sub(*A*) denotes *A*(*ia*:*ia*+*m*-1,*ja*:*ja*+*n*-1) and sub(*B*) denotes *B*(*ib*:*ib*+*m*-1,*jb*:*jb*+*n*-1).

Input Parameters

uplo (global). CHARACTER. Specifies the part of the distributed matrix sub(*A*) to be copied:
= 'U': Upper triangular part; the strictly lower triangular part of sub(*A*) is not referenced;
= 'L': Lower triangular part; the strictly upper triangular part of sub(*A*) is not referenced.
Otherwise: all of the matrix sub(*A*) is copied.

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥0).

a (local).
REAL for pslacpy
DOUBLE PRECISION for pdlacpy
COMPLEX for pclacpy
COMPLEX*16 for pzlacpy.

	Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the local pieces of the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.
ib, *jb*	(global) INTEGER. The row and column indices in the global array *B* indicating the first row and the first column of sub(*B*) respectively.
descb	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

Output Parameters

b	(local). REAL for pslacpy DOUBLE PRECISION for pdlacpy COMPLEX for pclacpy COMPLEX*16 for pzlacpy. Pointer into the local memory to an array of DIMENSION (*lld_b*, *LOCc*(*jb*+*n*-1)). This array contains on exit the local pieces of the distributed matrix sub(B) set as follows: if *uplo* = 'U', *B*(*ib*+*i*-1, *jb*+*j*-1) = *A*(*ia*+*i*-1, *ja*+*j*-1), 1≤*i*≤*j*, 1≤*j*≤*n*; if *uplo* = 'L', *B*(*ib*+*i*-1, *jb*+*j*-1) = *A*(*ia*+*i*-1, *ja*+*j*-1), *j*≤*i*≤*m*, 1≤*j*≤*n*; otherwise, *B*(*ib*+*i*-1, *jb*+*j*-1) = *A*(*ia*+*i*-1, *ja*+*j*-1), 1≤*i*≤*m*, 1≤*j*≤*n*.

p?laevswp

Moves the eigenvectors from where they are computed to ScaLAPACK standard block cyclic array.

Syntax

```
call pslaevswp(n, zin, ldzi, z, iz, jz, descz, nvs, key, rwork, lrwork)
call pdlaevswp(n, zin, ldzi, z, iz, jz, descz, nvs, key, rwork, lrwork)
call pclaevswp(n, zin, ldzi, z, iz, jz, descz, nvs, key, rwork, lrwork)
call pzlaevswp(n, zin, ldzi, z, iz, jz, descz, nvs, key, rwork, lrwork)
```

Include Files

- C: mkl_scalapack.h

Description

The p?laevswp routine moves the eigenvectors (potentially unsorted) from where they are computed, to a ScaLAPACK standard block cyclic array, sorted so that the corresponding eigenvalues are sorted.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

n	(global). INTEGER. The order of the matrix *A*. *n* ≥ 0.
zin	(local). REAL for pslaevswp DOUBLE PRECISION for pdlaevswp COMPLEX for pclaevswp COMPLEX*16 for pzlaevswp. Array, DIMENSION (*ldzi*, *nvs*(*iam*)). The eigenvectors on input. Each eigenvector resides entirely in one process. Each process holds a contiguous set of *nvs*(*iam*) eigenvectors. The first eigenvector which the process holds is: sum for *i*=[0, *iam*-1) of *nvs*(*i*).
ldzi	(local) INTEGER.The leading dimension of the *zin* array.
iz, *jz*	(global) INTEGER.The row and column indices in the global array *Z* indicating the first row and the first column of the submatrix *Z*, respectively.
descz	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix Z.
nvs	(global) INTEGER. Array, DIMENSION(*nprocs*+1) *nvs*(*i*) = number of processes number of eigenvectors held by processes [0, *i*-1) *nvs*(1) = number of eigen vectors held by[0, 1 -1) = 0 *nvs*(*nprocs*+1)= number of eigen vectors held by [0, *nprocs*)= total number of eigenvectors.
key	(global) INTEGER. Array, DIMENSION (*n*). Indicates the actual index (after sorting) for each of the eigenvectors.
rwork	(local). REAL for pslaevswp DOUBLE PRECISIONfor pdlaevswp COMPLEX for pclaevswp COMPLEX*16 for pzlaevswp. Array, DIMENSION (*lrwork*).
lrwork	(local) INTEGER. Dimension of *work*.

Output Parameters

z	(local). REAL for pslaevswp DOUBLE PRECISION for pdlaevswp COMPLEX for pclaevswp COMPLEX*16 for pzlaevswp. Array, global DIMENSION (*n*, *n*), local DIMENSION (*descz*(*dlen_*), *nq*). The eigenvectors on output. The eigenvectors are distributed in a block cyclic manner in both dimensions, with a block size of *nb*.

p?lahrd

Reduces the first nb columns of a general rectangular matrix A so that elements below the k-th subdiagonal are zero, by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A.

Syntax

```
call pslahrd(n, k, nb, a, ia, ja, desca, tau, t, y, iy, jy, descy, work)
call pdlahrd(n, k, nb, a, ia, ja, desca, tau, t, y, iy, jy, descy, work)
call pclahrd(n, k, nb, a, ia, ja, desca, tau, t, y, iy, jy, descy, work)
call pzlahrd(n, k, nb, a, ia, ja, desca, tau, t, y, iy, jy, descy, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lahrd routine reduces the first *nb* columns of a real general *n*-by-(*n*-*k*+1) distributed matrix *A*(*ia*:*ia*+*n*-1 , *ja*:*ja*+*n*-*k*) so that elements below the *k*-th subdiagonal are zero. The reduction is performed by an orthogonal/unitary similarity transformation *Q'*A*Q*. The routine returns the matrices *V* and *T* which determine *Q* as a block reflector *I*-*V*T*V'*, and also the matrix *Y* = *A*V*T*.

This is an auxiliary routine called by p?gehrd. In the following comments sub(*A*) denotes *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

Input Parameters

n
(global) INTEGER.
The order of the distributed submatrix sub(*A*). *n* ≥ 0.

k
(global) INTEGER.
The offset for the reduction. Elements below the *k*-th subdiagonal in the first *nb* columns are reduced to zero.

nb
(global) INTEGER.
The number of columns to be reduced.

a
(local).
REAL for pslahrd
DOUBLE PRECISION for pdlahrd
COMPLEX for pclahrd
COMPLEX*16 for pzlahrd.
Pointer into the local memory to an array of DIMENSION (*lld_a*, *LOCc*(*ja*+*n*-*k*)). On entry, this array contains the local pieces of the *n*-by-(*n*-*k*+1) general distributed matrix *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-*k*).

ia, *ja*
(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca
(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

iy, *jy*
(global) INTEGER. The row and column indices in the global array *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.

descy
(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *Y*.

work
(local).
REAL for pslahrd
DOUBLE PRECISION for pdlahrd
COMPLEX for pclahrd
COMPLEX*16 for pzlahrd.

Array, DIMENSION (*nb*).

Output Parameters

a (local).
On exit, the elements on and above the *k*-th subdiagonal in the first *nb* columns are overwritten with the corresponding elements of the reduced distributed matrix;the elements below the *k*-th subdiagonal, with the array *tau*, represent the matrix *Q* as a product of elementary reflectors. The other columns of *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-*k*) are unchanged. (See *Application Notes* below.)

tau (local)
REAL for pslahrd
DOUBLE PRECISION for pdlahrd
COMPLEX for pclahrd
COMPLEX*16 for pzlahrd.
Array, DIMENSION *LOCc*(*ja*+*n*-2). The scalar factors of the elementary reflectors (see *Application Notes* below). *tau* is tied to the distributed matrix *A*.

t (local)REAL for pslahrd
DOUBLE PRECISION for pdlahrd
COMPLEX for pclahrd
COMPLEX*16 for pzlahrd.
Array, DIMENSION (*nb_a*, *nb_a*) The upper triangular matrix *T*.

y (local).
REAL for pslahrd
DOUBLE PRECISION for pdlahrd
COMPLEX for pclahrd
COMPLEX*16 for pzlahrd.
Pointer into the local memory to an array of DIMENSION (*lld_y*, *nb_a*). On exit, this array contains the local pieces of the *n*-by-*nb* distributed matrix *Y*. *lld_y* ≥ *LOCr*(*ia*+*n*-1).

Application Notes

The matrix *Q* is represented as a product of *nb* elementary reflectors

Q = *H*(1)**H*(2)*...**H*(*nb*).

Each *H*(*i*) has the form

H(*i*) = *i*-*tau***v***v*',

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1: *i*+*k*-1)= 0, *v*(*i*+*k*)= 1; *v*(*i*+*k* +1:*n*) is stored on exit in *A*(*ia*+*i*+*k*:*ia*+*n*-1, *ja*+*i*-1), and *tau* in *TAU*(*ja*+*i*-1).

The elements of the vectors *v* together form the (*n*-*k*+1)-by-*nb* matrix *V* which is needed, with *T* and *Y*, to apply the transformation to the unreduced part of the matrix, using an update of the form: *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-*k*) := (*I*-*V***T***V*')*(*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-*k*)-*Y***V*'). The contents of *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-*k*) on exit are illustrated by the following example with *n* = 7, *k* = 3, and *nb* = 2:

$$\begin{bmatrix} a & h & a & a & a \\ a & h & a & a & a \\ a & h & a & a & a \\ h & h & a & a & a \\ v1 & h & a & a & a \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \end{bmatrix}$$

where *a* denotes an element of the original matrix *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-*k*), *h* denotes a modified element of the upper Hessenberg matrix *H*, and *vi* denotes an element of the vector defining *H*(*i*).

p?laiect

Exploits IEEE arithmetic to accelerate the computations of eigenvalues. (C interface function).

Syntax

```
void pslaiect(float *sigma, int *n, float *d, int *count);
void pdlaiectb(float *sigma, int *n, float *d, int *count);
void pdlaiectl(float *sigma, int *n, float *d, int *count);
```

Include Files

- C: mkl_scalapack.h

Description

The p?laiect routine computes the number of negative eigenvalues of ($A - \sigma I$). This implementation of the Sturm Sequence loop exploits IEEE arithmetic and has no conditionals in the innermost loop. The signbit for real routine pslaiect is assumed to be bit 32. Double precision routines pdlaiectb and pdlaiectl differ in the order of the double precision word storage and, consequently, in the signbit location. For pdlaiectb, the double precision word is stored in the big-endian word order and the signbit is assumed to be bit 32. For pdlaiectl, the double precision word is stored in the little-endian word order and the signbit is assumed to be bit 64.

Note that all arguments are call-by-reference so that this routine can be directly called from Fortran code.

This is a ScaLAPACK internal subroutine and arguments are not checked for unreasonable values.

Input Parameters

sigma	Real for pslaiect DOUBLE PRECISION for pdlaiectb/pdlaiectl. The shift. p?laiect finds the number of eigenvalues less than equal to *sigma*.
n	INTEGER. The order of the tridiagonal matrix *T*. $n \geq 1$.
d	Real for pslaiect DOUBLE PRECISION for pdlaiectb/pdlaiectl. Array of DIMENSION(2*n* -1). On entry, this array contains the diagonals and the squares of the off-diagonal elements of the tridiagonal matrix *T*. These elements are assumed to be interleaved in memory for better cache performance. The diagonal entries of *T* are in the entries *d*(1), *d*(3),..., *d*(2*n*-1), while the

squares of the off-diagonal entries are d(2), d(4), ..., d(2n-2). To avoid overflow, the matrix must be scaled so that its largest entry is no greater than $overflow^{(1/2)} * underflow^{(1/4)}$ in absolute value, and for greatest accuracy, it should not be much smaller than that.

Output Parameters

n INTEGER. The count of the number of eigenvalues of *T* less than or equal to *sigma*.

p?lange

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a general rectangular matrix.

Syntax

```
val = pslange(norm, m, n, a, ia, ja, desca, work)
val = pdlange(norm, m, n, a, ia, ja, desca, work)
val = pclange(norm, m, n, a, ia, ja, desca, work)
val = pzlange(norm, m, n, a, ia, ja, desca, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lange routine returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1).

Input Parameters

norm (global) CHARACTER. Specifies what value is returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*, it s not a matrix norm.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

m (global). INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(A). When *m* = 0, p?lange is set to zero. *m* ≥ 0.

n (global). INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). When *n* = 0, p?lange is set to zero. *n* ≥ 0.

a (local).
Real for pslange
DOUBLE PRECISION for pdlange
COMPLEX for pclange

COMPLEX*16 for pzlange.
Pointer into the local memory to an array of DIMENSION (*lld_a*, *LOCc*(*ja*+*n*-1)) containing the local pieces of the distributed matrix sub(*A*).

ia, *ja* — (global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca — (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

work — (local).
Real for pslange
DOUBLE PRECISION for pdlange
COMPLEX for pclange
COMPLEX*16 for pzlange.
Array DIMENSION (*lwork*).
lwork ≥ 0 if *norm* = 'M' or 'm' (not referenced),
*nq*0 if *norm* = '1', 'O' or 'o',
*mp*0 if *norm* = 'I' or 'i',
0 if *norm* = 'F', 'f', 'E' or 'e' (not referenced),
where
iroffa = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*),
indxg2p and numroc are ScaLAPACK tool routines; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

val — The value returned by the routine.

p?lanhs

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of an upper Hessenberg matrix.

Syntax

```
val = pslanhs(norm, n, a, ia, ja, desca, work)
val = pdlanhs(norm, n, a, ia, ja, desca, work)
val = pclanhs(norm, n, a, ia, ja, desca, work)
val = pzlanhs(norm, n, a, ia, ja, desca, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lanhs routine returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an upper Hessenberg distributed matrix sub(*A*) = *A*(*ia:ia+m-1*, *ja:ja+n-1*).

Input Parameters

norm CHARACTER*1. Specifies the value to be returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). When *n* = 0, p?lanhs is set to zero. *n* ≥ 0.

a (local).
Real for pslanhs
DOUBLE PRECISION for pdlanhs
COMPLEX for pclanhs
COMPLEX*16 for pzlanhs.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)) containing the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER.
The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

work (local).
Real for pslanhs
DOUBLE PRECISION for pdlanhs
COMPLEX for pclanhs
COMPLEX*16 for pzlanh.
Array, DIMENSION (*lwork*).
lwork ≥ 0 if *norm* = 'M' or 'm' (not referenced),
*nq*0 if *norm* = '1', 'O' or 'o',
*mp*0 if *norm* = 'I' or 'i',
0 if *norm* = 'F', 'f', 'E' or 'e' (not referenced),
where
iroffa = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*),
indxg2p and numroc are ScaLAPACK tool routines; *myrow*, *imycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

val The value returned by the fuction.

p?lansy, p?lanhe

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a real symmetric or a complex Hermitian matrix.

Syntax

```
val = pslansy(norm, uplo, n, a, ia, ja, desca, work)
val = pdlansy(norm, uplo, n, a, ia, ja, desca, work)
val = pclansy(norm, uplo, n, a, ia, ja, desca, work)
val = pzlansy(norm, uplo, n, a, ia, ja, desca, work)
val = pclanhe(norm, uplo, n, a, ia, ja, desca, work)
val = pzlanhe(norm, uplo, n, a, ia, ja, desca, work)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?lansy` and `p?lanhe` routines return the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a distributed matrix `sub(A) = A(ia:ia+m-1, ja:ja+n-1)`.

Input Parameters

norm — (global) `CHARACTER`. Specifies what value is returned by the routine:
= `'M'` or `'m'`: *val* = `max(abs(`A_{ij}`))`, largest absolute value of the matrix *A*, it s not a matrix norm.
= `'1'` or `'O'` or `'o'`: *val* = `norm1(A)`, 1-norm of the matrix *A* (maximum column sum),
= `'I'` or `'i'`: *val* = `normI(A)`, infinity norm of the matrix *A* (maximum row sum),
= `'F'`, `'f'`, `'E'` or `'e'`: *val* = `normF(A)`, Frobenius norm of the matrix *A* (square root of sum of squares).

uplo — (global) `CHARACTER`. Specifies whether the upper or lower triangular part of the symmetric matrix sub(*A*) is to be referenced.
= `'U'`: Upper triangular part of sub(*A*) is referenced,
= `'L'`: Lower triangular part of sub(*A*) is referenced.

n — (global) `INTEGER`.
The number of columns to be operated on i.e the number of columns of the distributed submatrix sub(*A*). When *n* = 0, `p?lansy` is set to zero. $n \geq 0$.

a — (local).
`REAL` for `pslansy`
`DOUBLE PRECISION` for `pdlansy`
`COMPLEX` for `pclansy`, `pclanhe`
`COMPLEX*16` for `pzlansy`, `pzlanhe`.
Pointer into the local memory to an array of `DIMENSION` (*lld_a*, *LOCc*(*ja*+*n*-1)) containing the local pieces of the distributed matrix sub(*A*).

	If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular matrix whose norm is to be computed, and the strictly lower triangular part of this matrix is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular matrix whose norm is to be computed, and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
work	(local). REAL for pslansy, pclansy, pclanhe DOUBLE PRECISION for pdlansy, pzlansy, pzlanhe Array DIMENSION (*lwork*). *lwork* ≥ 0 if *norm* = 'M' or 'm' (not referenced), 2**nq*0+*mp*0+*ldw* if *norm* = '1', 'O' or 'o', 'I' or 'i', where *ldw* is given by: if(*nprow*.*ne*.*npcol*) then *ldw* = *mb_a**iceil(iceil(*np*0,*mb_a*),(*lcm*/*nprow*)) else *ldw* = 0 end if 0 if *norm* = 'F', 'f', 'E' or 'e' (not referenced), where *lcm* is the least common multiple of *nprow* and *npcol*, *lcm* = ilcm(*nprow*, *npcol*) and iceil(*x*,*y*) is a ScaLAPACK function that returns ceiling (*x*/*y*). *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*), *iacol* = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*), *mp*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*), *nq*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*), ilcm, iceil, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

val	The value returned by the routine.

p?lantr

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a triangular matrix.

Syntax

```
val = pslantr(norm, uplo, diag, m, n, a, ia, ja, desca, work)
val = pdlantr(norm, uplo, diag, m, n, a, ia, ja, desca, work)
val = pclantr(norm, uplo, diag, m, n, a, ia, ja, desca, work)
val = pzlantr(norm, uplo, diag, m, n, a, ia, ja, desca, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lantr routine returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1).

Input Parameters

norm (global) CHARACTER. Specifies what value is returned by the routine:
= 'M' or 'm': *val* = max(abs(A_{ij})), largest absolute value of the matrix *A*, it s not a matrix norm.
= '1' or 'O' or 'o': *val* = norm1(*A*), 1-norm of the matrix *A* (maximum column sum),
= 'I' or 'i': *val* = normI(*A*), infinity norm of the matrix *A* (maximum row sum),
= 'F', 'f', 'E' or 'e': *val* = normF(*A*), Frobenius norm of the matrix *A* (square root of sum of squares).

uplo (global) CHARACTER.
Specifies whether the upper or lower triangular part of the symmetric matrix sub(*A*) is to be referenced.
= 'U': Upper trapezoidal,
= 'L': Lower trapezoidal.
Note that sub(*A*) is triangular instead of trapezoidal if *m* = *n*.

diag (global). CHARACTER.
Specifies whether the distributed matrix sub(*A*) has unit diagonal.
= 'N': Non-unit diagonal.
= 'U': Unit diagonal.

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(A). When *m* = 0, p?lantr is set to zero. *m* ≥ 0.

n (global) INTEGER.
The number of columns to be operated on i.e the number of columns of the distributed submatrix sub(*A*). When *n* = 0, p?lantr is set to zero. *n* ≥ 0.

a (local).
Real for pslantr
DOUBLE PRECISION for pdlantr
COMPLEX for pclantr
COMPLEX*16 for pzlantr.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)) containing the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER.
The row and column indices in the global array *a* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

work (local).
Real for pslantr
DOUBLE PRECISION for pdlantr
COMPLEX for pclantr

COMPLEX*16 for pzlantr.
Array DIMENSION (*lwork*).
lwork ≥ 0 if *norm* = 'M' or 'm' (not referenced),
*nq*0 if *norm* = '1', 'O' or 'o',
*mp*0 if *norm* = 'I' or 'i',
0 if *norm* = 'F', 'f', 'E' or 'e' (not referenced),
iroffa = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*),
indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

val — The value returned by the routine.

p?lapiv

Applies a permutation matrix to a general distributed matrix, resulting in row or column pivoting.

Syntax

```
call pslapiv(direc, rowcol, pivroc, m, n, a, ia, ja, desca, ipiv, ip, jp, descip, iwork)
call pdlapiv(direc, rowcol, pivroc, m, n, a, ia, ja, desca, ipiv, ip, jp, descip, iwork)
call pclapiv(direc, rowcol, pivroc, m, n, a, ia, ja, desca, ipiv, ip, jp, descip, iwork)
call pzlapiv(direc, rowcol, pivroc, m, n, a, ia, ja, desca, ipiv, ip, jp, descip, iwork)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lapiv routine applies either *P* (permutation matrix indicated by *ipiv*) or inv(*P*) to a general *m*-by-*n* distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1), resulting in row or column pivoting. The pivot vector may be distributed across a process row or a column. The pivot vector should be aligned with the distributed matrix *A*. This routine will transpose the pivot vector, if necessary.

For example, if the row pivots should be applied to the columns of sub(*A*), pass *rowcol*='C' and *pivroc*='C'.

Input Parameters

direc — (global) CHARACTER*1.
Specifies in which order the permutation is applied:
= 'F' (Forward). Applies pivots forward from top of matrix. Computes *P**sub(*A*).

= 'B' (Backward). Applies pivots backward from bottom of matrix.
Computes inv(*P*)*sub(*A*).

rowcol (global) CHARACTER*1.
Specifies if the rows or columns are to be permuted:
= 'R' Rows will be permuted,
= 'C' Columns will be permuted.

pivroc (global) CHARACTER*1.
Specifies whether *ipiv* is distributed over a process row or column:
= 'R'*ipiv* is distributed over a process row,
= 'C'*ipiv* is distributed over a process column.

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). When $m = 0$, p?lapiv is set to zero. $m \geq 0$.

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). When $n = 0$, p?lapiv is set to zero. $n \geq 0$.

a (local).
Real for pslapiv
DOUBLE PRECISION for pdlapiv
COMPLEX for pclapiv
COMPLEX*16 for pzlapiv.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)) containing the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER.
The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

ipiv (local). INTEGER.
Array, DIMENSION (*lipiv*) ;
when *rowcol*='R' or 'r':
lipiv ≥ *LOCr*(*ia*+*m*-1) + *mb_a* if *pivroc*='C' or 'c',
lipiv ≥ *LOCc*(*m* + *mod*(*jp*-1, *nb_p*)) if *pivroc*='R' or 'r', and,
when *rowcol*='C' or 'c':
lipiv ≥ *LOCr*(*n* + mod(*ip*-1, *mb_p*)) if *pivroc*='C' or 'c',
lipiv ≥ *LOCc*(*ja*+*n*-1) + *nb_a* if *pivroc*='R' or 'r'.
This array contains the pivoting information. *ipiv(i)* is the global row (column), local row (column) *i* was swapped with. When *rowcol*='R' or 'r' and *pivroc*='C' or 'c', or *rowcol*='C' or 'c' and *pivroc*='R' or 'r', the last piece of this array of size *mb_a* (resp. *nb_a*) is used as workspace. In those cases, this array is tied to the distributed matrix *A*.

ip, *jp* (global) INTEGER. The row and column indices in the global array *P* indicating the first row and the first column of the submatrix sub(*P*), respectively.

descip (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed vector *ipiv*.

iwork (local). INTEGER.
Array, DIMENSION (*ldw*), where *ldw* is equal to the workspace necessary for transposition, and the storage of the transposed *ipiv* :

Let *lcm* be the least common multiple of *nprow* and *npcol*.

```
If( rowcol.eq.'r' .and.  pivroc. eq.'r') then
      If( nprow.eq. npcol) then
ldw = LOCr( n_p + mod(jp-1, nb_p) ) + nb_p
     else
ldw = LOCr( n_p + mod(jp-1, nb_p) )+
nb_p * ceil( ceil(LOCc(n_p)/nb_p) / (lcm/npcol) )
   end if
 else if( rowcol.eq.'c' .and.  pivroc.eq.'c') then
       if( nprow.eq.
npcol ) then
ldw = LOCc( m_p + mod(ip-1, mb_p) ) + mb_p
 else
ldw = LOCc( m_p + mod(ip-1, mb_p) ) +
mb_p *ceil(ceil(LOCr(m_p)/mb_p) / (lcm/nprow) )
    end if
 else
iwork is not referenced.
 end if.
```

Output Parameters

a (local).
On exit, the local pieces of the permuted distributed submatrix.

p?laqge

Scales a general rectangular matrix, using row and column scaling factors computed by `p?geequ` *.*

Syntax

```
call pslaqge(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, equed)
call pdlaqge(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, equed)
call pclaqge(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, equed)
call pzlaqge(m, n, a, ia, ja, desca, r, c, rowcnd, colcnd, amax, equed)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?laqge` routine equilibrates a general *m*-by-*n* distributed matrix sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) using the row and scaling factors in the vectors *r* and *c* computed by `p?geequ`.

Input Parameters

m	(global). INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m* ≥0).
n	(global).INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n* ≥0).
a	(local). REAL for pslaqge DOUBLE PRECISION for pdlaqge COMPLEX for pclaqge COMPLEX*16 for pzlaqge. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, this array contains the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
r	(local). REAL for pslaqge DOUBLE PRECISION for pdlaqge COMPLEX for pclaqge COMPLEX*16 for pzlaqge. Array, DIMENSION *LOCr*(*m_a*). The row scale factors for sub(*A*). *r* is aligned with the distributed matrix *A*, and replicated across every process column. *r* is tied to the distributed matrix *A*.
c	(local). REAL for pslaqge DOUBLE PRECISION for pdlaqge COMPLEX for pclaqge COMPLEX*16 for pzlaqge. Array, DIMENSION *LOCc*(*n_a*). The row scale factors for sub(*A*). *c* is aligned with the distributed matrix *A*, and replicated across every process column. *c* is tied to the distributed matrix *A*.
rowcnd	(local). REAL for pslaqge DOUBLE PRECISION for pdlaqge COMPLEX for pclaqge COMPLEX*16 for pzlaqge. The global ratio of the smallest *r(i)* to the largest *r(i)*, *ia* ≤ *i* ≤ *ia*+*m*-1.
colcnd	(local). REAL for pslaqge DOUBLE PRECISION for pdlaqge COMPLEX for pclaqge COMPLEX*16 for pzlaqge. The global ratio of the smallest *c(i)* to the largest *c(i)*, *ia* ≤ *i* ≤ *ia*+*n*-1.
amax	(global). REAL for pslaqge DOUBLE PRECISION for pdlaqge

COMPLEX for pclaqge
COMPLEX*16 for pzlaqge.
Absolute value of largest distributed submatrix entry.

Output Parameters

a (local).
On exit, the equilibrated distributed matrix. See *equed* for the form of the equilibrated distributed submatrix.

equed (global) CHARACTER.
Specifies the form of equilibration that was done.
= 'N': No equilibration
= 'R': Row equilibration, that is, sub(*A*) has been pre-multiplied by diag(r(ia:ia+m-1)),
= 'C': column equilibration, that is, sub(*A*) has been post-multiplied by diag(c(ja:ja+n-1)),
= 'B': Both row and column equilibration, that is, sub(*A*) has been replaced by diag(r(ia:ia+m-1))* sub(*A*) * diag(c(ja:ja+n-1)).

p?laqsy

Scales a symmetric/Hermitian matrix, using scaling factors computed by p?poequ *.*

Syntax

```
call pslaqsy(uplo, n, a, ia, ja, desca, sr, sc, scond, amax, equed)
call pdlaqsy(uplo, n, a, ia, ja, desca, sr, sc, scond, amax, equed)
call pclaqsy(uplo, n, a, ia, ja, desca, sr, sc, scond, amax, equed)
call pzlaqsy(uplo, n, a, ia, ja, desca, sr, sc, scond, amax, equed)
```

Include Files

- C: mkl_scalapack.h

Description

The p?laqsy routine equilibrates a symmetric distributed matrix sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) using the scaling factors in the vectors *sr* and *sc*. The scaling factors are computed by p?poequ.

Input Parameters

uplo (global) CHARACTER. Specifies the upper or lower triangular part of the symmetric distributed matrix sub(*A*) is to be referenced:
= 'U': Upper triangular part;
= 'L': Lower triangular part.

n (global) INTEGER.
The order of the distributed submatrix sub(*A*). $n \geq 0$.

a (local).
REAL for pslaqsy
DOUBLE PRECISION for pdlaqsy
COMPLEX for pclaqsy
COMPLEX*16 for pzlaqsy.

Pointer into the local memory to an array of DIMENSION (*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the distributed matrix sub(*A*). On entry, the local pieces of the distributed symmetric matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and the strictly lower triangular part of sub(*A*) is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja*
(global) INTEGER.
The row and column indices in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca
(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

sr
(local)
REAL for pslaqsy
DOUBLE PRECISION for pdlaqsy
COMPLEX for pclaqsy
COMPLEX*16 for pzlaqsy.
Array, DIMENSION *LOCr*(*m_a*). The scale factors for *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1). *sr* is aligned with the distributed matrix *A*, and replicated across every process column. *sr* is tied to the distributed matrix *A*.

sc
(local)
REAL for pslaqsy
DOUBLE PRECISION for pdlaqsy
COMPLEX for pclaqsy
COMPLEX*16 for pzlaqsy.
Array, DIMENSION *LOCc*(*m_a*). The scale factors for *A* (*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1). *sr* is aligned with the distributed matrix *A*, and replicated across every process column. *sr* is tied to the distributed matrix *A*.

scond
(global). REAL for pslaqsy
DOUBLE PRECISION for pdlaqsy
COMPLEX for pclaqsy
COMPLEX*16 for pzlaqsy.
Ratio of the smallest *sr(i)* (respectively *sc(j)*) to the largest *sr(i)* (respectively *sc(j)*), with $ia \le i \le ia+n-1$ and $ja \le j \le ja+n-1$.

amax
(global).
REAL for pslaqsy
DOUBLE PRECISION for pdlaqsy
COMPLEX for pclaqsy
COMPLEX*16 for pzlaqsy.
Absolute value of largest distributed submatrix entry.

Output Parameters

a
On exit,
if *equed* = 'Y', the equilibrated matrix:
diag(*sr*(*ia*:*ia*+*n*-1)) * sub(*A*) * diag(*sc*(*ja*:*ja*+*n*-1)).

equed
(global) CHARACTER*1.
Specifies whether or not equilibration was done.

= 'N': No equilibration.
= 'Y': Equilibration was done, that is, sub(*A*) has been replaced by:
diag(*sr*(*ia*:*ia*+*n*-1))* sub(*A*) * diag(*sc*(*ja*:*ja*+*n*-1)).

p?lared1d

Redistributes an array assuming that the input array, bycol, is distributed across rows and that all process columns contain the same copy of bycol.

Syntax

```
call pslared1d(n, ia, ja, desc, bycol, byall, work, lwork)
call pdlared1d(n, ia, ja, desc, bycol, byall, work, lwork)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lared1d routine redistributes a 1D array. It assumes that the input array *bycol* is distributed across rows and that all process column contain the same copy of *bycol*. The output array *byall* is identical on all processes and contains the entire array.

Input Parameters

np = Number of local rows in *bycol*()

n	(global). INTEGER. The number of elements to be redistributed. $n \geq 0$.
ia, *ja*	(global) INTEGER. *ia*, *ja* must be equal to 1.
desc	(global and local) INTEGER array, DIMENSION 8. A 2D array descriptor, which describes *bycol*.
bycol	(local). REAL for pslared1d DOUBLE PRECISION for pdlared1d COMPLEX for pclared1d COMPLEX*16 for pzlared1d. Distributed block cyclic array global DIMENSION (*n*), local DIMENSION *np*. *bycol* is distributed across the process rows. All process columns are assumed to contain the same value.
work	(local). REAL for pslared1d DOUBLE PRECISION for pdlared1d COMPLEX for pclared1d COMPLEX*16 for pzlared1d. DIMENSION (*lwork*). Used to hold the buffers sent from one process to another.
lwork	(local) INTEGER. The size of the *work* array. *lwork* $\geq$ numroc(*n*, *desc*(*nb_*), 0, 0, *npcol*).

Output Parameters

byall	(global). REAL for pslared1d DOUBLE PRECISION for pdlared1d COMPLEX for pclared1d COMPLEX*16 for pzlared1d. Global DIMENSION (*n*), local DIMENSION (*n*). *byall* is exactly duplicated on all processes. It contains the same values as *bycol*, but it is replicated across all processes rather than being distributed.

p?lared2d

Redistributes an array assuming that the input array byrow is distributed across columns and that all process rows contain the same copy of byrow.

Syntax

```
call pslared2d(n, ia, ja, desc, byrow, byall, work, lwork)
call pdlared2d(n, ia, ja, desc, byrow, byall, work, lwork)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lared2d routine redistributes a 1D array. It assumes that the input array *byrow* is distributed across columns and that all process rows contain the same copy of *byrow*. The output array *byall* will be identical on all processes and will contain the entire array.

Input Parameters

np = Number of local rows in *byrow*()

n	(global) INTEGER. The number of elements to be redistributed. $n \geq 0$.
ia, *ja*	(global) INTEGER. *ia*, *ja* must be equal to 1.
desc	(global and local) INTEGER array, DIMENSION (*dlen_*). A 2D array descriptor, which describes *byrow*.
byrow	(local). REAL for pslared2d DOUBLE PRECISION for pdlared2d COMPLEX for pclared2d COMPLEX*16 for pzlared2d. Distributed block cyclic array global DIMENSION (*n*), local DIMENSION *np*. *bycol* is distributed across the process columns. All process rows are assumed to contain the same value.
work	(local). REAL for pslared2d DOUBLE PRECISION for pdlared2d COMPLEX for pclared2d COMPLEX*16 for pzlared2d. DIMENSION (*lwork*). Used to hold the buffers sent from one process to another.

lwork (local).INTEGER. The size of the *work* array. *lwork* ≥ numroc(*n*, *desc*(*nb_*), 0, 0, *npcol*).

Output Parameters

byall (global). REAL for pslared2d
DOUBLE PRECISION for pdlared2d
COMPLEX for pclared2d
COMPLEX*16 for pzlared2d.
Global DIMENSION(*n*), local DIMENSION (*n*). *byall* is exactly duplicated on all processes. It contains the same values as *bycol*, but it is replicated across all processes rather than being distributed.

p?larf

Applies an elementary reflector to a general rectangular matrix.

Syntax

```
call pslarf(side, m, n, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pdlarf(side, m, n, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pclarf(side, m, n, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pzlarf(side, m, n, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?larf routine applies a real/complex elementary reflector Q (or Q^T) to a real/complex m-by-n distributed matrix sub(C) = C(ic:ic+m-1, jc:jc+n-1), from either the left or the right. Q is represented in the form

```
Q = I-tau*v*v',
```

where *tau* is a real/complex scalar and v is a real/complex vector.

If *tau* = 0, then Q is taken to be the unit matrix.

Input Parameters

side (global). CHARACTER.
= 'L': form Q*sub(C),
= 'R': form sub(C)*Q, Q=Q^T.

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(A). ($m \geq 0$).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(A). ($n \geq 0$).

v (local).
REAL for pslarf
DOUBLE PRECISION for pdlarf
COMPLEX for pclarf

COMPLEX*16 for pzlarf.
Pointer into the local memory to an array of DIMENSION (*lld_v*,*) containing the local pieces of the distributed vectors *V* representing the Householder transformation *Q*,
v(*iv*:*iv*+*m*-1, *jv*) if *side* = 'L' and *incv* = 1,
v(*iv*, *jv*:*jv*+*m*-1) if *side* = 'L' and *incv* = *m_v*,
v(*iv*:*iv*+*n*-1, *jv*) if *side* = 'R' and *incv* = 1,
v(*iv*, *jv*:*jv*+*n*-1) if *side* = 'R' and *incv* = *m_v*.
The vector *v* is the representation of *Q*. *v* is not used if *tau* = 0.

iv, *jv* — (global) INTEGER. The row and column indices in the global array *V* indicating the first row and the first column of the submatrix sub(*V*), respectively.

descv — (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix V.

incv — (global) INTEGER.
The global increment for the elements of *v*. Only two values of *incv* are supported in this version, namely 1 and *m_v*.
incv must not be zero.

tau — (local).
REAL for pslarf
DOUBLE PRECISION for pdlarf
COMPLEX for pclarf
COMPLEX*16 for pzlarf.
Array, DIMENSION *LOCc*(*jv*) if *incv* = 1, and *LOCr*(*iv*) otherwise. This array contains the Householder scalars related to the Householder vectors. *tau* is tied to the distributed matrix *v*.

c — (local).
REAL for pslarf
DOUBLE PRECISION for pdlarf
COMPLEX for pclarf
COMPLEX*16 for pzlarf.
Pointer into the local memory to an array of DIMENSION(*lld_c*, *LOCc*(*jc*+*n*-1)), containing the local pieces of sub(*C*).

ic, *jc* — (global) INTEGER.
The row and column indices in the global array *c* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc — (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work — (local).
REAL for pslarf
DOUBLE PRECISION for pdlarf
COMPLEX for pclarf
COMPLEX*16 for pzlarf.
Array, DIMENSION (*lwork*).
If *incv* = 1,
  if *side* = 'L',
   if *ivcol* = *iccol*,
    lwork≥*nqc*0
   else
    lwork≥*mpc*0 + max(1, *nqc*0)
   end if

else if *side* = 'R' ,
lwork≥*nqc*0 + max(max(1, *mpc*0), numroc(numroc(*n*+
icoffc,*nb_v*,0,0,*npcol*),*nb_v*,0,0,*lcmq*))
end if
else if *incv* = *m_v*,
if *side* = 'L',
lwork≥*mpc*0 + max(max(*1*, *nqc*0), numroc(
numroc(*m*+*iroffc*,*mb_v*,0,0,*nprow*),*mb_v*,0,0, *lcmp*))
else if *side* = 'R',
if *ivrow* = *icrow*,
lwork≥*mpc*0
else
lwork≥*nqc*0 + max(*1*, *mpc*0)
end if
end if
end if,
where *lcm* is the least common multiple of *nprow* and *npcol* and *lcm* = *ilcm*(*nprow*, *npcol*), *lcmp* = *lcm*/*nprow*, *lcmq* = *lcm*/*npcol*,
iroffc = mod(*ic*-1, *mb_c*), *icoffc* = *mod*(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

c (local).
On exit, sub(*C*) is overwritten by the *Q**sub(*C*) if *side* = 'L',
or sub(*C*) * *Q* if *side* = 'R'.

p?larfb

Applies a block reflector or its transpose/conjugate-transpose to a general rectangular matrix.

Syntax

call pslarfb(*side*, *trans*, *direct*, *storev*, *m*, *n*, *k*, *v*, *iv*, *jv*, *descv*, *t*, *c*, *ic*, *jc*, *descc*, *work*)

call pdlarfb(*side*, *trans*, *direct*, *storev*, *m*, *n*, *k*, *v*, *iv*, *jv*, *descv*, *t*, *c*, *ic*, *jc*, *descc*, *work*)

call pclarfb(*side*, *trans*, *direct*, *storev*, *m*, *n*, *k*, *v*, *iv*, *jv*, *descv*, *t*, *c*, *ic*, *jc*, *descc*, *work*)

call pzlarfb(*side*, *trans*, *direct*, *storev*, *m*, *n*, *k*, *v*, *iv*, *jv*, *descv*, *t*, *c*, *ic*, *jc*, *descc*, *work*)

Include Files

- C: mkl_scalapack.h

Description

The p?larfb routine applies a real/complex block reflector Q or its transpose Q^T/conjugate transpose Q^H to a real/complex distributed m-by-n matrix sub(C) = $C(ic:ic+m-1, jc:jc+n-1)$ from the left or the right.

Input Parameters

side (global).CHARACTER.
if *side* = 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the Left;
if *side* = 'R': apply Q or $Q^T$for real flavors (Q^H for complex flavors) from the Right.

trans (global).CHARACTER.
if *trans* = 'N': no transpose, apply Q;
for real flavors, if *trans*='T': transpose, apply Q^T
for complex flavors, if *trans* = 'C': conjugate transpose, apply Q^H;

direct (global) CHARACTER. Indicates how Q is formed from a product of elementary reflectors.
if *direct* = 'F': $Q = H(1)*H(2)*...*H(k)$ (Forward)
if *direct* = 'B': $Q = H(k)*...*H(2)*H(1)$ (Backward)

storev (global) CHARACTER.
Indicates how the vectors that define the elementary reflectors are stored:
if *storev* = 'C': Columnwise
if *storev* = 'R': Rowwise.

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(C). ($m \geq 0$).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(C). ($n \geq 0$).

k (global) INTEGER.
The order of the matrix T.

v (local).
REAL for pslarfb
DOUBLE PRECISION for pdlarfb
COMPLEX for pclarfb
COMPLEX*16 for pzlarfb.
Pointer into the local memory to an array of DIMENSION
(*lld_v*, *LOCc*(*jv*+*k*-1)) if *storev* = 'C',
(*lld_v*, *LOCc*(*jv*+*m*-1)) if *storev* = 'R' and *side* = 'L',
(*lld_v*, *LOCc*(*jv*+*n*-1)) if *storev* = 'R' and *side* = 'R'.
It contains the local pieces of the distributed vectors V representing the Householder transformation.
if *storev* = 'C' and *side* = 'L', *lld_v* ≥ max(1,*LOCr*(*iv*+*m*-1));
if *storev* = 'C' and *side* = 'R', *lld_v* ≥ max(1,*LOCr*(*iv*+*n*-1));
if *storev* = 'R', *lld_v* ≥ *LOCr*(*jv*+*k*-1).

iv, *jv* (global) INTEGER.
The row and column indices in the global array V indicating the first row and the first column of the submatrix sub(V), respectively.

descv (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix V.

c (local).
REAL for pslarfb
DOUBLE PRECISION for pdlarfb
COMPLEX for pclarfb
COMPLEX*16 for pzlarfb.
Pointer into the local memory to an array of DIMENSION(*lld_c*, *LOCc*(*jc*+*n*-1)), containing the local pieces of sub(*C*).

ic, *jc* (global) INTEGER. The row and column indices in the global array *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local).
REAL for pslarfb
DOUBLE PRECISION for pdlarfb
COMPLEX for pclarfb
COMPLEX*16 for pzlarfb.
Workspace array, DIMENSION (*lwork*).
If *storev* = 'C',
 if *side* = 'L',
  lwork ≥ (*nqc*0 + *mpc*0) * *k*
 else if *side* = 'R',
  lwork ≥ (*nqc*0 + max(*npv*0 + numroc(numroc(*n* +
   icoffc, *nb_v*, 0, 0, *npcol*), *nb_v*, 0, 0, *lcmq*),
   *mpc*0)) * *k*
 end if
else if *storev* = 'R' ,
 if *side* = 'L' ,
  lwork≥ (*mpc*0 + max(*mqv*0 + numroc(numroc(*m* +
  iroffc, *mb_v*, 0, 0, *nprow*), *mb_v*, 0, 0, *lcmp*),
  *nqc*0)) * *k*
 else if *side* = 'R',
  lwork ≥ (*mpc*0 + *nqc*0) * *k*
 end if
end if,
where
lcmq = *lcm* / *npcol* with *lcm* = iclm(*nprow*, *npcol*),
iroffv = mod(*iv*-1, *mb_v*), *icoffv* = mod(*jv*-1, *nb_v*),
ivrow = indxg2p(*iv*, *mb_v*, *myrow*, *rsrc_v*, *nprow*),
ivcol = indxg2p(*jv*, *nb_v*, *mycol*, *csrc_v*, *npcol*),
MqV0 = numroc(*m*+*icoffv*, *nb_v*, *mycol*, *ivcol*, *npcol*),
NpV0 = numroc(*n*+*iroffv*, *mb_v*, *myrow*, *ivrow*, *nprow*),
iroffc = mod(*ic*-1, *mb_c*), *icoffc* = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_*c, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_*c, *npcol*),
MpC0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
NpC0 = numroc(*n*+*icoffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
NqC0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

t	(local). REAL for pslarfb DOUBLE PRECISION for pdlarfb COMPLEX for pclarfb COMPLEX*16 for pzlarfb. Array, DIMENSION(*mb_v*, *mb_v*) if *storev* = 'R', and (*nb_v*, *nb_v*) if *storev* = 'C'. The triangular matrix *t* is the representation of the block reflector.
c	(local). On exit, sub(*C*) is overwritten by the *Q**sub(*C*), or *Q*'*sub(*C*), or sub(*C*)**Q*, or sub(*C*)**Q*'. *Q*' is transpose (conjugate transpose) of *Q*.

p?larfc

Applies the conjugate transpose of an elementary reflector to a general matrix.

Syntax

```
call pclarfc(side, m, n, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
```

```
call pzlarfc(side, m, n, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?larfc routine applies a complex elementary reflector Q^H to a complex *m*-by-*n* distributed matrix sub(*C*) = *C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1), from either the left or the right. *Q* is represented in the form

Q = *i*-*tau***v***v*',

where *tau* is a complex scalar and *v* is a complex vector.

If *tau* = 0, then *Q* is taken to be the unit matrix.

Input Parameters

side	(global).CHARACTER. if *side* = 'L': form Q^H*sub(*C*) ; if *side* = 'R': form sub (*C*)*Q^H.
m	(global) INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*C*). (*m* ≥ 0).
n	(global) INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*C*). (*n* ≥ 0).
v	(local). COMPLEX for pclarfc COMPLEX*16 for pzlarfc. Pointer into the local memory to an array of DIMENSION (*lld_v*,*) containing the local pieces of the distributed vectors *v* representing the Householder transformation *Q*,

v(*iv*:*iv*+*m*-1, *jv*) if *side* = 'L' and *incv* = 1,
v(*iv*, *jv*:*jv*+*m*-1) if *side* = 'L' and *incv* = *m_v*,
v(*iv*:*iv*+*n*-1, *jv*) if *side* = 'R' and *incv* = 1,
v(*iv*, *jv*:*jv*+*n*-1) if *side* = 'R' and *incv* = *m_v*.
The vector *v* is the representation of *Q*. *v* is not used if *tau* = 0.

iv, *jv*
(global) INTEGER.
The row and column indices in the global array *V* indicating the first row and the first column of the submatrix sub(*V*), respectively.

descv
(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *V*.

incv
(global) INTEGER.
The global increment for the elements of *v*. Only two values of *incv* are supported in this version, namely 1 and *m_v*.
incv must not be zero.

tau
(local)
COMPLEX for pclarfc
COMPLEX*16 for pzlarfc.
Array, DIMENSION *LOCc*(*jv*) if *incv* = 1, and *LOCr*(*iv*) otherwise. This array contains the Householder scalars related to the Householder vectors. *tau* is tied to the distributed matrix *V*.

c
(local).
COMPLEX for pclarfc
COMPLEX*16 for pzlarfc.
Pointer into the local memory to an array of DIMENSION (*lld_c*, *LOCc*(*jc*+*n*-1)), containing the local pieces of sub(*C*).

ic, *jc*
(global) INTEGER.
The row and column indices in the global array *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc
(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work
(local).
COMPLEX for pclarfc
COMPLEX*16 for pzlarfc.
Workspace array, DIMENSION (*lwork*).
If *incv* = 1,
 if *side* = 'L' ,
  if *ivcol* = *iccol*,
   lwork ≥ *nqc*0
  else
   lwork ≥ *mpc*0 + max(1, *nqc*0)
  end if
 else if *side* = 'R',
  lwork ≥ *nqc*0 + max(max(1, *mpc*0), numroc(numroc(
   n+*icoffc*,*nb_v*,0,0,*npcol*), *nb_v*,0,0,*lcmq*))
 end if
else if *incv* = *m_v*,
 if *side* = 'L',
  lwork ≥ *mpc*0 + max(max(1, *nqc*0), numroc(numroc(
   m+*iroffc*,*mb_v*,0,0,*nprow*),*mb_v*,0,0,*lcmp*))
 else if *side* = 'R' ,
  if *ivrow* = *icrow*,

lwork ≥ *mpc*0
else
lwork ≥ *nqc*0 + max(1, *mpc*0)
end if
end if
end if,
where *lcm* is the least common multiple of *nprow* and *npcol* and *lcm* = ilcm(*nprow*, *npcol*),
lcmp = *lcm*/*nprow*, *lcmq* = *lcm*/*npcol*,
iroffc = mod(*ic*-1, *mb_c*), *icoffc* = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

c (local).
On exit, sub(*C*) is overwritten by the Q^H*sub(*C*) if *side* = 'L', or sub(*C*) * Q^H if *side* = 'R'.

p?larfg

Generates an elementary reflector (Householder matrix).

Syntax

```
call pslarfg(n, alpha, iax, jax, x, ix, jx, descx, incx, tau)
call pdlarfg(n, alpha, iax, jax, x, ix, jx, descx, incx, tau)
call pclarfg(n, alpha, iax, jax, x, ix, jx, descx, incx, tau)
call pzlarfg(n, alpha, iax, jax, x, ix, jx, descx, incx, tau)
```

Include Files

- C: mkl_scalapack.h

Description

The p?larfg routine generates a real/complex elementary reflector *H* of order *n*, such that

$$H * \mathrm{sub}(X) = H * \begin{pmatrix} x(iax, jax) \\ x \end{pmatrix} = \begin{pmatrix} alpha \\ 0 \end{pmatrix}, \quad H'* H = I,$$

where *alpha* is a scalar (a real scalar - for complex flavors), and sub(*X*) is an (*n*-1)-element real/complex distributed vector *X*(*ix*:*ix*+*n*-2, *jx*) if *incx* = 1 and *X*(*ix*, *jx*:*jx*+*n*-2) if *incx* = *descx*(*m_*). *H* is represented in the form

$$H = I - tau * \begin{pmatrix} 1 \\ v \end{pmatrix} * (1\ v')$$

where *tau* is a real/complex scalar and *v* is a real/complex (*n*-1)-element vector. Note that *H* is not Hermitian.

If the elements of sub(*X*) are all zero (and *X*(*iax*, *jax*) is real for complex flavors), then *tau* = 0 and *H* is taken to be the unit matrix.

Otherwise 1 ≤ real(*tau*) ≤ 2 and abs(*tau*-1) ≤ 1.

Input Parameters

n — (global) INTEGER.
The global order of the elementary reflector. *n* ≥ 0.

iax, *jax* — (global) INTEGER.
The global row and column indices in *x* of *X*(*iax*, *jax*).

x — (local).
Real for pslarfg
DOUBLE PRECISION for pdlarfg
COMPLEX for pclarfg
COMPLEX*16 for pzlarfg.
Pointer into the local memory to an array of DIMENSION (*lld_x*, *). This array contains the local pieces of the distributed vector sub(*X*). Before entry, the incremented array sub(*X*) must contain vector *x*.

ix, *jx* — (global) INTEGER.
The row and column indices in the global array *X* indicating the first row and the first column of sub(*X*), respectively.

descx — (global and local) INTEGER.
Array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *X*.

incx — (global) INTEGER.
The global increment for the elements of *x*. Only two values of *incx* are supported in this version, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

alpha — (local)
REAL for pslafg
DOUBLE PRECISION for pdlafg
COMPLEX for pclafg
COMPLEX*16 for pzlafg.
On exit, *alpha* is computed in the process scope having the vector sub(*X*).

x — (local).
On exit, it is overwritten with the vector *v*.

tau — (local).
Real for pslarfg
DOUBLE PRECISION for pdlarfg
COMPLEX for pclarfg
COMPLEX*16 for pzlarfg.

Array, DIMENSION *LOCc*(*jx*) if *incx* = 1, and *LOCr*(*ix*) otherwise. This array contains the Householder scalars related to the Householder vectors. *tau* is tied to the distributed matrix *X*.

p?larft

*Forms the triangular vector T of a block reflector H=I-V*T*V^H.*

Syntax

```
call pslarft(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
call pdlarft(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
call pclarft(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
call pzlarft(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?larft routine forms the triangular factor *T* of a real/complex block reflector *H* of order *n*, which is defined as a product of *k* elementary reflectors.

If *direct* = 'F', *H* = *H*(1)**H*(2)...**H*(*k*), and *T* is upper triangular;

If *direct* = 'B', *H* = *H*(*k*)*...**H*(2)**H*(1), and *T* is lower triangular.

If *storev* = 'C', the vector which defines the elementary reflector *H*(*i*) is stored in the *i*-th column of the distributed matrix *V*, and

H = *I*-*V***T***V*'

If *storev* = 'R', the vector which defines the elementary reflector *H*(*i*) is stored in the *i*-th row of the distributed matrix *V*, and

H = *I*-*V*'**T***V*.

Input Parameters

direct
(global) CHARACTER*1.
Specifies the order in which the elementary reflectors are multiplied to form the block reflector:
if *direct* = 'F': *H* = *H*(1)**H*(2)*...**H*(*k*) (forward)
if *direct* = 'B': *H* = *H*(*k*)*...**H*(2)**H*(1) (backward).

storev
(global) CHARACTER*1.
Specifies how the vectors that define the elementary reflectors are stored (See *Application Notes* below):
if *storev* = 'C': columnwise;
if *storev* = 'R': rowwise.

n
(global) INTEGER.
The order of the block reflector *H*. *n* ≥ 0.

k
(global) INTEGER.
The order of the triangular factor *T*, is equal to the number of elementary reflectors.
1 ≤ *k* ≤ *mb_v* (= *nb_v*).

v
REAL for pslarft
DOUBLE PRECISION for pdlarft
COMPLEX for pclarft
COMPLEX*16 for pzlarft.
Pointer into the local memory to an array of local DIMENSION (*LOCr*(*iv*+*n*-1), *LOCc*(*jv*+*k*-1)) if *storev* = 'C', and (*LOCr*(*iv*+*k*-1), *LOCc*(*jv*+*n*-1)) if *storev* = 'R'.
The distributed matrix *V* contains the Householder vectors. (See *Application Notes* below).

iv, *jv*
(global) INTEGER.
The row and column indices in the global array *v* indicating the first row and the first column of the submatrix sub(*V*), respectively.

descv
(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *V*.

tau
(local)
REAL for pslarft
DOUBLE PRECISION for pdlarft
COMPLEX for pclarft
COMPLEX*16 for pzlarft.
Array, DIMENSION*LOCr*(*iv*+*k*-1) if *incv* = *m_v*, and *LOCc*(*jv*+*k*-1) otherwise. This array contains the Householder scalars related to the Householder vectors.
tau is tied to the distributed matrix *V*.

work
(local).
REAL for pslarft
DOUBLE PRECISION for pdlarft
COMPLEX for pclarft
COMPLEX*16 for pzlarft.
Workspace array, DIMENSION (*k**(*k*-1)/2).

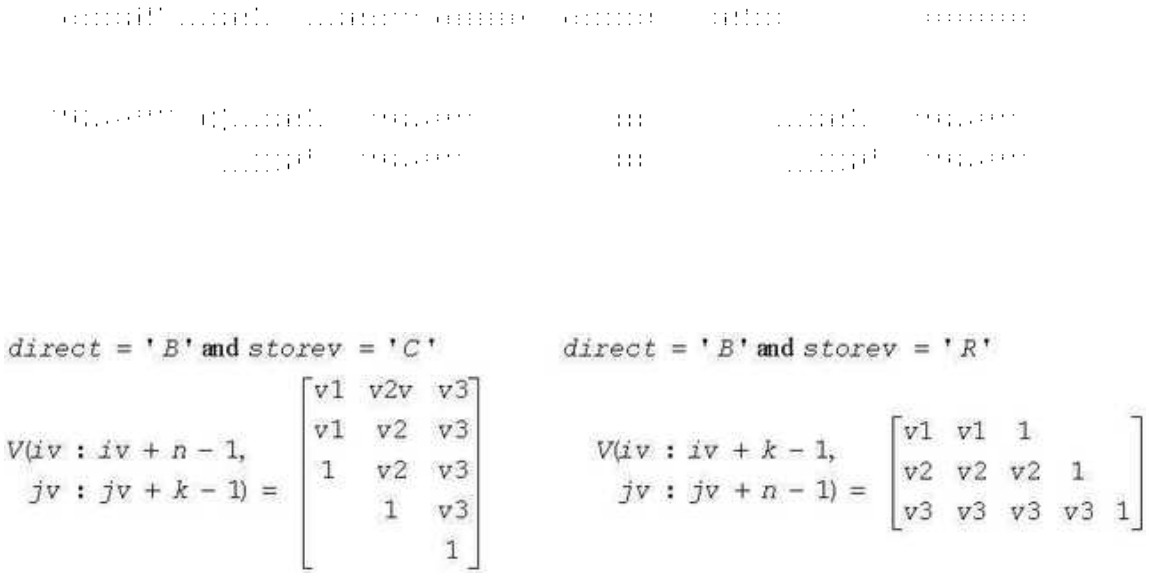
Output Parameters

v
REAL for pslarft
DOUBLE PRECISION for pdlarft
COMPLEX for pclarft
COMPLEX*16 for pzlarft.

t
(local)
REAL for pslarft
DOUBLE PRECISION for pdlarft
COMPLEX for pclarft
COMPLEX*16 for pzlarft.
Array, DIMENSION (*nb_v*,*nb_v*) if *storev* = 'C', and (*mb_v*,*mb_v*) otherwise. It contains the *k*-by-*k* triangular factor of the block reflector associated with *v*. If *direct* = 'F', *t* is upper triangular; if *direct* = 'B', *t* is lower triangular.

Application Notes

The shape of the matrix *V* and the storage of the vectors that define the *H*(*i*) is best illustrated by the following example with *n* = 5 and *k* = 3. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.



p?larz

Applies an elementary reflector as returned by `p?tzrzf` *to a general matrix.*

Syntax

```
call pslarz(side, m, n, l, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pdlarz(side, m, n, l, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pclarz(side, m, n, l, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pzlarz(side, m, n, l, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?larz` routine applies a real/complex elementary reflector Q (or Q^T) to a real/complex m-by-n distributed matrix `sub(C) = C(ic:ic+m-1, jc:jc+n-1)`, from either the left or the right. Q is represented in the form

Q = `I-tau*v*v'`,

where tau is a real/complex scalar and v is a real/complex vector.

If tau = 0, then Q is taken to be the unit matrix.

Q is a product of k elementary reflectors as returned by `p?tzrzf`.

Input Parameters

side	(global) CHARACTER. if *side* = 'L': form Q*sub(C), if *side* = 'R': form sub(C)*Q, $Q = Q^T$ (for real flavors).
m	(global) INTEGER.

The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*C*). (*m* ≥ 0).

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*C*). (*n* ≥ 0).

l (global). INTEGER.
The columns of the distributed submatrix sub(*A*) containing the meaningful part of the Householder reflectors. If *side* = 'L', *m* ≥ *l* ≥ 0, if *side* = 'R', *n* ≥ *l* ≥ 0.

v (local).
REAL for pslarz
DOUBLE PRECISION for pdlarz
COMPLEX for pclarz
COMPLEX*16 for pzlarz.
Pointer into the local memory to an array of DIMENSION (*lld_v*,*) containing the local pieces of the distributed vectors *v* representing the Householder transformation *Q*,
v(*iv*:*iv*+*l*-1, *jv*) if *side* = 'L' and *incv* = 1,
v(*iv*, *jv*:*jv*+*l*-1) if *side* = 'L' and *incv* = *m_v*,
v(*iv*:*iv*+*l*-1, *jv*) if *side* = 'R' and *incv* = 1,
v(*iv*, *jv*:*jv*+*l*-1) if *side* = 'R' and *incv* = *m_v*.
The vector *v* in the representation of *Q*. *v* is not used if *tau* = 0.

iv, *jv* (global) INTEGER. The row and column indices in the global array *V* indicating the first row and the first column of the submatrix sub(*V*), respectively.

descv (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *V*.

incv (global) INTEGER.
The global increment for the elements of *v*. Only two values of *incv* are supported in this version, namely 1 and *m_v*.
incv must not be zero.

tau (local)
REAL for pslarz
DOUBLE PRECISION for pdlarz
COMPLEX for pclarz
COMPLEX*16 for pzlarz.
Array, DIMENSION *LOCc*(*jv*) if *incv* = 1, and *LOCr*(*iv*) otherwise. This array contains the Householder scalars related to the Householder vectors. *tau* is tied to the distributed matrix *V*.

c (local).
REAL for pslarz
DOUBLE PRECISION for pdlarz
COMPLEX for pclarz
COMPLEX*16 for pzlarz.
Pointer into the local memory to an array of DIMENSION (*lld_c*, *LOCc*(*jc* +*n*-1)), containing the local pieces of sub(*C*).

ic, *jc* (global) INTEGER.
The row and column indices in the global array *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local).
REAL for pslarz
DOUBLE PRECISION for pdlarz
COMPLEX for pclarz
COMPLEX*16 for pzlarz.
Array, DIMENSION (*lwork*)
If *incv* = 1,
 if *side* = 'L' ,
  if *ivcol* = *iccol*,
   lwork ≥ *NqC*0
  else
   lwork ≥ *MpC*0 + max(1, *NqC*0)
  end if
 else if *side* = 'R' ,
  lwork ≥ *NqC*0 + max(max(1, *MpC*0), numroc(numroc(*n*+*icoffc*,*nb_v*, 0,0,*npcol*),*nb_v*,0,0,*lcmq*))
 end if
else if *incv* = *m_v*,
 if *side* = 'L' ,
  lwork ≥ *MpC*0 + max(max(1, *NqC*0), numroc(numroc(*m*+*iroffc*,*mb_v*, 0,0,*nprow*),*mb_v*,0,0,*lcmp*))
 else if *side* = 'R' ,
  if *ivrow* = *icrow*,
   lwork ≥ *MpC*0
  else
   lwork ≥ *NqC*0 + max(1, *MpC*0)
  end if
 end if
end if.
Here *lcm* is the least common multiple of *nprow* and *npcol* and
lcm = ilcm(*nprow*, *npcol*), *lcmp* = *lcm* / *nprow*,
lcmq = *lcm* / *npcol*,
iroffc = mod(*ic*-1, *mb_c*), *icoffc* = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_*c, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_*c, *npcol*),
*mpc*0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
*nqc*0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

c (local).
On exit, sub(*C*) is overwritten by the *Q**sub(*C*) if *side* = 'L', or sub(*C*)**Q* if *side* = 'R'.

p?larzb

Applies a block reflector or its transpose/conjugate-transpose as returned by `p?tzrzf` *to a general matrix.*

Syntax

```
call pslarzb(side, trans, direct, storev, m, n, k, l, v, iv, jv, descv, t, c, ic, jc, descc, work)
call pdlarzb(side, trans, direct, storev, m, n, k, l, v, iv, jv, descv, t, c, ic, jc, descc, work)
call pclarzb(side, trans, direct, storev, m, n, k, l, v, iv, jv, descv, t, c, ic, jc, descc, work)
call pzlarzb(side, trans, direct, storev, m, n, k, l, v, iv, jv, descv, t, c, ic, jc, descc, work)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?larzb` routine applies a real/complex block reflector Q or its transpose Q^T (conjugate transpose Q^H for complex flavors) to a real/complex distributed m-by-n matrix sub(C) = $C(ic:ic+m-1, jc:jc+n-1)$ from the left or the right.

Q is a product of k elementary reflectors as returned by `p?tzrzf`.

Currently, only *storev* = `'R'` and *direct* = `'B'` are supported.

Input Parameters

side
(global) `CHARACTER`.
if *side* = `'L'`: apply Q or Q^T (Q^H for complex flavors) from the Left;
if *side* = `'R'`: apply Q or Q^T (Q^H for complex flavors) from the Right.

trans
(global) `CHARACTER`.
if *trans* = `'N'`: No transpose, apply Q;
If *trans*=`'T'`: Transpose, apply Q^T (real flavors);
If *trans*=`'C'`: Conjugate transpose, apply Q^H (complex flavors).

direct
(global) `CHARACTER`.
Indicates how H is formed from a product of elementary reflectors.
if *direct* = `'F'`: $H = H(1)*H(2)*...*H(k)$ - forward (not supported) ;
if *direct* = `'B'`: $H = H(k)*...*H(2)*H(1)$ - backward.

storev
(global) `CHARACTER`.
Indicates how the vectors that define the elementary reflectors are stored:
if *storev* = `'C'`: columnwise (not supported).
if *storev* = `'R'`: rowwise.

m
(global) `INTEGER`.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(C). $(m \geq 0)$.

n
(global) `INTEGER`.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(C). $(n \geq 0)$.

k	(global) INTEGER. The order of the matrix *T*. (= the number of elementary reflectors whose product defines the block reflector).
l	(global) INTEGER. The columns of the distributed submatrix sub(*A*) containing the meaningful part of the Householder reflectors. If *side* = 'L', *m* ≥ *l* ≥ 0, if *side* = 'R', *n* ≥ *l* ≥ 0.
v	(local). REAL for pslarzb DOUBLE PRECISION for pdlarzb COMPLEX for pclarzb COMPLEX*16 for pzlarzb. Pointer into the local memory to an array of DIMENSION(*lld_v*, *LOCc*(*jv*+*m*-1)) if *side* = 'L', (*lld_v*, *LOCc*(*jv*+*m*-1)) if *side* = 'R'. It contains the local pieces of the distributed vectors *V* representing the Householder transformation as returned by p?tzrzf. *lld_v* ≥ *LOCr*(*iv*+*k*-1).
iv, *jv*	(global) INTEGER. The row and column indices in the global array *V* indicating the first row and the first column of the submatrix sub(*V*), respectively.
descv	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *V*.
t	(local) REAL for pslarzb DOUBLE PRECISION for pdlarzb COMPLEX for pclarzb COMPLEX*16 for pzlarzb. Array, DIMENSION *mb_v* by *mb_v*. The lower triangular matrix *T* in the representation of the block reflector.
c	(local). REAL for pslarfb DOUBLE PRECISION for pdlarfb COMPLEX for pclarfb COMPLEX*16 for pzlarfb. Pointer into the local memory to an array of DIMENSION(*lld_c*, *LOCc*(*jc*+*n*-1)). On entry, the *m*-by-*n* distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the global array *c* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.
work	(local). REAL for pslarzb DOUBLE PRECISION for pdlarzb COMPLEX for pclarzb COMPLEX*16 for pzlarzb.

Array, DIMENSION (*lwork*).

```
If storev = 'C' ,
  if side = 'L' ,
    lwork ≥(nqc0 + mpc0)* k
  else if side = 'R' ,
    lwork ≥ (nqc0 + max(npv0 + numroc(numroc(n+icoffc, nb_v, 0, 0,
npcol),
           nb_v, 0, 0, lcmq), mpc0))* k
  end if
else if storev = 'R' ,
  if side = 'L' ,
    lwork ≥ (mpc0 + max(mqv0 + numroc(numroc(m+iroffc, mb_v, 0, 0,
nprow),
             mb_v, 0, 0, lcmp), nqc0))* k
  else if side = 'R' ,
    lwork ≥ (mpc0 + nqc0) * k
  end if
  end if.
```

Here *lcmq* = *lcm*/*npcol* with *lcm* = iclm(*nprow*, *npcol*),
iroffv = mod(*iv*-1, *mb_v*), *icoffv* = mod(*jv*-*1*, *nb_v*),
ivrow = indxg2p(*iv*, *mb_v*, *myrow*, *rsrc_v*, *nprow*),
ivcol = indxg2p(*jv*, *nb_v*, *mycol*, *csrc_v*, *npcol*),
mqv0 = numroc(*m*+*icoffv*, *nb_v*, *mycol*, *ivcol*, *npcol*),
npv0 = numroc(*n*+*iroffv*, *mb_v*, *myrow*, *ivrow*, *nprow*),
iroffc = mod(*ic*-1, *mb_c*), *icoffc*= *mod*(*jc*-1, *nb_c*),
icrow= indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol= indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
mpc0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
npc0 = numroc(*n*+*icoffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
nqc0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

c (local).
On exit, sub(*C*) is overwritten by the *Q**sub(*C*), or *Q*'*sub(*C*), or sub(*C*)**Q*, or sub(*C*)**Q*', where *Q*' is the transpose (conjugate transpose) of *Q*.

p?larzc

Applies (multiplies by) the conjugate transpose of an elementary reflector as returned by p?tzrzf *to a general matrix.*

Syntax

```
call pclarzc(side, m, n, l, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
call pzlarzc(side, m, n, l, v, iv, jv, descv, incv, tau, c, ic, jc, descc, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?larzc routine applies a complex elementary reflector Q^H to a complex m-by-n distributed matrix sub(C) = C(ic:ic+m-1, jc:jc+n-1), from either the left or the right. Q is represented in the form

Q = i-tau*v*v',

where tau is a complex scalar and v is a complex vector.

If tau = 0, then Q is taken to be the unit matrix.

Q is a product of k elementary reflectors as returned by p?tzrzf.

Input Parameters

side	(global) CHARACTER. if *side* = 'L': form Q^H*sub(C); if *side* = 'R': form sub(C)*Q^H.
m	(global) INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(C). ($m \geq 0$).
n	(global) INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(C). ($n \geq 0$).
l	(global) INTEGER. The columns of the distributed submatrix sub(A) containing the meaningful part of the Householder reflectors. If *side* = 'L', $m \geq l \geq 0$, if *side* = 'R', $n \geq l \geq 0$.
v	(local). COMPLEX for pclarzc COMPLEX*16 for pzlarzc. Pointer into the local memory to an array of DIMENSION (*lld_v*,*) containing the local pieces of the distributed vectors v representing the Householder transformation Q, v(iv:iv+l-1, jv) if *side* = 'L' and *incv* = 1, v(iv, jv:jv+l-1) if *side* = 'L' and *incv* = *m_v*, v(iv:iv+l-1, jv) if *side* = 'R' and *incv* = 1, v(iv, jv:jv+l-1) if *side* = 'R' and *incv* = *m_v*. The vector v in the representation of Q. v is not used if *tau* = 0.
iv, *jv*	(global) INTEGER. The row and column indices in the global array V indicating the first row and the first column of the submatrix sub(V), respectively.
descv	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix V.
incv	(global). INTEGER.

	The global increment for the elements of *v*. Only two values of *incv* are supported in this version, namely 1 and *m_v*. *incv* must not be zero.
tau	(local) COMPLEX for pclarzc COMPLEX*16 for pzlarzc. Array, DIMENSION*LOCc*(*jv*) if *incv* = 1, and *LOCr*(*iv*) otherwise. This array contains the Householder scalars related to the Householder vectors. *tau* is tied to the distributed matrix *V*.
c	(local). COMPLEX for pclarzc COMPLEX*16 for pzlarzc. Pointer into the local memory to an array of DIMENSION(*lld_c*, *LOCc*(*jc* +*n*-1)), containing the local pieces of sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the global array *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local).

```
If incv = 1,
  if side = 'L' ,
    if ivcol = iccol,
      lwork ≥ nqc0
    else
      lwork ≥ mpc0 + max(1, nqc0)
    end if
  else if side = 'R' ,
    lwork ≥ nqc0 + max(max(1, mpc0), numroc(numroc(n+icoffc, nb_v, 0, 0,
npcol),
           nb_v, 0, 0, lcmq))
  end if
else if incv = m_v,
  if side = 'L' ,
    lwork ≥ mpc0 + max(max(1, nqc0), numroc(numroc(m+iroffc, mb_v, 0, 0,
nprow),
           mb_v, 0, 0, lcmp))
  else if side = 'R',
    if ivrow = icrow,
      lwork ≥ mpc0
    else
      lwork ≥ nqc0 + max(1, mpc0)
    end if
  end if
        end if
```

Here *lcm* is the least common multiple of *nprow* and *npcol*;
lcm = ilcm(*nprow*, *npcol*), *lcmp* = *lcm*/*nprow*, *lcmq*= *lcm*/*npcol*,
iroffc = mod(*ic*-1, *mb_c*), *icoffc*= mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
mpc0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
nqc0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions;
myrow, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

c (local).
On exit, sub(*C*) is overwritten by the Q^H*sub(*C*) if *side* = 'L', or sub(*C*)*Q^H if *side* = 'R'.

p?larzt

*Forms the triangular factor T of a block reflector H=I-V*T*V^H as returned by* `p?tzrzf`.

Syntax

```
call pslarzt(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
call pdlarzt(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
call pclarzt(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
call pzlarzt(direct, storev, n, k, v, iv, jv, descv, tau, t, work)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?larzt` routine forms the triangular factor *T* of a real/complex block reflector *H* of order greater than *n*, which is defined as a product of *k* elementary reflectors as returned by `p?tzrzf`.

If *direct* = 'F', *H* = *H*(1)**H*(2)*...**H*(*k*), and *T* is upper triangular;

If *direct* = 'B', *H* = *H*(*k*)*...**H*(2)**H*(1), and *T* is lower triangular.

If *storev* = 'C', the vector which defines the elementary reflector *H*(*i*), is stored in the *i*-th column of the array *v*, and

H = *i*-*v***t***v*'.

If *storev* = 'R', the vector, which defines the elementary reflector *H*(*i*), is stored in the *i*-th row of the array *v*, and

H = *i*-*v*'**t***v*

Currently, only *storev* = 'R' and *direct* = 'B' are supported.

Input Parameters

direct
(global) CHARACTER.
Specifies the order in which the elementary reflectors are multiplied to form the block reflector:
if *direct* = 'F': *H* = *H*(1)**H*(2)*...**H*(*k*) (Forward, not supported)
if *direct* = 'B': *H* = *H*(*k*)*...**H*(2)**H*(1) (Backward).

storev
(global) CHARACTER.
Specifies how the vectors which defines the elementary reflectors are stored:
if *storev* = 'C': columnwise (not supported);
if *storev* = 'R': rowwise.

n
(global). INTEGER.
The order of the block reflector *H*. *n* ≥ 0.

k
(global). INTEGER.
The order of the triangular factor *T* (= the number of elementary reflectors).
1≤*k*≤*mb_v*(= *nb_v*).

v
REAL for `pslarzt`
DOUBLE PRECISION for `pdlarzt`

	COMPLEX for pclarzt COMPLEX*16 for pzlarzt. Pointer into the local memory to an array of local DIMENSION(*LOCr*(*iv*+*k*-1), *LOCc*(*jv*+*n*-1)). The distributed matrix *V* contains the Householder vectors. See *Application Notes* below.
iv, *jv*	(global) INTEGER. The row and column indices in the global array *V* indicating the first row and the first column of the submatrix sub(*V*), respectively.
descv	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *V*.
tau	(local) REAL for pslarzt DOUBLE PRECISION for pdlarzt COMPLEX for pclarzt COMPLEX*16 for pzlarzt. Array, DIMENSION *LOCr*(*iv*+*k*-1) if *incv* = *m_v*, and *LOCc*(*jv*+*k*-1) otherwise. This array contains the Householder scalars related to the Householder vectors. *tau* is tied to the distributed matrix *V*.
work	(local). REAL for pslarzt DOUBLE PRECISION for pdlarzt COMPLEX for pclarzt COMPLEX*16 for pzlarzt. Workspace array, DIMENSION(*k**(*k*-1)/2).

Output Parameters

v	REAL for pslarzt DOUBLE PRECISION for pdlarzt COMPLEX for pclarzt COMPLEX*16 for pzlarzt.
t	(local) REAL for pslarzt DOUBLE PRECISION for pdlarzt COMPLEX for pclarzt COMPLEX*16 for pzlarzt. Array, DIMENSION (*mb_v*, *mb_v*). It contains the *k*-by-*k* triangular factor of the block reflector associated with *v*. *t* is lower triangular.

Application Notes

The shape of the matrix *V* and the storage of the vectors which define the *H*(*i*) is best illustrated by the following example with *n* = 5 and *k* = 3. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct='F' and *storev*='C'

$$V = \begin{bmatrix} v1 & v2 & v3 \\ v1 & v2 & v3 \\ v1 & v2 & v3 \\ v1 & v2 & v3 \\ v1 & v2 & v3 \end{bmatrix} \begin{matrix} . & . & . \\ . & . & . \\ 1 & . & . \\ & 1 & . \\ & & 1 \end{matrix}$$

direct='F' and *storev*='R':

$$\begin{bmatrix} \overbrace{v1 \;\; v1 \;\; v1 \;\; v1 \;\; v1}^{V} . \;\; . \;\; . \;\; . \;\; 1 \\ v2 \;\; v2 \;\; v2 \;\; v2 \;\; v2 . \;\; . \;\; . \;\; 1 \\ v3 \;\; v3 \;\; v3 \;\; v3 \;\; v3 . \;\; . \;\; 1 \end{bmatrix}$$

direct='B' and *storev*='C':

$$V = \begin{matrix} 1 \\ . \;\; 1 \\ . \;\; . \;\; 1 \\ . \;\; . \;\; . \\ . \;\; . \;\; . \end{matrix} \begin{bmatrix} v1 & v2 & v3 \\ v1 & v2 & v3 \\ v1 & v2 & v3 \\ v1 & v2 & v3 \\ v1 & v2 & v3 \end{bmatrix}$$

direct='B' and *storev*='R':

$$\begin{bmatrix} 1 \;\; . \;\; . \;\; . \;\; . \overbrace{v1 \;\; v1 \;\; v1 \;\; v1 \;\; v1}^{V} \\ . \;\; 1 \;\; . \;\; . \;\; . v2 \;\; v2 \;\; v2 \;\; v2 \;\; v2 \\ . \;\; . \;\; 1 \;\; . \;\; . v3 \;\; v3 \;\; v3 \;\; v3 \;\; v3 \end{bmatrix}$$

p?lascl

Multiplies a general rectangular matrix by a real scalar defined as C_{to}/C_{from}.

Syntax

```
call pslascl(type, cfrom, cto, m, n, a, ia, ja, desca, info)
call pdlascl(type, cfrom, cto, m, n, a, ia, ja, desca, info)
call pclascl(type, cfrom, cto, m, n, a, ia, ja, desca, info)
call pzlascl(type, cfrom, cto, m, n, a, ia, ja, desca, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lascl routine multiplies the *m*-by-*n* real/complex distributed matrix sub(*A*) denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) by the real/complex scalar *cto/cfrom*. This is done without over/underflow as long as the final result *cto***A*(*i*,*j*)/*cfrom* does not over/underflow. *type* specifies that sub(*A*) may be full, upper triangular, lower triangular or upper Hessenberg.

Input Parameters

type
(global) CHARACTER.
type indicates the storage type of the input distributed matrix.
if *type* = 'G': sub(*A*) is a full matrix,
if *type* = 'L': sub(*A*) is a lower triangular matrix,
if *type* = 'U': sub(*A*) is an upper triangular matrix,
if *type* = 'H': sub(*A*) is an upper Hessenberg matrix.

cfrom, *cto*
(global)
REAL for pslascl/pclascl
DOUBLE PRECISION for pdlascl/pzlascl.
The distributed matrix sub(*A*) is multiplied by *cto/cfrom* . *A*(*i*,*j*) is computed without over/underflow if the final result *cto***A*(*i*,*j*)/*cfrom* can be represented without over/underflow. *cfrom* must be nonzero.

m
(global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).

n
(global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥0).

a
(local input/local output)
REAL for pslascl
DOUBLE PRECISION for pdlascl
COMPLEX for pclascl
COMPLEX*16 for pzlascl.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
This array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja*
(global) INTEGER.
The column and row indices in the global array *A* indicating the first row and column of the submatrix sub(*A*), respectively.

desca
(global and local) INTEGER .
Array of DIMENSION (*dlen_*).The array descriptor for the distributed matrix *A*.

Output Parameters

a
(local).
On exit, this array contains the local pieces of the distributed matrix multiplied by *cto/cfrom*.

info
(local)
INTEGER.
if *info* = 0: the execution is successful.

if *info* < 0: If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*),
if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?laset

Initializes the offdiagonal elements of a matrix to alpha *and the diagonal elements to* beta.

Syntax

```
call pslaset(uplo, m, n, alpha, beta, a, ia, ja, desca)
call pdlaset(uplo, m, n, alpha, beta, a, ia, ja, desca)
call pclaset(uplo, m, n, alpha, beta, a, ia, ja, desca)
call pzlaset(uplo, m, n, alpha, beta, a, ia, ja, desca)
```

Include Files

- C: mkl_scalapack.h

Description

The p?laset routine initializes an *m*-by-*n* distributed matrix sub(*A*) denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) to *beta* on the diagonal and *alpha* on the offdiagonals.

Input Parameters

uplo — (global) CHARACTER.
Specifies the part of the distributed matrix sub(*A*) to be set:
if *uplo* = 'U': upper triangular part; the strictly lower triangular part of sub(*A*) is not changed;
if *uplo* = 'L': lower triangular part; the strictly upper triangular part of sub(*A*) is not changed.
Otherwise: all of the matrix sub(*A*) is set.

m — (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). (*m*≥0).

n — (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). (*n*≥0).

alpha — (global).
REAL for pslaset
DOUBLE PRECISION for pdlaset
COMPLEX for pclaset
COMPLEX*16 for pzlaset.
The constant to which the offdiagonal elements are to be set.

beta — (global).
REAL for pslaset
DOUBLE PRECISION for pdlaset
COMPLEX for pclaset
COMPLEX*16 for pzlaset.
The constant to which the diagonal elements are to be set.

Output Parameters

a	(local). REAL for pslaset DOUBLE PRECISION for pdlaset COMPLEX for pclaset COMPLEX*16 for pzlaset. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*) to be set. On exit, the leading *m*-by-*n* submatrix sub(*A*) is set as follows: if *uplo* = 'U', *A*(*ia*+*i*-1, *ja*+*j*-1) = *alpha*, 1≤*i*≤*j*-1, 1≤*j*≤*n*, if *uplo* = 'L', *A*(*ia*+*i*-1, *ja*+*j*-1) = *alpha*, *j*+1≤*i*≤ *m*, 1≤*j*≤ *n*, otherwise, *A*(*ia*+*i*-1, *ja*+*j*-1) = *alpha*, 1≤*i*≤*m*, 1≤*j*≤*n*, *ia*+*i*.ne.*ja*+*j*, and, for all *uplo*, *A*(*ia*+*i*-1, *ja*+*i*-1) = *beta*, 1≤*i*≤min(*m*,*n*).
ia, *ja*	(global) INTEGER. The column and row indices in the global array *A* indicating the first row and column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER . Array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

p?lasmsub

Looks for a small subdiagonal element from the bottom of the matrix that it can safely set to zero.

Syntax

```
call pslasmsub(a, desca, i, l, k, smlnum, buf, lwork)
call pdlasmsub(a, desca, i, l, k, smlnum, buf, lwork)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lasmsub routine looks for a small subdiagonal element from the bottom of the matrix that it can safely set to zero. This routine does a global maximum and must be called by all processes.

Input Parameters

a	(global) REAL for pslasmsub DOUBLE PRECISION for pdlasmsub Array, DIMENSION(*desca*(*lld_*),*). On entry, the Hessenberg matrix whose tridiagonal part is being scanned. Unchanged on exit.
desca	(global and local) INTEGER. Array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
i	(global) INTEGER.

	The global location of the bottom of the unreduced submatrix of *A*. Unchanged on exit.
l	(global) INTEGER. The global location of the top of the unreduced submatrix of *A*. Unchanged on exit.
smlnum	(global) REAL for pslasmsub DOUBLE PRECISION for pdlasmsub On entry, a "small number" for the given matrix. Unchanged on exit. A suggested value for *smlnum* is slamch('s') * (*n*/slamch('p') for pslasmsub or dlamch('s') * (*n*/dlamch('p') for pdlasmsub. See lamch.
lwork	(global) INTEGER. On exit, *lwork* is the size of the work buffer. This must be at least 2*ceil(ceil((*i*-*l*)/*hbl*)/ lcm(*nprow*,*npcol*)). Here lcm is least common multiple, and *nprow* x *npcol* is the logical grid size.

Output Parameters

k	(global) INTEGER. On exit, this yields the bottom portion of the unreduced submatrix. This will satisfy: $l \le m \le i-1$.
buf	(local). REAL for pslasmsub DOUBLE PRECISION for pdlasmsub Array of size *lwork*.

p?lassq

Updates a sum of squares represented in scaled form.

Syntax

```
call pslassq(n, x, ix, jx, descx, incx, scale, sumsq)
call pdlassq(n, x, ix, jx, descx, incx, scale, sumsq)
call pclassq(n, x, ix, jx, descx, incx, scale, sumsq)
call pzlassq(n, x, ix, jx, descx, incx, scale, sumsq)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lassq routine returns the values *scl* and *smsq* such that

$scl^2 * smsq = x(1)^2 + ... + x(n)^2 + scale^2*sumsq$,

where $x(i) = sub(X) = X(ix + (jx-1)*descx(m\_) + (i - 1)*incx)$ for pslassq/pdlassq ,

and $x(i) = sub(X) = abs(X(ix + (jx-1)*descx(m\_) + (i - 1)*incx)$ for pclassq/pzlassq.

For real routines pslassq/pdlassq the value of *sumsq* is assumed to be non-negative and *scl* returns the value

$scl = max(scale, abs(x(i)))$.

For complex routines pclassq/pzlassq the value of *sumsq* is assumed to be at least unity and the value of *ssq* will then satisfy

$$1.0 \leq ssq \leq sumsq + 2n$$

Value *scale* is assumed to be non-negative and *scl* returns the value

$$scl = \max_i \left(scale, \mathrm{abs}\left(\mathrm{real}\left(x\left(i\right)\right)\right), \mathrm{abs}\left(\mathrm{aimag}\left(x\left(i\right)\right)\right)\right)$$

For all routines p?lassq values *scale* and *sumsq* must be supplied in *scale* and *sumsq* respectively, and *scale* and *sumsq* are overwritten by *scl* and *ssq* respectively.

All routines p?lassq make only one pass through the vector sub(*x*).

Input Parameters

n — (global) INTEGER.
The length of the distributed vector sub(*x*).

x — REAL for pslassq
DOUBLE PRECISION for pdlassq
COMPLEX for pclassq
COMPLEX*16 for pzlassq.
The vector for which a scaled sum of squares is computed:
x(*ix* + (*jx*-1)*m_x + (*i* - 1)**incx*), 1 ≤ *i* ≤ *n*.

ix — (global) INTEGER.
The row index in the global array *X* indicating the first row of sub(*X*).

jx — (global) INTEGER.
The column index in the global array *X* indicating the first column of sub(*X*).

descx — (global and local) INTEGER array of DIMENSION (*dlen_*).
The array descriptor for the distributed matrix *X*.

incx — (global) INTEGER.
The global increment for the elements of *X*. Only two values of *incx* are supported in this version, namely 1 and *m_x*. The argument *incx* must not equal zero.

scale — (local).
REAL for pslassq/pclassq
DOUBLE PRECISION for pdlassq/pzlassq.
On entry, the value *scale* in the equation above.

sumsq — (local)
REAL for pslassq/pclassq
DOUBLE PRECISION for pdlassq/pzlassq.
On entry, the value *sumsq* in the equation above.

Output Parameters

scale — (local).
On exit, *scale* is overwritten with *scl* , the scaling factor for the sum of squares.

sumsq — (local).
On exit, *sumsq* is overwritten with the value *smsq*, the basic sum of squares from which *scl* has been factored out.

p?laswp

Performs a series of row interchanges on a general rectangular matrix.

Syntax

```
call pslaswp(direc, rowcol, n, a, ia, ja, desca, k1, k2, ipiv)
call pdlaswp(direc, rowcol, n, a, ia, ja, desca, k1, k2, ipiv)
call pclaswp(direc, rowcol, n, a, ia, ja, desca, k1, k2, ipiv)
call pzlaswp(direc, rowcol, n, a, ia, ja, desca, k1, k2, ipiv)
```

Include Files

- C: mkl_scalapack.h

Description

The p?laswp routine performs a series of row or column interchanges on the distributed matrix sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1). One interchange is initiated for each of rows or columns *k1* through *k2* of sub(*A*). This routine assumes that the pivoting information has already been broadcast along the process row or column. Also note that this routine will only work for *k1*-*k2* being in the same *mb* (or *nb*) block. If you want to pivot a full matrix, use p?lapiv.

Input Parameters

direc — (global) CHARACTER.
Specifies in which order the permutation is applied:
= 'F' - forward,
= 'B' - backward.

rowcol — (global) CHARACTER.
Specifies if the rows or columns are permuted:
= 'R' - rows,
= 'C' - columns.

n — (global) INTEGER.
If *rowcol*='R', the length of the rows of the distributed matrix *A*(*, *ja*:*ja*+*n*-1) to be permuted;
If *rowcol*='C', the length of the columns of the distributed matrix *A*(*ia*:*ia*+*n*-1 , *) to be permuted;

a — (local)
REAL for pslaswp
DOUBLE PRECISION for pdlaswp
COMPLEX for pclaswp
COMPLEX*16 for pzlaswp.
Pointer into the local memory to an array of DIMENSION (*lld_a*, *). On entry, this array contains the local pieces of the distributed matrix to which the row/columns interchanges will be applied.

ia — (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja — (global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca — (global and local) INTEGER array of DIMENSION (*dlen_*).
The array descriptor for the distributed matrix *A*.

k1	(global) INTEGER. The first element of *ipiv* for which a row or column interchange will be done.
k2	(global) INTEGER. The last element of *ipiv* for which a row or column interchange will be done.
ipiv	(local) INTEGER. Array, DIMENSION *LOCr(m_a)+mb_a* for row pivoting and *LOCr(n_a)+nb_a* for column pivoting. This array is tied to the matrix *A*, *ipiv(k)=l* implies rows (or columns) *k* and *l* are to be interchanged.

Output Parameters

A	(local) REAL for pslaswp DOUBLE PRECISION for pdlaswp COMPLEX for pclaswp COMPLEX*16 for pzlaswp. On exit, the permuted distributed matrix.

p?latra

Computes the trace of a general square distributed matrix.

Syntax

```
val = pslatra(n, a, ia, ja, desca)
val = pdlatra(n, a, ia, ja, desca)
val = pclatra(n, a, ia, ja, desca)
val = pzlatra(n, a, ia, ja, desca)
```

Include Files

- C: mkl_scalapack.h

Description

This function computes the trace of an *n*-by-*n* distributed matrix sub(*A*) denoting *A(ia:ia+n-1, ja:ja+n-1)*. The result is left on every process of the grid.

Input Parameters

n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*). *n* ≥0.
a	(local). Real for pslatra DOUBLE PRECISION for pdlatra COMPLEX for pclatra COMPLEX*16 for pzlatra. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc(ja+n-1)*) containing the local pieces of the distributed matrix, the trace of which is to be computed.

ia, *ja*	(global) INTEGER. The row and column indices respectively in the global array *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

val	The value returned by the fuction.

p?latrd

Reduces the first nb rows and columns of a symmetric/Hermitian matrix A to real tridiagonal form by an orthogonal/unitary similarity transformation.

Syntax

```
call pslatrd(uplo, n, nb, a, ia, ja, desca, d, e, tau, w, iw, jw, descw, work)
call pdlatrd(uplo, n, nb, a, ia, ja, desca, d, e, tau, w, iw, jw, descw, work)
call pclatrd(uplo, n, nb, a, ia, ja, desca, d, e, tau, w, iw, jw, descw, work)
call pzlatrd(uplo, n, nb, a, ia, ja, desca, d, e, tau, w, iw, jw, descw, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?latrd routine reduces *nb* rows and columns of a real symmetric or complex Hermitian matrix sub(*A*)= *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) to symmetric/complex tridiagonal form by an orthogonal/unitary similarity transformation *Q*'*sub(*A*)**Q*, and returns the matrices *V* and *W*, which are needed to apply the transformation to the unreduced part of sub(*A*).

If *uplo* = U, p?latrd reduces the last *nb* rows and columns of a matrix, of which the upper triangle is supplied;

if *uplo* = L, p?latrd reduces the first *nb* rows and columns of a matrix, of which the lower triangle is supplied.

This is an auxiliary routine called by p?sytrd/p?hetrd.

Input Parameters

uplo	(global) CHARACTER. Specifies whether the upper or lower triangular part of the symmetric/ Hermitian matrix sub(*A*) is stored: = 'U': Upper triangular = L: Lower triangular.
n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*). *n* ≥ 0.
nb	(global) INTEGER. The number of rows and columns to be reduced.
a	REAL for pslatrd DOUBLE PRECISION for pdlatrd

COMPLEX for pclatrd
COMPLEX*16 for pzlatrd.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the symmetric/Hermitian distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced.

ia
(global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja
(global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca
(global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

iw
(global) INTEGER.
The row index in the global array *W* indicating the first row of sub(*W*).

jw
(global) INTEGER.
The column index in the global array *W* indicating the first column of sub(*W*).

descw
(global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *W*.

work
(local)
REAL for pslatrd
DOUBLE PRECISION for pdlatrd
COMPLEX for pclatrd
COMPLEX*16 for pzlatrd.
Workspace array of DIMENSION (*nb_a*).

Output Parameters

a
(local)
On exit, if *uplo* = 'U', the last *nb* columns have been reduced to tridiagonal form, with the diagonal elements overwriting the diagonal elements of sub(*A*); the elements above the diagonal with the array *tau* represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors;
if *uplo* = 'L', the first *nb* columns have been reduced to tridiagonal form, with the diagonal elements overwriting the diagonal elements of sub(*A*); the elements below the diagonal with the array *tau* represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors.

d
(local)
REAL for pslatrd/pclatrd
DOUBLE PRECISION for pdlatrd/pzlatrd.
Array, DIMENSION *LOCc*(*ja*+*n*-1).
The diagonal elements of the tridiagonal matrix *T*: *d*(i) = *a*(i,i). *d* is tied to the distributed matrix *A*.

e
(local)
REAL for pslatrd/pclatrd

DOUBLE PRECISION for pdlatrd/pzlatrd.
Array, DIMENSION *LOCc*(*ja*+*n*-1) if *uplo* = 'U', *LOCc*(*ja*+*n*-2) otherwise.
The off-diagonal elements of the tridiagonal matrix *T*:
e(*i*) = *a*(*i*, *i* + 1) if *uplo* = 'U',
e(*i*) = *a*(*i* + 1, *i*) if *uplo* = L.
e is tied to the distributed matrix *A*.

tau (local)
REAL for pslatrd
DOUBLE PRECISION for pdlatrd
COMPLEX for pclatrd
COMPLEX*16 for pzlatrd.
Array, DIMENSION *LOCc*(*ja*+*n*-1). This array contains the scalar factors *tau* of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

w (local)
REAL for pslatrd
DOUBLE PRECISION for pdlatrd
COMPLEX for pclatrd
COMPLEX*16 for pzlatrd.
Pointer into the local memory to an array of DIMENSION (*lld_w*, *nb_w*). This array contains the local pieces of the *n*-by-*nb_w* matrix *w* required to update the unreduced part of sub(*A*).

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

```
Q = H(n)*H(n-1)*...*H(n-nb+1)
```

Each *H*(i) has the form

```
H(i) = I - tau*v*v' ,
```

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(*i*:*n*) = 0 and *v*(*i*-1) = 1; *v*(1:*i*-1) is stored on exit in *A*(*ia*:*ia*+*i*-1, *ja*+*i*), and *tau* in *tau*(*ja*+*i*-1).

If *uplo* = L, the matrix *Q* is represented as a product of elementary reflectors

```
Q = H(1)*H(2)*...*H(nb)
```

Each *H*(i) has the form

```
H(i) = I - tau*v*v' ,
```

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*) = 0 and *v*(*i*+1) = 1; *v*(*i*+2: *n*) is stored on exit in *A*(*ia*+*i*+1: *ia*+*n*-1, *ja*+*i*-1), and *tau* in *tau*(*ja*+*i*-1).

The elements of the vectors *v* together form the *n*-by-*nb* matrix *V* which is needed, with *W*, to apply the transformation to the unreduced part of the matrix, using a symmetric/Hermitian rank-2*k* update of the form:

sub(*A*) := sub(*A*)-*vw*'-*wv*'.

The contents of *a* on exit are illustrated by the following examples with

n = 5 and *nb* = 2:

if *uplo*='U':

$$\begin{bmatrix} a & a & a & v_4 & v_5 \\ & a & a & v_4 & v_5 \\ & & a & 1 & v_5 \\ & & & d & 1 \\ & & & & d \end{bmatrix}$$

if *uplo*='L':

$$\begin{bmatrix} d & & & & \\ 1 & d & & & \\ v_1 & 1 & a & & \\ v_1 & v_2 & a & a & \\ v_1 & v_2 & a & a & a \end{bmatrix}$$

where d denotes a diagonal element of the reduced matrix, a denotes an element of the original matrix that is unchanged, and v_i denotes an element of the vector defining H(i).

p?latrs

Solves a triangular system of equations with the scale factor set to prevent overflow.

Syntax

call pslatrs(*uplo*, *trans*, *diag*, *normin*, *n*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *scale*, *cnorm*, *work*)

call pdlatrs(*uplo*, *trans*, *diag*, *normin*, *n*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *scale*, *cnorm*, *work*)

call pclatrs(*uplo*, *trans*, *diag*, *normin*, *n*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *scale*, *cnorm*, *work*)

call pzlatrs(*uplo*, *trans*, *diag*, *normin*, *n*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *scale*, *cnorm*, *work*)

Include Files

- C: mkl_scalapack.h

Description

The p?latrs routine solves a triangular system of equations $Ax = sb$, $A^Tx = sb$ or $A^Hx = sb$, where s is a scale factor set to prevent overflow. The description of the routine will be extended in the future releases.

Input Parameters

uplo — CHARACTER*1.
Specifies whether the matrix A is upper or lower triangular.
= 'U': Upper triangular
= 'L': Lower triangular

trans — CHARACTER*1.
Specifies the operation applied to Ax.
= 'N': Solve $Ax = s*b$ (no transpose)
= 'T': Solve $A^Tx = s*b$ (transpose)
= 'C': Solve $A^Hx = s*b$ (conjugate transpose),
where s - is a scale factor

diag — CHARACTER*1.
Specifies whether or not the matrix A is unit triangular.
= 'N': Non-unit triangular
= 'U': Unit triangular

normin	CHARACTER*1. Specifies whether *cnorm* has been set or not. = 'Y': *cnorm* contains the column norms on entry; = 'N': *cnorm* is not set on entry. On exit, the norms will be computed and stored in *cnorm*.
n	INTEGER. The order of the matrix *A*. $n \geq 0$
a	REAL for pslatrs/pclatrs DOUBLE PRECISION for pdlatrs/pzlatrs Array, DIMENSION (*lda*, *n*). Contains the triangular matrix *A*. If *uplo* = U, the leading *n*-by-*n* upper triangular part of the array *a* contains the upper triangular matrix, and the strictly lower triangular part of *a* is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the array *a* contains the lower triangular matrix, and the strictly upper triangular part of *a* is not referenced. If *diag* = 'U', the diagonal elements of *a* are also not referenced and are assumed to be 1.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *a* indicating the first row and the first column of the submatrix *A*, respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
x	REAL for pslatrs/pclatrs DOUBLE PRECISION for pdlatrs/pzlatrs Array, DIMENSION (*n*). On entry, the right hand side *b* of the triangular system.
ix	(global) INTEGER.The row index in the global array *x* indicating the first row of sub(*x*).
jx	(global) INTEGER. The column index in the global array *x* indicating the first column of sub(*x*).
descx	(global and local) INTEGER. Array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *X*.
cnorm	REAL for pslatrs/pclatrs DOUBLE PRECISION for pdlatrs/pzlatrs. Array, DIMENSION (*n*). If *normin* = 'Y', *cnorm* is an input argument and *cnorm* (*j*) contains the norm of the off-diagonal part of the *j*-th column of *A*. If *trans* = 'N', *cnorm* (*j*) must be greater than or equal to the infinity-norm, and if *trans* = 'T' or 'C', *cnorm*(*j*) must be greater than or equal to the 1-norm.
work	(local). REAL for pslatrs DOUBLE PRECISION for pdlatrs COMPLEX for pclatrs COMPLEX*16 for pzlatrs. Temporary workspace.

Output Parameters

X	On exit, *x* is overwritten by the solution vector *x*.
scale	REAL for pslatrs/pclatrs

DOUBLE PRECISION for pdlatrs/pzlatrs.
Array, DIMENSION (*lda*, *n*). The scaling factor *s* for the triangular system as described above.
If *scale* = 0, the matrix *A* is singular or badly scaled, and the vector *x* is an exact or approximate solution to *Ax* = 0.

cnorm — If *normin* = 'N', *cnorm* is an output argument and *cnorm*(*j*) returns the 1-norm of the off-diagonal part of the *j*-th column of *A*.

p?latrz

Reduces an upper trapezoidal matrix to upper triangular form by means of orthogonal/unitary transformations.

Syntax

```
call pslatrz(m, n, l, a, ia, ja, desca, tau, work)
call pdlatrz(m, n, l, a, ia, ja, desca, tau, work)
call pclatrz(m, n, l, a, ia, ja, desca, tau, work)
call pzlatrz(m, n, l, a, ia, ja, desca, tau, work)
```

Include Files

- C: mkl_scalapack.h

Description

The p?latrz routine reduces the *m*-by-*n*(*m* ≤ *n*) real/complex upper trapezoidal matrix sub(*A*) = [*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*m*-1) *A*(*ia*:*ia*+*m*-1, *ja*+*n*-*l*:*ja*+*n*-1)] to upper triangular form by means of orthogonal/unitary transformations.

The upper trapezoidal matrix sub(*A*) is factored as

sub(*A*) = (*R* 0)**Z*,

where *Z* is an *n*-by-*n* orthogonal/unitary matrix and *R* is an *m*-by-*m* upper triangular matrix.

Input Parameters

m — (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*A*). *m* ≥ 0.

n — (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*A*). *n* ≥ 0.

l — (global) INTEGER.
The number of columns of the distributed submatrix sub(*A*) containing the meaningful part of the Householder reflectors. *l* > 0.

a — (local)
REAL for pslatrz
DOUBLE PRECISION for pdlatrz
COMPLEX for pclatrz
COMPLEX*16 for pzlatrz.

Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)). On entry, the local pieces of the *m*-by-*n* distributed matrix sub(*A*), which is to be factored.

ia (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja (global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca (global and local) INTEGER array of DIMENSION (*dlen_*).
The array descriptor for the distributed matrix *A*.

work (local)
REAL for pslatrz
DOUBLE PRECISION for pdlatrz
COMPLEX for pclatrz
COMPLEX*16 for pzlatrz.
Workspace array, DIMENSION (*lwork*).
lwork ≥ *nq*0 + max(1, *mp*0), where
iroff = mod(*ia*-1, *mb_a*),
icoff = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mp*0 = numroc(*m*+*iroff*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nq*0 = numroc(*n*+*icoff*, *nb_a*, *mycol*, *iacol*, *npcol*),
numroc, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.

Output Parameters

a On exit, the leading *m*-by-*m* upper triangular part of sub(*A*) contains the upper triangular matrix *R*, and elements *n*-*l*+1 to *n* of the first *m* rows of sub(*A*), with the array *tau*, represent the orthogonal/unitary matrix *Z* as a product of *m* elementary reflectors.

tau (local)
REAL for pslatrz
DOUBLE PRECISION for pdlatrz
COMPLEX for pclatrz
COMPLEX*16 for pzlatrz.
Array, DIMENSION(*LOCr*(*ja*+*m*-1)). This array contains the scalar factors of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

Application Notes

The factorization is obtained by Householder's method. The *k*-th transformation matrix, *Z*(*k*), which is used (or, in case of complex routines, whose conjugate transpose is used) to introduce zeros into the (*m* - *k* + 1)-th row of sub(*A*), is given in the form

$$Z(k)=\begin{bmatrix} I & 0 \\ 0 & T(k) \end{bmatrix},$$

where

$$T(k)=I - tau * u(k) * u(k)', \quad u(k)=\begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

tau is a scalar and *z*(*k*) is an (*n*–*m*)-element vector. *tau* and *z*(*k*) are chosen to annihilate the elements of the *k*-th row of sub(*A*). The scalar *tau* is returned in the *k*-th element of *tau* and the vector *u*(*k*) in the *k*-th row of sub(*A*), such that the elements of *z*(*k*) are in *a*(*k*, *m* + 1), ..., *a*(*k*, *n*). The elements of *R* are returned in the upper triangular part of sub(*A*).

Z is given by

```
Z =  Z(1)Z(2)...Z(m).
```

p?lauu2

*Computes the product U*U' or L'*L, where U and L are upper or lower triangular matrices (local unblocked algorithm).*

Syntax

```
call pslauu2(uplo, n, a, ia, ja, desca)
call pdlauu2(uplo, n, a, ia, ja, desca)
call pclauu2(uplo, n, a, ia, ja, desca)
call pzlauu2(uplo, n, a, ia, ja, desca)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lauu2 routine computes the product *U*U'* or *L'*L*, where the triangular factor *U* or *L* is stored in the upper or lower triangular part of the distributed matrix

sub(*A*)= *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

If *uplo* = 'U' or 'u', then the upper triangle of the result is stored, overwriting the factor *U* in sub(*A*).

If *uplo* = 'L' or 'l', then the lower triangle of the result is stored, overwriting the factor *L* in sub(*A*).

This is the unblocked form of the algorithm, calling BLAS Level 2 Routines. No communication is performed by this routine, the matrix to operate on should be strictly local to one process.

Input Parameters

uplo — (global) CHARACTER*1.
Specifies whether the triangular factor stored in the *matrix* sub(*A*) is upper or lower triangular:
= U: upper triangular
= L: lower triangular.

n — (global) INTEGER.
The number of rows and columns to be operated on, that is, the order of the triangular factor *U* or *L*. *n* ≥ 0.

a — (local)
REAL for pslauu2
DOUBLE PRECISION for pdlauu2

COMPLEX for pclauu2
COMPLEX*16 for pzlauu2.
Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1). On entry, the local pieces of the triangular factor *U* or *L*.

ia (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja (global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a (local)
On exit, if *uplo* = 'U', the upper triangle of the distributed matrix sub(*A*) is overwritten with the upper triangle of the product *U***U*'; if *uplo* = 'L', the lower triangle of sub(*A*) is overwritten with the lower triangle of the product *L*'**L*.

p?lauum

*Computes the product U*U' or L'*L, where U and L are upper or lower triangular matrices.*

Syntax

```
call pslauum(uplo, n, a, ia, ja, desca)
call pdlauum(uplo, n, a, ia, ja, desca)
call pclauum(uplo, n, a, ia, ja, desca)
call pzlauum(uplo, n, a, ia, ja, desca)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lauum routine computes the product *U***U*' or *L*'**L*, where the triangular factor *U* or *L* is stored in the upper or lower triangular part of the matrix sub(*A*)= *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

If *uplo* = 'U' or 'u', then the upper triangle of the result is stored, overwriting the factor *U* in sub(*A*). If *uplo* = 'L' or 'l', then the lower triangle of the result is stored, overwriting the factor *L* in sub(*A*).

This is the blocked form of the algorithm, calling Level 3 PBLAS.

Input Parameters

uplo (global) CHARACTER*1.
Specifies whether the triangular factor stored in the matrix sub(*A*) is upper or lower triangular:
= 'U': upper triangular
= 'L': lower triangular.

n (global) INTEGER.
The number of rows and columns to be operated on, that is, the order of the triangular factor *U* or *L*. *n* ≥ 0.

a	(local) REAL for pslauum DOUBLE PRECISION for pdlauum COMPLEX for pclauum COMPLEX*16 for pzlauum. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1). On entry, the local pieces of the triangular factor *U* or *L*.
ia	(global) INTEGER. The row index in the global array *A* indicating the first row of sub(*A*).
ja	(global) INTEGER. The column index in the global array *A* indicating the first column of sub(*A*).
desca	(global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a	(local) On exit, if *uplo* = 'U', the upper triangle of the distributed matrix sub(*A*) is overwritten with the upper triangle of the product *U***U*' ; if *uplo* = 'L', the lower triangle of sub(*A*) is overwritten with the lower triangle of the product *L*'**L*.

p?lawil

Forms the Wilkinson transform.

Syntax

```
call pslawil(ii, jj, m, a, desca, h44, h33, h43h34, v)
```

```
call pdlawil(ii, jj, m, a, desca, h44, h33, h43h34, v)
```

Include Files

- C: mkl_scalapack.h

Description

The p?lawil routine gets the transform given by *h44*, *h33*, and *h43h34* into *v* starting at row *m*.

Input Parameters

ii	(global) INTEGER. Row owner of *h*(*m*+2, *m*+2).
jj	(global) INTEGER. Column owner of *h*(*m*+2, *m*+2).
m	(global) INTEGER. On entry, the location from where the transform starts (row *m*). Unchanged on exit.
a	(global) REAL for pslawil DOUBLE PRECISION for pdlawil Array, DIMENSION (*desca(lld_)*,*). On entry, the Hessenberg matrix. Unchanged on exit.
desca	(global and local) INTEGER

Array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*. Unchanged on exit.

h43h34 (global)
REAL for pslawil
DOUBLE PRECISION for pdlawil
These three values are for the double shift *QR* iteration. Unchanged on exit.

Output Parameters

v (global)
REAL for pslawil
DOUBLE PRECISION for pdlawil
Array of size 3 that contains the transform on output.

p?org2l/p?ung2l

Generates all or part of the orthogonal/unitary matrix Q from a QL factorization determined by p?geqlf *(unblocked algorithm).*

Syntax

```
call psorg2l(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorg2l(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pcung2l(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzung2l(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?org2l/p?ung2l routine generates an *m*-by-*n* real/complex distributed matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal columns, which is defined as the last *n* columns of a product of *k* elementary reflectors of order *m*:

Q = *H*(*k*)*...*H*(2)*H*(1) as returned by p?geqlf.

Input Parameters

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix *Q*. *m* ≥ 0.

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix *Q*. *m* ≥ *n* ≥ 0.

k (global) INTEGER.
The number of elementary reflectors whose product defines the matrix *Q*. *n*≥ *k* ≥ 0.

a REAL for psorg2l
DOUBLE PRECISION for pdorg2l
COMPLEX for pcung2l
COMPLEX*16 for pzung2l.

Pointer into the local memory to an array, DIMENSION (*lld_a*, *LOCc*(*ja*+*n*-1).
On entry, the *j*-th column must contain the vector that defines the elementary reflector *H*(*j*), *ja*+*n*-*k* ≤ *j* ≤ *ja*+*n*-*k*, as returned by p?geqlf in the *k* columns of its *distributed matrix* argument *A*(*ia*:*,*ja*+*n*-*k*:*ja*+*n*-1).

ia (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja (global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
REAL for psorg2l
DOUBLE PRECISION for pdorg2l
COMPLEX for pcung2l
COMPLEX*16 for pzung2l.
Array, DIMENSION *LOCc*(*ja*+*n*-1).
This array contains the scalar factor *tau*(*j*) of the elementary reflector *H*(*j*), as returned by p?geqlf.

work (local)
REAL for psorg2l
DOUBLE PRECISION for pdorg2l
COMPLEX for pcung2l
COMPLEX*16 for pzung2l.
Workspace array, DIMENSION (*lwork*).

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least *lwork* ≥ *mpa*0 + max(1, *nqa*0), where
iroffa = mod(*ia*-1, *mb_a*),
icoffa = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*),
*mpa*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*),
*nqa*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*).
indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a On exit, this array contains the local pieces of the *m*-by-*n* distributed matrix *Q*.

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local) INTEGER.
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value,

then *info* = - (*i**100 +*j*),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

p?org2r/p?ung2r

Generates all or part of the orthogonal/unitary matrix Q from a QR factorization determined by p?geqrf *(unblocked algorithm).*

Syntax

```
call psorg2r(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorg2r(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pcung2r(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzung2r(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?org2r/p?ung2r routine generates an *m*-by-*n* real/complex matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal columns, which is defined as the first *n* columns of a product of *k* elementary reflectors of order *m*:

```
Q = H(1)*H(2)*...*H(k)
```

as returned by p?geqrf.

Input Parameters

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix *Q*. *m* ≥ 0.

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix *Q*. *m* ≥ *n* ≥ 0.

k (global) INTEGER.
The number of elementary reflectors whose product defines the matrix *Q*. *n* ≥ *k* ≥ 0.

a REAL for psorg2r
DOUBLE PRECISION for pdorg2r
COMPLEX for pcung2r
COMPLEX*16 for pzung2r.
Pointer into the local memory to an array, DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1).
On entry, the *j*-th column must contain the vector that defines the elementary reflector *H*(*j*), *ja* ≤ *j* ≤ *ja*+*k*-1, as returned by p?geqrf in the *k* columns of its *distributed matrix* argument *A*(*ia*:*,*ja*:*ja*+*k*-1).

ia (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja (global) INTEGER.

	The column index in the global array *A* indicating the first column of sub(*A*).
desca	(global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) REAL for psorg2r DOUBLE PRECISION for pdorg2r COMPLEX for pcung2r COMPLEX*16 for pzung2r. Array, DIMENSION *LOCc*(*ja*+*k*-1). This array contains the scalar factor *tau*(*j*) of the elementary reflector *H*(*j*), as returned by p?geqrf. This array is tied to the distributed matrix *A*.
work	(local) REAL for psorg2r DOUBLE PRECISION for pdorg2r COMPLEX for pcung2r COMPLEX*16 for pzung2r. Workspace array, DIMENSION (*lwork*).
lwork	(local or global) INTEGER. The dimension of the array *work*. *lwork* is local input and must be at least *lwork* ≥ *mpa*0 + max(1, *nqa*0), where *iroffa* = mod(*ia*-1, *mb_a* , *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*), *iacol* = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*), *mpa*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*), *nqa*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*). indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	On exit, this array contains the local pieces of the *m*-by-*n* distributed matrix *Q*.
work	On exit, *work*(1) returns the minimal and optimal *lwork*.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?orgl2/p?ungl2

Generates all or part of the orthogonal/unitary matrix Q from an LQ factorization determined by p?gelqf *(unblocked algorithm).*

Syntax

```
call psorgl2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorgl2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pcungl2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzungl2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?orgl2/p?ungl2 routine generates a *m*-by-*n* real/complex matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal rows, which is defined as the first *m* rows of a product of *k* elementary reflectors of order *n*

Q = *H*(*k*)*...**H*(2)**H*(1) (for real flavors),

Q = (*H*(*k*))^{H}*...*(*H*(2))^{H}*(*H*(1))^{H} (for complex flavors) as returned by p?gelqf.

Input Parameters

m — (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix *Q*. *m* ≥ 0.

n — (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix *Q*. *n* ≥ *m* ≥ 0.

k — (global) INTEGER.
The number of elementary reflectors whose product defines the matrix *Q*. *m* ≥ *k* ≥ 0.

a — REAL for psorgl2
DOUBLE PRECISION for pdorgl2
COMPLEX for pcungl2
COMPLEX*16 for pzungl2.
Pointer into the local memory to an array, DIMENSION (*lld_a*, *LOCc*(*ja*+*n*-1).
On entry, the *i*-th row must contain the vector that defines the elementary reflector *H*(*i*), *ia* ≤ *i* ≤ *ia*+*k*-1, as returned by p?gelqf in the *k* rows of its *distributed matrix* argument *A*(*ia*:*ia*+*k*-1, *ja*:*).

ia — (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja — (global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca — (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

tau — (local)
REAL for psorgl2
DOUBLE PRECISION for pdorgl2
COMPLEX for pcungl2
COMPLEX*16 for pzungl2.

	Array, DIMENSION *LOCr*(*ja*+*k*-1). This array contains the scalar factors *tau*(*i*) of the elementary reflectors *H*(*i*), as returned by p?gelqf. This array is tied to the distributed matrix *A*.
WORK	(local) REAL for psorgl2 DOUBLE PRECISION for pdorgl2 COMPLEX for pcungl2 COMPLEX*16 for pzungl2. Workspace array, DIMENSION (*lwork*).
lwork	(local or global) INTEGER. The dimension of the array *work*. *lwork* is local input and must be at least *lwork* ≥ *nqa*0 + max(1, *mpa*0), where *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*), *iacol* = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*), *mpa*0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*), *nqa*0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*). indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	On exit, this array contains the local pieces of the *m*-by-*n* distributed matrix *Q*.
work	On exit, *work*(1) returns the minimal and optimal *lwork*.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?orgr2/p?ungr2

Generates all or part of the orthogonal/unitary matrix Q from an RQ factorization determined by p?gerqf *(unblocked algorithm).*

Syntax

```
call psorgr2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pdorgr2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pcungr2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
call pzungr2(m, n, k, a, ia, ja, desca, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?orgr2/p?ungr2 routine generates an *m*-by-*n* real/complex matrix *Q* denoting *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1) with orthonormal rows, which is defined as the last *m* rows of a product of *k* elementary reflectors of order *n*

Q = *H*(1)**H*(2)*...**H*(*k*) (for real flavors);

Q = (*H*(1))^H*(*H*(2))^H...*(*H*(*k*))^H (for complex flavors) as returned by p?gerqf.

Input Parameters

m	(global) INTEGER. The number of rows to be operated on, that is, the number of rows of the distributed submatrix *Q*. *m* ≥ 0.
n	(global) INTEGER. The number of columns to be operated on, that is, the number of columns of the distributed submatrix *Q*. *n* ≥ *m* ≥ 0.
k	(global) INTEGER. The number of elementary reflectors whose product defines the matrix *Q*. *m* ≥ *k* ≥ 0.
a	REAL for psorgr2 DOUBLE PRECISION for pdorgr2 COMPLEX for pcungr2 COMPLEX*16 for pzungr2. Pointer into the local memory to an array, DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1). On entry, the *i*-th row must contain the vector that defines the elementary reflector *H*(*i*), *ia*+*m*-*k* ≤ *i* ≤ *ia*+*m*-1, as returned by p?gerqf in the *k* rows of its *distributed matrix* argument *A*(*ia*+*m*-*k*:*ia*+*m*-1, *ja*:*).
ia	(global) INTEGER. The row index in the global array *A* indicating the first row of sub(*A*).
ja	(global) INTEGER. The column index in the global array *A* indicating the first column of sub(*A*).
desca	(global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
tau	(local) REAL for psorgr2 DOUBLE PRECISION for pdorgr2 COMPLEX for pcungr2 COMPLEX*16 for pzungr2. Array, DIMENSION *LOCr*(*ja*+*m*-1). This array contains the scalar factors *tau*(*i*) of the elementary reflectors *H*(*i*), as returned by p?gerqf. This array is tied to the distributed matrix *A*.
work	(local) REAL for psorgr2 DOUBLE PRECISION for pdorgr2 COMPLEX for pcungr2 COMPLEX*16 for pzungr2. Workspace array, DIMENSION (*lwork*).

lwork	(local or global) INTEGER. The dimension of the array *work*. *lwork* is local input and must be at least *lwork* ≥ *nqa0* + max(1, *mpa0*), where *iroffa* = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*), *iarow* = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nprow*), *iacol* = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *npcol*), *mpa0* = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nprow*), *nqa0* = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *npcol*). indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo. If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

a	On exit, this array contains the local pieces of the *m*-by-*n* distributed matrix *Q*.
work	On exit, *work*(1) returns the minimal and optimal *lwork*.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?orm2l/p?unm2l

Multiplies a general matrix by the orthogonal/unitary matrix from a QL factorization determined by p?geqlf *(unblocked algorithm).*

Syntax

call psorm2l(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdorm2l(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pcunm2l(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunm2l(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?orm2l/p?unm2l routine overwrites the general real/complex *m*-by-*n* distributed matrix sub(*C*)=*C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) with

*Q**sub(*C*) if *side* = 'L' and *trans* = 'N', or

Q^T*sub(C) / Q^H*sub(C) if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

sub(C)*Q if *side* = 'R' and *trans* = 'N', or

sub(C)*Q^T / sub(C)*Q^H if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

where Q is a real orthogonal or complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(k)*...*H(2)*H(1)$ as returned by p?geqlf . Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

side (global) CHARACTER.
= 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the left,
= 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right.

trans (global) CHARACTER.
= 'N': apply Q (no transpose)
= 'T': apply Q^T (transpose, for real flavors)
= 'C': apply Q^H (conjugate transpose, for complex flavors)

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(C). $m \geq 0$.

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(C). $n \geq 0$.

k (global) INTEGER.
The number of elementary reflectors whose product defines the matrix Q.
If *side* = 'L', $m \geq k \geq 0$;
if *side* = 'R', $n \geq k \geq 0$.

a (local)
REAL for psorm2l
DOUBLE PRECISION for pdorm2l
COMPLEX for pcunm2l
COMPLEX*16 for pzunm2l.
Pointer into the local memory to an array, DIMENSION(*lld_a*, *LOCc*(*ja*+*k*-1).
On entry, the j-th row must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by p?geqlf in the k columns of its distributed matrix argument $A(ia{:}*, ja{:}ja+k-1)$. The argument $A(ia{:}*, ja{:}ja+k-1)$ is modified by the routine but restored on exit.
If *side* = 'L', lld_a $\geq$ max(1, *LOCr*(*ia*+*m*-1)),
if *side* = 'R', lld_a $\geq$ max(1, *LOCr*(*ia*+*n*-1)).

ia (global) INTEGER.
The row index in the global array A indicating the first row of sub(A).

ja (global) INTEGER.
The column index in the global array A indicating the first column of sub(A).

desca (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix A.

tau (local)

REAL for psorm2l
DOUBLE PRECISION for pdorm2l
COMPLEX for pcunm2l
COMPLEX*16 for pzunm2l.
Array, DIMENSION*LOCc*(*ja*+*n*-1). This array contains the scalar factor *tau*(*j*) of the elementary reflector *H*(*j*), as returned by p?geqlf. This array is tied to the distributed matrix *A*.

c (local)
REAL for psorm2l
DOUBLE PRECISION for pdorm2l
COMPLEX for pcunm2l
COMPLEX*16 for pzunm2l.
Pointer into the local memory to an array, DIMENSION(*lld_c*, *LOCc*(*jc* +*n*-1)).On entry, the local pieces of the distributed matrix sub (*C*).

ic (global) INTEGER.
The row index in the global array *C* indicating the first row of sub(*C*).

jc (global) INTEGER.
The column index in the global array *C* indicating the first column of sub(*C*).

descc (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psorm2l
DOUBLE PRECISION for pdorm2l
COMPLEX for pcunm2l
COMPLEX*16 for pzunm2l.
Workspace array, DIMENSION (*lwork*).
On exit, *work*(1) returns the minimal and optimal *lwork*.

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least
if *side* = 'L', *lwork* ≥ *mpc*0 + max(1, *nqc*0),
if *side* = 'R', *lwork* ≥ *nqc*0 + max(max(1, *mpc*0), numroc(numroc(*n* +*icoffc*, *nb_a*, 0, 0, *npcol*), *nb_a*, 0, 0, *lcmq*)),
where
lcmq = *lcm*/*npcol*,
lcm = iclm(*nprow*, *npcol*),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
Mqc0 = numroc(*m*+*icoffc*, *nb_c*, *mycol*, *icrow*, *nprow*),
Npc0 = numroc(*n*+*iroffc*, *mb_c*, *myrow*, *iccol*, *npcol*),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c	On exit, *c* is overwritten by *Q**sub(*C*), or Q^T*sub(*C*)/ Q^H*sub(*C*), or sub(*C*)**Q*, or sub(*C*)*Q^T / sub(*C*)*Q^H
work	On exit, *work*(1) returns the minimal and optimal *lwork*.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i*100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

NOTE The distributed submatrices *A*(*ia*:*, *ja*:*) and *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) must verify some alignment properties, namely the following expressions should be true:

If *side* = 'L', (*mb_a*.eq.*mb_c* .AND. *iroffa*.eq.*iroffc* .AND. *iarow*.eq.*icrow*)

If *side* = 'R', (*mb_a*.eq.*nb_c* .AND. *iroffa*.eq.*iroffc*).

p?orm2r/p?unm2r

Multiplies a general matrix by the orthogonal/unitary matrix from a QR factorization determined by p?geqrf *(unblocked algorithm).*

Syntax

call psorm2r(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdorm2r(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pcunm2r(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunm2r(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?orm2r/p?unm2r routine overwrites the general real/complex *m*-by-*n* distributed matrix sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) with

*Q**sub(*C*) if *side* = 'L' and *trans* = 'N', or

Q^T*sub(*C*) / Q^H*sub(*C*) if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

sub(*C*)**Q* if *side* = 'R' and *trans* = 'N', or

sub(*C*)*Q^T / sub(*C*)*Q^H if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

where *Q* is a real orthogonal or complex unitary matrix defined as the product of *k* elementary reflectors

$Q = H(k)*...*H(2)*H(1)$ as returned by p?geqrf . Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

side
(global) CHARACTER.
= 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the left,
= 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right.

trans
(global) CHARACTER.
= 'N': apply Q (no transpose)
= 'T': apply Q^T (transpose, for real flavors)
= 'C': apply Q^H (conjugate transpose, for complex flavors)

m
(global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(C). $m \geq 0$.

n
(global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(C). $n \geq 0$.

k
(global) INTEGER.
The number of elementary reflectors whose product defines the matrix Q.
If $side = 'L'$, $m \geq k \geq 0$;
if $side = 'R'$, $n \geq k \geq 0$.

a
(local)
REAL for psorm2r
DOUBLE PRECISION for pdorm2r
COMPLEX for pcunm2r
COMPLEX*16 for pzunm2r.
Pointer into the local memory to an array, DIMENSION(lld_a, $LOCc(ja+k-1)$).
On entry, the j-th column must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by p?geqrf in the k columns of its distributed matrix argument $A(ia:*,ja:ja+k-1)$. The argument $A(ia:*,ja:ja+k-1)$ is modified by the routine but restored on exit.
If $side = 'L'$, lld_a $\geq \max(1, LOCr(ia+m-1))$,
if $side = 'R'$, lld_a $\geq \max(1, LOCr(ia+n-1))$.

ia
(global) INTEGER.
The row index in the global array A indicating the first row of sub(A).

ja
(global) INTEGER.
The column index in the global array A indicating the first column of sub(A).

desca
(global and local) INTEGER array of DIMENSION ($dlen\_$). The array descriptor for the distributed matrix A.

tau
(local)
REAL for psorm2r
DOUBLE PRECISION for pdorm2r
COMPLEX for pcunm2r
COMPLEX*16 for pzunm2r.
Array, DIMENSION $LOCc(ja+k-1)$. This array contains the scalar factors $tau(j)$ of the elementary reflector $H(j)$, as returned by p?geqrf. This array is tied to the distributed matrix A.

c
(local)

REAL for psorm2r
DOUBLE PRECISION for pdorm2r
COMPLEX for pcunm2r
COMPLEX*16 for pzunm2r.
Pointer into the local memory to an array, DIMENSION(*lld_c*, *LOCc*(*jc*+*n*-1)).
On entry, the local pieces of the distributed matrix sub (*C*).

ic (global) INTEGER.
The row index in the global array *C* indicating the first row of sub(*C*).

jc (global) INTEGER.
The column index in the global array *C* indicating the first column of sub(*C*).

descc (global and local) INTEGER array of DIMENSION (*dlen_*).
The array descriptor for the distributed matrix *C*.

work (local)
REAL for psorm2r
DOUBLE PRECISION for pdorm2r
COMPLEX for pcunm2r
COMPLEX*16 for pzunm2r.
Workspace array, DIMENSION (*lwork*).

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least
if *side* = 'L', *lwork* ≥ *mpc*0 + max(1, *nqc*0),
if *side* = 'R', *lwork* ≥ *nqc*0 + max(max(1, *mpc*0), numroc(numroc(*n*+*icoffc*, *nb_a*, 0, 0, *npcol*), *nb_a*, 0, 0, *lcmq*)),
where
lcmq = *lcm*/*npcol* ,
lcm = *iclm*(*nprow*, *npcol*),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
Mqc0 = numroc(*m*+*icoffc*, *nb_c*, *mycol*, *icrow*, *nprow*),
Npc0 = numroc(*n*+*iroffc*, *mb_c*, *myrow*, *iccol*, *npcol*),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c On exit, *c* is overwritten by Q*sub(*C*), or Q^T*sub(*C*)/ Q^H*sub(*C*), or sub(*C*)*Q, or sub(*C*)*Q^T / sub(*C*)*Q^H

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local) INTEGER.
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*),

if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

NOTE The distributed submatrices *A*(*ia*:*, *ja*:*) and *C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) must verify some alignment properties, namely the following expressions should be true:

If *side* = 'L', (*mb_a*.eq.*mb_c* .AND. *iroffa*.eq.*iroffc* .AND. *iarow*.eq.*icrow*)

If *side* = 'R', (*mb_a*.eq.*nb_c* .AND. *iroffa*.eq.*iroffc*).

p?orml2/p?unml2

Multiplies a general matrix by the orthogonal/unitary matrix from an LQ factorization determined by p?gelqf *(unblocked algorithm).*

Syntax

call psorml2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdorml2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pcunml2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunml2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?orml2/p?unml2 routine overwrites the general real/complex *m*-by-*n* distributed matrix sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) with

*Q**sub(*C*) if *side* = 'L' and *trans* = 'N', or

Q^T*sub(*C*) / Q^H*sub(*C*) if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

sub(*C*)**Q* if *side* = 'R' and *trans* = 'N', or

sub(*C*)*Q^T / sub(*C*)*Q^H if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

where *Q* is a real orthogonal or complex unitary distributed matrix defined as the product of *k* elementary reflectors

Q = *H*(*k*)*...**H*(2)**H*(1) (for real flavors)

Q = $(H(k))^H$*...*$(H(2))^H$*$(H(1))^H$ (for complex flavors)

as returned by p?gelqf . *Q* is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side (global) CHARACTER.
= 'L': apply *Q* or Q^T for real flavors (Q^H for complex flavors) from the left,

= 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right.

trans (global) CHARACTER.
= 'N': apply Q (no transpose)
= 'T': apply Q^T (transpose, for real flavors)
= 'C': apply Q^H (conjugate transpose, for complex flavors)

m (global) INTEGER.
The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(C). $m \geq 0$.

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(C). $n \geq 0$.

k (global) INTEGER.
The number of elementary reflectors whose product defines the matrix Q.
If $side$ = 'L', $m \geq k \geq 0$;
if $side$ = 'R', $n \geq k \geq 0$.

a (local)
REAL for psorml2
DOUBLE PRECISION for pdorml2
COMPLEX for pcunml2
COMPLEX*16 for pzunml2.
Pointer into the local memory to an array, DIMENSION
(lld_a, $LOCc(ja+m-1)$ if $side$='L',
(lld_a, $LOCc(ja+n-1)$ if $side$='R',
where $lld\_a \geq \max(1, LOCr(ia+k-1))$.
On entry, the i-th row must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?gelqf in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. The argument $A(ia:ia+k-1, ja:*)$ is modified by the routine but restored on exit.

ia (global) INTEGER.
The row index in the global array A indicating the first row of sub(A).

ja (global) INTEGER.
The column index in the global array A indicating the first column of sub(A).

desca (global and local) INTEGER array of DIMENSION ($dlen\_$). The array descriptor for the distributed matrix A.

tau (local)
REAL for psorml2
DOUBLE PRECISION for pdorml2
COMPLEX for pcunml2
COMPLEX*16 for pzunml2.
Array, DIMENSION $LOCc(ia+k-1)$. This array contains the scalar factors $tau(i)$ of the elementary reflector $H(i)$, as returned by p?gelqf. This array is tied to the distributed matrix A.

c (local)
REAL for psorml2
DOUBLE PRECISION for pdorml2
COMPLEX for pcunml2
COMPLEX*16 for pzunml2.
Pointer into the local memory to an array, DIMENSION(lld_c, $LOCc(jc+n-1)$). On entry, the local pieces of the distributed matrix sub (C).

ic (global) INTEGER.

The row index in the global array *c* indicating the first row of sub(*C*).

jc (global) INTEGER.
The column index in the global array *c* indicating the first column of sub(*C*).

descc (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psorml2
DOUBLE PRECISION for pdorml2
COMPLEX for pcunml2
COMPLEX*16 for pzunml2.
Workspace array, DIMENSION (*lwork*).

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least
if *side* = 'L', *lwork* ≥ *mqc*0 + max(max(1, *npc*0), numroc(numroc(*m*+*icoffc*, *mb_a*, 0, 0, *nprow*), *mb_a*, 0, 0, *lcmp*)),
if *side* = 'R', *lwork* ≥ *npc*0 + max(1, *mqc*0),
where
lcmp = *lcm* / *nprow*,
lcm = *iclm*(*nprow*, *npcol*),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
Mpc0 = numroc(*m*+*icoffc*, *mb_c*, *mycol*, *icrow*, *nprow*),
Nqc0 = numroc(*n*+*iroffc*, *nb_c*, *myrow*, *iccol*, *npcol*),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c On exit, *c* is overwritten by Q*sub(*C*), or Q^T*sub(*C*)/ Q^H*sub(*C*), or sub(*C*)*Q, or sub(*C*)*Q^T / sub(*C*)*Q^H

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local) INTEGER.
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

NOTE The distributed submatrices *A*(*ia*:*, *ja*:*) and *C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) must verify some alignment properties, namely the following expressions should be true:

If *side* = 'L', (*nb_a*.eq.*mb_c* .AND. *icoffa*.eq.*iroffc*)

If *side* = 'R', (*nb_a*.eq.*nb_c* .AND. *icoffa*.eq.*icoffc* .AND. *iacol*.eq.*iccol*).

p?ormr2/p?unmr2

Multiplies a general matrix by the orthogonal/unitary matrix from an RQ factorization determined by p?gerqf *(unblocked algorithm).*

Syntax

call psormr2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pdormr2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pcunmr2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

call pzunmr2(*side*, *trans*, *m*, *n*, *k*, *a*, *ia*, *ja*, *desca*, *tau*, *c*, *ic*, *jc*, *descc*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?ormr2/p?unmr2 routine overwrites the general real/complex *m*-by-*n* distributed matrix sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1) with

*Q**sub(*C*) if *side* = 'L' and *trans* = 'N', or

Q^T*sub(*C*) / Q^H*sub(*C*) if *side* = 'L' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors), or

sub(*C*)**Q* if *side* = 'R' and *trans* = 'N', or

sub(*C*)*Q^T / sub(*C*)*Q^H if *side* = 'R' and *trans* = 'T' (for real flavors) or *trans* = 'C' (for complex flavors).

where *Q* is a real orthogonal or complex unitary distributed matrix defined as the product of *k* elementary reflectors

Q = *H*(1)**H*(2)**...**H*(*k*) (for real flavors)

Q = $(H(1))^H$*$(H(2))^H$*...*$(H(k))^H$ (for complex flavors)

as returned by p?gerqf . *Q* is of order *m* if *side* = 'L' and of order *n* if *side* = 'R'.

Input Parameters

side	(global) CHARACTER. = 'L': apply *Q* or Q^T for real flavors (Q^H for complex flavors) from the left, = 'R': apply *Q* or Q^T for real flavors (Q^H for complex flavors) from the right.
trans	(global) CHARACTER. = 'N': apply *Q* (no transpose) = 'T': apply Q^T (transpose, for real flavors) = 'C': apply Q^H (conjugate transpose, for complex flavors)
m	(global) INTEGER.

The number of rows to be operated on, that is, the number of rows of the distributed submatrix sub(*C*). *m* ≥ 0.

n (global) INTEGER.
The number of columns to be operated on, that is, the number of columns of the distributed submatrix sub(*C*). *n* ≥ 0.

k (global) INTEGER.
The number of elementary reflectors whose product defines the matrix *Q*.
If *side* = 'L', *m* ≥ *k* ≥ 0;
if *side* = 'R', *n* ≥ *k* ≥ 0.

a (local)
REAL for psormr2
DOUBLE PRECISION for pdormr2
COMPLEX for pcunmr2
COMPLEX*16 for pzunmr2.
Pointer into the local memory to an array, DIMENSION
(*lld_a*, *LOCc*(*ja*+*m*-1) if *side*='L',
(*lld_a*, *LOCc*(*ja*+*n*-1) if *side*='R',
where *lld_a* ≥ max (1, *LOCr*(*ia*+*k*-1)).
On entry, the *i*-th row must contain the vector that defines the elementary reflector *H*(*i*), *ia* ≤ *i* ≤ *ia*+*k*-1, as returned by p?gerqf in the *k* rows of its distributed matrix argument *A*(*ia*:*ia*+*k*-1, *ja*:*).
The argument *A*(*ia*:*ia*+*k*-1, *ja*:*) is modified by the routine but restored on exit.

ia (global) INTEGER.
The row index in the global array *A* indicating the first row of sub(*A*).

ja (global) INTEGER.
The column index in the global array *A* indicating the first column of sub(*A*).

desca (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

tau (local)
REAL for psormr2
DOUBLE PRECISION for pdormr2
COMPLEX for pcunmr2
COMPLEX*16 for pzunmr2.
Array, DIMENSION *LOCc*(*ia*+*k*-1). This array contains the scalar factors *tau*(*i*) of the elementary reflector *H*(*i*), as returned by p?gerqf. This array is tied to the distributed matrix *A*.

c (local)
REAL for psormr2
DOUBLE PRECISION for pdormr2
COMPLEX for pcunmr2
COMPLEX*16 for pzunmr2.
Pointer into the local memory to an array, DIMENSION(*lld_c*, *LOCc*(*jc*+*n*-1)). On entry, the local pieces of the distributed matrix sub (*C*).

ic (global) INTEGER.
The row index in the global array *C* indicating the first row of sub(*C*).

jc (global) INTEGER.
The column index in the global array *C* indicating the first column of sub(*C*).

descc (global and local) INTEGER array of DIMENSION (*dlen_*). The array descriptor for the distributed matrix *C*.

work (local)
REAL for psormr2
DOUBLE PRECISION for pdormr2
COMPLEX for pcunmr2
COMPLEX*16 for pzunmr2.
Workspace array, DIMENSION (*lwork*).

lwork (local or global) INTEGER.
The dimension of the array *work*.
lwork is local input and must be at least
if *side* = 'L', *lwork* ≥ *mpc*0 + max(max(1, *nqc*0), numroc(numroc(*m* +*iroffc*, *mb_a*, 0, 0, *nprow*), *mb_a*, 0, 0, *lcmp*)),
if *side* = 'R', *lwork* ≥ *nqc*0 + max(1, *mpc*0),
where *lcmp* = *lcm*/*nprow*,
lcm = iclm(*nprow*, *npcol*),
iroffc = mod(*ic*-1, *mb_c*),
icoffc = mod(*jc*-1, *nb_c*),
icrow = indxg2p(*ic*, *mb_c*, *myrow*, *rsrc_c*, *nprow*),
iccol = indxg2p(*jc*, *nb_c*, *mycol*, *csrc_c*, *npcol*),
Mpc0 = numroc(*m*+*iroffc*, *mb_c*, *myrow*, *icrow*, *nprow*),
Nqc0 = numroc(*n*+*icoffc*, *nb_c*, *mycol*, *iccol*, *npcol*),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the subroutine blacs_gridinfo.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

Output Parameters

c On exit, *c* is overwritten by Q*sub(*C*), or Q^T*sub(*C*)/ Q^H*sub(*C*), or sub(*C*)*Q, or sub(*C*)*Q^T / sub(*C*)*Q^H

work On exit, *work*(1) returns the minimal and optimal *lwork*.

info (local) INTEGER.
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100+*j*),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

NOTE The distributed submatrices *A*(*ia*:*, *ja*:*) and *C*(*ic*:*ic*+*m*-1,*jc*:*jc*+*n*-1) must verify some alignment properties, namely the following expressions should be true:

If *side* = 'L', (*nb_a*.eq.*mb_c* .AND. *icoffa*.eq.*iroffc*)

If *side* = 'R', (*nb_a*.eq.*nb_c* .AND. *icoffa*.eq.*icoffc* .AND. *iacol*.eq.*iccol*).

p?pbtrsv

Solves a single triangular linear system via frontsolve or backsolve where the triangular matrix is a factor of a banded matrix computed by p?pbtrf.

Syntax

call pspbtrsv(*uplo*, *trans*, *n*, *bw*, *nrhs*, *a*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

call pdpbtrsv(*uplo*, *trans*, *n*, *bw*, *nrhs*, *a*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

call pcpbtrsv(*uplo*, *trans*, *n*, *bw*, *nrhs*, *a*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

call pzpbtrsv(*uplo*, *trans*, *n*, *bw*, *nrhs*, *a*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?pbtrsv routine solves a banded triangular system of linear equations

A(1:*n*, *ja*:*ja*+*n*-1)***X* = *B*(*jb*:*jb*+*n*-1, 1:*nrhs*)

or

A(1:*n*, *ja*:*ja*+*n*-1)^T***X* = *B*(*jb*:*jb*+*n*-1, 1:*nrhs*) for real flavors,

A(1:*n*, *ja*:*ja*+*n*-1)^H***X* = *B*(*jb*:*jb*+*n*-1, 1:*nrhs*) for complex flavors,

where *A*(1:*n*, *ja*:*ja*+*n*-1) is a banded triangular matrix factor produced by the Cholesky factorization code p?pbtrf and is stored in *A*(1:*n*, *ja*:*ja*+*n*-1) and *af*. The matrix stored in *A*(1:*n*, *ja*:*ja*+*n*-1) is either upper or lower triangular according to *uplo*.

Routine p?pbtrf must be called first.

Input Parameters

uplo (global) CHARACTER. Must be 'U' or 'L'.
If *uplo* = 'U', upper triangle of *A*(1:*n*, *ja*:*ja*+*n*-1) is stored;
If *uplo* = 'L', lower triangle of *A*(1:*n*, *ja*:*ja*+*n*-1) is stored.

trans (global) CHARACTER. Must be 'N' or 'T' or 'C'.
If *trans* = 'N', solve with *A*(1:*n*, *ja*:*ja*+*n*-1);
If *trans* = 'T' or 'C' for real flavors, solve with *A*(1:*n*, *ja*:*ja*+*n*-1)^T.
If *trans* = 'C' for complex flavors, solve with conjugate transpose (*A*(1:*n*, *ja*:*ja*+*n*-1)^H.

n (global) INTEGER.
The number of rows and columns to be operated on, that is, the order of the distributed submatrix *A*(1:*n*, *ja*:*ja*+*n*-1). *n* ≥ 0.

bw (global) INTEGER.
The number of subdiagonals in 'L' or 'U', 0 ≤ *bw* ≤ *n*-1.

nrhs (global) INTEGER.
The number of right hand sides; the number of columns of the distributed submatrix *B*(*jb*:*jb*+*n*-1, 1:*nrhs*); *nrhs* ≥ 0.

a (local)
REAL for pspbtrsv
DOUBLE PRECISION for pdpbtrsv
COMPLEX for pcpbtrsv
COMPLEX*16 for pzpbtrsv.

Pointer into the local memory to an array with the first DIMENSION *lld_a* ≥ (*bw*+1), stored in *desca*.
On entry, this array contains the local pieces of the *n*-by-*n* symmetric banded distributed Cholesky factor *L* or L^T*A(1:*n*, *ja*:*ja*+*n*−1).
This local portion is stored in the packed banded format used in LAPACK.
See the *Application Notes* below and the ScaLAPACK manual for more detail on the format of distributed matrices.

ja (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
If 1D type (*dtype_a* = 501), then *dlen* ≥ 7;
If 2D type (*dtype_a* = 1), then *dlen* ≥ 9.
Contains information on mapping of *A* to memory. (See ScaLAPACK manual for full description and options.)

b (local)
REAL for pspbtrsv
DOUBLE PRECISION for pdpbtrsv
COMPLEX for pcpbtrsv
COMPLEX*16 for pzpbtrsv.
Pointer into the local memory to an array of local lead DIMENSION *lld_b* ≥ *nb*.
On entry, this array contains the local pieces of the right hand sides *B*(*jb*:*jb*+*n*−1, 1:*nrhs*).

ib (global) INTEGER. The row index in the global array *B* that points to the first row of the matrix to be operated on (which may be either all of *B* or a submatrix of *B*).

descb (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *B*.
If 1D type (*dtype_b* = 502), then *dlen* ≥ 7;
If 2D type (*dtype_b* = 1), then *dlen* ≥ 9.
Contains information on mapping of *B* to memory. Please, see ScaLAPACK manual for full description and options.

laf (local)
INTEGER. The size of user-input auxiliary Fillin space *af*. Must be *laf* ≥ (*nb*+2***bw*)***bw* . If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

work (local)
REAL for pspbtrsv
DOUBLE PRECISION for pdpbtrsv
COMPLEX for pcpbtrsv
COMPLEX*16 for pzpbtrsv.
The array *work* is a temporary workspace array of DIMENSION *lwork*. This space may be overwritten in between calls to routines.

lwork (local or global) INTEGER. The size of the user-input workspace *work*, must be at least *lwork* ≥ *bw*nrhs*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned.

Output Parameters

af	(local) REAL for pspbtrsv DOUBLE PRECISION for pdpbtrsv COMPLEX for pcpbtrsv COMPLEX*16 for pzpbtrsv. The array *af* is of DIMENSION *laf*. It contains auxiliary Fillin space. Fillin is created during the factorization routine p?pbtrf and this is stored in *af*. If a linear system is to be solved using p?pbtrs after the factorization routine, *af* must not be altered after the factorization.
b	On exit, this array contains the local piece of the solutions distributed matrix *X*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork*.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

If the factorization routine and the solve routine are to be called separately to solve various sets of right-hand sides using the same coefficient matrix, the auxiliary space *af* must not be altered between calls to the factorization routine and the solve routine.

The best algorithm for solving banded and tridiagonal linear systems depends on a variety of parameters, especially the bandwidth. Currently, only algorithms designed for the case N/P >> *bw* are implemented. These algorithms go by many names, including Divide and Conquer, Partitioning, domain decomposition-type, etc.

The Divide and Conquer algorithm assumes the matrix is narrowly banded compared with the number of equations. In this situation, it is best to distribute the input matrix *A* one-dimensionally, with columns atomic and rows divided amongst the processes. The basic algorithm divides the banded matrix up into *P* pieces with one stored on each processor, and then proceeds in 2 phases for the factorization or 3 for the solution of a linear system.

1. **Local Phase**: The individual pieces are factored independently and in parallel. These factors are applied to the matrix creating fill-in, which is stored in a non-inspectable way in auxiliary space *af*. Mathematically, this is equivalent to reordering the matrix *A* as PAP^T and then factoring the principal leading submatrix of size equal to the sum of the sizes of the matrices factored on each processor. The factors of these submatrices overwrite the corresponding parts of *A* in memory.
2. **Reduced System Phase**: A small (*bw**(*P*-1)) system is formed representing interaction of the larger blocks and is stored (as are its factors) in the space *af*. A parallel Block Cyclic Reduction algorithm is used. For a linear system, a parallel front solve followed by an analogous backsolve, both using the structure of the factored matrix, are performed.
3. **Back Subsitution Phase**: For a linear system, a local backsubstitution is performed on each processor in parallel.

p?pttrsv

Solves a single triangular linear system via frontsolve or backsolve where the triangular matrix is a factor of a tridiagonal matrix computed by p?pttrf *.*

Syntax

call pspttrsv(*uplo*, *n*, *nrhs*, *d*, *e*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

call pdpttrsv(*uplo*, *n*, *nrhs*, *d*, *e*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

call pcpttrsv(*uplo*, *trans*, *n*, *nrhs*, *d*, *e*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

call pzpttrsv(*uplo*, *trans*, *n*, *nrhs*, *d*, *e*, *ja*, *desca*, *b*, *ib*, *descb*, *af*, *laf*, *work*, *lwork*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?pttrsv routine solves a tridiagonal triangular system of linear equations

A(1:*n*, *ja*:*ja*+*n*-1)**X* = *B*(*jb*:*jb*+*n*-1, 1:*nrhs*)

or

A(1:*n*, *ja*:*ja*+*n*-1)^T**X* = *B*(*jb*:*jb*+*n*-1, 1:*nrhs*) for real flavors,

A(1:*n*, *ja*:*ja*+*n*-1)^H**X* = *B*(*jb*:*jb*+*n*-1, 1:*nrhs*) for complex flavors,

where *A*(1:*n*, *ja*:*ja*+*n*-1) is a tridiagonal triangular matrix factor produced by the Cholesky factorization code p?pttrf and is stored in *A*(1:*n*, *ja*:*ja*+*n*-1) and *af*. The matrix stored in *A*(1:*n*, *ja*:*ja*+*n*-1) is either upper or lower triangular according to *uplo*.

Routine p?pttrf must be called first.

Input Parameters

uplo (global) CHARACTER. Must be 'U' or 'L'.
If *uplo* = 'U', upper triangle of *A*(1:*n*, *ja*:*ja*+*n*-1) is stored;
If *uplo* = 'L', lower triangle of *A*(1:*n*, *ja*:*ja*+*n*-1) is stored.

trans (global) CHARACTER. Must be 'N' or 'C'.
If *trans* = 'N', solve with *A*(1:*n*, *ja*:*ja*+*n*-1);
If *trans* = 'C' (for complex flavors), solve with conjugate transpose (*A*(1:*n*, *ja*:*ja*+*n*-1))^H.

n (global) INTEGER.
The number of rows and columns to be operated on, that is, the order of the distributed submatrix *A*(1:*n*, *ja*:*ja*+*n*-1). *n* ≥ 0.

nrhs (global) INTEGER.
The number of right hand sides; the number of columns of the distributed submatrix *B*(*jb*:*jb*+*n*-1, 1:*nrhs*); *nrhs* ≥ 0.

d (local)
REAL for pspttrsv
DOUBLE PRECISION for pdpttrsv
COMPLEX for pcpttrsv
COMPLEX*16 for pzpttrsv.
Pointer to the local part of the global vector storing the main diagonal of the matrix; must be of size ≥ *desca*(*nb_*).

e (local)

REAL for pspttrsv
DOUBLE PRECISION for pdpttrsv
COMPLEX for pcpttrsv
COMPLEX*16 for pzpttrsv.
Pointer to the local part of the global vector storing the upper diagonal of the matrix; must be of size ≥ *desca*(*nb_*). Globally, *du*(*n*) is not referenced, and *du* must be aligned with *d*.

ja (global) INTEGER. The index in the global array *A* that points to the start of the matrix to be operated on (which may be either all of *A* or a submatrix of *A*).

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.
If 1D type (*dtype_a* = 501 or 502), then *dlen* ≥ 7;
If 2D type (*dtype_a* = 1), then *dlen* ≥ 9.
Contains information on mapping of *A* to memory. See ScaLAPACK manual for full description and options.

b (local)
REAL for pspttrsv
DOUBLE PRECISION for pdpttrsv
COMPLEX for pcpttrsv
COMPLEX*16 for pzpttrsv.
Pointer into the local memory to an array of local lead DIMENSION *lld_b* ≥ *nb*.
On entry, this array contains the local pieces of the right hand sides *B*(*jb*:*jb*+*n*-1, 1:*nrhs*).

ib (global) INTEGER. The row index in the global array *B* that points to the first row of the matrix to be operated on (which may be either all of *B* or a submatrix of *B*).

descb (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *B*.
If 1D type (*dtype_b* = 502), then *dlen* ≥ 7;
If 2D type (*dtype_b* = 1), then *dlen* ≥ 9.
Contains information on mapping of *B* to memory. See ScaLAPACK manual for full description and options.

laf (local)
INTEGER. The size of user-input auxiliary Fillin space *af*. Must be *laf* ≥ (*nb*+2**bw*)**bw*.
If *laf* is not large enough, an error code will be returned and the minimum acceptable size will be returned in *af*(1).

work (local)
REAL for pspttrsv
DOUBLE PRECISION for pdpttrsv
COMPLEX for pcpttrsv
COMPLEX*16 for pzpttrsv.
The array *work* is a temporary workspace array of DIMENSION *lwork*. This space may be overwritten in between calls to routines.

lwork (local or global) INTEGER. The size of the user-input workspace *work*, must be at least *lwork* ≥(10+2*min(100, *nrhs*))**npcol*+4**nrhs*. If *lwork* is too small, the minimal acceptable size will be returned in *work*(1) and an error code is returned.

Output Parameters

d, *e*	(local). REAL for pspttrsv DOUBLE PRECISION for pdpttrsv COMPLEX for pcpttrsv COMPLEX*16 for pzpttrsv. On exit, these arrays contain information on the factors of the matrix.
af	(local) REAL for pspttrsv DOUBLE PRECISION for pdpttrsv COMPLEX for pcpttrsv COMPLEX*16 for pzpttrsv. The array *af* is of DIMENSION *laf*. It contains auxiliary Fillin space. Fillin is created during the factorization routine p?pbtrf and this is stored in *af*. If a linear system is to be solved using p?pttrs after the factorization routine, *af* must not be altered after the factorization.
b	On exit, this array contains the local piece of the solutions distributed matrix *X*.
work(1)	On exit, *work*(1) contains the minimum value of *lwork*.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?potf2

Computes the Cholesky factorization of a symmetric/ Hermitian positive definite matrix (local unblocked algorithm).

Syntax

```
call pspotf2(uplo, n, a, ia, ja, desca, info)
call pdpotf2(uplo, n, a, ia, ja, desca, info)
call pcpotf2(uplo, n, a, ia, ja, desca, info)
call pzpotf2(uplo, n, a, ia, ja, desca, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?potf2 routine computes the Cholesky factorization of a real symmetric or complex Hermitian positive definite distributed matrix sub (*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

The factorization has the form

sub(*A*) = *U*'**U*, if *uplo* = 'U', or sub(*A*) = *L***L*', if *uplo* = 'L',

where *U* is an upper triangular matrix, *L* is lower triangular. *X*' denotes transpose (conjugate transpose) of *X*.

Input Parameters

uplo	(global) CHARACTER. Specifies whether the upper or lower triangular part of the symmetric/ Hermitian matrix *A* is stored. = 'U': upper triangle of sub (*A*) is stored; = 'L': lower triangle of sub (*A*) is stored.
n	(global) INTEGER. The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub (*A*). *n* ≥ 0.
a	(local) REAL for pspotf2 DOUBLE PRECISION or pdpotf2 COMPLEX for pcpotf2 COMPLEX*16 for pzpotf2. Pointer into the local memory to an array of DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)) containing the local pieces of the *n*-by-*n* symmetric distributed matrix sub(*A*) to be factored. If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular matrix and the strictly lower triangular part of this matrix is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular matrix and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the global array *A* indicating the first row and the first column of the sub(*A*), respectively.
desca	(global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a	(local) On exit, if *uplo* = 'U', the upper triangular part of the distributed matrix contains the Cholesky factor U; if *uplo* = 'L', the lower triangular part of the distributed matrix contains the Cholesky factor L.
info	(local) INTEGER. = 0: successful exit < 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*. > 0: if *info* = *k*, the leading minor of order *k* is not positive definite, and the factorization could not be completed.

p?rscl

Multiplies a vector by the reciprocal of a real scalar.

Syntax

```
call psrscl(n, sa, sx, ix, jx, descx, incx)
```

```
call pdrscl(n, sa, sx, ix, jx, descx, incx)
call pcsrscl(n, sa, sx, ix, jx, descx, incx)
call pzdrscl(n, sa, sx, ix, jx, descx, incx)
```

Include Files

- C: mkl_scalapack.h

Description

The p?rscl routine multiplies an *n*-element real/complex vector sub(*x*) by the real scalar 1/a. This is done without overflow or underflow as long as the final result sub(*x*)/a does not overflow or underflow.

sub(*x*) denotes *x*(*ix*:*ix*+*n*-1, *jx*:*jx*), if *incx* = 1,

and *x*(*ix*:*ix*, *jx*:*jx*+*n*-1), if *incx* = *m_x*.

Input Parameters

n	(global) INTEGER. The number of components of the distributed vector sub(*x*). *n* ≥ 0.
sa	REAL for psrscl/pcsrscl DOUBLE PRECISION for pdrscl/pzdrscl. The scalar a that is used to divide each component of the vector *x*. This parameter must be ≥ 0.
sx	REAL forpsrscl DOUBLE PRECISION for pdrscl COMPLEX for pcsrscl COMPLEX*16 for pzdrscl. Array containing the local pieces of a distributed matrix of DIMENSION of at least ((*jx*-1)*m_x + *ix* + (*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix	(global) INTEGER.The row index of the submatrix of the distributed matrix *X* to operate on.
jx	(global) INTEGER. The column index of the submatrix of the distributed matrix *X* to operate on.
descx	(global and local). INTEGER. Array of DIMENSION 8. The array descriptor for the distributed matrix *X*.
incx	(global) INTEGER. The increment for the elements of *X*. This version supports only two values of *incx*, namely 1 and *m_x*.

Output Parameters

sx	On exit, the result *x*/*a*.

p?sygs2/p?hegs2

Reduces a symmetric/Hermitian definite generalized eigenproblem to standard form, using the factorization results obtained from p?potrf *(local unblocked algorithm).*

Syntax

```
call pssygs2(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, info)
call pdsygs2(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, info)
call pchegs2(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, info)
call pzhegs2(ibtype, uplo, n, a, ia, ja, desca, b, ib, jb, descb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?sygs2/p?hegs2 routine reduces a real symmetric-definite or a complex Hermitian-definite generalized eigenproblem to standard form.

Here sub(*A*) denotes *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1), and sub(*B*) denotes *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*n*-1).

If *ibtype* = 1, the problem is

sub(*A*)**x* = λ*sub(*B*)**x*

and sub(*A*) is overwritten by

inv(U^T)*sub(*A*)*inv(*U*) or inv(*L*)*sub(*A*)*inv(L^T) - for real flavors, and

inv(U^H)*sub(*A*)*inv(*U*) or inv(*L*)*sub(*A*)*inv(L^H) - for complex flavors.

If *ibtype* = 2 or 3, the problem is

sub(*A*)*sub(*B*)*x* = λ**x* or sub(*B*)*sub(*A*)*x* =λ**x*

and sub(*A*) is overwritten by

*U**sub(*A*)*U^T or *L****T**sub(*A*)**L* - for real flavors and

*U**sub(*A*)*U^H or *L****H**sub(*A*)**L* - for complex flavors.

The matrix sub(*B*) must have been previously factorized as U^T**U* or *L**L^T (for real flavors), or as U^H**U* or *L**L^H (for complex flavors) by p?potrf.

Input Parameters

ibtype — (global) INTEGER.
= 1:
compute inv(U^T)*sub(*A*)*inv(*U*), or inv(*L*)*sub(*A*)*inv(L^T) for real subroutines,
and inv(U^H)*sub(*A*)*inv(*U*), or inv(*L*)*sub(*A*)*inv(L^H) for complex subroutines;
= 2 or 3:
compute *U**sub(*A*)*U^T, or L^T*sub(*A*)**L* for real subroutines,
and *U**sub(*A*)*U^H or L^H*sub(*A*)**L* for complex subroutines.

uplo — (global) CHARACTER
Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix sub(*A*) is stored, and how sub(*B*) is factorized.
= 'U': Upper triangular of sub(*A*) is stored and sub(*B*) is factorized as U^T**U* (for real subroutines) or as U^H**U* (for complex subroutines).
= 'L': Lower triangular of sub(*A*) is stored and sub(*B*) is factorized as *L**L^T (for real subroutines) or as *L**L^H (for complex subroutines)

n — (global) INTEGER.
The order of the matrices sub(*A*) and sub(*B*). *n* ≥ 0.

a (local)
REAL for pssygs2
DOUBLE PRECISION for pdsygs2
COMPLEX for pchegs2
COMPLEX*16 for pzhegs2.
Pointer into the local memory to an array, DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the *n*-by-*n* symmetric/Hermitian distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and the strictly lower triangular part of sub(*A*) is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* (global) INTEGER.
The row and column indices in the global array *A* indicating the first row and the first column of the sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

B (local)
REAL for pssygs2
DOUBLE PRECISION for pdsygs2
COMPLEX for pchegs2
COMPLEX*16 for pzhegs2.
Pointer into the local memory to an array, DIMENSION(*lld_b*, *LOCc*(*jb*+*n*-1)).
On entry, this array contains the local pieces of the triangular factor from the Cholesky factorization of sub(*B*) as returned by p?potrf.

ib, *jb* (global) INTEGER.
The row and column indices in the global array *B* indicating the first row and the first column of the sub(*B*), respectively.

descb (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *B*.

Output Parameters

a (local)
On exit, if *info* = 0, the transformed matrix is stored in the same format as sub(*A*).

info INTEGER.
= 0: successful exit.
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

p?sytd2/p?hetd2

Reduces a symmetric/Hermitian matrix to real symmetric tridiagonal form by an orthogonal/unitary similarity transformation (local unblocked algorithm).

Syntax

```
call pssytd2(uplo, n, a, ia, ja, desca, d, e, tau, work, lwork, info)
call pdsytd2(uplo, n, a, ia, ja, desca, d, e, tau, work, lwork, info)
call pchetd2(uplo, n, a, ia, ja, desca, d, e, tau, work, lwork, info)
call pzhetd2(uplo, n, a, ia, ja, desca, d, e, tau, work, lwork, info)
```

Include Files

- C: mkl_scalapack.h

Description

The p?sytd2/p?hetd2 routine reduces a real symmetric/complex Hermitian matrix sub(*A*) to symmetric/Hermitian tridiagonal form *T* by an orthogonal/unitary similarity transformation:

Q'*sub(*A*)**Q* = *T*, where sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

Input Parameters

uplo — (global) CHARACTER.
Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix sub(*A*) is stored:
= 'U': upper triangular
= 'L': lower triangular

n — (global) INTEGER.
The number of rows and columns to be operated on, that is, the order of the distributed submatrix sub(*A*). *n* ≥ 0.

a — (local)
REAL for pssytd2
DOUBLE PRECISION for pdsytd2
COMPLEX for pchetd2
COMPLEX*16 for pzhetd2.
Pointer into the local memory to an array, DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the *n*-by-*n* symmetric/Hermitian distributed matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of sub(*A*) contains the upper triangular part of the matrix, and the strictly lower triangular part of sub(*A*) is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* — (global) INTEGER.
The row and column indices in the global array *A* indicating the first row and the first column of the sub(*A*), respectively.

desca — (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

work — (local)
REAL for pssytd2
DOUBLE PRECISION for pdsytd2
COMPLEX for pchetd2
COMPLEX*16 for pzhetd2.

The array *work* is a temporary workspace array of DIMENSION *lwork*.

Output Parameters

a
On exit, if *uplo* = 'U', the diagonal and first superdiagonal of sub(*A*) are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements above the first superdiagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors;
if *uplo* = 'L', the diagonal and first subdiagonal of *A* are overwritten by the corresponding elements of the tridiagonal matrix *T*, and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix *Q* as a product of elementary reflectors. See the *Application Notes* below.

d
(local)
REAL for pssytd2/pchetd2
DOUBLE PRECISION for pdsytd2/pzhetd2.
Array, DIMENSION(*LOCc*(*ja*+*n*-1)). The diagonal elements of the tridiagonal matrix *T*:
d(i) = *a*(i,i); *d* is tied to the distributed matrix *A*.

e
(local)
REAL for pssytd2/pchetd2
DOUBLE PRECISION for pdsytd2/pzhetd2.
Array, DIMENSION(*LOCc*(*ja*+*n*-1)),
if *uplo* = 'U', *LOCc*(*ja*+*n*-2) otherwise.
The off-diagonal elements of the tridiagonal matrix *T*:
e(i) = *a*(i,i+1) if *uplo* = 'U',
e(i) = *a*(i+1,i) if *uplo* = 'L'.
e is tied to the distributed matrix *A*.

tau
(local)
REAL for pssytd2
DOUBLE PRECISION for pdsytd2
COMPLEX for pchetd2
COMPLEX*16 for pzhetd2.
Array, DIMENSION(*LOCc*(*ja*+*n*-1)).
The scalar factors of the elementary reflectors. *tau* is tied to the distributed matrix *A*.

work(1)
On exit, *work*(1) returns the minimal and optimal value of *lwork*.

lwork
(local or global) INTEGER.
The dimension of the workspace array *work*.
lwork is local input and must be at least *lwork* ≥ 3*n*.
If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla.

info
(local) INTEGER.
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = -(*i**100),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(*n*-1)*...*H*(2)*H*(1)

Each *H*(i) has the form

H(i) = *I* - *tau***v***v*',

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(*i*+1:*n*) = 0 and *v*(*i*) = 1; *v*(1:*i*-1) is stored on exit in *A*(*ia*:*ia*+*i*-2, *ja*+*i*), and *tau* in *TAU*(*ja*+*i*-1).

If *uplo* = 'L', the matrix *Q* is represented as a product of elementary reflectors

Q = *H*(1)*H*(2)*...*H*(*n*-1).

Each *H*(i) has the form

H(i) = *I* - *tau***v***v*' ,

where *tau* is a real/complex scalar, and *v* is a real/complex vector with *v*(1:*i*) = 0 and *v*(*i*+1) = 1; *v*(*i*+2:*n*) is stored on exit in *A*(*ia*+*i*+1:*ia*+*n*-1, *ja*+*i*-1), and *tau* in *TAU*(*ja*+*i*-1).

The contents of sub (*A*) on exit are illustrated by the following examples with *n* = 5:

if *uplo*='U':

$$\begin{bmatrix} d & e & v_2 & v_3 & v_4 \\ & d & e & v_3 & v_4 \\ & & d & e & v_4 \\ & & & d & e \\ & & & & d \end{bmatrix}$$

if *uplo*='L':

$$\begin{bmatrix} d & & & & \\ e & d & & & \\ v1 & e & d & & \\ v1 & v_2 & e & d & \\ v1 & v_2 & v_3 & e & d \end{bmatrix}$$

where *d* and *e* denotes diagonal and off-diagonal elements of *T*, and v_i denotes an element of the vector defining *H*(i).

NOTE The distributed submatrix sub(*A*) must verify some alignment properties, namely the following expression should be true:

(*mb_a*.eq.*nb_a* .AND. *iroffa*.eq.*icoffa*) with *iroffa* = mod(*ia* - 1, *mb_a*) and *icoffa* = mod(*ja* -1, *nb_a*).

p?trti2

Computes the inverse of a triangular matrix (local unblocked algorithm).

Syntax

call pstrti2(*uplo*, *diag*, *n*, *a*, *ia*, *ja*, *desca*, *info*)

call pdtrti2(*uplo*, *diag*, *n*, *a*, *ia*, *ja*, *desca*, *info*)

call pctrti2(*uplo*, *diag*, *n*, *a*, *ia*, *ja*, *desca*, *info*)

call pztrti2(*uplo*, *diag*, *n*, *a*, *ia*, *ja*, *desca*, *info*)

Include Files

- C: mkl_scalapack.h

Description

The p?trti2 routine computes the inverse of a real/complex upper or lower triangular block matrix sub (*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

This matrix should be contained in one and only one process memory space (local operation).

Input Parameters

uplo (global) CHARACTER*1.
Specifies whether the matrix sub (*A*) is upper or lower triangular.
= 'U': sub (*A*) is upper triangular
= 'L': sub (*A*) is lower triangular.

diag (global) CHARACTER*1.
Specifies whether or not the matrix *A* is unit triangular.
= 'N': sub (*A*) is non-unit triangular
= 'U': sub (*A*) is unit triangular.

n (global) INTEGER.
The number of rows and columns to be operated on, i.e., the order of the distributed submatrix sub(*A*). $n \geq 0$.

a (local)
REAL for pstrti2
DOUBLE PRECISION for pdtrti2
COMPLEX for pctrti2
COMPLEX*16 for pztrti2.
Pointer into the local memory to an array, DIMENSION(*lld_a*, *LOCc*(*ja*+*n*-1)).
On entry, this array contains the local pieces of the triangular matrix sub(*A*).
If *uplo* = 'U', the leading *n*-by-*n* upper triangular part of the matrix sub(*A*) contains the upper triangular part of the matrix, and the strictly lower triangular part of sub(*A*) is not referenced.
If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the matrix sub(*A*) contains the lower triangular part of the matrix, and the strictly upper triangular part of sub(*A*) is not referenced. If *diag* = 'U', the diagonal elements of sub(*A*) are not referenced either and are assumed to be 1.

ia, *ja* (global) INTEGER.
The row and column indices in the global array *A* indicating the first row and the first column of the sub(*A*), respectively.

desca (global and local) INTEGER array, DIMENSION (*dlen_*). The array descriptor for the distributed matrix *A*.

Output Parameters

a On exit, the (triangular) inverse of the original matrix, in the same storage format.

info INTEGER.
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = - (*i**100),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

?lamsh

Sends multiple shifts through a small (single node) matrix to maximize the number of bulges that can be sent through.

Syntax

```
call slamsh(s, lds, nbulge, jblk, h, ldh, n, ulp)
call dlamsh(s, lds, nbulge, jblk, h, ldh, n, ulp)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `?lamsh` routine sends multiple shifts through a small (single node) matrix to see how small consecutive subdiagonal elements are modified by subsequent shifts in an effort to maximize the number of bulges that can be sent through. The subroutine should only be called when there are multiple shifts/bulges (*nbulge* > 1) and the first shift is starting in the middle of an unreduced Hessenberg matrix because of two or more small consecutive subdiagonal elements.

Input Parameters

s	(local) INTEGER. REAL for slamsh DOUBLE PRECISION for dlamsh Array, DIMENSION (*lds*,*). On entry, the matrix of shifts. Only the 2x2 diagonal of *s* is referenced. It is assumed that *s* has *jblk* double shifts (size 2).
lds	(local) INTEGER. On entry, the leading dimension of *S*; unchanged on exit. 1<*nbulge* ≤ *jblk* ≤ *lds*/2.
nbulge	(local) INTEGER. On entry, the number of bulges to send through *h* (>1). *nbulge* should be less than the maximum determined (*jblk*). 1<*nbulge* ≤ *jblk* ≤ *lds*/2.
jblk	(local) INTEGER. On entry, the leading dimension of *S*; unchanged on exit.
h	(local) INTEGER. REAL for slamsh DOUBLE PRECISION for dlamsh Array, DIMENSION (*lds*, *n*). On entry, the local matrix to apply the shifts on. *h* should be aligned so that the starting row is 2.
ldh	(local) INTEGER. On entry, the leading dimension of *H*; unchanged on exit.
n	(local) INTEGER. On entry, the size of *H*. If all the bulges are expected to go through, *n* should be at least 4*nbulge*+2. Otherwise, *nbulge* may be reduced by this routine.
ulp	(local) REAL for slamsh

DOUBLE PRECISION for dlamsh
On entry, machine precision. Unchanged on exit.

Output Parameters

s	On exit, the data is rearranged in the best order for applying.
nbulge	On exit, the maximum number of bulges that can be sent through.
h	On exit, the data is destroyed.

?laref

Applies Householder reflectors to matrices on either their rows or columns.

Syntax

```
call slaref(type, a, lda, wantz, z, ldz, block, irow1, icol1, istart, istop, itmp1, itmp2, liloz, lihiz, vecs, v2, v3, t1, t2, t3)
call dlaref(type, a, lda, wantz, z, ldz, block, irow1, icol1, istart, istop, itmp1, itmp2, liloz, lihiz, vecs, v2, v3, t1, t2, t3)
```

Include Files

- C: mkl_scalapack.h

Description

The ?laref routine applies one or several Householder reflectors of size 3 to one or two matrices (if column is specified) on either their rows or columns.

Input Parameters

type	(global) CHRACTER*1. If *type* = 'R', apply reflectors to the rows of the matrix (apply from left). Otherwise, apply reflectors to the columns of the matrix. Unchanged on exit.
a	(global) REAL for slaref DOUBLE PRECISION for dlaref Array, DIMENSION (*lda*, *). On entry, the matrix to receive the reflections.
lda	(local) INTEGER. On entry, the leading dimension of *A*; unchanged on exit.
wantz	(global) LOGICAL. If *wantz* = .TRUE., apply any column reflections to *Z* as well. If *wantz* = .FALSE., do no additional work on *Z*.
z	(global) REAL for slaref DOUBLE PRECISION for dlaref Array, DIMENSION (*ldz*, *). On entry, the second matrix to receive column reflections.
ldz	(local) INTEGER. On entry, the leading dimension of *Z*; unchanged on exit.
block	(global). LOGICAL. = .TRUE. : apply several reflectors at once and read their data from the *vecs* array;

	= .FALSE. : apply the single reflector given by *v2*, *v3*, *t1*, *t2*, and *t3*.
irow1	(local) INTEGER. On entry, the local row element of the matrix A.
icol1	(local) INTEGER. On entry, the local column element of the matrix A.
istart	(global) INTEGER. Specifies the "number" of the first reflector. *istart* is used as an index into *vecs* if *block* is set. *istart* is ignored if *block* is .FALSE. .
istop	(global) INTEGER. Specifies the "number" of the last reflector. *istop* is used as an index into *vecs* if *block* is set. *istop* is ignored if *block* is .FALSE. .
itmp1	(local) INTEGER. Starting range into *A*. For rows, this is the local first column. For columns, this is the local first row.
itmp2	(local) INTEGER. Ending range into *A*. For rows, this is the local last column. For columns, this is the local last row.
liloz, *lihiz*	(local). INTEGER. Serve the same purpose as *itmp1*, *itmp2* but for *Z* when *wantz* is set.
vecs	(global) REAL for slaref DOUBLE PRECISION for dlaref. Array of size 3 *n (matrix size). This array holds the size 3 reflectors one after another and is only accessed when *block* is .TRUE.
v2,*v3*,*t1*,*t2*,*t3*	(global). INTEGER. REAL for slaref DOUBLE PRECISION for dlaref. These parameters hold information on a single size 3 Householder reflector and are read when *block* is .FALSE., and overwritten when *block* is .TRUE..

Output Parameters

a	On exit, the updated matrix.
z	Changed only if *wantz* is set. If *wantz* is .FALSE. , *z* is not referenced.
irow1	Undefined.
icol1	Undefined.
v2,*v3*,*t1*,*t2*,*t3*	These parameters are read when *block* is .FALSE., and overwritten when *block* is .TRUE..

?lasorte

Sorts eigenpairs by real and complex data types.

Syntax

```
call slasorte(s, lds, j, out, info)
call dlasorte(s, lds, j, out, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?lasorte routine sorts eigenpairs so that real eigenpairs are together and complex eigenpairs are together. This helps to employ 2x2 shifts easily since every second subdiagonal is guaranteed to be zero. This routine does no parallel work and makes no calls.

Input Parameters

s	(local) INTEGER. REAL for slasorte DOUBLE PRECISION for dlasorte Array, DIMENSION (*lds*). On entry, a matrix already in Schur form.
lds	(local) INTEGER. On entry, the leading dimension of the array *s*; unchanged on exit.
j	(local) INTEGER. On entry, the order of the matrix *S*; unchanged on exit.
out	(local) INTEGER. REAL for slasorte DOUBLE PRECISION for dlasorte Array, DIMENSION (2**j*). The work buffer required by the routine.
info	(local) INTEGER. Set, if the input matrix had an odd number of real eigenvalues and things could not be paired or if the input matrix *S* was not originally in Schur form. 0 indicates successful completion.

Output Parameters

s	On exit, the diagonal blocks of *S* have been rewritten to pair the eigenvalues. The resulting matrix is no longer similar to the input.
out	Work buffer.

?lasrt2

Sorts numbers in increasing or decreasing order.

Syntax

```
call slasrt2(id, n, d, key, info)
call dlasrt2(id, n, d, key, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?lasrt2 routine is modified LAPACK routine ?lasrt, which sorts the numbers in *d* in increasing order (if *id* = 'I') or in decreasing order (if *id* = 'D'). It uses Quick Sort, reverting to Insertion Sort on arrays of size ≤ 20. Dimension of STACK limits *n* to about 2^{32}.

Input Parameters

id	CHARACTER*1. = 'I': sort *d* in increasing order; = 'D': sort *d* in decreasing order.
n	INTEGER. The length of the array *d*.
d	REAL for slasrt2 DOUBLE PRECISION for dlasrt2. Array, DIMENSION (*n*). On entry, the array to be sorted.
key	INTEGER. Array, DIMENSION (*n*). On entry, *key* contains a key to each of the entries in *d*(). Typically, *key*(*i*) = *i* for all *i* .

Output Parameters

D	On exit, *d* has been sorted into increasing order (*d*(1) ≤ ... ≤ *d*(*n*)) or into decreasing order (*d*(1) ≥ ... ≥ *d*(*n*)), depending on *id*.
info	INTEGER. = 0: successful exit < 0: if *info* = -i, the *i*-th argument had an illegal value.
key	On exit, *key* is permuted in exactly the same manner as *d*() was permuted from input to output. Therefore, if *key*(*i*) = *i* for all *i* upon input, then *d*_out(*i*) = *d*_in(*key*(*i*)).

?stein2

Computes the eigenvectors corresponding to specified eigenvalues of a real symmetric tridiagonal matrix, using inverse iteration.

Syntax

```
call sstein2(n, d, e, m, w, iblock, isplit, orfac, z, ldz, work, iwork, ifail, info)
call dstein2(n, d, e, m, w, iblock, isplit, orfac, z, ldz, work, iwork, ifail, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?stein2 routine is a modified LAPACK routine ?stein. It computes the eigenvectors of a real symmetric tridiagonal matrix *T* corresponding to specified eigenvalues, using inverse iteration.

The maximum number of iterations allowed for each eigenvector is specified by an internal parameter *maxits* (currently set to 5).

Input Parameters

n	INTEGER. The order of the matrix *T* (*n* ≥ 0).
m	INTEGER. The number of eigenvectors to be found (0 ≤ *m* ≤ *n*).
d, *e*, *w*	REAL for single-precision flavors

DOUBLE PRECISION for double-precision flavors.
Arrays: *d*(*), DIMENSION (*n*). The *n* diagonal elements of the tridiagonal matrix *T*.
e(*), DIMENSION (*n*).
The (*n*-1) subdiagonal elements of the tridiagonal matrix *T*, in elements 1 to *n*-1. *e*(*n*) need not be set.
w(*), DIMENSION (*n*).
The first *m* elements of *w* contain the eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block. (The output array *w* from ?stebz with ORDER = 'B' is expected here).
The dimension of *w* must be at least max(1, *n*).

iblock — INTEGER.
Array, DIMENSION (*n*).
The submatrix indices associated with the corresponding eigenvalues in *w* ; *iblock*(*i*) = 1, if eigenvalue *w*(*i*) belongs to the first submatrix from the top,
iblock(*i*) = 2, if eigenvalue *w*(*i*) belongs to the second submatrix, etc. (The output array *iblock* from ?stebz is expected here).

isplit — INTEGER.
Array, DIMENSION (*n*).
The splitting points, at which *T* breaks up into submatrices. The first submatrix consists of rows/columns 1 to *isplit*(1), the second submatrix consists of rows/columns *isplit*(1)+1 through *isplit*(2), etc. (The output array *isplit* from ?stebz is expected here).

orfac — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
orfac specifies which eigenvectors should be orthogonalized. Eigenvectors that correspond to eigenvalues which are within *orfac**||*T*|| of each other are to be orthogonalized.

ldz — INTEGER. The leading dimension of the output array *z*; *ldz* ≥ max(1, *n*).

work — REAL for single-precision flavors
DOUBLE PRECISION for double-precision flavors.
Workspace array, DIMENSION (5*n*).

iwork — INTEGER. Workspace array, DIMENSION (*n*).

Output Parameters

z — REAL for sstein2
DOUBLE PRECISION for dstein2
Array, DIMENSION (*ldz*, *m*).
The computed eigenvectors. The eigenvector associated with the eigenvalue *w*(*i*) is stored in the *i*-th column of *z*. Any vector that fails to converge is set to its current iterate after *maxits* iterations.

ifail — INTEGER.
Array, DIMENSION (*m*).
On normal exit, all elements of *ifail* are zero. If one or more eigenvectors fail to converge after *maxits* iterations, then their indices are stored in the array *ifail*.

info — INTEGER.
info = 0, the exit is successful.

info < 0: if *info* = -*i*, the *i*-th had an illegal value.
info > 0: if *info* = *i*, then *i* eigenvectors failed to converge in *maxits* iterations. Their indices are stored in the array *ifail*.

?dbtf2

Computes an LU factorization of a general band matrix with no pivoting (local unblocked algorithm).

Syntax

```
call sdbtf2(m, n, kl, ku, ab, ldab, info)
call ddbtf2(m, n, kl, ku, ab, ldab, info)
call cdbtf2(m, n, kl, ku, ab, ldab, info)
call zdbtf2(m, n, kl, ku, ab, ldab, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?dbtf2 routine computes an *LU* factorization of a general real/complex *m*-by-*n* band matrix *A* without using partial pivoting with row interchanges.

This is the unblocked version of the algorithm, calling BLAS Routines and Functions.

Input Parameters

m	INTEGER. The number of rows of the matrix *A*(*m* ≥ 0).
n	INTEGER. The number of columns in *A*(*n* ≥ 0).
kl	INTEGER. The number of sub-diagonals within the band of *A*(*kl* ≥ 0).
ku	INTEGER. The number of super-diagonals within the band of *A*(*ku* ≥ 0).
ab	REAL for sdbtf2 DOUBLE PRECISION for ddbtf2 COMPLEX for cdbtf2 COMPLEX*16 for zdbtf2. Array, DIMENSION (*ldab*, *n*). The matrix *A* in band storage, in rows *kl*+1 to 2*kl*+*ku*+1; rows 1 to *kl* of the array need not be set. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*kl*+*ku*+1+*i*-*j*,*j*) = *A*(*i*,*j*) for max(1,*j*-*ku*) ≤ *i* ≤ min(*m*,*j*+*kl*).
ldab	INTEGER. The leading dimension of the array *ab*. (*ldab* ≥ 2*kl* + *ku* +1)

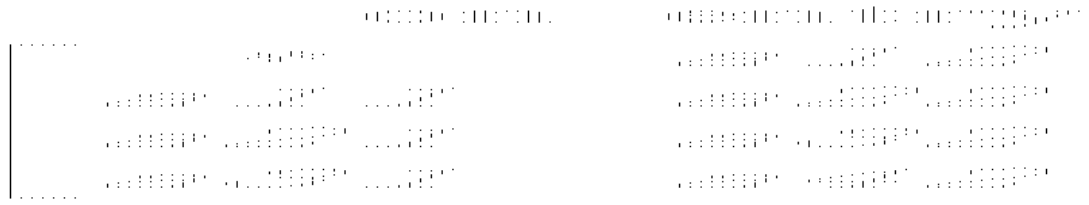
Output Parameters

ab	On exit, details of the factorization: *U* is stored as an upper triangular band matrix with *kl*+*ku* superdiagonals in rows 1 to *kl*+*ku*+1, and the multipliers used during the factorization are stored in rows *kl*+*ku*+2 to 2**kl*+*ku*+1. See the *Application Notes* below for further details.
info	INTEGER. = 0: successful exit < 0: if *info* = - *i*, the *i*-th argument had an illegal value,

> 0: if *info* = + *i*, *u*(*i*,*i*) is 0. The factorization has been completed, but the factor *U* is exactly singular. Division by 0 will occur if you use the factor *U* for solving a system of linear equations.

Application Notes

The band storage scheme is illustrated by the following example, when *m* = *n* = 6, *kl* = 2, *ku* = 1:



The routine does not use array elements marked *; elements marked + need not be set on entry, but the routine requires them to store elements of *U*, because of fill-in resulting from the row interchanges.

?dbtrf

Computes an LU factorization of a general band matrix with no pivoting (local blocked algorithm).

Syntax

```
call sdbtrf(m, n, kl, ku, ab, ldab, info)
call ddbtrf(m, n, kl, ku, ab, ldab, info)
call cdbtrf(m, n, kl, ku, ab, ldab, info)
call zdbtrf(m, n, kl, ku, ab, ldab, info)
```

Include Files

- C: mkl_scalapack.h

Description

This routine computes an LU factorization of a real *m*-by-*n* band matrix *A* without using partial pivoting or row interchanges.

This is the blocked version of the algorithm, calling BLAS Routines and Functions.

Input Parameters

m	INTEGER. The number of rows of the matrix *A* (*m* ≥ 0).
n	INTEGER. The number of columns in *A*(*n* ≥ 0).
kl	INTEGER. The number of sub-diagonals within the band of *A*(*kl* ≥ 0).
ku	INTEGER. The number of super-diagonals within the band of *A*(*ku* ≥ 0).
ab	REAL for sdbtrf DOUBLE PRECISION for ddbtrf COMPLEX for cdbtrf COMPLEX*16 for zdbtrf. Array, DIMENSION (*ldab*, *n*).

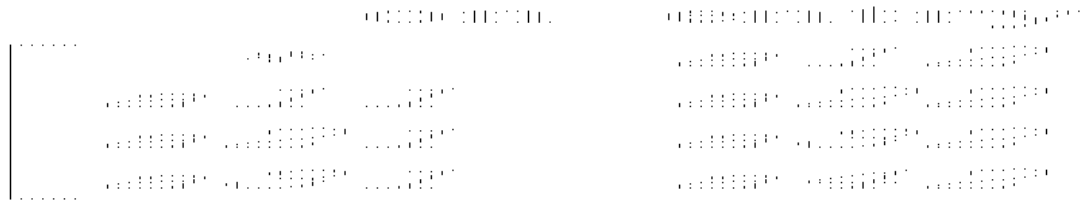
	The matrix *A* in band storage, in rows *kl*+1 to 2*kl*+*ku*+1; rows 1 to *kl* of the array need not be set. The *j*-th column of *A* is stored in the *j*-th column of the array *ab* as follows: *ab*(*kl*+*ku*+1+*i*–*j*,*j*) = *A*(*i*,*j*) for max(1,*j*–*ku*) ≤ *i* ≤ min(*m*,*j*+*kl*).
ldab	INTEGER. The leading dimension of the array *ab*. (*ldab* ≥ 2*kl* + *ku* +1)

Output Parameters

ab	On exit, details of the factorization: *U* is stored as an upper triangular band matrix with *kl*+*ku* superdiagonals in rows 1 to *kl*+*ku*+1, and the multipliers used during the factorization are stored in rows *kl*+*ku*+2 to 2**kl*+*ku*+1. See the *Application Notes* below for further details.
info	INTEGER. = 0: successful exit < 0: if *info* = – *i*, the *i*-th argument had an illegal value, > 0: if *info* = + *i*, *u*(*i*,*i*) is 0. The factorization has been completed, but the factor *U* is exactly singular. Division by 0 will occur if you use the factor *U* for solving a system of linear equations.

Application Notes

The band storage scheme is illustrated by the following example, when *m* = *n* = 6, *kl* = 2, *ku* = 1:



The routine does not use array elements marked *.

?dttrf

Computes an LU factorization of a general tridiagonal matrix with no pivoting (local blocked algorithm).

Syntax

```
call sdttrf(n, dl, d, du, info)
call ddttrf(n, dl, d, du, info)
call cdttrf(n, dl, d, du, info)
call zdttrf(n, dl, d, du, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?dttrf routine computes an *LU* factorization of a real or complex tridiagonal matrix *A* using elimination without partial pivoting.

The factorization has the form *A = L*U*, where *L* is a product of unit lower bidiagonal matrices and *U* is upper triangular with nonzeros only in the main diagonal and first superdiagonal.

Input Parameters

n	INTEGER. The order of the matrix *A*(*n* ≥ 0).
dl, *d*, *du*	REAL for sdttrf DOUBLE PRECISION for ddttrf COMPLEX for cdttrf COMPLEX*16 for zdttrf. Arrays containing elements of *A*. The array *dl* of DIMENSION(*n* - 1) contains the sub-diagonal elements of *A*. The array *d* of DIMENSION *n* contains the diagonal elements of *A*. The array *du* of DIMENSION(*n* - 1) contains the super-diagonal elements of *A*.

Output Parameters

dl	Overwritten by the (*n*-1) multipliers that define the matrix *L* from the *LU* factorization of *A*.
d	Overwritten by the *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*.
du	Overwritten by the (*n*-1) elements of the first super-diagonal of *U*.
info	INTEGER. = 0: successful exit < 0: if *info* = - *i*, the *i*-th argument had an illegal value, > 0: if *info* = *i*, *u*(*i*,*i*) is exactly 0. The factorization has been completed, but the factor *U* is exactly singular. Division by 0 will occur if you use the factor *U* for solving a system of linear equations.

?dttrsv

Solves a general tridiagonal system of linear equations using the LU factorization computed by ?dttrf.

Syntax

```
call sdttrsv(uplo, trans, n, nrhs, dl, d, du, b, ldb, info)
call ddttrsv(uplo, trans, n, nrhs, dl, d, du, b, ldb, info)
call cdttrsv(uplo, trans, n, nrhs, dl, d, du, b, ldb, info)
call zdttrsv(uplo, trans, n, nrhs, dl, d, du, b, ldb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?dttrsv routine solves one of the following systems of linear equations:

*L*X = B*, $L^T*X = B$, or $L^H*X = B$,

*U*X = B*, $U^T*X = B$, or $U^H*X = B$

with factors of the tridiagonal matrix *A* from the *LU* factorization computed by ?dttrf.

Input Parameters

uplo	CHARACTER*1. Specifies whether to solve with *L* or *U*.
trans	CHARACTER. Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If *trans* = 'N', then *A*X=B* is solved for *X* (no transpose). If *trans* = 'T', then $A^T*X = B$ is solved for *X* (transpose). If *trans* = 'C', then $A^H*X = B$ is solved for *X* (conjugate transpose).
n	INTEGER. The order of the matrix *A*($n \geq 0$).
nrhs	INTEGER. The number of right-hand sides, that is, the number of columns in the matrix *B*($nrhs \geq 0$).
dl,*d*,*du*,*b*	REAL for sdttrsv DOUBLE PRECISION for ddttrsv COMPLEX for cdttrsv COMPLEX*16 for zdttrsv. Arrays of DIMENSIONs: *dl*(*n* -1), *d*(*n*), *du*(*n* -1), *b*(*ldb*,*nrhs*). The array *dl* contains the (*n* – 1) multipliers that define the matrix *L* from the *LU* factorization of *A*. The array *d* contains *n* diagonal elements of the upper triangular matrix *U* from the *LU* factorization of *A*. The array *du* contains the (*n* – 1) elements of the first super-diagonal of *U*. On entry, the array *b* contains the right-hand side matrix *B*.
ldb	INTEGER. The leading dimension of the array *b*; $ldb \geq \max(1, n)$.

Output Parameters

b	Overwritten by the solution matrix *X*.
info	INTEGER. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

?pttrsv

*Solves a symmetric (Hermitian) positive-definite tridiagonal system of linear equations, using the L*D*L^H factorization computed by* ?pttrf.

Syntax

```
call spttrsv(trans, n, nrhs, d, e, b, ldb, info)
call dpttrsv(trans, n, nrhs, d, e, b, ldb, info)
call cpttrsv(uplo, trans, n, nrhs, d, e, b, ldb, info)
call zpttrsv(uplo, trans, n, nrhs, d, e, b, ldb, info)
```

Include Files

- C: mkl_scalapack.h

Description

The ?pttrsv routine solves one of the triangular systems:

$L^T*X = B$, or $L*X = B$ for real flavors,

or

$L*X = B$, or $L^H*X = B$,

$U*X = B$, or $U^H*X = B$ for complex flavors,

where L (or U for complex flavors) is the Cholesky factor of a Hermitian positive-definite tridiagonal matrix A such that

$A = L*D*L^H$ (computed by spttrf/dpttrf)

or

$A = U^H*D*U$ or $A = L*D*L^H$ (computed by cpttrf/zpttrf).

Input Parameters

uplo	CHARACTER*1. Must be 'U' or 'L'. Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix A is stored and the form of the factorization: If $uplo$ = 'U', e is the superdiagonal of U, and $A = U^H*D*U$ or $A = L*D*L^H$; if $uplo$ = 'L', e is the subdiagonal of L, and $A = L*D*L^H$. The two forms are equivalent, if A is real.
trans	CHARACTER. Specifies the form of the system of equations: for real flavors: if $trans$ = 'N': $L*X = B$ (no transpose) if $trans$ = 'T': $L^T*X = B$ (transpose) for complex flavors: if $trans$ = 'N': $U*X = B$ or $L*X = B$ (no transpose) if $trans$ = 'C': $U^H*X = B$ or $L^H*X = B$ (conjugate transpose).
n	INTEGER. The order of the tridiagonal matrix A. $n \geq 0$.
nrhs	INTEGER. The number of right hand sides, that is, the number of columns of the matrix B. $nrhs \geq 0$.
d	REAL array, DIMENSION (n). The n diagonal elements of the diagonal matrix D from the factorization computed by ?pttrf.
e	COMPLEX array, DIMENSION(n-1). The (n-1) off-diagonal elements of the unit bidiagonal factor U or L from the factorization computed by ?pttrf. See $uplo$.
b	COMPLEX array, DIMENSION (ldb, $nrhs$). On entry, the right hand side matrix B.
ldb	INTEGER. The leading dimension of the array b. $ldb \geq \max(1, n)$.

Output Parameters

b	On exit, the solution matrix X.
info	INTEGER. = 0: successful exit < 0: if $info = -i$, the i-th argument had an illegal value.

?steqr2

Computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method.

Syntax

```
call ssteqr2(compz, n, d, e, z, ldz, nr, work, info)
call dsteqr2(compz, n, d, e, z, ldz, nr, work, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

The `?steqr2` routine is a modified version of LAPACK routine `?steqr`. The `?steqr2` routine computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method. `?steqr2` is modified from `?steqr` to allow each ScaLAPACK process running `?steqr2` to perform updates on a distributed matrix Q. Proper usage of `?steqr2` can be gleaned from examination of ScaLAPACK routine `p?syev`.

Input Parameters

compz	CHARACTER*1. Must be 'N' or 'I'. If *compz* = 'N', the routine computes eigenvalues only. If *compz* = 'I', the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix *T*. *z* must be initialized to the identity matrix by `p?laset` or `?laset` prior to entering this subroutine.
n	INTEGER. The order of the matrix T ($n \geq 0$).
d, *e*, *work*	REAL for single-precision flavors DOUBLE PRECISION for double-precision flavors. Arrays: *d* contains the diagonal elements of *T*. The dimension of *d* must be at least max(1, *n*). *e* contains the (*n*-1) subdiagonal elements of *T*. The dimension of *e* must be at least max(1, *n*-1). *work* is a workspace array. The dimension of *work* is max(1, 2**n*-2). If *compz* = 'N', then *work* is not referenced.
z	(local) REAL for ssteqr2 DOUBLE PRECISION for dsteqr2 Array, global DIMENSION (*n*, *n*), local DIMENSION (*ldz*, *nr*). If *compz* = 'V', then *z* contains the orthogonal matrix used in the reduction to tridiagonal form.
ldz	INTEGER. The leading dimension of the array *z*. Constraints: *ldz* $\geq$ 1, *ldz* $\geq$ max(1, *n*), if eigenvectors are desired.
nr	INTEGER. *nr* = max(1, *numroc*(*n*, *nb*, *myprow*, 0, *nprocs*)). If *compz* = 'N', then *nr* is not referenced.

Output Parameters

d	REAL array, DIMENSION (*n*), for ssteqr2. DOUBLE PRECISION array, DIMENSION (*n*), for dsteqr2. On exit, the eigenvalues in ascending order, if *info* = 0. See also *info*.
e	REAL array, DIMENSION (*n*-1), for ssteqr2. DOUBLE PRECISION array, DIMENSION (*n*-1), for dsteqr2. On exit, *e* has been destroyed.
z	(local) REAL for ssteqr2 DOUBLE PRECISION for dsteqr2 Array, global DIMENSION (*n*, *n*), local DIMENSION (*ldz*, *nr*). On exit, if *info* = 0, then, if *compz* = 'V', *z* contains the orthonormal eigenvectors of the original symmetric matrix, and if *compz* = 'I', *z* contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If *compz* = 'N', then *z* is not referenced.
info	INTEGER. *info* = 0, the exit is successful. *info* < 0: if *info* = -*i*, the *i*-th had an illegal value. *info* > 0: the algorithm has failed to find all the eigenvalues in a total of 30*n* iterations; if *info* = *i*, then *i* elements of *e* have not converged to zero; on exit, *d* and *e* contain the elements of a symmetric tridiagonal matrix, which is orthogonally similar to the original matrix.

Utility Functions and Routines

This section describes ScaLAPACK utility functions and routines. Summary information about these routines is given in the following table:

ScaLAPACK Utility Functions and Routines

Routine Name	Data Types	Description
p?labad	s,d	Returns the square root of the underflow and overflow thresholds if the exponent-range is very large.
p?lachkieee	s,d	Performs a simple check for the features of the IEEE standard. (C interface function).
p?lamch	s,d	Determines machine parameters for floating-point arithmetic.
p?lasnbt	s,d	Computes the position of the sign bit of a floating-point number. (C interface function).
pxerbla		Error handling routine called by ScaLAPACK routines.

p?labad

Returns the square root of the underflow and overflow thresholds if the exponent-range is very large.

Syntax

```
call pslabad(ictxt, small, large)
```

```
call pdlabad(ictxt, small, large)
```

Include Files

- C: mkl_scalapack.h

Description

The p?labad routine takes as input the values computed by p?lamch for underflow and overflow, and returns the square root of each of these values if the log of *large* is sufficiently large. This subroutine is intended to identify machines with a large exponent range, such as the Crays, and redefine the underflow and overflow limits to be the square roots of the values computed by p?lamch. This subroutine is needed because p?lamch does not compensate for poor arithmetic in the upper half of the exponent range, as is found on a Cray.

In addition, this routine performs a global minimization and maximization on these values, to support heterogeneous computing networks.

Input Parameters

ictxt	(global) INTEGER. The BLACS context handle in which the computation takes place.
small	(local). REAL PRECISION for pslabad. DOUBLE PRECISION for pdlabad. On entry, the underflow threshold as computed by p?lamch.
large	(local). REAL PRECISION for pslabad. DOUBLE PRECISION for pdlabad. On entry, the overflow threshold as computed by p?lamch.

Output Parameters

small	(local). On exit, if *log10(large)* is sufficiently large, the square root of *small*, otherwise unchanged.
large	(local). On exit, if *log10(large)* is sufficiently large, the square root of *large*, otherwise unchanged.

p?lachkieee

Performs a simple check for the features of the IEEE standard. (C interface function).

Syntax

```
void pslachkieee(int *isieee, float *rmax, float *rmin);
```

```
void pdlachkieee(int *isieee, float *rmax, float *rmin);
```

Include Files

- C: mkl_scalapack.h

Description

The p?lachkieee routine performs a simple check to make sure that the features of the IEEE standard are implemented. In some implementations, p?lachkieee may not return.

Note that all arguments are call-by-reference so that this routine can be directly called from Fortran code.

This is a ScaLAPACK internal subroutine and arguments are not checked for unreasonable values.

Input Parameters

rmax (local).
REAL for pslachkieee
DOUBLE PRECISION for pdlachkieee
The overflow threshold (= ?lamch ('O')).

rmin (local).
REAL for pslachkieee
DOUBLE PRECISION for pdlachkieee
The underflow threshold (= ?lamch ('U')).

Output Parameters

isieee (local). INTEGER.
On exit, *isieee* = 1 implies that all the features of the IEEE standard that we rely on are implemented. On exit, *isieee* = 0 implies that some the features of the IEEE standard that we rely on are missing.

p?lamch

Determines machine parameters for floating-point arithmetic.

Syntax

val = pslamch(*ictxt*, *cmach*)

val = pdlamch(*ictxt*, *cmach*)

Include Files

- C: mkl_scalapack.h

Description

The p?lamch routine determines single precision machine parameters.

Input Parameters

ictxt (global). INTEGER.The BLACS context handle in which the computation takes place.

cmach (global) CHARACTER*1.
Specifies the value to be returned by p?lamch:
= 'E' or 'e', p?lamch := eps
= 'S' or 's' , p?lamch := sfmin
= 'B' or 'b', p?lamch := base
= 'P' or 'p', p?lamch := eps*base
= 'N' or 'n', p?lamch := t
= 'R' or 'r', p?lamch := rnd
= 'M' or 'm', p?lamch := emin
= 'U' or 'u', p?lamch := rmin
= 'L' or 'l', p?lamch := emax
= 'O' or 'o', p?lamch := rmax,
where

`eps` = relative machine precision
`sfmin` = safe minimum, such that 1/sfmin does not overflow
`base` = base of the machine
`prec` = `eps*base`
`t` = number of (base) digits in the mantissa
`rnd` = 1.0 when rounding occurs in addition, 0.0 otherwise
`emin` = minimum exponent before (gradual) underflow
`rmin` = underflow threshold - $base^{(emin-1)}$
`emax` = largest exponent before overflow
`rmax` = overflow threshold - $(base^{emax})*(1-eps)$

Output Parameters

val Value returned by the routine.

p?lasnbt

Computes the position of the sign bit of a floating-point number. (C interface function).

Syntax

```
void pslasnbt(int *ieflag);
void pdlasnbt(int *ieflag);
```

Include Files

- C: `mkl_scalapack.h`

Description

The `p?lasnbt` routine finds the position of the signbit of a single/double precision floating point number. This routine assumes IEEE arithmetic, and hence, tests only the 32-nd bit (for single precision) or 32-nd and 64-th bits (for double precision) as a possibility for the signbit. `sizeof(int)` is assumed equal to 4 bytes.

If a compile time flag (`NO_IEEE`) indicates that the machine does not have IEEE arithmetic, *ieflag* = `0` is returned.

Output Parameters

ieflag `INTEGER`.
This flag indicates the position of the signbit of any single/double precision floating point number.
ieflag = `0`, if the compile time flag `NO_IEEE` indicates that the machine does not have IEEE arithmetic, or if `sizeof(int)` is different from 4 bytes.
ieflag = `1` indicates that the signbit is the 32-nd bit for a single precision routine.
In the case of a double precision routine:
ieflag = `1` indicates that the signbit is the 32-nd bit (Big Endian).
ieflag = `2` indicates that the signbit is the 64-th bit (Little Endian).

pxerbla

Error handling routine called by ScaLAPACK routines.

Syntax

```
call pxerbla(ictxt, srname, info)
```

Include Files

- C: `mkl_scalapack.h`

Description

This routine is an error handler for the *ScaLAPACK* routines. It is called by a *ScaLAPACK* routine if an input parameter has an invalid value. A message is printed. Program execution is not terminated. For the ScaLAPACK driver and computational routines, a `RETURN` statement is issued following the call to `pxerbla`.

Control returns to the higher-level calling routine, and it is left to the user to determine how the program should proceed. However, in the specialized low-level ScaLAPACK routines (auxiliary routines that are Level 2 equivalents of computational routines), the call to `pxerbla()` is immediately followed by a call to `BLACS_ABORT()` to terminate program execution since recovery from an error at this level in the computation is not possible.

It is always good practice to check for a nonzero value of *`info`* on return from a ScaLAPACK routine. Installers may consider modifying this routine in order to call system-specific exception-handling facilities.

Input Parameters

`ictxt`	(global) `INTEGER` The BLACS context handle, indicating the global context of the operation. The context itself is global.
`srname`	(global) `CHARACTER*6` The name of the routine which called `pxerbla`.
`info`	(global) `INTEGER`. The position of the invalid parameter in the parameter list of the calling routine.

Sparse Solver Routines

8

Intel® Math Kernel Library (Intel® MKL) provides user-callable sparse solver software to solve real or complex, symmetric, structurally symmetric or non-symmetric, positive definite, indefinite or Hermitian sparse linear system of equations.

The terms and concepts required to understand the use of the Intel MKL sparse solver routines are discussed in the Appendix A "Linear Solvers Basics". If you are familiar with linear sparse solvers and sparse matrix storage schemes, you can skip these sections and go directly to the interface descriptions.

This chapter describes the direct sparse solver PARDISO* and the alternative interface for the direct sparse solver referred to here as DSS interface; iterative sparse solvers (ISS) based on the reverse communication interface (RCI); and two preconditioners based on the incomplete LU factorization technique.

PARDISO* - Parallel Direct Sparse Solver Interface

This section describes the interface to the shared-memory multiprocessing parallel direct sparse solver known as the PARDISO* solver. The interface is Fortran, but it can be called from C programs by observing Fortran parameter passing and naming conventions used by the supported compilers and operating systems. A discussion of the algorithms used in the PARDISO* software and more information on the solver can be found at http://www.pardiso-project.org.

The current implementation of the PARDISO solver additionally supports the out-of-core (OOC) version.

The PARDISO package is a high-performance, robust, memory efficient, and easy to use software package for solving large sparse symmetric and unsymmetric linear systems of equations on shared memory multiprocessors. The solver uses a combination of left- and right-looking Level-3 BLAS supernode techniques [Schenk00-2]. To improve sequential and parallel sparse numerical factorization performance, the algorithms are based on a Level-3 BLAS update and pipelining parallelism is used with a combination of left- and right-looking supernode techniques [Schenk00, Schenk01, Schenk02, Schenk03]. The parallel pivoting methods allow complete supernode pivoting to compromise numerical stability and scalability during the factorization process. For sufficiently large problem sizes, numerical experiments demonstrate that the scalability of the parallel algorithm is nearly independent of the shared-memory multiprocessing architecture.

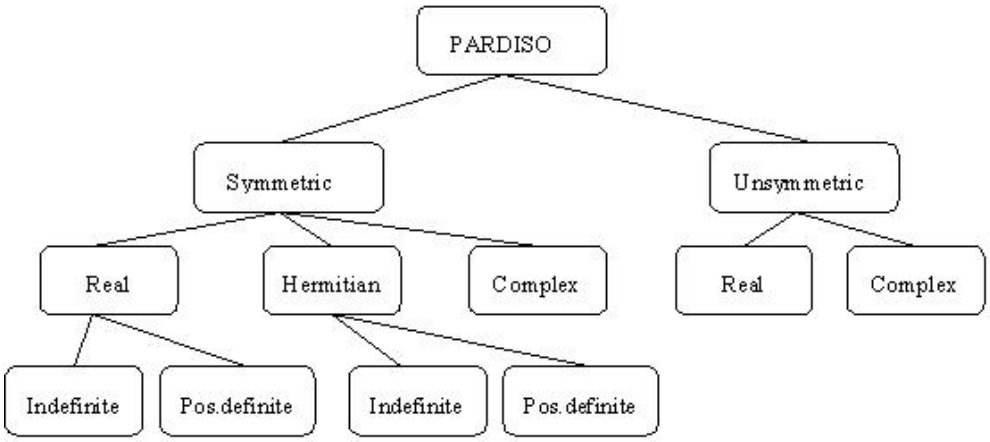
The following table lists the names of the PARDISO routines and describes their general use.

PARDISO Routines

Routine	Description
pardiso	Calculates the solution of a set of sparse linear equations with multiple right-hand sides.
pardisoinit	Initialize PARDISO with default parameters depending on the matrix type.
pardiso_64	Calculates the solution of a set of sparse linear equations with multiple right-hand sides, 64-bit integer version.
pardiso_getenv pardiso_setenv	Retrieves additional values from the PARDISO handle.
pardiso_getenv pardiso_setenv	Sets additional values in the PARDISO handle.

The PARDISO solver supports a wide range of sparse matrix types (see the figure below) and computes the solution of real or complex sparse linear system of equations on shared-memory multiprocessing architectures.

Sparse Matrices That Can Be Solved with the PARDISO* Solver



The PARDISO solver performs four tasks:

- analysis and symbolic factorization
- numerical factorization
- forward and backward substitution including iterative refinement
- termination to release all internal solver memory.

You can find a code example that uses the PARDISO interface routine to solve systems of linear equations in the `examples\solver\source` folder of your Intel MKL directory.

pardiso

Calculates the solution of a set of sparse linear equations with multiple right-hand sides.

Syntax

Fortran:

```
call pardiso (pt, maxfct, mnum, mtype, phase, n, a, ia, ja, perm, nrhs, iparm, msglvl, b, x, error)
```

C:

```
pardiso (pt, &maxfct, &mnum, &mtype, &phase, &n, a, ia, ja, perm, &nrhs, iparm, &msglvl, b, x, &error);
```

Include Files

- FORTRAN 77: `mkl_pardiso.f77`
- Fortran 90: `mkl_pardiso.f90`
- C: `mkl_pardiso.h`

Description

The routine `pardiso` calculates the solution of a set of sparse linear equations

```
A*X = B
```

with multiple right-hand sides, using a parallel LU, LDL or LL^T factorization, where A is an n-by-n matrix, and X and B are n-by-$nrhs$ matrices.

Supported Matrix Types.

The analysis steps performed by `pardiso` depends on the structure of the input matrix A.

Symmetric Matrices: The solver first computes a symmetric fill-in reducing permutation P based on either the minimum degree algorithm [Liu85] or the nested dissection algorithm from the METIS package [Karypis98] (both included with Intel MKL), followed by the parallel left-right looking numerical Cholesky factorization [Schenk00-2] of $PAP^T = LL^T$ for symmetric positive-definite matrices, or $PAP^T = LDL^T$ for symmetric indefinite matrices. The solver uses diagonal pivoting, or 1x1 and 2x2 Bunch and Kaufman pivoting for symmetric indefinite matrices, and an approximation of X is found by forward and backward substitution and iterative refinements.

Whenever numerically acceptable 1x1 and 2x2 pivots cannot be found within the diagonal supernode block, the coefficient matrix is perturbed. One or two passes of iterative refinements may be required to correct the effect of the perturbations. This restricting notion of pivoting with iterative refinements is effective for highly indefinite symmetric systems. Furthermore, for a large set of matrices from different applications areas, this method is as accurate as a direct factorization method that uses complete sparse pivoting techniques [Schenk04].

Another method of improving the pivoting accuracy is to use symmetric weighted matching algorithms. These algorithms identify large entries in the coefficient matrix A that, if permuted close to the diagonal, permit the factorization process to identify more acceptable pivots and proceed with fewer pivot perturbations. These algorithms are based on maximum weighted matchings and improve the quality of the factor in a complementary way to the alternative idea of using more complete pivoting techniques.

The inertia is also computed for real symmetric indefinite matrices.

Structurally Symmetric Matrices: The solver first computes a symmetric fill-in reducing permutation P followed by the parallel numerical factorization of $PAP^T = QLU^T$. The solver uses partial pivoting in the supernodes and an approximation of X is found by forward and backward substitution and iterative refinements.

Unsymmetric Matrices: The solver first computes a non-symmetric permutation P_{MPS} and scaling matrices D_r and D_c with the aim of placing large entries on the diagonal to enhance reliability of the numerical factorization process [Duff99]. In the next step the solver computes a fill-in reducing permutation P based on the matrix $P_{MPS}A + (P_{MPS}A)_T$ followed by the parallel numerical factorization

$$QLUR = PP_{MPS}D_rAD_cP$$

with supernode pivoting matrices Q and R. When the factorization algorithm reaches a point where it cannot factor the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [Li99]. The magnitude of the potential pivot is tested against a constant threshold of $\texttt{alpha} = \texttt{eps}*||A2||_{\texttt{inf}}$, where $\texttt{eps}$ is the machine precision, $A2 = P*P_{MPS}*D_r*A*D_c*P$, and $||A2||_{\texttt{inf}}$ is the infinity norm of the scaled and permuted matrix A. Any tiny pivots encountered during elimination are set to the sign $(l_{II})*\texttt{eps}*||A2||_{\texttt{inf}}$, which trades off some numerical stability for the ability to keep pivots from getting too small. Although many failures could render the factorization well-defined but essentially useless, in practice the diagonal elements are rarely modified for a large class of matrices. The result of this pivoting approach is that the factorization is, in general, not exact and iterative refinement may be needed.

Direct-Iterative Preconditioning for Unsymmetric Linear Systems.

The solver enables to use a combination of direct and iterative methods [Sonn89] to accelerate the linear solution process for transient simulation. Most of applications of sparse solvers require solutions of systems with gradually changing values of the nonzero coefficient matrix, but the same identical sparsity pattern. In these applications, the analysis phase of the solvers has to be performed only once and the numerical factorizations are the important time-consuming steps during the simulation. PARDISO uses a numerical factorization $A = LU$ for the first system and applies the factors L and U for the next steps in a preconditioned Krylow-Subspace iteration. If the iteration does not converge, the solver automatically switches back to the numerical factorization. This method can be applied to unsymmetric matrices in PARDISO. You can select the method using only one input parameter. For further details see the parameter description ($iparm(4)$, $iparm(20)$).

Single and Double Precision Computations.

PARDISO solves tasks using single or double precision. Each precision has its benefits and drawbacks. Double precision variables have more digits to store value, so the solver uses more memory for keeping data. But this mode solves matrices with better accuracy, and input matrices can have large condition numbers.

Single precision variables have fewer digits to store values, so the solver uses less memory than in the double precision mode. Additionally this mode usually takes less time. But as computations are made more roughly, only numerically stable process can use single precision.

Separate Forward and Backward Substitution.

The solver execution step (see parameter `phase = 33` below) can be divided into two or three separate substitutions: forward, backward, and possible diagonal . This separation can be explained by the examples of solving systems with different matrix types.

A real symmetric positive definite matrix `A` (`mtype = 2`) is factored by PARDISO as $A = L*L^T$. In this case the solution of the system $A*x=b$ can be found as sequence of substitutions: $L*y=b$ (forward substitution, `phase =331`) and $L^T*x=y$ (backward substitution, `phase =333`).

A real unsymmetric matrix `A` (`mtype = 11`) is factored by PARDISO as $A = L*U$. In this case the solution of the system $A*x=b$ can be found by the following sequence: $L*y=b$ (forward substitution, `phase =331`) and $U*x=y$ (backward substitution, `phase =333`).

Solving a system with a real symmetric indefinite matrix `A` (`mtype = -2`) is slightly different from the cases above. PARDISO factors this matrix as $A=LDL^T$, and the solution of the system $A*x=b$ can be calculated as the following sequence of substitutions: $L*y=b$ (forward substitution, `phase =331`) s: $D*v=y$ (diagonal substitution, `phase =332`) and, finally $L^T*x=v$ (backward substitution, `phase =333`). Diagonal substitution makes sense only for indefinite matrices (`mtype = -2, -4, 6`). For matrices of other types a solution can be found as described in the first two examples.

NOTE The number of refinement steps (`iparm(8)`) must be set to zero if a solution is calculated with separate substitutions (`phase = 331, 332, 333`), otherwise PARDISO produces the wrong result.

NOTE Different pivoting (`iparm(21)`) produces different LDL^T factorization. Therefore results of forward, diagonal and backward substitutions with diagonal pivoting can differ from results of the same steps with Bunch and Kaufman pivoting. Of course, the final results of sequential execution of forward, diagonal and backward substitution are equal to the results of the full solving step (`phase=33`) regardless of the pivoting used.

Sparse Data Storage.

Sparse data storage in PARDISO follows the scheme described in Sparse Matrix Storage Format with `ja` standing for `columns`, `ia` for `rowIndex`, and `a` for `values`. The algorithms in PARDISO require column indices `ja` to be in increasing order per row and that the diagonal element in each row be present for any structurally symmetric matrix. For symmetric or unsymmetric matrices the diagonal elements are not necessary: they may be present or not.

NOTE The presence of diagonal elements for symmetric matrices is not mandatory starting from the Intel MKL 10.3 beta release.

CAUTION It's recommended to set explicitly zero diagonal elements for symmetric matrices because in the opposite case PARDISO creates internal copies of arrays `ia`, `ja` and `a` full of diagonal elements that requires additional memory and computational time. However, in general, memory and time overheads are not significant comparing to the memory and the time needed to factor and solve the matrix.

Input Parameters

NOTE Parameters types in this section are specified in FORTRAN 77 notation. See PARDISO Parameters in Tabular Form for detailed description of types of PARDISO parameters in C/Fortran 90 notations.

pt

INTEGER
Array, DIMENSION (64)
Pointer to the address of solver internal data. These addresses are passed to the solver and all related internal memory management is organized through this pointer.

NOTE *pt* is an integer array with 64 entries. It is very important that the pointer is initialized with zero at the first call of pardiso. After that first do not modify the pointer, as a serious memory leak can occur. The integer length must be 4 bytes on 32-bit operating systems and 8 bytes on 64-bit operating systems.

maxfct

INTEGER
Maximum number of factors with identical nonzero sparsity structure that must be keep at the same time in memory. In most applications this value is equal to 1. It is possible to store several different factorizations with the same nonzero structure at the same time in the internal data management of the solver.
pardiso can process several matrices with an identical matrix sparsity pattern and it can store the factors of these matrices at the same time. Matrices with a different sparsity structure can be kept in memory with different memory address pointers *pt*.

mnum

INTEGER
Indicates the actual matrix for the solution phase. With this scalar you can define which matrix to factorize. The value must be: $1 \leq mnum \leq maxfct$. In most applications this value is 1.

mtype

INTEGER
Defines the matrix type, which influences the pivoting method. The PARDISO solver supports the following matrices:

1	real and structurally symmetric
2	real and symmetric positive definite
-2	real and symmetric indefinite
3	complex and structurally symmetric
4	complex and Hermitian positive definite
-4	complex and Hermitian indefinite
6	complex and symmetric
11	real and unsymmetric
13	complex and unsymmetric

phase

INTEGER
Controls the execution of the solver. Usually it is a two- or three-digit integer ij ($10i + j$, $1 \leq i \leq 3$, $i \leq j \leq 3$ for normal execution modes). The i digit indicates the starting phase of execution, j indicates the ending phase. PARDISO has the following phases of execution:

- Phase 1: Fill-reduction analysis and symbolic factorization
- Phase 2: Numerical factorization
- Phase 3: Forward and Backward solve including iterative refinements

  This phase can be divided into two or three separate substitutions: forward, backward, and diagonal (see above).
- Termination and Memory Release Phase (*phase*≤ 0)

If a previous call to the routine has computed information from previous phases, execution may start at any phase. The *phase* parameter can have the following values:

phase	**Solver Execution Steps**
11	Analysis
12	Analysis, numerical factorization
13	Analysis, numerical factorization, solve, iterative refinement
22	Numerical factorization
23	Numerical factorization, solve, iterative refinement
33	Solve, iterative refinement
331	like *phase*=33, but only forward substitution
332	like *phase*=33, but only diagonal substitution
333	like *phase*=33, but only backward substitution
0	Release internal memory for *L* and *U* matrix number *mnum*
-1	Release all internal memory for all matrices

n INTEGER
Number of equations in the sparse linear systems of equations *A*X = B*. Constraint: *n* > 0.

a DOUBLE PRECISION - for real types of matrices (*mtype*=1, 2, -2 and 11) and for double precision PARDISO (*iparm*(28)=0)
REAL - for real types of matrices (*mtype*=1, 2, -2 and 11) and for single precision PARDISO (*iparm*(28)=1)
DOUBLE COMPLEX - for complex types of matrices (*mtype*=3, 6, 13, 14 and -4) and for double precision PARDISO (*iparm*(28)=0)
COMPLEX - for complex types of matrices (*mtype*=3, 6, 13, 14 and -4) and for single precision PARDISO (*iparm*(28)=1)
Array. Contains the non-zero elements of the coefficient matrix *A* corresponding to the indices in *ja*. The size of *a* is the same as that of *ja* and the coefficient matrix can be either real or complex. The matrix must be stored in compressed sparse row format with increasing values of *ja* for each row. Refer to *values* array description in Storage Formats for the Direct Sparse Solvers for more details.

NOTE The non-zero elements of each row of the matrix *A* must be stored in increasing order. For symmetric or structural symmetric matrices, it is also important that the diagonal elements are available and stored in the matrix. If the matrix is symmetric, the array *a* is only accessed in the factorization phase, in the triangular solution and iterative refinement phase. Unsymmetric matrices are accessed in all phases of the solution process.

ia INTEGER

Array, dimension (*n*+1). For *i*≤*n*, *ia*(*i*) points to the first column index of row *i* in the array *ja* in compressed sparse row format. That is, *ia*(*I*) gives the index of the element in array *a* that contains the first non-zero element from row *i* of *A*. The last element *ia*(*n*+1) is taken to be equal to the number of non-zero elements in *A*, plus one. Refer to *rowIndex* array description in Storage Formats for the Direct Sparse Solvers for more details. The array *ia* is also accessed in all phases of the solution process. Indexing of *ia* is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter *iparm*(35).

ja INTEGER

Array *ja*(*) contains column indices of the sparse matrix *A* stored in compressed sparse row format. The indices in each row must be sorted in increasing order. The array *ja* is also accessed in all phases of the solution process. For structurally symmetric matrices it is assumed that diagonal elements, which are zero, are also stored in the list of non-zero elements in *a* and *ja*. For symmetric matrices, the solver needs only the upper triangular part of the system as is shown for *columns* array in Storage Formats for the Direct Sparse Solvers.
Indexing of *ja* is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter *iparm*(35).

perm INTEGER

Array, dimension (*n*). Holds the permutation vector of size *n*. You can use it to apply your own fill-in reducing ordering to the solver. The array *perm* is defined as follows. Let *A* be the original matrix and $B = P*A*P^T$ be the permuted matrix. Row (column) i of *A* is the *perm*(i) row (column) of *B*. The permutation vector *perm* is used by the solver if *iparm*(5) = 1.
The array *perm* is also used to return the permutation vector calculated during fill-in reducing ordering stage. The permutation vector is returned into the *perm* array if *iparm*(5) = 2.
Indexing of *perm* is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter *iparm*(35).

NOTE The first elements of row, column and permutation are numbered as array elements 1 by default (Fortran style, or one based array indexing), but these first elements can be numbered as array elements 0 (C style, or zero based array indexing) by setting the appropriate value to the parameter *iparm*(35).

nrhs INTEGER

Number of right-hand sides that need to be solved for.

iparm INTEGER

Array, dimension (64). This array is used to pass various parameters to PARDISO and to return some useful information after execution of the solver. If *iparm*(1) = 0, PARDISO uses default values for *iparm*(2) through *iparm*(64).
The individual components of the *iparm* array are described below (some of them are described in the Output Parameters section).

iparm(1)**- use default values.**
If *iparm*(1) = 0, *iparm*(2) through *iparm*(64) are filled with default values, otherwise you must set all values in *iparm* from *iparm*(2) to *iparm*(64).

iparm(2) **- fill-in reducing ordering.**
iparm(2) controls the fill-in reducing ordering for the input matrix.
If *iparm*(2) = 0, the minimum degree algorithm is applied [Li99].
If *iparm*(2) = 2, the solver uses the nested dissection algorithm from the METIS package [Karypis98].
If *iparm*(2) = 3, the parallel (OpenMP) version of the nested dissection algorithm is used. It can decrease the time of computations on multi-core computers, especially when PARDISO Phase 1 takes significant time.
The default value of *iparm*(2) is 2.

CAUTION You can control the parallel execution of the solver by explicitly setting the environment variable MKL_NUM_THREADS. If fewer processors are available than specified, the execution may slow down instead of speeding up. If the variable MKL_NUM_THREADS is not defined, then the solver uses all available processors.

iparm(3)**- currently is not used.**

iparm(4) **- preconditioned CGS.**
This parameter controls preconditioned CGS [Sonn89] for unsymmetric or structurally symmetric matrices and Conjugate-Gradients for symmetric matrices. *iparm*(4) has the form *iparm*(4) = 10*L*+*K*.
The *K* and *L* values have the meanings as follows.

Value of *K*	Description
0	The factorization is always computed as required by *phase*.
1	CGS iteration replaces the computation of *LU*. The preconditioner is *LU* that was computed at a previous step (the first step or last step with a failure) in a sequence of solutions needed for identical sparsity patterns.
2	CGS iteration for symmetric matrices replaces the computation of *LU*. The preconditioner is *LU* that was computed at a previous step (the first step or last step with a failure) in a sequence of solutions needed for identical sparsity patterns.

Value *L*:
The value *L* controls the stopping criterion of the Krylow-Subspace iteration:

$eps_{CGS} = 10^{-L}$ is used in the stopping criterion
$||dx_i|| / ||dx_i|| < eps_{CGS}$
with $||dx_i|| = ||inv(L*U)*r_i||$ and r_i is the residue at iteration i of the preconditioned Krylow-Subspace iteration.
Strategy: A maximum number of 150 iterations is fixed with the assumption that the iteration will converge before consuming half the factorization time. Intermediate convergence rates and residue excursions are checked and can terminate the iteration process. If *phase* =23, then the factorization for a given *A* is automatically recomputed in cases where the Krylow-Subspace iteration failed, and the corresponding direct solution is returned. Otherwise the solution from the preconditioned Krylow-Subspace iteration is returned. Using *phase* =33 results in an error message (*error* =-4) if the stopping criteria for the Krylow-Subspace iteration can not be reached. More information on the failure can be obtained from *iparm*(20).
The default is *iparm*(4)=0, and other values are only recommended for an advanced user. *iparm*(4) must be greater or equal to zero.
Examples:

iparm(4)	**Description**
31	*LU*-preconditioned CGS iteration with a stopping criterion of 1.0E-3 for unsymmetric matrices
61	*LU*-preconditioned CGS iteration with a stopping criterion of 1.0E-6 for unsymmetric matrices
62	*LU*-preconditioned CGS iteration with a stopping criterion of 1.0E-6 for symmetric matrices

iparm(5)**- user permutation.**
This parameter controls whether user supplied fill-in reducing permutation is used instead of the integrated multiple-minimum degree or nested dissection algorithms. Another possible use of this parameter is to control obtaining the fill-in reducing permutation vector calculated during the reordering stage of PARDISO.
This option is useful for testing reordering algorithms, adapting the code to special applications problems (for instance, to move zero diagonal elements to the end of $P*A*P^T$), or for using the permutation vector more than once for equal or similar matrices. For definition of the permutation, see the description of the *perm* parameter.
If *parm*(5)=0 (default value), then the array *perm* is not used by PARDISO;
if *parm*(5)=1, then the user supplied fill-in reducing permutation in the array *perm* is used;
if *parm*(5)=2, then PARDISO returns the permutation vector into the array *perm.*

iparm(6)**- write solution on** *x***.**
If *iparm*(6) = 0 (default value), then the array *x* contains the solution and the value of *b* is not changed.
If *iparm*(6) = 1, then the solver stores the solution in the right-hand side *b*.
Note that the array *x* is always used. The default value of *iparm*(6) is 0.

iparm(8) **- iterative refinement step.**
On entry to the solve and iterative refinement step, *iparm*(8) must be set to the maximum number of iterative refinement steps that the solver performs. The solver does not perform more than the absolute value of

iparm(8) steps of iterative refinement and stops the process if a satisfactory level of accuracy of the solution in terms of backward error is achieved.
If *iparm*(8) < 0, the accumulation of the residue uses extended precision real and complex data types. Perturbed pivots result in iterative refinement (independent of *iparm*(8)=0) and the number of executed iterations is reported in *iparm*(7).
The solver automatically performs two steps of iterative refinements when perturbed pivots are obtained during the numerical factorization and *iparm*(8) = 0.
The number of performed iterative refinement steps is reported in *iparm*(7).
The default value for *iparm*(8) is 0.

iparm(9)
This parameter is reserved for future use. Its value must be set to 0.

iparm(10)**- pivoting perturbation.**
This parameter instructs PARDISO how to handle small pivots or zero pivots for unsymmetric matrices (*mtype* =11 or *mtype* =13) and symmetric matrices (*mtype* =-2, *mtype* =-4, or *mtype* =6). For these matrices the solver uses a complete supernode pivoting approach. When the factorization algorithm reaches a point where it cannot factor the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [Li99], [Schenk04].
The magnitude of the potential pivot is tested against a constant threshold of
$alpha = eps*||A2||_{inf}$,
where $eps = 10^{(-iparm(10))}$, $A2 = P*P_{MPS}*D_r*A*D_c*P$, and $||A2||_{inf}$ is the infinity norm of the scaled and permuted matrix *A*. Any tiny pivots encountered during elimination are set to the sign $(l_{II})*eps*||A2||_{inf}$ - this trades off some numerical stability for the ability to keep pivots from getting too small. Small pivots are therefore perturbed with $eps = 10^{(-iparm(10))}$.
For unsymmetric matrices (*mtype* =11 or *mtype* =13) the default value of *iparm*(10) is 13 and therefore eps = 1.0E-13.
For symmetric indefinite matrices (*mtype* =-2, *mtype* =-4, or *mtype* =6) the default value of *iparm*(10) is 8, and therefore eps = 1.0E-8.

iparm(11)**- scaling vectors.**
PARDISO uses a maximum weight matching algorithm to permute large elements on the diagonal and to scale the matrix so that the diagonal elements are equal to 1 and the absolute values of the off-diagonal entries are less or equal to 1. This scaling method is applied only to unsymmetric matrices (*mtype* =11 or *mtype* =13). The scaling can also be used for symmetric indefinite matrices (*mtype* =-2, *mtype* =-4, or *mtype* =6) when the symmetric weighted matchings are applied (*iparm*(13)= 1).
Use *iparm*(11) = 1 (scaling) and *iparm*(13) = 1 (matching) for highly indefinite symmetric matrices, for example, from interior point optimizations or saddle point problems. Note that in the analysis phase (*phase*=11) you must provide the numerical values of the matrix *A* in case of scaling and symmetric weighted matching.

The default value of *iparm*(11) is 1 for unsymmetric matrices (*mtype* =11 or *mtype* =13). The default value of *iparm*(11) is 0 for symmetric indefinite matrices (*mtype* =-2, *mtype* =-4, or *mtype* =6).

iparm(12) **- solving with transposed or conjugate transposed matrix.**
If *iparm*(12)= 0, PARDISO solves a linear system $Ax = b$ (default value).
If *iparm*(12)= 1, PARDISO solves a conjugate transposed system $A^H x = b$ based on the factorization of the matrix A.
If *iparm*(12)= 2, PARDISO solves a transposed system $A^T x = b$ based on the factorization of the matrix A.

NOTE For real matrices the terms *conjugate transposed* and *transposed* are equivalent.

iparm(13) **- improved accuracy using (non-)symmetric weighted matchings.**
PARDISO can use a maximum weighted matching algorithm to permute large elements close the diagonal. This strategy adds an additional level of reliability to our factorization methods and can be seen as a complement to the alternative idea of using more complete pivoting techniques during the numerical factorization.
Use *iparm*(11)=1 (scalings) and *iparm*(13)=1 (matchings) for highly indefinite symmetric matrices, for example from interior point optimizations or saddle point problems. Note that in the analysis phase (*phase* =11) you must provide the numerical values of the matrix A in the case of scalings and symmetric weighted matchings.
The default value of *iparm*(13) is 1 for unsymmetric matrices (*mtype* =11 or *mtype* =13). The default value of *iparm*(13) is 0 for symmetric matrices (*mtype* =-2, *mtype* =-4, or *mtype* =6).

iparm(18) **- numbers of non-zero elements in the factors.**
If *iparm*(18)< 0 on entry, the solver reports the numbers of non-zero elements in the factors.
The default value of *iparm*(18)is -1.

iparm(19)**- MFLOPS of factorization.**
If *iparm*(19)< 0 on entry, the solver reports the number of MFLOPS (1.0E6) that are necessary to factor the matrix A. Reporting this number increases the reordering time.
The default value of *iparm*(19) is 0.

iparm(21) **- pivoting for symmetric indefinite matrices.**
iparm(21) controls the pivoting method for sparse symmetric indefinite matrices.
If *iparm*(21) = 0, then 1x1 diagonal pivoting is used.
If *iparm*(21) = 1, then 1x1 and 2x2 Bunch and Kaufman pivoting is used in the factorization process.

NOTE Use *iparm*(11) = 1 (scaling) and *iparm*(13) = 1 (matchings) for highly indefinite symmetric matrices, for example from interior point optimizations or saddle point problems.

The default value of *iparm*(21) is 1. Bunch and Kaufman pivoting is available for matrices: *mtype*=-2, *mtype*=-4, or *mtype*=6.

iparm(24) **- parallel factorization control.**
This parameter selects the scheduling method for the parallel numerical factorization.
If *iparm*(24) = 0 (default value), then PARDISO uses the previous parallel factorization.
If *iparm*(24) = 1, then PARDISO uses new two-level scheduling algorithm. This algorithm generally improves scalability in case of parallel factorization on many threads (more than eight).

The two-level scheduling factorization algorithm is enabled by default in previous MKL releases for matrices *mtype*=11. If you see performance degradation for such matrices with the default value, set manually *iparm*(24)=1.

iparm(25) **- parallel forward/backward solve control.**
If *iparm*(25) = 0 (default value), then PARDISO uses a parallel algorithm for the solve step.
If *iparm*(25) = 1, then PARDISO uses sequential forward and backward solve.

This feature is available only for *in-core* version.

iparm(27) **- matrix checker.**
If *iparm*(27)=0 (default value), PARDISO does not check the sparse matrix representation.
If *iparm*(27)=1, then PARDISO checks integer arrays *ia* and *ja*. In particular, PARDISO checks whether column indices are sorted in increasing order within each row.

iparm(28) **- sets single or double precision of PARDISO.**
If *iparm*(28)=0, then the input arrays (matrix *a*, vectors *x* and *b*) and all internal arrays must be presented in double precision.
If *iparm*(28)=1, then the input arrays must be presented in single precision. In this case all internal computations are performed in single precision.
Depending on the sign of *iparm*(8), refinement steps can be calculated in quad or double precision for double precision accuracy, and in double or single precision for single precision accuracy.
Default value of *iparm*(28) is 0 (double precision).

Important *iparm*(28) value is stored in the PARDISO handle between PARDISO calls, so the precision mode can be changed only during the solver's phase 1.

iparm(31) **- partial solution for sparse right-hand sides and sparse solution.**
This parameter controls the solution method if the right hand side contains a few nonzero components. It can be also used if only few components of the solution vector are needed, or if you want to reduce computation cost at solver step. To use this option define the input permutation vector *perm* so that *perm*(i) = 1 means that the i-the component in the right-hand side is nonzero or the i-th component in the solution vector is computed.
If *iparm*(31) =0 (default value), this option is disabled.
If *iparm*(31) =1, the right hand side must be sparse, and the *i*-th component in the solution vector is computed if *perm*(*i*) = 1. You can set *perm*(i) = 1 only if the i-th component of the right hand side is nonzero.
If *iparm*(31) =2, the right hand side must be sparse, all components of the solution vector are computed. *perm*(i) = 1 means that the *i*-th component of the right hand side is nonzero.
In the last case the computation cost at solver step is reduced due to reduced forward solver step.
To use *iparm*(31) =2, you must set the i-th component of the right hand side to zero explicitly if *perm*(i) is not equal to 1.
If *iparm*(31) =3, the right hand side can be of any type and you must set *perm*(i) = 1 to compute the *i*-th component in the solution vector.
The permutation vector *perm* must be present in all phases of Intel MKL PARDISO software. At the reordering step, the software overwrites the input vector *perm* by a permutation vector used by the software at the factorization and solver step. If *m* is the number of components such that *perm*(i) = 1, then the last *m* components of the output vector *perm* are a set of the indices i satisfying the condition *perm*(i) = 1 on input.

NOTE Turning on this option often increases time used by PARDISO for factorization and reordering steps, but it enables time to be reduced for the solver step.

Important This feature is available only for the *in-core* version, so to use it you must set *iparm*(60) =0. Set the parameters *iparm*(8) (iterative refinement steps), *iparm*(4) (preconditioned CGS), and *iparm*(5) (user permutation) to 0 as well.

iparm(32) - *iparm*(34) - these parameters are reserved for future use. Their values must be set to 0.

iparm(35) **- C or Fortran style array indexing.**
iparm(35) determines the indexing base for input matrices.
If *iparm*(35)=0 (default value), then PARDISO uses Fortran style indexing: first value is referenced as array element 1.
Otherwise PARDISO uses C style indexing: the first value is referenced as array element 0.

iparm(35) - *iparm*(59) - these parameters are reserved for future use. Their values must be set to 0.

iparm(60) **- version of PARDISO.**

iparm(60) controls what version of PARDISO - out-of-core (OC) version or in-core (IC) version - is used. The OC PARDISO can solve very large problems by holding the matrix factors in files on the disk. Because of that the amount of main memory required by OC PARDISO is significantly reduced.
If *iparm*(60) = 0 (default value), then IC PARDISO is used.
If *iparm*(60) = 1 - then IC PARDISO is used if the total memory of RAM (in megabytes) needed for storing the matrix factors is less than sum of two values of the environment variables: MKL_PARDISO_OOC_MAX_CORE_SIZE (its default value is 2000 MB) and MKL_PARDISO_OOC_MAX_SWAP_SIZE (its default value is 0 MB); otherwise OOC PARDISO is used. In this case amount of RAM used by OOC PARDISO can not exceed the value of MKL_PARDISO_OOC_MAX_CORE_SIZE.
If *iparm*(60) = 2 - then OOC PARDISO is used.
If *iparm*(60) is equal to 1 or 2, and the total peak memory needed for storing the local arrays is more than MKL_PARDISO_OOC_MAX_CORE_SIZE, the program stops with error -9. In this case, increase MKL_PARDISO_OOC_MAX_CORE_SIZE.
OOC parameters can be set in a configuration file. You can set the path to this file and its name using environmental variable MKL_PARDISO_OOC_CFG_PATH and MKL_PARDISO_OOC_CFG_FILE_NAME. Path and name are as follows:
<MKL_PARDISO_OOC_CFG_PATH >/< MKL_PARDISO_OOC_CFG_FILE_NAME>
for Linux* OS, and
<MKL_PARDISO_OOC_CFG_PATH >\< MKL_PARDISO_OOC_CFG_FILE_NAME>
for Windows* OS.
By default, the name of the file is pardiso_ooc.cfg and it is placed to the current directory.
All temporary data files can be deleted or stored when the calculations are completed in accordance with the value of the environmental variable MKL_PARDISO_OOC_KEEP_FILE. If it is set to 1 (default value), then all files are deleted, if it is set to 0, then all files are stored.
By default, the OOC PARDISO uses the current directory for storing data, and all work arrays associated with the matrix factors are stored in files named ooc_temp with different extensions. These default values can be changed by using the environmental variable MKL_PARDISO_OOC_PATH.
To set the environmental variables MKL_PARDISO_OOC_MAX_CORE_SIZE, MKL_PARDISO_OOC_MAX_SWAP_SIZE, MKL_PARDISO_OOC_KEEP_FILE, and MKL_PARDISO_OOC_PATH, create the configuration file with the following lines:

```
MKL_PARDISO_OOC_PATH = <path>\ooc_file
MKL_PARDISO_OOC_MAX_CORE_SIZE = N
MKL_PARDISO_OOC_MAX_SWAP_SIZE = K
MKL_PARDISO_OOC_KEEP_FILE = 0 (or 1)
```

where <*path*> is the directory for storing data, *ooc_file* is the file name without any extension, *N* is the maximum size of RAM in megabytes available for PARDISO (default value is 2000 MB), *K* is the maximum swap size in megabytes available for PARDISO (default value is 0 MB). Do not set *N* greater than the size of the RAM and *K* greater than the size of the swap.

WARNING The maximum length of the path lines in the configuration files is 1000 characters.

Alternatively the environment variables can be set via command line.
For Linux* OS:

```
export MKL_PARDISO_OOC_PATH = <path>/ooc_file
export MKL_PARDISO_OOC_MAX_CORE_SIZE = N
export MKL_PARDISO_OOC_MAX_CORE_SIZE = K
export MKL_PARDISO_OOC_KEEP_FILE = 0 (or 1)
```

For Windows* OS:

```
set MKL_PARDISO_OOC_PATH = <path>\ooc_file
set MKL_PARDISO_OOC_MAX_CORE_SIZE = N
set MKL_PARDISO_OOC_MAX_CORE_SIZE = K
set MKL_PARDISO_OOC_KEEP_FILE = 0 (or 1)
```

NOTE The values specified in a command line have higher priorities - it means that if a variable is changed in the configuration file and in the command line, OOC PARDISO uses only value defined in the command line.

NOTE You can switch between IC and OOC modes after the reordering phase. There are some recommendations and limitations:

- Set *iparm*(60) before reordering phase to get better PARDISO performance.
- Two-level factorization algorithm is not supported in the OOC mode. If you set two-level algorithm in the OOC mode then PARDISO returns error -1.
- Switching between IC and OOC modes after reordering phase is not available in sequential mode. The program returns error -1.

iparm(61), *iparm*(62), *iparm*(64) - these parameters are reserved for future use. Their values must be set to 0.

msglvl INTEGER
Message level information. If *msglvl* = 0 then PARDISO generates no output, if *msglvl* = 1 the solver prints statistical information to the screen.

b DOUBLE PRECISION - for real types of matrices (*mtype*=1, 2, -2 and 11) and for double precision PARDISO (*iparm*(28)=0)
REAL - for real types of matrices (*mtype*=1, 2, -2 and 11) and for single precision PARDISO (*iparm*(28)=1)
DOUBLE COMPLEX - for complex types of matrices (*mtype*=3, 6, 13, 14 and -4) and for double precision PARDISO (*iparm*(28)=0)
COMPLEX - for complex types of matrices (*mtype*=3, 6, 13, 14 and -4) and for single precision PARDISO (*iparm*(28)=1)
Array, dimension (*n*, *nrhs*). On entry, contains the right-hand side vector/matrix *B*, which is placed in memory contiguously. The *b*(i+(k-1)×*nrhs*) must hold the i-th component of k-th right-hand side vector. Note that *b* is only accessed in the solution phase.

Output Parameters

(See also PARDISO Parameters in Tabular Form.)

pt This parameter contains internal address pointers.

iparm On output, some *iparm* values report information such as the numbers of non-zero elements in the factors.

iparm(7)**- number of performed iterative refinement steps.**
The number of iterative refinement steps that are actually performed during the solve step.

iparm(14)**- number of perturbed pivots.**
After factorization, *iparm*(14) contains the number of perturbed pivots during the elimination process for *mtype* =11, *mtype* =13, *mtype* =-2, *mtype* =-4, or *mtype* =-6.

iparm(15) **- peak memory symbolic factorization.**
The parameter *iparm*(15) reports the total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase. This value is only computed in phase 1.

iparm(16) **- permanent memory symbolic factorization.**
The parameter *iparm*(16) reports the permanent memory in kilobytes from the analysis and symbolic factorization phase that the solver needs in the factorization and solve phases. This value is only computed in phase 1.

iparm(17) **- size of factors /memory numerical factorization and solution.**
The parameter *iparm*(17) provides the size in kilobytes of the total memory consumed by IC PARDISO for internal float point arrays. This parameter is computed in phase 1. See *iparm*(63) for the OOC mode. The total peak memory solver consumption for all phases is max(*iparm*(15), *iparm*(16)+*iparm*(17))

iparm(18) **- number of non-zero elements in factors.**
The solver reports the numbers of non-zero elements on the factors if *iparm*(18) < 0 on entry.

iparm(19) **- MFLOPS of factorization.**
The solver reports the number of operations in MFLOPS (1.0E6 operations) that are necessary to factor the matrix *A* if *iparm*(19) < 0 on entry.

iparm(20) **- CG/CGS diagnostics.**
The value is used to give CG/CGS diagnostics (for example, the number of iterations and cause of failure):
If *iparm*(20)> 0, CGS succeeded, and the number of iterations executed are reported in *iparm*(20).
If *iparm*(20)< 0, iterations executed, but CG/CGS failed. The error report details in *iparm*(20) are of the form: *iparm*(20)= – it_cgs*10 – cgs_error.
If *phase*= 23, then the factors *L* and *U* are recomputed for the matrix *A* and the error flag *error*=0 in case of a successful factorization. If *phase* = 33, then *error* = -4 signals failure.
Description of cgs_error is given in the table below:

cgs_error	Description
1	fluctuations of the residue are too large

cgs_error	Description
2	$\|\|dx_{max\_it\_cgs/2}\|\|$ is too large (slow convergence)
3	stopping criterion is not reached at max_it_cgs
4	perturbed pivots causes iterative refinement
5	factorization is too fast for this matrix. It is better to use the factorization method with *iparm*(4)=0

iparm(22) – **inertia: number of positive eigenvalues.**
The parameter *iparm*(22) reports the number of positive eigenvalues for symmetric indefinite matrices.

iparm(23) **- inertia: number of negative eigenvalues.**
The parameter *iparm*(23) reports the number of negative eigenvalues for symmetric indefinite matrices.

iparm(30) - **the number of the equation where PARDISO detects zero or negative pivot**
If the solver detects a zero or negative pivot for matrix types *mtype* = 2 (real positive definite matrix) and *mtype* = 4 (complex and Hermitian positive definite matrices), the factorization is stopped, PARDISO returns immediately with an error (*error* = -4) and *iparm*(30) contains the number of the equation where the first zero or negative pivot is detected.

iparm(63) **- size of the minimum OOC memory for numerical factorization and solution.**
The parameter *iparm*(63) provides the size in kilobytes of the minimum memory required by OOC PARDISO for internal float point arrays. This parameter is computed in phase 1.
Total peak memory consumption of OOC PARDISO can be estimated as max(*iparm*(15), *iparm*(16)+*iparm*(63))

b — On output, the array is replaced with the solution if *iparm*(6) = 1.

x — DOUBLE PRECISION - for real types of matrices (*mtype*=1, 2, -2 and 11) and for double precision PARDISO (*iparm*(28)=0)
REAL - for real types of matrices (*mtype*=1, 2, -2 and 11) and for single precision PARDISO (*iparm*(28)=1)
DOUBLE COMPLEX - for complex types of matrices (*mtype*=3, 6, 13, 14 and -4) and for double precision PARDISO (*iparm*(28)=0)
COMPLEX - for complex types of matrices (*mtype*=3, 6, 13, 14 and -4) and for single precision PARDISO (*iparm*(28)=1)
Array, dimension (*n*,*nrhs*). If *iparm*(6)=0 it contains solution vector/ matrix *X*, which is placed contiguously in memory. The *x*(i+(k–1)× *nrhs*) element must hold the i-th component of the k-th solution vector. Note that *x* is only accessed in the solution phase.

error — INTEGER
The error indicator according to the below table:

error	Information
0	no error
–1	input inconsistent
–2	not enough memory
–3	reordering problem

error	Information
-4	zero pivot, numerical factorization or iterative refinement problem
-5	unclassified (internal) error
-6	reordering failed (matrix types 11 and 13 only)
-7	diagonal matrix is singular
-8	32-bit integer overflow problem
-9	not enough memory for OOC
-10	problems with opening OOC temporary files
-11	read/write problems with the OOC data file

See Also

mkl_progress

pardisoinit

Initialize PARDISO with default parameters in accordance with the matrix type.

Syntax

Fortran:

```
call pardisoinit (pt, mtype, iparm)
```

C:

```
pardisoinit (pt, &mtype, iparm);
```

Include Files

- FORTRAN 77: `mkl_pardiso.f77`
- Fortran 90: `mkl_pardiso.f90`
- C: `mkl_pardiso.h`

Description

This function initializes PARDISO internal address pointer *pt* with zero values (as needed for the very first call of PARDISO) and sets default *iparm* values in accordance with the matrix type. Intel MKL supplies the `pardisoinit` routine to be compatible with PARDISO 3.2 or lower distributed by the University of Basel.

NOTE An alternative way to set default PARDISO *iparm* values is to call `pardiso` with *iparm*`(1)=0`. In this case you must initialize the internal address pointer *pt* with zero values manually.

NOTE The `pardisoinit` routine initializes only the in-core version of PARDISO. Switching on the out-of core version of PARDISO as well as changing default *iparm* values can be done after the call to `pardisoinit` but before the first call to `pardiso`.

Input Parameters

NOTE Parameters types in this section are specified in FORTRAN 77 notation. See PARDISO Parameters in Tabular Form section for detailed description of types of PARDISO parameters in C/Fortran 90 notations.

mtype — INTEGER
This scalar value defines the matrix type. Based on this value pardisoinit sets default values for the *iparm* array. Refer to the section PARDISO Parameters in Tabular Form for more details about the default values of PARDISO.

Output Parameters

pt — INTEGER for 32-bit architectures
INTEGER*8 for 64-bit architectures
Array, DIMENSION (64)
Solver internal data address pointer. These addresses are passed to the solver, and all related internal memory management is organized through this array. The pardisoinit routine nullifies the array *pt*.

NOTE It is very important that the pointer *pt* is initialized with zero before the first call of PARDISO. After that first call you should never modify the pointer, as a serious memory leak can occur.

iparm — INTEGER
Array, dimension (64). This array is used to pass various parameters to PARDISO and to return some useful information after execution of the solver. The pardisoinit routine fills-in the *iparm* array with the default values. Refer to the section PARDISO Parameters in Tabular Form for more details about the default values of PARDISO.

pardiso_64

Calculates the solution of a set of sparse linear equations with multiple right-hand sides, 64-bit integer version.

Syntax

Fortran:

```
call pardiso_64 (pt, maxfct, mnum, mtype, phase, n, a, ia, ja, perm, nrhs, iparm, msglvl, b, x, error)
```

C:

```
pardiso_64 (pt, &maxfct, &mnum, &mtype, &phase, &n, a, ia, ja, perm, &nrhs, iparm, &msglvl, b, x, &error);
```

Include Files

- FORTRAN 77: mkl_pardiso.f77
- Fortran 90: mkl_pardiso.f90
- C: mkl_pardiso.h

Description

pardiso_64 is an alternative ILP64 (64-bit integer) version of the pardiso routine (see Description section for more details). The interface of pardiso_64 is equal to the interface of pardiso, but it accepts and returns all INTEGER data as INTEGER*8.

Use pardiso_64 interface for solving large matrices (with the number of non-zero elements on the order of 500 million or more). You can use it together with the usual LP64 interfaces for the rest of Intel MKL functionality. In other words, if you use 64-bit integer version (pardiso_64), you do not need to re-link your applications with ILP64 libraries. Take into account that pardiso_64 may perform slower than regular pardiso on reordering and symbolic factorization phase.

NOTE pardiso_64 is supported only in the 64-bit libraries. If pardiso_64 is called from the 32-bit libraries, it returns *error* =-12.

Input Parameters

The input parameters of pardiso_64 are equal to the input parameters of pardiso, but pardiso_64 accepts all INTEGER data as INTEGER*8.

Output Parameters

The output parameters of pardiso_64 are equal to output parameters of pardiso, but pardiso_64 returns all INTEGER data as INTEGER*8.

See Also

mkl_progress

pardiso_getenv, pardiso_setenv

Retrieves additional values from the PARDISO handle or sets them in it.

Syntax

```
error = pardiso_getenv(handle, param, value)
error = pardiso_setenv(handle, param, value)
```

outputtext(Interface):

```
_INTEGER_t pardiso_getenv (const _MKL_DSS_HANDLE_t handle, const enum
PARDISO_ENV_PARAM* param, char* value);
_INTEGER_t pardiso_setenv (_MKL_DSS_HANDLE_t handle, const enum PARDISO_ENV_PARAM*
param, const char* value);
```

Include Files

- FORTRAN 77: mkl_pardiso.f77
- Fortran 90: mkl_pardiso.f90
- C: mkl_pardiso.h

NOTE pardiso_setenv requires the *value* parameter to be converted to the string in C notation if it is called from Fortran. You can do this using mkl_cvt_to_null_terminated_str subroutine declared in the mkl_dss.f77 or mkl_dss.f90 include files (see example below).

Description

These functions operate with the PARDISO handle. The `pardiso_getenv` routine retrieves additional values from the PARDISO handle, and `pardiso_setenv` sets specified values in the PARDISO handle.

These functions enable retrieving and setting the name of the PARDISO OOC file.

To retrieve the PARDISO OOC file name, you can apply this function to any non-empty handle.

To set the the PARDISO OOC file name in the handle you must apply the function before reordering stage. That is you must apply the function only for the empty handle. This is because OOC file name is stored in the handle after reordering stage and it is not changed during further computations.

NOTE 1024-byte internal buffer is used inside PARDISO for storing OOC file name. Allocate 1024-byte buffer (*value* parameter) for passing it to `pardiso_getenv` function.

Input Parameters

handle	Input parameter for `pardiso_getenv`. Data object of the `MKL_DSS_HANDLE` type (see DSS Interface Description).
param	`INTEGER`. Specifies the required parameter. The only value is `PARDISO_OCC_FILE_NAME`, defined in the corresponding include file.
value	Input parameter for `pardiso_setenv`. `STRING`. Contains the name of the OOC file that must be used in the handle.

Output Parameters

value	Output parameter for `pardiso_getenv`. `STRING`. Contains the name of the OOC file that is used in the handle.
handle	Output parameter for `pardiso_setenv`. Data object of the `MKL_DSS_HANDLE` type (see DSS Interface Description).

Example (FORTRAN 90)

```
INCLUDE 'mkl_pardiso.f90'
INCLUDE 'mkl_dss.f90'
PROGRAM pardiso_sym_f90
USE mkl_pardiso
USE mkl_dss

INTEGER*8 pt(64)
CHARACTER*1024 file_name
INTEGER buff(256), bufLen, error

pt(1:64) = 0

file_name = 'pardiso_ooc_file'
bufLen = len_trim(file_name)
call mkl_cvt_to_null_terminated_str(buff, bufLen, trim(file_name))
error = pardiso_setenv(pt, PARDISO_OOC_FILE_NAME, buff)

! call pardiso() here

END PROGRAM
```

PARDISO Parameters in Tabular Form

The following table lists all parameters of PARDISO and gives their brief descriptions.

Parameter	Type	Description	Values	Comments	In/Out
pt(64)	**FORTRAN 77:** INTEGER on 32-bit architectures, INTEGER*8 on 64-bit architectures **Fortran 90:** TYPE(MKL_PARDISO_HANDLE), INTENT(INOUT) **C:** void*	Solver internal data address pointer	0	Must be initialized by zeros and never be modified later	in/out
maxfct	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	Maximal number of factors in memory	>0	Generally used value is 1	in
mnum	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	The number of matrix (from 1 to *maxfct*) to solve	[1; *maxfct*]	Generally used value is 1	in
mtype	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	Matrix type	1 2 -2 3 4 -4 6 11 13	Real and structurally symmetric Real and symmetric positive definite Real and symmetric indefinite Complex and structurally symmetric Complex and Hermitian positive definite Complex and Hermitian indefinite Complex and symmetric matrix Real and unsymmetric matrix Complex and unsymmetric matrix	in
phase	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	Controls the execution of the solver	11 12 13 22	Analysis Analysis, numerical factorization Analysis, numerical factorization, solve Numerical factorization	in

Parameter	Type	Description	Values	Comments	In/Out
			23	Numerical factorization, solve	
			33	Solve, iterative refinement	
			331	*phase*=33, but only forward substitution	
			332	*phase*=33, but only diagonal substitution	
			333	*phase*=33, but only backward substitution	
			0	Release internal memory for L and U of the matrix number *mnum*	
			-1	Release all internal memory for all matrices	
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	Number of equations in the sparse linear system *A***X* = *B*	>0		in
a(*)	**FORTRAN 77:** PARDISO_DATA_TYPE[1)] **Fortran 90:** PARDISO_DATA_TYPE[1)], INTENT(IN) **C:** void*	Contains the non-zero elements of the coefficient matrix *A*	*	The size of *a* is the same as that of *ja*, and the coefficient matrix can be either real or complex. The matrix must be stored in CSR format with increasing values of *ja* for each row	in
ia(n+1)	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	*rowIndex* array in CSR format	>=0	0 < *ia*(*i*) <= *ia*(*i*+1) *ia*(*i*) gives the index of the element in array *a* that contains the first non-zero element from row *i* of *A*. The last element *ia*(*n*+1) is taken to be equal to the number of non-zero elements in *A*, plus one. Note: *iparm*(35) indicates whether row/column indexing starts from 1 or 0.	in
ja(*)	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	*columns* array in CSR format	>=0	The indices in each row must be sorted in increasing order. For symmetric and structurally symmetric matrices zero diagonal elements are also stored in *a* and *ja*. For symmetric	in

Parameter	Type	Description	Values	Comments	In/Out
				matrices, the solver needs only the upper triangular part of the system. Note: *iparm*(35) indicates whether row/column indexing starts from 1 or 0.	
perm(n)	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(INOUT) **C:** _INTEGER_t*	Holds the permutation vector of size *n*	>=0	Let *B = P*A*PT* be the permuted matrix. Row (column) *i* of *A* is the *perm*(*i*) row (column) of *B*. The numbering of the array must describe a permutation. You can apply your own fill-in reducing ordering (*iparm*(5)=1) or return the permutation from the solver (*iparm*(5)=2). Note: *iparm*(35) indicates whether row/column indexing starts from 1 or 0.	in/out
nrhs	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	Number of right-hand sides that need to be solved for	>=0	Generally used value is 1	in
iparm(64)	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(INOUT) **C:** _INTEGER_t*	This array is used to pass various parameters to PARDISO and to return some useful information after execution of the solver	*	If *iparm*(1)=0 , PARDISO fills *iparm*(2) through *iparm*(64) with default values and uses them.	in/out
msglvl	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t*	Message level information	0 1	PARDISO generates no output PARDISO prints statistical information	in
*b(n*nrhs)*	**FORTRAN 77:** PARDISO_DATA_TYPE[1)] **Fortran 90:** PARDISO_DATA_TYPE[1)], INTENT(INOUT) **C:** void*	Right-hand side vectors	*	On entry, contains the right hand side vector/matrix *B*, which is placed contiguously in memory. The *b*(i+(k-1)×*nrhs*) element must hold the *i*-th component of *k*-th right-hand side vector. Note that *b* is only accessed in the solution phase.	in/out

Parameter	Type	Description	Values	Comments	In/Out
				On output, the array is replaced with the solution if *iparm*(6)=1.	
*x(n*nrhs)*	**FORTRAN 77:** PARDISO_DATA_TYPE[1)] **Fortran 90:** PARDISO_DATA_TYPE[1)], INTENT(OUT) **C:** void*	Solution vectors	*	On output, if *iparm*(6)=1, contains solution vector/ matrix *X* which is placed contiguously in memory. The *x*(i+(k-1)×*nrhs*) element must hold the *i*-th component of *k*-th solution vector. Note that *x* is only accessed in the solution phase.	out
error	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(OUT) **C:** _INTEGER_t*	Error indicator	0	No error	out
			-1	Input inconsistent	
			-2	Not enough memory	
			-3	Reordering problem	
			-4	Zero pivot, numerical factorization or iterative refinement problem	
			-5	Unclassified (internal) error	
			-6	Reordering failed (matrix types 11 and 13 only)	
			-7	Diagonal matrix is singular	
			-8	32-bit integer overflow problem	
			-9	Not enough memory for OOC	
			-10	Problems with opening OOC temporary files	
			-11	Read/write problems with the OOC data file	

[1)] See description of PARDISO_DATA_TYPE in the table below.

The following table lists the values of PARDISO_DATA_TYPE depending on the matrix types and values of the parameter *iparm*(28).

Data type value	Matrix type *mtype*	*iparm*(28)	comments
DOUBLE PRECISION	1, 2, -2, 11	0	Real matrices, double precision
REAL		1	Real matrices, single precision
DOUBLE COMPLEX	3, 6, 13, 4, -4	0	Complex matrices, double precision

Data type value	Matrix type *mtype*	*iparm*(28)	comments
COMPLEX		1	Complex matrices, single precision

The following table lists all individual components of the PARDISO *iparm()* parameter and their brief descriptions. Components not listed in the table must be initialized with 0. Default values in the column **Values** are denoted as X*.

Component	Description	Values	Comments	In/ Out
INPUT, INPUT/OUTPUT PARAMETERS				
iparm(1)	Use default values	0	*iparm*(2) -*iparm*(64) are filled with default values.	in
		!=0	You must supply all values in components *iparm*(2) -*iparm*(64)	
iparm(2)	Fill-in reducing ordering for the input matrix	0	The minimum degree algorithm.	in
		2*	The nested dissection algorithm from the METIS package	
		3	The parallel (OpenMP) version of the nested dissection algorithm.	
iparm(4)	Preconditioned CGS/ CG	0*	Do not perform preconditioned Krylow-Subspace iterations.	in
		10*L*+1	CGS iteration replaces the computation of LU. The preconditioner is LU that is computed at the previous step (the first step or last step with a failure) in a sequence of solutions needed for identical sparsity patterns. *L* controls the stopping criterion of the Krylow-Subspace iteration: epsCGS = 10^(-*L*) is used in the stopping criterion ||dxi|| / ||dx0|| < epsCGS, with ||dxi|| = ||inv(L*U)*ri|| and ri is the residuum at iteration i of the preconditioned Krylow-Subspace iteration.	
		10*L*+2	Same as above, but CG iteration replaces the computation of LU. Designed for symmetric positive definite matrices.	
iparm(5)	User permutation	0*	User permutation in *perm* array is ignored.	in
		1	PARDISO uses the user supplied fill-in reducing permutation from *perm* array. *iparm*(2) is ignored.	
		2	PARDISO returns the permutation vector computed at phase 1 into *perm* array	
iparm(6)	Write solution on *x*	0*	The array *x* contains the solution; right-hand side vector *b* is kept unchanged.	in
		1	The solver stores the solution on the right-hand side *b*. Note that the array *x* is always used.	
iparm(8)	Iterative refinement step	0*	The solver automatically performs two steps of iterative refinements when perturbed pivots are obtained during the numerical factorization.	in

Component	Description	Values	Comments	In/Out
		>0	Maximum number of iterative refinement steps that the solver performs. The solver performs not more than the absolute value of *iparm*(8) steps of iterative refinement and stops the process if a satisfactory level of accuracy of the solution in terms of backward error is achieved. The number of executed iterations is reported in *iparm*(7).	
		<0	Same as above, but the accumulation of the residuum uses extended precision real and complex data types.	
iparm(10)	Pivoting perturbation	*	This parameter instructs PARDISO how to handle small pivots or zero pivots for unsymmetric matrices (*mtype* =11, *mtype* =13) and symmetric matrices (*mtype* =-2, *mtype* =-4, *mtype* =6). Small pivots are perturbed with eps = 10^(-*iparm*(10)).	in
		13*	The default value for unsymmetric matrices(*mtype* =11, *mtype*=13), eps = 10^(-13).	
		8*	The default value for symmetric indefinite matrices (*mtype* =-2, *mtype*=-4, *mtype*=6), eps = 10^(-8).	
iparm(11)	Scaling	0*	Disable scaling. Default for symmetric indefinite matrices.	in
		1*	Enable scaling. Default for unsymmetric matrices. Scale the matrix so that the diagonal elements are equal to 1 and the absolute values of the off-diagonal entries are less or equal to 1. This scaling method is applied to unsymmetric matrices (*mtype* = 11, *mtype* = 13). The scaling can also be used for symmetric indefinite matrices (*mtype* = -2, *mtype* = -4, *mtype* = 6) when the symmetric weighted matchings are applied (*iparm*(13) = 1). Note that in the analysis phase (*phase*=11) you must provide the numerical values of the matrix *A* in case of scaling.	
iparm(12)	Solving with transposed or conjugate transposed matrix *A*	0*	Solve a $Ax=b$ linear system.	
		1	Solve a conjugate transposed system $A^Hx = b$ based on the factorization of the matrix *A*.	
		2	Solve a transposed system $A^Tx = b$ based on the factorization of the matrix *A*.	
iparm(13)	Improved accuracy using (non-) symmetric weighted matching	0*	Disable matching. Default for symmetric indefinite matrices.	in
		1*	Enable matching. Default for symmetric indefinite matrices. Maximum weighted matching algorithm to permute large elements close to the diagonal.	

Component	Description	Values	Comments	In/ Out
			It is recommended to use *iparm*(11) = 1 (scaling) and *iparm*(13) = 1 (matching) for highly indefinite symmetric matrices, for example from interior point optimizations or saddle point problems. Note that in the analysis phase (*phase*=11) you must provide the numerical values of the matrix *A* in case of symmetric weighted matching.	
iparm(18)	Report the number of non-zero elements in the factors	<0	Enable reporting if *iparm*(18) < 0 on entry. The default value is -1.	in/ out
		>=0	Disable reporting.	
iparm(19)	Report Mflops that are necessary to factor the matrix *A*.	<0	Enable report if *iparm*(18) < 0 on entry. This increases the reordering time.	in/ out
		>=0	Disable report. 0 is a default value.	
iparm(21)	Pivoting for symmetric indefinite matrices	0	Apply 1x1 diagonal pivoting during the factorization process.	in
		1*	Apply 1x1 and 2x2 Bunch and Kaufman pivoting during the factorization process.	
iparm(24)	parallel factorization control	0	PARDISO uses the previous algorithm for factorization. Default value.	in
		1	PARDISO uses new two-level factorization algorithm.	
iparm(25)	Parallel forward/ backward solve control	0	PARDISO uses the parallel algorithm for solve step. Default value.	in
		1	PARDISO uses the sequential forward and backward solve.	
iparm(27)	Matrix checker	0*	PARDISO does not check the sparse matrix representation.	in
		1	PARDISO checks integer arrays *ia* and *ja*. In particular, PARDISO checks whether column indices are sorted in increasing order within each row.	
iparm(28)	Single or double precision of PARDISO	0*	Input arrays (*a*, *x* and *b*) and all internal arrays must be presented in double precision.	in
		1	Input arrays (*a*, *x* and *b*) must be presented in single precision. In this case all internal computations are performed in single precision.	
iparm(31)	Enables to solve partially for sparse right-hand sides and sparse solution	0	Disables this option. Default value.	in
		1	The right hand side is assumed to be sparse, *perm*(i)=1 means that the i-th component of the right hand side is nonzero, and this component of the solution vector is computed.	

Component	Description	Values	Comments	In/Out
		2	The right hand side is assumed to be sparse, *perm*(i)=1 means that the i-th component of the right hand side is nonzero, and all components of the solution vector are computed.	
		3	The right hand side can be of any type. If *perm*(i)=1, the i-th component of the solution vector is computed.	
iparm(35)	One- or zero-based indexing of columns and rows	0*	One-based indexing: columns and rows indexing in arrays *ia*, *ja*, and *perm* starts from 1. Default value.	in
		1	Zero-based indexing: columns and rows indexing in arrays *ia*, *ja*, and *perm* starts from 0.	
iparm(60)	PARDISO mode	0*	In-core PARDISO	in
		1	In-core PARDISO is used if the total memory needed for storing the matrix factors is less than the value of the environment variable MKL_PARDISO_OOC_MAX_CORE_SIZE. Otherwise out-of-core (OOC) PARDISO is used.	
		2	Out-of-core (OOC) PARDISO The OOC PARDISO can solve very large problems by holding the matrix factors in files on the disk. Hence the amount of RAM required by OOC PARDISO is significantly reduced.	
OUTPUT PARAMETERS				
iparm(7)	Number of performed iterative refinement steps	>=0	Reports the number of iterative refinement steps that were actually performed during the solve step.	out
iparm(14)	Number of perturbed pivots	>=0	After factorization, contains the number of perturbed pivots for the matrix types: 11, 13, -2, -4 and -6.	out
iparm(15)	Peak memory on symbolic factorization	>0 KB	The total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase. This value is only computed in phase 1.	out
iparm(16)	Permanent memory on symbolic factorization	>0 KB	Permanent memory from the analysis and symbolic factorization phase in kilobytes that the solver needs in the factorization and solve phases. This value is only computed in phase 1.	out
iparm(17)	Size of factors/Peak memory on numerical factorization and solution	>0 KB	This parameter provides the size in kilobytes of the total memory consumed by in-core PARDISO for internal float point arrays. This parameter is computed in phase 1. See *iparm*(63) for the OOC mode. The total peak memory consumed by PARDISO is max(*iparm*(15), *iparm*(16)+*iparm*(17))	out
iparm(20)	CG/CGS diagnostics	>0	CGS succeeded, reports the number of completed iterations.	out
		<0	CG/CGS failed (*error*=-4 after the solution phase)	

Component	Description	Values	Comments	In/Out
			iparm(20)= - it_cgs*10 - cgs_error. Possible values of cgs_error: 1 - fluctuations of the residuum are too large 2 - ||dx at max_it_cgs/2|| is too large (slow convergence) 3 - stopping criterion is not reached at max_it_cgs 4 - perturbed pivots causes iterative refinement	
iparm(22)	Inertia: number of positive eigenvalues	>=0	PARDISO reports the number of positive eigenvalues for symmetric indefinite matrices	out
iparm(23)	Inertia: number of negative eigenvalues	>=0	PARDISO reports the number of negative eigenvalues for symmetric indefinite matrices.	out
iparm(30)	Number of zero or negative pivots	>=0	If PARDISO detects zero or negative pivot for *mtype*=2 or *mtype*=4 types, the factorization is stopped, PARDISO returns immediately with an *error* = -4, and *iparm*(30) reports the number of the equation where the first zero or negative pivot is detected.	out
iparm(63)	Size of the minimum OOC memory for numerical factorization and solution	>0 KB	This parameter provides the size in kilobytes of the minimum memory required by OOC PARDISO for internal float point arrays. This parameter is computed in phase 1. Total peak memory consumption of OOC PARDISO can be estimated as max(*iparm*(15), *iparm*(16)+*iparm*(63))	

Direct Sparse Solver (DSS) Interface Routines

Intel MKL supports the DSS interface, an alternative to the PARDISO* interface for the direct sparse solver. The DSS interface implements a group of user-callable routines that are used in the step-by-step solving process and utilizes the general scheme described in Appendix A Linear Solvers Basics for solving sparse systems of linear equations. This interface also includes one routine for gathering statistics related to the solving process and an auxiliary routine for passing character strings from Fortran routines to C routines.

The current implementation of the DSS interface additionally supports the out-of-core (OOC) mode.

Table "DSS Interface Routines" lists the names of the routines and describes their general use.

DSS Interface Routines

Routine	Description
dss_create	Initializes the solver and creates the basic data structures necessary for the solver. This routine must be called before any other DSS routine.
dss_define_structure	Informs the solver of the locations of the non-zero elements of the array.
dss_reorder	Based on the non-zero structure of the matrix, computes a permutation vector to reduce fill-in during the factoring process.

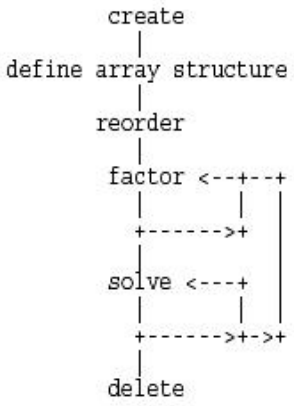
Routine	Description
dss_factor_real, dss_factor_complex	Computes the LU, LDL^T or LL^T factorization of a real or complex matrix.
dss_solve_real, dss_solve_complex	Computes the solution vector for a system of equations based on the factorization computed in the previous phase.
dss_delete	Deletes all data structures created during the solving process.
dss_statistics	Returns statistics about various phases of the solving process.
mkl_cvt_to_null_terminated_str	Passes character strings from Fortran routines to C routines.

To find a single solution vector for a single system of equations with a single right hand side, invoke the Intel MKL DSS interface routines in this order:

1. dss_create
2. dss_define_structure
3. dss_reorder
4. dss_factor_real, dss_factor_complex
5. dss_solve_real, dss_solve_complex
6. dss_delete

However, in certain applications it is necessary to produce solution vectors for multiple right-hand sides for a given factorization and/or factor several matrices with the same non-zero structure. Consequently, it is sometimes necessary to invoke the Intel MKL sparse routines in an order other than that listed, which is possible using the DSS interface. The solving process is conceptually divided into six phases. Figure "Typical order for invoking DSS interface routines" indicates the typical order in which the DSS interface routines can be invoked.

Typical order for invoking DSS interface routines



See the code examples that use the DSS interface routines to solve systems of linear equations in the `examples\solver\source` folder of your Intel MKL directory (`dss_sym_f.f`, `dss_sym_c.c`, `dss_sym_f90.f90`).

DSS Interface Description

Each DSS routine reads from or writes to a data object called a *handle*. Refer to Memory Allocation and Handles to determine the correct method for declaring a handle argument for each language. For simplicity, the descriptions in DSS routines refer to the data type as `MKL_DSS_HANDLE`.

C and C++ programmers should refer to Calling Sparse Solver and Preconditioner Routines from C C++ for information on mapping Fortran types to C/C++ types.

Routine Options

The DSS routines have an integer argument (referred below to as *opt*) for passing various options to the routines. The permissible values for *opt* should be specified using only the symbol constants defined in the language-specific header files (see Implementation Details). The routines accept options for setting the message and termination levels as described in Table "Symbolic Names for the Message and Termination Levels Options". Additionally, each routine accepts the option `MKL_DSS_DEFAULTS` that sets the default values (as documented) for *opt* to the routine.

Symbolic Names for the Message and Termination Levels Options

Message Level	Termination Level
MKL_DSS_MSG_LVL_SUCCESS	MKL_DSS_TERM_LVL_SUCCESS
MKL_DSS_MSG_LVL_INFO	MKL_DSS_TERM_LVL_INFO
MKL_DSS_MSG_LVL_WARNING	MKL_DSS_TERM_LVL_WARNING
MKL_DSS_MSG_LVL_ERROR	MKL_DSS_TERM_LVL_ERROR
MKL_DSS_MSG_LVL_FATAL	MKL_DSS_TERM_LVL_FATAL

The settings for message and termination levels can be set on any call to a DSS routine. However, once set to a particular level, they remain at that level until they are changed in another call to a DSS routine.

You can specify both message and termination level for a DSS routine by adding the options together. For example, to set the message level to `debug` and the termination level to `error` for all the DSS routines, use the following call:

```
CALL dss_create( handle, MKL_DSS_MSG_LVL_INFO + MKL_DSS_TERM_LVL_ERROR)
```

User Data Arrays

Many of the DSS routines take arrays of user data as input. For example, user arrays are passed to the routine `dss_define_structure` to describe the location of the non-zero entries in the matrix. To minimize storage requirements and improve overall run-time efficiency, the Intel MKL DSS routines do not make copies of the user input arrays.

WARNING Do not modify the contents of these arrays after they are passed to one of the solver routines.

DSS Routines

dss_create

Initializes the solver.

Syntax

C:

```
dss_create(handle, opt)
```

Fortran:

```
call dss_create(handle, opt)
```

Include Files

- FORTRAN 77: `mkl_dss.f77`
- Fortran 90: `mkl_dss.f90`
- C: `mkl_dss.h`

Description

The `dss_create` routine initializes the solver. After the call to `dss_create`, all subsequent invocations of the Intel MKL DSS routines must use the value of the handle returned by `dss_create`.

WARNING Do not write the value of handle directly.

The default value of the parameter *opt* is

`MKL_DSS_MSG_LVL_WARNING` + `MKL_DSS_TERM_LVL_ERROR`.

By default, the DSS routines use double precision for solving systems of linear equations. The precision used by the DSS routines can be set to single mode by adding the following value to the *opt* parameter:

`MKL_DSS_SINGLE_PRECISION`.

As for PARDISO, input data and internal arrays are required to have single precision.

By default, the DSS routines use Fortran style indexing for input arrays of integer types (the first value is referenced as array element 1). The indexing can be set to C style (the first value is referenced as array element 0) by adding the following value to the *opt* parameter:

`MKL_DSS_ZERO_BASED_INDEXING`.

This parameter can also control number of refinement steps used on the solution stage by specifying the two following values:

`MKL_DSS_REFINEMENT_OFF` - maximum number of refinement steps is set to zero;
`MKL_DSS_REFINEMENT_ON` (default value) - maximum number of refinement steps is set to 2.

By default, DSS uses in-core computations. To launch the out-of-core version of DSS (OOC DSS) you can add to this parameter one of two possible values: `MKL_DSS_OOC_STRONG` and `MKL_DSS_OOC_VARIABLE`.

`MKL_DSS_OOC_STRONG` - OOC DSS is used.

`MKL_DSS_OOC_VARIABLE` - if the memory needed for the matrix factors is less than the value of the environment variable `MKL_PARDISO_OOC_MAX_CORE_SIZE`, then the OOC DSS uses the in-core kernels of PARDISO, otherwise it uses the OOC computations.

The variable `MKL_PARDISO_OOC_MAX_CORE_SIZE` defines the maximum size of RAM allowed for storing work arrays associated with the matrix factors. It is ignored if `MKL_DSS_OOC_STRONG` is set. The default value of `MKL_PARDISO_OOC_MAX_CORE_SIZE` is 2000 MB. This value and default path and file name for storing temporary data can be changed using the configuration file `pardiso_ooc.cfg` or command line (See more details in the pardiso description above).

WARNING Do not change the OOC DSS settings after they are specified in the routine `dss_create`.

Input Parameters

Name	Type	Description
opt	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Parameter to pass the DSS options. The default value is MKL_DSS_MSG_LVL_WARNING + MKL_DSS_TERM_LVL_ERROR.

Output Parameters

Name	Type	Description
handle	**FORTRAN 77:** INTEGER*8 **Fortran 90:** TYPE (MKL_DSS_HANDLE), INTENT(OUT) **C:** _MKL_DSS_HANDLE_t*	Pointer to the data structure storing intermediate DSS results (MKL_DSS_HANDLE).

Return Values

MKL_DSS_SUCCESS

MKL_DSS_INVALID_OPTION

MKL_DSS_OUT_OF_MEMORY

MKL_DSS_MSG_LVL_ERR

MKL_DSS_TERM_LVL_ERR

dss_define_structure

Communicates locations of non-zero elements in the matrix to the solver.

Syntax

C:

```
dss_define_structure(handle, opt, rowIndex, nRows, nCols, columns, nNonZeros);
```

Fortran:

```
call dss_define_structure(handle, opt, rowIndex, nRows, nCols, columns, nNonZeros);
```

Include Files

- FORTRAN 77: mkl_dss.f77
- Fortran 90: mkl_dss.f90
- C: mkl_dss.h

Description

The routine dss_define_structure communicates the locations of the *nNonZeros* number of non-zero elements in a matrix of *nRows * nCols* size to the solver.

The Intel MKL DSS software operates only on square matrices, so *nRows* must be equal to *nCols*.

To communicate the locations of non-zero elements in the matrix, do the following:

1. Define the general non-zero structure of the matrix by specifying the value for the options argument *opt*. You can set the following values for real matrices:

- MKL_DSS_SYMMETRIC_STRUCTURE
- MKL_DSS_SYMMETRIC
- MKL_DSS_NON_SYMMETRIC

and for complex matrices:

- MKL_DSS_SYMMETRIC_STRUCTURE_COMPLEX
- MKL_DSS_SYMMETRIC_COMPLEX
- MKL_DSS_NON_SYMMETRIC_COMPLEX

The information about the matrix type must be defined in dss_define_structure.

2. Provide the actual locations of the non-zeros by means of the arrays *rowIndex* and *columns* (see Sparse Matrix Storage Format).

Input Parameters

Name	Type	Description
opt	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Parameter to pass the DSS options. The default value for the matrix structure is MKL_DSS_SYMMETRIC.
rowIndex	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Array of size min(*nRows*, *nCols*)+1. Defines the location of non-zero entries in the matrix.
nRows	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Number of rows in the matrix.
nCols	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Number of columns in the matrix.
columns	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Array of size *nNonZeros*. Defines the location of non-zero entries in the matrix.
nNonZeros	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Number of non-zero elements in the matrix.

Output Parameters

Name	Type	Description
handle	**FORTRAN 77:** INTEGER*8 **Fortran 90:** TYPE (MKL_DSS_HANDLE), INTENT(INOUT) **C:** _MKL_DSS_HANDLE_t*	Pointer to the data structure storing intermediate DSS results (MKL_DSS_HANDLE).

Return Values

MKL_DSS_SUCCESS

MKL_DSS_STATE_ERR

MKL_DSS_INVALID_OPTION

MKL_DSS_STRUCTURE_ERR

MKL_DSS_ROW_ERR

MKL_DSS_COL_ERR

MKL_DSS_NOT_SQUARE

MKL_DSS_TOO_FEW_VALUES

MKL_DSS_TOO_MANY_VALUES

MKL_DSS_OUT_OF_MEMORY

MKL_DSS_MSG_LVL_ERR

MKL_DSS_TERM_LVL_ERR

dss_reorder

Computes or sets a permutation vector that minimizes the fill-in during the factorization phase.

Syntax

C:

dss_reorder(*handle*, *opt*, *perm*)

Fortran:

call dss_reorder(*handle*, *opt*, *perm*)

Include Files

- FORTRAN 77: mkl_dss.f77
- Fortran 90: mkl_dss.f90
- C: mkl_dss.h

Description

If *opt* contains the option MKL_DSS_AUTO_ORDER, then the routine dss_reorder computes a permutation vector that minimizes the fill-in during the factorization phase. For this option, the routine ignores the contents of the *perm* array.

If *opt* contains the option MKL_DSS_METIS_OPENMP_ORDER, then the routine dss_reorder computes permutation vector using the parallel (OpenMP) nested dissections algorithm to minimize the fill-in during the factorization phase. This option can be used to decrease the time of dss_reorder call on multi-core computers. For this option, the routine ignores the contents of the *perm* array.

If *opt* contains the option MKL_DSS_MY_ORDER, then you must supply a permutation vector in the array *perm*. In this case, the array *perm* is of length *nRows*, where *nRows* is the number of rows in the matrix as defined by the previous call to dss_define_structure.

If *opt* contains the option MKL_DSS_GET_ORDER, then the permutation vector computed during the dss_reorder call is copied to the array *perm*. In this case you must allocate the array *perm* beforehand. The permutation vector is computed in the same way as if the option MKL_DSS_AUTO_ORDER is set.

Input Parameters

Name	Type	Description
opt	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Parameter to pass the DSS options. The default value for the permutation type is MKL_DSS_AUTO_ORDER.
perm	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Array of length *nRows*. Contains a user-defined permutation vector (accessed only if *opt* contains MKL_DSS_MY_ORDER or MKL_DSS_GET_ORDER).

Output Parameters

Name	Type	Description
handle	**FORTRAN 77:** INTEGER*8 **Fortran 90:** TYPE (MKL_DSS_HANDLE), INTENT(INOUT) **C:** _MKL_DSS_HANDLE_t*	Pointer to the data structure storing intermediate DSS results (MKL_DSS_HANDLE).

Return Values

MKL_DSS_SUCCESS

MKL_DSS_STATE_ERR

MKL_DSS_INVALID_OPTION

MKL_DSS_REORDER_ERR

MKL_DSS_REORDER1_ERR

MKL_DSS_I32BIT_ERR

MKL_DSS_FAILURE

MKL_DSS_OUT_OF_MEMORY

MKL_DSS_MSG_LVL_ERR

MKL_DSS_TERM_LVL_ERR

dss_factor_real, dss_factor_complex

Compute factorization of the matrix with previously specified location of non-zero elements.

Syntax

C:

```
dss_factor_real(handle, opt, rValues)
dss_factor_complex(handle, opt, cValues)
```

Fortran 77:

```
call dss_factor_real(handle, opt, rValues)
call dss_factor_complex(handle, opt, cValues)
```

Fortran 90:

outputtext(unified Fortran 90 interface):

```
call dss_factor(handle, opt, Values)
```

outputtext(or FORTRAN 77 like interface):

```
call dss_factor_real(handle, opt, rValues)
call dss_factor_complex(handle, opt, cValues)
```

Include Files

- FORTRAN 77: `mkl_dss.f77`
- Fortran 90: `mkl_dss.f90`
- C: `mkl_dss.h`

Description

These routines compute factorization of the matrix whose non-zero locations were previously specified by a call to `dss_define_structure` and whose non-zero values are given in the array *rValues*, *cValues* or *Values*. Data type These arrays must be of length *nNonZeros* as defined in a previous call to `dss_define_structure`.

NOTE The data type (single or double precision) of *rValues*, *cValues*, *Values* must be in correspondence with precision specified by the parameter *opt* in the routine `dss_create`.

The *opt* argument can contain one of the following options:

- `MKL_DSS_POSITIVE_DEFINITE`
- `MKL_DSS_INDEFINITE`
- `MKL_DSS_HERMITIAN_POSITIVE_DEFINITE`
- `MKL_DSS_HERMITIAN_INDEFINITE`

depending on your matrix's type.

NOTE This routine supports the Progress Routine feature. See Progress Function section for details.

Input Parameters

Name	Type	Description
handle	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `TYPE (MKL_DSS_HANDLE), INTENT(INOUT)` **C:** `_MKL_DSS_HANDLE_t*`	Pointer to the data structure storing intermediate DSS results (`MKL_DSS_HANDLE`).
opt	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `_INTEGER_t const*`	Parameter to pass the DSS options. The default value is `MKL_DSS_POSITIVE_DEFINITE`.
rValues	**FORTRAN 77:** `REAL*4` or `REAL*8`	Array of elements of the matrix *A*. Real data, single or double precision as it is specified by the parameter *opt* in the routine `dss_create`.

Name	Type	Description
	Fortran 90: `REAL(KIND=4), INTENT(IN)` or `REAL(KIND=8), INTENT(IN)` **C:** `VOID const*`	
cValues	**FORTRAN 77:** `COMPLEX*8` or `COMPLEX*16` **Fortran 90:** `COMPLEX(KIND=4), INTENT(IN)` or `COMPLEX(KIND=8), INTENT(IN)` **C:** `VOID const*`	Array of elements of the matrix *A*. Complex data, single or double precision as it is specified by the parameter *opt* in the routine `dss_create`.
Values	**Fortran 90:** `REAL(KIND=4), INTENT(OUT)`, or `REAL(KIND=8), INTENT(OUT)`, or `COMPLEX(KIND=4), INTENT(OUT)`, or `COMPLEX(KIND=8), INTENT(OUT)`	Array of elements of the matrix *A*. Real or complex data, single or double precision as it is specified by the parameter *opt* in the routine `dss_create`.

Return Values

`MKL_DSS_SUCCESS`

`MKL_DSS_STATE_ERR`

`MKL_DSS_INVALID_OPTION`

`MKL_DSS_OPTION_CONFLICT`

`MKL_DSS_VALUES_ERR`

`MKL_DSS_OUT_OF_MEMORY`

`MKL_DSS_ZERO_PIVOT`

`MKL_DSS_FAILURE`

`MKL_DSS_MSG_LVL_ERR`

`MKL_DSS_TERM_LVL_ERR`

`MKL_DSS_OOC_MEM_ERR`

`MKL_DSS_OOC_OC_ERR`

`MKL_DSS_OOC_RW_ERR`

See Also

mkl_progress

dss_solve_real, dss_solve_complex

Compute the corresponding solution vector and place it in the output array.

Syntax

C:

```
dss_solve_real(handle, opt, rRhsValues, nRhs, rSolValues)
dss_solve_complex(handle, opt, cRhsValues, nRhs, cSolValues)
```

Fortran 77:

```
call dss_solve_real(handle, opt, rRhsValues, nRhs, rSolValues)
call dss_solve_complex(handle, opt, cRhsValues, nRhs, cSolValues)
```

Fortran 90:

outputtext(unified Fortran 90 interface):

```
call dss_solve(handle, opt, RhsValues, nRhs, SolValues)
```

outputtext(or FORTRAN 77 like interface):

```
call dss_solve_real(handle, opt, rRhsValues, nRhs, rSolValues)
call dss_solve_complex(handle, opt, cRhsValues, nRhs, cSolValues)
```

Include Files

- FORTRAN 77: `mkl_dss.f77`
- Fortran 90: `mkl_dss.f90`
- C: `mkl_dss.h`

Description

For each right hand side column vector defined in the arrays *rRhsValues*, *cRhsValues*, or *RhsValues*, these routines compute the corresponding solution vector and place it in the arrays *rSolValues*, *cSolValues*, or *SolValues* respectively.

NOTE The data type (single or double precision) of all arrays must be in correspondence with precision specified by the parameter *opt* in the routine `dss_create`.

The lengths of the right-hand side and solution vectors, *nCols* and *nRows* respectively, must be defined in a previous call to dss_define_structure.

By default, both routines perform the full solution step (it corresponds to *phase* = 33 in PARDISO). The parameter *opt* enables you to calculate the final solution step-by-step, calling forward and backward substitutions.

If it is set to `MKL_DSS_FORWARD_SOLVE`, the forward substitution (corresponding to *phase* = 331 in PARDISO) is performed;

if it is set to `MKL_DSS_DIAGONAL_SOLVE`, the diagonal substitution (corresponding to *phase* = 332 in PARDISO) is performed;

if it is set to `MKL_DSS_BACKWARD_SOLVE`, the backward substitution (corresponding to *phase* = 333 in PARDISO) is performed.

For more details about using these substitutions for different types of matrices, see the description of the PARDISO solver.

This parameter also can control the number of refinement steps that is used on the solution stage: if it is set to `MKL_DSS_REFINEMENT_OFF`, the maximum number of refinement steps equal to zero, and if it is set to `MKL_DSS_REFINEMENT_ON` (default value), the maximum number of refinement steps is equal to 2.

MKL_DSS_CONJUGATE_SOLVE option added to the parameter *opt* enables solving a conjugate transposed system $A^H x = b$ based on the factorization of the matrix *A*. This option is equivalent to the parameter *iparm*(12)= 1 in PARDISO.

MKL_DSS_TRANSPOSE_SOLVE option added to the parameter *opt* enables solving a transposed system $A^T x = b$ based on the factorization of the matrix *A*. This option is equivalent to the parameter *iparm*(12)= 2 in PARDISO.

Input Parameters

Name	Type	Description
handle	**FORTRAN 77:** INTEGER*8 **Fortran 90:** TYPE (MKL_DSS_HANDLE), INTENT(INOUT) **C:** _MKL_DSS_HANDLE_t*	Pointer to the data structure storing intermediate DSS results (MKL_DSS_HANDLE).
opt	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Parameter to pass the DSS options.
nRhs	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Number of the right-hand sides in the linear equation.
rRhsValues	**FORTRAN 77:** REAL*4 or REAL*8 **Fortran 90:** REAL(KIND=4), INTENT(IN) or REAL(KIND=8), INTENT(IN) **C:** VOID const*	Array of size *nRows * nRhs*. Contains real right-hand side vectors. Real data, single or double precision as it is specified by the parameter *opt* in the routine dss_create.
cRhsValues	**FORTRAN 77:** COMPLEX*8 or COMPLEX*16 **Fortran 90:** COMPLEX(KIND=4), INTENT(IN) or COMPLEX(KIND=8), INTENT(IN) **C:** VOID const*	Array of size *nRows * nRhs*. Contains complex right-hand side vectors. Complex data, single or double precision as it is specified by the parameter *opt* in the routine dss_create.
RhsValues	**Fortran 90:** REAL(KIND=4), INTENT(IN), or REAL(KIND=8), INTENT(IN), or COMPLEX(KIND=4), INTENT(IN), or COMPLEX(KIND=8), INTENT(IN)	Array of size *nRows * nRhs*. Contains right-hand side vectors. Real or complex data, single or double precision as it is specified by the parameter *opt* in the routine dss_create.

Output Parameters

Name	Type	Description
rSolValues	**FORTRAN 77:** REAL*4 or REAL*8 **Fortran 90:** REAL(KIND=4), INTENT(OUT) or REAL(KIND=8), INTENT(OUT) **C:** VOID const*	Array of size *nCols * nRhs*. Contains real solution vectors. Real data, single or double precision as it is specified by the parameter *opt* in the routine dss_create.
cSolValues	**FORTRAN 77:** COMPLEX*8 or COMPLEX*16 **Fortran 90:** COMPLEX(KIND=4), INTENT(OUT) or COMPLEX(KIND=8), INTENT(OUT) **C:** VOID const*	Array of size *nCols * nRhs*. Contains complex solution vectors. Complex data, single or double precision as it is specified by the parameter *opt* in the routine dss_create.
SolValues	**Fortran 90:** REAL(KIND=4), INTENT(OUT), or REAL(KIND=8), INTENT(OUT), or COMPLEX(KIND=4), INTENT(OUT), or COMPLEX(KIND=8), INTENT(OUT)	Array of size *nCols * nRhs*. Contains solution vectors. Real or complex data, single or double precision as it is specified by the parameter *opt* in the routine dss_create.

Return Values

MKL_DSS_SUCCESS

MKL_DSS_STATE_ERR

MKL_DSS_INVALID_OPTION

MKL_DSS_OUT_OF_MEMORY

MKL_DSS_DIAG_ERR

MKL_DSS_FAILURE

MKL_DSS_MSG_LVL_ERR

MKL_DSS_TERM_LVL_ERR

MKL_DSS_OOC_MEM_ERR

MKL_DSS_OOC_OC_ERR

MKL_DSS_OOC_RW_ERR

dss_delete

Deletes all of the data structures created during the solutions process.

Syntax

C:

dss_delete(*handle*, *opt*)

Fortran:

```
call dss_delete(handle, opt)
```

Include Files

- FORTRAN 77: `mkl_dss.f77`
- Fortran 90: `mkl_dss.f90`
- C: `mkl_dss.h`

Description

The routine `dss_delete` deletes all data structures created during the solving process.

Input Parameters

Name	Type	Description
opt	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `_INTEGER_t const*`	Parameter to pass the DSS options. The default value is `MKL_DSS_MSG_LVL_WARNING + MKL_DSS_TERM_LVL_ERROR`.

Output Parameters

Name	Type	Description
handle	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `TYPE (MKL_DSS_HANDLE), INTENT(INOUT)` **C:** `_MKL_DSS_HANDLE_t*`	Pointer to the data structure storing intermediate DSS results (`MKL_DSS_HANDLE`).

Return Values

`MKL_DSS_SUCCESS`

`MKL_DSS_STATE_ERR`

`MKL_DSS_INVALID_OPTION`

`MKL_DSS_OUT_OF_MEMORY`

`MKL_DSS_MSG_LVL_ERR`

`MKL_DSS_TERM_LVL_ERR`

dss_statistics

Returns statistics about various phases of the solving process.

Syntax

C:

```
dss_statistics(handle, opt, statArr, retValues)
```

Fortran:

```
call dss_statistics(handle, opt, statArr, retValues)
```

Include Files

- FORTRAN 77: `mkl_dss.f77`
- Fortran 90: `mkl_dss.f90`
- C: `mkl_dss.h`

Description

The `dss_statistics` routine returns statistics about various phases of the solving process. This routine gathers the following statistics:

- time taken to do reordering,
- time taken to do factorization,
- duration of problem solving,
- determinant of the input matrix,
- inertia of the input matrix,
- number of floating point operations taken during factorization,
- total peak memory needed during the analysis and symbolic factorization,
- permanent memory needed from the analysis and symbolic factorization,
- memory consumption for the factorization and solve phases.

Statistics are returned in accordance with the input string specified by the parameter *statArr*. The value of the statistics is returned in double precision in a return array, which you must allocate beforehand.

For multiple statistics, multiple string constants separated by commas can be used as input. Return values are put into the return array in the same order as specified in the input string.

Statistics can only be requested at the appropriate stages of the solving process. For example, requesting `FactorTime` before a matrix is factored leads to errors.

The following table shows the point at which each individual statistics item can be requested:

Statistics Calling Sequences

Type of Statistics	When to Call
`ReorderTime`	After `dss_reorder` is completed successfully.
`FactorTime`	After `dss_factor_real` or `dss_factor_complex` is completed successfully.
`SolveTime`	After `dss_solve_real` or `dss_solve_complex` is completed successfully.
`Determinant`	After `dss_factor_real` or `dss_factor_complex` is completed successfully.
`Inertia`	After `dss_factor_real` is completed successfully and the matrix is real, symmetric, and indefinite.
`Flops`	After `dss_factor_real` or `dss_factor_complex` is completed successfully.
`Peakmem`	After `dss_reorder` is completed successfully.
`Factormem`	After `dss_reorder` is completed successfully.
`Solvemem`	After `dss_factor_real` or `dss_factor_complex` is completed successfully.

Input Parameters

Name	Type	Description
handle	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `TYPE (MKL_DSS_HANDLE), INTENT(IN)`	Pointer to the data structure storing intermediate DSS results (`MKL_DSS_HANDLE`).

Name	Type	Description
	C: _MKL_DSS_HANDLE_t*	
opt	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** _INTEGER_t const*	Parameter to pass the DSS options.
statArr	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** char const*	Input string that defines the type of the returned statistics. The parameter can include one or more of the following string constants (case of the input string has no effect): ReorderTime — Amount of time taken to do the reordering. FactorTime — Amount of time taken to do the factorization. SolveTime — Amount of time taken to solve the problem after factorization. Determinant — Determinant of the matrix *A*. For real matrices: the determinant is returned as det_pow, det_base in two consecutive return array locations, where 1.0 ≤ abs(*det_base*) < 10.0 and determinant = *det_base*$*10^{(det\_pow)}$. For complex matrices: the determinant is returned as *det_pow*, *det_re*, *det_im* in three consecutive return array locations, where 1.0 ≤abs(*det_re*) + abs(*det_im*) < 10.0 and determinant = *det_re*, *det_im*$*10^{(det\_pow)}$. Inertia — Inertia of a real symmetric matrix is defined as a triplet of nonnegative integers (*p*,*n*,*z*), where *p* is the number of positive eigenvalues, *n* is the number of negative eigenvalues, and *z* is the number of zero eigenvalues. Inertia is returned as three consecutive return array locations *p*, *n*, *z*. Computing inertia is recommended only for stable matrices. Unstable matrices can lead to incorrect results. Inertia of a *k*-by-*k* real symmetric positive definite matrix is always (*k*, 0, 0). Therefore Inertia is returned only in cases of real symmetric indefinite matrices. For all other matrix types, an error message is returned. Flops — Number of floating point operations performed during the factorization. Peakmem — Total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase.

Name	Type	Description
		`Factormem` Permanent memory in kilobytes that the solver needs from the analysis and symbolic factorization phase in the factorization and solve phases.
		`Solvemem` Total double precision memory consumption (kilobytes) of the solver for the factorization and solve phases.

NOTE To avoid problems in passing strings from Fortran to C, Fortran users must call the `mkl_cvt_to_null_terminated_str` routine before calling `dss_statistics`. Refer to the description of `mkl_cvt_to_null_terminated_str` for details.

Output Parameters

Name	Type	Description
retValues	**FORTRAN 77:** `REAL*8` **Fortran 90:** `REAL(KIND=8), INTENT(OUT)` **C:** `VOID const*`	Value of the statistics returned.

Finding 'time used to reorder' and 'inertia' of a matrix

The example below illustrates the use of the `dss_statistics` routine.

To find the above values, call `dss_statistics`(*handle*, *opt*, *statArr*, *retValue*), where *staArr* is "ReorderTime,Inertia"

In this example, *retValue* has the following values:

retValue[0]	Time to reorder.
retValue[1]	Positive Eigenvalues.
retValue[2]	Negative Eigenvalues.
retValue[3]	Zero Eigenvalues.

Return Values

`MKL_DSS_SUCCESS`

`MKL_DSS_INVALID_OPTION`

`MKL_DSS_STATISTICS_INVALID_MATRIX`

`MKL_DSS_STATISTICS_INVALID_STATE`

`MKL_DSS_STATISTICS_INVALID_STRING`

`MKL_DSS_MSG_LVL_ERR`

`MKL_DSS_TERM_LVL_ERR`

mkl_cvt_to_null_terminated_str

Passes character strings from Fortran routines to C routines.

Syntax

`mkl_cvt_to_null_terminated_str (destStr, destLen, srcStr)`

Include Files

- FORTRAN 77: `mkl_dss.f77`
- Fortran 90: `mkl_dss.f90`

Description

The routine `mkl_cvt_to_null_terminated_str` passes character strings from Fortran routines to C routines. The strings are converted into integer arrays before being passed to C. Using this routine avoids the problems that can occur on some platforms when passing strings from Fortran to C. The use of this routine is highly recommended.

Input Parameters

destLen	`INTEGER`. Length of the output array *destStr*.
srcStr	`STRING`. Input string.

Output Parameters

destStr	`INTEGER`. One-dimensional array of integers.

Implementation Details

Several aspects of the Intel MKL DSS interface are platform-specific and language-specific. To promote portability across platforms and ease of use across different languages, one of the following Intel MKL DSS language-specific header files can be included:

- `mkl_dss.f77` for F77 programs
- `mkl_dss.f90` for F90 programs
- `mkl_dss.h` for C programs

These header files define symbolic constants for returned error values, function options, certain defined data types, and function prototypes.

NOTE Constants for options, returned error values, and message severities must be referred only by the symbolic names that are defined in these header files. Use of the Intel MKL DSS software without including one of the above header files is not supported.

Memory Allocation and Handles

To simplify the use of the Intel MKL DSS routines, they do not require you to allocate any temporary working storage. The solver itself allocates any required storage. To enable multiple users to access the solver simultaneously, the solver keeps track of the storage allocated for a particular application by using a data object called a *handle*.

Each of the Intel MKL DSS routines creates, uses or deletes a handle. Consequently, each program must be able to allocate storage for a handle. The exact syntax for allocating storage for a handle varies from language to language. To standardize the handle declarations, the language-specific header files declare constants and defined data types that must be used when declaring a handle object in the user code.

Fortran 90 programmers must declare a handle as:

```
INCLUDE "mkl_dss.f90"

TYPE(MKL_DSS_HANDLE) handle
```

C and C++ programmers must declare a handle as:

```
#include "mkl_dss.h"

_MKL_DSS_HANDLE_t handle;
```

FORTRAN 77 programmers using compilers that support eight byte integers, must declare a handle as:

```
INCLUDE "mkl_dss.f77"

INTEGER*8 handle
```

Otherwise they can replace the `INTEGER*8` data types with the `DOUBLE PRECISION` data type.

In addition to the definition for the correct declaration of a handle, the include file also defines the following:

- function prototypes for languages that support prototypes
- symbolic constants that are used for the returned error values
- user options for the solver routines
- constants indicating message severity.

Iterative Sparse Solvers based on Reverse Communication Interface (RCI ISS)

Intel MKL supports the iterative sparse solvers (ISS) based on the reverse communication interface (RCI), referred to here as RCI ISS interface . The RCI ISS interface implements a group of user-callable routines that are used in the step-by-step solving process of a symmetric positive definite system (RCI conjugate gradient solver, or RCI CG), and of a non-symmetric indefinite (non-degenerate) system (RCI flexible generalized minimal residual solver, or RCI FGMRES) of linear algebraic equations. This interface uses the general RCI scheme described in [Dong95].

See the Appendix A Linear Solvers Basics for discussion of terms and concepts related to the ISS routines.

RCI means that when the solver needs the results of certain operations (for example, matrix-vector multiplications), the user must perform them and pass the result to the solver. This gives great universality to the solver as it is independent of the specific implementation of the operations like the matrix-vector multiplication. To perform such operations, the user can use the built-in sparse matrix-vector multiplications and triangular solvers routines (see Sparse BLAS Level 2 and Level 3 Routines).

NOTE The RCI CG solver is implemented in two versions: for system of equations with a single right hand side, and for system of equations with multiple right hand sides.

The CG method may fail to compute the solution or compute the wrong solution if the matrix of the system is not symmetric and positive definite.

The FGMRES method may fail if the matrix is degenerate.

Table "RCI CG Interface Routines" lists the names of the routines, and describes their general use.

RCI ISS Interface Routines

Routine	Description
dcg_init, dcgmrhs_init, dfgmres_init	Initializes the solver.
dcg_check, dcgmrhs_check, dfgmres_check	Checks the consistency and correctness of the user defined data.
dcg, dcgmrhs, dfgmres	Computes the approximate solution vector.
dcg_get, dcgmrhs_get, dfgmres_get	Retrieves the number of the current iteration.

The Intel MKL RCI ISS interface routines are normally invoked in this order:

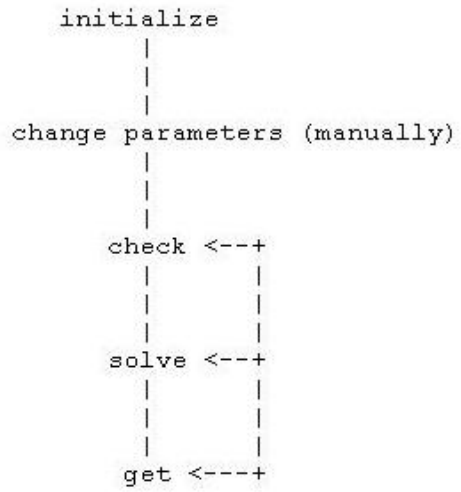
1. `<type>_init`
2. `<type>_check`
3. `<type>`

4. `<type>_get`

Advanced users can change that order if they need it. Others should follow the above order of calls.

The following diagram in Figure "Typical Order for Invoking RCI ISS Interface Routines" indicates the typical order in which the RCI ISS interface routines can be invoked.

Typical Order for Invoking RCI ISS interface Routines



Code examples that use the RCI ISS interface routines to solve systems of linear equations can be found in the `examples\solver\source` folder of your Intel MKL directory (`cg_no_precon.f`, `cg_no_precon_c.c`, `cg_mrhs.f`, `cg_mrhs_precond.f`, `cg_mrhs_stop_crt.f`, `fgmres_no_precon_f.f`, `fgmres_no_precon_c.c`).

CG Interface Description

All types in this documentation refer to the common Fortran types, `INTEGER`, and `DOUBLE PRECISION`.

C and C++ programmers should refer to the section Calling Sparse Solver and Preconditioner Routines from C C++ for information on mapping Fortran types to C/C++ types.

Each routine for the RCI CG solver is implemented in two versions: for a system of equations with a single right hand side (SRHS), and for a system of equations with multiple right hand sides (MRHS). The names of routines for a system with MRHS contain the suffix `mrhs`.

Routine Options

All of the RCI CG routines have common parameters for passing various options to the routines (see CG Common Parameters). The values for these parameters can be changed during computations.

User Data Arrays

Many of the RCI CG routines take arrays of user data as input. For example, user arrays are passed to the routine dcg to compute the solution of a system of linear algebraic equations. The Intel MKL RCI CG routines do not make copies of the user input arrays to minimize storage requirements and improve overall run-time efficiency.

CG Common Parameters

NOTE The default and initial values listed below are assigned to the parameters by calling the dcg_init/dcgmrhs_init routine.

n INTEGER, this parameter sets the size of the problem in the dcg_init/dcgmrhs_init routine. All the other routines uses the *ipar*(1) parameter instead. Note that the coefficient matrix *A* is a square matrix of size *n***n*.

x DOUBLE PRECISION array of size *n* for SRHS, or matrix of size (*n***nrhs*) for MRHS. This parameter contains the current approximation to the solution. Before the first call to the dcg/dcgmrhs routine, it contains the initial approximation to the solution.

nrhs INTEGER, this parameter sets the number of right-hand sides for MRHS routines.

b DOUBLE PRECISION array containing a single right-hand side vector, or matrix of size (*nrhs***n*) containing right-hand side vectors.

RCI_request INTEGER, this parameter gives information about the result of work of the RCI CG routines. Negative values of the parameter indicate that the routine completed with errors or warnings. The 0 value indicates successful completion of the task. Positive values mean that you must perform specific actions:

- *RCI_request*= 1 — multiply the matrix by *tmp* (1:*n*,1), put the result in *tmp*(1:*n*,2), and return the control to the dcg/dcgmrhs routine;
- *RCI_request*= 2 — to perform the stopping tests. If they fail, return the control to the dcg/dcgmrhs routine. If the stopping tests succeed, it indicates that the solution is found and stored in the *x* array;
- *RCI_request*= 3 — for SRHS: apply the preconditioner to *tmp*(1:*n*,3), put the result in *tmp*(1:*n*,4), and return the control to the dcg routine;
  for MRHS: apply the preconditioner to *tmp*(:, 3+*ipar*(3)), put the result in *tmp*(:,3), and return the control to the dcgmrhs routine.

Note that the dcg_get/dcgmrhs_get routine does not change the parameter *RCI_request*. This enables use of this routine inside the *reverse communication* computations.

ipar INTEGER array, of size 128 for SRHS, and of size (128+2**nrhs*) for MRHS. This parameter specifies the integer set of data for the RCI CG computations:

- *ipar*(1) — specifies the size of the problem. The dcg_init/dcgmrhs_init routine assigns *ipar*(1)=*n*. All the other routines use this parameter instead of *n*. There is no default value for this parameter.

ipar(2)	specifies the type of output for error and warning messages generated by the RCI CG routines. The default value 6 means that all messages are displayed on the screen. Otherwise, the error and warning messages are written to the newly created files `dcg_errors.txt` and `dcg_check_warnings.txt`, respectively. Note that if *ipar*(6) and *ipar*(7) parameters are set to 0, error and warning messages are not generated at all.
ipar(3)	for SRHS: contains the current stage of the RCI CG computations. The initial value is 1; for MRHS: contains the right-hand side for which the calculations are currently performed.  **WARNING** Avoid altering this variable during computations.
ipar(4)	contains the current iteration number. The initial value is 0.
ipar(5)	specifies the maximum number of iterations. The default value is `min(150, n)`.
ipar(6)	if the value is not equal to 0, the routines output error messages in accordance with the parameter *ipar*(2). Otherwise, the routines do not output error messages at all, but return a negative value of the parameter *RCI_request*. The default value is 1.
ipar(7)	if the value is not equal to 0, the routines output warning messages in accordance with the parameter *ipar*(2). Otherwise, the routines do not output warning messages at all, but they return a negative value of the parameter *RCI_request*. The default value is 1.
ipar(8)	if the value is not equal to 0, the `dcg`/`dcgmrhs` routine performs the stopping test for the maximum number of iterations: *ipar*(4) ≤ *ipar*(5). Otherwise, the method is stopped and the corresponding value is assigned to the *RCI_request*. If the value is 0, the routine does not perform this stopping test. The default value is 1.
ipar(9)	if the value is not equal to 0, the `dcg`/`dcgmrhs` routine performs the residual stopping test: *dpar*(5) ≤ *dpar*(4)= *dpar*(1)**dpar*(3)+ *dpar*(2). Otherwise, the method is stopped and corresponding value is assigned to the *RCI_request*. If the value is 0, the routine does not perform this stopping test. The default value is 0.
ipar(10)	if the value is not equal to 0, the `dcg`/`dcgmrhs` routine requests a user-defined stopping test by setting the output parameter *RCI_request*=2. If the value is 0, the routine does not perform the user defined stopping test. The default value is 1.

NOTE At least one of the parameters *ipar*(8)-*ipar*(10) must be set to 1.

ipar(11) — if the value is equal to 0, the dcg/dcgmrhs routine runs the non-preconditioned version of the corresponding CG method. Otherwise, the routine runs the preconditioned version of the CG method, and by setting the output parameter *RCI_request*=3, indicates that you must perform the preconditioning step. The default value is 0.

ipar(11:128), *ipar*(11:128+2**nrhs*) — are reserved and not used in the current RCI CG SRHS and MRHS routines.

NOTE You must declare the array *ipar* with length 128. While defining the array in the code using RCI CG SRHS as INTEGER *ipar*(11) works, there is no guarantee of future compatibility with Intel MKL.

dpar — DOUBLE PRECISION array, for SRHS of size 128, for MRHS of size (128+2**nrhs*); this parameter is used to specify the double precision set of data for the RCI CG computations, specifically:

dpar(1) — specifies the relative tolerance. The default value is 1.0D-6.

dpar(2) — specifies the absolute tolerance. The default value is 0.0D-0.

dpar(3) — specifies the square norm of the initial residual (if it is computed in the dcg/dcgmrhs routine). The initial value is 0.

dpar(4) — service variable equal to *dpar*(1)**dpar*(3)+*dpar*(2) (if it is computed in the dcg/dcgmrhs routine). The initial value is 0.

dpar(5) — - specifies the square norm of the current residual. The initial value is 0.0.

dpar(6) — specifies the square norm of residual from the previous iteration step (if available). The initial value is 0.0.

dpar(7) — contains the *alpha* parameter of the CG method. The initial value is 0.0.

dpar(8) — contains the *beta* parameter of the CG method, it is equal to *dpar*(5)/*dpar*(6) The initial value is 0.0.

dpar(9:128), *dpar*(9:128+2**nrhs*) — are reserved and not used in the current RCI CG SRHS and MRHS routines respectively.

NOTE You must declare the array *dpar* with length 128. While defining the array in the code using RCI CG SRHS as `DOUBLE PRECISION dpar(8)` works, there is no guarantee of future compatibility with Intel MKL.

tmp `DOUBLE PRECISION` array of size `(n,4)` for SRHS, and `(n,3+nrhs)` for MRHS. This parameter is used to supply the double precision temporary space for the RCI CG computations, specifically:

`tmp(:,1)`	specifies the current search direction. The initial value is 0.0.
`tmp(:,2)`	contains the matrix multiplied by the current search direction. The initial value is 0.0.
`tmp(:,3)`	contains the current residual. The initial value is 0.0.
`tmp(:,4)`	contains the inverse of the preconditioner applied to the current residual. There is no initial value for this parameter.

NOTE You can define this array in the code using RCI CG SRHS as `DOUBLE PRECISION tmp(n,3)` if you run only non-preconditioned CG iterations.

Schemes of Using the RCI CG Routines

The following pseudocode shows the general schemes of using the RCI CG routines.

```
...
generate matrix A
generate preconditioner C (optional)
   call dcg_init(n, x, b, RCI_request, ipar, dpar, tmp)
   change parameters in ipar, dpar if necessary
   call dcg_check(n, x, b, RCI_request, ipar, dpar, tmp)
1  call dcg(n, x, b, RCI_request, ipar, dpar, tmp)
   if (RCI_request.eq.1) then
    multiply the matrix A by tmp(1:n,1) and put the result in tmp(1:n,2)
   It is possible to use MKL Sparse BLAS Level 2 subroutines for this operation
c  proceed with CG iterations
     goto 1
   endif
   if (RCI_request.eq.2)then
    do the stopping test
    if (test not passed) then
c  proceed with CG iterations
```

```
     go to 1
    else
c  stop CG iterations
     goto 2
    endif
   endif
   if (RCI_request.eq.3) then (optional)
   apply the preconditioner C inverse to tmp(1:n,3) and put the result in tmp(1:n,4)
c  proceed with CG iterations
    goto 1
   end
```

2 call dcg_get(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*, *itercount*)

current iteration number is in *itercount*

the computed approximation is in the array *x*

FGMRES Interface Description

All types in this documentation refer to the common Fortran types, INTEGER, and DOUBLE PRECISION.

C and C++ programmers should refer to the section Calling Sparse Solver and Preconditioner Routines from C C++ for information on mapping Fortran types to C/C++ types.

Routine Options

All of the RCI FGMRES routines have common parameters for passing various options to the routines (see FGMRES Common Parameters). The values for these parameters can be changed during computations.

User Data Arrays

Many of the RCI FGMRES routines take arrays of user data as input. For example, user arrays are passed to the routine dfgmres to compute the solution of a system of linear algebraic equations. To minimize storage requirements and improve overall run-time efficiency, the Intel MKL RCI FGMRES routines do not make copies of the user input arrays.

FGMRES Common Parameters

NOTE The default and initial values listed below are assigned to the parameters by calling the dfgmres_init routine.

n — INTEGER, this parameter sets the size of the problem in the dfgmres_init routine. All the other routines uses *ipar*(1) parameter instead. Note that the coefficient matrix *A* is a square matrix of size *n***n*.

x DOUBLE PRECISION array, this parameter contains the current approximation to the solution vector. Before the first call to the dfgmres routine, it contains the initial approximation to the solution vector.

b DOUBLE PRECISION array, this parameter contains the right-hand side vector. Depending on user requests (see the parameter *ipar*(13), it may later contain the approximate solution.

RCI_request INTEGER, this parameter gives information about the result of work of the RCI FGMRES routines. Negative values of the parameter indicate that the routine completed with errors or warnings. The 0 value indicates successful completion of the task. Positive values mean that you must perform specific actions:

- *RCI_request*= 1 — multiply the matrix by *tmp*(*ipar*(22)), put the result in *tmp*(*ipar*(23)), and return the control to the dfgmres routine;
- *RCI_request*= 2 — perform the stopping tests. If they fail, return the control to the dfgres routine. Otherwise, the solution can be updated by a subsequent call to dfgmres_get routine;
- *RCI_request*= 3 — apply the preconditioner to *tmp*(*ipar*(22)), put the result in *tmp*(*ipar*(23)), and return the control to the dfgmres routine.
- *RCI_request*= 4 — check if the norm of the current orthogonal vector is zero, within the rounding or computational errors. Return the control to the dfgmres routine if it is not zero, otherwise complete the solution process by calling dfgmres_get routine.

ipar(128) INTEGER array, this parameter specifies the integer set of data for the RCI FGMRES computations:

- *ipar*(1) — specifies the size of the problem. The dfgmres_init routine assigns *ipar*(1)=*n*. All the other routines uses this parameter instead of *n*. There is no default value for this parameter.
- *ipar*(2) — specifies the type of output for error and warning messages that are generated by the RCI FGMRES routines. The default value 6 means that all messages are displayed on the screen. Otherwise the error and warning messages are written to the newly created file MKL_RCI_FGMRES_Log.txt. Note that if *ipar*(6) and *ipar*(7) parameters are set to 0, error and warning messages are not generated at all.
- *ipar*(3) — contains the current stage of the RCI FGMRES computations., The initial value is 1.

  WARNING Avoid altering this variable during computations.

- *ipar*(4) — contains the current iteration number. The initial value is 0.

ipar(5)	specifies the maximum number of iterations. The default value is *min* (150,*n*).
ipar(6)	if the value is not 0, the routines output error messages in accordance with the parameter *ipar*(2). If it is 0, the routines do not output error messages at all, but return a negative value of the parameter *RCI_request*. The default value is 1.
ipar(7)	if the value is not 0, the routines output warning messages in accordance with the parameter *ipar*(2). Otherwise, the routines do not output warning messages at all, but they return a negative value of the parameter *RCI_request*. The default value is 1.
ipar(8)	if the value is not equal to 0, the dfmres routine performs the stopping test for the maximum number of iterations: *ipar*(4) ≤ *ipar*(5). If the value is 0, the dfgmres routine does not perform this stopping test. The default value is 1.
ipar(9)	if the value is not 0, the dfgmres routine performs the residual stopping test: *dpar*(5) ≤ *dpar*(4).If the value is 0, the dfgmres routine does not perform this stopping test. The default value is 0.
ipar(10)	if the value is not 0, the dfgmres routine indicates that the user-defined stopping test be performed by setting *RCI_request*=2. If the value is 0, the dfgmres routine does not perform the user-defined stopping test. The default value is 1.

NOTE At least one of the parameters *ipar*(8)-*ipar*(10) must be set to 1.

ipar(11)	if the value is 0, the dfgmres routine runs the non-preconditioned version of the FGMRES method. Otherwise, the routine runs the preconditioned version of the FGMRES method, and requests that you perform the preconditioning step by setting the output parameter *RCI_request*=3. The default value is 0.
ipar(12)	if the value is not equal to 0, the dfgmres routine performs the automatic test for zero norm of the currently generated vector: *dpar*(7) ≤ *dpar*(8), where *dpar*(8) contains the tolerance value. Otherwise, the routine indicates that you must perform this check by setting the output parameter *RCI_request*=4. The default value is 0.
ipar(13)	if the value is equal to 0, the dfgmres_get routine updates the solution to the vector *x* according to the computations done by the dfgmres routine. If the value is positive, the routine writes the solution to the right hand side vector *b*. If the value is

negative, the routine returns only the number of the current iteration, and does not update the solution. The default value is 0.

NOTE It is possible to call the `dfgmres_get` routine at any place in the code, but you must pay special attention to the parameter *ipar*(13). The RCI FGMRES iterations can be continued after the call to `dfgmres_get` routine only if the parameter *ipar*(13) is not equal to zero. If *ipar*(13) is positive, then the updated solution overwrites the right hand side in the vector *b*. If you want to run the restarted version of FGMRES with the same right hand side, then it must be saved in a different memory location before the first call to the `dfgmres_get` routine with positive *ipar*(13).

ipar(14) contains the internal iteration counter that counts the number of iterations before the restart takes place. The initial value is 0.

WARNING Do not alter this variable during computations.

ipar(15) specifies the number of the non-restarted FGMRES iterations. To run the restarted version of the FGMRES method, assign the number of iterations to *ipar*(15) before the restart. The default value is *min*(150, *n*), which means that by default the non-restarted version of FGMRES method is used.

ipar(16) service variable specifying the location of the rotated Hessenberg matrix from which the matrix stored in the packed format (see Matrix Arguments in the Appendix B for details) is started in the *tmp* array.

ipar(17) service variable specifying the location of the rotation cosines from which the vector of cosines is started in the *tmp* array.

ipar(18) service variable specifying the location of the rotation sines from which the vector of sines is started in the *tmp* array.

ipar(19) service variable specifying the location of the rotated residual vector from which the vector is started in the *tmp* array.

ipar(20) service variable, specifies the location of the least squares solution vector from which the vector is started in the *tmp* array.

ipar(21) service variable specifying the location of the set of preconditioned vectors from which the set is started in the *tmp* array. The memory locations in the *tmp* array starting from *ipar*(21) are used only for the preconditioned FGMRES method.

ipar(22) specifies the memory location from which the first vector (source) used in operations requested via *RCI_request* is started in the *tmp* array.

ipar(23) specifies the memory location from which the second vector (source) used in operations requested via *RCI_request* is started in the *tmp* array.

ipar(24:128) are reserved and not used in the current RCI FGMRES routines.

NOTE You must declare the array *ipar* with length 128. While defining the array in the code as INTEGER *ipar*(23) works, there is no guarantee of future compatibility with Intel MKL.

dpar(128) DOUBLE PRECISION array, this parameter specifies the double precision set of data for the RCI CG computations, specifically:

dpar(1) specifies the relative tolerance. The default value is 1.0D-6.

dpar(2) specifies the absolute tolerance. The default value is 0.0D-0.

dpar(3) specifies the Euclidean norm of the initial residual (if it is computed in the dfgmres routine). The initial value is 0.0.

dpar(4) service variable equal to *dpar*(1)***dpar*(3)+*dpar*(2) (if it is computed in the dfgmres routine). The initial value is 0.0.

dpar(5) specifies the Euclidean norm of the current residual. The initial value is 0.0.

dpar(6) specifies the Euclidean norm of residual from the previous iteration step (if available). The initial value is 0.0.

dpar(7) contains the norm of the generated vector. The initial value is 0.0.

NOTE In terms of [Saad03] this parameter is the coefficient $h_{k+1,k}$ of the Hessenberg matrix.

dpar(8) contains the tolerance for the zero norm of the currently generated vector. The default value is 1.0D-12.

dpar(9:128) are reserved and not used in the current RCI FGMRES routines.

NOTE You must declare the array *dpar* with length 128. While defining the array in the code as DOUBLE PRECISION *dpar*(8) works, there is no guarantee of future compatibility with Intel MKL.

tmp DOUBLE PRECISION array of size ((2**ipar*(15)+1)**n* + *ipar*(15)*(*ipar*(15)+9)/2 + 1)) used to supply the double precision temporary space for the RCI FGMRES computations, specifically:

tmp(1:ipar(16)-1)	contains the sequence of vectors generated by the FGMRES method. The initial value is 0.0.
tmp(ipar(16):ipar(17)-1)	contains the rotated Hessenberg matrix generated by the FGMRES method; the matrix is stored in the packed format. There is no initial value for this part of *tmp* array.
tmp(ipar(17):ipar(18)-1)	contains the rotation cosines vector generated by the FGMRES method. There is no initial value for this part of *tmp* array.
tmp(ipar(18):ipar(19)-1)	contains the rotation sines vector generated by the FGMRES method. There is no initial value for this part of *tmp* array.
tmp(ipar(19):ipar(20)-1)	contains the rotated residual vector generated by the FGMRES method. There is no initial value for this part of *tmp* array.
tmp(ipar(20):ipar(21)-1)	contains the solution vector to the least squares problem generated by the FGMRES method. There is no initial value for this part of *tmp* array.
tmp(ipar(21):)	contains the set of preconditioned vectors generated for the FGMRES method by the user. This part of *tmp* array is not used if the non-preconditioned version of FGMRES method is called. There is no initial value for this part of *tmp* array.

NOTE You can define this array in the code as DOUBLE PRECISION *tmp*((2**ipar*(15)+1)**n* + *ipar*(15)*(*ipar*(15)+9)/2 + 1)) if you run only non-preconditioned FGMRES iterations.

Schemes of Using the RCI FGMRES Routines

The following pseudocode shows the general schemes of using the RCI FGMRES routines.

...

generate matrix *A*

generate preconditioner *C* (optional)

call dfgmres_init(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*)

change parameters in *ipar*, *dpar* if necessary

call dfgmres_check(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*)

1 call dfgmres(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*)

if (*RCI_request*.eq.1) then

multiply the matrix *A* by *tmp*(*ipar*(22)) and put the result in *tmp*(*ipar*(23))

It is possible to use MKL Sparse BLAS Level 2 subroutines for this operation

```
c  proceed with FGMRES iterations
    goto 1
   endif
   if (RCI_request.eq.2) then
    do the stopping test
    if (test not passed) then
c  proceed with FGMRES iterations
     go to 1
    else
c  stop FGMRES iterations
     goto 2
    endif
   endif
   if (RCI_request.eq.3) then (optional)
```

apply the preconditioner *C* inverse to *tmp*(*ipar*(22)) and put the result in *tmp*(*ipar*(23))

```
c  proceed with FGMRES iterations
    goto 1
   endif
   if (RCI_request.eq.4) then
```

check the norm of the next orthogonal vector, it is contained in *dpar*(7)

```
    if (the norm is not zero up to rounding/computational errors) then
c  proceed with FGMRES iterations
     goto 1
    else
c  stop FGMRES iterations
     goto 2
    endif
   endif
```

2 call dfgmres_get(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*, *itercount*)

current iteration number is in *itercount*

the computed approximation is in the array *x*

For the FGMRES method, the array *x* initially contains the current approximation to the solution. It can be updated only by calling the routine dfgmres_get, which updates the solution in accordance with the computations performed by the routine dfgmres.

The above pseudocode demonstrates two main differences in the use of RCI FGMRES interface comparing with the CG Interface Description. The first difference relates to *RCI_request*=3: it uses different locations in the *tmp* array, which is two-dimensional for CG and one-dimensional for FGMRES. The second difference relates to *RCI_request*=4: the RCI CG interface never produces *RCI_request*=4.

RCI ISS Routines

dcg_init

Initializes the solver.

Syntax

`dcg_init(n, x, b, RCI_request, ipar, dpar, tmp)`

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine `dcg_init` initializes the solver. After initialization, all subsequent invocations of the Intel MKL RCI CG routines use the values of all parameters returned by the routine `dcg_init`. Advanced users can skip this step and set these parameters directly in the appropriate routines.

WARNING You can modify the contents of these arrays after they are passed to the solver routine only if you are sure that the values are correct and consistent. You can perform a basic check for correctness and consistency by calling the `dcg_check` routine, but it does not guarantee that the method will work correctly.

Input Parameters

n	`INTEGER`.Sets the size of the problem.
x	`DOUBLE PRECISION` array of size *n*. Contains the initial approximation to the solution vector. Normally it is equal to 0 or to *b*.
b	`DOUBLE PRECISION` array of size *n*. Contains the right-hand side vector.

Output Parameters

RCI_request	`INTEGER`. Gives information about the result of the routine.
ipar	`INTEGER` array of size `128`. Refer to the CG Common Parameters.
dpar	`DOUBLE PRECISION` array of size `128`. Refer to the CG Common Parameters.
tmp	`DOUBLE PRECISION` array of size `(n,4)`. Refer to the CG Common Parameters.

Return Values

RCI_request= 0	Indicates that the task completed normally.
RCI_request= -10000	Indicates failure to complete the task.

dcg_check

Checks consistency and correctness of the user defined data.

Syntax

`dcg_check(n, x, b, RCI_request, ipar, dpar, tmp)`

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine `dcg_check` checks consistency and correctness of the parameters to be passed to the solver routine `dcg`. However this operation does not guarantee that the solver returns the correct result. It only reduces the chance of making a mistake in the parameters of the method. Skip this operation only if you are sure that the correct data is specified in the solver parameters.

The lengths of all vectors must be defined in a previous call to the `dcg_init` routine.

Input Parameters

n	`INTEGER`. Sets the size of the problem.
x	`DOUBLE PRECISION` array of size *n*. Contains the initial approximation to the solution vector. Normally it is equal to 0 or to *b*.
b	`DOUBLE PRECISION` array of size *n*. Contains the right-hand side vector.

Output Parameters

RCI_request	`INTEGER`. Gives information about result of the routine.
ipar	`INTEGER` array of size `128`. Refer to the CG Common Parameters.
dpar	`DOUBLE PRECISION` array of size `128`. Refer to the CG Common Parameters.
tmp	`DOUBLE PRECISION` array of size `(n,4)`. Refer to the CG Common Parameters.

Return Values

RCI_request= 0	Indicates that the task completed normally.
RCI_request= -1100	Indicates that the task is interrupted and the errors occur.
RCI_request= -1001	Indicates that there are some warning messages.
RCI_request= -1010	Indicates that the routine changed some parameters to make them consistent or correct.
RCI_request= -1011	Indicates that there are some warning messages and that the routine changed some parameters.

dcg

Computes the approximate solution vector.

Syntax

`dcg(n, x, b, RCI_request, ipar, dpar, tmp)`

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `dcg` routine computes the approximate solution vector using the CG method [Young71]. The routine `dcg` uses the vector in the array *x* before the first call as an initial approximation to the solution. The parameter *RCI_request* gives you information about the task completion and requests results of certain operations that are required by the solver.

Note that lengths of all vectors must be defined in a previous call to the `dcg_init` routine.

Input Parameters

n	`INTEGER`. Sets the size of the problem.
x	`DOUBLE PRECISION` array of size *n*. Contains the initial approximation to the solution vector.
b	`DOUBLE PRECISION` array of size *n*. Contains the right-hand side vector.
tmp	`DOUBLE PRECISION` array of size `(n,4)`. Refer to the CG Common Parameters.

Output Parameters

RCI_request	`INTEGER`. Gives information about result of work of the routine.
x	`DOUBLE PRECISION` array of size *n*. Contains the updated approximation to the solution vector.
ipar	`INTEGER` array of size *128*. Refer to the CG Common Parameters.
dpar	`DOUBLE PRECISION` array of size *128*. Refer to the CG Common Parameters.
tmp	`DOUBLE PRECISION` array of size `(n,4)`. Refer to the CG Common Parameters.

Return Values

RCI_request=0	Indicates that the task completed normally and the solution is found and stored in the vector *x*. This occurs only if the stopping tests are fully automatic. For the user defined stopping tests, see the description of the *RCI_request*= 2.
RCI_request=-1	Indicates that the routine was interrupted because the maximum number of iterations was reached, but the relative stopping criterion was not met. This situation occurs only if you request both tests.
RCI_request=-2	Indicates that the routine was interrupted because of an attempt to divide by zero. This situation happens if the matrix is non-positive definite or almost non-positive definite.
RCI_request=- 10	Indicates that the routine was interrupted because the residual norm is invalid. This usually happens because the value *dpar*(6) was altered outside of the routine, or the `dcg_check` routine was not called.

RCI_request=-11	Indicates that the routine was interrupted because it enters the infinite cycle. This usually happens because the values *ipar*(8), *ipar*(9), *ipar*(10) were altered outside of the routine, or the dcg_check routine was not called.
RCI_request= 1	Indicates that you must multiply the matrix by *tmp*(1:*n*, 1), put the result in the *tmp*(1:*n*,2), and return the control back to the routine dcg.
RCI_request= 2	Indicates that you must perform the stopping tests. If they fail, return control back to the dcg routine. Otherwise, the solution is found and stored in the vector *x*.
RCI_request= 3	Indicates that you must apply the preconditioner to *tmp*(:, 3), put the result in the *tmp*(:, 4), and return the control back to the routine dcg.

dcg_get

Retrieves the number of the current iteration.

Syntax

dcg_get(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*, *itercount*)

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine dcg_get retrieves the current iteration number of the solutions process.

Input Parameters

n	INTEGER. Sets the size of the problem.
x	DOUBLE PRECISION array of size *n*. Contains the initial approximation vector to the solution.
b	DOUBLE PRECISION array of size *n*. Contains the right-hand side vector.
RCI_request	INTEGER. This parameter is not used.
ipar	INTEGER array of size 128. Refer to the CG Common Parameters.
dpar	DOUBLE PRECISION array of size 128. Refer to the CG Common Parameters.
tmp	DOUBLE PRECISION array of size (*n*,4). Refer to the CG Common Parameters.

Output Parameters

itercount	INTEGER argument. Returns the current iteration number.

Return Values

The routine dcg_get has not return values.

dcgmrhs_init

Initializes the RCI CG solver with MHRS.

Syntax

`dcgmrhs_init(n, x, nrhs, b, method, RCI_request, ipar, dpar, tmp)`

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine `dcgmrhs_init` initializes the solver. After initialization all subsequent invocations of the Intel MKL RCI CG with multiple right hand sides (MRHS) routines use the values of all parameters that are returned by `dcgmrhs_init`. Advanced users may skip this step and set the values to these parameters directly in the appropriate routines.

WARNING You can modify the contents of these arrays after they are passed to the solver routine only if you are sure that the values are correct and consistent. You can perform a basic check for correctness and consistency by calling the `dcgmrhs_check` routine, but it does not guarantee that the method will work correctly.

Input Parameters

n	`INTEGER`. Sets the size of the problem.
x	`DOUBLE PRECISION` matrix of size *n*nrhs*. Contains the initial approximation to the solution vectors. Normally it is equal to 0 or to *b*.
nrhs	`INTEGER`. Sets the number of right-hand sides.
b	`DOUBLE PRECISION` matrix of size *nrhs*n*. Contains the right-hand side vectors.
method	`INTEGER`. Specifies the method of solution: A value of 1 indicates CG with multiple right hand sides (default value)

Output Parameters

RCI_request	`INTEGER`. Gives information about the result of the routine.
ipar	`INTEGER` array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
dpar	`DOUBLE PRECISION` array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
tmp	`DOUBLE PRECISION` array of size *(n, 3+nrhs)*. Refer to the CG Common Parameters.

Return Values

RCI_request= 0	Indicates that the task completed normally.
RCI_request= -10000	Indicates failure to complete the task.

dcgmrhs_check

Checks consistency and correctness of the user defined data.

Syntax

`dcgmrhs_check(n, x, nrhs, b, RCI_request, ipar, dpar, tmp)`

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine dcgmrhs_check checks the consistency and correctness of the parameters to be passed to the solver routine dcgmrhs. While this operation reduces the chance of making a mistake in the parameters, it does not guarantee that the solver returns the correct result.

If you are sure that the correct data is *specified* in the solver parameters, you can skip this operation.

The lengths of all vectors must be defined in a previous call to the dcgmrhs_init routine.

Input Parameters

n	INTEGER. Sets the size of the problem.
x	DOUBLE PRECISION matrix of size *n*nrhs*. THAT IS TEST Contains the initial approximation to the solution vectors. Normally it is equal to 0 or to *b*.
nrhs	INTEGER. This parameter sets the number of right-hand sides.
b	DOUBLE PRECISION matrix of size (*nrhs*,*n*). Contains the right-hand side vectors.

Output Parameters

RCI_request	INTEGER. Gives information about the results of the routine.
ipar	INTEGER array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
dpar	DOUBLE PRECISION array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
tmp	DOUBLE PRECISION array of size (*n*, 3+*nrhs*). Refer to the CG Common Parameters.

Return Values

RCI_request= 0	Indicates that the task completed normally.
RCI_request= -1100	Indicates that the task is interrupted and the errors occur.
RCI_request= -1001	Indicates that there are some warning messages.
RCI_request= -1010	Indicates that the routine changed some parameters to make them consistent or correct.
RCI_request= -1011	Indicates that there are some warning messages and that the routine changed some parameters.

dcgmrhs

Computes the approximate solution vectors.

Syntax

dcgmrhs(*n*, *x*, *nrhs*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*)

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine dcgmrhs computes approximate solution vectors using the CG with multiple right hand sides (MRHS) method [Young71]. The routine dcgmrhs uses the value that was in the *x* before the first call as an initial approximation to the solution. The parameter *RCI_request* gives information about task completion status and requests results of certain operations that are required by the solver.

Note that lengths of all vectors are assumed to have been defined in a previous call to the dcgmrhs_init routine.

Input Parameters

n	INTEGER. Sets the size of the problem, and the sizes of arrays *x* and *b*.
x	DOUBLE PRECISION matrix of size *n*nrhs*. Contains the initial approximation to the solution vectors.
nrhs	INTEGER. Sets the number of right-hand sides.
b	DOUBLE PRECISION matrix of size *(nrhs*n)*. Contains the right-hand side vectors.
tmp	DOUBLE PRECISION array of size *(n, 3+nrhs)*. Refer to the CG Common Parameters.

Output Parameters

RCI_request	INTEGER. Gives information about result of work of the routine.
x	DOUBLE PRECISION matrix of size *n*-by-*nrhs*. Contains the updated approximation to the solution vectors.
ipar	INTEGER array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
dpar	DOUBLE PRECISION array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
tmp	DOUBLE PRECISION array of size *(n, 3+nrhs)*. Refer to the CG Common Parameters.

Return Values

RCI_request=0	Indicates that the task completed normally and the solution is found and stored in the vector *x*. This occurs only if the stopping tests are fully automatic. For the user defined stopping tests, see the description of the *RCI_request= 2*.
RCI_request=-1	Indicates that the routine was interrupted because the maximum number of iterations was reached, but the relative stopping criterion was not met. This situation occurs only if both tests are requested by the user.
RCI_request=-2	The routine was interrupted because of an attempt to divide by zero. This situation happens if the matrix is non-positive definite or almost non-positive definite.
RCI_request=- 10	Indicates that the routine was interrupted because the residual norm is invalid. This usually happens because the value *dpar(6)* was altered outside of the routine, or the dcg_check routine was not called.
RCI_request=-11	Indicates that the routine was interrupted because it enters the infinite cycle. This usually happens because the values *ipar(8)*, *ipar(9)*, *ipar(10)* were altered outside of the routine, or the dcg_check routine was not called.

RCI_request= 1	Indicates that you must multiply the matrix by *tmp*(1:*n*, 1), put the result in the *tmp*(1:*n*,2), and return the control back to the routine dcg.
RCI_request= 2	Indicates that you must perform the stopping tests. If they fail, return control back to the dcg routine. Otherwise, the solution is found and stored in the vector *x*.
RCI_request= 3	Indicates that you must apply the preconditioner to *tmp*(:, 3), put the result in the *tmp*(:, 4), and return the control back to the routine dcg.

dcgmrhs_get

Retrieves the number of the current iteration.

Syntax

dcgmrhs_get(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*, *itercount*)

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine dcgmrhs_get retrieves the current iteration number of the solving process.

Input Parameters

n	INTEGER. Sets the size of the problem.
x	DOUBLE PRECISION matrix of size *n*nrhs*. Contains the initial approximation to the solution vectors.
nrhs	INTEGER. Sets the number of right-hand sides.
b	DOUBLE PRECISION matrix of size (*nrhs*,*n*). Contains the right-hand side .
RCI_request	INTEGER. This parameter is not used.
ipar	INTEGER array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
dpar	DOUBLE PRECISION array of size *(128+2*nrhs)*. Refer to the CG Common Parameters.
tmp	DOUBLE PRECISION array of size (*n*, 3+*nrhs*). Refer to the CG Common Parameters.

Output Parameters

itercount	INTEGER argument. Returns the current iteration number.

Return Values

The routine dcgmrhs_get has no return values.

dfgmres_init

Initializes the solver.

Syntax

dfgmres_init(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*)

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine `dfgmres_init` initializes the solver. After initialization all subsequent invocations of Intel MKL RCI FGMRES routines use the values of all parameters that are returned by `dfgmres_init`. Advanced users may skip this step and set the values to these parameters directly in the appropriate routines.

WARNING You can modify the contents of these arrays after they are passed to the solver routine only if you are sure that the values are correct and consistent. You can perform a basic check for correctness and consistency by calling the `dfgmres_check` routine, but it does not guarantee that the method will work correctly.

Input Parameters

n	`INTEGER`. Sets the size of the problem.
x	`DOUBLE PRECISION` array of size *n*. Contains the initial approximation to the solution vector. Normally it is equal to 0 or to *b*.
b	`DOUBLE PRECISION` array of size *n*. Contains the right-hand side vector.

Output Parameters

RCI_request	`INTEGER`. Gives information about the result of the routine.
ipar	`INTEGER` array of size `128`. Refer to the FGMRES Common Parameters.
dpar	`DOUBLE PRECISION` array of size `128`. Refer to the FGMRES Common Parameters.
tmp	`DOUBLE PRECISION` array of size `((2*`*ipar*`(15)+1)*`*n* `+`*ipar*`(15)*`*ipar*`(15)+9)/2 + 1)`. Refer to the FGMRES Common Parameters.

Return Values

RCI_request`= 0`	Indicates that the task completed normally.
RCI_request`= -10000`	Indicates failure to complete the task.

dfgmres_check

Checks consistency and correctness of the user defined data.

Syntax

`dfgmres_check(`*n*`, `*x*`, `*b*`, `*RCI_request*`, `*ipar*`, `*dpar*`, `*tmp*`)`

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine dfgmres_check checks consistency and correctness of the parameters to be passed to the solver routine dfgmres. However, this operation does not guarantee that the method gives the correct result. It only reduces the chance of making a mistake in the parameters of the routine. Skip this operation only if you are sure that the correct data is specified in the solver parameters.

The lengths of all vectors are assumed to have been defined in a previous call to the dfgmres_init routine.

Input Parameters

n	INTEGER. Sets the size of the problem.
x	DOUBLE PRECISION array of size *n*. Contains the initial approximation to the solution vector. Normally it is equal to 0 or to *b*.
b	DOUBLE PRECISION array of size *n*. Contains the right-hand side vector.

Output Parameters

RCI_request	INTEGER. Gives information about result of the routine.
ipar	INTEGER array of size *128*. Refer to the FGMRES Common Parameters.
dpar	DOUBLE PRECISION array of size *128*. Refer to the FGMRES Common Parameters.
tmp	DOUBLE PRECISION array of size ((2**ipar*(15)+1)**n* +*ipar*(15)**ipar*(15)+9)/2 + 1). Refer to the FGMRES Common Parameters.

Return Values

RCI_request= 0	Indicates that the task completed normally.
RCI_request= -1100	Indicates that the task is interrupted and the errors occur.
RCI_request= -1001	Indicates that there are some warning messages.
RCI_request= -1010	Indicates that the routine changed some parameters to make them consistent or correct.
RCI_request= -1011	Indicates that there are some warning messages and that the routine changed some parameters.

dfgmres

Makes the FGMRES iterations.

Syntax

dfgmres(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*)

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine dfgmres performs the FGMRES iterations [Saad03], using the value that was in the array *x* before the first call as an initial approximation of the solution vector. To update the current approximation to the solution, the dfgmres_get routine must be called. The RCI FGMRES iterations can be continued after the call to the dfgmres_get routine only if the value of the parameter *ipar*(13) is not equal to 0 (default value). Note that the updated solution overwrites the right hand side in the vector *b* if the parameter

ipar(13) is positive, and the restarted version of the FGMRES method can not be run. If you want to keep the right hand side, you must be save it in a different memory location before the first call to the dfgmres_get routine with a positive *ipar*(13).

The parameter *RCI_request* gives information about the task completion and requests results of certain operations that the solver requires.

The lengths of all the vectors must be defined in a previous call to the dfgmres_init routine.

Input Parameters

n	INTEGER. Sets the size of the problem.
x	DOUBLE PRECISION array of size *n*. Contains the initial approximation to the solution vector.
b	DOUBLE PRECISION array of size *n*. Contains the right-hand side vector.
tmp	DOUBLE PRECISION array of size ((2**ipar*(15)+1)**n* +*ipar*(15)**ipar*(15)+9)/2 + 1). Refer to the FGMRES Common Parameters.

Output Parameters

RCI_request	INTEGER. Informs about result of work of the routine.
ipar	INTEGER array of size *128*. Refer to the FGMRES Common Parameters.
dpar	DOUBLE PRECISION array of size *128*. Refer to the FGMRES Common Parameters.
tmp	DOUBLE PRECISION array of size ((2**ipar*(15)+1)**n* +*ipar*(15)**ipar*(15)+9)/2 + 1). Refer to the FGMRES Common Parameters.

Return Values

RCI_request=0	Indicates that the task completed normally and the solution is found and stored in the vector *x*. This occurs only if the stopping tests are fully automatic. For the user defined stopping tests, see the description of the *RCI_request*= 2 or 4.
RCI_request=-1	Indicates that the routine was interrupted because the maximum number of iterations was reached, but the relative stopping criterion was not met. This situation occurs only if you request both tests.
RCI_request= -10	Indicates that the routine was interrupted because of an attempt to divide by zero. Usually this happens if the matrix is degenerate or almost degenerate. However, it may happen if the parameter *dpar* is altered, or if the method is not stopped when the solution is found.
RCI_request= -11	Indicates that the routine was interrupted because it entered an infinite cycle. Usually this happens because the values *ipar*(8), *ipar*(9), *ipar*(10) were altered outside of the routine, or the dfgmres_check routine was not called.
RCI_request= -12	Indicates that the routine was interrupted because errors were found in the method parameters. Usually this happens if the parameters *ipar* and *dpar* were altered by mistake outside the routine.

RCI_request= 1	Indicates that you must multiply the matrix by *tmp*(*ipar*(22)), put the result in the *tmp*(*ipar*(23)), and return the control back to the routine dfgmres.
RCI_request= 2	Indicates that you must perform the stopping tests. If they fail, return control to the dfgmres routine. Otherwise, the FGMRES solution is found, and you can run the fgmres_get routine to update the computed solution in the vector *x*.
RCI_request= 3	Indicates that you must apply the inverse preconditioner to *ipar*(22), put the result in the *ipar*(23), and return the control back to the routine dfgmres.
RCI_request= 4	Indicates that you must check the norm of the currently generated vector. If it is not zero within the computational/ rounding errors, return control to the dfgmres routine. Otherwise, the FGMRES solution is found, and you can run the dfgmres_get routine to update the computed solution in the vector *x*.

dfgmres_get

Retrieves the number of the current iteration and updates the solution.

Syntax

dfgmres_get(*n*, *x*, *b*, *RCI_request*, *ipar*, *dpar*, *tmp*, *itercount*)

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine dfgmres_get retrieves the current iteration number of the solution process and updates the solution according to the computations performed by the dfgmres routine. To retrieve the current iteration number only, set the parameter *ipar*(13) = -1 beforehand. Normally, you should do this before proceeding further with the computations. If the intermediate solution is needed, the method parameters must be set properly. For details see FGMRES Common Parameters and the Iterative Sparse Solver code examples in the examples\solver\source folder of your Intel MKL directory (cg_no_precon.f, cg_no_precon_c.c, cg_mrhs.f, cg_mrhs_precond.f, cg_mrhs_stop_crt.f, fgmres_no_precon_f.f, fgmres_no_precon_c.c).

Input Parameters

n	INTEGER. Sets the size of the problem.
ipar	INTEGER array of size *128*. Refer to the FGMRES Common Parameters.
dpar	DOUBLE PRECISION array of size *128*. Refer to the FGMRES Common Parameters.
tmp	DOUBLE PRECISION array of size ((2**ipar*(15)+1)**n* +*ipar*(15)**ipar*(15)+9)/2 + 1). Refer to the FGMRES Common Parameters.

Output Parameters

x	DOUBLE PRECISION array of size *n*. If *ipar*(13) = 0, it contains the updated approximation to the solution according to the computations done in dfgmres routine. Otherwise, it is not changed.
b	DOUBLE PRECISION array of size *n*. If *ipar*(13) > 0, it contains the updated approximation to the solution according to the computations done in dfgmres routine. Otherwise, it is not changed.
RCI_request	INTEGER. Gives information about result of the routine.
itercount	INTEGER argument. Contains the value of the current iteration number.

Return Values

RCI_request = 0	Indicates that the task completed normally.
RCI_request = -12	Indicates that the routine was interrupted because errors were found in the routine parameters. Usually this happens if the parameters *ipar* and *dpar* are altered by mistake outside of the routine.
RCI_request = -10000	Indicates that the routine failed to complete the task.

Implementation Details

Several aspects of the Intel MKL RCI ISS interface are platform-specific and language-specific. To promote portability across platforms and ease of use across different languages, include one of the Intel MKL RCI ISS language-specific header files.

The C-language header file defines these function prototypes:

```
void dcg_init(int *n, double *x, double *b, int *rci_request, int *ipar, double dpar,
double *tmp);

void dcg_check(int *n, double *x, double *b, int *rci_request, int *ipar, double dpar,
double *tmp);

void dcg(int *n, double *x, double *b, int *rci_request, int *ipar, double dpar, double
*tmp);

void dcg_get(int *n, double *x, double *b, int *rci_request, int *ipar, double dpar,
double *tmp, int *itercount);

void dcgmrhs_init(int *n, double *x, int *nRhs, double *b, int *method, int
*rci_request, int *ipar, double dpar, double *tmp);

void dcgmrhs_check(int *n, double *x, int *nRhs, double *b, int *rci_request, int
*ipar, double dpar, double *tmp);

void dcgmrhs(int *n, double *x, int *nRhs, double *b, int *rci_request, int *ipar,
double dpar, double *tmp);

void dcgmrhs_get(int *n, double *x, int *nRhs, double *b, int *rci_request, int *ipar,
double dpar, double *tmp, int *itercount);

void dfgmres_init(int *n, double *x, double *b, int *rci_request, int *ipar, double
dpar, double *tmp);

void dfgmres_check(int *n, double *x, double *b, int *rci_request, int *ipar, double
dpar, double *tmp);

void dfgmres(int *n, double *x, double *b, int *rci_request, int *ipar, double dpar,
double *tmp);
```

```
void dfgmres_get(int *n, double *x, double *b, int *rci_request, int *ipar, double
dpar, double *tmp, int *itercount);
```

NOTE Intel MKL does not support the RCI ISS interface unless you include the language-specific header file.

Preconditioners based on Incomplete LU Factorization Technique

Preconditioners, or accelerators are used to accelerate an iterative solution process. In some cases, their use can reduce the number of iterations dramatically and thus lead to better solver performance. Although the terms *preconditioner* and *accelerator* are synonyms, hereafter only *preconditioner* is used.

Intel MKL provides two preconditioners, ILU0 and ILUT, for sparse matrices presented in the format accepted in the Intel MKL direct sparse solvers (three-array variation of the CSR storage format described in Sparse Matrix Storage Format). The used algorithms used are described in [Saad03].

The ILU0 preconditioner is based on a well-known factorization of the original matrix into a product of two triangular matrices: lower and upper triangular matrices. Usually, such decomposition leads to some fill-in in the resulting matrix structure in comparison with the original matrix. The distinctive feature of the ILU0 preconditioner is that it preserves the structure of the original matrix in the result.

Unlike the ILU0 preconditioner, the ILUT preconditioner preserves some resulting fill-in in the preconditioner matrix structure. The distinctive feature of the ILUT algorithm is that it calculates each element of the preconditioner and saves each one if it satisfies two conditions simultaneously: its value is greater than the product of the given tolerance and matrix row norm, and its value is in the given bandwidth of the resulting preconditioner matrix.

Both ILU0 and ILUT preconditioners can apply to any non-degenerate matrix. They can be used alone or together with the Intel MKL RCI FGMRES solver (see Sparse Solver Routines). Avoid using these preconditioners with MKL RCI CG solver because in general, they produce a non-symmetric resulting matrix even if the original matrix is symmetric. Usually, an inverse of the preconditioner is required in this case. To do this the Intel MKL triangular solver routine `mkl_dcsrtrsv` must be applied twice: for the lower triangular part of the preconditioner, and then for its upper triangular part.

NOTE Although ILU0 and ILUT preconditioners apply to any non-degenerate matrix, in some cases the algorithm may fail to ensure successful termination and the required result. Whether or not the preconditioner produces an acceptable result can only be determined in practice.

A preconditioner may increase the number of iterations for an arbitrary case of the system and the initial solution, and even ruin the convergence. It is your responsibility as a user to choose a suitable preconditioner.

General Scheme of Using ILUT and RCI FGMRES Routines

The general scheme for use is the same for both preconditioners. Some differences exist in the calling parameters of the preconditioners and in the subsequent call of two triangular solvers. You can see all these differences in the code examples for both preconditioners in the `examples\solver\source` folder of your Intel MKL directory (`dcsrilu0_expl1.c`, `dcsrilu0_expl2.f`, `dcsrilut_expl1.c`, `dcsrilut_expl2.f`).

The following pseudocode shows the general scheme of using the ILUT preconditioner in the RCI FGMRES context.

...

generate matrix *A*

generate preconditioner *C* (optional)

call `dfgmres_init(n, x, b, RCI_request, ipar, dpar, tmp)`

```
   change parameters in ipar, dpar if necessary
   call dcsrilut(n, a, ia, ja, bilut, ibilut, jbilut, tol, maxfil, ipar, dpar, ierr)
   call dfgmres_check(n, x, b, RCI_request, ipar, dpar, tmp)
1  call dfgmres(n, x, b, RCI_request, ipar, dpar, tmp)
   if (RCI_request.eq.1) then
    multiply the matrix A by tmp(ipar(22)) and put the result in tmp(ipar(23))
c  proceed with FGMRES iterations
    goto 1
   endif
   if (RCI_request.eq.2) then
    do the stopping test
    if (test not passed) then
c proceed with FGMRES iterations
     go to 1
    else
c  stop FGMRES iterations.
     goto 2
    endif
   endif
   if (RCI_request.eq.3) then
c   Below, trvec is an intermediate vector of length at least n
c   Here is the recommended use of the result produced by the ILUT routine.
c   via standard Intel MKL Sparse Blas solver routine mkl_dcsrtrsv.
   call mkl_dcsrtrsv('L','N','U', n, bilut, ibilut, jbilut, tmp(ipar(22)),trvec)
   call mkl_dcsrtrsv('U','N','N', n, bilut, ibilut, jbilut, trvec, tmp(ipar(23)))
c proceed with FGMRES iterations
    goto 1
   endif
   if (RCI_request.eq.4) then
   check the norm of the next orthogonal vector, it is contained in dpar(7)
    if (the norm is not zero up to rounding/computational errors) then
c  proceed with FGMRES iterations
     goto 1
    else
c  stop FGMRES iterations
     goto 2
    endif
```

```
endif
```

2 call `dfgmres_get(n, x, b, RCI_request, ipar, dpar, tmp, itercount)`

current iteration number is in *itercount*

the computed approximation is in the array *x*

ILU0 and ILUT Preconditioners Interface Description

The concepts required to understand the use of the Intel MKL preconditioner routines are discussed in the Appendix A Linear Solvers Basics.

In this section the FORTRAN style notations are used. All types refer to the standard Fortran types, `INTEGER`, and `DOUBLE PRECISION`.

C and C++ programmers must refer to the section Calling Sparse Solver and Preconditioner Routines from C C++ for information on mapping Fortran types to C/C++ types.

User Data Arrays

The preconditioner routines take arrays of user data as input. To minimize storage requirements and improve overall run-time efficiency, the Intel MKL preconditioner routines do not make copies of the user input arrays.

Common Parameters

Some parameters of the preconditioners are common with the FGMRES Common Parameters. The routine `dfgmres_init` specifies their default and initial values. However, some parameters can be redefined with other values. These parameters are listed below.

For the ILU0 preconditioner:

ipar(2) - specifies the destination of error messages generated by the ILU0 routine. The default value 6 means that all error messages are displayed on the screen. Otherwise routine creates a log file called `MKL_PREC_log.txt` and writes error messages to it. Note if the parameter *ipar*(6) is set to 0, then error messages are not generated at all.

ipar(6) - specifies whether error messages are generated. If its value is not equal to 0, the ILU0 routine returns error messages as specified by the parameter *ipar*(2). Otherwise, the routine does not generate error messages at all, but returns a negative value for the parameter *ierr*. The default value is 1.

For the ILUT preconditioner:

ipar(2) - specifies the destination of error messages generated by the ILUT routine. The default value 6 means that all messages are displayed on the screen. Otherwise routine creates a log file called `MKL_PREC_log.txt` and writes error messages to it. Note if the parameter *ipar*(6) is set to 0, then error messages are not generated at all.

ipar(6) - specifies whether error messages are generated. If its value is not equal to 0, the ILUT routine returns error messages as specified by the parameter *ipar*(2). Otherwise, the routine does not generate error messages at all, but returns a negative value for the parameter *ierr*. The default value is 1.

ipar(7) - if its value is greater than 0, the ILUT routine generates warning messages as specified by the parameter *ipar*(2) and continues calculations. If its value is equal to 0, the routine returns a positive value of the parameter *ierr*. If its value is less than 0, the routine generates a warning message as specified by the parameter *ipar*(2) and returns a positive value of the parameter *ierr*. The default value is 1.

dcsrilu0

ILU0 preconditioner based on incomplete LU factorization of a sparse matrix.

Syntax

Fortran:

```
call dcsrilu0(n, a, ia, ja, bilu0, ipar, dpar, ierr)
```

C:

```
dcsrilu0(&n, a, ia, ja, bilu0, ipar, dpar, &ierr);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine `dcsrilu0` computes a preconditioner *B* [Saad03] of a given sparse matrix *A* stored in the format accepted in the direct sparse solvers:

*A~B=L*U* , where *L* is a lower triangular matrix with a unit diagonal, *U* is an upper triangular matrix with a non-unit diagonal, and the portrait of the original matrix *A* is used to store the incomplete factors *L* and *U*.

Input Parameters

n — `INTEGER`. Size (number of rows or columns) of the original square *n*-by-*n* matrix *A*.

a — `DOUBLE PRECISION`. Array containing the set of elements of the matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to the *values* array description in the Sparse Matrix Storage Format for more details.

ia — `INTEGER`. Array of size `(n+1)` containing begin indices of rows of the matrix *A* such that *ia*`(i)` is the index in the array *A* of the first non-zero element from the row `i`. The value of the last element *ia*`(n+1)` is equal to the number of non-zero elements in the matrix *A* plus one. Refer to the *rowIndex* array description in the Sparse Matrix Storage Format for more details.

ja — `INTEGER`. Array containing the column indices for each non-zero element of the matrix *A*. Its size is equal to the size of the array *a*. Refer to the *columns* array description in the Sparse Matrix Storage Format for more details.

NOTE Column indices must be in ascending order for each row of matrix.

ipar — `INTEGER` array of size *128*. This parameter specifies the integer set of data for both the ILU0 and RCI FGMRES computations. Refer to the *ipar* array description in the FGMRES Common Parameters for more details on FGMRES parameter entries. The entries that are specific to ILU0 are listed below.

ipar(31) specifies how the routine operates when a zero diagonal element occurs during calculation. If this parameter is set to 0 (the default value set by the routine dfgmres_init), then the calculations are stopped and the routine returns a non-zero error value. Otherwise, the diagonal element is set to the value of *dpar*(32) and the calculations continue.

NOTE You must declare the array *ipar* with length 128. While defining the array in the code as INTEGER *ipar*(31) works, there is no guarantee of future compatibility with Intel MKL.

dpar DOUBLE PRECISION array of size *128*. This parameter specifies the double precision set of data for both the ILU0 and RCI FGMRES computations. Refer to the *dpar* array description in the FGMRES Common Parameters for more details on FGMRES parameter entries. The entries specific to ILU0 are listed below.

dpar(31) specifies a small value, which is compared with the computed diagonal elements. When *ipar*(31) is not 0, then diagonal elements less than *dpar*(31) are set to *dpar*(32). The default value is 1.0D-16.

NOTE This parameter can be set to the negative value, because the calculation uses its absolute value.

If this parameter is set to 0, the comparison with the diagonal element is not performed.

dpar(32) specifies the value that is assigned to the diagonal element if its value is less than *dpar*(31) (see above). The default value is 1.0D-10.

NOTE You must declare the array *dpar* with length 128. While defining the array in the code as DOUBLE PRECISION *ipar*(32) works, there is no guarantee of future compatibility with Intel MKL.

Output Parameters

bilu0 DOUBLE PRECISION. Array *B* containing non-zero elements of the resulting preconditioning matrix *B*, stored in the format accepted in direct sparse solvers. Its size is equal to the number of non-zero elements in the matrix *A*. Refer to the *values* array description in the Sparse Matrix Storage Format section for more details.

ierr INTEGER. Error flag, gives information about the routine completion.

NOTE To present the resulting preconditioning matrix in the CSR format the arrays *ia* (row indices) and *ja* (column indices) of the input matrix must be used.

Return Values

ierr=0	Indicates that the task completed normally.
ierr=-101	Indicates that the routine was interrupted and that error occurred: at least one diagonal element is omitted from the matrix in CSR format (see Sparse Matrix Storage Format).
ierr=-102	Indicates that the routine was interrupted because the matrix contains a diagonal element with the value of zero.
ierr=-103	Indicates that the routine was interrupted because the matrix contains a diagonal element which is so small that it could cause an overflow, or that it would cause a bad approximation to ILU0.
ierr=-104	Indicates that the routine was because the memory is insufficient for the internal work array.
ierr=-105	Indicates that the routine was because the input matrix size *n* is less than or equal to 0.
ierr=-106	Indicates that the routine was because the column indices *ja* are not in the ascending order.

Interfaces

FORTRAN 77 and Fortran 95:

```
SUBROUTINE dcsrilu0 (n, a, ia, ja, bilu0, ipar, dpar, ierr)
INTEGER n, ierr, ipar(128)
INTEGER ia(*), ja(*)
DOUBLE PRECISION a(*), bilu0(*), dpar(128)
```

C:

```
void dcsrilu0 (int *n, double *a, int *ia, int *ja, double  *bilu0, int *ipar, double *dpar, int *ierr);
```

dcsrilut

ILUT preconditioner based on the incomplete LU factorization with a threshold of a sparse matrix.

Syntax

Fortran:

```
call dcsrilut(n, a, ia, ja, bilut, bilut, ibilut, jbilut, tol, maxfil, ipar, dpar, ierr)
```

C:

```
dcsrilut(&n, a, ia, ja, bilut, bilut, ibilut, jbilut, &tol, &maxfil, ipar, dpar, &ierr);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The routine dcsrilut computes a preconditioner *B* [Saad03] of a given sparse matrix *A* stored in the format accepted in the direct sparse solvers:

*A~B=L*U* , where *L* is a lower triangular matrix with unit diagonal and *U* is an upper triangular matrix with non-unit diagonal.

The following threshold criteria are used to generate the incomplete factors *L* and *U*:

1) the resulting entry must be greater than the matrix current row norm multiplied by the parameter *tol*, and

2) the number of the non-zero elements in each row of the resulting *L* and *U* factors must not be greater than the value of the parameter *maxfil*.

Input Parameters

n — INTEGER. Size (number of rows or columns) of the original square *n*-by-*n* matrix *A*.

a — DOUBLE PRECISION. Array containing all non-zero elements of the matrix *A*. The length of the array is equal to their number. Refer to *values* array description in the Sparse Matrix Storage Format section for more details.

ia — INTEGER. Array of size (*n*+1) containing indices of non-zero elements in the array *A*. *ia*(i) is the index of the first non-zero element from the row i. The value of the last element *ia*(*n*+1) is equal to the number of non-zeros in the matrix *A* plus one. Refer to the *rowIndex* array description in the Sparse Matrix Storage Format for more details.

ja — INTEGER. Array of size equal to the size of the array *a*. This array contains the column numbers for each non-zero element of the matrix *A*. Refer to the *columns* array description in the Sparse Matrix Storage Format for more details.

NOTE Column numbers must be in ascending order for each row of matrix.

tol — DOUBLE PRECISION. Tolerance for threshold criterion for the resulting entries of the preconditioner.

maxfil — INTEGER. Maximum fill-in, which is half of the preconditioner bandwidth. The number of non-zero elements in the rows of the preconditioner can not exceed (2**maxfil*+1).

ipar — INTEGER array of size *128*. This parameter is used to specify the integer set of data for both the ILUT and RCI FGMRES computations. Refer to the *ipar* array description in the FGMRES Common Parameters for more details on FGMRES parameter entries. The entries specific to ILUT are listed below.

ipar(31) specifies how the routine operates if the value of the computed diagonal element is less than the current matrix row norm multiplied by the value of the parameter *tol*. If *ipar*(31) = 0, then the calculation is stopped and the routine returns non-zero error value. Otherwise, the value of the diagonal element is set to a value determined by *dpar*(31) (see its description below), and the calculations continue.

NOTE There is no default value for *ipar*(31) even if the preconditioner is used within the RCI ISS context. Always set the value of this entry.

NOTE You must declare the array *ipar* with length 128. While defining the array in the code as INTEGER *ipar*(31) works, there is no guarantee of future compatibility with Intel MKL.

dpar DOUBLE PRECISION array of size *128*. This parameter specifies the double precision set of data for both ILUT and RCI FGMRES computations. Refer to the *dpar* array description in the FGMRES Common Parameters for more details on FGMRES parameter entries. The entries that are specific to ILUT are listed below.

dpar(31) used to adjust the value of small diagonal elements. Diagonal elements with a value less than the current matrix row norm multiplied by *tol* are replaced with the value of *dpar*(31) multiplied by the matrix row norm.

NOTE There is no default value for *dpar*(31) entry even if the preconditioner is used within RCI ISS context. Always set the value of this entry.

NOTE You must declare the array *dpar* with length 128. While defining the array in the code as DOUBLE PRECISION *ipar*(31) works, there is no guarantee of future compatibility with Intel MKL.

Output Parameters

bilut DOUBLE PRECISION. Array containing non-zero elements of the resulting preconditioning matrix *B*, stored in the format accepted in the direct sparse solvers. Refer to the *values* array description in the Sparse Matrix Storage Format for more details. The size of the array is equal to (2**maxfil*+1)**n*-*maxfil**(*maxfil*+1)+1.

NOTE Provide enough memory for this array before calling the routine. Otherwise, the routine may fail to complete successfully with a correct result.

ibilut INTEGER. Array of size (*n*+1) containing indices of non-zero elements in the array *bilut*. *ibilut*(i) is the index of the first non-zero element from the row i. The value of the last element *ibilut*(*n*+1) is equal to the number of non-zeros in the matrix *B* plus one. Refer to the *rowIndex* array description in the Sparse Matrix Storage Format for more details.

jbilut	INTEGER. Array, its size is equal to the size of the array *bilut*. This array contains the column numbers for each non-zero element of the matrix *B*. Refer to the *columns* array description in the Sparse Matrix Storage Format for more details.
ierr	INTEGER. Error flag, informs about the routine completion.

Return Values

ierr=0	Indicates that the task completed normally.
ierr=-101	Indicates that the routine was interrupted because of an error: the number of elements in some matrix row specified in the sparse format is equal to or less than 0.
ierr=-102	Indicates that the routine was interrupted because the value of the computed diagonal element is less than the product of the given tolerance and the current matrix row norm, and it cannot be replaced as *ipar*(31)=0.
ierr=-103	Indicates that the routine was interrupted because the element *ia*(i+1) is less than or equal to the element *ia*(i) (see Sparse Matrix Storage Format).
ierr=-104	Indicates that the routine was interrupted because the memory is insufficient for the internal work arrays.
ierr=-105	Indicates that the routine was interrupted because the input value of *maxfil* is less than 0.
ierr=-106	Indicates that the routine was interrupted because the size *n* of the input matrix is less than 0.
ierr=-107	Indicates that the routine was interrupted because an element of the array *ja* is less than 0, or greater than *n* (see Sparse Matrix Storage Format).
ierr=101	The value of *maxfil* is greater than or equal to *n*. The calculation is performed with the value of *maxfil* set to (*n*-1).
ierr=102	The value of *tol* is less than 0. The calculation is performed with the value of the parameter set to (-*tol*)
ierr=103	The absolute value of *tol* is greater than value of *dpar*(31); it can result in instability of the calculation.
ierr=104	The value of *dpar*(31) is equal to 0. It can cause calculations to fail.

Interfaces

FORTRAN 77 and Fortran 95:

```
SUBROUTINE dcsrilut (n, a, ia, ja, bilut, ibilut, jbilut, tol, maxfil, ipar, dpar, ierr)
INTEGER n, ierr, ipar(*), maxfil
INTEGER ia(*), ja(*), ibilut(*), jbilut(*)
DOUBLE PRECISION a(*), bilut(*), dpar(*), tol
```

C:

```
void dcsrilut (int *n, double *a, int *ia, int *ja, double *bilut, int *ibilut, int *jbilut, double
*tol, int *maxfil, int *ipar, double *dpar, int *ierr);
```

Calling Sparse Solver and Preconditioner Routines from C/C++

All of the Intel MKL sparse solver and preconditioner routines is designed to be called easily from FORTRAN 77 or Fortran 90. However, any of these routines can be invoked directly from C or C++ if you are familiar with the inter-language calling conventions of your platforms. These conventions include, but are not limited to, the argument passing mechanisms for the language, the data type mappings from Fortran to C/C++, and the platform specific method of decoration for Fortran external names.

To promote portability, the C header files provide a set of macros and type definitions intended to hide the inter-language calling conventions and provide an interface to the Intel MKL sparse solver routines that appears natural for C/C++.

For example, consider a hypothetical library routine `foo` that takes a real vector of length *n*, and returns an integer status. Fortran users would access such a function as:

```
INTEGER n, status, foo
REAL x(*)
status = foo(x, n)
```

As noted above, to invoke `foo`, C users need to know what C data types correspond to Fortran types `INTEGER` and `REAL`; what argument passing mechanism the Fortran compiler uses; and what, if any, name decoration the Fortran compiler performs when generating the external symbol `foo`.

However, by using the C specific header file, for example `mkl_solver.h`, the invocation of `foo`, within a C program would look as follows:

```
#include "mkl_solver.h"
_INTEGER_t i, status;
_REAL_t x[];
status = foo( x, i );
```

Note that in the above example, the header file `mkl_solver.h` provides definitions for the types `_INTEGER_t` and `_REAL_t` that correspond to the Fortran types `INTEGER` and `REAL`.

To simplify calling of the Intel MKL sparse solver routines from C and C++, the following approach of providing C definitions of Fortran types is used: if an argument or a result from a sparse solver is documented as having the Fortran language specific type *XXX*, then the C and C++ header files provide an appropriate C language type definitions for the name *_XXX*_t.

Caveat for C Users

One of the key differences between C/C++ and Fortran is the argument passing mechanisms for the languages: Fortran programs pass arguments by reference and C/C++ programs pass arguments by value. In the above example, the header file `mkl_solver.h` attempts to hide this difference by defining a macro `foo`, which takes the address of the appropriate arguments. For example, on the Tru64 UNIX* operating system `mkl_solver.h` defines the macro as follows:

```
#define foo(a,b) foo_((a), &(b))
```

Note how constants are treated when using the macro form of `foo`. `foo( x, 10 )` is converted into `foo_( x, &10 )`. In a strictly ANSI compliant C compiler, taking the address of a constant is not permitted, so a strictly conforming program would look like:

```
INTEGER_t iTen = 10;

_REAL_t * x;

status = foo( x, iTen );
```

However, some C compilers in a non-ANSI compliant mode enable taking the address of a constant for ease of use with Fortran programs. The form `foo( x, 10 )` is acceptable for such compilers.

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9 Vector Mathematical Functions

This chapter describes Intel® MKL Vector Mathematical Functions Library (VML), which computes a mathematical function of each of the vector elements. VML includes a set of highly optimized functions (arithmetic, power, trigonometric, exponential, hyperbolic, special, and rounding) that operate on vectors of real and complex numbers.

Application programs that improve performance with VML include nonlinear programming software, computation of integrals, financial calculations, computer graphics, and many others.

VML functions fall into the following groups according to the operations they perform:

- VML Mathematical Functions compute values of mathematical functions, such as sine, cosine, exponential, or logarithm, on vectors stored contiguously in memory.
- VML Pack/Unpack Functions convert to and from vectors with positive increment indexing, vector indexing, and mask indexing (see Appendix B for details on vector indexing methods).
- VML Service Functions set/get the accuracy modes and the error codes.

The VML mathematical functions take an input vector as an argument, compute values of the respective function element-wise, and return the results in an output vector. All the VML mathematical functions can perform in-place operations, where the input and output arrays are at the same memory locations.

The Intel MKL interfaces are given in the following include files:

- `mkl_vml.f77`, which declares the FORTRAN 77 interfaces
- `mkl_vml.f90`, which declares the Fortran 90 interfaces; the `mkl_vml.fi` include file available in the previous versions of Intel MKL is retained for backward compatibility
- `mkl_vml_functions.h`, which declares the C interfaces

The following directories provide examples that demonstrate how to use the VML functions:

`${MKL}/examples/vmlc/source`

`${MKL}/examples/vmlf/source`

See VML performance and accuracy data in the online VML Performance and Accuracy Data document available at http://software.intel.com/en-us/articles/intel-math-kernel-library-documentation/

Data Types, Accuracy Modes, and Performance Tips

VML includes mathematical and pack/unpack vector functions for single and double precision vector arguments of real and compex types. The library provides Fortran- and C-interfaces for all functions, including the associated service functions. The Function Naming Conventions section below shows how to call these functions from different languages.

Performance depends on a number of factors, including vectorization and threading overhead. The recommended usage is as follows:

- Use VML for vector lengths larger than 40 elements.

- Use the Intel® Compiler for vector lengths less than 40 elements.

All VML vector functions support the following accuracy modes:

- High Accuracy (HA), the default mode
- Low Accuracy (LA), which improves performance by reducing accuracy of the two least significant bits
- Enhanced Performance (EP), which provides better performance at the cost of significantly reduced accuracy. Approximately half of the bits in the mantissa are correct.

Note that using the EP mode does not guarantee accurate processing of corner cases and special values. Although the default accuracy is HA, LA is sufficient in most cases. For applications that require less accuracy (for example, media applications, some Monte Carlo simulations, etc.), the EP mode may be sufficient.

VML handles special values in accordance with the C99 standard [C99].

Use the `vmlSetMode(mode)` function (see Table "Values of the *mode* Parameter") to switch between the HA, LA, and EP modes. The `vmlGetMode()` function returns the current mode.

Optimization Notice
Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice. Notice revision #20110804

See Also

Function Naming Conventions

Function Naming Conventions

The VML function names are lowercase for Fortran (`vsabs`) and of mixed (lower and upper) case for C (`vsAbs`).

The VML mathematical and pack/unpack function names have the following structure:

v[*m*]<*?*><name><*mod*>

where

- *v* is a prefix indicating vector operations.
- [*m*] is an optional prefix for mathematical functions that indicates additional argument to specify a VML mode for a given function call (see `vmlSetMode` for possible values and their description).
- <*?*> is a precision prefix that indicates one of the following the data types:

s	`REAL` for the Fortran interface, or `float` for the C interface
d	`DOUBLE PRECISION` for the Fortran interface, or `double` for the C interface.
c	`COMPLEX` for the Fortran interface, or `MKL_Complex8` for the C interface.
z	`DOUBLE COMPLEX` for the Fortran interface, or `MKL_Complex16` for the C interface.

- <name> indicates the function short name, with some of its letters in uppercase for the C interface. See examples in Table "VML Mathematical Functions".
- <*mod*> field (written in uppercase for the C interface) is present only in the pack/unpack functions and indicates the indexing method used:

i	indexing with a positive increment
v	indexing with an index vector
m	indexing with a mask vector.

The VML service function names have the following structure:

vml<name>

where

<name> indicates the function short name, with some of its letters in uppercase for the C interface. See examples in Table "VML Service Functions".

To call VML functions from an application program, use conventional function calls. For example, call the vector single precision real exponential function as

`call vsexp ( n, a, y )` for the Fortran interface, or

`call vmsexp ( n, a, y, mode )` for the Fortran interface with a specified mode, or

`vsExp ( n, a, y );` for the C interface.

Function Interfaces

VML interfaces include the function names and argument lists. The following sections describe the Fortran and C interfaces for the VML functions. Note that some of the functions have multiple input and output arguments

Some VML functions may also take scalar arguments as input. See the function description for the naming conventions of such arguments.

VML Mathematical Functions

Fortran:

```
call v<?><name>( n, a, [scalar input arguments,] y )
call v<?><name>( n, a, b, [scalar input arguments,] y )
call v<?><name>( n, a, y, z )
call vm<?><name>( n, a, [scalar input arguments,] y, mode )
call vm<?><name>( n, a, b, [scalar input arguments,] y, mode )
call vm<?><name>( n, a, y, z, mode )
```

C:

```
v<?><name>( n, a, [scalar input arguments,] y );
v<?><name>( n, a, b, [scalar input arguments,] y );
v<?><name>( n, a, y, z );
vm<?><name>( n, a, [scalar input arguments,] y, mode );
vm<?><name>( n, a, b, [scalar input arguments,] y, mode );
vm<?><name>( n, a, y, z, mode );
```

Pack Functions

Fortran:

```
call v<?>packi( n, a, inca, y )
call v<?>packv( n, a, ia, y )
call v<?>packm( n, a, ma, y )
```

C:

```
v<?>PackI( n, a, inca, y );
v<?>PackV( n, a, ia, y );
v<?>PackM( n, a, ma, y );
```

Unpack Functions

Fortran:

```
call v<?>unpacki( n, a, y, incy )
call v<?>unpackv( n, a, y, iy )
call v<?>unpackm( n, a, y, my )
```

C:

```
v<?>UnpackI( n, a, y, incy );
v<?>UnpackV( n, a, y, iy );
v<?>UnpackM( n, a, y, my );
```

Service Functions

Fortran:

```
oldmode = vmlsetmode( mode )
mode = vmlgetmode( )
olderr = vmlseterrstatus ( err )
err = vmlgeterrstatus( )
olderr = vmlclearerrstatus( )
oldcallback = vmlseterrorcallback( callback )
callback = vmlgeterrorcallback( )
oldcallback = vmlclearerrorcallback( )
```

C:

```
oldmode = vmlSetMode( mode );
mode = vmlGetMode( void );
olderr = vmlSetErrStatus ( err );
err = vmlGetErrStatus( void );
olderr = vmlClearErrStatus( void );
oldcallback = vmlSetErrorCallBack( callback );
callback = vmlGetErrorCallBack( void );
oldcallback = vmlClearErrorCallBack( void );
```

Note that *oldmode*, *oldcerr*, and *oldcallback* refer to settings prior to the call.

Input Parameters

n	number of elements to be calculated
a	first input vector
b	second input vector
inca	vector increment for the input vector *a*
ia	index vector for the input vector *a*
ma	mask vector for the input vector *a*
incy	vector increment for the output vector *y*
iy	index vector for the output vector *y*
my	mask vector for the output vector *y*
err	error code
mode	VML mode
callback	address of the callback function

Output Parameters

y	first output vector
z	second output vector
err	error code
mode	VML mode
olderr	former error code
oldmode	former VML mode
callback	address of the callback function
oldcallback	address of the former callback function

See the data types of the parameters used in each function in the respective function description section. All the Intel MKL VML mathematical functions can perform in-place operations.

Vector Indexing Methods

VML mathematical functions work only with unit stride. To accommodate arrays with other increments, or more complicated indexing, you can gather the elements into a contiguous vector and then scatter them after the computation is complete.

VML uses the three following indexing methods to do this task:

- positive increment
- index vector
- mask vector

The indexing method used in a particular function is indicated by the indexing modifier (see the description of the *<mod>* field in Function Naming Conventions). For more information on the indexing methods, see Vector Arguments in VML in Appendix B.

Error Diagnostics

The VML mathematical functions incorporate the error handling mechanism, which is controlled by the following service functions:

`vmlGetErrStatus`, `vmlSetErrStatus`, `vmlClearErrStatus`	These functions operate with a global variable called VML Error Status. The VML Error Status flags an error, a warning, or a successful execution of a VML function.
`vmlGetErrCallBack`, `vmlSetErrCallBack`, `vmlClearErrCallBack`	These functions enable you to customize the error handling. For example, you can identify a particular argument in a vector where an error occurred or that caused a warning.
`vmlSetMode`, `vmlGetMode`	These functions get and set a VML mode. If you set a new VML mode using the `vmlSetMode` function, you can store the previous VML mode returned by the routine and restore it at any point of your application.

If both an error and a warning situation occur during the function call, the VML Error Status variable keeps only the value of the error code. See Table "Values of the VML Error Status" for possible values. If a VML function does not encounter errors or warnings, it sets the VML Error Status to `VML_STATUS_OK`.

If you use the Fortran interface, call the error reporting function `XERBLA` to receive information about correctness of input arguments (`VML_STATUS_BADSIZE` and `VML_STATUS_BADMEM`). See Table "Values of the VML Error Status" for details.

You can use the `vmlSetMode` and `vmlGetMode` functions to modify error handling behavior. Depending on the VML mode, the error handling behavior includes the following operations:

- setting the VML Error Status to a value corresponding to the observed error or warning
- setting the *errno* variable to one of the values described in Table "Set Values of the *errno* Variable"
- writing error text information to the *stderr* stream
- raising the appropriate exception on an error, if necessary
- calling the additional error handler callback function that is set by `vmlSetErrorCallBack`.

Set Values of the *errno* Variable

Value of *errno*	Description
0	No errors are detected.
EINVAL	The array dimension is not positive.
EACCES	`NULL` pointer is passed.
EDOM	At least one of array values is out of a range of definition.
ERANGE	At least one of array values caused a singularity, overflow or underflow.

See Also

vmlGetErrStatus
vmlSetErrStatus
vmlClearErrStatus
vmlSetErrorCallBack
vmlGetErrorCallBack
vmlClearErrorCallBack
vmlGetMode
vmlSetMode

VML Mathematical Functions

This section describes VML functions that compute values of mathematical functions on real and complex vector arguments with unit increment.

Each function is introduced by its short name, a brief description of its purpose, and the calling sequence for each type of data both for Fortran- and C-interfaces, as well as a description of the input/output arguments.

The input range of parameters is equal to the mathematical range of the input data type, unless the function description specifies input threshold values, which mark off the precision overflow, as follows:

- *FLT_MAX* denotes the maximum number representable in single precision real data type
- *DBL_MAX* denotes the maximum number representable in double precision real data type

Table "VML Mathematical Functions" lists available mathematical functions and associated data types.

VML Mathematical Functions

Function	Data Types	Description
Arithmetic Functions		
v?Add	*s*, *d*, *c*, *z*	Addition of vector elements
v?Sub	*s*, *d*, *c*, *z*	Subtraction of vector elements
v?Sqr	*s*, *d*	Squaring of vector elements
v?Mul	*s*, *d*, *c*, *z*	Multiplication of vector elements
v?MulByConj	*c*, *z*	Multiplication of elements of one vector by conjugated elements of the second vector
v?Conj	*c*, *z*	Conjugation of vector elements
v?Abs	*s*, *d*, *c*, *z*	Computation of the absolute value of vector elements
v?Arg	*c*, *z*	Computation of the argument of vector elements
v?LinearFrac	*s*, *d*	Linear fraction transformation of vectors
Power and Root Functions		
v?Inv	*s*, *d*	Inversion of vector elements

Function	Data Types	Description
v?Div	*s*, *d*, *c*, *z*	Division of elements of one vector by elements of the second vector
v?Sqrt	*s*, *d*, *c*, *z*	Computation of the square root of vector elements
v?InvSqrt	*s*, *d*	Computation of the inverse square root of vector elements
v?Cbrt	*s*, *d*	Computation of the cube root of vector elements
v?InvCbrt	*s*, *d*	Computation of the inverse cube root of vector elements
v?Pow2o3	*s*, *d*	Raising each vector element to the power of 2/3
v?Pow3o2	*s*, *d*	Raising each vector element to the power of 3/2
v?Pow	*s*, *d*, *c*, *z*	Raising each vector element to the specified power
v?Powx	*s*, *d*, *c*, *z*	Raising each vector element to the constant power
v?Hypot	*s*, *d*	Computation of the square root of sum of squares
Exponential and Logarithmic Functions		
v?Exp	*s*, *d*, *c*, *z*	Computation of the exponential of vector elements
v?Expm1	*s*, *d*	Computation of the exponential of vector elements decreased by 1
v?Ln	*s*, *d*, *c*, *z*	Computation of the natural logarithm of vector elements
v?Log10	*s*, *d*, *c*, *z*	Computation of the denary logarithm of vector elements
v?Log1p	*s*, *d*	Computation of the natural logarithm of vector elements that are increased by 1
Trigonometric Functions		
v?Cos	*s*, *d*, *c*, *z*	Computation of the cosine of vector elements
v?Sin	*s*, *d*, *c*, *z*	Computation of the sine of vector elements
v?SinCos	*s*, *d*	Computation of the sine and cosine of vector elements
v?CIS	*c*, *z*	Computation of the complex exponent of vector elements (cosine and sine combined to complex value)
v?Tan	*s*, *d*, *c*, *z*	Computation of the tangent of vector elements
v?Acos	*s*, *d*, *c*, *z*	Computation of the inverse cosine of vector elements
v?Asin	*s*, *d*, *c*, *z*	Computation of the inverse sine of vector elements
v?Atan	*s*, *d*, *c*, *z*	Computation of the inverse tangent of vector elements
v?Atan2	*s*, *d*	Computation of the four-quadrant inverse tangent of elements of two vectors
Hyperbolic Functions		
v?Cosh	*s*, *d*, *c*, *z*	Computation of the hyperbolic cosine of vector elements
v?Sinh	*s*, *d*, *c*, *z*	Computation of the hyperbolic sine of vector elements
v?Tanh	*s*, *d*, *c*, *z*	Computation of the hyperbolic tangent of vector elements
v?Acosh	*s*, *d*, *c*, *z*	Computation of the inverse hyperbolic cosine of vector elements
v?Asinh	*s*, *d*, *c*, *z*	Computation of the inverse hyperbolic sine of vector elements
v?Atanh	*s*, *d*, *c*, *z*	Computation of the inverse hyperbolic tangent of vector elements.
Special Functions		
v?Erf	*s*, *d*	Computation of the error function value of vector elements
v?Erfc	*s*, *d*	Computation of the complementary error function value of vector elements
v?CdfNorm	*s*, *d*	Computation of the cumulative normal distribution function value of vector elements
v?ErfInv	*s*, *d*	Computation of the inverse error function value of vector elements
v?ErfcInv	*s*, *d*	Computation of the inverse complementary error function value of vector elements
v?CdfNormInv	*s*, *d*	Computation of the inverse cumulative normal distribution function value of vector elements
v?LGamma	*s*, *d*	Computation of the natural logarithm for the absolute value of the gamma function of vector elements
v?TGamma	*s*, *d*	Computation of the gamma function of vector elements
Rounding Functions		
v?Floor	*s*, *d*	Rounding towards minus infinity
v?Ceil	*s*, *d*	Rounding towards plus infinity
v?Trunc	*s*, *d*	Rounding towards zero infinity

Function	Data Types	Description
v?Round	*s*, *d*	Rounding to nearest integer
v?NearbyInt	*s*, *d*	Rounding according to current mode
v?Rint	*s*, *d*	Rounding according to current mode and raising inexact result exception
v?Modf	*s*, *d*	Computation of the integer and fraction parts

Special Value Notations

This section defines notations of special values for complex functions. The definitions are provided in text, tables, or formulas.

- `z`, `z1`, `z2`, etc. denote complex numbers.
- `i`, $i^2=-1$ is the imaginary unit.
- `x`, `X`, `x1`, `x2`, etc. denote real imaginary parts.
- `y`, `Y`, `y1`, `y2`, etc. denote imaginary parts.
- `X` and `Y` represent any finite positive IEEE-754 floating point values, if not stated otherwise.
- Quiet `NaN` and signaling `NaN` are denoted with `QNAN` and `SNAN`, respectively.
- The IEEE-754 positive infinities or floating-point numbers are denoted with a `+` sign before `X`, `Y`, etc.
- The IEEE-754 negative infinities or floating-point numbers are denoted with a – sign before `X`, `Y`, etc.

`CONJ(z)` and `CIS(z)` are defined as follows:

`CONJ(x+i·y)=x-i·y`

`CIS(y)=cos(y)+i·sin(y).`

The special value tables show the result of the function for the `z` argument at the intersection of the `RE(z)` column and the `i*IM(z)` row. If the function raises an exception on the argument `z`, the lower part of this cell shows the raised exception and the VML Error Status. An empty cell indicates that this argument is normal and the result is defined mathematically.

Arithmetic Functions

Arithmetic functions perform the basic mathematical operations like addition, subtraction, multiplication or computation of the absolute value of the vector elements.

v?Add

Performs element by element addition of vector `a` and vector `b`.

Syntax

Fortran:

```
call vsadd( n, a, b, y )
call vmsadd( n, a, b, y, mode )
call vdadd( n, a, b, y )
call vmdadd( n, a, b, y, mode )
call vcadd( n, a, b, y )
call vmcadd( n, a, b, y, mode )
call vzadd( n, a, b, y )
call vmzadd( n, a, b, y, mode )
```

C:

```
vsAdd( n, a, b, y );
vmsAdd( n, a, b, y, mode );
vdAdd( n, a, b, y );
vmdAdd( n, a, b, y, mode );
vcAdd( n, a, b, y );
vmcAdd( n, a, b, y, mode );
vzAdd( n, a, b, y );
vmzAdd( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a, *b*	**FORTRAN 77:** `REAL` for `vsadd`, `vmsadd` `DOUBLE PRECISION` for `vdadd`, `vmdadd` `COMPLEX` for `vcadd`, `vmcadd` `DOUBLE COMPLEX` for `vzadd`, `vmzadd` **Fortran 90:** `REAL, INTENT(IN)` for `vsadd`, `vmsadd` `DOUBLE PRECISION, INTENT(IN)` for `vdadd`, `vmdadd` `COMPLEX, INTENT(IN)` for `vcadd`, `vmcadd` `DOUBLE COMPLEX, INTENT(IN)` for `vzadd`, `vmzadd` **C:** `const float*` for `vsAdd`, `vmsadd` `const double*` for `vdAdd`, `vmdadd` `const MKL_Complex8*` for `vcAdd`, `vmcadd` `const MKL_Complex16*` for `vzAdd`, `vmzadd`	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.

Name	Type	Description
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsadd`, `vmsadd` `DOUBLE PRECISION` for `vdadd`, `vmdadd` `COMPLEX`, for `vcadd`, `vmcadd` `DOUBLE COMPLEX` for `vzadd`, `vmzadd` **Fortran 90:** `REAL, INTENT(OUT)` for `vsadd`, `vmsadd` `DOUBLE PRECISION, INTENT(OUT)` for `vdadd`, `vmdadd` `COMPLEX, INTENT(OUT)` for `vcadd`, `vmcadd` `DOUBLE COMPLEX, INTENT(OUT)` for `vzadd`, `vmzadd` **C:** `float*` for `vsAdd`, `vmsadd` `double*` for `vdAdd`, `vmdadd` `MKL_Complex8*` for `vcAdd`, `vmcadd` `MKL_Complex16*` for `vzAdd`, `vmzadd`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Add` function performs element by element addition of vector *a* and vector *b*.

Special values for Real Function v?Add(x)

Argument 1	Argument 2	Result	Exception
+0	+0	+0	
+0	-0	+0	
-0	+0	+0	
-0	-0	-0	
+∞	+∞	+∞	
+∞	-∞	QNAN	INVALID
-∞	+∞	QNAN	INVALID
-∞	-∞	-∞	
SNAN	any value	QNAN	INVALID
any value	SNAN	QNAN	INVALID
QNAN	non-SNAN	QNAN	

Argument 1	Argument 2	Result	Exception
non-SNAN	QNAN	QNAN	

Specifications for special values of the complex functions are defined according to the following formula

```
Add(x1+i*y1,x2+i*y2) = (x1+x2) + i*(y1+y2)
```

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when x1, x2, y1, y2 are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In this case, the function returns ∞ in that part of the result, raises the OVERFLOW exception, and sets the VML Error Status to VML_STATUS_OVERFLOW.

v?Sub

Performs element by element subtraction of vector b from vector a.

Syntax

Fortran:

```
call vssub( n, a, b, y )
call vmssub( n, a, b, y, mode )
call vdsub( n, a, b, y )
call vmdsub( n, a, b, y, mode )
call vcsub( n, a, b, y )
call vmcsub( n, a, b, y, mode )
call vzsub( n, a, b, y )
call vmzsub( n, a, b, y, mode )
```

C:

```
vsSub( n, a, b, y );
vmsSub( n, a, b, y, mode );
vdSub( n, a, b, y );
vmdSub( n, a, b, y, mode );
vcSub( n, a, b, y );
vmcSub( n, a, b, y, mode );
vzSub( n, a, b, y );
vmzSub( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a, *b*	**FORTRAN 77:** REAL for vssub, vmssub DOUBLE PRECISION for vdsub, vmdsub COMPLEX for vcsub, vmcsub DOUBLE COMPLEX for vzsub, vmzsub **Fortran 90:** REAL, INTENT(IN) for vssub, vmssub DOUBLE PRECISION, INTENT(IN) for vdsub, vmdsub COMPLEX, INTENT(IN) for vcsub, vmcsub DOUBLE COMPLEX, INTENT(IN) for vzsub, vmzsub **C:** const float* for vsSub, vmssub const double* for vdSub, vmdsub const MKL_Complex8* for vcSub, vmcsub const MKL_Complex16* for vzSub, vmzsub	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vssub, vmssub DOUBLE PRECISION for vdsub, vmdsub COMPLEX for vcsub, vmcsub DOUBLE COMPLEX for vzsub, vmzsub **Fortran 90:** REALINTENT(OUT) for vssub, vmssub	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE PRECISION, INTENT(OUT)` for `vdsub, vmdsub` `COMPLEX, INTENT(OUT)` for `vcsub, vmcsub` `DOUBLE COMPLEX, INTENT(OUT)` for `vzsub, vmzsub` **C:** `float*` for `vsSub, vmssub` `double*` for `vdSub, vmdsub` `MKL_Complex8*` for `vcSub, vmcsub` `MKL_Complex16*` for `vzSub, vmzsub`	

Description

The `v?Sub` function performs element by element subtraction of vector *b* from vector *a*.

Special values for Real Function v?Sub(x)

Argument 1	Argument 2	Result	Exception
+0	+0	+0	
+0	-0	+0	
-0	+0	-0	
-0	-0	+0	
+∞	+∞	QNAN	INVALID
+∞	-∞	+∞	
-∞	+∞	-∞	
-∞	-∞	QNAN	INVALID
SNAN	any value	QNAN	INVALID
any value	SNAN	QNAN	INVALID
QNAN	non-SNAN	QNAN	
non-SNAN	QNAN	QNAN	

Specifications for special values of the complex functions are defined according to the following formula

`Sub(x1+i*y1,x2+i*y2) = (x1-x2) + i*(y1-y2).`

Overflow in a complex function occurs (supported in the `HA`/`LA` accuracy modes only) when `x1`, `x2`, `y1`, `y2` are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In this case, the function returns ∞ in that part of the result, raises the `OVERFLOW` exception, and sets the VML Error Status to `VML_STATUS_OVERFLOW`.

v?Sqr

Performs element by element squaring of the vector.

Syntax

Fortran:

```
call vssqr( n, a, y )
call vmssqr( n, a, y, mode )
call vdsqr( n, a, y )
call vmdsqr( n, a, y, mode )
```

C:

```
vsSqr( n, a, y );
vmsSqr( n, a, y, mode );
vdSqr( n, a, y );
vmdSqr( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vssqr, vmssqr` `DOUBLE PRECISION` for `vdsqr, vmdsqr` **Fortran 90:** `REAL, INTENT(IN)` for `vssqr, vmssqr` `DOUBLE PRECISION, INTENT(IN)` for `vdsqr, vmdsqr` **C:** `const float*` for `vsSqr, vmssqr` `const double*` for `vdSqr, vmdsqr`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vssqr, vmssqr` `DOUBLE PRECISION` for `vdsqr, vmdsqr`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	Fortran 90: `REAL, INTENT(OUT)` for `vssqr, vmssqr` `DOUBLE PRECISION, INTENT(OUT)` for `vdsqr, vmdsqr` **C:** `float*` for `vsSqr, vmssqr` `double*` for `vdSqr, vmdsqr`	

Description

The `v?Sqr` function performs element by element squaring of the vector.

Special Values for Real Function v?Sqr(x)

Argument	Result	Exception
+0	+0	
-0	+0	
+∞	+∞	
-∞	+∞	
QNAN	QNAN	
SNAN	QNAN	INVALID

v?Mul

Performs element by element multiplication of vector `a` and vector `b`.

Syntax

Fortran:

```
call vsmul( n, a, b, y )
call vmsmul( n, a, b, y, mode )
call vdmul( n, a, b, y )
call vmdmul( n, a, b, y, mode )
call vcmul( n, a, b, y )
call vmcmul( n, a, b, y, mode )
call vzmul( n, a, b, y )
call vmzcmul( n, a, b, y, mode )
```

C:

```
vsMul( n, a, b, y );
vmsMul( n, a, b, y, mode );
vdMul( n, a, b, y );
vmdMul( n, a, b, y, mode );
vcMul( n, a, b, y );
vmcMul( n, a, b, y, mode );
vzMul( n, a, b, y );
```

```
vmzMul( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a, *b*	**FORTRAN 77:** `REAL` for `vsmul`, `vmsmul` `DOUBLE PRECISION` for `vdmul`, `vmdmul` `COMPLEX` for `vcmul`, `vmcmul` `DOUBLE COMPLEX` for `vzmul`, `vmzmul` **Fortran 90:** `REAL, INTENT(IN)` for `vsmul`, `vmsmul` `DOUBLE PRECISION, INTENT(IN)` for `vdmul`, `vmdmul` `COMPLEX, INTENT(IN)` for `vcmul`, `vmcmul` `DOUBLE COMPLEX, INTENT(IN)` for `vzmul`, `vmzmul` **C:** `const float*` for `vsMul`, `vmsmul` `const double*` for `vdMul`, `vmdmul` `const MKL_Complex8*` for `vcMul`, `vmcMul` `const MKL_Complex16*` for `vzMul`, `vmzMul`	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsmul`, `vmsmul`	**FORTRAN:** Array that specifies the output vector *y*.

Name	Type	Description
	`DOUBLE PRECISION` for `vdmul`, `vmdmul` `COMPLEX`, for `vcmul`, `vmcmul` `DOUBLE COMPLEX` for `vzmul`, `vmzmul` **Fortran 90:** `REAL, INTENT(OUT)` for `vsmul`, `vmsmul` `DOUBLE PRECISION, INTENT(OUT)` for `vdmul`, `vmdmul` `COMPLEX, INTENT(OUT)` for `vcmul`, `vmcmul` `DOUBLE COMPLEX, INTENT(OUT)` for `vzmul`, `vmzmul` **C:** `float*` for `vsMul`, `vmsmul` `double*` for `vdMul`, `vmdmul` `MKL_Complex8*` for `vcMul`, `vmcMul` `MKL_Complex16*` for `vzMul`, `vmzMul`	**C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Mul` function performs element by element multiplication of vector *a* and vector *b*.

Special values for Real Function v?Mul(x)

Argument 1	Argument 2	Result	Exception
+0	+0	+0	
+0	-0	-0	
-0	+0	-0	
-0	-0	+0	
+0	+∞	QNAN	INVALID
+0	-∞	QNAN	INVALID
-0	+∞	QNAN	INVALID
-0	-∞	QNAN	INVALID
+∞	+0	QNAN	INVALID
+∞	-0	QNAN	INVALID
-∞	+0	QNAN	INVALID
-∞	-0	QNAN	INVALID
+∞	+∞	+∞	
+∞	-∞	-∞	
-∞	+∞	-∞	
-∞	-∞	+∞	
SNAN	any value	QNAN	INVALID
any value	SNAN	QNAN	INVALID
QNAN	non-SNAN	QNAN	
non-SNAN	QNAN	QNAN	

Specifications for special values of the complex functions are defined according to the following formula

`Mul(x1+i*y1,x2+i*y2) = (x1*x2-y1*y2) + i*(x1*y2+y1*x2).`

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when x1, x2, y1, y2 are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In this case, the function returns ∞ in that part of the result, raises the OVERFLOW exception, and sets the VML Error Status to VML_STATUS_OVERFLOW.

v?MulByConj

Performs element by element multiplication of vector a element and conjugated vector b element.

Syntax

Fortran:

```
call vcmulbyconj( n, a, b, y )
call vmcmulbyconj( n, a, b, y, mode )
call vzmulbyconj( n, a, b, y )
call vmzmulbyconj( n, a, b, y, mode )
```

C:

```
vcMulByConj( n, a, b, y );
vmcMulByConj( n, a, b, y, mode );
vzMulByConj( n, a, b, y );
vmzMulByConj( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a, *b*	**FORTRAN 77:** COMPLEX for vcmulbyconj, vmcmulbyconj DOUBLE COMPLEX for vzmulbyconj, vmzmulbyconj **Fortran 90:** COMPLEX, INTENT(IN) for vcmulbyconj, vmcmulbyconj DOUBLE COMPLEX, INTENT(IN) for vzmulbyconj, vmzmulbyconj **C:** const MKL_Complex8* for vcMulByConj, vmcMulByConj	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.

Name	Type	Description
	`const MKL_Complex16*` for `vzMulByConj`, `vmzMulByConj`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `COMPLEX` for `vcmulbyconj`, `vmcmulbyconj` `DOUBLE COMPLEX` for `vzmulbyconj`, `vmzmulbyconj` **Fortran 90:** `COMPLEX, INTENT(OUT)` for `vcmulbyconj`, `vmcmulbyconj` `DOUBLE COMPLEX, INTENT(OUT)` for `vzmulbyconj`, `vmzmulbyconj` **C:** `MKL_Complex8*` for `vcMulByConj`, `vmcMulByConj` `MKL_Complex16*` for `vzMulByConj`, `vmzMulByConj`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?MulByConj` function performs element by element multiplication of vector *a* element and conjugated vector *b* element.

Specifications for special values of the functions are found according to the formula

`MulByConj(x1+i*y1,x2+i*y2) = Mul(x1+i*y1,x2-i*y2).`

Overflow in a complex function occurs (supported in the `HA`/`LA` accuracy modes only) when `x1`, `x2`, `y1`, `y2` are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In this case, the function returns ∞ in that part of the result, raises the `OVERFLOW` exception, and sets the VML Error Status to `VML_STATUS_OVERFLOW`.

v?Conj

Performs element by element conjugation of the vector.

Syntax

Fortran:

```
call vcconj( n, a, y )
call vmcconj( n, a, y, mode )
call vzconj( n, a, y )
call vmzconj( n, a, y, mode )
```

C:

```
vcConj( n, a, y );
vmcConj( n, a, y, mode );
vzConj( n, a, y );
vmzConj( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `COMPLEX, INTENT(IN)` for `vcconj, vmcconj` `DOUBLE COMPLEX, INTENT(IN)` for `vzconj, vmzconj` **Fortran 90:** `COMPLEX, INTENT(IN)` for `vcconj, vmcconj` `DOUBLE COMPLEX, INTENT(IN)` for `vzconj, vmzconj` **C:** `const MKL_Complex8*` for `vcConj, vmcconj` `const MKL_Complex16*` for `vzConj, vmzconj`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `COMPLEX`, for `vcconj, vmcconj` `DOUBLE COMPLEX` for `vzconj, vmzconj` **Fortran 90:** `COMPLEX, INTENT(OUT)` for `vcconj, vmcconj`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE COMPLEX`, `INTENT(OUT)` for `vzconj`, `vmzconj` **C:** `MKL_Complex8*` for `vcConj`, `vmcconj` `MKL_Complex16*` for `vzConj`, `vmzconj`	

Description

The `v?Conj` function performs element by element conjugation of the vector.

No special values are specified. The function does not raise floating-point exceptions.

v?Abs

Computes absolute value of vector elements.

Syntax

Fortran:

```
call vsabs( n, a, y )
call vmsabs( n, a, y, mode )
call vdabs( n, a, y )
call vmdabs( n, a, y, mode )
call vcabs( n, a, y )
call vmcabs( n, a, y, mode )
call vzabs( n, a, y )
call vmzabs( n, a, y, mode )
```

C:

```
vsAbs( n, a, y );
vmsAbs( n, a, y, mode );
vdAbs( n, a, y );
vmdAbs( n, a, y, mode );
vcAbs( n, a, y );
vmcAbs( n, a, y, mode );
vzAbs( n, a, y );
vmzAbs( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vsabs, vmsabs DOUBLE PRECISION for vdabs, vmdabs COMPLEX for vcabs, vmcabs DOUBLE COMPLEX for vzabs, vmzabs **Fortran 90:** REAL, INTENT(IN) for vsabs, vmsabs DOUBLE PRECISION, INTENT(IN) for vdabs, vmdabs COMPLEX, INTENT(IN) for vcabs, vmcabs DOUBLE COMPLEX, INTENT(IN) for vzabs, vmzabs **C:** const float* for vsabs, vmsabs const double* for vdabs, vmdabs const MKL_Complex8* for vcAbs, vmcAbs const MKL_Complex16* for vzAbs, vmzAbs	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsabs, vmsabs, vcabs, vmcabs DOUBLE PRECISION for vdabs, vmdabs, vzabs, vmzabs **Fortran 90:** REAL, INTENT(OUT) for vsabs, vmsabs, vcabs, vmcabs DOUBLE PRECISION, INTENT(OUT) for vdabs, vmdabs, vzabs, vmzabs	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	C: `float*` for `vsabs`, `vmsabs`, `vcAbs`, `vmcAbs` `double*` for `vdabs`, `vmdabs`, `vzAbs`, `vmzAbs`	

Description

The `v?Abs` function computes an absolute value of vector elements.

Special Values for Real Function v?Abs(x)

Argument	Result	Exception
+0	+0	
-0	+0	
+∞	+∞	
-∞	+∞	
QNAN	QNAN	
SNAN	QNAN	INVALID

Specifications for special values of the complex functions are defined according to the following formula

`Abs(z) = Hypot(RE(z),IM(z)).`

v?Arg

Computes argument of vector elements.

Syntax

Fortran:

```
call vcarg( n, a, y )
call vmcarg( n, a, y, mode )
call vzarg( n, a, y )
call vmzarg( n, a, y, mode )
```

C:

```
vcArg( n, a, y );
vmcArg( n, a, y, mode );
vzArg( n, a, y );
vmzArg( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN77:** COMPLEX for vcarg, vmcarg DOUBLE COMPLEX for vzarg, vmzarg **Fortran 90:** COMPLEX, INTENT(IN) for vcarg, vmcarg DOUBLE COMPLEX, INTENT(IN) for vzarg, vmzarg **C:** const MKL_Complex8* for vcArg, vmcArg const MKL_Complex16* for vzArg, vmcArg	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vcarg, vmcarg DOUBLE PRECISION for vzarg, vmzarg **Fortran 90:** REAL, INTENT(OUT) for vcarg, vmcarg DOUBLE PRECISION, INTENT(OUT) for vzarg, vmzarg **C:** float* for vcArg, vmcArg double* for vzArg, vmcArg	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Arg function computes argument of vector elements.

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Arg(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	+3·π/4	+π/2	+π/2	+π/2	+π/2	+π/4	NAN
+i·Y	+π		+π/2	+π/2		+0	NAN
+i·0	+π	+π	+π	+0	+0	+0	NAN
-i·0	-π	-π	-π	-0	-0	-0	NAN
-i·Y	-π		-π/2	-π/2		-0	NAN
-i·∞	-3·π/4	-π/2	-π/2	-π/2	-π/2	-π/4	NAN
+i·NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- `Arg(z)=Atan2(IM(z),RE(z))`.

v?LinearFrac

Performs linear fraction transformation of vectors `a` and `b` with scalar parameters.

Syntax

Fortran:

```
call vslinearfrac( n, a, b, scalea, shifta, scaleb, shiftb, y )
call vmslinearfrac( n, a, b, scalea, shifta, scaleb, shiftb, y, mode )
call vdlinearfrac( n, a, b, scalea, shifta, scaleb, shiftb, y )
call vmdlinearfrac( n, a, b, scalea, shifta, scaleb, shiftb, y, mode )
```

C:

```
vsLinearFrac( n, a, b, scalea, shifta, scaleb, shiftb, y );
vmsLinearFrac( n, a, b, scalea, shifta, scaleb, shiftb, y, mode );
vdLinearFrac( n, a, b, scalea, shifta, scaleb, shiftb, y )
vmdLinearFrac( n, a, b, scalea, shifta, scaleb, shiftb, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.

Name	Type	Description
a, *b*	**FORTRAN 77:** REAL for vslinearfrac DOUBLE PRECISION for vdlinearfrac **Fortran 90:** REAL, INTENT(IN) for vslinearfrac DOUBLE PRECISION, INTENT(IN) for vdlinearfrac **C:** const float* for vsLinearFrac const double* for vdLinearFrac	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.
scalea, *scaleb*	**FORTRAN 77:** REAL for vslinearfrac DOUBLE PRECISION for vdlinearfrac **Fortran 90:** REAL, INTENT(IN) for vslinearfrac DOUBLE PRECISION, INTENT(IN) for vdlinearfrac **C:** const float* for vsLinearFrac const double* for vdLinearFrac	Constant values for shifting addends of vectors *a* and *b*.
shifta, *shiftb*	**FORTRAN 77:** REAL for vslinearfrac DOUBLE PRECISION for vdlinearfrac **Fortran 90:** REAL, INTENT(IN) for vslinearfrac DOUBLE PRECISION, INTENT(IN) for vdlinearfrac **C:** const float* for vsLinearFrac const double* for vdLinearFrac	Constant values for scaling multipliers of vectors *a* and *b*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vslinearfrac DOUBLE PRECISION for vdlinearfrac **Fortran 90:** REAL, INTENT(OUT) for vslinearfrac	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	DOUBLE PRECISION, INTENT(OUT) for vdlinearfrac **C:** float* for vsLinearFrac double* for vdLinearFrac	

Description

The v?LinearFrac function performs linear fraction transformation of vectors *a* by vector *b* with scalar parameters: scaling multipliers *scalea*, *scaleb* and shifting addends *shifta*, *shiftb*:

$y[i]=(scalea \cdot a[i]+shifta)/(scaleb \cdot b[i]+shiftb)$, $i=1,2 \ldots n$

The v?LinearFrac function is implemented in the EP accuracy mode only, therefore no special values are defined for this function. Correctness is guaranteed within the threshold limitations defined for each input parameter (see the table below); otherwise, the behavior is unspecified.

Threshold Limitations on Input Parameters
$2^{E_{MIN}/2} \le \lvert scalea \rvert \le 2^{(E_{MAX}-2)/2}$
$2^{E_{MIN}/2} \le \lvert scaleb \rvert \le 2^{(E_{MAX}-2)/2}$
$\lvert shifta \rvert \le 2^{E_{MAX}-2}$
$\lvert shiftb \rvert \le 2^{E_{MAX}-2}$
$2^{E_{MIN}/2} \le a[i] \le 2^{(E_{MAX}-2)/2}$
$2^{E_{MIN}/2} \le b[i] \le 2^{(E_{MAX}-2)/2}$
$a[i] \ne -(shifta/scalea)*(1-\delta_1), \lvert\delta_1\rvert \le 2^{1-(p-1)/2}$
$b[i] \ne -(shiftb/scaleb)*(1-\delta_2), \lvert\delta_2\rvert \le 2^{1-(p-1)/2}$

E_{MIN} and E_{MAX} are the maximum and minimum exponents and p is the number of significant bits (precision) for corresponding data type according to the ANSI/IEEE Std 754-2008 standard ([IEEE754]):

- for single precision E_{MIN} = -126, E_{MAX} = 127, p = 24
- for double precision E_{MIN} = -1022, E_{MAX} = 1023, p = 53

The thresholds become less strict for common cases with *scalea*=0 and/or *scaleb*=0:

- if *scalea*=0, there are no limitations for the values of *a*[i] and *shifta*
- if *scaleb*=0, there are no limitations for the values of *b*[i] and *shiftb*

Power and Root Functions

v?Inv

Performs element by element inversion of the vector.

Syntax

Fortran:

```
call vsinv( n, a, y )
call vmsinv( n, a, y, mode )
call vdinv( n, a, y )
```

```
call vmdinv( n, a, y, mode )
```

C:

```
vsInv( n, a, y );
vmsInv( n, a, y, mode );
vdInv( n, a, y );
vmdInv( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsinv, vmsinv` `DOUBLE PRECISION` for `vdinv, vmdinv` **Fortran 90:** `REAL, INTENT(IN)` for `vsinv, vmsinv` `DOUBLE PRECISION, INTENT(IN)` for `vdinv, vmdinv` **C:** `const float*` for `vsInv, vmsInv` `const double*` for `vdInv, vmdInv`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsinv, vmsinv` `DOUBLE PRECISION` for `vdinv, vmdinv`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	Fortran 90: `REAL, INTENT(OUT)` for `vsinv, vmsinv` `DOUBLE PRECISION, INTENT(OUT)` for `vdinv, vmdinv` **C:** `float*` for `vsInv, vmsInv` `double*` for `vdInv, vmdInv`	

Description

The `v?Inv` function performs element by element inversion of the vector.

Special Values for Real Function v?Inv(x)

Argument	Result	VML Error Status	Exception
+0	+∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
+∞	+0		
-∞	-0		
QNAN	QNAN		
SNAN	QNAN		INVALID

v?Div

Performs element by element division of vector `a` by vector `b`

Syntax

Fortran:

```
call vsdiv( n, a, b, y )
call vmsdiv( n, a, b, y, mode )
call vddiv( n, a, b, y )
call vmddiv( n, a, b, y, mode )
call vcdiv( n, a, b, y )
call vmcdiv( n, a, b, y, mode )
call vzdiv( n, a, b, y )
call vmzdiv( n, a, b, y, mode )
```

C:

```
vsDiv( n, a, b, y );
vmsDiv( n, a, b, y, mode );
vdDiv( n, a, b, y );
vmdDiv( n, a, b, y, mode );
vcDiv( n, a, b, y );
vmcDiv( n, a, b, y, mode );
vzDiv( n, a, b, y );
```

```
vmzDiv( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a, *b*	**FORTRAN 77:** `REAL` for `vsdiv`, `vmsdiv` `DOUBLE PRECISION` for `vddiv`, `vmddiv` `COMPLEX` for `vcdiv`, `vmcdiv` `DOUBLE COMPLEX` for `vzdiv`, `vmzdiv` **Fortran 90:** `REAL, INTENT(IN)` for `vsdiv`, `vmsdiv` `DOUBLE PRECISION, INTENT(IN)` for `vddiv`, `vmddiv` `COMPLEX, INTENT(IN)` for `vcdiv`, `vmcdiv` `DOUBLE COMPLEX, INTENT(IN)` for `vzdiv`, `vmzdiv` **C:** `const float*` for `vsDiv`, `vmsDiv` `const double*` for `vdDiv`, `vmdDiv` `const MKL_Complex8*` for `vcDiv`, `vmcDiv` `const MKL_Complex16*` for `vzDiv`, `vmzDiv`	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Real v?Div Function

Data Type	Threshold Limitations on Input Parameters
single precision	`abs(a[i]) < abs(b[i]) * FLT_MAX`
double precision	`abs(a[i]) < abs(b[i]) * DBL_MAX`

Precision overflow thresholds for the complex `v?Div` function are beyond the scope of this document.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsdiv, vmsdiv DOUBLE PRECISION for vddiv, vmddiv COMPLEX for vcdiv, vmcdiv DOUBLE COMPLEX for vzdiv, vmzdiv **Fortran 90:** REAL, INTENT(OUT) for vsdiv, vmsdiv DOUBLE PRECISION, INTENT(OUT) for vddiv, vmddiv COMPLEX, INTENT(OUT) for vcdiv, vmcdiv DOUBLE COMPLEX, INTENT(OUT) for vzdiv, vmzdiv **C:** float* for vsDiv, vmsDiv double* for vdDiv, vmdDiv MKL_Complex8* for vcDiv, vmcDiv MKL_Complex16* for vzDiv, vmzDiv	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Div function performs element by element division of vector *a* by vector *b*.

Special values for Real Function v?Div(x)

Argument 1	Argument 2	Result	VML Error Status	Exception
X > +0	+0	+∞	VML_STATUS_SING	ZERODIVIDE
X > +0	-0	-∞	VML_STATUS_SING	ZERODIVIDE
X < +0	+0	-∞	VML_STATUS_SING	ZERODIVIDE
X < +0	-0	+∞	VML_STATUS_SING	ZERODIVIDE
+0	+0	QNAN	VML_STATUS_SING	
-0	-0	QNAN	VML_STATUS_SING	
X > +0	+∞	+0		
X > +0	-∞	-0		
+∞	+∞	QNAN		
-∞	-∞	QNAN		
QNAN	QNAN	QNAN		
SNAN	SNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

Div(x1+i*y1,x2+i*y2) = (x1+i*y1)*(x2-i*y2)/(x2*x2+y2*y2).

Overflow in a complex function occurs when x2+i*y2 is not zero, x1, x2, y1, y2 are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In that case, the function returns ∞ in that part of the result, raises the OVERFLOW exception, and sets the VML Error Status to VML_STATUS_OVERFLOW.

v?Sqrt

Computes a square root of vector elements.

Syntax

Fortran:

```
call vssqrt( n, a, y )
call vmssqrt( n, a, y, mode )
call vdsqrt( n, a, y )
call vmdsqrt( n, a, y, mode )
call vcsqrt( n, a, y )
call vmcsqrt( n, a, y, mode )
call vzsqrt( n, a, y )
call vmzsqrt( n, a, y, mode )
```

C:

```
vsSqrt( n, a, y );
vmsSqrt( n, a, y, mode );
vdSqrt( n, a, y );
vmdSqrt( n, a, y, mode );
vcSqrt( n, a, y );
vmcSqrt( n, a, y, mode );
vzSqrt( n, a, y );
vmzSqrt( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vssqrt`, `vmssqrt` `DOUBLE PRECISION` for `vdsqrt`, `vmdsqrt` `COMPLEX` for `vcsqrt`, `vmcsqrt`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`DOUBLE COMPLEX` for `vzsqrt`, `vmzsqrt` **Fortran 90:** `REAL, INTENT(IN)` for `vssqrt`, `vmssqrt` `DOUBLE PRECISION, INTENT(IN)` for `vdsqrt`, `vmdsqrt` `COMPLEX, INTENT(IN)` for `vcsqrt`, `vmcsqrt` `DOUBLE COMPLEX, INTENT(IN)` for `vzsqrt`, `vmzsqrt` **C:** `const float*` for `vsSqrt`, `vmsSqrt` `const double*` for `vdSqrt`, `vmdSqrt` `const MKL_Complex8*` for `vcSqrt`, `vmcSqrt` `const MKL_Complex16*` for `vzSqrt`, `vmzSqrt`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN:** `REAL` for `vssqrt`, `vmssqrt` `DOUBLE PRECISION` for `vdsqrt`, `vmdsqrt` `COMPLEX` for `vcsqrt`, `vmcsqrt` `DOUBLE COMPLEX` for `vzsqrt`, `vmzsqrt` **Fortran 90:** `REAL, INTENT(OUT)` for `vssqrt`, `vmssqrt` `DOUBLE PRECISION, INTENT(OUT)` for `vdsqrt`, `vmdsqrt` `COMPLEX, INTENT(OUT)` for `vcsqrt`, `vmcsqrt` `DOUBLE COMPLEX, INTENT(OUT)` for `vzsqrt`, `vmzsqrt` **C:** `float*` for `vsSqrt`, `vmsSqrt` `double*` for `vdSqrt`, `vmdSqrt`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`MKL_Complex8*` for `vcSqrt`, `vmcSqrt` `MKL_Complex16*` for `vzSqrt`, `vmzSqrt`	

Description

The `v?Sqrt` function computes a square root of vector elements.

Special Values for Real Function v?Sqrt(x)

Argument	Result	VML Error Status	Exception
X < +0	QNAN	VML_STATUS_ERRDOM	INVALID
+0	+0		
-0	-0		
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Sqrt(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	+∞+i·∞	+∞+i·∞	+∞+i·∞	+∞+i·∞	+∞+i·∞	+∞+i·∞	+∞+i·∞
+i·Y	+0+i·∞					+∞+i·0	QNAN+i·QNAN
+i·0	+0+i·∞		+0+i·0	+0+i·0		+∞+i·0	QNAN+i·QNAN
-i·0	+0-i·∞		+0-i·0	+0-i·0		+∞-i·0	QNAN+i·QNAN
-i·Y	+0-i·∞					+∞-i·0	QNAN+i·QNAN
-i·∞	+∞-i·∞	+∞-i·∞	+∞-i·∞	+∞-i·∞	+∞-i·∞	+∞-i·∞	+∞-i·∞
+i·NAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	+∞+i·QNAN	QNAN+i·QNAN

Notes:

- raises `INVALID` exception when the real or imaginary part of the argument is `SNAN`
- `Sqrt(CONJ(z))=CONJ(Sqrt(z))`.

v?InvSqrt

Computes an inverse square root of vector elements.

Syntax

Fortran:

```
call vsinvsqrt( n, a, y )
call vmsinvsqrt( n, a, y, mode )
call vdinvsqrt( n, a, y )
call vmdinvsqrt( n, a, y, mode )
```

C:

```
vsInvSqrt( n, a, y );
vmsInvSqrt( n, a, y, mode );
vdInvSqrt( n, a, y );
vmdInvSqrt( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsinvsqrt, vmsinvsqrt` `DOUBLE PRECISION` for `vdinvsqrt, vmdinvsqrt` **Fortran 90:** `REAL, INTENT(IN)` for `vsinvsqrt, vmsinvsqrt` `DOUBLE PRECISION, INTENT(IN)` for `vdinvsqrt, vmdinvsqrt` **C:** `const float*` for `vsInvSqrt, vmsInvSqrt` `const double*` for `vdInvSqrt, vmdInvSqrt`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsinvsqrt, vmsinvsqrt` `DOUBLE PRECISION` for `vdinvsqrt, vmdinvsqrt` **Fortran 90:** `REAL, INTENT(OUT)` for `vsinvsqrt, vmsinvsqrt`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	DOUBLE PRECISION, INTENT(OUT) for vdinvsqrt, vmdinvsqrt **C:** float* for vsInvSqrt, vmsInvSqrt double* for vdInvSqrt, vmdInvSqrt	

Description

The v?InvSqrt function computes an inverse square root of vector elements.

Special Values for Real Function v?InvSqrt(x)

Argument	Result	VML Error Status	Exception
X < +0	QNAN	VML_STATUS_ERRDOM	INVALID
+0	+∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+0		
QNAN	QNAN		
SNAN	QNAN		INVALID

v?Cbrt

Computes a cube root of vector elements.

Syntax

Fortran:

```
call vscbrt( n, a, y )
call vmscbrt( n, a, y, mode )
call vdcbrt( n, a, y )
call vmdcbrt( n, a, y, mode )
```

C:

```
vsCbrt( n, a, y );
vmsCbrt( n, a, y, mode );
vdCbrt( n, a, y );
vmdCbrt( n, a, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vscbrt, vmscbrt DOUBLE PRECISION for vdcbrt, vmdcbrt **Fortran 90:** REAL, INTENT(IN) for vscbrt, vmscbrt DOUBLE PRECISION, INTENT(IN) for vdcbrt, vmdcbrt **C:** const float* for vsCbrt, vmsCbrt const double* for vdCbrt, vmdCbrt	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vscbrt, vmscbrt DOUBLE PRECISION for vdcbrt, vmdcbrt **Fortran 90:** REAL, INTENT(OUT) for vscbrt, vmscbrt DOUBLE PRECISION, INTENT(OUT) for vdcbrt, vmdcbrt **C:** float* for vsCbrt, vmsCbrt double* for vdCbrt, vmdCbrt	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Cbrt function computes a cube root of vector elements.

Special Values for Real Function v?Cbrt(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	

Argument	Result	Exception
-∞	-∞	
QNAN	QNAN	
SNAN	QNAN	INVALID

v?InvCbrt

Computes an inverse cube root of vector elements.

Syntax

Fortran:

```
call vsinvcbrt( n, a, y )
call vmsinvcbrt( n, a, y, mode )
call vdinvcbrt( n, a, y )
call vmdinvcbrt( n, a, y, mode )
```

C:

```
vsInvCbrt( n, a, y );
vmsInvCbrt( n, a, y, mode );
vdInvCbrt( n, a, y );
vmdInvCbrt( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsinvcbrt`, `vmsinvcbrt` `DOUBLE PRECISION` for `vdinvcbrt`, `vmdinvcbrt` **Fortran 90:** `REAL, INTENT(IN)` for `vsinvcbrt`, `vmsinvcbrt` `DOUBLE PRECISION, INTENT(IN)` for `vdinvcbrt`, `vmdinvcbrt` **C:** `const float*` for `vsInvCbrt`, `vmsInvCbrt`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`const double*` for `vdInvCbrt`, `vmdInvCbrt`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsinvcbrt`, `vmsinvcbrt` `DOUBLE PRECISION` for `vdinvcbrt`, `vmdinvcbrt` **Fortran 90:** `REAL, INTENT(OUT)` for `vsinvcbrt, vmsinvcbrt` `DOUBLE PRECISION, INTENT(OUT)` for `vdinvcbrt, vmdinvcbrt` **C:** `float*` for `vsInvCbrt`, `vmsInvCbrt` `double*` for `vdInvCbrt, vmdInvCbrt`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?InvCbrt` function computes an inverse cube root of vector elements.

Special Values for Real Function v?InvCbrt(x)

Argument	Result	VML Error Status	Exception
+0	+∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
+∞	+0		
-∞	-0		
QNAN	QNAN		
SNAN	QNAN		INVALID

v?Pow2o3

Raises each element of a vector to the constant power 2/3.

Syntax

Fortran:

```
call vspow2o3( n, a, y )
call vmspow2o3( n, a, y, mode )
call vdpow2o3( n, a, y )
call vmdpow2o3( n, a, y, mode )
```

C:

```
vsPow2o3( n, a, y );
vmsPow2o3( n, a, y, mode );
vdPow2o3( n, a, y );
vmdPow2o3( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vspow2o3, vmspow2o3` `DOUBLE PRECISION` for `vdpow2o3, vmdpow2o3` **Fortran 90:** `REAL, INTENT(IN)` for `vspow2o3, vmspow2o3` `DOUBLE PRECISION, INTENT(IN)` for `vdpow2o3, vmdpow2o3` **C:** `const float*` for `vsPow2o3, vmsPow2o3` `const double*` for `vdPow2o3, vmdPow2o3`	**FORTRAN:** Arrays, specify the input vector *a*. **C:** Pointers to arrays that contain the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vspow2o3, vmspow2o3` `DOUBLE PRECISION` for `vdpow2o3, vmdpow2o3` **Fortran 90:** `REAL, INTENT(OUT)` for `vspow2o3, vmspow2o3`	**FORTRAN:** Array, specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE PRECISION, INTENT(OUT)` for `vdpow2o3, vmdpow2o3` **C:** `float*` for `vsPow2o3, vmsPow2o3` `double*` for `vdPow2o3, vmdPow2o3`	

Description

The `v?Pow2o3` function raises each element of a vector to the constant power 2/3.

Special Values for Real Function v?Pow2o3(x)

Argument	Result	Exception
+0	+0	
-0	+0	
+∞	+∞	
-∞	+∞	
QNAN	QNAN	
SNAN	QNAN	INVALID

v?Pow3o2

Raises each element of a vector to the constant power 3/2.

Syntax

Fortran:

```
call vspow3o2( n, a, y )
call vmspow3o2( n, a, y, mode )
call vdpow3o2( n, a, y )
call vmdpow3o2( n, a, y, mode )
```

C:

```
vsPow3o2( n, a, y );
vmsPow3o2( n, a, y, mode );
vdPow3o2( n, a, y );
vmdPow3o2( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)`	Specifies the number of elements to be calculated.

Name	Type	Description
	C: `const int`	
a	**FORTRAN 77:** `REAL` for `vspow3o2`, `vmspow3o2` `DOUBLE PRECISION` for `vdpow3o2`, `vmdpow3o2` **Fortran 90:** `REAL, INTENT(IN)` for `vspow3o2, vmspow3o2` `DOUBLE PRECISION, INTENT(IN)` for `vdpow3o2, vmdpow3o2` **C:** `const float*` for `vsPow3o2, vmsPow3o2` `const double*` for `vdPow3o2, vmdPow3o2`	**FORTRAN:** Arrays, specify the input vector *a*. **C:** Pointers to arrays that contain the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Pow3o2 Function

Data Type	Threshold Limitations on Input Parameters
single precision	`abs(a[i]) < ( FLT_MAX )`$^{2/3}$
double precision	`abs(a[i]) < ( DBL_MAX )`$^{2/3}$

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vspow3o2`, `vmspow3o2` `DOUBLE PRECISION` for `vdpow3o2`, `vmdpow3o2` **Fortran 90:** `REAL, INTENT(OUT)` for `vspow3o2, vmspow3o2` `DOUBLE PRECISION, INTENT(OUT)` for `vdpow3o2, vmdpow3o2` **C:** `float*` for `vsPow3o2, vmsPow3o2` `double*` for `vdPow3o2, vmdPow3o2`	**FORTRAN:** Array, specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Pow3o2` function raises each element of a vector to the constant power 3/2.

Special Values for Real Function v?Pow3o2(x)

Argument	Result	VML Error Status	Exception
X < +0	QNAN	VML_STATUS_ERRDOM	INVALID

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

v?Pow

Computes `a` *to the power* `b` *for elements of two vectors.*

Syntax

Fortran:

```
call vspow( n, a, b, y )
call vmspow( n, a, b, y, mode )
call vdpow( n, a, b, y )
call vmdpow( n, a, b, y, mode )
call vcpow( n, a, b, y )
call vmcpow( n, a, b, y, mode )
call vzpow( n, a, b, y )
call vmzpow( n, a, b, y, mode )
```

C:

```
vsPow( n, a, b, y );
vmsPow( n, a, b, y, mode );
vdPow( n, a, b, y );
vmdPow( n, a, b, y, mode );
vcPow( n, a, b, y );
vmcPow( n, a, b, y, mode );
vzPow( n, a, b, y );
vmzPow( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)`	Specifies the number of elements to be calculated.

Name	Type	Description
	C: `const int`	
a, *b*	**FORTRAN 77:** `REAL` for `vspow`, `vmspow` `DOUBLE PRECISION` for `vdpow`, `vmdpow` `COMPLEX` for `vcpow`, `vmcpow` `DOUBLE COMPLEX` for `vzpow`, `vmzpow` **Fortran 90:** `REAL, INTENT(IN)` for `vspow`, `vmspow` `DOUBLE PRECISION, INTENT(IN)` for `vdpow`, `vmdpow` `COMPLEX, INTENT(IN)` for `vcpow`, `vmcpow` `DOUBLE COMPLEX, INTENT(IN)` for `vzpow`, `vmzpow` **C:** `const float*` for `vsPow`, `vmsPow` `const double*` for `vdPow`, `vmdPow` `const MKL_Complex8*` for `vcPow`, `vmcPow` `const MKL_Complex16*` for `vzPow`, `vmzPow`	**FORTRAN:** Arrays that specify the input vectors *a* and *b*. **C:** Pointers to arrays that contain the input vectors *a* and *b*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Real v?Pow Function

Data Type	Threshold Limitations on Input Parameters
single precision	$abs(a[i]) < (FLT\_MAX)^{1/b[i]}$
double precision	$abs(a[i]) < (DBL\_MAX)^{1/b[i]}$

Precision overflow thresholds for the complex `v?Pow` function are beyond the scope of this document.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vspow`, `vmspow` `DOUBLE PRECISION` for `vdpow`, `vmdpow` `COMPLEX` for `vcpow`, `vmcpow` `DOUBLE COMPLEX` for `vzpow`, `vmzpow`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	Fortran 90: `REAL, INTENT(OUT)` for `vspow, vmspow` `DOUBLE PRECISION, INTENT(OUT)` for `vdpow, vmdpow` `COMPLEX, INTENT(OUT)` for `vcpow, vmcpow` `DOUBLE COMPLEX, INTENT(OUT)` for `vzpow, vmzpow` **C:** `float*` for `vsPow, vmsPow` `double*` for `vdPow, vmdPow` `MKL_Complex8*` for `vcPow, vmcPow` `MKL_Complex16*` for `vzPow, vmzPow`	

Description

The `v?Pow` function computes *a* to the power *b* for elements of two vectors.

The real function `v(s/d)Pow` has certain limitations on the input range of *a* and *b* parameters. Specifically, if *a[i]* is positive, then *b[i]* may be arbitrary. For negative *a[i]*, the value of *b[i]* must be an integer (either positive or negative).

The complex function `v(c/z)Pow` has no input range limitations.

Special values for Real Function v?Pow(x)

Argument 1	Argument 2	Result	VML Error Status	Exception
+0	neg. odd integer	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
-0	neg. odd integer	-∞	VML_STATUS_ERRDOM	ZERODIVIDE
+0	neg. even integer	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
-0	neg. even integer	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
+0	neg. non-integer	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
-0	neg. non-integer	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
-0	pos. odd integer	+0		
-0	pos. odd integer	-0		
+0	pos. even integer	+0		
-0	pos. even integer	+0		
+0	pos. non-integer	+0		
-0	pos. non-integer	+0		
-1	+∞	+1		
-1	-∞	+1		
+1	any value	+1		
+1	+0	+1		
+1	-0	+1		
+1	+∞	+1		
+1	-∞	+1		
+1	QNAN	+1		
any value	+0	+1		
+0	+0	+1		

Argument 1	Argument 2	Result	VML Error Status	Exception
-0	+0	+1		
+∞	+0	+1		
-∞	+0	+1		
QNAN	+0	+1		
any value	-0	+1		
+0	-0	+1		
-0	-0	+1		
+∞	-0	+1		
-∞	-0	+1		
QNAN	-0	+1		
X < +0	non-integer	QNAN	VML_STATUS_ERRDOM	INVALID
|X| < 1	-∞	+∞		
+0	-∞	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
-0	-∞	+∞	VML_STATUS_ERRDOM	ZERODIVIDE
|X| > 1	-∞	+0		
+∞	-∞	+0		
-∞	-∞	+0		
|X| < 1	+∞	+0		
+0	+∞	+0		
-0	+∞	+0		
|X| > 1	+∞	+∞		
+∞	+∞	+∞		
-∞	+∞	+∞		
-∞	neg. odd integer	-0		
-∞	neg. even integer	+0		
-∞	neg. non-integer	+0		
-∞	pos. odd integer	-∞		
-∞	pos. even integer	+∞		
-∞	pos. non-integer	+∞		
+∞	X < +0	+0		
+∞	X > +0	+∞		
QNAN	QNAN	QNAN		
QNAN	SNAN	QNAN		INVALID
SNAN	QNAN	QNAN		INVALID
SNAN	SNAN	QNAN		INVALID

Overflow in a complex function occurs (supported in the `HA`/`LA` accuracy modes only) when `x1`, `x2`, `y1`, `y2` are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In this case, the function returns ∞ in that part of the result, raises the `OVERFLOW` exception, and sets the VML Error Status to `VML_STATUS_OVERFLOW`.

v?Powx

Raises each element of a vector to the constant power.

Syntax

Fortran:

```
call vspowx( n, a, b, y )
```

```
call vmspowx( n, a, b, y, mode )
call vdpowx( n, a, b, y )
call vmdpowx( n, a, b, y, mode )
call vcpowx( n, a, b, y )
call vmcpowx( n, a, b, y, mode )
call vzpowx( n, a, b, y )
call vmzpowx( n, a, b, y, mode )
```

C:

```
vsPowx( n, a, b, y );
vmsPowx( n, a, b, y, mode );
vdPowx( n, a, b, y );
vmdPowx( n, a, b, y, mode );
vcPowx( n, a, b, y );
vmcPowx( n, a, b, y, mode );
vzPowx( n, a, b, y );
vmzPowx( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vspowx`, `vmspowx` `DOUBLE PRECISION` for `vdpowx`, `vmdpowx` `COMPLEX` for `vcpowx`, `vmcpowx` `DOUBLE COMPLEX` for `vzpowx`, `vmzpowx` **Fortran 90:** `REAL, INTENT(IN)` for `vspowx`, `vmspowx` `DOUBLE PRECISION, INTENT(IN)` for `vdpowx`, `vmdpowx` `COMPLEX, INTENT(IN)` for `vcpowx`, `vmcpowx`	**FORTRAN:** Array *a* that specifies the input vector **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	DOUBLE COMPLEX, INTENT(IN) for vzpowx, vmzpowx **C:** const float* for vsPowx, vmsPowx const double* for vdPowx, vmdPowx const MKL_Complex8* for vcPowx, vmcPowx const MKL_Complex16* for vzPowx, vmzPowx	
b	**FORTRAN 77:** REAL for vspowx, vmspowx DOUBLE PRECISION for vdpowx, vmdpowx COMPLEX for vcpowx, vmcpowx DOUBLE COMPLEX for vzpowx, vmzpowx **Fortran 90:** REAL, INTENT(IN) for vspowx, vmspowx DOUBLE PRECISION, INTENT(IN) for vdpowx, vmdpowx COMPLEX, INTENT(IN) for vcpowx, vmcpowx DOUBLE COMPLEX, INTENT(IN) for vzpowx, vmzpowx **C:** const float* for vsPowx, vmsPowx const double* for vdPowx, vmdPowx const MKL_Complex8* for vcPowx, vmcPowx const MKL_Complex16* for vzPowx, vmzPowx	**FORTRAN:** Scalar value *b* that is the constant power. **C:** Constant value for power *b*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Precision Overflow Thresholds for Real v?Powx Function

Data Type	Threshold Limitations on Input Parameters
single precision	abs(*a*[i]) < (FLT_MAX)$^{1/b}$
double precision	abs(*a*[i]) < (DBL_MAX)$^{1/b}$

Precision overflow thresholds for the complex v?Powx function are beyond the scope of this document.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vspowx, vmspowx DOUBLE PRECISION for vdpowx, vmdpowx COMPLEX for vcpowx, vmcpowx DOUBLE COMPLEX for vzpowx, vmzpowx **Fortran 90:** REAL, INTENT(OUT) for vspowx, vmspowx DOUBLE PRECISION, INTENT(OUT) for vdpowx, vmdpowx COMPLEX, INTENT(OUT) for vcpowx, vmcpowx DOUBLE COMPLEX, INTENT(OUT) for vzpowx, vmzpowx **C:** float* for vsPowx, vmsPowx double* for vdPowx, vmdPowx MKL_Complex8* for vcPowx, vmcPowx MKL_Complex16* for vzPowx, vmzPowx	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Powx function raises each element of a vector to the constant power.

The real function v(s/d)Powx has certain limitations on the input range of *a* and *b* parameters. Specifically, if *a[i]* is positive, then *b* may be arbitrary. For negative *a[i]*, the value of *b* must be an integer (either positive or negative).

The complex function v(c/z)Powx has no input range limitations.

Special values are the same as for the v?Pow function.

v?Hypot

Computes a square root of sum of two squared elements.

Syntax

Fortran:

```
call vshypot( n, a, b, y )
call vmshypot( n, a, b, y, mode )
call vdhypot( n, a, b, y )
call vmdhypot( n, a, b, y, mode )
```

C:

```
vsHypot( n, a, b, y );
vmsHypot( n, a, b, y, mode );
vdHypot( n, a, b, y );
vmdHypot( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of elements to be calculated.
a, *b*	**FORTRAN 77:** `REAL` for `vshypot`, `vmshypot` `DOUBLE PRECISION` for `vdhypot`, `vmdhypot` **Fortran 90:** `REAL, INTENT(IN)` for `vshypot, vmshypot` `DOUBLE PRECISION, INTENT(IN)` for `vdhypot, vmdhypot` **C:** `const float*` for `vsHypot`, `vmsHypot` `const double*` for `vdHypot`, `vmdHypot`	**FORTRAN:** Arrays that specify the input vectors *a* and *b* **C:** Pointers to arrays that contain the input vectors *a* and *b*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Hypot Function

Data Type	Threshold Limitations on Input Parameters
single precision	`abs(a[i]) < sqrt(FLT_MAX)` `abs(b[i]) < sqrt(FLT_MAX)`
double precision	`abs(a[i]) < sqrt(DBL_MAX)` `abs(b[i]) < sqrt(DBL_MAX)`

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vshypot, vmshypot DOUBLE PRECISION for vdhypot, vmdhypot **Fortran 90:** REAL, INTENT(OUT) for vshypot, vmshypot DOUBLE PRECISION, INTENT(OUT) for vdhypot, vmdhypot **C:** float* for vsHypot, vmsHypot double* for vdHypot, vmdHypot	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The function v?Hypot computes a square root of sum of two squared elements.

Special values for Real Function v?Hypot(x)

Argument 1	Argument 2	Result	Exception
+0	+0	+0	
-0	-0	+0	
+∞	any value	+∞	
any value	+∞	+∞	
SNAN	any value	QNAN	INVALID
any value	SNAN	QNAN	INVALID
QNAN	any value	QNAN	
any value	QNAN	QNAN	

Exponential and Logarithmic Functions

v?Exp

Computes an exponential of vector elements.

Syntax

Fortran:

```
call vsexp( n, a, y )
call vmsexp( n, a, y, mode )
call vdexp( n, a, y )
call vmdexp( n, a, y, mode )
call vcexp( n, a, y )
call vmcexp( n, a, y, mode )
call vzexp( n, a, y )
call vmzexp( n, a, y, mode )
```

C:

```
vsExp( n, a, y );
vmsExp( n, a, y, mode );
vdExp( n, a, y );
vmdExp( n, a, y, mode );
vcExp( n, a, y );
vmcExp( n, a, y, mode );
vzExp( n, a, y );
vmzExp( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsexp, vmsexp` `DOUBLE PRECISION` for `vdexp, vmdexp` `COMPLEX` for `vcexp, vmcexp` `DOUBLE COMPLEX` for `vzexp, vmzexp` **Fortran 90:** `REAL, INTENT(IN)` for `vsexp, vmsexp` `DOUBLE PRECISION, INTENT(IN)` for `vdexp, vmdexp` `COMPLEX, INTENT(IN)` for `vcexp, vmcexp` `DOUBLE COMPLEX, INTENT(IN)` for `vzexp, vmzexp` **C:** `const float*` for `vsExp, vmsExp` `const double*` for `vdExp, vmdExp` `const MKL_Complex8*` for `vcExp, vmcExp` `const MKL_Complex16*` for `vzExp, vmzExp`	**FORTRAN:** Array, specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Real v?Exp Function

Data Type	Threshold Limitations on Input Parameters
single precision	`a[i] < Ln( FLT_MAX )`
double precision	`a[i] < Ln( DBL_MAX )`

Precision overflow thresholds for the complex `v?Exp` function are beyond the scope of this document.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsexp`, `vmsexp` `DOUBLE PRECISION` for `vdexp`, `vmdexp` `COMPLEX` for `vcexp`, `vmcexp` `DOUBLE COMPLEX` for `vzexp`, `vmzexp` **Fortran 90:** `REAL, INTENT(OUT)` for `vsexp`, `vmsexp` `DOUBLE PRECISION, INTENT(OUT)` for `vdexp`, `vmdexp` `COMPLEX, INTENT(OUT)` for `vcexp`, `vmcexp` `DOUBLE COMPLEX, INTENT(OUT)` for `vzexp`, `vmzexp` **C:** `float*` for `vsExp`, `vmsExp` `double*` for `vdExp`, `vmdExp` `MKL_Complex8*` for `vcExp`, `vmcExp` `MKL_Complex16*` for `vzExp`, `vmzExp`	**FORTRAN:** Array, specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Exp` function computes an exponential of vector elements.

Special Values for Real Function v?Exp(x)

Argument	Result	VML Error Status	Exception
+0	+1		
-0	+1		
X > overflow	+∞	VML_STATUS_OVERFLOW	OVERFLOW
X < underflow	+0	VML_STATUS_UNDERFLOW	UNDERFLOW
+∞	+∞		

Argument	Result	VML Error Status	Exception
-∞	+0		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Exp(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	+0+i·0	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN INVALID	QNAN+i·QNAN INVALID
+i·Y	+0·CIS(Y)					+∞·CIS(Y)	QNAN+i·QNAN
+i·0	+0·CIS(0)		+1+i·0	+1+i·0		+∞+i·0	QNAN+i·0
-i·0	+0·CIS(0)		+1-i·0	+1-i·0		+∞-i·0	QNAN-i·0
-i·Y	+0·CIS(Y)					+∞·CIS(Y)	QNAN+i·QNAN
-i·∞	+0-i·0	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN INVALID	QNAN+i·QNAN
+i·NAN	+0+i·0	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN	QNAN+i·QNAN

Notes:

- raises the `INVALID` exception when real or imaginary part of the argument is `SNAN`
- raises the `INVALID` exception on argument `z=-∞+i·QNAN`
- raises the `OVERFLOW` exception and sets the VML Error Status to `VML_STATUS_OVERFLOW` in the case of overflow, that is, when `RE(z)`, `IM(z)` are finite non-zero numbers, but the real or imaginary part of the exact result is so large that it does not meet the target precision.

v?Expm1

Computes an exponential of vector elements decreased by 1.

Syntax

Fortran:

```
call vsexpm1( n, a, y )
call vmsexpm1( n, a, y, mode )
call vdexpm1( n, a, y )
call vdexpm1( n, a, y, mode )
```

C:

```
vsExpm1( n, a, y );
vmsExpm1( n, a, y, mode );
vdExpm1( n, a, y );
vmdExpm1( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsexpm1`, `vmsexpm1` `DOUBLE PRECISION` for `vdexpm1`, `vmdexpm1` **Fortran 90:** `REAL, INTENT(IN)` for `vsexpm1`, `vmsexpm1` `DOUBLE PRECISION, INTENT(IN)` for `vdexpm1`, `vmdexpm1` **C:** `const float*` for `vsExpm1`, `vmsExpm1` `const double*` for `vdExpm1`, `vmdExpm1`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Expm1 Function

Data Type	Threshold Limitations on Input Parameters
single precision	`a[i] < Ln( FLT_MAX )`
double precision	`a[i] < Ln( DBL_MAX )`

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsexpm1`, `vmsexpm1` `DOUBLE PRECISION` for `vdexpm1`, `vmdexpm1` **Fortran 90:** `REAL, INTENT(OUT)` for `vsexpm1`, `vmsexpm1` `DOUBLE PRECISION, INTENT(OUT)` for `vdexpm1`, `vmdexpm1`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	C: `float*` for `vsExpm1`, `vmsExpm1` `double*` for `vdExpm1`, `vmdExpm1`	

Description

The `v?Expm1` function computes an exponential of vector elements decreased by 1.

Special Values for Real Function v?Expm1(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	+0		
X > overflow	+∞	VML_STATUS_OVERFLOW	OVERFLOW
+∞	+∞		
-∞	-1		
QNAN	QNAN		
SNAN	QNAN		INVALID

v?Ln

Computes natural logarithm of vector elements.

Syntax

Fortran:

```
call vsln( n, a, y )
call vmsln( n, a, y, mode )
call vdln( n, a, y )
call vmdln( n, a, y, mode )
call vcln( n, a, y )
call vmcln( n, a, y, mode )
call vzln( n, a, y )
call vmzln( n, a, y, mode )
```

C:

```
vsLn( n, a, y );
vmsLn( n, a, y, mode );
vdLn( n, a, y );
vmdLn( n, a, y, mode );
vcLn( n, a, y );
vmcLn( n, a, y, mode );
vzLn( n, a, y );
vmzLn( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`

- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsln, vmsln` `DOUBLE PRECISION` for `vdln, vmdln` `COMPLEX` for `vcln, vmcln` `DOUBLE COMPLEX` for `vzln, vmzln` **Fortran 90:** `REAL, INTENT(IN)` for `vsln, vmsln` `DOUBLE PRECISION, INTENT(IN)` for `vdln, vmdln` `COMPLEX, INTENT(IN)` for `vcln, vmcln` `DOUBLE COMPLEX, INTENT(IN)` for `vzln, vmzln` **C:** `const float*` for `vsLn, vmsLn` `const double*` for `vdLn, vmdLn` `const MKL_Complex8*` for `vcLn, vmcLn` `const MKL_Complex16*` for `vzLn, vmzLn`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsln, vmsln` `DOUBLE PRECISION` for `vdln, vmdln` `COMPLEX` for `vcln, vmcln` `DOUBLE COMPLEX` for `vzln, vmzln` **Fortran 90:** `REAL, INTENT(OUT)` for `vsln, vmsln` `DOUBLE PRECISION, INTENT(OUT)` for `vdln, vmdln`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`COMPLEX`, `INTENT(OUT)` for `vcln`, `vmcln`	
	`DOUBLE COMPLEX`, `INTENT(OUT)` for `vzln`, `vmzln`	
	C: `float*` for `vsLn`, `vmsLn`	
	`double*` for `vdLn`, `vmdLn`	
	`MKL_Complex8*` for `vcLn`, `vmcLn`	
	`MKL_Complex16*` for `vzLn`, `vmzLn`	

Description

The `v?Ln` function computes natural logarithm of vector elements.

Special Values for Real Function v?Ln(x)

Argument	Result	VML Error Status	Exception
+1	+0		
X < +0	QNAN	VML_STATUS_ERRDOM	INVALID
+0	-∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Ln(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	$+\infty + i \cdot \frac{3\pi}{4}$	+∞+i·π/2	+∞+i·π/2	+∞+i·π/2	+∞+i·π/2	+∞+i·π/4	+∞+i·QNAN
+i·Y	+∞+i·π					+∞+i·0	QNAN+i·QNAN INVALID
+i·0	+∞+i·π		-∞+i·π ZERODIVIDE	-∞+i·0 ZERODIVIDE		+∞+i·0	QNAN+i·QNAN INVALID
-i·0	+∞-i·π		-∞-i·π ZERODIVIDE	-∞-i·0 ZERODIVIDE		+∞-i·0	QNAN+i·QNAN INVALID
-i·Y	+∞-i·π					+∞-i·0	QNAN+i·QNAN INVALID
-i·∞	$+\infty - i \cdot \frac{3\pi}{4}$	+∞-i·π/2	+∞-i·π/2	+∞-i·π/2	+∞-i·π/2	+∞-i·π/4	+∞+i·QNAN
+i·NAN	+∞+i·QNAN	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN	QNAN+i·QNAN INVALID

Notes:

- raises INVALID exception when real or imaginary part of the argument is SNAN

v?Log10

Computes denary logarithm of vector elements.

Syntax

Fortran:

```
call vslog10( n, a, y )
call vmslog10( n, a, y, mode )
call vdlog10( n, a, y )
call vmdlog10( n, a, y, mode )
call vclog10( n, a, y )
call vmclog10( n, a, y, mode )
call vzlog10( n, a, y )
call vmzlog10( n, a, y, mode )
```

C:

```
vsLog10( n, a, y );
vmsLog10( n, a, y, mode );
vdLog10( n, a, y );
vmdLog10( n, a, y, mode );
vcLog10( n, a, y );
vmcLog10( n, a, y, mode );
vzLog10( n, a, y );
vmzLog10( n, a, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vslog10, vmslog10 DOUBLE PRECISION for vdlog10, vmdlog10 COMPLEX for vclog10, vmclog10	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`DOUBLE COMPLEX` for `vzlog10`, `vmzlog10` **Fortran 90:** `REAL, INTENT(IN)` for `vslog10, vmslog10` `DOUBLE PRECISION, INTENT(IN)` for `vdlog10, vmdlog10` `COMPLEX, INTENT(IN)` for `vclog10, vmclog10` `DOUBLE COMPLEX, INTENT(IN)` for `vzlog10, vmzlog10` **C:** `const float*` for `vsLog10, vmsLog10` `const double*` for `vdLog10, vmdLog10` `const MKL_Complex8*` for `vcLog10, vmcLog10` `const MKL_Complex16*` for `vzLog10, vmzLog10`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vslog10, vmslog10` `DOUBLE PRECISION` for `vdlog10, vmdlog10` `COMPLEX` for `vclog10, vmclog10` `DOUBLE COMPLEX` for `vzlog10, vmzlog10` **Fortran 90:** `REAL, INTENT(OUT)` for `vslog10, vmslog10` `DOUBLE PRECISION, INTENT(OUT)` for `vdlog10, vmdlog10` `COMPLEX, INTENT(OUT)` for `vclog10, vmclog10` `DOUBLE COMPLEX, INTENT(OUT)` for `vzlog10, vmzlog10` **C:** `float*` for `vsLog10, vmsLog10`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`double*` for `vdLog10`, `vmdLog10`	
	`MKL_Complex8*` for `vcLog10`, `vmcLog10`	
	`MKL_Complex16*` for `vzLog10`, `vmzLog10`	

Description

The `v?Log10` function computes a denary logarithm of vector elements.

Special Values for Real Function v?Log10(x)

Argument	Result	VML Error Status	Exception
+1	+0		
X < +0	QNAN	VML_STATUS_ERRDOM	INVALID
+0	-∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Log10(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	$+\infty + i\frac{3}{4}\frac{\pi}{\ln(10)}$	$+\infty + i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty + i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty + i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty + i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty + i\frac{\pi}{4}\frac{1}{\ln(10)}$	+∞+i·QNAN INVALID
+i·Y	$+\infty + i\frac{\pi}{\ln(10)}$					+∞+i·0	QNAN+i·QNAN INVALID
+i·0	$+\infty + i\frac{\pi}{\ln(10)}$		$-\infty + i\frac{\pi}{\ln(10)}$ ZERODRIVE	-∞+i·0 ZERODRIVE		+∞+i·0	QNAN+i·QNAN INVALID
-i·0	$+\infty - i\frac{\pi}{\ln(10)}$		$-\infty - i\frac{\pi}{\ln(10)}$ ZERODIVIDE	-∞-i·0 ZERODIVIDE		+∞-i·0	QNAN-i·QNAN INVALID
-i·Y	$+\infty - i\frac{\pi}{\ln(10)}$					+∞-i·0	QNAN+i·QNAN INVALID
-i·∞	$+\infty + i\frac{3}{4}\frac{\pi}{\ln(10)}$	$+\infty - i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty - i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty - i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty - i\frac{\pi}{2}\frac{1}{\ln(10)}$	$+\infty - i\frac{\pi}{4}\frac{1}{\ln(10)}$	+∞+i·QNAN
+i·NAN	+∞+i·QNAN	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN	QNAN+i·QNAN INVALID

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`

v?Log1p

Computes a natural logarithm of vector elements that are increased by 1.

Syntax

Fortran:

```
call vslog1p( n, a, y )
call vmslog1p( n, a, y, mode )
call vdlog1p( n, a, y )
call vmdlog1p( n, a, y, mode )
```

C:

```
vsLog1p( n, a, y );
vmsLog1p( n, a, y, mode );
vdLog1p( n, a, y );
vmdLog1p( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vslog1p`, `vmslog1p` `DOUBLE PRECISION` for `vdlog1p`, `vmdlog1p` **Fortran 90:** `REAL, INTENT(IN)` for `vslog1p`, `vmslog1p` `DOUBLE PRECISION, INTENT(IN)` for `vdlog1p`, `vmdlog1p` **C:** `const float*` for `vsLog1p`, `vmsLog1p` `const double*` for `vdLog1p`, `vmdLog1p`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Name	Type	Description
	C: `const MKL_INT64`	

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vslog1p`, `vmslog1p` `DOUBLE PRECISION` for `vdlog1p`, `vmdlog1p` **Fortran 90:** `REAL, INTENT(OUT)` for `vslog1p, vmslog1p` `DOUBLE PRECISION, INTENT(OUT)` for `vdlog1p, vmdlog1p` **C:** `float*` for `vsLog1p, vmsLog1p` `double*` for `vdLog1p, vmdLog1p`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Log1p` function computes a natural logarithm of vector elements that are increased by 1.

Special Values for Real Function v?Log1p(x)

Argument	Result	VML Error Status	Exception
-1	-∞	VML_STATUS_SING	ZERODIVIDE
X < -1	QNAN	VML_STATUS_ERRDOM	INVALID
+0	+0		
-0	-0		
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

Trigonometric Functions

v?Cos

Computes cosine of vector elements.

Syntax

Fortran:

```
call vscos( n, a, y )
call vmscos( n, a, y, mode )
call vdcos( n, a, y )
call vmdcos( n, a, y, mode )
call vccos( n, a, y )
call vmccos( n, a, y, mode )
```

```
call vzcos( n, a, y )
call vmzcos( n, a, y, mode )
```

C:

```
vsCos( n, a, y );
vmsCos( n, a, y, mode );
vdCos( n, a, y );
vmdCos( n, a, y, mode );
vcCos( n, a, y );
vmcCos( n, a, y, mode );
vzCos( n, a, y );
vmzCos( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vscos, vmscos` `DOUBLE PRECISION` for `vdcos, vmdcos` `COMPLEX` for `vccos, vmccos` `DOUBLE PRECISION` for `vzcos, vmzcos` **Fortran 90:** `REAL, INTENT(IN)` for `vscos, vmscos` `DOUBLE PRECISION, INTENT(IN)` for `vdcos, vmdcos` `COMPLEX, INTENT(IN)` for `vccos, vmccos` `DOUBLE PRECISION, INTENT(IN)` for `vzcos, vmzcos` **C:** `const float*` for `vsCos, vmsCos` `const double*` for `vdCos, vmdCos` `const MKL_Complex8*` for `vcCos, vmcCos`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`const MKL_Complex16*` for `vzCos`, `vmzCos`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vscos`, `vmscos` `DOUBLE PRECISION` for `vdcos`, `vmdcos` `COMPLEX` for `vccos`, `vmccos` `DOUBLE PRECISION` for `vzcos`, `vmzcos` **Fortran 90:** `REAL, INTENT(OUT)` for `vscos`, `vmscos` `DOUBLE PRECISION, INTENT(OUT)` for `vdcos`, `vmdcos` `COMPLEX, INTENT(OUT)` for `vccos`, `vmccos` `DOUBLE PRECISION, INTENT(OUT)` for `vzcos`, `vmzcos` **C:** `float*` for `vsCos`, `vmsCos` `double*` for `vdCos`, `vmdCos` `MKL_Complex8*` for `vcCos`, `vmcCos` `MKL_Complex16*` for `vzCos`, `vmzCos`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Cos` function computes cosine of vector elements.

Note that arguments `abs(a[i])` $\leq 2^{13}$ and `abs(a[i])` $\leq 2^{16}$ for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VML provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VML High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VML Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

Special Values for Real Function v?Cos(x)

Argument	Result	VML Error Status	Exception
+0	+1		
-0	+1		
+∞	QNAN	VML_STATUS_ERRDOM	INVALID

Argument	Result	VML Error Status	Exception
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

`Cos(z) = Cosh(i*z).`

v?Sin

Computes sine of vector elements.

Syntax

Fortran:

```
call vssin( n, a, y )
call vmssin( n, a, y, mode )
call vdsin( n, a, y )
call vmdsin( n, a, y, mode )
call vcsin( n, a, y )
call vmcsin( n, a, y, mode )
call vzsin( n, a, y )
call vmzsin( n, a, y, mode )
```

C:

```
vsSin( n, a, y );
vmsSin( n, a, y, mode );
vdSin( n, a, y );
vmdSin( n, a, y, mode );
vcSin( n, a, y );
vmcSin( n, a, y, mode );
vzSin( n, a, y );
vmzSin( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.

Name	Type	Description
a	**FORTRAN 77:** REAL for vssin, vmssin DOUBLE PRECISION for vdsin, vmdsin COMPLEX for vcsin, vmcsin DOUBLE PRECISION for vzsin, vmzsin **Fortran 90:** REAL, INTENT(IN) for vssin, vmssin DOUBLE PRECISION, INTENT(IN) for vdsin, vmdsin COMPLEX, INTENT(IN) for vcsin, vmcsin DOUBLE PRECISION, INTENT(IN) for vzsin, vmzsin **C:** const float* for vsSin, vmsSin const double* for vdSin, vmdSin const MKL_Complex8* for vcSin, vmcSin const MKL_Complex16* for vzSin, vmzSin	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vssin, vmssin DOUBLE PRECISION for vdsin, vmdsin COMPLEX for vcsin, vmcsin DOUBLE PRECISION for vzsin, vmzsin **Fortran 90:** REAL, INTENT(OUT) for vssin, vmssin DOUBLE PRECISION, INTENT(OUT) for vdsin, vmdsin	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`COMPLEX, INTENT(OUT)` for `vcsin, vmcsin` `DOUBLE PRECISION, INTENT(OUT)` for `vzsin, vmzsin` **C:** `float*` for `vsSin, vmsSin` `double*` for `vdSin, vmdSin` `MKL_Complex8*` for `vcSin, vmcSin` `MKL_Complex16*` for `vzSin, vmzSin`	

Description

This function is declared in `mkl_vml.f77` for FORTRAN 77 interface, in `mkl_vml.f90` for Fortran 90 interface, and in `mkl_vml_functions.h` for C interface.

The function computes sine of vector elements.

Note that arguments `abs(a[i])` $\leq 2^{13}$ and `abs(a[i])` $\leq 2^{16}$ for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VML provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VML High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VML Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

Special Values for Real Function v?Sin(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

`Sin(z) = –i*Sinh(i*z).`

v?SinCos

Computes sine and cosine of vector elements.

Syntax

Fortran:

```
call vssincos( n, a, y, z )
call vmssincos( n, a, y, z, mode )
call vdsincos( n, a, y, z )
call vmdsincos( n, a, y, z, mode )
```

C:

```
vsSinCos( n, a, y, z );
vmsSinCos( n, a, y, z, mode );
vdSinCos( n, a, y, z );
```

```
vmdSinCos( n, a, y, z, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vssincos, vmssincos` `DOUBLE PRECISION` for `vdsincos, vmdsincos` **Fortran 90:** `REAL, INTENT(IN)` for `vssincos, vmssincos` `DOUBLE PRECISION, INTENT(IN)` for `vdsincos, vmdsincos` **C:** `const float*` for `vsSinCos, vmsSinCos` `const double*` for `vdSinCos, vmdSinCos`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y, *z*	**FORTRAN 77:** `REAL` for `vssincos, vmssincos` `DOUBLE PRECISION` for `vdsincos, vmdsincos` **Fortran 90:** `REAL, INTENT(OUT)` for `vssincos, vmssincos` `DOUBLE PRECISION, INTENT(OUT)` for `vdsincos, vmdsincos` **C:** `float*` for `vsSinCos, vmsSinCos` `double*` for `vdSinCos, vmdSinCos`	**FORTRAN:** Arrays that specify the output vectors *y* (for sine values) and *z* (for cosine values). **C:** Pointers to arrays that contain the output vectors *y* (for sinevalues) and *z*(for cosine values).

Description

This function is declared in `mkl_vml.f77` for FORTRAN 77 interface, in `mkl_vml.f90` for Fortran 90 interface, and in `mkl_vml_functions.h` for C interface.

The function computes sine and cosine of vector elements.

Note that arguments `abs(a[i])` $\leq 2^{13}$ and `abs(a[i])` $\leq 2^{16}$ for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VML provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VML High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VML Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

Special Values for Real Function v?SinCos(x)

Argument	Result 1	Result 2	VML Error Status	Exception
+0	+0	+1		
-0	-0	+1		
+∞	QNAN	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN	QNAN		
SNAN	QNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

`Sin(z) = -i*Sinh(i*z).`

v?CIS

Computes complex exponent of real vector elements (cosine and sine of real vector elements combined to complex value).

Syntax

Fortran:

```
call vccis( n, a, y )
call vmccis( n, a, y, mode )
call vzcis( n, a, y )
call vmzcis( n, a, y, mode )
```

C:

```
vcCIS( n, a, y );
vmcCIS( n, a, y, mode );
vzCIS( n, a, y );
vmzCIS( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vccis, vmccis DOUBLE PRECISION for vzcis, vmzcis **Fortran 90:** REAL, INTENT(IN) for vccis, vmccis DOUBLE PRECISION, INTENT(IN) for vzcis, vmzcis **C:** const float* for vcCIS, vmcCIS const double* for vzCIS, vmzCIS	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** COMPLEX for vccis, vmccis DOUBLE COMPLEX for vzcis, vmzcis **Fortran 90:** COMPLEX, INTENT(OUT) for vccis, vmccis DOUBLE COMPLEX, INTENT(OUT) for vzcis, vmzcis **C:** MKL_Complex8* for vcCIS, vmcCIS MKL_Complex16* for vzCIS, vmzCIS	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?CIS function computes complex exponent of real vector elements (cosine and sine of real vector elements combined to complex value).

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?CIS(x)

x	CIS(x)
+ ∞	QNAN+i·QNAN

x	CIS(x)
	INVALID
+ 0	+1+i·0
- 0	+1-i·0
- ∞	QNAN+i·QNAN INVALID
NAN	QNAN+i·QNAN

Notes:

- raises `INVALID` exception when the argument is `SNAN`
- raises `INVALID` exception and sets the VML Error Status to `VML_STATUS_ERRDOM` for `x=+∞`, `x=-∞`

v?Tan

Computes tangent of vector elements.

Syntax

Fortran:

```
call vstan( n, a, y )
call vmstan( n, a, y, mode )
call vdtan( n, a, y )
call vmdtan( n, a, y, mode )
call vctan( n, a, y )
call vmctan( n, a, y, mode )
call vztan( n, a, y )
call vmztan( n, a, y, mode )
```

C:

```
vsTan( n, a, y );
vmsTan( n, a, y, mode );
vdTan( n, a, y );
vmdTan( n, a, y, mode );
vcTan( n, a, y );
vmcTan( n, a, y, mode );
vzTan( n, a, y );
vmzTan( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vstan, vmstan DOUBLE PRECISION for vdtan, vmdtan COMPLEX for vctan, vmctan DOUBLE COMPLEX for vztan, vmztan **Fortran 90:** REAL, INTENT(IN) for vstan, vmstan DOUBLE PRECISION, INTENT(IN) for vdtan, vmdtan COMPLEX, INTENT(IN) for vctan, vmctan DOUBLE COMPLEX, INTENT(IN) for vztan, vmztan **C:** const float* for vsTan, vmsTan const double* for vdTan, vmdTan const MKL_Complex8* for vcTan, vmcTan const MKL_Complex16* for vzTan, vmzTan	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vstan, vmstan DOUBLE PRECISION for vdtan, vmdtan COMPLEX for vctan, vmctan DOUBLE COMPLEX for vztan, vmztan **Fortran 90:** REAL, INTENT(OUT) for vstan, vmstan	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE PRECISION, INTENT(OUT)` for `vdtan, vmdtan`	
	`COMPLEX, INTENT(OUT)` for `vctan, vmctan`	
	`DOUBLE COMPLEX, INTENT(OUT)` for `vztan, vmztan`	
	C: `float*` for `vsTan, vmsTan`	
	`double*` for `vdTan, vmdTan`	
	`MKL_Complex8*` for `vcTan, vmcTan`	
	`MKL_Complex16*` for `vzTan, vmzTan`	

Description

The `v?Tan` function computes tangent of vector elements.

Note that arguments `abs(a[i])` $\leq 2^{13}$ and `abs(a[i])` $\leq 2^{16}$ for single and double precisions respectively are called fast computational path. These are trigigonometric function arguments for which VML provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VML High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VML Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

Special Values for Real Function v?Tan(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

`Tan(z) = -i*Tanh(i*z).`

v?Acos

Computes inverse cosine of vector elements.

Syntax

Fortran:

```
call vsacos( n, a, y )
call vmsacos( n, a, y, mode )
call vdacos( n, a, y )
call vmdacos( n, a, y, mode )
call vcacos( n, a, y )
call vmcacos( n, a, y, mode )
call vzacos( n, a, y )
call vmzacos( n, a, y, mode )
```

C:

```
vsAcos( n, a, y );
vmsAcos( n, a, y, mode );
vdAcos( n, a, y );
vmdAcos( n, a, y, mode );
vcAcos( n, a, y );
vmcAcos( n, a, y, mode );
vzAcos( n, a, y );
vmzAcos( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsacos`, `vmsacos` `DOUBLE PRECISION` for `vdacos`, `vmdacos` `COMPLEX` for `vcacos`, `vmcacos` `DOUBLE COMPLEX` for `vzacos`, `vmzacos` **Fortran 90:** `REAL, INTENT(IN)` for `vsacos`, `vmsacos` `DOUBLE PRECISION, INTENT(IN)` for `vdacos`, `vmdacos` `COMPLEX, INTENT(IN)` for `vcacos`, `vmcacos` `DOUBLE COMPLEX, INTENT(IN)` for `vzacos`, `vmzacos` **C:** `const float*` for `vsAcos`, `vmsAcos` `const double*` for `vdAcos`, `vmdAcos` `const MKL_Complex8*` for `vcAcos`, `vmcAcos`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`const MKL_Complex16*` for `vzAcos`, `vmzAcos`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsacos`, `vmsacos` `DOUBLE PRECISION` for `vdacos`, `vmdacos` `COMPLEX` for `vcacos`, `vmcacos` `DOUBLE COMPLEX` for `vzacos`, `vmzacos` **Fortran 90:** `REAL, INTENT(OUT)` for `vsacos`, `vmsacos` `DOUBLE PRECISION, INTENT(OUT)` for `vdacos`, `vmdacos` `COMPLEX, INTENT(OUT)` for `vcacos`, `vmcacos` `DOUBLE COMPLEX, INTENT(OUT)` for `vzacos`, `vmzacos` **C:** `float*` for `vsAcos`, `vmsAcos` `double*` for `vdAcos`, `vmdAcos` `MKL_Complex8*` for `vcAcos`, `vmcAcos` `MKL_Complex16*` for `vzAcos`, `vmzAcos`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Acos` function computes inverse cosine of vector elements.

Special Values for Real Function v?Acos(x)

Argument	Result	VML Error Status	Exception
+0	+π/2		
-0	+π/2		
+1	+0		
-1	+π		
|X| > 1	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID

Argument	Result	VML Error Status	Exception
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Acos(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	$+\frac{3\pi}{4} - i \cdot \infty$	$+\frac{\pi}{2} - i \cdot \infty$	$+\frac{\pi}{2} - i \cdot \infty$	$+\frac{\pi}{2} - i \cdot \infty$	$+\frac{\pi}{2} - i \cdot \infty$	$+\frac{\pi}{4} - i \cdot \infty$	QNAN-i·∞
+i·Y	+π-i·∞					+0-i·∞	QNAN+i·QNAN
+i·0	+π-i·∞		$+\frac{\pi}{2} - i \cdot 0$	$+\frac{\pi}{2} - i \cdot 0$		+0-i·∞	QNAN+i·QNAN
-i·0	+π+i·∞		$+\frac{\pi}{2} + i \cdot \infty$	$+\frac{\pi}{2} + i \cdot \infty$		+0+i·∞	QNAN+i·QNAN
-i·Y	+π+i·∞					+0+i·∞	QNAN+i·QNAN
-i·∞	$+\frac{3\pi}{4} + i \cdot \infty$	$+\frac{\pi}{2} + i \cdot \infty$	$+\frac{\pi}{2} + i \cdot \infty$	$+\frac{\pi}{2} + i \cdot \infty$	$+\frac{\pi}{2} + i \cdot \infty$	$+\frac{\pi}{4} + i \cdot \infty$	QNAN+i·∞
+i·NAN	QNAN+i·∞	QNAN+i·QNAN	$+\frac{\pi}{2} + i \cdot QNAN$	$+\frac{\pi}{2} + i \cdot QNAN$	QNAN+i·QNAN	QNAN+i·∞	QNAN+i·QNAN

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- `Acos(CONJ(z))=CONJ(Acos(z))`.

v?Asin

Computes inverse sine of vector elements.

Syntax

Fortran:

```
call vsasin( n, a, y )
call vmsasin( n, a, y, mode )
call vdasin( n, a, y )
call vmdasin( n, a, y, mode )
call vcasin( n, a, y )
call vmcasin( n, a, y, mode )
call vzasin( n, a, y )
call vmzasin( n, a, y, mode )
```

C:

```
vsAsin( n, a, y );
vmsAsin( n, a, y, mode );
vdAsin( n, a, y );
vmdAsin( n, a, y, mode );
```

```
vcAsin( n, a, y );
vmcAsin( n, a, y, mode );
vzAsin( n, a, y );
vmzAsin( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsasin, vmsasin` `DOUBLE PRECISION` for `vdasin, vmdasin` `COMPLEX` for `vcasin, vmcasin` `DOUBLE COMPLEX` for `vzasin, vmzasin` **Fortran 90:** `REAL, INTENT(IN)` for `vsasin, vmsasin` `DOUBLE PRECISION, INTENT(IN)` for `vdasin, vmdasin` `COMPLEX, INTENT(IN)` for `vcasin, vmcasin` `DOUBLE COMPLEX, INTENT(IN)` for `vzasin, vmzasin` **C:** `const float*` for `vsAsin, vmsAsin` `const double*` for `vdAsin, vmdAsin` `const MKL_Complex8*` for `vcAsin, vmcAsin` `const MKL_Complex16*` for `vzAsin, vmzAsin`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsasin, vmsasin DOUBLE PRECISION for vdasin, vmdasin COMPLEX for vcasin, vmcasin DOUBLE COMPLEX for vzasin, vmzasin **Fortran 90:** REAL, INTENT(OUT) for vsasin, vmsasin DOUBLE PRECISION, INTENT(OUT) for vdasin, vmdasin COMPLEX, INTENT(OUT) for vcasin, vmcasin DOUBLE COMPLEX, INTENT(OUT) for vzasin, vmzasin **C:** float* for vsAsin, vmsAsin double* for vdAsin, vmdAsin MKL_Complex8* for vcAsin, vmcAsin MKL_Complex16* for vzAsin, vmzAsin	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Asin function computes inverse sine of vector elements.

Special Values for Real Function v?Asin(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
+1	+π/2		
-1	-π/2		
|X| > 1	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

```
Asin(z) = -i*Asinh(i*z).
```

v?Atan

Computes inverse tangent of vector elements.

Syntax

Fortran:

```
call vsatan( n, a, y )
call vmsatan( n, a, y, mode )
call vdatan( n, a, y )
call vmdatan( n, a, y, mode )
call vcatan( n, a, y )
call vmcatan( n, a, y, mode )
call vzatan( n, a, y )
call vmzatan( n, a, y, mode )
```

C:

```
vsAtan( n, a, y );
vmsAtan( n, a, y, mode );
vdAtan( n, a, y );
vmdAtan( n, a, y, mode );
vcAtan( n, a, y );
vmcAtan( n, a, y, mode );
vzAtan( n, a, y );
vmzAtan( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsatan`, `vmsatan` `DOUBLE PRECISION` for `vdatan`, `vmdatan` `COMPLEX` for `vcatan`, `vmcatan` `DOUBLE COMPLEX` for `vzatan`, `vmzatan` **Fortran 90:** `REAL, INTENT(IN)` for `vsatan`, `vmsatan`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	DOUBLE PRECISION, INTENT(IN) for vdatan, vmdatan COMPLEX, INTENT(IN) for vcatan, vmcatan DOUBLE COMPLEX, INTENT(IN) for vzatan, vmzatan **C:** const float* for vsAtan, vmsAtan const double* for vdAsin, vmdAtan const MKL_Complex8* for vcAtan, vmcAtan const MKL_Complex16* for vzAsin, vmzAtan	
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsatan, vmsatan DOUBLE PRECISION for vdatan, vmdatan COMPLEX for vcatan, vmcatan DOUBLE COMPLEX for vzatan, vmzatan **Fortran 90:** REAL, INTENT(OUT) for vsatan, vmsatan DOUBLE PRECISION, INTENT(OUT) for vdatan, vmdatan COMPLEX, INTENT(OUT) for vcatan, vmcatan DOUBLE COMPLEX, INTENT(OUT) for vzatan, vmzatan **C:** float* for vsAtan, vmsAtan double* for vdAsin, vmdAtan MKL_Complex8* for vcAtan, vmcAtan MKL_Complex16* for vzAsin, vmzAtan	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Atan function computes inverse tangent of vector elements.

Special Values for Real Function v?Atan(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
+∞	+π/2		
-∞	-π/2		
QNAN	QNAN		
SNAN	QNAN		INVALID

Specifications for special values of the complex functions are defined according to the following formula

Atan(z) = -i*Atanh(i*z).

v?Atan2

Computes four-quadrant inverse tangent of elements of two vectors.

Syntax

Fortran:

```
call vsatan2( n, a, b, y )
call vmsatan2( n, a, b, y, mode )
call vdatan2( n, a, b, y )
call vmdatan2( n, a, b, y, mode )
```

C:

```
vsAtan2( n, a, b, y );
vmsAtan2( n, a, b, y, mode );
vdAtan2( n, a, b, y );
vmdAtan2( n, a, b, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a, *b*	**FORTRAN 77:** REAL for vsatan2, vmsatan2	**FORTRAN:** Arrays that specify the input vectors *a* and *b*.

Name	Type	Description
	`DOUBLE PRECISION` for `vdatan2`, `vmdatan2` **Fortran 90:** `REAL, INTENT(IN)` for `vsatan2`, `vmsatan2` `DOUBLE PRECISION, INTENT(IN)` for `vdatan2`, `vmdatan2` **C:** `const float*` for `vsAtan2`, `vmsAtan2` `const double*` for `vdAtan2`, `vmdAtan2`	**C:** Pointers to arrays that contain the input vectors *a* and *b*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsatan2`, `vmsatan2` `DOUBLE PRECISION` for `vdatan2`, `vmdatan2` **Fortran 90:** `REAL, INTENT(OUT)` for `vsatan2`, `vmsatan2` `DOUBLE PRECISION, INTENT(OUT)` for `vdatan2`, `vmdatan2` **C:** `float*` for `vsAtan2`, `vmsAtan2` `double*` for `vdAtan2`, `vmdAtan2`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Atan2` function computes four-quadrant inverse tangent of elements of two vectors.

The elements of the output vector*y* are computed as the four-quadrant arctangent of `a[i] / b[i]`.

Special values for Real Function v?Atan2(x)

Argument 1	Argument 2	Result	Exception
-∞	-∞	-3*π/4	
-∞	X < +0	-π/2	
-∞	-0	-π/2	
-∞	+0	-π/2	
-∞	X > +0	-π/2	
-∞	+∞	-π/4	
X < +0	-∞	-π	
X < +0	-0	-π/2	

Argument 1	Argument 2	Result	Exception
X < +0	+0	-π/2	
X < +0	+∞	-0	
-0	-∞	-π	
-0	X < +0	-π	
-0	-0	-π	
-0	+0	-0	
-0	X > +0	-0	
-0	+∞	-0	
+0	-∞	+π	
+0	X < +0	+π	
+0	-0	+π	
+0	+0	+0	
+0	X > +0	+0	
+0	+∞	+0	
X > +0	-∞	+π	
X > +0	-0	+π/2	
X > +0	+0	+π/2	
X > +0	+∞	+0	
+∞	-∞	-3*π/4	
+∞	X < +0	+π/2	
+∞	-0	+π/2	
+∞	+0	+π/2	
+∞	X > +0	+π/2	
+∞	+∞	+π/4	
X > +0	QNAN	QNAN	
X > +0	SNAN	QNAN	INVALID
QNAN	X > +0	QNAN	
SNAN	X > +0	QNAN	INVALID
QNAN	QNAN	QNAN	
QNAN	SNAN	QNAN	INVALID
SNAN	QNAN	QNAN	INVALID
SNAN	SNAN	QNAN	INVALID

Hyperbolic Functions

v?Cosh

Computes hyperbolic cosine of vector elements.

Syntax

Fortran:

```
call vscosh( n, a, y )
call vmscosh( n, a, y, mode )
call vdcosh( n, a, y )
call vmdcosh( n, a, y, mode )
```

```
call vccosh( n, a, y )
call vmccosh( n, a, y, mode )
call vzcosh( n, a, y )
call vmzcosh( n, a, y, mode )
```

C:

```
vsCosh( n, a, y );
vmsCosh( n, a, y, mode );
vdCosh( n, a, y );
vmdCosh( n, a, y, mode );
vcCosh( n, a, y );
vmcCosh( n, a, y, mode );
vzCosh( n, a, y );
vmzCosh( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vscosh, vmscosh` `DOUBLE PRECISION` for `vdcosh, vmdcosh` `COMPLEX` for `vccosh, vmccosh` `DOUBLE COMPLEX` for `vzcosh, vmzcosh` **Fortran 90:** `REAL, INTENT(IN)` for `vscosh, vmscosh` `DOUBLE PRECISION, INTENT(IN)` for `vdcosh, vmdcosh` `COMPLEX, INTENT(IN)` for `vccosh, vmccosh` `DOUBLE COMPLEX, INTENT(IN)` for `vzcosh, vmzcosh` **C:** `const float*` for `vsCosh, vmsCosh`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`const double*` for `vdCosh`, `vmdCosh` `const MKL_Complex8*` for `vcCosh`, `vmcCosh` `const MKL_Complex16*` for `vzCosh`, `vmzCosh`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Real v?Cosh Function

Data Type	Threshold Limitations on Input Parameters
single precision	`-Ln(FLT_MAX)-Ln2 <a[i] < Ln(FLT_MAX)+Ln2`
double precision	`-Ln(DBL_MAX)-Ln2 <a[i] < Ln(DBL_MAX)+Ln2`

Precision overflow thresholds for the complex `v?Cosh` function are beyond the scope of this document.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vscosh`, `vmscosh` `DOUBLE PRECISION` for `vdcosh`, `vmdcosh` `COMPLEX` for `vccosh`, `vmccosh` `DOUBLE COMPLEX` for `vzcosh`, `vmzcosh` **Fortran 90:** `REAL, INTENT(OUT)` for `vscosh`, `vmscosh` `DOUBLE PRECISION, INTENT(OUT)` for `vdcosh`, `vmdcosh` `COMPLEX, INTENT(OUT)` for `vccosh`, `vmccosh` `DOUBLE COMPLEX, INTENT(OUT)` for `vzcosh`, `vmzcosh` **C:** `float*` for `vsCosh`, `vmsCosh` `double*` for `vdCosh`, `vmdCosh` `MKL_Complex8*` for `vcCosh`, `vmcCosh` `MKL_Complex16*` for `vzCosh`, `vmzCosh`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Cosh` function computes hyperbolic cosine of vector elements.

Special Values for Real Function v?Cosh(x)

Argument	Result	VML Error Status	Exception
+0	+1		
-0	+1		
X > overflow	+∞	VML_STATUS_OVERFLOW	OVERFLOW
X < -overflow	+∞	VML_STATUS_OVERFLOW	OVERFLOW
+∞	+∞		
-∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Cosh(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	+∞+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN-i·0 INVALID	QNAN+i·0 INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN INVALID	QNAN+i·QNAN
+i·Y	+∞·Cos(Y)-i·∞·Sin(Y)					+∞·CIS(Y)	QNAN+i·QNAN
+i·0	+∞-i·0		+1-i·0	+1+i·0		+∞+i·0	QNAN+i·0
-i·0	+∞+i·0		+1+i·0	+1-i·0		+∞-i·0	QNAN-i·0
-i·Y	+∞·Cos(Y)-i·∞·Sin(Y)					+∞·CIS(Y)	QNAN+i·QNAN
-i·∞	+∞+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·0 INVALID	QNAN-i·0 INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN INVALID	QNAN+i·QNAN
+i·NAN	+∞+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN-i·QNAN	QNAN+i·QNAN	+∞+i·QNAN	QNAN+i·QNAN

Notes:

- raises the `INVALID` exception when the real or imaginary part of the argument is `SNAN`
- raises the `OVERFLOW` exception and sets the VML Error Status to `VML_STATUS_OVERFLOW` in the case of overflow, that is, when `RE(z)`, `IM(z)` are finite non-zero numbers, but the real or imaginary part of the exact result is so large that it does not meet the target precision.
- `Cosh(CONJ(z))=CONJ(Cosh(z))`
- `Cosh(-z)=Cosh(z).`

v?Sinh

Computes hyperbolic sine of vector elements.

Syntax

Fortran:

```
call vssinh( n, a, y )
call vmssinh( n, a, y, mode )
call vdsinh( n, a, y )
call vmdsinh( n, a, y, mode )
```

```
call vcsinh( n, a, y )
call vmcsinh( n, a, y, mode )
call vzsinh( n, a, y )
call vmzsinh( n, a, y, mode )
```

C:

```
vsSinh( n, a, y );
vmsSinh( n, a, y, mode );
vdSinh( n, a, y );
vmdSinh( n, a, y, mode );
vcSinh( n, a, y );
vmcSinh( n, a, y, mode );
vzSinh( n, a, y );
vmzSinh( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vssinh, vmssinh` `DOUBLE PRECISION` for `vdsinh, vmdsinh` `COMPLEX` for `vcsinh, vmcsinh` `DOUBLE COMPLEX` for `vzsinh, vmzsinh` **Fortran 90:** `REAL, INTENT(IN)` for `vssinh, vmssinh` `DOUBLE PRECISION, INTENT(IN)` for `vdsinh, vmdsinh` `COMPLEX, INTENT(IN)` for `vcsinh, vmcsinh` `DOUBLE COMPLEX, INTENT(IN)` for `vzsinh, vmzsinh` **C:** `const float*` for `vsSinh, vmsSinh`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`const double*` for `vdSinh`, `vmdSinh` `const MKL_Complex8*` for `vcSinh`, `vmcSinh` `const MKL_Complex16*` for `vzSinh`, `vmzSinh`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Precision Overflow Thresholds for Real v?Sinh Function

Data Type	Threshold Limitations on Input Parameters
single precision	`-Ln(FLT_MAX)-Ln2 <a[i] < Ln(FLT_MAX)+Ln2`
double precision	`-Ln(DBL_MAX)-Ln2 <a[i] < Ln(DBL_MAX)+Ln2`

Precision overflow thresholds for the complex `v?Sinh` function are beyond the scope of this document.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vssinh`, `vmssinh` `DOUBLE PRECISION` for `vdsinh`, `vmdsinh` `COMPLEX` for `vcsinh`, `vmcsinh` `DOUBLE COMPLEX` for `vzsinh`, `vmzsinh` **Fortran 90:** `REAL, INTENT(OUT)` for `vssinh`, `vmssinh` `DOUBLE PRECISION, INTENT(OUT)` for `vdsinh`, `vmdsinh` `COMPLEX, INTENT(OUT)` for `vcsinh`, `vmcsinh` `DOUBLE COMPLEX, INTENT(OUT)` for `vzsinh`, `vmzsinh` **C:** `float*` for `vsSinh`, `vmsSinh` `double*` for `vdSinh`, `vmdSinh` `MKL_Complex8*` for `vcSinh`, `vmcSinh` `MKL_Complex16*` for `vzSinh`, `vmzSinh`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Sinh` function computes hyperbolic sine of vector elements.

Special Values for Real Function v?Sinh(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
X > overflow	+∞	VML_STATUS_OVERFLOW	OVERFLOW
X < -overflow	-∞	VML_STATUS_OVERFLOW	OVERFLOW
+∞	+∞		
-∞	-∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Sinh(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	-∞+i·QNAN INVALID	QNAN+i·QNAN INVALID	-0+i·QNAN INVALID	+0+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN INVALID	QNAN+i·QNAN
+i·Y	-∞·Cos(Y)+ i·∞·Sin(Y)					+∞·CIS(Y)	QNAN+i·QNAN
+i·0	-∞+i·0		-0+i·0	+0+i·0		+∞+i·0	QNAN+i·0
-i·0	-∞-i·0		-0-i·0	+0-i·0		+∞-i·0	QNAN-i·0
-i·Y	-∞·Cos(Y)+ i·∞·Sin(Y)					+∞·CIS(Y)	QNAN+i·QNAN
-i·∞	-∞+i·QNAN INVALID	QNAN+i·QNAN INVALID	-0+i·QNAN INVALID	+0+i·QNAN INVALID	QNAN+i·QNAN INVALID	+∞+i·QNAN INVALID	QNAN+i·QNAN
+i·NAN	-∞+i·QNAN	QNAN+i·QNAN	-0+i·QNAN	+0+i·QNAN	QNAN+i·QNAN	+∞+i·QNAN	QNAN+i·QNAN

Notes:

- raises the `INVALID` exception when the real or imaginary part of the argument is `SNAN`
- raises the `OVERFLOW` exception and sets the VML Error Status to `VML_STATUS_OVERFLOW` in the case of overflow, that is, when `RE(z)`, `IM(z)` are finite non-zero numbers, but the real or imaginary part of the exact result is so large that it does not meet the target precision.
- `Sinh(CONJ(z))=CONJ(Sinh(z))`
- `Sinh(-z)=-Sinh(z)`.

v?Tanh

Computes hyperbolic tangent of vector elements.

Syntax

Fortran:

```
call vstanh( n, a, y )
call vmstanh( n, a, y, mode )
call vdtanh( n, a, y )
call vmdtanh( n, a, y, mode )
call vctanh( n, a, y )
```

```
call vmctanh( n, a, y, mode )
call vztanh( n, a, y )
call vmztanh( n, a, y, mode )
```

C:

```
vsTanh( n, a, y );
vmsTanh( n, a, y, mode );
vdTanh( n, a, y );
vmdTanh( n, a, y, mode );
vcTanh( n, a, y );
vmcTanh( n, a, y, mode );
vzTanh( n, a, y );
vmzTanh( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vstanh, vmstanh` `DOUBLE PRECISION` for `vdtanh, vmdtanh` `COMPLEX` for `vctanh, vmctanh` `DOUBLE COMPLEX` for `vztanh, vmztanh` **Fortran 90:** `REAL, INTENT(IN)` for `vstanh, vmstanh` `DOUBLE PRECISION, INTENT(IN)` for `vdtanh, vmdtanh` `COMPLEX, INTENT(IN)` for `vctanh, vmctanh` `DOUBLE COMPLEX, INTENT(IN)` for `vztanh, vmztanh` **C:** `const float*` for `vsTanh, vmsTanh` `const double*` for `vdTanh, vmdTanh`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	`const MKL_Complex8*` for `vcTanh`, `vmcTanh` `const MKL_Complex16*` for `vzTanh`, `vmzTanh`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vstanh`, `vmstanh` `DOUBLE PRECISION` for `vdtanh`, `vmdtanh` `COMPLEX` for `vctanh`, `vmctanh` `DOUBLE COMPLEX` for `vztanh`, `vmztanh` **Fortran 90:** `REAL, INTENT(OUT)` for `vstanh`, `vmstanh` `DOUBLE PRECISION, INTENT(OUT)` for `vdtanh`, `vmdtanh` `COMPLEX, INTENT(OUT)` for `vctanh`, `vmctanh` `DOUBLE COMPLEX, INTENT(OUT)` for `vztanh`, `vmztanh` **C:** `float*` for `vsTanh`, `vmsTanh` `double*` for `vdTanh`, `vmdTanh` `MKL_Complex8*` for `vcTanh`, `vmcTanh` `MKL_Complex16*` for `vzTanh`, `vmzTanh`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `v?Tanh` function computes hyperbolic tangent of vector elements.

Special Values for Real Function v?Tanh(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+1	
-∞	-1	
QNAN	QNAN	

Argument	Result	Exception
SNAN	QNAN	INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Tanh(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	-1+i·0	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+1+i·0	QNAN+i·QNAN
+i·Y	-1+i·0·Tan(Y)					+1+i·0·Tan(Y)	QNAN+i·QNAN
+i·0	-1+i·0		-0+i·0	+0+i·0		+1+i·0	QNAN+i·0
-i·0	-1-i·0		-0-i·0	+0-i·0		+1-i·0	QNAN-i·0
-i·Y	-1+i·0·Tan(Y)					+1+i·0·Tan(Y)	QNAN+i·QNAN
-i·∞	-1-i·0	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	QNAN+i·QNAN INVALID	+1-i·0	QNAN+i·QNAN
+i·NAN	-1+i·0	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	+1+i·0	QNAN+i·QNAN

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- `Tanh(CONJ(z))=CONJ(Tanh(z))`
- `Tanh(-z)=-Tanh(z).`

v?Acosh

Computes inverse hyperbolic cosine (nonnegative) of vector elements.

Syntax

Fortran:

```
call vsacosh( n, a, y )
call vmsacosh( n, a, y, mode )
call vdacosh( n, a, y )
call vmdacosh( n, a, y, mode )
call vcacosh( n, a, y )
call vmcacosh( n, a, y, mode )
call vzacosh( n, a, y )
call vmzacosh( n, a, y, mode )
```

C:

```
vsAcosh( n, a, y );
vmsAcosh( n, a, y, mode );
vdAcosh( n, a, y );
vmdAcosh( n, a, y, mode );
```

```
vcAcosh( n, a, y );
vmcAcosh( n, a, y, mode );
vzAcosh( n, a, y );
vmzAcosh( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsacosh, vmsacosh` `DOUBLE PRECISION` for `vdacosh, vmdacosh` `COMPLEX` for `vcacosh, vmcacosh` `DOUBLE COMPLEX` for `vzacosh, vmzacosh` **Fortran 90:** `REAL, INTENT(IN)` for `vsacosh, vmsacosh` `DOUBLE PRECISION, INTENT(IN)` for `vdacosh, vmdacosh` `COMPLEX, INTENT(IN)` for `vcacosh, vmcacosh` `DOUBLE COMPLEX, INTENT(IN)` for `vzacosh, vmzacosh` **C:** `const float*` for `vsAcosh, vmsAcosh` `const double*` for `vdAcosh, vmdAcosh` `const MKL_Complex8*` for `vcAcosh, vmcAcosh` `const MKL_Complex16*` for `vzAcosh, vmzAcosh`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsacosh, vmsacosh DOUBLE PRECISION for vdacosh, vmdacosh COMPLEX for vcacosh, vmcacosh DOUBLE COMPLEX for vzacosh, vmzacosh **Fortran 90:** REAL, INTENT(OUT) for vsacosh, vmsacosh DOUBLE PRECISION, INTENT(OUT) for vdacosh, vmdacosh COMPLEX, INTENT(OUT) for vcacosh, vmcacosh DOUBLE COMPLEX, INTENT(OUT) for vzacosh, vmzacosh **C:** float* for vsAcosh, vmsAcosh double* for vdAcosh, vmdAcosh MKL_Complex8* for vcAcosh, vmcAcosh MKL_Complex16* for vzAcosh, vmzAcosh	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Acosh function computes inverse hyperbolic cosine (nonnegative) of vector elements.

Special Values for Real Function v?Acosh(x)

Argument	Result	VML Error Status	Exception
+1	+0		
X < +1	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Acosh(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	$+\infty + i \cdot \frac{3\pi}{4}$	+∞+i·π/2	+∞+i·π/2	+∞+i·π/2	+∞+i·π/2	+∞+i·π/4	+∞+i·QNAN
+i·Y	+∞+i·π					+∞+i·0	QNAN+i·QNAN

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·0	+∞+i·π		+0+i·π/2	+0+i·π/2		+∞+i·0	QNAN+i·QNAN
-i·0	+∞+i·π		+0+i·π/2	+0+i·π/2		+∞+i·0	QNAN+i·QNAN
-i·Y	+∞+i·π					+∞+i·0	QNAN+i·QNAN
-i·∞	$+\infty - i \cdot \frac{3\pi}{4}$	+∞-i·π/2	+∞-i·π/2	+∞-i·π/2	+∞-i·π/2	+∞-i·π/4	+∞+i·QNAN
+i·NAN	+∞+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	QNAN+i·QNAN	+∞+i·QNAN	QNAN+i·QNAN

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- `Acosh(CONJ(z))=CONJ(Acosh(z))`.

v?Asinh

Computes inverse hyperbolic sine of vector elements.

Syntax

Fortran:

```
call vsasinh( n, a, y )
call vmsasinh( n, a, y, mode )
call vdasinh( n, a, y )
call vmdasinh( n, a, y, mode )
call vcasinh( n, a, y )
call vmcasinh( n, a, y, mode )
call vzasinh( n, a, y )
call vmzasinh( n, a, y, mode )
```

C:

```
vsAsinh( n, a, y );
vmsAsinh( n, a, y, mode );
vdAsinh( n, a, y );
vmdAsinh( n, a, y, mode );
vcAsinh( n, a, y );
vmcAsinh( n, a, y, mode );
vzAsinh( n, a, y );
vmzAsinh( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vsasinh, vmsasinh DOUBLE PRECISION for vdasinh, vmdasinh COMPLEX for vcasinh, vmcasinh DOUBLE COMPLEX for vzasinh, vmzasinh **Fortran 90:** REAL, INTENT(IN) for vsasinh, vmsasinh DOUBLE PRECISION, INTENT(IN) for vdasinh, vmdasinh COMPLEX, INTENT(IN) for vcasinh, vmcasinh DOUBLE COMPLEX, INTENT(IN) for vzasinh, vmzasinh **C:** const float* for vsAsinh, vmsAsinh const double* for vdAsinh, vmdAsinh const MKL_Complex8* for vcAsinh, vmcAsinh const MKL_Complex16* for vzAsinh, vmzAsinh	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsasinh, vmsasinh DOUBLE PRECISION for vdasinh, vmdasinh COMPLEX for vcasinh, vmcasinh	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE COMPLEX` for `vzasinh`, `vmzasinh` **Fortran 90:** `REAL, INTENT(OUT)` for `vsasinh, vmsasinh` `DOUBLE PRECISION, INTENT(OUT)` for `vdasinh, vmdasinh` `COMPLEX, INTENT(OUT)` for `vcasinh, vmcasinh` `DOUBLE COMPLEX, INTENT(OUT)` for `vzasinh, vmzasinh` **C:** `float*` for `vsAsinh, vmsAsinh` `double*` for `vdAsinh, vmdAsinh` `MKL_Complex8*` for `vcAsinh, vmcAsinh` `MKL_Complex16*` for `vzAsinh, vmzAsinh`	

Description

The `v?Asinh` function computes inverse hyperbolic sine of vector elements.

Special Values for Real Function v?Asinh(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
QNAN	QNAN	
SNAN	QNAN	INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Asinh(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	-∞+i·π/4	-∞+i·π/2	+∞+i·π/2	+∞+i·π/2	+∞+i·π/2	+∞+i·π/4	+∞+i·QNAN
+i·Y	-∞+i·0					+∞+i·0	QNAN+i·QNAN
+i·0	+∞+i·0		+0+i·0	+0+i·0		+∞+i·0	QNAN+i·QNAN
-i·0	-∞-i·0		-0-i·0	+0-i·0		+∞-i·0	QNAN-i·QNAN
-i·Y	-∞-i·0					+∞-i·0	QNAN+i·QNAN
-i·∞	-∞-i·π/4	-∞-i·π/2	-∞-i·π/2	+∞-i·π/2	+∞-i·π/2	+∞-i·π/4	+∞+i·QNAN
+i·NAN	-∞+i·QNAN	QNAN +i·QNAN	QNAN +i·QNAN	QNAN +i·QNAN	QNAN +i·QNAN	+∞+i·QNAN	QNAN+i·QNAN

Notes:

- raises INVALID exception when real or imaginary part of the argument is SNAN
- Asinh(CONJ(z))=CONJ(Asinh(z))
- Asinh(-z)=-Asinh(z).

v?Atanh

Computes inverse hyperbolic tangent of vector elements.

Syntax

Fortran:

```
call vsatanh( n, a, y )
call vmsatanh( n, a, y, mode )
call vdatanh( n, a, y )
call vmdatanh( n, a, y, mode )
call vcatanh( n, a, y )
call vmcatanh( n, a, y, mode )
call vzatanh( n, a, y )
call vmzatanh( n, a, y, mode )
```

C:

```
vsAtanh( n, a, y );
vmsAtanh( n, a, y, mode );
vdAtanh( n, a, y );
vmdAtanh( n, a, y, mode );
vcAtanh( n, a, y );
vmcAtanh( n, a, y, mode );
vzAtanh( n, a, y );
vmzAtanh( n, a, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vsatanh, vmsatanh	**FORTRAN:** Array that specifies the input vector *a*.

Name	Type	Description
	DOUBLE PRECISION for vdatanh, vmdatanh COMPLEX for vcatanh, vmcatanh DOUBLE COMPLEX for vzatanh, vmzatanh **Fortran 90:** REAL, INTENT(IN) for vsatanh, vmsatanh DOUBLE PRECISION, INTENT(IN) for vdatanh, vmdatanh COMPLEX, INTENT(IN) for vcatanh, vmcatanh DOUBLE COMPLEX, INTENT(IN) for vzatanh, vmzatanh **C:** const float* for vsAtanh, vmsAtanh const double* for vdAtanh, vmdAtanh const MKL_Complex8* for vcAtanh, vmcAtanh const MKL_Complex16* for vzAtanh, vmzAtanh	**C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsatanh, vmsatanh DOUBLE PRECISION for vdatanh, vmdatanh COMPLEX for vcatanh, vmcatanh DOUBLE COMPLEX for vzatanh, vmzatanh **Fortran 90:** REAL, INTENT(OUT) for vsatanh, vmsatanh DOUBLE PRECISION, INTENT(OUT) for vdatanh, vmdatanh COMPLEX, INTENT(OUT) for vcatanh, vmcatanh	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE COMPLEX`, `INTENT(OUT)` for `vzatanh`, `vmzatanh` **C:** `float*` for `vsAtanh`, `vmsAtanh` `double*` for `vdAtanh`, `vmdAtanh` `MKL_Complex8*` for `vcAtanh`, `vmcAtanh` `MKL_Complex16*` for `vzAtanh`, `vmzAtanh`	

Description

The `v?Atanh` function computes inverse hyperbolic tangent of vector elements.

Special Values for Real Function v?Atanh(x)

Argument	Result	VML Error Status	Exception
+1	+∞	VML_STATUS_SING	ZERODIVIDE
-1	-∞	VML_STATUS_SING	ZERODIVIDE
|X| > 1	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

See the Special Value Notations section for the conventions used in the table below.

Special Values for Complex Function v?Atanh(z)

RE(z) i·IM(z)	-∞	-X	-0	+0	+X	+∞	NAN
+i·∞	-0+i·π/2	-0+i·π/2	-0+i·π/2	+0+i·π/2	+0+i·π/2	+0+i·π/2	+0+i·π/2
+i·Y	-0+i·π/2					+0+i·π/2	QNAN+i·QNAN
+i·0	-0+i·π/2		-0+i·0	+0+i·0		+0+i·π/2	QNAN+i·QNAN
-i·0	-0-i·π/2		-0-i·0	+0-i·0		+0-i·π/2	QNAN-i·QNAN
-i·Y	-0-i·π/2					+0-i·π/2	QNAN+i·QNAN
-i·∞	-0-i·π/2	-0-i·π/2	-0-i·π/2	+0-i·π/2	+0-i·π/2	+0-i·π/2	+0-i·π/2
+i·NAN	-0+i·QNAN	QNAN +i·QNAN	-0+i·QNAN	+0+i·QNAN	QNAN +i·QNAN	+0+i·QNAN	QNAN+i·QNAN

Notes:

- `Atanh(+-1+-i*0)=+-∞+-i*0`, and `ZERODIVIDE` exception is raised
- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- `Atanh(CONJ(z))=CONJ(Atanh(z))`
- `Atanh(-z)=-Atanh(z)`.

Special Functions

v?Erf

Computes the error function value of vector elements.

Syntax

Fortran:

```
call vserf( n, a, y )
call vmserf( n, a, y, mode )
call vderf( n, a, y )
call vmderf( n, a, y, mode )
```

C:

```
vsErf( n, a, y );
vmsErf( n, a, y, mode );
vdErf( n, a, y );
vmdErf( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vserf, vmserf` `DOUBLE PRECISION` for `vderf, vmderf` **Fortran 90:** `REAL, INTENT(IN)` for `vserf, vmserf` `DOUBLE PRECISION, INTENT(IN)` for `vderf, vmderf` **C:** `const float*` for `vsErf, vmsErf` `const double*` for `vdErf, vmdErf`	**FORTRAN:** Array, specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Name	Type	Description
	Fortran 90: `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vserf`, `vmserf` `DOUBLE PRECISION` for `vderf`, `vmderf` **Fortran 90:** `REAL, INTENT(OUT)` for `vserf, vmserf` `DOUBLE PRECISION, INTENT(OUT)` for `vderf, vmderf` **C:** `float*` for `vsErf, vmsErf` `double*` for `vdErf, vmdErf`	**FORTRAN:** Array, specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `Erf` function computes the error function values for elements of the input vector *a* and writes them to the output vector *y*.

The error function is defined as given by:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \ .$$

Useful relations:

1. $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$,

where `erfc` is the complementary error function.

2. $\Phi(x) = \frac{1}{2} \mathrm{erf}\,(x/\sqrt{2})$,

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp(-t^2/2) dt$$

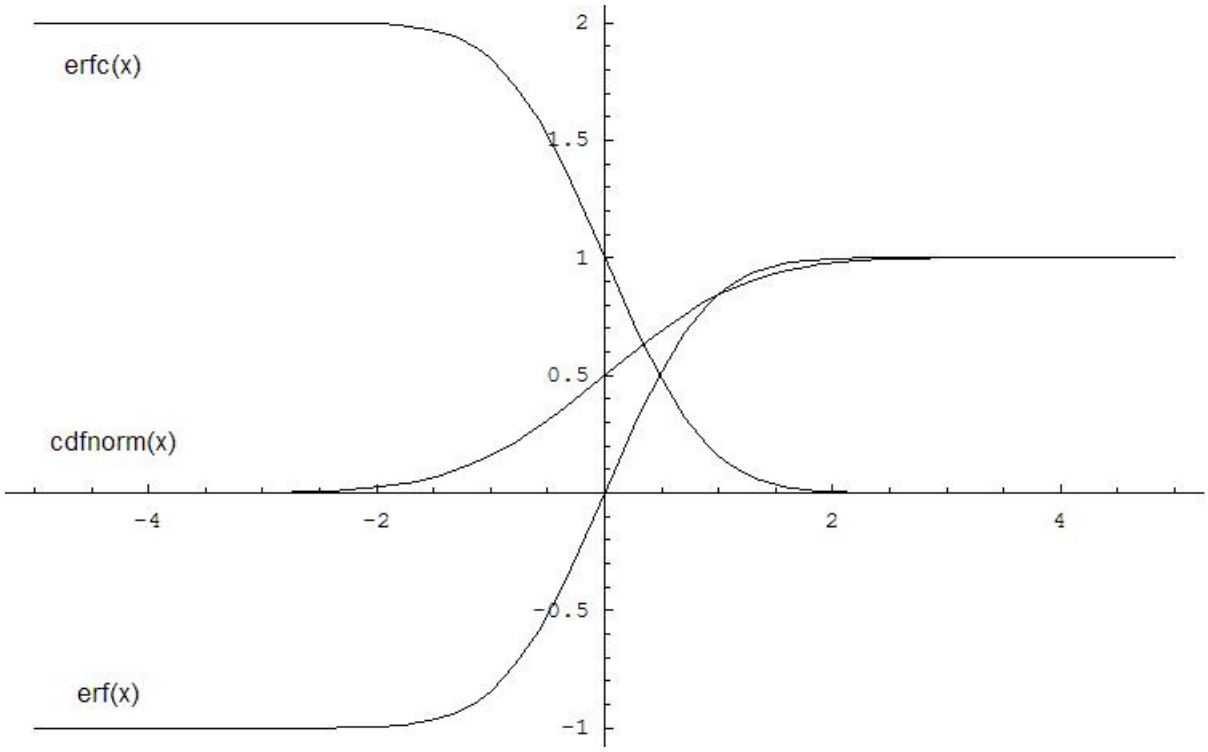
is the cumulative normal distribution function.

3. $\Phi^{-1}(x) = \sqrt{2}\,\mathrm{erf}^{-1}(2x - 1)$,

where $\Phi^{-1}(x)$ and $\mathtt{erf}^{-1}(x)$ are the inverses to $\Phi(x)$ and $\mathtt{erf}(x)$ respectively.

The following figure illustrates the relationships among `Erf` family functions (`Erf`, `Erfc`, `CdfNorm`).

Erf Family Functions Relationship



Useful relations for these functions:

$$\mathrm{erf}(x) + \mathrm{erfc}(x) = 1$$

$$\mathrm{cdfnorm}(x) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right) = 1 - \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

Special Values for Real Function v?Erf(x)

Argument	Result	Exception
+∞	+1	
-∞	-1	
QNAN	QNAN	
SNAN	QNAN	INVALID

See Also

v?Erfc
v?CdfNorm

v?Erfc

Computes the complementary error function value of vector elements.

Syntax

Fortran:

```
call vserfc( n, a, y )
call vmserfc( n, a, y, mode )
call vderfc( n, a, y )
call vmderfc( n, a, y, mode )
```

C:

```
vsErfc( n, a, y );
vmsErfc( n, a, y, mode );
vdErfc( n, a, y );
vmdErfc( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vserfc`, `vmserfc` `DOUBLE PRECISION` for `vderfc`, `vmderfc` **Fortran 90:** `REAL, INTENT(IN)` for `vserfc, vmserfc` `DOUBLE PRECISION, INTENT(IN)` for `vderfc, vmderfc` **C:** `const float*` for `vsErfc`, `vmsErfc` `const double*` for `vdErfc, vmdErfc`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Name	Type	Description
	C: `const MKL_INT64`	

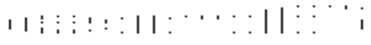
Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vserfc`, `vmserfc` `DOUBLE PRECISION` for `vderfc`, `vmderfc` **Fortran 90:** `REAL, INTENT(OUT)` for `vserfc, vmserfc` `DOUBLE PRECISION, INTENT(OUT)` for `vderfc, vmderfc` **C:** `float*` for `vsErfc, vmsErfc` `double*` for `vdErfc, vmdErfc`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `Erfc` function computes the complementary error function values for elements of the input vector *a* and writes them to the output vector *y*.

The complementary error function is defined as follows:



Useful relations:

1. $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$.

2. $\Phi(x) = \frac{1}{2}\,\mathrm{erf}\,(x/\sqrt{2})$,

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp\,(-t^2/2)dt$$

is the cumulative normal distribution function.

3. $\Phi^{-1}(x) = \sqrt{2}\,\mathrm{erf}^{-1}(2x - 1)$,

where $\Phi^{-1}(x)$ and $\mathrm{erf}^{-1}(x)$ are the inverses to $\Phi(x)$ and $\mathrm{erf}(x)$ respectively.

See also Figure "Erf Family Functions Relationship" in `Erf` function description for `Erfc` function relationship with the other functions of `Erf` family.

Special Values for Real Function v?Erfc(x)

Argument	Result	VML Error Status	Exception
X > underflow	+0	VML_STATUS_UNDERFLOW	UNDERFLOW
+∞	+0		
-∞	+2		
QNAN	QNAN		
SNAN	QNAN		INVALID

See Also

v?Erf
v?CdfNorm

v?CdfNorm

Computes the cumulative normal distribution function values of vector elements.

Syntax

Fortran:

```
call vscdfnorm( n, a, y )
call vmscdfnorm( n, a, y, mode )
call vdcdfnorm( n, a, y )
call vmdcdfnorm( n, a, y, mode )
```

C:

```
vsCdfNorm( n, a, y );
vmsCdfNorm( n, a, y, mode );
vdCdfNorm( n, a, y );
vmdCdfNorm( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vscdfnorm`, `vmscdfnorm` `DOUBLE PRECISION` for `vdcdfnorm`, `vmdcdfnorm`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	Fortran 90: `REAL, INTENT(IN)` for `vscdfnorm, vmscdfnorm` `DOUBLE PRECISION, INTENT(IN)` for `vdcdfnorm, vmdcdfnorm` **C:** `const float*` for `vsCdfNorm, vmsCdfNorm` `const double*` for `vdCdfNorm, vmdCdfNorm`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vscdfnorm, vmscdfnorm` `DOUBLE PRECISION` for `vdcdfnorm, vmdcdfnorm` **Fortran 90:** `REAL, INTENT(OUT)` for `vscdfnorm, vmscdfnorm` `DOUBLE PRECISION, INTENT(OUT)` for `vdcdfnorm, vmdcdfnorm` **C:** `float*` for `vsCdfNorm, vmsCdfNorm` `double*` for `vdCdfNorm, vmdCdfNorm`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `CdfNorm` function computes the cumulative normal distribution function values for elements of the input vector *a* and writes them to the output vector *y*.

The cumulative normal distribution function is defined as given by:

$$\mathrm{CdfNorm}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \ .$$

Useful relations:

$$\mathrm{cdfnorm}(x) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right) = 1 - \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

where Erf and Erfc are the error and complementary error functions.

See also Figure "Erf Family Functions Relationship" in Erf function description for CdfNorm function relationship with the other functions of Erf family.

Special Values for Real Function v?CdfNorm(x)

Argument	Result	VML Error Status	Exception
X < underflow	+0	VML_STATUS_UNDERFLOW	UNDERFLOW
+∞	+1		
-∞	+0		
QNAN	QNAN		
SNAN	QNAN		INVALID

See Also

v?Erf
v?Erfc

v?ErfInv

Computes inverse error function value of vector elements.

Syntax

Fortran:

```
call vserfinv( n, a, y )
call vmserfinv( n, a, y, mode )
call vderfinv( n, a, y )
call vmderfinv( n, a, y, mode )
```

C:

```
vsErfInv( n, a, y );
vmsErfInv( n, a, y, mode );
vdErfInv( n, a, y );
vmdErfInv( n, a, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.

Name	Type	Description
a	**FORTRAN 77:** REAL for vserfinv, vmserfinv DOUBLE PRECISION for vderfinv, vmderfinv **Fortran 90:** REAL, INTENT(IN) for vserfinv, vmserfinv DOUBLE PRECISION, INTENT(IN) for vderfinv, vmderfinv **C:** const float* for vsErfInv, vmsErfInv const double* for vdErfInv, vmdErfInv	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vserfinv, vmserfinv DOUBLE PRECISION for vderfinv, vmderfinv **Fortran 90:** REAL, INTENT(OUT) for vserfinv, vmserfinv DOUBLE PRECISION, INTENT(OUT) for vderfinv, vmderfinv **C:** float* for vsErfInv, vmsErfInv double* for vdErfInv, vmdErfInv	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The ErfInv function computes the inverse error function values for elements of the input vector *a* and writes them to the output vector *y*

y = erf^{-1}(*a*),

where erf(*x*) is the error function defined as given by:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \ .$$

Useful relations:

1. $\mathrm{erf}^{-1}(x) = \mathrm{erfc}^{-1}(1 - x)$,

where `erfc` is the complementary error function.

2. $\Phi(x) = \frac{1}{2}\,\mathrm{erf}\,(x/\sqrt{2})$,

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp\,(-t^2/2)dt$$

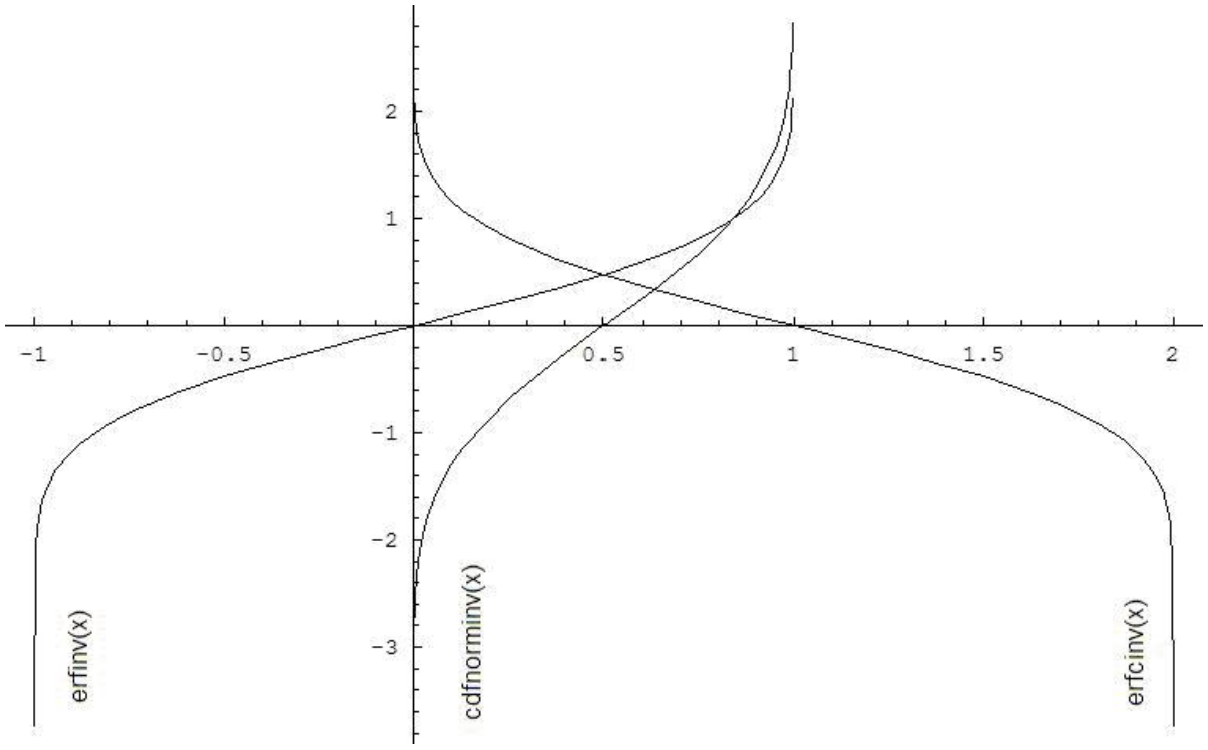
is the cumulative normal distribution function.

3. $\Phi^{-1}(x) = \sqrt{2}\,\mathrm{erf}^{-1}(2x - 1)$,

where $\Phi^{-1}(x)$ and $\mathrm{erf}^{-1}(x)$ are the inverses to $\Phi(x)$ and $\mathrm{erf}(x)$ respectively.

Figure "ErfInv Family Functions Relationship" illustrates the relationships among `ErfInv` family functions (`ErfInv`, `ErfcInv`, `CdfNormInv`).

ErfInv Family Functions Relationship



Useful relations for these functions:

$$\mathrm{erfcinv}(x) = \mathrm{erfinv}(1 - x)$$

$$\text{cdfnorminv}(x) = \sqrt{2}\,\text{erfinv}(2x - 1) = \sqrt{2}\,\text{erfcinv}(2 - 2x)$$

Special Values for Real Function v?ErfInv(x)

Argument	Result	VML Error Status	Exception
+0	+0		
-0	-0		
+1	+∞	VML_STATUS_SING	ZERODIVIDE
-1	-∞	VML_STATUS_SING	ZERODIVIDE
|X| > 1	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

See Also

v?ErfcInv
v?CdfNormInv

v?ErfcInv

Computes the inverse complementary error function value of vector elements.

Syntax

Fortran:

```
call vserfcinv( n, a, y )
call vmserfcinv( n, a, y, mode )
call vderfcinv( n, a, y )
call vmderfcinv( n, a, y, mode )
```

C:

```
vsErfcInv( n, a, y );
vmsErfcInv( n, a, y, mode );
vdErfcInv( n, a, y );
vmdErfcInv( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.

Name	Type	Description
a	**FORTRAN 77:** REAL for vserfcinv, vmserfcinv DOUBLE PRECISION for vderfcinv, vmderfcinv **Fortran 90:** REAL, INTENT(IN) for vserfcinv, vmserfcinv DOUBLE PRECISION, INTENT(IN) for vderfcinv, vmderfcinv **C:** const float* for vsErfcInv, vmsErfcInv const double* for vdErfcInv, vmdErfcInv	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vserfcinv, vmserfcinv DOUBLE PRECISION for vderfcinv, vmderfcinv **Fortran 90:** REAL, INTENT(OUT) for vserfcinv, vmserfcinv DOUBLE PRECISION, INTENT(OUT) for vderfcinv, vmderfcinv **C:** float* for vsErfcInv, vmsErfcInv double* for vdErfcInv, vmdErfcInv	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The ErfcInv function computes the inverse complimentary error function values for elements of the input vector *a* and writes them to the output vector *y*.

The inverse complementary error function is defined as given by:

$$\text{erfcinv}(x) = \text{erfinv}(1 - x)$$

$$\text{erfinv}(x) = \text{erf}^{-1}(x)$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

where `erf(x)` denotes the error function and `erfinv(x)` denotes the inverse error function.

See also Figure "ErfInv Family Functions Relationship" in `ErfInv` function description for `ErfcInv` function relationship with the other functions of `ErfInv` family.

Special Values for Real Function v?ErfcInv(x)

Argument	Result	VML Error Status	Exception
+1	+0		
+2	-∞	VML_STATUS_SING	ZERODIVIDE
-0	+∞	VML_STATUS_SING	ZERODIVIDE
+0	+∞	VML_STATUS_SING	ZERODIVIDE
X < -0	QNAN	VML_STATUS_ERRDOM	INVALID
X > +2	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

See Also

v?ErfInv
v?CdfNormInv

v?CdfNormInv

Computes the inverse cumulative normal distribution function values of vector elements.

Syntax

Fortran:

```
call vscdfnorminv( n, a, y )
call vmscdfnorminv( n, a, y, mode )
call vdcdfnorminv( n, a, y )
call vmdcdfnorminv( n, a, y, mode )
```

C:

```
vsCdfNormInv( n, a, y );
vmsCdfNormInv( n, a, y, mode );
vdCdfNormInv( n, a, y );
vmdCdfNormInv( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`

- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vscdfnorminv, vmscdfnorminv` `DOUBLE PRECISION` for `vdcdfnorminv, vmdcdfnorminv` **Fortran 90:** `REAL, INTENT(IN)` for `vscdfnorminv, vmscdfnorminv` `DOUBLE PRECISION, INTENT(IN)` for `vdcdfnorminv, vmdcdfnorminv` **C:** `const float*` for `vsCdfNormInv, vmsCdfNormInv` `const double*` for `vdCdfNormInv, vmdCdfNormInv`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vscdfnorminv, vmscdfnorminv` `DOUBLE PRECISION` for `vdcdfnorminv, vmdcdfnorminv` **Fortran 90:** `REAL, INTENT(OUT)` for `vscdfnorminv, vmscdfnorminv` `DOUBLE PRECISION, INTENT(OUT)` for `vdcdfnorminv, vmdcdfnorminv` **C:** `float*` for `vsCdfNormInv, vmsCdfNormInv` `double*` for `vdCdfNormInv, vmdCdfNormInv`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The `CdfNormInv` function computes the inverse cumulative normal distribution function values for elements of the input vector *a* and writes them to the output vector *y*.

The inverse cumulative normal distribution function is defined as given by:

$$\mathrm{CdfNormInv}(x) = \mathrm{CdfNorm}^{-1}(x) ,$$

where `CdfNorm(x)` denotes the cumulative normal distribution function.

Useful relations:

$$\mathrm{cdfnorminv}(x) = \sqrt{2}\,\mathrm{erfinv}(2x - 1) = \sqrt{2}\,\mathrm{erfcinv}(2 - 2x)$$

where `erfinv(x)` denotes the inverse error function and `erfcinv(x)` denotes the inverse complementary error functions.

See also Figure "ErfInv Family Functions Relationship" in `ErfInv` function description for `CdfNormInv` function relationship with the other functions of `ErfInv` family.

Special Values for Real Function v?CdfNormInv(x)

Argument	Result	VML Error Status	Exception
+0.5	+0		
+1	+∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
+0	-∞	VML_STATUS_SING	ZERODIVIDE
X < -0	QNAN	VML_STATUS_ERRDOM	INVALID
X > +1	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
QNAN	QNAN		
SNAN	QNAN		INVALID

See Also

v?ErfInv
v?ErfcInv

v?LGamma

Computes the natural logarithm of the absolute value of gamma function for vector elements.

Syntax

Fortran:

```
call vslgamma( n, a, y )
call vmslgamma( n, a, y, mode )
call vdlgamma( n, a, y )
call vmdlgamma( n, a, y, mode )
```

C:

```
vsLGamma( n, a, y );
vmsLGamma( n, a, y, mode );
vdLGamma( n, a, y );
```

```
vmdLGamma( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vslgamma, vmslgamma` `DOUBLE PRECISION` for `vdlgamma, vmdlgamma` **Fortran 90:** `REAL, INTENT(IN)` for `vslgamma, vmslgamma` `DOUBLE PRECISION, INTENT(IN)` for `vdlgamma, vmdlgamma` **C:** `const float*` for `vsLGamma, vmsLGamma` `const double*` for `vdLGamma, vmdLGamma`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vslgamma, vmslgamma` `DOUBLE PRECISION` for `vdlgamma, vmdlgamma` **Fortran 90:** `REAL, INTENT(OUT)` for `vslgamma, vmslgamma` `DOUBLE PRECISION, INTENT(OUT)` for `vdlgamma, vmdlgamma` **C:** `float*` for `vsLGamma, vmsLGamma` `double*` for `vdLGamma, vmdLGamma`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?LGamma function computes the natural logarithm of the absolute value of gamma function for elements of the input vector *a* and writes them to the output vector *y*. Precision overflow thresholds for the v?LGamma function are beyond the scope of this document. If the result does not meet the target precision, the function raises the OVERFLOW exception and sets the VML Error Status to VML_STATUS_OVERFLOW.

Special Values for Real Function v?LGamma(x)

Argument	Result	VML Error Status	Exception
+1	+0		
+2	+0		
+0	+?	VML_STATUS_SING	ZERODIVIDE
-0	+?	VML_STATUS_SING	ZERODIVIDE
negative integer	+?	VML_STATUS_SING	ZERODIVIDE
-?	+?		
+?	+?		
X > overflow	+?	VML_STATUS_OVERFLOW	OVERFLOW
QNAN	QNAN		
SNAN	QNAN		INVALID

v?TGamma

Computes the gamma function of vector elements.

Syntax

Fortran:

```
call vstgamma( n, a, y )
call vmstgamma( n, a, y, mode )
call vdtgamma( n, a, y )
call vmdtgamma( n, a, y, mode )
```

C:

```
vsTGamma( n, a, y );
vmsTGamma( n, a, y, mode );
vdTGamma( n, a, y );
vmdTGamma( n, a, y, mode );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.

Name	Type	Description
a	**FORTRAN 77:** REAL for vstgamma, vmstgamma DOUBLE PRECISION for vdtgamma, vmdtgamma **Fortran 90:** REAL, INTENT(IN) for vstgamma, vmstgamma DOUBLE PRECISION, INTENT(IN) for vdtgamma, vmdtgamma **C:** const float* for vsTGamma, vmsTGamma const double* for vdTGamma, vmdTGamma	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vstgamma, vmstgamma DOUBLE PRECISION for vdtgamma, vmdtgamma **Fortran 90:** REAL, INTENT(OUT) for vstgamma, vmstgamma DOUBLE PRECISION, INTENT(OUT) for vdtgamma, vmdtgamma **C:** float* for vsTGamma, vmsTGamma double* for vdTGamma, vmdTGamma	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?TGamma function computes the gamma function for elements of the input vector *a* and writes them to the output vector *y*. Precision overflow thresholds for the v?TGamma function are beyond the scope of this document. If the result does not meet the target precision, the function raises the OVERFLOW exception and sets the VML Error Status to VML_STATUS_OVERFLOW.

Special Values for Real Function v?TGamma(x)

Argument	Result	VML Error Status	Exception
+0	+∞	VML_STATUS_SING	ZERODIVIDE
-0	-∞	VML_STATUS_SING	ZERODIVIDE
negative integer	QNAN	VML_STATUS_ERRDOM	INVALID
-∞	QNAN	VML_STATUS_ERRDOM	INVALID
+∞	+∞		

Argument	Result	VML Error Status	Exception
X > overflow	+∞	VML_STATUS_OVERFLOW	OVERFLOW
QNAN	QNAN		
SNAN	QNAN		INVALID

Rounding Functions

v?Floor

Computes an integer value rounded towards minus infinity for each vector element.

Syntax

Fortran:

```
call vsfloor( n, a, y )
call vmsfloor( n, a, y, mode )
call vdfloor( n, a, y )
call vmdfloor( n, a, y, mode )
```

C:

```
vsFloor( n, a, y );
vmsFloor( n, a, y, mode );
vdFloor( n, a, y );
vmdFloor( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsfloor, vmsfloor` `DOUBLE PRECISION` for `vdfloor, vmdfloor` **Fortran 90:** `REAL, INTENT(IN)` for `vsfloor, vmsfloor` `DOUBLE PRECISION, INTENT(IN)` for `vdfloor, vmdfloor`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	C: `const float*` for `vsFloor`, `vmsfloor` `const double*` for `vdFloor`, `vmdfloor`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8)`, `INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsfloor`, `vmsfloor` `DOUBLE PRECISION` for `vdfloor`, `vmdfloor` **Fortran 90:** `REAL`, `INTENT(OUT)` for `vsfloor`, `vmsfloor` `DOUBLE PRECISION`, `INTENT(OUT)` for `vdfloor`, `vmdfloor` **C:** `float*` for `vsFloor`, `vmsfloor` `double*` for `vdFloor`, `vmdfloor`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The function computes an integer value rounded towards minus infinity for each vector element.

Special Values for Real Function v?Floor(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
SNAN	QNAN	INVALID
QNAN	QNAN	

v?Ceil

Computes an integer value rounded towards plus infinity for each vector element.

Syntax

Fortran:

```
call vsceil( n, a, y )
call vmsceil( n, a, y, mode )
call vdceil( n, a, y )
```

```
call vmdceil( n, a, y, mode )
```

C:

```
vsCeil( n, a, y );
vmsCeil( n, a, y, mode );
vdCeil( n, a, y );
vmdCeil( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsceil, vmsceil` `DOUBLE PRECISION` for `vdceil, vmdceil` **Fortran 90:** `REAL, INTENT(IN)` for `vsceil, vmsceil` `DOUBLE PRECISION, INTENT(IN)` for `vdceil, vmdceil` **C:** `const float*` for `vsCeil, vmsceil` `const double*` for `vdCeil, vmdceil`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsceil, vmsceil` `DOUBLE PRECISION` for `vdceil, vmdceil` **Fortran 90:** `REAL, INTENT(OUT)` for `vsceil, vmsceil`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE PRECISION, INTENT(OUT)` for `vdceil, vmdceil` **C:** `float*` for `vsCeil, vmsceil` `double*` for `vdCeil, vmdceil`	

Description

The function computes an integer value rounded towards plus infinity for each vector element.

Special Values for Real Function v?Ceil(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
SNAN	QNAN	INVALID
QNAN	QNAN	

v?Trunc

Computes an integer value rounded towards zero for each vector element.

Syntax

Fortran:

```
call vstrunc( n, a, y )
call vmstrunc( n, a, y, mode )
call vdtrunc( n, a, y )
call vmdtrunc( n, a, y, mode )
```

C:

```
vsTrunc( n, a, y );
vmsTrunc( n, a, y, mode );
vdTrunc( n, a, y );
vmdTrunc( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)`	Specifies the number of elements to be calculated.

Name	Type	Description
	C: `const int`	
a	**FORTRAN 77:** `REAL` for `vstrunc`, `vmstrunc` `DOUBLE PRECISION` for `vdtrunc`, `vmdtrunc` **Fortran 90:** `REAL, INTENT(IN)` for `vstrunc, vmstrunc` `DOUBLE PRECISION, INTENT(IN)` for `vdtrunc, vmdtrunc` **C:** `const float*` for `vsTrunc`, `vmstrunc` `const double*` for `vdTrunc`, `vmdtrunc`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vstrunc`, `vmstrunc` `DOUBLE PRECISION` for `vdtrunc`, `vmdtrunc` **Fortran 90:** `REAL, INTENT(OUT)` for `vstrunc, vmstrunc` `DOUBLE PRECISION, INTENT(OUT)` for `vdtrunc, vmdtrunc` **C:** `float*` for `vsTrunc, vmstrunc` `double*` for `vdTrunc, vmdtrunc`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The function computes an integer value rounded towards zero for each vector element.

Special Values for Real Function v?Trunc(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
SNAN	QNAN	INVALID
QNAN	QNAN	

v?Round

Computes a value rounded to the nearest integer for each vector element.

Syntax

Fortran:

```
call vsround( n, a, y )
call vmsround( n, a, y, mode )
call vdround( n, a, y )
call vmdround( n, a, y, mode )
```

C:

```
vsRound( n, a, y );
vmsRound( n, a, y, mode );
vdRound( n, a, y );
vmdRound( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsround`, `vmsround` `DOUBLE PRECISION` for `vdround`, `vmdround` **Fortran 90:** `REAL, INTENT(IN)` for `vsround, vmsround` `DOUBLE PRECISION, INTENT(IN)` for `vdround, vmdround` **C:** `const float*` for `vsRound`, `vmsround` `const double*` for `vdRound`, `vmdround`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Name	Type	Description
	C: `const MKL_INT64`	

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsround`, `vmsround` `DOUBLE PRECISION` for `vdround`, `vmdround` **Fortran 90:** `REAL, INTENT(OUT)` for `vsround, vmsround` `DOUBLE PRECISION, INTENT(OUT)` for `vdround, vmdround` **C:** `float*` for `vsRound, vmsround` `double*` for `vdRound, vmdround`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The function computes a value rounded to the nearest integer for each vector element.

The resulting mode affects the results computed for inputs that fall half-way between consecutive integres. For example:

- `f(0.5) = 0`, for rounding modes set to round to nearest round toward zero or to minus infinity.
- `f(0.5) = 1`, for rounding modes set to plus infinity.
- `f(-1.5) = -2`, for rounding modes set to round to nearest or to minus infinity.
- `f(-1.5) = -1`, for rounding modes set to round toward zero or to plus infinity.

Special Values for Real Function v?Round(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
SNAN	QNAN	INVALID
QNAN	QNAN	

v?NearbyInt

Computes a rounded integer value in the current rounding mode for each vector element.

Syntax

Fortran:

```
call vsnearbyint( n, a, y )
call vmsnearbyint( n, a, y, mode )
call vdnearbyint( n, a, y )
call vmdnearbyint( n, a, y, mode )
```

C:

```
vsNearbyInt( n, a, y );
vmsNearbyInt( n, a, y, mode );
vdNearbyInt( n, a, y );
vmdNearbyInt( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsnearbyint, vmsnearbyint` `DOUBLE PRECISION` for `vdnearbyint, vmdnearbyint` **Fortran 90:** `REAL, INTENT(IN)` for `vsnearbyint, vmsnearbyint` `DOUBLE PRECISION, INTENT(IN)` for `vdnearbyint, vmdnearbyint` **C:** `const float*` for `vsNearbyInt, vmsnearbyint` `const double*` for `vdNearbyInt, vmdnearbyint`	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** `REAL` for `vsnearbyint, vmsnearbyint` `DOUBLE PRECISION` for `vdnearbyint, vmdnearbyint` **Fortran 90:** `REAL, INTENT(OUT)` for `vsnearbyint, vmsnearbyint`	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Name	Type	Description
	`DOUBLE PRECISION, INTENT(OUT)` for `vdnearbyint, vmdnearbyint` **C:** `float*` for `vsNearbyInt, vmsnearbyint` `double*` for `vdNearbyInt, vmdnearbyint`	

Description

The `v?NearbyInt` function computes a rounded integer value in a current rounding mode for each vector element.

Halfway values, that is, 0.5, -1.5, and the like, are rounded off towards even values.

Special Values for Real Function v?NearbyInt(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
SNAN	QNAN	INVALID
QNAN	QNAN	

v?Rint

Computes a rounded integer value in the current rounding mode for each vector element with inexact result exception raised for each changed value.

Syntax

Fortran:

```
call vsrint( n, a, y )
call vmsrint( n, a, y, mode )
call vdrint( n, a, y )
call vmdrint( n, a, y, mode )
```

C:

```
vsRint( n, a, y );
vmsRint( n, a, y, mode );
vdRint( n, a, y );
vmdRint( n, a, y, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** REAL for vsrint, vmsrint DOUBLE PRECISION for vdrint, vmdrint **Fortran 90:** REAL, INTENT(IN) for vsrint, vmsrint DOUBLE PRECISION, INTENT(IN) for vdrint, vmdrint **C:** const float* for vsRint, vmsrint const double* for vdRint, vmdrint	**FORTRAN:** Array that specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Overrides global VML mode setting for this function call. See vmlSetMode for possible values and their description.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsrint, vmsrint DOUBLE PRECISION for vdrint, vmdrint **Fortran 90:** REAL, INTENT(OUT) for vsrint, vmsrint DOUBLE PRECISION, INTENT(OUT) for vdrint, vmdrint **C:** float* for vsRint, vmsrint double* for vdRint, vmdrint	**FORTRAN:** Array that specifies the output vector *y*. **C:** Pointer to an array that contains the output vector *y*.

Description

The v?Rint function computes a rounded floating-point integer value using the current rounding mode for each vector element.

The resulting mode affects the results computed for inputs that fall half-way between consecutive integres. For example:

- f(0.5) = 0, for rounding modes set to round to nearest round toward zero or to minus infinity.
- f(0.5) = 1, for rounding modes set to plus infinity.

- `f(-1.5) = -2`, for rounding modes set to round to nearest or to minus infinity.
- `f(-1.5) = -1`, for rounding modes set to round toward zero or to plus infinity.

Special Values for Real Function v?Rint(x)

Argument	Result	Exception
+0	+0	
-0	-0	
+∞	+∞	
-∞	-∞	
SNAN	QNAN	INVALID
QNAN	QNAN	

v?Modf

Computes a truncated integer value and the remaining fraction part for each vector element.

Syntax

Fortran:

```
call vsmodf( n, a, y, z )
call vmsmodf( n, a, y, z, mode )
call vdmodf( n, a, y, z )
call vmdmodf( n, a, y, z, mode )
```

C:

```
vsModf( n, a, y, z );
vmsModf( n, a, y, z, mode );
vdModf( n, a, y, z );
vmdModf( n, a, y, z, mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsmodf`, `vmsmodf` `DOUBLE PRECISION` for `vdmodf`, `vmdmodf`	**FORTRAN:** Array, specifies the input vector *a*. **C:** Pointer to an array that contains the input vector *a*.

Name	Type	Description
	Fortran 90: `REAL, INTENT(IN)` for `vsmodf, vmsmodf` `DOUBLE PRECISION, INTENT(IN)` for `vdmodf, vmdmodf` **C:** `const float*` for `vsModf, vmsmodf` `const double*` for `vdModf, vmdmodf`	
mode	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const MKL_INT64`	Overrides global VML mode setting for this function call. See `vmlSetMode` for possible values and their description.

Output Parameters

Name	Type	Description
y, *z*	**FORTRAN 77:** `REAL` for `vsmodf, vmsmodf` `DOUBLE PRECISION` for `vdmodf, vmdmodf` **Fortran 90:** `REAL, INTENT(OUT)` for `vsmodf, vmsmodf` `DOUBLE PRECISION, INTENT(OUT)` for `vdmodf, vmdmodf` **C:** `float*` for `vsModf, vmsmodf` `double*` for `vdModf, vmdmodf`	**FORTRAN:** Array, specifies the output vector *y* and *z*. **C:** Pointer to an array that contains the output vector *y* and *z*.

Description

The function computes a truncated integer value and the remaining fraction part for each vector element.

Halfway values, such as 0.5, -1.5, are rounded off towards even values. An inexact result exception is raised for each changed value.

Special Values for Real Function v?Modf(x)

Argument	Result 1	Result 2	Exception
+0	+0	+0	
-0	-0	-0	
+∞	+∞	+0	
-∞	-∞	-0	
SNAN	QNAN	QNAN	INVALID
QNAN	QNAN	QNAN	

VML Pack/Unpack Functions

This section describes VML functions that convert vectors with unit increment to and from vectors with positive increment indexing, vector indexing, and mask indexing (see Appendix B for details on vector indexing methods).

The table below lists available VML Pack/Unpack functions, together with data types and indexing methods associated with them.

VML Pack/Unpack Functions

Function Short Name	Data Types	Indexing Methods	Description
v?Pack	s, d, c, z	I,V,M	Gathers elements of arrays, indexed by different methods.
v?Unpack	s, d, c, z	I,V,M	Scatters vector elements to arrays with different indexing.

See Also

Vector Arguments in VML

v?Pack

Copies elements of an array with specified indexing to a vector with unit increment.

Syntax

Fortran:

```
call vspacki( n, a, inca, y )
call vspackv( n, a, ia, y )
call vspackm( n, a, ma, y )
call vdpacki( n, a, inca, y )
call vdpackv( n, a, ia, y )
call vdpackm( n, a, ma, y )
call vcpacki( n, a, inca, y )
call vcpackv( n, a, ia, y )
call vcpackm( n, a, ma, y )
call vzpacki( n, a, inca, y )
call vzpackv( n, a, ia, y )
call vzpackm( n, a, ma, y )
```

C:

```
vsPackI( n, a, inca, y );
vsPackV( n, a, ia, y );
vsPackM( n, a, ma, y );
vdPackI( n, a, inca, y );
vdPackV( n, a, ia, y );
vdPackM( n, a, ma, y );
vcPackI( n, a, inca, y );
```

```
vcPackV( n, a, ia, y );
vcPackM( n, a, ma, y );
vzPackI( n, a, inca, y );
vzPackV( n, a, ia, y );
vzPackM( n, a, ma, y );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vspacki, vspackv, vspackm` `DOUBLE PRECISION` for `vdpacki, vdpackv, vdpackm` `COMPLEX` for `vcpacki, vcpackv, vcpackm` `DOUBLE COMPLEX` for `vzpacki, vzpackv, vzpackm` **Fortran 90:** `REAL, INTENT(IN)` for `vspacki, vspackv, vspackm` `DOUBLE PRECISION, INTENT(IN)` for `vdpacki, vdpackv, vdpackm` `COMPLEX, INTENT(IN)` for `vcpacki, vcpackv, vcpackm` `DOUBLE COMPLEX, INTENT(IN)` for `vzpacki, vzpackv, vzpackm` **C:** `const float*` for `vsPackI, vsPackV, vsPackM` `const double*` for `vdPackI, vdPackV, vdPackM` `const MKL_Complex8*` for `vcPackI, vcPackV, vcPackM` `const MKL_Complex16*` for `vzPackI, vzPackV, vzPackM`	**FORTRAN:** Array, `DIMENSION` at least `(1 + (n-1)*inca)` for `v?packi`, Array, `DIMENSION` at least `max( n,max(ia[j]) ), j=0, …, n-1` for `v?packv`, Array, `DIMENSION` at least `n` for `v?packm`. Specifies the input vector `a`. **C:** Specifies pointer to an array that contains the input vector `a`. The arrays must be: for `v?PackI`, at least `(1 + (n-1)*inca)` for `v?PackV`, at least `max( n,max(ia[j]) ), j=0, …, n-1` for `v?PackM`, at least `n`.

Name	Type	Description
inca	**FORTRAN 77:** INTEGER for vspacki, vdpacki, vcpacki, vzpacki **Fortran 90:** INTEGER, INTENT(IN) for vspacki, vdpacki, vcpacki, vzpacki **C:** const int for vsPackI, vdPackI, vcPackI, vzPackI	Specifies the increment for the elements of *a*.
ia	**FORTRAN 77:** INTEGER for vspackv, vdpackv, vcpackv, vzpackv **Fortran 90:** INTEGER, INTENT(IN) for vspackv, vdpackv, vcpackv, vzpackv **C:** const int* for vsPackV, vdPackV, vcPackV, vzPackV	**FORTRAN:** Array, DIMENSION at least *n*. Specifies the index vector for the elements of *a*. **C:** Specifies the pointer to an array of size at least *n* that contains the index vector for the elements of *a*.
ma	**FORTRAN 77:** INTEGER for vspackm, vdpackm, vcpackm, vzpackm **Fortran 90:** INTEGER, INTENT(IN) for vspackm, vdpackm, vcpackm, vzpackm **C:** const int* for vsPackM, vdPackM, vcPackM, vzPackM	**FORTRAN:** Array, DIMENSION at least *n*, Specifies the mask vector for the elements of *a*. **C:** Specifies the pointer to an array of size at least *n* that contains the mask vector for the elements of *a*.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vspacki, vspackv, vspackm DOUBLE PRECISION for vdpacki, vdpackv, vdpackm COMPLEX for vcpacki, vcpackv, vcpackm DOUBLE COMPLEX for vzpacki, vzpackv, vzpackm **Fortran 90:** REAL, INTENT(OUT) for vspacki, vspackv, vspackm DOUBLE PRECISION, INTENT(OUT) for vdpacki, vdpackv, vdpackm COMPLEX, INTENT(OUT) for vcpacki, vcpackv, vcpackm	**FORTRAN:** Array, DIMENSION at least *n*. Specifies the output vector *y*. **C:** Pointer to an array of size at least *n* that contains the output vector *y*.

Name	Type	Description
	`DOUBLE COMPLEX`, `INTENT(OUT)` for `vzpacki`, `vzpackv`, `vzpackm` **C:** `float*` for `vsPackI`, `vsPackV`, `vsPackM` `double*` for `vdPackI`, `vdPackV`, `vdPackM` `const MKL_Complex8*` for `vcPackI`, `vcPackV`, `vcPackM` `const MKL_Complex16*` for `vzPackI`, `vzPackV`, `vzPackM`	

v?Unpack

Copies elements of a vector with unit increment to an array with specified indexing.

Syntax

Fortran:

```
call vsunpacki( n, a, y, incy )
call vsunpackv( n, a, y, iy )
call vsunpackm( n, a, y, my )
call vdunpacki( n, a, y, incy )
call vdunpackv( n, a, y, iy )
call vdunpackm( n, a, y, my )
call vcunpacki( n, a, y, incy )
call vcunpackv( n, a, y, iy )
call vcunpackm( n, a, y, my )
call vzunpacki( n, a, y, incy )
call vzunpackv( n, a, y, iy )
call vzunpackm( n, a, y, my )
```

C:

```
vsUnpackI( n, a, y, incy );
vsUnpackV( n, a, y, iy );
vsUnpackM( n, a, y, my );
vdUnpackI( n, a, y, incy );
vdUnpackV( n, a, y, iy );
vdUnpackM( n, a, y, my );
vcUnpackI( n, a, y, incy );
vcUnpackV( n, a, y, iy );
```

```
vcUnpackM( n, a, y, my );
vzUnpackI( n, a, y, incy );
vzUnpackV( n, a, y, iy );
vzUnpackM( n, a, y, my );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the number of elements to be calculated.
a	**FORTRAN 77:** `REAL` for `vsunpacki, vsunpackv, vsunpackm` `DOUBLE PRECISION` for `vdunpacki, vdunpackv, vdunpackm` `COMPLEX` for `vcunpacki, vcunpackv, vcunpackm` `DOUBLE COMPLEX` for `vzunpacki, vzunpackv, vznpackm` **Fortran 90:** `REAL, INTENT(IN)` for `vsunpacki, vsunpackv, vsunpackm` `DOUBLE PRECISION, INTENT(IN)` for `vdunpacki, vdunpackv, vdunpackm` `COMPLEX, INTENT(IN)` for `vcunpacki, vcunpackv, vcunpackm` `DOUBLE COMPLEX, INTENT(IN)` for `vzunpacki, vzunpackv, vzunpackm` **C:** `const float*` for `vsUnpackI, vsUnpackV, vsUnpackM` `const double*` for `vdUnpackI, vdUnpackV, vdUnpackM` `const MKL_Complex8*` for `vcUnpackI, vcUnpackV, vcUnpackM` `const MKL_Complex16*` for `vzUnpackI, vzUnpackV, vzUnpackM`	**FORTRAN:** Array, `DIMENSION` at least *n*. Specifies the input vector *a*. **C:** Specifies the pointer to an array of size at least *n* that contains the input vector *a*.
incy	**FORTRAN 77:** `INTEGER` for `vsunpacki, vdunpacki, vcunpacki, vzunpacki`	Specifies the increment for the elements of *y*.

Name	Type	Description
	Fortran 90: INTEGER, INTENT(IN) for vsunpacki, vdunpacki, vcunpacki, vzunpacki **C:** const int for vsUnpackI, vdUnpackI, vcUnpackI, vzUnpackI	
iy	**FORTRAN 77:** INTEGER for vsunpackv, vdunpackv, vcunpackv, vzunpackv **Fortran 90:** INTEGER, INTENT(IN) for vsunpackv, vdunpackv, vcunpackv, vzunpackv **C:** const int* for vsUnpackV, vdUnpackV, vcUnpackV, vzUnpackV	**FORTRAN:** Array, DIMENSION at least *n*. Specifies the index vector for the elements of *y*. **C:** Specifies the pointer to an array of size at least *n* that contains the index vector for the elements of *a*.
my	**FORTRAN 77:** INTEGER for vsunpackm, vdunpackm, vcunpackm, vzunpackm **Fortran 90:** INTEGER, INTENT(IN) for vsunpackm, vdunpackm, vcunpackm, vzunpackm **C:** const int* for vsUnpackM, vdUnpackM, vcUnpackM, vzUnpackM	**FORTRAN:** Array, DIMENSION at least *n*, Specifies the mask vector for the elements of *y*. **C:** Specifies the pointer to an array of size at least *n* that contains the mask vector for the elements of *a*.

Output Parameters

Name	Type	Description
y	**FORTRAN 77:** REAL for vsunpacki, vsunpackv, vsunpackm DOUBLE PRECISION for vdunpacki, vdunpackv, vdunpackm COMPLEX, INTENT(IN) for vcunpacki, vcunpackv, vcunpackm DOUBLE COMPLEX, INTENT(IN) for vzunpacki, vzunpackv, vzunpackm **Fortran 90:** REAL, INTENT(OUT) for vsunpacki, vsunpackv, vsunpackm DOUBLE PRECISION, INTENT(OUT) for vdunpacki, vdunpackv, vdunpackm COMPLEX, INTENT(OUT) for vcunpacki, vcunpackv, vcunpackm DOUBLE COMPLEX, INTENT(OUT) for vzunpacki, vzunpackv, vzunpackm	**FORTRAN:** Array, DIMENSION for v?unpacki, at least (1 + (*n*-1)**incy*) for v?unpackv, at least max(*n*,max(*iy*[j])),j=0,..., *n*-1 for v?unpackm, at least *n* **C:** Specifies the pointer to an array that contains the output vector *y*. The array must be: for v?UnpackI, at least (1 + (*n*-1)**incy*) for v?UnpackV, at least max(*n*,max(*ia*[j])),j=0,..., *n*-1, for v?UnpackM, at least *n*.

Name	Type	Description
	C: `float*` for `vsUnpackI`, `vsUnpackV`, `vsUnpackM` `double*` for `vdUnpackI`, `vdUnpackV`, `vdUnpackM` `const MKL_Complex8*` for `vcUnpackI`, `vcUnpackV`, `vcUnpackM` `const MKL_Complex16*` for `vzUnpackI`, `vzUnpackV`, `vzUnpackM`	

VML Service Functions

The VML Service functions enable you to set/get the accuracy mode and error code. These functions are available both in the Fortran and C interfaces. The table below lists available VML Service functions and their short description.

VML Service Functions

Function Short Name	Description
`vmlSetMode`	Sets the VML mode
`vmlGetMode`	Gets the VML mode
`vmlSetErrStatus`	Sets the VML Error Status
`vmlGetErrStatus`	Gets the VML Error Status
`vmlClearErrStatus`	Clears the VML Error Status
`vmlSetErrorCallBack`	Sets the additional error handler callback function
`vmlGetErrorCallBack`	Gets the additional error handler callback function
`vmlClearErrorCallBack`	Deletes the additional error handler callback function

vmlSetMode

Sets a new mode for VML functions according to the `mode` *parameter and stores the previous VML mode to* `oldmode`*.*

Syntax

Fortran:

```
oldmode = vmlsetmode( mode )
```

C:

```
oldmode = vmlSetMode( mode );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
mode	**FORTRAN 77:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8), INTENT(IN) **C:** const MKL_INT64	Specifies the VML mode to be set.

Output Parameters

Name	Type	Description
oldmode	**FORTRAN:** INTEGER*8 **Fortran 90:** INTEGER(KIND=8) **C:** MKL_INT64	Specifies the former VML mode.

Description

The vmlSetMode function sets a new mode for VML functions according to the *mode* parameter and stores the previous VML mode to *oldmode*. The mode change has a global effect on all the VML functions within a thread.

NOTE You can override the global mode setting and change the mode for a given VML function call by using a respective vm[s,d]<Func> variant of the function.

The *mode* parameter is designed to control accuracy, handling of denormalized numbers, and error handling. Table "Values of the *mode* Parameter" lists values of the *mode* parameter. You can obtain all other possible values of the *mode* parameter from the *mode* parameter values by using bitwise OR (|) operation to combine one value for accuracy, one value for handling of denormalized numbers, and one vlaue for error control options. The default value of the *mode* parameter is VML_HA | VML_FTZDAZ_OFF | VML_ERRMODE_DEFAULT.

The VML_FTZDAZ_ON mode is specifically designed to improve the performance of computations that involve denormalized numbers at the cost of reasonable accuracy loss. This mode changes the numeric behavior of the functions: denormalized input values are treated as zeros (DAZ = denormals-are-zero) and denormalized results are flushed to zero (FTZ = flush-to-zero). Accuracy loss may occur if input and/or output values are close to denormal range.

Values of the *mode* Parameter

Value of *mode*	Description
Accuracy Control	
VML_HA	high accuracy versions of VML functions
VML_LA	low accuracy versions of VML functions
VML_EP	enhanced performance accuracy versions of VML functions
Denormalized Numbers Handling Control	
VML_FTZDAZ_ON	Faster processing of denormalized inputs is enabled.
VML_FTZDAZ_OFF	Faster processing of denormalized inputs is disabled.
Error Mode Control	
VML_ERRMODE_IGNORE	No action is set for computation errors.
VML_ERRMODE_ERRNO	On error, the *errno* variable is set.

Value of *mode*	Description
VML_ERRMODE_STDERR	On error, the error text information is written to *stderr*.
VML_ERRMODE_EXCEPT	On error, an exception is raised.
VML_ERRMODE_CALLBACK	On error, an additional error handler function is called.
VML_ERRMODE_DEFAULT	On error, the *errno* variable is set, an exception is raised, and an additional error handler function is called.

Examples

The following example shows how to set low accuracy, fast processing for denormalized numbers and errno error mode in the Fortran and C languages:

```
oldmode = vmlsetmode( VML_LA )
call vmlsetmode( IOR(VML_LA, VML_FTZDAZ_ON, VML_ERRMODE_ERRNO) )

vmlSetMode( VML_LA );
vmlSetMode( VML_LA | VML_FTZDAZ_ON | VML_ERRMODE_ERRNO );
```

vmlGetMode

Gets the VML mode.

Syntax

Fortran:

```
mod = vmlgetmode()
```

C:

```
mod = vmlGetMode( void );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Output Parameters

Name	Type	Description
mod	**FORTRAN:** INTEGER **C:** int	Specifies the packed *mode* parameter.

Description

The function vmlGetMode() returns the VML *mode* parameter that controls accuracy, handling of denormalized numbers, and error handling options. The *mod* variable value is a combination of the values listed in the table "Values of the *mode* Parameter". You can obtain these values using the respective mask from the table "Values of Mask for the *mode* Parameter".

Values of Mask for the *mode* Parameter

Value of mask	Description
VML_ACCURACY_MASK	Specifies mask for accuracy *mode* selection.

Value of mask	Description
VML_FTZDAZ_MASK	Specifies mask for FTZDAZ *mode* selection.
VML_ERRMODE_MASK	Specifies mask for error *mode* selection.

See example below:

Examples

```
mod = vmlgetmode()
accm = IAND(mod, VML_ACCURACY_MASK)
denm = IAND(mod, VML_FTZDAZ_MASK)
errm = IAND(mod, VML_ERRMODE_MASK)

accm = vmlGetMode(void )& VML_ACCURACY_MASK;
denm = vmlGetMode(void )& VML_FTZDAZ_MASK;
errm = vmlGetMode(void )& VML_ERRMODE_MASK;
```

vmlSetErrStatus

Sets the new VML Error Status according to `err` *and stores the previous VML Error Status to* `olderr`.

Syntax

Fortran:

```
olderr = vmlseterrstatus( err )
```

C:

```
olderr = vmlSetErrStatus( err );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name	Type	Description
err	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Specifies the VML error status to be set.

Output Parameters

Name	Type	Description
olderr	**FORTRAN:** `INTEGER` **C:** `int`	Specifies the former VML error status.

Description

Table "Values of the VML Status" lists possible values of the *err* parameter.

Values of the VML Status

Status	Description
Successful Execution	
VML_STATUS_OK	The execution was completed successfully.
Warnings	
VML_STATUS_ACCURACYWARNING	The execution was completed successfully in a different accuracy mode.
Errors	
VML_STATUS_BADSIZE	The function does not support the preset accuracy mode. The Low Accuracy mode is used instead.
VML_STATUS_BADMEM	NULL pointer is passed.
VML_STATUS_ERRDOM	At least one of array values is out of a range of definition.
VML_STATUS_SING	At least one of the input array values causes a divide-by-zero exception or produces an invalid (QNaN) result.
VML_STATUS_OVERFLOW	An overflow has happened during the calculation process.
VML_STATUS_UNDERFLOW	An underflow has happened during the calculation process.

Examples

```
olderr = vmlSetErrStatus( VML_STATUS_OK );
olderr = vmlSetErrStatus( VML_STATUS_ERRDOM );
olderr = vmlSetErrStatus( VML_STATUS_UNDERFLOW );
```

vmlGetErrStatus

Gets the VML Error Status.

Syntax

Fortran:

```
err = vmlgeterrstatus( )
```

C:

```
err = vmlGetErrStatus( void );
```

Include Files

- FORTRAN 77: mkl_vml.f77
- Fortran 90: mkl_vml.f90
- C: mkl_vml_functions.h

Output Parameters

Name	Type	Description
err	**FORTRAN:** INTEGER **C:** int	Specifies the VML error status.

vmlClearErrStatus

Sets the VML Error Status to `VML_STATUS_OK` *and stores the previous VML Error Status to* `olderr`*.*

Syntax

Fortran:

```
olderr = vmlclearerrstatus( )
```

C:

```
olderr = vmlClearErrStatus( void );
```

Include Files

- FORTRAN 77: `mkl_vml.f77`
- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Output Parameters

Name	Type	Description
olderr	**FORTRAN:** `INTEGER` **C:** `int`	Specifies the former VML error status.

vmlSetErrorCallBack

Sets the additional error handler callback function and gets the old callback function.

Syntax

Fortran:

```
oldcallback = vmlseterrorcallback( callback )
```

C:

```
oldcallback = vmlSetErrorCallBack( callback );
```

Include Files

- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Input Parameters

Name		Description
FORTRAN: *callback*	Address of the callback function.	The callback function has the following format: `INTEGER FUNCTION ERRFUNC(par)` `TYPE (ERROR_STRUCTURE) par` `! ...` `! user error processing` `! ...` `ERRFUNC = 0` `! if ERRFUNC= 0 - standard VML error handler` `! is called after the callback` `! if ERRFUNC != 0 - standard VML error handler` `! is not called` `END` The passed error structure is defined as follows: `TYPE ERROR_STRUCTURE SEQUENCE` `INTEGER*4 ICODE` `INTEGER*4 IINDEX` `REAL*8 DBA1` `REAL*8 DBA2` `REAL*8 DBR1` `REAL*8 DBR2` `CHARACTER(64) CFUNCNAME` `INTEGER*4 IFUNCNAMELEN` `REAL*8 DBA1IM` `REAL*8 DBA2IM` `REAL*8 DBR1IM` `REAL*8 DBR2IM` `END TYPE ERROR_STRUCTURE`
C: *callback*	Pointer to the callback function.	The callback function has the following format: `static int __stdcall MyHandler(DefVmlErrorContext* pContext)` `{` `/* Handler body */` `};`

Name	Description
	The passed error structure is defined as follows: `typedef struct _DefVmlErrorContext` `{` `int iCode;/* Error status value */` `int iIndex;/* Index for bad array` `element, or bad array` `dimension, or bad` `array pointer */` `double dbA1; /* Error argument 1 */` `double dbA2; /* Error argument 2 */` `double dbR1; /* Error result 1 */` `double dbR2; /* Error result 2 */` `char cFuncName[64]; /* Function name */` `int iFuncNameLen; /* Length of functionname*/` `double dbA1Im; /* Error argument 1, imag part*/` `double dbA2Im; /* Error argument 2, imag part*/` `double dbR1Im; /* Error result 1, imag part*/` `double dbR2Im; /* Error result 2, imag part*/` `} DefVmlErrorContext;`

Output Parameters

Name	Type	Description
oldcallback	**Fortran 90:** `INTEGER` **C:** `int`	**FORTRAN:** Address of the former callback function. **C:** Pointer to the former callback function.

NOTE This function does not have a FORTRAN 77 interface due to the use of internal structures.

Description

The callback function is called on each VML mathematical function error if `VML_ERRMODE_CALLBACK` error mode is set (see "Values of the *mode* Parameter").

Use the `vmlSetErrorCallBack()` function if you need to define your own callback function instead of default empty callback function.

The input structure for a callback function contains the following information about the error encountered:

- the input value that caused an error
- location (array index) of this value
- the computed result value
- error code
- name of the function in which the error occurred.

You can insert your own error processing into the callback function. This may include correcting the passed result values in order to pass them back and resume computation. The standard error handler is called after the callback function only if it returns *0*.

vmlGetErrorCallBack

Gets the additional error handler callback function.

Syntax

Fortran:

```
callback = vmlgeterrorcallback( )
```

C:

```
callback = vmlGetErrorCallBack( void );
```

Include Files

- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Output Parameters

Name	Description
callback	**Fortran 90:** Address of the callback function **C:** Pointer to the callback function

vmlClearErrorCallBack

Deletes the additional error handler callback function and retrieves the former callback function.

Syntax

Fortran:

```
oldcallback = vmlclearerrorcallback( )
```

C:

```
oldcallback = vmlClearErrorCallBack( void );
```

Include Files

- Fortran 90: `mkl_vml.f90`
- C: `mkl_vml_functions.h`

Output Parameters

Name	Type	Description
oldcallback	**Fortran 90:** `INTEGER` **C:** `int`	**FORTRAN:** Address of the former callback function **C:** Pointer to the former callback function

10 Statistical Functions

Statistical functions in Intel® MKL are known as Vector Statistical Library (VSL) that is designed for the purpose of

- generating vectors of pseudorandom and quasi-random numbers
- performing mathematical operations of convolution and correlation
- computing basic statistical estimates for single and double precision multi-dimensional datasets

The corresponding functionality is described in the respective Random Number Generators, Convolution and Correlation, and VSL Summary Statistics sections.

See VSL performance data in the online VSL Performance Data document available at http://software.intel.com/en-us/articles/intel-math-kernel-library-documentation/

The basic notion in VSL is a task. The task object is a data structure or descriptor that holds the parameters related to a specific statistical operation: random number generation, convolution and correlation, or summary statistics estimation. Such parameters can be an identifier of a random number generator, its internal state and parameters, data arrays, their shape and dimensions, an identifier of the operation and so forth. You can modify the VSL task parameters using the respective service functionality of the library.

Random Number Generators

Intel MKL VSL provides a set of routines implementing commonly used pseudo- or quasi-random number generators with continuous and discrete distribution. To improve performance, all these routines were developed using the calls to the highly optimized *Basic Random Number Generators* (BRNGs) and the library of vector mathematical functions (VML, see Chapter 9, "Vector Mathematical Functions").

VSL provides interfaces both for Fortran and C languages. For users of the C and C++ languages the `mkl_vsl.h` header file is provided. For users of the Fortran 90 or Fortran 95 language the `mkl_vsl.f90` header file is provided. The `mkl_vsl.fi` header file available in the previous versions of Intel MKL is retained for backward compatibility. For users of the FORTRAN 77 language the `mkl_vsl.f77` header file is provided. All header files are found in the following directory:

```
${MKL}/include
```

The `mkl_vsl.f90` header is intended for using via the Fortran `include` clause and is compatible with both standard forms of F90/F95 sources - the free and 72-columns fixed forms. If you need to use the VSL interface with 80- or 132-columns fixed form sources, you may add a new file to your project. That file is formatted as a 72-columns fixed-form source and consists of a single `include` clause as follows:

```
include 'mkl_vsl.f90'
```

This `include` clause causes the compiler to generate the module files `mkl_vsl.mod` and `mkl_vsl_type.mod`, which are used to process the Fortran use clauses referencing to the VSL interface:

```
use mkl_vsl_type
```

```
use mkl_vsl
```

Because of this specific feature, you do not need to include the `mkl_vsl.f90` header into each source of your project. You only need to include the header into some of the sources. In any case, make sure that the sources that depend on the VSL interface are compiled after those that include the header so that the module files `mkl_vsl.mod` and `mkl_vsl_type.mod` are generated prior to using them.

The `mkl_vsl.f77` header is intended for using via the Fortran `include` clause as follows:

```
include 'mkl_vsl.f77'
```

NOTE For Fortran 90 interface, VSL provides both subroutine-style interface and function-style interface. Default interface in this case is a function-style interface. Function-style interface, unlike subroutine-style interface, allows the user to get error status of each routine. Subroutine-style interface is provided for backward compatibility only. To use subroutine-style interface, manually include `mkl_vsl_subroutine.fi` file instead of `mkl_vsl.f90` by changing the line `include 'mkl_vsl.f90'` in `include\mkl.fi` with the line `include 'mkl_vsl_subroutine.fi'`.

For the FORTRAN 77 interface, VSL provides only function-style interface.

All VSL routines can be classified into three major categories:

- Transformation routines for different types of statistical distributions, for example, uniform, normal (Gaussian), binomial, etc. These routines indirectly call basic random number generators, which are either pseudorandom number generators or quasi-random number generators. Detailed description of the generators can be found in Distribution Generators section.
- Service routines to handle random number streams: create, initialize, delete, copy, save to a binary file, load from a binary file, get the index of a basic generator. The description of these routines can be found in Service Routines section.
- Registration routines for basic pseudorandom generators and routines that obtain properties of the registered generators (see Advanced Service Routines section).

The last two categories are referred to as service routines.

Conventions

This document makes no specific differentiation between random, pseudorandom, and quasi-random numbers, nor between random, pseudorandom, and quasi-random number generators unless the context requires otherwise. For details, refer to the '*Random Numbers*' section in VSL Notes document provided at the Intel® MKL web page.

All generators of nonuniform distributions, both discrete and continuous, are built on the basis of the uniform distribution generators, called Basic Random Number Generators (BRNGs). The pseudorandom numbers with nonuniform distribution are obtained through an appropriate transformation of the uniformly distributed pseudorandom numbers. Such transformations are referred to as *generation methods*. For a given distribution, several generation methods can be used. See VSL Notes for the description of methods available for each generator.

An RNG task determines environment in which random number generation is performed, in particular parameters of the BRNG and its internal state. Output of VSL generators is a stream of random numbers that are used in Monte Carlo simulations. A *random stream descriptor* and a *random stream* are used as synonyms of an *RNG task* in the document unless the context requires otherwise.

The *random stream descriptor* specifies which BRNG should be used in a given transformation method. See the *Random Streams and RNGs in Parallel Computation* section of VSL Notes.

The term *computational node* means a logical or physical unit that can process data in parallel.

Mathematical Notation

The following notation is used throughout the text:

N	The set of natural numbers $N = \{1, 2, 3 \ldots\}$.
Z	The set of integers $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$.
R	The set of real numbers.
$\lfloor a \rfloor$	The floor of a (the largest integer less than or equal to a).
$\oplus$ or **xor**	Bitwise exclusive OR.
C_α^κ or $\binom{\alpha}{\kappa}$	Binomial coefficient or combination $(\alpha \in R,\ \alpha \geq 0;\ k \in N \cup \{0\})$.

$$C_\alpha^0 = 1$$

For $\alpha \geq k$ binomial coefficient is defined as

$$C_\alpha^\kappa = \frac{\alpha(\alpha - 1) \ldots (\alpha - \kappa + 1)}{\kappa!}$$

If $\alpha < k$, then

$$C_\alpha^\kappa = 0$$

$\Phi(x)$	Cumulative Gaussian distribution function

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy$$

defined over $-\infty < x < +\infty$.
$\Phi(-\infty) = 0$, $\Phi(+\infty) = 1$.

$\Gamma(\alpha)$	The complete gamma function

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt$$

where $\alpha > 0$.

$B(p, q)$	The complete beta function

$$B(p,q) = \int_0^1 t^{p-1}(1 - t)^{q-1} dt$$

where $p>0$ and $q>0$.

LCG(*a*,*c*, *m*)	Linear Congruential Generator $x_{n+1} = (ax_n + c) \bmod m$, where *a* is called the *multiplier*, *c* is called the *increment*, and *m* is called the *modulus* of the generator.
MCG(*a*,*m*)	Multiplicative Congruential Generator $x_{n+1} = (ax_n) \bmod m$ is a special case of Linear Congruential Generator, where the increment *c* is taken to be 0.
GFSR(*p*, *q*)	Generalized Feedback Shift Register Generator $x_n = x_{n-p} \oplus x_{n-q}$.

Naming Conventions

The names of the Fortran routines in VSL random number generators are lowercase (`virnguniform`). The names are not case-sensitive.

In C, the names of the routines, types, and constants are case-sensitive and can be lowercase and uppercase `viRngUniform`).

The names of generator routines have the following structure:

`v<type of result>rng<distribution>` for the Fortran interface

`v<type of result>Rng<distribution>` for the C interface,

where

- `v` is the prefix of a VSL vector function.
- <*type of result*> is either `s`, `d`, or `i` and specifies one of the following types:

`s`	`REAL` for the Fortran interface `float` for the C interface
`d`	`DOUBLE PRECISION` for the Fortran interface `double` for the C interface
`i`	`INTEGER` for the Fortran interface `int` for the C interface Prefixes `s` and `d` apply to continuous distributions only, prefix `i` applies only to discrete case.

- `rng` indicates that the routine is a random generator.
- <*distribution*> specifies the type of statistical distribution.

Names of service routines follow the template below:

```
vsl<name>
```

where

- `vsl` is the prefix of a VSL service function.
- <*name*> contains a short function name.

For a more detailed description of service routines, refer to Service Routines and Advanced Service Routines sections.

Prototype of each generator routine corresponding to a given probability distribution fits the following structure:

```
status = <function name>( method, stream, n, r, [<distribution parameters>] )
```

where

- *method* defines the method of generation. A detailed description of this parameter can be found in table "Values of <*method*> in *method* parameter". See the next page, where the structure of the *method* parameter name is explained.
- *stream* defines the descriptor of the random stream and must have a non-zero value. Random streams, descriptors, and their usage are discussed further in Random Streams and Service Routines.
- *n* defines the number of random values to be generated. If *n* is less than or equal to zero, no values are generated. Furthermore, if *n* is negative, an error condition is set.

- *r* defines the destination array for the generated numbers. The dimension of the array must be large enough to store at least *n* random numbers.
- *status* defines the error status of a VSL routine. See Error Reporting section for a detailed description of error status values.

Additional parameters included into *<distribution parameters>* field are individual for each generator routine and are described in detail in Distribution Generators section.

To invoke a distribution generator, use a call to the respective VSL routine. For example, to obtain a vector *r*, composed of *n* independent and identically distributed random numbers with normal (Gaussian) distribution, that have the mean value *a* and standard deviation *sigma*, write the following:

for the Fortran interface

```
status = vsrnggaussian( method, stream, n, r, a, sigma )
```

for the C interface

```
status = vsRngGaussian( method, stream, n, r, a, sigma )
```

The name of a *method* parameter has the following structure:

```
VSL_RNG_METHOD_method<distribution>_<method>
```

```
VSL_RNG_METHOD_<distribution>_<method>_ACCURATE
```

where

- *<distribution>* is the probability distribution.
- *<method>* is the method name.

Type of the name structure for the *method* parameter corresponds to fast and accurate modes of random number generation (see "Distribution Generators" section and VSL Notes for details).

Method names `VSL_RNG_METHOD_<distribution>_<method>`

and

`VSL_RNG_METHOD_<distribution>_<method>_ACCURATE`

should be used with

`v<precision>Rng<distribution>`

function only, where

- *<precision>* is

s	for single precision continuous distribution
d	for double precision continuous distribution
i	for discrete distribution

- *<distribution>* is the probability distribution.

is the probability distribution.Table "Values of *<method>* in *method* parameter" provides specific predefined values of the *method* name. The third column contains names of the functions that use the given method.

Values of *<method>* in *method* parameter

Method	Short Description	Functions
STD	Standard method. Currently there is only one method for these functions.	Uniform (continuous), Uniform (discrete), UniformBits, UniformBits32, UniformBits64

Method	Short Description	Functions
BOXMULLER	BOXMULLER generates normally distributed random number x thru the pair of uniformly distributed numbers u_1 and u_2 according to the formula: $x = \sqrt{-2 \ln u_1} \sin 2\pi u_2$	Gaussian, GaussianMV
BOXMULLER2	BOXMULLER2 generates normally distributed random numbers x_1 and x_2 thru the pair of uniformly distributed numbers u_1 and u_2 according to the formulas: $x_1 = \sqrt{-2 \ln u_1} \sin 2\pi u_2$ $x_2 = \sqrt{-2 \ln u_1} \cos 2\pi u_2$	Gaussian, GaussianMV, Lognormal
ICDF	Inverse cumulative distribution function method.	Exponential, Laplace, Weibull, Cauchy, Rayleigh, Gumbel, Bernoulli, Geometric, Gaussian, GaussianMV
GNORM	For $\alpha > 1$, a gamma distributed random number is generated as a cube of properly scaled normal random number; for $0.6 \le \alpha < 1$, a gamma distributed random number is generated using rejection from Weibull distribution; for $\alpha < 0.6$, a gamma distributed random number is obtained using transformation of exponential power distribution; for $\alpha = 1$, gamma distribution is reduced to exponential distribution.	Gamma
CJA	For $min(p, q) > 1$, Cheng method is used; for $min(p, q) < 1$, Johnk method is used, if $q + K \cdot p^2 + C \le 0$ (K = 0.852..., C=-0.956...) otherwise, Atkinson switching algorithm is used; for $max(p, q) < 1$, method of Johnk is used; for $min(p, q) < 1$, $max(p, q) > 1$, Atkinson switching algorithm is used (CJA stands for the first letters of Cheng, Johnk, Atkinson); for $p = 1$ or $q = 1$, inverse cumulative distribution function method is used;for $p = 1$ and $q = 1$, beta distribution is reduced to uniform distribution.	Beta
BTPE	Acceptance/rejection method for $ntrial \cdot \min(p, 1 - p) \ge 30$ with decomposition into 4 regions: – 2 parallelograms – triangle – left exponential tail – right exponential tail	Binomial

Method	Short Description	Functions
H2PE	Acceptance/rejection method for large mode of distribution with decomposition into 3 regions: – rectangular – left exponential tail – right exponential tail	Hypergeometric
PTPE	Acceptance/rejection method for $\lambda \geq 27$ with decomposition into 4 regions: – 2 parallelograms – triangle – left exponential tail – right exponential tail; otherwise, table lookup method is used.	Poisson
POISNORM	for $\lambda \geq 1$, method based on Poisson inverse CDF approximation by Gaussian inverse CDF; for $\lambda < 1$, table lookup method is used.	Poisson, PoissonV
NBAR	Acceptance/rejection method for , $\frac{(a-1)\cdot(1-p)}{p} \geq 100$ with decomposition into 5 regions: – rectangular – 2 trapezoid – left exponential tail – right exponential tail	NegBinomial

NOTE In this document, routines are often referred to by their base name (`Gaussian`) when this does not lead to ambiguity. In the routine reference, the full name (`vsrnggaussian`, `vsRngGaussian`) is always used in prototypes and code examples.

Basic Generators

VSL provides the following BRNGs, which differ in speed and other properties:

- the 32-bit multiplicative congruential pseudorandom number generator *MCG(1132489760, 2^{31} -1)* [L'Ecuyer99]
- the 32-bit generalized feedback shift register pseudorandom number generator *GFSR(250,103)* [Kirkpatrick81]
- the combined multiple recursive pseudorandom number generator *MRG-32k3a* [L'Ecuyer99a]
- the 59-bit multiplicative congruential pseudorandom number generator *MCG(13^{13}, 2^{59})* from NAG Numerical Libraries [NAG]
- Wichmann-Hill pseudorandom number generator (a set of 273 basic generators) from NAG Numerical Libraries [NAG]
- Mersenne Twister pseudorandom number generator *MT19937* [Matsumoto98] with period length 2^{19937}-1 of the produced sequence

- Set of 6024 Mersenne Twister pseudorandom number generators *MT2203* [Matsumoto98], [Matsumoto00]. Each of them generates a sequence of period length equal to 2^{2203}-1. Parameters of the generators provide mutual independence of the corresponding sequences.
- SIMD-oriented Fast Mersenne Twister pseudorandom number generator *SFMT19937* [Saito08] with a period length equal to 2^{19937}-1 of the produced sequence.

Besides these pseudorandom number generators, VSL provides two basic quasi-random number generators:

- Sobol quasi-random number generator [Sobol76], [Bratley88], which works in arbitrary dimension. For dimensions greater than 40 the user should supply initialization parameters (initial direction numbers and primitive polynomials or direction numbers) by using `vslNewStreamEx` function. See additional details on interface for registration of the parameters in the library in VSL Notes.
- Niederreiter quasi-random number generator [Bratley92], which works in arbitrary dimension. For dimensions greater than 318 the user should supply initialization parameters (irreducible polynomials or direction numbers) by using `vslNewStreamEx` function. See additional details on interface for registration of the parameters in the library in VSL Notes.

See some testing results for the generators in VSL Notes and comparative performance data at http://software.intel.com/sites/products/documentation/hpc/mkl/vsl/vsl_data/vsl_performance_data.htm.

VSL provides means of registration of such user-designed generators through the steps described in Advanced Service Routines section.

For some basic generators, VSL provides two methods of creating independent random streams in multiprocessor computations, which are the leapfrog method and the block-splitting method. These sequence splitting methods are also useful in sequential Monte Carlo.

In addition, MT2203 pseudorandom number generator is a set of 6024 generators designed to create up to 6024 independent random sequences, which might be used in parallel Monte Carlo simulations. Another generator that has the same feature is Wichmann-Hill. It allows creating up to 273 independent random streams. The properties of the generators designed for parallel computations are discussed in detail in [Coddington94].

You may want to design and use your own basic generators. VSL provides means of registration of such user-designed generators through the steps described in Advanced Service Routines section.

There is also an option to utilize externally generated random numbers in VSL distribution generator routines. For this purpose VSL provides three additional basic random number generators:

- for external random data packed in 32-bit integer array
- for external random data stored in double precision floating-point array; data is supposed to be uniformly distributed over (`a`,`b`) interval
- for external random data stored in single precision floating-point array; data is supposed to be uniformly distributed over (`a`,`b`) interval.

Such basic generators are called the abstract basic random number generators.

See VSL Notes for a more detailed description of the generator properties.

BRNG Parameter Definition

Predefined values for the *brng* input parameter are as follows:

Values of *brng* parameter

Value	Short Description
VSL_BRNG_MCG31	A 31-bit multiplicative congruential generator.
VSL_BRNG_R250	A generalized feedback shift register generator.
VSL_BRNG_MRG32K3A	A combined multiple recursive generator with two components of order 3.
VSL_BRNG_MCG59	A 59-bit multiplicative congruential generator.
VSL_BRNG_WH	A set of 273 Wichmann-Hill combined multiplicative congruential generators.
VSL_BRNG_MT19937	A Mersenne Twister pseudorandom number generator.
VSL_BRNG_MT2203	A set of 6024 Mersenne Twister pseudorandom number generators.
VSL_BRNG_SFMT19937	A SIMD-oriented Fast Mersenne Twister pseudorandom number generator.
VSL_BRNG_SOBOL	A 32-bit Gray code-based generator producing low-discrepancy sequences for dimensions $1 \le s \le 40$; user-defined dimensions are also available.
VSL_BRNG_NIEDERR	A 32-bit Gray code-based generator producing low-discrepancy sequences for dimensions $1 \le s \le 318$; user-defined dimensions are also available.
VSL_BRNG_IABSTRACT	An abstract random number generator for integer arrays.
VSL_BRNG_DABSTRACT	An abstract random number generator for double precision floating-point arrays.
VSL_BRNG_SABSTRACT	An abstract random number generator for single precision floating-point arrays.

See VSL Notes for detailed description.

Random Streams

Random stream (or *stream*) is an abstract source of pseudo- and quasi-random sequences of uniform distribution. You can operate with stream state descriptors only. A stream state descriptor, which holds state descriptive information for a particular BRNG, is a necessary parameter in each routine of a distribution generator. Only the distribution generator routines operate with random streams directly. See VSL Notes for details.

NOTE Random streams associated with abstract basic random number generator are called the abstract random streams. See VSL Notes for detailed description of abstract streams and their use.

You can create unlimited number of random streams by VSL Service Routines like NewStream and utilize them in any distribution generator to get the sequence of numbers of given probability distribution. When they are no longer needed, the streams should be deleted calling service routine DeleteStream.

VSL provides service functions SaveStreamF and LoadStreamF to save random stream descriptive data to a binary file and to read this data from a binary file respectively. See VSL Notes for detailed description.

Data Types

FORTRAN 77:

```
INTEGER*4 vslstreamstate(2)
```

Fortran 90:

```
TYPE
        VSL_STREAM_STATE

INTEGER*4 descriptor1

INTEGER*4 descriptor2

END
        TYPE VSL_STREAM_STATE
```

C:

```
typedef (void*) VSLStreamStatePtr;
```

See Advanced Service Routines for the format of the stream state structure for user-designed generators.

Error Reporting

VSL RNG routines return status codes of the performed operation to report errors to the calling program. The application should perform error-related actions and/or recover from the error. The status codes are of integer type and have the following format:

VSL_ERROR_<*ERROR_NAME*> - indicates VSL errors common for all VSL domains.

VSL_RNG_ERROR_<*ERROR_NAME*> - indicates VSL RNG errors.

VSL RNG errors are of negative values while warnings are of positive values. The status code of zero value indicates successful completion of the operation: VSL_ERROR_OK (or synonymic VSL_STATUS_OK).

Status Codes

Status Code	Description
Common VSL	
VSL_ERROR_OK, VSL_STATUS_OK	No error, execution is successful.
VSL_ERROR_BADARGS	Input argument value is not valid.
VSL_ERROR_CPU_NOT_SUPPORTED	CPU version is not supported.
VSL_ERROR_FEATURE_NOT_IMPLEMENTED	Feature invoked is not implemented.
VSL_ERROR_MEM_FAILURE	System cannot allocate memory.
VSL_ERROR_NULL_PTR	Input pointer argument is NULL.
VSL_ERROR_UNKNOWN	Unknown error.
VSL RNG Specific	
VSL_RNG_ERROR_BAD_FILE_FORMAT	File format is unknown.

Status Code	Description
VSL_RNG_ERROR_BAD_MEM_FORMAT	Descriptive random stream format is unknown.
VSL_RNG_ERROR_BAD_NBITS	The value in NBits field is bad.
VSL_RNG_ERROR_BAD_NSEEDS	The value in NSeeds field is bad.
VSL_RNG_ERROR_BAD_STREAM	The random stream is invalid.
VSL_RNG_ERROR_BAD_STREAM_STATE_SIZE	The value in StreamStateSize field is bad.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or >nmax.
VSL_RNG_ERROR_BAD_WORD_SIZE	The value in WordSize field is bad.
VSL_RNG_ERROR_BRNG_NOT_SUPPORTED	BRNG is not supported by the function.
VSL_RNG_ERROR_BRNG_TABLE_FULL	Registration cannot be completed due to lack of free entries in the table of registered BRNGs.
VSL_RNG_ERROR_BRNGS_INCOMPATIBLE	Two BRNGs are not compatible for the operation.
VSL_RNG_ERROR_FILE_CLOSE	Error in closing the file.
VSL_RNG_ERROR_FILE_OPEN	Error in opening the file.
VSL_RNG_ERROR_FILE_READ	Error in reading the file.
VSL_RNG_ERROR_FILE_WRITE	Error in writing the file.
VSL_RNG_ERROR_INVALID_ABSTRACT_STREAM	The abstract random stream is invalid.
VSL_RNG_ERROR_INVALID_BRNG_INDEX	BRNG index is not valid.
VSL_RNG_ERROR_LEAPFROG_UNSUPPORTED	BRNG does not support Leapfrog method.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns zero as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator is exceeded.
VSL_RNG_ERROR_SKIPAHEAD_UNSUPPORTED	BRNG does not support Skip-Ahead method.
VSL_RNG_ERROR_UNSUPPORTED_FILE_VER	File format version is not supported.

VSL RNG Usage Model

A typical algorithm for VSL random number generators is as follows:

1. Create and initialize stream/streams. Functions vslNewStream, vslNewStreamEx, vslCopyStream, vslCopyStreamState, vslLeapfrogStream, vslSkipAheadStream.
2. Call one or more RNGs.
3. Process the output.
4. Delete the stream/streams. Function vslDeleteStream.

NOTE You may reiterate steps 2-3. Random number streams may be generated for different threads.

The following C example demonstrates generation of a random stream that is output of basic generator MT19937. The seed is equal to 777. The stream is used to generate 10,000 normally distributed random numbers in blocks of 1,000 random numbers with parameters *a* = 5 and *sigma* = 2. Delete the streams after completing the generation. The purpose of the example is to calculate the sample mean for normal distribution with the given parameters.

C Example of VSL RNG Usage

```
#include <stdio.h>
#include "mkl_vsl.h"

int main()
{
   double r[1000]; /* buffer for random numbers */
   double s; /* average */
   VSLStreamStatePtr stream;
   int i, j;

   /* Initializing */
   s = 0.0;
   vslNewStream( &stream, VSL_BRNG_MT19937, 777 );

   /* Generating */
   for ( i=0; i<10; i++ );
   {
      vdRngGaussian( VSL_RNG_METHOD_GAUSSIAN_ICDF, stream, 1000, r, 5.0, 2.0 );
      for ( j=0; j<1000; j++ );
      {
         s += r[j];
      }
   }
   s /= 10000.0;

   /* Deleting the stream */
   vslDeleteStream( &stream );

   /* Printing results */
   printf( "Sample mean of normal distribution = %f\n", s );

   return 0;
}
```

The Fortran version of the same example is below:

Fortran Example of VSL RNG Usage

```
      include 'mkl_vsl.f90'

      program MKL_VSL_GAUSSIAN

      USE MKL_VSL_TYPE
      USE MKL_VSL

      real(kind=8) r(1000)  ! buffer for random numbers
      real(kind=8) s        ! average
      real(kind=8) a, sigma ! parameters of normal distribution

      TYPE (VSL_STREAM_STATE) :: stream

      integer(kind=4) errcode
      integer(kind=4) i,j
      integer brng,method,seed,n

      n = 1000
      s = 0.0
      a = 5.0
```

```
      sigma  = 2.0
      brng=VSL_BRNG_MT19937
      method=VSL_RNG_METHOD_GAUSSIAN_ICDF
      seed=777

!     ***** Initializing *****
      errcode=vslnewstream( stream, brng,  seed )

!     ***** Generating *****
      do i = 1,10
          errcode=vdrnggaussian( method, stream, n, r, a, sigma )
          do j = 1, 1000
              s = s + r(j)
          end do
      end do

      s = s / 10000.0

!     ***** Deinitialize *****
      errcode=vsldeletestream( stream )

!     ***** Printing results *****
      print *,"Sample mean of normal distribution = ", s

      end
```

Additionally, examples that demonstrate usage of VSL random number generators are available in the following directories:

`${MKL}/examples/vslc/source`

`${MKL}/examples/vslf/source`

Service Routines

Stream handling comprises routines for creating, deleting, or copying the streams and getting the index of a basic generator. A random stream can also be saved to and then read from a binary file. Table "Service Routines" lists all available service routines

Service Routines

Routine	Short Description
vslNewStream	Creates and initializes a random stream.
vslNewStreamEx	Creates and initializes a random stream for the generators with multiple initial conditions.
vsliNewAbstractStream	Creates and initializes an abstract random stream for integer arrays.
vsldNewAbstractStream	Creates and initializes an abstract random stream for double precision floating-point arrays.
vslsNewAbstractStream	Creates and initializes an abstract random stream for single precision floating-point arrays.
vslDeleteStream	Deletes previously created stream.
vslCopyStream	Copies a stream to another stream.
vslCopyStreamState	Creates a copy of a random stream state.
vslSaveStreamF	Writes a stream to a binary file.
vslLoadStreamF	Reads a stream from a binary file.

Routine	Short Description
vslSaveStreamM	Writes a random stream descriptive data, including state, to a memory buffer.
vslLoadStreamM	Creates a new stream and reads stream descriptive data, including state, from the memory buffer.
vslGetStreamSize	Computes size of memory necessary to hold the random stream.
vslLeapfrogStream	Initializes the stream by the leapfrog method to generate a subsequence of the original sequence.
vslSkipAheadStream	Initializes the stream by the skip-ahead method.
vslGetStreamStateBrng	Obtains the index of the basic generator responsible for the generation of a given random stream.
vslGetNumRegBrngs	Obtains the number of currently registered basic generators.

Most of the generator-based work comprises three basic steps:

1. Creating and initializing a stream (vslNewStream, vslNewStreamEx, vslCopyStream, vslCopyStreamState, vslLeapfrogStream, vslSkipAheadStream).
2. Generating random numbers with given distribution, see Distribution Generators.
3. Deleting the stream (vslDeleteStream).

Note that you can concurrently create multiple streams and obtain random data from one or several generators by using the stream state. You must use the vslDeleteStream function to delete all the streams afterwards.

vslNewStream

Creates and initializes a random stream.

Syntax

Fortran:

```
status = vslnewstream( stream, brng, seed )
```

C:

```
status = vslNewStream( &stream, brng, seed );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
brng	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Index of the basic generator to initialize the stream. See Table Values of *brng* parameter for specific value.

Name	Type	Description
seed	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const unsigned int`	Initial condition of the stream. In the case of a quasi-random number generator seed parameter is used to set the dimension. If the dimension is greater than the dimension that brng can support or is less than 1, then the dimension is assumed to be equal to 1.

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr*`	Stream state descriptor

Description

For a basic generator with number *brng*, this function creates a new stream and initializes it with a 32-bit seed. The seed is an initial value used to select a particular sequence generated by the basic generator *brng*. The function is also applicable for generators with multiple initial conditions. See VSL Notes for a more detailed description of stream initialization for different basic generators.

NOTE This function is not applicable for abstract basic random number generators. Please use `vsliNewAbstractStream`, `vslsNewAbstractStream` or `vsldNewAbstractStream` to utilize integer, single-precision or double-precision external random data respectively.

Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_RNG_ERROR_INVALID_BRNG_INDEX`	BRNG index is invalid.
`VSL_ERROR_MEM_FAILURE`	System cannot allocate memory for *stream*.

vslNewStreamEx

Creates and initializes a random stream for generators with multiple initial conditions.

Syntax

Fortran:

```
status = vslnewstreamex( stream, brng, n, params )
```

C:

```
status = vslNewStreamEx( &stream, brng, n, params );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
brng	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Index of the basic generator to initialize the stream. See Table "Values of *brng* parameter" for specific value.
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of initial conditions contained in *params*
params	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER(KIND=4), INTENT(IN) **C:** const unsigned int	Array of initial conditions necessary for the basic generator *brng* to initialize the stream. In the case of a quasi-random number generator only the first element in *params* parameter is used to set the dimension. If the dimension is greater than the dimension that *brng* can support or is less than 1, then the dimension is assumed to be equal to 1.

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE(VSL_STREAM_STATE), INTENT(OUT) **C:** VSLStreamStatePtr*	Stream state descriptor

Description

The vslNewStreamEx function provides an advanced tool to set the initial conditions for a basic generator if its input arguments imply several initialization parameters. Initial values are used to select a particular sequence generated by the basic generator *brng*. Whenever possible, use vslNewStream, which is analogous to vslNewStreamEx except that it takes only one 32-bit initial condition. In particular, vslNewStreamEx may be used to initialize the state table in Generalized Feedback Shift Register Generators (GFSRs). A more detailed description of this issue can be found in VSL Notes.

This function is also used to pass user-defined initialization parameters of quasi-random number generators into the library. See VSL Notes for the format for their passing and registration in VSL.

NOTE This function is not applicable for abstract basic random number generators. Please use vsliNewAbstractStream, vslsNewAbstractStream or vsldNewAbstractStream to utilize integer, single-precision or double-precision external random data respectively.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_RNG_ERROR_INVALID_BRNG_INDEX	BRNG index is invalid.

VSL_ERROR_MEM_FAILURE	System cannot allocate memory for *stream*.

vsliNewAbstractStream

Creates and initializes an abstract random stream for integer arrays.

Syntax

Fortran:

```
status = vslinewabstractstream( stream, n, ibuf, icallback )
```

C:

```
status = vsliNewAbstractStream( &stream, n, ibuf, icallback );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Size of the array *ibuf*
ibuf	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER(KIND=4), INTENT(IN) **C:** const unsigned int	Array of *n* 32-bit integers
icallback	See *Note* below	**Fortran:** Address of the callback function used for *ibuf* update **C:** Pointer to the callback function used for *ibuf* update

NOTE Format of the callback function in FORTRAN 77:

```
INTEGER FUNCTION IUPDATEFUNC( stream, n, ibuf, nmin, nmax, idx )
INTEGER*4 stream(2)
INTEGER n
INTEGER*4 ibuf(n)
INTEGER nmin
INTEGER nmax
INTEGER idx
```

Format of the callback function in Fortran 90:

```
INTEGER FUNCTION IUPDATEFUNC[C]( stream, n, ibuf, nmin, nmax, idx )
TYPE(VSL_STREAM_STATE),POINTER :: stream[reference]
INTEGER(KIND=4),INTENT(IN)     :: n[reference]
INTEGER(KIND=4),INTENT(OUT)    :: ibuf[reference](0:n-1)
INTEGER(KIND=4),INTENT(IN)     :: nmin[reference]
INTEGER(KIND=4),INTENT(IN)     :: nmax[reference]
INTEGER(KIND=4),INTENT(IN)     :: idx[reference]
```

Format of the callback function in C:

```
int iUpdateFunc( VSLStreamStatePtr stream, int* n, unsigned int ibuf[], int* nmin, int* nmax,
int* idx );
```

The callback function returns the number of elements in the array actually updated by the function. Table `icallback` Callback Function Parameters gives the description of the callback function parameters.

icallback Callback Function Parameters

Parameters	Short Description
stream	Abstract random stream descriptor
n	Size of *ibuf*
ibuf	Array of random numbers associated with the stream *stream*
nmin	Minimal quantity of numbers to update
nmax	Maximal quantity of numbers that can be updated
idx	Position in cyclic buffer *ibuf* to start update $0 \le idx < n$.

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), TINTENT(OUT)` **C:** `VSLStreamStatePtr*`	Descriptor of the stream state structure

Description

The `vsliNewAbstractStream` function creates a new abstract stream and associates it with an integer array *ibuf* and your callback function *icallback* that is intended for updating of *ibuf* content.

Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_BADARGS`	Parameter *n* is not positive.
`VSL_ERROR_MEM_FAILURE`	System cannot allocate memory for *stream*.
`VSL_ERROR_NULL_PTR`	Either buffer or callback function parameter is a `NULL` pointer.

vsldNewAbstractStream

Creates and initializes an abstract random stream for double precision floating-point arrays.

Syntax

Fortran:

```
status = vsldnewabstractstream( stream, n, dbuf, a, b, dcallback )
```

C:

```
status = vsldNewAbstractStream( &stream, n, dbuf, a, b, dcallback );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Size of the array *dbuf*
dbuf	**FORTRAN 77:** `DOUBLE PRECISION` **Fortran 90:** `REAL(KIND=8), INTENT(IN)` **C:** `const double`	Array of *n* double precision floating-point random numbers with uniform distribution over interval (`a`,`b`)
a	**FORTRAN 77:** `DOUBLE PRECISION` **Fortran 90:** `REAL(KIND=8), INTENT(IN)` **C:** `const double`	Left boundary `a`
b	**FORTRAN 77:** `DOUBLE PRECISION` **Fortran 90:** `REAL(KIND=8), INTENT(IN)` **C:** `const double`	Right boundary `b`
dcallback	See *Note* below	**Fortran:** Address of the callback function used for update of the array *dbuf* **C:** Pointer to the callback function used for update of the array *dbuf*

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr*`	Descriptor of the stream state structure

NOTE Format of the callback function in FORTRAN 77:

```
INTEGER FUNCTION DUPDATEFUNC( stream, n, dbuf, nmin, nmax, idx )
INTEGER*4 stream(2)
INTEGER n
DOUBLE PRECISION dbuf(n)
INTEGER nmin
INTEGER nmax
INTEGER idx
```

Format of the callback function in Fortran 90:

```
INTEGER FUNCTION DUPDATEFUNC[C]( stream, n, dbuf, nmin, nmax, idx )
TYPE(VSL_STREAM_STATE),POINTER   :: stream[reference]
INTEGER(KIND=4),INTENT(IN)       :: n[reference]
REAL(KIND=8),INTENT(OUT)         :: dbuf[reference](0:n-1)
INTEGER(KIND=4),INTENT(IN)       :: nmin[reference]
INTEGER(KIND=4),INTENT(IN)       :: nmax[reference]
INTEGER(KIND=4),INTENT(IN)       :: idx[reference]
```

Format of the callback function in C:

```
int dUpdateFunc( VSLStreamStatePtr stream, int* n, double dbuf[], int* nmin, int* nmax, int*
idx );
```

The callback function returns the number of elements in the array actually updated by the function. Table `dcallback` Callback Function Parameters gives the description of the callback function parameters.

dcallback Callback Function Parameters

Parameters	Short Description
stream	Abstract random stream descriptor
n	Size of *dbuf*
dbuf	Array of random numbers associated with the stream *stream*
nmin	Minimal quantity of numbers to update
nmax	Maximal quantity of numbers that can be updated
idx	Position in cyclic buffer *dbuf* to start update $0 \le idx < n$.

Description

The vsldNewAbstractStream function creates a new abstract stream for double precision floating-point arrays with random numbers of the uniform distribution over interval (a,b). The function associates the stream with a double precision array *dbuf* and your callback function *dcallback* that is intended for updating of *dbuf* content.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_BADARGS	Parameter *n* is not positive.
VSL_ERROR_MEM_FAILURE	System cannot allocate memory for *stream*.
VSL_ERROR_NULL_PTR	Either buffer or callback function parameter is a NULL pointer.

vslsNewAbstractStream

Creates and initializes an abstract random stream for single precision floating-point arrays.

Syntax

Fortran:

```
status = vslsnewabstractstream( stream, n, sbuf, a, b, scallback )
```

C:

```
status = vslsNewAbstractStream( &stream, n, sbuf, a, b, scallback );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Size of the array *sbuf*
sbuf	**FORTRAN 77:** REAL **Fortran 90:** REAL(KIND=4), INTENT(IN) **C:** const float	Array of *n* single precision floating-point random numbers with uniform distribution over interval (a,b)
a	**FORTRAN 77:** REAL **Fortran 90:** REAL(KIND=4), INTENT(IN) **C:** const float	Left boundary a

Name	Type	Description
b	**FORTRAN 77:** `REAL` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` **C:** `const float`	Right boundary `b`
scallback	See *Note* below	**Fortran:** Address of the callback function used for update of the array *sbuf* **C:** Pointer to the callback function used for update of the array *sbuf*

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr*`	Descriptor of the stream state structure

NOTE Format of the callback function in FORTRAN 77:

```
INTEGER FUNCTION SUPDATEFUNC( stream, n, ibuf, nmin, nmax, idx )
INTEGER*4 stream(2)
INTEGER n
REAL sbuf(n)
INTEGER nmin
INTEGER nmax
INTEGER idx
```

Format of the callback function in Fortran 90:

```
INTEGER FUNCTION SUPDATEFUNC[C]( stream, n, sbuf, nmin, nmax, idx )
TYPE(VSL_STREAM_STATE),POINTER :: stream[reference]
INTEGER(KIND=4),INTENT(IN)     :: n[reference]
REAL(KIND=4),   INTENT(OUT)    :: sbuf[reference](0:n-1)
INTEGER(KIND=4),INTENT(IN)     :: nmin[reference]
INTEGER(KIND=4),INTENT(IN)     :: nmax[reference]
INTEGER(KIND=4),INTENT(IN)     :: idx[reference]
```

Format of the callback function in C:

```
int sUpdateFunc( VSLStreamStatePtr stream, int* n, float sbuf[], int* nmin, int* nmax, int* idx );
```

The callback function returns the number of elements in the array actually updated by the function. Table `scallback` Callback Function Parameters gives the description of the callback function parameters.

scallback Callback Function Parameters

Parameters	Short Description
stream	Abstract random stream descriptor
n	Size of *sbuf*
sbuf	Array of random numbers associated with the stream *stream*
nmin	Minimal quantity of numbers to update
nmax	Maximal quantity of numbers that can be updated
idx	Position in cyclic buffer *sbuf* to start update $0 \le idx < n$.

Description

The vslsNewAbstractStream function creates a new abstract stream for single precision floating-point arrays with random numbers of the uniform distribution over interval (a,b). The function associates the stream with a single precision array *sbuf* and your callback function *scallback* that is intended for updating of *sbuf* content.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_BADARGS	Parameter *n* is not positive.
VSL_ERROR_MEM_FAILURE	System cannot allocate memory for *stream*.
VSL_ERROR_NULL_PTR	Either buffer or callback function parameter is a NULL pointer.

vslDeleteStream

Deletes a random stream.

Syntax

Fortran:

```
status = vsldeletestream( stream )
```

C:

```
status = vslDeleteStream( &stream );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input/Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** INTEGER*4 stream(2)	**Fortran:** Stream state descriptor. Must have non-zero value. After the stream is successfully deleted, the descriptor becomes invalid.

Name	Type	Description
	Fortran 90: `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr*`	**C:** Stream state descriptor. Must have non-zero value. After the stream is successfully deleted, the pointer is set to `NULL`.

Description

The function deletes the random stream created by one of the initialization functions.

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*`stream`* parameter is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*`stream`* is not a valid random stream.

vslCopyStream

Creates a copy of a random stream.

Syntax

Fortran:

```
status = vslcopystream( newstream, srcstream )
```

C:

```
status = vslCopyStream( &newstream, srcstream );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
`srcstream`	**FORTRAN 77:** `INTEGER*4 srcstream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(IN)` **C:** `const VSLStreamStatePtr`	**Fortran:** Descriptor of the stream to be copied **C:** Pointer to the stream state structure to be copied

Output Parameters

Name	Type	Description
`newstream`	**FORTRAN 77:** `INTEGER*4 newstream(2)`	Copied random stream descriptor

Name	Type	Description
	Fortran 90: `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr*`	

Description

The function creates an exact copy of *srcstream* and stores its descriptor to *newstream*.

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*srcstream* parameter is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*srcstream* is not a valid random stream.
`VSL_ERROR_MEM_FAILURE`	System cannot allocate memory for *newstream*.

vslCopyStreamState

Creates a copy of a random stream state.

Syntax

Fortran:

```
status = vslcopystreamstate( deststream, srcstream )
```

C:

```
status = vslCopyStreamState( deststream, srcstream );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
srcstream	**FORTRAN 77:** `INTEGER*4 srcstream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(IN)` **C:** `const VSLStreamStatePtr`	**Fortran:** Descriptor of the destination stream where the state of *scrstream* stream is copied **C:** Pointer to the stream state structure, from which the state structure is copied

Output Parameters

Name	Type	Description
deststream	**FORTRAN 77:** `INTEGER*4 deststream(2)`	**Fortran:** Descriptor of the stream with the state to be copied

Name	Type	Description
	Fortran 90: `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr`	**C:** Pointer to the stream state structure where the stream state is copied

Description

The `vslCopyStreamState` function copies a stream state from *srcstream* to the existing *deststream* stream. Both the streams should be generated by the same basic generator. An error message is generated when the index of the BRNG that produced *deststream* stream differs from the index of the BRNG that generated *srcstream* stream.

Unlike `vslCopyStream` function, which creates a new stream and copies both the stream state and other data from *srcstream*, the function `vslCopyStreamState` copies only *srcstream* stream state data to the generated *deststream* stream.

Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	Either *srcstream* or *deststream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	Either *srcstream* or *deststream* is not a valid random stream.
`VSL_RNG_ERROR_BRNGS_INCOMPATIBLE`	BRNG associated with *srcstream* is not compatible with BRNG associated with *deststream*.

vslSaveStreamF

Writes random stream descriptive data, including stream state, to binary file.

Syntax

Fortran:

```
errstatus = vslsavestreamf( stream, fname )
```

C:

```
errstatus = vslSaveStreamF( stream, fname );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(IN)`	Random stream to be written to the file

Name	Type	Description
	C: `const VSLStreamStatePtr`	
fname	**FORTRAN 77:** `CHARACTER(*)` **Fortran 90:** `CHARACTER(*), INTENT(IN)` **C:** `const char*`	**Fortran:** File name specified as a C-style null-terminated string **C:** File name specified as a Fortran-style character string

Output Parameters

Name	Type	Description
errstatus	**Fortran:** `INTEGER` **C:** `int`	Error status of the operation

Description

The `vslSaveStreamF` function writes the random stream descriptive data, including the stream state, to the binary file. Random stream descriptive data is saved to the binary file with the name *fname*. The random stream *stream* must be a valid stream created by `vslNewStream`-like or `vslCopyStream`-like service routines. If the stream cannot be saved to the file, *errstatus* has a non-zero value. The random stream can be read from the binary file using the `vslLoadStreamF` function.

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	Either *fname* or *stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is not a valid random stream.
`VSL_RNG_ERROR_FILE_OPEN`	Indicates an error in opening the file.
`VSL_RNG_ERROR_FILE_WRITE`	Indicates an error in writing the file.
`VSL_RNG_ERROR_FILE_CLOSE`	Indicates an error in closing the file.
`VSL_ERROR_MEM_FAILURE`	System cannot allocate memory for internal needs.

vslLoadStreamF

Creates new stream and reads stream descriptive data, including stream state, from binary file.

Syntax

Fortran:

```
errstatus = vslloadstreamf( stream, fname )
```

C:

```
errstatus = vslLoadStreamF( &stream, fname );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
fname	**FORTRAN 77:** CHARACTER(*) **Fortran 90:** CHARACTER(*), INTENT(IN) **C:** const char*	**Fortran:** File name specified as a C-style null-terminated string **C:** File name specified as a Fortran-style character string

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE(VSL_STREAM_STATE), INTENT(OUT) **C:** VSLStreamStatePtr*	**Fortran:** Descriptor of a new random stream **C:** Pointer to a new random stream
errstatus	**Fortran:** INTEGER **C:** int	Error status of the operation

Description

The vslLoadStreamF function creates a new stream and reads stream descriptive data, including the stream state, from the binary file. A new random stream is created using the stream descriptive data from the binary file with the name *fname*. If the stream cannot be read (for example, an I/O error occurs or the file format is invalid), *errstatus* has a non-zero value. To save random stream to the file, use vslSaveStreamF function.

CAUTION Calling vslLoadStreamF with a previously initialized *stream* pointer can have unintended consequences such as a memory leak. To initialize a stream which has been in use until calling vslLoadStreamF, you should call the vslDeleteStream function first to deallocate the resources.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*fname* is a NULL pointer.
VSL_RNG_ERROR_FILE_OPEN	Indicates an error in opening the file.
VSL_RNG_ERROR_FILE_WRITE	Indicates an error in writing the file.
VSL_RNG_ERROR_FILE_CLOSE	Indicates an error in closing the file.
VSL_ERROR_MEM_FAILURE	System cannot allocate memory for internal needs.
VSL_RNG_ERROR_BAD_FILE_FORMAT	Unknown file format.
VSL_RNG_ERROR_UNSUPPORTED_FILE_VER	File format version is unsupported.

vslSaveStreamM

Writes random stream descriptive data, including stream state, to a memory buffer.

Syntax

Fortran:

```
errstatus = vslsavestreamm( stream, memptr )
```

C:

```
errstatus = vslSaveStreamM( stream, memptr );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(IN)` **C:** `const VSLStreamStatePtr`	Random stream to be written to the memory
memptr	**FORTRAN 77:** `INTEGER*1` **Fortran 90:** `INTEGER(KIND=1), DIMENSION(*), INTENT(IN)` **C:** `char*`	**Fortran:** Memory buffer to save random stream descriptive data to **C:** Memory buffer to save random stream descriptive data to

Output Parameters

Name	Type	Description
errstatus	**Fortran:** `INTEGER` **C:** `int`	Error status of the operation

Description

The `vslSaveStreamM` function writes the random stream descriptive data, including the stream state, to the memory at *memptr*. Random stream *stream* must be a valid stream created by `vslNewStream`-like or `vslCopyStream`-like service routines. The *memptr* parameter must be a valid pointer to the memory of size sufficient to hold the random stream *stream*. Use the service routine `vslGetStreamSize` to determine this amount of memory.

If the stream cannot be saved to the memory, *errstatus* has a non-zero value. The random stream can be read from the memory pointed by *memptr* using the `vslLoadStreamM` function.

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	Either *memptr* or *stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is a `NULL` pointer.
`VSL_ERROR_BAD_STREAM`	*stream* is not a valid random stream.

vslLoadStreamM

Creates a new stream and reads stream descriptive data, including stream state, from the memory buffer.

Syntax

Fortran:

```
errstatus = vslloadstreamm( stream, memptr )
```

C:

```
errstatus = vslLoadStreamM( &stream, memptr );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
memptr	**FORTRAN 77:** `INTEGER*1` **Fortran 90:** `INTEGER(KIND=1), DIMENSION(*), INTENT(IN)` **C:** `const char*`	**Fortran:** Memory buffer to load random stream descriptive data from **C:** Memory buffer to load random stream descriptive data from

Output Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(OUT)` **C:** `VSLStreamStatePtr*`	**Fortran:** Descriptor of a new random stream **C:** Pointer to a new random stream
errstatus	**Fortran:** `INTEGER` **C:** `int`	Error status of the operation

Description

The `vslLoadStreamM` function creates a new stream and reads stream descriptive data, including the stream state, from the memory buffer. A new random stream is created using the stream descriptive data from the memory pointer by *memptr*. If the stream cannot be read (for example, *memptr* is invalid), *errstatus* has a non-zero value. To save random stream to the memory, use `vslSaveStreamM` function. Use the service routine `vslGetStreamSize` to determine the amount of memory sufficient to hold the random stream.

CAUTION Calling `LoadStreamM` with a previously initialized *stream* pointer can have unintended consequences such as a memory leak. To initialize a stream which has been in use until calling `vslLoadStreamM`, you should call the `vslDeleteStream` function first to deallocate the resources.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*memptr* is a NULL pointer.
VSL_ERROR_MEM_FAILURE	System cannot allocate memory for internal needs.
VSL_RNG_ERROR_BAD_MEM_FORMAT	Descriptive random stream format is unknown.

vslGetStreamSize

Computes size of memory necessary to hold the random stream.

Syntax

Fortran:

```
memsize = vslgetstreamsize( stream )
```

C:

```
memsize = vslGetStreamSize( stream );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE(VSL_STREAM_STATE), INTENT(IN) **C:** const VSLStreamStatePtr	Random stream

Output Parameters

Name	Type	Description
memsize	**Fortran:** INTEGER **C:** int	Amount of memory in bytes necessary to hold descriptive data of random stream *stream*

Description

The vslGetStreamSize function returns the size of memory in bytes which is necessary to hold the given random stream. Use the output of the function to allocate the buffer to which you will save the random stream by means of the vslSaveStreamM function.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_RNG_ERROR_BAD_STREAM	*stream* is a NULL pointer.

VSL_ERROR_BAD_STREAM	*stream* is not a valid random stream.

vslLeapfrogStream

Initializes a stream using the leapfrog method.

Syntax

Fortran:

```
status = vslleapfrogstream( stream, k, nstreams )
```

C:

```
status = vslLeapfrogStream( stream, k, nstreams );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

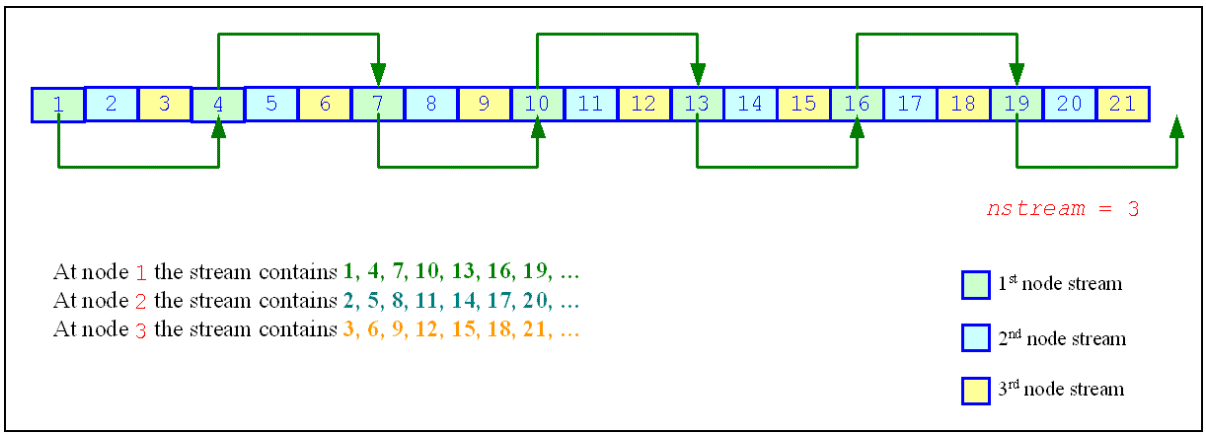
Name	Type	Description
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE(VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream to which leapfrog method is applied **C:** Pointer to the stream state structure to which leapfrog method is applied
k	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Index of the computational node, or stream number
nstreams	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Largest number of computational nodes, or stride

Description

The vslLeapfrogStream function generates random numbers in a random stream with non-unit stride. This feature is particularly useful in distributing random numbers from the original stream across the *nstreams* buffers without generating the original random sequence with subsequent manual distribution.

One of the important applications of the leapfrog method is splitting the original sequence into non-overlapping subsequences across *nstreams* computational nodes. The function initializes the original random stream (see Figure "Leapfrog Method") to generate random numbers for the computational node k, $0 \le k < nstreams$, where *nstreams* is the largest number of computational nodes used.

Leapfrog Method



The leapfrog method is supported only for those basic generators that allow splitting elements by the leapfrog method, which is more efficient than simply generating them by a generator with subsequent manual distribution across computational nodes. See VSL Notes for details.

For quasi-random basic generators, the leapfrog method allows generating individual components of quasi-random vectors instead of whole quasi-random vectors. In this case *nstreams* parameter should be equal to the dimension of the quasi-random vector while *k* parameter should be the index of a component to be generated ($0 \le k < nstreams$). Other parameters values are not allowed.

The following code examples illustrate the initialization of three independent streams using the leapfrog method:

Fortran 90 Code for Leapfrog Method

```
...
TYPE(VSL_STREAM_STATE)   ::stream1
TYPE(VSL_STREAM_STATE)   ::stream2
TYPE(VSL_STREAM_STATE)   ::stream3

! Creating 3 identical streams
status = vslnewstream(stream1, VSL_BRNG_MCG31, 174)
status = vslcopystream(stream2, stream1)
status = vslcopystream(stream3, stream1)

! Leapfrogging the streams
status = vslleapfrogstream(stream1, 0, 3)
status = vslleapfrogstream(stream2, 1, 3)
status = vslleapfrogstream(stream3, 2, 3)

! Generating random numbers
...
! Deleting the streams
status = vsldeletestream(stream1)
status = vsldeletestream(stream2)
status = vsldeletestream(stream3)
...
```

C Code for Leapfrog Method

```
...
VSLStreamStatePtr stream1;
VSLStreamStatePtr stream2;
VSLStreamStatePtr stream3;

/* Creating 3 identical streams */
status = vslNewStream(&stream1, VSL_BRNG_MCG31, 174);
status = vslCopyStream(&stream2, stream1);
status = vslCopyStream(&stream3, stream1);

/* Leapfrogging the streams
*/
status = vslLeapfrogStream(stream1, 0, 3);
status = vslLeapfrogStream(stream2, 1, 3);
status = vslLeapfrogStream(stream3, 2, 3);

/* Generating random numbers
*/
...
/* Deleting the streams
*/
status = vslDeleteStream(&stream1);
status = vslDeleteStream(&stream2);
status = vslDeleteStream(&stream3);
...
```

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_LEAPFROG_UNSUPPORTED	BRNG does not support Leapfrog method.

vslSkipAheadStream

Initializes a stream using the block-splitting method.

Syntax

Fortran:

```
status = vslskipaheadstream( stream, nskip )
```

C:

```
status = vslSkipAheadStream( stream, nskip);
```

Include Files

- FORTRAN 77: mkl_vsl.f77

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

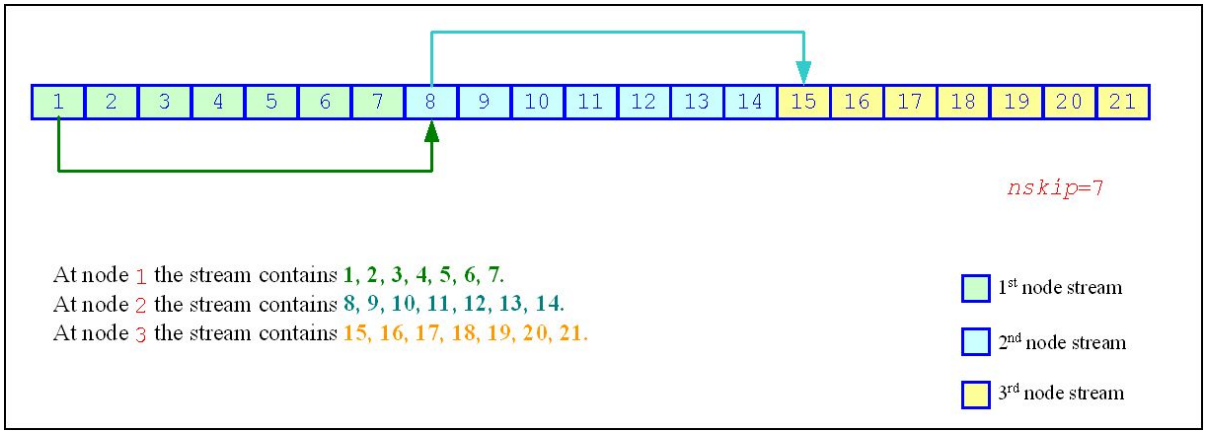
Input Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream to which block-splitting method is applied **C:** Pointer to the stream state structure to which block-splitting method is applied
nskip	**FORTRAN 77:** `INTEGER*4 nskip(2)` **Fortran 90:** `INTEGER(KIND=8), INTENT(IN)` **C:** `const long long int`	Number of skipped elements

Description

The `vslSkipAheadStream` function skips a given number of elements in a random stream. This feature is particularly useful in distributing random numbers from original random stream across different computational nodes. If the largest number of random numbers used by a computational node is *nskip*, then the original random sequence may be split by `vslSkipAheadStream` into non-overlapping blocks of *nskip* size so that each block corresponds to the respective computational node. The number of computational nodes is unlimited. This method is known as the block-splitting method or as the skip-ahead method. (see Figure "Block-Splitting Method").

Block-Splitting Method



The skip-ahead method is supported only for those basic generators that allow skipping elements by the skip-ahead method, which is more efficient than simply generating them by generator with subsequent manual skipping. See VSL Notes for details.

Please note that for quasi-random basic generators the skip-ahead method works with components of quasi-random vectors rather than with whole quasi-random vectors. Therefore, to skip NS quasi-random vectors, set the *nskip* parameter equal to the NS*DIMEN, where DIMEN is the dimension of the quasi-random vector. If this operation results in exceeding the period of the quasi-random number generator, which is 2^{32}-1, the library returns the VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED error code.

The following code examples illustrate how to initialize three independent streams using the vslSkipAheadStream function:

Fortran 90 Code for Block-Splitting Method

```
...
type(VSL_STREAM_STATE)  ::stream1
type(VSL_STREAM_STATE)  ::stream2
type(VSL_STREAM_STATE)  ::stream3

! Creating the 1st stream
status = vslnewstream(stream1, VSL_BRNG_MCG31, 174)

! Skipping ahead by 7 elements the 2nd stream
status = vslcopystream(stream2, stream1);
status = vslskipaheadstream(stream2, 7);

! Skipping ahead by 7 elements the 3rd stream
status = vslcopystream(stream3, stream2);
status = vslskipaheadstream(stream3, 7);

! Generating random numbers
...
! Deleting the streams
status = vsldeletestream(stream1)
status = vsldeletestream(stream2)
status = vsldeletestream(stream3)
...
```

C Code for Block-Splitting Method

```
VSLStreamStatePtr stream1;
VSLStreamStatePtr stream2;
VSLStreamStatePtr stream3;

/* Creating the 1st stream
*/
status = vslNewStream(&stream1, VSL_BRNG_MCG31, 174);

/* Skipping ahead by 7 elements the 2nd stream */
status = vslCopyStream(&stream2, stream1);
status = vslSkipAheadStream(stream2, 7);

/* Skipping ahead by 7 elements the 3rd stream */
status = vslCopyStream(&stream3, stream2);
status = vslSkipAheadStream(stream3, 7);

/* Generating random numbers
*/
...
/* Deleting the streams
*/
status = vslDeleteStream(&stream1);
status = vslDeleteStream(&stream2);
status = vslDeleteStream(&stream3);
...
```

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	BRNG does not support the Skip-Ahead method.

vslGetStreamStateBrng

Returns index of a basic generator used for generation of a given random stream.

Syntax

Fortran:

```
brng = vslgetstreamstatebrng( stream )
```

C:

```
brng = vslGetStreamStateBrng( stream );
```

Include Files

- FORTRAN 77: mkl_vsl.f77

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE(VSL_STREAM_STATE), TINTENT(IN)` **C:** `const VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state **C:** Pointer to the stream state structure

Output Parameters

Name	Type	Description
brng	**Fortran:** `INTEGER` **C:** `int`	Index of the basic generator assigned for the generation of *stream* ; negative in case of an error

Description

The `vslGetStreamStateBrng` function retrieves the index of a basic generator used for generation of a given random stream.

Return Values

`VSL_ERROR_NULL_PTR`	*stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is not a valid random stream.

vslGetNumRegBrngs

Obtains the number of currently registered basic generators.

Syntax

Fortran:

```
nregbrngs = vslgetnumregbrngs( )
```

C:

```
nregbrngs = vslGetNumRegBrngs( void );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Output Parameters

Name	Type	Description
nregbrngs	**Fortran:** `INTEGER`	Number of basic generators registered at the moment of the function call

Name	Type	Description
	C: int	

Description

The vslGetNumRegBrngs function obtains the number of currently registered basic generators. Whenever user registers a user-designed basic generator, the number of registered basic generators is incremented. The maximum number of basic generators that can be registered is determined by the VSL_MAX_REG_BRNGS parameter.

Distribution Generators

Intel MKL VSL routines are used to generate random numbers with different types of distribution. Each function group is introduced below by the type of underlying distribution and contains a short description of its functionality, as well as specifications of the call sequence for both Fortran and C-interface and the explanation of input and output parameters. Table "Continuous Distribution Generators" and Table "Discrete Distribution Generators" list the random number generator routines with data types and output distributions, and sets correspondence between data types of the generator routines and the basic random number generators.

Continuous Distribution Generators

Type of Distribution	Data Types	BRNG Data Type	Description
vRngUniform	s, d	s, d	Uniform continuous distribution on the interval [*a*,*b*]
vRngGaussian	s, d	s, d	Normal (Gaussian) distribution
vRngGaussianMV	s, d	s, d	Multivariate normal (Gaussian) distribution
vRngExponential	s, d	s, d	Exponential distribution
vRngLaplace	s, d	s, d	Laplace distribution (double exponential distribution)
vRngWeibull	s, d	s, d	Weibull distribution
vRngCauchy	s, d	s, d	Cauchy distribution
vRngRayleigh	s, d	s, d	Rayleigh distribution
vRngLognormal	s, d	s, d	Lognormal distribution
vRngGumbel	s, d	s, d	Gumbel (extreme value) distribution
vRngGamma	s, d	s, d	Gamma distribution
vRngBeta	s, d	s, d	Beta distribution

Discrete Distribution Generators

Type of Distribution	Data Types	BRNG Data Type	Description
vRngUniform	i	d	Uniform discrete distribution on the interval [*a*,*b*)
vRngUniformBits	i	i	Underlying BRNG integer recurrence

Type of Distribution	Data Types	BRNG Data Type	Description
vRngUniformBits32	i	i	Uniformly distributed bits in 32-bit chunks
vRngUniformBits64	i	i	Uniformly distributed bits in 64-bit chunks
vRngBernoulli	i	s	Bernoulli distribution
vRngGeometric	i	s	Geometric distribution
vRngBinomial	i	d	Binomial distribution
vRngHypergeometric	i	d	Hypergeometric distribution
vRngPoisson	i	s (for VSL_RNG_METHOD_POISSON_POISNORM) s (for distribution parameter $\lambda \geq 27$) and d (for $\lambda < 27$) (for VSL_RNG_METHOD_POISSON_PTPE)	Poisson distribution
vRngPoissonV	i	s	Poisson distribution with varying mean
vRngNegBinomial	i	d	Negative binomial distribution, or Pascal distribution

Modes of random number generation

The library provides two modes of random number generation, accurate and fast. Accurate generation mode is intended for the applications that are highly demanding to accuracy of calculations. When used in this mode, the generators produce random numbers lying completely within definitional domain for all values of the distribution parameters. For example, random numbers obtained from the generator of continuous distribution that is uniform on interval [*a*,*b*] belong to this interval irrespective of what *a* and *b* values may be. Fast mode provides high performance of generation and also guaranties that generated random numbers belong to the definitional domain except for some specific values of distribution parameters. The generation mode is set by specifying relevant value of the method parameter in generator routines. List of distributions that support accurate mode of generation is given in the table below.

Distribution Generators Supporting Accurate Mode

Type of Distribution	Data Types
vRngUniform	s, d
vRngExponential	s, d
vRngWeibull	s, d
vRngRayleigh	s, d
vRngLognormal	s, d
vRngGamma	s, d
vRngBeta	s, d

See additional details about accurate and fast mode of random number generation in VSL Notes.

New method names

The current version of Intel MKL has a modified structure of VSL RNG method names. (See RNG Naming Conventions for details.) The old names are kept for backward compatibility. The set correspondence between the new and legacy method names for VSL random number generators.

Method Names for Continuous Distribution Generators

RNG	Legacy Method Name	New Method Name
vRngUniform	VSL_METHOD_SUNIFORM_STD, VSL_METHOD_DUNIFORM_STD, VSL_METHOD_SUNIFORM_STD_ACCURATE, VSL_METHOD_DUNIFORM_STD_ACCURATE	VSL_RNG_METHOD_UNIFORM_STD, VSL_RNG_METHOD_UNIFORM_STD_ACCURATE
vRngGaussian	VSL_METHOD_SGAUSSIAN_BOXMULLER, VSL_METHOD_SGAUSSIAN_BOXMULLER2, VSL_METHOD_SGAUSSIAN_ICDF, VSL_METHOD_DGAUSSIAN_BOXMULLER, VSL_METHOD_DGAUSSIAN_BOXMULLER2, VSL_METHOD_DGAUSSIAN_ICDF	VSL_RNG_METHOD_GAUSSIAN_BOXMULLER, VSL_RNG_METHOD_GAUSSIAN_BOXMULLER2, VSL_RNG_METHOD_GAUSSIAN_ICDF
vRngGaussianMV	VSL_METHOD_SGAUSSIANMV_BOXMULLER, VSL_METHOD_SGAUSSIANMV_BOXMULLER2, VSL_METHOD_SGAUSSIANMV_ICDF, VSL_METHOD_DGAUSSIANMV_BOXMULLER, VSL_METHOD_DGAUSSIANMV_BOXMULLER2, VSL_METHOD_DGAUSSIANMV_ICDF	VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER, VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER2, VSL_RNG_METHOD_GAUSSIANMV_ICDF
vRngExponential	VSL_METHOD_SEXPONENTIAL_ICDF, VSL_METHOD_DEXPONENTIAL_ICDF, VSL_METHOD_SEXPONENTIAL_ICDF_ACCURATE, VSL_METHOD_DEXPONENTIAL_ICDF_ACCURATE	VSL_RNG_METHOD_EXPONENTIAL_ICDF, VSL_RNG_METHOD_EXPONENTIAL_ICDF_ACCURATE
vRngLaplace	VSL_METHOD_SLAPLACE_ICDF, VSL_METHOD_DLAPLACEL_ICDF	VSL_RNG_METHOD_LAPLACE_ICDF
vRngWeibull	VSL_METHOD_SWEIBULL_ICDF, VSL_METHOD_DWEIBULL_ICDF, VSL_METHOD_SWEIBULL_ICDF_ACCURATE, VSL_METHOD_DWEIBULL_ICDF_ACCURATE	VSL_RNG_METHOD_WEIBULL_ICDF, VSL_RNG_METHOD_WEIBULL_ICDF_ACCURATE
vRngCauchy	VSL_METHOD_SCAUCHY_ICDF, VSL_METHOD_DCAUCHY_ICDF	VSL_RNG_METHOD_CAUCHY_ICDF
vRngRayleigh	VSL_METHOD_SRAYLEIGH_ICDF, VSL_METHOD_DRAYLEIGH_ICDF, VSL_METHOD_SRAYLEIGH_ICDF_ACCURATE, VSL_METHOD_DRAYLEIGH_ICDF_ACCURATE	VSL_RNG_METHOD_RAYLEIGH_ICDF, VSL_RNG_METHOD_RAYLEIGH_ICDF_ACCURATE
vRngLognormal	VSL_METHOD_SLOGNORMAL_BOXMULLER2, VSL_METHOD_DLOGNORMAL_BOXMULLER2, VSL_METHOD_SLOGNORMAL_BOXMULLER2_ACCURATE, VSL_METHOD_DLOGNORMAL_BOXMULLER2_ACCURATE	VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2, VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2_ACCURATE
vRngGumbel	VSL_METHOD_SGUMBEL_ICDF, VSL_METHOD_DGUMBEL_ICDF	VSL_RNG_METHOD_GUMBEL_ICDF

RNG	Legacy Method Name	New Method Name
vRngGamma	VSL_METHOD_SGAMMA_GNORM, VSL_METHOD_DGAMMA_GNORM, VSL_METHOD_SGAMMA_GNORM_ACCURATE, VSL_METHOD_DGAMMA_GNORM_ACCURATE	VSL_RNG_METHOD_GAMMA_GNORM, VSL_RNG_METHOD_GAMMA_GNORM_ACCURATE
vRngBeta	VSL_METHOD_SBETA_CJA, VSL_METHOD_DBETA_CJA, VSL_METHOD_SBETA_CJA_ACCURATE, VSL_METHOD_DBETA_CJA_ACCURATE	VSL_RNG_METHOD_BETA_CJA, VSL_RNG_METHOD_BETA_CJA_ACCURATE

Method Names for Discrete Distribution Generators

RNG	Legacy Method Name	New Method Name
vRngUniform	VSL_METHOD_IUNIFORM_STD	VSL_RNG_METHOD_UNIFORM_STD
vRngUniformBits	VSL_METHOD_IUNIFORMBITS_STD	VSL_RNG_METHOD_UNIFORMBITS_STD
vRngBernoulli	VSL_METHOD_IBERNOULLI_ICDF	VSL_RNG_METHOD_BERNOULLI_ICDF
vRngGeometric	VSL_METHOD_IGEOMETRIC_ICDF	VSL_RNG_METHOD_GEOMETRIC_ICDF
vRngBinomial	VSL_METHOD_IBINOMIAL_BTPE	VSL_RNG_METHOD_BINOMIAL_BTPE
vRngHypergeometric	VSL_METHOD_IHYPERGEOMETRIC_H2PE	VSL_RNG_METHOD_HYPERGEOMETRIC_H2PE
vRngPoisson	VSL_METHOD_IPOISSON_PTPE, VSL_METHOD_IPOISSON_POISNORM	VSL_RNG_METHOD_POISSON_PTPE, VSL_RNG_METHOD_POISSON_POISNORM
vRngPoissonV	VSL_METHOD_IPOISSONV_POISNORM	VSL_RNG_METHOD_POISSONV_POISNORM
vRngNegBinomial	VSL_METHOD_INEGBINOMIAL_NBAR	VSL_RNG_METHOD_NEGBINOMIAL_NBAR

Continuous Distributions

This section describes routines for generating random numbers with continuous distribution.

vRngUniform

Generates random numbers with uniform distribution.

Syntax

Fortran:

```
status = vsrnguniform( method, stream, n, r, a, b )
status = vdrnguniform( method, stream, n, r, a, b )
```

C:

```
status = vsRngUniform( method, stream, n, r, a, b );
status = vdRngUniform( method, stream, n, r, a, b );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Generation method; the specific values are as follows: `VSL_RNG_METHOD_UNIFORM_STD` `VSL_RNG_METHOD_UNIFORM_STD_ACCURATE` Standard method.
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of random values to be generated
a	**FORTRAN 77:** `REAL` for `vsrnguniform` `DOUBLE PRECISION` for `vdrnguniform` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnguniform` `REAL(KIND=8), INTENT(IN)` for `vdrnguniform` **C:** `const float` for `vsRngUniform` `const double` for `vdRngUniform`	Left bound `a`
b	**FORTRAN 77:** `REAL` for `vsrnguniform` `DOUBLE PRECISION` for `vdrnguniform` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnguniform` `REAL(KIND=8), INTENT(IN)` for `vdrnguniform`	Right bound `b`

Name	Type	Description
	C: `const float` for `vsRngUniform` `const double` for `vdRngUniform`	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `REAL` for `vsrnguniform` `DOUBLE PRECISION` for `vdrnguniform` **Fortran 90:** `REAL(KIND=4), INTENT(OUT)` for `vsrnguniform` `REAL(KIND=8), INTENT(OUT)` for `vdrnguniform` **C:** `float*` for `vsRngUniform` `double*` for `vdRngUniform`	Vector of `n` random numbers uniformly distributed over the interval `[a,b]`

Description

The `vRngUniform` function generates random numbers uniformly distributed over the interval `[a, b]`, where `a, b` are the left and right bounds of the interval, respectively, and `a, b ∈ R ; a < b`.

The probability density function is given by:

$$f_{a,b}(x) = \begin{cases} \dfrac{1}{b-a}, & x \in [a,b] \\ 0, & x \notin [a,b] \end{cases}, \quad -\infty < x < +\infty$$

The cumulative distribution function is as follows:

$$f_{a,b}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ 1, & x \ge b \end{cases} \quad -\infty < x < +\infty .$$

Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*stream* is a `NULL` pointer.

VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngGaussian

Generates normally distributed random numbers.

Syntax

Fortran:

```
status = vsrnggaussian( method, stream, n, r, a, sigma )
status = vdrnggaussian( method, stream, n, r, a, sigma )
```

C:

```
status = vsRngGaussian( method, stream, n, r, a, sigma );
status = vdRngGaussian( method, stream, n, r, a, sigma );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_GAUSSIAN_BOXMULLER VSL_RNG_METHOD_GAUSSIAN_BOXMULLER2 VSL_RNG_METHOD_GAUSSIAN_ICDF See brief description of the methods BOXMULLER, BOXMULLER2, and ICDF in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated

Name	Type	Description
a	**FORTRAN 77:** REAL for vsrnggaussian DOUBLE PRECISION for vdrnggaussian **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnggaussian REAL(KIND=8), INTENT(IN) for vdrnggaussian **C:** const float for vsRngGaussian const double for vdRngGaussian	Mean value *a*.
sigma	**FORTRAN 77:** REAL for vsrnggaussian DOUBLE PRECISION for vdrnggaussian **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnggaussian REAL(KIND=8), INTENT(IN) for vdrnggaussian **C:** const float for vsRngGaussian const double for vdRngGaussian	Standard deviation σ.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** REAL for vsrnggaussian DOUBLE PRECISION for vdrnggaussian **Fortran 90:** REAL(KIND=4), INTENT(OUT) for vsrnggaussian REAL(KIND=8), INTENT(OUT) for vdrnggaussian **C:** float* for vsRngGaussian double* for vdRngGaussian	Vector of n normally distributed random numbers

Description

The vRngGaussian function generates random numbers with normal (Gaussian) distribution with mean value a and standard deviation σ, where

$a, \sigma \in R ; \sigma > 0.$

The probability density function is given by:

$$f_{a,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-a)^2}{2\sigma^2}\right), \quad -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\sigma}(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-a)^2}{2\sigma^2}\right) dy, \quad -\infty < x < +\infty.$$

The cumulative distribution function $F_{a,\sigma}(x)$ can be expressed in terms of standard normal distribution $\Phi(x)$ as

$F_{a,\sigma}(x) = \Phi((x - a)/\sigma)$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngGaussianMV

Generates random numbers from multivariate normal distribution.

Syntax

Fortran:

```
status = vsrnggaussianmv( method, stream, n, r, dimen, mstorage, a, t )
status = vdrnggaussianmv( method, stream, n, r, dimen, mstorage, a, t )
```

C:

```
status = vsRngGaussianMV( method, stream, n, r, dimen, mstorage, a, t );
status = vdRngGaussianMV( method, stream, n, r, dimen, mstorage, a, t );
```

Include Files

- FORTRAN 77: mkl_vsl.f77

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Generation method. The specific values are as follows: `VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER` `VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER2` `VSL_RNG_METHOD_GAUSSIANMV_ICDF` See brief description of the methods `BOXMULLER`, `BOXMULLER2`, and `ICDF` in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of d-dimensional vectors to be generated
dimen	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Dimension d ($d \geq 1$) of output random vectors
mstorage	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	**Fortran:** Matrix storage scheme for upper triangular matrix T^T. The routine supports three matrix storage schemes: • `VSL_MATRIX_STORAGE_FULL`— all d x d elements of the matrix T^T are passed, however, only the upper triangle part is actually used in the routine. • `VSL_MATRIX_STORAGE_PACKED`— upper triangle elements of T^T are packed by rows into a one-dimensional array. • `VSL_MATRIX_STORAGE_DIAGONAL`— only diagonal elements of T^T are passed. **C:** Matrix storage scheme for lower triangular matrix T. The routine supports three matrix storage schemes: • `VSL_MATRIX_STORAGE_FULL`— all d x d elements of the matrix T are passed, however, only the lower triangle part is actually used in the routine.

Name	Type	Description
		• VSL_MATRIX_STORAGE_PACKED— lower triangle elements of *T* are packed by rows into a one-dimensional array. • VSL_MATRIX_STORAGE_DIAGONAL— only diagonal elements of *T* are passed.
a	**FORTRAN 77:** REAL for vsrnggaussianmv DOUBLE PRECISION for vdrnggaussianmv **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnggaussianmv REAL(KIND=8), INTENT(IN) for vdrnggaussianmv **C:** const float* for vsRngGaussianMV const double* for vdRngGaussianMV	Mean vector *a* of dimension *d*
t	**FORTRAN 77:** REAL for vsrnggaussianmv DOUBLE PRECISION for vdrnggaussianmv **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnggaussianmv REAL(KIND=8), INTENT(IN) for vdrnggaussianmv **C:** const float* for vsRngGaussianMV const double* for vdRngGaussianMV	**Fortran:** Elements of the upper triangular matrix passed according to the matrix T^T storage scheme *mstorage*. **C:** Elements of the lower triangular matrix passed according to the matrix *T* storage scheme *mstorage*.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** REAL for vsrnggaussianmv DOUBLE PRECISION for vdrnggaussianmv **Fortran 90:** REAL(KIND=4), INTENT(OUT) for vsrnggaussianmv	Array of *n* random vectors of dimension *dimen*

Name	Type	Description
	REAL(KIND=8), INTENT(OUT) for vdrnggaussianmv **C:** float* for vsRngGaussianMV double* for vdRngGaussianMV	

Description

The vRngGaussianMV function generates random numbers with d-variate normal (Gaussian) distribution with mean value a and variance-covariance matrix C, where $a \in R^d$; C is a $d \times d$ symmetric positive-definite matrix.

The probability density function is given by:

$$f_{a,C}(x) = \frac{1}{\sqrt{\det(2\pi C)}} \exp\left(-1/2(x-a)^T C^{-1}(x-a)\right),$$

where $x \in R^d$.

Matrix C can be represented as $C = TT^T$, where T is a lower triangular matrix - Cholesky factor of C.

Instead of variance-covariance matrix C the generation routines require Cholesky factor of C in input. To compute Cholesky factor of matrix C, the user may call MKL LAPACK routines for matrix factorization: ?potrf or ?pptrf for v?RngGaussianMV/v?rnggaussianmv routines (? means either s or d for single and double precision respectively). See Application Notes for more details.

Application Notes

Since matrices are stored in Fortran by columns, while in C they are stored by rows, the usage of MKL factorization routines (assuming Fortran matrices storage) in combination with multivariate normal RNG (assuming C matrix storage) is slightly different in C and Fortran. The following tables help in using these routines in C and Fortran. For further information please refer to the appropriate VSL example file.

Using Cholesky Factorization Routines in Fortran

Matrix Storage Scheme	Variance-Covariance Matrix Argument	Factorization Routine	UPLO Parameter in Factorization Routine	Result of Factorization as Input Argument for RNG
VSL_MATRIX_STORAGE_FULL	C in Fortran two-dimensional array	spotrf for vsrnggaussianmv dpotrf for vdrnggaussianmv	'U'	Upper triangle of T^T. Lower triangle is not used.
VSL_MATRIX_STORAGE_PACKED	Lower triangle of C packed by columns into one-dimensional array	spptrf for vsrnggaussianmv dpptrf for vdrnggaussianmv	'L'	Upper triangle of T^T packed by rows into one-dimensional array.

Using Cholesky Factorization Routines in C

Matrix Storage Scheme	Variance-Covariance Matrix Argument	Factorization Routine	UPLO Parameter in Factorization Routine	Result of Factorization as Input Argument for RNG
VSL_MATRIX_STORAGE_FULL	*C* in C two-dimensional array	spotrf for vsRngGaussianMV dpotrf for vdRngGaussianMV	'U'	Upper triangle of T^T. Lower triangle is not used.
VSL_MATRIX_STORAGE_PACKED	Lower triangle of *C* packed by columns into one-dimensional array	spptrf for vsRngGaussianMV dpptrf for vdRngGaussianMV	'L'	Upper triangle of T^T packed by rows into one-dimensional array.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngExponential

Generates exponentially distributed random numbers.

Syntax

Fortran:

```
status = vsrngexponential( method, stream, n, r, a, beta )
status = vdrngexponential( method, stream, n, r, a, beta )
```

C:

```
status = vsRngExponential( method, stream, n, r, a, beta );
status = vdRngExponential( method, stream, n, r, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_EXPONENTIAL_ICDF VSL_RNG_METHOD_EXPONENTIAL_ICDF_ACCURATE Inverse cumulative distribution function method
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
a	**FORTRAN 77:** REAL for vsrngexponential DOUBLE PRECISION for vdrngexponential **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngexponential REAL(KIND=8), INTENT(IN) for vdrngexponential **C:** const float for vsRngExponential **C:** const double for vdRngExponential	Displacement *a*
beta	**FORTRAN 77:** REAL for vsrngexponential DOUBLE PRECISION for vdrngexponential **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngexponential REAL(KIND=8), INTENT(IN) for vdrngexponential **C:** const float for vsRngExponential	Scalefactor β.

Name	Type	Description
	`const double` for `vdRngExponential`	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `REAL` for `vsrngexponential` `DOUBLE PRECISION` for `vdrngexponential` **Fortran 90:** `REAL(KIND=4), INTENT(OUT)` for `vsrngexponential` `REAL(KIND=8), INTENT(OUT)` for `vdrngexponential` **C:** `float*` for `vsRngExponential` `double*` for `vdRngExponential`	Vector of *n* exponentially distributed random numbers

Description

The `vRngExponential` function generates random numbers with exponential distribution that has displacement `a` and scalefactor β, where *a*, β∈*R* ; β > 0.

The probability density function is given by:

$$f_{a,\beta}(x) = \begin{cases} \frac{1}{\beta} \exp\left((-(x-a)) / \beta \right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = \begin{cases} 1 - \exp\left((-(x-a)) / \beta \right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is not a valid random stream.
`VSL_RNG_ERROR_BAD_UPDATE`	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, `< 0` or `> nmax`.

VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngLaplace

Generates random numbers with Laplace distribution.

Syntax

Fortran:

```
status = vsrnglaplace( method, stream, n, r, a, beta )
status = vdrnglaplace( method, stream, n, r, a, beta )
```

C:

```
status = vsRngLaplace( method, stream, n, r, a, beta );
status = vdRngLaplace( method, stream, n, r, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_LAPLACE_ICDF Inverse cumulative distribution function method
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
a	**FORTRAN 77:** REAL for vsrnglaplace DOUBLE PRECISION for vdrnglaplace **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnglaplace	Mean value *a*

Name	Type	Description
	`REAL(KIND=8), INTENT(IN)` for `vdrnglaplace` **C:** `const float` for `vsRngLaplace` `const double` for `vdRngLaplace`	
beta	**FORTRAN 77:** `REAL` for `vsrnglaplace` `DOUBLE PRECISION` for `vdrnglaplace` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnglaplace` `REAL(KIND=8), INTENT(IN)` for `vdrnglaplace` **C:** `const float` for `vsRngLaplace` `const double` for `vdRngLaplace`	Scalefactor β.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `REAL` for `vsrnglaplace` `DOUBLE PRECISION` for `vdrnglaplace` **Fortran 90:** `REAL(KIND=4), INTENT(OUT)` for `vsrnglaplace` `REAL(KIND=8), INTENT(OUT)` for `vdrnglaplace` **C:** `float*` for `vsRngLaplace` `double*` for `vdRngLaplace`	Vector of *n* Laplace distributed random numbers

Description

The `vRngLaplace` function generates random numbers with Laplace distribution with mean value (or average) *a* and scalefactor β, where *a*, $\beta \in R$; $\beta > 0$. The scalefactor value determines the standard deviation as

$\sqrt{2}\beta$

The probability density function is given by:

$$f_{a,\beta}(x) = \frac{1}{\sqrt{2\beta}} \exp\left(-\frac{|x-a|}{\beta}\right), -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$f_{a,\beta}(x) = \begin{cases} \frac{1}{2}\exp\left(-\frac{|x-a|}{\beta}\right), & x \geq a \\ 1 - \frac{1}{2}\exp\left(-\frac{|x-a|}{\beta}\right), & x < a \end{cases}, -\infty < x < +\infty.$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngWeibull

Generates Weibull distributed random numbers.

Syntax

Fortran:

```
status = vsrngweibull( method, stream, n, r, alpha, a, beta )
status = vdrngweibull( method, stream, n, r, alpha, a, beta )
```

C:

```
status = vsRngWeibull( method, stream, n, r, alpha, a, beta );
status = vdRngWeibull( method, stream, n, r, alpha, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_WEIBULL_ICDF VSL_RNG_METHOD_WEIBULL_ICDF_ACCURATE Inverse cumulative distribution function method
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
alpha	**FORTRAN 77:** REAL for vsrngweibull DOUBLE PRECISION for vdrngweibull **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngweibull REAL(KIND=8), INTENT(IN) for vdrngweibull **C:** const float for vsRngWeibull const double for vdRngWeibull	Shape α.
a	**FORTRAN 77:** REAL for vsrngweibull DOUBLE PRECISION for vdrngweibull **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngweibull REAL(KIND=8), INTENT(IN) for vdrngweibull **C:** const float for vsRngWeibull const double for vdRngWeibull	Displacement *a*

Name	Type	Description
beta	**FORTRAN 77:** REAL for vsrngweibull DOUBLE PRECISION for vdrngweibull **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngweibull REAL(KIND=8), INTENT(IN) for vdrngweibull **C:** const float for vsRngWeibull const double for vdRngWeibull	Scalefactor β.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** REAL for vsrngweibull DOUBLE PRECISION for vdrngweibull **Fortran 90:** REAL(KIND=4), INTENT(OUT) for vsrngweibull REAL(KIND=8), INTENT(OUT) for vdrngweibull **C:** float* for vsRngWeibull double* for vdRngWeibull	Vector of *n* Weibull distributed random numbers

Description

The vRngWeibull function generates Weibull distributed random numbers with displacement *a*, scalefactor β, and shape α, where α, β, *a*∈*R* ; α > 0, β > 0.

The probability density function is given by:

$$f_{a,\alpha,\beta}(x) = \begin{cases} \frac{\alpha}{\beta^{\alpha}}(x-a)^{\alpha-1}\exp\left(-\left(\frac{x-a}{\beta}\right)^{\alpha}\right), & x \geq a \\ 0, & x < a \end{cases}$$

The cumulative distribution function is as follows:

$$F_{a,\alpha,\beta}(x) = \begin{cases} 1 - \exp\left(-\left(\frac{x-a}{\beta}\right)^{\alpha}\right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngCauchy

Generates Cauchy distributed random values.

Syntax

Fortran:

```
status = vsrngcauchy( method, stream, n, r, a, beta )
status = vdrngcauchy( method, stream, n, r, a, beta )
```

C:

```
status = vsRngCauchy( method, stream, n, r, a, beta );
status = vdRngCauchy( method, stream, n, r, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_CAUCHY_ICDF Inverse cumulative distribution function method
stream	**FORTRAN 77:** INTEGER*4 stream(2)	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure

Name	Type	Description
	Fortran 90: TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
a	**FORTRAN 77:** REAL for vsrngcauchy DOUBLE PRECISION for vdrngcauchy **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngcauchy REAL(KIND=8), INTENT(IN) for vdrngcauchy **C:** const float for vsRngCauchy const double for vdRngCauchy	Displacement *a*.
beta	**FORTRAN 77:** REAL for vsrngcauchy DOUBLE PRECISION for vdrngcauchy **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngcauchy REAL(KIND=8), INTENT(IN) for vdrngcauchy **C:** const float for vsRngCauchy const double for vdRngCauchy	Scalefactor β.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** REAL for vsrngcauchy DOUBLE PRECISION for vdrngcauchy **Fortran 90:** REAL(KIND=4), INTENT(OUT) for vsrngcauchy	Vector of *n* Cauchy distributed random numbers

Name	Type	Description
	REAL(KIND=8), INTENT(OUT) for vdrngcauchy **C:** float* for vsRngCauchy double* for vdRngCauchy	

Description

The function generates Cauchy distributed random numbers with displacement *a* and scalefactor β, where *a*, β∈*R* ; β > 0.

The probability density function is given by:

$$f_{a,\beta}(x) = \frac{1}{\pi\beta\left(1+\left(\frac{x-a}{\beta}\right)^2\right)}, -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{x-a}{\beta}\right), -\infty < x < +\infty.$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngRayleigh

Generates Rayleigh distributed random values.

Syntax

Fortran:

```
status = vsrngrayleigh( method, stream, n, r, a, beta )
status = vdrngrayleigh( method, stream, n, r, a, beta )
```

C:

```
status = vsRngRayleigh( method, stream, n, r, a, beta );
```

```
status = vdRngRayleigh( method, stream, n, r, a, beta );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Generation method. The specific values are as follows: `VSL_RNG_METHOD_RAYLEIGH_ICDF` `VSL_RNG_METHOD_RAYLEIGH_ICDF_ACCURATE` Inverse cumulative distribution function method
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of random values to be generated
a	**FORTRAN 77:** `REAL` for `vsrngrayleigh` `DOUBLE PRECISION` for `vdrngrayleigh` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrngrayleigh` `REAL(KIND=8), INTENT(IN)` for `vdrngrayleigh` **C:** `const float` for `vsRngRayleigh` `const double` for `vdRngRayleigh`	Displacement *a*
beta	**FORTRAN 77:** `REAL` for `vsrngrayleigh` `DOUBLE PRECISION` for `vdrngrayleigh` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrngrayleigh`	Scalefactor β.

Name	Type	Description
	`REAL(KIND=8), INTENT(IN)` for `vdrngrayleigh` **C:** `const float` for `vsRngRayleigh` `const double` for `vdRngRayleigh`	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `REAL` for `vsrngrayleigh` `DOUBLE PRECISION` for `vdrngrayleigh` **Fortran 90:** `REAL(KIND=4), INTENT(OUT)` for `vsrngrayleigh` `REAL(KIND=8), INTENT(OUT)` for `vdrngrayleigh` **C:** `float*` for `vsRngRayleigh` `double*` for `vdRngRayleigh`	Vector of *n* Rayleigh distributed random numbers

Description

The `vRngRayleigh` function generates Rayleigh distributed random numbers with displacement *a* and scalefactor β, where *a*, $\beta \in R$; $\beta > 0$.

The Rayleigh distribution is a special case of the `Weibull` distribution, where the shape parameter $\alpha = 2$.

The probability density function is given by:

$$f_{a,\beta}(x) = \begin{cases} \frac{2(x-a)}{\beta^2} \exp\left(-\frac{(x-a)^2}{\beta^2}\right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = \begin{cases} 1 - \exp\left(-\frac{(x-a)^2}{\beta^2}\right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngLognormal

Generates lognormally distributed random numbers.

Syntax

Fortran:

```
status = vsrnglognormal( method, stream, n, r, a, sigma, b, beta )
status = vdrnglognormal( method, stream, n, r, a, sigma, b, beta )
```

C:

```
status = vsRngLognormal( method, stream, n, r, a, sigma, b, beta );
status = vdRngLognormal( method, stream, n, r, a, sigma, b, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2 VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2_ACCURATE Inverse cumulative distribution function method
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN)	Number of random values to be generated

Name	Type	Description
	C: `const int`	
a	**FORTRAN 77:** `REAL` for `vsrnglognormal` `DOUBLE PRECISION` for `vdrnglognormal` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnglognormal` `REAL(KIND=8), INTENT(IN)` for `vdrnglognormal` **C:** `const float` for `vsRngLognormal` `const double` for `vdRngLognormal`	Average *a* of the subject normal distribution
sigma	**FORTRAN 77:** `REAL` for `vsrnglognormal` `DOUBLE PRECISION` for `vdrnglognormal` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnglognormal` `REAL(KIND=8), INTENT(IN)` for `vdrnglognormal` **C:** `const float` for `vsRngLognormal` `const double` for `vdRngLognormal`	Standard deviation σ of the subject normal distribution
b	**FORTRAN 77:** `REAL` for `vsrnglognormal` `DOUBLE PRECISION` for `vdrnglognormal` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnglognormal` `REAL(KIND=8), INTENT(IN)` for `vdrnglognormal` **C:** `const float` for `vsRngLognormal` `const double` for `vdRngLognormal`	Displacement *b*

Name	Type	Description
beta	**FORTRAN 77:** REAL for vsrnglognormal DOUBLE PRECISION for vdrnglognormal **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnglognormal REAL(KIND=8), INTENT(IN) for vdrnglognormal **C:** const float for vsRngLognormal const double for vdRngLognormal	Scalefactor β.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** REAL for vsrnglognormal DOUBLE PRECISION for vdrnglognormal **Fortran 90:** REAL(KIND=4), INTENT(OUT) for vsrnglognormal REAL(KIND=8), INTENT(OUT) for vdrnglognormal **C:** float* for vsRngLognormal double* for vdRngLognormal	Vector of *n* lognormally distributed random numbers

Description

The vRngLognormal function generates lognormally distributed random numbers with average of distribution *a* and standard deviation σ of subject normal distribution, displacement *b*, and scalefactor β, where *a*, σ, *b*, β∈*R* ; σ > 0 , β > 0.

The probability density function is given by:

$$f_{a,\sigma,b,\beta}(x) = \begin{cases} \dfrac{1}{\sigma(x-b)\sqrt{2\pi}} \exp\left(-\dfrac{\left[\ln((x-b)/\beta) - a\right]^2}{2\sigma^2}\right), & x > b \\ 0, & x \le b \end{cases}$$

The cumulative distribution function is as follows:

$$F_{a,\sigma,b,\beta}(x) = \begin{cases} \Phi(\ (\ln(\ (x-b)/\beta) - a)/\sigma), & x > b \\ 0, & x \le b \end{cases}$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngGumbel

Generates Gumbel distributed random values.

Syntax

Fortran:

```
status = vsrnggumbel( method, stream, n, r, a, beta )
status = vdrnggumbel( method, stream, n, r, a, beta )
```

C:

```
status = vsRngGumbel( method, stream, n, r, a, beta );
status = vdRngGumbel( method, stream, n, r, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_GUMBEL_ICDF Inverse cumulative distribution function method
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN)	**Fortran:** Descriptor of the stream state structure **C:** Pointer to the stream state structure

Name	Type	Description
	C: VSLStreamStatePtr	
n	**FORTRAN 77:** INTEGER	Number of random values to be generated
	Fortran 90: INTEGER, INTENT(IN)	
	C: const int	
a	**FORTRAN 77:** REAL for vsrnggumbel	Displacement *a*.
	DOUBLE PRECISION for vdrnggumbel	
	Fortran 90: REAL(KIND=4), INTENT(IN) for vsrnggumbel	
	REAL(KIND=8), INTENT(IN) for vdrnggumbel	
	C: const float for vsRngGumbel	
	const double for vdRngGumbel	
beta	**FORTRAN 77:** REAL for vsrnggumbel	Scalefactor β.
	DOUBLE PRECISION for vdrnggumbel	
	Fortran 90: REAL(KIND=4), INTENT(IN) for vsrnggumbel	
	REAL(KIND=8), INTENT(IN) for vdrnggumbel	
	C: const float for vsRngGumbel	
	const double for vdRngGumbel	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** REAL for vsrnggumbel	Vector of *n* random numbers with Gumbel distribution
	DOUBLE PRECISION for vdrnggumbel	
	Fortran 90: REAL(KIND=4), INTENT(OUT) for vsrnggumbel	
	REAL(KIND=8), INTENT(OUT) for vdrnggumbel	
	C: float* for vsRngGumbel	
	double* for vdRngGumbel	

Description

The vRngGumbel function generates Gumbel distributed random numbers with displacement a and scalefactor β, where a, $\beta \in R$; β > 0.

The probability density function is given by:

$$f_{a,\beta}(x) = \left\{ \frac{1}{\beta} \exp\left(\frac{x-a}{\beta} \right) \exp\left(-\exp\left((x-a)/\beta\right)\right), -\infty < x < +\infty \right.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = 1 - \exp\left(-\exp\left((x-a)/\beta\right)\right), -\infty < x < +\infty$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngGamma

Generates gamma distributed random values.

Syntax

Fortran:

```
status = vsrnggamma( method, stream, n, r, alpha, a, beta )
status = vdrnggamma( method, stream, n, r, alpha, a, beta )
```

C:

```
status = vsRngGamma( method, stream, n, r, alpha, a, beta );
status = vdRngGamma( method, stream, n, r, alpha, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_GAMMA_GNORM VSL_RNG_METHOD_GAMMA_GNORM_ACCURATE Acceptance/rejection method using random numbers with Gaussian distribution. See brief description of the method GNORM in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
alpha	**FORTRAN 77:** REAL for vsrnggamma DOUBLE PRECISION for vdrnggamma **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnggamma REAL(KIND=8), INTENT(IN) for vdrnggamma **C:** const float for vsRngGamma const double for vdRngGamma	Shape α.
a	**FORTRAN 77:** REAL for vsrnggamma DOUBLE PRECISION for vdrnggamma **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrnggamma REAL(KIND=8), INTENT(IN) for vdrnggamma **C:** const float for vsRngGamma const double for vdRngGamma	Displacement a.

Name	Type	Description
beta	**FORTRAN 77:** `REAL` for `vsrnggamma` `DOUBLE PRECISION` for `vdrnggamma` **Fortran 90:** `REAL(KIND=4), INTENT(IN)` for `vsrnggamma` `REAL(KIND=8), INTENT(IN)` for `vdrnggamma` **C:** `const float` for `vsRngGamma` `const double` for `vdRngGamma`	Scalefactor β.

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `REAL` for `vsrnggamma` `DOUBLE PRECISION` for `vdrnggamma` **Fortran 90:** `REAL(KIND=4), INTENT(OUT)` for `vsrnggamma` `REAL(KIND=8), INTENT(OUT)` for `vdrnggamma` **C:** `float*` for `vsRngGamma` `double*` for `vdRngGamma`	Vector of *n* random numbers with gamma distribution

Description

The `vRngGamma` function generates random numbers with gamma distribution that has shape parameter α, displacement *a*, and scale parameter β, where α, β, and $a \in R$; $\alpha > 0$, $\beta > 0$.

The probability density function is given by:

$$f_{\alpha,a,\beta}(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^{\alpha}}(x-a)^{\alpha-1}e^{-(x-a)/\beta}, & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty$$

where Γ(α) is the complete gamma function.

The cumulative distribution function is as follows:

$$F_{\alpha,a,\beta}(x) = \begin{cases} \int_{a}^{x} \frac{1}{\Gamma(\alpha)\beta^{\alpha}}(y-a)^{\alpha-1}e^{-(y-a)/\beta}dy, & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngBeta

Generates beta distributed random values.

Syntax

Fortran:

```
status = vsrngbeta( method, stream, n, r, p, q, a, beta )
status = vdrngbeta( method, stream, n, r, p, q, a, beta )
```

C:

```
status = vsRngBeta( method, stream, n, r, p, q, a, beta );
status = vdRngBeta( method, stream, n, r, p, q, a, beta );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_BETA_CJA VSL_RNG_METHOD_BETA_CJA_ACCURATE See brief description of the method CJA in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER	Number of random values to be generated

Name	Type	Description
	Fortran 90: INTEGER, INTENT(IN) **C:** const int	
p	**FORTRAN 77:** REAL for vsrngbeta DOUBLE PRECISION for vdrngbeta **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngbeta REAL(KIND=8), INTENT(IN) for vdrngbeta **C:** const float for vsRngBeta const double for vdRngBeta	Shape *p*
q	**FORTRAN 77:** REAL for vsrngbeta DOUBLE PRECISION for vdrngbeta **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngbeta REAL(KIND=8), INTENT(IN) for vdrngbeta **C:** const float for vsRngBeta const double for vdRngBeta	Shape *q*
a	**FORTRAN 77:** REAL for vsrngbeta DOUBLE PRECISION for vdrngbeta **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngbeta REAL(KIND=8), INTENT(IN) for vdrngbeta **C:** const float for vsRngBeta const double for vdRngBeta	Displacement *a*.
beta	**FORTRAN 77:** REAL for vsrngbeta DOUBLE PRECISION for vdrngbeta **Fortran 90:** REAL(KIND=4), INTENT(IN) for vsrngbeta	Scalefactor β.

Name	Type	Description
	`REAL(KIND=8), INTENT(IN)` for `vdrngbeta` **C:** `const float` for `vsRngBeta` `const double` for `vdRngBeta`	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `REAL` for `vsrngbeta` `DOUBLE PRECISION` for `vdrngbeta` **Fortran 90:** `REAL(KIND=4), INTENT(OUT)` for `vsrngbeta` `REAL(KIND=8), INTENT(OUT)` for `vdrngbeta` **C:** `float*` for `vsRngBeta` `double*` for `vdRngBeta`	Vector of *n* random numbers with beta distribution

Description

The `vRngBeta` function generates random numbers with beta distribution that has shape parameters `p` and *q*, displacement *a*, and scale parameter β, where *p*, *q*, *a*, and β∈R ; $p > 0$, $q > 0$, $\beta > 0$.

The probability density function is given by:

$$f_{p,q,a,\beta}(x) = \begin{cases} \frac{1}{B(p,q)\beta^{p+q-1}}(x-a)^{p-1}(\beta+a-x)^{q-1}, & a \le x < a+\beta \\ 0, & x < a, x \ge a+\beta \end{cases}, -\infty < x < \infty,$$

where $B(p, q)$ is the complete beta function.

The cumulative distribution function is as follows:

$$F_{p,q,a,\beta}(x) = \begin{cases} 0, & x < a \\ \int_a^x \frac{1}{B(p,q)\beta^{p+q-1}}(y-a)^{p-1}(\beta+a-y)^{q-1}dy, & a \le x < a+\beta \\ 1, & x \ge a+\beta \end{cases}, -\infty < x < \infty.$$

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is not a valid random stream.

VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

Discrete Distributions

This section describes routines for generating random numbers with discrete distribution.

vRngUniform

Generates random numbers uniformly distributed over the interval [a, b).

Syntax

Fortran:

```
status = virnguniform( method, stream, n, r, a, b )
```

C:

```
status = viRngUniform( method, stream, n, r, a, b );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method; the specific value is as follows: VSL_RNG_METHOD_UNIFORM_STD Standard method. Currently there is only one method for this distribution generator.
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
a	**FORTRAN 77:** INTEGER*4	Left interval bound a

Name	Type	Description
	Fortran 90: INTEGER(KIND=4), INTENT(IN) **C:** const int	
b	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(IN) **C:** const int	Right interval bound b

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(OUT) **C:** int*	Vector of *n* random numbers uniformly distributed over the interval [*a*,*b*)

Description

The vRngUniform function generates random numbers uniformly distributed over the interval [*a*, *b*), where *a*, *b* are the left and right bounds of the interval respectively, and *a*, *b*∈*Z*; *a* < *b*.

The probability distribution is given by:

$$P(X = k) = \frac{1}{b-a}, k \in \{a, a+1, \ldots, b-1\}.$$

The cumulative distribution function is as follows:

$$F_{a,b}(x) = \begin{cases} 0, & x < a \\ \dfrac{\lfloor x - a + 1 \rfloor}{b-a}, & a \le x < b, x \in R. \\ 1, & x \ge b \end{cases}$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.

VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngUniformBits

Generates bits of underlying BRNG integer recurrence.

Syntax

Fortran:

```
status = virnguniformbits( method, stream, n, r )
```

C:

```
status = viRngUniformBits( method, stream, n, r );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method; the specific value is VSL_RNG_METHOD_UNIFORMBITS_STD
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(OUT) **C:** unsigned int*	**Fortran:** Vector of *n* random integer numbers. If the *stream* was generated by a 64 or a 128-bit generator, each integer value is represented by two or four elements of *r* respectively. The number of bytes occupied by each integer is contained in the field *wordsize* of the structure VSL_BRNG_PROPERTIES. The total number of bits that are

Name	Type	Description
		actually used to store the value are contained in the field *nbits* of the same structure. See Advanced Service Routines for a more detailed discussion of VSLBRngProperties. **C:** Vector of *n* random integer numbers. If the *stream* was generated by a 64 or a 128-bit generator, each integer value is represented by two or four elements of *r* respectively. The number of bytes occupied by each integer is contained in the field *WordSize* of the structure VSLBRngProperties. The total number of bits that are actually used to store the value are contained in the field *NBits* of the same structure.See Advanced Service Routines for a more detailed discussion of VSLBRngProperties.

Description

The vRngUniformBits function generates integer random values with uniform bit distribution. The generators of uniformly distributed numbers can be represented as recurrence relations over integer values in modular arithmetic. Apparently, each integer can be treated as a vector of several bits. In a truly random generator, these bits are random, while in pseudorandom generators this randomness can be violated. For example, a well known drawback of linear congruential generators is that lower bits are less random than higher bits (for example, see [Knuth81]). For this reason, care should be taken when using this function. Typically, in a 32-bit *LCG* only 24 higher bits of an integer value can be considered random. See VSL Notes for details.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngUniformBits32

Generates uniformly distributed bits in 32-bit chunks.

Syntax

Fortran:

```
status = virnguniformbits32( method, stream, n, r )
```

C:

```
status = viRngUniformBits32( method, stream, n, r );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90

- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Generation method; the specific value is `VSL_RNG_METHOD_UNIFORMBITS32_STD`
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of random values to be generated

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `INTEGER*4` **Fortran 90:** `INTEGER (KIND=4), INTENT(OUT)` **C:** `unsigned int*`	**Fortran:** Vector of *n* 32-bit random integer numbers with uniform bit distribution. **C:** Vector of *n* 32-bit random integer numbers with uniform bit distribution.

Description

The `vRngUniformBits32` function generates uniformly distributed bits in 32-bit chunks. Unlike `vRngUniformBits`, which provides the output of underlying integer recurrence and does not guarantee uniform distribution across bits, `vRngUniformBits32` is designed to ensure each bit in the 32-bit chunk is uniformly distributed. See VSL Notes for details.

Return Values

`VSL_ERROR_OK, VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is not a valid random stream.
`VSL_RNG_ERROR_BRNG_NOT_SUPPORTED`	BRNG is not supported by the function.
`VSL_RNG_ERROR_NO_NUMBERS`	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
`VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED`	Period of the generator has been exceeded.

vRngUniformBits64

Generates uniformly distributed bits in 64-bit chunks.

Syntax

Fortran:

```
status = virnguniformbits64( method, stream, n, r )
```

C:

```
status = viRngUniformBits64( method, stream, n, r );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Generation method; the specific value is `VSL_RNG_METHOD_UNIFORMBITS64_STD`
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of random values to be generated

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `INTEGER*8` **Fortran 90:** `INTEGER (KIND=8), INTENT(OUT)` **C:** `unsigned long long*`	**Fortran:** Vector of *n* 64-bit random integer numbers with uniform bit distribution. **C:** Vector of *n* 64-bit random integer numbers with uniform bit distribution.

Description

The `vRngUniformBits64` function generates uniformly distributed bits in 64-bit chunks. Unlike `vRngUniformBits`, which provides the output of underlying integer recurrence and does not guarantee uniform distribution across bits, `vRngUniformBits64` is designed to ensure each bit in the 64-bit chunk is uniformly distributed. See VSL Notes for details.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BRNG_NOT_SUPPORTED	BRNG is not supported by the function.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngBernoulli

Generates Bernoulli distributed random values.

Syntax

Fortran:

```
status = virngbernoulli( method, stream, n, r, p )
```

C:

```
status = viRngBernoulli( method, stream, n, r, p );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific value is as follows: VSL_RNG_METHOD_BERNOULLI_ICDF Inverse cumulative distribution function method.
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
p	**FORTRAN 77:** DOUBLE PRECISION	Success probability *p* of a trial

Name	Type	Description
	Fortran 90: REAL(KIND=8), INTENT(IN) **C:** const double	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(OUT) **C:** int*	Vector of *n* Bernoulli distributed random values

Description

The vRngBernoulli function generates Bernoulli distributed random numbers with probability *p* of a single trial success, where

$p \in R;\ 0 \le p \le 1.$

A variate is called Bernoulli distributed, if after a trial it is equal to 1 with probability of success *p*, and to 0 with probability 1 - *p*.

The probability distribution is given by:

$P(X = 1) = p$

$P(X = 0) = 1 - p$

The cumulative distribution function is as follows:

$$F_p(x) = \begin{cases} 0, x < 0 \\ 1 - p, 0 \le x < 1, x \in R. \\ 1, x \ge 1 \end{cases}$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngGeometric

Generates geometrically distributed random values.

Syntax

Fortran:

```
status = virnggeometric( method, stream, n, r, p )
```

C:

```
status = viRngGeometric( method, stream, n, r, p );
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Generation method. The specific value is as follows: `VSL_RNG_METHOD_GEOMETRIC_ICDF` Inverse cumulative distribution function method.
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of random values to be generated
p	**FORTRAN 77:** `DOUBLE PRECISION` **Fortran 90:** `REAL(KIND=8), INTENT(IN)` **C:** `const double`	Success probability *p* of a trial

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `INTEGER*4` **Fortran 90:** `INTEGER(KIND=4), INTENT(OUT)` **C:** `int*`	Vector of *n* geometrically distributed random values

Description

The vRngGeometric function generates geometrically distributed random numbers with probability p of a single trial success, where $p \in R;\ 0 < p < 1$.

A geometrically distributed variate represents the number of independent Bernoulli trials preceding the first success. The probability of a single Bernoulli trial success is p.

The probability distribution is given by:

$P(X = k) = p \cdot (1 - p)^k$, $k \in \{0,1,2, \ldots\}$.

The cumulative distribution function is as follows:

$$F_p(x) = \begin{cases} 0, & x < 0 \\ 1 - (1 - p)^{\lfloor x+1 \rfloor}, & 0 \geq x \end{cases} \qquad x \in R.$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngBinomial

Generates binomially distributed random numbers.

Syntax

Fortran:

```
status = virngbinomial( method, stream, n, r, ntrial, p )
```

C:

```
status = viRngBinomial( method, stream, n, r, ntrial, p );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific value is as follows: VSL_RNG_METHOD_BINOMIAL_BTPE See brief description of the BTPE method in Table "Values of <*method*> in *method* parameter".
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
ntrial	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(IN) **C:** const int	Number of independent trials m
p	**FORTRAN 77:** DOUBLE PRECISION **Fortran 90:** REAL(KIND=8), INTENT(IN) **C:** const double	Success probability p of a single trial

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(OUT) **C:** int*	Vector of n binomially distributed random values

Description

The vRngBinomial function generates binomially distributed random numbers with number of independent Bernoulli trials m, and with probability p of a single trial success, where $p \in R;\ 0 \le p \le 1,\ m \in N$.

A binomially distributed variate represents the number of successes in m independent Bernoulli trials with probability of a single trial success p.

The probability distribution is given by:

$$P(X = k) = C_m^k p^k (1 - p)^{m-k}, k \in \{0, 1, \ldots, m\}.$$

The cumulative distribution function is as follows:

$$F_{m,p}(x) = \begin{cases} 0, & x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} C_m^k p^k (1 - p)^{m-k}, & 0 \le x < m, x \in R \\ 1, & x \ge m \end{cases}$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngHypergeometric

Generates hypergeometrically distributed random values.

Syntax

Fortran:

```
status = virnghypergeometric( method, stream, n, r, l, s, m )
```

C:

```
status = viRngHypergeometric( method, stream, n, r, l, s, m );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER	Generation method. The specific value is as follows: VSL_RNG_METHOD_HYPERGEOMETRIC_H2PE

Name	Type	Description
	Fortran 90: `INTEGER, INTENT(IN)` **C:** `const int`	See brief description of the `H2PE` method in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** `INTEGER*4 stream(2)` **Fortran 90:** `TYPE (VSL_STREAM_STATE), INTENT(IN)` **C:** `VSLStreamStatePtr`	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, INTENT(IN)` **C:** `const int`	Number of random values to be generated
l	**FORTRAN 77:** `INTEGER*4` **Fortran 90:** `INTEGER(KIND=4), INTENT(IN)` **C:** `const int`	Lot size `l`
s	**FORTRAN 77:** `INTEGER*4` **Fortran 90:** `INTEGER(KIND=4), INTENT(IN)` **C:** `const int`	Size of sampling without replacement `s`
m	**FORTRAN 77:** `INTEGER*4` **Fortran 90:** `INTEGER(KIND=4), INTENT(IN)` **C:** `const int`	Number of marked elements `m`

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `INTEGER*4` **Fortran 90:** `INTEGER(KIND=4), INTENT(OUT)` **C:** `int*`	Vector of *n* hypergeometrically distributed random values

Description

The `vRngHypergeometric` function generates hypergeometrically distributed random values with lot size *l*, size of sampling *s*, and number of marked elements in the lot *m*, where $l,\ m,\ s \in N \cup \{0\};\ l \geq \max(s,\ m)$.

Consider a lot of *l* elements comprising *m* "marked" and *l*-*m* "unmarked" elements. A trial sampling without replacement of exactly *s* elements from this lot helps to define the hypergeometric distribution, which is the probability that the group of *s* elements contains exactly *k* marked elements.

The probability distribution is given by:)

$$P(X = k) = \frac{C_m^k C_{l-m}^{s-k}}{C_l^s}$$

, $k \in$ {max(0, s + m - l), ..., min(s, m)}

The cumulative distribution function is as follows:

$$F_{l,s,m}(x) = \begin{cases} 0, & x < \max(0, s + m - l) \\ \sum_{k=\max(0,s+m-l)}^{\lfloor x \rfloor} \frac{C_m^k C_{l-m}^{s-k}}{C_l^s}, & \max(0, s + m - l) \le x \le \min(s, m) \\ 1, & x > \min(s, m) \end{cases}$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngPoisson

Generates Poisson distributed random values.

Syntax

Fortran:

```
status = virngpoisson( method, stream, n, r, lambda )
```

C:

```
status = viRngPoisson( method, stream, n, r, lambda );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific values are as follows: VSL_RNG_METHOD_POISSON_PTPE VSL_RNG_METHOD_POISSON_POISNORM See brief description of the PTPE and POISNORM methods in Table "Values of <method> in method parameter".
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
lambda	**FORTRAN 77:** DOUBLE PRECISION **Fortran 90:** REAL(KIND=8), INTENT(IN) **C:** const double	Distribution parameter λ.

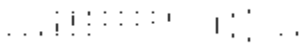
Output Parameters

Name	Type	Description
r	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(OUT) **C:** int*	Vector of *n* Poisson distributed random values

Description

The vRng"Poisson function generates Poisson distributed random numbers with distribution parameter λ, where $\lambda \in R;\ \lambda > 0$.

The probability distribution is given by:



$k \in \{0, 1, 2, \ldots\}$.

The cumulative distribution function is as follows:

$$F_\lambda(x) = \begin{cases} \sum_{k=0}^{\lfloor x \rfloor} \frac{\lambda^k e^{-\lambda}}{k!}, & x \geq 0 \\ 0, & x < 0 \end{cases}, x \in R$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngPoissonV

Generates Poisson distributed random values with varying mean.

Syntax

Fortran:

```
status = virngpoissonv( method, stream, n, r, lambda )
```

C:

```
status = viRngPoissonV( method, stream, n, r, lambda );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific value is as follows: VSL_RNG_METHOD_POISSONV_POISNORM See brief description of the POISNORM method in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN)	**Fortran:** Descriptor of the stream state structure. **C:** Pointer to the stream state structure

Name	Type	Description
	C: `VSLStreamStatePtr`	
n	**FORTRAN 77:** `INTEGER`	Number of random values to be generated
	Fortran 90: `INTEGER, INTENT(IN)`	
	C: `const int`	
lambda	**FORTRAN 77:** `DOUBLE PRECISION`	Array of *n* distribution parameters λ_i.
	Fortran 90: `REAL(KIND=8), INTENT(IN)`	
	C: `const double*`	

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** `INTEGER*4`	Vector of *n* Poisson distributed random values
	Fortran 90: `INTEGER(KIND=4), INTENT(OUT)`	
	C: `int*`	

Description

The `vRngPoissonV` function generates *n* Poisson distributed random numbers $x_i (i = 1, ..., n)$ with distribution parameter λ_i, where $\lambda_i \in R$; $\lambda_i > 0$.

The probability distribution is given by:

$$P(X_i = k) = \frac{\lambda_i^k \exp(-\lambda_i)}{k!}, k \in \{0, 1, 2, ...\}.$$

The cumulative distribution function is as follows:

$$F_{\lambda_i}(x) = \begin{cases} \sum_{k=0}^{\lfloor x \rfloor} \frac{\lambda_i^k e^{-\lambda_i}}{k!}, & x \geq 0 \\ 0, & x < 0 \end{cases}, x \in R$$

Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_ERROR_NULL_PTR`	*stream* is a `NULL` pointer.
`VSL_RNG_ERROR_BAD_STREAM`	*stream* is not a valid random stream.

VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

vRngNegBinomial

Generates random numbers with negative binomial distribution.

Syntax

Fortran:

```
status = virngnegbinomial( method, stream, n, r, a, p )
```

C:

```
status = viRngNegbinomial( method, stream, n, r, a, p );
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
method	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Generation method. The specific value is: VSL_RNG_METHOD_NEGBINOMIAL_NBAR See brief description of the NBAR method in Table "Values of <*method*> in *method* parameter"
stream	**FORTRAN 77:** INTEGER*4 stream(2) **Fortran 90:** TYPE (VSL_STREAM_STATE), INTENT(IN) **C:** VSLStreamStatePtr	**Fortran:** descriptor of the stream state structure. **C:** pointer to the stream state structure
n	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, INTENT(IN) **C:** const int	Number of random values to be generated
a	**FORTRAN 77:** DOUBLE PRECISION **Fortran 90:** REAL(KIND=8), INTENT(IN) **C:** const double	The first distribution parameter *a*

Name	Type	Description
p	**FORTRAN 77:** DOUBLE PRECISION **Fortran 90:** REAL(KIND=8), INTENT(IN) **C:** const double	The second distribution parameter p

Output Parameters

Name	Type	Description
r	**FORTRAN 77:** INTEGER*4 **Fortran 90:** INTEGER(KIND=4), INTENT(OUT) **C:** int*	Vector of *n* random values with negative binomial distribution.

Description

The vRngNegBinomial function generates random numbers with negative binomial distribution and distribution parameters *a* and *p*, where *p*, *a*∈*R*; 0 < *p* < 1; *a* > 0.

If the first distribution parameter *a*∈*N*, this distribution is the same as Pascal distribution. If *a*∈*N*, the distribution can be interpreted as the expected time of *a*-th success in a sequence of Bernoulli trials, when the probability of success is *p*.

The probability distribution is given by:

$$P(X = k) = C_{a+k-1}^{k} p^{a}(1-p)^{k}, k \in \{0, 1, 2, \ldots\}.$$

The cumulative distribution function is as follows:

$$F_{a,p}(x) = \begin{cases} \sum_{k=0}^{\lfloor x \rfloor} C_{a+k-1}^{k} p^{a}(1-p)^{k}, & x \geq 0 \\ 0, & x < 0 \end{cases}, x \in R$$

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_ERROR_NULL_PTR	*stream* is a NULL pointer.
VSL_RNG_ERROR_BAD_STREAM	*stream* is not a valid random stream.
VSL_RNG_ERROR_BAD_UPDATE	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax.
VSL_RNG_ERROR_NO_NUMBERS	Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.
VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED	Period of the generator has been exceeded.

Advanced Service Routines

This section describes service routines for registering a user-designed basic generator (vslRegisterBrng) and for obtaining properties of the previously registered basic generators (vslGetBrngProperties). See VSL Notes ("Basic Generators" section of VSL Structure chapter) for substantiation of the need for several basic generators including user-defined BRNGs.

Data types

The Advanced Service routines refer to a structure defining the properties of the basic generator. This structure is described in Fortran 90 as follows:

```
TYPE VSL_BRNG_PROPERTIES
    INTEGER streamstatesize
    INTEGER nseeds
    INTEGER includeszero
    INTEGER wordsize
    INTEGER nbits
    INTEGER nitstream
    INTEGER sbrng
    INTEGER dbrng
    INTEGER ibrng
END TYPE VSL_BRNG_PROPERTIES
```

The C version is as follows:

```
typedef struct _VSLBRngProperties {
       int StreamStateSize;
       int NSeeds;
       int IncludesZero;
       int WordSize;
       int NBits;
       InitStreamPtr InitStream;
       sBRngPtr sBRng;
       dBRngPtr dBRng;
       iBRngPtr iBRng;
} VSLBRngProperties;
```

The following table provides brief descriptions of the fields engaged in the above structure:

Field Descriptions

Field	Short Description
Fortran: `streamstatesize` **C:** `StreamStateSize`	The size, in bytes, of the stream state structure for a given basic generator.
Fortran: `nseeds` **C:** `NSeeds`	The number of 32-bit initial conditions (seeds) necessary to initialize the stream state structure for a given basic generator.

Field	Short Description
Fortran: includeszero **C:** IncludesZero	Flag value indicating whether the generator can produce a random 0.
Fortran: wordsize **C:** WordSize	Machine word size, in bytes, used in integer-value computations. Possible values: 4, 8, and 16 for 32, 64, and 128-bit generators, respectively.
Fortran: nbits **C:** NBits	The number of bits required to represent a random value in integer arithmetic. Note that, for instance, 48-bit random values are stored to 64-bit (8 byte) memory locations. In this case, wordsize/WordSize is equal to 8 (number of bytes used to store the random value), while nbits/NBits contains the actual number of bits occupied by the value (in this example, 48).
Fortran: initstream **C:** InitStream	Contains the pointer to the initialization routine of a given basic generator.
Fortran: sbrng **C:** sBRng	Contains the pointer to the basic generator of single precision real numbers uniformly distributed over the interval (*a*,*b*) (real in Fortran and float in C).
Fortran: dbrng **C:** dBRng	Contains the pointer to the basic generator of double precision real numbers uniformly distributed over the interval (*a*,*b*) (double PRECISION in Fortran and double in C).
Fortran: ibrng **C:** iBRng	Contains the pointer to the basic generator of integer numbers with uniform bit distribution[1] (INTEGER in Fortran and unsigned int in C).

[1]A specific generator that permits operations over single bits and bit groups of random numbers.

vslRegisterBrng

Registers user-defined basic generator.

Syntax

Fortran:

```
brng = vslregisterbrng( properties )
```

C:

```
brng = vslRegisterBrng( &properties );
```

Include Files

- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
properties	**Fortran:** TYPE(VSL_BRNG_PROPERTIES), INTENT(IN) **C:** const VSLBRngProperties*	Pointer to the structure containing properties of the basic generator to be registered

NOTE FORTRAN 77 support is unavailable for this function.

Output Parameters

Name	Type	Description
brng	**Fortran:** INTEGER, INTENT(OUT) **C:** int	Number (index) of the registered basic generator; used for identification. Negative values indicate the registration error.

Description

An example of a registration procedure can be found in the respective directory of the VSL examples.

Return Values

VSL_ERROR_OK, VSL_STATUS_OK	Indicates no error, execution is successful.
VSL_RNG_ERROR_BRNG_TABLE_FULL	Registration cannot be completed due to lack of free entries in the table of registered BRNGs.
VSL_RNG_ERROR_BAD_STREAM_STATE_SIZE	Bad value in StreamStateSize field.
VSL_RNG_ERROR_BAD_WORD_SIZE	Bad value in WordSize field.
VSL_RNG_ERROR_BAD_NSEEDS	Bad value in NSeeds field.
VSL_RNG_ERROR_BAD_NBITS	Bad value in NBits field.
VSL_ERROR_NULL_PTR	At least one of the fields iBrng, dBrng, sBrng or InitStream is a NULL pointer.

vslGetBrngProperties

Returns structure with properties of a given basic generator.

Syntax

Fortran:

```
status = vslgetbrngproperties( brng, properties )
```

C:

```
status = vslGetBrngProperties( brng, &properties );
```

Include Files

- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
brng	**Fortran:** INTEGER(KIND=4), INTENT(IN) **C:** const int	Number (index) of the registered basic generator; used for identification. See specific values in Table "Values of *brng* parameter". Negative values indicate the registration error.

NOTE FORTRAN 77 support is unavailable for this function.

Output Parameters

Name	Type	Description
properties	**Fortran:** `TYPE(VSL_BRNG_PROPERTIES), INTENT(OUT)` **C:** `VSLBRngProperties*`	Pointer to the structure containing properties of the generator with number *brng*

Description

The `vslGetBrngProperties` function returns a structure with properties of a given basic generator.

Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK`	Indicates no error, execution is successful.
`VSL_RNG_ERROR_INVALID_BRNG_INDEX`	BRNG index is invalid.

Formats for User-Designed Generators

To register a user-designed basic generator using `vslRegisterBrng` function, you need to pass the pointer `iBrng` to the integer-value implementation of the generator; the pointers `sBrng` and `dBrng` to the generator implementations for single and double precision values, respectively; and pass the pointer `InitStream` to the stream initialization routine. See below recommendations on defining such functions with input and output arguments. An example of the registration procedure for a user-designed generator can be found in the respective directory of VSL examples.

The respective pointers are defined as follows:

```
typedef int(*InitStreamPtr)( int method, VSLStreamStatePtr stream, int n, const unsigned int params[] );
typedef int(*sBRngPtr)( VSLStreamStatePtr stream, int n, float r[], float a, float b );
typedef int(*dBRngPtr)( VSLStreamStatePtr stream, int n, double r[], double a, double b );
typedef int(*iBRngPtr)( VSLStreamStatePtr stream, int n, unsigned int r[] );
```

InitStream

C:

```
int MyBrngInitStream( int method, VSLStreamStatePtr stream, int n, const unsigned int params[] )
{
      /* Initialize the stream */
      ...
} /* MyBrngInitStream */
```

Description

The initialization routine of a user-designed generator must initialize *stream* according to the specified initialization *method*, initial conditions *params* and the argument *n*. The value of *method* determines the initialization method to be used.

- If *method* is equal to `1`, the initialization is by the standard generation method, which must be supported by all basic generators. In this case the function assumes that the *stream* structure was not previously initialized. The value of *n* is used as the actual number of 32-bit values passed as initial conditions

through *params*. Note, that the situation when the actual number of initial conditions passed to the function is not sufficient to initialize the generator is not an error. Whenever it occurs, the basic generator must initialize the missing conditions using default settings.

- If *method* is equal to 2, the generation is by the leapfrog method, where *n* specifies the number of computational nodes (independent streams). Here the function assumes that the *stream* was previously initialized by the standard generation method. In this case *params* contains only one element, which identifies the computational node. If the generator does not support the leapfrog method, the function must return the error code VSL_ERROR_LEAPFROG_UNSUPPORTED.
- If *method* is equal to 3, the generation is by the block-splitting method. Same as above, the *stream* is assumed to be previously initialized by the standard generation method; *params* is not used, *n* identifies the number of skipped elements. If the generator does not support the block-splitting method, the function must return the error code VSL_ERROR_SKIPAHEAD_UNSUPPORTED.

For a more detailed description of the leapfrog and the block-splitting methods, refer to the description of vslLeapfrogStream and vslSkipAheadStream, respectively.

Stream state structure is individual for every generator. However, each structure has a number of fields that are the same for all the generators:

C:

```
typedef struct
{
    unsigned int Reserved1[2];
    unsigned int Reserved2[2];
    [fields specific for the given generator]
} MyStreamState;
```

The fields *Reserved1* and *Reserved2* are reserved for private needs only, and must not be modified by the user. When including specific fields into the structure, follow the rules below:

- The fields must fully describe the current state of the generator. For example, the state of a linear congruential generator can be identified by only one initial condition;
- If the generator can use both the leapfrog and the block-splitting methods, additional fields should be introduced to identify the independent streams. For example, in *LCG*(*a*, *c*, *m*), apart from the initial conditions, two more fields should be specified: the value of the multiplier a^k and the value of the increment $(a^k-1)c/(a-1)$.

For a more detailed discussion, refer to [Knuth81], and [Gentle98]. An example of the registration procedure can be found in the respective directory of VSL examples.

iBRng

C:

```
int iMyBrng( VSLStreamStatePtr stream, int n, unsigned int r[] )
{
    int i; /* Loop variable */
    /* Generating integer random numbers */
    /* Pay attention to word size needed to
    store only random number */
    for( i = 0; i < n; i++)
    {
        r[i] = ...;
    }
    /* Update stream state */
    ...
    return errcode;
} /* iMyBrng */
```

NOTE When using 64 and 128-bit generators, consider digit capacity to store the numbers to the random vector *r* correctly. For example, storing one 64-bit value requires two elements of *r*, the first to store the lower 32 bits and the second to store the higher 32 bits. Similarly, use 4 elements of *r* to store a 128-bit value.

sBRng

C:

```
int sMyBrng( VSLStreamStatePtr stream, int n, float r[], float a, float b )
{
    int   i;    /* Loop variable */
    /* Generating float (a,b) random numbers */
    for ( i = 0; i < n; i++ )
    {
        r[i] = ...;
    }
    /* Update stream state */
    ...
    return errcode;
} /* sMyBrng */
```

dBRng

C:

```
int dMyBrng( VSLStreamStatePtr stream, int n, double r[], double a, double b )
{
    int i;   /* Loop variable */
    /* Generating double (a,b) random numbers */
    for ( i = 0; i < n; i++ )
    {
        r[i] = ...;
    }
    /* Update stream state */
    ...
    return errcode;
} /* dMyBrng */
```

Convolution and Correlation

Intel MKL VSL provides a set of routines intended to perform linear convolution and correlation transformations for single and double precision real and complex data.

For correct definition of implemented operations, see the Mathematical Notation and Definitions section.

The current implementation provides:

- Fourier algorithms for one-dimensional single and double precision real and complex data
- Fourier algorithms for multi-dimensional single and double precision real and complex data
- Direct algorithms for one-dimensional single and double precision real and complex data
- Direct algorithms for multi-dimensional single and double precision real and complex data

One-dimensional algorithms cover the following functions from the IBM* ESSL library:

```
SCONF, SCORF
SCOND, SCORD
SDCON, SDCOR
DDCON, DDCOR
SDDCON, SDDCOR.
```

Special wrappers are designed to simulate these ESSL functions. The wrappers are provided as sample sources for Fortran and C. To reuse them, use the following directories:

```
${MKL}/examples/vslc/essl/vsl_wrappers
${MKL}/examples/vslf/essl/vsl_wrappers
```

Additionally, you can browse the examples demonstrating the calculation of the ESSL functions through the wrappers. You can find the examples in the following directories:

```
${MKL}/examples/vslc/essl
${MKL}/examples/vslf/essl
```

Convolution and correlation API provides interfaces for FORTRAN 77, Fortran 90 and C/89 languages. You may use the C/89 interface also with later versions of C or C++, or Fortran 90 interface with programs written in Fortran 95.

For users of the C/C++ and Fortran languages, the `mkl_vsl.h`, `mkl_vsl.f90`, and `mkl_vsl.f77` headers are provided. All header files are found under the directory:

`${MKL}/include`

See more details about the Fortran header in Random Number Generators section of this chapter.

Convolution and correlation API is implemented through task objects, or tasks. Task object is a data structure, or descriptor, which holds parameters that determine the specific convolution or correlation operation. Such parameters may be precision, type, and number of dimensions of user data, an identifier of the computation algorithm to be used, shapes of data arrays, and so on.

All the Intel MKL VSL convolution and correlation routines process task objects in one way or another: either create a new task descriptor, change the parameter settings, compute mathematical results of the convolution or correlation using the stored parameters, or perform other operations. Accordingly, all routines are split into the following groups:

Task Constructors - routines that create a new task object descriptor and set up most common parameters.

Task Editors - routines that can set or modify some parameter settings in the existing task descriptor.

Task Execution Routines - compute results of the convolution or correlation operation over the actual input data, using the operation parameters held in the task descriptor.

Task Copy - routines used to make several copies of the task descriptor.

Task Destructors - routines that delete task objects and free the memory.

When the task is executed or copied for the first time, a special process runs which is called task commitment. During this process, consistency of task parameters is checked and the required work data are prepared. If the parameters are consistent, the task is tagged as committed successfully. The task remains committed until you edit its parameters. Hence, the task can be executed multiple times after a single commitment process. Since the task commitment process may include costly intermediate calculations such as preparation of Fourier transform of input data, launching the process only once can help speed up overall performance.

Naming Conventions

The names of Fortran routines in the convolution and correlation API are written in lowercase (`vslsconvexec`), while the names of Fortran types and constants are written in uppercase. The names are not case-sensitive.

In C, the names of routines, types, and constants are case-sensitive and can be lowercase and uppercase (`vslsConvExec`).

The names of routines have the following structure:

`vsl[`*`datatype`*`]{Conv|Corr}<base name>` for the C interface

`vsl[`*`datatype`*`]{conv|corr}<base name>` for the Fortran interface

where

- `vsl` is a prefix indicating that the routine belongs to Vector Statistical Library of Intel® MKL.
- `[`*`datatype`*`]` is optional. If present, the symbol specifies the type of the input and output data and can be `s` (for single precision real type), `d` (for double precision real type), `c` (for single precision complex type), or `z` (for double precision complex type).
- `Conv` or `Corr` specifies whether the routine refers to convolution or correlation task, respectively.
- `<base name>` field specifies a particular functionality that the routine is designed for, for example, `NewTask`, `DeleteTask`.

Data Types

All convolution or correlation routines use the following types for specifying data objects:

Type	Data Object
FORTRAN 77: `INTEGER*4 task(2)`	Pointer to a task descriptor for convolution

Type	Data Object
Fortran 90: `TYPE(VSL_CONV_TASK)` **C:** `VSLConvTaskPtr`	
FORTRAN 77: `INTEGER*4 task(2)` **Fortran 90:** `TYPE(VSL_CORR_TASK)` **C:** `VSLCorrTaskPtr`	Pointer to a task descriptor for correlation
FORTRAN 77: `REAL*4` **Fortran 90:** `REAL(KIND=4)` **C:** `float`	Input/output user real data in single precision
FORTRAN 77: `REAL*8` **Fortran 90:** `REAL(KIND=8)` **C:** `double`	Input/output user real data in double precision
FORTRAN 77: `COMLEX*8` **Fortran 90:** `COMPLEX(KIND=4)` **C:** `MKL_Complex8`	Input/output user complex data in single precision
FORTRAN 77: `COMPLEX*16` **Fortran 90:** `COMPLEX(KIND=8)` **C:** `MKL_Complex16`	Input/output user complex data in double precision
FORTRAN 77: `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	All other data

Generic integer type (without specifying the byte size) is used for all integer data.

NOTE The actual size of the generic integer type is platform-dependent. Before you compile your application, set an appropriate byte size for integers. See details in the 'Using the ILP64 Interface vs. LP64 Interface' section of the *Intel© MKL User's Guide*.

Parameters

Basic parameters held by the task descriptor are assigned values when the task object is created, copied, or modified by task editors. Parameters of the correlation or convolution task are initially set up by task constructors when the task object is created. Parameter changes or additional settings are made by task editors. More parameters which define location of the data being convolved need to be specified when the task execution routine is invoked.

According to how the parameters are passed or assigned values, all of them can be categorized as either explicit (directly passed as routine parameters when a task object is created or executed) or optional (assigned some default or implicit values during task construction).

The following table lists all applicable parameters used in the Intel MKL convolution and correlation API.

Convolution and Correlation Task Parameters

Name	Category	Type	Default Value Label	Description
job	explicit	integer	Implied by the constructor name	Specifies whether the task relates to convolution or correlation
type	explicit	integer	Implied by the constructor name	Specifies the type (real or complex) of the input/output data. Set to real in the current version.
precision	explicit	integer	Implied by the constructor name	Specifies precision (single or double) of the input/output data to be provided in arrays *x*,*y*,*z*.
mode	explicit	integer	None	Specifies whether the convolution/ correlation computation should be done via Fourier transforms, or by a direct method, or by automatically choosing between the two. See SetMode for the list of named constants for this parameter.
method	optional	integer	"auto"	Hints at a particular computation method if several methods are available for the given *mode*. Setting this parameter to "auto" means that software will choose the best available method.
internal_precision	optional	integer	Set equal to the value of *precision*	Specifies precision of internal calculations. Can enforce double precision calculations even when input/output data are single precision. See SetInternalPrecision for the list of named constants for this parameter.
dims	explicit	integer	None	Specifies the rank (number of dimensions) of the user data provided in arrays *x*,*y*,*z*. Can be in the range from 1 to 7.
x,*y*	explicit	real arrays	None	Specify input data arrays. See Data Allocation for more information.
z	explicit	real array	None	Specifies output data array. See Data Allocation for more information.
xshape, *yshape*, *zshape*	explicit	integer arrays	None	Define shapes of the arrays *x*, *y*, *z*. See Data Allocation for more information.
xstride, *ystride*, *zstride*	explicit	integer arrays	None	Define strides within arrays *x*, *y*, *z*, that is specify the physical location of the input and output data in these arrays. See Data Allocation for more information.
start	optional	integer array	Undefined	Defines the first element of the mathematical result that will be stored to output array *z*. See SetStart and Data Allocation for more information.

Name	Category	Type	Default Value Label	Description
decimation	optional	integer array	Undefined	Defines how to thin out the mathematical result that will be stored to output array *z*. See SetDecimation and Data Allocation for more information.

Users of the C or C++ language may pass the NULL pointer instead of either or all of the parameters *xstride*, *ystride*, or *zstride* for multi-dimensional calculations. In this case, the software assumes the dense data allocation for the arrays *x*, *y*, or *z* due to the Fortran-style "by columns" representation of multi-dimensional arrays.

Task Status and Error Reporting

The task status is an integer value, which is zero if no error has been detected while processing the task, or a specific non-zero error code otherwise. Negative status values indicate errors, and positive values indicate warnings.

An error can be caused by invalid parameter values, a system fault like a memory allocation failure, or can be an internal error self-detected by the software.

Each task descriptor contains the current status of the task. When creating a task object, the constructor assigns the VSL_STATUS_OK status to the task. When processing the task afterwards, other routines such as editors or executors can change the task status if an error occurs and write a corresponding error code into the task status field.

Note that at the stage of creating a task or editing its parameters, the set of parameters may be inconsistent. The parameter consistency check is only performed during the task commitment operation, which is implicitly invoked before task execution or task copying. If an error is detected at this stage, task execution or task copying is terminated and the task descriptor saves the corresponding error code. Once an error occurs, any further attempts to process that task descriptor is terminated and the task keeps the same error code.

Normally, every convolution or correlation function (except DeleteTask) returns the status assigned to the task while performing the function operation.

The status codes are given symbolic names defined in the respective header files. For the C/C++ interface, these names are defined as macros via the #define statements, and for the Fortran interface as integer constants via the PARAMETER operators.

If there is no error, the VSL_STATUS_OK status is returned, which is defined as zero:

```
C/C++:          #define VSL_STATUS_OK 0
F90/F95:        INTEGER(KIND=4) VSL_STATUS_OK
                PARAMETER(VSL_STATUS_OK = 0)

F77:            INTEGER*4 VSL_STATUS_OK
                PARAMETER(VSL_STATUS_OK = 0)
```

In case of an error, a non-zero error code is returned, which indicates the origin of the failure. The following status codes for the convolution/correlation error codes are pre-defined in the header files for both C/C++ and Fortran languages.

Convolution/Correlation Status Codes

Status Code	Description
VSL_CC_ERROR_NOT_IMPLEMENTED	Requested functionality is not implemented.
VSL_CC_ERROR_ALLOCATION_FAILURE	Memory allocation failure.
VSL_CC_ERROR_BAD_DESCRIPTOR	Task descriptor is corrupted.

Status Code	Description
VSL_CC_ERROR_SERVICE_FAILURE	A service function has failed.
VSL_CC_ERROR_EDIT_FAILURE	Failure while editing the task.
VSL_CC_ERROR_EDIT_PROHIBITED	You cannot edit this parameter.
VSL_CC_ERROR_COMMIT_FAILURE	Task commitment has failed.
VSL_CC_ERROR_COPY_FAILURE	Failure while copying the task.
VSL_CC_ERROR_DELETE_FAILURE	Failure while deleting the task.
VSL_CC_ERROR_BAD_ARGUMENT	Bad argument or task parameter.
VSL_CC_ERROR_JOB	Bad parameter: *job*.
SL_CC_ERROR_KIND	Bad parameter: *kind*.
VSL_CC_ERROR_MODE	Bad parameter: *mode*.
VSL_CC_ERROR_METHOD	Bad parameter: *method*.
VSL_CC_ERROR_TYPE	Bad parameter: *type*.
VSL_CC_ERROR_EXTERNAL_PRECISION	Bad parameter: *external_precision*.
VSL_CC_ERROR_INTERNAL_PRECISION	Bad parameter: *internal_precision*.
VSL_CC_ERROR_PRECISION	Incompatible external/internal precisions.
VSL_CC_ERROR_DIMS	Bad parameter: *dims*.
VSL_CC_ERROR_XSHAPE	Bad parameter: *xshape*.
VSL_CC_ERROR_YSHAPE	Bad parameter: *yshape*.
	Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or >nmax.
VSL_CC_ERROR_ZSHAPE	Bad parameter: *zshape*.
VSL_CC_ERROR_XSTRIDE	Bad parameter: *xstride*.
VSL_CC_ERROR_YSTRIDE	Bad parameter: *ystride*.
VSL_CC_ERROR_ZSTRIDE	Bad parameter: *zstride*.
VSL_CC_ERROR_X	Bad parameter: *x*.
VSL_CC_ERROR_Y	Bad parameter: *y*.
VSL_CC_ERROR_Z	Bad parameter: *z*.
VSL_CC_ERROR_START	Bad parameter: *start*.
VSL_CC_ERROR_DECIMATION	Bad parameter: *decimation*.
VSL_CC_ERROR_OTHER	Another error.

Task Constructors

Task constructors are routines intended for creating a new task descriptor and setting up basic parameters. No additional parameter adjustment is typically required and other routines can use the task object.

Intel® MKL implementation of the convolution and correlation API provides two different forms of constructors: a general form and an X-form. X-form constructors work in the same way as the general form constructors but also assign particular data to the first operand vector used in the convolution or correlation operation (stored in array *x*).

Using X-form constructors is recommended when you need to compute multiple convolutions or correlations with the same data vector held in array *x* against different vectors held in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

Each constructor routine has an associated one-dimensional version that provides algorithmic and computational benefits.

NOTE If the constructor fails to create a task descriptor, it returns the `NULL` task pointer.

The Table "Task Constructors" lists available task constructors:

Task Constructors

Routine	Description
vslConvNewTask/vslCorrNewTask	Creates a new convolution or correlation task descriptor for a multidimensional case.
vslConvNewTask1D/ vslCorrNewTask1D	Creates a new convolution or correlation task descriptor for a one-dimensional case.
vslConvNewTaskX/vslCorrNewTaskX	Creates a new convolution or correlation task descriptor as an X-form for a multidimensional case.
vslConvNewTaskX1D/ vslCorrNewTaskX1D	Creates a new convolution or correlation task descriptor as an X-form for a one-dimensional case.

vslConvNewTask/vslCorrNewTask

Creates a new convolution or correlation task descriptor for multidimensional case.

Syntax

Fortran:

```
status = vslsconvnewtask(task, mode, dims, xshape, yshape, zshape)
status = vsldconvnewtask(task, mode, dims, xshape, yshape, zshape)
status = vslcconvnewtask(task, mode, dims, xshape, yshape, zshape)
status = vslzconvnewtask(task, mode, dims, xshape, yshape, zshape)
status = vslscorrnewtask(task, mode, dims, xshape, yshape, zshape)
status = vsldcorrnewtask(task, mode, dims, xshape, yshape, zshape)
status = vslccorrnewtask(task, mode, dims, xshape, yshape, zshape)
status = vslzcorrnewtask(task, mode, dims, xshape, yshape, zshape)
```

C:

```
status = vslsConvNewTask(task, mode, dims, xshape, yshape, zshape);
```

```
status = vsldConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslcConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslzConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslsCorrNewTask(task, mode, dims, xshape, yshape, zshape);
status = vsldCorrNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslcCorrNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslzCorrNewTask(task, mode, dims, xshape, yshape, zshape);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
mode	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See Table "Values of mode parameter" for a list of possible values.
dims	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Rank of user data. Specifies number of dimensions for the input and output arrays *x*, *y*, and *z* used during the execution stage. Must be in the range from 1 to 7. The value is explicitly assigned by the constructor.
xshape	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION(*)` **C:** `const int[]`	Defines the shape of the input data for the source array *x*. See Data Allocation for more information.
yshape	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION(*)` **C:** `const int[]`	Defines the shape of the input data for the source array *y*. See Data Allocation for more information.
zshape	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION(*)` **C:** `const int[]`	Defines the shape of the output data to be stored in array *z*. See Data Allocation for more information.

Output Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvnewtask`, `vsldconvnewtask`, `vslcconvnewtask`, `vslzconvnewtask` `INTEGER*4 task(2)` for `vslscorrnewtask`, `vsldcorrnewtask`, `vslccorrnewtask`, `vslzcorrnewtask` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvnewtask`, `vsldconvnewtask`, `vslcconvnewtask`, `vslzconvnewtask` `TYPE(VSL_CORR_TASK)` for `vslscorrnewtask`, `vsldcorrnewtask`, `vslccorrnewtask`, `vslzcorrnewtask` **C:** `VSLConvTaskPtr*` for `vslsConvNewTask`, `vsldConvNewTask`, `vslcConvNewTask`, `vslzConvNewTask` `VSLCorrTaskPtr*` for `vslsCorrNewTask`, `vsldCorrNewTask`, `vslcConvNewTask`, `vslzConvNewTask`	Pointer to the task descriptor if created successfully or `NULL` pointer otherwise.
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Set to `VSL_STATUS_OK` if the task is created successfully or set to non-zero error code otherwise.

Description

Each `vslConvNewTask`/`vslCorrNewTask` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see Table "Convolution and Correlation Task Parameters").

The parameters *xshape*, *yshape*, and *zshape* define the shapes of the input and output data provided by the arrays *x*, *y*, and *z*, respectively. Each shape parameter is an array of integers with its length equal to the value of *dims*. You explicitly assign the shape parameters when calling the constructor. If the value of the parameter *dims* is 1, then *xshape*, *yshape*, *zshape* are equal to the number of elements read from the arrays *x* and *y* or stored to the array *z*. Note that values of shape parameters may differ from physical shapes of arrays *x*, *y*, and *z* if non-trivial strides are assigned.

If the constructor fails to create a task descriptor, it returns a NULL task pointer.

vslConvNewTask1D/vslCorrNewTask1D

Creates a new convolution or correlation task descriptor for one-dimensional case.

Syntax

Fortran:

```
status = vslsconvnewtask1d(task, mode, xshape, yshape, zshape)
status = vsldconvnewtask1d(task, mode, xshape, yshape, zshape)
status = vslcconvnewtask1d(task, mode, xshape, yshape, zshape)
status = vslzconvnewtask1d(task, mode, xshape, yshape, zshape)
status = vslscorrnewtask1d(task, mode, xshape, yshape, zshape)
status = vsldcorrnewtask1d(task, mode, xshape, yshape, zshape)
status = vslccorrnewtask1d(task, mode, xshape, yshape, zshape)
status = vslzcorrnewtask1d(task, mode, xshape, yshape, zshape)
```

C:

```
status = vslsConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vsldConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vslcConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vslzConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vslsCorrNewTask1D(task, mode, xshape, yshape, zshape);
status = vsldCorrNewTask1D(task, mode, xshape, yshape, zshape);
status = vslcCorrNewTask1D(task, mode, xshape, yshape, zshape);
status = vslzCorrNewTask1D(task, mode, xshape, yshape, zshape);
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
mode	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** const int	Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See Table "Values of mode parameter" for a list of possible values.
xshape	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** const int	Defines the length of the input data sequence for the source array *x*. See Data Allocation for more information.

Name	Type	Description
yshape	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** const int	Defines the length of the input data sequence for the source array *y*. See Data Allocation for more information.
zshape	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** const int	Defines the length of the output data sequence to be stored in array *z*. See Data Allocation for more information.

Output Parameters

Name	Type	Description
task	**FORTRAN 77:** INTEGER*4 task(2) for vslsconvnewtask1d, vsldconvnewtask1d, vslcconvnewtask1d, vslzconvnewtask1d INTEGER*4 task(2) for vslscorrnewtask1d, vsldcorrnewtask1d, vslccorrnewtask1d, vslzcorrnewtask1d **Fortran 90:** TYPE(VSL_CONV_TASK) for vslsconvnewtask1d, vsldconvnewtask1d, vslcconvnewtask1d, vslzconvnewtask1d TYPE(VSL_CORR_TASK) for vslscorrnewtask1d, vsldcorrnewtask1d, vslccorrnewtask1d, vslzcorrnewtask1d **C:** VSLConvTaskPtr* for vslsConvNewTask1D, vsldConvNewTask1D, vslcConvNewTask1D, vslzConvNewTask1D VSLCorrTaskPtr* for vslsCorrNewTask1D, vsldCorrNewTask1D, vslcCorrNewTask1D, vslzCorrNewTask1D	Pointer to the task descriptor if created successfully or NULL pointer otherwise.
status	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER	Set to VSL_STATUS_OK if the task is created successfully or set to non-zero error code otherwise.

Name	Type	Description
	C: `int`	

Description

Each `vslConvNewTask1D`/`vslCorrNewTask1D` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see Table "Convolution and Correlation Task Parameters"). Unlike `vslConvNewTask`/`vslCorrNewTask`, these routines represent a special one-dimensional version of the constructor which assumes that the value of the parameter *dims* is 1. The parameters *xshape*, *yshape*, and *zshape* are equal to the number of elements read from the arrays *x* and *y* or stored to the array *z*. You explicitly assign the shape parameters when calling the constructor.

vslConvNewTaskX/vslCorrNewTaskX

Creates a new convolution or correlation task descriptor for multidimensional case and assigns source data to the first operand vector.

Syntax

Fortran:

```
status = vslsconvnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vsldconvnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vslcconvnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vslzconvnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vslscorrnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vsldcorrnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vslccorrnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
status = vslzcorrnewtaskx(task, mode, dims, xshape, yshape, zshape, x, xstride)
```

C:

```
status = vslsConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vsldConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslcConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslzConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslsCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vsldCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslcCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslzCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
mode	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** const int	Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See Table "Values of mode parameter" for a list of possible values.
dims	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** const int	Rank of user data. Specifies number of dimensions for the input and output arrays *x*, *y*, and *z* used during the execution stage. Must be in the range from 1 to 7. The value is explicitly assigned by the constructor.
xshape	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, DIMENSION(*) **C:** const int[]	Defines the shape of the input data for the source array *x*. See Data Allocation for more information.
yshape	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, DIMENSION(*) **C:** const int[]	Defines the shape of the input data for the source array *y*. See Data Allocation for more information.
zshape	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER, DIMENSION(*) **C:** const int[]	Defines the shape of the output data to be stored in array *z*.See Data Allocation for more information.
x	**FORTRAN 77:** REAL*4 for real data in single precision flavors, REAL*8 for real data in double precision flavors, COMPLEX*8 for complex data in single precision flavors, COMPLEX*16 for complex data in double precision flavors **Fortran 90:** REAL(KIND=4), DIMENSION (*) for real data in single precision flavors, REAL(KIND=8), DIMENSION (*) for real data in double precision flavors, COMPLEX(KIND=4), DIMENSION (*) for complex data in single precision flavors, COMPLEX(KIND=8), DIMENSION (*) for complex data in double precision flavors	Pointer to the array containing input data for the first operand vector.See Data Allocation for more information.

Name	Type	Description
	C: `const float[]` for real data in single precision flavors, `const double[]` for real data in double precision flavors, `const MKL_Complex8[]` for complex data in single precision flavors, `const MKL_Complex16[]` for complex data in double precision flavors	
xstride	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION(*)` **C:** `const int[]`	Strides for input data in the array *x*.

Output Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvnewtaskx`, `vsldconvnewtaskx`, `vslcconvnewtaskx`, `vslzconvnewtaskx` `INTEGER*4 task(2)` for `vslscorrnewtaskx`, `vsldcorrnewtaskx`, `vslccorrnewtaskx`, `vslzcorrnewtaskx` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvnewtaskx`, `vsldconvnewtaskx`, `vslcconvnewtaskx`, `vslzconvnewtaskx` `TYPE(VSL_CORR_TASK)` for `vslscorrnewtaskx`, `vsldcorrnewtaskx`, `vslccorrnewtaskx`, `vslzcorrnewtaskx` **C:** `VSLConvTaskPtr*` for `vslsConvNewTaskX`, `vsldConvNewTaskX`, `vslcConvNewTaskX`, `vslzConvNewTaskX`	Pointer to the task descriptor if created successfully or `NULL` pointer otherwise.

Name	Type	Description
	VSLCorrTaskPtr* for vslsCorrNewTaskX, vsldCorrNewTaskX, vslcCorrNewTaskX, vslzCorrNewTaskX	
status	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** int	Set to VSL_STATUS_OK if the task is created successfully or set to non-zero error code otherwise.

Description

Each vslConvNewTaskX/vslCorrNewTaskX constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see Table "Convolution and Correlation Task Parameters").

Unlike vslConvNewTask/vslCorrNewTask, these routines represent the so called X-form version of the constructor, which means that in addition to creating the task descriptor they assign particular data to the first operand vector in array *x* used in convolution or correlation operation. The task descriptor created by the vslConvNewTaskX/vslCorrNewTaskX constructor keeps the pointer to the array *x* all the time, that is, until the task object is deleted by one of the destructor routines (see vslConvDeleteTask/ vslCorrDeleteTask).

Using this form of constructors is recommended when you need to compute multiple convolutions or correlations with the same data vector in array *x* against different vectors in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

The parameters *xshape*, *yshape*, and *zshape* define the shapes of the input and output data provided by the arrays *x*, *y*, and *z*, respectively. Each shape parameter is an array of integers with its length equal to the value of *dims*. You explicitly assign the shape parameters when calling the constructor. If the value of the parameter *dims* is 1, then *xshape*, *yshape*, and *zshape* are equal to the number of elements read from the arrays *x* and *y* or stored to the array *z*. Note that values of shape parameters may differ from physical shapes of arrays *x*, *y*, and *z* if non-trivial strides are assigned.

The stride parameter *xstride* specifies the physical location of the input data in the array *x*. In a one-dimensional case, stride is an interval between locations of consecutive elements of the array. For example, if the value of the parameter *xstride* is s, then only every s^{th} element of the array *x* will be used to form the input sequence. The stride value must be positive or negative but not zero.

vslConvNewTaskX1D/vslCorrNewTaskX1D

Creates a new convolution or correlation task descriptor for one-dimensional case and assigns source data to the first operand vector.

Syntax

Fortran:

```
status = vslsconvnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
status = vsldconvnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
status = vslcconvnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
status = vslzconvnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
```

```
status = vslscorrnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
status = vsldcorrnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
status = vslccorrnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
status = vslzcorrnewtaskx1d(task, mode, xshape, yshape, zshape, x, xstride)
```

C:

```
status = vslsConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vsldConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslcConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslzConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslsCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vsldCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslcCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslzCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
mode	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See Table "Values of mode parameter" for a list of possible values.
xshape	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Defines the length of the input data sequence for the source array *x*. See Data Allocation for more information.
yshape	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Defines the length of the input data sequence for the source array *y*. See Data Allocation for more information.
zshape	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Defines the length of the output data sequence to be stored in array *z*. See Data Allocation for more information.
x	**FORTRAN 77:** `REAL*4` for real data in single precision flavors, `REAL*8` for real data in double precision flavors, `COMPLEX*8` for complex data in single precision flavors,	Pointer to the array containing input data for the first operand vector. See Data Allocation for more information.

Name	Type	Description
	`COMPLEX*16` for complex data in double precision flavors **Fortran 90:** `REAL(KIND=4), DIMENSION (*)` for real data in single precision flavors, `REAL(KIND=8), DIMENSION (*)` for real data in double precision flavors, `COMPLEX(KIND=4), DIMENSION (*)` for complex data in single precision flavors, `COMPLEX(KIND=8), DIMENSION (*)` for complex data in double precision flavors **C:** `const float[]` for real data in single precision flavors, `const double[]` for real data in double precision flavors, `const MKL_Complex8[]` for complex data in single precision flavors, `const MKL_Complex16[]` for complex data in double precision flavors	
xstride	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Stride for input data sequence in the array *x*.

Output Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvnewtaskx1d`, `vsldconvnewtaskx1d`, `vslcconvnewtaskx1d`, `vslzconvnewtaskx1d` `INTEGER*4 task(2)` for `vslscorrnewtaskx1d`, `vsldcorrnewtaskx1d`, `vslccorrnewtaskx1d`, `vslzcorrnewtaskx1d` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvnewtaskx1d`,	Pointer to the task descriptor if created successfully or `NULL` pointer otherwise.

Name	Type	Description
	`vsldconvnewtaskx1d`, `vslcconvnewtaskx1d`, `vslzconvnewtaskx1d` `TYPE(VSL_CORR_TASK)` for `vslscorrnewtaskx1d`, `vsldcorrnewtaskx1d`, `vslccorrnewtaskx1d`, `vslzcorrnewtaskx1d` **C:** `VSLConvTaskPtr*` for `vslsConvNewTaskX1D`, `vsldConvNewTaskX1D`, `vslcConvNewTaskX1D`, `vslzConvNewTaskX1D` `VSLCorrTaskPtr*` for `vslsCorrNewTaskX1D`, `vsldCorrNewTaskX1D`, `vslcCorrNewTaskX1D`, `vslzCorrNewTaskX1D`	
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Set to `VSL_STATUS_OK` if the task is created successfully or set to non-zero error code otherwise.

Description

Each `vslConvNewTaskX1D`/`vslCorrNewTaskX1D` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see Table "Convolution and Correlation Task Parameters").

These routines represent a special one-dimensional version of the so called X-form of the constructor. This assumes that the value of the parameter *dims* is 1 and that in addition to creating the task descriptor, constructor routines assign particular data to the first operand vector in array *x* used in convolution or correlation operation. The task descriptor created by the `vslConvNewTaskX1D`/`vslCorrNewTaskX1D` constructor keeps the pointer to the array *x* all the time, that is, until the task object is deleted by one of the destructor routines (see `vslConvDeleteTask`/`vslCorrDeleteTask`).

Using this form of constructors is recommended when you need to compute multiple convolutions or correlations with the same data vector in array *x* against different vectors in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

The parameters *xshape*, *yshape*, and *zshape* are equal to the number of elements read from the arrays *x* and *y* or stored to the array *z*. You explicitly assign the shape parameters when calling the constructor.

The stride parameters *xstride* specifies the physical location of the input data in the array *x* and is an interval between locations of consecutive elements of the array. For example, if the value of the parameter *xstride* is `s`, then only every s^{th} element of the array *x* will be used to form the input sequence. The stride value must be positive or negative but not zero.

Task Editors

Task editors in convolution and correlation API of Intel MKL are routines intended for setting up or changing the following task parameters (see Table "Convolution and Correlation Task Parameters"):

- *mode*
- *internal_precision*
- *start*
- *decimation*

For setting up or changing each of the above parameters, a separate routine exists.

NOTE Fields of the task descriptor structure are accessible only through the set of task editor routines provided with the software.

The work data computed during the last commitment process may become invalid with respect to new parameter settings. That is why after applying any of the editor routines to change the task descriptor settings, the task loses its commitment status and goes through the full commitment process again during the next execution or copy operation. For more information on task commitment, see the Introduction to Convolution and Correlation.

Table "Task Editors" lists available task editors.

Task Editors

Routine	Description
vslConvSetMode/vslCorrSetMode	Changes the value of the parameter *mode* for the operation of convolution or correlation.
vslConvSetInternalPrecision/ vslCorrSetInternalPrecision	Changes the value of the parameter *internal_precision* for the operation of convolution or correlation.
vslConvSetStart/vslCorrSetStart	Sets the value of the parameter *start* for the operation of convolution or correlation.
vslConvSetDecimation/ vslCorrSetDecimation	Sets the value of the parameter *decimation* for the operation of convolution or correlation.

NOTE You can use the NULL task pointer in calls to editor routines. In this case, the routine is terminated and no system crash occurs.

vslConvSetMode/vslCorrSetMode

Changes the value of the parameter mode in the convolution or correlation task descriptor.

Syntax

Fortran:

```
status = vslconvsetmode(task, newmode)
status = vslcorrsetmode(task, newmode)
```

C:

```
status = vslConvSetMode(task, newmode);
status = vslCorrSetMode(task, newmode);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslconvsetmode` `INTEGER*4 task(2)` for `vslcorrsetmode` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvsetmode` `TYPE(VSL_CORR_TASK)` for `vslcorrsetmode` **C:** `VSLConvTaskPtr` for `vslConvSetMode` `VSLCorrTaskPtr` for `vslCorrSetMode`	Pointer to the task descriptor.
newmode	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	New value of the parameter `mode`.

Output Parameters

Name	Type	Description
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Current status of the task.

Description

This function is declared in `mkl_vsl.f77` for FORTRAN 77 interface, in `mkl_vsl.f90` for Fortran 90 interface, and in `mkl_vsl_functions.h` for C interface.

The function routine changes the value of the parameter `mode` for the operation of convolution or correlation. This parameter defines whether the computation should be done via Fourier transforms of the input/output data or using a direct algorithm. Initial value for `mode` is assigned by a task constructor.

Predefined values for the `mode` parameter are as follows:

Values of *mode* parameter

Value	Purpose
`VSL_CONV_MODE_FFT`	Compute convolution by using fast Fourier transform.
`VSL_CORR_MODE_FFT`	Compute correlation by using fast Fourier transform.

Value	Purpose
VSL_CONV_MODE_DIRECT	Compute convolution directly.
VSL_CORR_MODE_DIRECT	Compute correlation directly.
VSL_CONV_MODE_AUTO	Automatically choose direct or Fourier mode for convolution.
VSL_CORR_MODE_AUTO	Automatically choose direct or Fourier mode for correlation.

vslConvSetInternalPrecision/vslCorrSetInternalPrecision

Changes the value of the parameter `internal_precision` *in the convolution or correlation task descriptor.*

Syntax

Fortran:

```
status = vslconvsetinternalprecision(task, precision)
status = vslcorrsetinternalprecision(task, precision)
```

C:

```
status = vslConvSetInternalPrecision(task, precision);
status = vslCorrSetInternalPrecision(task, precision);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslconvsetinternalprecision` `INTEGER*4 task(2)` for `vslcorrsetinternalprecision` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvsetinternalprecision` `TYPE(VSL_CORR_TASK)` for `vslcorrsetinternalprecision` **C:** `VSLConvTaskPtr` for `vslConvSetInternalPrecision`	Pointer to the task descriptor.

Name	Type	Description
	`VSLCorrTaskPtr` for `vslCorrSetInternalPrecision`	
precision	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	New value of the parameter *internal_precision*.

Output Parameters

Name	Type	Description
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Current status of the task.

Description

The `vslConvSetInternalPrecision`/`vslCorrSetInternalPrecision` routine changes the value of the parameter *internal_precision* for the operation of convolution or correlation. This parameter defines whether the internal computations of the convolution or correlation result should be done in single or double precision. Initial value for *internal_precision* is assigned by a task constructor and set to either "single" or "double" according to the particular flavor of the constructor used.

Changing the *internal_precision* can be useful if the default setting of this parameter was "single" but you want to calculate the result with double precision even if input and output data are represented in single precision.

Predefined values for the *internal_precision* input parameter are as follows:

Values of *internal_precision* Parameter

Value	Purpose
`VSL_CONV_PRECISION_SINGLE`	Compute convolution with single precision.
`VSL_CORR_PRECISION_SINGLE`	Compute correlation with single precision.
`VSL_CONV_PRECISION_DOUBLE`	Compute convolution with double precision.
`VSL_CORR_PRECISION_DOUBLE`	Compute correlation with double precision.

vslConvSetStart/vslCorrSetStart

Changes the value of the parameter `start` in the convolution or correlation task descriptor.

Syntax

Fortran:

```
status = vslconvsetstart(task, start)
status = vslcorrsetstart(task, start)
```

C:

```
status = vslConvSetStart(task, start);
```

```
status = vslCorrSetStart(task, start);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslconvsetstart` `INTEGER*4 task(2)` for `vslcorrsetstart` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvsetstart` `TYPE(VSL_CORR_TASK)` for `vslcorrsetstart` **C:** `VSLConvTaskPtr` for `vslConvSetStart` `VSLCorrTaskPtr` for `vslCorrSetStart`	Pointer to the task descriptor.
start	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION (*)` **C:** `const int[]`	New value of the parameter *start*.

Output Parameters

Name	Type	Description
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Current status of the task.

Description

The `vslConvSetStart`/`vslCorrSetStart` routine sets the value of the parameter *start* for the operation of convolution or correlation. In a one-dimensional case, this parameter points to the first element in the mathematical result that should be stored in the output array. In a multidimensional case, *start* is an array of indices and its length is equal to the number of dimensions specified by the parameter *dims*. For more information about the definition and effect of this parameter, see Data Allocation.

During the initial task descriptor construction, the default value for *start* is undefined and this parameter is not used. Therefore the only way to set and use the *start* parameter is via assigning it some value by one of the `vslConvSetStart`/`vslCorrSetStart` routines.

vslConvSetDecimation/vslCorrSetDecimation

Changes the value of the parameter `decimation` *in the convolution or correlation task descriptor.*

Syntax

Fortran:

```
status = vslconvsetdecimation(task, decimation)
status = vslcorrsetdecimation(task, decimation)
```

C:

```
status = vslConvSetDecimation(task, decimation);
status = vslCorrSetDecimation(task, decimation);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslconvsetdecimation` `INTEGER*4 task(2)` for `vslcorrsetdecimation` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvsetdecimation` `TYPE(VSL_CORR_TASK)` for `vslcorrsetdecimation` **C:** `VSLConvTaskPtr` for `vslConvSetDecimation` `VSLCorrTaskPtr` for `vslCorrSetDecimation`	Pointer to the task descriptor.
decimation	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION (*)` **C:** `const int[]`	New value of the parameter *decimation*.

Output Parameters

Name	Type	Description
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER`	Current status of the task.

Name	Type	Description
	C: `int`	

Description

The routine sets the value of the parameter *decimation* for the operation of convolution or correlation. This parameter determines how to thin out the mathematical result of convolution or correlation before writing it into the output data array. For example, in a one-dimensional case, if *decimation* = *d* > 1, only every *d*-th element of the mathematical result is written to the output array *z*. In a multidimensional case, *decimation* is an array of indices and its length is equal to the number of dimensions specified by the parameter *dims*. For more information about the definition and effect of this parameter, see Data Allocation.

During the initial task descriptor construction, the default value for *decimation* is undefined and this parameter is not used. Therefore the only way to set and use the *decimation* parameter is via assigning it some value by one of the `vslSetDecimation` routines.

Task Execution Routines

Task execution routines compute convolution or correlation results based on parameters held by the task descriptor and on the user data supplied for input vectors.

After you create and adjust a task, you can execute it multiple times by applying to different input/output data of the same type, precision, and shape.

Intel MKL provides the following forms of convolution/correlation execution routines:

- **General form** executors that use the task descriptor created by the general form constructor and expect to get two source data arrays *x* and *y* on input
- **X-form** executors that use the task descriptor created by the X-form constructor and expect to get only one source data array *y* on input because the first array *x* has been already specified on the construction stage

When the task is executed for the first time, the execution routine includes a task commitment operation, which involves two basic steps: parameters consistency check and preparation of auxiliary data (for example, this might be the calculation of Fourier transform for input data).

Each execution routine has an associated one-dimensional version that provides algorithmic and computational benefits.

NOTE You can use the `NULL` task pointer in calls to execution routines. In this case, the routine is terminated and no system crash occurs.

If the task is executed successfully, the execution routine returns the zero status code. If an error is detected, the execution routine returns an error code which signals that a specific error has occurred. In particular, an error status code is returned in the following cases:

- if the task pointer is `NULL`
- if the task descriptor is corrupted
- if calculation has failed for some other reason.

NOTE Intel® MKL does not control floating-point errors, like overflow or gradual underflow, or operations with NaNs, etc.

If an error occurs, the task descriptor stores the error code.

The table below lists all task execution routines.

Task Execution Routines

Routine	Description
vslConvExec/vslCorrExec	Computes convolution or correlation for a multidimensional case.
vslConvExec1D/vslCorrExec1D	Computes convolution or correlation for a one-dimensional case.
vslConvExecX/vslCorrExecX	Computes convolution or correlation as X-form for a multidimensional case.
vslConvExecX1D/vslCorrExecX1D	Computes convolution or correlation as X-form for a one-dimensional case.

vslConvExec/vslCorrExec

Computes convolution or correlation for multidimensional case.

Syntax

Fortran:

```
status = vslsconvexec(task, x, xstride, y, ystride, z, zstride)
status = vsldconvexec(task, x, xstride, y, ystride, z, zstride)
status = vslcconvexec(task, x, xstride, y, ystride, z, zstride)
status = vslzconvexec(task, x, xstride, y, ystride, z, zstride)
status = vslscorrexec(task, x, xstride, y, ystride, z, zstride)
status = vsldcorrexec(task, x, xstride, y, ystride, z, zstride)
status = vslccorrexec(task, x, xstride, y, ystride, z, zstride)
status = vslzcorrexec(task, x, xstride, y, ystride, z, zstride)
```

C:

```
status = vslsConvExec(task, x, xstride, y, ystride, z, zstride);
status = vsldConvExec(task, x, xstride, y, ystride, z, zstride);
status = vslcConvExec(task, x, xstride, y, ystride, z, zstride);
status = vslzConvExec(task, x, xstride, y, ystride, z, zstride);
status = vslsCorrExec(task, x, xstride, y, ystride, z, zstride);
status = vsldCorrExec(task, x, xstride, y, ystride, z, zstride);
status = vslcCorrExec(task, x, xstride, y, ystride, z, zstride);
status = vslzCorrExec(task, x, xstride, y, ystride, z, zstride);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvexec`, `vsldconvexec`, `vslcconvexec`, `vslzconvexec` `INTEGER*4 task(2)` for `vslscorrexec`, `vsldcorrexec`, `vslccorrexec`, `vslzcorrexec` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvexec`, `vsldconvexec`, `vslcconvexec`, `vslzconvexec` `TYPE(VSL_CORR_TASK)` for `vslscorrexec`, `vsldcorrexec`, `vslccorrexec`, `vslzcorrexec` **C:** `VSLConvTaskPtr` for `vslsConvExec`, `vsldConvExec`, `vslcConvExec`, `vslzConvExec` `VSLCorrTaskPtr` for `vslsCorrExec`, `vsldCorrExec`, `vslcCorrExec`, `vslzCorrExec`	Pointer to the task descriptor
x, *y*	**FORTRAN 77:** `REAL*4` for `vslsconvexec` and `vslscorrexec`, `REAL*8` for `vsldconvexec` and `vsldcorrexec`, `COMPLEX*8` for`vslcconvexec` and `vslccorrexec`, `COMPLEX*16` for`vslzconvexec` and `vslzcorrexec` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexec` and `vslscorrexec`, `REAL(KIND=8), DIMENSION(*)` for `vsldconvexec` and `vsldcorrexec`, `COMPLEX(KIND=4), DIMENSION(*)` for`vslcconvexec` and `vslccorrexec`, `COMPLEX(KIND=8), DIMENSION(*)` for `vslzconvexec` and `vslzcorrexec`	Pointers to arrays containing input data. See Data Allocation for more information.

Name	Type	Description
	C: `const float[]` for `vslsConvExec` and `vslsCorrExec`, `const double[]` for `vsldConvExec` and `vsldCorrExec`, `const MKL_Complex8[]` for `vslcConvExec` and `vslcCorrExec`, `const MKL_Complex16[]` for `vslzConvExec` and `vslzCorrExec`	
xstride, *ystride*, *zstride*	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION (*)` **C:** `const int[]`	Strides for input and output data. For more information, see stride parameters.

Output Parameters

Name	Type	Description
z	**FORTRAN 77:** `REAL*4` for `vslsconvexec` and `vslscorrexec`, `REAL*8` for `vsldconvexec` and `vsldcorrexec`, `COMPLEX*8` for`vslcconvexec` and `vslccorrexec`, `COMPLEX*16` for`vslzconvexec` and `vslzcorrexec` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexec` and `vslscorrexec`, `REAL(KIND=8), DIMENSION(*)` for `vsldconvexec` and `vsldcorrexec`, `COMPLEX(KIND=4), DIMENSION (*)` for`vslcconvexec` and `vslccorrexec`, `COMPLEX(KIND=8), DIMENSION (*)` for `vslzconvexec` and `vslzcorrexec`	Pointer to the array that stores output data. See Data Allocation for more information.

Name	Type	Description
	C: `const float[]` for `vslsConvExec` and `vslsCorrExec`, `const double[]` for `vsldConvExec` and `vsldCorrExec`, `const MKL_Complex8[]` for `vslcConvExec` and `vslcCorrExec`, `const MKL_Complex16[]` for `vslzConvExec` and `vslzCorrExec`	
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Set to `VSL_STATUS_OK` if the task is executed successfully or set to non-zero error code otherwise.

Description

Each of the `vslConvExec`/`vslCorrExec` routines computes convolution or correlation of the data provided by the arrays *x* and *y* and then stores the results in the array *z*. Parameters of the operation are read from the task descriptor created previously by a corresponding `vslConvNewTask`/`vslCorrNewTask` constructor and pointed to by *task*. If *task* is `NULL`, no operation is done.

The stride parameters *xstride*, *ystride*, and *zstride* specify the physical location of the input and output data in the arrays *x*, *y*, and *z*, respectively. In a one-dimensional case, stride is an interval between locations of consecutive elements of the array. For example, if the value of the parameter *zstride* is `s`, then only every s^{th} element of the array *z* will be used to store the output data. The stride value must be positive or negative but not zero.

vslConvExec1D/vslCorrExec1D

Computes convolution or correlation for one-dimensional case.

Syntax

Fortran:

```
status = vslsconvexec1d(task, x, xstride, y, ystride, z, zstride)
status = vsldconvexec1d(task, x, xstride, y, ystride, z, zstride)
status = vslcconvexec1d(task, x, xstride, y, ystride, z, zstride)
status = vslzconvexec1d(task, x, xstride, y, ystride, z, zstride)
status = vslscorrexec1d(task, x, xstride, y, ystride, z, zstride)
status = vsldcorrexec1d(task, x, xstride, y, ystride, z, zstride)
status = vslccorrexec1d(task, x, xstride, y, ystride, z, zstride)
status = vslzcorrexec1d(task, x, xstride, y, ystride, z, zstride)
```

C:

```
status = vslsConvExec1D(task, x, xstride, y, ystride, z, zstride);
```

```
status = vsldConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslcConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslzConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslsCorrExec1D(task, x, xstride, y, ystride, z, zstride);
status = vsldCorrExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslcCorrExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslzCorrExec1D(task, x, xstride, y, ystride, z, zstride);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvexec1d`, `vsldconvexec1d`, `vslcconvexec1d`, `vslzconvexec1d` `INTEGER*4 task(2)` for `vslscorrexec1d`, `vsldcorrexec1d`, `vslccorrexec1d`, `vslzcorrexec1d` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvexec1d`, `vsldconvexec1d`, `vslcconvexec1d`, `vslzconvexec1d` `TYPE(VSL_CORR_TASK)` for `vslscorrexec1d`, `vsldcorrexec1d`, `vslccorrexec1d`, `vslzcorrexec1d` **C:** `VSLConvTaskPtr` for `vslsConvExec1D`, `vsldConvExec1D`, `vslcConvExec1D`, `vslzConvExec1D` `VSLCorrTaskPtr` for `vslsCorrExec1D`, `vsldCorrExec1D`, `vslcCorrExec1D`, `vslzCorrExec1D`	Pointer to the task descriptor.

Name	Type	Description
x, *y*	**FORTRAN 77:** `REAL*4` for `vslsconvexec1d` and `vslscorrexec1d`, `REAL*8` for `vsldconvexec1d` and `vsldcorrexec1d`, `COMPLEX*8` for`vslcconvexec1d` and `vslccorrexec1d`, `COMPLEX*16` for`vslzconvexec1d` and `vslzcorrexec1d` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexec1d` and `vslscorrexec1d`, `REAL(KIND=8), DIMENSION(*)` for `vsldconvexec1d` and `vsldcorrexec1d`, `COMPLEX(KIND=4), DIMENSION(*)` for`vslcconvexec1d` and `vslccorrexec1d`, `COMPLEX(KIND=8), DIMENSION(*)` for `vslzconvexec1d` and `vslzcorrexec1d` **C:** `const float[]` for `vslsConvExec1D` and `vslsCorrExec1D`, `const double[]` for `vsldConvExec1D` and `vsldCorrExec1D`, `const MKL_Complex8[]` for `vslcConvExec1D` and `vslcCorrExec1D`, `const MKL_Complex16[]` for `vslzConvExec1D` and `vslzCorrExec1D`	Pointers to arrays containing input data. See Data Allocation for more information.
xstride, *ystride*, *zstride*	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Strides for input and output data. For more information, see stride parameters.

Output Parameters

Name	Type	Description
z	**FORTRAN 77:** `REAL*4` for `vslsconvexec1d` and `vslscorrexec1d`, `REAL*8` for `vsldconvexec1d` and `vsldcorrexec1d`, `COMPLEX*8` for`vslcconvexec1d` and `vslccorrexec1d`, `COMPLEX*16` for`vslzconvexec1d` and `vslzcorrexec1d` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexec1d` and `vslscorrexec1d`, `REAL(KIND=8), DIMENSION(*)` for `vsldconvexec1d` and `vsldcorrexec1d`, `COMPLEX(KIND=4), DIMENSION(*)` for`vslcconvexec1d` and `vslccorrexec1d`, `COMPLEX(KIND=8), DIMENSION(*)` for `vslzconvexec1d` and `vslzcorrexec1d` **C:** `const float[]` for `vslsConvExec1D` and `vslsCorrExec1D`, `const double[]` for `vsldConvExec1D` and `vsldCorrExec1D`, `const MKL_Complex8[]` for `vslcConvExec1D` and `vslcCorrExec1D`, `const MKL_Complex16[]` for `vslzConvExec1D` and `vslzCorrExec1D`	Pointer to the array that stores output data. See Data Allocation for more information.
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Set to `VSL_STATUS_OK` if the task is executed successfully or set to non-zero error code otherwise.

Description

Each of the vslConvExec1D/vslCorrExec1D routines computes convolution or correlation of the data provided by the arrays *x* and *y* and then stores the results in the array *z*. These routines represent a special one-dimensional version of the operation, assuming that the value of the parameter *dims* is 1. Using this version of execution routines can help speed up performance in case of one-dimensional data.

Parameters of the operation are read from the task descriptor created previously by a corresponding vslConvNewTask1D/vslCorrNewTask1D constructor and pointed to by *task*. If *task* is NULL, no operation is done.

vslConvExecX/vslCorrExecX

Computes convolution or correlation for multidimensional case with the fixed first operand vector.

Syntax

Fortran:

```
status = vslsconvexecx(task, y, ystride, z, zstride)
status = vsldconvexecx(task, y, ystride, z, zstride)
status = vslcconvexecx(task, y, ystride, z, zstride)
status = vslzconvexecx(task, y, ystride, z, zstride)
status = vslscorrexecx(task, y, ystride, z, zstride)
status = vsldcorrexecx(task, y, ystride, z, zstride)
status = vslccorrexecx(task, y, ystride, z, zstride)
status = vslzcorrexecx(task, y, ystride, z, zstride)
```

C:

```
status = vslsConvExecX(task, y, ystride, z, zstride);
status = vsldConvExecX(task, y, ystride, z, zstride);
status = vslcConvExecX(task, y, ystride, z, zstride);
status = vslzConvExecX(task, y, ystride, z, zstride);
status = vslsCorrExecX(task, y, ystride, z, zstride);
status = vslcCorrExecX(task, y, ystride, z, zstride);
status = vslzCorrExecX(task, y, ystride, z, zstride);
status = vsldCorrExecX(task, y, ystride, z, zstride);
```

Include Files

- FORTRAN 77: mkl_vsl.f77
- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvexecx`, `vsldconvexecx`, `vslcconvexecx`, `vslzconvexecx` `INTEGER*4 task(2)` for `vslscorrexecx`, `vsldcorrexecx`, `vslccorrexecx`, `vslzcorrexecx` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvexecx`, `vsldconvexecx`, `vslcconvexecx`, `vslzconvexecx` `TYPE(VSL_CORR_TASK)` for `vslscorrexecx`, `vsldcorrexecx`, `vslccorrexecx`, `vslzcorrexecx` **C:** `VSLConvTaskPtr` for `vslsConvExecX`, `vsldConvExecX`, `vslcConvExecX`, `vslzConvExecX` `VSLCorrTaskPtr` for `vslsCorrExecX`, `vsldCorrExecX`, `vslcCorrExecX`, `vslzCorrExecX`	Pointer to the task descriptor.
x ,*y*	**FORTRAN 77:** `REAL*4` for `vslsconvexecx` and `vslscorrexecx`, `REAL*8` for `vsldconvexecx` and `vsldcorrexecx`, `COMPLEX*8` for`vslcconvexecx` and `vslccorrexecx`, `COMPLEX*16` for`vslzconvexecx` and `vslzcorrexecx` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexecx` and `vslscorrexecx`,	Pointer to array containing input data (for the second operand vector). See Data Allocation for more information.

Name	Type	Description
	`REAL(KIND=8), DIMENSION(*)` for `vsldconvexecx` and `vsldcorrexecx`, `COMPLEX(KIND=4), DIMENSION (*)` for`vslcconvexecx` and `vslccorrexecx`, `COMPLEX(KIND=8), DIMENSION (*)` for `vslzconvexecx` and `vslzcorrexecx` **C:** `const float[]` for `vslsConvExecX` and `vslsCorrExecX`, `const double[]` for `vsldConvExecX` and `vsldCorrExecX`, `const MKL_Complex8[]` for `vslcConvExecX` and `vslcCorrExecX`, `const MKL_Complex16[]` for `vslzConvExecX` and `vslzCorrExecX`	
ystride ,*z stride*	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER, DIMENSION (*)` **C:** `const int[]`	Strides for input and output data. For more information, see stride parameters.

Output Parameters

Name	Type	Description
z	**FORTRAN 77:** `REAL*4` for `vslsconvexecx` and `vslscorrexecx`, `REAL*8` for `vsldconvexecx` and `vsldcorrexecx`, `COMPLEX*8` for`vslcconvexecx` and `vslccorrexecx`, `COMPLEX*16` for`vslzconvexecx` and `vslzcorrexecx` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexecx` and `vslscorrexecx`,	Pointer to the array that stores output data. See Data Allocation for more information.

Name	Type	Description
	`REAL(KIND=8), DIMENSION(*)` for `vsldconvexecx` and `vsldcorrexecx`, `COMPLEX(KIND=4), DIMENSION (*)` for`vslcconvexecx` and `vslccorrexecx`, `COMPLEX(KIND=8), DIMENSION (*)` for `vslzconvexecx` and `vslzcorrexecx` **C:** `const float[]` for `vslsConvExecX` and `vslsCorrExecX`, `const double[]` for `vsldConvExecX` and `vsldCorrExecX`, `const MKL_Complex8[]` for `vslcConvExecX` and `vslcCorrExecX`, `const MKL_Complex16[]` for `vslzConvExecX` and `vslzCorrExecX`	
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Set to `VSL_STATUS_OK` if the task is executed successfully or set to non-zero error code otherwise.

Description

Each of the `vslConvExecX`/`vslCorrExecX` routines computes convolution or correlation of the data provided by the arrays *x* and *y* and then stores the results in the array *z*. These routines represent a special version of the operation, which assumes that the first operand vector was set on the task construction stage and the task object keeps the pointer to the array *x*.

Parameters of the operation are read from the task descriptor created previously by a corresponding `vslConvNewTaskX`/`vslCorrNewTaskX` constructor and pointed to by *task*. If *task* is `NULL`, no operation is done.

Using this form of execution routines is recommended when you need to compute multiple convolutions or correlations with the same data vector in array *x* against different vectors in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

vslConvExecX1D/vslCorrExecX1D

Computes convolution or correlation for one-dimensional case with the fixed first operand vector.

Syntax

Fortran:

```
status = vslsconvexecx1d(task, y, ystride, z, zstride)
```

```
status = vsldconvexecx1d(task, y, ystride, z, zstride)
status = vslcconvexecx1d(task, y, ystride, z, zstride)
status = vslzconvexecx1d(task, y, ystride, z, zstride)
status = vslscorrexecx1d(task, y, ystride, z, zstride)
status = vsldcorrexecx1d(task, y, ystride, z, zstride)
status = vslccorrexecx1d(task, y, ystride, z, zstride)
status = vslzcorrexecx1d(task, y, ystride, z, zstride)
```

C:

```
status = vslsConvExecX1D(task, y, ystride, z, zstride);
status = vsldConvExecX1D(task, y, ystride, z, zstride);
status = vslcConvExecX1D(task, y, ystride, z, zstride);
status = vslzConvExecX1D(task, y, ystride, z, zstride);
status = vslsCorrExecX1D(task, y, ystride, z, zstride);
status = vslcCorrExecX1D(task, y, ystride, z, zstride);
status = vslzCorrExecX1D(task, y, ystride, z, zstride);
status = vsldCorrExecX1D(task, y, ystride, z, zstride);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslsconvexecx1d`, `vsldconvexecx1d`, `vslcconvexecx1d`, `vslzconvexecx1d` `INTEGER*4 task(2)` for `vslscorrexecx1d`, `vsldcorrexecx1d`, `vslccorrexecx1d`, `vslzcorrexecx1d` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslsconvexecx1d`, `vsldconvexecx1d`, `vslcconvexecx1d`, `vslzconvexecx1d`	Pointer to the task descriptor.

Name	Type	Description
	`TYPE(VSL_CORR_TASK)` for `vslscorrexecx1d`, `vsldcorrexecx1d`, `vslccorrexecx1d`, `vslzcorrexecx1d` **C:** `VSLConvTaskPtr` for `vslsConvExecX1D`, `vsldConvExecX1D`, `vslcConvExecX1D`, `vslzConvExecX1D` `VSLCorrTaskPtr` for `vslsCorrExecX1D`, `vsldCorrExecX1D`, `vslcCorrExecX1D`, `vslzCorrExecX1D`	
x, *y*	**FORTRAN 77:** `REAL*4` for `vslsconvexecx1d` and `vslscorrexecx1d`, `REAL*8` for `vsldconvexecx1d` and `vsldcorrexecx1d`, `COMPLEX*8` for`vslcconvexecx1d` and `vslccorrexecx1d`, `COMPLEX*16` for`vslzconvexecx1d` and `vslzcorrexecx1d` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexecx1d` and `vslscorrexecx1d`, `REAL(KIND=8), DIMENSION(*)` for `vsldconvexecx1d` and `vsldcorrexecx1d`, `COMPLEX(KIND=4), DIMENSION(*)` for`vslcconvexecx1d` and `vslccorrexecx1d`, `COMPLEX(KIND=8), DIMENSION(*)` for `vslzconvexecx1d` and `vslzcorrexecx1d` **C:** `const float[]` for `vslsConvExecX1D` and `vslsCorrExecX1D`, `const double[]` for `vsldConvExecX1D` and `vsldCorrExecX1D`,	Pointer to array containing input data (for the second operand vector). See Data Allocation for more information.

Name	Type	Description
	`const MKL_Complex8[]` for `vslcConvExecX1D` and `vslcCorrExecX1D`, `const MKL_Complex16[]` for `vslzConvExecX1D` and `vslzCorrExecX1D`	
ystride, *zstride*	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `const int`	Strides for input and output data. For more information, see stride parameters.

Output Parameters

Name	Type	Description
z	**FORTRAN 77:** `REAL*4` for `vslsconvexecx1d` and `vslscorrexecx1d`, `REAL*8` for `vsldconvexecx1d` and `vsldcorrexecx1d`, `COMPLEX*8` for`vslcconvexecx1d` and `vslccorrexecx1d`, `COMPLEX*16` for`vslzconvexecx1d` and `vslzcorrexecx1d` **Fortran 90:** `REAL(KIND=4), DIMENSION(*)` for `vslsconvexecx1d` and `vslscorrexecx1d`, `REAL(KIND=8), DIMENSION(*)` for `vsldconvexecx1d` and `vsldcorrexecx1d`, `COMPLEX(KIND=4), DIMENSION(*)` for`vslcconvexecx1d` and `vslccorrexecx1d`, `COMPLEX(KIND=8), DIMENSION(*)` for `vslzconvexecx1d` and `vslzcorrexecx1d` **C:** `const float[]` for `vslsConvExecX1D` and `vslsCorrExecX1D`, `const double[]` for `vsldConvExecX1D` and `vsldCorrExecX1D`,	Pointer to the array that stores output data. See Data Allocation for more information.

Name	Type	Description
	`const MKL_Complex8[]` for `vslcConvExecX1D` and `vslcCorrExecX1D`, `const MKL_Complex16[]` for `vslzConvExecX1D` and `vslzCorrExecX1D`	
status	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Set to `VSL_STATUS_OK` if the task is executed successfully or set to non-zero error code otherwise.

Description

Each of the `vslConvExecX1D`/`vslCorrExecX1D` routines computes convolution or correlation of one-dimensional (assuming that *dims* =1) data provided by the arrays *x* and *y* and then stores the results in the array *z*. These routines represent a special version of the operation, which expects that the first operand vector was set on the task construction stage.

Parameters of the operation are read from the task descriptor created previously by a corresponding `vslConvNewTaskX1D`/`vslCorrNewTaskX1D` constructor and pointed to by *task*. If *task* is `NULL`, no operation is done.

Using this form of execution routines is recommended when you need to compute multiple one-dimensional convolutions or correlations with the same data vector in array *x* against different vectors in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

Task Destructors

Task destructors are routines designed for deleting task objects and deallocating memory.

vslConvDeleteTask/vslCorrDeleteTask

Destroys the task object and frees the memory.

Syntax

Fortran:

```
errcode = vslconvdeletetask(task)
errcode = vslcorrdeletetask(task)
```

C:

```
errcode = vslConvDeleteTask(task);
errcode = vslCorrDeleteTask(task);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**FORTRAN 77:** `INTEGER*4 task(2)` for `vslconvdeletetask` `INTEGER*4 task(2)` for `vslcorrdeletetask` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvdeletetask` `TYPE(VSL_CORR_TASK)` for `vslcorrdeletetask` **C:** `VSLConvTaskPtr*` for `vslConvDeleteTask` `VSLCorrTaskPtr*` for `vslCorrDeleteTask`	Pointer to the task descriptor.

Output Parameters

Name	Type	Description
errcode	**FORTRAN 77:** `INTEGER` **Fortran 90:** `INTEGER` **C:** `int`	Contains 0 if the task object is deleted successfully. Contains an error code if an error occurred.

Description

The `vslConvDeleteTask`/`vslCorrvDeleteTask` routine deletes the task descriptor object and frees any working memory and the memory allocated for the data structure. The task pointer is set to `NULL`.

Note that if the `vslConvDeleteTask`/`vslCorrvDeleteTask` routine does not delete the task successfully, the routine returns an error code. This error code has no relation to the task status code and does not change it.

NOTE You can use the `NULL` task pointer in calls to destructor routines. In this case, the routine terminates with no system crash.

Task Copy

The routines are designed for copying convolution and correlation task descriptors.

vslConvCopyTask/vslCorrCopyTask

Copies a descriptor for convolution or correlation task.

Syntax

Fortran:

```
status = vslconvcopytask(newtask, srctask)
```

```
status = vslcorrcopytask(newtask, srctask)
```

C:

```
status = vslConvCopyTask(newtask, srctask);
status = vslCorrCopyTask(newtask, srctask);
```

Include Files

- FORTRAN 77: `mkl_vsl.f77`
- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
srctask	**FORTRAN 77:** `INTEGER*4 srctask(2)` for `vslconvcopytask` `INTEGER*4 srctask(2)` for `vslcorrcopytask` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvcopytask` `TYPE(VSL_CORR_TASK)` for `vslcorrcopytask` **C:** `const VSLConvTaskPtr` for `vslConvCopyTask` `const VSLCorrTaskPtr` for `vslCorrCopyTask`	Pointer to the source task descriptor.

Output Parameters

Name	Type	Description
newtask	**FORTRAN 77:** `INTEGER*4 srctask(2)` for `vslconvcopytask` `INTEGER*4 srctask(2)` for `vslcorrcopytask` **Fortran 90:** `TYPE(VSL_CONV_TASK)` for `vslconvcopytask` `TYPE(VSL_CORR_TASK)` for `vslcorrcopytask` **C:** `VSLConvTaskPtr*` for `vslConvCopyTask` `VSLCorrTaskPtr*` for `vslCorrCopyTask`	Pointer to the new task descriptor.

Name	Type	Description
status	**FORTRAN 77:** INTEGER **Fortran 90:** INTEGER **C:** int	Current status of the source task.

Description

If a task object *srctask* already exists, you can use an appropriate vslConvCopyTask/vslCorrCopyTask routine to make its copy in *newtask*. After the copy operation, both source and new task objects will become committed (see Introduction to Convolution and Correlation for information about task commitment). If the source task was not previously committed, the commitment operation for this task is implicitly invoked before copying starts. If an error occurs during source task commitment, the task stores the error code in the status field. If an error occurs during copy operation, the routine returns a NULL pointer instead of a reference to a new task object.

Usage Examples

This section demonstrates how you can use the Intel MKL routines to perform some common convolution and correlation operations both for single-threaded and multithreaded calculations. The following two sample functions scond1 and sconf1 simulate the convolution and correlation functions SCOND and SCONF found in IBM ESSL* library. The functions assume single-threaded calculations and can be used with C or C++ compilers.

Function scond1 for Single-Threaded Calculations

```
#include "mkl_vsl.h"

int scond1(
    float h[], int inch,
    float x[], int incx,
    float y[], int incy,
    int nh, int nx, int iy0, int ny)
{
    int status;
    VSLConvTaskPtr task;
    vslsConvNewTask1D(&task,VSL_CONV_MODE_DIRECT,nh,nx,ny);
    vslConvSetStart(task, &iy0);
    status = vslsConvExec1D(task, h,inch, x,incx, y,incy);
    vslConvDeleteTask(&task);
    return status;
}
```

Function sconf1 for Single-Threaded Calculations

```
#include "mkl_vsl.h"
int sconf1(
    int init,
    float h[], int inc1h,
    float x[], int inc1x, int inc2x,
    float y[], int inc1y, int inc2y,
    int nh, int nx, int m, int iy0, int ny,
    void* aux1, int naux1, void* aux2, int naux2)
{
    int status;
    /* assume that aux1!=0 and naux1 is big enough */
    VSLConvTaskPtr* task = (VSLConvTaskPtr*)aux1;
    if (init != 0)
        /* initialization: */
        status = vslsConvNewTaskX1D(task,VSL_CONV_MODE_FFT,
         nh,nx,ny, h,inc1h);
    if (init == 0) {
        /* calculations: */
        int i;
        vslConvSetStart(*task, &iy0);
        for (i=0; i<m; i++) {
         float* xi = &x[inc2x * i];
         float* yi = &y[inc2y * i];
         /* task is implicitly committed at i==0 */
         status = vslsConvExecX1D(*task, xi, inc1x, yi, inc1y);
        };
    };
    vslConvDeleteTask(task);
    return status;
}
```

Using Multiple Threads

For functions such as `sconf1` described in the previous example, parallel calculations may be more preferable instead of cycling. If `m>1`, you can use multiple threads for invoking the task execution against different data sequences. For such cases, use task copy routines to create `m` copies of the task object before the calculations stage and then run these copies with different threads. Ensure that you make all necessary parameter adjustments for the task (using Task Editors) before copying it.

The sample code in this case may look as follows:

```
if (init == 0) {
    int i, status, ss[M];
    VSLConvTaskPtr tasks[M];
    /* assume that M is big enough */
    . . .
    vslConvSetStart(*task, &iy0);
    . . .
    for (i=0; i<m; i++)
        /* implicit commitment at i==0 */
        vslConvCopyTask(&tasks[i],*task);
    . . .
```

Then, `m` threads may be started to execute different copies of the task:

```
. . .
        float* xi = &x[inc2x * i];
        float* yi = &y[inc2y * i];
        ss[i]=vslsConvExecX1D(tasks[i], xi,inc1x, yi,inc1y);
    . . .
```

And finally, after all threads have finished the calculations, overall status should be collected from all task objects. The following code signals the first error found, if any:

```
    . . .
    for (i=0; i<m; i++) {
        status = ss[i];
        if (status != 0) /* 0 means "OK" */
            break;
    };
    return status;
}; /* end if init==0 */
```

Execution routines modify the task internal state (fields of the task structure). Such modifications may conflict with each other if different threads work with the same task object simultaneously. That is why different threads must use different copies of the task.

Mathematical Notation and Definitions

The following notation is necessary to explain the underlying mathematical definitions used in the text:

$R = (-\infty, +\infty)$	The set of real numbers.
$Z = \{0, \pm1, \pm2, \ldots\}$	The set of integer numbers.
$Z^N = Z\times \ldots \times Z$	The set of N-dimensional series of integer numbers.
$p = (p_1, \ldots, p_N) \in Z^N$	N-dimensional series of integers.
u:$\mathbf{Z}^N \rightarrow \mathbf{R}$	Function u with arguments from $\mathbf{Z}^N$ and values from **R**.
$u(p) = u(p_1, \ldots, p_N)$	The value of the function u for the argument $(p_1, \ldots, p_N)$.
$w = u*v$	Function w is the convolution of the functions u, v.

$w = u \bullet v$	Function w is the correlation of the functions u, v.

Given series $p, q \in \mathbf{Z}^N$:

- series $r = p + q$ is defined as $r^n = p^n + q^n$ for every $n=1,\ldots,N$
- series $r = p - q$ is defined as $r^n = p^n - q^n$ for every $n=1,\ldots,N$
- series $r = \sup\{p, q\}$ is defines as $r^n = \max\{p^n, q^n\}$ for every $n=1,\ldots,N$
- series $r = \inf\{p, q\}$ is defined as $r^n = \min\{p^n, q^n\}$ for every $n=1,\ldots,N$
- inequality $p \le q$ means that $p^n \le q^n$ for every $n=1,\ldots,N$.

A function $u(p)$ is called a finite function if there exist series P^{min}, $P^{max} \in \mathbf{Z}^N$ such that:

$$u(p) \neq 0$$

implies

$$P^{min} \le p \le P^{max}.$$

Operations of convolution and correlation are only defined for finite functions.

Consider functions u, v and series P^{min}, $P^{max}Q^{min}$, $Q^{max} \in \mathbf{Z}^N$ such that:

$u(p) \neq 0$ implies $P^{min} \le p \le P^{max}$.
$v(q) \neq 0$ implies $Q^{min} \le q \le Q^{max}$.

Definitions of linear correlation and linear convolution for functions u and v are given below.

Linear Convolution

If function $w = u*v$ is the convolution of u and v, then:

$w(r) \neq 0$ implies $R^{min} \le r \le R^{max}$,
where $R^{min} = P^{min} + Q^{min}$ and $R^{max} = P^{max} + Q^{max}$.

If $R^{min} \le r \le R^{max}$, then:

$w(r) = \sum u(t) \cdot v(r-t)$ is the sum for all $t \in \mathbf{Z}^N$ such that $T^{min} \le t \le T^{max}$,
where $T^{min} = \sup\{P^{min}, r - Q^{max}\}$ and $T^{max} = \inf\{P^{max}, r - Q^{min}\}$.

Linear Correlation

If function $w = u \bullet v$ is the correlation of u and v, then:

$w(r) \neq 0$ implies $R^{min} \le r \le R^{max}$,
where $R^{min} = Q^{min} - P^{max}$ and $R^{max} = Q^{max} - P^{min}$.

If $R^{min} \le r \le R^{max}$, then:

$w(r) = \sum u(t) \cdot v(r+t)$ is the sum for all $t \in Z^N$ such that $T^{min} \le t \le T^{max}$,
where $T^{min} = \sup\{P^{min}, Q^{min} - r\}$ and $T^{max} = \inf\{P^{max}, Q^{max} - r\}$.

Representation of the functions u, v, w as the input/output data for the Intel MKL convolution and correlation functions is described in the Data Allocation section below.

Data Allocation

This section explains the relation between:

- mathematical finite functions u, v, w introduced in the section Mathematical Notation and Definitions;
- multi-dimensional input and output data vectors representing the functions u, v, w;
- arrays u, v, w used to store the input and output data vectors in computer memory

The convolution and correlation routine parameters that determine the allocation of input and output data are the following:

- Data arrays x, y, z

- Shape arrays *xshape*, *yshape*, *zshape*
- Strides within arrays *xstride*, *ystride*, *zstride*
- Parameters *start*, *decimation*

Finite Functions and Data Vectors

The finite functions *u(p)*, *v(q)*, and *w(r)* introduced above are represented as multi-dimensional vectors of input and output data:

inputu(i_1,...,i_{dims}) for *u*(p_1,...,p_N)

inputv(j_1,...,j_{dims}) for *v*(q_1,...,q_N)

output(k_1,...,k_{dims}) for *w*(r_1,...,r_N).

Parameter *dims* represents the number of dimensions and is equal to N.

The parameters *xshape*, *yshape*, and *zshape* define the shapes of input/output vectors:

inputu(i_1,...,i_{dims}) is defined if $1 \le i_n \le xshape(n)$ for every n=1,...,*dims*

inputv(j_1,...,j_{dims}) is defined if $1 \le j_n \le yshape(n)$ for every n=1,...,*dims*

output(k_1,...,k_{dims}) is defined if $1 \le k_n \le zshape(n)$ for every n=1,...,*dims*.

Relation between the input vectors and the functions *u* and *v* is defined by the following formulas:

inputu(i_1,...,i_{dims})= *u*(p_1,...,p_N), where $p_n = P_n^{min} + (i_n-1)$ for every n

inputv(j_1,...,j_{dims})= *v*(q_1,...,q_N), where $q_n=Q_n^{min} + (j_n-1)$ for every n.

The relation between the output vector and the function *w(r)* is similar (but only in the case when parameters *start* and *decimation* are not defined):

output(k_1,...,k_{dims})= w(r_1,...,r_N), where $r_n=R_n^{min} + (k_n-1)$ for every n.

If the parameter *start* is defined, it must belong to the interval $R_n^{min} \le start(n) \le R_n^{max}$. If defined, the *start* parameter replaces R^{min} in the formula:

output(k_1,...,k_{dims})=w(r_1,...,r_N), where r_n=start(n) + (k_n-1)

If the parameter *decimation* is defined, it changes the relation according to the following formula:

output(k_1,...,k_{dims})=w(r_1,...,r_N), where $r_n= R_n^{min} + (k_n-1)$*decimation(n)

If both parameters *start* and *decimation* are defined, the formula is as follows:

output(k_1,...,k_{dims})=w(r_1,...,r_N), where r_n=start(n) + (k_n-1)*decimation(n)

The convolution and correlation software checks the values of *zshape*, *start*, and *decimation* during task commitment. If r_n exceeds R_n^{max} for some k_n,n=1,...,dims, an error is raised.

Allocation of Data Vectors

Both parameter arrays *x* and *y* contain input data vectors in memory, while array *z* is intended for storing output data vector. To access the memory, the convolution and correlation software uses only pointers to these arrays and ignores the array shapes.

For parameters *x*, *y*, and *z*, you can provide one-dimensional arrays with the requirement that actual length of these arrays be sufficient to store the data vectors.

The allocation of the input and output data inside the arrays *x*, *y*, and *z* is described below assuming that the arrays are one-dimensional. Given multi-dimensional indices $i, j, k \in \mathbf{Z}^N$, one-dimensional indices $e, f, g \in \mathbf{Z}$ are defined such that:

inputu(i_1,...,i_{dims}) is allocated at *x(e)*

inputv(j_1,...,j_{dims}) is allocated at *y(f)*

output(k_1,...,k_{dims}) is allocated at *z(g)*.

The indices *e*, *f*, and *g* are defined as follows:

e = 1 + ∑*xstride*(n)·*dx*(n) (the sum is for all n=1,...,dims)

f = 1 + ∑*ystride*(n)·*dy*(n) (the sum is for all n=1,...,dims)

g = 1 + ∑*zstride*(n)·*dz*(n) (the sum is for all n=1,...,dims)

The distances dx(n), dy(n), and dz(n) depend on the signum of the stride:

dx(n) = i_n-1 if xstride(n)>0, or dx(n) = i_n-xshape(n) if xstride(n)<0

dy(n) = j_n-1 if ystride(n)>0, or dy(n) = j_n-yshape(n) if ystride(n)<0

dz(n) = k_n-1 if zstride(n)>0, or dz(n) = k_n-zshape(n) if zstride(n)<0

The definitions of indices *e*, *f*, and *g* assume that indexes for arrays *x*, *y*, and *z* are started from unity:

x(e) is defined for e=1,...,length(x)

y(f) is defined for f=1,...,length(y)

z(g) is defined for g=1,...,length(z)

Below is a detailed explanation about how elements of the multi-dimensional output vector are stored in the array *z* for one-dimensional and two-dimensional cases.

One-dimensional case. If *dims*=1, then *zshape* is the number of the output values to be stored in the array *z*. The actual length of array *z* may be greater than *zshape* elements.

If zstride>1, output values are stored with the stride: output(1) is stored to z(1), output(2) is stored to z(1+zstride), and so on. Hence, the actual length of *z* must be at least 1+zstride*(zshape-1) elements or more.

If zstride<0, it still defines the stride between elements of array *z*. However, the order of the used elements is the opposite. For the *k*-th output value, output(k) is stored in z(1+|zstride|*(zshape-k)), where |zstride| is the absolute value of zstride. The actual length of the array *z* must be at least 1+|zstride|*(zshape - 1) elements.

Two-dimensional case. If *dims*=2, the output data is a two-dimensional matrix. The value zstride(1) defines the stride inside matrix columns, that is, the stride between the output(k_1, k_2) and output(k_1+1, k_2) for every pair of indices k_1, k_2. On the other hand, zstride(2) defines the stride between columns, that is, the stride between output(k_1,k_2) and output(k_1,k_2+1).

If zstride(2) is greater than zshape(1), this causes sparse allocation of columns. If the value of zstride(2) is smaller than zshape(1), this may result in the transposition of the output matrix. For example, if zshape = (2,3), you can define zstride = (3,1) to allocate output values like transposed matrix of the shape 3x2.

Whether zstride assumes this kind of transformations or not, you need to ensure that different elements output (k_1, ...,k_{dims}) will be stored in different locations *z(g)*.

VSL Summary Statistics

The VSL Summary Statistics domain comprises a set of routines that compute basic statistical estimates for single and double precision multi-dimensional datasets.

See the definition of the supported operations in the Mathematical Notation and Definitions section.

The VSL Summary Statistics routines calculate:

- raw and central moments up to the fourth order
- skewness and excess kurtosis (further referred to as *kurtosis* for brevity)

- variation coefficient
- quantiles and order statistics
- minimum and maximum
- variance-covariance/correlation matrix
- pooled/group variance-covariance matrix and mean
- partial variance-covariance/correlation matrix
- robust estimators for variance-covariance matrix and mean in presence of outliers

The library also contains functions to perform the following tasks:

- Detect outliers in datasets
- Support missing values in datasets
- Parameterize correlation matrices
- Compute quantiles for streaming data

You can access the VSL Summary Statistics routines through the Fortran 90 and C89 language interfaces. You can also use the C89 interface with later versions of the C/C++, or Fortran 90 interface with programs written in Fortran 95.

For users of the C/C++ and Fortran languages, Intel MKL provides the `mkl_vsl.h`, `mkl_vsl.f90`, and `mkl_vsl.f77` header files. All the header files are in the directory

```
${MKL}/include
```

See more details about the Fortran header in the Random Number Generators section of this chapter.

You can find examples that demonstrate calculation of the VSL Summary Statistics estimates in the following directories:

```
${MKL}/examples/vslc
```

```
${MKL}/examples/vslf
```

The VSL Summary Statistics API is implemented through task objects, or tasks. A task object is a data structure, or a descriptor, holding parameters that determine a specific VSL Summary Statistics operation. For example, such parameters may be precision, dimensions of user data, the matrix of the observations, or shapes of data arrays.

All the VSL Summary Statistics routines process a task object as follows:

1. Create a task.
2. Modify settings of the task parameters.
3. Compute statistical estimates.
4. Destroy the task.

The VSL Summary Statistics functions fall into the following categories:

Task Constructors - routines that create a new task object descriptor and set up most common parameters (dimension, number of observations, and matrix of the observations).

Task Editors - routines that can set or modify some parameter settings in the existing task descriptor.

Task Computation Routine - a routine that computes specified statistical estimates.

Task Destructor - a routine that deletes the task object and frees the memory.

A VSL Summary Statistics task object contains a series of pointers to the input and output data arrays. You can read and modify the datasets and estimates at any time but you should allocate and release memory for such data.

See detailed information on the algorithms, API, and their usage in the *Intel® MKL Summary Statistics Library Application Notes* on the Intel® MKL web page.

Naming Conventions

The names of the Fortran routines in the VSL Summary Statistics are in lowercase (`vslsseditquantiles`), while the names of types and constants are in uppercase. The names are not case-sensitive.

In C, the names of the routines, types, and constants are case-sensitive and can be lowercase and uppercase (`vslsSSEditQuantiles`).

The names of routines have the following structure:

`vsl[datatype]SS<base name>` for the C interface

`vsl[datatype]ss<base name>` for the Fortran interface

where

- `vsl` is a prefix indicating that the routine belongs to the Vector Statistical Library of Intel MKL.
- `[datatype]` specifies the type of the input and/or output data and can be `s` (single precision real type), `d` (double precision real type), or `i` (integer type).
- `SS`/`ss` indicates that the routine is intended for calculations of the VSL Summary Statistics estimates.
- `<base name>` specifies a particular functionality that the routine is designed for, for example, `NewTask`, `Compute`, `DeleteTask`.

NOTE The VSL Summary Statistics routine `vslDeleteTask` for deletion of the task is independent of the data type and its name omits the `[datatype]` field.

Data Types

The VSL Summary Statistics routines use the following data types for the calculations:

Type	Data Object
Fortran 90: `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Pointer to a VSL Summary Statistics task
Fortran 90: `REAL(KIND=4)` **C:** `float`	Input/output user data in single precision
Fortran 90: `REAL(KIND=8)` **C:** `double`	Input/output user data in double precision
Fortran 90: `INTEGER` or `INTEGER(KIND=8)` **C:** `MKL_INT` or `long long`	Other data

NOTE The actual size of the generic integer type is platform-specific and can be 32 or 64 bits in length. Before you compile your application, set an appropriate size for integers. See details in the 'Using the ILP64 Interface vs. LP64 Interface' section of the *Intel® MKL User's Guide*.

Parameters

The basic parameters in the task descriptor (addresses of dimensions, number of observations, and datasets) are assigned values when the task editors create or modify the task object. Other parameters are determined by the specific task and changed by the task editors.

Task Status and Error Reporting

The task status is an integer value, which is zero if no error is detected, or a specific non-zero error code otherwise. Negative status values indicate errors, and positive values indicate warnings. An error can be caused by invalid parameter values or a memory allocation failure.

The status codes have symbolic names defined in the respective header files. For the C/C++ interface, these names are defined as macros via the `#define` statements, and for the Fortran interface as integer constants via the `PARAMETER` operators.

If no error is detected, the function returns the `VSL_STATUS_OK` code, which is defined as zero:

```
C/C++:          #define VSL_STATUS_OK 0
F90/F95:        INTEGER, PARAMETER::VSL_STATUS_OK = 0
```

In the case of an error, the function returns a non-zero error code that specifies the origin of the failure. The header files for both C/C++ and Fortran languages define the following status codes for the VSL Summary Statistics error codes:

VSL Summary Statistics Status Codes

Status Code	Description
VSL_STATUS_OK	Operation is successfully completed.
VSL_SS_ERROR_ALLOCATION_FAILURE	Memory allocation has failed.
VSL_SS_ERROR_BAD_DIMEN	Dimension value is invalid.
VSL_SS_ERROR_BAD_OBSERV_N	Invalid number (zero or negative) of observations was obtained.
VSL_SS_ERROR_STORAGE_NOT_SUPPORTED	Storage format is not supported.
VSL_SS_ERROR_BAD_INDC_ADDR	Array of indices is not defined.
VSL_SS_ERROR_BAD_WEIGHTS	Array of weights contains negative values.
VSL_SS_ERROR_BAD_MEAN_ADDR	Array of means is not defined.
VSL_SS_ERROR_BAD_2R_MOM_ADDR	Array of the second order raw moments is not defined.
VSL_SS_ERROR_BAD_3R_MOM_ADDR	Array of the third order raw moments is not defined.
VSL_SS_ERROR_BAD_4R_MOM_ADDR	Array of the fourth order raw moments is not defined.
VSL_SS_ERROR_BAD_2C_MOM_ADDR	Array of the second order central moments is not defined.
VSL_SS_ERROR_BAD_3C_MOM_ADDR	Array of the third order central moments is not defined.
VSL_SS_ERROR_BAD_4C_MOM_ADDR	Array of the fourth order central moments is not defined.
VSL_SS_ERROR_BAD_KURTOSIS_ADDR	Array of kurtosis values is not defined.
VSL_SS_ERROR_BAD_SKEWNESS_ADDR	Array of skewness values is not defined.
VSL_SS_ERROR_BAD_MIN_ADDR	Array of minimum values is not defined.
VSL_SS_ERROR_BAD_MAX_ADDR	Array of maximum values is not defined.
VSL_SS_ERROR_BAD_VARIATION_ADDR	Array of variation coefficients is not defined.
VSL_SS_ERROR_BAD_COV_ADDR	Covariance matrix is not defined.
VSL_SS_ERROR_BAD_COR_ADDR	Correlation matrix is not defined.
VSL_SS_ERROR_BAD_QUANT_ORDER_ADDR	Array of quantile orders is not defined.

Status Code	Description
VSL_SS_ERROR_BAD_QUANT_ORDER	Quantile order value is invalid.
VSL_SS_ERROR_BAD_QUANT_ADDR	Array of quantiles is not defined.
VSL_SS_ERROR_BAD_ORDER_STATS_ADDR	Array of order statistics is not defined.
VSL_SS_ERROR_MOMORDER_NOT_SUPPORTED	Moment of requested order is not supported.
VSL_SS_NOT_FULL_RANK_MATRIX	Correlation matrix is not of full rank.
VSL_SS_ERROR_ALL_OBSERVS_OUTLIERS	All observations are outliers. (At least one observation must not be an outlier.)
VSL_SS_ERROR_BAD_ROBUST_COV_ADDR	Robust covariance matrix is not defined.
VSL_SS_ERROR_BAD_ROBUST_MEAN_ADDR	Array of robust means is not defined.
VSL_SS_ERROR_METHOD_NOT_SUPPORTED	Requested method is not supported.
VSL_SS_ERROR_NULL_TASK_DESCRIPTOR	Task descriptor is null.
VSL_SS_ERROR_BAD_OBSERV_ADDR	Dataset matrix is not defined.
VSL_SS_ERROR_BAD_ACCUM_WEIGHT_ADDR	Pointer to the variable that holds the value of accumulated weight is not defined.
VSL_SS_ERROR_SINGULAR_COV	Covariance matrix is singular.
VSL_SS_ERROR_BAD_POOLED_COV_ADDR	Pooled covariance matrix is not defined.
VSL_SS_ERROR_BAD_POOLED_MEAN_ADDR	Array of pooled means is not defined.
VSL_SS_ERROR_BAD_GROUP_COV_ADDR	Group covariance matrix is not defined.
VSL_SS_ERROR_BAD_GROUP_MEAN_ADDR	Array of group means is not defined.
VSL_SS_ERROR_BAD_GROUP_INDC_ADDR	Array of group indices is not defined.
VSL_SS_ERROR_BAD_GROUP_INDC	Group indices have improper values.
VSL_SS_ERROR_BAD_OUTLIERS_PARAMS_ADDR	Array of parameters for the outlier detection algorithm is not defined.
VSL_SS_ERROR_BAD_OUTLIERS_PARAMS_N_ADDR	Pointer to size of the parameter array for the outlier detection algorithm is not defined.
VSL_SS_ERROR_BAD_OUTLIERS_WEIGHTS_ADDR	Output of the outlier detection algorithm is not defined.
VSL_SS_ERROR_BAD_ROBUST_COV_PARAMS_ADDR	Array of parameters of the robust covariance estimation algorithm is not defined.
VSL_SS_ERROR_BAD_ROBUST_COV_PARAMS_N_ADDR	Pointer to the number of parameters of the algorithm for robust covariance is not defined.
VSL_SS_ERROR_BAD_STORAGE_ADDR	Pointer to the variable that holds the storage format is not defined.
VSL_SS_ERROR_BAD_PARTIAL_COV_IDX_ADDR	Array that encodes sub-components of a random vector for the partial covariance algorithm is not defined.

Status Code	Description
VSL_SS_ERROR_BAD_PARTIAL_COV_IDX	Array that encodes sub-components of a random vector for partial covariance has improper values.
VSL_SS_ERROR_BAD_PARTIAL_COV_ADDR	Partial covariance matrix is not defined.
VSL_SS_ERROR_BAD_PARTIAL_COR_ADDR	Partial correlation matrix is not defined.
VSL_SS_ERROR_BAD_MI_PARAMS_ADDR	Array of parameters for the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_PARAMS_N_ADDR	Pointer to number of parameters for the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_BAD_PARAMS_N	Size of the parameter array of the Multiple Imputation method is invalid.
VSL_SS_ERROR_BAD_MI_PARAMS	Parameters of the Multiple Imputation method are invalid.
VSL_SS_ERROR_BAD_MI_INIT_ESTIMATES_N_ADDR	Pointer to the number of initial estimates in the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_INIT_ESTIMATES_ADDR	Array of initial estimates for the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_SIMUL_VALS_ADDR	Array of simulated missing values in the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_SIMUL_VALS_N_ADDR	Pointer to the size of the array of simulated missing values in the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_ESTIMATES_N_ADDR	Pointer to the number of parameter estimates in the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_ESTIMATES_ADDR	Array of parameter estimates in the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_SIMUL_VALS_N	Invalid size of the array of simulated values in the Multiple Imputation method.
VSL_SS_ERROR_BAD_MI_ESTIMATES_N	Invalid size of an array to hold parameter estimates obtained using the Multiple Imputation method.
VSL_SS_ERROR_BAD_MI_OUTPUT_PARAMS	Array of output parameters in the Multiple Imputation method is not defined.
VSL_SS_ERROR_BAD_MI_PRIOR_N_ADDR	Pointer to the number of prior parameters is not defined.
VSL_SS_ERROR_BAD_MI_PRIOR_ADDR	Array of prior parameters is not defined.
VSL_SS_ERROR_BAD_MI_MISSING_VALS_N	Invalid number of missing values was obtained.
VSL_SS_SEMIDEFINITE_COR	Correlation matrix passed into the parameterization function is semi-definite.
VSL_SS_ERROR_BAD_PARAMTR_COR_ADDR	Correlation matrix to be parameterized is not defined.

Status Code	Description
VSL_SS_ERROR_BAD_COR	All eigenvalues of the correlation matrix to be parameterized are non-positive.
VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS_N_ADDR	Pointer to the number of parameters for the quantile computation algorithm for streaming data is not defined.
VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS_ADDR	Array of parameters of the quantile computation algorithm for streaming data is not defined.
VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS_N	Invalid number of parameters of the quantile computation algorithm for streaming data has been obtained.
VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS	Invalid parameters of the quantile computation algorithm for streaming data have been passed.
VSL_SS_ERROR_BAD_STREAM_QUANT_ORDER_ADDR	Array of the quantile orders for streaming data is not defined.
VSL_SS_ERROR_BAD_STREAM_QUANT_ORDER	Invalid quantile order for streaming data is defined.
VSL_SS_ERROR_BAD_STREAM_QUANT_ADDR	Array of quantiles for streaming data is not defined.

Routines for robust covariance estimation, outlier detection, partial covariance estimation, multiple imputation, and parameterization of a correlation matrix can return internal error codes that are related to a specific implementation. Such error codes indicate invalid input data or other bugs in the Intel MKL routines other than the VSL Summary Statistics routines.

Task Constructors

Task constructors are routines intended for creating a new task descriptor and setting up basic parameters.

NOTE If the constructor fails to create a task descriptor, it returns the NULL task pointer.

vslSSNewTask

Creates and initializes a new summary statistics task descriptor.

Syntax

Fortran:

```
status = vslsssnewtask(task, p, n, xstorage, x, w, indices)
status = vsldssnewtask(task, p, n, xstorage, x, w, indices)
```

C:

```
status = vslsSSNewTask(&task, p, n, xstorage, x, w, indices);
status = vsldSSNewTask(&task, p, n, xstorage, x, w, indices);
```

Include Files

- Fortran 90: mkl_vsl.f90

- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
p	**Fortran:** INTEGER **C:** MKL_INT	Dimension of the task, number of variables
n	**Fortran:** INTEGER **C:** MKL_INT	Number of observations
xstorage	**Fortran:** INTEGER **C:** MKL_INT	Storage format of matrix of observations
x	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslsssnewtask REAL(KIND=8) DIMENSION(*) for vsldssnewtask **C:** float* for vslsSSNewTask double* for vsldSSNewTask	Matrix of observations
w	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslsssnewtask REAL(KIND=8) DIMENSION(*) for vsldssnewtask **C:** float* for vslsSSNewTask double* for vsldSSNewTask	Array of weights of size *n*. Elements of the arrays are non-negative numbers. If a NULL pointer is passed, each observation is assigned weight equal to 1.
indices	**Fortran:** INTEGER, DIMENSION(*) **C:** MKL_INT*	Array of vector components that will be processed. Size of array is *p*. If a NULL pointer is passed, all components of random vector are processed.

Output Parameters

Name	Type	Description
task	**Fortran:** TYPE(VSL_SS_TASK) **C:** VSLSSTaskPtr*	Descriptor of the task
status	**Fortran:** INTEGER **C:** int	Set to VSL_STATUS_OK if the task is created successfully, otherwise a non-zero error code is returned.

Description

Each vslSSNewTask constructor routine creates a new summary statistics task descriptor with the user-specified value for a required parameter, dimension of the task. The optional parameters (matrix of observations, its storage format, number of observations, weights of observations, and indices of the random vector components) are set to their default values.

The observations of random p-dimensional vector $\xi = (\xi_1, ..., \xi_i, ..., \xi_p)$, which are n vectors of dimension p, are passed as a one-dimensional array x. The parameter *xstorage* defines the storage format of the observations and takes one of the possible values listed in Table "Storage format of matrix of observations and order statistics".

NOTE Since matrices in Fortran are stored by columns while in C they are stored by rows, initialization of the *xstorage* variable in Fortran is opposite to that in C. Set *xstorage* to VSL_SS_MATRIX_STORAGE_COLS, if the dataset is stored as a two-dimensional matrix that consists of p rows and n columns; otherwise, use the VSL_SS_MATRIX_STORAGE_ROWS constant.

Storage format of matrix of observations and order statistics

Parameter	Description
VSL_SS_MATRIX_STORAGE_ROWS	The observations of random vector ξ are packed by rows: n data points for the vector component ξ_1 come first, n data points for the vector component ξ_2 come second, and so forth.
VSL_SS_MATRIX_STORAGE_COLS	The observations of random vector ξ are packed by columns: the first p-dimensional observation of the vector ξ comes first, the second p-dimensional observation of the vector comes second, and so forth.

A one-dimensional array w of size n contains non-negative weights assigned to the observations. You can pass a NULL array into the constructor. In this case, each observation is assigned the default value of the weight.

You can choose vector components for which you wish to compute statistical estimates. If an element of the vector indices of size p contains 0, the observations that correspond to this component are excluded from the calculations. If you pass the NULL value of the parameter into the constructor, statistical estimates for all random variables are computed.

If the constructor fails to create a task descriptor, it returns the NULL task pointer.

Task Editors

Task editors are intended to set up or change the task parameters listed in Table "Parameters of VSL Summary Statistics Task to Be Initialized or Modified". As an example, to compute the sample mean for a one-dimensional dataset, initialize a variable for the mean value, and pass its address into the task as shown in the example below:

```
#define DIM    1
#define N   1000

int main()
{
     VSLSSTaskPtr task;
     double x[N];
     double mean;
     MKL_INT p, n, xstorage;
     int status;
     /* initialize variables used in the computations of sample mean */
     p = DIM;
     n = N;
     xstorage = VSL_SS_MATRIX_STORAGE_ROWS;
     mean = 0.0;

     /* create task */
     status = vsldSSNewTask( &task, &p, &n, &xstorage, x, 0, 0 );

     /* initialize task parameters */
```

```
    status = vsldSSEditTask( task, VSL_SS_ED_MEAN, &mean );

    /* compute mean using SS fast method */
    status = vsldSSCompute(task, VSL_SS_MEAN, VSL_SS_METHOD_FAST );

    /* deallocate task resources */
    status = vslSSDeleteTask( &task );

    return 0;
}
```

Use the single (`vslsssedittask`) or double (`vsldsedittask`) version of an editor, to initialize single or double precision version task parameters, respectively. Use an integer version of an editor (`vslisseditask`) to initialize parameters of the integer type.

Table "VSL Summary Statistics Task Editors" lists the task editors for VSL Summary Statistics. Each of them initializes and/or modifies a respective group of related parameters.

VSL Summary Statistics Task Editors

Editor	Description
vslSSEditTask	Changes a pointer in the task descriptor.
vslSSEditMoments	Changes pointers to arrays associated with raw and central moments.
vslSSEditCovCor	Changes pointers to arrays associated with covariance and/or correlation matrices.
vslSSEditPartialCovCor	Changes pointers to arrays associated with partial covariance and/or correlation matrices.
vslSSEditQuantiles	Changes pointers to arrays associated with quantile/order statistics calculations.
vslSSEditStreamQuantiles	Changes pointers to arrays for quantile related calculations for streaming data.
vslSSEditPooledCovariance	Changes pointers to arrays associated with algorithms related to a pooled covariance matrix.
vslSSEditRobustCovariance	Changes pointers to arrays for robust estimation of a covariance matrix and mean.
vslSSEditOutliersDetection	Changes pointers to arrays for detection of outliers.
vslSSEditMissingValues	Changes pointers to arrays associated with the method of supporting missing values in a dataset.
vslSSEditCorParameterization	Changes pointers to arrays associated with the algorithm for parameterization of a correlation matrix.

NOTE You can use the `NULL` task pointer in calls to editor routines. In this case, the routine is terminated and no system crash occurs.

vslSSEditTask

Modifies address of an input/output parameter in the task descriptor.

Syntax

Fortran:

```
status = vslsssedittask(task, parameter, par_addr)
```

```
status = vsldssedittask(task, parameter, par_addr)
status = vslissedittask(task, parameter, par_addr)
```

C:

```
status = vslsSSEditTask(task, parameter, par_addr);
status = vsldSSEditTask(task, parameter, par_addr);
status = vsliSSEditTask(task, parameter, par_addr);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Descriptor of the task
parameter	**Fortran:** `INTEGER` **C:** `MKL_INT`	Parameter to change
par_addr	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssedittask` `REAL(KIND=8) DIMENSION(*)` for `vsldssedittask` `INTEGER DIMENSION(*)` for `vslissedittask` **C:** `float*` for `vslsSSEditTask` `double*` for `vsldSSEditTask` `MKL_INT*` for `vsliSSEditTask`	Address of the new parameter

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditTask` routine replaces the pointer to the parameter stored in the VSL Summary Statistics task descriptor with the *par_addr* pointer. If you pass the `NULL` pointer to the editor, no changes take place in the task and a corresponding error code is returned. See Table "Parameters of VSL Summary Statistics Task to Be Initialized or Modified" for the predefined values of the parameter.

Use the single (`vslsssedittask`) or double (`vsldssedittask`) version of the editor, to initialize single or double precision version task parameters, respectively. Use an integer version of the editor (`vslissedittask`) to initialize parameters of the integer type.

Parameters of VSL Summary Statistics Task to Be Initialized or Modified

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_DIMEN	i	Address of a variable that holds the task dimension	Required. Positive integer value.
VSL_SS_ED_OBSERV_N	i	Address of a variable that holds the number of observations	Required. Positive integer value.
VSL_SS_ED_OBSERV	d, s	Address of the observation matrix	Required. Provide the matrix containing your observations.
VSL_SS_ED_OBSERV_STORAGE	i	Address of a variable that holds the storage format for the observation matrix	Required. Provide a storage format supported by the library whenever you pass a matrix of observations.[1]
VSL_SS_ED_INDC	i	Address of the array of indices	Optional. Provide this array if you need to process individual components of the random vector. Set entry *i* of the array to one to include the *i*th coordinate in the analysis. Set entry *i* of the array to zero to exclude the *i*th coordinate from the analysis.
VSL_SS_ED_WEIGHTS	d, s	Address of the array of observation weights	Optional. If the observations have weights different from the default weight (one), set entries of the array to non-negative floating point values.
VSL_SS_ED_MEAN	d, s	Address of the array of means	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.
VSL_SS_ED_2R_MOM	d, s	Address of an array of raw moments of the second order	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.
VSL_SS_ED_3R_MOM	d, s	Address of an array of raw moments of the third order	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.
VSL_SS_ED_4R_MOM	d, s	Address of an array of raw moments of the fourth order	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_2C_MOM	d, s	Address of an array of central moments of the second order	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first and second order.
VSL_SS_ED_3C_MOM	d, s	Address of an array of central moments of the third order	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, and third order.
VSL_SS_ED_4C_MOM	d, s	Address of an array of central moments of the fourth order	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, third, and fourth order.
VSL_SS_ED_KURTOSIS	d, s	Address of the array of kurtosis estimates	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, third, and fourth order.
VSL_SS_ED_SKEWNESS	d, s	Address of the array of skewness estimates	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, and third order.
VSL_SS_ED_MIN	d, s	Address of the array of minimum estimates	Optional. Set entries of array to meaningful values, such as the values of the first observation.
VSL_SS_ED_MAX	d, s	Address of the array of maximum estimates	Optional. Set entries of array to meaningful values, such as the values of the first observation.

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_VARIATION	d, s	Address of the array of variation coefficients	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first and second order.
VSL_SS_ED_COV	d, s	Address of a covariance matrix	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Make sure you also provide an array for the mean.
VSL_SS_ED_COV_STORAGE	i	Address of the variable that holds the storage format for a covariance matrix	Required. Provide a storage format supported by the library whenever you intend to compute the covariance matrix.[2]
VSL_SS_ED_COR	d, s	Address of a correlation matrix	Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. If you initialize the matrix in non-trivial way, make sure that the main diagonal contains variance values. Also, provide an array for the mean.
VSL_SS_ED_COR_STORAGE	i	Address of the variable that holds the correlation storage format for a correlation matrix	Required. Provide a storage format supported by the library whenever you intend to compute the correlation matrix.[2]
VSL_SS_ED_ACCUM_WEIGHT	d, s	Address of the array of size 2 that holds the accumulated weight (sum of weights) in the first position and the sum of weights squared in the second position	Optional. Set the entries of the matrix to meaningful values (typically zero) if you intend to do progressive processing of the dataset or need the sum of weights and sum of squared weights assigned to observations.
VSL_SS_ED_QUANT_ORDER_N	i	Address of the variable that holds the number of quantile orders	Required. Positive integer value. Provide the number of quantile orders whenever you compute quantiles.
VSL_SS_ED_QUANT_ORDER	d, s	Address of the array of quantile orders	Required. Set entries of array to values from the interval (0,1). Provide this parameter whenever you compute quantiles.

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_QUANT_QUANTILES	d, s	Address of the array of quantiles	None.
VSL_SS_ED_ORDER_STATS	d, s	Address of the array of order statistics	None.
VSL_SS_ED_GROUP_INDC	i	Address of the array of group indices used in computation of a pooled covariance matrix	Required. Set entry i to integer value k if the observation belongs to group k. Values of k take values in the range [0, g-1], where g is the number of groups.
VSL_SS_ED_POOLED_COV_STORAGE	i	Address of a variable that holds the storage format for a pooled covariance matrix	Required. Provide a storage format supported by the library whenever you intend to compute pooled covariance.[2]
VSL_SS_ED_POOLED_MEAN	i	Address of an array of pooled means	None.
VSL_SS_ED_POOLED_COV	d, s	Address of pooled covariance matrices	None.
VSL_SS_ED_GROUP_COV_INDC	d, s	Address of an array of indices for which covariance/means should be computed	Optional. Set the kth entry of the array to 1 if you need group covariance and mean for group k; otherwise set it to zero.
VSL_SS_ED_GROUP_MEANS	i	Address of an array of group means	None.
VSL_SS_ED_GROUP_COV_STORAGE	d, s	Address of a variable that holds the storage format for a group covariance matrix	Required. Provide a storage format supported by the library whenever you intend to get group covariance.[2]
VSL_SS_ED_GROUP_COV	i	Address of group covariance matrices	None.
VSL_SS_ED_ROBUST_COV_STORAGE	d, s	Address of a variable that holds the storage format for a robust covariance matrix	Required. Provide a storage format supported by the library whenever you compute robust covariance[2].
VSL_SS_ED_ROBUST_COV_PARAMS_N	i	Address of a variable that holds the number of algorithmic parameters of the method for robust covariance estimation	Required. Set to the number of TBS parameters, *VSL_SS_TBS_PARAMS_N*.
VSL_SS_ED_ROBUST_COV_PARAMS	d, s	Address of an array of parameters of the method for robust estimation of a covariance	Required. Set the entries of the array according to the description in EditRobustCovariance.

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_ROBUST_MEAN	i	Address of an array of robust means	None.
VSL_SS_ED_ROBUST_COV	d, s	Address of a robust covariance matrix	None.
VSL_SS_ED_OUTLIERS_PARAMS_N	d, s	Address of a variable that holds the number of parameters of the outlier detection method	Required. Set to the number of outlier detection parameters, *VSL_SS_BACON_PARAMS_N*.
VSL_SS_ED_OUTLIERS_PARAMS	i	Address of an array of algorithmic parameters for the outlier detection method	Required. Set the entries of the array according to the description in EditOutliersDetection.
VSL_SS_ED_OUTLIERS_WEIGHT	d, s	Address of an array of weights assigned to observations by the outlier detection method	None.
VSL_SS_ED_ORDER_STATS_STORAGE	d, s	Address of a variable that holds the storage format of an order statistics matrix	Required. Provide a storage format supported by the library whenever you compute a matrix of order statistics.[1]
VSL_SS_ED_PARTIAL_COV_IDX	i	Address of an array that encodes subcomponents of a random vector	Required. Set the entries of the array according to the description in EditPartialCovCor.
VSL_SS_ED_PARTIAL_COV	d, s	Address of a partial covariance matrix	None.
VSL_SS_ED_PARTIAL_COV_STORAGE	i	Address of a variable that holds the storage format of a partial covariance matrix	Required. Provide a storage format supported by the library whenever you compute the partial covariance.[2]
VSL_SS_ED_PARTIAL_COR	d, s	Address of a partial correlation matrix	None.
VSL_SS_ED_PARTIAL_COR_STORAGE	i	Address of a variable that holds the storage format for a partial correlation matrix	Required. Provide a storage format supported by the library whenever you compute the partial correlation.[2]
VSL_SS_ED_MI_PARAMS_N	i	Address of a variable that holds the number of algorithmic parameters for the Multiple Imputation method	Required. Set to the number of MI parameters, *VSL_SS_MI_PARAMS_SIZE*.
VSL_SS_ED_MI_PARAMS	d, s	Address of an array of algorithmic parameters for the Multiple Imputation method	Required. Set entries of the array according to the description in EditMissingValues.

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_MI_INIT_ESTIMATES_N	i	Address of a variable that holds the number of initial estimates for the Multiple Imputation method	Optional. Set to $p+p*(p+1)/2$, where p is the task dimension.
VSL_SS_ED_MI_INIT_ESTIMATES	d, s	Address of an array of initial estimates for the Multiple Imputation method	Optional. Set the values of the array according to the description in "Basic Components of the Multiple Imputation Function in Summary Statistics Library" in the *Intel® MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page.
VSL_SS_ED_MI_SIMUL_VALS_N	i	Address of a variable that holds the number of simulated values in the Multiple Imputation method	Optional. Positive integer indicating the number of missing points in the observation matrix.
VSL_SS_ED_MI_SIMUL_VALS	d, s	Address of an array of simulated values in the Multiple Imputation method	None.
VSL_SS_ED_MI_ESTIMATES_N	i	Address of a variable that holds the number of estimates obtained as a result of the Multiple Imputation method	Optional. Positive integer number defined according to the description in "Basic Components of the Multiple Imputation Function in Summary Statistics Library" in the *Intel® MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page.
VSL_SS_ED_MI_ESTIMATES	d, s	Address of an array of estimates obtained as a result of the Multiple Imputation method	None.
VSL_SS_ED_MI_PRIOR_N	i	Address of a variable that holds the number of prior parameters for the Multiple Imputation method	Optional. If you pass a user-defined array of prior parameters, set this parameter to $(p^2+3*p+4)/2$, where p is the task dimension.
VSL_SS_ED_MI_PRIOR	d, s	Address of an array of prior parameters for the Multiple Imputation method	Optional. Set entries of the array of prior parameters according to the description in "Basic Components of the Multiple Imputation Function in Summary Statistics Library" in the *Intel® MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page.

Parameter Value	Type	Purpose	Initialization
VSL_SS_ED_PARAMTR_COR	d, s	Address of a parameterized correlation matrix	None.
VSL_SS_ED_PARAMTR_COR_STORAGE	i	Address of a variable that holds the storage format of a parameterized correlation matrix	Required. Provide a storage format supported by the library whenever you compute the parameterized correlation matrix.[2]
VSL_SS_ED_STREAM_QUANT_PARAMS_N	i	Address of a variable that holds the number of parameters of a quantile computation method for streaming data	Required. Set to the number of quantile computation parameters, *VSL_SS_SQUANTS_ZW_PARAMS_N*.
VSL_SS_ED_STREAM_QUANT_PARAMS	d, s	Address of an array of parameters of a quantile computation method for streaming data	Required. Set the entries of the array according to the description in "Computing Quantiles for Streaming Data" in the *Intel® MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page.
VSL_SS_ED_STREAM_QUANT_ORDER_N	i	Address of a variable that holds the number of quantile orders for streaming data	Required. Positive integer value.
VSL_SS_ED_STREAM_QUANT_ORDER	d, s	Address of an array of quantile orders for streaming data	Required. Set entries of the array to values from the interval (0,1). Provide this parameter whenever you compute quantiles.
VSL_SS_ED_STREAM_QUANT_QUANTILES	d, s	Address of an array of quantiles for streaming data	None.

1. See Table: "Storage format of matrix of observations and order statistics" for storage formats.
2. See Table: "Storage formats of a variance-covariance/correlation matrix" for storage formats.

vslSSEditMoments

Modifies the pointers to arrays that hold moment estimates.

Syntax

Fortran:

```
status = vslssseditmoments(task, mean, r2m, r3m, r4m, c2m, c3m, c4m)
status = vsldsseditmoments(task, mean, r2m, r3m, r4m, c2m, c3m, c4m)
```

C:

```
status = vslsSSEditMoments(task, mean, r2m, r3m, r4m, c2m, c3m, c4m);
status = vsldSSEditMoments(task, mean, r2m, r3m, r4m, c2m, c3m, c4m);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Descriptor of the task
mean	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	Pointer to the array of means
r2m	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	Pointer to the array of raw moments of the 2nd order
r3m	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	Pointer to the array of raw moments of the 3rd order
r4m	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	Pointer to the array of raw moments of the 4th order
c2m	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	Pointer to the array of central moments of the 2nd order
c3m	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments`	Pointer to the array of central moments of the 3rd order

Name	Type	Description
	`REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	
c4m	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditmoments` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmoments` **C:** `float*` for `vslsSSEditMoments` `double*` for `vsldSSEditMoments`	Pointer to the array of central moments of the 4th order

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditMoments` routine replaces pointers to the arrays that hold estimates of raw and central moments with values passed as corresponding parameters of the routine. If an input parameter is `NULL`, the value of the relevant parameter remains unchanged.

vslSSEditCovCor

Modifies the pointers to covariance/correlation parameters.

Syntax

Fortran:

```
status = vslsssseditcovcor(task, mean, cov, cov_storage, cor, cor_storage)
status = vsldsseditcovcor(task, mean, cov, cov_storage, cor, cor_storage)
```

C:

```
status = vslsSSEditCovCor(task, mean, cov, cov_storage, cor, cor_storage);
status = vsldSSEditCovCor(task, mean, cov, cov_storage, cor, cor_storage);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)`	Descriptor of the task

Name	Type	Description
	C: `VSLSSTaskPtr`	
mean	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsseditcovcor` `REAL(KIND=8) DIMENSION(*)` for `vsldseditcovcor` **C:** `float*` for `vslsSSEditCovCor` `double*` for `vsldSSEditCovCor`	Pointer to the array of means
cov	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsseditcovcor` `REAL(KIND=8) DIMENSION(*)` for `vsldseditcovcor` **C:** `float*` for `vslsSSEditCovCor` `double*` for `vsldSSEditCovCor`	Pointer to a covariance matrix
cov_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the covariance matrix
cor	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsseditcovcor` `REAL(KIND=8) DIMENSION(*)` for `vsldseditcovcor` **C:** `float*` for `vslsSSEditCovCor` `double*` for `vsldSSEditCovCor`	Pointer to a correlation matrix
cor_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the correlation matrix

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditCovCor` routine replaces pointers to the array of means, covariance/correlation arrays, and their storage format with values passed as corresponding parameters of the routine. See Table "Storage formats of a variance-covariance/correlation matrix" for possible values of the *cov_storage* and *cor_storage* parameters. If an input parameter is `NULL`, the old value of the parameter remains unchanged in the VSL Summary Statistics task descriptor.

Storage formats of a variance-covariance/correlation matrix

Parameter	Description
VSL_SS_MATRIX_STORAGE_FULL	A symmetric variance-covariance/correlation matrix is a one-dimensional array with elements $c(i,j)$ stored as $cp(i*p + j)$. The size of the array is $p*p$.
VSL_SS_MATRIX_STORAGE_L_PACKED	A symmetric variance-covariance/correlation matrix with elements $c(i,j)$ is packed as a one-dimensional array $cp(i + (2n - j)*(j - 1)/2)$ for $j \le i$. The size of the array is $p*(p+ 1)/2$.
VSL_SS_MATRIX_STORAGE_U_PACKED	A symmetric variance-covariance/correlation matrix with elements $c(i,j)$ is packed as a one-dimensional array $cp(i + j*(j - 1)/2)$ for $i \le j$. The size of the array is $p*(p+ 1)/2$.

vslSSEditPartialCovCor

Modifies the pointers to partial covariance/correlation parameters.

Syntax

Fortran:

```
status = vslssseditpartialcovcor(task, p_idx_array, cov, cov_storage, cor,
cor_storage, p_cov, p_cov_storage, p_cor, p_cor_storage)
```

```
status = vsldsseditpartialcovcor(task, p_idx_array, cov, cov_storage, cor,
cor_storage, p_cov, p_cov_storage, p_cor, p_cor_storage)
```

C:

```
status = vslsSSEditPartialCovCor(task, p_idx_array, cov, cov_storage, cor,
cor_storage, p_cov, p_cov_storage, p_cor, p_cor_storage);
```

```
status = vsldSSEditPartialCovCor(task, p_idx_array, cov, cov_storage, cor,
cor_storage, p_cov, p_cov_storage, p_cor, p_cor_storage);
```

Include Files

- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(VSL_SS_TASK) **C:** VSLSSTaskPtr	Descriptor of the task
p_idx_array	**Fortran:** INTEGER **C:** MKL_INT*	Pointer to the array that encodes indices of subcomponents z and Y of the random vector as described in section Mathematical Notation and Definitions. *p_idx_array[i]* equals to

Name	Type	Description
		-1 if the *i*-th component of the random vector belongs to *Z* 1, if the *i*-th component of the random vector belongs to *Y*.
cov	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditpartialcovcor` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpartialcovcor` **C:** `float*` for `vslsSSEditPartialCovCor` `double*` for `vsldSSEditPartialCovCor`	Pointer to a covariance matrix
cov_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the covariance matrix
cor	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditpartialcovcor` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpartialcovcor` **C:** `float*` for `vslsSSEditPartialCovCor` `double*` for `vsldSSEditPartialCovCor`	Pointer to a correlation matrix
cor_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the correlation matrix
p_cov	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditpartialcovcor` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpartialcovcor` **C:** `float*` for `vslsSSEditPartialCovCor` `double*` for `vsldSSEditPartialCovCor`	Pointer to a partial covariance matrix
p_cov_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the partial covariance matrix
p_cor	**Fortran:** `REAL(KIND=4) DIMENSION(*)`	Pointer to a partial correlation matrix

Name	Type	Description
	for `vslsssEditPartialCovCor` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpartialcovcor` **C:** `float*` for `vslsSSEditPartialCovCor` `double*` for `vsldSSEditPartialCovCor`	
p_cor_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the partial correlation matrix

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditPartialCovCor` routine replaces pointers to covariance/correlation arrays, partial covariance/correlation arrays, and their storage format with values passed as corresponding parameters of the routine. See Table "Storage formats of a variance-covariance/correlation matrix" for possible values of the *cov_storage*, *cor_storage*, *p_cov_storage*, and *p_cor_storage* parameters. If an input parameter is `NULL`, the old value of the parameter remains unchanged in the VSL Summary Statistics task descriptor.

vslSSEditQuantiles

Modifies the pointers to parameters related to quantile computations.

Syntax

Fortran:

```
status = vslsssEditQuantiles(task, quant_order_n, quant_order, quants, order_stats, order_stats_storage)
status = vsldsseditquantiles(task, quant_order_n, quant_order, quants, order_stats, order_stats_storage)
```

C:

```
status = vslsSSEditQuantiles(task, quant_order_n, quant_order, quants, order_stats, order_stats_storage);
status = vsldSSEditQuantiles(task, quant_order_n, quant_order, quants, order_stats, order_stats_storage);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(VSL_SS_TASK) **C:** VSLSSTaskPtr	Descriptor of the task
quant_order_n	**Fortran:** INTEGER **C:** MKL_INT*	Pointer to the number of quantile orders
quant_order	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslssseditquantiles REAL(KIND=8) DIMENSION(*) for vsldsseditquantiles **C:** float* for vslsSSEditQuantiles double* for vsldSSEditQuantiles	Pointer to the array of quantile orders
quants	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslssseditquantiles REAL(KIND=8) DIMENSION(*) for vsldsseditquantiles **C:** float* for vslsSSEditQuantiles double* for vsldSSEditQuantiles	Pointer to the array of quantiles
order_stats	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslssseditquantiles REAL(KIND=8) DIMENSION(*) for vsldsseditquantiles **C:** float* for vslsSSEditQuantiles double* for vsldSSEditQuantiles	Pointer to the array of order statistics
order_stats_storage	**Fortran:** INTEGER **C:** MKL_INT*	Pointer to the storage format of the order statistics array

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER	Current status of the task

Name	Type	Description
	C: `int`	

Description

The `vslSSEditQuantiles` routine replaces pointers to the number of quantile orders, the array of quantile orders, the array of quantiles, the array that holds order statistics, and the storage format for the order statistics with values passed into the routine. See Table "Storage format of matrix of observations and order statistics" for possible values of the *order_statistics_storage* parameter. If an input parameter is `NULL`, the corresponding parameter in the VSL Summary Statistics task descriptor remains unchanged.

vslSSEditStreamQuantiles

Modifies the pointers to parameters related to quantile computations for streaming data.

Syntax

Fortran:

status = vslsssseditstreamquantiles(*task*, *quant_order_n*, *quant_order*, *quants*, *nparams*, *params*)

status = vsldsseditstreamquantiles(*task*, *quant_order_n*, *quant_order*, *quants*, *nparams*, *params*)

C:

status = vslsSSEditStreamQuantiles(*task*, *quant_order_n*, *quant_order*, *quants*, *nparams*, *params*);

status = vsldSSEditStreamQuantiles(*task*, *quant_order_n*, *quant_order*, *quants*, *nparams*, *params*);

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)`	Descriptor of the task
	C: `VSLSSTaskPtr`	
quant_order_n	**Fortran:** `INTEGER`	Pointer to the number of quantile orders
	C: `MKL_INT*`	
quant_order	**Fortran:** `REAL(KIND=4) DIMENSION(*)`	Pointer to the array of quantile orders
	for `vslsssseditstreamquantiles`	
	`REAL(KIND=8) DIMENSION(*)`	
	for `vsldsseditstreamquantiles`	
	C: `float*`	
	for `vslsSSEditStreamQuantiles`	

Name	Type	Description
	`double*` for `vsldSSEditStreamQuantiles`	
quants	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditstreamquantiles` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditstreamquantiles` **C:** `float*` for `vslsSSEditStreamQuantiles` `double*` for `vsldSSEditStreamQuantiles`	Pointer to the array of quantiles
nparams	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the number of the algorithm parameters
params	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditstreamquantiles` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditstreamquantiles` **C:** `float*` for `vslsSSEditStreamQuantiles` `double*` for `vsldSSEditStreamQuantiles`	Pointer to the array of the algorithm parameters

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditStreamQuantiles` routine replaces pointers to the number of quantile orders, the array of quantile orders, the array of quantiles, the number of the algorithm parameters, and the array of the algorithm parameters with values passed into the routine. If an input parameter is `NULL`, the corresponding parameter in the VSL Summary Statistics task descriptor remains unchanged.

vslSSEditPooledCovariance

Modifies pooled/group covariance matrix array pointers.

Syntax

Fortran:

```
status = vslsssEditPooledCovariance(task, grp_indices, pld_mean, pld_cov,
grp_cov_indices, grp_means, grp_cov)
status = vsldsseditpooledcovariance(task, grp_indices, pld_mean, pld_cov,
grp_cov_indices, grp_means, grp_cov)
```

C:

```
status = vslsSSEditPooledCovariance(task, grp_indices, pld_mean, pld_cov,
grp_cov_indices, grp_means, grp_cov);
status = vsldSSEditPooledCovariance(task, grp_indices, pld_mean, pld_cov,
grp_cov_indices, grp_means, grp_cov);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Descriptor of the task
grp_indices	**Fortran:** `INTEGER DIMENSION(*)` **C:** `MKL_INT*`	Pointer to an array of size *n*. The *i*-th element of the array contains the number of the group the observation belongs to.
pld_mean	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssEditPooledCovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpooledcovariance` **C:** `float*` for `vslsSSEditPooledCovariance` `double*` for `vsldSSEditPooledCovariance`	Pointer to the array of pooled means
pld_cov	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssEditPooledCovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpooledcovariance` **C:** `float*` for `vslsSSEditPooledCovariance` `double*` for	Pointer to the array that holds a pooled covariance matrix

Name	Type	Description
	`vsldSSEditPooledCovariance`	
grp_cov_indices	**Fortran:** `INTEGER DIMENSION(*)` **C:** `MKL_INT*`	Pointer to the array that contains indices of group covariance matrices to return
grp_means	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditpooledcovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpooledcovariance` **C:** `float*` for `vslsSSEditPooledCovariance` `double*` for `vsldSSEditPooledCovariance`	Pointer to the array of group means
grp_cov	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditpooledcovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditpooledcovariance` **C:** `float*` for `vslsSSEditPooledCovariance` `double*` for `vsldSSEditPooledCovariance`	Pointer to the array that holds group covariance matrices

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditPooledCovariance` routine replaces pointers to the array of group indices, the array of pooled means, the array for a pooled covariance matrix, and pointers to the array of indices of group matrices, the array of group means, and the array for group covariance matrices with values passed in the editors. If an input parameter is `NULL`, the corresponding parameter in the VSL Summary Statistics task descriptor remains unchanged. Use the `vslSSEditTask` routine to replace the storage format for pooled and group covariance matrices.

vslSSEditRobustCovariance

Modifies pointers to arrays related to a robust covariance matrix.

Syntax

Fortran:

```
status = vslsssEditRobustCovariance(task, rcov_storage, nparams, params, rmean, rcov)
status = vsldsseditrobustcovariance(task, rcov_storage, nparams, params, rmean, rcov)
```

C:

```
status = vslsSSEditRobustCovariance(task, rcov_storage, nparams, params, rmean, rcov);
status = vsldSSEditRobustCovariance(task, rcov_storage, nparams, params, rmean, rcov);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Descriptor of the task
rcov_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of a robust covariance matrix
nparams	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the number of method parameters
params	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssEditRobustCovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditrobustcovariance` **C:** `float*` for `vslsSSEditRobustCovariance` `double*` for `vsldSSEditRobustCovariance`	Pointer to the array of method parameters
rmean	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssEditRobustCovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditrobustcovariance` **C:** `float*` for `vslsSSEditRobustCovariance` `double*` for	Pointer to the array of robust means

Name	Type	Description
	`vsldSSEditRobustCovariance`	
rcov	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsssseditrobustcovariance` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditrobustcovariance` **C:** `float*` for `vslsSSEditRobustCovariance` `double*` for `vsldSSEditRobustCovariance`	Pointer to a robust covariance matrix

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditRobustCovariance` routine uses values passed as parameters of the routine to replace:

- pointers to covariance matrix storage
- pointers to the number of method parameters and to the array of the method parameters of size *nparams*
- pointers to the arrays that hold robust means and covariance

See Table "Storage formats of a variance-covariance/correlation matrix" for possible values of the *rcov_storage* parameter. If an input parameter is `NULL`, the corresponding parameter in the task descriptor remains unchanged.

Intel MKL provides a Translated Biweight S-estimator (TBS) for robust estimation of a variance-covariance matrix and mean [Rocke96]. Use one iteration of the Maronna algorithm with the reweighting step [Maronna02] to compute the initial point of the algorithm. Pack the parameters of the TBS algorithm into the *params* array and pass them into the editor. Table "Structure of the Array of TBS Parameters" describes the *params* structure.

Structure of the Array of TBS Parameters

Array Position	Algorithm Parameter	Description
0	ε	Breakdown point, the number of outliers the algorithm can hold. By default, the value is $(n-p)/(2n)$.
1	α	Asymptotic rejection probability, see details in [Rocke96]. By default, the value is 0.001.
2	δ	Stopping criterion: the algorithm is terminated if weights are changed less than δ. By default, the value is 0.001.
3	`max_iter`	Maximum number of iterations. The algorithm terminates after `max_iter` iterations. By default, the value is 10.

Array Position	Algorithm Parameter	Description
		If you set this parameter to zero, the function returns a robust estimate of the variance-covariance matrix computed using the Maronna method [Maronna02] only.

See additional details of the algorithm usage model in the *Intel© MKL Summary Statistics Library Application Notes* document on the Intel© MKL web page.

vslSSEditOutliersDetection

Modifies array pointers related to multivariate outliers detection.

Syntax

Fortran:

```
status = vslssseditoutliersdetection(task, nparams, params, w)
status = vsldsseditoutliersdetection(task, nparams, params, w)
```

C:

```
status = vslsSSEditOutliersDetection(task, nparams, params, w);
status = vsldSSEditOutliersDetection(task, nparams, params, w);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Descriptor of the task
nparams	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the number of method parameters
params	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslssseditoutliersdetection` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditoutliersdetection` **C:** `float*` for `vslsSSEditOutliersDetection` `double*` for `vsldSSEditOutliersDetection`	Pointer to the array of method parameters

Name	Type	Description
w	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslssseditoutliersdetection` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditoutliersdetection` **C:** `float*` for `vslsSSEditOutliersDetection` `double*` for `vsldSSEditOutliersDetection`	Pointer to an array of size *n*. The array holds the weights of observations to be marked as outliers.

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditOutliersDetection` routine uses the parameters passed to replace

- the pointers to the number of method parameters and to the array of the method parameters of size *nparams*
- the pointer to the array that holds the calculated weights of the observations

If an input parameter is `NULL`, the corresponding parameter in the task descriptor remains unchanged.

Intel MKL provides the BACON algorithm ([Billor00]) for the detection of multivariate outliers. Pack the parameters of the BACON algorithm into the *params* array and pass them into the editor. Table "Structure of the Array of BACON Parameters" describes the *params* structure.

Structure of the Array of BACON Parameters

Array Position	Algorithm Parameter	Description
0	Method to start the algorithm	The parameter takes one of the following possible values: `VSL_SS_METHOD_BACON_MEDIAN_INIT`, if the algorithm is started using the median estimate. This is the default value of the parameter. `VSL_SS_METHOD_BACON_MAHALANOBIS_INIT`, if the algorithm is started using the Mahalanobis distances.
1	α	One-tailed probability that defines the $(1 - \alpha)$ quantile of χ^2 distribution with p degrees of freedom. The recommended value is α / n, where n is the number of observations. By default, the value is 0.05.
2	δ	Stopping criterion; the algorithm is terminated if the size of the basic subset is changed less than δ. By default, the value is 0.005.

Output of the algorithm is the vector of weights, `BaconWeights`, such that `BaconWeights(i)` = 0 if *i*-th observation is detected as an outlier. Otherwise `BaconWeights(i)` = *w*(*i*), where *w* is the vector of input weights. If you do not provide the vector of input weights, `BaconWeights(i)` is set to 1 if the *i*-th observation is not detected as an outlier.

See additional details about usage model of the algorithm in the *Intel(R) MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page.

vslSSEditMissingValues

Modifies pointers to arrays associated with the method of supporting missing values in a dataset.

Syntax

Fortran:

```
status = vslssseditmissingvalues(task, nparams, params, init_estimates_n,
init_estimates, prior_n, prior, simul_missing_vals_n, simul_missing_vals, estimates_n,
estimates)
```

```
status = vsldsseditmissingvalues(task, nparams, params, init_estimates_n,
init_estimates, prior_n, prior, simul_missing_vals_n, simul_missing_vals, estimates_n,
estimates)
```

C:

```
status = vslsSSEditMissingValues(task, nparams, params, init_estimates_n,
init_estimates, prior_n, prior, simul_missing_vals_n, simul_missing_vals, estimates_n,
estimates);
```

```
status = vsldSSEditMissingValues(task, nparams, params, init_estimates_n,
init_estimates, prior_n, prior, simul_missing_vals_n, simul_missing_vals, estimates_n,
estimates);
```

Include Files

- Fortran 90: `mkl_vsl.f90`
- C: `mkl_vsl_functions.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(VSL_SS_TASK)` **C:** `VSLSSTaskPtr`	Descriptor of the task
nparams	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the number of method parameters
params	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslssseditmissingvalues` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmissingvalues` **C:** `float*` for	Pointer to the array of method parameters

Name	Type	Description
	vslsSSEditMissingValues double* for vsldSSEditMissingValues	
init_estimates_n	**Fortran:** INTEGER **C:** MKL_INT*	Pointer to the number of initial estimates for mean and a variance-covariance matrix
init_estimates	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslsssseditmissingvalues REAL(KIND=8) DIMENSION(*) for vsldsseditmissingvalues **C:** float* for vslsSSEditMissingValues double* for vsldSSEditMissingValues	Pointer to the array that holds initial estimates for mean and a variance-covariance matrix
prior_n	**Fortran:** INTEGER **C:** MKL_INT*	Pointer to the number of prior parameters
prior	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslsssseditmissingvalues REAL(KIND=8) DIMENSION(*) for vsldsseditmissingvalues **C:** float* for vslsSSEditMissingValues double* for vsldSSEditMissingValues	Pointer to the array of prior parameters
simul_missing_vals_n	**Fortran:** INTEGER **C:** MKL_INT*	Pointer to the size of the array that holds output of the Multiple Imputation method
simul_missing_vals	**Fortran:** REAL(KIND=4) DIMENSION(*) for vslsssseditmissingvalues REAL(KIND=8) DIMENSION(*) for vsldsseditmissingvalues **C:** float* for vslsSSEditMissingValues	Pointer to the array of size $k*m$, where k is the total number of missing values, and m is number of copies of missing values. The array holds m sets of simulated missing values for the matrix of observations.

Name	Type	Description
	`double*` for `vsldSSEditMissingValues`	
estimates_n	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the number of estimates to be returned by the routine
estimates	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsseditmissingvalues` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditmissingvalues` **C:** `float*` for `vslsSSEditMissingValues` `double*` for `vsldSSEditMissingValues`	Pointer to the array that holds estimates of the mean and a variance-covariance matrix.

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditMissingValues` routine uses values passed as parameters of the routine to replace pointers to the number and the array of the method parameters, pointers to the number and the array of initial mean/variance-covariance estimates, the pointer to the number and the array of prior parameters, pointers to the number and the array of simulated missing values, and pointers to the number and the array of the intermediate mean/covariance estimates. If an input parameter is `NULL`, the corresponding parameter in the task descriptor remains unchanged.

Before you call the VSL Summary Statistics routines to process missing values, preprocess the dataset and denote missing observations with one of the following predefined constants:

- `VSL_SS_SNAN`, if the dataset is stored in single precision floating-point arithmetic
- `VSL_SS_DNAN`, if the dataset is stored in double precision floating-point arithmetic

Intel MKL provides the `VSL_SS_METHOD_MI` method to support missing values in the dataset based on the Multiple Imputation (MI) approach described in [Schafer97]. The following components support Multiple Imputation:

- Expectation Maximization (EM) algorithm to compute the start point for the Data Augmentation (DA) procedure
- DA function

NOTE The DA component of the MI procedure is simulation-based and uses the `VSL_BRNG_MCG59` basic random number generator with predefined *seed* = 2^{50} and the Gaussian distribution generator (`ICDF` method) available in Intel MKL [Gaussian].

Pack the parameters of the MI algorithm into the *params* array. Table "Structure of the Array of MI Parameters" describes the *params* structure.

Structure of the Array of MI Parameters

Array Position	Algorithm Parameter	Description
0	em_iter_num	Maximal number of iterations for the EM algorithm. By default, this value is 50.
1	da_iter_num	Maximal number of iterations for the DA algorithm. By default, this value is 30.
2	ε	Stopping criterion for the EM algorithm. The algorithm terminates if the maximal module of the element-wise difference between the previous and current parameter values is less than ε. By default, this value is 0.001.
3	m	Number of sets to impute
4	missing_vals_num	Total number of missing values in the datasets

You can also pass initial estimates into the EM algorithm by packing both the vector of means and the variance-covariance matrix as a one-dimensional array *init_estimates*. The size of the array should be at least $p + p(p + 1)/2$. For i=0, .., p-1, the *init_estimates*[i] array contains the initial estimate of means. The remaining positions of the array are occupied by the upper triangular part of the variance-covariance matrix.

If you provide no initial estimates for the EM algorithm, the editor uses the default values, that is, the vector of zero means and the unitary matrix as a variance-covariance matrix. You can also pass *prior* parameters for μ and Σ into the library: μ_0, τ, m, and Λ^{-1}. Pack these parameters as a one-dimensional array *prior* with a size of at least

$(p^2 + 3p + 4)/2$.

The storage format is as follows:

- *prior*[0], ..., *prior*[p–1] contain the elements of the vector μ_0.
- *prior*[p] contains the parameter τ.
- *prior*[p+1] contains the parameter m.
- The remaining positions are occupied by the upper-triangular part of the inverted matrix Λ^{-1}.

If you provide no *prior* parameters, the editor uses their default values:

- The array of p zeros is used as μ_0.
- τ is set to 0.
- m is set to p.
- The zero matrix is used as an initial approximate of Λ^{-1}.

The EditMissingValues editor returns m sets of imputed values and/or a sequence of parameter estimates drawn during the DA procedure.

The editor returns the imputed values as the *simul_missing_vals* array. The size of the array should be sufficient to hold m sets each of the missing_vals_num size, that is, at least m*missing_vals_num in total. The editor packs the imputed values one by one in the order of their appearance in the matrix of observations.

For example, consider a task of dimension 4. The total number of observations n is 10. The second observation vector misses variables 1 and 2, and the seventh observation vector lacks variable 1. The number of sets to impute is m=2. Then, *simul_missing_vals*[0] and *simul_missing_vals*[1] contains the first and the second points for the second observation vector, and *simul_missing_vals*[2] holds the first point for the seventh observation. Positions 3, 4, and 5 are formed similarly.

To estimate convergence of the DA algorithm and choose a proper value of the number of DA iterations, request the sequence of parameter estimates that are produced during the DA procedure. The editor returns the sequence of parameters as a single array. The size of the array is

m*da_iter_num*(p+(p^2+p)/2)

where

- m is the number of sets of values to impute.
- da_iter_num is the number of DA iterations.
- The value p+(p^2+p)/2 determines the size of the memory to hold one set of the parameter estimates.

In each set of the parameters, the vector of means occupies the first p positions and the remaining (p^2+p)/2 positions are intended for the upper triangular part of the variance-covariance matrix.

Upon successful generation of m sets of imputed values, you can place them in cells of the data matrix with missing values and use the VSL Summary Statistics routines to analyze and get estimates for each of the m complete datasets.

NOTE Intel MKL implementation of the MI algorithm rewrites cells of the dataset that contain the VSL_SS_SNAN/VSL_SS_DNAN values. If you want to use the VSL Summary Statistics routines to process the data with missing values again, mask the positions of the empty cells.

See additional details of the algorithm usage model in the *Intel© MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page.

vslSSEditCorParameterization

Modifies pointers to arrays related to the algorithm of correlation matrix parameterization.

Syntax

Fortran:

```
status = vslsssseditcorparameterization(task, cor, cor_storage, pcor, pcor_storage)
status = vsldsseditcorparameterization(task, cor, cor_storage, pcor, pcor_storage)
```

C:

```
status = vslsSSEditCorParameterization(task, cor, cor_storage, pcor, pcor_storage);
status = vsldSSEditCorParameterization(task, cor, cor_storage, pcor, pcor_storage);
```

Include Files

- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(VSL_SS_TASK)	Descriptor of the task

Name	Type	Description
	C: `VSLSSTaskPtr`	
cor	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsseditcorparameterization` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditcorparameterization` **C:** `float*` for `vslsSSEditCorParameterization` `double*` for `vsldSSEditCorParameterization`	Pointer to the correlation matrix
cor_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the correlation matrix
pcor	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `vslsseditcorparameterization` `REAL(KIND=8) DIMENSION(*)` for `vsldsseditcorparameterization` **C:** `float*` for `vslsSSEditCorParameterization` `double*` for `vsldSSEditCorParameterization`	Pointer to the parameterized correlation matrix
por_storage	**Fortran:** `INTEGER` **C:** `MKL_INT*`	Pointer to the storage format of the parameterized correlation matrix

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Current status of the task

Description

The `vslSSEditCorParameterization` routine uses values passed as parameters of the routine to replace pointers to the correlation matrix, pointers to the correlation matrix storage format, a pointer to the parameterized correlation matrix, and a pointer to the parameterized correlation matrix storage format. See Table "Storage formats of a variance-covariance/correlation matrix" for possible values of the *cor_storage* and *pcor_storage* parameters. If an input parameter is `NULL`, the corresponding parameter in the VSL Summary Statistics task descriptor remains unchanged.

Task Computation Routines

Task computation routines calculate statistical estimates on the data provided and parameters held in the task descriptor. After you create the task and initialize its parameters, you can call the computation routines as many times as necessary. Table "VSL Summary Statistics Estimates Obtained with `vslSSCompute` Routine" lists the statistical estimates that you can obtain using the `vslSSCompute` routine.

NOTE The VSL Summary Statistics computation routines do not signal floating-point errors, such as overflow or gradual underflow, or operations with `NaNs` (except for the missing values in the observations).

VSL Summary Statistics Estimates Obtained with vslSSCompute Routine

Estimate	Support of Observations Available in Blocks	Description
`VSL_SS_MEAN`	Yes	Computes the array of means.
`VSL_SS_2R_MOM`	Yes	Computes the array of the 2nd order raw moments.
`VSL_SS_3R_MOM`	Yes	Computes the array of the 3rd order raw moments.
`VSL_SS_4R_MOM`	Yes	Computes the array of the 4th order raw moments.
`VSL_SS_2C_MOM`	Yes	Computes the array of the 2nd order central moments.
`VSL_SS_3C_MOM`	Yes	Computes the array of the 3rd order central moments.
`VSL_SS_4C_MOM`	Yes	Computes the array of the 4th order central moments.
`VSL_SS_KURTOSIS`	Yes	Computes the array of kurtosis values.
`VSL_SS_SKEWNESS`	Yes	Computes the array of skewness values.
`VSL_SS_MIN`	Yes	Computes the array of minimum values.
`VSL_SS_MAX`	Yes	Computes the array of maximum values.
`VSL_SS_VARIATION`	Yes	Computes the array of variation coefficients.
`VSL_SS_COV`	Yes	Computes a covariance matrix.
`VSL_SS_COR`	Yes	Computes a correlation matrix.
`VSL_SS_POOLED_COV`	No	Computes a pooled covariance matrix.
`VSL_SS_GROUP_COV`	No	Computes group covariance matrices.
`VSL_SS_QUANTS`	No	Computes quantiles.
`VSL_SS_ORDER_STATS`	No	Computes order statistics.
`VSL_SS_ROBUST_COV`	No	Computes a robust covariance matrix.
`VSL_SS_OUTLIERS`	No	Detects outliers in the dataset.

Estimate	Support of Observations Available in Blocks	Description
VSL_SS_PARTIAL_COV	No	Computes a partial covariance matrix.
VSL_SS_PARTIAL_COR	No	Computes a partial correlation matrix.
VSL_SS_MISSING_VALS	No	Supports missing values in datasets.
VSL_SS_PARAMTR_COR	No	Computes a parameterized correlation matrix.
VSL_SS_STREAM_QUANTS	Yes	Computes quantiles for streaming data.

Table "VSL Summary Statistics Computation Methods" lists estimate calculation methods supported by Intel MKL. See the *Intel(R) MKL Summary Statistics Library Application Notes* document on the Intel® MKL web page for a detailed description of the methods.

VSL Summary Statistics Computation Method

Method	Description
VSL_SS_METHOD_FAST	Fast method for calculation of the estimates
VSL_SS_METHOD_1PASS	One-pass method for calculation of estimates
VSL_SS_METHOD_TBS	TBS method for robust estimation of covariance and mean
VSL_SS_METHOD_BACON	BACON method for detection of multivariate outliers
VSL_SS_METHOD_MI	Multiple imputation method for support of missing values
VSL_SS_METHOD_SD	Spectral decomposition method for parameterization of a correlation matrix
VSL_SS_METHOD_SQUANTS_ZW	Zhang-Wang (ZW) method for quantile estimation for streaming data
VSL_SS_METHOD_SQUANTS_ZW_FAST	Fast ZW method for quantile estimation for streaming data

You can calculate all requested estimates in one call of the routine. For example, to compute a kurtosis and covariance matrix using a fast method, pass a combination of the pre-defined parameters into the `Compute` routine as shown in the example below:

```
...
method = VSL_SS_METHOD_FAST;
task_params = VSL_SS_KURTOSIS|VSL_SS_COV;
…
status = vsldSSCompute( task, task_params, method );
```

To compute statistical estimates for the next block of observations, you can do one of the following:

- copy the observations to memory, starting with the address available to the task
- use one of the appropriate Editors to modify the pointer to the new dataset in the task.

The library does not detect your changes of the dataset and computed statistical estimates. To obtain statistical estimates for a new matrix, change the observations and initialize relevant arrays. You can follow this procedure to compute statistical estimates for observations that come in portions. See Table "VSL Summary Statistics Estimates Obtained with `vslSSCompute` Routine" for information on such observations supported by the Intel MKL VSL Summary Statistics estimators.

To modify parameters of the task using the Task Editors, set the address of the targeted matrix of the observations or change the respective vector component indices. After you complete editing the task parameters, you can compute statistical estimates in the modified environment.

If the task completes successfully, the computation routine returns the zero status code. If an error is detected, the computation routine returns an error code. In particular, an error status code is returned in the following cases:

- the task pointer is NULL
- memory allocation has failed
- the calculation has failed for some other reason

NOTE You can use the NULL task pointer in calls to editor routines. In this case, the routine is terminated and no system crash occurs.

vslSSCompute

Computes VSL Summary Statistics estimates.

Syntax

Fortran:

```
status = vslssscompute(task, estimates, method)
status = vsldsscompute(task, estimates, method)
```

C:

```
status = vslsSSCompute(task, estimates, method);
status = vsldSSCompute(task, estimates, method);
```

Include Files

- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(VSL_SS_TASK) **C:** VSLSSTaskPtr	Descriptor of the task
estimates	**Fortran:** INTEGER (KIND=8) **C:** unsigned long long	List of statistical estimates to compute
method	**Fortran:** INTEGER **C:** MKL_INT	Method to be used in calculations

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Current status of the task

Description

The vslSSCompute routine calculates statistical estimates passed as the *estimates* parameter using the algorithms passed as the *method* parameter of the routine. The computations are done in the context of the task descriptor that contains pointers to all required and optional, if necessary, properly initialized arrays. In

one call of the function, you can compute several estimates using proper methods for their calculation. See Table "VSL Summary Statistics Estimates Obtained with Compute Routine" for the list of the estimates that you can calculate with the vslSSCompute routine. See Table "VSL Summary Statistics Computation Methods" for the list of possible values of the *method* parameter.

To initialize single or double precision version task parameters, use the single (vslssscompute) or double (vsldsscompute) version of the editor, respectively. To initialize parameters of the integer type, use an integer version of the editor (vslisscompute).

NOTE Requesting a combination of the VSL_SS_MISSING_VALS value and any other estimate parameter in the Compute function results in processing only the missing values.

Task Destructor

Task destructor is the vslSSDeleteTask routine intended to delete task objects and release memory.

vslSSDeleteTask

Destroys the task object and releases the memory.

Syntax

Fortran:

```
status = vslssdeletetask(task)
```

C:

```
status = vslSSDeleteTask(&task);
```

Include Files

- Fortran 90: mkl_vsl.f90
- C: mkl_vsl_functions.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(VSL_SS_TASK) **C:** VSLSSTaskPtr*	Descriptor of the task to destroy

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Sets to VSL_STATUS_OK if the task is deleted; otherwise a non-zero code is returned.

Description

The vslSSDeleteTask routine deletes the task descriptor object, releases the memory allocated for the structure, and sets the task pointer to NULL. If vslSSDeleteTask fails to delete the task successfully, it returns an error code.

NOTE Call of the destructor with the `NULL` pointer as the parameter results in termination of the function with no system crash.

Usage Examples

The following examples show various standard operations with Summary Statistics routines.

Calculating Fixed Estimates for Fixed Data

The example shows recurrent calculation of the same estimates with a given set of variables for the complete life cycle of the task in the case of a variance-covariance matrix. The set of vector components to process remains unchanged, and the data comes in blocks. Before you call the `vslSSCompute` routine, initialize pointers to arrays for mean and covariance and set buffers.

```
….
double w[2];
double indices[DIM] = {1, 0, 1};

/* calculating mean for 1st and 3d random vector components */

/* Initialize parameters of the task */
p = DIM;
n = N;

xstorage   = VSL_SS_MATRIX_STORAGE_ROWS;
covstorage = VSL_SS_MATRIX_STORAGE_FULL;

w[0] = 0.0; w[1] = 0.0;

for ( i = 0; i < p; i++ ) mean[i] = 0.0;
for ( i = 0; i < p*p; i++ ) cov[i] = 0.0;

status = vsldSSNewTask( &task, &p, &n, &xstorage, x, 0, indices );

status = vsldSSEditTask  ( task, VSL_SS_ED_ACCUM_WEIGHT, w    );
status = vsldSSEditCovCor( task, mean, cov, &covstorage, 0, 0 );
```

You can process data arrays that come in blocks as follows:

```
for ( i = 0; i < num_of_blocks; i++ )
{
    status = vsldSSCompute( task, VSL_SS_COV, VSL_SS_METHOD_FAST );
    /* Read new data block into array x */
}
…
```

Calculating Different Estimates for Variable Data

The context of your calculation may change in the process of data analysis. The example below shows the data that comes in two blocks. You need to estimate a covariance matrix for the complete data, and the third central moment for the second block of the data using the weights that were accumulated for the previous datasets. The second block of the data is stored in another array. You can proceed as follows:

```
/* Set parameters for the task */
p = DIM;
n = N;
xstorage   = VSL_SS_MATRIX_STORAGE_ROWS;
covstorage = VSL_SS_MATRIX_STORAGE_FULL;

w[0] = 0.0; w[1] = 0.0;

for ( i = 0; i < p; i++ ) mean[i] = 0.0;
for ( i = 0; i < p*p; i++ ) cov[i] = 0.0;

/* Create task */
status = vsldSSNewTask( &task, &p, &n, &xstorage, x1, 0, indices );
```

```
/* Initialize the task parameters */
status = vsldSSEditTask( task, VSL_SS_ED_ACCUM_WEIGHT, w );
status = vsldSSEditCovCor( task, mean, cov, &covstorage, 0, 0 );

/* Calculate covariance for the x1 data */
status = vsldSSCompute( task, VSL_SS_COV, VSL_SS_METHOD_FAST );

/* Initialize array of the 3d central moments and pass the pointer to the task */
for ( i = 0; i < p; i++ ) c3_m[i] = 0.0;

/* Modify task context */
status = vsldSSEditTask( task, VSL_SS_ED_3C_MOM, c3_m );
status = vsldSSEditTask( task, VSL_SS_ED_OBSERV, x2 );

/* Calculate covariance for the x1 & x2 data block */
/* Calculate the 3d central moment for the 2nd data block using earlier accumulated weight */
status = vsldSSCompute(task, VSL_SS_COV|VSL_SS_3C_MOM, VSL_SS_METHOD_FAST );
…
status = vslSSDeleteTask( &task );
```

Similarly, you can modify indices of the variables to be processed for the next data block.

Mathematical Notation and Definitions

The following notations are used in the mathematical definitions and the description of the Intel MKL VSL Summary Statistics functions.

Matrix and Weights of Observations

For a random p-dimensional vector $\xi = (\xi_1,..., \xi_i,..., \xi_p)$, this manual denotes the following:

- $(X)_i=(x_{ij})_{j=1..n}$ is the result of n independent observations for the i-th component ξ_i of the vector ξ.
- The two-dimensional array $X=(x_{ij})_{p \times n}$ is the matrix of observations.
- The column $[X]_j=(x_{ij})_{i=1..p}$ of the matrix X is the j-th observation of the random vector ξ.

Each observation $[X]_j$ is assigned a non-negative weight w_j , where

- The vector $(w_j)_{j=1..n}$ is a vector of weights corresponding to n observations of the random vector ξ.
-

$$W = \sum_{i=1}^{n} w_i$$

  is the accumulated weight corresponding to observations X.

Vector of sample means

$$M(X) = (M_1(X),\ldots,M_p(X)) \text{ with } M_i(X) = \frac{1}{w}\sum_{j=1}^{n} w_j x_{ij}$$

for all i = 1, ..., p.

Vector of sample variances

$$V(X) = (V_1(X),\ldots,V_p(X)) \text{ with } V_i(X) = \frac{1}{\mathrm{B}}\sum_{j=1}^{n} w_j (x_{ij} - M_i(X))^2,\ \mathrm{B} = W - \sum_{j=1}^{n} w_j^2 / W$$

for all $i = 1, \ldots, p$.

Vector of sample raw/algebraic moments of k-th order, $k \geq 1$

$$R^{(k)}(X) = (R_1^{(k)}(X), \ldots, R_p^{(k)}(X)) \text{ with } R_i^{(k)}(X) = \frac{1}{W}\sum_{j=1}^{n} w_j x_{ij}^k$$

for all $i = 1, \ldots, p$.

Vector of sample central moments of the third and the fourth order

$$C^{(k)}(X) = (C_1^{(k)}(X), \ldots, C_p^{(k)}(X)) \text{ with } C_i^{(k)}(X) = \frac{1}{B}\sum_{j=1}^{n} w_j (x_{ij} - M_i(X))^k,\ B = \sum_{j=1}^{n} w_j$$

for all $i = 1, \ldots, p$ and $k = 3, 4$.

Vector of sample excess kurtosis values

$$\mathrm{B}(X) = (\mathrm{B}_1(X), \ldots, \mathrm{B}_p(X)) \text{ with } \mathrm{B}_i(X) = \frac{C_i^{(4)}(X)}{V_i^2(X)} - 3$$

for all $i = 1, \ldots, p$.

Vector of sample skewness values

$$\Gamma(X) = (\Gamma_1(X), \ldots, \Gamma_p(X)) \text{ with } \Gamma_i(X) = \frac{C_i^{(3)}(X)}{V_i^{1.5}(X)}$$

for all $i = 1, \ldots, p$.

Vector of sample variation coefficients

$$VC(X) = (VC_1(X), \ldots, VC_p(X)) \text{ with } VC_i(X) = \frac{V_i^{0.5}(X)}{M_i(X)}$$

for all $i = 1, \ldots, p$.

Matrix of order statistics

Matrix $Y = (y_{ij})_{pxn}$, in which the i-th row $(Y)_i = (y_{ij})_{j=1..n}$ is obtained as a result of sorting in the ascending order of row $(X)_i = (x_{ij})_{j=1..n}$ in the original matrix of observations.

Vector of sample minimum values

$$Min(X) = (Min_1(X), \ldots, Min_p(X)), \text{ where } Min_i(X) = y_{i1}$$

for all i = 1, ..., p.

Vector of sample maximum values

$$Max(X) = (Max_1(X), \ldots, Max_p(X)), \text{ where } Max_i(X) = y_{in}$$

for all i = 1, ..., p.

Vector of sample median values

$$Med(X) = (Med_1(X), \ldots, Med_p(X)), \text{ where } Med_i(X) = \begin{cases} y_{i,(n+1)/2}, & \text{if } n \text{ is odd} \\ \left(y_{i,n/2} + y_{i,n/2+1}\right)/2, & \text{if } n \text{ is even} \end{cases}$$

for all i = 1, ..., p.

Vector of sample quantile values

For a positive integer number q and k belonging to the interval [0, q-1], point z_i is the k-th q quantile of the random variable ξ_i if $P\{\xi_i \le z_i\} \ge \beta$ and $P\{\xi_i \le z_i\} \ge 1 - \beta$, where

- P is the probability measure.
- $\beta = k/n$ is the quantile order.

The calculation of quantiles is as follows:

$j = [(n\text{-}1)\beta]$ and $f = \{(n\text{-}1)\beta\}$ as integer and fractional parts of the number $(n\text{-}1)\beta$, respectively, and the vector of sample quantile values is

$Q(X,\beta) = (Q_1(X,\beta), \ldots, Q_p(X,\beta))$

where

$(Q_i(X,\beta) = y_{i,j+1} + f(y_{i,j+2} - y_{i,j+1})$

for all i = 1, ..., p.

Variance-covariance matrix

$C(X) = (c_{ij}(X))_{p \times p}$

where

$$c_{ij}(X) = \frac{1}{B}\sum_{k=1}^{n} w_k (x_{ik} - M_i(X))(x_{jk} - M_j(X)), B = W - \sum_{j=1}^{n} w_j^2 / W$$

Pooled and group variance-covariance matrices

The set $N = \{1, ..., n\}$ is partitioned into non-intersecting subsets

$$G_i, i = 1..g, N = \bigcup_{i=1}^{g} G_i$$

The observation $[X]_j = (x_{ij})_{i=1..p}$ belongs to the group r if $j \in G_r$. One observation belongs to one group only. The group mean and variance-covariance matrices are calculated similarly to the formulas above:

$$M^{(r)}(X) = (M_1^{(r)}(X), \ldots, M_p^{(r)}(X)) \text{ with } M_i^{(r)}(X) = \frac{1}{W^{(r)}} \sum_{j \in G_r} w_j x_{ij}, W^{(r)} = \sum_{j \in G_r} w_j$$

for all $i = 1, ..., p$,

$$C^{(r)}(X) = (c_{ij}^{(r)}(X))_{p \times p}$$

where

$$c_{ij}^{(r)}(X) = \frac{1}{B^{(r)}} \sum_{k \in G_r} w_k (x_{ik} - M_i^{(r)}(X))(x_{jk} - M_j^{(r)}(X)), \; B^{(r)} = W^{(r)} - \sum_{j \in G_r} w_j^2 / W^{(r)}$$

for all $i = 1, ..., p$ and $j = 1, ..., p$.

A pooled variance-covariance matrix and a pooled mean are computed as weighted mean over group covariance matrices and group means, correspondingly:

$$M^{pooled}(X) = (M_1^{pooled}(X), \ldots, M_p^{pooled}(X)) \text{ with } M_i^{pooled}(X) = \frac{1}{W^{(1)} + \ldots + W^{(g)}} \sum_{r=1}^{g} W^{(r)} M_i^{(r)}(X)$$

for all $i = 1, ..., p$,

$$C^{pooled}(X) = (c_{ij}^{pooled}(X))_{p \times p}, c_{ij}^{pooled}(X) = \frac{1}{B^{(1)} + \ldots + B^{(g)}} \sum_{r=1}^{g} B^{(r)} c_{ij}^{(r)}(X)$$

for all $i = 1, ..., p$ and $j = 1, ..., p$.

Correlation matrix

$$R(X) = (r_{ij}(X))_{p \times p}, \text{ where } r_{ij}(X) = \frac{c_{ij}}{\sqrt{c_{ii} c_{jj}}}$$

for all i = 1, ..., p and j = 1, ..., p.

Partial variance-covariance matrix

For a random vector ξ partitioned into two components Z and Y, a variance-covariance matrix C describes the structure of dependencies in the vector ξ:

$$C(X)=\begin{bmatrix} C_Z(X) & C_{ZY}(X) \\ C_{YZ}(X) & C_Y(X) \end{bmatrix}.$$

The partial covariance matrix $P(X) = (p_{ij}(X))_{kxk}$ is defined as

$$P(X) = C_Y(X) - C_{YZ}(X)C_Z^{-1}(X)C_{ZY}(X).$$

where k is the dimension of Y.

Partial correlation matrix

The following is a partial correlation matrix for all i = 1, ..., k and j = 1, ..., k:

$$RP(X) = (rp_{ij}(X))_{kxk}, \text{ where } rp_{ij}(X) = \frac{p_{ij}(X)}{\sqrt{p_{ii}(X)p_{jj}(X)}}$$

where

- k is the dimension of Y.
- $p_{ij}(X)$ are elements of the partial variance-covariance matrix.

11 Fourier Transform Functions

The general form of the discrete Fourier transform is

$$Z_{k_1,k_2,\ldots,k_d} = \sigma \times \sum_{j_d=0}^{n_d-1} \ldots \sum_{j_2=0}^{n_2-1} \sum_{j_1=0}^{n_1-1} w_{j_1,j_2,\ldots,j_d} \exp\left(\delta i 2\pi \sum_{l=1}^{d} j_l k_l / n_l\right)$$

for $k_l = 0, \ldots n_l-1$ $(l = 1, \ldots, d)$, where σ is a scale factor, $\delta = -1$ for the forward transform, and $\delta = +1$ for the inverse (backward) transform. In the forward transform, the input (periodic) sequence $\{w_{j_1, j_2, \ldots, j_d}\}$ typically belongs to the set of complex-valued sequences and real-valued sequences (forward domain). Respective domains for the backward transform, or backward domains, are represented by complex-valued sequences and complex-valued conjugate-even sequences.

Math Kernel Library (Intel® MKL) provides an interface for computing a discrete Fourier transform through the fast Fourier transform algorithm. This chapter describes the following implementations of the fast Fourier transform functions available in Intel MKL:

- Fast Fourier transform (FFT) functions for single-processor or shared-memory systems (see FFT Functions below)
- Cluster FFT functions for distributed-memory architectures (available with Intel® MKL for the Linux* and Windows* operating systems only).

NOTE Intel MKL also supports the FFTW3* interfaces to the fast Fourier transform functionality for symmetric multiprocessing (SMP) systems.

Both FFT and Cluster FFT functions support a five-stage usage model for computing an FFT:

1. Allocate a fresh descriptor for the problem with a call to the `DftiCreateDescriptor` or `DftiCreateDescriptorDM` function. The descriptor captures the configuration of the transform, such as the dimensionality (or rank), sizes, number of transforms, memory layout of the input/output data (defined by strides), and scaling factors. Many of the configuration settings are assigned default values in this call and may need modification depending on your application.
2. Optionally adjust the descriptor configuration with a call to the `DftiSetValue` or `DftiSetValueDM` function as needed. Typically, you must carefully define the data storage layout for an FFT or the data distribution among processes for a Cluster FFT. The configuration settings of the descriptor, such as the default values, can be obtained with the `DftiGetValue` or `DftiGetValueDM` function.
3. Commit the descriptor with a call to the `DftiCommitDescriptor` or `DftiCommitDescriptorDM` function, that is, make the descriptor ready for the transform computation. Once the descriptor is committed, the parameters of the transform, such as the type and number of transforms, strides and distances, the type and storage layout of the data, and so on, are "frozen" in the descriptor.
4. Compute the transform with a call to the `DftiComputeForward`/`DftiComputeBackward` or `DftiComputeForwardDM`/`DftiComputeBackwardDM` functions as many times as needed. With the committed descriptor, the compute functions only accept pointers to the input/output data and compute the transform as defined. To modify any configuration parameters later on, use `DftiSetValue` followed by `DftiCommitDescriptor` (`DftiSetValueDM` followed by `DftiCommitDescriptorDM`) or create and commit another descriptor.
5. Deallocate the descriptor with a call to the `DftiFreeDescriptor` or `DftiFreeDescriptorDM` function. This will return the memory internally consumed by the descriptor to the operating system.

All the above functions return an integer status value, which is zero upon successful completion of the operation. You can interpret a non-zero status with the help of the `DftiErrorClass` or `DftiErrorMessage` function.

The FFT functions support lengths with arbitrary factors. You can improve performance of the Intel MKL FFT if the length of your data vector permits factorization into powers of optimized radices. See the *Intel MKL User's Guide* for specific radices supported efficiently and the length constraints.

NOTE The FFT functions assume the Cartesian representation of complex data (that is, the real and imaginary parts define a complex number). The Intel MKL Vector Mathematical Functions provide an efficient tool for conversion to and from the polar representation (see Example "Conversion from Cartesian to polar representation of complex data" and Example "Conversion from polar to Cartesian representation of complex data").

FFT Functions

The fast Fourier transform function library of Intel MKL provides one-dimensional, two-dimensional, and multi-dimensional transforms (of up to seven dimensions); and both Fortran and C interfaces for all transform functions.

Table "FFT Functions in Intel MKL" lists FFT functions implemented in Intel MKL:

FFT Functions in Intel MKL

Function Name	Operation
Descriptor Manipulation Functions	
DftiCreateDescriptor	Allocates the descriptor data structure and initializes it with default configuration values.
DftiCommitDescriptor	Performs all initialization for the actual FFT computation.
DftiFreeDescriptor	Frees memory allocated for a descriptor.
DftiCopyDescriptor	Makes a copy of an existing descriptor.
FFT Computation Functions	
DftiComputeForward	Computes the forward FFT.
DftiComputeBackward	Computes the backward FFT.
Descriptor Configuration Functions	
DftiSetValue	Sets one particular configuration parameter with the specified configuration value.
DftiGetValue	Gets the value of one particular configuration parameter.
Status Checking Functions	
DftiErrorClass	Checks if the status reflects an error of a predefined class.
DftiErrorMessage	Translates the numeric value of an error status into a message.

Computing an FFT

The FFT functions described later in this chapter are provided with the Fortran and C interfaces. Fortran 95 is required because it offers features that have no counterpart in FORTRAN 77.

NOTE The Fortran interface of the FFT computation functions requires one-dimensional data arrays for any dimension of FFT problem. For multidimensional transforms, you can pass the address of the first column of the multidimensional data to the computation functions.

The materials presented in this chapter assume the availability of native complex types in C as they are specified in C9X.

You can find code examples that use FFT interface functions to compute transform results in the Fourier Transform Functions Code Examples section in the Appendix C.

For most common situations, an FFT computation can be effected by four function calls (refer to the usage model for details). A single data structure, the descriptor, stores configuration parameters that can be changed independently.

The descriptor data structure, when created, contains information about the length and domain of the FFT to be computed, as well as the setting of several configuration parameters. Default settings for some of these parameters are as follows:

- The FFT to be computed does not have a scale factor;
- There is only one set of data to be transformed;
- The data is stored contiguously in memory;
- The computed result overwrites the input data (the transform is in-place);

The default settings can be changed one-at-a-time through the function `DftiSetValue` as illustrated in the Example "Changing Default Settings (Fortran)" and Example "Changing Default Settings (C)".

FFT Interface

To use the FFT functions, you need to access the module `MKL_DFTI` through the "`use`" statement in Fortran; or include the header file `mkl_dfti.h` in C.

The Fortran interface provides a derived type `DFTI_DESCRIPTOR`, named constants representing various names of configuration parameters and their possible values, and overloaded functions through the generic functionality of Fortran 95.

The C interface provides the `DFTI_DESCRIPTOR_HANDLE` type, named constants of two enumeration types `DFTI_CONFIG_PARAM` and `DFTI_CONFIG_VALUE`, and functions, some of which accept different numbers of input arguments.

NOTE The current version of the library may not support some of the FFT functions or functionality described in the subsequent sections of this chapter. You can find the complete list of the implementation-specific exceptions in the Intel MKL Release Notes.

For the main categories of Intel MKL FFT functions, see FFT Functions.

Descriptor Manipulation Functions

There are four functions in this category: create a descriptor, commit a descriptor, copy a descriptor, and free a descriptor.

DftiCreateDescriptor

Allocates the descriptor data structure and initializes it with default configuration values.

Syntax

Fortran:

```
status = DftiCreateDescriptor( desc_handle, precision, forward_domain, dimension,
length )
```

C:

```
status = DftiCreateDescriptor(&desc_handle, precision, forward_domain, dimension,
length);
```

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Input Parameters

Name	Type	Description
precision	**FORTRAN:** `INTEGER` **C:** `enum`	Precision of the transform: `DFTI_SINGLE` or `DFTI_DOUBLE`.
forward_domain	**FORTRAN:** `INTEGER` **C:** `enum`	Forward domain of the transform: `DFTI_COMPLEX` or `DFTI_REAL`.
dimension	**FORTRAN:** `INTEGER` **C:** `MKL_LONG`	Dimension of the transform.
length	**FORTRAN:** `INTEGER` if *dimension* = 1. Array `INTEGER, DIMENSION(*)` otherwise. **C:** `MKL_LONG` if *dimension* == 1. Array of type `MKL_LONG` otherwise.	Length of the transform for a one-dimensional transform. Lengths of each dimension for a multi-dimensional transform.

Output Parameters

Name	Type	Description
desc_handle	**FORTRAN:** `DFTI_DESCRIPTOR` **C:** `DFTI_DESCRIPTOR_HANDLE`	FFT descriptor.
status	**FORTRAN:** `INTEGER` **C:** `MKL_LONG`	Function completion status.

Description

This function allocates memory for the descriptor data structure and instantiates it with all the default configuration settings with respect to the precision, forward domain, dimension, and length of the desired transform. Because memory is allocated dynamically, the result is actually a pointer to the created descriptor. This function is slightly different from the "initialization" function that can be found in software packages or libraries that implement more traditional algorithms for computing FFT. This function does not perform any significant computational work such as computation of twiddle factors. The function `DftiCommitDescriptor` does this work after the function `DftiSetValue` has set values of all needed parameters.

The function returns the zero status when it completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface.
! Note that the body provided below only illustrates the list of different
! parameters and the types of dummy parameters. You can rely only on the function
! name following keyword INTERFACE. For the precise definition of the
! interface, see the include/mkl_dfti.f90 file in the Intel MKL directory.

  INTERFACE DftiCreateDescriptor

     FUNCTION some_actual_function_1d(desc, precision, domain, dim, length)
       INTEGER :: some_actual_function_1d
       ...
       INTEGER, INTENT(IN) :: length
     END FUNCTION some_actual_function_1d

     FUNCTION some_actual_function_md(desc, precision, domain, dim, lengths)
       INTEGER :: some_actual_function_md
       ...
       INTEGER, INTENT(IN), DIMENSION(*) :: lengths
     END FUNCTION some_actual_function_md

     ...

  END INTERFACE DftiCreateDescriptor
```

Note that the function is overloaded, because the actual parameter for the formal parameter *length* can be a scalar or a rank-one array.

The function is also overloaded with respect to the type of the *precision* parameter to provide an option of using a precision-specific function for the generic name. Using more specific functions can reduce the size of statically linked executable for the applications using only single-precision FFTs or only double-precision FFTs. To use this option, change the "`USE MKL_DFTI`" statement in your program unit to one of the following:

```
USE MKL_DFTI, FORGET=>DFTI_SINGLE, DFTI_SINGLE=>DFTI_SINGLE_R
```

```
USE MKL_DFTI, FORGET=>DFTI_DOUBLE, DFTI_DOUBLE=>DFTI_DOUBLE_R
```

where the name "`FORGET`" can be replaced with any name that is not used in the program unit.

```
/* C prototype.
 * Note that the preprocessor definition provided below only illustrates
 * that the actual function called may be determined at compile time.
 * You can rely only on the declaration of the function.
 * For precise definition of the preprocessor macro, see the include/mkl_dfti.h
 * file in the Intel MKL directory.
 */
MKL_LONG DftiCreateDescriptor(DFTI_DESCRIPTOR_HANDLE * pHandle,
     enum DFTI_CONFIG_VALUE precision,
     enum DFTI_CONFIG_VALUE domain,
     MKL_LONG dimension, ... /* length/lengths */ );
```

```
#define DftiCreateDescriptor(desc,prec,domain,dim,sizes) \
     ((prec)==DFTI_SINGLE && (dim)==1) ? \
     some_actual_function_s1d((desc),(domain),(MKL_LONG)(sizes)) : \
     ...
```

Variable *length/lengths* is interpreted as a scalar (MKL_LONG) or an array (MKL_LONG*), depending on the value of parameter *dimension*. If the value of parameter *precision* is known at compile time, an optimizing compiler retains only the call to the respective specific function, thereby reducing the size of the statically linked application. Avoid direct calls to the specific functions used in the preprocessor macro definition, because their interface may change in future releases of the library. If the use of the macro is undesirable, you can safely undefine it after inclusion of the Intel MKL FFT header file, as follows:

```
#include "mkl_dfti.h"
#undef DftiCreateDescriptor
```

See Also

DFTI_PRECISION
DFTI_FORWARD_DOMAIN
DFTI_DIMENSION, DFTI_LENGTHS
Configuration Parameters

DftiCommitDescriptor

Performs all initialization for the actual FFT computation.

Syntax

Fortran:

```
status = DftiCommitDescriptor( desc_handle )
```

C:

```
status = DftiCommitDescriptor(desc_handle);
```

Include Files

- FORTRAN 90: mkl_dfti.f90
- C: mkl_dfti.h

Input Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	FFT descriptor.

Output Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	Updated FFT descriptor.
status	**FORTRAN:** INTEGER **C:** MKL_LONG	Function completion status.

Description

This function completes initialization of a previously created descriptor, which is required before the descriptor can be used for FFT computations. Typically, this committal performs all initialization that facilitates the actual FFT computation. This initialization may involve exploring many different factorizations of the input length to find the optimal computation method.

Any changes of configuration parameters of a committed descriptor via the set value function (see Descriptor Configuration) requires a re-committal of the descriptor before a computation function can be invoked. Typically, this committal function call is immediately followed by a computation function call (see FFT Computation).

The function returns the zero status when it completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface
INTERFACE DftiCommitDescriptor
!Note that the body provided here is to illustrate the different
!argument list and types of dummy arguments. The interface
!does not guarantee what the actual function names are.
!Users can only rely on the function name following the
!keyword INTERFACE
 FUNCTION some_actual function_1 ( Desc_Handle )
 INTEGER :: some_actual function_1
 TYPE(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 END FUNCTION some_actual function_1
END INTERFACE DftiCommitDescriptor
```

```
/* C prototype */
MKL_LONG DftiCommitDescriptor( DFTI_DESCRIPTOR_HANDLE );
```

DftiFreeDescriptor

Frees the memory allocated for a descriptor.

Syntax

Fortran:

```
status = DftiFreeDescriptor( desc_handle )
```

C:

```
status = DftiFreeDescriptor(&desc_handle);
```

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Input Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DESCRIPTOR_HANDLE **C:** DFTI_DESCRIPTOR_HANDLE	FFT descriptor.

Output Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DESCRIPTOR_HANDLE **C:** DFTI_DESCRIPTOR_HANDLE	Memory for the FFT descriptor is released.
status	**FORTRAN:** INTEGER **C:** MKL_LONG	Function completion status.

Description

This function frees all memory allocated for a descriptor.

NOTE Memory allocation/deallocation inside Intel MKL is managed by Intel MKL memory management software. So, even after successful completion of FreeDescriptor, the memory space may continue being allocated for the application because the memory management software sometimes does not return the memory space to the OS, but considers the space free and can reuse it for future memory allocation. See Example "mkl_free_buffers Usage with FFT Functions" in the description of the service function FreeBuffers on how to use Intel MKL memory management software and release memory to the OS.

The function returns the zero status when it completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface
INTERFACE DftiFreeDescriptor
//Note that the body provided here is to illustrate the different
//argument list and types of dummy arguments. The interface
//does not guarantee what the actual function names are.
//Users can only rely on the function name following the
//keyword INTERFACE
 FUNCTION some_actual_function_3( Desc_Handle )
 INTEGER :: some_actual_function_3
 TYPE(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 END FUNCTION some_actual_function_3
END INTERFACE DftiFreeDescriptor
```

```
/* C prototype */
MKL_LONG DftiFreeDescriptor( DFTI_DESCRIPTOR_HANDLE * );
```

DftiCopyDescriptor

Makes a copy of an existing descriptor.

Syntax

Fortran:

```
status = DftiCopyDescriptor( desc_handle_original, desc_handle_copy )
```

C:

```
status = DftiCopyDescriptor(desc_handle_original, &desc_handle_copy);
```

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Input Parameters

Name	Type	Description
desc_handle_original	**FORTRAN:** DESCRIPTOR_HANDLE **C:** DFTI_DESCRIPTOR_HANDLE	The FFT descriptor to make a copy of.

Output Parameters

Name	Type	Description
desc_handle_copy	**FORTRAN:** DESCRIPTOR_HANDLE **C:** DFTI_DESCRIPTOR_HANDLE	The copy of the FFT descriptor.
status	**FORTRAN:** INTEGER **C:** MKL_LONG	Function completion status.

Description

This function makes a copy of an existing descriptor and provides a pointer to it. The purpose is that all information of the original descriptor will be maintained even if the original is destroyed via the free descriptor function `DftiFreeDescriptor`.

The function returns the zero status when it completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface
INTERFACE DftiCopyDescriptor
! Note that the body provided here is to illustrate the different
!argument list and types of dummy arguments. The interface
!does not guarantee what the actual function names are.
!Users can only rely on the function name following the
!keyword INTERFACE
 FUNCTION some_actual_function_2( Desc_Handle_Original,
 Desc_Handle_Copy )
 INTEGER :: some_actual_function_2
 TYPE(DFTI_DESCRIPTOR), POINTER :: Desc_Handle_Original, Desc_Handle_Copy
 END FUNCTION some_actual_function_2
END INTERFACE DftiCopyDescriptor
```

```
/* C prototype */
MKL_LONG DftiCopyDescriptor( DFTI_DESCRIPTOR_HANLDE, DFTI_DESCRIPTOR_HANDLE * );
```

FFT Computation Functions

There are two functions in this category: compute the forward transform, and compute the backward transform.

DftiComputeForward

Computes the forward FFT.

Syntax

Fortran:

```
status = DftiComputeForward( desc_handle, x_inout )
status = DftiComputeForward( desc_handle, x_in, y_out )
status = DftiComputeForward( desc_handle, xre_inout, xim_inout )
status = DftiComputeForward( desc_handle, xre_in, xim_in, yre_out, yim_out )
```

C:

```
status = DftiComputeForward(desc_handle, x_inout);
status = DftiComputeForward(desc_handle, x_in, y_out);
status = DftiComputeForward(desc_handle, xre_inout, xim_inout);
status = DftiComputeForward(desc_handle, xre_in, xim_in, yre_out, yim_out);
```

Input Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	FFT descriptor.
x_inout, x_in	**FORTRAN:** Array REAL(KIND=WP) or COMPLEX(KIND=WP), DIMENSION(*), where type and working precision WP must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type float or double depending on the precision of the transform, specified in the DFTI_PRECISION configuration setting.	Data to be transformed in case of a real forward domain, specified in the DFTI_FORWARD_DOMAIN configuration setting.
xre_inout, xim_inout, xre_in, xim_in	**FORTRAN:** Array REAL(KIND=WP) or COMPLEX(KIND=WP), DIMENSION(*), where type and working precision WP must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type float or double depending on the precision of the transform.	Real and imaginary parts of the data to be transformed in case of a complex forward domain, specified in the DFTI_FORWARD_DOMAIN configuration setting.

The suffix in parameter names corresponds to the value of the configuration parameter DFTI_PLACEMENT as follows:

- *_inout* to DFTI_INPLACE
- *_in* or *_out* to DFTI_NOT_INPLACE

Output Parameters

Name	Type	Description
y_out	**FORTRAN:** Array `REAL(KIND=WP)` or `COMPLEX(KIND=WP), DIMENSION(*)`, where type and working precision `WP` must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type `float` or `double` depending on the precision of the transform.	The transformed data in case of a real backward domain, determined by the `DFTI_FORWARD_DOMAIN` configuration setting.
xre_inout, *xim_inout*, *yre_out*, *yim_out*	**FORTRAN:** Array `REAL(KIND=WP)` or `COMPLEX(KIND=WP), DIMENSION(*)`, where type and working precision `WP` must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type `float` or `double` depending on the precision of the transform.	Real and imaginary parts of the transformed data in case of a complex backward domain, determined by the `DFTI_FORWARD_DOMAIN` configuration setting.
status	**FORTRAN:** `INTEGER` **C:** `MKL_LONG`	Function completion status.

The suffix in parameter names corresponds to the value of the configuration parameter `DFTI_PLACEMENT` as follows:

- *_inout* to `DFTI_INPLACE`
- *_in* or *_out* to `DFTI_NOT_INPLACE`

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Description

The `DftiComputeForward` function accepts the descriptor handle parameter and one or more data parameters. Provided the descriptor is configured and committed successfully, this function computes the forward FFT, that is, the transform with the minus sign in the exponent, $\delta = -1$.

The number and types of the data parameters that the function requires may vary depending on the configuration of the descriptor. This variation is accommodated by variable parameters in C and the generic interface in Fortran. The generic Fortran interface to the function is based on a set of specific functions. These functions can check for inconsistency between the required and actual number of parameters. However, the specific functions disregard the type of the actual parameters and instead use the interpretation defined in the descriptor by configuration parameters `FTI_FORWARD_DOMAIN`, `DFTI_INPUT_STRIDES`, `DFTI_INPUT_DISTANCE`, and so on.

The function returns the zero status when completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface.
! Note that the body provided below only illustrates the list of different
! parameters and the types of dummy parameters. You can rely only on the function
```

```
! name following keyword INTERFACE. For the precise definition of the
! interface, see the include/mkl_dfti.f90 file in the Intel MKL directory.
INTERFACE DftiComputeForward

  FUNCTION some_actual_function_1(desc,sSrcDst)
    INTEGER some_actual_function_1
    REAL(4), INTENT(INOUT), DIMENSION(*) :: sSrcDst
    ...
  END FUNCTION some_actual_function_1

  FUNCTION some_actual_function_2(desc,cSrcDst)
    INTEGER some_actual_function_2
    COMPLEX(8), INTENT(INOUT), DIMENSION(*) :: cSrcDst
    ...
  END FUNCTION some_actual_function_2

  FUNCTION some_actual_function_3(desc,sSrcDstRe,sSrcDstIm)
    INTEGER some_actual_function_3
    REAL(4), INTENT(INOUT), DIMENSION(*) :: sSrcDstRe
    REAL(4), INTENT(INOUT), DIMENSION(*) :: sSrcDstIm
    ...
  END FUNCTION some_actual_function_3
  ...
END INTERFACE DftiComputeForward
```

The Fortran interface requires that the data parameters have the type of assumed-size rank-1 array, even for multidimensional transforms. The implementations of the FFT interface require the data stored linearly in memory with a regular "stride" pattern capable of describing multidimensional array layout (discussed more fully in `DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES`, see also [3]), and the function requires that the data parameters refer to the first element of the data. Consequently, the data arrays should be specified with the `DIMENSION(*)` attribute and the storage associated with the actual multidimensional arrays via the `EQUIVALENCE` statement.

```
/* C prototype */
MKL_LONG DftiComputeForward( DFTI_DESCRIPTOR_HANDLE, void*, ... );
```

See Also

DFTI_FORWARD_DOMAIN
DFTI_PLACEMENT
DFTI_PACKED_FORMAT
DFTI_DIMENSION, DFTI_LENGTHS
DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE
DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES
DftiComputeBackward

DftiComputeBackward

Computes the backward FFT.

Syntax

Fortran:

```
status = DftiComputeBackward( desc_handle, x_inout )

status = DftiComputeBackward( desc_handle, y_in, x_out )

status = DftiComputeBackward( desc_handle, xre_inout, xim_inout )

status = DftiComputeBackward( desc_handle, yre_in, yim_in, xre_out, xim_out )
```

C:

```
status = DftiComputeBackward(desc_handle, x_inout);
```

```
status = DftiComputeBackward(desc_handle, y_in, x_out);
status = DftiComputeBackward(desc_handle, xre_inout, xim_inout);
status = DftiComputeBackward(desc_handle, yre_in, yim_in, xre_out, xim_out);
```

Input Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	FFT descriptor.
x_inout, y_in	**FORTRAN:** Array REAL(KIND=WP) or COMPLEX(KIND=WP), DIMENSION(*), where type and working precision WP must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type float or double depending on the precision of the transform, specified in the DFTI_PRECISION configuration setting.	Data to be transformed in case of a real backward domain, determined by the DFTI_FORWARD_DOMAIN configuration setting.
xre_inout, xim_inout, yre_in, yim_in	**FORTRAN:** Array REAL(KIND=WP) or COMPLEX(KIND=WP), DIMENSION(*), where type and working precision WP must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type float or double depending on the precision of the transform.	Real and imaginary parts of the data to be transformed in case of a complex backward domain, determined by the DFTI_FORWARD_DOMAIN configuration setting.

The suffix in parameter names corresponds to the value of the configuration parameter DFTI_PLACEMENT as follows:

- *_inout* to DFTI_INPLACE
- *_in* or *_out* to DFTI_NOT_INPLACE

Output Parameters

Name	Type	Description
x_out	**FORTRAN:** Array REAL(KIND=WP) or COMPLEX(KIND=WP), DIMENSION(*), where type and working precision WP must be consistent with the forward domain and precision specified in the descriptor. **C:** Array of type float or double depending on the precision of the transform.	The transformed data in case of a real forward domain, specified in the DFTI_FORWARD_DOMAIN configuration setting.
xre_inout, xim_inout, xre_out, xim_out	**FORTRAN:** Array REAL(KIND=WP) or COMPLEX(KIND=WP), DIMENSION(*), where type and working precision WP must be consistent with the forward domain and precision specified in the descriptor.	Real and imaginary parts of the transformed data in case of a complex forward domain, specified in the DFTI_FORWARD_DOMAIN configuration setting.

Name	Type	Description
	C: Array of type `float` or `double` depending on the precision of the transform.	
status	**FORTRAN:** `INTEGER` **C:** `MKL_LONG`	Function completion status.

The suffix in parameter names corresponds to the value of the configuration parameter `DFTI_PLACEMENT` as follows:

- *_inout* to `DFTI_INPLACE`
- *_in* or *_out* to `DFTI_NOT_INPLACE`

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Description

The function accepts the descriptor handle parameter and one or more data parameters. Provided the descriptor is configured and committed successfully, the `DftiComputeBackward` function computes the inverse FFT, that is, the transform with the plus sign in the exponent, $\delta = +1$.

The number and types of the data parameters that the function requires may vary depending on the configuration of the descriptor. This variation is accommodated by variable parameters in C and the generic interface in Fortran. The generic Fortran interface to the computation function is based on a set of specific functions. These functions can check for inconsistency between the required and actual number of parameters. However, the specific functions disregard the type of the actual parameters and instead use the interpretation defined in the descriptor by configuration parameters `DFTI_FORWARD_DOMAIN`, `DFTI_INPUT_STRIDES`, `DFTI_INPUT_DISTANCE`, and so on.

The function returns the zero status when completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface.
! Note that the body provided below only illustrates the list of different
! parameters and the types of dummy parameters. You can rely only on the function
! name following keyword INTERFACE. For the precise definition of the
! interface, see the include/mkl_dfti.f90 file in the Intel MKL directory.
INTERFACE DftiComputeBackward

  FUNCTION some_actual_function_1(desc,sSrcDst)
    INTEGER some_actual_function_1
    REAL(4), INTENT(INOUT), DIMENSION(*) :: sSrcDst
    ...
  END FUNCTION some_actual_function_1

  FUNCTION some_actual_function_2(desc,cSrcDst)
    INTEGER some_actual_function_2
    COMPLEX(8), INTENT(INOUT), DIMENSION(*) :: cSrcDst
    ...
  END FUNCTION some_actual_function_2

  FUNCTION some_actual_function_3(desc,sSrcDstRe,sSrcDstIm)
    INTEGER some_actual_function_3
    REAL(4), INTENT(INOUT), DIMENSION(*) :: sSrcDstRe
    REAL(4), INTENT(INOUT), DIMENSION(*) :: sSrcDstIm
    ...
  END FUNCTION some_actual_function_3
```

```
  ...
END INTERFACE DftiComputeBackward
```

The Fortran interface requires that the data parameters have the type of assumed-size rank-1 array, even for multidimensional transforms. The implementations of the FFT interface require the data stored linearly in memory with a regular "stride" pattern capable of describing multidimensional array layout (discussed more fully in DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES, see also [3]), and the function requires that the data parameters refer to the first element of the data. Consequently, the data arrays should be specified with the DIMENSION(*) attribute and the storage associated with the actual multidimensional arrays via the EQUIVALENCE statement.

```
/* C prototype */
MKL_LONG DftiComputeBackward( DFTI_DESCRIPTOR_HANDLE, void *, ... );
```

See Also

DFTI_FORWARD_DOMAIN
DFTI_PLACEMENT
DFTI_PACKED_FORMAT
DFTI_DIMENSION, DFTI_LENGTHS
DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE
DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES
DftiComputeForward

Descriptor Configuration Functions

There are two functions in this category: the value setting function DftiSetValue sets one particular configuration parameter to an appropriate value, and the value getting function DftiGetValue reads the value of one particular configuration parameter. While all configuration parameters are readable, you cannot set a few of them. Some of these contain fixed information of a particular implementation such as version number, or dynamic information, which is derived by the implementation during execution of one of the functions. See Configuration Settings for details.

DftiSetValue

Sets one particular configuration parameter with the specified configuration value.

Syntax

Fortran:

```
status = DftiSetValue( desc_handle, config_param, config_val )
```

C:

```
status = DftiSetValue(desc_handle, config_param, config_val);
```

Include Files

- FORTRAN 90: mkl_dfti.f90
- C: mkl_dfti.h

Input Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	FFT descriptor.
config_param	**FORTRAN:** INTEGER **C:** enum	Configuration parameter.
config_val	Depends on the configuration parameter.	Configuration value.

Output Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	Updated FFT descriptor.
status	**FORTRAN:** INTEGER **C:** MKL_LONG	Function completion status.

Description

This function sets one particular configuration parameter with the specified configuration value. Each configuration parameter is a named constant, and the configuration value must have the corresponding type, which can be a named constant or a native type. For available configuration parameters and the corresponding configuration values, see:

- DFTI_PRECISION
- DFTI_FORWARD_DOMAIN
- DFTI_DIMENSION, DFTI_LENGTH
- DFTI_PLACEMENT
- DFTI_FORWARD_SCALE, DFTI_BACKWARD_SCALE
- DFTI_NUMBER_OF_USER_THREADS
- DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES
- DFTI_NUMBER_OF_TRANSFORMS
- DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE
- DFTI_COMPLEX_STORAGE, DFTI_REAL_STORAGE, DFTI_CONJUGATE_EVEN_STORAGE
- DFTI_PACKED_FORMAT
- DFTI_WORKSPACE
- DFTI_ORDERING

The DftiSetValue function cannot be used to change configuration parameters DFTI_FORWARD_DOMAIN, DFTI_PRECISION, DFTI_DIMENSION, and DFTI_LENGTHS. Use the DftiCreateDescriptor function to set them.

The function returns the zero status when it completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface
INTERFACE DftiSetValue
//Note that the body provided here is to illustrate the different
```

```
//argument list and types of dummy arguments. The interface
//does not guarantee what the actual function names are.
//Users can only rely on the function name following the
//keyword INTERFACE
 FUNCTION some_actual_function_6_INTVAL( Desc_Handle, Config_Param, INTVAL )
 INTEGER :: some_actual_function_6_INTVAL
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 INTEGER, INTENT(IN) :: INTVAL
 END FUNCTION some_actual_function_6_INTVAL
 FUNCTION some_actual_function_6_SGLVAL( Desc_Handle, Config_Param, SGLVAL )
 INTEGER :: some_actual_function_6_SGLVAL
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 REAL, INTENT(IN) :: SGLVAL
END FUNCTION some_actual_function_6_SGLVAL
 FUNCTION some_actual_function_6_DBLVAL( Desc_Handle, Config_Param, DBLVAL )
 INTEGER :: some_actual_function_6_DBLVAL
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 REAL (KIND(0D0)), INTENT(IN) :: DBLVAL
 END FUNCTION some_actual_function_6_DBLVAL
 FUNCTION some_actual_function_6_INTVEC( Desc_Handle, Config_Param, INTVEC )
 INTEGER :: some_actual_function_6_INTVEC
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 INTEGER, INTENT(IN) :: INTVEC(*)
 END FUNCTION some_actual_function_6_INTVEC
 FUNCTION some_actual_function_6_CHARS( Desc_Handle, Config_Param, CHARS )
 INTEGER :: some_actual_function_6_CHARS
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 CHARCTER(*), INTENT(IN) :: CHARS
 END FUNCTION some_actual_function_6_CHARS
END INTERFACE DftiSetValue
```

```
/* C prototype */
MKL_LONG DftiSetValue( DFTI_DESCRIPTOR_HANDLE, DFTI_CONFIG_PARAM , ... );
```

See Also

Configuration Settings
DftiCreateDescriptor
DftiGetValue

DftiGetValue

Gets the configuration value of one particular configuration parameter.

Syntax

Fortran:

```
status = DftiGetValue( desc_handle, config_param, config_val )
```

C:

```
status = DftiGetValue(desc_handle, config_param, &config_val);
```

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Input Parameters

Name	Type	Description
desc_handle	**FORTRAN:** DFTI_DESCRIPTOR **C:** DFTI_DESCRIPTOR_HANDLE	FFT descriptor.
config_param	**FORTRAN:** INTEGER **C:** enum	Configuration parameter. See Table "Configuration Parameters" for allowable values of *config_param*.

Output Parameters

Name	Type	Description
config_val	Depends on the configuration parameter.	Configuration value.
status	**FORTRAN:** INTEGER **C:** MKL_LONG	Function completion status.

Description

This function gets the configuration value of one particular configuration parameter. Each configuration parameter is a named constant, and the configuration value must have the corresponding type, which can be a named constant or a native type. For available configuration parameters and the corresponding configuration values, see:

- DFTI_PRECISION
- DFTI_FORWARD_DOMAIN
- DFTI_DIMENSION, DFTI_LENGTH
- DFTI_PLACEMENT
- DFTI_FORWARD_SCALE, DFTI_BACKWARD_SCALE
- DFTI_NUMBER_OF_USER_THREADS
- DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES
- DFTI_NUMBER_OF_TRANSFORMS
- DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE
- DFTI_COMPLEX_STORAGE, DFTI_REAL_STORAGE, DFTI_CONJUGATE_EVEN_STORAGE
- DFTI_PACKED_FORMAT
- DFTI_WORKSPACE
- DFTI_COMMIT_STATUS
- DFTI_ORDERING

The function returns the zero status when it completes successfully. See Status Checking Functions for more information on the returned status.

Interface and Prototype

```
! Fortran interface
INTERFACE DftiGetValue
//Note that the body provided here is to illustrate the different
//argument list and types of dummy arguments. The interface
//does not guarantee what the actual function names are.
//Users can only rely on the function name following the
//keyword INTERFACE
 FUNCTION some_actual_function_7_INTVAL( Desc_Handle, Config_Param, INTVAL )
 INTEGER :: some_actual_function_7_INTVAL
```

```
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 INTEGER, INTENT(OUT) :: INTVAL
 END FUNCTION DFTI_GET_VALUE_INTVAL
 FUNCTION some_actual_function_7_SGLVAL( Desc_Handle, Config_Param, SGLVAL )
 INTEGER :: some_actual_function_7_SGLVAL
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 REAL, INTENT(OUT) :: SGLVAL
 END FUNCTION some_actual_function_7_SGLVAL
 FUNCTION some_actual_function_7_DBLVAL( Desc_Handle, Config_Param, DBLVAL )
 INTEGER :: some_actual_function_7_DBLVAL
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 REAL (KIND(0D0)), INTENT(OUT) :: DBLVAL
 END FUNCTION some_actual_function_7_DBLVAL
 FUNCTION some_actual_function_7_INTVEC( Desc_Handle, Config_Param, INTVEC )
 INTEGER :: some_actual_function_7_INTVEC
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 INTEGER, INTENT(OUT) :: INTVEC(*)
 END FUNCTION some_actual_function_7_INTVEC
 FUNCTION some_actual_function_7_INTPNT( Desc_Handle, Config_Param, INTPNT )
 INTEGER :: some_actual_function_7_INTPNT
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 INTEGER, DIMENSION(*), POINTER :: INTPNT
 END FUNCTION some_actual_function_7_INTPNT
 FUNCTION some_actual_function_7_CHARS( Desc_Handle, Config_Param, CHARS )
 INTEGER :: some_actual_function_7_CHARS
 Type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle
 INTEGER, INTENT(IN) :: Config_Param
 CHARCTER(*), INTENT(OUT):: CHARS
 END FUNCTION some_actual_function_7_CHARS
END INTERFACE DftiGetValue
```

```
/* C prototype */
MKL_LONG DftiGetValue( DFTI_DESCRIPTOR_HANDLE,
      DFTI_CONFIG_PARAM ,
      ... );
```

See Also

Configuration Settings
DftiSetValue

Status Checking Functions

All of the descriptor manipulation, FFT computation, and descriptor configuration functions return an integer value denoting the status of the operation. Two functions serve to check the status. The first function is a logical function that checks if the status reflects an error of a predefined class, and the second is an error message function that returns a character string.

DftiErrorClass

Checks whether the status reflects an error of a predefined class.

Syntax

Fortran:

```
predicate = DftiErrorClass( status, error_class )
```

C:

```
predicate = DftiErrorClass(status, error_class);
```

Include Files

- FORTRAN 90: `mkl_dfti.f90`
- C: `mkl_dfti.h`

Input Parameters

Name	Type	Description
status	**FORTRAN:** `INTEGER` **C:** `MKL_LONG`	Completion status of an FFT function.
error_class	**FORTRAN:** `INTEGER` **C:** `MKL_LONG`	Predefined error class.

Output Parameters

Name	Type	Description
predicate	**FORTRAN:** `LOGICAL` **C:** `MKL_LONG`	Result of checking.

Description

The FFT interface in Intel MKL provides a set of predefined error classes listed in Table "Predefined Error Classes". They are named constants and have the type `INTEGER` in Fortran and `MKL_LONG` in C.

Predefined Error Classes

Named Constants	Comments
`DFTI_NO_ERROR`	No error. The zero status belongs to this class.
`DFTI_MEMORY_ERROR`	Usually associated with memory allocation
`DFTI_INVALID_CONFIGURATION`	Invalid settings of one or more configuration parameters
`DFTI_INCONSISTENT_CONFIGURATION`	Inconsistent configuration or input parameters
`DFTI_NUMBER_OF_THREADS_ERROR`	Number of OMP threads in the computation function is not equal to the number of OMP threads in the initialization stage (commit function)
`DFTI_MULTITHREADED_ERROR`	Usually associated with a value that OMP routines return in case of errors
`DFTI_BAD_DESCRIPTOR`	Descriptor is unusable for computation
`DFTI_UNIMPLEMENTED`	Unimplemented legitimate settings; implementation dependent
`DFTI_MKL_INTERNAL_ERROR`	Internal library error
`DFTI_1D_LENGTH_EXCEEDS_INT32`	Length of one of dimensions exceeds 2^{32} -1 (4 bytes).

The DftiErrorClass function returns a non-zero value in C or the value of .TRUE. in Fortran if the status belongs to a predefined error class. To check whether a function call was successful, call DftiErrorClass with a specific error class. However, the zero value of the status belongs to the DFTI_NO_ERROR class and thus the zero status indicates successful completion of an operation. See Example "Using Status Checking Functions" for an illustration of correct use of the status checking functions.

NOTE It is incorrect to directly compare a status with a predefined class.

Interface and Prototype

```
//Fortran interface
INTERFACE DftiErrorClass
//Note that the body provided here is to illustrate the different
//argument list and types of dummy arguments. The interface
//does not guarantee what the actual function names are.
//Users can only rely on the function name following the
//keyword INTERFACE
 FUNCTION some_actual_function_8( Status, Error_Class )
 LOGICAL some_actual_function_8
 INTEGER, INTENT(IN) :: Status, Error_Class
 END FUNCTION some_actual_function_8
END INTERFACE DftiErrorClass
```

```
/* C prototype */
MKL_LONG DftiErrorClass( MKL_LONG , MKL_LONG );
```

DftiErrorMessage

Generates an error message.

Syntax

Fortran:

```
error_message = DftiErrorMessage( status )
```

C:

```
error_message = DftiErrorMessage(status);
```

Include Files

- FORTRAN 90: mkl_dfti.f90
- C: mkl_dfti.h

Input Parameters

Name	Type	Description
status	**FORTRAN:** INTEGER **C:** MKL_LONG	Completion status of a function.

Output Parameters

Name	Type	Description
error_message	**FORTRAN:** CHARACTER(LEN=DFTI_MAX_MESSAGE_LENGTH) **C:** Array of char	The character string with the error message.

Description

The error message function generates an error message character string. In Fortran, use a character string of length DFTI_MAX_MESSAGE_LENGTH as a target for the error message. In C, the function returns a pointer to a constant character string, that is, a character array with terminating '\0' character, and you do not need to free this pointer.

Example "Using Status Checking Function" shows how this function can be used.

Interface and Prototype

```
//Fortran interface
INTERFACE DftiErrorMessage
//Note that the body provided here is to illustrate the different
//argument list and types of dummy arguments. The interface
//does not guarantee what the actual function names are.
//Users can only rely on the function name following the
//keyword INTERFACE
 FUNCTION some_actual_function_9( Status )
 CHARACTER(LEN=DFTI_MAX_MESSAGE_LENGTH) some_actual_function_9( Status )
 INTEGER, INTENT(IN) :: Status
 END FUNCTION some_actual_function_9
END INTERFACE DftiErrorMessage
```

```
/* C prototype */
char *DftiErrorMessage( MKL_LONG );
```

Configuration Settings

Each of the configuration parameters is identified by a named constant in the MKL_DFTI module. In C, these named constants have the enumeration type DFTI_CONFIG_PARAM.

All the Intel MKL FFT configuration parameters are readable. Some of them are read-only, while others can be set using the DftiCreateDescriptor or DftiSetValue function.

Values of the configuration parameters fall into the following groups:

- Values that have native data types. For example, the number of simultaneous transforms requested has an integer value, while the scale factor for a forward transform is a single-precision number.
- Values that are discrete in nature and are provided in the MKL_DFTI module as named constants. For example, the domain of the forward transform requires values to be named constants. In C, the named constants for configuration values have the enumeration type DFTI_CONFIG_VALUE.

Table "Configuration Parameters" summarises the information on configuration parameters, along with their types and values. For more details of each configuration parameter, see the subsection describing this parameter.

Configuration Parameters

Configuration Parameter	Type/Value	Comments
Most common configuration parameters, no default, must be set explicitly by `DftiCreateDescriptor`		
`DFTI_PRECISION`	Named constant `DFTI_SINGLE` or `DFTI_DOUBLE`	Precision of the computation.
`DFTI_FORWARD_DOMAIN`	Named constant `DFTI_COMPLEX` or `DFTI_REAL`	Type of the transform.
`DFTI_DIMENSION`	Integer scalar	Dimension of the transform.
`DFTI_LENGTH`	Integer scalar/array	Lengths of each dimension.
Common configuration parameters, settable by `DftiSetValue`		
`DFTI_PLACEMENT`	Named constant `DFTI_INPLACE` or `DFTI_NOT_INPLACE`	Defines whether the result overwrites the input data. Default value: `DFTI_INPLACE`.
`DFTI_FORWARD_SCALE`	Floating-point scalar	Scale factor for the forward transform. Default value: 1.0. Precision of the value should be the same as defined by `DFTI_PRECISION`.
`DFTI_BACKWARD_SCALE`	Floating-point scalar	Scale factor for the backward transform. Default value: 1.0. Precision of the value should be the same as defined by `DFTI_PRECISION`.
`DFTI_NUMBER_OF_USER_THREADS`	Integer scalar	Number of threads that concurrently use the same descriptor to compute FFT.
`DFTI_DESCRIPTOR_NAME`	Character string	Assigns a name to a descriptor. Assumed length of the string is `DFTI_MAX_NAME_LENGTH`. Default value: empty string.
Data layout configuration parameters for single and multiple transforms. Settable by `DftiSetValue`		
`DFTI_INPUT_STRIDES`	Integer array	Defines the input data layout.
`DFTI_OUTPUT_STRIDES`	Integer array	Defines the output data layout.
`DFTI_NUMBER_OF_TRANSFORMS`	Integer scalar	Number of transforms. Default value: 1.
`DFTI_INPUT_DISTANCE`	Integer scalar	Defines the distance between input data sets for multiple transforms. Default value: 0.
`DFTI_OUTPUT_DISTANCE`	Integer scalar	Defines the distance between output data sets for multiple transforms. Default value: 0.

Configuration Parameter	Type/Value	Comments
DFTI_COMPLEX_STORAGE	Named constant DFTI_COMPLEX_COMPLEX or DFTI_REAL_REAL	Defines whether the real and imaginary parts of data for a complex transform are interleaved in one array or split in two arrays. Default value: DFTI_COMPLEX_COMPLEX.
DFTI_REAL_STORAGE	Named constant DFTI_REAL_REAL	Defines how real data for a real transform is stored. Only the DFTI_REAL_REAL value is supported.
DFTI_CONJUGATE_EVEN_STORAGE	Named constant DFTI_COMPLEX_COMPLEX or DFTI_COMPLEX_REAL	Defines whether the complex data in the backward domain of a real transform is stored as complex elements or as real elements. For the default value, see the detailed description.
DFTI_PACKED_FORMAT	Named constant DFTI_CCE_FORMAT, DFTI_CCS_FORMAT, DFTI_PACK_FORMAT, or DFTI_PERM_FORMAT	Defines the layout of real elements in the backward domain of a one-dimensional or two-dimensional real transform.
Advanced configuration parameters, settable by DftiSetValue		
DFTI_WORKSPACE	Named constant DFTI_ALLOW or DFTI_AVOID	Defines whether the library should prefer algorithms using additional memory. Default value: DFTI_ALLOW.
DFTI_ORDERING	Named constant DFTI_ORDERED or DFTI_BACKWARD_SCRAMBLED	Defines whether the result of a complex transform is ordered or permuted. Default value: DFTI_ORDERED.
Read-Only configuration parameters		
DFTI_COMMIT_STATUS	Named constant DFTI_UNCOMMITTED or DFTI_COMMITTED	Readiness of the descriptor for computation.
DFTI_VERSION	String	Version of Intel MKL. Assumed length of the string is DFTI_VERSION_LENGTH.

DFTI_PRECISION

The configuration parameter DFTI_PRECISION denotes the floating-point precision in which the transform is to be carried out. A setting of DFTI_SINGLE stands for single precision, and a setting of DFTI_DOUBLE stands for double precision. The data must be presented in this precision, the computation is carried out in this precision, and the result is delivered in this precision.

DFTI_PRECISION does not have a default value. Set it explicitly by calling the DftiCreateDescriptor function.

NOTE Fortran module MKL_DFTI also defines named constants DFTI_SINGLE_R and DFTI_DOUBLE_R, with the same semantics as DFTI_SINGLE and DFTI_DOUBLE, respectively. Do not use these constants to set the DFTI_PRECISION configuration parameter. Use them only as described in section DftiCreateDescriptor.

See Also

DFTI_FORWARD_DOMAIN
DFTI_DIMENSION, DFTI_LENGTHS
DftiCreateDescriptor

DFTI_FORWARD_DOMAIN

The general form of a discrete Fourier transform is

$$Z_{k_1,k_2,\ldots,k_d} = \sigma \times \sum_{j_d=0}^{n_d-1} \cdots \sum_{j_2=0}^{n_2-1} \sum_{j_1=0}^{n_1-1} w_{j_1,j_2,\ldots,j_d} \exp\left(\delta i 2\pi \sum_{l=1}^{d} j_l k_l / n_l\right)$$

where w is the input sequence, z is the output sequence, both indexed by $k_l = 0, \ldots n_l-1$, for $l = 1, \ldots, d$, scale factor σ is an arbitrary real number with the default value of 1.0, δ is the sign in the exponent, and $\delta = -1$ for the forward transform and $\delta = +1$ for the backward transform.

The Intel MKL implementation of the FFT algorithm, used for fast computation of discrete Fourier transforms, supports forward transforms on input sequences of two domains, as specified by configuration parameter DFTI_FORWARD_DOMAIN: general complex-valued sequences (DFTI_COMPLEX domain) and general real-valued sequences (DFTI_REAL domain). The forward transform maps the forward domain to the corresponding backward domain, as shown in Table "Correspondence of Forward and Backward Domain".

The conjugate-even domain covers complex-valued sequences with the symmetry property:

$$x(k_1, k_2, \ldots, k_d) = conjugate(x(n_1 - k_1, n_2 - k_2, \ldots, n_d - k_d)),$$

where the index arithmetic is performed modulo respective size, that is,

$$x(\ldots, expr_s, \ldots) \equiv x(\ldots, \mathrm{mod}(expr_s, n_s), \ldots),$$

and therefore

$$x(\ldots, n_s, \ldots) \equiv x(\ldots, 0, \ldots).$$

Due to this property of conjugate-even sequences, only a part of such sequence is stored in the computer memory, as described in DFTI_CONJUGATE_EVEN_STORAGE.

Correspondence of Forward and Backward Domain

Forward Domain	Implied Backward Domain
Complex (DFTI_COMPLEX)	Complex (DFTI_COMPLEX)
Real (DFTI_REAL)	Conjugate-even

DFTI_FORWARD_DOMAIN does not have a default value. Set it explicitly by calling the DftiCreateDescriptor function.

See Also

DFTI_PRECISION
DFTI_DIMENSION, DFTI_LENGTHS
DftiCreateDescriptor

DFTI_DIMENSION, DFTI_LENGTHS

The dimension of the transform is a positive integer value represented in an integer scalar of `Integer` data type in Fortran and `MKL_LONG` data type in C. For a one-dimensional transform, the transform length is specified by a positive integer value represented in an integer scalar of `Integer` data type in Fortran and `MKL_LONG` data type in C. For multi-dimensional (≥ 2) transform, the lengths of each of the dimensions are supplied in an integer array (`Integer` data type in Fortran and `MKL_LONG` data type in C).

`DFTI_DIMENSION` and `DFTI_LENGTHS` do not have a default value. To set them, use the `DftiCreateDescriptor` function and not the `DftiSetValue` function.

See Also

DFTI_FORWARD_DOMAIN
DFTI_PRECISION
DftiCreateDescriptor
DftiSetValue

DFTI_PLACEMENT

By default, the computational functions overwrite the input data with the output result. That is, the default setting of the configuration parameter `DFTI_PLACEMENT` is `DFTI_INPLACE`. You can change that by setting it to `DFTI_NOT_INPLACE`.

NOTE The data sets have no common elements.

See Also

DftiSetValue

DFTI_FORWARD_SCALE, DFTI_BACKWARD_SCALE

The forward transform and backward transform are each associated with a scale factor σ of its own having the default value of 1. You can specify the scale factors using one or both of the configuration parameters `DFTI_FORWARD_SCALE` and `DFTI_BACKWARD_SCALE`. For example, for a one-dimensional transform of length n, you can use the default scale of 1 for the forward transform and set the scale factor for the backward transform to be $1/n$, thus making the backward transform the inverse of the forward transform.

Set the scale factor configuration parameter using a real floating-point data type of the same precision as the value for `DFTI_PRECISION`.

NOTE For inquiry of the scale factor with the `DftiGetValue` function in C, the *`config_val`* parameter must have the same floating-point precision as the descriptor.

See Also

DftiSetValue
DFTI_PRECISION
DftiGetValue

DFTI_NUMBER_OF_USER_THREADS

Use one of the following techniques to parallelize your application:

a. You specify the parallel mode within the FFT module of Intel MKL instead of creating threads in your application. See *Intel MKL User's Guide* for more information on how to do this. See also Example "Using Intel MKL Internal Threading Mode".

b. You create threads in the application yourself and have each thread perform all stages of FFT implementation, including descriptor initialization, FFT computation, and descriptor deallocation. In this case, each descriptor is used only within its corresponding thread. In this case, set single-threaded mode for Intel MKL. See Example "Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region".

c. You create threads in the application yourself after initializing all FFT descriptors. This implies that threading is employed for parallel FFT computation only, and the descriptors are released upon return from the parallel region. In this case, each descriptor is used only within its corresponding thread. You must explicitly set the single-threaded mode for Intel MKL, otherwise, the actual number of threads may differ from one, because the `DftiCommitDescriptor` function is not in a parallel region. See Example "Using Parallel Mode with Multiple Descriptors Initialized in One Thread".

d. You create threads in the application yourself after initializing the only FFT descriptor. This implies that threading is employed for parallel FFT computation only, and the descriptor is released upon return from the parallel region. In this case, each thread uses the same descriptor. See Example "Using Parallel Mode with a Common Descriptor".

In cases "a", "b", and "c", listed above, set the parameter `DFTI_NUMBER_OF_USER_THREADS` to 1 (its default value), since each particular descriptor instance is used only in a single thread.

In case "d", use the `DftiSetValue()` function to set the `DFTI_NUMBER_OF_USER_THREADS` to the actual number of FFT computation threads, because multiple threads will be using the same descriptor. If this setting is not done, your program will work incorrectly or fail, since the descriptor contains individual data for each thread.

WARNING

- Avoid parallelizing your program and employing the Intel MKL internal threading simultaneously because this will slow down the performance. Note that in case "d" above, FFT computation is automatically initiated in a single-threading mode.
- Do not change the number of threads after the `DftiCommitDescriptor()` function completes FFT initialization.

See Also

DftiSetValue

DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES

The FFT interface provides configuration parameters that define the layout of multidimensional data in the computer memory. For d-dimensional data set X defined by dimensions N_1 x N_2 x ... x N_d, the layout describes where a particular element $X(k_1, k_2, ..., k_d)$ of the data set is located. The memory address of the element $X(k_1, k_2, ..., k_d)$ is expressed by the formula

$$\text{address of } X(k_1, k_2, ..., k_d) = \text{address of } X(0, 0, ..., 0) + \text{offset}$$

$$= \text{address of } X(0, 0, ..., 0) + s_0 + k_1*s_1 + k_2*s_2 + ...+ k_d*s_d,$$

where s_0 is the displacement and s_1, ..., s_d are generalized strides. The configuration parameters `DFTI_INPUT_STRIDES` and `DFTI_OUTPUT_STRIDES` enable you to get and set these values. The configuration value is an array of values (s_0, s_1, ..., s_d) of `INTEGER` data type in Fortran and `MKL_LONG` data type in C.

The offset is counted in elements of the data type defined by the descriptor configuration (rather than by the type of the variable passed to the computation functions). Specifically, the `DFTI_FORWARD_DOMAIN`, `DFTI_COMPLEX_STORAGE`, and `DFTI_CONJUGATE_EVEN_STORAGE` configuration parameters define the type of the elements as shown in Table "Assumed Element Types of the Input/Output Data":

Assumed Element Types of the Input/Output Data

Descriptor Configuration	Element Type in the Forward Domain	Element Type in the Backward Domain
DFTI_FORWARD_DOMAIN=DFTI_COMPLEX DFTI_COMPLEX_STORAGE=DFTI_COMPLEX_COMPLEX	Complex	Complex
DFTI_FORWARD_DOMAIN=DFTI_COMPLEX DFTI_COMPLEX_STORAGE=DFTI_REAL_REAL	Real	Real
DFTI_FORWARD_DOMAIN=DFTI_REAL DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_REAL	Real	Real
DFTI_FORWARD_DOMAIN=DFTI_REAL DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_COMPLEX	Real	Complex

The DFTI_INPUT_STRIDES configuration parameter describes the layout of the input data, and the element type is defined by the forward domain for the DftiComputeForward function, and by the backward domain for the DftiComputeBackward function. The DFTI_OUTPUT_STRIDES configuration parameter describes the layout of the output data, and the element type is defined by the backward domain for the DftiComputeForward function, and by the forward domain for DftiComputeBackward function.

For in-place transforms, the configuration set by DFTI_OUTPUT_STRIDES is ignored except when the element types in forward and backward domains are different. If they are different, set DFTI_OUTPUT_STRIDES explicitly (even though the transform is in-place). For in-place transforms, the configuration must be consistent, that is, the locations of the first elements in input and output must coincide in each dimension. The DFTI_PLACEMENT configuration parameter defines whether the transform is in-place or out-of-place.

The configuration parameters define the layout of input and output data, and not the forward-domain and backward-domain data. If the data layouts in forward domain and backward domain differ, set DFTI_INPUT_STRIDES and DFTI_OUTPUT_STRIDES explicitly and then commit the descriptor before calling computation functions.

The FFT interface supports both positive and negative stride values. If you use negative strides, set the displacement of the data as follows:

$$s_0 = \sum_{i=1}^{d} (N_i - 1) \cdot \max(-s_i, 0).$$

The default setting of strides in a general multi-dimensional case assumes that the array that contains the data has no padding. The order of the strides depends on the programming language. For example:

```
/* C/C++ */
MKL_LONG dims[] = { nd, …, n2, n1 };
DftiCreateDescriptor( &hand, precision, domain, d, dims );
// The above call assumes data declaration:  type X[nd]…[n2][n1]
// Default strides are { 0, nd*…*n2*n1, …,  n2*n1, n1, 1 }

! Fortran
INTEGER :: dims(d) = [n1, n2, …, nd]
status = DftiCreateDescriptor( hand, precision, domain, d, dims)
! The above call assumes data declaration:  type X(n1,n2,…,nd)
! Default strides are [ 0, 1, n1, n1*n2, …, n1*n2*…*nd]
```

Note that in case of a real FFT (DFTI_DOMAIN=DFTI_REAL), where different data layouts in the backward domain are available (see DFTI_PACKED_FORMAT), the default value of the strides is not intuitive for the recommended CCE format (configuration setting DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_COMPLEX). In case of an *in-place* real transform with the CCE format, set the strides explicitly, as follows:

```
/* C/C++ */
MKL_LONG dims[] = { nd, …, n2, n1 };
MKL_LONG rstrides[] = { 0, nd*…*n2*(n1/2+1), …, 2*n2*(n1/2+1), 2*(n1/2+1), 1 };
MKL_LONG cstrides[] = { 0, nd*…*n2*(n1/2+1), …, n2*(n1/2+1), (n1/2+1), 1 };
DftiCreateDescriptor( &hand, precision,  DFTI_REAL, d, dims );
DftiSetValue(hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX);
// Set the strides appropriately for forward/backward transform

! Fortran
INTEGER :: dims(d) = [n1, n2, …, nd]
INTEGER :: rstrides(1+d) = [0, 1, 2*(n1/2+1), 2*(n1/2+1)*n2, … ]
INTEGER :: cstrides(1+d) = [0, 1, (n1/2+1), (n1/2+1)*n2, … ]
status = DftiCreateDescriptor( hand, precision, domain, d, dims)
status = DftiSetValue( hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX)
! Set the strides appropriately for forward/backward transform
```

See Also

DFTI_FORWARD_DOMAIN
DFTI_PLACEMENT
DftiSetValue
DftiCommitDescriptor
DftiComputeForward
DftiComputeBackward

DFTI_NUMBER_OF_TRANSFORMS

In some situations, you may need to perform a number of FFTs of the same dimension and lengths. For example, you may need to transform a number of one-dimensional data sets of the same length. To specify this number, use the DFTI_NUMBER_OF_TRANSFORMS parameter, which has the default value of 1. You can set this parameter to a positive integer value using the Integer data type in Fortran and MKL_LONG data type in C.

NOTE The data sets to be transformed must not have common elements. Therefore one (or both) of the configuration parameters DFTI_INPUT_DISTANCE and DFTI_OUTPUT_DISTANCE is required if DFTI_NUMBER_OF_TRANSFORMS is greater than one.

See Also

DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE
DftiSetValue

DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE

The FFT interface in Intel MKL enables computation of multiple transforms. To compute multiple transforms, you need to specify the data distribution of the multiple sets of data. The distance between the first data elements of consecutive data sets, DFTI_INPUT_DISTANCE for input data or DFTI_OUTPUT_DISTANCE for output data, specifies the distribution. The configuration setting is a value of INTEGER data type in Fortran and MKL_LONG data type in C.

The default value for both configuration settings is one. You must set this parameter explicitly if the number of transforms is greater than one (see DFTI_NUMBER_OF_TRANSFORMS).

The distance is counted in elements of the data type defined by the descriptor configuration (rather than by the type of the variable passed to the computation functions). Specifically, the DFTI_FORWARD_DOMAIN, DFTI_COMPLEX_STORAGE, and DFTI_CONJUGATE_EVEN_STORAGE configuration parameters define the type of the elements as shown in Table "Assumed Element Types of the Input/Output Data".

For in-place transforms, the configuration set by DFTI_OUTPUT_DISTANCE is ignored except when the element types in forward and backward domains are different. If they are different, set DFTI_OUTPUT_DISTANCE explicitly (even though the transform is in-place). For in-place transforms, the configuration must be consistent, that is, the locations of the data sets on input and output must coincide. The DFTI_PLACEMENT configuration parameter defines whether the transform is in-place or out-of-place.

The configuration parameters define the distance within input and output data, and not within the forward-domain and backward-domain data. If the distances in the forward and backward domains differ, set DFTI_INPUT_DISTANCE and DFTI_OUTPUT_DISTANCE explicitly and then commit the descriptor before calling computation functions.

The following examples illustrate setting of the DFTI_INPUT_DISTANCE configuration parameter:

```
MKL_LONG dims[] = { nd, …, n2, n1 };
MKL_LONG distance = nd*…*n2*n1;
DftiCreateDescriptor( &hand, precision,  DFTI_COMPLEX, d, dims );
DftiSetValue( hand, DFTI_NUMBER_OF_TRANSFORMS, (MLK_LONG)howmany );
DftiSetValue( hand, DFTI_INPUT_DISTANCE,  distance );

! Fortran
INTEGER :: dims(d) = [n1, n2, …, nd]
INTEGER :: distance = n1*n2*…*nd
status = DftiCreateDescriptor( hand, precision, DFTI_COMPLEX, d, dims)
status = DftiSetValue( hand, DFTI_NUMBER_OF_TRANSFORMS, howmany )
status = DftiSetValue( hand, DFTI_INPUT_DISTANCE,  distance );
```

See Also

DFTI_PLACEMENT
DftiSetValue
DftiCommitDescriptor
DftiComputeForward
DftiComputeBackward

DFTI_COMPLEX_STORAGE, DFTI_REAL_STORAGE, DFTI_CONJUGATE_EVEN_STORAGE

Depending on the value of configuration parameter DFTI_FORWARD_DOMAIN, the implementation of FFT supports several storage schemes for input and output data (see document [3] for the rationale behind the definition of the storage schemes). The data elements are placed within contiguous memory blocks, defined with generalized strides (see DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES). For multiple transforms, each nth set of data (where nth≥0) should be located within the same memory block, and the data sets should be placed at a distance from each other (see DFTI_NUMBER_OF_TRANSFORMS and DFTI_INPUT_DISTANCE, DFTI_OUTPUT_DISTANCE).

NOTE In C/C++, avoid setting up multidimensional arrays with lists of pointers to one-dimensional arrays. Instead use a one-dimensional array with the explicit indexing to access the data elements.

C notation is used in this section to describe association of mathematical entities with the data elements stored in memory. FFT Examples demonstrate the usage of storage formats in both C and Fortran.

Storage schemes for complex domain. For the DFTI_COMPLEX forward domain, both input and output sequences belong to the complex domain. In this case, the configuration parameter DFTI_COMPLEX_STORAGE can have one of the two values: DFTI_COMPLEX_COMPLEX (default) or DFTI_REAL_REAL.

NOTE In the Intel MKL FFT implementation, storage schemes for a forward complex domain and the respective backward complex domain are the same.

With DFTI_COMPLEX_COMPLEX storage, the complex-valued data sequence is referenced by a single complex parameter Z so that complex-valued element $z_{k_1, k_2, \ldots, k_d}$ of the sequence is located at Z[nth*distance + $stride_0$ + k_1*$stride_1$ + k_2*$stride_2$+ ... k_d*$stride_d$] as a structure consisting of the real and imaginary parts.

The following example illustrates a typical usage of the DFTI_COMPLEX_COMPLEX storage:

```
complex :: x(n)
...
! on input, for i=1,...,N: x(i) = r_{i-1}
status = DftiComputeForward( desc_handle, x )
! on output, for i=1,...,N: x(i) = z_{i-1}
```

With the DFTI_REAL_REAL storage, the complex-valued data sequence is referenced by two real parameters ZRe and ZIm so that complex-valued element $z_{k_1, k_2, \ldots, k_d}$ of the sequence is computed as ZRe[nth*distance + stride$_0$ + k_1*stride$_1$ + k_2*stride$_2$+ ... k_d*stride$_d$] + √(-1) × ZIm[nth*distance + stride$_0$ + k_1*stride$_1$ + k_2*stride$_2$+ ... k_d*stride$_d$].

A typical usage of the DFTI_REAL_REAL storage is illustrated by the following example:

```
real :: xre(n), xim(n)
...
status = DftiSetValue( desc_handle, DFTI_COMPLEX_STORAGE, DFTI_REAL_REAL)
! on input, for i=1,...,N: cmplx(xre(i),xim(i)) = r_{i-1}
status = DftiComputeForward( desc_handle, xre, xim )
! on output, for i=1,...,N: cmplx(xre(i),xim(i)) = z_{i-1}
```

Storage scheme for the real and conjugate-even domains. The setting for the storage schemes for real and conjugate-even domains is recorded in the configuration parameters DFTI_REAL_STORAGE and DFTI_CONJUGATE_EVEN_STORAGE. Since a forward real domain corresponds to a conjugate-even backward domain, they are considered together. The example below uses one-, two- and three-dimensional real to conjugate-even transforms. In-place computation is assumed whenever possible (that is, when the input data type matches the output data type).

One-Dimensional Transform

Consider a one-dimensional n-length transform of the form

$$z_k = \sum_{j=0}^{n-1} r_j e^{-i2\pi jk/n}, \qquad r_j \in R, z_k \in C.$$

There is a symmetry:

For even n: z(n/2+i) = conjg(z(n/2-i)), $1 \le i \le n/2-1$, and moreover z(0) and z(n/2) are real values.

For odd n: z(m+i) = conjg(z(m-i+1)), m = floor($n/2$), $1 \le i \le m$, and moreover z(0) is real value.

Comparison of the Storage Effects of Complex-to-Complex and Real-to-Complex FFTs for a Forward Transform

N=8							
Input Vectors			**Output Vectors**				
Complex FFT		**Real FFT**	**Complex FFT**		**Real FFT**		
Complex Data		**Real Data**	**Complex Data**		**Real Data**		
Real	**Imaginary**		**Real**	**Imaginary**	**CCS**	**Pack**	**Perm**

N=8							
r0	0.000000	r0	z0	0.000000	z0	z0	z0
r1	0.000000	r1	Re(z1)	Im(z1)	0.000000	Re(z1)	z4
r2	0.000000	r2	Re(z2)	Im(z2)	Re(z1)	Im(z1)	Re(z1)
r3	0.000000	r3	Re(z3)	Im(z3)	Im(z1)	Re(z2)	Im(z1)
r4	0.000000	r4	z4	0.000000	Re(z2)	Im(z2)	Re(z2)
r5	0.000000	r5	Re(z3)	-Im(z3)	Im(z2)	Re(z3)	Im(z2)
r6	0.000000	r6	Re(z2)	-Im(z2)	Re(z3)	Im(z3)	Re(z3)
r7	0.000000	r7	Re(z1)	-Im(z1)	Im(z3)	z4	Im(z3)
					z4		
					0.000000		

N=7							
Input Vectors			**Output Vectors**				
Complex FFT		**Real FFT**	**Complex FFT**		**Real FFT**		
Complex Data		**Real Data**	**Complex Data**		**Real Data**		
Real	**Imaginary**		**Real**	**Imaginary**	**CCS**	**Pack**	**Perm**
r0	0.000000	r0	z0	0.000000	z0	z0	z0
r1	0.000000	r1	Re(z1)	Im(z1)	0.000000	Re(z1)	Re(z1)
r2	0.000000	r2	Re(z2)	Im(z2)	Re(z1)	Im(z1)	Im(z1)
r3	0.000000	r3	Re(z3)	Im(z3)	Im(z1)	Re(z2)	Re(z2)
r4	0.000000	r4	Re(z3)	-Im(z3)	Re(z2)	Im(z2)	Im(z2)
r5	0.000000	r5	Re(z2)	-Im(z2)	Im(z2)	Re(z3)	Re(z3)
r6	0.000000	r6	Re(z1)	-Im(z1)	Re(z3)	Im(z3)	Im(z3)
					Im(z3)		

Comparison of the Storage Effects of Complex-to-Complex and Complex-to-Real FFTs for Backward Transform

N=8							
Input Vectors			**Output Vectors**				
Complex FFT		**Real FFT**	**Complex FFT**		**Real FFT**		
Complex Data		**Real Data**	**Complex Data**		**Real Data**		

N=8

Real	Imaginary		Real	Imaginary	CCS	Pack	Perm
r0	0.000000	r0	z0	0.000000	z0	z0	z0
r1	0.000000	r1	Re(z1)	Im(z1)	0.000000	Re(z1)	z4
r2	0.000000	r2	Re(z2)	Im(z2)	Re(z1)	Im(z1)	Re(z1)
r3	0.000000	r3	Re(z3)	Im(z3)	Im(z1)	Re(z2)	Im(z1)
r4	0.000000	r4	z4		Re(z2)	Im(z2)	Re(z2)
r5	0.000000	r5	Re(z3)	-Im(z3)	Im(z2)	Re(z3)	Im(z2)
r6	0.000000	r6	Re(z2)	-Im(z2)	Re(z3)	Im(z3)	Re(z3)
r7	0.00000	r7	Re(z1)	-Im(z1)	Im(z3)	z4	Im(z3)
					z4		
					0.000000		

N=7

Input Vectors			**Output Vectors**				
Complex FFT		**Real FFT**	**Complex FFT**		**Real FFT**		
Complex Data		**Real Data**	**Complex Data**		**Real Data**		
Real	**Imaginary**		**Real**	**Imaginary**	**CCS**	**Pack**	**Perm**
r0	0.000000	r0	z0	0.000000	z0	z0	z0
r1	0.000000	r1	Re(z1)	Im(z1)	0.000000	Re(z1)	Re(z1)
r2	0.000000	r2	Re(z2)	Im(z2)	Re(z1)	Im(z1)	Im(z1)
r3	0.000000	r3	Re(z3)	Im(z3)	Im(z1)	Re(z2)	Re(z2)
r4	0.000000	r4	Re(z3)	-Im(z3)	Re(z2)	Im(z2)	Im(z2)
r5	0.000000	r5	Re(z2)	-Im(z2)	Im(z2)	Re(z3)	Re(z3)
r6	0.000000	r6	Re(z1)	-Im(z1)	Re(z3)	Im(z3)	Im(z3)
					Im(z3)		

Assume that the stride has the default value of one.

This complex conjugate symmetric vector can be stored in the complex array of size m+1 or in the real array of size 2m+2 or 2m depending on which packed format is used.

Two-Dimensional Transform

Each of the real-to-complex functions computes the forward FFT of a two-dimensional real matrix according to the mathematical equation

$$z_{j,p} = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} r_{k,l} * e^{-i2\pi jk/m} * e^{-i2\pi pl/n},$$

$$0 \le j \le m\text{-}1, 0 \le p \le n\text{-}1$$

The mathematical result $z_{j,p}$, $0 \le j \le m-1$, $0 \le p \le n-1$, is the complex matrix of size `(m,n)`.

This mathematical result can be stored in the real two-dimensional array of size:

`(m+2,n+2)` (`CCS` format), or
`(m,n)` (`Pack` or `Perm` formats), or
`(2*(m/2+1), n)` (`CCE` format, Fortran interface),
`(m, 2*(n/2+1))` (`CCE` format, C interface)

or in the complex two-dimensional array of size:

`(m/2+1, n)` (`CCE` format, Fortran interface),
`(m, n/2+1)` (`CCE` format, C interface)

Since the multidimensional array data are arranged differently in Fortran and C (see `DFTI_INPUT_STRIDES`, `DFTI_OUTPUT_STRIDES`), the output array that holds the computational result contains complex conjugate-symmetric columns (for Fortran) or complex conjugate-symmetric rows (for C).

The following tables give examples of output data layout in `Pack` format for a forward two-dimensional real-to-complex FFT of a 6-by-4 real matrix. Note that the same layout is used for the input data of the corresponding backward complex-to-real FFT.

Fortran-interface Data Layout for a 6-by-4 Matrix

z(1,1)	Re z(1,2)	Im z(1,2)	z(1,3)
Re z(2,1)	Re z(2,2)	Re z(2,3)	Re z(2,4)
Im z(2,1)	Im z(2,2)	Im z(2,3)	Im z(2,4)
Re z(3,1)	Re z(3,2)	Re z(3,3)	Re z(3,4)
Im z(3,1)	Im z(3,2)	Im z(3,3)	Im z(3,4)
z(4,1)	Re z(4,2)	Im z(4,2)	z(4,3)

For the above example, the stride array is (0, 1, 6).

C-interface Data Layout for a 6-by-4 Matrix

z(1,1)	Re z(1,2)	Im z(1,2)	z(1,3)
Re z(2,1)	Re z(2,2)	Im z(2,2)	Re z(2,3)
Im z(2,1)	Re z(3,2)	Im z(3,2)	Im z(2,3)
Re z(3,1)	Re z(4,2)	Im z(4,2)	Re z(3,3)
Im z(3,1)	Re z(5,2)	Im z(5,2)	Im z(3,3)
z(4,1)	Re z(6,2)	Im z(6,2)	z(4,3)

For the second example, the stride array is (0, 4, 1). See `DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES` for details.

See also `DFTI_PACKED_FORMAT`.

Three-Dimensional Transform

Each of the real-to-complex functions computes the forward FFT of a three-dimensional real matrix according to the mathematical equation

$$z_{j,t,q} = \sum_{p=0}^{m-1} \sum_{l=0}^{n-1} \sum_{s=0}^{k-1} r_{p,l,s} * e^{-i2\pi jp/m} * e^{-i2\pi tl/n} * e^{-i2\pi qs/k},$$

$$0 \le j \le m\text{-}1, 0 \le t \le n\text{-}1, 0 \le q \le k\text{-}1$$

The mathematical result $z_{j,t,q}$, $0 \le j \le m-1$, $0 \le t \le n-1$, $0 \le q \le k-1$ is the complex matrix of size (m,n,k), which is a complex conjugate-symmetric, or conjugate-even, matrix as follows:

$z_{m1,n1,k1} = conjg(z_{m-m1,n-n1,k-k1})$, where each dimension is periodic.

This mathematical result can be stored in the real three-dimensional array of size:

(m/2+1,n,k) (CCE format, Fortran interface),
(m,n,k/2+1) (CCE format, C interface).

Since the multidimensional array data are arranged differently in Fortran and C (see DFTI_INPUT_STRIDES, DFTI_OUTPUT_STRIDES), the output array that holds the computational result contains complex conjugate-symmetric columns (for Fortran) or complex conjugate-symmetric rows (for C).

NOTE CCE is the only packed format for a three-dimensional real FFT. In both in-place and out-of-place REAL FFT, for real data, the stride and distance parameters are in REAL units and for complex data, they are in COMPLEX units. So elements of the input and output data can be placed in different elements of input-output array of the in-place FFT.

1. DFTI_REAL_REAL for real domain, DFTI_COMPLEX_REAL for conjugate-even domain (by default). It is used for 1D and 2D REAL FFT.
   - A typical usage of in-place transform is as follows:

```
// m = floor( n/2 )
REAL :: X(0:2*m+1)
...some other code...
...assuming inplace transform...
Status = DftiComputeForward( Desc_Handle, X )
```

   On input,

   X(p) = r_p, p = 0,1,...,n-1.

   On output,

   Output data stored in one of formats: Pack, Perm or CCS (see DFTI_PACKED_FORMAT).

   CCS format: X(2*k) = Re(z_k), X(2*k+1) = Im(z_k), k = 0,1,...,m.

   Pack format:

   even n: X(0) = Re(z_0), X(2*k-1) = Re(z_k), X(2*k) = Im(z_k), k = 1,...,m-1, and X(n-1) = Re(z_m)

   odd n: X(0) = Re(z_0), X(2*k-1) = Re(z_k), X(2*k) = Im(z_k), k = 1,...,m

   Perm format:

   even n: X(0) = Re(z_0), X(1) = Re(z_m), X(2*k) = Re(z_k), X(2*k+1) = Im(z_k), k = 1,...,m-1,

odd n: $X(0) = Re(z_0), X(2*k-1) = Re(z_k), X(2*k) = Im(z_k), k = 1,...,m$.

See Example "One-dimensional In-place FFT (Fortran Interface)", Example "One-dimensional In-place FFT (C Interface)", Example "Two-dimensional FFT (Fortran Interface)", and Example "Two-dimensional FFT (C Interface)".

Input and output data exchange roles in the backward transform.

- A typical usage of out-of-place transform is as follows:

```
// m = floor( n/2 )
REAL :: X(0:n-1)
REAL :: Y(0:2*m+1)
...some other code...
...assuming out-of-place transform...
Status = DftiComputeForward( Desc_Handle, X, Y )
```

On input, $X(p) = r_p, p = 0,1,...,n-1$.

On output,

Output data stored in one of formats: `Pack`, `Perm` or `CCS` (see `DFTI_PACKED_FORMAT`).

CCS format: $Y(2*k) = Re(z_k), Y(2*k+1) = Im(z_k), k = 0,1,...,m$.

Pack format:

even n: $Y(0) = Re(z_0), Y(2*k-1) = Re(z_k), Y(2*k) = Im(z_k), k = 1,...,m-1,$ and $Y(n-1) = Re(z_m)$

odd n: $Y(0) = Re(z_0), Y(2*k-1) = Re(z_k), Y(2*k) = Im(z_k), k = 1,...,m$

Perm format:

even n: $Y(0) = Re(z_0), Y(1) = Re(z_m), Y(2*k) = Re(z_k)$, $Y(2*k+1) = Im(z_k), k = 1,...,m-1,$

odd n: $Y(0) = Re(z_0), Y(2*k-1) = Re(z_k), Y(2*k) = Im(z_k), k = 1,...,m$.

Notice that if the stride of the output array is not set to the default value unit stride, the real and imaginary parts of one complex element will be placed with this stride.

For example:

CCS format: $Y(2*k*s) = Re(z_k), Y((2*k+1)*s) = Im(z_k), k = 0,1, ..., m,$ *s* - stride.

See Example "One-dimensional Out-of-place FFT (Fortran Interface)" and Example "One-dimensional Out-of-place FFT (C Interface)".

Input and output data exchange roles in the backward transform.

2. `DFTI_REAL_REAL` for real domain, `DFTI_COMPLEX_COMPLEX` for conjugate-even domain. It is used for 1D, 2D and 3D REAL FFT. The CCE format is set by default. You must explicitly set the storage scheme in this case, because its value is not the default one.

- A typical usage of in-place transform is as follows:

```
// m = floor( n/2 )
REAL :: X(0:m*2)
...some other code...
...assuming in-place transform...
Status = DftiSetValue( Desc_Handle,  DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX)
...
Status = DftiComputeForward( Desc_Handle, X)
```

  On input,

  $X(p) = r_p$, $p = 0,1,\ldots,n-1$.

  On output,

  $X(2*k) = Re(z_k)$, $X(2*k+1) = Im(z_k)$, $k = 0,1,\ldots,m$.

  See Example "Two-Dimensional REAL In-place FFT (Fortran Interface)".

  Input and output data exchange roles in the backward transform.

- A typical usage of out-of-place transform is as follows:

```
// m = floor( n/2 )
REAL :: X(0:n-1)
COMPLEX :: Y(0:m)
...some other code...
...assuming out-of-place transform...
Status = DftiSetValue( Desc_Handle,  DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX)
...
Status = DftiComputeForward( Desc_Handle, X, Y )
```

  On input,

  $X(p) = r_p$, $p = 0,1,\ldots,n-1$.

  On output,

  $Y(k) = z_k$, $k = 0,1,\ldots,m$.

  See Example "Two-Dimensional REAL Out-of-place FFT (Fortran Interface)" and Example "Three-Dimensional REAL FFT (C Interface)"

  Input and output data exchange roles in the backward transform.

See Also

DftiSetValue

DFTI_PACKED_FORMAT

The result of the forward transform (that is, in the frequency domain) of real data is represented in several possible packed formats: Pack, Perm, CCS, or CCE. The data can be packed due to the symmetry property of the FFT of real data.

Use the following non-default settings for real transforms of all ranks:

- The configuration parameter `DFTI_CONJUGATE_EVEN_STORAGE` has the value of `DFTI_COMPLEX_COMPLEX`.
- Elements of the result in the conjugate-even domain have a complex type.
- The configuration parameter `DFTI_PACKED_FORMAT` has the value of `DFTI_CCE_FORMAT`.

The following setting is the default for one-dimensional and two-dimensional real transforms:

- The configuration parameter `DFTI_CONJUGATE_EVEN_STORAGE` has the value of `DFTI_COMPLEX_REAL`.
- Data elements in the frequency domain have a real type.
- The value of `DFTI_PACKED_FORMAT` defines how real and imaginary parts of the data are laid out in the result.

NOTE This setting does not apply to three-dimensional and higher-rank transforms. Though not recommended, it is the default for backward compatibility.

The CCE format stores the values of the first half of the output complex conjugate-even signal resulting from the forward FFT. For a multi-dimensional real transform, `n1 * n2 * n3 * ... * nk` the size of complex matrix in CCE format is `(n1/2+1)* n2 * n3 * ... * nk` for Fortran and `n1 * n2 * ... * (nk/2+1)` for C.

The CCS format is similar to the CCE format and is the same format for one-dimensional transform. It differs slightly for multi-dimensional real transforms. In CCS format, the output samples of the FFT are arranged as shown in Table "Packed Format Output Samples" for a one-dimensional FFT and in Table "CCS Format Output Samples (Two-Dimensional Matrix (m+2)-by-(n+2))" for a two-dimensional FFT.

The Pack format is a compact representation of a complex conjugate-symmetric sequence, but the elements are arranged intuitively for complex FFT algorithms rather than for real FFTs. In the Pack format, the output samples of the FFT are arranged as shown in Table "Packed Format Output Samples" for one-dimensional FFT and in Table "Pack Format Output Samples (Two-Dimensional Matrix m-by-n)" for two-dimensional FFT.

The Perm format is a permutation of the Pack format for even lengths and is the same as the Pack format for odd lengths. In Perm format, the output samples of the FFT are arranged as shown in Table "Packed Format Output Samples" for a one-dimensional FFT and in Table "Perm Format Output Samples (Two-Dimensional Matrix m-by-n)" for a two-dimensional FFT.

Packed Format Output Samples

For n = 2*s

FFT Real	0	1	2	3	...	n-2	n-1	n	n+1
CCS	R_0	0	R_1	I_1	...	$R_{n/2-1}$	$I_{n/2-1}$	$R_{n/2}$	0
Pack	R_0	R_1	I_1	R_2	...	$I_{n/2-1}$	$R_{n/2}$		
Perm	R_0	$R_{n/2}$	R_1	I_1	...	$R_{n/2-1}$	$I_{n/2-1}$		

For n = 2*s + 1

FFT Real	0	1	2	3	...	n-4	n-3	n-2	n-1	n	n+1
CCS	R_0	0	R_1	I_1	...	I_{s-2}	R_{s-1}	I_{s-1}	R_s	I_s	
Pack	R_0	R_1	I_1	R_2	...	R_{s-1}	I_{s-1}	R_s	I_s		
Perm	R_0	R_1	I_1	R_2	...	R_{s-1}	I_{s-1}	R_s	I_s		

Note that Table "Packed Format Output Samples" uses the following notation for complex data entries:

R_j = Re z_j

I_j = Im z_j

See also Table "Comparison of the Storage Effects of Complex-to-Complex and Real-to-Complex FFTs for Forward Transform" and Table "Comparison of the Storage Effects of Complex-to-Complex and Complex-to-Real FFTs for Backward Transform".

CCS Format Output Samples (Two-Dimensional Matrix (*m*+2)-by-(*n*+2))

For m = 2*s, n = 2*k

z(1,1)	0	REz(1,2)	IMz(1,2)	...	REz(1,k)	IMz(1,k)	z(1,k+1)	0
0	0	0	0	...	0	0	0	0
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)	n/u*	n/u
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)	n/u	n/u
...	...	...	...	...	...	...	n/u	n/u
REz(m/2,1)	REz(m/2,2)	REz(m/2,3)	REz(m/2,4)	...	REz(m/2,n-1)	REz(m/2,n)	n/u	n/u
IMz(m/2,1)	IMz(m/2,2)	IMz(m/2,3)	IMz(m/2,4)	...	IMz(m/2,n-1)	IMz(m/2,n)	n/u	n/u
z(m/2+1,1)	0	REz(m/2+1,2)	IMz(m/2+1,2)	...	REz(m/2+1,k)	IMz(m/2+1,k)	z(m/2+1,k+1)	0
0	0	0	0	...	0	0	n/u	n/u

For m = 2*s+1, n = 2*k

z(1,1)	0	REz(1,2)	IMz(1,2)	...	REz(1,k)	IMz(1,k)	z(1,k+1)	0
0	0	0	0	...	0	0	0	0
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)	n/u	n/u
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)	n/u	n/u
...	...	...	...	...	...	...	n/u	n/u
REz(s,1)	REz(s,2)	REz(s,3)	REz(s,4)	...	REz(s,n-1)	REz(s,n)	n/u	n/u
IMz(s,1)	IMz(s,2)	IMz(s,3)	IMz(s,4)	...	IMz(s,n-1)	IMz(s,n)	n/u	n/u

For m = 2*s, n = 2*k+1

z(1,1)	0	REz(1,2)	IMz(1,2)	...	IMz(1,k-1)	REz(1,k)	IM z(1,k)
0	0	0	0	...	0	0	0
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)	n/u*

For m = 2*s, n = 2*k+1							
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)	n/u
...	...	...	...	...	...	...	n/u
REz(m/2,1)	REz(m/2,2)	REz(m/2,3)	REz(m/2,4)	...	REz(m/2,n-1)	REz(m/2,n)	n/u
IMz(m/2,1)	IMz(m/2,2)	IMz(m/2,3)	IMz(m/2,4)	...	IMz(m/2,n-1)	IMz(m/2,n)	n/u
z(m/2+1,1)	0	REz(m/2+1,2)	IMz(m/2+1,2)	...	IMz(m/2+1,k-1)	REz(m/2+1,k)	IMz(m/2+1,k)
0	0	0	0	...	0	0	n/u

For m = 2*s+1, n = 2*k+1							
z(1,1)	0	REz(1,2)	IMz(1,2)	...	IMz(1,k-1)	REz(1,k)	IMz(1,k)
0	0	0	0	...	0	0	0
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)	n/u
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)	n/u
...	...	...	...	...	...	...	n/u
REz(s,1)	REz(s,2)	REz(s,3)	REz(s,4)	...	REz(s,n-1)	REz(s,n)	n/u
IMz(s,1)	IMz(s,2)	IMz(s,3)	IMz(s,4)	...	IMz(s,n-1)	IMz(s,n)	n/u

* n/u - not used.

Note that in the Table "CCS Format Output Samples (Two-Dimensional Matrix (*m*+2)-by-(*n*+2))", (*n*+2) columns are used for even `n = k*2`, while *n* columns are used for odd `n = k*2+1`.

Pack Format Output Samples (Two-Dimensional Matrix *m*-by-*n*)

For m = 2*s, n = 2*k						
z(1,1)	REz(1,2)	IMz(1,2)	REz(1,3)	...	IMz(1,k)	z(1,k+1)
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)
...	...	...	...	...	...	...
REz(m/2,1)	REz(m/2,2)	REz(m/2,3)	REz(m/2,4)	...	REz(m/2,n-1)	REz(m/2,n)
IMz(m/2,1)	IMz(m/2,2)	IMz(m/2,3)	IMz(m/2,4)	...	IMz(m/2,n-1)	IMz(m/2,n)
z(m/2+1,1)	REz(m/2+1,2)	IMz(m/2+1,2)	REz(m/2+1,3)	...	IMz(m/2+1,k)	z(m/2+1,k+1)

For m = 2*s+1, n = 2*k						
z(1,1)	REz(1,2)	IMz(1,2)	REz(1,3)	...	IMz(1,k)	z(1,n/2+1)
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)
...	...	...	...	...	...	...
REz(s,1)	REz(s,2)	REz(s,3)	REz(s,4)	...	REz(s,n-1)	REz(s,n)
IMz(s,1)	IMz(s,2)	IMz(s,3)	IMz(s,4)	...	IMz(s,n-1)	IMz(s,n)

Perm Format Output Samples (Two-Dimensional Matrix *m*-by-*n*)

For m = 2*s, n = 2*k+1						
z(1,1)	z(1,k+1)	REz(1,2)	IMz(1,2)	...	REz(1,k)	IMz(1,k)
z(m/2+1,1)	z(m/2+1,k+1)	REz(m/2+1,2)	IMz(m/2+1,2)	...	REz(m/2+1,k)	IMz(m/2+1,k)
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)
...	...	...	...	...	...	...
REz(m/2,1)	REz(m/2,2)	REz(m/2,3)	REz(m/2,4)	...	REz(m/2,n-1)	REz(m/2,n)
IMz(m/2,1)	IMz(m/2,2)	IMz(m/2,3)	IMz(m/2,4)	...	IMz(m/2,n-1)	IMz(m/2,n)

For m = 2*s+1, n = 2*k+1						
z(1,1)	z(1,k+1)	REz(1,2)	IMz(1,2)	...	REz(1,k)	IMz(1,k)
REz(2,1)	REz(2,2)	REz(2,3)	REz(2,4)	...	REz(2,n-1)	REz(2,n)
IMz(2,1)	IMz(2,2)	IMz(2,3)	IMz(2,4)	...	IMz(2,n-1)	IMz(2,n)
...	...	...	...	...	...	...
REz(s,1)	REz(s,2)	REz(s,3)	REz(s,4)	...	REz(s,n-1)	REz(s,n)
IMz(s,1)	IMz(s,2)	IMz(s,3)	IMz(s,4)	...	IMz(s,n-1)	IMz(s,n)

The tables for two-dimensional FFT use Fortran-interface conventions. For C-interface specifics in storing packed data, see `DFTI_COMPLEX_STORAGE, DFTI_REAL_STORAGE, DFTI_CONJUGATE_EVEN_STORAGE`. See also Table "Fortran-interface Data Layout for a 6-by-4 Matrix" and Table "C-interface Data Layout for a 6-by-4 Matrix" for examples of Fortran-interface and C-interface formats.

To better understand packed formats for two-dimensional transforms, refer to these examples in your Intel MKL directory:

C:	`./examples/dftc/source/config_conjugate_even_storage.c`
Fortran:	`./examples/dftf/source/config_conjugate_even_storage.f90`

See Also

DftiSetValue

DFTI_WORKSPACE

The computation step for some FFT algorithms requires a scratch space for permutation or other purposes. To manage the use of the auxiliary storage, Intel MKL enables you to set the configuration parameter `DFTI_WORKSPACE` with the following values:

DFTI_ALLOW	(default) Permits the use of the auxiliary storage.
DFTI_AVOID	Instructs Intel MKL to avoid using the auxiliary storage if possible.

See Also
DftiSetValue

DFTI_COMMIT_STATUS

The DFTI_COMMIT_STATUS configuration parameter indicates whether the descriptor is ready for computation. The parameter has two possible values:

DFTI_UNCOMMITTED	Default value, set after a successful call of DftiCreateDescriptor.
DFTI_COMMITTED	The value after a successful call to DftiCommitDescriptor.

A computation function called with an uncommitted descriptor returns an error.

You cannot directly set this configuration parameter in a call to DftiSetValue, but a change in the configuration of a committed descriptor may change the commit status of the descriptor to DFTI_UNCOMMITTED.

See Also
DftiCreateDescriptor
DftiCommitDescriptor
DftiSetValue

DFTI_ORDERING

Some FFT algorithms apply an explicit permutation stage that is time consuming [4]. The exclusion of this step is similar to applying an FFT to input data whose order is scrambled, or allowing a scrambled order of the FFT results. In applications such as convolution and power spectrum calculation, the order of result or data is unimportant and thus using scrambled data is acceptable if it leads to better performance. The following options are available in Intel MKL:

- DFTI_ORDERED: Forward transform data ordered, backward transform data ordered (default option).
- DFTI_BACKWARD_SCRAMBLED: Forward transform data ordered, backward transform data scrambled.

Table "Scrambled Order Transform" tabulates the effect of this configuration setting.

Scrambled Order Transform

	DftiComputeForward	DftiComputeBackward
DFTI_ORDERING	Input → Output	Input → Output
DFTI_ORDERED	ordered → ordered	ordered → ordered
DFTI_BACKWARD_SCRAMBLED	ordered → scrambled	scrambled → ordered

NOTE The word "scrambled" in this table means "permit scrambled order if possible". In some situations permitting out-of-order data gives no performance advantage and an implementation may choose to ignore the suggestion.

See Also
DftiSetValue

Cluster FFT Functions

This section describes the cluster Fast Fourier Transform (FFT) functions implemented in Intel® MKL.

NOTE These functions are available only for the Linux* and Windows* operating systems.

The cluster FFT function library was designed to perform fast Fourier transforms on a cluster, that is, a group of computers interconnected via a network. Each computer (node) in the cluster has its own memory and processor(s). Data interchanges between the nodes are provided by the network.

One or more processes may be running in parallel on each cluster node. To organize communication between different processes, the cluster FFT function library uses the Message Passing Interface (MPI). To avoid dependence on a specific MPI implementation (for example, MPICH, Intel® MPI, and others), the library works with MPI via a message-passing library for linear algebra called BLACS.

Cluster FFT functions of Intel MKL provide one-dimensional, two-dimensional, and multi-dimensional (up to the order of 7) functions and both Fortran and C interfaces for all transform functions.

To develop applications using the cluster FFT functions, you should have basic skills in MPI programming.

The interfaces for the Intel MKL cluster FFT functions are similar to the corresponding interfaces for the conventional Intel MKL FFT functions, described earlier in this chapter. Refer there for details not explained in this section.

Table "Cluster FFT Functions in Intel MKL" lists cluster FFT functions implemented in Intel MKL:

Cluster FFT Functions in Intel MKL

Function Name	Operation
Descriptor Manipulation Functions	
DftiCreateDescriptorDM	Allocates memory for the descriptor data structure and preliminarily initializes it.
DftiCommitDescriptorDM	Performs all initialization for the actual FFT computation.
DftiFreeDescriptorDM	Frees memory allocated for a descriptor.
FFT Computation Functions	
DftiComputeForwardDM	Computes the forward FFT.
DftiComputeBackwardDM	Computes the backward FFT.
Descriptor Configuration Functions	
DftiSetValueDM	Sets one particular configuration parameter with the specified configuration value.
DftiGetValueDM	Gets the value of one particular configuration parameter.

Computing Cluster FFT

The cluster FFT functions described later in this section are provided with Fortran and C interfaces. Fortran stands for Fortran 95.

Cluster FFT computation is performed by DftiComputeForwardDM and DftiComputeBackwardDM functions, called in a program using MPI, which will be referred to as MPI program. After an MPI program starts, a number of processes are created. MPI identifies each process by its rank. The processes are independent of one another and communicate via MPI. A function called in an MPI program is invoked in all the processes. Each process manipulates data according to its rank. Input or output data for a cluster FFT transform is a sequence of real or complex values. A cluster FFT computation function operates local part of the input data, i.e. some part of the data to be operated in a particular process, as well as generates local part of the output data. While each process performs its part of computations, running in parallel and communicating through MPI, the processes perform the entire FFT computation. FFT computations using the Intel MKL cluster FFT functions are typically effected by a number of steps listed below:

1. Initiate MPI by calling MPI_Init in C/C++ or MPI_INIT in Fortran (the function must be called prior to calling any FFT function and any MPI function).

2. Allocate memory for the descriptor and create it by calling DftiCreateDescriptorDM.
3. Specify one of several values of configuration parameters by one or more calls to DftiSetValueDM.
4. Obtain values of configuration parameters needed to create local data arrays; the values are retrieved by calling DftiGetValueDM.
5. Initialize the descriptor for the FFT computation by calling DftiCommitDescriptorDM.
6. Create arrays for local parts of input and output data and fill the local part of input data with values. (For more information, see Distributing Data among Processes.)
7. Compute the transform by calling DftiComputeForwardDM or DftiComputeBackwardDM.
8. Gather local output data into the global array using MPI functions. (This step is optional because you may need to immediately employ the data differently.)
9. Release memory allocated for the descriptor by calling DftiFreeDescriptorDM.
10. Finalize communication through MPI by calling MPI_Finalize in C/C++ or MPI_FINALIZE in Fortran (the function must be called after the last call to a cluster FFT function and the last call to an MPI function).

Several code examples in the "Examples for Cluster FFT Functions" section in Appendix C illustrate cluster FFT computations.

Distributing Data among Processes

The Intel MKL cluster FFT functions store all input and output multi-dimensional arrays (matrices) in one-dimensional arrays (vectors). The arrays are stored in the row-major order in C/C++ and in the column-major order in Fortran. For example, a two-dimensional matrix A of size (*m*,*n*) is stored in a vector B of size *m***n* so that

- B[i*n+j]=A[i][j] in C/C++ (*i=0, ..., m-1, j=0, ..., n-1*)
- B(j*m+i)=A(i,j) in Fortran (*i=1, ..., m, j=1, ..., n*).

NOTE Order of FFT dimensions is the same as the order of array dimensions in the programming language. For example, a 3-dimensional FFT with Lengths=(*m*,*n*,*1*) can be computed over an array Ar[m][n][l] in C/C++ or AR(m,n,l) in Fortran.

All MPI processes involved in cluster FFT computation operate their own portions of data. These local arrays make up the virtual global array that the fast Fourier transform is applied to. It is your responsibility to properly allocate local arrays (if needed), fill them with initial data and gather resulting data into an actual global array or process the resulting data differently. To be able do this, see sections below on how the virtual global array is composed of the local ones.

Multi-dimensional transforms

If the dimension of transform is greater than one, the cluster FFT function library splits data in the dimension whose index changes most slowly, so that the parts contain all elements with several consecutive values of this index. It is the first dimension in C and the last one in Fortran. If the global array is two-dimensional, in C, it gives each process several consecutive rows. The term "rows" will be used regardless of the array dimension and programming language. Local arrays are placed in memory allocated for the virtual global array consecutively, in the order determined by process ranks. For example, in case of two processes, during the computation of a three-dimensional transform whose matrix has size (11,15,12), the processes may store local arrays of sizes (6,15,12) and (5,15,12), respectively.

If p is the number of MPI processes and the matrix of a transform to be computed has size (*m*,*n*,*1*), in C, each MPI process works with local data array of size (m_q , *n*, *1*), where $\Sigma m_q=m$, $q=0, \ldots, p-1$. Local input arrays must contain appropriate parts of the actual global input array, and then local output arrays will contain appropriate parts of the actual global output array. You can figure out which particular rows of the global array the local array must contain from the following configuration parameters of the cluster FFT interface: CDFT_LOCAL_NX, CDFT_LOCAL_START_X, and CDFT_LOCAL_SIZE. To retrieve values of the parameters, use the DftiGetValueDM function:

- CDFT_LOCAL_NX specifies how many rows of the global array the current process receives.

- CDFT_LOCAL_START_X specifies which row of the global input or output array corresponds to the first row of the local input or output array. If A is a global array and L is the appropriate local array, then
  - L[i][j][k]=A[i+cdft_local_start_x][j][k], where *i=0, ..., m_q-1, j=0, ..., n-1, k=0, ..., l-1* for C/C++
  - L(i,j,k)=A(i,j,k+cdft_local_start_x-1), where *i=1, ..., m, j=1, ..., n, k=1, ..., l_q* for Fortran.

Example "2D Out-of-place Cluster FFT Computation" in Appendix C shows how the data is distributed among processes for a two-dimensional cluster FFT computation.

One-dimensional transforms

In this case, input and output data are distributed among processes differently and even the numbers of elements stored in a particular process before and after the transform may be different. Each local array stores a segment of consecutive elements of the appropriate global array. Such segment is determined by the number of elements and a shift with respect to the first array element. So, to specify segments of the global input and output arrays that a particular process receives, *four* configuration parameters are needed: CDFT_LOCAL_NX, CDFT_LOCAL_START_X, CDFT_LOCAL_OUT_NX, and CDFT_LOCAL_OUT_START_X. Use the DftiGetValueDM function to retrieve their values. The meaning of the four configuration parameters depends upon the type of the transform, as shown in Table "Data Distribution Configuration Parameters for 1D Transforms":

Data Distribution Configuration Parameters for 1D Transforms

Meaning of the Parameter	Forward Transform	Backward Transform
Number of elements in input array	CDFT_LOCAL_NX	CDFT_LOCAL_OUT_NX
Elements shift in input array	CDFT_LOCAL_START_X	CDFT_LOCAL_OUT_START_X
Number of elements in output array	CDFT_LOCAL_OUT_NX	CDFT_LOCAL_NX
Elements shift in output array	CDFT_LOCAL_OUT_START_X	CDFT_LOCAL_START_X

Memory size for local data

The memory size needed for local arrays cannot be just calculated from CDFT_LOCAL_NX (CDFT_LOCAL_OUT_NX), because the cluster FFT functions sometimes require allocating a little bit more memory for local data than just the size of the appropriate sub-array. The configuration parameter CDFT_LOCAL_SIZE specifies the size of the local input and output array in data elements. Each local input and output arrays must have size not less than CDFT_LOCAL_SIZE*size_of_element. Note that in the current implementation of the cluster FFT interface, data elements can be real or complex values, each complex value consisting of the real and imaginary parts. If you employ a user-defined workspace for in-place transforms (for more information, refer to Table "Settable configuration Parameters"), it must have the same size as the local arrays. Example "1D In-place Cluster FFT Computations" in Appendix C illustrates how the cluster FFT functions distribute data among processes in case of a one-dimensional FFT computation effected with a user-defined workspace.

Available Auxiliary Functions

If a global input array is located on one MPI process and you want to obtain its local parts or you want to gather the global output array on one MPI process, you can use functions MKL_CDFT_ScatterData and MKL_CDFT_GatherData to distribute or gather data among processes, respectively. These functions are defined in a file that is delivered with Intel MKL and located in the following subdirectory of the Intel MKL installation directory: examples/cdftc/source/cdft_example_support.c for C/C++ and examples/cdftf/source/cdft_example_support.f90 for Fortran.

Restriction on Lengths of Transforms

The algorithm that the Intel MKL cluster FFT functions use to distribute data among processes imposes a restriction on lengths of transforms with respect to the number of MPI processes used for the FFT computation:

- For a multi-dimensional transform, lengths of the first two dimensions in C/C++ or of the last two dimensions in Fortran must be not less than the number of MPI processes.
- Length of a one-dimensional transform must be the product of two integers each of which is not less than the number of MPI processes.

Non-compliance with the restriction causes an error `CDFT_SPREAD_ERROR` (refer to Error Codes for details). To achieve the compliance, you can change the transform lengths and/or the number of MPI processes, which is specified at start of an MPI program. MPI-2 enables changing the number of processes during execution of an MPI program.

Cluster FFT Interface

To use the cluster FFT functions, you need to access the module `MKL_CDFT` through the "`use`" statement in Fortran; or access the header file `mkl_cdft.h` through "`include`" in C/C++.

The Fortran interface provides a derived type `DFTI_DESCRIPTOR_DM`; a number of named constants representing various names of configuration parameters and their possible values; and a number of overloaded functions through the generic functionality of Fortran 95.

The C interface provides a structure type `DFTI_DESCRIPTOR_DM_HANDLE` and a number of functions, some of which accept a different number of input arguments.

To provide communication between parallel processes through MPI, the following include statement must be present in your code:

- Fortran:

  ```
  INCLUDE "mpif.h"
  ```

  (for some MPI versions, "`mpif90.h`" header may be used instead).
- C/C++:

  ```
  #include "mpi.h"
  ```

There are three main categories of the cluster FFT functions in Intel MKL:

1. **Descriptor Manipulation**. There are three functions in this category. The `DftiCreateDescriptorDM` function creates an FFT descriptor whose storage is allocated dynamically. The `DftiCommitDescriptorDM` function "commits" the descriptor to all its settings. The `DftiFreeDescriptorDM` function frees up the memory allocated for the descriptor.
2. **FFT Computation**. There are two functions in this category. The `DftiComputeForwardDM` function performs the forward FFT computation, and the `DftiComputeBackwardDM` function performs the backward FFT computation.
3. **Descriptor Configuration**. There are two functions in this category. The `DftiSetValueDM` function sets one specific configuration value to one of the many configuration parameters. The `DftiGetValueDM` function gets the current value of any of these configuration parameters, all of which are readable. These parameters, though many, are handled one at a time.

Descriptor Manipulation Functions

There are three functions in this category: create a descriptor, commit a descriptor, and free a descriptor.

DftiCreateDescriptorDM

Allocates memory for the descriptor data structure and preliminarily initializes it.

Syntax

Fortran:

```
Status = DftiCreateDescriptorDM(comm, handle, v1, v2, dim, size)
Status = DftiCreateDescriptorDM(comm, handle, v1, v2, dim, sizes)
```

C:

```
status = DftiCreateDescriptorDM(comm, &handle, v1, v2, dim, size );
status = DftiCreateDescriptorDM(comm, &handle, v1, v2, dim, sizes );
```

Include Files

- FORTRAN 90: `mkl_cdft.f90`
- C: `mkl_cdft.h`

Input Parameters

comm	MPI communicator, e.g. `MPI_COMM_WORLD`.
v1	Precision of the transform.
v2	Type of the forward domain. Must be `DFTI_COMPLEX` for complex-to-complex transforms or `DFTI_REAL` for real-to-complex transforms.
dim	Dimension of the transform.
size	Length of the transform in a one-dimensional case.
sizes	Lengths of the transform in a multi-dimensional case.

Output Parameters

handle	Pointer to the descriptor handle of transform. If the function completes successfully, the pointer to the created handle is stored in the variable.

Description

This function allocates memory in a particular MPI process for the descriptor data structure and instantiates it with default configuration settings with respect to the precision, domain, dimension, and length of the desired transform. The domain is understood to be the domain of the forward transform. The result is a pointer to the created descriptor. This function is slightly different from the "initialization" function `DftiCommitDescriptorDM` in a more traditional software packages or libraries used for computing the FFT. This function does not perform any significant computation work, such as twiddle factors computation, because the default configuration settings can still be changed using the function `DftiSetValueDM`.

The value of the parameter *v1* is specified through named constants `DFTI_SINGLE` and `DFTI_DOUBLE`. It corresponds to precision of input data, output data, and computation. A setting of `DFTI_SINGLE` indicates single-precision floating-point data type and a setting of `DFTI_DOUBLE` indicates double-precision floating-point data type.

The parameter *dim* is a simple positive integer indicating the dimension of the transform.

In C/C++, for one-dimensional transforms, length is a single integer value of the parameter *size* having type `MKL_LONG`; for multi-dimensional transforms, length is supplied with the parameter *sizes*, which is an array of integers having type `MKL_LONG`. In Fortran, length is an integer or an array of integers.

Return Values

The function returns DFTI_NO_ERROR when completes successfully. In this case, the pointer to the created descriptor handle is stored in *handle*. If the function fails, it returns a value of another error class constant

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiCreateDescriptorDM
    INTEGER(4) FUNCTION DftiCreateDescriptorDMn(C,H,P1,P2,D,L)
        TYPE(DFTI_DESCRIPTOR_DM), POINTER :: H
        INTEGER(4) C,P1,P2,D,L(*)
    END FUNCTION
    INTEGER(4) FUNCTION DftiCreateDescriptorDM1(C,H,P1,P2,D,L)
        TYPE(DFTI_DESCRIPTOR_DM), POINTER :: H
        INTEGER(4) C,P1,P2,D,L
    END FUNCTION
END INTERFACE


/* C/C++ prototype */
MKL_LONG DftiCreateDescriptorDM(MPI_Comm,DFTI_DESCRIPTOR_DM_HANDLE*,
   enum DFTI_CONFIG_VALUE,enum DFTI_CONFIG_VALUE,MKL_LONG,...);
```

DftiCommitDescriptorDM

Performs all initialization for the actual FFT computation.

Syntax

Fortran:

```
Status = DftiCommitDescriptorDM(handle)
```

C:

```
status = DftiCommitDescriptorDM(handle);
```

Include Files

- FORTRAN 90: mkl_cdft.f90
- C: mkl_cdft.h

Input Parameters

handle	The descriptor handle. Must be valid, that is, created in a call to DftiCreateDescriptorDM.

Description

The cluster FFT interface requires a function that completes initialization of a previously created descriptor before the descriptor can be used for FFT computations in a particular MPI process. The `DftiCommitDescriptorDM` function performs all initialization that facilitates the actual FFT computation. For the current implementation, it may involve exploring many different factorizations of the input length to search for highly efficient computation method.

Any changes of configuration parameters of a committed descriptor via the set value function (see Descriptor Configuration) requires a re-committal of the descriptor before a computation function can be invoked. Typically, this committal function is called right before a computation function call (see FFT Computation).

Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the Error Codes section).

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiCommitDescriptorDM
   INTEGER(4) FUNCTION DftiCommitDescriptorDM(handle);
     TYPE(DFTI_DESCRIPTOR_DM), POINTER :: handle
 END FUNCTION
END INTERFACE


/* C/C++ prototype */
MKL_LONG DftiCommitDescriptorDM(DFTI_DESCRIPTOR_DM_HANDLE handle);
```

DftiFreeDescriptorDM

Frees memory allocated for a descriptor.

Syntax

Fortran:

```
Status = DftiFreeDescriptorDM(handle)
```

C:

```
status = DftiFreeDescriptorDM(&handle);
```

Include Files

- FORTRAN 90: `mkl_cdft.f90`
- C: `mkl_cdft.h`

Input Parameters

handle	The descriptor handle. Must be valid, that is, created in a call to `DftiCreateDescriptorDM`.

Output Parameters

handle	The descriptor handle. Memory allocated for the handle is released on output.

Description

This function frees up all memory allocated for a descriptor in a particular MPI process. Call the DftiFreeDescriptorDM function to delete the descriptor handle. Upon successful completion of DftiFreeDescriptorDM the descriptor handle is no longer valid.

Return Values

The function returns DFTI_NO_ERROR when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the Error Codes section).

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiFreeDescriptorDM
 INTEGER(4) FUNCTION DftiFreeDescriptorDM(handle)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: handle
   END FUNCTION
END INTERFACE


/* C/C++ prototype */
MKL_LONG DftiFreeDescriptorDM(DFTI_DESCRIPTOR_DM_HANDLE *handle);
```

FFT Computation Functions

There are two functions in this category: compute the forward transform and compute the backward transform.

DftiComputeForwardDM

Computes the forward FFT.

Syntax

Fortran:

```
Status = DftiComputeForwardDM(handle, in_X, out_X)
Status = DftiComputeForwardDM(handle, in_out_X)
```

C:

```
status = DftiComputeForwardDM(handle, in_X, out_X);
status = DftiComputeForwardDM(handle, in_out_X);
```

Include Files

- FORTRAN 90: mkl_cdft.f90

- C: `mkl_cdft.h`

Input Parameters

handle	The descriptor handle.
in_X, in_out_X	Local part of input data. Array of complex values. Refer to the Distributing Data among Processes section on how to allocate and initialize the array.

Output Parameters

out_X, in_out_X	Local part of output data. Array of complex values. Refer to the Distributing Data among Processes section on how to allocate the array.

Description

The `DftiComputeForwardDM` function computes the forward FFT. Forward FFT is the transform using the factor $e^{-i2\pi/n}$.

Before you call the function, the valid descriptor, created by `DftiCreateDescriptorDM`, must be configured and committed using the `DftiCommitDescriptorDM` function.

The computation is carried out by calling the `DftiComputeForward` function. So, the functions have very much in common, and details not explicitly mentioned below can be found in the description of `DftiComputeForward`.

Local part of input data, as well as local part of the output data, is an appropriate sequence of complex values (each complex value consists of two real numbers: real part and imaginary part) that a particular process stores. See the Distributing Data Among Processes section for details.

Refer to the Configuration Settings section for the list of configuration parameters that the descriptor passes to the function.

The configuration parameter `DFTI_PRECISION` determines the precision of input data, output data, and transform: a setting of `DFTI_SINGLE` indicates single-precision floating-point data type and a setting of `DFTI_DOUBLE` indicates double-precision floating-point data type.

The configuration parameter `DFTI_PLACEMENT` informs the function whether the computation should be in-place. If the value of this parameter is `DFTI_INPLACE` (default), you must call the function with two parameters, otherwise you must supply three parameters. If `DFTI_PLACEMENT = DFTI_INPLACE` and three parameters are supplied, then the third parameter is ignored.

CAUTION Even in case of an out-of-place transform, local array of input data *in_X* may be changed. To save data, make its copy before calling `DftiComputeForwardDM`.

In case of an in-place transform, `DftiComputeForwardDM` dynamically allocates and deallocates a work buffer of the same size as the local input/output array requires.

NOTE You can specify your own workspace of the same size through the configuration parameter `CDFT_WORKSPACE` to avoid redundant memory allocation.

Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the Error Codes section).

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiComputeForwardDM
   INTEGER(4) FUNCTION DftiComputeForwardDM(h, in_X, out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
   COMPLEX(8), DIMENSION(*) :: in_x, out_X
   END FUNCTION DftiComputeForwardDM
   INTEGER(4) FUNCTION DftiComputeForwardDMi(h, in_out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(8), DIMENSION(*) :: in_out_X
   END FUNCTION DftiComputeForwardDMi
   INTEGER(4) FUNCTION DftiComputeForwardDMs(h, in_X, out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(4), DIMENSION(*) :: in_x, out_X
   END FUNCTION DftiComputeForwardDMs
   INTEGER(4) FUNCTION DftiComputeForwardDMis(h, in_out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(4), DIMENSION(*) :: in_out_X
   END FUNCTION DftiComputeForwardDMis
END INTERFACE


/*   C/C++ prototype */
MKL_LONG DftiComputeForwardDM(DFTI_DESCRIPTOR_DM_HANDLE handle, void *in_X,...);
```

DftiComputeBackwardDM

Computes the backward FFT.

Syntax

Fortran:

```
Status = DftiComputeBackwardDM(handle, in_X, out_X)
Status = DftiComputeBackwardDM(handle, in_out_X)
```

C:

```
status = DftiComputeBackwardDM(handle, in_X, out_X);
status = DftiComputeBackwardDM(handle, in_out_X);
```

Include Files

- FORTRAN 90: `mkl_cdft.f90`
- C: `mkl_cdft.h`

Input Parameters

handle	The descriptor handle.
in_X, in_out_X	Local part of input data. Array of complex values. Refer to the Distributing Data among Processes section on how to allocate and initialize the array.

Output Parameters

out_X, in_out_X	Local part of output data. Array of complex values. Refer to the Distributing Data among Processes section on how to allocate the array.

Description

The DftiComputeBackwardDM function computes the backward FFT. Backward FFT is the transform using the factor $e^{i2\pi/n}$.

Before you call the function, the valid descriptor, created by DftiCreateDescriptorDM, must be configured and committed using the DftiCommitDescriptorDM function.

The computation is carried out by calling the DftiComputeBackward function. So, the functions have very much in common, and details not explicitly mentioned below can be found in the description of DftiComputeBackward.

Local part of input data, as well as local part of the output data, is an appropriate sequence of complex values (each complex value consists of two real numbers: real part and imaginary part) that a particular process stores. See the Distributing Data among Processes section for details.

Refer to the Configuration Settings section for the list of configuration parameters that the descriptor passes to the function.

The configuration parameter DFTI_PRECISION determines the precision of input data, output data, and transform: a setting of DFTI_SINGLE indicates single-precision floating-point data type and a setting of DFTI_DOUBLE indicates double-precision floating-point data type.

The configuration parameter DFTI_PLACEMENT informs the function whether the computation should be in-place. If the value of this parameter is DFTI_INPLACE (default), you must call the function with two parameters, otherwise you must supply three parameters. If DFTI_PLACEMENT = DFTI_INPLACE and three parameters are supplied, then the third parameter is ignored.

CAUTION Even in case of an out-of-place transform, local array of input data *in_X* may be changed. To save data, make its copy before calling DftiComputeBackwardDM.

In case of an in-place transform, DftiComputeBackwardDM dynamically allocates and deallocates a work buffer of the same size as the local input/output array requires.

NOTE You can specify your own workspace of the same size through the configuration parameter CDFT_WORKSPACE to avoid redundant memory allocation.

Return Values

The function returns DFTI_NO_ERROR when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the Error Codes section).

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiComputeBackwardDM
   INTEGER(4) FUNCTION DftiComputeBackwardDM(h, in_X, out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(8), DIMENSION(*) :: in_x, out_X
   END FUNCTION DftiComputeBackwardDM
   INTEGER(4) FUNCTION DftiComputeBackwardDMi(h, in_out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(8), DIMENSION(*) :: in_out_X
   END FUNCTION DftiComputeBackwardDMi
   INTEGER(4) FUNCTION DftiComputeBackwardDMs(h, in_X, out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(4), DIMENSION(*) :: in_x, out_X
   END FUNCTION DftiComputeBackwardDMs
   INTEGER(4) FUNCTION DftiComputeBackwardDMis(h, in_out_X)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      COMPLEX(4), DIMENSION(*) :: in_out_X
   END FUNCTION DftiComputeBackwardDMis
END INTERFACE


/* C/C++ prototype */
MKL_LONG DftiComputeBackwardDM(DFTI_DESCRIPTOR_DM_HANDLE handle, void *in_X,...);
```

Descriptor Configuration Functions

There are two functions in this category: the value setting function `DftiSetValueDM` sets one particular configuration parameter to an appropriate value, the value getting function `DftiGetValueDM` reads the value of one particular configuration parameter.

Some configuration parameters used by cluster FFT functions originate from the conventional FFT interface (see Configuration Settings subsection in the "FFT Functions" section for details).

Other configuration parameters are specific to the cluster FFT. Integer values of these parameters have type `MKL_LONG` in C/C++ and `INTEGER(4)` in Fortran. The exact type of the configuration parameters being floating-point scalars is `float` or `double` in C/C++ and `REAL(4)` or `REAL(8)` in Fortran. The configuration parameters whose values are named constants have the `enum` type in C/C++ and `INTEGER` in Fortran. They are defined in the `mkl_cdft.h` header file in C/C++ and `MKL_CDFT` module in Fortran.

Names of the configuration parameters specific to the cluster FFT interface have prefix `CDFT`.

DftiSetValueDM

Sets one particular configuration parameter with the specified configuration value.

Syntax

Fortran:

```
Status = DftiSetValueDM (handle, param, value)
```

C:

```
status = DftiSetValueDM (handle, param, value);
```

Include Files

- FORTRAN 90: `mkl_cdft.f90`
- C: `mkl_cdft.h`

Input Parameters

handle	The descriptor handle. Must be valid, that is, created in a call to `DftiCreateDescriptorDM`.
param	Name of a parameter to be set up in the descriptor handle. See Table "Settable Configuration Parameters" for the list of available parameters.
value	Value of the parameter.

Description

This function sets one particular configuration parameter with the specified configuration value. The configuration parameter is one of the named constants listed in the table below, and the configuration value must have the corresponding type. See Configuration Settings for details of the meaning of each setting and for possible values of the parameters whose values are named constants.

Settable Configuration Parameters

Parameter Name	Data Type	Description	Default Value
`DFTI_FORWARD_SCALE`	Floating-point scalar	Scale factor of forward transform.	1.0
`DFTI_BACKWARD_SCALE`	Floating-point scalar	Scale factor of backward transform.	1.0
`DFTI_PLACEMENT`	Named constant	Placement of the computation result.	`DFTI_INPLACE`
`DFTI_ORDERING`	Named constant	Scrambling of data order.	`DFTI_ORDERED`
`DFTI_WORKSPACE`	Array of an appropriate type	Auxiliary buffer, a user-defined workspace. Enables saving memory during in-place computations.	`NULL` (allocate workspace dynamically).
`DFTI_PACKED_FORMAT`	Named constant	Packed format, real data.	• `DFTI_PERM_FORMAT` — default and the only available value for one-dimensional transforms

Parameter Name	Data Type	Description	Default Value
			• DFTI_CCE_FORMAT — default and the only available value for multi-dimensional transforms
DFTI_TRANSPOSE	Named constant	This parameter determines how the output data is located for multi-dimensional transforms. If the parameter value is DFTI_NONE, the data is located in a usual manner described in this manual. If the value is DFTI_ALLOW, the last (first) global transposition is not performed for a forward (backward) transform.	DFTI_NONE

Return Values

The function returns DFTI_NO_ERROR when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the Error Codes section).

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiSetValueDM
   INTEGER(4) FUNCTION DftiSetValueDM(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p, v
   END FUNCTION
   INTEGER(4) FUNCTION DftiSetValueDMd(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p
      REAL(8) :: v
   END FUNCTION
   INTEGER(4) FUNCTION DftiSetValueDMs(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p
      REAL(4) :: v
   END FUNCTION
   INTEGER(4) FUNCTION DftiSetValueDMsw(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p
      COMPLEX(4) :: v(*)
   END FUNCTION
   INTEGER(4) FUNCTION DftiSetValueDMdw(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p
      COMPLEX(8) :: v(*)
   END FUNCTION
END INTERFACE


/* C/C++ prototype */
MKL_LONG DftiSetValueDM(DFTI_DESCRIPTOR_DM_HANDLE handle, int param,...);
```

DftiGetValueDM

Gets the value of one particular configuration parameter.

Syntax

Fortran:

```
Status = DftiGetValueDM(handle, param, value)
```

C:

```
status = DftiGetValueDM(handle, param, &value);
```

Include Files

- FORTRAN 90: `mkl_cdft.f90`
- C: `mkl_cdft.h`

Input Parameters

handle	The descriptor handle. Must be valid, that is, created in a call to DftiCreateDescriptorDM.
param	Name of a parameter to be retrieved from the descriptor. See Table "Retrievable Configuration Parameters" for the list of available parameters.

Output Parameters

value	Value of the parameter.

Description

This function gets the configuration value of one particular configuration parameter. The configuration parameter is one of the named constants listed in the table below, and the configuration value is the corresponding appropriate type, which can be a named constant or a native type. Possible values of the named constants can be found in Table "Configuration Parameters" and relevant subsections of the Configuration Settings section.

Retrievable Configuration Parameters

Parameter Name	Data Type	Description
DFTI_PRECISION	Named constant	Precision of computation, input data and output data.
DFTI_DIMENSION	Integer scalar	Dimension of the transform
DFTI_LENGTHS	Array of integer values	Array of lengths of the transform. Number of lengths corresponds to the dimension of the transform.
DFTI_FORWARD_SCALE	Floating-point scalar	Scale factor of forward transform.
DFTI_BACKWARD_SCALE	Floating-point scalar	Scale factor of backward transform.
DFTI_PLACEMENT	Named constant	Placement of the computation result.
DFTI_COMMIT_STATUS	Named constant	Shows whether descriptor has been committed.
DFTI_FORWARD_DOMAIN	Named constant	Forward domain of transforms, has the value of DFTI_COMPLEX or DFTI_REAL.
DFTI_ORDERING	Named constant	Scrambling of data order.

Parameter Name	Data Type	Description
CDFT_MPI_COMM	Type of MPI communicator	MPI communicator used for transforms.
CDFT_LOCAL_SIZE	Integer scalar	Necessary size of input, output, and buffer arrays in data elements.
CDFT_LOCAL_X_START	Integer scalar	Row/element number of the global array that corresponds to the first row/element of the local array. For more information, see Distributing Data among Processes.
CDFT_LOCAL_NX	Integer scalar	The number of rows/elements of the global array stored in the local array. For more information, see Distributing Data among Processes.
CDFT_LOCAL_OUT_X_START	Integer scalar	Element number of the appropriate global array that corresponds to the first element of the input or output local array in a 1D case. For details, see Distributing Data among Processes.
CDFT_LOCAL_OUT_NX	Integer scalar	The number of elements of the appropriate global array that are stored in the input or output local array in a 1D case. For details, see Distributing Data among Processes.

Return Values

The function returns DFTI_NO_ERROR when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the Error Codes section).

Interface and Prototype

```
! Fortran Interface
INTERFACE DftiGetValueDM
   INTEGER(4) FUNCTION DftiGetValueDM(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p, v
   END FUNCTION
   INTEGER(4) FUNCTION DftiGetValueDMar(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p, v(*)
   END FUNCTION
   INTEGER(4) FUNCTION DftiGetValueDMd(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p
      REAL(8) :: v
   END FUNCTION
   INTEGER(4) FUNCTION DftiGetValueDMs(h, p, v)
      TYPE(DFTI_DESCRIPTOR_DM), POINTER :: h
      INTEGER(4) :: p
      REAL(4) :: v
   END FUNCTION
END INTERFACE


/* C/C++ prototype */
MKL_LONG DftiGetValueDM(DFTI_DESCRIPTOR_DM_HANDLE handle, int param,...);
```

Error Codes

All the cluster FFT functions return an integer value denoting the status of the operation. These values are identified by named constants. Each function returns `DFTI_NO_ERROR` if no errors were encountered during execution. Otherwise, a function generates an error code. In addition to FFT error codes, the cluster FFT interface has its own ones. Named constants specific to the cluster FFT interface have prefix "`CDFT`" in names. Table "Error Codes that Cluster FFT Functions Return" lists error codes that the cluster FFT functions may return.

Error Codes that Cluster FFT Functions Return

Named Constants	Comments
`DFTI_NO_ERROR`	No error.
`DFTI_MEMORY_ERROR`	Usually associated with memory allocation.

Named Constants	Comments
DFTI_INVALID_CONFIGURATION	Invalid settings of one or more configuration parameters.
DFTI_INCONSISTENT_CONFIGURATION	Inconsistent configuration or input parameters.
DFTI_NUMBER_OF_THREADS_ERROR	Number of OMP threads in the computation function is not equal to the number of OMP threads in the initialization stage (commit function).
DFTI_MULTITHREADED_ERROR	Usually associated with a value that OMP routines return in case of errors.
DFTI_BAD_DESCRIPTOR	Descriptor is unusable for computation.
DFTI_UNIMPLEMENTED	Unimplemented legitimate settings; implementation dependent.
DFTI_MKL_INTERNAL_ERROR	Internal library error.
DFTI_1D_LENGTH_EXCEEDS_INT32	Length of one of dimensions exceeds 2^{32} -1 (4 bytes).
CDFT_SPREAD_ERROR	Data cannot be distributed (For more information, see Distributing Data among Processes.)
CDFT_MPI_ERROR	MPI error. Occurs when calling MPI.

PBLAS Routines

12

This chapter describes the Intel® Math Kernel Library implementation of the PBLAS (Parallel Basic Algebra Subprograms) routines from the ScaLAPACK package for distributed-memory architecture. PBLAS is intended for using in vector-vector, matrix-vector, and matrix-matrix operations to simplify the parallelization of linear codes. The design of PBLAS is as consistent as possible with that of the BLAS. The routine descriptions are arranged in several sections according to the PBLAS level of operation:

- PBLAS Level 1 Routines (distributed vector-vector operations)
- PBLAS Level 2 Routines (distributed matrix-vector operations)
- PBLAS Level 3 Routines (distributed matrix-matrix operations)

Each section presents the routine and function group descriptions in alphabetical order by the routine group name; for example, the `p?asum` group, the `p?axpy` group. The question mark in the group name corresponds to a character indicating the data type (`s`, `d`, `c`, and `z` or their combination); see *Routine Naming Conventions*.

NOTE PBLAS routines are provided only with Intel® MKL versions for Linux* and Windows* OSs.

Generally, PBLAS runs on a network of computers using MPI as a message-passing layer and a set of prebuilt communication subprograms (BLACS), as well as a set of PBLAS optimized for the target architecture. The Intel MKL version of PBLAS is optimized for Intel® processors. For the detailed system and environment requirements see *Intel® MKL Release Notes* and *Intel® MKL User's Guide*.

For full reference on PBLAS routines and related information, see http://www.netlib.org/scalapack/html/pblas_qref.html.

Overview

The model of the computing environment for PBLAS is represented as a one-dimensional array of processes or also a two-dimensional process grid. To use PBLAS, all global matrices or vectors must be distributed on this array or grid prior to calling the PBLAS routines.

PBLAS uses the two-dimensional block-cyclic data distribution as a layout for dense matrix computations. This distribution provides good work balance between available processors, as well as gives the opportunity to use PBLAS Level 3 routines for optimal local computations. Information about the data distribution that is required to establish the mapping between each global array and its corresponding process and memory location is contained in the so called *array descriptor* associated with each global array. Table "Content of the array descriptor for dense matrices" gives an example of an array descriptor structure.

Content of Array Descriptor for Dense Matrices

Array Element #	Name	Definition
1	*dtype*	Descriptor type (=1 for dense matrices)
2	*ctxt*	BLACS context handle for the process grid

Array Element #	Name	Definition
3	*m*	Number of rows in the global array
4	*n*	Number of columns in the global array
5	*mb*	Row blocking factor
6	*nb*	Column blocking factor
7	*rsrc*	Process row over which the first row of the global array is distributed
8	*csrc*	Process column over which the first column of the global array is distributed
9	*lld*	Leading dimension of the local array

The number of rows and columns of a global dense matrix that a particular process in a grid receives after data distributing is denoted by *LOCr*() and *LOCc*(), respectively. To compute these numbers, you can use the ScaLAPACK tool routine numroc.

After the block-cyclic distribution of global data is done, you may choose to perform an operation on a submatrix of the global matrix *A*, which is contained in the global subarray sub(*A*), defined by the following 6 values (for dense matrices):

m	The number of rows of sub(*A*)
n	The number of columns of sub(*A*)
a	A pointer to the local array containing the entire global array *A*
ia	The row index of sub(*A*) in the global array
ja	The column index of sub(*A*) in the global array
desca	The array descriptor for the global array *A*

Intel MKL provides the PBLAS routines with interface similar to the interface used in the Netlib PBLAS (see http://www.netlib.org/scalapack/html/pblas_qref.html).

Routine Naming Conventions

The naming convention for PBLAS routines is similar to that used for BLAS routines (see Routine Naming Conventions in Chapter 2). A general rule is that each routine name in PBLAS, which has a BLAS equivalent, is simply the BLAS name prefixed by initial letter p that stands for "parallel".

The Intel MKL PBLAS routine names have the following structure:

```
p <character> <name> <mod> ( )
```

The <*character*> field indicates the Fortran data type:

s	real, single precision
c	complex, single precision
d	real, double precision
z	complex, double precision
i	integer

Some routines and functions can have combined character codes, such as sc or dz.

For example, the function pscasum uses a complex input array and returns a real value.

The <*name*> field, in PBLAS level 1, indicates the operation type. For example, the PBLAS level 1 routines p?dot, p?swap, p?copy compute a vector dot product, vector swap, and a copy vector, respectively.

In PBLAS level 2 and 3, <*name*> reflects the matrix argument type:

ge	general matrix
sy	symmetric matrix
he	Hermitian matrix
tr	triangular matrix

In PBLAS level 3, the <*name*>=tran indicates the transposition of the matrix.

The <mod> field, if present, provides additional details of the operation. The PBLAS level 1 names can have the following characters in the <mod> field:

c	conjugated vector
u	unconjugated vector

The PBLAS level 2 names can have the following additional characters in the <mod> field:

mv	matrix-vector product
sv	solving a system of linear equations with matrix-vector operations
r	rank-1 update of a matrix
r2	rank-2 update of a matrix.

The PBLAS level 3 names can have the following additional characters in the <mod> field:

mm	matrix-matrix product
sm	solving a system of linear equations with matrix-matrix operations
rk	rank-*k* update of a matrix
r2k	rank-2*k* update of a matrix.

The examples below show how to interpret PBLAS routine names:

pddot	<p> <d> <dot>: double-precision real distributed vector-vector dot product
pcdotc	<p> <c> <dot> <c>: complex distributed vector-vector dot product, conjugated
pscasum	<p> <sc> <asum>: sum of magnitudes of distributed vector elements, single precision real output and single precision complex input
pcdotu	<p> <c> <dot> <u>: distributed vector-vector dot product, unconjugated, complex
psgemv	<p> <s> <ge> <mv>: distributed matrix-vector product, general matrix, single precision
pztrmm	<p> <z> <tr> <mm>: distributed matrix-matrix product, triangular matrix, double-precision complex.

PBLAS Level 1 Routines

PBLAS Level 1 includes routines and functions that perform distributed vector-vector operations. Table "PBLAS Level 1 Routine Groups and Their Data Types" lists the PBLAS Level 1 routine groups and the data types associated with them.

PBLAS Level 1 Routine Groups and Their Data Types

Routine or Function Group	Data Types	Description
p?amax	s, d, c, z	Calculates an index of the distributed vector element with maximum absolute value
p?asum	s, d, sc, dz	Calculates sum of magnitudes of a distributed vector
p?axpy	s, d, c, z	Calculates distributed vector-scalar product
p?copy	s, d, c, z	Copies a distributed vector
p?dot	s, d	Calculates a dot product of two distributed real vectors
p?dotc	c, z	Calculates a dot product of two distributed complex vectors, one of them is conjugated

Routine or Function Group	Data Types	Description
p?dotu	c, z	Calculates a dot product of two distributed complex vectors
p?nrm2	s, d, sc, dz	Calculates the 2-norm (Euclidean norm) of a distributed vector
p?scal	s, d, c, z, cs, zd	Calculates a product of a distributed vector by a scalar
p?swap	s, d, c, z	Swaps two distributed vectors

p?amax

Computes the global index of the element of a distributed vector with maximum absolute value.

Syntax

```
call psamax(n, amax, indx, x, ix, jx, descx, incx)
call pdamax(n, amax, indx, x, ix, jx, descx, incx)
call pcamax(n, amax, indx, x, ix, jx, descx, incx)
call pzamax(n, amax, indx, x, ix, jx, descx, incx)
```

Include Files

- C: `mkl_pblas.h`

Description

The functions `p?amax` compute global index of the maximum element in absolute value of a distributed vector `sub(x)`,

where `sub(x)` denotes `X(ix, jx:jx+n-1)` if `incx=m_x`, and `X(ix: ix+n-1, jx)` if `incx= 1`.

Input Parameters

n	(global) `INTEGER`. The length of distributed vector `sub(x)`, *n*≥0.
x	(local) `REAL` for `psamax` `DOUBLE PRECISION` for `pdamax` `COMPLEX` for `pcamax` `DOUBLE COMPLEX` for `pzamax` Array, `DIMENSION (jx-1)*m_x + ix+(n-1)*abs(incx))`. This array contains the entries of the distributed vector `sub(x)`.
ix, *jx*	(global) `INTEGER`. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix `sub(X)`, respectively.
descx	(global and local) `INTEGER` array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) `INTEGER`. Specifies the increment for the elements of `sub(x)`. Only two values are supported, namely 1 and `m_x`. *incx* must not be zero.

Output Parameters

amax	(global) `REAL` for `psamax`.

DOUBLE PRECISION for pdamax.
COMPLEX for pcamax.
DOUBLE COMPLEX for pzamax.
Maximum absolute value (magnitude) of elements of the distributed vector only in its scope.

indx (global) INTEGER. The global index of the maximum element in absolute value of the distributed vector sub(*x*) only in its scope.

p?asum

Computes the sum of magnitudes of elements of a distributed vector.

Syntax

```
call psasum(n, asum, x, ix, jx, descx, incx)
call pscasum(n, asum, x, ix, jx, descx, incx)
call pdasum(n, asum, x, ix, jx, descx, incx)
call pdzasum(n, asum, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_pblas.h

Description

The functions p?asum compute the sum of the magnitudes of elements of a distributed vector sub(*x*), where sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1.

Input Parameters

n (global) INTEGER. The length of distributed vector sub(*x*), *n*≥0.

x (local) REAL for psasum
DOUBLE PRECISION for pdasum
COMPLEX for pscasum
DOUBLE COMPLEX for pdzasum
Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)).
This array contains the entries of the distributed vector sub(*x*).

ix, *jx* (global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.

descx (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx (global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

asum (local) REAL for psasum and pscasum.
DOUBLE PRECISION for pdasum and pdzasum
Contains the sum of magnitudes of elements of the distributed vector only in its scope.

p?axpy

Computes a distributed vector-scalar product and adds the result to a distributed vector.

Syntax

```
call psaxpy(n, a, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pdaxpy(n, a, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pcaxpy(n, a, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pzaxpy(n, a, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
```

Include Files

- C: `mkl_pblas.h`

Description

The `p?axpy` routines perform the following operation with distributed vectors:

```
sub(y) := sub(y) + a*sub(x)
```

where:

a is a scalar;

`sub(x)` and `sub(y)` are *n*-element distributed vectors.

`sub(x)` denotes `X(ix, jx:jx+n-1)` if *incx*=*m_x*, and `X(ix: ix+n-1, jx)` if *incx*= 1;

`sub(y)` denotes `Y(iy, jy:jy+n-1)` if *incy*=*m_y*, and `Y(iy: iy+n-1, jy)` if *incy*= 1.

Input Parameters

n	(global) `INTEGER`. The length of distributed vectors, *n*≥0.
a	(local) `REAL` for `psaxpy` `DOUBLE PRECISION` for `pdaxpy` `COMPLEX` for `pcaxpy` `DOUBLE COMPLEX` for `pzaxpy` Specifies the scalar *a*.
x	(local) `REAL` for `psaxpy` `DOUBLE PRECISION` for `pdaxpy` `COMPLEX` for `pcaxpy` `DOUBLE COMPLEX` for `pzaxpy` Array, `DIMENSION (jx-1)*m_x + ix+(n-1)*abs(incx))`. This array contains the entries of the distributed vector `sub(x)`.
ix, *jx*	(global) `INTEGER`. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix `sub(X)`, respectively.
descx	(global and local) `INTEGER` array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) `INTEGER`. Specifies the increment for the elements of `sub(x)`. Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) `REAL` for `psaxpy` `DOUBLE PRECISION` for `pdaxpy` `COMPLEX` for `pcaxpy`

	DOUBLE COMPLEX for pzaxpy Array, DIMENSION (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global)INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten by sub(*y*) := sub(*y*) + *a**sub(*x*).

p?copy

Copies one distributed vector to another vector.

Syntax

```
call pscopy(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pdcopy(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pccopy(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pzcopy(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call picopy(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The p?copy routines perform a copy operation with distributed vectors defined as

```
sub(y) = sub(x),
```

where sub(*x*) and sub(*y*) are *n*-element distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1;

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy*=*m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vectors, *n*≥0.
x	(local) REAL for pscopy DOUBLE PRECISION for pdcopy COMPLEX for pccopy DOUBLE COMPLEX for pzcopy INTEGER for picopy Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).

ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) REAL for pscopy DOUBLE PRECISION for pdcopy COMPLEX for pccopy DOUBLE COMPLEX for pzcopy INTEGER for picopy Array, DIMENSION (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global)INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten with the distributed vector sub(*x*).

p?dot

Computes the dot product of two distributed real vectors.

Syntax

```
call psdot(n, dot, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pddot(n, dot, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The ?dot functions compute the dot product *dot* of two distributed real vectors defined as

```
dot  = sub(x)'*sub(y)
```

where sub(*x*) and sub(*y*) are *n*-element distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1;

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy*=*m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vectors, *n*≥0.
x	(local) REAL for psdot

DOUBLE PRECISION for pddot
Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)).
This array contains the entries of the distributed vector sub(*x*).

ix, *jx* (global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.

descx (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx (global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

y (local) REAL for psdot
DOUBLE PRECISION for pddot
Array, DIMENSION (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)).
This array contains the entries of the distributed vector sub(*y*).

iy, *jy* (global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.

descy (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.

incy (global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

dot (local) REAL for psdot
DOUBLE PRECISION for pddot
Dot product of sub(*x*) and sub(*y*) only in their scope.

p?dotc

Computes the dot product of two distributed complex vectors, one of them is conjugated.

Syntax

```
call pcdotc(n, dotu, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pzdotc(n, dotu, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The p?dotu functions compute the dot product *dotc* of two distributed vectors one of them is conjugated:

```
dotc  = conjg(sub(x)')*sub(y)
```

where sub(*x*) and sub(*y*) are *n*-element distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1;

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy*=*m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vectors, *n*≥0.
x	(local) COMPLEX for pcdotc DOUBLE COMPLEX for pzdotc Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) COMPLEX for pcdotc DOUBLE COMPLEX for pzdotc Array, DIMENSION (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

dotc	(local) COMPLEX for pcdotc DOUBLE COMPLEX for pzdotc Dot product of sub(*x*) and sub(*y*) only in their scope.

p?dotu

Computes the dot product of two distributed complex vectors.

Syntax

```
call pcdotu(n, dotu, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pzdotu(n, dotu, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The p?dotu functions compute the dot product *dotu* of two distributed vectors defined as

```
dotu  = sub(x)'*sub(y)
```

where sub(*x*) and sub(*y*) are *n*-element distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1;

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy*=*m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vectors, $n \geq 0$.
x	(local) COMPLEX for pcdotu DOUBLE COMPLEX for pzdotu Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) COMPLEX for pcdotu DOUBLE COMPLEX for pzdotu Array, DIMENSION (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

dotu	(local) COMPLEX for pcdotu DOUBLE COMPLEX for pzdotu Dot product of sub(*x*) and sub(*y*) only in their scope.

p?nrm2

Computes the Euclidean norm of a distributed vector.

Syntax

```
call psnrm2(n, norm2, x, ix, jx, descx, incx)
call pdnrm2(n, norm2, x, ix, jx, descx, incx)
call pscnrm2(n, norm2, x, ix, jx, descx, incx)
call pdznrm2(n, norm2, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_pblas.h

Description

The p?nrm2 functions compute the Euclidean norm of a distributed vector sub(*x*),

where sub(*x*) is an *n*-element distributed vector.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vector sub(*x*), *n*≥0.
x	(local) REAL for psnrm2 DOUBLE PRECISION for pdnrm2 COMPLEX for pscnrm2 DOUBLE COMPLEX for pdznrm2 Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

norm2	(local) REAL for psnrm2 and pscnrm2. DOUBLE PRECISION for pdnrm2 and pdznrm2 Contains the Euclidean norm of a distributed vector only in its scope.

p?scal

Computes a product of a distributed vector by a scalar.

Syntax

```
call psscal(n, a, x, ix, jx, descx, incx)
call pdscal(n, a, x, ix, jx, descx, incx)
call pcscal(n, a, x, ix, jx, descx, incx)
call pzscal(n, a, x, ix, jx, descx, incx)
call pcsscal(n, a, x, ix, jx, descx, incx)
call pzdscal(n, a, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_pblas.h

Description

The p?scal routines multiplies a *n*-element distributed vector sub(*x*) by the scalar *a*:

```
sub(x) = a*sub(x),
```

where sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vector sub(*x*), *n*≥0.
a	(global) REAL for psscal and pcsscal DOUBLE PRECISION for pdscal and pzdscal

	COMPLEX for pcscal DOUBLE COMPLEX for pzscal Specifies the scalar *a*.
x	(local) REAL for psscal DOUBLE PRECISION for pdscal COMPLEX for pcscal and pcsscal DOUBLE COMPLEX for pzscal and pzdscal Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

x	Overwritten by the updated distributed vector sub(*x*)

p?swap

Swaps two distributed vectors.

Syntax

```
call psswap(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pdswap(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pcswap(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
call pzswap(n, x, ix, jx, descx, incx, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

Given two distributed vectors sub(*x*) and sub(*y*), the p?swap routines return vectors sub(*y*) and sub(*x*) swapped, each replacing the other.

Here sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx*=*m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx*= 1;

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy*=*m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy*= 1.

Input Parameters

n	(global) INTEGER. The length of distributed vectors, *n*≥0.
x	(local) REAL for psswap DOUBLE PRECISION for pdswap COMPLEX for pcswap DOUBLE COMPLEX for pzswap Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).

ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*X*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) REAL for psswap DOUBLE PRECISION for pdswap COMPLEX for pcswap DOUBLE COMPLEX for pzswap Array, DIMENSION (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*Y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

x	Overwritten by distributed vector sub(*y*).
y	Overwritten by distributed vector sub(*x*).

PBLAS Level 2 Routines

This section describes PBLAS Level 2 routines, which perform distributed matrix-vector operations. Table "PBLAS Level 2 Routine Groups and Their Data Types" lists the PBLAS Level 2 routine groups and the data types associated with them.

PBLAS Level 2 Routine Groups and Their Data Types

Routine Groups	Data Types	Description
p?gemv	s, d, c, z	Matrix-vector product using a distributed general matrix
p?agemv	s, d, c, z	Matrix-vector product using absolute values for a distributed general matrix
p?ger	s, d	Rank-1 update of a distributed general matrix
p?gerc	c, z	Rank-1 update (conjugated) of a distributed general matrix
p?geru	c, z	Rank-1 update (unconjugated) of a distributed general matrix
p?hemv	c, z	Matrix-vector product using a distributed Hermitian matrix
p?ahemv	c, z	Matrix-vector product using absolute values for a distributed Hermitian matrix
p?her	c, z	Rank-1 update of a distributed Hermitian matrix
p?her2	c, z	Rank-2 update of a distributed Hermitian matrix

Routine Groups	Data Types	Description
p?symv	s, d	Matrix-vector product using a distributed symmetric matrix
p?asymv	s, d	Matrix-vector product using absolute values for a distributed symmetric matrix
p?syr	s, d	Rank-1 update of a distributed symmetric matrix
p?syr2	s, d	Rank-2 update of a distributed symmetric matrix
p?trmv	s, d, c, z	Distributed matrix-vector product using a triangular matrix
p?atrmv	s, d, c, z	Distributed matrix-vector product using absolute values for a triangular matrix
p?trsv	s, d, c, z	Solves a system of linear equations whose coefficients are in a distributed triangular matrix

p?gemv

Computes a distributed matrix-vector product using a general matrix.

Syntax

```
call psgemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pdgemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pcgemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pzgemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

Include Files

- C: `mkl_pblas.h`

Description

The `p?gemv` routines perform a distributed matrix-vector operation defined as

```
sub(y)  := alpha*sub(A)*sub(x) + beta*sub(y),
```

or

```
sub(y)  := alpha*sub(A)'*sub(x) + beta*sub(y),
```

or

```
sub(y)  := alpha*conjg(sub(A)')*sub(x) + beta*sub(y),
```

where

alpha and *beta* are scalars,

sub(*A*) is a *m*-by-*n* submatrix, sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1),

sub(*x*) and sub(*y*) are subvectors.

When *trans* = 'N' or 'n', sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1, sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*m*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*m*-1, *jy*) if *incy* = 1.

When *trans* = 'T' or 't', or 'C', or 'c', sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*m*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*m*-1, *jx*) if *incx* = 1, sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*m*-1, *jy*) if *incy* = 1.

Input Parameters

trans
(global) CHARACTER*1. Specifies the operation:
if *trans*= 'N' or 'n', then sub(*y*) := *alpha**sub(*A*)'*sub(*x*) + *beta**sub(*y*);
if *trans*= 'T' or 't', then sub(*y*) := *alpha**sub(*A*)'*sub(*x*) + *beta**sub(*y*);
if *trans*= 'C' or 'c', then sub(*y*) := *alpha**conjg(subA)')*sub(*x*) + *beta**sub(*y*).

m
(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*A*), *m*≥0.

n
(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*A*), *n*≥0.

alpha
(global) REAL for psgemv
DOUBLE PRECISION for pdgemv
COMPLEX for pcgemv
DOUBLE COMPLEX for pzgemv
Specifies the scalar *alpha*.

a
(local) REAL for psgemv
DOUBLE PRECISION for pdgemv
COMPLEX for pcgemv
DOUBLE COMPLEX for pzgemv
Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*A*).

ia, *ja*
(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca
(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

x
(local) REAL for psgemv
DOUBLE PRECISION for pdgemv
COMPLEX for pcgemv
DOUBLE COMPLEX for pzgemv
Array, DIMENSION (*jx*-1)**m_x* + *ix*+(*n*-1)*abs(*incx*)) when *trans* = 'N' or 'n', and (*jx*-1)**m_x* + *ix*+(*m*-1)*abs(*incx*)) otherwise.
This array contains the entries of the distributed vector sub(*x*).

ix, *jx*
(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.

descx
(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx
(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

beta
(global) REAL for psgemv

	DOUBLE PRECISION for pdgemv COMPLEX for pcgemv DOUBLE COMPLEX for pzgemv Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.
y	(local) REAL for psgemv DOUBLE PRECISION for pdgemv COMPLEX for pcgemv DOUBLE COMPLEX for pzgemv Array, DIMENSION (*jy*-1)**m_y* + *iy*+(*m*-1)*abs(*incy*)) when *trans* = 'N' or 'n', and (*jy*-1)**m_y* + *iy*+(*n*-1)*abs(*incy*)) otherwise. This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten by the updated distributed vector sub(*y*).

p?agemv

Computes a distributed matrix-vector product using absolute values for a general matrix.

Syntax

```
call psagemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pdagemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pcagemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pzagemv(trans, m, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The p?agemv routines perform a distributed matrix-vector operation defined as

```
sub(y)  := abs(alpha)*abs(sub(A)')*abs(sub(x)) + abs(beta*sub(y)),
```

or

```
sub(y)  := abs(alpha)*abs(sub(A)')*abs(sub(x)) + abs(beta*sub(y)),
```

or

```
sub(y)  := abs(alpha)*abs(conjg(sub(A)'))*abs(sub(x)) + abs(beta*sub(y)),
```

where

alpha and *beta* are scalars,

sub(*A*) is a *m*-by-*n* submatrix, sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1),

sub(*x*) and sub(*y*) are subvectors.

When *trans* = 'N' or 'n',

sub(*x*) denotes *X*(*ix*:*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and

X(*ix*:*ix*+*n*-1, *jx*:*jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*:*iy*, *jy*:*jy*+*m*-1) if *incy* = *m_y*, and

Y(*iy*:*iy*+*m*-1, *jy*:*jy*) if *incy* = 1.

When *trans* = 'T' or 't', or 'C', or 'c',

sub(*x*) denotes *X*(*ix*:*ix*, *jx*:*jx*+*m*-1) if *incx* = *m_x*, and

X(*ix*:*ix*+*m*-1, *jx*:*jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*:*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and

Y(*iy*:*iy*+*m*-1, *jy*:*jy*) if *incy* = 1.

Input Parameters

trans	(global) CHARACTER*1. Specifies the operation: if *trans*= 'N' or 'n', then sub(*y*) := |*alpha*|*|sub(*A*)|*|sub(*x*)| + |*beta**sub(*y*)| if *trans*= 'T' or 't', then sub(*y*) := |*alpha*|*|sub(*A*)'|*|sub(*x*)| + |*beta**sub(*y*)| if *trans*= 'C' or 'c', then sub(*y*) := |*alpha*|*|sub(*A*)'|*|sub(*x*)| + |*beta**sub(*y*)|.
m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*A*), *m*≥0.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*A*), *n*≥0.
alpha	(global) REAL for psagemv DOUBLE PRECISION for pdagemv COMPLEX for pcagemv DOUBLE COMPLEX for pzagemv Specifies the scalar *alpha*.
a	(local) REAL for psagemv DOUBLE PRECISION for pdagemv COMPLEX for pcagemv DOUBLE COMPLEX for pzagemv Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
x	(local) REAL for psagemv DOUBLE PRECISION for pdagemv

	COMPLEX for pcagemv DOUBLE COMPLEX for pzagemv Array, DIMENSION (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)) when *trans* = 'N' or 'n', and (*jx*-1)*m_x + *ix*+(*m*-1)*abs(*incx*)) otherwise. This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
beta	(global) REAL for psagemv DOUBLE PRECISION for pdagemv COMPLEX for pcagemv DOUBLE COMPLEX for pzagemv Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.
y	(local) REAL for psagemv DOUBLE PRECISION for pdagemv COMPLEX for pcagemv DOUBLE COMPLEX for pzagemv Array, DIMENSION (*jy*-1)*m_y + *iy*+(*m*-1)*abs(*incy*)) when *trans* = 'N' or 'n', and (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)) otherwise. This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten by the updated distributed vector sub(*y*).

p?ger

Performs a rank-1 update of a distributed general matrix.

Syntax

```
call psger(m, n, alpha, x, ix, jx, descx, incx, y, iy, jy, descy, incy, a, ia, ja, desca)
```

```
call pdger(m, n, alpha, x, ix, jx, descx, incx, y, iy, jy, descy, incy, a, ia, ja, desca)
```

Include Files

- C: mkl_pblas.h

Description

The p?ger routines perform a distributed matrix-vector operation defined as

```
sub(A) := alpha*sub(x)*sub(y)' + sub(A),
```

where:

alpha is a scalar,

sub(*A*) is a *m*-by-*n* distributed general matrix, sub(*A*)=*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1),

sub(*x*) is an *m*-element distributed vector, sub(*y*) is an *n*-element distributed vector,

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*m*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*m*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*A*), *m*≥0.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*A*), *n*≥0.
alpha	(global) REAL for psger DOUBLE REAL for pdger Specifies the scalar *alpha*.
x	(local) REAL for psger DOUBLE REAL for pdger Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*m*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) REAL for psger DOUBLE REAL for pdger Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.
a	(local) REAL for psger DOUBLE REAL for pdger Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). Before entry this array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	Overwritten by the updated distributed matrix sub(*A*).

p?gerc

Performs a rank-1 update (conjugated) of a distributed general matrix.

Syntax

```
call pcgerc(m, n, alpha, x, ix, jx, descx, incx, y, iy, jy, descy, incy, a, ia, ja, desca)
```

```
call pzgerc(m, n, alpha, x, ix, jx, descx, incx, y, iy, jy, descy, incy, a, ia, ja, desca)
```

Include Files

- C: mkl_pblas.h

Description

The p?gerc routines perform a distributed matrix-vector operation defined as

```
sub(A) := alpha*sub(x)*conjg(sub(y)') + sub(A),
```

where:

alpha is a scalar,

sub(*A*) is a *m*-by-*n* distributed general matrix, sub(*A*) = *A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1),

sub(*x*) is an *m*-element distributed vector, sub(*y*) is an *n*-element distributed vector,

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*m*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*m*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*A*), $m \geq 0$.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*A*), $n \geq 0$.
alpha	(global) COMPLEX for pcgerc DOUBLE COMPLEX for pzgerc Specifies the scalar *alpha*.
x	(local) COMPLEX for pcgerc DOUBLE COMPLEX for pzgerc Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).

ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) COMPLEX for pcgerc DOUBLE COMPLEX for pzgerc Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.
a	(local) COMPLEX for pcgerc DOUBLE COMPLEX for pzgerc Array, DIMENSION at least (*lld_a*, LOCq(*ja*+*n*-1)). Before entry this array contains the local pieces of the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	Overwritten by the updated distributed matrix sub(*A*).

p?geru

Performs a rank-1 update (unconjugated) of a distributed general matrix.

Syntax

```
call pcgeru(m, n, alpha, x, ix, jx, descx, incx, y, iy, jy, descy, incy, a, ia, ja, desca)
```

```
call pzgeru(m, n, alpha, x, ix, jx, descx, incx, y, iy, jy, descy, incy, a, ia, ja, desca)
```

Include Files

- C: mkl_pblas.h

Description

The p?geru routines perform a matrix-vector operation defined as

```
sub(A) := alpha*sub(x)*sub(y)' + sub(A),
```

where:

alpha is a scalar,

sub(*A*) is a *m*-by-*n* distributed general matrix, sub(*A*)=*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*n*-1),

sub(*x*) is an *m*-element distributed vector, sub(*y*) is an *n*-element distributed vector,

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*m*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*m*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

m (global) INTEGER. Specifies the number of rows of the distributed matrix sub(*A*), $m \geq 0$.

n (global) INTEGER. Specifies the number of columns of the distributed matrix sub(*A*), $n \geq 0$.

alpha (global) COMPLEX for pcgeru
DOUBLE COMPLEX for pzgeru
Specifies the scalar *alpha*.

x (local) COMPLEX for pcgeru
DOUBLE COMPLEX for pzgeru
Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).

ix, *jx* (global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.

descx (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx (global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

y (local) COMPLEX for pcgeru
DOUBLE COMPLEX for pzgeru
Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).

iy, *jy* (global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.

descy (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.

incy (global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

a (local) COMPLEX for pcgeru
DOUBLE COMPLEX for pzgeru
Array, DIMENSION at least (*lld_a*, LOCq(*ja*+*n*-1)). Before entry this array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	Overwritten by the updated distributed matrix sub(*A*).

p?hemv

Computes a distributed matrix-vector product using a Hermitian matrix.

Syntax

call pchemv(*uplo*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *incx*, *beta*, *y*, *iy*, *jy*, *descy*, *incy*)

call pzhemv(*uplo*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *incx*, *beta*, *y*, *iy*, *jy*, *descy*, *incy*)

Include Files

- C: mkl_pblas.h

Description

The p?hemv routines perform a distributed matrix-vector operation defined as

sub(*y*) := *alpha**sub(*A*)*sub(*x*) + *beta**sub(*y*),

where:

alpha and *beta* are scalars,

sub(*A*) is a *n*-by-*n* Hermitian distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) ,

sub(*x*) and sub(*y*) are distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian distributed matrix sub(*A*) is used: If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), $n \geq 0$.
alpha	(global) COMPLEX for pchemv DOUBLE COMPLEX for pzhemv Specifies the scalar *alpha*.
a	(local) COMPLEX for pchemv DOUBLE COMPLEX for pzhemv Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*). Before entry when *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and when *uplo* = 'L' or 'l', the *n*-by-*n* lower

	triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local)INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
x	(local) COMPLEX for pchemv DOUBLE COMPLEX for pzhemv Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
beta	(global) COMPLEX for pchemv DOUBLE COMPLEX for pzhemv Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.
y	(local) COMPLEX for pchemv DOUBLE COMPLEX for pzhemv Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten by the updated distributed vector sub(*y*).

p?ahemv

Computes a distributed matrix-vector product using absolute values for a Hermitian matrix.

Syntax

```
call pcahemv(uplo, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

```
call pzahemv(uplo, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The p?ahemv routines perform a distributed matrix-vector operation defined as

```
sub(y) := abs(alpha)*abs(sub(A))*abs(sub(x)) + abs(beta*sub(y)),
```

where:

alpha and *beta* are scalars,

sub(*A*) is a *n*-by-*n* Hermitian distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) ,

sub(*x*) and sub(*y*) are distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

uplo
(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian distributed matrix sub(*A*) is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used.
If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.

n
(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), $n \geq 0$.

alpha
(global) COMPLEX for pcahemv
DOUBLE COMPLEX for pzahemv
Specifies the scalar *alpha*.

a
(local) COMPLEX for pcahemv
DOUBLE COMPLEX for pzahemv
Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*).
Before entry when *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and when *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja*
(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca
(global and local)INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

x
(local) COMPLEX for pcahemv
DOUBLE COMPLEX for pzahemv
Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)).
This array contains the entries of the distributed vector sub(*x*).

ix, *jx*
(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.

descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
beta	(global) COMPLEX for pcahemv DOUBLE COMPLEX for pzahemv Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.
y	(local) COMPLEX for pcahemv DOUBLE COMPLEX for pzahemv Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten by the updated distributed vector sub(*y*).

p?her

Performs a rank-1 update of a distributed Hermitian matrix.

Syntax

```
call pcher(uplo, n, alpha, x, ix, jx, descx, incx, a, ia, ja, desca)
call pzher(uplo, n, alpha, x, ix, jx, descx, incx, a, ia, ja, desca)
```

Include Files

- C: mkl_pblas.h

Description

The p?her routines perform a distributed matrix-vector operation defined as

```
sub(A) := alpha*sub(x)*conjg(sub(x)') + sub(A),
```

where:

alpha is a real scalar,

sub(*A*) is a *n*-by-*n* distributed Hermitian matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1),

sub(*x*) is distributed vector.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian distributed matrix sub(*A*) is used:

	If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), $n \geq 0$.
alpha	(global) REAL for pcher DOUBLE REAL for pzher Specifies the scalar *alpha*.
x	(local) COMPLEX for pcher DOUBLE COMPLEX for pzher Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
a	(local) COMPLEX for pcher DOUBLE COMPLEX for pzher Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*). Before entry with *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and with *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated distributed matrix sub(*A*). With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated distributed matrix sub(*A*).

p?her2

Performs a rank-2 update of a distributed Hermitian matrix.

Syntax

call pcher2(*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *descx*, *incx*, *y*, *iy*, *jy*, *descy*, *incy*, *a*, *ia*, *ja*, *desca*)

call pzher2(*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *descx*, *incx*, *y*, *iy*, *jy*, *descy*, *incy*, *a*, *ia*, *ja*, *desca*)

Include Files

- C: mkl_pblas.h

Description

The p?her2 routines perform a distributed matrix-vector operation defined as

sub(*A*) := *alpha**sub(*x*)*conj(sub(*y*)')+ conj(*alpha*)*sub(*y*)*conj(sub(*x*)') + sub(*A*),

where:

alpha is a scalar,

sub(*A*) is a *n*-by-*n* distributed Hermitian matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1),

sub(*x*) and sub(*y*) are distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

uplo — (global) CHARACTER*1. Specifies whether the upper or lower triangular part of the distributed Hermitian matrix sub(*A*) is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used.
If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.

n — (global) INTEGER. Specifies the order of the distributed matrix sub(*A*), *n* ≥ 0.

alpha — (global) COMPLEX for pcher2
DOUBLE COMPLEX for pzher2
Specifies the scalar *alpha*.

x — (local) COMPLEX for pcher2
DOUBLE COMPLEX for pzher2
Array, DIMENSION at least (*jx*-1)**m_x* + *ix*+(*n*-1)*abs(*incx*)).
This array contains the entries of the distributed vector sub(*x*).

ix, *jx* — (global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.

descx — (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx — (global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

y — (local) COMPLEX for pcher2
DOUBLE COMPLEX for pzher2
Array, DIMENSION at least (*jy*-1)**m_y* + *iy*+(*n*-1)*abs(*incy*)).
This array contains the entries of the distributed vector sub(*y*).

iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.
a	(local) COMPLEX for pcher2 DOUBLE COMPLEX for pzher2 Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*). Before entry with *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and with *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated distributed matrix sub(*A*). With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated distributed matrix sub(*A*).

p?symv

Computes a distributed matrix-vector product using a symmetric matrix.

Syntax

call pssymv(*uplo*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *incx*, *beta*, *y*, *iy*, *jy*, *descy*, *incy*)

call pdsymv(*uplo*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *incx*, *beta*, *y*, *iy*, *jy*, *descy*, *incy*)

Include Files

- C: mkl_pblas.h

Description

The p?symv routines perform a distributed matrix-vector operation defined as

sub(*y*) := *alpha**sub(*A*)*sub(*x*) + *beta**sub(*y*),

where:

alpha and *beta* are scalars,

sub(*A*) is a *n*-by-*n* symmetric distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) ,

sub(*x*) and sub(*y*) are distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

uplo (global) CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric distributed matrix sub(*A*) is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used.
If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.

n (global) INTEGER. Specifies the order of the distributed matrix sub(*A*), $n \geq 0$.

alpha (global) REAL for pssymv
DOUBLE REAL for pdsymv
Specifies the scalar *alpha*.

a (local) REAL for pssymv
DOUBLE REAL for pdsymv
Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*).
Before entry when *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and when *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local)INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

x (local) REAL for pssymv
DOUBLE REAL for pdsymv
Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)).
This array contains the entries of the distributed vector sub(*x*).

ix, *jx* (global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.

descx (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx (global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

beta (global) REAL for pssymv
DOUBLE REAL for pdsymv
Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.

y	(local) REAL for pssymv DOUBLE REAL for pdsymv Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y	Overwritten by the updated distributed vector sub(*y*).

p?asymv

Computes a distributed matrix-vector product using absolute values for a symmetric matrix.

Syntax

```
call psasymv(uplo, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
call pdasymv(uplo, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

Include Files

- C: mkl_pblas.h

Description

The p?symv routines perform a distributed matrix-vector operation defined as

```
sub(y)  := abs(alpha)*abs(sub(A))*abs(sub(x)) + abs(beta*sub(y)),
```

where:

alpha and *beta* are scalars,

sub(*A*) is a *n*-by-*n* symmetric distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) ,

sub(*x*) and sub(*y*) are distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric distributed matrix sub(*A*) is used: If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), *n* ≥ 0.

alpha (global) REAL for psasymv
DOUBLE REAL for pdasymv
Specifies the scalar *alpha*.

a (local) REAL for psasymv
DOUBLE REAL for pdasymv
Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*).
Before entry when *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and when *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local)INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

x (local) REAL for psasymv
DOUBLE PRECISION for pdasymv
Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)).
This array contains the entries of the distributed vector sub(*x*).

ix, *jx* (global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.

descx (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.

incx (global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

beta (global) REAL for psasymv
DOUBLE PRECISION for pdasymv
Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.

y (local) REAL for psasymv
DOUBLE PRECISION for pdasymv
Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)).
This array contains the entries of the distributed vector sub(*y*).

iy, *jy* (global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.

descy (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.

incy (global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

y Overwritten by the updated distributed vector sub(*y*).

p?syr

Performs a rank-1 update of a distributed symmetric matrix.

Syntax

```
call pssyr(uplo, n, alpha, x, ix, jx, descx, incx, a, ia, ja, desca)
call pdsyr(uplo, n, alpha, x, ix, jx, descx, incx, a, ia, ja, desca)
```

Include Files

- C: mkl_pblas.h

Description

The p?syr routines perform a distributed matrix-vector operation defined as

```
sub(A) := alpha*sub(x)*sub(x)' + sub(A),
```

where:

alpha is a scalar,

sub(*A*) is a *n*-by-*n* distributed symmetric matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) ,

sub(*x*) is distributed vector.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric distributed matrix sub(*A*) is used: If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), *n* ≥ 0.
alpha	(global) REAL for pssyr DOUBLE REAL for pdsyr Specifies the scalar *alpha*.
x	(local) REAL for pssyr DOUBLE REAL for pdsyr Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
a	(local) REAL for pssyr DOUBLE REAL for pdsyr Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*).

	Before entry with *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and with *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated distributed matrix sub(*A*). With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated distributed matrix sub(*A*).

p?syr2

Performs a rank-2 update of a distributed symmetric matrix.

Syntax

call pssyr2(*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *descx*, *incx*, *y*, *iy*, *jy*, *descy*, *incy*, *a*, *ia*, *ja*, *desca*)

call pdsyr2(*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *descx*, *incx*, *y*, *iy*, *jy*, *descy*, *incy*, *a*, *ia*, *ja*, *desca*)

Include Files

- C: mkl_pblas.h

Description

The p?syr2 routines perform a distributed matrix-vector operation defined as

sub(*A*) := *alpha**sub(*x*)*sub(*y*)'+ *alpha**sub(*y*)*sub(*x*)' + sub(*A*),

where:

alpha is a scalar,

sub(*A*) is a *n*-by-*n* distributed symmetric matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) ,

sub(*x*) and sub(*y*) are distributed vectors.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

sub(*y*) denotes *Y*(*iy*, *jy*:*jy*+*n*-1) if *incy* = *m_y*, and *Y*(*iy*: *iy*+*n*-1, *jy*) if *incy* = 1.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the distributed symmetric matrix sub(*A*) is used: If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*A*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*A*) is used.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), $n \geq 0$.
alpha	(global) REAL for pssyr2 DOUBLE REAL for pdsyr2 Specifies the scalar *alpha*.
x	(local) REAL for pssyr2 DOUBLE REAL for pdsyr2 Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
y	(local) REAL for pssyr2 DOUBLE REAL for pdsyr2 Array, DIMENSION at least (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)). This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.
a	(local) REAL for pssyr2 DOUBLE REAL for pdsyr2 Array, DIMENSION (*lld_a*, LOCq(*ja*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*A*). Before entry with *uplo* = 'U' or 'u', the *n*-by-*n* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the distributed symmetric matrix and the strictly lower triangular part of sub(*A*) is not referenced, and with *uplo* = 'L' or 'l', the *n*-by-*n* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the distributed symmetric matrix and the strictly upper triangular part of sub(*A*) is not referenced.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local)INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

Output Parameters

a	With *uplo* = 'U' or 'u', the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated distributed matrix sub(*A*). With *uplo* = 'L' or 'l', the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated distributed matrix sub(*A*).

p?trmv

Computes a distributed matrix-vector product using a triangular matrix.

Syntax

```
call pstrmv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
call pdtrmv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
call pctrmv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
call pztrmv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_pblas.h

Description

The p?trmv routines perform one of the following distributed matrix-vector operations defined as

sub(*x*) := sub(*A*)*sub(*x*), or sub(*x*) :=sub(*A*)'*sub(*x*), or sub(*x*) := conjg(sub(*A*)')*sub(*x*),

where:

sub(*A*) is a *n*-by-*n* unit, or non-unit, upper or lower triangular distributed matrix, sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1),

sub(*x*) is an *n*-element distributed vector.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the distributed matrix sub(*A*) is upper or lower triangular: if *uplo* = 'U' or 'u', then the matrix is upper triangular; if *uplo* = 'L' or 'l', then the matrix is low triangular.
trans	(global) CHARACTER*1. Specifies the form of op(sub(*A*)) used in the matrix equation: if *transa* = 'N' or 'n', then sub(*x*) := sub(*A*)*sub(*x*); if *transa* = 'T' or 't', then sub(*x*) :=sub(*A*)'*sub(*x*); if *transa* = 'C' or 'c', then sub(*x*) := conjg(sub(*A*)')*sub(*x*).
diag	(global) CHARACTER*1. Specifies whether the matrix sub(*A*) is unit triangular: if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), *n*≥0.

a	(local) REAL for pstrmv DOUBLE PRECISION for pdtrmv COMPLEX for pctrmv DOUBLE COMPLEX for pztrmv Array, DIMENSION at least (*lld_a*, LOCq(1, *ja*+*n*-1)). Before entry with *uplo* = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix sub(*A*) is not referenced. Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix sub(*A*) is not referenced . When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix sub(*A*) are not referenced either, but are assumed to be unity.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
x	(local) REAL for pstrmv DOUBLE PRECISION for pdtrmv COMPLEX for pctrmv DOUBLE COMPLEX for pztrmv Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

x	Overwritten by the transformed distributed vector sub(*x*).

p?atrmv

Computes a distributed matrix-vector product using absolute values for a triangular matrix.

Syntax

```
call psatrmv(uplo, trans, diag, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

```
call pdatrmv(uplo, trans, diag, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

```
call pcatrmv(uplo, trans, diag, n, alpha, a, ia, ja, desca, x, ix, jx, descx, incx, beta, y, iy, jy, descy, incy)
```

call pzatrmv(*uplo*, *trans*, *diag*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *x*, *ix*, *jx*, *descx*, *incx*, *beta*, *y*, *iy*, *jy*, *descy*, *incy*)

Include Files

- C: mkl_pblas.h

Description

The p?atrmv routines perform one of the following distributed matrix-vector operations defined as

sub(*y*) := abs(*alpha*)*abs(sub(*A*))*abs(sub(*x*))+ abs(*beta**sub(*y*)), or

sub(*y*) := abs(*alpha*)*abs(sub(*A*)')*abs(sub(*x*))+ abs(*beta**sub(*y*)), or

sub(*y*) := abs(*alpha*)*abs(conjg(sub(*A*)'))*abs(sub(*x*))+ abs(*beta**sub(*y*)),

where:

alpha and *beta* are scalars,

sub(*A*) is a *n*-by-*n* unit, or non-unit, upper or lower triangular distributed matrix, sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1),

sub(*x*) is an *n*-element distributed vector.

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1.

Input Parameters

uplo — (global) CHARACTER*1. Specifies whether the distributed matrix sub(*A*) is upper or lower triangular:
if *uplo* = 'U' or 'u', then the matrix is upper triangular;
if *uplo* = 'L' or 'l', then the matrix is low triangular.

trans — (global) CHARACTER*1. Specifies the form of op(sub(*A*)) used in the matrix equation:
if *trans* = 'N' or 'n', then sub(*y*) := |*alpha*|*|sub(*A*)|*|sub(*x*)|+|*beta**sub(*y*)|;
if *trans* = 'T' or 't', then sub(*y*) := |*alpha*|*|sub(*A*)'|*|sub(*x*)|+|*beta**sub(*y*)|;
if *trans* = 'C' or 'c', then sub(*y*) := |*alpha*|*|conjg(sub(*A*)')|*|sub(*x*)|+|*beta**sub(*y*)|.

diag — (global) CHARACTER*1. Specifies whether the matrix sub(*A*) is unit triangular:
if *diag* = 'U' or 'u' then the matrix is unit triangular;
if *diag* = 'N' or 'n', then the matrix is not unit triangular.

n — (global) INTEGER. Specifies the order of the distributed matrix sub(*A*), $n \geq 0$.

alpha — (global) REAL for psatrmv
DOUBLE PRECISION for pdatrmv
COMPLEX for pcatrmv
DOUBLE COMPLEX for pzatrmv
Specifies the scalar *alpha*.

a — (local) REAL for psatrmv
DOUBLE PRECISION for pdatrmv
COMPLEX for pcatrmv
DOUBLE COMPLEX for pzatrmv
Array, DIMENSION at least (*lld_a*, LOCq(1, *ja*+*n*-1)).

	Before entry with *uplo* = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix sub(*A*) is not referenced. Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix sub(*A*) is not referenced. When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix sub(*A*) are not referenced either, but are assumed to be unity.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
x	(local) REAL for psatrmv DOUBLE PRECISION for pdatrmv COMPLEX for pcatrmv DOUBLE COMPLEX for pzatrmv Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*).
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.
beta	(global) REAL for psatrmv DOUBLE PRECISION for pdatrmv COMPLEX for pcatrmv DOUBLE COMPLEX for pzatrmv Specifies the scalar *beta*. When *beta* is set to zero, then sub(*y*) need not be set on input.
y	(local) REAL for psatrmv DOUBLE PRECISION for pdatrmv COMPLEX for pcatrmv DOUBLE COMPLEX for pzatrmv Array, DIMENSION (*jy*-1)*m_y + *iy*+(*m*-1)*abs(*incy*)) when *trans* = 'N' or 'n', and (*jy*-1)*m_y + *iy*+(*n*-1)*abs(*incy*)) otherwise. This array contains the entries of the distributed vector sub(*y*).
iy, *jy*	(global) INTEGER. The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix sub(*y*), respectively.
descy	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *Y*.
incy	(global) INTEGER. Specifies the increment for the elements of sub(*y*). Only two values are supported, namely 1 and *m_y*. *incy* must not be zero.

Output Parameters

x	Overwritten by the transformed distributed vector sub(*x*).

p?trsv

Solves a system of linear equations whose coefficients are in a distributed triangular matrix.

Syntax

```
call pstrsv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
call pdtrsv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
call pctrsv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
call pztrsv(uplo, trans, diag, n, a, ia, ja, desca, x, ix, jx, descx, incx)
```

Include Files

- C: mkl_pblas.h

Description

The p?trsv routines solve one of the systems of equations:

sub(*A*)*sub(*x*) = *b*, or sub(*A*)'*sub(*x*) = *b*, or conjg(sub(*A*)')*sub(*x*) = *b*,

where:

sub(*A*) is a *n*-by-*n* unit, or non-unit, upper or lower triangular distributed matrix, sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1),

b and sub(*x*) are *n*-element distributed vectors,

sub(*x*) denotes *X*(*ix*, *jx*:*jx*+*n*-1) if *incx* = *m_x*, and *X*(*ix*: *ix*+*n*-1, *jx*) if *incx* = 1,.

The routine does not test for singularity or near-singularity. Such tests must be performed before calling this routine.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the distributed matrix sub(*A*) is upper or lower triangular: if *uplo* = 'U' or 'u', then the matrix is upper triangular; if *uplo* = 'L' or 'l', then the matrix is low triangular.
trans	(global) CHARACTER*1. Specifies the form of the system of equations: if *transa* = 'N' or 'n', then sub(*A*)*sub(*x*) = *b*; if *transa* = 'T' or 't', then sub(*A*)'*sub(*x*) = *b*; if *transa* = 'C' or 'c', then conjg(sub(*A*)')*sub(*x*) = *b*.
diag	(global) CHARACTER*1. Specifies whether the matrix sub(*A*) is unit triangular: if *diag* = 'U' or 'u' then the matrix is unit triangular; if *diag* = 'N' or 'n', then the matrix is not unit triangular.
n	(global) INTEGER. Specifies the order of the distributed matrix sub(*A*), *n* ≥ 0.
a	(local) REAL for pstrsv DOUBLE PRECISION for pdtrsv COMPLEX for pctrsv

	DOUBLE COMPLEX for pztrsv Array, DIMENSION at least (*lld_a*, LOCq(1, *ja*+*n*-1)). Before entry with *uplo* = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix sub(*A*) is not referenced. Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix sub(*A*) is not referenced . When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix sub(*A*) are not referenced either, but are assumed to be unity.
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
x	(local) REAL for pstrsv DOUBLE PRECISION for pdtrsv COMPLEX for pctrsv DOUBLE COMPLEX for pztrsv Array, DIMENSION at least (*jx*-1)*m_x + *ix*+(*n*-1)*abs(*incx*)). This array contains the entries of the distributed vector sub(*x*). Before entry, sub(*x*) must contain the *n*-element right-hand side distributed vector *b*.
ix, *jx*	(global) INTEGER. The row and column indices in the distributed matrix *X* indicating the first row and the first column of the submatrix sub(*x*), respectively.
descx	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *X*.
incx	(global) INTEGER. Specifies the increment for the elements of sub(*x*). Only two values are supported, namely 1 and *m_x*. *incx* must not be zero.

Output Parameters

x	Overwritten with the solution vector.

PBLAS Level 3 Routines

The PBLAS Level 3 routines perform distributed matrix-matrix operations. Table "PBLAS Level 3 Routine Groups and Their Data Types" lists the PBLAS Level 3 routine groups and the data types associated with them.

PBLAS Level 3 Routine Groups and Their Data Types

Routine Group	Data Types	Description
p?geadd	s, d, c, z	Distributed matrix-matrix sum of general matrices
p?tradd	s, d, c, z	Distributed matrix-matrix sum of triangular matrices
p?gemm	s, d, c, z	Distributed matrix-matrix product of general matrices

Routine Group	Data Types	Description
p?hemm	c, z	Distributed matrix-matrix product, one matrix is Hermitian
p?herk	c, z	Rank-k update of a distributed Hermitian matrix
p?her2k	c, z	Rank-2k update of a distributed Hermitian matrix
p?symm	s, d, c, z	Matrix-matrix product of distributed symmetric matrices
p?syrk	s, d, c, z	Rank-k update of a distributed symmetric matrix
p?syr2k	s, d, c, z	Rank-2k update of a distributed symmetric matrix
p?tran	s, d	Transposition of a real distributed matrix
p?tranc	c, z	Transposition of a complex distributed matrix (conjugated)
p?tranu	c, z	Transposition of a complex distributed matrix
p?trmm	s, d, c, z	Distributed matrix-matrix product, one matrix is triangular
p?trsm	s, d, c, z	Solution of a distributed matrix equation, one matrix is triangular

p?geadd

Performs sum operation for two distributed general matrices.

Syntax

```
call psgeadd(trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pdgeadd(trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pcgeadd(trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pzgeadd(trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: `mkl_pblas.h`

Description

The `p?geadd` routines perform sum operation for two distributed general matrices. The operation is defined as

```
sub(C):=beta*sub(C) + alpha*op(sub(A)),
```

where:

`op(x)` is one of `op(x) = x`, or `op(x) = x'`,

alpha and *beta* are scalars,

`sub(C)` is an *m*-by-*n* distributed matrix, `sub(C)=C(ic:ic+m-1, jc:jc+n-1)`.

`sub(A)` is a distributed matrix, `sub(A)=A(ia:ia+n-1, ja:ja+m-1)`.

Input Parameters

trans (global) `CHARACTER*1`. Specifies the operation:

	if *trans* = 'N' or 'n', then op(sub(*A*)) := sub(*A*); if *trans* = 'T' or 't', then op(sub(*A*)) := sub(*A*)'; if *trans* = 'C' or 'c', then op(sub(*A*)) := sub(*A*)'.
m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*C*) and the number of columns of the submatrix sub(*A*), $m \geq 0$.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*C*) and the number of rows of the submatrix sub(*A*), $n \geq 0$.
alpha	(global) REAL for psgeadd DOUBLE PRECISION for pdgeadd COMPLEX for pcgeadd DOUBLE COMPLEX for pzgeadd Specifies the scalar *alpha*.
a	(local) REAL for psgeadd DOUBLE PRECISION for pdgeadd COMPLEX for pcgeadd DOUBLE COMPLEX for pzgeadd Array, DIMENSION (*lld_a*, LOCq(*ja*+*m*-1)). This array contains the local pieces of the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
beta	(global) REAL for psgeadd DOUBLE PRECISION for pdgeadd COMPLEX for pcgeadd DOUBLE COMPLEX for pzgeadd Specifies the scalar *beta*. When *beta* is equal to zero, then sub(*C*) need not be set on input.
c	(local) REAL for psgeadd DOUBLE PRECISION for pdgeadd COMPLEX for pcgeadd DOUBLE COMPLEX for pzgeadd Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the updated submatrix.

p?tradd

Performs sum operation for two distributed triangular matrices.

Syntax

```
call pstradd(uplo, trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pdtradd(uplo, trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pctradd(uplo, trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pztradd(uplo, trans, m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: `mkl_pblas.h`

Description

The `p?tradd` routines perform sum operation for two distributed triangular matrices. The operation is defined as

```
sub(C):=beta*sub(C) + alpha*op(sub(A)),
```

where:

`op(x)` is one of `op(x) = x`, or `op(x) = x'`, or `op(x) = conjg(x')`.

alpha and *beta* are scalars,

`sub(C)` is an *m*-by-*n* distributed matrix, `sub(C)=C(ic:ic+m-1, jc:jc+n-1)`.

`sub(A)` is a distributed matrix, `sub(A)=A(ia:ia+n-1, ja:ja+m-1)`.

Input Parameters

uplo
(global) `CHARACTER*1`. Specifies whether the distributed matrix `sub(C)` is upper or lower triangular:
if *uplo* = `'U'` or `'u'`, then the matrix is upper triangular;
if *uplo* = `'L'` or `'l'`, then the matrix is low triangular.

trans
(global) `CHARACTER*1`. Specifies the operation:
if *trans* = `'N'` or `'n'`, then `op(sub(A)) := sub(A)`;
if *trans* = `'T'` or `'t'`, then `op(sub(A)) := sub(A)'`;
if *trans* = `'C'` or `'c'`, then `op(sub(A)) := conjg(sub(A)')`.

m
(global) `INTEGER`. Specifies the number of rows of the distributed matrix `sub(C)` and the number of columns of the submatrix `sub(A)`, $m \geq 0$.

n
(global) `INTEGER`. Specifies the number of columns of the distributed matrix `sub(C)` and the number of rows of the submatrix `sub(A)`, $n \geq 0$.

alpha
(global) `REAL` for `pstradd`
`DOUBLE PRECISION` for `pdtradd`
`COMPLEX` for `pctradd`
`DOUBLE COMPLEX` for `pztradd`
Specifies the scalar *alpha*.

a
(local) `REAL` for `pstradd`
`DOUBLE PRECISION` for `pdtradd`
`COMPLEX` for `pctradd`
`DOUBLE COMPLEX` for `pztradd`
Array, `DIMENSION (lld_a, LOCq(ja+m-1))`. This array contains the local pieces of the distributed matrix `sub(A)`.

ia, *ja*
(global) `INTEGER`. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix `sub(A)`, respectively.

desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
beta	(global) REAL for pstradd DOUBLE PRECISION for pdtradd COMPLEX for pctradd DOUBLE COMPLEX for pztradd Specifies the scalar *beta*. When *beta* is equal to zero, then sub(*C*) need not be set on input.
c	(local) REAL for pstradd DOUBLE PRECISION for pdtradd COMPLEX for pctradd DOUBLE COMPLEX for pztradd Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the updated submatrix.

p?gemm

Computes a scalar-matrix-matrix product and adds the result to a scalar-matrix product for distributed matrices.

Syntax

```
call psgemm(transa, transb, m, n, k, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
call pdgemm(transa, transb, m, n, k, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
call pcgemm(transa, transb, m, n, k, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
call pzgemm(transa, transb, m, n, k, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
```

Include Files

- C: mkl_pblas.h

Description

The p?gemm routines perform a matrix-matrix operation with general distributed matrices. The operation is defined as

```
sub(C) := alpha*op(sub(A))*op(sub(B)) + beta*sub(C),
```

where:

op(*x*) is one of op(*x*) = *x*, or op(*x*) = *x*',

alpha and *beta* are scalars,

sub(*A*)=*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*k*-1), sub(*B*)=*B*(*ib*:*ib*+*k*-1, *jb*:*jb*+*n*-1), and sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1), are distributed matrices.

Input Parameters

transa (global) CHARACTER*1. Specifies the form of op(sub(*A*)) used in the matrix multiplication:
if *transa* = 'N' or 'n', then op(sub(*A*)) = sub(*A*);
if *transa* = 'T' or 't', then op(sub(*A*)) = sub(*A*)';
if *transa* = 'C' or 'c', then op(sub(*A*)) = sub(*A*)'.

transb (global) CHARACTER*1. Specifies the form of op(sub(*B*)) used in the matrix multiplication:
if *transb* = 'N' or 'n', then op(sub(*B*)) = sub(*B*);
if *transb* = 'T' or 't', then op(sub(*B*)) = sub(*B*)';
if *transb* = 'C' or 'c', then op(sub(*B*)) = sub(*B*)'.

m (global) INTEGER. Specifies the number of rows of the distributed matrices op(sub(*A*)) and sub(*C*), $m \geq 0$.

n (global) INTEGER. Specifies the number of columns of the distributed matrices op(sub(*B*)) and sub(*C*), $n \geq 0$.
The value of *n* must be at least zero.

k (global) INTEGER. Specifies the number of columns of the distributed matrix op(sub(*A*)) and the number of rows of the distributed matrix op(sub(*B*)).
The value of *k* must be greater than or equal to 0.

alpha (global) REAL for psgemm
DOUBLE PRECISION for pdgemm
COMPLEX for pcgemm
DOUBLE COMPLEX for pzgemm
Specifies the scalar *alpha*.
When *alpha* is equal to zero, then the local entries of the arrays *a* and *b* corresponding to the entries of the submatrices sub(*A*) and sub(*B*) respectively need not be set on input.

a (local) REAL for psgemm
DOUBLE PRECISION for pdgemm
COMPLEX for pcgemm
DOUBLE COMPLEX for pzgemm
Array, DIMENSION (*lld_a*, *kla*), where *kla* is LOCc(*ja*+*k*-1) when *transa* = 'N' or 'n', and is LOCq(*ja*+*m*-1) otherwise. Before entry this array must contain the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b (local)REAL for psgemm
DOUBLE PRECISION for pdgemm
COMPLEX for pcgemm
DOUBLE COMPLEX for pzgemm
Array, DIMENSION (*lld_b*, *klb*), where *klb* is LOCc(*jb*+*n*-1) when *transb* = 'N' or 'n', and is LOCq(*jb*+*k*-1) otherwise. Before entry this array must contain the local pieces of the distributed matrix sub(*B*).

ib, *jb*	(global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively
descb	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.
beta	(global) REAL for psgemm DOUBLE PRECISION for pdgemm COMPLEX for pcgemm DOUBLE COMPLEX for pzgemm Specifies the scalar *beta*. When *beta* is equal to zero, then sub(*C*) need not be set on input.
c	(local)REAL for psgemm DOUBLE PRECISION for pdgemm COMPLEX for pcgemm DOUBLE COMPLEX for pzgemm Array, DIMENSION (*lld_a*, LOCq(*jc*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the *m*-by-*n* distributed matrix *alpha**op(sub(*A*))*op(sub(*B*)) + *beta**sub(*C*).

p?hemm

Performs a scalar-matrix-matrix product (one matrix operand is Hermitian) and adds the result to a scalar-matrix product.

Syntax

call pchemm(*side*, *uplo*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

call pzhemm(*side*, *uplo*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

Include Files

- C: mkl_pblas.h

Description

The p?hemm routines perform a matrix-matrix operation with distributed matrices. The operation is defined as

sub(*C*):=*alpha**sub(*A*)*sub(*B*)+ *beta**sub(*C*),

or

sub(*C*):=*alpha**sub(*B*)*sub(*A*)+ *beta**sub(*C*),

where:

alpha and *beta* are scalars,

sub(*A*) is a Hermitian distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*m*-1), if *side* = 'L', and sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1), if *side* = 'R'.

sub(*B*) and sub(*C*) are *m*-by-*n* distributed matrices.

sub(*B*)=*B*(*ib*:*ib*+*m*-1, *jb*:*jb*+*n*-1), sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1).

Input Parameters

side (global) CHARACTER*1. Specifies whether the Hermitian distributed matrix sub(*A*) appears on the left or right in the operation:
if *side* = 'L' or 'l', then sub(*C*) := *alpha**sub(*A*) *sub(*B*) + *beta**sub(*C*);
if *side* = 'R' or 'r', then sub(*C*) := *alpha**sub(*B*) *sub(*A*) + *beta**sub(*C*).

uplo (global) CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian distributed matrix sub(*A*) is used:
if *uplo* = 'U' or 'u', then the upper triangular part is used;
if *uplo* = 'L' or 'l', then the lower triangular part is used.

m (global) INTEGER. Specifies the number of rows of the distribute submatrix sub(*C*), $m \geq 0$.

n (global) INTEGER. Specifies the number of columns of the distribute submatrix sub(*C*), $n \geq 0$.

alpha (global) COMPLEX for pchemm
DOUBLE COMPLEX for pzhemm
Specifies the scalar *alpha*.

a (local) COMPLEX for pchemm
DOUBLE COMPLEX for pzhemm
Array, DIMENSION (*lld_a*, LOCq(*ja*+*na*-1)).
Before entry this array must contain the local pieces of the symmetric distributed matrix sub(*A*), such that when *uplo* = 'U' or 'u', the *na*-by-*na* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and when *uplo* = 'L' or 'l', the *na*-by-*na* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b (local) COMPLEX for pchemm
DOUBLE COMPLEX for pzhemm
Array, DIMENSION (*lld_b*, LOCq(*jb*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*B*).

ib, *jb* (global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.

beta (global) COMPLEX for pchemm

	DOUBLE COMPLEX for pzhemm Specifies the scalar *beta*. When *beta* is set to zero, then sub(*C*) need not be set on input.
c	(local) COMPLEX for pchemm DOUBLE COMPLEX for pzhemm Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively
descc	(global and local)INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the *m*-by-*n* updated distributed matrix.

p?herk

Performs a rank-k update of a distributed Hermitian matrix.

Syntax

```
call pcherk(uplo, trans, n, k, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pzherk(uplo, trans, n, k, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: mkl_pblas.h

Description

The p?herk routines perform a distributed matrix-matrix operation defined as

```
sub(C):=alpha*sub(A)*conjg(sub(A)')+ beta*sub(C),
```

or

```
sub(C):=alpha*conjg(sub(A)')*sub(A)+ beta*sub(C),
```

where:

alpha and *beta* are scalars,

sub(*C*) is an *n*-by-*n* Hermitian distributed matrix, sub(*C*)=C(*ic*:*ic*+*n*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*k*-1), if *trans* = 'N' or 'n', and sub(*A*)=*A*(*ia*:*ia*+*k*-1, *ja*:*ja*+*n*-1) otherwise.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian distributed matrix sub(*C*) is used: If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*C*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*C*) is used.
trans	(global) CHARACTER*1. Specifies the operation:

if *trans* = 'N' or 'n', then sub(*C*) := *alpha**sub(*A*)*conjg(sub(*A*)') + *beta**sub(*C*);
if *trans* = 'C' or 'c', then sub(*C*) := *alpha**conjg(sub(*A*)')*sub(*A*) + *beta**sub(*C*).

n (global) INTEGER. Specifies the order of the distributed matrix sub(*C*), $n \geq 0$.

k (global) INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the distributed matrix sub(*A*) , and on entry with *trans* = 'T' or 't' or 'C' or 'c', *k* specifies the number of rows of the distributed matrix sub(*A*), $k \geq 0$.

alpha (global) REAL for pcherk
DOUBLE PRECISION for pzherk
Specifies the scalar *alpha*.

a (local) COMPLEX for pcherk
DOUBLE COMPLEX for pzherk
Array, DIMENSION (*lld_a*, *kla*), where *kla* is LOCq(*ja*+*k*-1) when *trans* = 'N' or 'n', and is LOCq(*ja*+*n*-1) otherwise. Before entry with *trans* = 'N' or 'n', this array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

beta (global) REAL for pcherk
DOUBLE PRECISION for pzherk
Specifies the scalar *beta*.

c (local) COMPLEX for pcherk
DOUBLE COMPLEX for pzherk
Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)).
Before entry with *uplo* = 'U' or 'u', this array contains *n*-by-*n* upper triangular part of the symmetric distributed matrix sub(*C*) and its strictly lower triangular part is not referenced.
Before entry with *uplo* = 'L' or 'l', this array contains *n*-by-*n* lower triangular part of the symmetric distributed matrix sub(*C*) and its strictly upper triangular part is not referenced.

ic, *jc* (global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c With *uplo* = 'U' or 'u', the upper triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.
With *uplo* = 'L' or 'l', the lower triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.

p?her2k

Performs a rank-2k update of a Hermitian distributed matrix.

Syntax

Fortran 77:

call pcher2k(*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

call pzher2k(*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

Include Files

- C: mkl_pblas.h

Description

The p?her2k routines perform a distributed matrix-matrix operation defined as

sub(*C*):=*alpha**sub(*A*)*conjg(sub(*B*)')+ conjg(*alpha*)*sub(*B*)*conjg(sub(*A*)')+*beta**sub(*C*),

or

sub(*C*):=*alpha**conjg(sub(*A*)')*sub(*A*)+ conjg(*alpha*)*conjg(sub(*B*)')*sub(*A*) + *beta**sub(*C*),

where:

alpha and *beta* are scalars,

sub(*C*) is an *n*-by-*n* Hermitian distributed matrix, sub(*C*) = C(*ic*:*ic*+*n*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*k*-1), if *trans* = 'N' or 'n', and sub(*A*) = *A*(*ia*:*ia*+*k*-1, *ja*:*ja*+*n*-1) otherwise.

sub(*B*) is a distributed matrix, sub(*B*) = *B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*k*-1), if *trans* = 'N' or 'n', and sub(*B*)=*B*(*ib*:*ib*+*k*-1, *jb*:*jb*+*n*-1) otherwise.

Input Parameters

uplo — (global) CHARACTER*1. Specifies whether the upper or lower triangular part of the Hermitian distributed matrix sub(*C*) is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*C*) is used.
If *uplo* = 'L' or 'l', then the low triangular part of the sub(*C*) is used.

trans — (global) CHARACTER*1. Specifies the operation:
if *trans* = 'N' or 'n', then sub(*C*) := *alpha**sub(*A*)*conjg(sub(*B*)') + conjg(*alpha*)*sub(*B*)*conjg(sub(*A*)') + *beta**sub(*C*);
if *trans* = 'C' or 'c', then sub(*C*) := *alpha**conjg(sub(*A*)')*sub(*A*) + conjg(*alpha*)*conjg(sub(*B*)')*sub(*A*) + *beta**sub(*C*).

n — (global) INTEGER. Specifies the order of the distributed matrix sub(*C*), $n \geq 0$.

k — (global) INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the distributed matrices sub(*A*) and sub(*B*), and on entry with *trans* = 'C' or 'c' , *k* specifies the number of rows of the distributed matrices sub(*A*) and sub(*B*), $k \geq 0$.

alpha — (global) COMPLEX for pcher2k

DOUBLE COMPLEX for pzher2k
Specifies the scalar *alpha*.

a (local) COMPLEX for pcher2k
DOUBLE COMPLEX for pzher2k
Array, DIMENSION (*lld_a*, *kla*), where *kla* is LOCq(*ja*+*k*-1) when *trans* = 'N' or 'n', and is LOCq(*ja*+*n*-1) otherwise. Before entry with *trans* = 'N' or 'n', this array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b (local) COMPLEX for pcher2k
DOUBLE COMPLEX for pzher2k
Array, DIMENSION (*lld_b*, *klb*), where *klb* is LOCq(*jb*+*k*-1) when *trans* = 'N' or 'n', and is LOCq(*jb*+*n*-1) otherwise. Before entry with *trans* = 'N' or 'n', this array contains the local pieces of the distributed matrix sub(*B*).

ib, *jb* (global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.

beta (global) REAL for pcher2k
DOUBLE PRECISION for pzher2k
Specifies the scalar *beta*.

c (local) COMPLEX for pcher2k
DOUBLE COMPLEX for pzher2k
Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)).
Before entry with *uplo* = 'U' or 'u', this array contains *n*-by-*n* upper triangular part of the symmetric distributed matrix sub(*C*) and its strictly lower triangular part is not referenced.
Before entry with *uplo* = 'L' or 'l', this array contains *n*-by-*n* lower triangular part of the symmetric distributed matrix sub(*C*) and its strictly upper triangular part is not referenced.

ic, *jc* (global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c With *uplo* = 'U' or 'u', the upper triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.
With *uplo* = 'L' or 'l', the lower triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.

p?symm

Performs a scalar-matrix-matrix product (one matrix operand is symmetric) and adds the result to a scalar-matrix product for distribute matrices.

Syntax

Fortran 77:

```
call pssymm(side, uplo, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
call pdsymm(side, uplo, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
call pcsymm(side, uplo, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
call pzsymm(side, uplo, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb, beta, c, ic, jc, descc)
```

Include Files

- C: `mkl_pblas.h`

Description

The `p?symm` routines perform a matrix-matrix operation with distributed matrices. The operation is defined as

`sub(C):=alpha*sub(A)*sub(B)+ beta*sub(C),`

or

`sub(C):=alpha*sub(B)*sub(A)+ beta*sub(C),`

where:

alpha and *beta* are scalars,

`sub(A)` is a symmetric distributed matrix, `sub(A)=A(ia:ia+m-1, ja:ja+m-1)`, if *side* `='L'`, and `sub(A)=A(ia:ia+n-1, ja:ja+n-1)`, if *side* `='R'`.

`sub(B)` and `sub(C)` are *m*-by-*n* distributed matrices.

`sub(B)=B(ib:ib+m-1, jb:jb+n-1)`, `sub(C)=C(ic:ic+m-1, jc:jc+n-1)`.

Input Parameters

side	(global) `CHARACTER*1`. Specifies whether the symmetric distributed matrix `sub(A)` appears on the left or right in the operation: if *side* `= 'L'` or `'l'`, then `sub(C) := alpha*sub(A) *sub(B) + beta*sub(C)`; if *side* `= 'R'` or `'r'`, then `sub(C) := alpha*sub(B) *sub(A) + beta*sub(C)`.
uplo	(global) `CHARACTER*1`. Specifies whether the upper or lower triangular part of the symmetric distributed matrix `sub(A)` is used: if *uplo* `= 'U'` or `'u'`, then the upper triangular part is used; if *uplo* `= 'L'` or `'l'`, then the lower triangular part is used.
m	(global) `INTEGER`. Specifies the number of rows of the distribute submatrix `sub(C)`, *m* ≥ 0.

n (global) INTEGER. Specifies the number of columns of the distribute submatrix sub(*C*), $m \geq 0$.

alpha (global) REAL for pssymm
DOUBLE PRECISION for pdsymm
COMPLEX for pcsymm
DOUBLE COMPLEX for pzsymm
Specifies the scalar *alpha*.

a (local) REAL for pssymm
DOUBLE PRECISION for pdsymm
COMPLEX for pcsymm
DOUBLE COMPLEX for pzsymm
Array, DIMENSION (*lld_a*, LOCq(*ja*+*na*-1)).
Before entry this array must contain the local pieces of the symmetric distributed matrix sub(*A*), such that when *uplo* = 'U' or 'u', the *na*-by-*na* upper triangular part of the distributed matrix sub(*A*) must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of sub(*A*) is not referenced, and when *uplo* = 'L' or 'l', the *na*-by-*na* lower triangular part of the distributed matrix sub(*A*) must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of sub(*A*) is not referenced.

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b (local) REAL for pssymm
DOUBLE PRECISION for pdsymm
COMPLEX for pcsymm
DOUBLE COMPLEX for pzsymm
Array, DIMENSION (*lld_b*, LOCq(*jb*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*B*).

ib, *jb* (global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.

beta (global) REAL for pssymm
DOUBLE PRECISION for pdsymm
COMPLEX for pcsymm
DOUBLE COMPLEX for pzsymm
Specifies the scalar *beta*.
When *beta* is set to zero, then sub(*C*) need not be set on input.

c (local) REAL for pssymm
DOUBLE PRECISION for pdsymm
COMPLEX for pcsymm
DOUBLE COMPLEX for pzsymm
Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). Before entry this array must contain the local pieces of the distributed matrix sub(*C*).

ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the *m*-by-*n* updated matrix.

p?syrk

Performs a rank-k update of a symmetric distributed matrix.

Syntax

```
call pssyrk(uplo, trans, n, k, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pdsyrk(uplo, trans, n, k, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pcsyrk(uplo, trans, n, k, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pzsyrk(uplo, trans, n, k, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: mkl_pblas.h

Description

The p?syrk routines perform a distributed matrix-matrix operation defined as

sub(*C*):=*alpha**sub(*A*)*sub(*A*)'+ *beta**sub(*C*),

or

sub(*C*):=*alpha**sub(*A*)'*sub(*A*)+ *beta**sub(*C*),

where:

alpha and *beta* are scalars,

sub(*C*) is an *n*-by-*n* symmetric distributed matrix, sub(*C*)=C(*ic*:*ic*+*n*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*k*-1), if *trans* = 'N' or 'n', and sub(*A*)=*A*(*ia*:*ia*+*k*-1, *ja*:*ja*+*n*-1) otherwise.

Input Parameters

uplo	(global) CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric distributed matrix sub(*C*) is used: If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*C*) is used. If *uplo* = 'L' or 'l', then the low triangular part of the sub(*C*) is used.
trans	(global) CHARACTER*1. Specifies the operation: if *trans* = 'N' or 'n', then sub(*C*) := *alpha**sub(*A*)*sub(*A*)' + *beta**sub(*C*); if *trans* = 'T' or 't', then sub(*C*) := *alpha**sub(*A*)'*sub(*A*) + *beta**sub(*C*).

n (global) INTEGER. Specifies the order of the distributed matrix sub(*C*), $n \geq 0$.

k (global) INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the distributed matrix sub(*A*) , and on entry with *trans* = 'T' or 't' , *k* specifies the number of rows of the distributed matrix sub(*A*), $k \geq 0$.

alpha (global) REAL for pssyrk
DOUBLE PRECISION for pdsyrk
COMPLEX for pcsyrk
DOUBLE COMPLEX for pzsyrk
Specifies the scalar *alpha*.

a (local) REAL for pssyrk
DOUBLE PRECISION for pdsyrk
COMPLEX for pcsyrk
DOUBLE COMPLEX for pzsyrk
Array, DIMENSION (*lld_a*, *kla*), where *kla* is LOCq(*ja*+*k*-1) when *trans* = 'N' or 'n', and is LOCq(*ja*+*n*-1) otherwise. Before entry with *trans* = 'N' or 'n', this array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

beta (global) REAL for pssyrk
DOUBLE PRECISION for pdsyrk
COMPLEX for pcsyrk
DOUBLE COMPLEX for pzsyrk
Specifies the scalar *beta*.

c (local) REAL for pssyrk
DOUBLE PRECISION for pdsyrk
COMPLEX for pcsyrk
DOUBLE COMPLEX for pzsyrk
Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)).
Before entry with *uplo* = 'U' or 'u', this array contains *n*-by-*n* upper triangular part of the symmetric distributed matrix sub(*C*) and its strictly lower triangular part is not referenced.
Before entry with *uplo* = 'L' or 'l', this array contains *n*-by-*n* lower triangular part of the symmetric distributed matrix sub(*C*) and its strictly upper triangular part is not referenced.

ic, *jc* (global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.

descc (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c With *uplo* = 'U' or 'u', the upper triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.

With *uplo* = 'L' or 'l', the lower triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.

p?syr2k

Performs a rank-2k update of a symmetric distributed matrix.

Syntax

call pssyr2k(*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

call pdsyr2k(*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

call pcsyr2k(*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

call pzsyr2k(*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desca*, *b*, *ib*, *jb*, *descb*, *beta*, *c*, *ic*, *jc*, *descc*)

Include Files

- C: mkl_pblas.h

Description

The p?syr2k routines perform a distributed matrix-matrix operation defined as

sub(*C*):=*alpha**sub(*A*)*sub(*B*)'+*alpha**sub(*B*)*sub(*A*)'+ *beta**sub(*C*),

or

sub(*C*):=*alpha**sub(*A*)'*sub(*B*) +*alpha**sub(*B*)'*sub(*A*) + *beta**sub(*C*),

where:

alpha and *beta* are scalars,

sub(*C*) is an *n*-by-*n* symmetric distributed matrix, sub(*C*)=C(*ic*:*ic*+*n*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*k*-1), if *trans* = 'N' or 'n', and sub(*A*)=*A*(*ia*:*ia*+*k*-1, *ja*:*ja*+*n*-1) otherwise.

sub(*B*) is a distributed matrix, sub(*B*)=*B*(*ib*:*ib*+*n*-1, *jb*:*jb*+*k*-1), if *trans* = 'N' or 'n', and sub(*B*)=*B*(*ib*:*ib*+*k*-1, *jb*:*jb*+*n*-1) otherwise.

Input Parameters

uplo — (global) CHARACTER*1. Specifies whether the upper or lower triangular part of the symmetric distributed matrix sub(*C*) is used:
If *uplo* = 'U' or 'u', then the upper triangular part of the sub(*C*) is used.
If *uplo* = 'L' or 'l', then the low triangular part of the sub(*C*) is used.

trans — (global) CHARACTER*1. Specifies the operation:
if *trans* = 'N' or 'n', then sub(*C*) := *alpha**sub(*A*)*sub(*B*)' + *alpha**sub(*B*)*sub(*A*)' + *beta**sub(*C*);
if *trans* = 'T' or 't', then sub(*C*) := *alpha**sub(*B*)'*sub(*A*) + *alpha**sub(*A*)'*sub(*B*) + *beta**sub(*C*).

n — (global) INTEGER. Specifies the order of the distributed matrix sub(*C*), $n \geq 0$.

k (global) INTEGER. On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the distributed matrices sub(*A*) and sub(*B*), and on entry with *trans* = 'T' or 't' , *k* specifies the number of rows of the distributed matrices sub(*A*) and sub(*B*), $k \geq 0$.

alpha (global) REAL for pssyr2k
DOUBLE PRECISION for pdsyr2k
COMPLEX for pcsyr2k
DOUBLE COMPLEX for pzsyr2k
Specifies the scalar *alpha*.

a (local) REAL for pssyr2k
DOUBLE PRECISION for pdsyr2k
COMPLEX for pcsyr2k
DOUBLE COMPLEX for pzsyr2k
Array, DIMENSION (*lld_a*, *kla*), where *kla* is LOCq(*ja*+*k*-1) when *trans* = 'N' or 'n', and is LOCq(*ja*+*n*-1) otherwise. Before entry with *trans* = 'N' or 'n', this array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja* (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b (local) REAL for pssyr2k
DOUBLE PRECISION for pdsyr2k
COMPLEX for pcsyr2k
DOUBLE COMPLEX for pzsyr2k
Array, DIMENSION (*lld_b*, *klb*), where *klb* is LOCq(*jb*+*k*-1) when *trans* = 'N' or 'n', and is LOCq(*jb*+*n*-1) otherwise. Before entry with *trans* = 'N' or 'n', this array contains the local pieces of the distributed matrix sub(*B*).

ib, *jb* (global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.

beta (global) REAL for pssyr2k
DOUBLE PRECISION for pdsyr2k
COMPLEX for pcsyr2k
DOUBLE COMPLEX for pzsyr2k
Specifies the scalar *beta*.

c (local) REAL for pssyr2k
DOUBLE PRECISION for pdsyr2k
COMPLEX for pcsyr2k
DOUBLE COMPLEX for pzsyr2k
Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)).
Before entry with *uplo* = 'U' or 'u', this array contains *n*-by-*n* upper triangular part of the symmetric distributed matrix sub(*C*) and its strictly lower triangular part is not referenced.

	Before entry with *uplo* = 'L' or 'l', this array contains *n*-by-*n* lower triangular part of the symmetric distributed matrix sub(*C*) and its strictly upper triangular part is not referenced.
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	With *uplo* = 'U' or 'u', the upper triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix. With *uplo* = 'L' or 'l', the lower triangular part of sub(*C*) is overwritten by the upper triangular part of the updated distributed matrix.

p?tran

Transposes a real distributed matrix.

Syntax

```
call pstran(m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pdtran(m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: mkl_pblas.h

Description

The p?tran routines transpose a real distributed matrix. The operation is defined as

```
sub(C):=beta*sub(C) + alpha*sub(A)',
```

where:

alpha and *beta* are scalars,

sub(*C*) is an *m*-by-*n* distributed matrix, sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*m*-1).

Input Parameters

m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*C*), $m \geq 0$.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*C*) , $n \geq 0$.
alpha	(global) REAL for pstran DOUBLE PRECISION for pdtran Specifies the scalar *alpha*.
a	(local) REAL for pstran DOUBLE PRECISION for pdtran Array, DIMENSION (*lld_a*, LOCq(*ja*+*m*-1)). This array contains the local pieces of the distributed matrix sub(*A*).

ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
beta	(global) REAL for pstran DOUBLE PRECISION for pdtran Specifies the scalar *beta*. When *beta* is equal to zero, then sub(*C*) need not be set on input.
c	(local) REAL for pstran DOUBLE PRECISION for pdtran Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the updated submatrix.

p?tranu

Transposes a distributed complex matrix.

Syntax

```
call pctranu(m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pztranu(m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: mkl_pblas.h

Description

The p?tranu routines transpose a complex distributed matrix. The operation is defined as

```
sub(C):=beta*sub(C) + alpha*sub(A)',
```

where:

alpha and *beta* are scalars,

sub(*C*) is an *m*-by-*n* distributed matrix, sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*m*-1).

Input Parameters

m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*C*), $m \geq 0$.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*C*) , $n \geq 0$.
alpha	(global) COMPLEX for pctranu

	DOUBLE COMPLEX for pztranu Specifies the scalar *alpha*.
a	(local) COMPLEX for pctranu DOUBLE COMPLEX for pztranu Array, DIMENSION (*lld_a*, LOCq(*ja*+*m*-1)). This array contains the local pieces of the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
beta	(global) COMPLEX for pctranu DOUBLE COMPLEX for pztranu Specifies the scalar *beta*. When *beta* is equal to zero, then sub(*C*) need not be set on input.
c	(local) COMPLEX for pctranu DOUBLE COMPLEX for pztranu Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the updated submatrix.

p?tranc

Transposes a complex distributed matrix, conjugated.

Syntax

```
call pctranc(m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
call pztranc(m, n, alpha, a, ia, ja, desca, beta, c, ic, jc, descc)
```

Include Files

- C: mkl_pblas.h

Description

The p?tranc routines transpose a complex distributed matrix. The operation is defined as

```
sub(C):=beta*sub(C) + alpha*conjg(sub(A)'),
```

where:

alpha and *beta* are scalars,

sub(*C*) is an *m*-by-*n* distributed matrix, sub(*C*)=*C*(*ic*:*ic*+*m*-1, *jc*:*jc*+*n*-1).

sub(*A*) is a distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*m*-1).

Input Parameters

m	(global) INTEGER. Specifies the number of rows of the distributed matrix sub(*C*), $m \geq 0$.
n	(global) INTEGER. Specifies the number of columns of the distributed matrix sub(*C*), $n \geq 0$.
alpha	(global) COMPLEX for pctranc DOUBLE COMPLEX for pztranc Specifies the scalar *alpha*.
a	(local) COMPLEX for pctranc DOUBLE COMPLEX for pztranc Array, DIMENSION (*lld_a*, LOCq(*ja*+*m*-1)). This array contains the local pieces of the distributed matrix sub(*A*).
ia, *ja*	(global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.
desca	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.
beta	(global) COMPLEX for pctranc DOUBLE COMPLEX for pztranc Specifies the scalar *beta*. When *beta* is equal to zero, then sub(*C*) need not be set on input.
c	(local) COMPLEX for pctranc DOUBLE COMPLEX for pztranc Array, DIMENSION (*lld_c*, LOCq(*jc*+*n*-1)). This array contains the local pieces of the distributed matrix sub(*C*).
ic, *jc*	(global) INTEGER. The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix sub(*C*), respectively.
descc	(global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *C*.

Output Parameters

c	Overwritten by the updated submatrix.

p?trmm

Computes a scalar-matrix-matrix product (one matrix operand is triangular) for distributed matrices.

Syntax

```
call pstrmm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
call pdtrmm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
call pctrmm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
call pztrmm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
```

Include Files

- C: mkl_pblas.h

Description

The p?trmm routines perform a matrix-matrix operation using triangular matrices. The operation is defined as

```
sub(B) := alpha*op(sub(A))*sub(B)
```

or

```
sub(B) := alpha*sub(B)*op(sub(A))
```

where:

alpha is a scalar,

sub(*B*) is an *m*-by-*n* distributed matrix, sub(*B*)=*B*(*ib*:*ib*+*m*-1, *jb*:*jb*+*n*-1).

A is a unit, or non-unit, upper or lower triangular distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*m*-1), if *side* = 'L' or 'l', and sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1), if *side* = 'R' or 'r'.

op(sub(*A*)) is one of op(sub(*A*)) = sub(*A*), or op(sub(*A*)) = sub(*A*)', or op(sub(*A*)) = conjg(sub(*A*)').

Input Parameters

side — (global)CHARACTER*1. Specifies whether op(sub(*A*)) appears on the left or right of sub(*B*) in the operation:
if *side* = 'L' or 'l', then sub(*B*) := *alpha**op(sub(*A*))*sub(*B*);
if *side* = 'R' or 'r', then sub(*B*) := *alpha**sub(*B*)*op(sub(*A*)).

uplo — (global) CHARACTER*1. Specifies whether the distributed matrix sub(*A*) is upper or lower triangular:
if *uplo* = 'U' or 'u', then the matrix is upper triangular;
if *uplo* = 'L' or 'l', then the matrix is low triangular.

transa — (global) CHARACTER*1. Specifies the form of op(sub(*A*)) used in the matrix multiplication:
if *transa* = 'N' or 'n', then op(sub(*A*)) = sub(*A*);
if *transa* = 'T' or 't', then op(sub(*A*)) = sub(*A*)' ;
if *transa* = 'C' or 'c', then op(sub(*A*)) = conjg(sub(*A*)').

diag — (global) CHARACTER*1. Specifies whether the matrix sub(*A*) is unit triangular:
if *diag* = 'U' or 'u' then the matrix is unit triangular;
if *diag* = 'N' or 'n', then the matrix is not unit triangular.

m — (global) INTEGER. Specifies the number of rows of the distributed matrix sub(*B*), $m \geq 0$.

n — (global) INTEGER. Specifies the number of columns of the distributed matrix sub(*B*), $n \geq 0$.

alpha — (global) REAL for pstrmm
DOUBLE PRECISION for pdtrmm
COMPLEX for pctrmm
DOUBLE COMPLEX for pztrmm
Specifies the scalar *alpha*.
When *alpha* is zero, then the array*b* need not be set before entry.

a — (local) REAL for pstrmm
DOUBLE PRECISION for pdtrmm
COMPLEX for pctrmm
DOUBLE COMPLEX for pztrmm

Array, DIMENSION (*lld_a*,*ka*), where *ka* is at least LOCq(1, *ja*+*m*-1) when *side* = 'L' or 'l' and is at least LOCq(1, *ja*+*n*-1) when *side* = 'R' or 'r'.
Before entry with *uplo* = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix sub(*A*) is not referenced.
Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix sub(*A*) is not referenced .
When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix sub(*A*) are not referenced either, but are assumed to be unity.

ia, *ja* — (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca — (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b — (local) REAL for pstrmm
DOUBLE PRECISION for pdtrmm
COMPLEX for pctrmm
DOUBLE COMPLEX for pztrmm
Array, DIMENSION (*lld_b*, LOCq(1, *jb*+*n*-1)).
Before entry, this array contains the local pieces of the distributed matrix sub(*B*).

ib, *jb* — (global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb — (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.

Output Parameters

b — Overwritten by the transformed distributed matrix.

p?trsm

Solves a distributed matrix equation (one matrix operand is triangular).

Syntax

```
call pstrsm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
call pdtrsm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
call pctrsm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
call pztrsm(side, uplo, transa, diag, m, n, alpha, a, ia, ja, desca, b, ib, jb, descb)
```

Include Files

- C: mkl_pblas.h

Description

The p?trsm routines solve one of the following distributed matrix equations:

op(sub(*A*))***X* = *alpha**sub(*B*),

or

*X**op(sub(*A*)) = *alpha**sub(*B*),

where:

alpha is a scalar,

X and sub(*B*) are *m*-by-*n* distributed matrices, sub(*B*)=*B*(*ib*:*ib*+*m*-1, *jb*:*jb*+*n*-1);

A is a unit, or non-unit, upper or lower triangular distributed matrix, sub(*A*)=*A*(*ia*:*ia*+*m*-1, *ja*:*ja*+*m*-1), if *side* = 'L' or 'l', and sub(*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1), if *side* = 'R' or 'r';

op(sub(*A*)) is one of op(sub(*A*)) = sub(*A*), or op(sub(*A*)) = sub(*A*)', or op(sub(*A*)) = conjg(sub(*A*)').

The distributed matrix sub(*B*) is overwritten by the solution matrix *X*.

Input Parameters

side (global)CHARACTER*1. Specifies whether op(sub(*A*)) appears on the left or right of *X* in the equation:
if *side* = 'L' or 'l', then op(sub(*A*))**X* = *alpha**sub(*B*);
if *side* = 'R' or 'r', then *X**op(sub(*A*)) = *alpha**sub(*B*).

uplo (global) CHARACTER*1. Specifies whether the distributed matrix sub(*A*) is upper or lower triangular:
if *uplo* = 'U' or 'u', then the matrix is upper triangular;
if *uplo* = 'L' or 'l', then the matrix is low triangular.

transa (global) CHARACTER*1. Specifies the form of op(sub(*A*)) used in the matrix equation:
if *transa* = 'N' or 'n', then op(sub(*A*)) = sub(*A*);
if *transa* = 'T' or 't', then op(sub(*A*)) = sub(*A*)';
if *transa* = 'C' or 'c', then op(sub(*A*)) = conjg(sub(*A*)').

diag (global) CHARACTER*1. Specifies whether the matrix sub(*A*) is unit triangular:
if *diag* = 'U' or 'u' then the matrix is unit triangular;
if *diag* = 'N' or 'n', then the matrix is not unit triangular.

m (global) INTEGER. Specifies the number of rows of the distributed matrix sub(*B*), $m \geq 0$.

n (global) INTEGER. Specifies the number of columns of the distributed matrix sub(*B*), $n \geq 0$.

alpha (global) REAL for pstrsm
DOUBLE PRECISION for pdtrsm
COMPLEX for pctrsm
DOUBLE COMPLEX for pztrsm
Specifies the scalar *alpha*.
When *alpha* is zero, then *a* is not referenced and *b* need not be set before entry.

a (local) REAL for pstrsm
DOUBLE PRECISION for pdtrsm
COMPLEX for pctrsm
DOUBLE COMPLEX for pztrsm

Array, DIMENSION (*lld_a*, *ka*), where *ka* is at least LOCq(1, *ja*+*m*-1) when *side* = 'L' or 'l' and is at least LOCq(1, *ja*+*n*-1) when *side* = 'R' or 'r'.
Before entry with *uplo* = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix sub(*A*) is not referenced.
Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix sub(*A*), and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix sub(*A*) is not referenced .
When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix sub(*A*) are not referenced either, but are assumed to be unity.

ia, *ja* — (global) INTEGER. The row and column indices in the distributed matrix *A* indicating the first row and the first column of the submatrix sub(*A*), respectively.

desca — (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *A*.

b — (local) REAL for pstrsm
DOUBLE PRECISION for pdtrsm
COMPLEX for pctrsm
DOUBLE COMPLEX for pztrsm
Array, DIMENSION (*lld_b*, LOCq(1, *jb*+*n*-1)).
Before entry, this array contains the local pieces of the distributed matrix sub(*B*).

ib, *jb* — (global) INTEGER. The row and column indices in the distributed matrix *B* indicating the first row and the first column of the submatrix sub(*B*), respectively.

descb — (global and local) INTEGER array of dimension 8. The array descriptor of the distributed matrix *B*.

Output Parameters

b — Overwritten by the solution distributed matrix *X*.

Partial Differential Equations Support 13

The Intel® Math Kernel Library (Intel® MKL) provides tools for solving Partial Differential Equations (PDE). These tools are Trigonometric Transform interface routines (see Trigonometric Transform Routines) and Poisson Library (see Poisson Library Routines).

Poisson Library is designed for fast solving of simple Helmholtz, Poisson, and Laplace problems. The solver is based on the Trigonometric Transform interface, which is, in turn, based on the Intel MKL Fast Fourier Transform (FFT) interface (refer to Fourier Transform Functions), optimized for Intel® processors.

Direct use of the Trigonometric Transform routines may be helpful to those who have already implemented their own solvers similar to the one that the Poisson Library provides. As it may be hard enough to modify the original code so as to make it work with Poisson Library, you are encouraged to use fast (staggered) sine/cosine transforms implemented in the Trigonometric Transform interface to improve performance of your solver.

Both Trigonometric Transform and Poisson Library routines can be called from C and Fortran 90, although the interfaces description uses C convention. Fortran 90 users can find routine calls specifics in the "Calling PDE Support Routines from Fortran 90" section.

Trigonometric Transform Routines

In addition to the Fast Fourier Transform (FFT) interface, described in chapter "Fast Fourier Transforms", Intel® MKL supports the Real Discrete Trigonometric Transforms (sometimes called real-to-real Discrete Fourier Transforms) interface. In this manual, the interface is referred to as TT interface. It implements a group of routines (TT routines) used to compute sine/cosine, staggered sine/cosine, and twice staggered sine/cosine transforms (referred to as staggered2 sine/cosine transforms, for brevity). The TT interface provides much flexibility of use: you can adjust routines to your particular needs at the cost of manual tuning routine parameters or just call routines with default parameter values. The current Intel MKL implementation of the TT interface can be used in solving partial differential equations and contains routines that are helpful for Fast Poisson and similar solvers.

To describe the Intel MKL TT interface, the C convention is used. Fortran users should refer to Calling PDE Support Routines from Fortran 90.

For the list of Trigonometric Transforms currently implemented in Intel MKL TT interface, see Transforms Implemented.

If you have got used to the FFTW interface (www.fftw.org), you can call the TT interface functions through real-to-real FFTW to Intel MKL wrappers without changing FFTW function calls in your code (refer to the "FFTW to Intel® MKL Wrappers for FFTW 3.x" section in Appendix F for details). However, you are strongly encouraged to use the native TT interface for better performance. Another reason why you should use the wrappers cautiously is that TT and the real-to-real FFTW interfaces are not fully compatible and some features of the real-to-real FFTW, such as strides and multidimensional transforms, are not available through wrappers.

Transforms Implemented

TT routines allow computing the following transforms:

Forward sine transform

$$F(k) = \frac{2}{n}\sum_{i=1}^{n-1} f(i)\sin\frac{ki\pi}{n},\ k = 1,\ldots,n-1$$

Backward sine transform

$$f(i) = \sum_{k=1}^{n-1} F(k)\sin\frac{ki\pi}{n},\ i = 1,\ldots,n-1$$

Forward staggered sine transform

$$F(k) = \frac{1}{n}\sin\frac{(2k-1)\pi}{2} f(n) + \frac{2}{n}\sum_{i=1}^{n-1} f(i)\sin\frac{(2k-1)i\pi}{2n},\ k = 1,\ldots,n$$

Backward staggered sine transform

$$f(i) = \sum_{k=1}^{n} F(k)\sin\frac{(2k-1)i\pi}{2n},\ i = 1,\ldots,n$$

Forward staggered2 sine transform

$$F(k) = \frac{2}{n}\sum_{i=1}^{n} f(i)\sin\frac{(2k-1)(2i-1)\pi}{4n},\ k = 1,\ldots,n$$

Backward staggered2 sine transform

$$f(i) = \sum_{k=1}^{n} F(k)\sin\frac{(2k-1)(2i-1)\pi}{4n},\ i = 1,\ldots,n$$

Forward cosine transform

$$F(k) = \frac{1}{n}\left[f(0) + f(n)\cos k\pi\right] + \frac{2}{n}\sum_{i=1}^{n-1} f(i)\cos\frac{ki\pi}{n},\ k = 0,\ldots,n$$

Backward cosine transform

$$f(i)=\frac{1}{2}\left[F(0)+F(n)\cos i\pi\right]+\sum_{k=1}^{n-1}F(k)\cos\frac{ki\pi}{n},\ i=0,\ldots,n$$

Forward staggered cosine transform

$$F(k)=\frac{1}{n}f(0)+\frac{2}{n}\sum_{i=1}^{n-1}f(i)\cos\frac{(2k+1)i\pi}{2n},\ k=0,\ldots,n-1$$

Backward staggered cosine transform

$$f(i)=\sum_{k=0}^{n-1}F(k)\cos\frac{(2k+1)i\pi}{2n},\ i=0,\ldots,n-1$$

Forward staggered2 cosine transform

$$F(k)=\frac{2}{n}\sum_{i=1}^{n}f(i)\cos\frac{(2k-1)(2i-1)\pi}{4n},\ k=1,\ldots,n$$

Backward staggered2 cosine transform

$$f(i)=\sum_{k=1}^{n}F(k)\cos\frac{(2k-1)(2i-1)\pi}{4n},\ i=1,\ldots,n$$

NOTE The size of the transform n can be any integer greater or equal to 2.

Sequence of Invoking TT Routines

Computation of a transform using TT interface is conceptually divided into four steps, each of which is performed via a dedicated routine. Table "TT Interface Routines" lists the routines and briefly describes their purpose and use.

Most TT routines have versions operating with single-precision and double-precision data. Names of such routines begin respectively with "s" and "d". The wildcard "?" stands for either of these symbols in routine names.

TT Interface Routines

Routine	Description
?_init_trig_transform	Initializes basic data structures of Trigonometric Transforms.
?_commit_trig_transform	Checks consistency and correctness of user-defined data as well as creates a data structure to be used by Intel MKL FFT interface[1].

Routine	Description
?_forward_trig_transform ?_backward_trig_transform	Computes a forward/backward Trigonometric Transform of a specified type using the appropriate formula (see Transforms Implemented).
free_trig_transform	Cleans the memory used by a data structure needed for calling FFT interface[1].

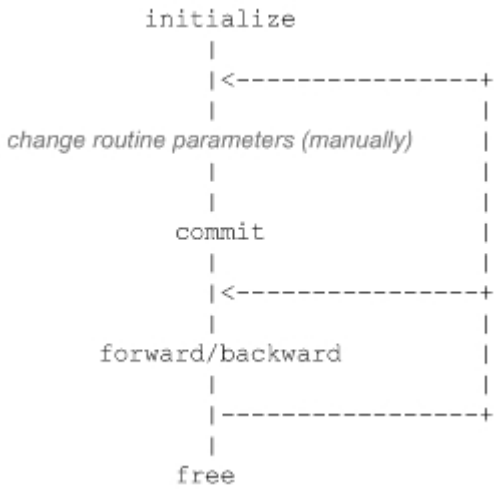
[1]TT routines call Intel MKL FFT interface for better performance.

To find a transformed vector for a particular input vector only once, the Intel MKL TT interface routines are normally invoked in the order in which they are listed in Table "TT Interface Routines".

NOTE Though the order of invoking TT routines may be changed, it is highly recommended to follow the above order of routine calls.

The diagram in Figure "Typical Order of Invoking TT Interface Routines" indicates the typical order in which TT interface routines can be invoked in a general case (prefixes and suffixes in routine names are omitted).

Typical Order of Invoking TT Interface Routines



A general scheme of using TT routines for double-precision computations is shown below. A similar scheme holds for single-precision computations with the only difference in the initial letter of routine names.

```
...
    d_init_trig_transform(&n, &tt_type, ipar, dpar, &ir);
/* Change parameters in ipar if necessary. */
/* Note that the result of the Transform will be in f ! If you want to preserve the data stored in f,
save them before this place in your code */
    d_commit_trig_transform(f, &handle, ipar, dpar, &ir);
    d_forward_trig_transform(f, &handle, ipar, dpar, &ir);
    d_backward_trig_transform(f, &handle, ipar, dpar, &ir);
    free_trig_transform(&handle, ipar, &ir);
/* here the user may clean the memory used by f, dpar, ipar */
...
```

You can find examples of Fortran 90 and C code that use TT interface routines to solve one-dimensional Helmholtz problem in the `examples\pdettf\source` and `examples\pdettc\source` folders of your Intel MKL directory.

Interface Description

All types in this documentation are standard C types: `int`, `float`, and `double`. Fortran 90 users can call the routines with `INTEGER`, `REAL`, and `DOUBLE PRECISION` Fortran types, respectively (see examples in the `examples\pdettf\source` and `examples\pdettc\source` folders of your Intel MKL directory).

The interface description uses the built-in type `int` for integer values. If you employ the ILP64 interface, read this type as `long long int` (or `INTEGER*8` for Fortran). For more information, refer to the *Intel MKL User's Guide*.

Routine Options

All TT routines use parameters to pass various options to one another. These parameters are arrays *ipar*, *dpar* and *spar*. Values for these parameters should be specified very carefully (see Common Parameters). You can change these values during computations to meet your needs.

WARNING To avoid failure or wrong results, you must provide correct and consistent parameters to the routines.

User Data Arrays

TT routines take arrays of user data as input. For example, user arrays are passed to the routine `d_forward_trig_transform` to compute a forward Trigonometric Transform. To minimize storage requirements and improve the overall run-time efficiency, Intel MKL TT routines do not make copies of user input arrays.

NOTE If you need a copy of your input data arrays, save them yourself.

TT Routines

The section gives detailed description of TT routines, their syntax, parameters and values they return. Double-precision and single-precision versions of the same routine are described together.

TT routines call Intel MKL FFT interface (described in section "FFT Functions" in chapter "Fast Fourier Transforms"), which enhances performance of the routines.

?_init_trig_transform

Initializes basic data structures of a Trigonometric Transform.

Syntax

```
void d_init_trig_transform(int *n, int *tt_type, int ipar[], double dpar[], int *stat);
void s_init_trig_transform(int *n, int *tt_type, int ipar[], float spar[], int *stat);
```

Include Files

- FORTRAN 90: `mkl_trig_transforms.f90`
- C: `mkl_trig_transforms.h`

Input Parameters

n	int*. Contains the size of the problem, which should be a positive integer greater than 1. Note that data vector of the transform, which other TT routines will use, must have size *n*+1 for all but staggered2 transforms. Staggered2 transforms require the vector of size *n*.
tt_type	int*. Contains the type of transform to compute, defined via a set of named constants. The following constants are available in the current implementation of TT interface: MKL_SINE_TRANSFORM, MKL_STAGGERED_SINE_TRANSFORM, MKL_STAGGERED2_SINE_TRANSFORM; MKL_COSINE_TRANSFORM, MKL_STAGGERED_COSINE_TRANSFORM, MKL_STAGGERED2_COSINE_TRANSFORM.

Output Parameters

ipar	int array of size 128. Contains integer data needed for Trigonometric Transform computations.
dpar	double array of size 5*n*/2+2. Contains double-precision data needed for Trigonometric Transform computations.
spar	float array of size 5*n*/2+2. Contains single-precision data needed for Trigonometric Transform computations.
stat	int*. Contains the routine completion status, which is also written to *ipar[6]*. The status should be 0 to proceed to other TT routines.

Description

The ?_init_trig_transform routine initializes basic data structures for Trigonometric Transforms of appropriate precision. After a call to ?_init_trig_transform, all subsequently invoked TT routines use values of *ipar* and *dpar* (*spar*) array parameters returned by ?_init_trig_transform. The routine initializes the entire array *ipar*. In the *dpar* or *spar* array, ?_init_trig_transform initializes elements that do not depend upon the type of transform. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the Common Parameters section. You can skip calling the initialization routine in your code. For more information, see Caveat on Parameter Modifications.

Return Values

stat= 0	The routine successfully completed the task. In general, to proceed with computations, the routine should complete with this *stat* value.
stat= -99999	The routine failed to complete the task.

?_commit_trig_transform

Checks consistency and correctness of user's data as well as initializes certain data structures required to perform the Trigonometric Transform.

Syntax

```
void d_commit_trig_transform(double f[], DFTI_DESCRIPTOR_HANDLE *handle, int ipar[],
double dpar[], int *stat);
```

```
void s_commit_trig_transform(float f[], DFTI_DESCRIPTOR_HANDLE *handle, int ipar[],
float spar[], int *stat);
```

Include Files

- FORTRAN 90: mkl_trig_transforms.f90

- C: `mkl_trig_transforms.h`

Input Parameters

f	`double` for `d_commit_trig_transform`, `float` for `s_commit_trig_transform`, array of size *n* for staggered2 transforms and of size *n*+1 for all other transforms, where *n* is the size of the problem. Contains data vector to be transformed. Note that the following values should be 0.0 up to rounding errors: • *f*[0] and *f*[*n*] for sine transforms • *f*[*n*] for staggered cosine transforms • *f*[0] for staggered sine transforms. Otherwise, the routine will produce a warning, and the result of the computations for sine transforms may be wrong. These restrictions meet the requirements of the Poisson Library (described in the Poisson Library Routines section), which the TT interface is primarily designed for.
ipar	`int` array of size 128. Contains integer data needed for Trigonometric Transform computations.
dpar	`double` array of size 5*n*/2+2. Contains double-precision data needed for Trigonometric Transform computations. The routine initializes most elements of this array.
spar	`float` array of size 5*n*/2+2. Contains single-precision data needed for Trigonometric Transform computations. The routine initializes most elements of this array.

Output Parameters

handle	`DFTI_DESCRIPTOR_HANDLE*`. The data structure used by Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms").
ipar	Contains integer data needed for Trigonometric Transform computations. On output, *ipar[6]* is updated with the *stat* value.
dpar	Contains double-precision data needed for Trigonometric Transform computations. On output, the entire array is initialized.
spar	Contains single-precision data needed for Trigonometric Transform computations. On output, the entire array is initialized.
stat	`int*`. Contains the routine completion status, which is also written to *ipar[6]*.

Description

The routine `?_commit_trig_transform` checks consistency and correctness of the parameters to be passed to the transform routines `?_forward_trig_transform` and/or `?_backward_trig_transform`. The routine also initializes the following data structures: *handle*, *dpar* in case of `d_commit_trig_transform`, and *spar* in case of `s_commit_trig_transform`. The `?_commit_trig_transform` routine initializes only those elements of *dpar* or *spar* that depend upon the type of transform, defined in the `?_init_trig_transform` routine and passed to `?_commit_trig_transform` with the *ipar* array. The size of the problem *n*, which determines sizes of the array parameters, is also passed to the routine with the *ipar* array and defined in the previously called `?_init_trig_transform` routine. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the Common Parameters section. The routine performs only a basic check for correctness and

consistency of the parameters. If you are going to modify parameters of TT routines, see the Caveat on Parameter Modifications section. Unlike `?_init_trig_transform`, the `?_commit_trig_transform` routine is mandatory, and you cannot skip calling it in your code.

Return Values

stat= 11	The routine produced some warnings and made some changes in the parameters to achieve their correctness and/or consistency. You may proceed with computations by assigning *ipar[6]*=0 if you are sure that the parameters are correct.
stat= 10	The routine made some changes in the parameters to achieve their correctness and/or consistency. You may proceed with computations by assigning *ipar[6]*=0 if you are sure that the parameters are correct.
stat= 1	The routine produced some warnings. You may proceed with computations by assigning *ipar[6]*=0 if you are sure that the parameters are correct.
stat= 0	The routine completed the task normally.
stat= -100	The routine stopped for any of the following reasons: • An error in the user's data was encountered. • Data in *ipar*, *dpar* or *spar* parameters became incorrect and/or inconsistent as a result of modifications.
stat= -1000	The routine stopped because of an FFT interface error.
stat= -10000	The routine stopped because the initialization failed to complete or the parameter *ipar[0]* was altered by mistake.

NOTE Although positive values of *stat* usually indicate minor problems with the input data and Trigonometric Transform computations can be continued, you are highly recommended to investigate the problem first and achieve *stat*=0.

?_forward_trig_transform

Computes the forward Trigonometric Transform of type specified by the parameter.

Syntax

```
void d_forward_trig_transform(double f[], DFTI_DESCRIPTOR_HANDLE *handle, int ipar[], double dpar[], int *stat);
```

```
void s_forward_trig_transform(float f[], DFTI_DESCRIPTOR_HANDLE *handle, int ipar[], float spar[], int *stat);
```

Include Files

- FORTRAN 90: `mkl_trig_transforms.f90`
- C: `mkl_trig_transforms.h`

Input Parameters

f	`double` for `d_forward_trig_transform`,

	`float` for `s_forward_trig_transform`, array of size *n* for staggered2 transforms and of size *n*+1 for all other transforms, where *n* is the size of the problem. On input, contains data vector to be transformed. Note that the following values should be 0.0 up to rounding errors: • *f*[0] and *f*[*n*] for sine transforms • *f*[*n*] for staggered cosine transforms • *f*[0] for staggered sine transforms. Otherwise, the routine will produce a warning, and the result of the computations for sine transforms may be wrong. The above restrictions meet the requirements of the Poisson Library (described in the Poisson Library Routines section), which the TT interface is primarily designed for.
handle	`DFTI_DESCRIPTOR_HANDLE*`. The data structure used by Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms").
ipar	`int` array of size 128. Contains integer data needed for Trigonometric Transform computations.
dpar	`double` array of size 5*n*/2+2. Contains double-precision data needed for Trigonometric Transform computations.
spar	`float` array of size 5*n*/2+2. Contains single-precision data needed for Trigonometric Transform computations.

Output Parameters

f	Contains the transformed vector on output.
ipar	Contains integer data needed for Trigonometric Transform computations. On output, *ipar[6]* is updated with the *stat* value.
stat	`int*`. Contains the routine completion status, which is also written to *ipar[6]*.

Description

The routine computes the forward Trigonometric Transform of type defined in the `?_init_trig_transform` routine and passed to `?_forward_trig_transform` with the *ipar* array. The size of the problem *n*, which determines sizes of the array parameters, is also passed to the routine with the *ipar* array and defined in the previously called `?_init_trig_transform` routine. The other data that facilitates the computation is created by `?_commit_trig_transform` and supplied in *dpar* or *spar*. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the Common Parameters section. The routine has a commit step, which calls the `?_commit_trig_transform` routine. The transform is computed according to formulas given in the Transforms Implemented section. The routine replaces the input vector *f* with the transformed vector.

NOTE If you need a copy of the data vector *f* to be transformed, make the copy before calling the `?_forward_trig_transform` routine.

Return Values

stat= 0	The routine completed the task normally.
stat= -100	The routine stopped for any of the following reasons: • An error in the user's data was encountered.

	• Data in *ipar*, *dpar* or *spar* parameters became incorrect and/or inconsistent as a result of modifications.
stat= -1000	The routine stopped because of an FFT interface error.
stat= -10000	The routine stopped because its commit step failed to complete or the parameter *ipar[0]* was altered by mistake.

?_backward_trig_transform

Computes the backward Trigonometric Transform of type specified by the parameter.

Syntax

```
void d_backward_trig_transform(double f[], DFTI_DESCRIPTOR_HANDLE *handle, int ipar[], double dpar[], int *stat);
```

```
void s_backward_trig_transform(float f[], DFTI_DESCRIPTOR_HANDLE *handle, int ipar[], float spar[], int *stat);
```

Include Files

- FORTRAN 90: `mkl_trig_transforms.f90`
- C: `mkl_trig_transforms.h`

Input Parameters

f	`double` for `d_backward_trig_transform`, `float` for `s_backward_trig_transform`, array of size *n* for staggered2 transforms and of size *n*+1 for all other transforms, where *n* is the size of the problem. On input, contains data vector to be transformed. Note that the following values should be 0.0 up to rounding errors: • *f*[0] and *f*[*n*] for sine transforms • *f*[*n*] for staggered cosine transforms • *f*[0] for staggered sine transforms. Otherwise, the routine will produce a warning, and the result of the computations for sine transforms may be wrong. The above restrictions meet the requirements of the Poisson Library (described in the Poisson Library Routines section), which the TT interface is primarily designed for.
handle	`DFTI_DESCRIPTOR_HANDLE*`. The data structure used by Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms").
ipar	`int` array of size 128. Contains integer data needed for Trigonometric Transform computations.
dpar	`double` array of size 5*n*/2+2. Contains double-precision data needed for Trigonometric Transform computations.
spar	`float` array of size 5*n*/2+2. Contains single-precision data needed for Trigonometric Transform computations.

Output Parameters

f	Contains the transformed vector on output.
ipar	Contains integer data needed for Trigonometric Transform computations. On output, *ipar[6]* is updated with the *stat* value.
stat	int*. Contains the routine completion status, which is also written to *ipar[6]*.

Description

The routine computes the backward Trigonometric Transform of type defined in the ?_init_trig_transform routine and passed to ?_backward_trig_transform with the *ipar* array. The size of the problem *n*, which determines sizes of the array parameters, is also passed to the routine with the *ipar* array and defined in the previously called ?_init_trig_transform routine. The other data that facilitates the computation is created by ?_commit_trig_transform and supplied in *dpar* or *spar*. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the Common Parameters section. The routine has a commit step, which calls the ?_commit_trig_transform routine. The transform is computed according to formulas given in the Transforms Implemented section. The routine replaces the input vector *f* with the transformed vector.

NOTE If you need a copy of the data vector *f* to be transformed, make the copy before calling the ?_backward_trig_transform routine.

Return Values

stat= 0	The routine completed the task normally.
stat= -100	The routine stopped for any of the following reasons: • An error in the user's data was encountered. • Data in *ipar*, *dpar* or *spar* parameters became incorrect and/or inconsistent as a result of modifications.
stat= -1000	The routine stopped because of an FFT interface error.
stat= -10000	The routine stopped because its commit step failed to complete or the parameter *ipar[0]* was altered by mistake.

free_trig_transform

Cleans the memory allocated for the data structure used by the FFT interface.

Syntax

```
void free_trig_transform(DFTI_DESCRIPTOR_HANDLE *handle, int ipar[], int *stat);
```

Include Files

- FORTRAN 90: mkl_trig_transforms.f90
- C: mkl_trig_transforms.h

Input Parameters

ipar	int array of size 128. Contains integer data needed for Trigonometric Transform computations.

handle	DFTI_DESCRIPTOR_HANDLE*. The data structure used by Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms").

Output Parameters

handle	The data structure used by Intel MKL FFT interface. Memory allocated for the structure is released on output.
ipar	Contains integer data needed for Trigonometric Transform computations. On output, *ipar[6]* is updated with the *stat* value.
stat	int*. Contains the routine completion status, which is also written to *ipar[6]*.

Description

The free_trig_transform routine cleans the memory used by the *handle* structure, needed for Intel MKL FFT functions. To release the memory allocated for other parameters, include cleaning of the memory in your code.

Return Values

stat= 0	The routine completed the task normally.
stat= -1000	The routine stopped because of an FFT interface error.
stat= -99999	The routine failed to complete the task.

Common Parameters

This section provides description of array parameters that hold TT routine options: *ipar*, *dpar* and *spar*.

NOTE Initial values are assigned to the array parameters by the appropriate ?_init_trig_transform and ?_commit_trig_transform routines.

ipar	int array of size 128, holds integer data needed for Trigonometric Transform computations. Its elements are described in Table "Elements of the ipar Array":

Elements of the ipar Array

Index	Description
0	Contains the size of the problem to solve. The ?_init_trig_transform routine sets *ipar[0]*=*n*, and all subsequently called TT routines use *ipar[0]* as the size of the transform.
1	Contains error messaging options: • *ipar[1]*=-1 indicates that all error messages will be printed to the file MKL_Trig_Transforms_log.txt in the folder from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device. • *ipar[1]*=0 indicates that no error messages will be printed. • *ipar[1]*=1 (default) indicates that all error messages will be printed to the preconnected default output device (usually, screen). In case of errors, each TT routine assigns a non-zero value to *stat* regardless of the *ipar[1]* setting.

Index	Description
2	Contains warning messaging options: • *ipar[2]*=-1 indicates that all warning messages will be printed to the file MKL_Trig_Transforms_log.txt in the directory from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device. • *ipar[2]*=0 indicates that no warning messages will be printed. • *ipar[2]*=1 (default) indicates that all warning messages will be printed to the preconnected default output device (usually, screen). In case of warnings, the *stat* parameter will acquire a non-zero value regardless of the *ipar[2]* setting.
3 through 4	Reserved for future use.
5	Contains the type of the transform. The ?_init_trig_transform routine sets *ipar[5]*=*tt_type*, and all subsequently called TT routines use *ipar[5]* as the type of the transform.
6	Contains the *stat* value returned by the last completed TT routine. Used to check that the previous call to a TT routine completed with *stat*=0.
7	Informs the ?_commit_trig_transform routines whether to initialize data structures *dpar* (*spar*) and *handle*. *ipar[7]*=0 indicates that the routine should skip the initialization and only check correctness and consistency of the parameters. Otherwise, the routine initializes the data structures. The default value is 1. The possibility to check correctness and consistency of input data without initializing data structures *dpar*, *spar* and *handle* enables avoiding performance losses in a repeated use of the same transform for different data vectors. Note that you can benefit from the opportunity that *ipar[7]* gives only if you are sure to have supplied proper tolerance value in the *dpar* or *spar* array. Otherwise, avoid tuning this parameter.
8	Contains message style options for TT routines. If *ipar[8]*=0 then TT routines print all error and warning messages in Fortran-style notations. Otherwise, TT routines print the messages in C-style notations. The default value is 1. When selecting between these notations, mind that by default, numbering of elements in C arrays starts from 0 and in Fortran, it starts from 1. For example, for a C-style message "*parameter ipar[0]=3 should be an even integer*", the corresponding Fortran-style message will be "*parameter ipar(1)=3 should be an even integer*". The use of *ipar[8]* enables you to view messages in a more convenient style.
9	Specifies the number of OpenMP threads to run TT routines in the OpenMP environment of the Poisson Library. The default value is 1. You are highly recommended not to alter this value. See also Caveat on Parameter Modifications.
10	Specifies the mode of compatibility with FFTW. The default value is 0. Set the value to 1 to invoke compatibility with FFTW. In the latter case, results will not be normalized, because FFTW does not do this. It is highly recommended not to alter this value, but rather use real-to-real FFTW to MKL wrappers, described in the "FFTW to Intel® MKL Wrappers for FFTW 3.x" section in Appendix F. See also Caveat on Parameter Modifications.
11 through 127	Reserved for future use.

NOTE You may declare the *ipar* array in your code as `int ipar[11]`. However, for compatibility with later versions of Intel MKL TT interface, which may require more *ipar* values, it is highly recommended to declare *ipar* as `int ipar[128]`.

Arrays *dpar* and *spar* are the same except in the data precision:

dpar	`double` array of size 5*n*/2+2, holds data needed for double-precision routines to perform TT computations. This array is initialized in the `d_init_trig_transform` and `d_commit_trig_transform` routines.
spar	`float` array of size 5*n*/2+2, holds data needed for single-precision routines to perform TT computations. This array is initialized in the `s_init_trig_transform` and `s_commit_trig_transform` routines.

As *dpar* and *spar* have similar elements in respective positions, the elements are described together in Table "Elements of the dpar and spar Arrays":

Elements of the dpar and spar Arrays

Index	Description
0	Contains the first absolute tolerance used by the appropriate `?_commit_trig_transform` routine. For a staggered cosine or a sine transform, *f[n]* should be equal to 0.0 and for a staggered sine or a sine transform, *f[0]* should be equal to 0.0. The `?_commit_trig_transform` routine checks whether absolute values of these parameters are below *dpar[0]*n* or *spar[0]*n*, depending on the routine precision. To suppress warnings resulting from tolerance checks, set *dpar[0]* or *spar[0]* to a sufficiently large number.
1	Reserved for future use.
2 through 5*n*/2+1	Contain tabulated values of trigonometric functions. Contents of the elements depend upon the type of transform *tt_type*, set up in the `?_commit_trig_transform` routine: • If *tt_type*=MKL_SINE_TRANSFORM, the transform uses only the first *n*/2 array elements, which contain tabulated sine values. • If *tt_type*=MKL_STAGGERED_SINE_TRANSFORM, the transform uses only the first 3*n*/2 array elements, which contain tabulated sine and cosine values. • If *tt_type*=MKL_STAGGERED2_SINE_TRANSFORM, the transform uses all the 5*n*/2 array elements, which contain tabulated sine and cosine values. • If *tt_type*=MKL_COSINE_TRANSFORM, the transform uses only the first *n* array elements, which contain tabulated cosine values. • If *tt_type*=MKL_STAGGERED_COSINE_TRANSFORM, the transform uses only the first 3*n*/2 elements, which contain tabulated sine and cosine values. • If *tt_type*=MKL_STAGGERED2_COSINE_TRANSFORM, the transform uses all the 5*n*/2 elements, which contain tabulated sine and cosine values.

NOTE To save memory, you can define the array size depending upon the type of transform.

Caveat on Parameter Modifications

Flexibility of the TT interface enables you to skip calling the `?_init_trig_transform` routine and to initialize the basic data structures explicitly in your code. You may also need to modify the contents of *ipar*, *dpar* and *spar* arrays after initialization. When doing so, provide correct and consistent data in the arrays. Mistakenly altered arrays cause errors or wrong computation. You can perform a basic check for correctness and consistency of parameters by calling the `?_commit_trig_transform` routine; however, this does not ensure the correct result of a transform but only reduces the chance of errors or wrong results.

NOTE To supply correct and consistent parameters to TT routines, you should have considerable experience in using the TT interface and good understanding of elements that the *ipar*, *spar* and *dpar* arrays contain and dependencies between values of these elements.

However, in rare occurrences, even advanced users might fail to compute a transform using TT routines after the parameter modifications. In cases like these, refer for technical support at http://www.intel.com/software/products/support/ .

WARNING The only way that ensures proper computation of the Trigonometric Transforms is to follow a typical sequence of invoking the routines and not change the default set of parameters. So, avoid modifications of *ipar*, *dpar* and *spar* arrays unless a strong need arises.

Implementation Details

Several aspects of the Intel MKL TT interface are platform-specific and language-specific. To promote portability across platforms and ease of use across different languages, users are provided with the TT language-specific header files to include in their code. Currently, the following of them are available:

- `mkl_trig_transforms.h`, to be used together with `mkl_dfti.h`, for C programs.
- `mkl_trig_transforms.f90`, to be used together with `mkl_dfti.f90`, for Fortran 90 programs.

NOTE Use of the Intel MKL TT software without including one of the above header files is not supported.

C-specific Header File

The C-specific header file defines the following function prototypes:

```
void d_init_trig_transform(int *, int *, int *, double *, int *);
void d_commit_trig_transform(double *, DFTI_DESCRIPTOR_HANDLE *, int *, double *, int *);
void d_forward_trig_transform(double *, DFTI_DESCRIPTOR_HANDLE *, int *, double *, int *);
void d_backward_trig_transform(double *, DFTI_DESCRIPTOR_HANDLE *, int *, double *, int *);

void s_init_trig_transform(int *, int *, int *, float *, int *);
void s_commit_trig_transform(float *, DFTI_DESCRIPTOR_HANDLE *, int *, float *, int *);
void s_forward_trig_transform(float *, DFTI_DESCRIPTOR_HANDLE *, int *, float *, int *);
void s_backward_trig_transform(float *, DFTI_DESCRIPTOR_HANDLE *, int *, float *, int *);

void free_trig_transform(DFTI_DESCRIPTOR_HANDLE *, int *, int *);
```

Fortran-Specific Header File

The Fortran90-specific header file defines the following function prototypes:

```
SUBROUTINE D_INIT_TRIG_TRANSFORM(n, tt_type, ipar, dpar, stat)
    INTEGER, INTENT(IN) :: n, tt_type
    INTEGER, INTENT(INOUT) :: ipar(*)
    REAL(8), INTENT(INOUT) :: dpar(*)
    INTEGER, INTENT(OUT) :: stat
END SUBROUTINE D_INIT_TRIG_TRANSFORM

SUBROUTINE D_COMMIT_TRIG_TRANSFORM(f, handle, ipar, dpar, stat)
    REAL(8), INTENT(INOUT) :: f(*)
    TYPE(DFTI_DESCRIPTOR), POINTER :: handle
    INTEGER, INTENT(INOUT) :: ipar(*)
    REAL(8), INTENT(INOUT) :: dpar(*)
    INTEGER, INTENT(OUT) :: stat
END SUBROUTINE D_COMMIT_TRIG_TRANSFORM

SUBROUTINE D_FORWARD_TRIG_TRANSFORM(f, handle, ipar, dpar, stat)
    REAL(8), INTENT(INOUT) :: f(*)
    TYPE(DFTI_DESCRIPTOR), POINTER :: handle
    INTEGER, INTENT(INOUT) :: ipar(*)
    REAL(8), INTENT(INOUT) :: dpar(*)
    INTEGER, INTENT(OUT) :: stat
END SUBROUTINE D_FORWARD_TRIG_TRANSFORM

SUBROUTINE D_BACKWARD_TRIG_TRANSFORM(f, handle, ipar, dpar, stat)
    REAL(8), INTENT(INOUT) :: f(*)
    TYPE(DFTI_DESCRIPTOR), POINTER :: handle
    INTEGER, INTENT(INOUT) :: ipar(*)
    REAL(8), INTENT(INOUT) :: dpar(*)
    INTEGER, INTENT(OUT) :: stat
END SUBROUTINE D_BACKWARD_TRIG_TRANSFORM

SUBROUTINE S_INIT_TRIG_TRANSFORM(n, tt_type, ipar, spar, stat)
    INTEGER, INTENT(IN) :: n, tt_type
    INTEGER, INTENT(INOUT) :: ipar(*)
    REAL(4), INTENT(INOUT) :: spar(*)
    INTEGER, INTENT(OUT) :: stat
END SUBROUTINE S_INIT_TRIG_TRANSFORM

SUBROUTINE S_COMMIT_TRIG_TRANSFORM(f, handle, ipar, spar, stat)
    REAL(4), INTENT(INOUT) :: f(*)
    TYPE(DFTI_DESCRIPTOR), POINTER :: handle
```

Fortran 90 specifics of the TT routines usage are similar for all Intel MKL PDE support tools and described in the Calling PDE Support Routines from Fortran 90 section.

Poisson Library Routines

In addition to Real Discrete Trigonometric Transforms (TT) interface (refer to Trigonometric Transform Routines), Intel® MKL supports the Poisson Library interface, referred to as PL interface. The interface implements a group of routines (PL routines) used to compute a solution of Laplace, Poisson, and Helmholtz problems of special kind using discrete Fourier transforms. Laplace and Poisson problems are special cases of a more general Helmholtz problem. The problems being solved are defined more exactly in the Poisson Library Implemented subsection. The PL interface provides much flexibility of use: you can adjust routines to your particular needs at the cost of manual tuning routine parameters or just call routines with default parameter values. The interface can adjust style of error and warning messages to C or Fortran notations by setting up a dedicated parameter. This adds convenience to debugging, because users can read information in the way that is natural for their code. The Intel MKL PL interface currently contains only routines that implement the following solvers:

- Fast Laplace, Poisson and Helmholtz solvers in a Cartesian coordinate system
- Fast Poisson and Helmholtz solvers in a spherical coordinate system.

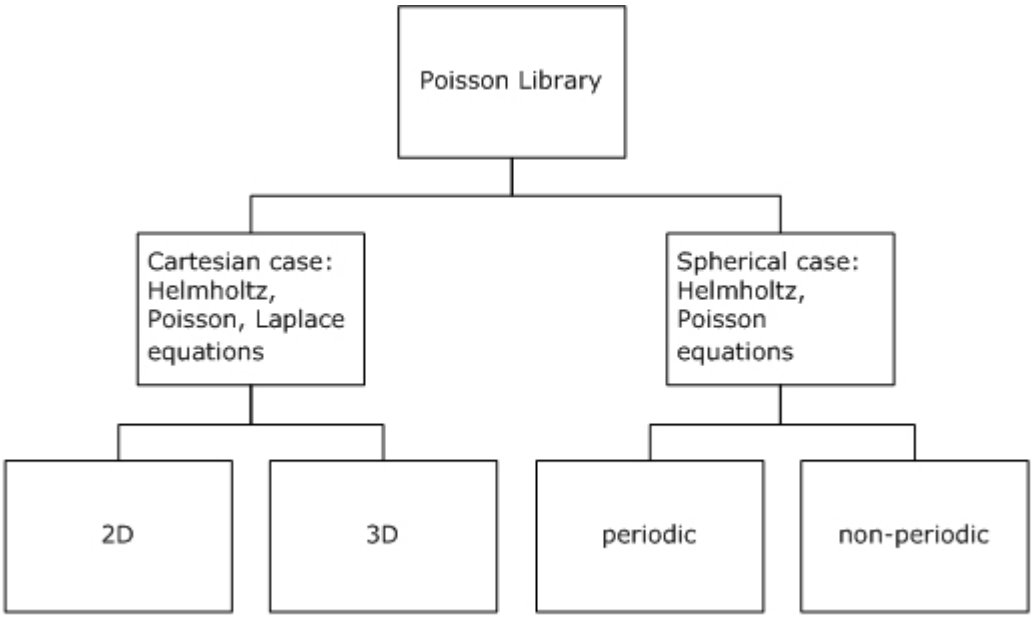
To describe the Intel MKL PL interface, the C convention is used. Fortran usage specifics can be found in the Calling PDE Support Routines from Fortran 90 section.

NOTE Fortran users should mind that respective array indices in Fortran increase by 1.

Poisson Library Implemented

PL routines enable approximate solving of certain two-dimensional and three-dimensional problems. Figure "Structure of the Poisson Library" shows the general structure of the Poisson Library.

Structure of the Poisson Library



Sections below provide details of the problems that can be solved using Intel MKL PL.

Two-Dimensional Problems

Notational Conventions

The PL interface description uses the following notation for boundaries of a rectangular domain $a_x < x < b_x$, $a_y < y < b_y$ on a Cartesian plane:

$bd\_a_x = \{x = a_x, a_y \le y \le b_y\}$, $bd\_b_x = \{x = b_x, a_y \le y \le b_y\}$

$bd\_a_y = \{a_x \le x \le b_x, y = a_y\}$, $bd\_b_y = \{a_x \le x \le b_x, y = b_y\}$.

The wildcard "+" may stand for any of the symbols a_x, b_x, a_y, b_y, so that bd_+ denotes any of the above boundaries.

The PL interface description uses the following notation for boundaries of a rectangular domain $a_\varphi < \varphi < b_\varphi$, $a_\theta < \theta < b_\theta$ on a sphere $0 \le \varphi \le 2\pi$, $0 \le \theta \le \pi$:

$bd\_a_\varphi = \{\varphi = a_\varphi, a_\theta \le \theta \le b_\theta\}$, $bd\_b_\varphi = \{\varphi = b_\varphi, a_\theta \le \theta \le b_\theta\}$

$bd\_a_\theta = \{a_\varphi \le \varphi \le b_\varphi, \theta = a_\theta\}$, $bd\_b_\theta = \{a_\varphi \le \varphi \le b_\varphi, \theta = b_\theta\}$.

The wildcard "~" may stand for any of the symbols a_φ, b_φ, a_θ, b_θ, so that $bd\_\sim$ denotes any of the above boundaries.

Two-dimensional (2D) Helmholtz problem on a Cartesian plane

The 2D Helmholtz problem is to find an approximate solution of the Helmholtz equation

$$-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}+qu=f(x,y), q=const \ge 0$$

in a rectangle, that is, a rectangular domain $a_x < x < b_x$, $a_y < y < b_y$, with one of the following boundary conditions on each boundary bd_+:

- The Dirichlet boundary condition

$$u(x,y)=G(x,y)$$

- The Neumann boundary condition

$$\frac{\partial u}{\partial n}(x,y)=g(x,y)$$

where

$n = -x$ on bd_a_x, $n = x$ on bd_b_x,

$n = -y$ on bd_a_y, $n = y$ on bd_b_y.

Two-dimensional (2D) Poisson problem on a Cartesian plane

The Poisson problem is a special case of the Helmholtz problem, when q=0. The 2D Poisson problem is to find an approximate solution of the Poisson equation

$$-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=f(x,y)$$

in a rectangle $a_x < x < b_x$, $a_y < y < b_y$ with the Dirichlet or Neumann boundary condition on each boundary `bd_+`. In case of a problem with the Neumann boundary condition on the entire boundary, you can find the solution of the problem only up to a constant. In this case, the Poisson Library will compute the solution that provides the minimal Euclidean norm of a residual.

Two-dimensional (2D) Laplace problem on a Cartesian plane

The Laplace problem is a special case of the Helmholtz problem, when $q=0$ and `f(x, y)`=0. The 2D Laplace problem is to find an approximate solution of the Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

in a rectangle $a_x < x < b_x$, $a_y < y < b_y$ with the Dirichlet or Neumann boundary condition on each boundary `bd_+`.

Helmholtz problem on a sphere

The Helmholtz problem on a sphere is to find an approximate solution of the Helmholtz equation

$$-\Delta_s u+qu=f,\ q=const\geq 0,$$
$$\Delta_s=\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

in a spherical rectangle that is, a domain bounded by angles $a_\varphi \leq \varphi \leq b_\varphi$, $a_\theta \leq \theta \leq b_\theta$, with boundary conditions for particular domains listed in Table "Details of Helmholtz Problem on a Sphere".

Details of Helmholtz Problem on a Sphere

Domain on a sphere	Boundary condition	Periodic/non-periodic case
Rectangular, that is, $b_\varphi - a_\varphi < 2\pi$ and $b_\theta - a_\theta < \pi$	Homogeneous Dirichlet boundary conditions on each boundary `bd_~`	*non-periodic*
Where $a_\varphi = 0$, $b_\varphi = 2\pi$, and $b_\theta - a_\theta < \pi$	Homogeneous Dirichlet boundary conditions on the boundaries `bd_`a_θ and `bd_`b_θ	*periodic*
Entire sphere, that is, $a_\varphi = 0$, $b_\varphi = 2\pi$, $a_\theta = 0$, and $b_\theta = \pi$	Boundary condition $\left(\sin\theta\frac{\partial u}{\partial\theta}\right)_{\substack{\theta\to 0\\ \theta\to\pi}}=0$ at the poles.	*periodic*

Poisson problem on a sphere

The Poisson problem is a special case of the Helmholtz problem, when q=0. The Poisson problem on a sphere is to find an approximate solution of the Poisson equation

$$-\Delta_s u = f,\ \Delta_s = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

in a spherical rectangle $a_\varphi \le \varphi \le b_\varphi$, $a_\theta \le \theta \le b_\theta$ in cases listed in Table "Details of Helmholtz Problem on a Sphere". The solution to the Poisson problem on the entire sphere can be found up to a constant only. In this case, Poisson Library will compute the solution that provides the minimal Euclidean norm of a residual.

Approximation of 2D problems

To find an approximate solution for any of the 2D problems, a uniform mesh is built in the rectangular domain:

$$\left\{x_i = a_x + ih_x, y_j = a_y + jh_y\right\},$$
$$i = 0,\ldots,n_x, j = 0,\ldots,n_y, h_x = \frac{b_x - a_x}{n_x}, h_y = \frac{b_y - a_y}{n_y}$$

in the Cartesian case and

$$\left\{\varphi_i = a_\varphi + ih_\varphi, \theta_j = a_\theta + jh_\theta\right\},$$
$$i = 0,\ldots,n_\varphi, j = 0,\ldots,n_\theta, h_\varphi = \frac{b_\varphi - a_\varphi}{n_\varphi}, h_\theta = \frac{b_\theta - a_\theta}{n_\theta}$$

in the spherical case.

Poisson Library uses the standard five-point finite difference approximation on this mesh to compute the approximation to the solution:

- In the Cartesian case, the values of the approximate solution will be computed in the mesh points (x_i , y_j) provided that the user knows the values of the right-hand side `f(x, y)` in these points and the values of the appropriate boundary functions `G(x, y)` and/or `g(x,y)` in the mesh points laying on the boundary of the rectangular domain.
- In the spherical case, the values of the approximate solution will be computed in the mesh points (φ_i , θ_j) provided that the user knows the values of the right-hand side $f(\varphi, \theta)$ in these points.

NOTE The number of mesh intervals n_φ in the φ direction of a spherical mesh must be even in the periodic case. The current implementation of the Poisson Library does not support meshes with the number of intervals that does not meet this condition.

Three-Dimensional Problems

Notational Conventions

The PL interface description uses the following notation for boundaries of a parallelepiped domain $a_x < x < b_x$, $a_y < y < b_y$, $a_z < z < b_z$:

$bd\_a_x = \{x = a_x, a_y \le y \le b_y, a_z \le z \le b_z\}$, $bd\_b_x = \{x = b_x, a_y \le y \le b_y, a_z \le z \le b_z\}$

$bd\_a_y = \{a_x \le x \le b_x, y = a_y, a_z \le z \le b_z\}$, $bd\_b_y = \{a_x \le x \le b_x, y = b_y, a_z \le z \le b_z\}$

$bd\_a_z = \{a_x \le x \le b_x, a_y \le y \le b_y, z = a_z\}$, $bd\_b_x = \{a_x \le x \le b_x, a_y \le y \le b_y, z = b_z\}$.

The wildcard "+" may stand for any of the symbols a_x, b_x, a_y, b_y, a_z, b_z, so that bd_+ denotes any of the above boundaries.

Three-dimensional (3D) Helmholtz problem

The 3D Helmholtz problem is to find an approximate solution of the Helmholtz equation

$$-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}+qu=f(x,y,z), q=const \ge 0$$

in a parallelepiped, that is, a parallelepiped domain $a_x < x < b_x$, $a_y < y < b_y$, $a_z < z < b_z$, with one of the following boundary conditions on each boundary bd_+:

- The Dirichlet boundary condition

$$u(x,y,z)=G(x,y,z)$$

- The Neumann boundary condition

$$\frac{\partial u}{\partial n}(x,y,z)=g(x,y,z)$$

where

$n = -x$ on bd_a_x, $n = x$ on bd_b_x,

$n = -y$ on bd_a_y, $n = y$ on bd_b_y,

$n = -z$ on bd_a_z, $n = z$ on bd_b_z.

Three-dimensional (3D) Poisson problem

The Poisson problem is a special case of the Helmholtz problem, when $q=0$. The 3D Poisson problem is to find an approximate solution of the Poisson equation

$$-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}=f(x,y,z)$$

in a parallelepiped $a_x < x < b_x$, $a_y < y < b_y$, $a_z < z < b_z$ with Dirichlet or Neumann boundary condition on each boundary bd_+.

Three-dimensional (3D) Laplace problem

The Laplace problem is a special case of the Helmholtz problem, when $q=0$ and $f(x, y, z)=0$. The 3D Laplace problem is to find an approximate solution of the Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$$

in a parallelepiped $a_x < x < b_x$, $a_y < y < b_y$, $a_z < z < b_z$ with the Dirichlet or Neumann boundary condition on each boundary `bd_+`.

Approximation of 3D problems

To find an approximate solution for each of the 3D problems, a uniform mesh is built in the parallelepiped domain

$$\left\{x_i = a_x + ih_x, y_j = a_y + jh_y, z_k = a_z + kh_z\right\}$$

where

$$i = 0,\ldots,n_x, j = 0,\ldots,n_y, k = 0,\ldots,n_z$$

$$h_x = \frac{b_x - a_x}{n_x}, h_y = \frac{b_y - a_y}{n_y}, h_z = \frac{b_z - a_z}{n_z}$$

The Poisson Library uses the standard seven-point finite difference approximation on this mesh to compute the approximation to the solution. The values of the approximate solution will be computed in the mesh points (x_i, y_j, z_k), provided that the user knows the values of the right-hand side `f(x, y, z)` in these points and the values of the appropriate boundary functions `G(x, y, z)` and/or `g(x, y, z)` in the mesh points laying on the boundary of the parallelepiped domain.

Sequence of Invoking PL Routines

NOTE This description always considers the solution process for the Helmholtz problem, because Fast Poisson Solver and Fast Laplace Solver are special cases of Fast Helmholtz Solver (see Poisson Library Implemented).

Computation of a solution of the Helmholtz problem using the PL interface is conceptually divided into four steps, each of which is performed via a dedicated routine. Table "PL Interface Routines" lists the routines and briefly describes their purpose.

Most PL routines have versions operating with single-precision and double-precision data. Names of such routines begin respectively with "`s`" and "`d`". The wildcard "`?`" stands for either of these symbols in routine names. The routines for Cartesian coordinate system have 2D and 3D versions. Their names end respectively in "`2D`" and "`3D`". The routines for spherical coordinate system have periodic and non-periodic versions. Their names end respectively in "`p`" and "`np`"

PL Interface Routines

Routine	Description
?_init_Helmholtz_2D/?_init_Helmholtz_3D/ ?_init_sph_p/?_init_sph_np	Initializes basic data structures of Poisson Library for Fast Helmholtz Solver in the 2D/3D/periodic/non-periodic case, respectively.
?_commit_Helmholtz_2D/?_commit_Helmholtz_3D/ ?_commit_sph_p/?_commit_sph_np	Checks consistency and correctness of user's data, creates and initializes data structures to be used by the Intel MKL FFT interface[1], as well as other data structures needed for the solver.
?_Helmholtz_2D/?_Helmholtz_3D/ ?_sph_p/?_sph_np	Computes an approximate solution of 2D/3D/periodic/non-periodic Helmholtz problem (see Poisson Library Implemented) specified by the parameters.
free_Helmholtz_2D/free_Helmholtz_3D/ free_sph_p/free_sph_np	Cleans the memory used by the data structures needed for calling the Intel MKL FFT interface[1].

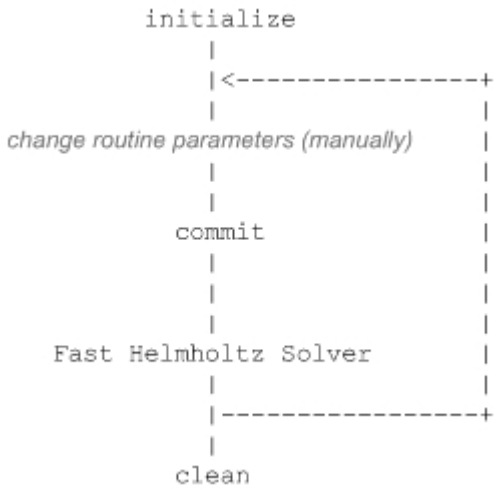
[1] PL routines call the Intel MKL FFT interface for better performance.

To find an approximate solution of Helmholtz problem only once, the Intel MKL PL interface routines are normally invoked in the order in which they are listed in Table "PL Interface Routines".

NOTE Though the order of invoking PL routines may be changed, it is highly recommended to follow the above order of routine calls.

The diagram in Figure "Typical Order of Invoking PL Routines" indicates the typical order in which PL routines can be invoked in a general case.

Typical Order of Invoking PL Routines



A general scheme of using PL routines for double-precision computations in a 3D Cartesian case is shown below. A similar scheme holds for single-precision computations with the only difference in the initial letter of routine names. The general scheme in a 2D Cartesian case differs form the one below in the set of routine parameters and the ending of routine names: "2D" instead of "3D".

```
...
d_init_Helmholtz_3D(&ax, &bx, &ay, &by, &az, &bz, &nx, &ny, &nz, BCtype, ipar, dpar, &stat);
/* change parameters in ipar and/or dpar if necessary. */
/* note that the result of the Fast Helmholtz Solver will be in f! If you want to keep the data stored
in f,
save it before the function call below */
d_commit_Helmholtz_3D(f, bd_ax, bd_bx, bd_ay, bd_by, bd_az, bd_bz, &xhandle, &yhandle, ipar, dpar,
&stat);
d_Helmholtz_3D(f, bd_ax, bd_bx, bd_ay, bd_by, bd_az, bd_bz, &xhandle, &yhandle, ipar, dpar, &stat);
free_Helmholtz_3D (&xhandle, &yhandle, ipar, &stat);
/* here you may clean the memory used by f, dpar, ipar */
...
```

A general scheme of using PL routines for double-precision computations in a spherical periodic case is shown below. Similar scheme holds for single-precision computations with the only difference in the initial letter of routine names. The general scheme in a spherical non-periodic case differs from the one below in the set of routine parameters and the ending of routine names: "np" instead of "p".

```
...
d_init_sph_p(&ap,&bp,&at,&bt,&np,&nt,&q,ipar,dpar,&stat);
/* change parameters in ipar and/or dpar if necessary. */
/* note that the result of the Fast Helmholtz Solver will be in f! If you want to
keep the data stored in f, save it before the function call below */
d_commit_sph_p(f,&handle_s,&handle_c,ipar,dpar,&stat);
d_sph_p(f,&handle_s,&handle_c,ipar,dpar,&stat);
free_sph_p(&handle_s,&handle_c,ipar,&stat);
/* here you may clean the memory used by f, dpar, ipar */
...
```

You can find examples of Fortran 90 and C code that use PL routines to solve Helmholtz problem (in both Cartesian and spherical cases) in the `examples\pdepoissonf\source` and `examples\pdepoissonc\source` folders of your Intel MKL directory.

Interface Description

All types in this documentation are standard C types: `int`, `float`, and `double`. Fortran 90 users can call the routines with `INTEGER`, `REAL`, and `DOUBLE PRECISION` Fortran types, respectively (see examples in the `examples\pdepoissonf\source` and `examples\pdepoissonc\source` folders of your Intel MKL directory.

The interface description uses the built-in type `int` for integer values. If you employ the ILP64 interface, read this type as `long long int` (or `INTEGER*8` for Fortran). For more information, refer to the *Intel(R) MKL User's Guide*.

Routine Options

All PL routines use parameters for passing various options to the routines. These parameters are arrays *ipar*, *dpar* and *spar*. Values for these parameters should be specified very carefully (see Common Parameters). You can change these values during computations to meet your needs.

WARNING To avoid failure or wrong results, you must provide correct and consistent parameters to the routines.

User Data Arrays

PL routines take arrays of user data as input. For example, user arrays are passed to the routine `d_Helmholtz_3D` to compute an approximate solution to 3D Helmholtz problem. To minimize storage requirements and improve the overall run-time efficiency, Intel MKL PL routines do not make copies of user input arrays.

NOTE If you need a copy of your input data arrays, save them yourself.

PL Routines for the Cartesian Solver

The section gives detailed description of Cartesian PL routines, their syntax, parameters and values they return. All flavors of the same routine, namely, double-precision and single-precision, 2D and 3D, are described together.

NOTE Some of the routine parameters are used only in the 3D Fast Helmholtz Solver.

PL routines call the Intel MKL FFT interface (described in section "FFT Functions" in chapter "Fast Fourier Transforms"), which enhances performance of the routines.

?_init_Helmholtz_2D/?_init_Helmholtz_3D

Initializes basic data structures of the Fast 2D/3D Helmholtz Solver.

Syntax

```
void d_init_Helmholtz_2D(double* ax, double* bx, double* ay, double* by, int* nx, int* ny, char* BCtype, double* q, int* ipar, double* dpar, int* stat);
```

```
void s_init_Helmholtz_2D(float* ax, float* bx, float* ay, float* by, int* nx, int* ny, char* BCtype, float* q, int* ipar, float* spar, int* stat);
```

```
void d_init_Helmholtz_3D(double* ax, double* bx, double* ay, double* by, double* az, double* bz, int* nx, int* ny, int* nz, char* BCtype, double* q, int* ipar, double* dpar, int* stat);
```

```
void s_init_Helmholtz_3D(float* ax, float* bx, float* ay, float* by, float* az, float* bz, int* nx, int* ny, int* nz, char* BCtype, float* q, int* ipar, float* spar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

ax	`double*` for `d_init_Helmholtz_2D/d_init_Helmholtz_3D`, `float*` for `s_init_Helmholtz_2D/s_init_Helmholtz_3D`. The coordinate of the leftmost boundary of the domain along x-axis.
bx	`double*` for `d_init_Helmholtz_2D/d_init_Helmholtz_3D`,

	float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D. The coordinate of the rightmost boundary of the domain along x-axis.
ay	double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D, float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D. The coordinate of the leftmost boundary of the domain along y-axis.
by	double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D, float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D. The coordinate of the rightmost boundary of the domain along y-axis.
az	double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D, float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D. The coordinate of the leftmost boundary of the domain along z-axis. This parameter is needed only for the ?_init_Helmholtz_3D routine.
bz	double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D, float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D. The coordinate of the rightmost boundary of the domain along z-axis. This parameter is needed only for the ?_init_Helmholtz_3D routine.
nx	int*. The number of mesh intervals along x-axis.
ny	int*. The number of mesh intervals along y-axis.
nz	int*. The number of mesh intervals along z-axis. This parameter is needed only for the ?_init_Helmholtz_3D routine.
BCtype	char*. Contains the type of boundary conditions on each boundary. Must contain four characters for ?_init_Helmholtz_2D and six characters for ?_init_Helmholtz_3D. Each of the characters can be 'N' (Neumann boundary condition) or 'D' (Dirichlet boundary condition). Types of boundary conditions for the boundaries should be specified in the following order: bd_a_x, bd_b_x, bd_a_y, bd_b_y, bd_a_z, bd_b_z. Boundary condition types for the last two boundaries should be specified only in the 3D case.
q	double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D, float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D . The constant Helmholtz coefficient. Note that to solve Poisson or Laplace problem, you should set the value of *q* to 0.

Output Parameters

ipar	int array of size 128. Contains integer data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).
dpar	double array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case. Contains double-precision data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).
spar	float array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case. Contains single-precision data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).
stat	int*. Routine completion status, which is also written to *ipar[0]*. The status should be 0 to proceed to other PL routines.

Description

The ?_init_Helmholtz_2D/?_init_Helmholtz_3D routines initialize basic data structures for Poisson Library computations of the appropriate precision. All routines invoked after a call to a ?_init_Helmholtz_2D/?_init_Helmholtz_3D routine use values of the *ipar*, *dpar* and *spar* array parameters returned by the routine. Detailed description of the array parameters can be found in Common Parameters.

WARNING Data structures initialized and created by 2D/3D flavors of the routine cannot be used by 3D/2D flavors of any PL routines, respectively.

You can skip calling this routine in your code. However, see Caveat on Parameter Modifications before doing so.

Return Values

stat= 0	The routine successfully completed the task. In general, to proceed with computations, the routine should complete with this *stat* value.
stat= -99999	The routine failed to complete the task because of a fatal error.

?_commit_Helmholtz_2D/?_commit_Helmholtz_3D

Checks consistency and correctness of user's data as well as initializes certain data structures required to solve 2D/3D Helmholtz problem.

Syntax

```
void d_commit_Helmholtz_2D(double* f, double* bd_ax, double* bd_bx, double* bd_ay,
double* bd_by, DFTI_DESCRIPTOR_HANDLE* xhandle, int* ipar, double* dpar, int* stat);
```

```
void s_commit_Helmholtz_2D(float* f, float* bd_ax, float* bd_bx, float* bd_ay, float*
bd_by, DFTI_DESCRIPTOR_HANDLE* xhandle, int* ipar, float* spar, int* stat);
```

```
void d_commit_Helmholtz_3D(double* f, double* bd_ax, double* bd_bx, double* bd_ay,
double* bd_by, double* bd_az, double* bd_bz, DFTI_DESCRIPTOR_HANDLE* xhandle,
DFTI_DESCRIPTOR_HANDLE* yhandle, int* ipar, double* dpar, int* stat);
```

```
void s_commit_Helmholtz_3D(float* f, float* bd_ax, float* bd_bx, float* bd_ay, float*
bd_by, float* bd_az, float* bd_bz, DFTI_DESCRIPTOR_HANDLE* xhandle,
DFTI_DESCRIPTOR_HANDLE* yhandle, int* ipar, float* spar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

f	`double*` for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D`, `float*` for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D`. Contains the right-hand side of the problem packed in a single vector. The size of the vector for the 2D problem is (*nx*+1)*(*ny*+1). In this case, the value of the right-hand side in the mesh point (*i, j*) is stored in *f*[*i+j*(*nx*+1)] . The size of the vector for the 3D problem is (*nx*+1)*(*ny*+1)*(*nz*+1). In this case, value of the right-hand side in the mesh point (*i, j, k*) is stored in *f*[*i+j*(*nx*+1)+*k*(*nx*+1)*(*ny*+1)]. Note that to solve the Laplace problem, you should set all the elements of the array *f* to 0. Note also that the array *f* may be altered by the routine. To preserve the vector, save it in another memory location.
ipar	`int` array of size 128. Contains integer data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).

dpar — `double` array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case. Contains double-precision data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).

spar — `float` array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case. Contains single-precision data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).

bd_ax — `double*` for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D`, `float*` for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D`. Contains values of the boundary condition on the leftmost boundary of the domain along x-axis.
For `?_commit_Helmholtz_2D`, the size of the array is ny+1. In case of the Dirichlet boundary condition (value of *BCtype*[0] is 'D'), it contains values of the function $G(ax, y_j)$, j=0, ..., ny. In case of the Neumann boundary condition (value of *BCtype*[0] is 'N'), it contains values of the function $g(ax, y_j)$, j=0, ..., ny. The value corresponding to the index j is placed in *bd_ax*[j].
For `?_commit_Helmholtz_3D`, the size of the array is (ny+1)*(nz+1). In case of the Dirichlet boundary condition (value of *BCtype*[0] is 'D'), it contains values of the function $G(ax, y_j, z_k)$, j=0, ..., ny, k=0, ..., nz. In case of the Neumann boundary condition (value of *BCtype*[0] is 'N'), it contains the values of the function $g(ax, y_j, z_k)$, j=0, ..., ny, k=0, ..., nz. The values are packed in the array so that the value corresponding to indices (j, k) is placed in *bd_ax*[j+k*(ny+1)].

bd_bx — `double*` for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D`, `float*` for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D`. Contains values of the boundary condition on the rightmost boundary of the domain along x-axis.
For `?_commit_Helmholtz_2D`, the size of the array is ny+1. In case of the Dirichlet boundary condition (value of *BCtype*[1] is 'D'), it contains values of the function $G(bx, y_j)$, j=0, ..., ny. In case of the Neumann boundary condition (value of *BCtype*[1] is 'N'), it contains values of the function $g(bx, y_j)$, j=0, ..., ny. The value corresponding to the index j is placed in *bd_bx*[j].
For `?_commit_Helmholtz_3D`, the size of the array is (ny+1)*(nz+1). In case of the Dirichlet boundary condition (value of *BCtype*[1] is 'D'), it contains values of the function $G(bx, y_j, z_k)$, j=0, ..., ny, k=0, ..., nz. In case of the Neumann boundary condition (value of *BCtype*[1] is 'N'), it contains the values of the function $g(bx, y_j, z_k)$, j=0, ..., ny, k=0, ..., nz. The values are packed in the array so that the value corresponding to indices (j, k) is placed in *bd_bx*[j+k*(ny+1)].

bd_ay — `double*` for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D`, `float*` for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D`. Contains values of the boundary condition on the leftmost boundary of the domain along y-axis.
For `?_commit_Helmholtz_2D`, the size of the array is nx+1. In case of the Dirichlet boundary condition (value of *BCtype*[2] is 'D'), it contains values of the function $G(x_i, ay)$, i=0, ..., nx. In case of the Neumann boundary condition (value of *BCtype*[2] is 'N'), it contains values of the function $g(x_i, ay)$, i=0, ..., nx. The value corresponding to the index i is placed in *bd_ay*[i].

For ?_commit_Helmholtz_3D, the size of the array is (*nx*+1)*(*nz*+1). In case of the Dirichlet boundary condition (value of *BCtype*[2] is 'D'), it contains values of the function $G(x_i, ay, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. In case of the Neumann boundary condition (value of *BCtype*[2] is 'N'), it contains the values of the function $g(x_i, ay, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. The values are packed in the array so that the value corresponding to indices *(i, k)* is placed in *bd_ay*[*i*+*k**(*nx*+1)].

bd_by — double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D, float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.
Contains values of the boundary condition on the rightmost boundary of the domain along *y*-axis.
For ?_commit_Helmholtz_2D, the size of the array is *nx*+1. In case of the Dirichlet boundary condition (value of *BCtype*[3] is 'D'), it contains values of the function $G(x_i, by)$, i=0, ..., *nx*. In case of the Neumann boundary condition (value of *BCtype*[3] is 'N'), it contains values of the function $g(x_i, by)$, i=0, ..., *nx*. The value corresponding to the index *i* is placed in *bd_by*[*i*].
For ?_commit_Helmholtz_3D, the size of the array is (*nx*+1)*(*nz*+1). In case of the Dirichlet boundary condition (value of *BCtype*[3] is 'D'), it contains values of the function $G(x_i, by, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. In case of the Neumann boundary condition (value of *BCtype*[3] is 'N'), it contains the values of the function $g(x_i, by, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. The values are packed in the array so that the value corresponding to indices *(i, k)* is placed in *bd_by*[*i*+*k**(*nx*+1)].

bd_az — double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D, float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.
This parameter is needed only for ?_commit_Helmholtz_3D. Contains values of the boundary condition on the leftmost boundary of the domain along *z*-axis.
The size of the array is (*nx*+1)*(*ny*+1). In case of the Dirichlet boundary condition (value of *BCtype*[4] is 'D'), it contains values of the function $G(x_i, y_j, az)$, i=0, ..., *nx*, j=0, ..., *ny*. In case of the Neumann boundary condition (value of *BCtype*[4] is 'N'), it contains the values of the function $g(x_i, y_j, az)$, i=0, ..., *nx*, j=0, ..., *ny*. The values are packed in the array so that the value corresponding to indices *(i, j)* is placed in *bd_az*[*i*+*j**(*nx*+1)].

bd_bz — double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D, float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.
This parameter is needed only for ?_commit_Helmholtz_3D. Contains values of the boundary condition on the rightmost boundary of the domain along *z*-axis.
The size of the array is (*nx*+1)*(*ny*+1). In case of the Dirichlet boundary condition (value of *BCtype*[5] is 'D'), it contains values of the function $G(x_i, y_j, bz)$, i=0, ..., *nx*, j=0, ..., *ny*. In case of the Neumann boundary condition (value of *BCtype*[5] is 'N'), it contains the values of the function $g(x_i, y_j, bz)$, i=0, ..., *nx*, j=0, ..., *ny*. The values are packed in the array so that the value corresponding to indices *(i, j)* is placed in *bd_bz*[*i*+*j**(*nx*+1)].

Output Parameters

f — Vector of the right-hand side of the problem. Possibly, altered on output.

ipar	Contains integer data to be used by Fast Helmholtz Solver. Modified on output as explained in Common Parameters.
dpar	Contains double-precision data to be used by Fast Helmholtz Solver. Modified on output as explained in Common Parameters.
spar	Contains single-precision data to be used by Fast Helmholtz Solver. Modified on output as explained in Common Parameters.
xhandle, yhandle	DFTI_DESCRIPTOR_HANDLE*. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). *yhandle* is used only by ?_commit_Helmholtz_3D.
stat	int*. Routine completion status, which is also written to *ipar[0]*. The status should be 0 to proceed to other PL routines.

Description

The ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routines check consistency and correctness of the parameters to be passed to the solver routines ?_Helmholtz_2D/?_Helmholtz_3D. They also initialize data structures *xhandle*, *yhandle* as well as arrays *ipar* and *dpar/spar*, depending upon the routine precision. Refer to Common Parameters to find out which particular array elements the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routines initialize and what values are written there.

The routines perform only a basic check for correctness and consistency. If you are going to modify parameters of PL routines, see the Caveat on Parameter Modifications section. Unlike ?_init_Helmholtz_2D/?_init_Helmholtz_3D, the routines ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D are mandatory, and you cannot skip calling them in your code. Values of *ax*, *bx*, *ay*, *by*, *az*, *bz*, *nx*, *ny*, *nz*, and *BCtype* are passed to each of the routines with the *ipar* array and defined in a previous call to the appropriate ?_init_Helmholtz_2D/?_init_Helmholtz_3D routine.

Return Values

stat= 1	The routine completed without errors and produced some warnings.
stat= 0	The routine successfully completed the task.
stat= -100	The routine stopped because an error in the user's data was found or the data in the *dpar*, *spar* or *ipar* array was altered by mistake.
stat= -1000	The routine stopped because of an Intel MKL FFT or TT interface error.
stat= -10000	The routine stopped because the initialization failed to complete or the parameter *ipar*[0] was altered by mistake.
stat= -99999	The routine failed to complete the task because of a fatal error.

?_Helmholtz_2D/?_Helmholtz_3D

Computes the solution of 2D/3D Helmholtz problem specified by the parameters.

Syntax

```
void d_Helmholtz_2D(double* f, double* bd_ax, double* bd_bx, double* bd_ay, double* bd_by, DFTI_DESCRIPTOR_HANDLE* xhandle, int* ipar, double* dpar, int* stat);
```

```
void s_Helmholtz_2D(float* f, float* bd_ax, float* bd_bx, float* bd_ay, float* bd_by, DFTI_DESCRIPTOR_HANDLE* xhandle, int* ipar, float* spar, int* stat);
```

```
void d_Helmholtz_3D(double* f, double* bd_ax, double* bd_bx, double* bd_ay, double*
bd_by, double* bd_az, double* bd_bz, DFTI_DESCRIPTOR_HANDLE* xhandle,
DFTI_DESCRIPTOR_HANDLE* yhandle, int* ipar, double* dpar, int* stat);
```

```
void s_Helmholtz_3D(float* f, float* bd_ax, float* bd_bx, float* bd_ay, float* bd_by,
float* bd_az, float* bd_bz, DFTI_DESCRIPTOR_HANDLE* xhandle, DFTI_DESCRIPTOR_HANDLE*
yhandle, int* ipar, float* spar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

f	`double*` for `d_Helmholtz_2D/d_Helmholtz_3D`, `float*` for `s_Helmholtz_2D/s_Helmholtz_3D`. Contains the right-hand side of the problem packed in a single vector and modified by the appropriate `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine. Note that an attempt to substitute the original right-hand side vector at this point will result in a wrong solution. The size of the vector for the 2D problem is (nx+1)*(ny+1). In this case, value of the right-hand side in the mesh point (i, j) is stored in f[i+j*(nx+1)] . The size of the vector for the 3D problem is (nx+1)*(ny+1)*(nz+1). In this case, value of the right-hand side in the mesh point (i, j, k) is stored in f[i+j*(nx+1)+k*(nx+1)*(ny+1)].
xhandle, yhandle	`DFTI_DESCRIPTOR_HANDLE*`. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). *yhandle* is used only by `?_Helmholtz_3D`.
ipar	`int` array of size 128. Contains integer data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).
dpar	`double` array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case. Contains double-precision data to be used by Fast Helmholtz Solver (for details, refer toCommon Parameters
spar	float array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case. Contains single-precision data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).
bd_ax	`double*` for `d_Helmholtz_2D/d_Helmholtz_3D`, `float*` for `s_Helmholtz_2D/s_Helmholtz_3D`. Contains values of the boundary condition on the leftmost boundary of the domain along x-axis. For `?_Helmholtz_2D`, the size of the array is ny+1. In case of the the Dirichlet boundary condition (value of $BCtype$[0] is 'D'), it contains values of the function $G(ax, y_j)$, j=0, ..., ny. In case of the the Neumann boundary condition (value of $BCtype$[0] is 'N'), it contains values of the function $g(ax, y_j)$, j=0, ..., ny. The value corresponding to the index j is placed in bd_ax[j]. For `?_Helmholtz_3D`, the size of the array is (ny+1)*(nz+1). In case of the the Dirichlet boundary condition (value of $BCtype$[0] is 'D'), it contains values of the function $G(ax, y_j, z_k)$, j=0, ..., ny, k=0, ..., nz. In case of the Neumann boundary condition (value of $BCtype$[0] is 'N'), it contains the values of the function $g(ax, y_j, z_k)$, j=0, ..., ny, k=0, ..., nz. The values are packed in the array so that the value corresponding to indices (j, k) is placed in bd_ax[j+k*(ny+1)].

bd_bx

double* for d_Helmholtz_2D/d_Helmholtz_3D,
float* for s_Helmholtz_2D/s_Helmholtz_3D.
Contains values of the boundary condition on the rightmost boundary of the domain along *x*-axis.
For ?_Helmholtz_2D, the size of the array is *ny*+1. In case of the Dirichlet boundary condition (value of *BCtype*[1] is 'D'), it contains values of the function $G(bx, y_j)$, j=0, ..., *ny*. In case of the Neumann boundary condition (value of *BCtype*[1] is 'N'), it contains values of the function $g(bx, y_j)$, j=0, ..., *ny*. The value corresponding to the index j is placed in *bd_bx*[j].
For ?_Helmholtz_3D, the size of the array is (*ny*+1)*(*nz*+1). In case of the Dirichlet boundary condition (value of *BCtype*[1] is 'D'), it contains values of the function $G(bx, y_j, z_k)$, j=0, ..., *ny*, k=0, ..., *nz*. In case of the Neumann boundary condition (value of *BCtype*[1] is 'N'), it contains the values of the function $g(bx, y_j, z_k)$, j=0, ..., *ny*, k=0, ..., *nz*. The values are packed in the array so that the value corresponding to indices *(j, k)* is placed in *bd_bx*[$j+k$*(*ny*+1)].

bd_ay

double* for d_Helmholtz_2D/d_Helmholtz_3D,
float* for s_Helmholtz_2D/s_Helmholtz_3D.
Contains values of the boundary condition on the leftmost boundary of the domain along *y*-axis.
For ?_Helmholtz_2D, the size of the array is *nx*+1. In case of the Dirichlet boundary condition (value of *BCtype*[2] is 'D'), it contains values of the function $G(x_i, ay)$, i=0, ..., *nx*. In case of the Neumann boundary condition (value of *BCtype*[2] is 'N'), it contains values of the function $g(x_i, ay)$, i=0, ..., *nx*. The value corresponding to the index i is placed in *bd_ay*[i].
For ?_Helmholtz_3D, the size of the array is (*nx*+1)*(*nz*+1). In case of the Dirichlet boundary condition (value of *BCtype*[2] is 'D'), it contains values of the function $G(x_i,ay, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. In case of the Neumann boundary condition (value of *BCtype*[2] is 'N'), it contains the values of the function $g(x_i,ay, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. The values are packed in the array so that the value corresponding to indices *(i, k)* is placed in *bd_ay*[$i+k$*(*nx*+1)].

bd_by

double* for d_Helmholtz_2D/d_Helmholtz_3D,
float* for s_Helmholtz_2D/s_Helmholtz_3D.
Contains values of the boundary condition on the rightmost boundary of the domain along *y*-axis.
For ?_Helmholtz_2D, the size of the array is *nx*+1. In case of the Dirichlet boundary condition (value of *BCtype*[3] is 'D'), it contains values of the function $G(x_i, by)$, i=0, ..., *nx*. In case of the Neumann boundary condition (value of *BCtype*[3] is 'N'), it contains values of the function $g(x_i, by)$, i=0, ..., *nx*. The value corresponding to the index i is placed in *bd_by*[i].
For ?_Helmholtz_3D, the size of the array is (*nx*+1)*(*nz*+1). In case of the Dirichlet boundary condition (value of *BCtype*[3] is 'D'), it contains values of the function $G(x_i,by, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. In case of the Neumann boundary condition (value of *BCtype*[3] is 'N'), it contains the values of the function $g(x_i,by, z_k)$, i=0, ..., *nx*, k=0, ..., *nz*. The values are packed in the array so that the value corresponding to indices *(i, k)* is placed in *bd_by*[$i+k$*(*nx*+1)].

bd_az

double* for d_Helmholtz_2D/d_Helmholtz_3D,
float* for s_Helmholtz_2D/s_Helmholtz_3D.
This parameter is needed only for ?_Helmholtz_3D. Contains values of the boundary condition on the leftmost boundary of the domain along *z*-axis.

The size of the array is (*nx*+1)*(*ny*+1). In case of the Dirichlet boundary condition (value of *BCtype*[4] is 'D'), it contains values of the function *G*(x_i, y_j,*az*), *i*=0, ..., *nx*, *j*=0, ..., *ny*. In case of the Neumann boundary condition (value of *BCtype*[4] is 'N'), it contains the values of the function *g*(x_i, y_j,*az*), *i*=0, ..., *nx*, *j*=0, ..., *ny*. The values are packed in the array so that the value corresponding to indices *(i, j)* is placed in *bd_az*[*i* +*j**(*nx*+1)].

bd_bz — `double*` for `d_Helmholtz_2D/d_Helmholtz_3D`, `float*` for `s_Helmholtz_2D/s_Helmholtz_3D`.
This parameter is needed only for `?_Helmholtz_3D`. Contains values of the boundary condition on the rightmost boundary of the domain along *z*-axis. The size of the array is (*nx*+1)*(*ny*+1). In case of the Dirichlet boundary condition (value of *BCtype*[5] is 'D'), it contains values of the function *G*(x_i, y_j,*bz*), *i*=0, ..., *nx*, *j*=0, ..., *ny*. In case of the Neumann boundary condition (value of *BCtype*[5] is 'N'), it contains the values of the function *g*(x_i, y_j,*bz*), *i*=0, ..., *nx*, *j*=0, ..., *ny*. The values are packed in the array so that the value corresponding to indices *(i, j)* is placed in *bd_bz*[*i* +*j**(*nx*+1)].

NOTE To avoid wrong computation results, do not change arrays *bd_ax*, *bd_bx*, *bd_ay*, *bd_by*, *bd_az*, *bd_bz* between a call to the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine and a subsequent call to the appropriate `?_Helmholtz_2D/?_Helmholtz_3D` routine.

Output Parameters

f — On output, contains the approximate solution to the problem packed the same way as the right-hand side of the problem was packed on input.

xhandle, yhandle — Data structures used by the Intel MKL FFT interface.

ipar — Contains integer data to be used by Fast Helmholtz Solver. Modified on output as explained in Common Parameters.

dpar — Contains double-precision data to be used by Fast Helmholtz Solver. Modified on output as explained in Common Parameters

spar — Contains single-precision data to be used by Fast Helmholtz Solver. Modified on output as explained in Common Parameters.

stat — `int*`. Routine completion status, which is also written to *ipar[0]*. The status should be 0 to proceed to other PL routines.

Description

The `Helmholtz_2D-Helmholtz_3D` routines compute the approximate solution of Helmholtz problem defined in the previous calls to the corresponding initialization and commit routines. The solution is computed according to formulas given in the Poisson Library Implemented section. The *f* parameter, which initially holds the packed vector of the right-hand side of the problem, is replaced by the computed solution packed in the same way. Values of *ax*, *bx*, *ay*, *by*, *az*, *bz*, *nx*, *ny*, *nz*, and *BCtype* are passed to each of the routines with the *ipar* array and defined in the previous call to the appropriate `?_init_Helmholtz_2D/?_init_Helmholtz_3D` routine.

Return Values

stat= 1 — The routine completed without errors and produced some warnings.

stat= 0 — The routine successfully completed the task.

stat= -2	The routine stopped because division by zero occurred. It usually happens if the data in the *dpar* or *spar* array was altered by mistake.
stat= -3	The routine stopped because the memory was insufficient to complete the computations.
stat= -100	The routine stopped because an error in the user's data was found or the data in the *dpar*, *spar* or *ipar* array was altered by mistake.
stat= -1000	The routine stopped because of the Intel MKL FFT or TT interface error.
stat= -10000	The routine stopped because the initialization failed to complete or the parameter *ipar*[0] was altered by mistake.
stat= -99999	The routine failed to complete the task because of a fatal error.

free_Helmholtz_2D/free_Helmholtz_3D

Cleans the memory allocated for the data structures used by the FFT interface.

Syntax

```
void free_Helmholtz_2D(DFTI_DESCRIPTOR_HANDLE* xhandle, int* ipar, int* stat);
void free_Helmholtz_3D(DFTI_DESCRIPTOR_HANDLE* xhandle, DFTI_DESCRIPTOR_HANDLE*
yhandle, int* ipar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

xhandle, yhandle	`DFTI_DESCRIPTOR_HANDLE*`. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). The structure *yhandle* is used only by `free_Helmholtz_3D`.
ipar	`int` array of size 128. Contains integer data to be used by Fast Helmholtz Solver (for details, refer to Common Parameters).

Output Parameters

xhandle, yhandle	Data structures used by the Intel MKL FFT interface. Memory allocated for the structures is released on output.
ipar	Contains integer data to be used by Fast Helmholtz Solver. Status of the routine call is written to *ipar[0]*.
stat	`int*`. Routine completion status, which is also written to *ipar[0]*.

Description

The `free_Helmholtz_2D-free_Helmholtz_3D` routine cleans the memory used by the *xhandle* and *yhandle* structures, needed for calling the Intel MKL FFT functions. To release memory allocated for other parameters, include cleaning of the memory in your code.

Return Values

stat= 0	The routine successfully completed the task.
stat= -1000	The routine stopped because of an Intel MKL FFT or TT interface error.
stat= -99999	The routine failed to complete the task because of a fatal error.

PL Routines for the Spherical Solver

The section gives detailed description of spherical PL routines, their syntax, parameters and values they return. All flavors of the same routine, namely, double-precision and single-precision, periodic (having names ending in "p") and non-periodic (having names ending in "np"), are described together.

These PL routines also call the Intel MKL FFT interface (described in section "FFT Functions" in chapter "Fast Fourier Transforms"), which enhances performance of the routines.

?_init_sph_p/?_init_sph_np

Initializes basic data structures of the Fast periodic and non-periodic Helmholtz Solver on a sphere.

Syntax

```
void d_init_sph_p(double* ap, double* at, double* bp, double* bt, int* np, int* nt, double* q, int* ipar, double* dpar, int* stat);
void s_init_sph_p(float* ap, float* at, float* bp, float* bt, int* np, int* nt, float* q, int* ipar, float* spar, int* stat);
void d_init_sph_np(double* ap, double* at, double* bp, double* bt, int* np, int* nt, double* q, int* ipar, double* dpar, int* stat);
void s_init_sph_np(float* ap, float* at, float* bp, float* bt, int* np, int* nt, float* q, int* ipar, float* spar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

ap	`double*` for `d_init_sph_p/d_init_sph_np`, `float*` for `s_init_sph_p/s_init_sph_np`. The coordinate (angle) of the leftmost boundary of the domain along φ-axis.
bp	`double*` for `d_init_sph_p/d_init_sph_np`, `float*` for `s_init_sph_p/s_init_sph_np`. The coordinate (angle) of the rightmost boundary of the domain along φ-axis.
at	`double*` for `d_init_sph_p/d_init_sph_np`, `float*` for `s_init_sph_p/s_init_sph_np`. The coordinate (angle) of the leftmost boundary of the domain along θ-axis.
bt	`double*` for `d_init_sph_p/d_init_sph_np`, `float*` for `s_init_sph_p/s_init_sph_np`. The coordinate (angle) of the rightmost boundary of the domain along θ-axis.

np	int*. The number of mesh intervals along φ-axis. Must be even in the periodic case.
nt	int*. The number of mesh intervals along θ-axis.
q	double* for d_init_sph_p/d_init_sph_np, float* for s_init_sph_p/s_init_sph_np. The constant Helmholtz coefficient. Note that to solve Poisson problem, you should set the value of *q* to 0.

Output Parameters

ipar	int array of size 128. Contains integer data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
dpar	double array of size 5**np*/2+*nt*+10. Contains double-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
spar	float array of size 5**np*/2+*nt*+10. Contains single-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
stat	int*. Routine completion status, which is also written to *ipar[0]*. The status should be 0 to proceed to other PL routines.

Description

The ?_init_sph_p/?_init_sph_np routines initialize basic data structures for Poisson Library computations of the appropriate precision. All routines invoked after a call to a ?_init_Helmholtz_2D/?_init_Helmholtz_3D routine use values of the *ipar*, *dpar* and *spar* array parameters returned by the routine. Detailed description of the array parameters can be found in Common Parameters.

WARNING Data structures initialized and created by periodic/non-periodic flavors of the routine cannot be used by non-periodic/periodic flavors of any PL routines for Helmholtz Solver on a sphere, respectively.

You can skip calling this routine in your code. However, see Caveat on Parameter Modifications before doing so.

Return Values

stat= 0	The routine successfully completed the task. In general, to proceed with computations, the routine should complete with this *stat* value.
stat= -99999	The routine failed to complete the task because of fatal error.

?_commit_sph_p/?_commit_sph_np

Checks consistency and correctness of user's data as well as initializes certain data structures required to solve periodic/non-periodic Helmholtz problem on a sphere.

Syntax

```
void d_commit_sph_p(double* f, DFTI_DESCRIPTOR_HANDLE* handle_s,
DFTI_DESCRIPTOR_HANDLE* handle_c, int* ipar, double* dpar, int* stat);
```

```
void s_commit_sph_p(float* f, DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE*
handle_c, int* ipar, float* spar, int* stat);
```

```
void d_commit_sph_np(double* f, DFTI_DESCRIPTOR_HANDLE* handle, int* ipar, double* dpar, int* stat);
```

```
void s_commit_sph_np(float* f, DFTI_DESCRIPTOR_HANDLE* handle, int* ipar, float* spar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

f	`double*` for `d_commit_sph_p/d_commit_sph_np`, `float*` for `s_commit_sph_p/s_commit_sph_np`. Contains the right-hand side of the problem packed in a single vector. The size of the vector is (*np*+1)*(*nt*+1) and value of the right-hand side in the mesh point (*i, j*) is stored in *f*[*i*+*j**(*np*+1)] . Note that the array *f* may be altered by the routine. Please save this vector in another memory location if you want to preserve it.
ipar	`int` array of size 128. Contains integer data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
dpar	`double` array of size 5**np*/2+*nt*+10. Contains double-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
spar	`float` array of size 5**np*/2+*nt*+10. Contains single-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).

Output Parameters

f	Vector of the right-hand side of the problem. Possibly, altered on output.
ipar	Contains integer data to be used by Fast Helmholtz Solver on a sphere. Modified on output as explained in Common Parameters.
dpar	Contains double-precision data to be used by Fast Helmholtz Solver on a sphere. Modified on output as explained in Common Parameters.
spar	Contains single-precision data to be used by Fast Helmholtz Solver on a sphere. Modified on output as explained in Common Parameters.
handle_s, handle_c, handle	`DFTI_DESCRIPTOR_HANDLE*`. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). *handle_s* and *handle_c* are used only in `?_commit_sph_p` and *handle* is used only in `?_commit_sph_np`.
stat	`int*`. Routine completion status, which is also written to *ipar[0]*. The status should be 0 to proceed to other PL routines.

Description

The `?_commit_sph_p/?_commit_sph_np` routines check consistency and correctness of the parameters to be passed to the solver routines `?_sph_p/?_sph_np`, respectively. They also initialize certain data structures. The routine `?_commit_sph_p` initializes structures *handle_s* and *handle_c*, and `?_commit_sph_np` initializes *handle*. The routines also initialize arrays *ipar* and *dpar/spar*, depending upon the routine precision. Refer to Common Parameters to find out which particular array elements the `?_commit_sph_p/?_commit_sph_np` routines initialize and what values are written there.

The routines perform only a basic check for correctness and consistency. If you are going to modify parameters of PL routines, see the Caveat on Parameter Modifications section. Unlike ?_init_sph_p/?_init_sph_np, the routines ?_commit_sph_p/?_commit_sph_np are mandatory, and you cannot skip calling them in your code. Values of *np* and *nt* are passed to each of the routines with the *ipar* array and defined in a previous call to the appropriate ?_init_sph_p/?_init_sph_np routine.

Return Values

stat= 1	The routine completed without errors and produced some warnings.
stat= 0	The routine successfully completed the task.
stat= -100	The routine stopped because an error in the user's data was found or the data in the *dpar*, *spar* or *ipar* array was altered by mistake.
stat= -1000	The routine stopped because of an Intel MKL FFT or TT interface error.
stat= -10000	The routine stopped because the initialization failed to complete or the parameter *ipar*[0] was altered by mistake.
stat= -99999	The routine failed to complete the task because of a fatal error.

?_sph_p/?_sph_np

Computes the solution of a spherical Helmholtz problem specified by the parameters.

Syntax

```
void d_sph_p(double* f, DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE* handle_c, int* ipar, double* dpar, int* stat);
```

```
void s_sph_p(float* f, DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE* handle_c, int* ipar, float* spar, int* stat);
```

```
void d_sph_np(double* f, DFTI_DESCRIPTOR_HANDLE* handle, int* ipar, double* dpar, int* stat);
```

```
void s_sph_np(float* f, DFTI_DESCRIPTOR_HANDLE* handle, int* ipar, float* spar, int* stat);
```

Include Files

- FORTRAN 90: mkl_poisson.f90
- C: mkl_poisson.h

Input Parameters

f	double* for d_sph_p/d_sph_np, float* for s_sph_p/s_sph_np. Contains the right-hand side of the problem packed in a single vector and modified by the appropriate ?_commit_sph_p/?_commit_sph_np routine. Note that an attempt to substitute the original right-hand side vector at this point will result in a wrong solution. The size of the vector is (*np*+1)*(*nt*+1) and value of the right-hand side in the mesh point (*i, j*) is stored in *f*[*i*+*j**(*np*+1)] .

handle_s, handle_c, handle	DFTI_DESCRIPTOR_HANDLE*. Data structures used by Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). *handle_s* and *handle_c* are used only in ?_sph_p and *handle* is used only in ?_sph_np.
ipar	int array of size 128. Contains integer data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
dpar	double array of size 5**np*/2+*nt*+10. Contains double-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).
spar	float array of size 5**np*/2+*nt*+10. Contains single-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).

Output Parameters

f	On output, contains the approximate solution to the problem packed the same way as the right-hand side of the problem was packed on input.
handle_s, handle_c, handle	Data structures used by the Intel MKL FFT interface.
ipar	Contains integer data to be used by Fast Helmholtz Solver on a sphere. Modified on output as explained in Common Parameters.
dpar	Contains double-precision data to be used by Fast Helmholtz Solver on a sphere. Modified on output as explained in Common Parameters.
spar	Contains single-precision data to be used by Fast Helmholtz Solver on a sphere. Modified on output as explained in Common Parameters.
stat	int*. Routine completion status, which is also written to *ipar[0]*. The status should be 0 to proceed to other PL routines.

Description

The sph_p-sph_np routines compute the approximate solution on a sphere of the Helmholtz problem defined in the previous calls to the corresponding initialization and commit routines. The solution is computed according to formulas given in the Poisson Library Implemented section. The *f* parameter, which initially holds the packed vector of the right-hand side of the problem, is replaced by the computed solution packed in the same way. Values of *np* and *nt* are passed to each of the routines with the *ipar* array and defined in the previous call to the appropriate ?_init_sph_p/?_init_sph_np routine.

Return Values

stat= 1	The routine completed without errors and produced some warnings.
stat= 0	The routine successfully completed the task.
stat= -2	The routine stopped because division by zero occurred. It usually happens if the data in the *dpar* or *spar* array was altered by mistake.
stat= -3	The routine stopped because the memory was insufficient to complete the computations.
stat= -100	The routine stopped because an error in the user's data was found or the data in the *dpar*, *spar* or *ipar* array was altered by mistake.
stat= -1000	The routine stopped because of an Intel MKL FFT or TT interface error.

stat= -10000	The routine stopped because the initialization failed to complete or the parameter *ipar*[0] was altered by mistake.
stat= -99999	The routine failed to complete the task because of a fatal error.

free_sph_p/free_sph_np

Cleans the memory allocated for the data structures used by the FFT interface.

Syntax

```
void free_sph_p(DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE* handle_c,
int* ipar, int* stat);
```

```
void free_sph_np(DFTI_DESCRIPTOR_HANDLE* handle, int* ipar, int* stat);
```

Include Files

- FORTRAN 90: `mkl_poisson.f90`
- C: `mkl_poisson.h`

Input Parameters

handle_s, *handle_c*, *handle*	`DFTI_DESCRIPTOR_HANDLE*`. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). The structures *handle_s* and *handle_c* are used only in `free_sph_p`, and *handle* is used only in `free_sph_np`.
ipar	`int` array of size 128. Contains integer data to be used by Fast Helmholtz Solver on a sphere (for details, refer to Common Parameters).

Output Parameters

handle_s, *handle_c*, *handle*	Data structures used by the Intel MKL FFT interface. Memory allocated for the structures is released on output.
ipar	Contains integer data to be used by Fast Helmholtz Solver on a sphere. Status of the routine call is written to *ipar[0]*.
stat	`int*`. Routine completion status, which is also written to *ipar[0]*.

Description

The `free_sph_p-free_sph_np` routine cleans the memory used by the *handle_s*, *handle_c* or *handle* structures, needed for calling the Intel MKL FFT functions. To release memory allocated for other parameters, include cleaning of the memory in your code.

Return Values

stat= 0	The routine successfully completed the task.
stat= -1000	The routine stopped because of the Intel MKL FFT or TT interface error.
stat= -99999	The routine failed to complete the task because of a fatal error.

Common Parameters

This section provides description of array parameters *ipar*, *dpar* and *spar*, which hold PL routine options in both Cartesian and spherical cases.

NOTE Initial values are assigned to the array parameters by the appropriate ?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np and ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D/?_commit_sph_p/?_commit_sph_np routines.

ipar — `int` array of size 128, holds integer data needed for Fast Helmholtz Solver (both for Cartesian and spherical coordinate systems). Its elements are described in Table "Elements of the ipar Array":

Elements of the ipar Array

Index	Description
0	Contains status value of the last called PL routine. In general, it should be 0 to proceed with Fast Helmholtz Solver. The element has no predefined values. This element can also be used to inform the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D/?_commit_sph_p/?_commit_sph_np routines of how the Commit step of the computation should be carried out (see Figure "Typical Order of Invoking PL Routines"). A non-zero value of *ipar[0]* with decimal representation $\overline{abc} = 100a + 10b + c$ where each of *a*, *b*, and *c* is equal to 0 or 9, indicates that some parts of the Commit step should be omitted. • If *c*=9, the routine omits checking of parameters and initialization of the data structures. • If *b*=9, then in the Cartesian case, the routine omits the adjustment of the right-hand side vector *f* to the Neumann boundary condition (multiplication of boundary values by 0.5 as well as incorporation of the boundary function *g*) and/or the Dirichlet boundary condition (setting boundary values to 0 as well as incorporation of the boundary function *G*). In this case, the routine also omits the adjustment of the right-hand side vector *f* to the particular boundary functions. For the Helmholtz solver on a sphere, the routine omits computation of the spherical weights for the *dpar*/*spar* array. • If *a*=9, then the routine omits the normalization of the right-hand side vector *f*. In the 2D Cartesian case, it is the multiplication by hy^2, where *hy* is the mesh size in the *y* direction (for details, see Poisson Library Implemented). In the 3D (Cartesian) case, it is the multiplication by hz^2, where *hz* is the mesh size in the *z* direction. For the Helmholtz solver on a sphere, it is the multiplication by h_θ^2, where h_θ is the mesh size in the θ direction (for details, see Poisson Library Implemented). Using *ipar[0]* you can adjust the routine to your needs and gain efficiency in solving multiple Helmholtz problems that differ only in the right-hand side. You must be cautious using this opportunity, because misunderstanding of the commit process may cause wrong results or program failure (see also Caveat on Parameter Modifications).
1	Contains error messaging options: • *ipar[1]*=-1 indicates that all error messages will be printed to the file MKL_Poisson_Library_log.txt in the folder from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device. • *ipar[1]*=0 indicates that no error messages will be printed. • *ipar[1]*=1 is the default value. It indicates that all error messages will be printed to the preconnected default output device (usually, screen).

Index	Description
	In case of errors, the *stat* parameter will acquire a non-zero value regardless of the *ipar[1]* setting.
2	Contains warning messaging options: • *ipar[2]*=-1 indicates that all warning messages will be printed to the file MKL_Poisson_Library_log.txt in the directory from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device. • *ipar[2]*=0 indicates that no warning messages will be printed. • *ipar[2]*=1 is the default value. It indicates that all warning messages will be printed to the preconnected default output device (usually, screen). In case of warnings, the *stat* parameter will acquire a non-zero value regardless of the *ipar[2]* setting.
3	Contains the number of the combination of boundary conditions. In the Cartesian case, it corresponds to the value that the *BCtype* parameter holds: • In the 2D case, 0 corresponds to 'DDDD' 1 corresponds to 'DDDN' … 15 corresponds to 'NNNN' • In the 3D case, 0 corresponds to 'DDDDDD' 1 corresponds to 'DDDDDN' ... 63 corresponds to 'NNNNNN'. The Helmholtz solver on a sphere uses this parameter only in a periodic case. The *bp* and *bt* parameters of the ?_init_sph_p/?_init_sph_np routine, which initializes *ipar[3]*, determine its value: • 0 corresponds to the problem without poles. • 1 corresponds to the problem on the entire sphere.
Parameters 4 through 9 are used only in Cartesian case.	
4	Takes the value of 1 if *BCtype[0]*='N', 0 if *BCtype[0]*='D', and -1 otherwise.
5	Takes the value of 1 if *BCtype[1]*='N', 0 if *BCtype[1]*='D', and -1 otherwise.
6	Takes the value of 1 if *BCtype[2]*='N', 0 if *BCtype[2]*='D', and -1 otherwise.
7	Takes the value of 1 if *BCtype[3]*='N', 0 if *BCtype[3]*='D', and -1 otherwise.
8	Takes the value of 1 if *BCtype[4]*='N', 0 if *BCtype[4]*='D', and -1 otherwise. This parameter is used only in the 3D case.
9	Takes the value of 1 if *BCtype[5]*='N', 0 if *BCtype[5]*='D', and -1 otherwise. This parameter is used only in the 3D case.
10	Takes the value of • *nx*, that is, the number of intervals along x-axis, in the Cartesian case. • *np*, that is, the number of intervals along φ-axis, in the spherical case.
11	Takes the value of

Index	Description
	• *ny*, that is, the number of intervals along *y*-axis, in the Cartesian case • *nt*, that is, the number of intervals along θ-axis, in the spherical case.
12	Takes the value of *nz*, the number of intervals along *z*-axis. This parameter is used only in the 3D case (Cartesian).
13	Takes the value of 6, which specifies the internal partitioning of the *dpar/spar* array.
14	Takes the value of *ipar[13]*+*ipar[10]*+1, which specifies the internal partitioning of the *dpar/spar* array.

Subsequent values of *ipar* depend upon the dimension of the problem or upon whether the solver on a sphere is periodic.

Index	2D case	3D case	Periodic case	Non-periodic case
15	Unused	Takes the value of *ipar[14]*+1, which specifies the internal partitioning of the *dpar/spar* array.		
16	Unused	Takes the value of *ipar[14]*+ *ipar[11]*+1, which specifies the internal partitioning of the *dpar/spar* array.		
17	Takes the value of *ipar[14]*+1, which specifies the internal partitioning of the *dpar/spar* array.	Takes the value of *ipar[16]*+1, which specifies the internal partitioning of the *dpar/spar* array.		
18	Takes the value of *ipar[14]*+3**ipar[10]* /2+1, which specifies the internal partitioning of the *dpar/spar* array.	Takes the value of *ipar[16]*+3* *ipar[10]*/2+1, which specifies the internal partitioning of the *dpar/spar* array.	Takes the value of *ipar[16]*+3 **ipar[10]*/4+1, which specifies the internal partitioning of the *dpar/spar* array.	Takes the value of *ipar[16]* +3**ipar[10]* / 2+1, which specifies the internal partitioning of the *dpar/spar* array.
19	Unused	Takes the value of *ipar[18]*+1, which specifies the internal partitioning of the *dpar/spar* array.		Unused
20	Unused	Takes the value of *ipar[18]*+ 3**ipar[11]*/2+1, which specifies the internal partitioning of the dpar/spar array.	Takes the value of *ipar[18]*+3**ipar[10]* /4+1, which specifies the internal partitioning of the dpar/spar array.	Unused

Subsequent values of *ipar* are assigned regardless.

21	Contains message style options: • *ipar[21]*=0 indicates that PL routines print all error and warning messages in Fortran-style notations. • *ipar[21]*=1 (default) indicates that PL routines print the messages in C-style notations.
22	Contains the number of threads to be used for computations in a multithreaded environment. The default value is 1 in the serial mode, and the result returned by the mkl_get_max_threads function otherwise.

Index	Description
23 through 39	Unused in the current implementation of the Poisson Library.
40 through 59	Contain the first twenty elements of the *ipar* array of the first Trigonometric Transform that the Solver uses. (For details, see Common Parameters in the "Trigonometric Transform Routines" chapter.)
60 through 79	Contain the first twenty elements of the *ipar* array of the second Trigonometric Transform that the 3D and periodic solvers use. (For details, see Common Parameters in the "Trigonometric Transform Routines" chapter.)

NOTE You may declare the *ipar* array in your code as `int ipar[80]`. However, for compatibility with later versions of Intel MKL Poisson Library, which may require more *ipar* values, it is highly recommended to declare *ipar* as `int ipar[128]`.

Arrays *dpar* and *spar* are the same except in the data precision:

dpar Holds data needed for double-precision Fast Helmholtz Solver computations.

- For the Cartesian solver, `double` array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case; initialized in the `d_init_Helmholtz_2D/d_init_Helmholtz_3D` and `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D` routines.
- For the spherical solver, `double` array of size 5*np/2+nt+10; initialized in the `d_init_sph_p/d_init_sph_np` and `d_commit_sph_p/d_commit_sph_np` routines.

spar Holds data needed for single-precision Fast Helmholtz Solver computations.

- For the Cartesian solver, `float` array of size 5*nx/2+7 in the 2D case or 5*(nx+ny)/2+9 in the 3D case; initialized in the `s_init_Helmholtz_2D/s_init_Helmholtz_3D` and `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D` routines.
- For the spherical solver, `float` array of size 5*np/2+nt+10; initialized in the `s_init_sph_p/s_init_sph_np` and `s_commit_sph_p/s_commit_sph_np` routines.

As *dpar* and *spar* have similar elements in respective positions, the elements are described together in Table "Elements of the dpar and spar Arrays":

Elements of the dpar and spar Arrays

Index	Description
0	In the Cartesian case, contains the length of the interval along x-axis right after a call to the `?_init_Helmholtz_2D/?_init_Helmholtz_3D` routine or the mesh size h_x in the x direction (for details, see Poisson Library Implemented) after a call to the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine. In the spherical case, contains the length of the interval along φ-axis right after a call to the `?_init_sph_p/?_init_sph_np` routine or the mesh size h_φ in the φ direction (for details, see Poisson Library Implemented) after a call to the `?_commit_sph_p/?_commit_sph_np` routine.
1	In the Cartesian case, contains the length of the interval along y-axis right after a call to the `?_init_Helmholtz_2D/?_init_Helmholtz_3D` routine or the mesh size h_y in the y direction (for details, see Poisson Library Implemented) after a call to the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine.

Index	Description
	In the spherical case, contains the length of the interval along θ-axis right after a call to the ?_init_sph_p/?_init_sph_np routine or the mesh size h_θ in the θ direction (for details, see Poisson Library Implemented) after a call to the ?_commit_sph_p/?_commit_sph_np routine.
2	In the Cartesian case, contains the length of the interval along z-axis right after a call to the ?_init_Helmholtz_2D/?_init_Helmholtz_3D routine or the mesh size h_z in the z direction (for details, see Poisson Library Implemented) after a call to the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routine. In the Cartesian solver, this parameter is used only in the 3D case. In the spherical solver, contains the coordinate of the leftmost boundary along θ-axis after a call to the ?_init_sph_p/?_init_sph_np routine.
3	Contains the value of the coefficient q after a call to the ?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np routine.
4	Contains the tolerance parameter after a call to the ?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np routine. • In the Cartesian case, this value is used only for the pure Neumann boundary conditions (*BCtype*="NNNN" in the 2D case; *BCtype*="NNNNNN" in the 3D case). This is a special case, because the right-hand side of the problem cannot be arbitrary if the coefficient q is zero. Poisson Library verifies that the classical solution exists (up to rounding errors) using this tolerance. In any case, Poisson Library computes the normal solution, that is, the solution that has the minimal Euclidean norm of residual. Nevertheless, the ?_Helmholtz_2D/?_Helmholtz_3D routine informs the user that the solution may not exist in a classical sense (up to rounding errors). • In the spherical case, the value is used for the special case of a periodic problem on the entire sphere. This special case is similar to the above described Cartesian case with pure Neumann boundary conditions. So, here Poisson Library computes the normal solution as well. The parameter is also used to detect whether the problem is periodic up to rounding errors. The default value for this parameter is 1.0E-10 in case of double-precision computations or 1.0E-4 in case of single-precision computations. You can increase the value of the tolerance, for instance, to avoid the warnings that may appear.
ipar[13]-1 through *ipar[14]*-1	In the Cartesian case, contain the spectrum of the 1D problem along x-axis after a call to the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routine. In the spherical case, contains the spectrum of the 1D problem along φ-axis after a call to the ?_commit_sph_p/?_commit_sph_np routine.
ipar[15]-1 through *ipar[16]*-1	In the Cartesian case, contain the spectrum of the 1D problem along y-axis after a call to the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routine. These elements are used only in the 3D case. In the spherical case, contains the spherical weights after a call to the ?_commit_sph_p/?_commit_sph_np routine.
ipar[17]-1 through *ipar[18]*-1	Take the values of the (staggered) sine/cosine in the mesh points: • along x-axis after a call to the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routine for a Cartesian solver • along φ-axis after a call to the ?_commit_sph_p/?_commit_sph_np routine for a spherical solver.

Index	Description
ipar[19]-1 through *ipar[20]*-1	Take the values of the (staggered) sine/cosine in the mesh points: • along *y*-axis after a call to the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routine for a Cartesian 3D solver • along φ-axis after a call to the ?_commit_sph_p routine for a spherical periodic solver. These elements are not used in the 2D Cartesian case and in the non-periodic spherical case.

NOTE You may define the array size depending upon the type of the problem to solve.

Caveat on Parameter Modifications

Flexibility of the PL interface enables you to skip calling the ?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np routine and to initialize the basic data structures explicitly in your code. You may also need to modify contents of *ipar*, *dpar* and *spar* arrays after initialization. When doing so, provide correct and consistent data in the arrays. Mistakenly altered arrays cause errors or wrong computation. You can perform a basic check for correctness and consistency of parameters by calling the ?_commit_Helmholtz_2D/?_commit_Helmholtz_3D routine; however, this does not ensure the correct solution but only reduces the chance of errors or wrong results.

NOTE To supply correct and consistent parameters to PL routines, you should have considerable experience in using the PL interface and good understanding of the solution process as well as elements that the *ipar*, *spar* and *dpar* arrays contain and dependencies between values of these elements.

However, in rare occurrences, even advanced users might fail in tuning parameters for the Fast Helmholtz Solver. In cases like these, refer for technical support at http://www.intel.com/software/products/support/ .

WARNING The only way that ensures a proper solution of a Helmholtz problem is to follow a typical sequence of invoking the routines and not change the default set of parameters. So, avoid modifications of *ipar*, *dpar* and *spar* arrays unless a strong need arises.

Implementation Details

Several aspects of the Intel MKL PL interface are platform-specific and language-specific. To promote portability across platforms and ease of use across different languages, users are provided with the PL language-specific header files to include in their code. Currently, the following header files are available:

- mkl_poisson.h, to be used together with mkl_dfti.h, for C programs.
- mkl_poisson.f90, to be used together with mkl_dfti.f90, for Fortran 90 programs.

Use of the Intel MKL PL software without including one of the above header files is not supported.

The include files define function prototypes for appropriate languages.

C-specific Header File

The C-specific header file defines the following function prototypes for the Cartesian solver:

```
void d_init_Helmholtz_2D(double*, double*, double*, double*, int*, int*, char*, double*, int*, double*,
int*);

void d_commit_Helmholtz_2D(double*, double*, double*, double*, double*, DFTI_DESCRIPTOR_HANDLE*, int*,
double*, int*);

void d_Helmholtz_2D(double*, double*, double*, double*, double*, DFTI_DESCRIPTOR_HANDLE*, int*,
double*, int*);


void s_init_Helmholtz_2D(float*, float*, float*, float*, int*, int*, char*, float*, int*, float*, int*);

void s_commit_Helmholtz_2D(float*, float*, float*, float*, float*, DFTI_DESCRIPTOR_HANDLE*, int*,
float*, int*);

void s_Helmholtz_2D(float*, float*, float*, float*, float*, DFTI_DESCRIPTOR_HANDLE*, int*, float*,
int*);


void free_Helmholtz_2D(DFTI_DESCRIPTOR_HANDLE*, int*, int*);


void d_init_Helmholtz_3D(double*, double*, double*, double*, double*, double*, int*, int*, int*, char*,
double*, int*, double*, int*);

void d_commit_Helmholtz_3D(double*, double*, double*, double*, double*, double*, double*,
DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, double*, int*);

void d_Helmholtz_3D(double*, double*, double*, double*, double*, double*, double*,
DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, double*, int*);


void s_init_Helmholtz_3D(float*, float*, float*, float*, float*, float*, int*, int*, int*, char*,
float*, int*, float*, int*);

void s_commit_Helmholtz_3D(float*, float*, float*, float*, float*, float*, float*,
DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, float*, int*);

void s_Helmholtz_3D(float*, float*, float*, float*, float*, float*, float*, DFTI_DESCRIPTOR_HANDLE*,
DFTI_DESCRIPTOR_HANDLE*, int*, float*, int*);

void free_Helmholtz_3D(DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, int*);
```

The C-specific header file defines the following function prototypes for the spherical solver:

```
void d_init_sph_p(double*, double*, double*, double*, int*, int*, double*, int*, double*, int*);

void d_commit_sph_p(double*, DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, double*, int*);

void d_sph_p(double*, DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, double*, int*);

void s_init_sph_p(float*, float*, float*, float*, int*, int*, float*, int*, float*, int*);

void s_commit_sph_p(float*, DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, float*, int*);

void s_sph_p(float*, DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, float*, int*);

void free_sph_p(DFTI_DESCRIPTOR_HANDLE*, DFTI_DESCRIPTOR_HANDLE*, int*, int*);


void d_init_sph_np(double*, double*, double*, double*, int*, int*, double*, int*, double*, int*);

void d_commit_sph_np(double*, DFTI_DESCRIPTOR_HANDLE*, int*, double*, int*);

void d_sph_np(double*, DFTI_DESCRIPTOR_HANDLE*, int*, double*, int*);

void s_init_sph_np(float*, float*, float*, float*, int*, int*, float*, int*, float*, int*);

void s_commit_sph_np(float*, DFTI_DESCRIPTOR_HANDLE*, int*, float*, int*);

void s_sph_np(float*, DFTI_DESCRIPTOR_HANDLE*, int*, float*, int*);

void free_sph_np(DFTI_DESCRIPTOR_HANDLE*, int*, int*);
```

Fortran-Specific Header File

The Fortran90-specific header file defines the following function prototypes for the Cartesian solver:

```
SUBROUTINE D_INIT_HELMHOLTZ_2D (AX, BX, AY, BY, NX, NY, BCTYPE, Q, IPAR, DPAR, STAT)
   USE MKL_DFTI

   INTEGER NX, NY, STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION AX, BX, AY, BY, Q
   DOUBLE PRECISION DPAR(*)
   CHARACTER(4) BCTYPE
END SUBROUTINE


SUBROUTINE D_COMMIT_HELMHOLTZ_2D (F, BD_AX, BD_BX, BD_AY, BD_BY, XHANDLE, IPAR, DPAR, STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION F(IPAR(11)+1,*)
   DOUBLE PRECISION DPAR(*)
   DOUBLE PRECISION BD_AX(*), BD_BX(*), BD_AY(*), BD_BY(*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: XHANDLE
END SUBROUTINE


SUBROUTINE D_HELMHOLTZ_2D (F, BD_AX, BD_BX, BD_AY, BD_BY, XHANDLE, IPAR, DPAR, STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION F(IPAR(11)+1,*)
   DOUBLE PRECISION DPAR(*)
   DOUBLE PRECISION BD_AX(*), BD_BX(*), BD_AY(*), BD_BY(*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: XHANDLE
END SUBROUTINE


SUBROUTINE S_INIT_HELMHOLTZ_2D (AX, BX, AY, BY, NX, NY, BCTYPE, Q, IPAR,
SPAR,
STAT)
   USE MKL_DFTI

   INTEGER NX, NY, STAT
   INTEGER IPAR(*)
```

```
DPAR, STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION F(IPAR(11)+1,IPAR(12)+1,*)
   DOUBLE PRECISION DPAR(*)
   DOUBLE PRECISION BD_AX(IPAR(12)+1,*), BD_BX(IPAR(12)+1,*), BD_AY(IPAR(11)+1,*)
   DOUBLE PRECISION BD_BY(IPAR(11)+1,*), BD_AZ(IPAR(11)+1,*), BD_BZ(IPAR(11)+1,*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: XHANDLE, YHANDLE
END SUBROUTINE


SUBROUTINE D_HELMHOLTZ_3D (F, BD_AX, BD_BX, BD_AY, BD_BY, BD_AZ, BD_BZ, XHANDLE, YHANDLE, IPAR, DPAR,
STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION F(IPAR(11)+1,IPAR(12)+1,*)
   DOUBLE PRECISION DPAR(*)
   DOUBLE PRECISION BD_AX(IPAR(12)+1,*), BD_BX(IPAR(12)+1,*), BD_AY(IPAR(11)+1,*)
   DOUBLE PRECISION BD_BY(IPAR(11)+1,*), BD_AZ(IPAR(11)+1,*), BD_BZ(IPAR(11)+1,*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: XHANDLE, YHANDLE
END SUBROUTINE


SUBROUTINE S_INIT_HELMHOLTZ_3D (AX, BX, AY, BY, AZ, BZ, NX, NY, NZ, BCTYPE, Q, IPAR, SPAR, STAT)
   USE MKL_DFTI

   INTEGER NX, NY, NZ, STAT
   INTEGER IPAR(*)
   REAL AX, BX, AY, BY, AZ, BZ, Q
   REAL SPAR(*)
   CHARACTER(6) BCTYPE
END SUBROUTINE


SUBROUTINE S_COMMIT_HELMHOLTZ_3D (F, BD_AX, BD_BX, BD_AY, BD_BY, BD_AZ, BD_BZ, XHANDLE, YHANDLE, IPAR,
SPAR, STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
```

```
STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   REAL F(IPAR(11)+1,IPAR(12)+1,*)
   REAL SPAR(*)
   REAL BD_AX(IPAR(12)+1,*), BD_BX(IPAR(12)+1,*), BD_AY(IPAR(11)+1,*)
   REAL BD_BY(IPAR(11)+1,*), BD_AZ(IPAR(11)+1,*), BD_BZ(IPAR(11)+1,*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: XHANDLE, YHANDLE
END SUBROUTINE


SUBROUTINE FREE_HELMHOLTZ_3D (XHANDLE, YHANDLE, IPAR, STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: XHANDLE, YHANDLE
END SUBROUTINE
```

The Fortran90-specific header file defines the following function prototypes for the spherical solver:

```
SUBROUTINE D_INIT_SPH_P(AP,BP,AT,BT,NP,NT,Q,IPAR,DPAR,STAT)
   USE MKL_DFTI

   INTEGER NP, NT, STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION AP,BP,AT,BT,Q
   DOUBLE PRECISION DPAR(*)
END SUBROUTINE


SUBROUTINE D_COMMIT_SPH_P(F,HANDLE_S,HANDLE_C,IPAR,DPAR,STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION DPAR(*)
   DOUBLE PRECISION F(IPAR(11)+1,*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: HANDLE_C, HANDLE_S
END SUBROUTINE


SUBROUTINE D_SPH_P(F,HANDLE_S,HANDLE_C,IPAR,DPAR,STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   DOUBLE PRECISION DPAR(*)
   DOUBLE PRECISION F(IPAR(11)+1,*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: HANDLE_C, HANDLE_S
END SUBROUTINE


SUBROUTINE S_INIT_SPH_P(AP,BP,AT,BT,NP,NT,Q,IPAR,SPAR,STAT)
   USE MKL_DFTI

   INTEGER NP, NT, STAT
   INTEGER IPAR(*)
   REAL AP,BP,AT,BT,Q
   REAL SPAR(*)
END SUBROUTINE
```

```
F(IPAR(11)+1,*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: HANDLE
END SUBROUTINE


SUBROUTINE S_SPH_NP(F,HANDLE,IPAR,SPAR,STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   REAL SPAR(*)
   REAL F(IPAR(11)+1,*)

TYPE(DFTI_DESCRIPTOR), POINTER :: HANDLE
END SUBROUTINE


SUBROUTINE FREE_SPH_NP(HANDLE,IPAR,STAT)
   USE MKL_DFTI

   INTEGER STAT
   INTEGER IPAR(*)
   TYPE(DFTI_DESCRIPTOR), POINTER :: HANDLE
END SUBROUTINE
```

Fortran 90 specifics of the PL routines usage are similar for all Intel MKL PDE support tools and described in the Calling PDE Support Routines from Fortran 90 section.

Calling PDE Support Routines from Fortran 90

The calling interface for all the Intel MKL TT and PL routines is designed to be easily used in C. However, you can invoke each TT or PL routine directly from Fortran 90 if you are familiar with the inter-language calling conventions of your platform.

The TT or PL interface cannot be invoked from Fortran 77 due to restrictions imposed by the use of the Intel MKL FFT interface.

The inter-language calling conventions include, but are not limited to, the argument passing mechanisms for the language, the data type mappings from C to Fortran 90 and how C external names are decorated on the platform.

To promote portability and relieve a user of dealing with the calling conventions specifics, Fortran 90 header file `mkl_trig_transforms.f90` for TT routines and `mkl_poisson.f90` for PL routines, used together with `mkl_dfti.f90`, declare a set of macros and introduce type definitions intended to hide the inter-language calling conventions and provide an interface to the routines that looks natural in Fortran 90.

For example, consider a hypothetical library routine, `foo`, which takes a double-precision vector of length `n`. C users access such a function as follows:

```
int n;
double *x;
…
foo(x, &n);
```

As noted above, to invoke `foo`, Fortran 90 users would need to know what Fortran 90 data types correspond to C types `int` and `double` (or `float` in case of single-precision), what argument-passing mechanism the C compiler uses and what, if any, name decoration is performed by the C compiler when generating the external symbol `foo`. However, with the Fortran 90 header files `mkl_trig_transforms.f90` / `mkl_poisson.f90` and `mkl_dfti.f90` included, the invocation of `foo` within a Fortran 90 program will look as follows:

- For TT interface,

```
use mkl_dfti
use mkl_trig_transforms
INTEGER n
DOUBLE PRECISION, ALLOCATABLE :: x
…
CALL FOO(x,n)
```

- For PL interface,

```
use mkl_dfti
use mkl_poisson
INTEGER n
DOUBLE PRECISION, ALLOCATABLE :: x
…
CALL FOO(x,n)
```

Note that in the above example, the header files `mkl_trig_transforms.f90 / mkl_poisson.f90` and `mkl_dfti.f90` provide a definition for the subroutine `FOO`. To ease the use of PL or TT routines in Fortran 90, the general approach of providing Fortran 90 definitions of names is used throughout the libraries. Specifically, if a name from a PL or TT interface is documented as having the C-specific name `foo`, then the Fortran 90 header files provide an appropriate Fortran 90 language type definition `FOO`.

One of the key differences between Fortran 90 and C is the language argument-passing mechanism: C programs use pass-by-value semantics and Fortran 90 programs use pass-by-reference semantics. The Fortran 90 headers ensure proper treatment of this difference. In particular, in the above example, the header files `mkl_trig_transforms.f90 / mkl_poisson.f90` and `mkl_dfti.f90` hide the difference by defining a macro `FOO` that takes the address of the appropriate arguments.

Nonlinear Optimization Problem Solvers

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Intel® Math Kernel Library (Intel® MKL) provides tools for solving nonlinear least squares problems using the Trust-Region (TR) algorithms. The solver routines are grouped according to their purpose as follows:

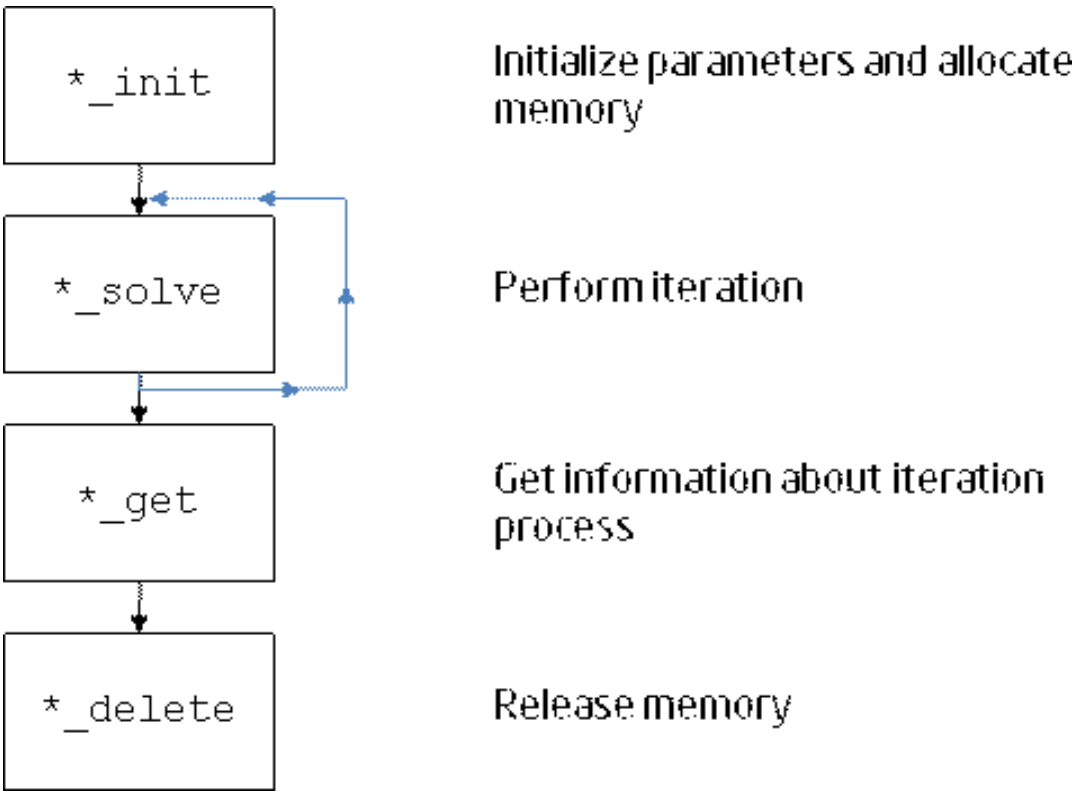
- Nonlinear Least Squares Problem without Constraints
- Nonlinear Least Squares Problem with Linear (Boundary) Constraints
- Jacobian Matrix Calculation Routines

For more information on the key concepts required to understand the use of the Intel MKL nonlinear least squares problem solver routines, see [Conn00].

Organization and Implementation

The Intel MKL solver routines for nonlinear least squares problems use reverse communication interfaces (RCI). That means you need to provide the solver with information required for the iteration process, for example, the corresponding Jacobian matrix, or values of the objective function. RCI removes the dependency of the solver on specific implementation of the operations. However, it does require that you organize a computational loop.

Typical order for invoking RCI solver routines



The nonlinear least squares problem solver routines, or Trust-Region (TR) solvers, are implemented with the OpenMP* support. You can manage the threads using threading control functions.

Memory Allocation and Handles

To make the TR solver routines easy to use, you are not required to allocate temporary working storage. The solver allocates any required memory. To allow multiple users to access the solver simultaneously, the solver keeps track of the storage allocated for a particular application by using a data object called a *handle*. Each TR solver routine creates, uses, or deletes a handle. To declare a handle, include `mkl_rci.h` or `mkl_rci.fi`.

For a C and C++ program, declare a handle as one of the following:

```
#include "mkl_rci.h"
_TRNSP_HANDLE_t handle;
```

or

```
_TRNSPBC_HANDLE_t handle;
```

The first declaration is used for nonlinear least squares problems without boundary constraints, and the second is used for nonlinear least squares problems with boundary constraints.

For a Fortran program using compilers that support eight byte integers, declare a handle as:

```
INCLUDE "mkl_rci.fi"
INTEGER*8 handle
```

Routine Naming Conventions

The TR routine names have the following structure:

```
      <character><name>_<action>( )
```

where

- <*character*> indicates the data type:

`s`	real, single precision
`d`	real, double precision

- <*name*> indicates the task type:

`trnlsp`	nonlinear least squares problem without constraints
`trnlspbc`	nonlinear least squares problem with boundary constraints
`jacobi`	computation of the Jacobian matrix using central differences

- <*action*> indicates an action on the task:

`init`	initializes the solver
`check`	checks correctness of the input parameters
`solve`	solves the problem
`get`	retrieves the number of iterations, the stop criterion, the initial residual, and the final residual
`delete`	releases the allocated data

Nonlinear Least Squares Problem without Constraints

The nonlinear least squares problem without constraints can be described as follows:

$$\min_{x \in R^n} \|F(x)\|_2^2 = \min_{x \in R^n} \|y - f(x)\|_2^2, \; y \in R^m, \; x \in R^n, \; f : R^n \to R^m, \; m \geq n,$$

where

$F(x) : R^n \to R^m$ is a twice differentiable function in R^n.

Solving a nonlinear least squares problem means searching for the best approximation to the vector y with the model function $f_i(x)$ and nonlinear variables x. The best approximation means that the sum of squares of residuals $y_i - f_i(x)$ is the minimum.

See usage examples in FORTRAN and C in the `examples\solver\source` folder of your Intel MKL directory (`ex_nlsqp_f.f` and `ex_nlsqp_c.c`, respectively).

RCI TR Routines

Routine Name	Operation
`?trnlsp_init`	Initializes the solver.
`?trnlsp_check`	Checks correctness of the input parameters.
`?trnlsp_solve`	Solves a nonlinear least squares problem using the Trust-Region algorithm.
`?trnlsp_get`	Retrieves the number of iterations, stop criterion, initial residual, and final residual.
`?trnlsp_delete`	Releases allocated data.

?trnlsp_init

Initializes the solver of a nonlinear least squares problem.

Syntax

Fortran:

```
res = strnlsp_init(handle, n, m, x, eps, iter1, iter2, rs)
res = dtrnlsp_init(handle, n, m, x, eps, iter1, iter2, rs)
```

C:

```
res = strnlsp_init(&handle, &n, &m, x, eps, &iter1, &iter2, &rs);
res = dtrnlsp_init(&handle, &n, &m, x, eps, &iter1, &iter2, &rs);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?trnlsp_init` routine initializes the solver.

After initialization, all subsequent invocations of the `?trnlsp_solve` routine should use the values of the handle returned by `?trnlsp_init`.

The *eps* array contains the stopping criteria:

eps Value	Description
1	$\Delta < eps(1)$
2	$\|\|F(x)\|\|_2 < eps(2)$
3	The Jacobian matrix is singular. $\|\|J(x)_{(1:m,j)}\|\|_2 < eps(3),\ j = 1, \ldots, n$

eps Value	Description
4	$\|\|s\|\|_2 < eps(4)$
5	$\|\|F(x)\|\|_2 - \|\|F(x) - J(x)s\|\|_2 < eps(5)$
6	The trial step precision. If $eps(6) = 0$, then the trial step meets the required precision ($\leq$ 1.0D-10).

Note:

- $J(x)$ is the Jacobian matrix.
- Δ is the trust-region area.
- $F(x)$ is the value of the functional.
- s is the trial step.

Input Parameters

n — INTEGER. Length of x.

m — INTEGER. Length of $F(x)$.

x — REAL for strnlsp_init
DOUBLE PRECISION for dtrnlsp_init
Array of size n. Initial guess.

eps — REAL for strnlsp_init
DOUBLE PRECISION for dtrnlsp_init
Array of size 6; contains stopping criteria. See the values in the Description section.

iter1 — INTEGER. Specifies the maximum number of iterations.

iter2 — INTEGER. Specifies the maximum number of iterations of trial step calculation.

rs — REAL for strnlsp_init
DOUBLE PRECISION for dtrnlsp_init
Definition of initial size of the trust region (boundary of the trial step). The minimum value is 0.1, and the maximum value is 100.0. Based on your knowledge of the objective function and initial guess you can increase or decrease the initial trust region. It can influence the iteration process, for example, the direction of the iteration process and the number of iterations. The default value is 100.0.

Output Parameters

handle — Type _TRNSP_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.

res — INTEGER. Indicates task completion status.

- *res* = TR_SUCCESS - the routine completed the task normally.
- *res* = TR_INVALID_OPTION - there was an error in the input parameters.
- *res* = TR_OUT_OF_MEMORY - there was a memory error.

TR_SUCCESS, TR_INVALID_OPTION, and TR_OUT_OF_MEMORY are defined in mkl_rci.fi (Fortran) and mkl_rci.h (C) include files.

See Also

?trnlsp_solve

?trnlsp_check

Checks the correctness of handle and arrays containing Jacobian matrix, objective function, and stopping criteria.

Syntax

Fortran:

```
res = strnlsp_check(handle, n, m, fjac, fvec, eps, info)
res = dtrnlsp_check(handle, n, m, fjac, fvec, eps, info)
```

C:

```
res = strnlsp_check(&handle, &n, &m, fjac, fvec, eps, info);
res = dtrnlsp_check(&handle, &n, &m, fjac, fvec, eps, info);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?trnlsp_check` routine checks the arrays passed into the solver as input parameters. If an array contains any `INF` or `NaN` values, the routine sets the flag in output array *info* (see the description of the values returned in the Output Parameters section for the *info* array).

Input Parameters

handle	Type `_TRNSPBC_HANDLE_t` in C/C++ and `INTEGER*8` in FORTRAN.
n	`INTEGER`. Length of x.
m	`INTEGER`. Length of $F(x)$.
fjac	`REAL` for `strnlsp_check` `DOUBLE PRECISION` for `dtrnlsp_check` Array of size m by n. Contains the Jacobian matrix of the function.
fvec	`REAL` for `strnlsp_check` `DOUBLE PRECISION` for `dtrnlsp_check` Array of size m. Contains the function values at `X`, where $fvec(i) = (y_i - f_i(x))$.
eps	`REAL` for `strnlsp_check` `DOUBLE PRECISION` for `dtrnlsp_check` Array of size 6; contains stopping criteria. See the values in the Description section of the `?trnlsp_init`.

Output Parameters

info	`INTEGER` Array of size 6. Results of input parameter checking:

Parameter		Used for	Value	Description
C Language	Fortran Language			
info(0)	*info*(1)	Flags for *handle*	0	The handle is valid.
			1	The handle is not allocated.
info(1)	*info*(2)	Flags for *fjac*	0	The *fjac* array is valid.
			1	The *fjac* array is not allocated
			2	The *fjac* array contains NaN.
			3	The *fjac* array contains Inf.
info(2)	*info*(3)	Flags for *fvec*	0	The *fvec* array is valid.
			1	The *fvec* array is not allocated
			2	The *fvec* array contains NaN.
			3	The *fvec* array contains Inf.
info(3)	*info*(4)	Flags for *eps*	0	The *eps* array is valid.
			1	The *eps* array is not allocated
			2	The *eps* array contains NaN.
			3	The *eps* array contains Inf.
			4	The *eps* array contains a value less than or equal to zero.

res INTEGER. Information about completion of the task.
res = TR_SUCCESS - the routine completed the task normally.
TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

?trnlsp_solve

Solves a nonlinear least squares problem using the TR algorithm.

Syntax

Fortran:

```
res = strnlsp_solve(handle, fvec, fjac, RCI_Request)
res = dtrnlsp_solve(handle, fvec, fjac, RCI_Request)
```

C:

```
res = strnlsp_solve(&handle, fvec, fjac, &RCI_Request);
res = dtrnlsp_solve(&handle, fvec, fjac, &RCI_Request);
```

Include Files

- Fortran: mkl_rci.fi

- C: `mkl_rci.h`

Description

The `?trnlsp_solve` routine uses the TR algorithm to solve nonlinear least squares problems.

The problem is stated as follows:

$$\min_{x \in R^n} \|F(x)\|_2^2 = \min_{x \in R^n} \|y - f(x)\|_2^2, \; y \in R^m, \; x \in R^n, \; f : R^n \to R^m, \; m \geq n,$$

where

- $F(x) : R^n \to R^m$
- $m \geq n$

From a current point $x_{current}$, the algorithm uses the trust-region approach:

$$\min_{x \in R^n} \|F(x_{current}) + J(x_{current})(x_{new} - x_{current})\|_2^2 \quad \text{subject to} \quad \|x_{new} - x_{current}\| \leq \Delta_{current}$$

to get $x_{new} = x_{current} + s$ that satisfies

$$\min_{x \in R^n} \|J^T(x)J(x)s + J^T F(x)\|_2^2$$

where

- $J(x)$ is the Jacobian matrix
- s is the trial step
- $||s||_2 \leq \Delta_{current}$

The *RCI_Request* parameter provides additional information:

RCI_Request **Value**	**Description**
2	Request to calculate the Jacobian matrix and put the result into *fjac*
1	Request to recalculate the function at vector `X` and put the result into *fvec*
0	One successful iteration step on the current trust-region radius (that does not mean that the value of x has changed)
-1	The algorithm has exceeded the maximum number of iterations
-2	$\Delta < eps(1)$
-3	$\|\|F(x)\|\|_2 < eps(2)$
-4	The Jacobian matrix is singular. $\|\|J(x)_{(1:m,j)}\|\|_2 < eps(3), \; j = 1, \ldots, n$
-5	$\|\|s\|\|_2 < eps(4)$
-6	$\|\|F(x)\|\|_2 - \|\|F(x) - J(x)s\|\|_2 < eps(5)$

Note:

- $J(x)$ is the Jacobian matrix.
- Δ is the trust-region area.
- $F(x)$ is the value of the functional.
- s is the trial step.

Input Parameters

handle	Type _TRNSP_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.
fvec	REAL for strnlsp_solve DOUBLE PRECISION for dtrnlsp_solve Array of size m. Contains the function values at X, where $fvec(i) = (y_i - f_i(x))$.
fjac	REAL for strnlsp_solve DOUBLE PRECISION for dtrnlsp_solve Array of size (m,n). Contains the Jacobian matrix of the function.

Output Parameters

fvec	REAL for strnlsp_solve DOUBLE PRECISION for dtrnlsp_solve Array of size m. Updated function evaluated at x.
RCI_Request	INTEGER. Informs about the task stage. See the Description section for the parameter values and their meaning.
res	INTEGER. Indicates the task completion. *res* = TR_SUCCESS - the routine completed the task normally. TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

?trnlsp_get

Retrieves the number of iterations, stop criterion, initial residual, and final residual.

Syntax

Fortran:

```
res = strnlsp_get(handle, iter, st_cr, r1, r2)
res = dtrnlsp_get(handle, iter, st_cr, r1, r2)
```

C:

```
res = strnlsp_get(&handle, &iter, &st_cr, &r1, &r2);
res = dtrnlsp_get(&handle, &iter, &st_cr, &r1, &r2);
```

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine retrieves the current number of iterations, the stop criterion, the initial residual, and final residual.

The initial residual is the value of the functional $(||y - f(x)||)$ of the initial x values provided by the user.

The final residual is the value of the functional $(||y - f(x)||)$ of the final x resulting from the algorithm operation.

The *st_cr* parameter contains the stop criterion:

st_cr **Value**	**Description**
1	The algorithm has exceeded the maximum number of iterations
2	$\Delta < eps(1)$
3	$\|\|F(x)\|\|_2 < eps(2)$
4	The Jacobian matrix is singular. $\|\|J(x)_{(1:m,j)}\|\|_2 < eps(3),\ j = 1, \ldots, n$
5	$\|\|s\|\|_2 < eps(4)$
6	$\|\|F(x)\|\|_2 - \|\|F(x) - J(x)s\|\|_2 < eps(5)$

Note:

- $J(x)$ is the Jacobian matrix.
- Δ is the trust-region area.
- $F(x)$ is the value of the functional.
- s is the trial step.

Input Parameters

handle	Type `_TRNSP_HANDLE_t` in C/C++ and `INTEGER*8` in FORTRAN.

Output Parameters

iter	`INTEGER`. Contains the current number of iterations.
st_cr	`INTEGER`. Contains the stop criterion. See the Description section for the parameter values and their meanings.
r1	`REAL` for `strnlsp_get` `DOUBLE PRECISION` for `dtrnlsp_get` Contains the residual, $(||y - f(x)||)$ given the initial x.
r2	`REAL` for `strnlsp_get` `DOUBLE PRECISION` for `dtrnlsp_get` Contains the final residual, that is, the value of the functional $(||y - f(x)||)$ of the final x resulting from the algorithm operation.
res	`INTEGER`. Indicates the task completion. *res* = `TR_SUCCESS` - the routine completed the task normally. `TR_SUCCESS` is defined in the `mkl_rci.h` and `mkl_rci.fi` include files.

?trnlsp_delete

Releases allocated data.

Syntax

Fortran:

```
res = strnlsp_delete(handle)
res = dtrnlsp_delete(handle)
```

C:

```
res = strnlsp_delete(&handle);
res = dtrnlsp_delete(&handle);
```

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The ?trnlsp_delete routine releases all memory allocated for the handle.

This routine flags memory as not used, but to actually release all memory you must call the support function mkl_free_buffers.

Input Parameters

handle	Type _TRNSP_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.

Output Parameters

res	INTEGER. Indicates the task completion. *res* = TR_SUCCESS means the routine completed the task normally. TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

Nonlinear Least Squares Problem with Linear (Bound) Constraints

The nonlinear least squares problem with linear bound constraints is very similar to the nonlinear least squares problem without constraints but it has the following constraints:

$$l_i \le x_i \le u_i, i = 1,\ldots,n, \quad l,u \in R^n.$$

See usage examples in FORTRAN and C in the examples\solver\source folder of your Intel MKL directory (ex_nlsqp_bc_f.f and ex_nlsqp_bc_c.c, respectively).

RCI TR Routines for Problem with Bound Constraints

Routine Name	Operation
?trnlspbc_init	Initializes the solver.
?trnlspbc_check	Checks correctness of the input parameters.
?trnlspbc_solve	Solves a nonlinear least squares problem using RCI and the Trust-Region algorithm.
?trnlspbc_get	Retrieves the number of iterations, stop criterion, initial residual, and final residual.
?trnlspbc_delete	Releases allocated data.

?trnlspbc_init

Initializes the solver of nonlinear least squares problem with linear (boundary) constraints.

Syntax

Fortran:

```
res = strnlspbc_init(handle, n, m, x, LW, UP, eps, iter1, iter2, rs)
res = dtrnlspbc_init(handle, n, m, x, LW, UP, eps, iter1, iter2, rs)
```

C:

```
res = strnlspbc_init(&handle, &n, &m, x, LW, UP, eps, &iter1, &iter2, &rs);
res = dtrnlspbc_init(&handle, &n, &m, x, LW, UP, eps, &iter1, &iter2, &rs);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?trnlspbc_init` routine initializes the solver.

After initialization all subsequent invocations of the `?trnlspbc_solve` routine should use the values of the handle returned by `?trnlspbc_init`.

The *eps* array contains the stopping criteria:

eps **Value**	**Description**
1	$\Delta < eps(1)$
2	$\|\|F(x)\|\|_2 < eps(2)$
3	The Jacobian matrix is singular. $\|\|J(x)_{(1:m,j)}\|\|_2 < eps(3),\ j = 1, \ldots, n$
4	$\|\|s\|\|_2 < eps(4)$
5	$\|\|F(x)\|\|_2 - \|\|F(x) - J(x)s\|\|_2 < eps(5)$
6	The trial step precision. If $eps(6) = 0$, then the trial step meets the required precision ($\leq$ 1.0D-10).

Note:

- $J(x)$ is the Jacobian matrix.
- Δ is the trust-region area.
- $F(x)$ is the value of the functional.
- s is the trial step.

Input Parameters

n	INTEGER. Length of x.
m	INTEGER. Length of $F(x)$.
x	REAL for `strnlspbc_init`

DOUBLE PRECISION for dtrnlspbc_init
Array of size n. Initial guess.

LW
REAL for strnlspbc_init
DOUBLE PRECISION for dtrnlspbc_init
Array of size n.
Contains low bounds for x ($lw_i < x_i$).

UP
REAL for strnlspbc_init
DOUBLE PRECISION for dtrnlspbc_init
Array of size n.
Contains upper bounds for x ($up_i > x_i$).

eps
REAL for strnlspbc_init
DOUBLE PRECISION for dtrnlspbc_init
Array of size 6; contains stopping criteria. See the values in the Description section.

iter1
INTEGER. Specifies the maximum number of iterations.

iter2
INTEGER. Specifies the maximum number of iterations of trial step calculation.

rs
REAL for strnlspbc_init
DOUBLE PRECISION for dtrnlspbc_init
Definition of initial size of the trust region (boundary of the trial step). The minimum value is 0.1, and the maximum value is 100.0. Based on your knowledge of the objective function and initial guess you can increase or decrease the initial trust region. It can influence the iteration process, for example, the direction of the iteration process and the number of iterations. The default value is 100.0.

Output Parameters

handle
Type _TRNSPBC_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.

res
INTEGER. Informs about the task completion.

- *res* = TR_SUCCESS - the routine completed the task normally.
- *res* = TR_INVALID_OPTION - there was an error in the input parameters.
- *res* = TR_OUT_OF_MEMORY - there was a memory error.

TR_SUCCESS, TR_INVALID_OPTION, and TR_OUT_OF_MEMORY are defined in mkl_rci.fi (Fortran) and mkl_rci.h (C) include files.

?trnlspbc_check

Checks the correctness of handle and arrays containing Jacobian matrix, objective function, lower and upper bounds, and stopping criteria.

Syntax

Fortran:

```
res = strnlspbc_check(handle, n, m, fjac, fvec, LW, UP, eps, info)
res = dtrnlspbc_check(handle, n, m, fjac, fvec, LW, UP, eps, info)
```

C:

```
res = strnlspbc_check(&handle, &n, &m, fjac, fvec, LW, UP, eps, info);
res = dtrnlspbc_check(&handle, &n, &m, fjac, fvec, LW, UP, eps, info);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?trnlspbc_check` routine checks the arrays passed into the solver as input parameters. If an array contains any `INF` or `NaN` values, the routine sets the flag in output array *info* (see the description of the values returned in the Output Parameters section for the *info* array).

Input Parameters

handle Type `_TRNSPBC_HANDLE_t` in C/C++ and `INTEGER*8` in FORTRAN.

n `INTEGER`. Length of x.

m `INTEGER`. Length of $F(x)$.

fjac
`REAL` for `strnlspbc_check`
`DOUBLE PRECISION` for `dtrnlspbc_check`
Array of size m by n. Contains the Jacobian matrix of the function.

fvec
`REAL` for `strnlspbc_check`
`DOUBLE PRECISION` for `dtrnlspbc_check`
Array of size m. Contains the function values at `X`, where $fvec(i) = (y_i - f_i(x))$.

LW
`REAL` for `strnlspbc_check`
`DOUBLE PRECISION` for `dtrnlspbc_check`
Array of size n.
Contains low bounds for x ($lw_i < x_i$).

UP
`REAL` for `strnlspbc_check`
`DOUBLE PRECISION` for `dtrnlspbc_check`
Array of size n.
Contains upper bounds for x ($up_i > x_i$).

eps
`REAL` for `strnlspbc_check`
`DOUBLE PRECISION` for `dtrnlspbc_check`
Array of size 6; contains stopping criteria. See the values in the Description section of the `?trnlspbc_init`.

Output Parameters

info `INTEGER`
Array of size 6.
Results of input parameter checking:

Parameter		Used for	Value	Description
C Language	Fortran Language			
info(0)	*info*(1)	Flags for *handle*	0	The handle is valid.
			1	The handle is not allocated.
info(1)	*info*(2)	Flags for *fjac*	0	The *fjac* array is valid.
			1	The *fjac* array is not allocated
			2	The *fjac* array contains `NaN`.

Parameter		Used for	Value	Description
C Language	Fortran Language			
			3	The *fjac* array contains Inf.
info(2)	*info*(3)	Flags for *fvec*	0	The *fvec* array is valid.
			1	The *fvec* array is not allocated
			2	The *fvec* array contains NaN.
			3	The *fvec* array contains Inf.
info(3)	*info*(4)	Flags for *LW*	0	The *LW* array is valid.
			1	The *LW* array is not allocated
			2	The *LW* array contains NaN.
			3	The *LW* array contains Inf.
			4	The lower bound is greater than the upper bound.
info(4)	*info*(5)	Flags for *up*	0	The *up* array is valid.
			1	The *up* array is not allocated
			2	The *up* array contains NaN.
			3	The *up* array contains Inf.
			4	The upper bound is less than the lower bound.
info(5)	*info*(6)	Flags for *eps*	0	The *eps* array is valid.
			1	The *eps* array is not allocated
			2	The *eps* array contains NaN.
			3	The *eps* array contains Inf.
			4	The *eps* array contains a value less than or equal to zero.

res INTEGER. Information about completion of the task.
res = TR_SUCCESS - the routine completed the task normally.
TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

?trnlspbc_solve

Solves a nonlinear least squares problem with linear (bound) constraints using the Trust-Region algorithm.

Syntax

Fortran:

```
res = strnlspbc_solve(handle, fvec, fjac, RCI_Request)
res = dtrnlspbc_solve(handle, fvec, fjac, RCI_Request)
```

C:

```
res = strnlspbc_solve(&handle, fvec, fjac, &RCI_Request);
res = dtrnlspbc_solve(&handle, fvec, fjac, &RCI_Request);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?trnlspbc_solve` routine, based on RCI, uses the Trust-Region algorithm to solve nonlinear least squares problems with linear (bound) constraints. The problem is stated as follows:

$$\min_{x \in R^n} \left\| F(x) \right\|_2^2 = \min_{x \in R^n} \left\| y - f(x) \right\|_2^2, y \in R^m, x \in R^n, f : R^n \to R^m, m \geq n$$

where

$l_i \leq x_i \leq u_i$

$i = 1, \ldots, n$.

The *RCI_Request* parameter provides additional information:

RCI_Request **Value**	**Description**
2	Request to calculate the Jacobian matrix and put the result into *fjac*
1	Request to recalculate the function at vector `X` and put the result into *fvec*
0	One successful iteration step on the current trust-region radius (that does not mean that the value of x has changed)
-1	The algorithm has exceeded the maximum number of iterations
-2	$\Delta < eps(1)$
-3	$\|\|F(x)\|\|_2 < eps(2)$
-4	The Jacobian matrix is singular. $\|\|J(x)_{(1:m,j)}\|\|_2 < eps(3), j = 1, \ldots, n$
-5	$\|\|s\|\|_2 < eps(4)$
-6	$\|\|F(x)\|\|_2 - \|\|F(x) - J(x)s\|\|_2 < eps(5)$

Note:

- $J(x)$ is the Jacobian matrix.
- Δ is the trust-region area.
- $F(x)$ is the value of the functional.

- s is the trial step.

Input Parameters

handle	Type _TRNSPBC_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.
fvec	REAL for strnlspbc_solve DOUBLE PRECISION for dtrnlspbc_solve Array of size *m*. Contains the function values at X, where *fvec(i)* = (y_i – $f_i(x)$).
fjac	REAL for strnlspbc_solve DOUBLE PRECISION for dtrnlspbc_solve Array of size *m* by *n*. Contains the Jacobian matrix of the function.

Output Parameters

fvec	REAL for strnlspbc_solve DOUBLE PRECISION for dtrnlspbc_solve Array of size *m*. Updated function evaluated at x.
RCI_Request	INTEGER. Informs about the task stage. See the Description section for the parameter values and their meaning.
res	INTEGER. Informs about the task completion. *res* = TR_SUCCESS means the routine completed the task normally. TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

?trnlspbc_get

Retrieves the number of iterations, stop criterion, initial residual, and final residual.

Syntax

Fortran:

```
res = strnlspbc_get(handle, iter, st_cr, r1, r2)
res = dtrnlspbc_get(handle, iter, st_cr, r1, r2)
```

C:

```
res = strnlspbc_get(&handle, &iter, &st_cr, &r1, &r2);
res = dtrnlspbc_get(&handle, &iter, &st_cr, &r1, &r2);
```

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine retrieves the current number of iterations, the stop criterion, the initial residual, and final residual.

The *st_cr* parameter contains the stop criterion:

st_cr Value	Description
1	The algorithm has exceeded the maximum number of iterations

st_cr Value	Description
2	$\Delta < eps(1)$
3	$\|\|F(x)\|\|_2 < eps(2)$
4	The Jacobian matrix is singular. $\|\|J(x)_{(1:m,j)}\|\|_2 < eps(3),\ j = 1, \ldots, n$
5	$\|\|s\|\|_2 < eps(4)$
6	$\|\|F(x)\|\|_2 - \|\|F(x) - J(x)s\|\|_2 < eps(5)$

Note:

- $J(x)$ is the Jacobian matrix.
- Δ is the trust-region area.
- $F(x)$ is the value of the functional.
- s is the trial step.

Input Parameters

handle	Type _TRNSPBC_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.

Output Parameters

iter	INTEGER. Contains the current number of iterations.
st_cr	INTEGER. Contains the stop criterion. See the Description section for the parameter values and their meanings.
r1	REAL for strnlspbc_get DOUBLE PRECISION for dtrnlspbc_get Contains the residual, $(\|\|y - f(x)\|\|)$ given the initial x.
r2	REAL for strnlspbc_get DOUBLE PRECISION for dtrnlspbc_get Contains the final residual, that is, the value of the function $(\|\|y - f(x)\|\|)$ of the final x resulting from the algorithm operation.
res	INTEGER. Informs about the task completion. *res* = TR_SUCCESS - the routine completed the task normally. TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

?trnlspbc_delete

Releases allocated data.

Syntax

Fortran:

```
res = strnlspbc_delete(handle)
res = dtrnlspbc_delete(handle)
```

C:

```
res = strnlspbc_delete(&handle);
res = dtrnlspbc_delete(&handle);
```

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The ?trnlspbc_delete routine releases all memory allocated for the handle.

NOTE This routine flags memory as not used, but to actually release all memory you must call the support function mkl_free_buffers.

Input Parameters

handle	Type _TRNSPBC_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.

Output Parameters

res	INTEGER. Informs about the task completion. *res* = TR_SUCCESS means the routine completed the task normally. TR_SUCCESS is defined in the mkl_rci.h and mkl_rci.fi include files.

Jacobian Matrix Calculation Routines

This section describes routines that compute the Jacobian matrix using the central difference algorithm. Jacobian matrix calculation is required to solve a nonlinear least squares problem and systems of nonlinear equations (with or without linear bound constraints). Routines for calculation of the Jacobian matrix have the "Black-Box" interfaces, where you pass the objective function via parameters. Your objective function must have a fixed interface.

Jacobian Matrix Calculation Routines

Routine Name	Operation
?jacobi_init	Initializes the solver.
?jacobi_solve	Computes the Jacobian matrix of the function on the basis of RCI using the central difference algorithm.
?jacobi_delete	Removes data.
?jacobi	Computes the Jacobian matrix of the fcn function using the central difference algorithm.
?jacobix	Presents an alternative interface for the ?jacobi function enabling you to pass additional data into the objective function.

?jacobi_init

Initializes the solver for Jacobian calculations.

Syntax

Fortran:

```
res = sjacobi_init(handle, n, m, x, fjac, esp)
res = djacobi_init(handle, n, m, x, fjac, esp)
```

C:

```
res = sjacobi_init(&handle, &n, &m, x, fjac, &eps);
res = djacobi_init(&handle, &n, &m, x, fjac, &eps);
```

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The routine initializes the solver.

Input Parameters

n	INTEGER. Length of *x*.
m	INTEGER. Length of *F*.
x	REAL for sjacobi_init DOUBLE PRECISION for djacobi_init Array of size *n*. Vector, at which the function is evaluated.
eps	REAL for sjacobi_init DOUBLE PRECISION for djacobi_init Precision of the Jacobian matrix calculation.
fjac	REAL for sjacobi_init DOUBLE PRECISION for djacobi_init Array of size (*m*,*n*). Contains the Jacobian matrix of the function.

Output Parameters

handle	Data object of the _JACOBIMATRIX_HANDLE_t type in C/C++ and INTEGER*8 in FORTRAN.
res	INTEGER. Indicates task completion status. • *res* = TR_SUCCESS - the routine completed the task normally. • *res* = TR_INVALID_OPTION - there was an error in the input parameters. • *res* = TR_OUT_OF_MEMORY - there was a memory error. TR_SUCCESS, TR_INVALID_OPTION, and TR_OUT_OF_MEMORY are defined in mkl_rci.fi (Fortran) and mkl_rci.h (C) include files.

?jacobi_solve

Computes the Jacobian matrix of the function using RCI and the central difference algorithm.

Syntax

Fortran:

```
res = sjacobi_solve(handle, f1, f2, RCI_Request)
res = djacobi_solve(handle, f1, f2, RCI_Request)
```

C:

```
res = sjacobi_solve(&handle, f1, f2, &RCI_Request);
res = djacobi_solve(&handle, f1, f2, &RCI_Request);
```

Include Files

- Fortran: mkl_rci.fi
- C: mkl_rci.h

Description

The ?jacobi_solve routine computes the Jacobian matrix of the function using RCI and the central difference algorothm.

See usage examples in FORTRAN and C in the examples\solver\source folder of your Intel MKL directory (sjacobi_rci_f.f, djacobi_rci_f.f and sjacobi_rci_c.c, djacobi_rci_c.c, respectively).

Input Parameters

handle	Type _JACOBIMATRIX_HANDLE_t in C/C++ and INTEGER*8 in FORTRAN.

Output Parameters

f1	REAL for sjacobi_solve DOUBLE PRECISION for djacobi_solve Contains the updated function values at *x* + *eps*.
f2	REAL for sjacobi_solve DOUBLE PRECISION for djacobi_solve Array of size *m*. Contains the updated function values at *x* - *eps*.
RCI_Request	INTEGER. Informs about the task completion. When equal to 0, the task has completed successfully. *RCI_Request*= 1 indicates that you should compute the function values at the current *x* point and put the results into *f1*. *RCI_Request*= 2 indicates that you should compute the function values at the current *x* point and put the results into *f2*.
res	INTEGER. Indicates the task completion status. • *res* = TR_SUCCESS - the routine completed the task normally. • *res* = TR_INVALID_OPTION - there was an error in the input parameters. TR_SUCCESS and TR_INVALID_OPTION are defined in mkl_rci.fi (Fortran) and mkl_rci.h (C) include files.

See Also

?jacobi_init

?jacobi_delete

Releases allocated data.

Syntax

Fortran:

```
res = sjacobi_delete(handle)
res = djacobi_delete(handle)
```

C:

```
res = sjacobi_delete(&handle);
```

```
res = djacobi_delete(&handle);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?jacobi_delete` routine releases all memory allocated for the handle.

This routine flags memory as not used, but to actually release all memory you must call the support function mkl_free_buffers.

Input Parameters

handle	Type `_JACOBIMATRIX_HANDLE_t` in C/C++ and `INTEGER*8` in FORTRAN.

Output Parameters

res	`INTEGER`. Informs about the task completion. *res* = `TR_SUCCESS` means the routine completed the task normally. `TR_SUCCESS` is defined in the `mkl_rci.h` and `mkl_rci.fi` include files.

?jacobi

Computes the Jacobian matrix of the objective function using the central difference algorithm.

Syntax

Fortran:

```
res = sjacobi(fcn, n, m, fjac, x, jac_eps)
res = djacobi(fcn, n, m, fjac, x, jac_eps)
```

C:

```
res = sjacobi(fcn, &n, &m, fjac, x, &jac_eps);
res = djacobi(fcn, &n, &m, fjac, x, &jac_eps);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?jacobi` routine computes the Jacobian matrix for function *fcn* using the central difference algorithm. This routine has a "Black-Box" interface, where you input the objective function via parameters. Your objective function must have a fixed interface.

See calling and usage examples in FORTRAN and C in the `examples\solver\source` folder of your Intel MKL directory (`ex_nlsqp_f.f`, `ex_nlsqp_bc_f.f` and `ex_nlsqp_c.c`, `ex_nlsqp_bc_c.c`, respectively).

Input Parameters

fcn	User-supplied subroutine to evaluate the function that defines the least squares problem. Call `fcn` (*m*, *n*, *x*, *f*) with the following parameters:

Parameter	Type	Description
Input Parameters		
m	INTEGER	Length of *f*
n	INTEGER	Length of *x*
x	REAL for sjacobi DOUBLE PRECISION for djacobi	Array of size *n*. Vector, at which the function is evaluated. The fcn function should not change this parameter.
Output Parameters		
f	REAL for sjacobix DOUBLE PRECISION for djacobix	Array of size *m*; contains the function values at *x*.

You need to declare fcn as EXTERNAL in the calling program.

n INTEGER. Length of *X*.

m INTEGER. Length of *F*.

x REAL for sjacobi
DOUBLE PRECISION for djacobi
Array of size *n*. Vector at which the function is evaluated.

eps REAL for sjacobi
DOUBLE PRECISION for djacobi
Precision of the Jacobian matrix calculation.

Output Parameters

fjac REAL for sjacobi
DOUBLE PRECISION for djacobi
Array of size (*m*,*n*). Contains the Jacobian matrix of the function.

res INTEGER. Indicates task completion status.

- *res* = TR_SUCCESS - the routine completed the task normally.
- *res* = TR_INVALID_OPTION - there was an error in the input parameters.
- *res* = TR_OUT_OF_MEMORY - there was a memory error.

TR_SUCCESS, TR_INVALID_OPTION, and TR_OUT_OF_MEMORY are defined in mkl_rci.fi (Fortran) and mkl_rci.h (C) include files.

See Also

?jacobix

?jacobix

Alternative interface for ?jacobi function for passing additional data into the objective function.

Syntax

Fortran:

```
res = sjacobix(fcn, n, m, fjac, x, jac_eps, user_data)
res = djacobix(fcn, n, m, fjac, x, jac_eps, user_data)
```

C:

```
res = sjacobix(fcn, &n, &m, fjac, x, &jac_eps, user_data);
res = djacobix(fcn, &n, &m, fjac, x, &jac_eps, user_data);
```

Include Files

- Fortran: `mkl_rci.fi`
- C: `mkl_rci.h`

Description

The `?jacobix` routine presents an alternative interface for the `?jacobi` function that enables you to pass additional data into the objective function *fcn*.

See calling and usage examples in FORTRAN and C in the `examples\solver\source` folder of your Intel MKL directory (`ex_nlsqp_f_x.f`, `ex_nlsqp_bc_f_x.f` and `ex_nlsqp_c_x.c`, `ex_nlsqp_bc_c_x.c`, respectively).

Input Parameters

fcn

User-supplied subroutine to evaluate the function that defines the least squares problem. Call `fcn` (*m*, *n*, *x*, *f*, *user_data*) with the following parameters:

Parameter	Type	Description
Input Parameters		
m	`INTEGER`	Length of *f*
n	`INTEGER`	Length of *x*
x	`REAL` for `sjacobix` `DOUBLE PRECISION` for `djacobix`	Array of size *n*. Vector, at which the function is evaluated. The `fcn` function should not change this parameter.
user_data	`INTEGER*8`, for Fortran	(Fortran) Your additional data, if any. Otherwise, a dummy argument.
	`void*`, for C	(C) Pointer to your additional data, if any. Otherwise, a dummy argument.
Output Parameters		
f	`REAL` for `sjacobix` `DOUBLE PRECISION` for `djacobix`	Array of size *m*; contains the function values at *x*.

You need to declare `fcn` as `EXTERNAL` in the calling program.

n

`INTEGER`. Length of *X*.

m

`INTEGER`. Length of *F*.

x

`REAL` for `sjacobix`
`DOUBLE PRECISION` for `djacobix`
Array of size *n*. Vector at which the function is evaluated.

eps

`REAL` for `sjacobix`
`DOUBLE PRECISION` for `djacobix`
Precision of the Jacobian matrix calculation.

user_data (Fortran) INTEGER*8. Contains your additional data. If there is no additional data, this is a dummy argument.
(C) void*. Pointer to your additional data. If there is no additional data, this is a dummy argument.

Output Parameters

fjac REAL for sjacobix
DOUBLE PRECISION for djacobix
Array of size (*m*,*n*). Contains the Jacobian matrix of the function.

res INTEGER. Indicates task completion status.

- *res* = TR_SUCCESS - the routine completed the task normally.
- *res* = TR_INVALID_OPTION - there was an error in the input parameters.
- *res* = TR_OUT_OF_MEMORY - there was a memory error.

TR_SUCCESS, TR_INVALID_OPTION, and TR_OUT_OF_MEMORY are defined in mkl_rci.fi (Fortran) and mkl_rci.h (C) include files.

See Also

?jacobi

15 Support Functions

Intel® Math Kernel Library (Intel® MKL) support functions are used to:

- retrieve information about the current Intel MKL version
- additionally control the number of threads
- handle errors
- test characters and character strings for equality
- measure user time for a process and elapsed CPU time
- measure CPU frequency
- free memory allocated by Intel MKL memory management software
- facilitate easy linking

Functions described below are subdivided according to their purpose into the following groups:

Version Information Functions

Threading Control Functions

Error Handling Functions

Equality Test Functions

Timing Functions

Memory Functions

Miscellaneous Utility Functions

Functions Supporting the Single Dynamic Library

Table "Intel MKL Support Functions" contains the list of support functions common for Intel MKL.

Intel MKL Support Functions

Function Name	Operation
Version Information Functions	
mkl_get_version	Returns information about the active library version.
mkl_get_version_string	Returns information about the library version string.
Threading Control Functions	
mkl_set_num_threads	Suggests the number of threads to use.
mkl_domain_set_num_threads	Suggests the number of threads for a particular function domain.
mkl_set_dynamic	Enables Intel MKL to dynamically change the number of threads.
mkl_get_max_threads	Inquires about the number of threads targeted for parallelism.
mkl_domain_get_max_threads	Inquires about the number of threads targeted for parallelism in different domains.
mkl_get_dynamic	Returns the current value of the MKL_DYNAMIC variable.
Error Handling Functions	

Function Name	Operation
xerbla	Handles error conditions for the BLAS, LAPACK, VSL, VML routines.
pxerbla	Handles error conditions for the ScaLAPACK routines.
Equality Test Functions	
lsame	Tests two characters for equality regardless of the case.
lsamen	Tests two character strings for equality regardless of the case.
Timing Functions	
second/dsecnd	Returns user time for a process.
mkl_get_cpu_clocks	Returns full precision elapsed CPU clocks.
mkl_get_cpu_frequency	Returns CPU frequency value in GHz.
mkl_get_max_cpu_frequency	Returns the maximum CPU frequency value in GHz.
mkl_get_clocks_frequency	Returns the frequency value in GHz based on constant-rate Time Stamp Counter.
Memory Functions	
mkl_free_buffers	Frees memory buffers.
mkl_thread_free_buffers	Frees memory buffers allocated only in the current thread.
mkl_mem_stat	Reports an amount of memory utilized by Intel MKL memory management software.
mkl_disable_fast_mm	Enables Intel MKL to dynamically turn off memory management.
mkl_malloc	Allocates the aligned memory buffer.
mkl_free	Frees the aligned memory buffer allocated by MKL_malloc.
Miscellaneous Utility Functions	
mkl_progress	Tracks computational progress of selective MKL routines.
mkl_enable_instructions	Allows Intel MKL to dispatch Intel® Advanced Vector Extensions (Intel® AVX) if run on the respective hardware (or simulation).
Functions Supporting the Single Dynamic Library (SDL)	
mkl_set_interface_layer	Sets the interface layer for Intel MKL at run time.
mkl_set_threading_layer	Sets the threading layer for Intel MKL at run time.
mkl_set_xerbla	Replaces the error handling routine. Use with SDL on Windows* OS.
mkl_set_progress	Replaces the progress information routine. Use with SDL on Windows* OS.

Version Information Functions

Intel® MKL provides two methods for extracting information about the library version number:

- extracting a version string using the `mkl_get_version_string` function
- using the `mkl_get_version` function to obtain an `MKLVersion` structure that contains the version information

A makefile is also provided to automatically build the examples and output summary files containing the version information for the current library.

mkl_get_version

Returns information about the active library C version.

Syntax

```
void mkl_get_version( MKLVersion* pVersion );
```

Include Files

- C: `mkl_service.h`

Output Parameters

pVersion	Pointer to the `MKLVersion` structure.

Description

The `mkl_get_version` function collects information about the active C version of the Intel MKL software and returns this information in a structure of `MKLVersion` type by the *pVersion* address. The `MKLVersion` structure type is defined in the `mkl_types.h` file. The following fields of the `MKLVersion` structure are available:

`MajorVersion`	is the major number of the current library version.
`MinorVersion`	is the minor number of the current library version.
`UpdateVersion`	is the update number of the current library version.
`ProductStatus`	is the status of the current library version. Possible variants could be “Beta”, “Product”.
`Build`	is the string that contains the build date and the internal build number.
`Processor`	is the processor optimization that is targeted for the specific processor. It is not the definition of the processor installed in the system, rather the MKL library detection that is optimal for the processor installed in the system.

NOTE `MKLGetVersion` is an obsolete name for the `mkl_get_version` function that is referenced in the library for back compatibility purposes but is deprecated and subject to removal in subsequent releases.

Optimization Notice
use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice. Notice revision #20110804

mkl_get_version Usage

```
--------------------------------------------------------------------------------
#include <stdio.h>
#include <stdlib.h>
#include "mkl_service.h"

int main(void)
  {
    MKLVersion Version;

    mkl_get_version(&Version);

  //  MKL_Get_Version(&Version);

    printf("Major version:           %d\n",Version.MajorVersion);
    printf("Minor version:           %d\n",Version.MinorVersion);
    printf("Update version:          %d\n",Version.UpdateVersion);
    printf("Product status:          %s\n",Version.ProductStatus);
    printf("Build:                   %s\n",Version.Build);
    printf("Processor optimization:  %s\n",Version.Processor);
    printf("================================================================\n");
    printf("\n");

    return 0;
  }
```

Output:

Major Version	9
Minor Version	0
Update Version	0
Product status	Product
Build	061909.09
Processor optimization	Intel® Xeon® Processor with Intel® 64 architecture

mkl_get_version_string

Gets the library version string.

Syntax

Fortran:

```
call mkl_get_version_string( buf )
```

C:

```
mkl_get_version_string( buf, len );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Output Parameters

Name	Type	Description
buf	**FORTRAN:** `CHARACTER*198` **C:** `char*`	Source string
len	**FORTRAN:** `INTEGER` **C:** `int`	Length of the source string

Description

The function returns a string that contains the library version information.

NOTE `MKLGetVersionString` is an obsolete name for the `mkl_get_version_string` function that is referenced in the library for back compatibility purposes but is deprecated and subject to removal in subsequent releases.

See example below:

Examples

Fortran Example

```
program mkl_get_version_string

character*198  buf

call mkl_get_version_string(buf)
write(*,'(a)') buf

end
```

C Example

```
#include <stdio.h>
#include "mkl_service.h"

int main(void)
{
  int len=198;
  char buf[198];
  mkl_get_version_string(buf, len);
  printf("%s\n",buf);
```

```
  printf("\n");
  return 0;
}
```

Threading Control Functions

Intel® MKL provides optional threading control functions that take precedence over OpenMP* environment variable settings with the same purpose (see *Intel® MKL User's Guide* for details).

These functions enable you to specify the number of threads for Intel MKL independently of the OpenMP* settings and takes precedence over them. Although Intel MKL may actually use a different number of threads from the number suggested, the controls also enable you to instruct the library to try using the suggested number when the number used in the calling application is unavailable.

See the following examples of Fortran and C usage:

Fortran Usage

```
call mkl_set_num_threads( foo )
ierr = mkl_domain_set_num_threads( num, MKL_DOMAIN_BLAS )
call mkl_set_dynamic ( 1 )
num = mkl_get_max_threads()
num = mkl_domain_get_max_threads( MKL_DOMAIN_BLAS );
ret = mkl_get_dynamic()
```

C Usage

```
#include "mkl.h"  // Mandatory to make these definitions work!
mkl_set_num_threads(num);
return_code = mkl_domain_set_num_threads( num, MKL_DOMAIN_FFT );
mkl_set_dynamic( 1 );
num = mkl_get_max_threads();
num = mkl_domain_get_max_threads( MKL_DOMAIN_FFT );
return_code = mkl_get_dynamic();
```

NOTE Always remember to add `#include "mkl.h"` to use the C usage syntax.

mkl_set_num_threads

Suggests the number of threads to use.

Syntax

Fortran:

```
call mkl_set_num_threads( number )
```

C:

```
void mkl_set_num_threads( number );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
number	**FORTRAN:** INTEGER **C:** int	Number of threads suggested by user

Description

This function allows you to specify how many threads Intel MKL should use. The number is a hint, and there is no guarantee that exactly this number of threads will be used. Enter a positive integer. This routine takes precedence over the MKL_NUM_THREADS environment variable.

NOTE Always remember to add #include "mkl.h" to use the C usage syntax.

See *Intel MKL User's Guide* for implementation details.

mkl_domain_set_num_threads

Suggests the number of threads for a particular function domain.

Syntax

Fortran:

```
ierr = mkl_domain_set_num_threads( num, mask )
```

C:

```
ierr = mkl_domain_set_num_threads( num, mask );
```

Include Files

- FORTRAN 77: mkl_service.fi
- C: mkl_service.h

Input Parameters

Name	Type	Description
num	**FORTRAN:** INTEGER **C:** int	Number of threads suggested by user
mask	**FORTRAN:** INTEGER **C:** int	Name of the targeted domain

Description

This function allows you to request different domains of Intel MKL to use different numbers of threads. The currently supported domains are:

- MKL_DOMAIN_BLAS - BLAS
- MKL_DOMAIN_FFT - FFT (excluding cluster FFT)
- MKL_DOMAIN_VML - Vector Math Library
- MKL_DOMAIN_PARDISO - PARDISO
- MKL_DOMAIN_ALL - another way to do what mkl_set_num_threads does

This is only a hint, and use of this number of threads is not guaranteed. Enter a valid domain and a positive integer for the number of threads. This routine has precedence over the MKL_DOMAIN_NUM_THREADS environment variable.

See *Intel MKL User's Guide* for implementation details.

Return Values

1(true)	Indicates no error, execution is successful.
0(false)	Indicates failure, possibly because the inputs were invalid.

mkl_set_dynamic

Enables Intel MKL to dynamically change the number of threads.

Syntax

Fortran:

```
call mkl_set_dynamic( boolean_var )
```

C:

```
void mkl_set_dynamic( boolean_var );
```

Include Files

- FORTRAN 77: mkl_service.fi
- C: mkl_service.h

Input Parameters

Name	Type	Description
boolean_var	**FORTRAN:** INTEGER **C:** int	The parameter that determines whether dynamic adjustment of the number of threads is enabled or disabled.

Description

This function indicates whether or not Intel MKL can dynamically change the number of threads. The default for this is true, regardless of how the OMP_DYNAMIC variable is set. This will also hold precedent over the OMP_DYNAMIC variable.

A value of false does not guarantee that the user's requested number of threads will be used. But it means that Intel MKL will attempt to use that value. This routine takes precedence over the environment variable MKL_DYNAMIC.

Note that if Intel MKL is called from within a parallel region, Intel MKL may not thread unless MKL_DYNAMIC is set to false, either with the environment variable or by this routine call.

See *Intel MKL User's Guide* for implementation details.

mkl_get_max_threads

Inquires about the number of threads targeted for parallelism.

Syntax

Fortran:

```
num = mkl_get_max_threads()
```

C:

```
num = mkl_get_max_threads();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Description

This function allows you to inquire independently of OpenMP* how many threads Intel MKL is targeting for parallelism. The number is a hint, and there is no guarantee that exactly this number of threads will be used.

See *Intel MKL User's Guide* for implementation details.

Return Values

The output is `INTEGER` equal to the number of threads.

mkl_domain_get_max_threads

Inquires about the number of threads targeted for parallelism in different domains.

Syntax

Fortran:

```
ierr = mkl_domain_get_max_threads( mask )
```

C:

```
ierr = mkl_domain_get_max_threads( mask );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
mask	**FORTRAN:** `INTEGER` **C:** `int`	The name of the targeted domain

Description

This function allows the user of different domains of Intel MKL to inquire what number of threads is being used as a hint. The inquiry does not imply that this is the actual number of threads used. The number may vary depending on the value of the `MKL_DYNAMIC` variable and/or problem size, system resources, etc. But the function returns the value that MKL is targeting for a given domain.

The currently supported domains are:

- `MKL_DOMAIN_BLAS` - BLAS

- MKL_DOMAIN_FFT - FFT (excluding cluster FFT)
- MKL_DOMAIN_VML - Vector Math Library
- MKL_DOMAIN_PARDISO - PARDISO
- MKL_DOMAIN_ALL - another way to do what mkl_get_max_threads does.

You are supposed to enter a valid domain.

See *Intel MKL User's Guide* for implementation details.

Return Values

Returns the hint about the number of threads for a given domain.

mkl_get_dynamic

Returns current value of MKL_DYNAMIC *variable.*

Syntax

Fortran:

```
ret = mkl_get_dynamic()
```

C:

```
ret = mkl_get_dynamic();
```

Include Files

- FORTRAN 77: mkl_service.fi
- C: mkl_service.h

Description

This function returns the current value of the MKL_DYNAMIC variable. This variable can be changed by manipulating the MKL_DYNAMIC environment variable before the Intel MKL run is launched or by calling mkl_set_dynamic(). Doing the latter has precedence over the former.

The function returns a value of 0 or 1: 1 indicates that MKL_DYNAMIC is true, 0 indicates that MKL_DYNAMIC is false. This variable indicates whether or not Intel MKL can dynamically change the number of threads. A value of false does not guarantee that the number of threads you requested will be used. But it means that Intel MKL will attempt to use that value.

Note that if Intel MKL is called from within a parallel region, Intel MKL may not thread unless MKL_DYNAMIC is set to false, either with the environment variable or by this routine call.

See *Intel MKL User's Guide* for implementation details.

Return Values

1	Indicates MKL_DYNAMIC is true.
0	Indicates MKL_DYNAMIC is false.

Error Handling Functions

xerbla

Error handling routine called by BLAS, LAPACK, VML, VSL routines.

Syntax

Fortran:

```
call xerbla( srname, info )
```

C:

```
xerbla( srname, info, len );
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- C: `mkl_blas.h`

Input Parameters

Name	Type	Description
srname	**FORTRAN:** `CHARACTER*(*)` **C:** `char*`	The name of the routine that called `xerbla`
info	**FORTRAN:** `INTEGER` **C:** `int*`	The position of the invalid parameter in the parameter list of the calling routine
len	**C:** `int`	Length of the source string

Description

The routine `xerbla` is an error handler for the BLAS, LAPACK, VSL, and VML routines. It is called by a BLAS, LAPACK, VSL or VML routine if an input parameter has an invalid value. If an issue is found with an input parameter, `xerbla` prints a message similar to the following:

```
MKL ERROR: Parameter 6 was incorrect on entry to DGEMM
```

and then returns to your application. Comments in the LAPACK reference code (http://www.netlib.org/lapack/explore-html/xerbla.f.html) suggest this behavior though the LAPACK User's Guide recommends that the execution should stop when an error is found.

Note that `xerbla` is an internal function. You can change or disable printing of an error message by providing your own `xerbla` function. See the FORTRAN and C examples below.

Examples

```
subroutine xerbla (srname, info)

character*(*) srname  !Name of subprogram that called xerbla

integer*4     info    !Position of the invalid parameter in the
parameter list

return                !Return to the calling subprogram

end
```

```
void xerbla(char* srname, int* info, int len){
// srname - name of the function that called xerbla
// info - position of the invalid parameter in the parameter list
// len - length of the name in bytes
printf("\nXERBLA is called :%s: %d\n",srname,*info);
}
```

pxerbla

Error handling routine called by ScaLAPACK routines.

Syntax

`call pxerbla(`*`ictxt`*`, `*`srname`*`, `*`info`*`)`

Include Files

- C: `mkl_scalapack.h`

Input Parameters

`ictxt`	(global) `INTEGER` The BLACS context handle, indicating the global context of the operation. The context itself is global.
`srname`	(global) `CHARACTER*6` The name of the routine which called `pxerbla`.
`info`	(global) `INTEGER`. The position of the invalid parameter in the parameter list of the calling routine.

Description

This routine is an error handler for the *ScaLAPACK* routines. It is called if an input parameter has an invalid value. A message is printed and program execution continues. For ScaLAPACK driver and computational routines, a `RETURN` statement is issued following the call to `pxerbla`.

Control returns to the higher-level calling routine, and you can determine how the program should proceed. However, in the specialized low-level ScaLAPACK routines (auxiliary routines that are Level 2 equivalents of computational routines), the call to `pxerbla()` is immediately followed by a call to `BLACS_ABORT()` to terminate program execution since recovery from an error at this level in the computation is not possible.

It is always good practice to check for a nonzero value of *`info`* on return from a ScaLAPACK routine. Installers may consider modifying this routine in order to call system-specific exception-handling facilities.

Equality Test Functions

lsame

Tests two characters for equality regardless of the case.

Syntax

Fortran:

```
val = lsame( ca, cb )
```

C:

```
val = lsame( ca, cb );
```

Include Files

- FORTRAN 77: `mkl_blas.fi`
- C: `mkl_blas.h`

Input Parameters

Name	Type	Description
ca, *cb*	**FORTRAN:** `CHARACTER*1`	**FORTRAN:** The single characters to be compared
	C: `const char*`	**C:** Pointers to the single characters to be compared

Output Parameters

Name	Type	Description
val	**FORTRAN:** `LOGICAL`	Result of the comparison
	C: `int`	

Description

This logical function returns `.TRUE.` if *ca* is the same letter as *cb* regardless of the case, and `.FALSE.` otherwise.

lsamen

Tests two character strings for equality regardless of the case.

Syntax

Fortran:

```
val = lsamen( n, ca, cb )
```

C:

```
val = lsamen( n, ca, cb );
```

Include Files

- FORTRAN 77: `mkl_lapack.fi`
- C: `mkl_lapack.h`

Input Parameters

Name	Type	Description
n	**FORTRAN:** `INTEGER`	**FORTRAN:** The number of characters in *ca* and *cb* to be compared.

Name	Type	Description
	C: `const int*`	**C:** Pointer to the number of characters in *ca* and *cb* to be compared.
ca, *cb*	**FORTRAN:** `CHARACTER*(*)` **C:** `const char*`	Specify two character strings of length at least *n* to be compared. Only the first *n* characters of each string will be accessed.

Output Parameters

Name	Type	Description
val	**FORTRAN:** `LOGICAL` **C:** `int`	**FORTRAN:** Result of the comparison. `.TRUE.` if *ca* and *cb* are equivalent except for the case, and `.FALSE.` otherwise. The function also returns `.FALSE.` if `len`(*ca*) or `len`(*cb*) is less than *n*. **C:** Result of the comparison. Non-zero if *ca* and *cb* are equivalent except for the case, and zero otherwise.

Description

This logical function tests if the first *n* letters of one string are the same as the first *n* letters of another string, regardless of the case.

Timing Functions

second/dsecnd

Returns elapsed CPU time in seconds.

Syntax

Fortran:

```
val = second()
val = dsecnd()
```

C:

```
val = second();
val = dsecnd();
```

Include Files

- FORTRAN 77: `mkl_lapack.fi`
- C: `mkl_lapack.h`

Output Parameters

Name	Type	Description
val	**FORTRAN:** `REAL` for `second` `DOUBLE PRECISION` for `dsecnd`	Elapsed CPU time in seconds

Name	Type	Description
	C: `float` for `second` `double` for `dsecnd`	

Description

The `second`/`dsecnd` functions return the elapsed CPU time in seconds. The difference between these functions is that `dsecnd` returns the result with double precision.

Apply each function in pairs: the first time, directly before a call to the routine to be measured, and the second time - after the measurement. The difference between the returned values is the time spent in the routine.

The `second`/`dsecnd` functions get the time from the elapsed CPU clocks divided by frequency. Obtaining the frequency may take some time when the `second`/`dsecnd` function runs for the first time. To eliminate the effect of this extra time on your measurements, make the first call to `second`/`dsecnd` in advance.

Do not use `second` for measuring short time intervals because the single-precision format is not capable of holding sufficient timer precision.

mkl_get_cpu_clocks

Returns full precision elapsed CPU clocks.

Syntax

Fortran:

```
call mkl_get_cpu_clocks( clocks )
```

C:

```
mkl_get_cpu_clocks( &clocks );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Output Parameters

Name	Type	Description
clocks	**FORTRAN:** `INTEGER*8`	Elapsed CPU clocks
	C: `unsigned MKL_INT64`	

Description

The `mkl_get_cpu_clocks` function returns the elapsed CPU clocks.

This may be useful when timing short intervals with high resolution. The `mkl_get_cpu_clocks` function is also applied in pairs like `second/dsecnd`. Note that out-of-order code execution on IA-32 or Intel® 64 architecture processors may disturb the exact elapsed CPU clocks value a little bit, which may be important while measuring extremely short time intervals.

NOTE `getcpuclocks` is an obsolete name for the `mkl_get_cpu_clocks` function that is referenced in the library for back compatibility purposes but is deprecated and subject to removal in subsequent releases.

mkl_get_cpu_frequency

Returns the current CPU frequency value in GHz.

Syntax

Fortran:

```
freq = mkl_get_cpu_frequency()
```

C:

```
freq = mkl_get_cpu_frequency();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Output Parameters

Name	Type	Description
freq	**FORTRAN:** `DOUBLE PRECISION` **C:** `double`	Current CPU frequency value in GHz

Description

The function `mkl_get_cpu_frequency` returns the current CPU frequency in GHz.

NOTE `getcpufrequency` is an obsolete name for the `mkl_get_cpu_frequency` function that is referenced in the library for back compatibility purposes but is deprecated and subject to removal in subsequent releases.

mkl_get_max_cpu_frequency

Returns the maximum CPU frequency value in GHz.

Syntax

Fortran:

```
freq = mkl_get_max_cpu_frequency()
```

C:

```
freq = mkl_get_max_cpu_frequency();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Output Parameters

Name	Type	Description
freq	**FORTRAN:** `DOUBLE PRECISION`	Maximum CPU frequency value in GHz
	C: `double`	

Description

The function `mkl_get_max_cpu_frequency` returns the maximum CPU frequency in GHz.

mkl_get_clocks_frequency

Returns the frequency value in GHz based on constant-rate Time Stamp Counter.

Syntax

Fortran:

```
freq = mkl_get_clocks_frequency()
```

C:

```
freq = mkl_get_clocks_frequency();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Output Parameters

Name	Type	Description
freq	**FORTRAN:** `DOUBLE PRECISION`	Frequency value in GHz
	C: `double`	

Description

The function `mkl_get_clocks_frequency` returns the CPU frequency value (in GHz) based on constant-rate Time Stamp Counter (TSC). Use of the constant-rate TSC ensures that each clock tick is constant even if the CPU frequency changes. Therefore, the returned frequency is constant.

NOTE Obtaining the frequency may take some time when `mkl_get_clocks_frequency` is called for the first time. The same holds for functions `second/dsecnd`, which call `mkl_get_clocks_frequency`.

See Also

second/dsecnd

Memory Functions

This section describes the Intel MKL memory support functions. See the *Intel© MKL User's Guide* for details of the Intel MKL memory management.

mkl_free_buffers

Frees memory buffers.

Syntax

Fortran:

```
call mkl_free_buffers
```

C:

```
mkl_free_buffers();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Description

The `mkl_free_buffers` function frees the memory allocated by the Intel MKL memory management software. The memory management software allocates new buffers if no free buffers are currently available. Call `mkl_free_buffers()` to free all memory buffers and to avoid memory leaking on completion of work with the Intel MKL functions, that is, after the last call of an Intel MKL function from your application.

See *Intel© MKL User's Guide* for details.

NOTE `MKL_FreeBuffers` is an obsolete name for the `mkl_free_buffers` function that is referenced in the library for back compatibility purposes but is deprecated and subject to removal in subsequent releases.

mkl_free_buffers Usage with FFT Functions

```
-------------------------------------------------------------------------------------------
DFTI_DESCRIPTOR_HANDLE hand1;
DFTI_DESCRIPTOR_HANDLE hand2;
void mkl_free_buffers(void);
. . . . . .
/* Using MKL FFT */
Status = DftiCreateDescriptor(&hand1, DFTI_SINGLE, DFTI_COMPLEX, dim, m1);
Status = DftiCommitDescriptor(hand1);
Status = DftiComputeForward(hand1, s_array1);
. . . . . .
Status = DftiCreateDescriptor(&hand2, DFTI_SINGLE, DFTI_COMPLEX, dim, m2);
Status = DftiCommitDescriptor(hand2);
. . . . . .
Status = DftiFreeDescriptor(&hand1);
/* Do not call mkl_free_buffers() here as the hand2 descriptor will be corrupted!  */
. . . . . .
Status = DftiComputeBackward(hand2, s_array2));
Status = DftiFreeDescriptor(&hand2);

/* Here user finishes the MKL FFT usage */
/* Memory leak will be triggered by any memory control tool */

/* Use mkl_free_buffers() to avoid memory leaking */
mkl_free_buffers();

-------------------------------------------------------------------------------------------
```

If the memory space is sufficient, use `mkl_free_buffers` after the last call of the MKL functions. Otherwise, a drop in performance can occur due to reallocation of buffers for the subsequent MKL functions.

WARNING For FFT calls, do not use `mkl_free_buffers` between `DftiCreateDescriptor(hand)` and `DftiFreeDescriptor(&hand)`.

mkl_thread_free_buffers

Frees memory buffers allocated in the current thread.

Syntax

Fortran:

```
call mkl_thread_free_buffers
```

C:

```
mkl_thread_free_buffers();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Description

The `mkl_thread_free_buffers` function frees the memory allocated by the Intel MKL memory management in the current thread only. Memory buffers allocated in other threads are not affected. Call `mkl_thread_free_buffers()` to avoid memory leaking if you are unable to call the `mkl_free_buffers` function in the multi-threaded application when you are not sure if all the other running Intel MKL functions completed operation.

mkl_disable_fast_mm

Enables Intel MKL to dynamically turn off memory management.

Syntax

Fortran:

```
mm = mkl_disable_fast_mm
```

C:

```
mm = mkl_disable_fast_mm();
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Description

The Intel MKL memory management software is turned on by default. To turn it off dynamically before any Intel MKL function call, you can use the `mkl_disable_fast_mm` function similarly to the `MKL_DISABLE_FAST_MM` environment variable (See *Intel© MKL User's Guide* for details.) Run `mkl_disable_fast_mm` function to allocate and free memory from call to call. Note that disabling the Intel MKL memory management software negatively impacts performance of some Intel MKL routines, especially for small problem sizes.

The function return value 1 indicates that the Intel MKL memory management was turned off successfully. The function return value 0 indicates a failure.

mkl_mem_stat

Reports amount of memory utilized by Intel MKL memory management software.

Syntax

Fortran:

```
AllocatedBytes = mkl_mem_stat( AllocatedBuffers )
```

C:

```
AllocatedBytes = mkl_mem_stat( &AllocatedBuffers );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Output Parameters

Name	Type	Description
AllocatedBytes	**FORTRAN:** `INTEGER*8` **C:** `MKL_INT64`	Amount of allocated bytes
AllocatedBuffers	**FORTRAN:** `INTEGER*4`, **C:** `int`	Number of allocated buffers

Description

The function returns the amount of the allocated memory in the *AllocatedBuffers* buffers. If there are no allocated buffers at the moment, the function returns 0. Call the `mkl_mem_stat()` function to check the Intel MKL memory status.

Note that after calling `mkl_free_buffers` there should not be any allocated buffers.

See Example "mkl_malloc(), mkl_free(), mkl_mem_stat() Usage".

NOTE `MKL_MemStat` is an obsolete name for the `MKL_Mem_Stat` function that is referenced in the library for back compatibility purposes but is deprecated and subject to removal in subsequent releases.

mkl_malloc

Allocates the aligned memory buffer.

Syntax

Fortran:

```
a_ptr = mkl_malloc( alloc_size, alignment )
```

C:

```
a_ptr = mkl_malloc( alloc_size, alignment );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
alloc_size	**FORTRAN:** `INTEGER*4` **C:** `size_t`	Size of the buffer to be allocated Note that Fortran type `INTEGER*4` is given for the 32-bit systems. Otherwise, it is `INTEGER*8`.
alignment	**FORTRAN:** `INTEGER*4`	Alignment of the allocated buffer

Name	Type	Description
	C: `int`	

Output Parameters

Name	Type	Description
a_ptr	**FORTRAN:** `POINTER`	Pointer to the allocated buffer
	C: `void*`	

Description

The function allocates a *size*-bytes buffer, aligned on the *alignment* boundary, and returns a pointer to this buffer.

The function returns `NULL` if *size* < 1. If alignment is not power of 2, the alignment 32 is used.

See Example "mkl_malloc(), mkl_free(), mkl_mem_stat() Usage".

mkl_free

Frees the aligned memory buffer allocated by `mkl_malloc`.

Syntax

Fortran:

```
call mkl_free( a_ptr )
```

C:

```
mkl_free( a_ptr );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
a_ptr	**FORTRAN:** `POINTER`	Pointer to the buffer to be freed
	C: `void*`	

Description

The function frees the buffer pointed by *ptr* and allocated by `mkl_malloc()`.

See Example "mkl_malloc(), mkl_free(), mkl_mem_stat() Usage".

Examples of mkl_malloc(), mkl_free(), mkl_mem_stat() Usage

Usage Example in Fortran

```
 PROGRAM FOO

 REAL*8     A,B,C
```

```
  POINTER    (A_PTR,A(1)), (B_PTR,B(1)), (C_PTR,C(1)
  INTEGER    N, I
  REAL*8     ALPHA, BETA
  INTEGER*8  ALLOCATED_BYTES
  INTEGER*4  ALLOCATED_BUFFERS

  #ifdef _SYSTEM_BITS32
        INTEGER*4 MKL_MALLOC
        INTEGER*4 ALLOC_SIZE
  #else
        INTEGER*8 MKL_MALLOC
        INTEGER*8 ALLOC_SIZE
  #endif

  INTEGER    MKL_MEM_STAT
  EXTERNAL   MKL_MALLOC, MKL_FREE, MKL_MEM_STAT

  ALPHA = 1.1; BETA = -1.2
  N = 1000
  ALLOC_SIZE = 8*N*N
  A_PTR = MKL_MALLOC(ALLOC_SIZE,64)
  B_PTR = MKL_MALLOC(ALLOC_SIZE,64)
  C_PTR = MKL_MALLOC(ALLOC_SIZE,64)
  DO I=1,N*N
    A(I) = I
    B(I) = -I
    C(I) = 0.0
  END DO

  CALL DGEMM('N','N',N,N,N,ALPHA,A,N,B,N,BETA,C,N);

  ALLOCATED_BYTES = MKL_MEM_STAT(ALLOCATED_BUFFERS)
  PRINT *,'DGEMM uses ',ALLOCATED_BYTES,' bytes in ',
 $  ALLOCATED_BUFFERS,' buffers '

  CALL MKL_FREE_BUFFERS

  ALLOCATED_BYTES = MKL_MEM_STAT(ALLOCATED_BUFFERS)
  IF (ALLOCATED_BYTES > 0) THEN
    PRINT *,'MKL MEMORY LEAK!'
    PRINT *,'AFTER MKL_FREE_BUFFERS there are ',
 $    ALLOCATED_BYTES,' bytes in ',
 $    ALLOCATED_BUFFERS,' buffers'
  END IF

  CALL MKL_FREE(A_PTR)
  CALL MKL_FREE(B_PTR)
  CALL MKL_FREE(C_PTR)

  STOP
  END
```

Usage Example in C

```
#include <stdio.h>
#include <mkl.h>
int main(void) {
  double *a, *b, *c;
  int n, i;
  double alpha, beta;
  MKL_INT64 AllocatedBytes;
  int N_AllocatedBuffers;

  alpha = 1.1; beta = -1.2;
  n = 1000;
  a = (double*)mkl_malloc(n*n*sizeof(double),64);
  b = (double*)mkl_malloc(n*n*sizeof(double),64);
  c = (double*)mkl_malloc(n*n*sizeof(double),64);
  for (i=0;i<(n*n);i++) {
     a[i] = (double)(i+1);
     b[i] = (double)(-i-1);
```

```
    c[i] = 0.0;
  }

  dgemm("N","N",&n,&n,&n,&alpha,a,&n,b,&n,&beta,c,&n);

  AllocatedBytes = mkl_mem_stat(&N_AllocatedBuffers);
  printf("\nDGEMM uses %ld bytes in %d buffers",(long)AllocatedBytes,N_AllocatedBuffers);

  mkl_free_buffers();

  AllocatedBytes = mkl_mem_stat(&N_AllocatedBuffers);
  if (AllocatedBytes > 0) {
      printf("\nMKL memory leak!");
      printf("\nAfter mkl_free_buffers there are %ld bytes in %d buffers",
         (long)AllocatedBytes,N_AllocatedBuffers);
  }

  mkl_free(a);
  mkl_free(b);
  mkl_free(c);

  return 0;
}
```

Miscellaneous Utility Functions

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

mkl_progress

Provides progress information.

Syntax

Fortran:

```
stopflag = mkl_progress( thread, step, stage )
```

C:

```
stopflag = mkl_progress( thread, step, stage, lstage );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_lapack.h` and `mkl_service.h`

Input Parameters

Name	Type	Description
thread	**FORTRAN:** `INTEGER*4` **C:** `const int*`	**FORTRAN:** The number of the thread the progress routine is called from. 0 is passed for sequential code.

Name	Type	Description
		C: Pointer to the number of the thread the progress routine is called from. 0 is passed for sequential code.
step	**FORTRAN:** INTEGER*4 **C:** const int*	**FORTRAN:** The linear progress indicator that shows the amount of work done. Increases from 0 to the linear size of the problem during the computation. **C:** Pointer to the linear progress indicator that shows the amount of work done. Increases from 0 to the linear size of the problem during the computation.
stage	**FORTRAN:** CHARACTER*(*) **C:** const char*	Message indicating the name of the routine or the name of the computation stage the progress routine is called from.
lstage	**C:** int	The length of a stage string excluding the trailing NULL character.

Output Parameters

Name	Type	Description
stopflag	**FORTRAN:** INTEGER **C:** int	The stopping flag. A non-zero flag forces the routine to be interrupted. The zero flag is the default return value.

Description

The mkl_progress function is intended to track progress of a lengthy computation and/or interrupt the computation. By default this routine does nothing but the user application can redefine it to obtain the computation progress information. You can set it to perform certain operations during the routine computation, for instance, to print a progress indicator. A non-zero return value may be supplied by the redefined function to break the computation.

The progress function mkl_progress is regularly called from some LAPACK and DSS/PARDISO functions during the computation. Refer to a specific LAPACK or DSS/PARDISO function description to see whether the function supports this feature or not.

Application Notes

Note that mkl_progress is a Fortran routine, that is, to redefine the progress routine from C, the name should be spelt differently, parameters should be passed by reference, and an extra parameter meaning the length of the stage string should be considered. The stage string is not terminated with the NULL character. The C interface of the progress routine is as follows:

```
int mkl_progress_( int* thread, int* step, char* stage, int lstage ); // Linux, Mac
int MKL_PROGRESS( int* thread, int* step, char* stage, int lstage ); // Windows
```

See further the examples of printing a progress information on the standard output in Fortran and C languages:

Examples

Fortran example:

```
integer function mkl_progress( thread, step, stage )
integer*4 thread, step
character*(*) stage
print*,'Thread:',thread,',stage:',stage,',step:',step
mkl_progress = 0
return
end
```

C example:

```
#include <stdio.h>
#include <string.h>
#define BUFLEN 16
int mkl_progress_( int* ithr, int* step, char* stage, int lstage )
{
  char buf[BUFLEN];
  if( lstage >= BUFLEN ) lstage = BUFLEN-1;
  strncpy( buf, stage, lstage );
  buf[lstage] = '\0';
  printf( "In thread %i, at stage %s, steps passed %i\n", *ithr, buf, *step );
  return 0;
}
```

mkl_enable_instructions

Allows dispatching Intel® Advanced Vector Extensions.

Syntax

Fortran:

```
irc = mkl_enable_instructions(MKL_AVX_ENABLE)
```

C:

```
irc = mkl_enable_instructions(MKL_AVX_ENABLE);
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

MKL_AVX_ENABLE	Parameter indicating which new instructions the user needs to enable.

Output Parameters

Name	Type	Description	
irc	**FORTRAN:** `INTEGER*4` **C:** `int`	Value reflecting AVX usage status:	
		=1	MKL uses the AVX code, if the hardware supports Intel® AVX.
		=0	The request is rejected. Most likely, `mkl_enable_instructions` has been called after another Intel MKL function.

Description

This function is currently void and deprecated but can be used in future Intel MKL releases.

NOTE Always remember to add `#include "mkl.h"` to use the C usage syntax.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on

Functions Supporting the Single Dynamic Library

Intel® MKL provides the Single Dynamic Library (SDL), which enables setting the interface and threading layer for Intel MKL at run time. See *Intel® MKL User's Guide* for details of SDL and layered model concept. This section describes the functions supporting SDL.

mkl_set_interface_layer

Sets the interface layer for Intel MKL at run time. Use with the Single Dynamic Library.

Syntax

Fortran:

```
interface = mkl_set_interface_layer( required_interface )
```

C:

```
interface = mkl_set_interface_layer( required_interface );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
required_interface	**FORTRAN:** `INTEGER` **C:** `int`	Determines the interface layer. Possible values: `MKL_INTERFACE_LP64` for the LP64 interface. `MKL_INTERFACE_ILP64` for the ILP64 interface.

Description

If you are using the Single Dynamic Library (SDL), the `mkl_set_interface_layer` function sets LP64 or ILP64 interface for Intel MKL at run time.

Call this function prior to calling any other Intel MKL function in your application except `mkl_set_threading_layer`. You can call `mkl_set_interface_layer` and `mkl_set_threading_layer` in any order.

The `mkl_set_interface_layer` function takes precedence over the `MKL_INTERFACE_LAYER` environment variable.

See *Intel MKL User's Guide* for the layered model concept and usage details of SDL.

mkl_set_threading_layer

Sets the threading layer for Intel MKL at run time. Use with the Single Dynamic Library (SDL).

Syntax

Fortran:

```
threading = mkl_set_threading_layer( required_threading )
```

C:

```
threading = mkl_set_threading_layer( required_threading );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
required_threading	**FORTRAN:** `INTEGER` **C:** `int`	Determines the threading layer. Possible values: `MKL_THREADING_INTEL` for Intel threading. `MKL_THREADING_SEQUENTIAL` for the sequential mode of Intel MKL. `MKL_THREADING_PGI` for PGI threading on Windows* or Linux* operating system only. `MKL_THREADING_GNU` for GNU threading on Linux* operating system only.

Description

If you are using the Single Dynamic Library (SDL), the `mkl_set_threading_layer` function sets the specified threading layer for Intel MKL at run time.

Call this function prior to calling any other Intel MKL function in your application except `mkl_set_interface_layer`.

You can call `mkl_set_threading_layer` and `mkl_set_interface_layer` in any order.

The `mkl_set_threading_layer` function takes precedence over the `MKL_THREADING_LAYER` environment variable.

See *Intel MKL User's Guide* for the layered model concept and usage details of SDL.

mkl_set_xerbla

Replaces the error handling routine. Use with the Single Dynamic Library on Windows OS.*

Syntax

Fortran:

```
old_xerbla_ptr = mkl_set_xerbla( new_xerbla_ptr )
```

C:

```
old_xerbla_ptr = mkl_set_xerbla( new_xerbla_ptr );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
new_xerbla_ptr	XerblaEntry	Pointer to the error handling routine to be used.

Description

If you are linking with the Single Dynamic Library (SDL) `mkl_rt.lib` on Windows* OS, the `mkl_set_xerbla` function replaces the error handling routine that is called by Intel MKL functions with the routine specified by the parameter.

See *Intel MKL User's Guide* for details of SDL.

Return Values

The function returns the pointer to the replaced error handling routine.

See Also

xerbla

mkl_set_progress

Replaces the progress information routine. Use with the Single Dynamic Library (SDL) on Windows OS.*

Syntax

Fortran:

```
old_progress_ptr mkl_set_progress( new_progress_ptr )
```

C:

```
old_progress_ptr mkl_set_progress( new_progress_ptr );
```

Include Files

- FORTRAN 77: `mkl_service.fi`
- C: `mkl_service.h`

Input Parameters

Name	Type	Description
new_progress_ptr	ProgressEntry	Pointer to the progress information routine to be used.

Description

If you are linking with the Single Dynamic Library (SDL) `mkl_rt.lib` on Windows* OS, the `mkl_set_progress` function replaces the currently used progress information routine with the routine specified by the parameter.

See *Intel MKL User's Guide* for details of SDL.

Return Values

The function returns the pointer to the replaced progress information routine.

See Also

mkl_progress

BLACS Routines 16

This chapter describes the Intel® Math Kernel Library implementation of FORTRAN 77 routines from the BLACS (Basic Linear Algebra Communication Subprograms) package. These routines are used to support a linear algebra oriented message passing interface that may be implemented efficiently and uniformly across a large range of distributed memory platforms.

The BLACS routines make linear algebra applications both easier to program and more portable. For this purpose, they are used in Intel MKL intended for the Linux* and Windows* OSs as the communication layer of ScaLAPACK and Cluster FFT.

On computers, a linear algebra matrix is represented by a two dimensional array (2D array), and therefore the BLACS operate on 2D arrays. See description of the basic matrix shapes in a special section.

The BLACS routines implemented in Intel MKL are of four categories:

- Combines
- Point to Point Communication
- Broadcast
- Support.

The Combines take data distributed over processes and combine the data to produce a result. The Point to Point routines are intended for point-to-point communication and Broadcast routines send data possessed by one process to all processes within a scope.

The Support routines perform distinct tasks that can be used for initialization, destruction, information, and miscellaneous tasks.

Matrix Shapes

The BLACS routines recognize the two most common classes of matrices for dense linear algebra. The first of these classes consists of general rectangular matrices, which in machine storage are 2D arrays consisting of *m* rows and *n* columns, with a leading dimension, *lda*, that determines the distance between successive columns in memory.

The *general rectangular* matrices take the following parameters as input when determining what array to operate on:

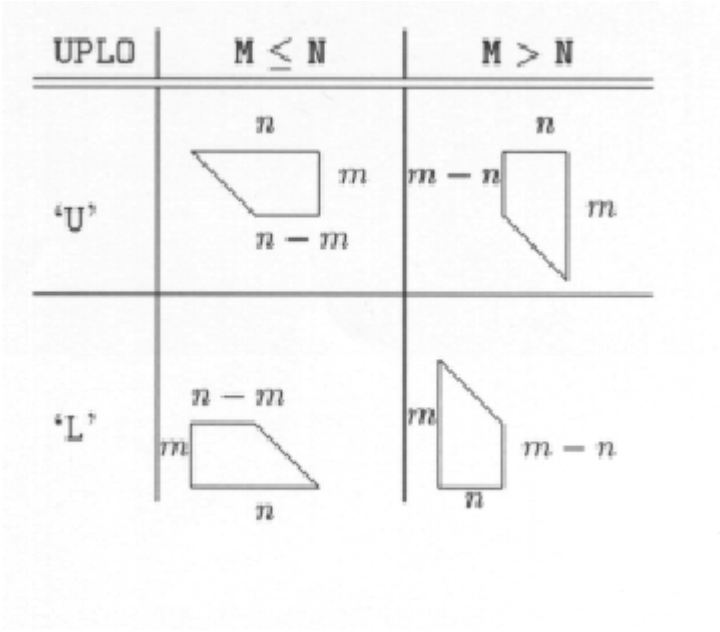
m	(input) INTEGER. The number of matrix rows to be operated on.
n	(input) INTEGER. The number of matrix columns to be operated on.
a	(input/output) TYPE (depends on routine), array of dimension (*lda*,*n*). A pointer to the beginning of the (sub)array to be sent.
lda	(input) INTEGER. The distance between two elements in matrix row.

The second class of matrices recognized by the BLACS are *trapezoidal* matrices (triangular matrices are a sub-class of trapezoidal). Trapezoidal arrays are defined by *m*, *n*, and *lda*, as above, but they have two additional parameters as well. These parameters are:

uplo	(input) CHARACTER*1 . Indicates whether the matrix is upper or lower trapezoidal, as discussed below.
diag	(input) CHARACTER*1 . Indicates whether the diagonal of the matrix is unit diagonal (will not be operated on) or otherwise (will be operated on).

The shape of the trapezoidal arrays is determined by these parameters as follows:

Trapezoidal Arrays Shapes



The packing of arrays, if required, so that they may be sent efficiently is hidden, allowing the user to concentrate on the logical matrix, rather than on how the data is organized in the system memory.

BLACS Combine Operations

This section describes BLACS routines that combine the data to produce a result.

In a combine operation, each participating process contributes data that is combined with other processes' data to produce a result. This result can be given to a particular process (called the *destination* process), or to all participating processes. If the result is given to only one process, the operation is referred to as a *leave-on-one* combine, and if the result is given to all participating processes the operation is referenced as a *leave-on-all* combine.

At present, three kinds of combines are supported. They are:

- element-wise summation
- element-wise absolute value maximization
- element-wise absolute value minimization

of general rectangular arrays.

Note that a combine operation combines data between processes. By definition, a combine performed across a scope of only one process does not change the input data. This is why the operations (`max`/`min`/`sum`) are specified as *element-wise*. Element-wise indicates that each element of the input array will be combined with the corresponding element from all other processes' arrays to produce the result. Thus, a 4 x 2 array of inputs produces a 4 x 2 answer array.

When the `max`/`min` comparison is being performed, absolute value is used. For example, -5 and 5 are equivalent. However, the returned value is unchanged; that is, it is not the absolute value, but is a signed value instead. Therefore, if you performed a BLACS absolute value maximum combine on the numbers -5, 3, 1, 8 the result would be -8.

The initial symbol `?` in the routine names below masks the data type:

`i`	integer
`s`	single precision real

d	double precision real
c	single precision complex
z	double precision complex.

BLACS Combines

Routine name	Results of operation
gamx2d	Entries of result matrix will have the value of the greatest absolute value found in that position.
gamn2d	Entries of result matrix will have the value of the smallest absolute value found in that position.
gsum2d	Entries of result matrix will have the summation of that position.

?gamx2d

Performs element-wise absolute value maximization.

Syntax

```
call igamx2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call sgamx2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call dgamx2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call cgamx2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call zgamx2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the combine should proceed on. Limited to ROW, COLUMN, or ALL.
top	CHARACTER*1. Communication pattern to use during the combine operation.
m	INTEGER. The number of matrix rows to be combined.
n	INTEGER. The number of matrix columns to be combined.
a	TYPE array (*lda*, *n*). Matrix to be compared with to produce the maximum.
lda	INTEGER. The leading dimension of the matrix *A*, that is, the distance between two successive elements in a matrix row.
rcflag	INTEGER. If *rcflag* = -1, the arrays *ra* and *ca* are not referenced and need not exist. Otherwise, *rcflag* indicates the leading dimension of these arrays, and so must be $\geq m$.
rdest	INTEGER. The process row coordinate of the process that should receive the result. If *rdest* or *cdest* = -1, all processes within the indicated scope receive the answer.
cdest	INTEGER. The process column coordinate of the process that should receive the result. If *rdest* or *cdest* = -1, all processes within the indicated scope receive the answer.

Output Parameters

a	TYPE array (*lda*, *n*). Contains the result if this process is selected to receive the answer, or intermediate results if the process is not selected to receive the result.
ra	INTEGER array (*rcflag*, *n*). If *rcflag* = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least *rcflag* x *n*) indicating the row index of the process that provided the maximum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results.
ca	INTEGER array (*rcflag*, *n*). If *rcflag* = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least *rcflag* x *n*) indicating the row index of the process that provided the maximum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results.

Description

This routine performs element-wise absolute value maximization, that is, each element of matrix *A* is compared with the corresponding element of the other process's matrices. Note that the value of *A* is returned, but the absolute value is used to determine the maximum (the 1-norm is used for complex numbers). Combines may be globally-blocking, so they must be programmed as if no process returns until all have called the routine.

See Also

BLACS Routines Usage Example

?gamn2d

Performs element-wise absolute value minimization.

Syntax

```
call igamn2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call sgamn2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call dgamn2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call cgamn2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
call zgamn2d( icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the combine should proceed on. Limited to ROW, COLUMN, or ALL.
top	CHARACTER*1. Communication pattern to use during the combine operation.
m	INTEGER. The number of matrix rows to be combined.
n	INTEGER. The number of matrix columns to be combined.
a	TYPE array (*lda*, *n*). Matrix to be compared with to produce the minimum.

lda INTEGER. The leading dimension of the matrix *A*, that is, the distance between two successive elements in a matrix row.

rcflag INTEGER.
If *rcflag* = -1, the arrays *ra* and *ca* are not referenced and need not exist. Otherwise, *rcflag* indicates the leading dimension of these arrays, and so must be ≥ *m*.

rdest INTEGER.
The process row coordinate of the process that should receive the result. If *rdest* or *cdest* = -1, all processes within the indicated scope receive the answer.

cdest INTEGER.
The process column coordinate of the process that should receive the result. If *rdest* or *cdest* = -1, all processes within the indicated scope receive the answer.

Output Parameters

a TYPE array (*lda*, *n*). Contains the result if this process is selected to receive the answer, or intermediate results if the process is not selected to receive the result.

ra INTEGER array (*rcflag*, *n*).
If *rcflag* = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least *rcflag* x *n*) indicating the row index of the process that provided the minimum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results.

ca INTEGER array (*rcflag*, *n*).
If *rcflag* = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least *rcflag* x *n*) indicating the row index of the process that provided the minimum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results.

Description

This routine performs element-wise absolute value minimization, that is, each element of matrix *A* is compared with the corresponding element of the other process's matrices. Note that the value of *A* is returned, but the absolute value is used to determine the minimum (the 1-norm is used for complex numbers). Combines may be globally-blocking, so they must be programmed as if no process returns until all have called the routine.

See Also

BLACS Routines Usage Example

?gsum2d

Performs element-wise summation.

Syntax

```
call igsum2d( icontxt, scope, top, m, n, a, lda, rdest, cdest )
call sgsum2d( icontxt, scope, top, m, n, a, lda, rdest, cdest )
call dgsum2d( icontxt, scope, top, m, n, a, lda, rdest, cdest )
call cgsum2d( icontxt, scope, top, m, n, a, lda, rdest, cdest )
```

```
call zgsum2d( icontxt, scope, top, m, n, a, lda, rdest, cdest )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the combine should proceed on. Limited to ROW, COLUMN, or ALL.
top	CHARACTER*1. Communication pattern to use during the combine operation.
m	INTEGER. The number of matrix rows to be combined.
n	INTEGER. The number of matrix columns to be combined.
a	TYPE array (*lda*, *n*). Matrix to be added to produce the sum.
lda	INTEGER. The leading dimension of the matrix *A*, that is, the distance between two successive elements in a matrix row.
rdest	INTEGER. The process row coordinate of the process that should receive the result. If *rdest* or *cdest* = -1, all processes within the indicated scope receive the answer.
cdest	INTEGER. The process column coordinate of the process that should receive the result. If *rdest* or *cdest* = -1, all processes within the indicated scope receive the answer.

Output Parameters

a	TYPE array (*lda*, *n*). Contains the result if this process is selected to receive the answer, or intermediate results if the process is not selected to receive the result.

Description

This routine performs element-wise summation, that is, each element of matrix *A* is summed with the corresponding element of the other process's matrices. Combines may be globally-blocking, so they must be programmed as if no process returns until all have called the routine.

See Also

BLACS Routines Usage Example

BLACS Point To Point Communication

This section describes BLACS routines for point to point communication.

Point to point communication requires two complementary operations. The *send* operation produces a message that is then consumed by the *receive* operation. These operations have various resources associated with them. The main such resource is the buffer that holds the data to be sent or serves as the area where the incoming data is to be received. The level of *blocking* indicates what correlation the return from a send/receive operation has with the availability of these resources and with the status of message.

Non-blocking

The return from the *send* or *receive* operations does not imply that the resources may be reused, that the message has been sent/received or that the complementary operation has been called. Return means only that the send/receive has been started, and will be completed at some later date. Polling is required to determine when the operation has finished.

In non-blocking message passing, the concept of *communication/computation overlap* (abbreviated C/C overlap) is important. If a system possesses C/C overlap, independent computation can occur at the same time as communication. That means a nonblocking operation can be posted, and unrelated work can be done while the message is sent/received in parallel. If C/C overlap is not present, after returning from the routine call, computation will be interrupted at some later date when the message is actually sent or received.

Locally-blocking

Return from the *send* or *receive* operations indicates that the resources may be reused. However, since this only depends on local information, it is unknown whether the complementary operation has been called. There are no locally-blocking receives: the send must be completed before the receive buffer is available for re-use.

If a receive has not been posted at the time a locally-blocking send is issued, buffering will be required to avoid losing the message. Buffering can be done on the sending process, the receiving process, or not done at all, losing the message.

Globally-blocking

Return from a globally-blocking procedure indicates that the operation resources may be reused, and that complement of the operation has at least been posted. Since the receive has been posted, there is no buffering required for globally-blocking sends: the message is always sent directly into the user's receive buffer.

Almost all processors support non-blocking communication, as well as some other level of blocking sends. What level of blocking the send possesses varies between platforms. For instance, the Intel® processors support locally-blocking sends, with buffering done on the receiving process. This is a very important distinction, because codes written assuming locally-blocking sends will hang on platforms with globally-blocking sends. Below is a simple example of how this can occur:

```
IAM = MY_PROCESS_ID()
 IF (IAM .EQ. 0) THEN
   SEND TO PROCESS 1
   RECV FROM PROCESS 1
ELSE IF (IAM .EQ. 1) THEN
   SEND TO PROCESS 0
   RECV FROM PROCESS 0
END IF
```

If the send is globally-blocking, process 0 enters the send, and waits for process 1 to start its receive before continuing. In the meantime, process 1 starts to send to 0, and waits for 0 to receive before continuing. Both processes are now waiting on each other, and the program will never continue.

The solution for this case is obvious. One of the processes simply reverses the order of its communication calls and the hang is avoided. However, when the communication is not just between two processes, but rather involves a hierarchy of processes, determining how to avoid this kind of difficulty can become problematic.

For this reason, it was decided the BLACS would support locally-blocking sends. On systems natively supporting globally-blocking sends, non-blocking sends coupled with buffering is used to simulate locally-blocking sends. The BLACS support globally-blocking receives.

In addition, the BLACS specify that point to point messages between two given processes will be strictly ordered. If process 0 sends three messages (label them A, B, and C) to process 1, process 1 must receive A before it can receive B, and message C can be received only after both A and B. The main reason for this restriction is that it allows for the computation of message identifiers.

Note, however, that messages from different processes are not ordered. If processes 0, . . ., 3 send messages A, . . ., D to process 4, process 4 may receive these messages in any order that is convenient.

Convention

The convention used in the communication routine names follows the template `?xxyy2d`, where the letter in the `?` position indicates the data type being sent, `xx` is replaced to indicate the shape of the matrix, and the `yy` positions are used to indicate the type of communication to perform:

`i`	integer
`s`	single precision real
`d`	double precision real
`c`	single precision complex
`z`	double precision complex
`ge`	The data to be communicated is stored in a general rectangular matrix.
`tr`	The data to be communicated is stored in a trapezoidal matrix.
`sd`	Send. One process sends to another.
`rv`	Receive. One process receives from another.

BLACS Point To Point Communication

Routine name	Operation performed
gesd2d trsd2d	Take the indicated matrix and send it to the destination process.
gerv2d trrv2d	Receive a message from the process into the matrix.

As a simple example, the pseudo code given above is rewritten below in terms of the BLACS. It is further specifed that the data being exchanged is the double precision vector *X*, which is 5 elements long.

```
CALL GRIDINFO(NPROW, NPCOL, MYPROW, MYPCOL)

IF (MYPROW.EQ.0 .AND. MYPCOL.EQ.0) THEN
   CALL DGESD2D(5, 1, X, 5, 1, 0)
   CALL DGERV2D(5, 1, X, 5, 1, 0)
ELSE IF (MYPROW.EQ.1 .AND. MYPCOL.EQ.0) THEN
   CALL DGESD2D(5, 1, X, 5, 0, 0)
   CALL DGERV2D(5, 1, X, 5, 0, 0)
END IF
```

?gesd2d

Takes a general rectangular matrix and sends it to the destination process.

Syntax

```
call igesd2d( icontxt, m, n, a, lda, rdest, cdest )
call sgesd2d( icontxt, m, n, a, lda, rdest, cdest )
call dgesd2d( icontxt, m, n, a, lda, rdest, cdest )
call cgesd2d( icontxt, m, n, a, lda, rdest, cdest )
call zgesd2d( icontxt, m, n, a, lda, rdest, cdest )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
m, *n*, *a*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.

rdest	INTEGER. The process row coordinate of the process to send the message to.
cdest	INTEGER. The process column coordinate of the process to send the message to.

Description

This routine takes the indicated general rectangular matrix and sends it to the destination process located at {RDEST, CDEST} in the process grid. Return from the routine indicates that the buffer (the matrix *A*) may be reused. The routine is locally-blocking, that is, it will return even if the corresponding receive is not posted.

See Also

BLACS Routines Usage Example

?trsd2d

Takes a trapezoidal matrix and sends it to the destination process.

Syntax

```
call itrsd2d( icontxt, uplo, diag, m, n, a, lda, rdest, cdest )
call strsd2d( icontxt, uplo, diag, m, n, a, lda, rdest, cdest )
call dtrsd2d( icontxt, uplo, diag, m, n, a, lda, rdest, cdest )
call ctrsd2d( icontxt, uplo, diag, m, n, a, lda, rdest, cdest )
call ztrsd2d( icontxt, uplo, diag, m, n, a, lda, rdest, cdest )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
uplo, *diag*, *m*, *n*, *a*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.
rdest	INTEGER. The process row coordinate of the process to send the message to.
cdest	INTEGER. The process column coordinate of the process to send the message to.

Description

This routine takes the indicated trapezoidal matrix and sends it to the destination process located at {RDEST, CDEST} in the process grid. Return from the routine indicates that the buffer (the matrix *A*) may be reused. The routine is locally-blocking, that is, it will return even if the corresponding receive is not posted.

?gerv2d

Receives a message from the process into the general rectangular matrix.

Syntax

```
call igerv2d( icontxt, m, n, a, lda, rsrc, csrc )
call sgerv2d( icontxt, m, n, a, lda, rsrc, csrc )
call dgerv2d( icontxt, m, n, a, lda, rsrc, csrc )
call cgerv2d( icontxt, m, n, a, lda, rsrc, csrc )
```

```
call zgerv2d( icontxt, m, n, a, lda, rsrc, csrc )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
m, *n*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.
rsrc	INTEGER. The process row coordinate of the source of the message.
csrc	INTEGER. The process column coordinate of the source of the message.

Output Parameters

a	An array of dimension (*lda*,*n*) to receive the incoming message into.

Description

This routine receives a message from process {RSRC, CSRC} into the general rectangular matrix *A*. This routine is globally-blocking, that is, return from the routine indicates that the message has been received into *A*.

See Also

BLACS Routines Usage Example

?trrv2d

Receives a message from the process into the trapezoidal matrix.

Syntax

```
call itrrv2d( icontxt, uplo, diag, m, n, a, lda, rsrc, csrc )
call strrv2d( icontxt, uplo, diag, m, n, a, lda, rsrc, csrc )
call dtrrv2d( icontxt, uplo, diag, m, n, a, lda, rsrc, csrc )
call ctrrv2d( icontxt, uplo, diag, m, n, a, lda, rsrc, csrc )
call ztrrv2d( icontxt, uplo, diag, m, n, a, lda, rsrc, csrc )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
uplo, *diag*, *m*, *n*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.
rsrc	INTEGER. The process row coordinate of the source of the message.
csrc	INTEGER. The process column coordinate of the source of the message.

Output Parameters

a	An array of dimension (*lda*,*n*) to receive the incoming message into.

Description

This routine receives a message from process {RSRC, CSRC} into the trapezoidal matrix *A*. This routine is globally-blocking, that is, return from the routine indicates that the message has been received into *A*.

BLACS Broadcast Routines

This section describes BLACS broadcast routines.

A broadcast sends data possessed by one process to all processes within a scope. Broadcast, much like point to point communication, has two complementary operations. The process that owns the data to be broadcast issues a *broadcast*/*send*. All processes within the same scope must then issue the complementary *broadcast*/*receive*.

The BLACS define that both broadcast/send and broadcast/receive are *globally-blocking*. Broadcasts/receives cannot be locally-blocking since they must post a receive. Note that receives cannot be locally-blocking. When a given process can leave, a broadcast/receive operation is topology dependent, so, to avoid a hang as topology is varied, the broadcast/receive must be treated as if no process can leave until all processes have called the operation.

Broadcast/sends could be defined to be *locally-blocking*. Since no information is being received, as long as locally-blocking point to point sends are used, the broadcast/send will be locally blocking. However, defining one process within a scope to be locally-blocking while all other processes are globally-blocking adds little to the programmability of the code. On the other hand, leaving the option open to have globally-blocking broadcast/sends may allow for optimization on some platforms.

The fact that broadcasts are defined as globally-blocking has several important implications. The first is that scoped operations (broadcasts or combines) must be strictly ordered, that is, all processes within a scope must agree on the order of calls to separate scoped operations. This constraint falls in line with that already in place for the computation of message IDs, and is present in point to point communication as well.

A less obvious result is that scoped operations with `SCOPE = 'ALL'` must be ordered with respect to any other scoped operation. This means that if there are two broadcasts to be done, one along a column, and one involving the entire process grid, all processes within the process column issuing the column broadcast must agree on which broadcast will be performed first.

The convention used in the communication routine names follows the template `?xxyy2d`, where the letter in the `?` position indicates the data type being sent, `xx` is replaced to indicate the shape of the matrix, and the `yy` positions are used to indicate the type of communication to perform:

`i`	integer
`s`	single precision real
`d`	double precision real
`c`	single precision complex
`z`	double precision complex
`ge`	The data to be communicated is stored in a general rectangular matrix.
`tr`	The data to be communicated is stored in a trapezoidal matrix.
`bs`	Broadcast/send. A process begins the broadcast of data within a scope.
`br`	Broadcast/receive A process receives and participates in the broadcast of data within a scope.

BLACS Broadcast Routines

Routine name	Operation performed
gebs2d trbs2d	Start a broadcast along a scope.
gebr2d trbr2d	Receive and participate in a broadcast along a scope.

?gebs2d

Starts a broadcast along a scope for a general rectangular matrix.

Syntax

```
call igebs2d( icontxt, scope, top, m, n, a, lda )
call sgebs2d( icontxt, scope, top, m, n, a, lda )
call dgebs2d( icontxt, scope, top, m, n, a, lda )
call cgebs2d( icontxt, scope, top, m, n, a, lda )
call zgebs2d( icontxt, scope, top, m, n, a, lda )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'.
top	CHARACTER*1. Indicates the communication pattern to use for the broadcast.
m, *n*, *a*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.

Description

This routine starts a broadcast along a scope. All other processes within the scope must call broadcast/receive for the broadcast to proceed. At the end of a broadcast, all processes within the scope will possess the data in the general rectangular matrix *A*.

Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

See Also

BLACS Routines Usage Example

?trbs2d

Starts a broadcast along a scope for a trapezoidal matrix.

Syntax

```
call itrbs2d( icontxt, scope, top, uplo, diag, m, n, a, lda )
call strbs2d( icontxt, scope, top, uplo, diag, m, n, a, lda )
call dtrbs2d( icontxt, scope, top, uplo, diag, m, n, a, lda )
call ctrbs2d( icontxt, scope, top, uplo, diag, m, n, a, lda )
call ztrbs2d( icontxt, scope, top, uplo, diag, m, n, a, lda )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'.
top	CHARACTER*1. Indicates the communication pattern to use for the broadcast.
uplo, *diag*, *m*, *n*, *a*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.

Description

This routine starts a broadcast along a scope. All other processes within the scope must call broadcast/receive for the broadcast to proceed. At the end of a broadcast, all processes within the scope will possess the data in the trapezoidal matrix *A*.

Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

?gebr2d

Receives and participates in a broadcast along a scope for a general rectangular matrix.

Syntax

```
call igebr2d( icontxt, scope, top, m, n, a, lda, rsrc, csrc )
call sgebr2d( icontxt, scope, top, m, n, a, lda, rsrc, csrc )
call dgebr2d( icontxt, scope, top, m, n, a, lda, rsrc, csrc )
call cgebr2d( icontxt, scope, top, m, n, a, lda, rsrc, csrc )
call zgebr2d( icontxt, scope, top, m, n, a, lda, rsrc, csrc )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'.
top	CHARACTER*1. Indicates the communication pattern to use for the broadcast.
m, *n*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.
rsrc	INTEGER. The process row coordinate of the process that called broadcast/send.
csrc	INTEGER. The process column coordinate of the process that called broadcast/send.

Output Parameters

a	An array of dimension (*lda*,*n*) to receive the incoming message into.

Description

This routine receives and participates in a broadcast along a scope. At the end of a broadcast, all processes within the scope will possess the data in the general rectangular matrix *A*. Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

See Also
BLACS Routines Usage Example

?trbr2d
Receives and participates in a broadcast along a scope for a trapezoidal matrix.

Syntax
```
call itrbr2d( icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc )
call strbr2d( icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc )
call dtrbr2d( icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc )
call ctrbr2d( icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc )
call ztrbr2d( icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'.
top	CHARACTER*1. Indicates the communication pattern to use for the broadcast.
uplo, *diag*, *m*, *n*, *lda*	Describe the matrix to be sent. See Matrix Shapes for details.
rsrc	INTEGER. The process row coordinate of the process that called broadcast/send.
csrc	INTEGER. The process column coordinate of the process that called broadcast/send.

Output Parameters

a	An array of dimension (*lda*,*n*) to receive the incoming message into.

Description
This routine receives and participates in a broadcast along a scope. At the end of a broadcast, all processes within the scope will possess the data in the trapezoidal matrix *A*. Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

BLACS Support Routines
The support routines perform distinct tasks that can be used for:

Initialization

Destruction

Information Purposes

Miscellaneous Tasks.

Initialization Routines
This section describes BLACS routines that deal with grid/context creation, and processing before the grid/context has been defined.

BLACS Initialization Routines

Routine name	Operation performed
blacs_pinfo	Returns the number of processes available for use.
blacs_setup	Allocates virtual machine and spawns processes.
blacs_get	Gets values that BLACS use for internal defaults.
blacs_set	Sets values that BLACS use for internal defaults.
blacs_gridinit	Assigns available processes into BLACS process grid.
blacs_gridmap	Maps available processes into BLACS process grid.

blacs_pinfo

Returns the number of processes available for use.

Syntax

```
call blacs_pinfo( mypnum, nprocs )
```

Output Parameters

mypnum	INTEGER. An integer between 0 and (*nprocs* - 1) that uniquely identifies each process.
nprocs	INTEGER.The number of processes available for BLACS use.

Description

This routine is used when some initial system information is required before the BLACS are set up. On all platforms except PVM, *nprocs* is the actual number of processes available for use, that is, *nprows * npcols <= nprocs*. In PVM, the virtual machine may not have been set up before this call, and therefore no parallel machine exists. In this case, *nprocs* is returned as less than one. If a process has been spawned via the keyboard, it receives *mypnum* of 0, and all other processes get *mypnum* of -1. As a result, the user can distinguish between processes. Only after the virtual machine has been set up via a call to BLACS_SETUP, this routine returns the correct values for *mypnum* and *nprocs*.

See Also

BLACS Routines Usage Example

blacs_setup

Allocates virtual machine and spawns processes.

Syntax

```
call blacs_setup( mypnum, nprocs )
```

Input Parameters

nprocs	INTEGER. On the process spawned from the keyboard rather than from pvmspawn, this parameter indicates the number of processes to create when building the virtual machine.

Output Parameters

mypnum	INTEGER. An integer between 0 and (*nprocs* - 1) that uniquely identifies each process.

nprocs	INTEGER. For all processes other than spawned from the keyboard, this parameter means the number of processes available for BLACS use.

Description

This routine only accomplishes meaningful work in the PVM BLACS. On all other platforms, it is functionally equivalent to blacs_pinfo. The BLACS assume a static system, that is, the given number of processes does not change. PVM supplies a dynamic system, allowing processes to be added to the system on the fly.

blacs_setup is used to allocate the virtual machine and spawn off processes. It reads in a file called blacs_setup.dat, in which the first line must be the name of your executable. The second line is optional, but if it exists, it should be a PVM spawn flag. Legal values at this time are 0 (PvmTaskDefault), 4 (PvmTaskDebug), 8 (PvmTaskTrace), and 12 (PvmTaskDebug + PvmTaskTrace). The primary reason for this line is to allow the user to easily turn on and off PVM debugging. Additional lines, if any, specify what machines should be added to the current configuration before spawning *nprocs*-1 processes to the machines in a round robin fashion.

nprocs is input on the process which has no PVM parent (that is, *mypnum*=0), and both parameters are output for all processes. So, on PVM systems, the call to blacs_pinfo informs you that the virtual machine has not been set up, and a call to blacs_setup then sets up the machine and returns the real values for *mypnum* and *nprocs*.

Note that if the file blacs_setup.dat does not exist, the BLACS prompt the user for the executable name, and processes are spawned to the current PVM configuration.

See Also

BLACS Routines Usage Example

blacs_get

Gets values that BLACS use for internal defaults.

Syntax

```
call blacs_get( icontxt, what, val )
```

Input Parameters

icontxt	INTEGER. On values of *what* that are tied to a particular context, this parameter is the integer handle indicating the context. Otherwise, ignored.
what	INTEGER. Indicates what BLACS internal(s) should be returned in *val*. Present options are: • *what* = 0 : Handle indicating default system context • *what* = 1 : The BLACS message ID range • *what* = 2 : The BLACS debug level the library was compiled with • *what* = 10 : Handle indicating the system context used to define the BLACS context whose handle is *icontxt* • *what* = 11 : Number of rings multiring topology is presently using • *what* = 12 : Number of branches general tree topology is presently using.

Output Parameters

val	INTEGER. The value of the BLACS internal.

Description

This routine gets the values that the BLACS are using for internal defaults. Some values are tied to a BLACS context, and some are more general. The most common use is in retrieving a default system context for input into `blacs_gridinit` or `blacs_gridmap`.

Some systems, such as MPI*, supply their own version of context. For those users who mix system code with BLACS code, a BLACS context should be formed in reference to a system context. Thus, the grid creation routines take a system context as input. If you wish to have strictly portable code, you may use `blacs_get` to retrieve a default system context that will include all available processes. This value is not tied to a BLACS context, so the parameter *icontxt* is unused.

`blacs_get` returns information on three quantities that are tied to an individual BLACS context, which is passed in as *icontxt*. The information that may be retrieved is:

- The handle of the system context upon which this BLACS context was defined
- The number of rings for `TOP = 'M'` (multiring broadcast)
- The number of branches for `TOP = 'T'` (general tree broadcast/general tree gather).

See Also

BLACS Routines Usage Example

blacs_set

Sets values that BLACS use for internal defaults.

Syntax

```
call blacs_set( icontxt, what, val )
```

Input Parameters

icontxt	`INTEGER`. For values of *what* that are tied to a particular context, this parameter is the integer handle indicating the context. Otherwise, ignored.
what	`INTEGER`. Indicates what BLACS internal(s) should be set. Present values are: • 1 = The BLACS message ID range • 11 = Number of rings for multiring topology to use • 12 = Number of branches for general tree topology to use.
val	`INTEGER`. Array of dimension (*). Indicates the value(s) the internals should be set to. The specific meanings depend on *what* values, as discussed in Description below.

Description

This routine sets the BLACS internal defaults depending on *what* values:

what = 1	Setting the BLACS message ID range. If you wish to mix the BLACS with other message-passing packages, restrict the BLACS to a certain message ID range not to be used by the non-BLACS routines. The message ID range must be set before the first call to `blacs_gridinit` or `blacs_gridmap`. Subsequent calls will have no effect. Because the message ID range is not tied to a particular context, the parameter *icontxt* is ignored, and *val* is defined as: `VAL (input) INTEGER` array of dimension (2) `VAL(1)` : The smallest message ID (also called message type or message tag) the BLACS should use.

	VAL(2) : The largest message ID (also called message type or message tag) the BLACS should use.
what = 11	Set number of rings for TOP = 'M' (multiring broadcast).This quantity is tied to a context, so *icontxt* is used, and *val* is defined as: VAL (input) INTEGER array of dimension (1) VAL(1) : The number of rings for multiring topology to use.
what = 12	Set number of rings for TOP = 'T' (general tree broadcast/general tree gather). This quantity is tied to a context, so *icontxt* is used, and *val* is defined as: VAL (input) INTEGER array of dimension (1) VAL(1) : The number of branches for general tree topology to use.

blacs_gridinit

Assigns available processes into BLACS process grid.

Syntax

```
call blacs_gridinit( icontxt, order, nprow, npcol )
```

Input Parameters

icontxt	INTEGER. Integer handle indicating the system context to be used in creating the BLACS context. Call blacs_get to obtain a default system context.
order	CHARACTER*1. Indicates how to map processes to BLACS grid. Options are: • 'R' : Use row-major natural ordering • 'C' : Use column-major natural ordering • ELSE : Use row-major natural ordering
nprow	INTEGER. Indicates how many process rows the process grid should contain.
npcol	INTEGER. Indicates how many process columns the process grid should contain.

Output Parameters

icontxt	INTEGER. Integer handle to the created BLACS context.

Description

All BLACS codes must call this routine, or its sister routine blacs_gridmap. These routines take the available processes, and assign, or map, them into a BLACS process grid. In other words, they establish how the BLACS coordinate system maps into the native machine process numbering system. Each BLACS grid is contained in a context, so that it does not interfere with distributed operations that occur within other grids/contexts. These grid creation routines may be called repeatedly to define additional contexts/grids.

The creation of a grid requires input from all processes that are defined to be in this grid. Processes belonging to more than one grid have to agree on which grid formation will be serviced first, much like the globally blocking sum or broadcast.

These grid creation routines set up various internals for the BLACS, and one of them must be called before any calls are made to the non-initialization BLACS.

Note that these routines map already existing processes to a grid: the processes are not created dynamically. On most parallel machines, the processes are actual processors (hardware), and they are "created" when you run your executable. When using the PVM BLACS, if the virtual machine has not been set up yet, the routine blacs_setup should be used to create the virtual machine.

This routine creates a simple nprow x npcol process grid. This process grid uses the first nprow * npcol processes, and assigns them to the grid in a row- or column-major natural ordering. If these process-to-grid mappings are unacceptable, call blacs_gridmap.

See Also

BLACS Routines Usage Example
blacs_get
blacs_gridmap
blacs_setup

blacs_gridmap

Maps available processes into BLACS process grid.

Syntax

```
call blacs_gridmap( icontxt, usermap, ldumap, nprow, npcol )
```

Input Parameters

icontxt	INTEGER. Integer handle indicating the system context to be used in creating the BLACS context. Call blacs_get to obtain a default system context.
usermap	INTEGER. Array, dimension (*ldumap*, *npcol*), indicating the process-to-grid mapping.
ldumap	INTEGER. Leading dimension of the 2D array *usermap*. *ldumap* ≥ *nprow*.
nprow	INTEGER. Indicates how many process rows the process grid should contain.
npcol	INTEGER. Indicates how many process columns the process grid should contain.

Output Parameters

icontxt	INTEGER. Integer handle to the created BLACS context.

Description

All BLACS codes must call this routine, or its sister routine blacs_gridinit. These routines take the available processes, and assign, or map, them into a BLACS process grid. In other words, they establish how the BLACS coordinate system maps into the native machine process numbering system. Each BLACS grid is contained in a context, so that it does not interfere with distributed operations that occur within other grids/contexts. These grid creation routines may be called repeatedly to define additional contexts/grids.

The creation of a grid requires input from all processes that are defined to be in this grid. Processes belonging to more than one grid have to agree on which grid formation will be serviced first, much like the globally blocking sum or broadcast.

These grid creation routines set up various internals for the BLACS, and one of them must be called before any calls are made to the non-initialization BLACS.

Note that these routines map already existing processes to a grid: the processes are not created dynamically. On most parallel machines, the processes are actual processors (hardware), and they are "created" when you run your executable. When using the PVM BLACS, if the virtual machine has not been set up yet, the routine blacs_setup should be used to create the virtual machine.

This routine allows the user to map processes to the process grid in an arbitrary manner. usermap(i,j) holds the process number of the process to be placed in {i, j} of the process grid. On most distributed systems, this process number is a machine defined number between 0 ... *nprow*-1. For PVM, these node numbers are the PVM TIDS (Task IDs). The blacs_gridmap routine is intended for an experienced user. The blacs_gridinit routine is much simpler. blacs_gridinit simply performs a gridmap where the first

nprow * npcol processes are mapped into the current grid in a row-major natural ordering. If you are an experienced user, blacs_gridmap allows you to take advantage of your system's actual layout. That is, you can map nodes that are physically connected to be neighbors in the BLACS grid, etc. The blacs_gridmap routine also opens the way for *multigridding*: you can separate your nodes into arbitrary grids, join them together at some later date, and then re-split them into new grids. blacs_gridmap also provides the ability to make arbitrary grids or subgrids (for example, a "nearest neighbor" grid), which can greatly facilitate operations among processes that do not fall on a row or column of the main process grid.

See Also

BLACS Routines Usage Example
blacs_get
blacs_gridinit
blacs_setup

Destruction Routines

This section describes BLACS routines that destroy grids, abort processes, and free resources.

BLACS Destruction Routines

Routine name	Operation performed
blacs_freebuff	Frees BLACS buffer.
blacs_gridexit	Frees a BLACS context.
blacs_abort	Aborts all processes.
blacs_exit	Frees all BLACS contexts and releases all allocated memory.

blacs_freebuff

Frees BLACS buffer.

Syntax

```
call blacs_freebuff( icontxt, wait )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the BLACS context.
wait	INTEGER. Parameter indicating whether to wait for non-blocking operations or not. If equals 0, the operations should not be waited for; free only unused buffers. Otherwise, wait in order to free all buffers.

Description

This routine releases the BLACS buffer.

The BLACS have at least one internal buffer that is used for packing messages. The number of internal buffers depends on what platform you are running the BLACS on. On systems where memory is tight, keeping this buffer or buffers may become expensive. Call freebuff to release the buffer. However, the next call of a communication routine that requires packing reallocates the buffer.

The *wait* parameter determines whether the BLACS should wait for any non-blocking operations to be completed or not. If *wait* = 0, the BLACS free any buffers that can be freed without waiting. If *wait* is not 0, the BLACS free all internal buffers, even if non-blocking operations must be completed first.

blacs_gridexit

Frees a BLACS context.

Syntax

```
call blacs_gridexit( icontxt )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the BLACS context to be freed.

Description

This routine frees a BLACS context.

Release the resources when contexts are no longer needed. After freeing a context, the context no longer exists, and its handle may be re-used if new contexts are defined.

blacs_abort

Aborts all processes.

Syntax

```
call blacs_abort( icontxt, errornum )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the BLACS context to be aborted.
errornum	INTEGER. User-defined integer error number.

Description

This routine aborts all the BLACS processes, not only those confined to a particular context.

Use `blacs_abort` to abort all the processes in case of a serious error. Note that both parameters are input, but the routine uses them only in printing out the error message. The context handle passed in is not required to be a valid context handle.

blacs_exit

Frees all BLACS contexts and releases all allocated memory.

Syntax

```
call blacs_exit( continue )
```

Input Parameters

continue	INTEGER. Flag indicating whether message passing continues after the BLACS are done. If *continue* is non-zero, the user is assumed to continue using the machine after completing the BLACS. Otherwise, no message passing is assumed after calling this routine.

Description

This routine frees all BLACS contexts and releases all allocated memory.

This routine should be called when a process has finished all use of the BLACS. The *continue* parameter indicates whether the user will be using the underlying communication platform after the BLACS are finished. This information is most important for the PVM BLACS. If *continue* is set to 0, then `pvm_exit` is called;

otherwise, it is not called. Setting *continue* not equal to 0 indicates that explicit PVM send/recvs will be called after the BLACS routines are used. Make sure your code calls pvm_exit. PVM users should either call blacs_exit or explicitly call pvm_exit to avoid PVM problems.

See Also
BLACS Routines Usage Example

Informational Routines

This section describes BLACS routines that return information involving the process grid.

BLACS Informational Routines

Routine name	Operation performed
blacs_gridinfo	Returns information on the current grid.
blacs_pnum	Returns the system process number of the process in the process grid.
blacs_pcoord	Returns the row and column coordinates in the process grid.

blacs_gridinfo
Returns information on the current grid.

Syntax
```
call blacs_gridinfo( icontxt, nprow, npcol, myprow, mypcol )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.

Output Parameters

nprow	INTEGER. Number of process rows in the current process grid.
npcol	INTEGER. Number of process columns in the current process grid.
myprow	INTEGER. Row coordinate of the calling process in the process grid.
mypcol	INTEGER. Column coordinate of the calling process in the process grid.

Description
This routine returns information on the current grid. If the context handle does not point at a valid context, all quantities are returned as -1.

See Also
BLACS Routines Usage Example

blacs_pnum
Returns the system process number of the process in the process grid.

Syntax
```
call blacs_pnum( icontxt, prow, pcol )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.

prow	INTEGER. Row coordinate of the process the system process number of which is to be determined.
pcol	INTEGER. Column coordinate of the process the system process number of which is to be determined.

Description

This function returns the system process number of the process at {PROW, PCOL} in the process grid.

See Also

BLACS Routines Usage Example

blacs_pcoord

Returns the row and column coordinates in the process grid.

Syntax

```
call blacs_pcoord( icontxt, pnum, prow, pcol )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
pnum	INTEGER. Process number the coordinates of which are to be determined. This parameter stand for the process number of the underlying machine, that is, it is a tid for PVM.

Output Parameters

prow	INTEGER. Row coordinates of the *pnum* process in the BLACS grid.
pcol	INTEGER. Column coordinates of the *pnum* process in the BLACS grid.

Description

Given the system process number, this function returns the row and column coordinates in the BLACS process grid.

See Also

BLACS Routines Usage Example

Miscellaneous Routines

This section describes blacs_barrier routine.

BLACS Informational Routines

Routine name	Operation performed
blacs_barrier	Holds up execution of all processes within the indicated scope until they have all called the routine.

blacs_barrier

Holds up execution of all processes within the indicated scope.

Syntax

```
call blacs_barrier( icontxt, scope )
```

Input Parameters

icontxt	INTEGER. Integer handle that indicates the context.
scope	CHARACTER*1. Parameter that indicates whether a process row (*scope*='R'), column ('C'), or entire grid ('A') will participate in the barrier.

Description

This routine holds up execution of all processes within the indicated scope until they have all called the routine.

Examples of BLACS Routines Usage

Example. BLACS Usage. Hello World

The following routine takes the available processes, forms them into a process grid, and then has each process check in with the process at {0,0} in the process grid.

```
      PROGRAM HELLO
*     -- BLACS example code --
*     Written by Clint Whaley 7/26/94
*     Performs a simple check-in type hello world
*     ..
*     .. External Functions ..
      INTEGER BLACS_PNUM
      EXTERNAL BLACS_PNUM
*     ..
*     .. Variable Declaration ..
      INTEGER CONTXT, IAM, NPROCS, NPROW, NPCOL, MYPROW, MYPCOL
      INTEGER ICALLER, I, J, HISROW, HISCOL
*
*     Determine my process number and the number of processes in
*     machine
*
      CALL BLACS_PINFO(IAM, NPROCS)
*
*     If in PVM, create virtual machine if it doesn't exist
*
      IF (NPROCS .LT. 1) THEN
         IF (IAM .EQ. 0) THEN
            WRITE(*, 1000)
            READ(*, 2000) NPROCS
         END IF
         CALL BLACS_SETUP(IAM, NPROCS)
      END IF
*
*     Set up process grid that is as close to square as possible
*
      NPROW = INT( SQRT( REAL(NPROCS) ) )
      NPCOL = NPROCS / NPROW
*
*     Get default system context, and define grid

*
      CALL BLACS_GET(0, 0, CONTXT)
      CALL BLACS_GRIDINIT(CONTXT, 'Row', NPROW, NPCOL)
      CALL BLACS_GRIDINFO(CONTXT, NPROW, NPCOL, MYPROW, MYPCOL)
*
*     If I'm not in grid, go to end of program
*
      IF ( (MYPROW.GE.NPROW) .OR. (MYPCOL.GE.NPCOL) ) GOTO 30

*
*     Get my process ID from my grid coordinates
*
```

```
      ICALLER = BLACS_PNUM(CONTXT, MYPROW, MYPCOL)
*
*     If I am process {0,0}, receive check-in messages from
*     all nodes
*
      IF ( (MYPROW.EQ.0) .AND. (MYPCOL.EQ.0) ) THEN

         WRITE(*,*) ' '

         DO 20 I = 0, NPROW-1
            DO 10 J = 0, NPCOL-1

               IF ( (I.NE.0) .OR. (J.NE.0) ) THEN
                  CALL IGERV2D(CONTXT, 1, 1, ICALLER, 1, I, J)
               END IF
*
*              Make sure ICALLER is where we think in process grid

*
               CALL BLACS_PCOORD(CONTXT, ICALLER, HISROW, HISCOL)
               IF ( (HISROW.NE.I) .OR. (HISCOL.NE.J) ) THEN
                  WRITE(*,*) 'Grid error!  Halting . . .'

                  STOP
               END IF
               WRITE(*, 3000) I, J, ICALLER



10          CONTINUE
20       CONTINUE
         WRITE(*,*) ' '
         WRITE(*,*) 'All processes checked in.  Run finished.'
*
*     All processes but {0,0} send process ID as a check-in


*
      ELSE

         CALL IGESD2D(CONTXT, 1, 1, ICALLER, 1, 0, 0)
      END IF


30    CONTINUE



      CALL BLACS_EXIT(0)

1000  FORMAT('How many processes in machine?')
2000  FORMAT(I)
3000  FORMAT('Process {',i2,',',i2,'} (node number =',I,
     $       ') has checked in.')

      STOP
      END
```

Example. BLACS Usage. PROCMAP

This routine maps processes to a grid using `blacs_gridmap`.

```
      SUBROUTINE PROCMAP(CONTEXT, MAPPING, BEGPROC, NPROW, NPCOL, IMAP)
*
*     -- BLACS example code --

*     Written by Clint Whaley 7/26/94
*     ..
*     .. Scalar Arguments ..
      INTEGER CONTEXT, MAPPING, BEGPROC, NPROW, NPCOL
```

```
*     ..
*     .. Array Arguments ..
      INTEGER IMAP(NPROW, *)
*     ..
*
*  Purpose
*  =======
*  PROCMAP maps NPROW*NPCOL processes starting from process BEGPROC to
*  the grid in a variety of ways depending on the parameter MAPPING.

*
*  Arguments

*  =========
*
*  CONTEXT      (output) INTEGER
*               This integer is used by the BLACS to indicate a context.
*               A context is a universe where messages exist and do not
*               interact with other context's messages.  The context
*               includes the definition of a grid, and each process's
*               coordinates in it.
*
*  MAPPING      (input) INTEGER
*               Way to map processes to grid.  Choices are:
*               1 : row-major natural ordering
*               2 : column-major natural ordering
*
*  BEGPROC      (input) INTEGER
*               The process number (between 0 and NPROCS-1) to use as

*               {0,0}. From this process, processes will be assigned
*               to the grid as indicated by MAPPING.
*
*  NPROW        (input) INTEGER
*               The number of process rows the created grid

*               should have.
*
*  NPCOL        (input) INTEGER
*               The number of process columns the created grid

*               should have.
*
*  IMAP         (workspace) INTEGER array of dimension (NPROW, NPCOL)
*               Workspace, where the array which maps the

*               processes to the grid will be stored for the
*               call to GRIDMAP.
*
*     ============================================================
*
*     ..
*     .. External Functions ..
      INTEGER  BLACS_PNUM

      EXTERNAL BLACS_PNUM

*     ..
*     .. External Subroutines ..
      EXTERNAL BLACS_PINFO, BLACS_GRIDINIT, BLACS_GRIDMAP
*     ..
*     .. Local Scalars ..
      INTEGER TMPCONTXT, NPROCS, I, J, K

*     ..
*     .. Executable Statements ..
*
*     See how many processes there are in the system
*
      CALL BLACS_PINFO( I, NPROCS )
```

```
      IF (NPROCS-BEGPROC .LT. NPROW*NPCOL) THEN
         WRITE(*,*) 'Not enough processes for grid'
         STOP
      END IF
*
*     Temporarily map all processes into 1 x NPROCS grid


*
      CALL BLACS_GET( 0, 0, TMPCONTXT )
      CALL BLACS_GRIDINIT( TMPCONTXT, 'Row', 1, NPROCS )
      K = BEGPROC


*
*     If we want a row-major natural ordering


*
      IF (MAPPING .EQ. 1) THEN

         DO I = 1, NPROW
            DO J = 1, NPCOL
               IMAP(I, J) = BLACS_PNUM(TMPCONTXT, 0, K)
               K = K + 1W
            END DO
         END DO
*
*     If we want a column-major natural ordering


*
      ELSE IF (MAPPING .EQ. 2) THEN

         DO J = 1, NPCOL
            DO I = 1, NPROW
               IMAP(I, J) = BLACS_PNUM(TMPCONTXT, 0, K)

               K = K + 1

            END DO
         END DO
      ELSE

         WRITE(*,*) 'Unknown mapping.'
         STOP
      END IF

*
*     Free temporary context
*
      CALL BLACS_GRIDEXIT(TMPCONTXT)
*
*     Apply the new mapping to form desired context
*
      CALL BLACS_GET( 0, 0, CONTEXT )
      CALL BLACS_GRIDMAP( CONTEXT, IMAP, NPROW, NPROW, NPCOL )




      RETURN
      END
```

Example. BLACS Usage. PARALLEL DOT PRODUCT

This routine does a bone-headed parallel double precision dot product of two vectors. Arguments are input on process {0,0}, and output everywhere else.

```
      DOUBLE PRECISION FUNCTION PDDOT( CONTEXT, N, X, Y )
*
*     -- BLACS example code --

*     Written by Clint Whaley 7/26/94
*     ..
*     .. Scalar Arguments ..
      INTEGER CONTEXT, N

*     ..
*     .. Array Arguments ..
      DOUBLE PRECISION X(*), Y(*)
*     ..
*
*  Purpose
*  =======
*  PDDOT is a restricted parallel version of the BLAS routine
*  DDOT.  It assumes that the increment on both vectors is one,
*  and that process {0,0} starts out owning the vectors and

*  has N.  It returns the dot product of the two N-length vectors
*  X and Y, that is, PDDOT = X' Y.
*
*  Arguments

*  =========
*
*  CONTEXT      (input) INTEGER
*               This integer is used by the BLACS to indicate a context.
*               A context is a universe where messages exist and do not
*               interact with other context's messages.  The context
*               includes the definition of a grid, and each process's
*               coordinates in it.
*
*  N            (input/output) INTEGER
*               The length of the vectors X and Y. Input
*               for {0,0}, output for everyone else.
*
*  X            (input/output) DOUBLE PRECISION array of dimension (N)
*               The vector X of PDDOT = X' Y. Input for {0,0},
*               output for everyone else.
*
*  Y            (input/output) DOUBLE PRECISION array of dimension (N)
*               The vector Y of PDDOT = X' Y. Input for {0,0},
*               output for everyone else.
*
*     =================================================================
*
*     ..
*     .. External Functions ..
      DOUBLE PRECISION DDOT

      EXTERNAL DDOT

*     ..
*     .. External Subroutines ..
      EXTERNAL BLACS_GRIDINFO, DGEBS2D, DGEBR2D, DGSUM2D
*     ..
*     .. Local Scalars ..
      INTEGER IAM, NPROCS, NPROW, NPCOL, MYPROW, MYPCOL, I, LN


      DOUBLE PRECISION LDDOT


*     ..
```

```
*     .. Executable Statements ..
*
*     Find out what grid has been set up, and pretend it is 1-D
*
      CALL BLACS_GRIDINFO( CONTXT, NPROW, NPCOL, MYPROW, MYPCOL )

      IAM = MYPROW*NPCOL + MYPCOL
      NPROCS = NPROW * NPCOL
*
*     Temporarily map all processes into 1 x NPROCS grid


*
      CALL BLACS_GET( 0, 0, TMPCONTXT )
      CALL BLACS_GRIDINIT( TMPCONTXT, 'Row', 1, NPROCS )
      K = BEGPROC


*
*     Do bone-headed thing, and just send entire X and Y to


*     everyone


*
      IF ( (MYPROW.EQ.0) .AND. (MYPCOL.EQ.0) ) THEN

         CALL IGEBS2D(CONTXT, 'All', 'i-ring', 1, 1, N, 1 )

         CALL DGEBS2D(CONTXT, 'All', 'i-ring', N, 1, X, N )
         CALL DGEBS2D(CONTXT, 'All', 'i-ring', N, 1, Y, N )
      ELSE
         CALL IGEBR2D(CONTXT, 'All', 'i-ring', 1, 1, N, 1, 0, 0 )
         CALL DGEBR2D(CONTXT, 'All', 'i-ring', N, 1, X, N, 0, 0 )
         CALL DGEBR2D(CONTXT, 'All', 'i-ring', N, 1, Y, N, 0, 0 )
      ENDIF
*
*     Find out the number of local rows to multiply (LN), and


*     where in vectors to start (I)


*
      LN = N / NPROCS

      I = 1 + IAM * LN
*
*     Last process does any extra rows
*
      IF (IAM .EQ. NPROCS-1) LN = LN + MOD(N, NPROCS)
*
*     Figure dot product of my piece of X and Y

*
      LDDOT = DDOT( LN, X(I), 1, Y(I), 1 )
*
*     Add local dot products to get global dot product;


*     give all procs the answer


*
      CALL DGSUM2D( CONTXT, 'All', '1-tree', 1, 1, LDDOT, 1, -1, 0 )


      PDDOT = LDDOT

      RETURN
```

```
      END
```

Example. BLACS Usage. PARALLEL MATRIX INFINITY NORM

This routine does a parallel infinity norm on a distributed double precision matrix. Unlike the PDDOT example, this routine assumes the matrix has already been distributed.

```
      DOUBLE PRECISION FUNCTION PDINFNRM(CONTXT, LM, LN, A, LDA, WORK)
*
*     -- BLACS example code --

*     Written by Clint Whaley.
*     ..
*     .. Scalar Arguments ..
      INTEGER CONTEXT, LM, LN, LDA


*     ..
*     .. Array Arguments ..
      DOUBLE PRECISION A(LDA, *), WORK(*)
*     ..
*
*  Purpose
*  =======
*  Compute the infinity norm of a distributed matrix, where
*  the matrix is spread across a 2D process grid.  The result is
*  left on all processes.

*
*  Arguments

*  =========
*
*  CONTEXT      (input) INTEGER
*               This integer is used by the BLACS to indicate a context.
*               A context is a universe where messages exist and do not
*               interact with other context's messages.  The context
*               includes the definition of a grid, and each process's
*               coordinates in it.
*
*  LM           (input) INTEGER
*               Number of rows of the global matrix owned by this
*               process.
*
*  LN           (input) INTEGER
*               Number of columns of the global matrix owned by this
*               process.
*
*  A            (input) DOUBLE PRECISION, dimension (LDA,N)
*               The matrix whose norm you wish to compute.

*
*  LDA          (input) INTEGER
*               Leading Dimension of A.

*
*  WORK         (temporary) DOUBLE PRECISION array, dimension (LM)
*               Temporary work space used for summing rows.


*
*     .. External Subroutines ..
      EXTERNAL BLACS_GRIDINFO, DGEBS2D, DGEBR2D, DGSUM2D, DGAMX2D

*     ..
*     .. External Functions ..
      INTEGER IDAMAX
      DOUBLE PRECISION DASUM
*
```

```
*     .. Local Scalars ..
      INTEGER NPROW, NPCOL, MYROW, MYCOL,  I, J



      DOUBLE PRECISION MAX


*
*     .. Executable Statements ..
*
*     Get process grid information
*
      CALL BLACS_GRIDINFO( CONTXT, NPROW, NPCOL, MYPROW, MYPCOL )

*
*     Add all local rows together


*
      DO 20 I = 1, LM

         WORK(I) = DASUM(LN, A(I,1), LDA)
20    CONTINUE


*
*     Find sum of global matrix rows and store on column 0 of


*     process grid


*
      CALL DGSUM2D(CONTXT, 'Row', '1-tree', LM, 1, WORK, LM, MYROW, 0)


*
*     Find maximum sum of rows for supnorm


*
      IF (MYCOL .EQ. 0) THEN

         MAX = WORK(IDAMAX(LM,WORK,1))

         IF (LM .LT. 1) MAX = 0.0D0


         CALL DGAMX2D(CONTXT, 'Col', 'h', 1, 1, MAX, 1, I, I, -1, -1, 0)
      END IF

*
*     Process column 0 has answer; send answer to all nodes

*
      IF (MYCOL .EQ. 0) THEN

         CALL DGEBS2D(CONTXT, 'Row', ' ', 1, 1, MAX, 1)

      ELSE



         CALL DGEBR2D(CONTXT, 'Row', ' ', 1, 1, MAX, 1, 0, 0)

      END IF

*
      PDINFNRM = MAX
```

```
*
      RETURN
*
*     End of PDINFNRM


*
      END
```

17 Data Fitting Functions

Data Fitting functions in Intel® MKL provide spline-based interpolation capabilities that you can use to approximate functions, function derivatives or integrals, and perform cell search operations.

The Data Fitting component is task based. The task is a data structure or descriptor that holds the parameters related to a specific Data Fitting operation. You can modify the task parameters using the task editing functionality of the library.

For definition of the implemented operations, see Mathematical Conventions.

Data Fitting routines use the following workflow to process a task:

1. Create a task or multiple tasks.
2. Modify the task parameters.
3. Perform a Data Fitting computation.
4. Destroy the task or tasks.

All Data Fitting functions fall into the following categories:

Task Creation and Initialization Routines - routines that create a new Data Fitting task descriptor and initialize the most common parameters, such as partition of the interpolation interval, values of the vector-valued function, and the parameters describing their structure.

Task Editors - routines that set or modify parameters in an existing Data Fitting task.

Computational Routines - routines that perform Data Fitting computations, such as construction of a spline, interpolation, computation of derivatives and integrals, and search.

Task Destructors - routines that delete Data Fitting task descriptors and deallocate resources.

You can access the Data Fitting routines through the Fortran and C89/C99 language interfaces. You can also use the C89 interface with more recent versions of C/C++, or the Fortran 90 interface with programs written in Fortran 95

The `${MKL}/include` directory of the Intel® MKL contains the following Data Fitting header files:

- C/C++: `mkl_df.h`
- Fortran: `mkl_df.f90` and `mkl_df.f77`

You can find examples that demonstrate C/C++ and Fortran usage of Data Fitting routines in the `${MKL}/examples/datafittingc` and `${MKL}/examples/datafittingf` directories, respectively.

Naming Conventions

The Fortran interfaces of the Data Fitting functions are in lowercase, while the names of the types and constants are in uppercase.

The C/C++ interface of the Data Fitting functions, types, and constants are case-sensitive and can be in lowercase, uppercase, and mixed case.

The names of all routines have the following structure:

`df[`*`datatype`*`]<`*`base_name`*`>`

where

- `df` is a prefix indicating that the routine belongs to the Data Fitting component of Intel MKL.
- `[`*`datatype`*`]` field specifies the type of the input and/or output data and can be `s` (for the single precision real type), `d` (for the double precision real type), or `i` (for the integer type). This field is omitted in the names of the routines that are not data type dependent.
- `<`*`base_name`*`>` field specifies the functionality the routine performs. For example, this field can be `NewTask1D`, `Interpolate1D`, or `DeleteTask`.

Data Types

The Data Fitting component provides routines for processing single and double precision real data types. The results of cell search operations are returned as a generic integer data type.

All Data Fitting routines use the following data type:

Type	Data Object
Fortran: `TYPE(DF_TASK)`	Pointer to a task
C: `DFTaskPtr`	

NOTE The actual size of the generic integer type is platform-dependent. Before compiling your application, you need to set an appropriate byte size for integers. For details, see section *Using the ILP64 Interface vs. LP64 Interface* of the Intel® MKL User's Guide.

Mathematical Conventions

This section explains the notation used for Data Fitting function descriptions. Spline notations are based on the terminology and definitions of [deBoor2001]. The definition of Subbotin quadratic splines follows the conventions of [StechSub76].

Mathematical Notation in the Data Fitting Component

Concept	Mathematical Notation
Partition of interpolation interval $[a, b]$, where • x_i denotes breakpoints. • $[x_i, x_{i+1})$ denotes a sub-interval (cell) of size Δx_{i+1}-x_i .	$\{x_i\}_{i=1,\ldots,n}$, where $a = x_1 < x_2 < \ldots < x_n = b$
Vector-valued function of dimension p being fit	$f(x) = (f_1(x),\ldots, f_p(x))$
Piecewise polynomial (PP) function f of order k+1	$f(x) := P_i(x)$, if $x \in [x_i, x_{i+1})$, $i = 1,\ldots, n$-1 where • $\{x_i\}_{i= 1,\ldots, n}$ is a strictly increasing sequence of breakpoints. • $P_i(x) = c_{i,0} + c_{i,1}(x - x_i) + \ldots + c_{i,k}(x - x_i)^k$ is a polynomial of degree k (order k+1) over the interval $x \in [x_i, x_{i+1})$.
Function p agrees with function g at the points $\{x_i\}_{i=1,\ldots,n}$.	For every point ζ in sequence $\{x_i\}_{i=1,\ldots,n}$ that occurs m times, the equality $p^{(i-1)}(\zeta) = g^{(i-1)}(\zeta)$ holds for all $i = 1,\ldots,m$, where $p^{(i)}(t)$ is the derivative of the i-th order.
The k-th divided difference of function g at points $x_i,\ldots, x_{i+k}$. This difference is the leading coefficient of the polynomial of order k+1 that agrees with g at $x_i,\ldots, x_{i+k}$.	$[x_i,\ldots, x_{i+k}]\, g$ In particular, • $[x_1]g = g(x_1)$ • $[x_1, x_2]\, g = (g(x_1) - g(x_2)) / (x_1 - x_2)$
A k-order derivative of interpolant $f(x)$ at interpolation site τ .	$f^{(k)}(\tau)$

Interpolants to the Function g at $x_i, \ldots, x_n$ and Boundary Conditions

Concept	Mathematical Notation
Linear interpolant	$P_i(x) = c_{1,i} + c_{2,i}(x - x_i)$, where • $x \in [x_i, x_{i+1})$ • $c_{1,i} = g(x_i)$ • $c_{2,i} = [x_i, x_{i+1}]g$ • $i = 1,\ldots, n\text{-}1$
Piecewise parabolic interpolant	$P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2$, $x \in [x_i, x_{i+1})$ Coefficients $c_{1,i}$, $c_{2,i}$, and $c_{3,i}$ depend on the conditions: • $P_i(x_i) = g(x_i)$ • $P_i(x_{i+1}) = g(x_{i+1})$ • $P_i((x_{i+1} + x_i) / 2) = v_{i+1}$ where parameter v_{i+1} depends on the interpolant being continuously differentiable: $P_{i\text{-}1}^{(1)}(x_i) = P_i^{(1)}(x_i)$
Piecewise parabolic Subbotin interpolant	$P(x) = P_i(x) = c_{1,i}+c_{2,i}(x-x_i)+c_{3,i}(x-x_i)^2+d_{3,i}((x-t_i)_+)^2$, where • $x \in [t_i, t_{i+1})$ • $\{t_i\}_{i=1,\ldots,n+1}$ is a sequence of knots such that • $t_1 = x_1$, $t_{n+1} = x_n$ • $t_i \in (x_{i\text{-}1}, x_i)$, $i = 2,\ldots, n$ • $x_+ = f(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$ Coefficients $c_{1,i}$, $c_{2,i}$, $c_{3,i}$, and $d_{3,i}$ depend on the following conditions: • $P_i(x_i) = g(x_i)$, $P_i(x_{i+1}) = g(x_{i+1})$ • $P(x)$ is a continuously differentiable polynomial of the second degree on $[t_i, t_{i+1})$, $i = 1,\ldots, n$.
Piecewise cubic Hermite interpolant	$P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3$, where • $x \in [x_i, x_{i+1})$ • $c_{1,i} = g(x_i)$ • $c_{2,i} = s_i$ • $c_{3,i} = ([x_i, x_{i+1}]g - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)$ • $c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]g) / (\Delta x_i)^2$ • $i = 1,\ldots, n\text{-}1$ • $s_i = g^{(1)}(x_i)$
Piecewise cubic Bessel interpolant	$P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3$, where • $x \in [x_i, x_{i+1})$ • $c_{1,i} = g(x_i)$

Concept	Mathematical Notation
	• $c_{2,i} = s_i$ • $c_{3,i} = ([x_i, x_{i+1}]g - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)$ • $c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]g) / (\Delta x_i)^2$ • $i = 1,..., n-1$ • $s_i = (\Delta x_i[x_{i-1}, x_i]g + \Delta x_{i-1}[x_i, x_{i+1}]g) / (\Delta x_i + \Delta x_{i+1})$
Piecewise cubic Akima interpolant	$P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3$, where • $x \in [x_i, x_{i+1})$ • $c_{1,i} = g(x_i)$ • $c_{2,i} = s_i$ • $c_{3,i} = ([x_i, x_{i+1}]g - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)$ • $c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]g) / (\Delta x_i)^2$ • $i = 1,..., n-1$ • $s_i = (w_{i+1}[x_{i-1}, x_i]g + w_{i-1}[x_i, x_{i+1}]g) / (w_{i+1} + w_{i-1})$, where $w_i = \lvert[x_i, x_{i+1}]g - [x_{i-1}, x_i]g\rvert$
Piecewise natural cubic interpolant	$P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3$, where • $x \in [x_i, x_{i+1})$ • $c_{1,i} = g(x_i)$ • $c_{2,i} = s_i$ • $c_{3,i} = ([x_i, x_{i+1}]g - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)$ • $c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]g) / (\Delta x_i)^2$ • $i = 1,..., n-1$ • Parameter s_i depends on the condition that the interpolant is twice continuously differentiable: $P_{i-1}^{(2)}(x_i) = P_i^{(2)}(x_i)$.
Not-a-knot boundary condition.	Parameters s_1 and s_n provide $P_1 = P_2$ and $P_{n-1} = P_n$, so that the first and the last interior breakpoints are inactive.
Free-end boundary condition.	$f''(x_1) = f''(x_n) = 0$
Look-up interpolator for discrete set of points $(x_1, y_1),..., (x_n, y_n)$.	$$y(x) = \begin{cases} y_1, & if\ x = x_1 \\ y_2, & if\ x = x_2 \\ \quad\dots & \\ y_n, & if\ x = x_n \\ error, & otherwise \end{cases}$$
Step-wise constant continuous right interpolator.	$$y(x) = \begin{cases} y_1, & \text{if } x_1 \le x < x_2 \\ y_2, & \text{if } x_2 \le x < x_3 \\ \quad\dots & \\ y_{n-1}, & \text{if } x_{n-1} \le x < x_n \\ y_n, & if\ x = x_n \end{cases}$$

Concept	Mathematical Notation
Step-wise constant continuous left interpolator.	$y(x)=\begin{cases} y_1, & \text{if } x = x_1 \\ y_2, & \text{if } x_1 < x \le x_2 \\ y_3, & \text{if } x_2 < x \le x_3 \\ \dots & \\ y_n, & \text{if } x_{n-1} < x \le x_n \end{cases}$

Data Fitting Usage Model

Consider an algorithm that uses the Data Fitting functions. Typically, such algorithms consist of four steps or stages:

1. Create a task. You can call the Data Fitting function several times to create multiple tasks.

```
status = dfdNewTask1D( &task, nx, x, xhint, ny, y, yhint );
```

2. Modify the task parameters.

```
status = dfdEditPPSpline1D( task, s_order, c_type, bc_type, bc, ic_type, ic, scoeff,
scoeffhint );
```

3. Perform Data Fitting spline-based computations. You may reiterate steps 2-3 as needed.

```
status = dfdInterpolate1D(task, estimate, method, nsite, site, sitehint, ndorder,
dorder, datahint, r, rhint, cell );
```

4. Destroy the task or tasks.

```
status = dfDeleteTask( &task );
```

See Also

Data Fitting Usage Examples

Data Fitting Usage Examples

The examples below illustrate several operations that you can perform with Data Fitting routines. If you want to run or reuse similar examples, you can get both C and Fortran source code in the `.\examples\datafittingc` and `.\examples\datafittingf` subdirectories of the Intel MKL installation directory.

The following C example demonstrates the construction of a linear spline using Data Fitting routines. The spline approximates a scalar function defined on non-uniform partition. The coefficients of the spline are returned as a one-dimensional array:

C Example of Linear Spline Construction

```
#include "mkl.h"
#define N 500                    /* Size of partition, number of breakpoints */
#define SPLINE_ORDER DF_PP_LINEAR /* Linear spline to construct */

int main()
{
    int status;          /* Status of a Data Fitting operation */
    DFTaskPtr task;      /* Data Fitting operations are task based */

    /* Parameters describing the partition */
    MKL_INT nx;          /* The size of partition x */
    double x[N];         /* Partition x */
    MKL_INT xhint;       /* Additional information about the structure of breakpoints */

    /* Parameters describing the function */
    MKL_INT ny;          /* Function dimension */
    double y[N];         /* Function values at the breakpoints */
```

```
    MKL_INT yhint;       /* Additional information about the function */

    /* Parameters describing the spline */
    MKL_INT  s_order;    /* Spline order */
    MKL_INT  s_type;     /* Spline type */
    MKL_INT  ic_type;    /* Type of internal conditions */
    MKL_INT* ic;         /* Array of internal conditions */
    MKL_INT  bc_type;    /* Type of boundary conditions */
    MKL_INT* bc;         /* Array of boundary conditions */

    double scoeff[(N-1)* ORDER];   /* Array of spline coefficients */
    MKL_INT scoeffhint;            /* Additional information about the coefficients */

    /* Initialize the partition */
    nx = N;
    /* Set values of partition x */
    ...
    xhint = DF_NO_HINT;    /* No additional information about the function is provided.
                              By default, the partition is non-uniform. */
    /* Initialize the function */
     ny = 1;               /* The function is scalar. */

    /* Set function values */
    ...
    yhint = DF_NO_HINT;    /* No additional information about the function is provided. */

    /* Create a Data Fitting task */
    status = dfdNewTask1D( &task, nx, x, xhint, ny, y, yhint );

    /* Check the Data Fitting operation status */
    ...

    /* Initialize spline parameters */
    s_order = DF_PP_LINEAR;    /* Spline is of the second order. */
    s_type = DF_PP_DEFAULT;    /* Spline is of the default type. */

    /* Define internal conditions for linear spline construction (none in this example) */
    ic_type = DF_NO_IC;
    ic = NULL;

    /* Define boundary conditions for linear spline construction (none in this example) */
    bc_type = DF_NO_BC;
    bc = NULL;
    scoeffhint = DF_NO_HINT;    /* No additional information about the spline. */

    /* Set spline parameters  in the Data Fitting task */
    status = dfdEditPPSpline1D( task, s_order, s_type, bc_type, bc, ic_type,
                                ic, scoeff, scoeffhint );

    /* Check the Data Fitting operation status */
    ...

    /* Use a standard computation method to construct a linear spline: */
    /* P_i(x) = c_1,i+c_2,i(x-x_i), i=0,..., N-2 */
    /* The library packs spline coefficients to array scoeff. */
    /* scoeff[2*i+0]=c_1,i and scoeff[2*i+1]=c_2,i, i=0,..., N-2 */
    status = dfdConstruct1D( task, DF_PP_SPLINE, DF_METHOD_STD );

    /* Check the Data Fitting operation status */
    ...

    /* Process spline coefficients */
    ...

    /* Deallocate Data Fitting task resources */
    status = dfDeleteTask( &task ) ;

    /* Check the Data Fitting operation status */
    ...
    return 0 ;
}
```

The following C example demonstrates cubic spline-based interpolation using Data Fitting routines. In this example, a scalar function defined on non-uniform partition is approximated by Bessel cubic spline using not-a-knot boundary conditions. Once the spline is constructed, you can use the spline to compute spline values at the given sites. Computation results are packed by the Data Fitting routine in row-major format.

C Example of Cubic Spline-Based Interpolation

```
#include "mkl.h"

#define NX 100                      /* Size of partition, number of breakpoints */
#define NSITE 1000                  /* Number of interpolation sites */
#define SPLINE_ORDER DF_PP_CUBIC    /* A cubic spline to construct */

int main()
{
    int status;          /* Status of a Data Fitting operation */
    DFTaskPtr task;      /* Data Fitting operations are task based */

    /* Parameters describing the partition */
    MKL_INT nx;          /* The size of partition x */
    double x[N];         /* Partition x */
    MKL_INT xhint;       /* Additional information about the structure of breakpoints */

    /* Parameters describing the function */
    MKL_INT ny;          /* Function dimension */
    double y[N];         /* Function values at the breakpoints */
    MKL_INT yhint;       /* Additional information about the function */

    /* Parameters describing the spline */
    MKL_INT  s_order;    /* Spline order */
    MKL_INT  s_type;     /* Spline type */
    MKL_INT  ic_type;    /* Type of internal conditions */
    MKL_INT* ic;         /* Array of internal conditions */
    MKL_INT  bc_type;    /* Type of boundary conditions */
    MKL_INT* bc;         /* Array of boundary conditions */

    double scoeff[(N-1)* ORDER];   /* Array of spline coefficients */
    MKL_INT scoeffhint;            /* Additional information about the coefficients */

    /* Parameters describing interpolation computations */
    MKL_INT nsite;        /* Number of interpolation sites */
    double site[NSITE];   /* Array of interpolation sites */
    MKL_INT sitehint;     /* Additional information about the structure of
                             interpolation sites */

    MKL_INT ndorder, dorder;    /* Parameters defining the type of interpolation */

    double* datahint;   /* Additional information on partition and interpolation sites */

    double r[NSITE];    /* Array of interpolation results */
    MKL_INT* rhint;     /* Additional information on the structure of the results */
    MKL_INT* cell;      /* Array of cell indices */

    /* Initialize the partition */
    nx = N;

    /* Set values of partition x */
    ...
    xhint = DF_NON_UNIFORM_PARTITION;  /* The partition is non-uniform. */

    /* Initialize the function */
     ny = 1;               /* The function is scalar. */

    /* Set function values */
    ...
    yhint = DF_NO_HINT;    /* No additional information about the function is provided. */

    /* Create a Data Fitting task */
    status = dfdNewTask1D( &task, nx, x, xhint, ny, y, yhint );

    /* Check the Data Fitting operation status */
```

```
    ...

    /* Initialize spline parameters */
    s_order = DF_PP_CUBIC;     /* Spline is of the fourth order (cubic spline). */
    s_type = DF_PP_BESSEL;     /* Spline is of the Bessel cubic type. */

    /* Define internal conditions for linear spline construction (none in this example) */
    ic_type = DF_NO_IC;
    ic = NULL;

    /* Use not-a-knot boundary conditions. In this case, the is first and the last
     interior breakpoints are inactive, no additional values are provided. */
    bc_type = DF_BC_NOT_A_KNOT;
    bc = NULL;
    scoeffhint = DF_NO_HINT;    /* No additional information about the spline. */

    /* Set spline parameters  in the Data Fitting task */
    status = dfdEditPPSpline1D( task, s_order, s_type, bc_type, bc, ic_type,
                                ic, scoeff, scoeffhint );

    /* Check the Data Fitting operation status */
    ...

    /* Use a standard method to construct a cubic Bessel spline: */
    /* P_i(x) = c_1,i + c_2,i(x - x_i) + c_3,i(x - x_i)^2 + c_4,i(x - x_i)^3, */
    /* The library packs spline coefficients to array scoeff: */
    /* scoeff[4*i+0] = c_1,i, scoef[4*i+1] = c_2,i,          */
    /* scoeff[4*i+2] = c_3,i, scoef[4*i+1] = c_4,i,          */
    /* i=0,...,N-2  */
    status = dfdConstruct1D( task, DF_PP_SPLINE, DF_METHOD_STD );

    /* Check the Data Fitting operation status */
    ...

    /* Initialize interpolation parameters */
    nsite = NSITE;

    /* Set site values */
    ...

    sitehint = DF_NON_UNIFORM_PARTITION; /* Partition of sites is non-uniform */

    /* Request to compute spline values */
    ndorder = 1;
    dorder = 1;
    datahint = DF_NO_APRIORI_INFO;  /* No additional information about breakpoints or
                                       sites is provided. */
    rhint = DF_MATRIX_STORAGE_ROWS; /* The library packs interpolation results
                                       in row-major format. */
    cell = NULL;                   /* Cell indices are not required. */

    /* Solve interpolation problem using the default method: compute the sline values
       at the points site(i), i=0,..., nsite-1 and place the results to array r */
    status = dfdInterpolate1D( task, DF_INTERP, DF_METHOD_STD, nsite, site,
    sitehint, ndorder, &dorder, datahint, r, rhint, cell );

    /* Check Data Fitting operation status */
     ...

    /* De-allocate Data Fitting task resources */
     status = dfDeleteTask( &task );
    /* Check Data Fitting operation status */
     ...
    return 0;
}
```

The following C example demonstrates how to compute indices of cells containing given sites. This example uses uniform partition presented with two boundary points. The sites are in the ascending order.

C Example of Cell Search

```
#include "mkl.h"

#define NX 100                    /* Size of partition, number of breakpoints */
#define NSITE 1000                /* Number of interpolation sites */

int main()
{
    int status;          /* Status of a Data Fitting operation */
    DFTaskPtr task;      /* Data Fitting operations are task based */

    /* Parameters describing the partition */
    MKL_INT nx;          /* The size of partition x */
    double x[2];         /* Partition x is uniform and holds endpoints
                            of interpolation interval [a, b] */
    MKL_INT xhint;       /* Additional information about the structure of breakpoints */

    /* Parameters describing the function */
    MKL_INT ny;          /* Function dimension */
    float   *y;          /* Function values at the breakpoints */
    MKL_INT yhint;       /* Additional information about the function */

    /* Parameters describing cell search */
    MKL_INT nsite;       /* Number of interpolation sites */
    double  site[NSITE]; /* Array of interpolation sites  */
    MKL_INT sitehint;    /* Additional information about the structure of sites */

    float* datahint;     /* Additional information on partition and interpolation sites */

    MKL_INT cell[NSITE]; /* Array for cell indices */

    /* Initialize a uniform partition */
    nx = N;
    /* Set values of partition x: for uniform partition,           */
    /* provide end-points of the interpolation interval [-1.0,1.0] */
    x[0] = -1.0f; x[1] = 1.0f;
    xhint = DF_UNIFORM_PARTITION; /* Partition is uniform */

    /* Initialize function parameters */
    /* In cell search, function values are not necessary and are set to zero/NULL values */
    ny    = 0;
    y     = NULL;
    yhint = DF_NO_HINT;

    /* Create a Data Fitting task */
    status = dfdNewTask1D( &task, nx, x, xhint, ny, y, yhint );

    /* Check Data Fitting operation status */
    ...

    /* Initialize interpolation (cell search) parameters */
    nsite = NSITE;

   /* Set sites in the ascending order */
    ...
    sitehint = DF_SORTED_DATA;      /* Sites are provided in the ascending order. */
    datahint = DF_NO_APRIORI_INFO;  /* No additional information
                                       about breakpoints/sites is provided.*/

    /* Use a standard method to compute indices of the cells that contain
       interpolation sites. The library places the index of the cell containing
       site(i) to the cell(i), i=0,...,nsite-1 */
    status = dfsSearchCells1D( task, DF_METHOD_STD, nsite, site, sitehint,
                               datahint, cell );
    /* Check Data Fitting operation status */
     ...

    /* Process cell indices */
     ...
```

```
    /* Deallocate Data Fitting task resources */
    status = dfDeleteTask( &task );

    /* Check Data Fitting operation status */
     ...
    return 0;
}
```

Task Status and Error Reporting

The Data Fitting routines report a task status through integer values. Negative status values indicate errors, while positive values indicate warnings. An error can be caused by invalid parameter values or a memory allocation failure.

The status codes have symbolic names predefined in the respective header files. For the C/C++ interface, these names are defined as macros via the `#define` statements. For the Fortran interface, the names are defined as integer constants via the `PARAMETER` operators.

If no error occurred, the function returns the `DF_STATUS_OK` code defined as zero:

```
C/C++:          #define DF_STATUS_OK 0
F90/F95:        INTEGER, PARAMETER::DF_STATUS_OK = 0
```

In case of an error, the function returns a non-zero error code that specifies the origin of the failure. Header files for both C/C++ and Fortran languages define the following status codes:

Status Codes in the Data Fitting Component

Status Code	Description
Common Status Codes	
DF_STATUS_OK	Operation completed successfully.
DF_ERROR_NULL_TASK	Data Fitting task is a NULL pointer.
DF_ERROR_MEM_FAILURE	Memory allocation failure.
DF_ERROR_METHOD_NOT_SUPPORTED	Requested method is not supported.
DF_ERROR_COMP_TYPE_NOT_SUPPORTED	Requested computation type is not supported.
Data Fitting Task Creation and Initialization, and Generic Editing Operations	
DF_ERROR_BAD_NX	Invalid number of breakpoints.
DF_ERROR_BAD_X	Array of breakpoints is invalid.
DF_ERROR_BAD_X_HINT	Invalid hint describing the platform structure.
DF_ERROR_BAD_NY	Invalid dimension of vector-valued function *y*.
DF_ERROR_BAD_Y	Array of function values is invalid.
DF_ERROR_BAD_Y_HINT	Invalid flag describing the structure of function *y*
Data Fitting Task-Specific Editing Operations	
DF_ERROR_BAD_SPLINE_ORDER	Invalid spline order.
DF_ERROR_BAD_SPLINE_TYPE	Invalid spline type.
DF_ERROR_BAD_IC_TYPE	Type of internal conditions used for spline construction is invalid.

Status Code	Description
DF_ERROR_BAD_IC	Array of internal conditions for spline construction is not defined.
DF_ERROR_BAD_BC_TYPE	Type of boundary conditions used in spline construction is invalid.
DF_ERROR_BAD_BC	Array of boundary conditions for spline construction is not defined.
DF_ERROR_BAD_PP_COEFF	Array of piecewise polynomial spline coefficients is not defined.
DF_ERROR_BAD_PP_COEFF_HINT	Invalid flag describing the structure of the piecewise polynomial spline coefficients.
DF_ERROR_BAD_PERIODIC_VAL	Function values at the endpoints of the interpolation interval are not equal as required in periodic boundary conditions.
DF_ERROR_BAD_DATA_ATTR	Invalid attribute of the pointer to be set or modified in Data Fitting task descriptor with the `df?editidxptr` task editor.
DF_ERROR_BAD_DATA_IDX	Index of the pointer to be set or modified in the Data Fitting task descriptor with the `df?editidxptr` task editor is out of the pre-defined range.
Data Fitting Computation Operations	
DF_ERROR_BAD_NSITE	Invalid number of interpolation sites.
DF_ERROR_BAD_SITE	Array of interpolation sites is not defined.
DF_ERROR_BAD_SITE_HINT	Invalid flag describing the structure of interpolation sites.
DF_ERROR_BAD_NDORDER	Invalid size of the array defining derivative orders to be computed at interpolation sites.
DF_ERROR_BAD_DORDER	Array defining derivative orders to be computed at interpolation sites is not defined.
DF_ERROR_BAD_DATA_HINT	Invalid flag providing additional information about partition or interpolation sites.
DF_ERROR_BAD_INTERP	Array of spline-based interpolation results is not defined.
DF_ERROR_BAD_INTERP_HINT	Invalid flag defining the structure of spline-based interpolation results.
DF_ERROR_BAD_CELL_IDX	Array of indices of partition cells containing interpolation sites is not defined.
DF_ERROR_BAD_NLIM	Invalid size of arrays containing integration limits.
DF_ERROR_BAD_LLIM	Array of the left-side integration limits is not defined.
DF_ERROR_BAD_RLIM	Array of the right-side integration limits is not defined.

Status Code	Description
DF_ERROR_BAD_INTEGR	Array of spline-based integration results is not defined.
DF_ERROR_BAD_INTEGR_HINT	Invalid flag providing the structure of the array of spline-based integration results.
DF_ERROR_BAD_LOOKUP_INTERP_SITE	Bad site provided for interpolation with look-up interpolator.

NOTE The routine that estimates piecewise polynomial cubic spline coefficients can return internal error codes related to the specifics of the implementation. Such error codes indicate invalid input data or other issues unrelated to Data Fitting routines.

Task Creation and Initialization Routines

Task creation and initialization routines are functions used to create a new task descriptor and initialize its parameters. The Data Fitting component provides the df?newtask1d routine that creates and initializes a new task descriptor for a one-dimensional Data Fitting task.

df?newtask1d

Creates and initializes a new task descriptor for a one-dimensional Data Fitting task.

Syntax

Fortran:

```
status = dfsnewtask1d(task, nx, x, xhint, ny, y, yhint)
status = dfdnewtask1d(task, nx, x, xhint, ny, y, yhint)
```

C:

```
status = dfsNewTask1D(&task, nx, x, xhint, ny, y, yhint)
status = dfdNewTask1D(&task, nx, x, xhint, ny, y, yhint)
```

Include Files

- Fortran: mkl_df.f90 and mkl_df.f77
- C: mkl_df.h

Input Parameters

Name	Type	Description
nx	**Fortran:** INTEGER **C:** MKL_INT*	Number of breakpoints representing partition of interpolation interval [a, b].
x	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfsnewtask1d REAL(KIND=8) DIMENSION(*) for dfdnewtask1d	One-dimensional array containing the sorted breakpoints from interpolation interval [a, b]. The structure of the array is defined by parameter *xhint*: • If partition is non-uniform or quasi-uniform, the array should contain *nx* ordered values.

Name	Type	Description
	C: `float*` for `dfsNewTask1D` `double*` for `dfdNewTask1D`	• If partition is uniform, the array should contain two entries that represent endpoints of interpolation interval [`a`, `b`].
xhint	**Fortran:** `INTEGER` **C:** `MKL_INT`	A flag describing the structure of partition *x*. For the list of possible values of *xhint*, see table "Hint Values for Partition *x*". If you set the flag to the `DF_NO_HINT` value, the library interprets the partition as non-uniform.
ny	**Fortran:** `INTEGER` **C:** `MKL_INT`	Dimension of vector-valued function *y*.
y	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsnewtask1d` `REAL(KIND=8) DIMENSION(*)` for `dfdnewtask1d` **C:** `float*` for `dfsNewTask` `double*` for `dfdNewTask`	Vector-valued function *y*, array of size *nx***ny*. The storage format of function values in the array is defined by the value of flag *yhint*.
yhint	**Fortran:** `INTEGER` **C:** `MKL_INT`	A flag describing the structure of array *y*. Valid hint values are listed in table "Hint Values for Vector-Valued Function *y*". If you set the flag to the `DF_NO_HINT` value, the library assumes that all *ny* coordinates of the vector-valued function *y* are provided and stored in row-major format.

Output Parameters

Name	Type	Description
task	**Fortran:** `TYPE(DF_TASK)` **C:** `DFTaskPtr`	Descriptor of the task.
status	**Fortran:** `INTEGER` **C:** `int`	Status of the routine: • `DF_STATUS_OK` if the task is created successfully. • Non-zero error code if the task creation failed. See "Task Status and Error Reporting" for error code definitions.

Description

The `df?newtask1d` routine creates and initializes a new Data Fitting task descriptor with user-specified parameters for a one-dimensional Data Fitting task. The *x* and *nx* parameters representing the partition of interpolation interval [*a*, *b*] are mandatory. If you provide invalid values for these parameters, such as a `NULL` pointer *x* or the number of breakpoints smaller than two, the routine does not create the Data Fitting task and returns an error code.

If you provide a vector-valued function *y*, make sure that the function dimension *ny* and the array of function values *y* are both valid. If any of these parameters are invalid, the routine does not create the Data Fitting task and returns an error code.

If you store coordinates of the vector-valued function *y* in non-contiguous memory locations, you can set the *yhint* flag to DF_1ST_COORDINATE, and pass only the first coordinate of the function into the task creation routine. After successful creation of the Data Fitting task, you can pass the remaining coordinates using the df?editidxptr task editor.

If the routine fails to create the task descriptor, it returns a NULL task pointer.

The routine supports the following hint values for partition *x*:

Hint Values for Partition *x*

Value	Description
DF_NON_UNIFORM_PARTITION	Partition is non-uniform.
DF_QUASI_UNIFORM_PARTITION	Partition is quasi-uniform.
DF_UNIFORM_PARTITION	Partition is uniform.
DF_NO_HINT	No hint is provided. By default, partition is interpreted as non-uniform.

The routine supports the following hint values for the vector-valued function:

Hint Values for Vector-Valued Function *y*

Value	Description
DF_MATRIX_STORAGE_ROWS	Data is stored in row-major format according to C conventions.
DF_MATRIX_STORAGE_COLS	Data is stored in column-major format according to Fortran conventions.
DF_1ST_COORDINATE	The first coordinate of vector-valued data is provided.
DF_NO_HINT	No hint is provided. By default, the coordinates of vector-valued function *y* are provided and stored in row-major format.

Task Editors

Task editors initialize or change the predefined Data Fitting task parameters. You can use task editors to initialize or modify pointers to arrays or parameter values.

Task editors can be task-specific and generic. Task-specific editors can modify more than one parameter related to a specific task. Generic editors modify a single parameter at a time.

The Data Fitting component of the Intel MKL provides the following task editors:

Data Fitting Task Editors

Editor	Description	Type
df? editppspline1d	Changes parameters of the piecewise polynomial spline.	Task-specific
df?editptr	Changes a pointer in the task descriptor.	Generic
dfieditval	Changes a value in the task descriptor.	Generic
df?editidxptr	Changes a coordinate of data represented in matrix format. For example, a vector-valued function or spline coefficients.	Generic

df?editppspline1d

Modifies parameters representing a spline in a Data Fitting task descriptor.

Syntax

Fortran:

```
status = dfseditppspline1d(task, s_order, s_type, bc_type, bc, ic_type, ic, scoeff, scoeffhint)
status = dfdeditppspline1d(task, s_order, s_type, bc_type, bc, ic_type, ic, scoeff, scoeffhint)
```

C:

```
status = dfsEditPPSpline1D(task, s_order, s_type, bc_type, bc, ic_type, ic, scoeff, scoeffhint)
status = dfdEditPPSpline1D(task, s_order, s_type, bc_type, bc, ic_type, ic, scoeff, scoeffhint)
```

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(DF_TASK)` **C:** `DFTaskPtr`	Descriptor of the task.
s_order	**Fortran:** `INTEGER` **C:** `MKL_INT`	Spline order. The parameter takes one of the values described in table "Spline Orders Supported by Data Fitting Functions".
s_type	**Fortran:** `INTEGER` **C:** `MKL_INT`	Spline type. The parameter takes one of the values described in table "Spline Types Supported by Data Fitting Functions".
bc_type	**Fortran:** `INTEGER` **C:** `MKL_INT`	Type of boundary conditions. The parameter takes one of the values described in table "Boundary Conditions Supported by Data Fitting Functions".
bc	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfseditppspline1d` `REAL(KIND=8) DIMENSION(*)` for `dfdeditppspline1d` **C:** `float*` for `dfsEditPPSpline1D` `double*` for `dfdEditPPSpline1D`	Pointer to boundary conditions. The size of the array is defined by the value of parameter *bc_type*: • If you set free-end or not-a-knot boundary conditions, pass the `NULL` pointer to this parameter. • If you combine boundary conditions at the endpoints of the interpolation interval, pass an array of two elements. • If you set a boundary condition for the default quadratic spline or a periodic condition for Hermite or the default cubic spline, pass an array of one element.

Name	Type	Description
ic_type	**Fortran:** INTEGER **C:** MKL_INT	Type of internal conditions. The parameter takes one of the values described in table "Internal Conditions Supported by Data Fitting Functions".
ic	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfseditppspline1d REAL(KIND=8) DIMENSION(*) for dfdeditppspline1d **C:** float* for dfsEditPPSpline1D double* for dfdEditPPSpline1D	A non-NULL pointer to the array of internal conditions. The size of the array is defined by the value of parameter *ic_type*: • If you set first derivatives or second derivatives internal conditions (*ic_type*=DF_IC_1ST_DER or *ic_type*=DF_IC_2ND_DER), pass an array of n-1 derivative values at the internal points of the interpolation interval. • If you set the knot values internal condition for Subbotin spline (*ic_type*=DF_IC_Q_KNOT) and the knot partition is non-uniform, pass an array of n+1 elements. • If you set the knot values internal condition for Subbotin spline (*ic_type*=DF_IC_Q_KNOT) and the knot partition is uniform, pass an array of four elements.
scoeff	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfseditppspline1d REAL(KIND=8) DIMENSION(*) for dfdeditppspline1d **C:** float* for dfsEditPPSpline1D double* for dfdEditPPSpline1D	Spline coefficients. An array of size *s_order**(*nx*-1). The storage format of the coefficients in the array is defined by the value of flag *scoeffhint*.
scoeffhint	**Fortran:** INTEGER **C:** MKL_INT	A flag describing the structure of the array of spline coefficients. For valid hint values, see table "Hint Values for Spline Coefficients". The library stores the coefficients in row-major format. The default value is DF_NO_HINT.

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Status of the routine: • DF_STATUS_OK if the routine execution completed successfully. • Non-zero error code if the routine execution failed. See "Task Status and Error Reporting" for error code definitions.

Description

The editor modifies parameters that describe the order, type, boundary conditions, internal conditions, and coefficients of a spline. The spline order definition is provided in the "Mathematical Conventions" section. You can set the spline order to any value supported by Data Fitting functions. The table below lists the available values:

Spline Orders Supported by the Data Fitting Functions

Order	Description
DF_PP_STD	Artificial value. Use this value for look-up and step-wise constant interpolants only.
DF_PP_LINEAR	Piecewise polynomial spline of the second order (linear spline).
DF_PP_QUADRATIC	Piecewise polynomial spline of the third order (quadratic spline).
DF_PP_CUBIC	Piecewise polynomial spline of the fourth order (cubic spline).

To perform computations with a spline not supported by Data Fitting routines, set the parameter defining the spline order and pass the spline coefficients to the library in one of the supported formats. For formats description, see table "Storage Formats for Spline Coefficients".

The table below lists the supported spline types:

Spline Types Supported by Data Fitting Functions

Type	Description
DF_PP_DEFAULT	The default spline type. You can use this type with linear, quadratic, or user-defined splines.
DF_PP_SUBBOTIN	Quadratic splines based on Subbotin algorithm, [StechSub76].
DF_PP_NATURAL	Natural cubic spline.
DF_PP_HERMITE	Hermite cubic spline.
DF_PP_BESSEL	Bessel cubic spline.
DF_PP_AKIMA	Akima cubic spline.
DF_LOOKUP_INTERPOLANT	Look-up interpolant.
DF_CR_STEPWISE_CONST_INTERPOLANT	Continuous right step-wise constant interpolant.
DF_CL_STEPWISE_CONST_INTERPOLANT	Continuous left step-wise constant interpolant.

If you perform computations with look-up or step-wise constant interpolants, set the spline order to the DF_PP_STD value.

Construction of specific splines may require boundary or internal conditions. To compute coefficients of such splines, you should pass boundary or internal conditions to the library by specifying the type of the conditions and providing the necessary values. For splines that do not require additional conditions, such as linear splines, set condition types to DF_NO_BC and DF_NO_IC, and pass NULL pointers to the conditions. The table below defines the supported boundary conditions:

Boundary Conditions Supported by Data Fitting Functions

Boundary Condition	Description	Spline
DF_BC_NOT_A_KNOT	Not-a-knot boundary conditions.	Akima, Bessel, Hermite, natural cubic
DF_BC_FREE_END	Free-end boundary conditions.	Akima, Bessel, Hermite, natural cubic, quadratic Subbotin
DF_BC_1ST_LEFT_DER	The first derivative at the left endpoint is zero.	Akima, Bessel, Hermite, natural cubic, quadratic Subbotin
DF_BC_1ST_RIGHT_DER	The first derivative at the right endpoint is zero.	Akima, Bessel, Hermite, natural cubic, quadratic Subbotin
DF_BC_2ST_LEFT_DER	The second derivative at the left endpoint is zero.	Akima, Bessel, Hermite, natural cubic, quadratic Subbotin
DF_BC_2ND_RIGHT_DER	The second derivative at the right endpoint is zero.	Akima, Bessel, Hermite, natural cubic, quadratic Subbotin
DF_BC_PERIODIC	Periodic boundary conditions.	Linear, all cubic splines
DF_BC_Q_VAL	Function value at point $(x_0 + x_1)/2$	Default quadratic

You can combine the values of boundary conditions with a bitwise OR operation. This permits you to pass combinations of first and second derivatives at the endpoints of the interpolation interval into the library. To pass a first derivative at the left endpoint and a second derivative at the right endpoint, set the boundary conditions to DF_BC_1ST_LEFT_DER OR DF_BC_2ND_RIGHT_DER.

You should pass the combined boundary conditions as an array of two elements. The first entry of the array contains the value of the boundary condition for the left endpoint of the interpolation interval, and the second entry - for the right endpoint. Pass other boundary conditions as arrays of one element.

For the conditions defined as a combination of valid values, the library applies the following rules to identify the boundary condition type:

- If not required for spline construction, the value of boundary conditions is ignored.
- Not-a-knot condition has the highest priority. If set, other boundary conditions are ignored.
- Free-end condition has the second priority after the not-a-knot condition. If set, other boundary conditions are ignored.
- Periodic boundary condition has the next priority after the free-end condition.
- The first derivative has higher priority than the second derivative at the right and left endpoints.

If you set the periodic boundary condition, make sure that function values at the endpoints of the interpolation interval are identical. Otherwise, the library returns an error code. The table below specifies the values to be provided for each type of spline if the periodic boundary condition is set.

Boundary Requirements for Periodic Conditions

Spline Type	Periodic Boundary Condition Support	Boundary Value
Linear	Yes	Not required
Default quadratic	No	
Subbotin quadratic	No	
Natural cubic	Yes	Not required

Spline Type	Periodic Boundary Condition Support	Boundary Value
Bessel	Yes	Not required
Akima	Yes	Not required
Hermite cubic	Yes	First derivative
Default cubic	Yes	Second derivative

Internal conditions supported in the Data Fitting domain that you can use for the *ic_type* parameter are the following:

Internal Conditions Supported by Data Fitting Functions

Internal Condition	Description	Spline
DF_IC_1ST_DER	Array of first derivatives of size n-2, where n is the number of breakpoints. Derivatives are applicable to each coordinate of the vector-valued function.	Hermite cubic
DF_IC_2ND_DER	Array of second derivatives of size n-2, where n is the number of breakpoints. Derivatives are applicable to each coordinate of the vector-valued function.	Default cubic
DF_IC_Q_KNOT	Knot array of size n+1, where n is the number of breakpoints.	Subbotin quadratic

To construct a Subbotin quadratic spline, you have three options to get the array of knots in the library:

- If you do not provide the knots, the library uses the default values of knots $t = \{t_i\}$, $i = 0, \ldots, n$ according to the rule:

  $t_0 = x_0$, $t_n = x_{n-1}$, $t_i = (x_i + x_{i-1})/2$, $i = 1, \ldots, n - 1$.
- If you provide the knots in an array of size $n + 1$, the knots form a non-uniform partition. Make sure that the knot values you provide meet the following conditions:

  $t_0 = x_0$, $t_n = x_{n-1}$, $t_i \in (x_{i-1}, x_i)$, $i = 1,\ldots, n - 1$.
- If you provide the knots in an array of size 4, the knots form a uniform partition

  $t_0 = x_0$, $t_1 = l$, $t_2 = r$, $t_3 = x_{n-1}$, where $l \in (x_0, x_1)$ and $r \in (x_{n-2}, x_{n-1})$.

  In this case, you need to set the value of the *ic_type* parameter holding the type of internal conditions to DF_IC_Q_KNOT OR DF_UNIFORM_PARTITION.

NOTE Since the partition is uniform, perform an OR operation with the DF_UNIFORM_PARTITION partition hint value described in Table Hint Values for Partition x.

For computations based on look-up and step-wise constant interpolants, you can avoid calling the df?editppspline1d editor and directly call one of the routines for spline-based computation of spline values, derivatives, or integrals. For example, you can call the df?construct1d routine to construct the required spline with the given attributes, such as order or type.

The memory location of the spline coefficients is defined by the *scoeff* parameter. Make sure that the size of the array is sufficient to hold *s_order* * (*nx*-1) values.

The df?editppspline1d routine supports the following hint values for spline coefficients:

Hint Values for Spline Coefficients

Order	Description
DF_1ST_COORDINATE	The first coordinate of vector-valued data is provided.
DF_NO_HINT	No hint is provided. By default, all sets of spline coefficients are stored in row-major format.

The coefficients for all coordinates of the vector-valued function are packed in memory one by one in successive order, from function y_1 to function y_{ny}.

Within each coordinate, the library stores the coefficients as an array, in row-major format:

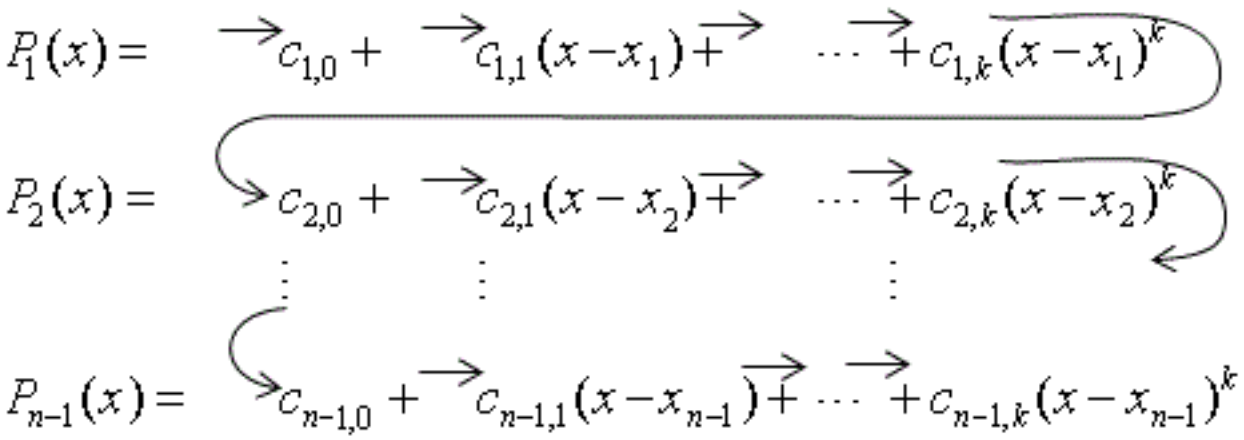
$c_{1,0}, c_{1,1}, \ldots, c_{1,k}, c_{2,0}, c_{2,1}, \ldots, c_{2,k}, \ldots, c_{n-1,0}, c_{n-1,1}, \ldots, c_{n-1,k}$

Mapping of the coefficients to storage in the *scoeff* array is described below, where $c_{i,j}$ is the jth coefficient of the function

$$P_i(x) = c_{i,0} + c_{i,1}(x - x_i) + \ldots + c_{i,k}(x - x_i)^k.$$

See Mathematical Conventions for more details on nomenclature and interpolants.

Row-major Coefficient Storage Format



If you store splines corresponding to different coordinates of the vector-valued function at non-contiguous memory locations, do the following:

1. Set the *scoeffhint* flag to DF_1ST_COORDINATE and provide the spline for the first coordinate.
2. Pass the spline coefficients for the remaining coordinates into the Data Fitting task using the df?editidxptr task editor.

Using the df?editppspline1d task editor, you can provide to the Data Fitting task an already constructed spline that you want to use in computations. To ensure correct interpretation of the memory content, you should set the following parameters:

- Spline order and type, if appropriate. If the spline is not supported by the library, set the *s_type* parameter to DF_PP_DEFAULT.
- Pointer to the array of spline coefficients in row-major format.
- The *scoeffhint* parameter describing the structure of the array:

- Set the *scoeffhint* flag to the DF_1ST_COORDINATE value to pass spline coefficients stored at different memory locations. In this case, you can set the parameters that describe boundary and internal conditions to zero.
- Use the default value DF_NO_HINT for all other cases.

After you provide the spline to the Data Fitting task, you can run computations that use this spline.

df?editptr

Modifies a pointer to an array held in a Data Fitting task descriptor.

Syntax

Fortran:

```
status = dfseditptr(task, ptr_type, ptr)
status = dfdeditptr(task, ptr_type, ptr)
status = dfieditptr(task, ptr_type, ptr)
```

C:

```
status = dfsEditPtr(task, ptr_type, ptr)
status = dfdEditPtr(task, ptr_type, ptr)
status = dfiEditPtr(task, ptr_type, ptr)
```

Include Files

- Fortran: mkl_df.f90 and mkl_df.f77
- C: mkl_df.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(DF_TASK) **C:** DFTaskPtr	Descriptor of the task.
ptr_type	**Fortran:** INTEGER **C:** MKL_INT	The parameter to change. For details, see the *Pointer Type* column in table "Pointers Supported by the df?editptr Task Editor".
ptr	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfseditptr REAL(KIND=8) DIMENSION(*) for dfdeditptr INTEGER DIMENSION(*) for dfieditptr **C:** float* for dfsEditPtr double* for dfdEditPtr MKL_INT* for dfiEditPtr	New pointer. For details, see the *Purpose* column in table "Pointers Supported by the df?editptr Task Editor".

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Status of the routine: • DF_STATUS_OK if the routine execution completed successfully. • Non-zero error code otherwise. See "Task Status and Error Reporting" for error code definitions.

Description

The df?editptr editor replaces the pointer of type *ptr_type* stored in a Data Fitting task descriptor with a new pointer *ptr*. The table below describes types of pointers supported by the editor:

Pointers Supported by the df?editptr Task Editor

Pointer Type	Purpose
DF_X	Partition *x* of the interpolation interval
DF_Y	Vector-valued function *y*
DF_IC	Internal conditions for spline construction. For details, see table "Internal Conditions Supported by Data Fitting Functions".
DF_BC	Boundary conditions for spline construction. For details, see table "Boundary Conditions Supported by Data Fitting Functions".
DF_PP_SCOEFF	Spline coefficients

You can use df?editptr to modify different types of pointers including pointers to the vector-valued function and spline coefficients stored in contiguous memory. Use the df?editidxptr editor if you need to modify pointers to coordinates of the vector-valued function or spline coefficients stored at non-contiguous memory locations.

If you pass a NULL pointer to the df?editptr task editor, the task remains unchanged and the routine returns an error code. For the predefined error codes, please see "Task Status and Error Reporting".

dfieditval

Modifies a parameter value in a Data Fitting task descriptor.

Syntax

Fortran:

```
status = dfieditval(task, val_type, val)
```

C:

```
status = dfiEditVal(task, val_type, val)
```

Include Files

- Fortran: mkl_df.f90 and mkl_df.f77
- C: mkl_df.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(DF_TASK) **C:** DFTaskPtr	Descriptor of the task.
val_type	**Fortran:** INTEGER **C:** MKL_INT	The parameter to change. See table "Parameters Supported by the dfieditval Task Editor".
val	**Fortran:** INTEGER **C:** MKL_INT	A new parameter value. See table "Parameters Supported by the dfieditval Task Editor".

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Status of the routine: • DF_STATUS_OK if the routine execution completed successfully. • Non-zero error code otherwise. See "Task Status and Error Reporting" for error code definitions.

Description

The dfieditval task editor replaces the parameter of type *val_type* stored in a Data Fitting task descriptor with a new value *val*. The table below describes valid types of parameter *val_type* supported by the editor:

Parameters Supported by the dfieditval Task Editor

Parameter	Purpose
DF_NX	Number of breakpoints
DF_XHINT	A flag describing the structure of partition. See table "Hint Values for Partition *x*" for the list of available values.
DF_NY	Dimension of the vector-valued function
DF_YHINT	A flag describing the structure of the vector-valued function. See table "Hint Values for Vector Function *y*" for the list of available values.
DF_SPLINE_ORDER	Spline order. See table "Spline Orders Supported by Data Fitting Functions" for the list of available values.
DF_SPLINE_TYPE	Spline type. See table "Spline Types Supported by Data Fitting Functions" for the list of available values.
DF_BC_TYPE	Type of boundary conditions used in spline construction. See table "Boundary Conditions Supported by Data Fitting Functions" for the list of available values.
DF_IC_TYPE	Type of internal conditions used in spline construction. See table "Internal Conditions Supported by Data Fitting Functions" for the list of available values.

Parameter	Purpose
DF_PP_COEFF_HINT	A flag describing the structure of spline coefficients. See table "Hint Values for Spline Coefficients" for the list of available values.

If you pass a zero value for the parameter describing the size of the arrays that hold coefficients for a partition, a vector-valued function, or a spline, the parameter held in the Data fitting task remains unchanged and the routine returns an error code. For the predefined error codes, see "Task Status and Error Reporting".

If you modify the parameter describing dimensions of the arrays that hold the vector-valued function or spline coefficients in contiguous memory, you should call the df?editptr task editor with the corresponding pointers to the vector-valued function or spline coefficients even when this pointer remains unchanged. Call the df?editidxptr editor if those arrays are stored in non-contiguous memory locations.

df?editidxptr

Modifies a pointer to the memory representing a coordinate of the data stored in matrix format.

Syntax

Fortran:

```
status = dfseditidxptr(task, type, idx, ptr)
status = dfdeditidxptr(task, type, idx, ptr)
```

C:

```
status = dfsEditIdxPtr(task, type, idx, ptr)
status = dfdEditIdxPtr(task, type, idx, ptr)
```

Include Files

- Fortran: mkl_df.f90 and mkl_df.f77
- C: mkl_df.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(DF_TASK) **C:** DFTaskPtr	Descriptor of the task.
type	**Fortran:** INTEGER **C:** MKL_INT	Type of the data to be modified. The parameter takes one of the values described in "Data Attributes Supported by the df?editidxptr Task Editor".
idx	**Fortran:** INTEGER **C:** MKL_INT	Index of the coordinate whose pointer is to be modified.
ptr	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfseditidxptr REAL(KIND=8) DIMENSION(*) for dfdeditidxptr	Pointer to the data that holds values of coordinate *idx*. For details, see table "Data Attributes Supported by the df?editidxptr Task Editor".

Name	Type	Description
	C: `float*` for `dfsEditIdxPtr` `double*` for `dfdEditIdxPtr`	

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Status of the routine: • `DF_STATUS_OK` if the routine execution completed successfully. • Non-zero error code otherwise. See "Task Status and Error Reporting" for error code definitions.

Description

The routine modifies a pointer to the array that holds the *idx* coordinate of vector-valued function *y* or the pointer to the array of spline coefficients corresponding to the given coordinate.

You can use the editor if you need to pass into a Data Fitting task or modify the pointer to coordinates of the vector-valued function or spline coefficients held at non-contiguous memory locations.

Before calling this editor, make sure that you have created and initialized the task using a task creation function or a relevant editor such as the generic or specific `df?editppspline1d` editor.

Data Attributes Supported by the df?editidxptr Task Editor

Data Attribute	Description
`DF_Y`	Vector-valued function *y*
`DF_PP_SCOEFF`	Piecewise polynomial spline coefficients

When using `df?editidxptr`, you might receive an error code in the following cases:

- You passed an unsupported parameter value into the editor.
- The value of the index exceeds the predefined value that equals the dimension *ny* of the vector-valued function.
- You pass a `NULL` pointer to the editor. In this case, the task remains unchanged.

The code example below demonstrates how to use the editor for providing values of a vector-valued function stored in two non-contiguous arrays:

```
#define NX 1000  /* number of break points       */
#define NY 2     /* dimension of vector-valued function */
int main()
{
    DFTaskPtr task;
    double x[NX];
    double y1[NX], y2[NX]; /* vector-valued function is stored as two arrays */
    /* Provide first coordinate of two-dimensional function y into creation routine */
    status = dfdNewTask1D( &task, NX, x, DF_NON_UNIFORM_PARTITION, NY, y1,
                           DF_1ST_COORDINATE );
    /* Provide second coordiante of two-dimensional function */
    status = dfdEditIdxPtr(task, DF_Y, 1, y2 );
    ...
}
```

Computational Routines

Data Fitting computational routines are functions used to perform spline-based computations, such as:

- spline construction
- computation of values, derivatives, and integrals of the predefined order
- cell search

Once you create a Data Fitting task and initialize the required parameters, you can call computational routines as many times as necessary.

The table below lists the available computational routines:

Data Fitting Computational Routines

Routine	Description
df?construct1d	Constructs a spline for a one-dimensional Data Fitting task.
df?interpolate1d	Computes spline values and derivatives.
df?interpolateex1d	Computes spline values and derivatives by calling user-provided interpolants.
df?integrate1d	Computes spline-based integrals.
df?integrateex1d	Computes spline-based integrals by calling user-provided integrators.
df?searchcells1d	Finds indices of cells containing interpolation sites.
df?searchcellsex1d	Finds indices of cells containing interpolation sites by calling user-provided cell searchers.

If a Data Fitting computation completes successfully, the computational routines return the `DF_STATUS_OK` code. If an error occurs, the routines return an error code specifying the origin of the failure. Some possible errors are the following:

- The task pointer is `NULL`.
- Memory allocation failed.
- The computation failed for another reason.

For the list of available status codes, see "Task Status and Error Reporting".

NOTE Data Fitting computational routines do not control errors for floating-point conditions, such as overflow, gradual underflow, or operations with Not a Number (NaN) values.

df?construct1d

Constructs a spline of the given type.

Syntax

Fortran:

```
status = dfsconstruct1d(task, s_format, method)
status = dfdconstruct1d(task, s_format, method)
```

C:

```
status = dfsConstruct1D(task, s_format, method)
status = dfdConstruct1D(task, s_format, method)
```

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(DF_TASK)` **C:** `DFTaskPtr`	Descriptor of the task.
s_format	**Fortran:** `INTEGER` **C:** `MKL_INT`	Spline format. The supported value is `DF_PP_SPLINE`.
method	**Fortran:** `INTEGER` **C:** `MKL_INT`	Construction method. The supported value is `DF_METHOD_STD`.

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Status of the routine: • `DF_STATUS_OK` if the routine execution completed successfully. • Non-zero error code if the routine execution failed. See "Task Status and Error Reporting" for error code definitions.

Description

Before calling `df?construct1d`, you need to create and initialize the task, and set the parameters representing the spline. Then you can call the `df?construct1d` routine to construct the spline. The format of the spline is defined by parameter *s_format*. The method for spline construction is defined by parameter *method*. Upon successful construction, the spline coefficients are available in the user-provided memory location in the format you set through the Data Fitting editor. For the available storage formats, see table "Hint Values for Spline Coefficients".

df?interpolate1d/df?interpolateex1d

Runs data fitting computations.

Syntax

Fortran:

status = dfsinterpolate1d(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*)

status = dfdinterpolate1d(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*)

status = dfsinterpolateex1d(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

status = dfdinterpolateex1d(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

C:

status = dfsInterpolate1D(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*)

status = dfdInterpolate1D(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*)

status = dfsInterpolateEx1D(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

status = dfdInterpolateEx1D(*task*, *type*, *method*, *nsite*, *site*, *sitehint*, *ndorder*, *dorder*, *datahint*, *r*, *rhint*, *cell*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(DF_TASK)` **C:** `DFTaskPtr`	Descriptor of the task.
type	**Fortran:** `INTEGER` **C:** `MKL_INT`	Type of spline-based computations. The parameter takes one or more values combined with an `OR` operation. For the list of possible values, see table "Computation Types Supported by the `df?interpolate1d`/ `df?interpolate1d` Routines".
method	**Fortran:** `INTEGER` **C:** `MKL_INT`	Computation method. The supported value is `DF_METHOD_PP`.
nsite	**Fortran:** `INTEGER` **C:** `MKL_INT`	Number of interpolation sites.
site	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsinterpolate1d`/ `dfsinterpolateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdinterpolate1d`/ `dfdinterpolateex1d` **C:** `float*` for `dfsInterpolate1D`/ `dfsInterpolateEx1D`	Array of interpolation sites of size *nsite*. The structure of the array is defined by the *sitehint* parameter: • If sites form a non-uniform partition, the array should contain *nsite* values. • If sites form a uniform partition, the array should contain two entries that represent the left and the right interpolation sites. The first entry of the array contains the left-most interpolation point. The second entry of the array contains the right-most interpolation point.

Name	Type	Description
	`double*` for `dfdInterpolate1D`/ `dfdInterpolateEx1D`	
sitehint	**Fortran:** `INTEGER` **C:** `MKL_INT`	A flag describing the structure of the interpolation sites. For the list of possible values of *sitehint*, see table "Hint Values for Interpolation Sites". If you set the flag to `DF_NO_HINT`, the library interprets the site-defined partition as non-uniform.
ndorder	**Fortran:** `INTEGER` **C:** `MKL_INT`	Maximal derivative order increased by one to be computed at interpolation sites.
dorder	**Fortran:** `INTEGER DIMENSION(*)` **C:** `MKL_INT*`	Array of size *ndorder* that defines the order of the derivatives to be computed at the interpolation sites. If all the elements in *dorder* are zero, the library computes the spline values only. If you do not need interpolation computations, set *ndorder* to zero and pass a `NULL` pointer to *dorder*.
datahint	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsinterpolate1d`/ `dfsinterpolateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdinterpolate1d`/ `dfdinterpolateex1d` **C:** `float*` for `dfsInterpolate1D`/ `dfsInterpolateEx1D` `double*` for `dfdInterpolate1D`/ `dfdInterpolateEx1D`	Array that contains additional information about the structure of partition *x* and interpolation sites. This data helps to speed up the computation. If you provide a `NULL` pointer, the routine uses the default settings for computations. For details on the *datahint* array, see table "Structure of the *datahint* Array".
r	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsinterpolate1d`/ `dfsinterpolateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdinterpolate1d`/ `dfdinterpolateex1d` **C:** `float*` for `dfsInterpolate1D`/ `dfsInterpolateEx1D` `double*` for `dfdInterpolate1D`/ `dfdInterpolateEx1D`	Array that contains results of computations at the interpolation sites. If you do not need spline-based interpolation or integration, set this pointer to `NULL`.

Name	Type	Description
rhint	**Fortran:** INTEGER **C:** MKL_INT	A flag describing the structure of the results. For the list of possible values of *rhint*, see table "Hint Values for the *rhint* Parameter". If you set the flag to DF_NO_HINT, the library stores the result in row-major format.
cell	**Fortran:** INTEGER DIMENSION(*) **C:** MKL_INT*	Array of cell indices in partition *x* that contain the interpolation sites. If you do not need cell indices, set this parameter to NULL.
le_cb	**Fortran:** INTEGER **C:** dfsInterpCallBack for dfsInterpolateEx1D dfdInterpCallBack for dfdInterpolateEx1D	User-defined callback function for extrapolation at the sites to the left of the interpolation interval.
le_params	**Fortran:** INTEGER DIMENSION(*) **C:** void*	Pointer to additional user-defined parameters passed by the library to the *le_cb* function.
re_cb	**Fortran:** INTEGER **C:** dfsInterpCallBack for dfsInterpolateEx1D dfdInterpCallBack for dfdInterpolateEx1D	User-defined callback function for extrapolation at the sites to the right of the interpolation interval.
re_params	**Fortran:** INTEGER DIMENSION(*) **C:** void*	Pointer to additional user-defined parameters passed by the library to the *re_cb* function.
i_cb	**Fortran:** INTEGER **C:** dfsInterpCallBack for dfsInterpolateEx1D dfdInterpCallBack for dfdInterpolateEx1D	User-defined callback function for interpolation within the interpolation interval.
i_params	**Fortran:** INTEGER DIMENSION(*) **C:** void*	Pointer to additional user-defined parameters passed by the library to the *i_cb* function.
search_cb	**Fortran:** INTEGER **C:** dfsSearchCellsCallBack for dfsInterpolateEx1D dfdSearchCellsCallBack for dfdInterpolateEx1D	User-defined callback function for computing indices of cells that can contain interpolation sites.
search_params	**Fortran:** INTEGER DIMENSION(*)	Pointer to additional user-defined parameters passed by the library to the *search_cb* function.

Name	Type	Description
	C: `void*`	

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Status of the routine: • `DF_STATUS_OK` if the routine execution completed successfully. • Non-zero error code if the routine execution failed. See "Task Status and Error Reporting" for error code definitions.

Description

The `df?interpolate1d`/`df?interpolateex1d` routine performs spline-based computations with user-defined settings. The routine supports two types of computations for interpolation sites provided in array *site*:

Computation Types Supported by the df?interpolate1d/df?interpolateex1d Routines

Type	Description
`DF_INTERP`	Compute derivatives of predefined order. The derivative of the zero order is the spline value.
`DF_CELL`	Compute indices of cells in partition x that contain the sites.

If the sites do not belong to interpolation interval $[a, b]$, the library uses:

- polynomial P_0 of the spline constructed on interval $[x_0, x_1]$ for computations at the sites to the left of a.
- polynomial P_{n-2} of the spline constructed on interval $[x_{n-2}, x_{n-1}]$ for computations at the sites to the right of b.

Interpolation sites support the following hints:

Hint Values for Interpolation Sites

Value	Description
`DF_NON_UNIFORM_PARTITION`	Partition is non-uniform.
`DF_UNIFORM_PARTITION`	Partition is uniform.
`DF_SORTED_DATA`	Interpolation sites are sorted in the ascending order and define a non-uniform partition.
`DF_NO_HINT`	No hint is provided. By default, the partition defined by interpolation sites is interpreted as non-uniform.

NOTE If you pass a sorted array of interpolation sites to the Intel MKL, set the *sitehint* parameter to the `DF_SORTED_DATA` value. The library uses this information when choosing the search algorithm and ignores any other data hints about the structure of the interpolation sites.

Data Fitting computation routines can use the following hints to speed up the computation:

- `DF_UNIFORM_PARTITION` describes the structure of breakpoints and the interpolation sites.

- DF_QUASI_UNIFORM_PARTITION describes the structure of breakpoints.

Pass the above hints to the library when appropriate.

The *r* pointer defines the memory location for the sets of interpolation and integration results for all coordinates of function *y*. The sets are stored one by one, in the successive order of the function coordinates from y_1 to y_{ny}.

You can define the following settings for packing the results within each set:

- Computation type: interpolation, integration, or both.
- Computation parameters: derivative orders.
- Storage format for the results. You can specify the format using the *rhint* parameter values described in the table below:

Hint Values for the *rhint* Parameter

Value	Description
DF_MATRIX_STORAGE_ROWS	Data is stored in row-major format according to C conventions.
DF_MATRIX_STORAGE_COLS	Data is stored in column-major format according to Fortran conventions.
DF_NO_HINT	No hint is provided. By default, the results are stored in row-major format.

For spline-based interpolation, you should set the derivatives whose values are required for the computation. You can provide the derivatives by setting the *dorder* array of size *ndorder* as follows:

$$dorder[i] = \begin{cases} 1, \text{if derivative of the } i-\text{th order is required} \\ 0, \text{otherwise} \end{cases} \qquad i = 0, \ldots, ndorder-1$$

See below a common structure of the storage formats of the interpolation results within each set *r* for computing derivatives of order $i_1, i_2, \ldots, i_m$ at *nsite* interpolation sites. In this description, *j* is the coordinate of the vector-valued function:

- Row-major format

$r_j(i_1, 0)$	$r_j(i_2, 0)$	...	$r_j(i_m, 0)$
$r_j(i_1, 1)$	$r_j(i_2, 1)$	...	$r_j(i_m, 1)$
...	...	...	...
$r_j(i_1, nsite - 1)$	$r_j(i_2, nsite - 1)$	...	$r_j(i_m, nsite - 1)$

- Column-major format

$r_j(i_1, 0)$	$r_j(i_1, 1)$	...	$r_j(i_1, nsite - 1)$
$r_j(i_2, 0)$	$r_j(i_2, 1)$	...	$r_j(i_2, nsite - 1)$
...	...	...	...
$r_j(i_m, 0)$	$r_j(i_m, 1)$	...	$r_j(i_m, nsite - 1)$

To speed up Data Fitting computations, use the *datahint* parameter that provides additional information about the structure of the partition and interpolation sites. This data represents a floating-point or a double array with the following structure:

Structure of the *datahint* Array

Element Number	Description
0	Task dimension
1	Type of additional information
2	Reserved field
3	The total number *q* of elements containing additional information.
4	Element (1)
...	...
q+3	Element (*q*)

Data Fitting computation functions support the following types of additional information for *datahint*[1]:

Types of Additional Information

Type	Element Number	Parameter
DF_NO_APRIORI_INFO	0	No parameters are provided. Information about the data structure is absent.
DF_APRIORI_MOST_LIKELY_CELL	1	Index of the cell that is likely to contain interpolation sites.

To compute indices of the cells that contain interpolation sites, provide the pointer to the array of size *nsite* for the results. The library supports the following scheme of cell indexing for the given partition$\{x_i\}$, $i=1,...,nx$:

$cell[j] = i$, if $site[j] \in [x_i, x_{i+1})$, $i = 0,..., nx$,

where

- $x_0 = -\infty$
- $x_{nx+1} = +\infty$
- $j = 0,..., nsite-1$

To perform interpolation computations with spline types unsupported in the Data Fitting component, use the extended version of the routine `df?interpolateex1d`. With this routine, you can provide user-defined callback functions for computations within, to the left of, or to the right of interpolaton interval [*a*, *b*]. The callback functions compute indices of the cells that contain the specified interpolation sites or can serve as an approximation for computing the exact indices of such cells.

If you do not pass any function for computations at the sites outside the interval [*a*, *b*], the routine uses the default settings.

See Also

df?interpcallback
df?searchcellscallback

df?integrate1d/df?integrateex1d

Computes a spline-based integral.

Syntax

Fortran:

status = dfsintegrate1d(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*)

status = dfdintegrate1d(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*)

status = dfsintegrateex1d(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

status = dfdintegrateex1d(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

C:

status = dfsIntegrate1D(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*)

status = dfdIntegrate1D(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*)

status = dfsIntegrateEx1D(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

status = dfdIntegrateEx1D(*task*, *method*, *nlim*, *llim*, *llimhint*, *rlim*, *rlimhint*, *ldatahint*, *rdatahint*, *r*, *rhint*, *le_cb*, *le_params*, *re_cb*, *re_params*, *i_cb*, *i_params*, *search_cb*, *search_params*)

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(DF_TASK)` **C:** `DFTaskPtr`	Descriptor of the task.
method	**Fortran:** `INTEGER` **C:** `MKL_INT`	Integration method. The supported value is `DF_METHOD_PP`.
nlim	**Fortran:** `INTEGER` **C:** `MKL_INT`	Number of pairs of integraion limits.
llim	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrate1d`/ `dfsintegrateex1d`	Array of size *nlim* that defines the left-side integration limits.

Name	Type	Description
	`REAL(KIND=8) DIMENSION(*)` for `dfdintegrate1d`/ `dfdintegrateex1d` **C:** `float*` for `dfsIntegrate1D`/ `dfsIntegrateEx1D` `double*` for `dfdIntegrate1D`/ `dfdIntegrateEx1D`	
llimhint	**Fortran:** `INTEGER` **C:** `MKL_INT`	A flag describing the structure of the left-side integration limits *llim*. For the list of possible values of *llimhint*, see table "Hint Values for Integration Limits". If you set the flag to the `DF_NO_HINT` value, the library assumes that the left-side integration limits define a non-uniform partition.
rlim	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrate1d`/ `dfsintegrateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrate1d`/ `dfdintegrateex1d` **C:** `float*` for `dfsIntegrate1D`/ `dfsIntegrateEx1D` `double*` for `dfdIntegrate1D`/ `dfdIntegrateEx1D`	Array of size *nlim* that defines the right-side integration limits.
rlimhint	**Fortran:** `INTEGER` **C:** `MKL_INT`	A flag describing the structure of the right-side integration limits *rlim*. For the list of possible values of *rlimhint*, see table "Hint Values for Integration Limits". If you set the flag to the `DF_NO_HINT` value, the library assumes that the right-side integration limits define a non-uniform partition.
ldatahint	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrate1d`/ `dfsintegrateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrate1d`/ `dfdintegrateex1d` **C:** `float*` for `dfsIntegrate1D`/ `dfsIntegrateEx1D`	Array that contains additional information about the structure of partition *x* and left-side integration limits. For details on the *ldatahint* array, see table "Structure of the *datahint* Array" in the description of the `df?Intepolate1D` function.

Name	Type	Description
	`double*` for `dfdIntegrate1D`/ `dfdIntegrateEx1D`	
rdatahint	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrate1d`/ `dfsintegrateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrate1d`/ `dfdintegrateex1d` **C:** `float*` for `dfsIntegrate1D`/ `dfsIntegrateEx1D` `double*` for `dfdIntegrate1D`/ `dfdIntegrateEx1D`	Array that contains additional information about the structure of partition *x* and right-side integration limits. For details on the *rdatahint* array, see table "Structure of the *datahint* Array" in the description of the `df? Intepolate1D` function.
r	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrate1d`/ `dfsintegrateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrate1d`/ `dfdintegrateex1d` **C:** `float*` for `dfsIntegrate1D`/ `dfsIntegrateEx1D` `double*` for `dfdIntegrate1D`/ `dfdIntegrateEx1D`	Array of `integration` results. The size of the array should be sufficient to hold *nlim***ny* values, where *ny* is the dimension of the vector-valued function. The integration results are packed according to the settings in *rhint*.
rhint	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrate1d`/ `dfsintegrateex1d` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrate1d`/ `dfdintegrateex1d` **C:** `float*` for `dfsIntegrate1D`/ `dfsIntegrateEx1D`	A flag describing the structure of the results. For the list of possible values of *rhint*, see table "Hint Values for Integration Results". If you set the flag to the `DF_NO_HINT` value, the library stores the results in row-major format.

Name	Type	Description
	`double*` for `dfdIntegrate1D`/ `dfdIntegrateEx1D`	
le_cb	**Fortran:** `INTEGER` **C:** `dfsIntegrCallBack` for `dfsIntegrateEx1D` `dfdIntegrCallBack` for `dfdIntegrateEx1D`	User-defined callback function for integration on interval [*llim*[*i*], `min`(*rlim*[*i*], *a*)) for *llim*[*i*] < *a* .
le_params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to additional user-defined parameters passed by the library to the *le_cb* function.
re_cb	**Fortran:** `INTEGER` **C:** `dfsInterpCallBack` for `dfsIntegrateEx1D` `dfdInterpCallBack` for `dfdIntegrateEx1D`	User-defined callback function for integration on interval [`max`(*llim*[*i*], *b*), *rlim*[*i*]) for *rlim*[*i*] ≥ *b*.
re_params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to additional user-defined parameters passed by the library to the *re_cb* function.
i_cb	**Fortran:** `INTEGER` **C:** `dfsIntegrCallBack` for `dfsIntegrateEx1D` `dfdIntegrCallBack` for `dfdIntegrateEx1D`	User-defined callback function for integration on interval [`max`(*a*, *llim*[*i*],), `min`(*rlim*[*i*], *b*)).
i_params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to additional user-defined parameters passed by the library to the *i_cb* function.
search_cb	**Fortran:** `INTEGER` **C:** `dfsSearchCellsCallBack` for `dfsIntegrateEx1D` `dfdSearchCellsCallBack` for `dfdIntegrateEx1D`	User-defined callback function for computing indices of cells that can contain interpolation sites.
search_params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to additional user-defined parameters passed by the library to the *search_cb* function.

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Status of the routine: • DF_STATUS_OK if the routine execution completed successfully. • Non-zero error code if the routine execution failed. See "Task Status and Error Reporting" for error code definitions.

Description

The df?integrate1d/df?integrateex1d routine computes spline-based integral on user-defined intervals

$$I(i,j) = \int_{llim[i]}^{rlim[i]} f_j(x)dx,\ i = 0,\ldots,nlim-1,\ j = 0,\ldots,ny-1$$

If *rlim*[*i*] < *llim*[*i*], the routine returns

$$I(i,j) = -\int_{rlim[i]}^{llim[i]} f_j(x)dx$$

The routine supports the following hint values for integration results:

Hint Values for Integration Results

Value	Description
DF_MATRIX_STORAGE_ROWS	Data is stored in row-major format according to C conventions.
DF_MATRIX_STORAGE_COLS	Data is stored in column-major format according to Fortran conventions.
DF_NO_HINT	No hint is provided. By default, the coordinates of vector-valued function *y* are provided and stored in row-major format.

A common structure of the storage formats for the integration results is as follows:

- Row-major format

I (0, 0)	...	*I* (0, *nlim* - 1])
...	...	...
I (*ny* – 1, 0)	...	*I* (*ny* - 1, *nlim* - 1])

- Column-major format

I (0, 0)	...	*I* (*ny* – 1, 0)
...	...	...
I (0, *nlim* - 1])	...	*I* (*ny* - 1, *nlim* - 1])

Using the *llimhint* and *rlimhint* parameters, you can provide the following hint values for integration limits:

Hint Values for Integration Limits

Value	Description
DF_SORTED_DATA	Integration limits are sorted in the ascending order and define a non-uniform partition.
DF_NON_UNIFORM_PARTITION	Partition defined by integration limits is non-uniform.
DF_UNIFORM_PARTITION	Partition defined by integration limits is uniform.
DF_NO_HINT	No hint is provided. By default, partition defined by integration limits is interpreted as non-uniform.

To compute integration with splines unsupported in the Data Fitting component, use the extended version of the routine df?integrateex1d. With this routine, you can provide user-defined callback functions that compute:

- integrals within, to the left of, or to the right of the interpolation interval [*a*, *b*]
- indices of cells that contain the provided integration limits or can serve as an approximation for computing the exact indices of such cells

If you do not pass a callback function, the routine uses the default settings.

See Also

df?interpolate1d/df?interpolateex1d
df?integrcallback
df?searchcellscallback

df?searchcells1d/df?searchcellsex1d

Searches sub-intervals containing interpolation sites.

Syntax

Fortran:

status = dfssearchcells1d(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*)

status = dfdsearchcells1d(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*)

status = dfssearchcellsex1d(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*, *search_cb*, *search_params*)

status = dfdsearchcellsex1d(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*, *search_cb*, *search_params*)

C:

status = dfsSearchCells1D(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*)

status = dfdSearchCells1D(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*)

status = dfsSearchCellsEx1D(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*, *search_cb*, *search_params*)

status = dfdSearchCellsEx1D(*task*, *method*, *nsite*, *site*, *sitehint*, *datahint*, *cell*, *search_cb*, *search_params*)

Include Files

- Fortran: mkl_df.f90 and mkl_df.f77
- C: mkl_df.h

Input Parameters

Name	Type	Description
task	**Fortran:** TYPE(DF_TASK) **C:** DFTaskPtr	Descriptor of the task.
method	**Fortran:** INTEGER **C:** MKL_INT	Search method. The supported value is DF_METHOD_STD.
nsite	**Fortran:** INTEGER **C:** MKL_INT*	Number of interpolation sites.
site	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfssearchcells1d/ dfssearchcellsex1d REAL(KIND=8) DIMENSION(*) for dfdsearchcells1d/ dfdsearchcellsex1d **C:** float* for dfsSearchCells1D/ dfsSearchCellsEx1D double* for dfdSearchCells1D/ dfdSearchCellsEx1D	Array of interpolation sites of size *nsite*. The structure of the array is defined by the *sitehint* parameter: • If the sites form a non-uniform partition, the array should contain *nsite* values. • If the sites form a uniform partition, the array should contain two entries that represent the left-most and the right-most interpolation sites. The first entry of the array contains the left-most interpolation point. The second entry of the array contains the right-most interpolation point.
sitehint	**Fortran:** INTEGER **C:** MKL_INT	A flag describing the structure of the interpolation sites. For the list of possible values of *sitehint*, see table "Hint Values for Interpolation Sites". If you set the flag to DF_NO_HINT, the library interprets the site-defined partition as non-uniform.
datahint	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfssearchcells1d/ dfssearchcellsex1d REAL(KIND=8) DIMENSION(*) for dfdsearchcells1d/ dfdsearchcellsex1d **C:** float* for dfsSearchCells1D/ dfsSearchCellsEx1D double* for dfdSearchCells1D/ dfdSearchCellsEx1D	Array that contains additional information about the structure of partition *x* and interpolation sites. This data helps to speed up the computation. If you provide a NULL pointer, the routine uses the default settings for computations. For details on the *datahint* array, see table "Structure of the *datahint* Array".
cell	**Fortran:** INTEGER DIMENSION(*) **C:** MKL_INT*	Array of cell indices in partition *x* that contain the interpolation sites.

Name	Type	Description
search_cb	**Fortran:** INTEGER **C:** dfsSearchCellsCallBack for dfsSearchCellsEx1D dfdSearchCellsCallBack for dfdSearchCellsEx1D	User-defined callback function for computing indices of cells that can contain interpolation sites.
search_params	**Fortran:** INTEGER DIMENSION(*) **C:** void*	Pointer to additional user-defined parameters passed by the library to the *search_cb* function.

Output Parameters

Name	Type	Description
status	**Fortran:** INTEGER **C:** int	Status of the routine: • DF_STATUS_OK if the routine execution completed successfully. • Non-zero error code if the routine execution failed. See "Task Status and Error Reporting" for error code definitions.

Description

The df?searchcells1d/df?searchcellsex1d routines return array *cell* of indices of sub-intervals (cells) in partition *x* that contain interpolation sites available in array *site*. For details on the cell indexing scheme, see the description of the df?interpolate1d/df?interpolateex1d computation routines.

Use the *datahint* parameter to provide additional information about the structure of the partition and/or interpolation sites. The definition of the *datahint* parameter is availalbe in the description of the df?interpolate1d/df?interpolateex1d computation routines.

For description of the user-defined callback for computation of cell indices, see df?searchcellscallback.

See Also

df?interpolate1d/df?interpolateex1d
df?searchcellscallback

df?interpcallback

A callback function for user-defined interpolation to be passed into df?interpolateex1d.

Syntax

Fortran:

```
status = dfsinterpcallback(n, cell, site, r, params)
status = dfdinterpcallback(n, cell, site, r, params)
```

C:

```
status = dfsInterpCallBack(n, cell, site, r, params)
status = dfdInterpCallBack(n, cell, site, r, params)
```

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
n	**Fortran:** `INTEGER(KIND=8)` **C:** `long long*`	Number of interpolation sites.
cell	**Fortran:** `INTEGER(KIND=8) DIMENSION(*)` **C:** `long long*`	Array of size *n* containing indices of the cells to which the interpolation sites in array *site* belong.
site	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsinterpcallback` `REAL(KIND=8) DIMENSION(*)` for `dfdinterpcallback` **C:** `float*` for `dfsInterpCallBack` `double*` for `dfdInterpCallBack`	Array of interpolation sites of size *n*.
r	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsinterpcallback` `REAL(KIND=8) DIMENSION(*)` for `dfdinterpcallback` **C:** `float*` for `dfsInterpCallBack` `double*` for `dfdInterpCallBack`	Array of the computed interpolation results packed in row-major format.
params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to user-defined parameters of the callback function.

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	The status returned by the callback function: • Zero indicates successful completion of the callback operation. • A negative value indicates an error. • A positive value indicates a warning.

Name	Type	Description
		See "Task Status and Error Reporting" for error code definitions.

Description

When passed into the `df?interpolateex1d` routine, this function performs user-defined interpolation operation.

See Also

df?interpolate1d/df?interpolateex1d
df?searchcellscallback

df?integrcallback

A callback function that you can pass into `df?integrateex1d` *to define integration computations.*

Syntax

Fortran:

```
status = dfsintegrcallback(n, lcell, llim, rcell, rlim, r, params)
status = dfdintegrcallback(n, lcell, llim, rcell, rlim, r, params)
```

C:

```
status = dfsIntegrCallBack(n, lcell, llim, rcell, rlim, r, params)
status = dfdIntegrCallBack(n, lcell, llim, rcell, rlim, r, params)
```

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
n	**Fortran:** `INTEGER(KIND=8)` **C:** `long long*`	Number of pairs of integration limits.
lcell	**Fortran:** `INTEGER(KIND=8) DIMENSION(*)` **C:** `long long*`	Array of size *n* with indices of the cells that contain the left-side integration limits in array *llim*.
llim	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrcallback` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrcallback` **C:** `float*` for `dfsIntegrCallBack`	Array of size *n* that holds the left-side integration limits.

Name	Type	Description
	`double*` for `dfdIntegrCallBack`	
rcell	**Fortran:** `INTEGER(KIND=8) DIMENSION(*)` **C:** `long long*`	Array of size *n* with indices of the cells that contain the right-side integration limits in array *rlim*.
rlim	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrcallback` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrcallback` **C:** `float*` for `dfsIntegrCallBack` `double*` for `dfdIntegrCallBack`	Array of size *n* that holds the right-side integration limits.
r	**Fortran:** `REAL(KIND=4) DIMENSION(*)` for `dfsintegrcallback` `REAL(KIND=8) DIMENSION(*)` for `dfdintegrcallback` **C:** `float*` for `dfsIntegrCallBack` `double*` for `dfdIntegrCallBack`	Array of integration results. For packing the results in row-major format, follow the instructions described in `df?interpolate1d/df?interpolateex1d`.
params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to user-defined parameters of the callback function.

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	The status returned by the callback function: • Zero indicates successful completion of the callback operation. • A negative value indicates an error. • A positive value indicates a warning. See "Task Status and Error Reporting" for error code definitions.

Description

When passed into the df?integrateex1d routine, this function defines integration computations. If at least one of the integration limits is outside the interpolation interval [*a*, *b*], the library decomposes the integration into sub-intervals that belong to the extrapolation range to the left of *a*, the extrapolation range to the right of *b*, and the interpolation interval [*a*, *b*], as follows:

- If the left integration limit is to the left of the interpolation interval (*llim*< *a*), the df?integrateex1d routine passes *llim* as the left integration limit and min(*rlim*, *a*) as the right integration limit to the user-defined callback function.
- If the right integration limit is to the right of the interpolation interval (*rlim*> *b*), the df?integrateex1d routine passes max(*llim*, *b*) as the left integration limit and *rlim* as the right integration limit to the user-defined callback function.
- If the left and the right integration limits belong to the interpolation interval, the df?integrateex1d routine passes them to the user-defined callback function unchanged.

The value of the integral is the sum of integral values obtained on the sub-intervals.

See Also

df?integrate1d/df?integrateex1d
df?integrcallback
df?searchcellscallback

df?searchcellscallback

A callback function for user-defined search to be passed into df?interpolateex1d *or* df?searchcellsex1d*.*

Syntax

Fortran:

status = dfssearchcellscallback(*n*, *site*, *cell*, *flag*, *params*)

status = dfdsearchcellscallback(*n*, *site*, *cell*, *flag*, *params*)

C:

status = dfsSearchCellsCallBack(*n*, *site*, *cell*, *flag*, *params*)

status = dfdSearchCellsCallBack(*n*, *site*, *cell*, *flag*, *params*)

Include Files

- Fortran: mkl_df.f90 and mkl_df.f77
- C: mkl_df.h

Input Parameters

Name	Type	Description
n	**Fortran:** INTEGER(KIND=8) **C:** long long*	Number of interpolation sites.
site	**Fortran:** REAL(KIND=4) DIMENSION(*) for dfssearchcellscallback	Array of interpolation sites of size *n*.

Name	Type	Description
	`REAL(KIND=8) DIMENSION(*)` for `dfdsearchcellscallback` **C:** `float*` for `dfsSearchCellsCallBack` `double*` for `dfdSearchCellsCallBack`	
cell	**Fortran:** `INTEGER(KIND=8) DIMENSION(*)` **C:** `long long*`	Array of size *n* that returns indices of the cells computed by the callback function.
flag	**Fortran:** `INTEGER(KIND=4) DIMENSION(*)` **C:** `int*`	Array of size *n*, with values set as follows: • If the cell with index *cell*[*i*] contains *site*[*i*], set *flag*[*i*] to 1. • Otherwise, set *flag*[*i*] to zero. In this case, the library interprets the index as an approximation and computes the index of the cell containing *site*[*i*] by using the provided index as a starting point for the search.
params	**Fortran:** `INTEGER DIMENSION(*)` **C:** `void*`	Pointer to user-defined parameters of the callback function.

Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	The status returned by the callback function: • Zero indicates successful completion of the callback operation. • A negative value indicates an error. • The `DF_STATUS_EXACT_RESULT` status indicates that cell indices returned by the callback function are exact. In this case, you do not need to initialize entries of the *flag* array. • A positive value indicates a warning. See "Task Status and Error Reporting" for error code definitions.

Description

When passed into the `df?interpolateex1d` or `df?searchcellsex1d` routine, this function performs a user-defined search.

See Also

df?interpolate1d/df?interpolateex1d
df?interpcallback

Task Destructors

Task destructors are routines used to delete task descriptors and deallocate the corresponding memory resources. The Data Fitting task destructor `dfdeletetask` destroys a Data Fitting task and frees the memory.

dfdeletetask

Destroys a Data Fitting task object and frees the memory.

Syntax

Fortran:

```
status = dfdeletetask(task)
```

C:

```
status = dfDeleteTask(&task)
```

Include Files

- Fortran: `mkl_df.f90` and `mkl_df.f77`
- C: `mkl_df.h`

Input Parameters

Name	Type	Description
task	**Fortran:** `TYPE(DF_TASK)` **C:** `DFTaskPtr`	Descriptor of the task to destroy.

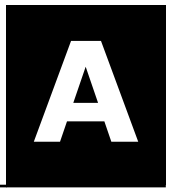
Output Parameters

Name	Type	Description
status	**Fortran:** `INTEGER` **C:** `int`	Status of the routine: • `DF_STATUS_OK` if the task is deleted successfully. • Non-zero error code if the operation failed. See "Task Status and Error Reporting" for error code definitions.

Description

Given a pointer to a task descriptor, this routine deletes the Data Fitting task descriptor and frees the memory allocated for the structure. If the task is deleted successfully, the routine sets the task pointer to `NULL`. Otherwise, the routine returns an error code.

Linear Solvers Basics



Many applications in science and engineering require the solution of a system of linear equations. This problem is usually expressed mathematically by the matrix-vector equation, $Ax = b$, where A is an m-by-n matrix, x is the n element column vector and b is the m element column vector. The matrix A is usually referred to as the coefficient matrix, and the vectors x and b are referred to as the solution vector and the right-hand side, respectively.

Basic concepts related to solving linear systems with sparse matrices are described in section Sparse Linear Systems that follows.

Sparse Linear Systems

In many real-life applications, most of the elements in A are zero. Such a matrix is referred to as sparse. Conversely, matrices with very few zero elements are called dense. For sparse matrices, computing the solution to the equation $Ax = b$ can be made much more efficient with respect to both storage and computation time, if the sparsity of the matrix can be exploited. The more an algorithm can exploit the sparsity without sacrificing the correctness, the better the algorithm.

Generally speaking, computer software that finds solutions to systems of linear equations is called a solver. A solver designed to work specifically on sparse systems of equations is called a sparse solver. Solvers are usually classified into two groups - direct and iterative.

Iterative Solvers start with an initial approximation to a solution and attempt to estimate the difference between the approximation and the true result. Based on the difference, an iterative solver calculates a new approximation that is closer to the true result than the initial approximation. This process is repeated until the difference between the approximation and the true result is sufficiently small. The main drawback to iterative solvers is that the rate of convergence depends greatly on the values in the matrix A. Consequently, it is not possible to predict how long it will take for an iterative solver to produce a solution. In fact, for ill-conditioned matrices, the iterative process will not converge to a solution at all. However, for well-conditioned matrices it is possible for iterative solvers to converge to a solution very quickly. Consequently for the right applications, iterative solvers can be very efficient.

Direct Solvers, on the other hand, often factor the matrix A into the product of two triangular matrices and then perform a forward and backward triangular solve.

This approach makes the time required to solve a systems of linear equations relatively predictable, based on the size of the matrix. In fact, for sparse matrices, the solution time can be predicted based on the number of non-zero elements in the array A.

Matrix Fundamentals

A matrix is a rectangular array of either real or complex numbers. A matrix is denoted by a capital letter; its elements are denoted by the same lower case letter with row/column subscripts. Thus, the value of the element in row i and column j in matrix A is denoted by $a(i,j)$. For example, a 3 by 4 matrix A, is written as follows:

$$A = \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(2,1) & a(2,2) & a(2,3) & a(2,4) \\ a(3,1) & a(3,2) & a(3,3) & a(3,4) \end{bmatrix}$$

Note that with the above notation, we assume the standard Fortran programming language convention of starting array indices at 1 rather than the C programming language convention of starting them at 0.

A matrix in which all of the elements are real numbers is called a real matrix. A matrix that contains at least one complex number is called a complex matrix. A real or complex matrix A with the property that $a(i,j) = a(j,i)$, is called a symmetric matrix. A complex matrix A with the property that $a(i,j) = conj(a(j,i))$, is called a Hermitian matrix. Note that programs that manipulate symmetric and Hermitian matrices need only store half of the matrix values, since the values of the non-stored elements can be quickly reconstructed from the stored values.

A matrix that has the same number of rows as it has columns is referred to as a square matrix. The elements in a square matrix that have same row index and column index are called the diagonal elements of the matrix, or simply the diagonal of the matrix.

The transpose of a matrix A is the matrix obtained by "flipping" the elements of the array about its diagonal. That is, we exchange the elements $a(i,j)$ and $a(j,i)$. For a complex matrix, if we both flip the elements about the diagonal and then take the complex conjugate of the element, the resulting matrix is called the Hermitian transpose or conjugate transpose of the original matrix. The transpose and Hermitian transpose of a matrix A are denoted by A^T and A^H respectively.

A column vector, or simply a vector, is a $n \times 1$ matrix, and a row vector is a $1 \times n$ matrix. A real or complex matrix A is said to be positive definite if the vector-matrix product x^TAx is greater than zero for all non-zero vectors x. A matrix that is not positive definite is referred to as indefinite.

An upper (or lower) triangular matrix, is a square matrix in which all elements below (or above) the diagonal are zero. A unit triangular matrix is an upper or lower triangular matrix with all 1's along the diagonal.

A matrix P is called a permutation matrix if, for any matrix A, the result of the matrix product PA is identical to A except for interchanging the rows of A. For a square matrix, it can be shown that if PA is a permutation of the rows of A, then AP^T is the same permutation of the columns of A. Additionally, it can be shown that the inverse of P is P^T.

In order to save space, a permutation matrix is usually stored as a linear array, called a permutation vector, rather than as an array. Specifically, if the permutation matrix maps the i-th row of a matrix to the j-th row, then the i-th element of the permutation vector is j.

A matrix with non-zero elements only on the diagonal is called a diagonal matrix. As is the case with a permutation matrix, it is usually stored as a vector of values, rather than as a matrix.

Direct Method

For solvers that use the direct method, the basic technique employed in finding the solution of the system $Ax = b$ is to first factor A into triangular matrices. That is, find a lower triangular matrix L and an upper triangular matrix U, such that $A = LU$. Having obtained such a factorization (usually referred to as an LU decomposition or LU factorization), the solution to the original problem can be rewritten as follows.

$$Ax = b$$
$$\Rightarrow \quad LUx = b$$
$$\Rightarrow \quad L(Ux) = b$$

This leads to the following two-step process for finding the solution to the original system of equations:

1. Solve the systems of equations $Ly = b$.
2. Solve the system $Ux = y$.

Solving the systems $Ly = b$ and $Ux = y$ is referred to as a forward solve and a backward solve, respectively.

If a symmetric matrix A is also positive definite, it can be shown that A can be factored as LL^T where L is a lower triangular matrix. Similarly, a Hermitian matrix, A, that is positive definite can be factored as $A = LL^H$. For both symmetric and Hermitian matrices, a factorization of this form is called a Cholesky factorization.

In a Cholesky factorization, the matrix U in an LU decomposition is either L^T or L^H. Consequently, a solver can increase its efficiency by only storing L, and one-half of A, and not computing U. Therefore, users who can express their application as the solution of a system of positive definite equations will gain a significant performance improvement over using a general representation.

For matrices that are symmetric (or Hermitian) but not positive definite, there are still some significant efficiencies to be had. It can be shown that if A is symmetric but not positive definite, then A can be factored as $A = LDL^T$, where D is a diagonal matrix and L is a lower unit triangular matrix. Similarly, if A is Hermitian, it can be factored as $A = LDL^H$. In either case, we again only need to store L, D, and half of A and we need not compute U. However, the backward solve phases must be amended to solving $L^Tx = D^{-1}y$ rather than $L^Tx = y$.

Fill-In and Reordering of Sparse Matrices

Two important concepts associated with the solution of sparse systems of equations are fill-in and reordering. The following example illustrates these concepts.

Consider the system of linear equation $Ax = b$, where A is a symmetric positive definite sparse matrix, and A and b are defined by the following:

$$A = \begin{bmatrix} 9 & \frac{3}{2} & 6 & \frac{3}{4} & 3 \\ \frac{3}{2} & \frac{1}{2} & * & * & * \\ 6 & * & 12 & * & * \\ \frac{3}{4} & * & * & \frac{5}{8} & * \\ 3 & * & * & * & 16 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

A star (*) is used to represent zeros and to emphasize the sparsity of A. The Cholesky factorization of A is: $A = LL^T$, where L is the following:

Notice that even though the matrix A is relatively sparse, the lower triangular matrix L has no zeros below the diagonal. If we computed L and then used it for the forward and backward solve phase, we would do as much computation as if A had been dense.

The situation of L having non-zeros in places where A has zeros is referred to as fill-in. Computationally, it would be more efficient if a solver could exploit the non-zero structure of A in such a way as to reduce the fill-in when computing L. By doing this, the solver would only need to compute the non-zero entries in L. Toward this end, consider permuting the rows and columns of A. As described in Matrix Fundamentals section , the permutations of the rows of A can be represented as a permutation matrix, P. The result of permuting the rows is the product of P and A. Suppose, in the above example, we swap the first and fifth row

of A, then swap the first and fifth columns of A, and call the resulting matrix B. Mathematically, we can express the process of permuting the rows and columns of A to get B as $B = PAP^T$. After permuting the rows and columns of A, we see that B is given by the following:

$$B = \begin{bmatrix} 16 & * & * & * & 3 \\ * & \frac{1}{2} & * & * & \frac{3}{2} \\ * & * & 12 & * & 6 \\ * & * & * & \frac{5}{8} & \frac{3}{4} \\ 3 & \frac{3}{2} & 6 & \frac{3}{4} & 9 \end{bmatrix}$$

Since B is obtained from A by simply switching rows and columns, the numbers of non-zero entries in A and B are the same. However, when we find the Cholesky factorization, $B = LL^T$, we see the following:

$$L = \begin{bmatrix} 4 & * & * & * & * \\ * & \frac{1}{\sqrt{2}} & * & * & * \\ * & * & 2(\sqrt{3}) & * & * \\ * & * & * & \frac{\sqrt{10}}{4} & * \\ \frac{3}{4} & \frac{3}{\sqrt{2}} & \sqrt{3} & \frac{3}{\sqrt{10}} & \frac{\sqrt{\frac{3}{5}}}{4} \end{bmatrix}$$

The fill-in associated with B is much smaller than the fill-in associated with A. Consequently, the storage and computation time needed to factor B is much smaller than to factor A. Based on this, we see that an efficient sparse solver needs to find permutation P of the matrix A, which minimizes the fill-in for factoring $B = PAP^T$, and then use the factorization of B to solve the original system of equations.

Although the above example is based on a symmetric positive definite matrix and a Cholesky decomposition, the same approach works for a general LU decomposition. Specifically, let P be a permutation matrix, $B = PAP^T$ and suppose that B can be factored as $B = LU$. Then

$Ax = b$

$\Rightarrow$ $PA(P^{-1}P)x = Pb$

$\Rightarrow$ $PA(P^TP)x = Pb$

$\Rightarrow$ $(PAP^T)(Px) = Pb$

$\Rightarrow$ $B(Px) = Pb$

$\Rightarrow$ $LU(Px) = Pb$

It follows that if we obtain an LU factorization for B, we can solve the original system of equations by a three step process:

1. Solve $Ly = Pb$.
2. Solve $Uz = y$.
3. Set $x = P^T z$.

If we apply this three-step process to the current example, we first need to perform the forward solve of the systems of equation $Ly = Pb$:

$$Ly = \begin{bmatrix} 4 & * & * & * & * \\ * & \frac{1}{\sqrt{2}} & * & * & * \\ * & * & 2(\sqrt{3}) & * & * \\ * & * & * & \frac{\sqrt{10}}{4} & * \\ \frac{3}{4} & \frac{3}{\sqrt{2}} & \sqrt{3} & \frac{3}{\sqrt{10}} & \frac{\sqrt{\frac{3}{5}}}{4} \end{bmatrix} * \begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \\ y5 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

This gives: $y^T = \frac{5}{4}, 2\sqrt{2}, \frac{\sqrt{3}}{2}, \frac{16}{\sqrt{10}}, \frac{-979\sqrt{\frac{3}{5}}}{12}$.

The second step is to perform the backward solve, $Uz = y$. Or, in this case, since a Cholesky factorization is used, $L^T z = y$.

$$\begin{bmatrix} 4 & * & * & * & * \\ * & \frac{1}{\sqrt{2}} & * & * & * \\ * & * & 2(\sqrt{3}) & * & * \\ * & * & * & \frac{\sqrt{10}}{4} & * \\ \frac{3}{4} & \frac{3}{\sqrt{2}} & \sqrt{3} & \frac{3}{\sqrt{10}} & \frac{\sqrt{\frac{3}{5}}}{4} \end{bmatrix}^T * \begin{bmatrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ 2(\sqrt{2}) \\ \frac{\sqrt{3}}{2} \\ \frac{16}{\sqrt{10}} \\ \frac{-979\sqrt{\frac{3}{5}}}{12} \end{bmatrix}$$

This gives $z^T = \frac{123}{2}, 983, \frac{1961}{12}, 398, \frac{-979}{3}$.

The third and final step is to set $x = P^T z$. This gives $X^T = \frac{-979}{3}, 983, \frac{1961}{12}, 398, \frac{123}{2}$.

Sparse Matrix Storage Formats

As discussed above, it is more efficient to store only the non-zero elements of a sparse matrix. There are a number of common storage formats used for sparse matrices, but most of them employ the same basic technique. That is, store all non-zero elements of the matrix into a linear array and provide auxiliary arrays to describe the locations of the non-zero elements in the original matrix.

Storage Formats for the Direct Sparse Solvers

The storing the non-zero elements of a sparse matrix into a linear array is done by walking down each column (column-major format) or across each row (row-major format) in order, and writing the non-zero elements to a linear array in the order they appear in the walk.

For symmetric matrices, it is necessary to store only the upper triangular half of the matrix (upper triangular format) or the lower triangular half of the matrix (lower triangular format).

The Intel MKL direct sparse solvers use a row-major upper triangular storage format: the matrix is compressed row-by-row and for symmetric matrices only non-zero elements in the upper triangular half of the matrix are stored.

The Intel MKL sparse matrix storage format for direct sparse solvers is specified by three arrays: *values*, *columns*, and *rowIndex*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix.

values	A real or complex array that contains the non-zero elements of a sparse matrix. The non-zero elements are mapped into the *values* array using the row-major upper triangular storage mapping described above.
columns	Element *i* of the integer array *columns* is the number of the column that contains the *i*-th element in the *values* array.
rowIndex	Element *j* of the integer array *rowIndex* gives the index of the element in the *values* array that is first non-zero element in a row *j*.

The length of the *values* and *columns* arrays is equal to the number of non-zero elements in the matrix.

As the *rowIndex* array gives the location of the first non-zero element within a row, and the non-zero elements are stored consecutively, the number of non-zero elements in the *i*-th row is equal to the difference of *rowIndex*(*i*) and *rowIndex*(*i*+1).

To have this relationship hold for the last row of the matrix, an additional entry (dummy entry) is added to the end of *rowIndex*. Its value is equal to the number of non-zero elements plus one. This makes the total length of the *rowIndex* array one larger than the number of rows in the matrix.

NOTE The Intel MKL sparse storage scheme for the direct sparse solvers supports both with one-based indexing and zero-based indexing.

Consider the symmetric matrix *A*:

$$A = \begin{bmatrix} 1 & -1 & * & -3 & * \\ -1 & 5 & * & * & * \\ * & * & 4 & 6 & 4 \\ -3 & * & 6 & 7 & * \\ * & * & 4 & * & -5 \end{bmatrix}$$

Only elements from the upper triangle are stored. The actual arrays for the matrix *A* are as follows:

Storage Arrays for a Symmetric Matrix

one-based indexing									
values	=	(1	-1	-3	5	4	6	4	7	-5)
columns	=	(1	2	4	2	3	4	5	4	5)
rowIndex	=	(1	4	5	8	9	10)			
zero-based indexing									
values	=	(1	-1	-3	5	4	6	4	7	-5)
columns	=	(0	1	3	1	2	3	4	3	4)
rowIndex	=	(0	3	4	7	8	9)			

For a non-symmetric or non-Hermitian matrix, all non-zero elements need to be stored. Consider the non-symmetric matrix *B*:

$$B = \begin{bmatrix} 1 & -1 & * & -3 & * \\ -2 & 5 & * & * & * \\ * & * & 4 & 6 & 4 \\ -4 & * & 2 & 7 & * \\ * & 8 & * & * & -5 \end{bmatrix}$$

The matrix *B* has 13 non-zero elements, and all of them are stored as follows:

Storage Arrays for a Non-Symmetric Matrix

one-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
columns	=	(1	2	4	1	2	3	4	5	1	3	4	2	5)
rowIndex	=	(1	4	6	9	12	14)							
zero-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
columns	=	(0	1	3	0	1	2	3	4	0	2	3	1	4)
rowIndex	=	(0	3	5	8	11	13)							

Direct sparse solvers can also solve symmetrically structured systems of equations. A symmetrically structured system of equations is one where the pattern of non-zero elements is symmetric. That is, a matrix has a symmetric structure if *a*(*j*,*i*) is not zero if and only if *a*(*j*, *i*) is not zero. From the point of view of the solver software, a "non-zero" element of a matrix is any element stored in the *values* array, even if its value

is equal to 0. In that sense, any non-symmetric matrix can be turned into a symmetrically structured matrix by carefully adding zeros to the *values* array. For example, the above matrix *B* can be turned into a symmetrically structured matrix by adding two non-zero entries:

$$B = \begin{bmatrix} 1 & -1 & * & -3 & * \\ -2 & 5 & * & * & 0 \\ * & * & 4 & 6 & 4 \\ -4 & * & 2 & 7 & * \\ * & 8 & 0 & * & -5 \end{bmatrix}$$

The matrix *B* can be considered to be symmetrically structured with 15 non-zero elements and represented as:

Storage Arrays for a Symmetrically Structured Matrix

one-based indexing															
values	=	(1	-1	-3	-2	5	0	4	6	4	-4	2	7	8	0	-5)
columns	=	(1	2	4	1	2	5	3	4	5	1	3	4	2	3	5)
rowIndex	=	(1	4	7	10	13	16)									
zero-based indexing															
values	=	(1	-1	-3	-2	5	0	4	6	4	-4	2	7	8	0	-5)
columns	=	(0	1	3	0	1	4	2	3	4	0	2	3	1	2	4)
rowIndex	=	(0	3	6	9	12	15)									

Storage Format Restrictions

The storage format for the sparse solver must conform to two important restrictions:

- the non-zero values in a given row must be placed into the *values* array in the order in which they occur in the row (from left to right);

- no diagonal element can be omitted from the *values* array for any symmetric or structurally symmetric matrix.

The second restriction implies that if symmetric or structurally symmetric matrices have zero diagonal elements, then they must be explicitly represented in the *values* array.

Sparse Matrix Storage Formats for Sparse BLAS Levels 2 and Level 3

This section describes in detail the sparse matrix storage formats supported in the current version of the Intel MKL Sparse BLAS Level 2 and Level 3.

CSR Format

The Intel MKL compressed sparse row (CSR) format is specified by four arrays: the *values*, *columns*, *pointerB*, and *pointerE*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix *A*.

values	A real or complex array that contains the non-zero elements of *A*. Values of the non-zero elements of *A* are mapped into the *values* array using the row-major storage mapping described above.
columns	Element *i* of the integer array *columns* is the number of the column in *A* that contains the *i*-th value in the *values* array.

pointerB	Element *j* of this integer array gives the index of the element in the *values* array that is first non-zero element in a row *j* of *A*. Note that this index is equal to *pointerB*(*j*) – *pointerB*(1)+1 .
pointerE	An integer array that contains row indices, such that *pointerE*(*j*)–*pointerB*(1) is the index of the element in the *values* array that is last non-zero element in a row *j* of *A*.

The length of the *values* and *columns* arrays is equal to the number of non-zero elements in *A*.The length of the *pointerB* and *pointerE* arrays is equal to the number of rows in *A*.

NOTE Note that the Intel MKL Sparse BLAS routines support the CSR format both with one-based indexing and zero-based indexing.

The matrix *B*

can be represented in the CSR format as:

Storage Arrays for a Matrix in CSR Format

one-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
columns	=	(1	2	4	1	2	3	4	5	1	3	4	2	5)
pointerB	=	(1	4	6	9	12)								
pointerE	=	(4	6	9	12	14)								
zero-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
columns	=	(0	1	3	0	1	2	3	4	0	2	3	1	4)
pointerB	=	(0	3	5	8	11)								
pointerE	=	(3	5	8	11	13)								

This storage format is used in the NIST Sparse BLAS library [Rem05].

Note that the storage format accepted for the direct sparse solvers and described above (see Storage Formats for the Direct Sparse Solvers) is a variation of the CSR format. It also is used in the Intel MKL Sparse BLAS Level 2 both with one-based indexing and zero-based indexing. The above matrix *B* can be represented in this format (referred to as the 3-array variation of the CSR format) as:

Storage Arrays for a Matrix in CSR Format (3-Array Variation)

one-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
columns	=	(1	2	4	1	2	3	4	5	1	3	4	2	5)
rowIndex	=	(1	4	6	9	12	14)							
zero-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)

columns	=	(0	1	3	0	1	2	3	4	0	2	3	1	4)
rowIndex	=	(0	3	5	8	11	13)							

The 3-array variation of the CSR format has a restriction: all non-zero elements are stored continuously, that is the set of non-zero elements in the row *J* goes just after the set of non-zero elements in the row *J-1* .

There are no such restrictions in the general (NIST) CSR format. This may be useful, for example, if there is a need to operate with different submatrices of the matrix at the same time. In this case, it is enough to define the arrays *pointerB* and *pointerE* for each needed submatrix so that all these arrays are pointers to the same array *values*.

Comparing the array *rowIndex* from the Table "Storage Arrays for a Non-Symmetric Example Matrix" with the arrays *pointerB* and *pointerE* from the Table "Storage Arrays for an Example Matrix in CSR Format" it is easy to see that

```
pointerB(i) = rowIndex(i) for i=1, ..5;

pointerE(i) = rowIndex(i+1) for i=1, ..5.
```

This enables calling a routine that has *values*, *columns*, *pointerB* and *pointerE* as input parameters for a sparse matrix stored in the format accepted for the direct sparse solvers. For example, a routine with the interface:

```
Subroutine name_routine(.... ,  values, columns, pointerB, pointerE, ...)
```

can be called with parameters *values*, *columns*, *rowIndex* as follows:

```
call name_routine(.... ,  values, columns, rowIndex, rowindex(2), ...).
```

CSC Format

The compressed sparse column format (CSC) is similar to the CSR format, but the columns are used instead the rows. In other words, the CSC format is identical to the CSR format for the transposed matrix. The CSR format is specified by four arrays: *values*, *columns*, *pointerB*, and *pointerE*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix *A*.

values	A real or complex array that contains the non-zero elements of *A*. Values of the non-zero elements of *A* are mapped into the *values* array using the column-major storage mapping.
rows	Element *i* of the integer array *rows* is the number of the row in *A* that contains the *i*-th value in the *values* array.
pointerB	Element *j* of this integer array gives the index of the element in the *values* array that is first non-zero element in a column *j* of *A*. Note that this index is equal to *pointerB*(*j*) – *pointerB*(1)+1 .
pointerE	An integer array that contains column indices, such that *pointerE*(*j*)–*pointerB*(1) is the index of the element in the *values* array that is last non-zero element in a column *j* of *A*.

The length of the *values* and *columns* arrays is equal to the number of non-zero elements in *A*.The length of the *pointerB* and *pointerE* arrays is equal to the number of columns in *A*.

NOTE Note that the Intel MKL Sparse BLAS routines support the CSC format both with one-based indexing and zero-based indexing.

The above matrix *B* can be represented in the CSC format as:

Storage Arrays for a Matrix in CSC Format

one-based indexing

values	=	(1	-2	-4	-1	5	8	4	2	-3	6	7	4	-5)
rows	=	(1	2	4	1	2	5	3	4	1	3	4	2	5)
pointerB	=	(1	4	7	9	12)								
pointerE	=	(4	7	9	12	14)								
zero-based indexing														
values	=	(1	-2	-4	-1	5	8	4	2	-3	6	7	4	-5)
rows	=	(0	1	3	0	1	4	2	3	0	2	3	1	4)
pointerB	=	(0	3	6	8	11)								
pointerE	=	(3	6	8	11	13)								

Coordinate Format

The coordinate format is the most flexible and simplest format for the sparse matrix representation. Only non-zero elements are stored, and the coordinates of each non-zero element are given explicitly. Many commercial libraries support the matrix-vector multiplication for the sparse matrices in the coordinate format.

The Intel MKL coordinate format is specified by three arrays: *values*, *rows*, and *column*, and a parameter *nnz* which is number of non-zero elements in *A*. All three arrays have dimension *nnz*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix *A*.

values	A real or complex array that contains the non-zero elements of *A* in any order.
rows	Element *i* of the integer array *rows* is the number of the row in *A* that contains the *i*-th value in the *values* array.
columns	Element *i* of the integer array *columns* is the number of the column in *A* that contains the *i*-th value in the *values* array.

NOTE Note that the Intel MKL Sparse BLAS routines support the coordinate format both with one-based indexing and zero-based indexing.

For example, the sparse matrix *C*

$$C = \begin{bmatrix} 1 & -1 & -3 & 0 & 0 \\ -2 & 5 & 0 & 0 & 0 \\ 0 & 0 & 4 & 6 & 4 \\ -4 & 0 & 2 & 7 & 0 \\ 0 & 8 & 0 & 0 & -5 \end{bmatrix}$$

can be represented in the coordinate format as follows:

Storage Arrays for an Example Matrix in case of the coordinate format

one-based indexing													
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
rows	=	(1	1	1	2	2	3	3	3	4	4	4	5	5)
columns	=	(1	2	3	1	2	3	4	5	1	3	4	2	5)
zero-based indexing														
values	=	(1	-1	-3	-2	5	4	6	4	-4	2	7	8	-5)
rows	=	(0	0	0	1	1	2	2	2	3	3	3	4	4)

columns	=	(0	1	2	0	1	2	3	4	0	2	3	1	4)

Diagonal Storage Format

If the sparse matrix has diagonals containing only zero elements, then the diagonal storage format can be used to reduce the amount of information needed to locate the non-zero elements. This storage format is particularly useful in many applications where the matrix arises from a finite element or finite difference discretization. The Intel MKL diagonal storage format is specified by two arrays: *values* and *distance*, and two parameters: *ndiag*, which is the number of non-empty diagonals, and *lval*, which is the declared leading dimension in the calling (sub)programs. The following table describes the arrays *values* and *distance*:

values	A real or complex two-dimensional array is dimensioned as *lval* by *ndiag*. Each column of it contains the non-zero elements of certain diagonal of *A*. The key point of the storage is that each element in *values* retains the row number of the original matrix. To achieve this diagonals in the lower triangular part of the matrix are padded from the top, and those in the upper triangular part are padded from the bottom. Note that the value of *distance*(*i*) is the number of elements to be padded for diagonal *i*.
distance	An integer array with dimension *ndiag*. Element *i* of the array *distance* is the distance between *i*-diagonal and the main diagonal. The distance is positive if the diagonal is above the main diagonal, and negative if the diagonal is below the main diagonal. The main diagonal has a distance equal to zero.

The above matrix *C* can be represented in the diagonal storage format as follows:

where the asterisks denote padded elements.

When storing symmetric, Hermitian, or skew-symmetric matrices, it is necessary to store only the upper or the lower triangular part of the matrix.

For the Intel MKL triangular solver routines elements of the array *distance* must be sorted in increasing order. In all other cases the diagonals and distances can be stored in arbitrary order.

Skyline Storage Format

The skyline storage format is important for the direct sparse solvers, and it is well suited for Cholesky or LU decomposition when no pivoting is required.

The skyline storage format accepted in Intel MKL can store only triangular matrix or triangular part of a matrix. This format is specified by two arrays: *values* and *pointers*. The following table describes these arrays:

values	A scalar array. For a lower triangular matrix it contains the set of elements from each row of the matrix starting from the first non-zero element to and including the diagonal element. For an upper triangular matrix it contains the set of elements from each column of the matrix starting with the first non-zero element down to and including the diagonal element. Encountered zero elements are included in the sets.
pointers	An integer array with dimension *(m+1)*, where *m* is the number of rows for lower triangle (columns for the upper triangle). *pointers(i) – pointers(1)+1* gives the index of element in *values* that is first non-zero element in row (column) *i*. The value of *pointers(m+1)* is set to *nnz+pointers(1)*, where *nnz* is the number of elements in the array *values*.

For example, the low triangle of the matrix *C* given above can be stored as follows:

```
values  = ( 1  -2   5   4  -4   0   2   7   8   0   0   -5 )

pointers = ( 1   2   4   5   9   13 )
```

and the upper triangle of this matrix *C* can be stored as follows:

```
values   = ( 1  -1   5  -3   0   4   6   7  4   0   -5 )

pointers = ( 1   2   4   7   9   12 )
```

This storage format is supported by the NIST Sparse BLAS library [Rem05].

Note that the Intel MKL Sparse BLAS routines operating with the skyline storage format does not support general matrices.

BSR Format

The Intel MKL block compressed sparse row (BSR) format for sparse matrices is specified by four arrays: *values*, *columns*, *pointerB*, and *pointerE*. The following table describes these arrays.

values	A real array that contains the elements of the non-zero blocks of a sparse matrix. The elements are stored block-by-block in row-major order. A non-zero block is the block that contains at least one non-zero element. All elements of non-zero blocks are stored, even if some of them is equal to zero. Within each non-zero block elements are stored in column-major order in the case of one-based indexing, and in row-major order in the case of the zero-based indexing.
columns	Element *i* of the integer array *columns* is the number of the column in the block matrix that contains the *i*-th non-zero block.
pointerB	Element *j* of this integer array gives the index of the element in the *columns* array that is first non-zero block in a row *j* of the block matrix.
pointerE	Element *j* of this integer array gives the index of the element in the *columns* array that contains the last non-zero block in a row *j* of the block matrix plus 1.

The length of the *values* array is equal to the number of all elements in the non-zero blocks, the length of the *columns* array is equal to the number of non-zero blocks. The length of the *pointerB* and *pointerE* arrays is equal to the number of block rows in the block matrix.

NOTE Note that the Intel MKL Sparse BLAS routines support BSR format both with one-based indexing and zero-based indexing.

For example, consider the sparse matrix *D*

$$D = \begin{bmatrix} 1 & 0 & 6 & 7 & * & * \\ 2 & 1 & 8 & 2 & * & * \\ * & * & 1 & 4 & * & * \\ * & * & 5 & 1 & * & * \\ * & * & 4 & 3 & 7 & 2 \\ * & * & 0 & 0 & 0 & 0 \end{bmatrix}$$

If the size of the block equals 2, then the sparse matrix D can be represented as a 3x3 block matrix E with the following structure:

$$E = \begin{bmatrix} L & M & * \\ * & N & * \\ * & P & Q \end{bmatrix}$$

where

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, M = \begin{bmatrix} 6 & 7 \\ 8 & 2 \end{bmatrix}, N = \begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix}, P = \begin{bmatrix} 4 & 3 \\ 0 & 0 \end{bmatrix}, Q = \begin{bmatrix} 7 & 2 \\ 0 & 0 \end{bmatrix}$$

The matrix D can be represented in the BSR format as follows:

one-based indexing

```
values  = (1 2 0 1 6 8 7 2 1 5 4 1 4 0 3 0 7 0 2 0)

columns  = (1   2   2   2   3)

pointerB = (1   3   4)

pointerE = (3   4   6)
```

zero-based indexing

```
values  = (1 0 2 1 6 7 8 2 1 4 5 1 4 3 0 0 7 2 0 0)

columns  = (0   1   1   1   2)

pointerB = (0   2   3)

pointerE = (2   3   5)
```

This storage format is supported by the NIST Sparse BLAS library [Rem05].

Intel MKL supports the variation of the BSR format that is specified by three arrays: *values*, *columns*, and *rowIndex*. The following table describes these arrays.

values	A real array that contains the elements of the non-zero blocks of a sparse matrix. The elements are stored block by block in row-major order. A non-zero block is the block that contains at least one non-zero element. All elements of non-zero blocks are stored, even if some of them is equal to zero. Within each non-zero block the elements are stored in column major order in the case of the one-based indexing, and in row major order in the case of the zero-based indexing.
columns	Element *i* of the integer array *columns* is the number of the column in the block matrix that contains the *i*-th non-zero block.
rowIndex	Element *j* of this integer array gives the index of the element in the *columns* array that is first non-zero block in a row *j* of the block matrix.

The length of the *values* array is equal to the number of all elements in the non-zero blocks, the length of the *columns* array is equal to the number of non-zero blocks.

As the *rowIndex* array gives the location of the first non-zero block within a row, and the non-zero blocks are stored consecutively, the number of non-zero blocks in the *i*-th row is equal to the difference of *rowIndex*(*i*) and *rowIndex*(*i*+1).

To retain this relationship for the last row of the block matrix, an additional entry (dummy entry) is added to the end of *rowIndex* with value equal to the number of non-zeros blocks plus one. This makes the total length of the *rowIndex* array one larger than the number of rows of the block matrix.

The above matrix *D* can be represented in this 3-array variation of the BSR format as follows:

one-based indexing

```
values  =  (1 2 0 1 6 8 7 2 1 5 4 2 4 0 3 0 7 0 2 0)

columns  = (1   2   2   2   3)

rowIndex = (1   3   4   6)
```

zero-based indexing

```
values  =  (1 0 2 1 6 7 8 2 1 4 5 1 4 3 0 0 7 2 0 0)

columns  = (0   1   1   1   2)

rowIndex = (0   2   3 5)
```

When storing symmetric matrices, it is necessary to store only the upper or the lower triangular part of the matrix.

For example, consider the symmetric sparse matrix *F*:

$$F = \begin{bmatrix} 1 & 0 & 6 & 7 & * & * \\ 2 & 1 & 8 & 2 & * & * \\ 6 & 8 & 1 & 4 & * & * \\ 7 & 2 & 5 & 2 & * & * \\ * & * & * & * & 7 & 2 \\ * & * & * & * & 0 & 0 \end{bmatrix}$$

If the size of the block equals 2, then the sparse matrix F can be represented as a 3x3 block matrix G with the following structure:

$$G = \begin{bmatrix} L & M & * \\ M' & N & * \\ * & * & Q \end{bmatrix}$$

where

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, M = \begin{bmatrix} 6 & 7 \\ 8 & 2 \end{bmatrix}, M' = \begin{bmatrix} 6 & 8 \\ 7 & 2 \end{bmatrix}, N = \begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}, Q = \begin{bmatrix} 7 & 2 \\ 0 & 0 \end{bmatrix}$$

The symmetric matrix F can be represented in this 3-array variation of the BSR format (storing only upper triangular) as follows:

one-based indexing

```
values  =  (1 2 0 1 6 8 7 2 1 5 4 2 7 0 2 0)

columns  = (1   2   2   3)

rowIndex = (1   3   4 5)
```

zero-based indexing

```
values  =  (1 0 2 1 6 7 8 2 1 4 5 2 7 2 0 0)

columns  = (0   1   1   2)

rowIndex = (0   2   3 4)
```

Routine and Function Arguments

B

The major arguments in the BLAS routines are vector and matrix, whereas VML functions work on vector arguments only. The sections that follow discuss each of these arguments and provide examples.

Vector Arguments in BLAS

Vector arguments are passed in one-dimensional arrays. The array dimension (length) and vector increment are passed as integer variables. The length determines the number of elements in the vector. The increment (also called stride) determines the spacing between vector elements and the order of the elements in the array in which the vector is passed.

A vector of length *n* and increment *incx* is passed in a one-dimensional array *x* whose values are defined as

```
x(1), x(1+|incx|), ..., x(1+(n-1)* |incx|)
```

If *incx* is positive, then the elements in array *x* are stored in increasing order. If *incx* is negative, the elements in array *x* are stored in decreasing order with the first element defined as *x*(1+(*n*-1)* |*incx*|). If *incx* is zero, then all elements of the vector have the same value, *x*(1). The dimension of the one-dimensional array that stores the vector must always be at least

```
idimx = 1 + (n-1)* |incx |
```

Example. One-dimensional Real Array

Let *x*(1:7) be the one-dimensional real array

```
x = (1.0, 3.0, 5.0, 7.0, 9.0, 11.0, 13.0).
```

If *incx* =2 and *n* = 3, then the vector argument with elements in order from first to last is (1.0, 5.0, 9.0).

If *incx* = -2 and *n* = 4, then the vector elements in order from first to last is (13.0, 9.0, 5.0, 1.0).

If *incx* = 0 and *n* = 4, then the vector elements in order from first to last is (1.0, 1.0, 1.0, 1.0).

One-dimensional substructures of a matrix, such as the rows, columns, and diagonals, can be passed as vector arguments with the starting address and increment specified. In Fortran, storing the *m*-by-*n* matrix is based on column-major ordering where the increment between elements in the same column is 1, the increment between elements in the same row is *m*, and the increment between elements on the same diagonal is *m* + 1.

Example. Two-dimensional Real Matrix

Let *a* be the real 5 x 4 matrix declared as REAL A (5,4).

To scale the third column of *a* by 2.0, use the BLAS routine sscal with the following calling sequence:

```
callsscal (5, 2.0, a(1,3), 1)
```

To scale the second row, use the statement:

```
callsscal (4, 2.0, a(2,1), 5)
```

To scale the main diagonal of A by 2.0, use the statement:

```
callsscal (5, 2.0, a(1,1), 6)
```

NOTE The default vector argument is assumed to be 1.

Vector Arguments in VML

Vector arguments of VML mathematical functions are passed in one-dimensional arrays with unit vector increment. It means that a vector of length n is passed contiguously in an array a whose values are defined as `a[0], a[1], ..., a[n-1]` (for the C interface).

To accommodate for arrays with other increments, or more complicated indexing, VML contains auxiliary pack/unpack functions that gather the array elements into a contiguous vector and then scatter them after the computation is complete.

Generally, if the vector elements are stored in a one-dimensional array a as

```
a[m0], a[m1], ..., a[mn-1]
```

and need to be regrouped into an array y as

```
y[k0], y[k1], ..., y[kn-1],
```

VML pack/unpack functions can use one of the following indexing methods:

Positive Increment Indexing

```
kj = incy * j, mj = inca * j, j = 0 ,..., n-1
```

Constraint: `incy > 0` and `inca > 0`.

For example, setting incy = 1 specifies gathering array elements into a contiguous vector.

This method is similar to that used in BLAS, with the exception that negative and zero increments are not permitted.

Index Vector Indexing

```
kj = iy[j], mj = ia[j], j = 0 ,..., n-1,
```

where ia and iy are arrays of length n that contain index vectors for the input and output arrays a and y, respectively.

Mask Vector Indexing

Indices `kj` , `mj` are such that:

```
my[kj] ≠ 0, ma[mj] ≠ 0 , j = 0,..., n-1,
```

where `ma` and `my` are arrays that contain mask vectors for the input and output arrays a and y, respectively.

Matrix Arguments

Matrix arguments of the Intel® Math Kernel Library routines can be stored in either one- or two-dimensional arrays, using the following storage schemes:

- conventional full storage (in a two-dimensional array)
- packed storage for Hermitian, symmetric, or triangular matrices (in a one-dimensional array)
- band storage for band matrices (in a two-dimensional array)
- rectangular full packed storage for symmetric, Hermitian, or triangular matrices as compact as the Packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels.

Full storage is the following obvious scheme: a matrix A is stored in a two-dimensional array a, with the matrix element a_{ij} stored in the array element $a(i,j)$.

If a matrix is triangular (upper or lower, as specified by the argument *uplo*), only the elements of the relevant triangle are stored; the remaining elements of the array need not be set.

Routines that handle symmetric or Hermitian matrices allow for either the upper or lower triangle of the matrix to be stored in the corresponding elements of the array:

if *uplo* ='U', a_{ij} is stored in *a*(*i*,*j*) for $i \le j$, other elements of *a* need not be set.

if *uplo* ='L', a_{ij} is stored in *a*(*i*,*j*) for $j \le i$, other elements of *a* need not be set.

Packed storage allows you to store symmetric, Hermitian, or triangular matrices more compactly: the relevant triangle (again, as specified by the argument *uplo*) is packed by columns in a one-dimensional array *ap*:

if *uplo* ='U', a_{ij} is stored in *ap*(*i*+*j*(*j*-1)/2) for $i \le j$

if *uplo* ='L', a_{ij} is stored in *ap*(*i*+(2**n*-*j*)*(*j*-1)/2) for $j \le i$.

In descriptions of LAPACK routines, arrays with packed matrices have names ending in *p*.

Band storage is as follows: an *m*-by-*n* band matrix with *kl* non-zero sub-diagonals and *ku* non-zero super-diagonals is stored compactly in a two-dimensional array *ab* with *kl*+*ku*+1 rows and *n* columns. Columns of the matrix are stored in the corresponding columns of the array, and diagonals of the matrix are stored in rows of the array. Thus,

a_{ij} is stored in *ab*(*ku*+1+*i*-*j*,*j*) for $\max(1,j-ku) \le i \le \min(n,j+kl)$.

Use the band storage scheme only when *kl* and *ku* are much less than the matrix size *n*. Although the routines work correctly for all values of *kl* and *ku*, using the band storage is inefficient if your matrices are not really banded.

The band storage scheme is illustrated by the following example, when

m = *n* = 6, *kl* = 2, *ku* = 1

Array elements marked * are not used by the routines:

Banded matrix A

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & a_{66} \end{bmatrix}$$

Band storage of A

$$\begin{matrix} * & a_{12} & a_{23} & a_{34} & a_{45} & a_{56} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} & a_{66} \\ a_{21} & a_{32} & a_{43} & a_{54} & a_{65} & * \\ a_{31} & a_{42} & a_{53} & a_{64} & * & * \end{matrix}$$

When a general band matrix is supplied for *LU factorization*, space must be allowed to store *kl* additional super-diagonals generated by fill-in as a result of row interchanges. This means that the matrix is stored according to the above scheme, but with *kl* + *ku* super-diagonals. Thus,

a_{ij} is stored in *ab*(*kl*+*ku*+1+*i*-*j*,*j*) for $\max(1,j-ku) \le i \le \min(n,j+kl)$.

The band storage scheme for LU factorization is illustrated by the following example, when*m* = *n* = 6, *kl* = 2, *ku* = 1:

Banded matrix A

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & a_{66} \end{bmatrix}$$

Band storage of A

$$\begin{matrix} * & * & * & + & + & + \\ * & * & + & + & + & + \\ * & a_{12} & a_{23} & a_{34} & a_{45} & a_{56} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} & a_{66} \\ a_{21} & a_{32} & a_{43} & a_{54} & a_{65} & * \\ a_{31} & a_{42} & a_{53} & a_{64} & * & * \end{matrix}$$

Array elements marked * are not used by the routines; elements marked + need not be set on entry, but are required by the LU factorization routines to store the results. The input array will be overwritten on exit by the details of the LU factorization as follows:

where u_{ij} are the elements of the upper triangular matrix U, and m_{ij} are the multipliers used during factorization.

Triangular band matrices are stored in the same format, with either $kl = 0$ if upper triangular, or $ku = 0$ if lower triangular. For symmetric or Hermitian band matrices with k sub-diagonals or super-diagonals, you need to store only the upper or lower triangle, as specified by the argument *uplo*:

if *uplo* ='U', a_{ij} is stored in $ab(k+1+i-j,j)$ for $\max(1,j-k) \le i \le j$

if *uplo* ='L', a_{ij} is stored in $ab(1+i-j,j)$ for $j \le i \le \min(n,j+k)$.

In descriptions of LAPACK routines, arrays that hold matrices in band storage have names ending in *b*.

In Fortran, column-major ordering of storage is assumed. This means that elements of the same column occupy successive storage locations.

Three quantities are usually associated with a two-dimensional array argument: its leading dimension, which specifies the number of storage locations between elements in the same row, its number of rows, and its number of columns. For a matrix in full storage, the leading dimension of the array must be at least as large as the number of rows in the matrix.

A character transposition parameter is often passed to indicate whether the matrix argument is to be used in normal or transposed form or, for a complex matrix, if the conjugate transpose of the matrix is to be used.

The values of the transposition parameter for these three cases are the following:

'N' or *'n'*	normal (no conjugation, no transposition)
'T' or *'t'*	transpose
'C' or *'c'*	conjugate transpose.

Example. Two-Dimensional Complex Array

Suppose `A (1:5, 1:4)` is the complex two-dimensional array presented by matrix

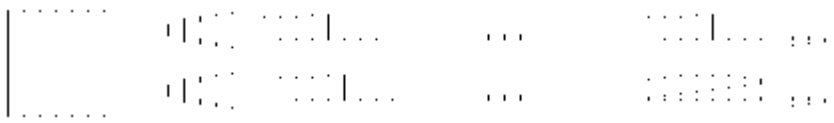
$$\begin{bmatrix} (1.1,0.11) & (1.2,0.12) & (1.3,0.13) & (1.4,0.14) \\ (2.1,0.21) & (2.2,0.22) & (2.3,0.23) & (1.4,0.24) \\ (3.1,0.31) & (3.2,0.32) & (3.3,0.33) & (1.4,0.34) \\ (4.1,0.41) & (4.2,0.42) & (4.3,0.43) & (1.4,0.44) \\ (5.1,0.51) & (5.2,0.52) & (5.3,0.53) & (1.4,0.54) \end{bmatrix}$$

Let *transa* be the transposition parameter, *m* be the number of rows, *n* be the number of columns, and *lda* be the leading dimension. Then if

transa = '*N*', *m* = 4, *n* = 2, and *lda* = 5, the matrix argument would be

$$\begin{bmatrix} (1.1,0.11) & (1.2,0.12) \\ (2.1,0.21) & (2.2,0.22) \\ (3.1,0.31) & (3.2,0.32) \\ (4.1,0.41) & (4.2,0.42) \end{bmatrix}$$

If *transa* = '*T*', *m* = 4, *n* = 2, and *lda* =5, the matrix argument would be



If *transa* = '*C*', *m* = 4, *n* = 2, and *lda* =5, the matrix argument would be

$$\begin{bmatrix} (1.1,-0.11) & (2.1,-0.21) & (3.1,-0.31) & (4.1,-0.41) \\ (1.2,-0.12) & (2.2,-0.22) & (3.2,-0.32) & (4.2,-0.42) \end{bmatrix}$$

Note that care should be taken when using a leading dimension value which is different from the number of rows specified in the declaration of the two-dimensional array. For example, suppose the array *A* above is declared as `COMPLEX A (5,4)`.

Then if *transa* = '*N*', *m* = 3, *n* = 4, and *lda* = 4, the matrix argument will be

$$\begin{bmatrix} (1.1, 0.11) & (5.1, 0.51) & (4.2, 0.42) & (3.3, 0.33) \\ (2.1, 0.21) & (1.2, 0.12) & (5.2, 0.52) & (4.3, 0.43) \\ (3.1, 0.31) & (2.2, 0.22) & (1.3, 0.13) & (5.3, 0.53) \end{bmatrix}$$

Rectangular Full Packed storage allows you to store symmetric, Hermitian, or triangular matrices as compact as the Packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels. To store an n-by-n triangle (and suppose for simplicity that n is even), you partition the triangle into three parts: two $n/2$-by-$n/2$ triangles and an $n/2$-by-$n/2$ square, then pack this as an n-by-$n/2$ rectangle (or $n/2$-by-n rectangle), by transposing (or transpose-conjugating) one of the triangles and packing it next to the other triangle. Since the two triangles are stored in full storage, you can use existing efficient routines on them.

There are eight cases of RFP storage representation: when n is even or odd, the packed matrix is transposed or not, the triangular matrix is lower or upper. See below for all the eight storage schemes illustrated:

n is odd, A is lower triangular

Full format

$$\begin{matrix} a_{11} & x & x & x & x & x & x \\ a_{21} & a_{22} & x & x & x & x & x \\ a_{31} & a_{32} & a_{33} & x & x & x & x \\ a_{41} & a_{42} & a_{43} & a_{44} & x & x & x \\ \mathbf{a_{51}} & \mathbf{a_{52}} & \mathbf{a_{53}} & \mathbf{a_{54}} & a_{55} & x & x \\ \mathbf{a_{61}} & \mathbf{a_{62}} & \mathbf{a_{63}} & \mathbf{a_{64}} & a_{65} & a_{66} & x \\ \mathbf{a_{71}} & \mathbf{a_{72}} & \mathbf{a_{73}} & \mathbf{a_{74}} & a_{75} & a_{76} & a_{77} \end{matrix}$$

RFP (not transposed)

$$\begin{matrix} a_{11} & a_{55} & a_{65} & a_{75} \\ a_{21} & a_{22} & a_{66} & a_{76} \\ a_{31} & a_{32} & a_{33} & a_{77} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ \mathbf{a_{51}} & \mathbf{a_{52}} & \mathbf{a_{53}} & \mathbf{a_{54}} \\ \mathbf{a_{61}} & \mathbf{a_{62}} & \mathbf{a_{63}} & \mathbf{a_{64}} \\ \mathbf{a_{71}} & \mathbf{a_{72}} & \mathbf{a_{73}} & \mathbf{a_{74}} \end{matrix}$$

RFP (transposed)

$$\begin{matrix} a_{11} & a_{21} & a_{31} & a_{41} & \mathbf{a_{51}} & \mathbf{a_{61}} & \mathbf{a_{71}} \\ a_{55} & a_{22} & a_{32} & a_{42} & \mathbf{a_{52}} & \mathbf{a_{62}} & \mathbf{a_{72}} \\ a_{65} & a_{66} & a_{33} & a_{43} & \mathbf{a_{53}} & \mathbf{a_{63}} & \mathbf{a_{73}} \\ a_{75} & a_{76} & a_{77} & a_{44} & \mathbf{a_{54}} & \mathbf{a_{64}} & \mathbf{a_{74}} \end{matrix}$$

n is even, A is lower triangular

Full format

$$\begin{matrix} a_{11} & x & x & x & x & x \\ a_{21} & a_{22} & x & x & x & x \\ a_{31} & a_{32} & a_{33} & x & x & x \\ \mathbf{a_{41}} & \mathbf{a_{42}} & \mathbf{a_{43}} & a_{44} & x & x \\ \mathbf{a_{51}} & \mathbf{a_{52}} & \mathbf{a_{53}} & a_{54} & a_{55} & x \\ \mathbf{a_{61}} & \mathbf{a_{62}} & \mathbf{a_{63}} & a_{64} & a_{65} & a_{66} \end{matrix}$$

RFP (not transposed)

$$\begin{matrix} a_{44} & a_{54} & a_{64} \\ a_{11} & a_{55} & a_{65} \\ a_{21} & a_{22} & a_{66} \\ a_{31} & a_{32} & a_{33} \\ \mathbf{a_{41}} & \mathbf{a_{42}} & \mathbf{a_{43}} \\ \mathbf{a_{51}} & \mathbf{a_{52}} & \mathbf{a_{53}} \\ \mathbf{a_{61}} & \mathbf{a_{62}} & \mathbf{a_{63}} \end{matrix}$$

RFP (transposed)

$$\begin{matrix} a_{44} & a_{11} & a_{21} & a_{31} & \mathbf{a_{41}} & \mathbf{a_{51}} & \mathbf{a_{61}} \\ a_{54} & a_{55} & a_{22} & a_{32} & \mathbf{a_{42}} & \mathbf{a_{52}} & \mathbf{a_{62}} \\ a_{64} & a_{65} & a_{66} & a_{33} & \mathbf{a_{43}} & \mathbf{a_{53}} & \mathbf{a_{63}} \end{matrix}$$

n is odd, A is upper triangular

Full format

$$\begin{matrix} a_{11} & a_{12} & a_{13} & \mathbf{a_{14}} & \mathbf{a_{15}} & \mathbf{a_{16}} & \mathbf{a_{17}} \\ x & a_{22} & a_{23} & \mathbf{a_{24}} & \mathbf{a_{25}} & \mathbf{a_{26}} & \mathbf{a_{27}} \\ x & x & a_{33} & \mathbf{a_{34}} & \mathbf{a_{35}} & \mathbf{a_{36}} & \mathbf{a_{37}} \\ x & x & x & a_{44} & a_{45} & a_{46} & a_{47} \\ x & x & x & x & a_{55} & a_{56} & a_{57} \\ x & x & x & x & x & a_{66} & a_{67} \\ x & x & x & x & x & x & a_{77} \end{matrix}$$

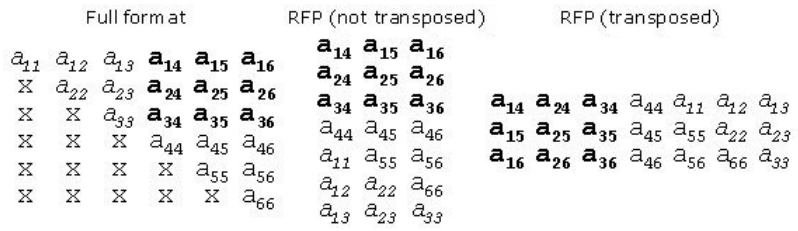
RFP (not transposed)

$$\begin{matrix} \mathbf{a_{14}} & \mathbf{a_{15}} & \mathbf{a_{16}} & \mathbf{a_{17}} \\ \mathbf{a_{24}} & \mathbf{a_{25}} & \mathbf{a_{26}} & \mathbf{a_{27}} \\ \mathbf{a_{34}} & \mathbf{a_{35}} & \mathbf{a_{36}} & \mathbf{a_{37}} \\ a_{44} & a_{45} & a_{46} & a_{47} \\ a_{11} & a_{55} & a_{56} & a_{57} \\ a_{12} & a_{22} & a_{66} & a_{67} \\ a_{13} & a_{23} & a_{33} & a_{77} \end{matrix}$$

RFP (transposed)

$$\begin{matrix} \mathbf{a_{14}} & \mathbf{a_{24}} & \mathbf{a_{34}} & a_{44} & a_{11} & a_{12} & a_{13} \\ \mathbf{a_{15}} & \mathbf{a_{25}} & \mathbf{a_{35}} & a_{45} & a_{55} & a_{22} & a_{23} \\ \mathbf{a_{16}} & \mathbf{a_{26}} & \mathbf{a_{36}} & a_{46} & a_{56} & a_{66} & a_{33} \\ \mathbf{a_{17}} & \mathbf{a_{27}} & \mathbf{a_{37}} & a_{47} & a_{57} & a_{67} & a_{77} \end{matrix}$$

n is even, A is upper triangular



Intel MKL provides a number of routines such as `?hfrk`, `?sfrk` performing BLAS operations working directly on RFP matrices, as well as some conversion routines, for instance, `?tpttf` goes from the standard packed format to RFP and `?trttf` goes from the full format to RFP.

Please refer to the Netlib site for more information.

Note that in the descriptions of LAPACK routines, arrays with RFP matrices have names ending in `fp`.

Code Examples

C

This appendix presents code examples of using some Intel MKL routines and functions. You can find here example code written in both Fortran and C.

Please refer to respective chapters in the manual for detailed descriptions of function parameters and operation.

BLAS Code Examples

Example. Using BLAS Level 1 Function

The following example illustrates a call to the BLAS Level 1 function `sdot`. This function performs a vector-vector operation of computing a scalar product of two single-precision real vectors *x* and *y*.

Parameters

n	Specifies the number of elements in vectors *x* and *y*.
incx	Specifies the increment for the elements of *x*.
incy	Specifies the increment for the elements of *y*.

```
program dot_main
real x(10), y(10), sdot, res
integer n, incx, incy, i
external sdot
n = 5
incx = 2
incy = 1
do i = 1, 10
  x(i) = 2.0e0
  y(i) = 1.0e0
end do
res = sdot (n, x, incx, y, incy)
print*, `SDOT = `, res
end
```

As a result of this program execution, the following line is printed:

SDOT = 10.000

Example. Using BLAS Level 1 Routine

The following example illustrates a call to the BLAS Level 1 routine `scopy`. This routine performs a vector-vector operation of copying a single-precision real vector *x* to a vector *y*.

Parameters

n	Specifies the number of elements in vectors *x* and *y*.
incx	Specifies the increment for the elements of *x*.
incy	Specifies the increment for the elements of *y*.

```
program copy_main
real x(10), y(10)
integer n, incx, incy, i
n = 3
```

```
incx = 3
incy = 1
do i = 1, 10
  x(i) = i
end do
call scopy (n, x, incx, y, incy)
print*, `Y = `, (y(i), i = 1, n)
end
```

As a result of this program execution, the following line is printed:

Y = 1.00000 4.00000 7.00000

Example. Using BLAS Level 2 Routine

The following example illustrates a call to the BLAS Level 2 routine `sger`. This routine performs a matrix-vector operation

```
a :=  alpha*x*y' + a.
```

Parameters

alpha	Specifies a scalar *alpha*.
x	*m*-element vector.
y	*n*-element vector.
a	*m*-by-*n* matrix.

```
program ger_main
real a(5,3), x(10), y(10), alpha
integer m, n, incx, incy, i, j, lda
m = 2
 n = 3
lda = 5
incx = 2
incy = 1
alpha = 0.5
do i = 1, 10
  x(i) = 1.0
  y(i) = 1.0
end do
do i = 1, m
  do j = 1, n
    a(i,j) = j
  end do
end do
call sger (m, n, alpha, x, incx, y, incy, a, lda)
print*, `Matrix A: `
do i = 1, m
  print*, (a(i,j), j = 1, n)
end do
end
```

As a result of this program execution, matrix *a* is printed as follows:

Matrix A:

1.50000 2.50000 3.50000

1.50000 2.50000 3.50000

Example. Using BLAS Level 3 Routine

The following example illustrates a call to the BLAS Level 3 routine `ssymm`. This routine performs a matrix-matrix operation

```
c :=  alpha*a*b' + beta*c.
```

Parameters

alpha	Specifies a scalar *alpha*.
beta	Specifies a scalar *beta*.
a	Symmetric matrix
b	*m*-by-*n* matrix
c	*m*-by-*n* matrix

```
program symm_main
real a(3,3), b(3,2), c(3,3), alpha, beta
integer m, n, lda, ldb, ldc, i, j
character uplo, side
uplo = 'u'
side = 'l'
m = 3
n = 2
lda = 3
ldb = 3
ldc = 3
alpha = 0.5
beta = 2.0
do i = 1, m
  do j = 1, m
    a(i,j) = 1.0
  end do
end do
do i = 1, m
  do j = 1, n
    c(i,j) = 1.0
    b(i,j) = 2.0
  end do
end do
call ssymm (side, uplo, m, n, alpha,
a, lda, b, ldb, beta, c, ldc)
print*, `Matrix C: `
do i = 1, m
  print*, (c(i,j), j = 1, n)
end do
end
```

As a result of this program execution, matrix *c* is printed as follows:

Matrix C:

5.00000 5.00000

5.00000 5.00000

5.00000 5.00000

The following example illustrates a call from a C program to the complex BLAS Level 1 function `zdotc()`. This function computes the dot product of two double-precision complex vectors.

Example. Calling a Complex BLAS Level 1 Function from C

In this example, the complex dot product is returned in the structure *c*.

```
#include <cstdio>
#include "mkl_blas.h"
#define N 5
void main()
{
  int n, inca = 1, incb = 1, i;
  MKL_Complex16 a[N], b[N], c;
  void zdotc();
  n = N;
  for( i = 0; i < n; i++ ){
    a[i].real = (double)i; a[i].imag = (double)i * 2.0;
    b[i].real = (double)(n - i); b[i].imag = (double)i * 2.0;
  }
```

```
  zdotc( &c, &n, a, &inca, b, &incb );
  printf( "The complex dot product is: ( %6.2f, %6.2f )\n", c.real, c.imag );
}
```

NOTE Instead of calling BLAS directly from C programs, you might wish to use the CBLAS interface; this is the supported way of calling BLAS from C. For more information about CBLAS, see Appendix D , which presents CBLAS, the C interface to the Basic Linear Algebra Subprograms (BLAS) implemented in Intel® MKL.

Fourier Transform Functions Code Examples

This section presents code examples of functions described in the "FFT Functions" and "Cluster FFT Functions" sections in the "Fourier Transform Functions" chapter. The examples are grouped in subsections

- Examples for FFT Functions, including Examples of Using Multi-Threading for FFT Computation
- Examples for Cluster FFT Functions
- Auxiliary data transformations.

FFT Code Examples

This section presents code examples of using the FFT interface functions described in "Fourier Transform Functions" chapter. Here are the examples of two one-dimensional computations. These examples use the default settings for all of the configuration parameters, which are specified in "Configuration Settings".

One-dimensional In-place FFT (Fortran Interface)

```
! Fortran example.
! 1D complex to complex, and real to conjugate-even
Use MKL_DFTI
Complex :: X(32)
Real :: Y(34)
type(DFTI_DESCRIPTOR), POINTER :: My_Desc1_Handle, My_Desc2_Handle
Integer :: Status
!...put input data into X(1),...,X(32); Y(1),...,Y(32)

! Perform a complex to complex transform
Status = DftiCreateDescriptor( My_Desc1_Handle, DFTI_SINGLE,&
   DFTI_COMPLEX, 1, 32 )
Status = DftiCommitDescriptor( My_Desc1_Handle )
Status = DftiComputeForward( My_Desc1_Handle, X )
Status = DftiFreeDescriptor(My_Desc1_Handle)
! result is given by {X(1),X(2),...,X(32)}

! Perform a real to complex conjugate-even transform
Status = DftiCreateDescriptor(My_Desc2_Handle, DFTI_SINGLE,&
   DFTI_REAL, 1, 32)
Status = DftiCommitDescriptor(My_Desc2_Handle)
Status = DftiComputeForward(My_Desc2_Handle, Y)
Status = DftiFreeDescriptor(My_Desc2_Handle)
! result is given in CCS format.
```

One-dimensional Out-of-place FFT (Fortran Interface)

```
! Fortran example.
! 1D complex to complex, and real to conjugate-even
Use MKL_DFTI
Complex :: X_in(32)
Complex :: X_out(32)
Real :: Y_in(32)
Real :: Y_out(34)
type(DFTI_DESCRIPTOR), POINTER :: My_Desc1_Handle, My_Desc2_Handle
```

```
Integer :: Status
...put input data into X_in(1),...,X_in(32); Y_in(1),...,Y_in(32)
! Perform a complex to complex transform
Status = DftiCreateDescriptor( My_Desc1_Handle, DFTI_SINGLE,
DFTI_COMPLEX, 1, 32 )
Status = DftiSetValue( My_Desc1_Handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE)
Status = DftiCommitDescriptor( My_Desc1_Handle )
Status = DftiComputeForward( My_Desc1_Handle, X_in, X_out )
Status = DftiFreeDescriptor(My_Desc1_Handle)
! result is given by {X_out(1),X_out(2),...,X_out(32)}
! Perform a real to complex conjugate-even transform
Status = DftiCreateDescriptor(My_Desc2_Handle, DFTI_SINGLE,
DFTI_REAL, 1, 32)
Status = DftiSetValue( My_Desc2_Handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE)
Status = DftiCommitDescriptor(My_Desc2_Handle)
Status = DftiComputeForward(My_Desc2_Handle, Y_in, Y_out)
Status = DftiFreeDescriptor(My_Desc2_Handle)
! result is given by Y_out in CCS format.
```

One-dimensional In-place FFT (C Interface)

```
/* C example, float _Complex is defined in C9X */
#include "mkl_dfti.h"
float _Complex x[32];
float y[34];
DFTI_DESCRIPTOR_HANDLE my_desc1_handle;
DFTI_DESCRIPTOR_HANDLE my_desc2_handle;
MKL_LONG status;
//...put input data into x[0],...,x[31]; y[0],...,y[31]
status = DftiCreateDescriptor( &my_desc1_handle, DFTI_SINGLE,
          DFTI_COMPLEX, 1, 32);
status = DftiCommitDescriptor( my_desc1_handle );
status = DftiComputeForward( my_desc1_handle, x);
status = DftiFreeDescriptor(&my_desc1_handle);
/* result is x[0], ..., x[31]*/
status = DftiCreateDescriptor( &my_desc2_handle, DFTI_SINGLE,
          DFTI_REAL, 1, 32);
status = DftiCommitDescriptor( my_desc2_handle);
status = DftiComputeForward( my_desc2_handle, y);
status = DftiFreeDescriptor(&my_desc2_handle);
/* result is given in CCS format*/
```

One-dimensional Out-of-place FFT (C Interface)

```
/* C example, float _Complex is defined in C9X */
#include "mkl_dfti.h"
float _Complex x_in[32];
float _Complex x_out[32];
float y_in[32];
float y_out[34];

DFTI_DESCRIPTOR_HANDLE my_desc1_handle;
DFTI_DESCRIPTOR_HANDLE my_desc2_handle;
MKL_LONG status;
//...put input data into x_in[0],...,x_in[31]; y_in[0],...,y_in[31]
status = DftiCreateDescriptor( &my_desc1_handle, DFTI_SINGLE,
          DFTI_COMPLEX, 1, 32);
status = DftiSetValue( my_desc1_handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
status = DftiCommitDescriptor( my_desc1_handle );
status = DftiComputeForward( my_desc1_handle, x_in, x_out);
status = DftiFreeDescriptor(&my_desc1_handle);
/* result is x_out[0], ..., x_out[31]*/
status = DftiCreateDescriptor( &my_desc2_handle, DFTI_SINGLE,
          DFTI_REAL, 1, 32);
Status = DftiSetValue( My_Desc2_Handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
status = DftiCommitDescriptor( my_desc2_handle);
```

```
status = DftiComputeForward( my_desc2_handle, y_in, y_out);
status = DftiFreeDescriptor(&my_desc2_handle);
/* result is given by y_out in CCS format*/
```

Two-dimensional FFT (Fortran Interface)

The following is an example of two simple two-dimensional transforms. Notice that the data and result parameters in computation functions are all declared as assumed-size rank-1 array `DIMENSION(0:*)`. Therefore two-dimensional array must be transformed to one-dimensional array by `EQUIVALENCE` statement or other facilities of Fortran.

```
! Fortran example.
! 2D complex to complex, and real to conjugate-even
Use MKL_DFTI
Complex ::  X_2D(32,100)
Real :: Y_2D(34, 102)
Complex ::  X(3200)
Real :: Y(3468)
Equivalence (X_2D, X)
Equivalence (Y_2D, Y)
type(DFTI_DESCRIPTOR), POINTER :: My_Desc1_Handle, My_Desc2_Handle
Integer :: Status, L(2)
!...put input data into X_2D(j,k), Y_2D(j,k), 1<=j=32,1<=k<=100
!...set L(1) = 32, L(2) = 100
!...the transform is a 32-by-100

! Perform a complex to complex transform
Status = DftiCreateDescriptor( My_Desc1_Handle, DFTI_SINGLE,&
          DFTI_COMPLEX, 2, L)
Status = DftiCommitDescriptor( My_Desc1_Handle)
Status = DftiComputeForward( My_Desc1_Handle, X)
Status = DftiFreeDescriptor(My_Desc1_Handle)
! result is given by X_2D(j,k), 1<=j<=32, 1<=k<=100

! Perform a real to complex conjugate-even transform
Status = DftiCreateDescriptor( My_Desc2_Handle, DFTI_SINGLE,&
          DFTI_REAL, 2, L)
Status = DftiCommitDescriptor( My_Desc2_Handle)
Status = DftiComputeForward( My_Desc2_Handle, Y)
Status = DftiFreeDescriptor(My_Desc2_Handle)
! result is given by the complex value z(j,k) 1<=j<=32; 1<=k<=100
! and is stored in CCS format
```

Two-dimensional FFT (C Interface)

```
/* C99 example */
#include "mkl_dfti.h"
float _Complex x[32][100];
float y[34][102];
DFTI_DESCRIPTOR_HANDLE my_desc1_handle;
DFTI_DESCRIPTOR_HANDLE my_desc2_handle;
MKL_LONG status, l[2];
//...put input data into x[j][k] 0<=j<=31, 0<=k<=99
//...put input data into y[j][k] 0<=j<=31, 0<=k<=99
l[0] = 32; l[1] = 100;
status = DftiCreateDescriptor( &my_desc1_handle, DFTI_SINGLE,
          DFTI_COMPLEX, 2, l);
status = DftiCommitDescriptor( my_desc1_handle);
status = DftiComputeForward( my_desc1_handle, x);
status = DftiFreeDescriptor(&my_desc1_handle);
/* result is the complex value x[j][k], 0<=j<=31, 0<=k<=99 */
status = DftiCreateDescriptor( &my_desc2_handle, DFTI_SINGLE,
          DFTI_REAL, 2, l);
status = DftiCommitDescriptor( my_desc2_handle);
status = DftiComputeForward( my_desc2_handle, y);
```

```
status = DftiFreeDescriptor(&my_desc2_handle);
/* result is the complex value z(j,k) 0<=j<=31; 0<=k<=99
/* and is stored in CCS format*/
```

The following examples demonstrate how you can change the default configuration settings by using the `DftiSetValue` function.

For instance, to preserve the input data after the FFT computation, the configuration of the `DFTI_PLACEMENT` should be changed to "not in place" from the default choice of "in place."

Changing Default Settings (Fortran)

The code below illustrates how this can be done:

```
! Fortran example
! 1D complex to complex, not in place
Use MKL_DFTI
Complex :: X_in(32), X_out(32)
type(DFTI_DESCRIPTOR), POINTER :: My_Desc_Handle
Integer :: Status
!...put input data into X_in(j), 1<=j<=32
Status = DftiCreateDescriptor( My_Desc_Handle,& DFTI_SINGLE, DFTI_COMPLEX, 1, 32)
Status = DftiSetValue( My_Desc_Handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE)
Status = DftiCommitDescriptor( My_Desc_Handle)
Status = DftiComputeForward( My_Desc_Handle, X_in, X_out)
Status = DftiFreeDescriptor (My_Desc_Handle)
! result is X_out(1),X_out(2),...,X_out(32)
```

Changing Default Settings (C)

```
/* C99 example */
#include "mkl_dfti.h"
float _Complex x_in[32], x_out[32];
DFTI_DESCRIPTOR_HANDLE my_desc_handle;
MKL_LONG status;
//...put input data into x_in[j], 0 <= j < 32
status = DftiCreateDescriptor( &my_desc_handle, DFTI_SINGLE,
   DFTI_COMPLEX, 1, 32);
status = DftiSetValue( my_desc_handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
status = DftiCommitDescriptor( my_desc_handle);
status = DftiComputeForward( my_desc_handle, x_in, x_out);
status = DftiFreeDescriptor(&my_desc_handle);
/* result is x_out[0], x_out[1], ..., x_out[31] */
```

Using Status Checking Functions

The example illustrates the use of status checking functions described in Chapter 11.

```
/* C */
DFTI_DESCRIPTOR_HANDLE desc;
MKL_LONG status;
// . . . descriptor creation and other code
status = DftiCommitDescriptor(desc);
if (status && !DftiErrorClass(status,DFTI_NO_ERROR))
{
   printf ('Error: %s\n', DftiErrorMessage(status));
}

! Fortran
type(DFTI_DESCRIPTOR), POINTER :: desc
integer status
! ...descriptor creation and other code
status =  DftiCommitDescriptor(desc)
```

```
if (status .ne. 0) then
  if (.not. DftiErrorClass(status,DFTI_NO_ERROR) then
     print *, 'Error: ', DftiErrorMessage(status)
  endif
endif
```

Computing 2D FFT by One-Dimensional Transforms

Below is an example where a 20-by-40 two-dimensional FFT is computed explicitly using one-dimensional transforms. Notice that the data and result parameters in computation functions are all declared as assumed-size rank-1 array `DIMENSION(0:*)`. Therefore two-dimensional array must be transformed to one-dimensional array by `EQUIVALENCE` statement or other facilities of Fortran.

```
! Fortran
use mkl_dfti
Complex ::  X_2D(20,40)
Complex ::  X(800)
Equivalence (X_2D, X)
INTEGER :: STRIDE(2)
type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle_Dim1
type(DFTI_DESCRIPTOR), POINTER :: Desc_Handle_Dim2
! ...
Status = DftiCreateDescriptor(Desc_Handle_Dim1, DFTI_SINGLE,&
                              DFTI_COMPLEX, 1, 20 )
Status = DftiCreateDescriptor(Desc_Handle_Dim2, DFTI_SINGLE,&
                              DFTI_COMPLEX, 1, 40 )
! perform 40 one-dimensional transforms along 1st dimension
Status = DftiSetValue( Desc_Handle_Dim1, DFTI_NUMBER_OF_TRANSFORMS, 40 )
Status = DftiSetValue( Desc_Handle_Dim1, DFTI_INPUT_DISTANCE, 20 )
Status = DftiSetValue( Desc_Handle_Dim1, DFTI_OUTPUT_DISTANCE, 20 )
Status = DftiCommitDescriptor( Desc_Handle_Dim1 )
Status = DftiComputeForward( Desc_Handle_Dim1, X )
! perform 20 one-dimensional transforms along 2nd dimension
Stride(1) = 0; Stride(2) = 20
Status = DftiSetValue( Desc_Handle_Dim2, DFTI_NUMBER_OF_TRANSFORMS, 20 )
Status = DftiSetValue( Desc_Handle_Dim2, DFTI_INPUT_DISTANCE, 1 )
Status = DftiSetValue( Desc_Handle_Dim2, DFTI_OUTPUT_DISTANCE, 1 )
Status = DftiSetValue( Desc_Handle_Dim2, DFTI_INPUT_STRIDES, Stride )
Status = DftiSetValue( Desc_Handle_Dim2, DFTI_OUTPUT_STRIDES, Stride )
Status = DftiCommitDescriptor( Desc_Handle_Dim2 )
Status = DftiComputeForward( Desc_Handle_Dim2, X )
Status = DftiFreeDescriptor( Desc_Handle_Dim1 )
Status = DftiFreeDescriptor( Desc_Handle_Dim2 )

/* C */
#include "mkl_dfti.h"
float _Complex x[20][40];
MKL_LONG stride[2];
MKL_LONG status;
DFTI_DESCRIPTOR_HANDLE desc_handle_dim1;
DFTI_DESCRIPTOR_HANDLE desc_handle_dim2;
//...
status = DftiCreateDescriptor( &desc_handle_dim1, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 20 );
status = DftiCreateDescriptor( &desc_handle_dim2, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 40 );

/* perform 40 one-dimensional transforms along 1st dimension */
/* note that the 1st dimension data are not unit-stride */
stride[0] = 0; stride[1] = 40;
status = DftiSetValue( desc_handle_dim1, DFTI_NUMBER_OF_TRANSFORMS, 40 );
status = DftiSetValue( desc_handle_dim1, DFTI_INPUT_DISTANCE, 1 );
status = DftiSetValue( desc_handle_dim1, DFTI_OUTPUT_DISTANCE, 1 );
status = DftiSetValue( desc_handle_dim1, DFTI_INPUT_STRIDES, stride );
status = DftiSetValue( desc_handle_dim1, DFTI_OUTPUT_STRIDES, stride );
status = DftiCommitDescriptor( desc_handle_dim1 );
status = DftiComputeForward( desc_handle_dim1, x );
```

```
/* perform 20 one-dimensional transforms along 2nd dimension */
/* note that the 2nd dimension is unit stride */
status = DftiSetValue( desc_handle_dim2, DFTI_NUMBER_OF_TRANSFORMS, 20 );
status = DftiSetValue( desc_handle_dim2,
DFTI_INPUT_DISTANCE, 40 );
status = DftiSetValue( desc_handle_dim2,
DFTI_OUTPUT_DISTANCE, 40 );
status = DftiCommitDescriptor( desc_handle_dim2 );
status = DftiComputeForward( desc_handle_dim2, x );
status = DftiFreeDescriptor( &desc_handle_dim1 );
status = DftiFreeDescriptor( &desc_handle_dim2 );
```

The following are examples of real multi-dimensional transforms with CCE format storage of conjugate-even complex matrix. Example "Two-Dimensional REAL In-place FFT (Fortran Interface)" is two-dimensional in-place transform and Example "Two-Dimensional REAL Out-of-place FFT (Fortran Interface)" is two-dimensional out-of-place transform in Fortran interface. Example "Three-Dimensional REAL FFT (C Interface)" is three-dimensional out-of-place transform in C interface. Note that the data and result parameters in computation functions are all declared as assumed-size rank-1 array `DIMENSION(0:*)`. Therefore two-dimensional array must be transformed to one-dimensional array by `EQUIVALENCE` statement or other facilities of Fortran.

Two-Dimensional REAL In-place FFT (Fortran Interface)

```
! Fortran example.
! 2D and real to conjugate-even
Use MKL_DFTI
Real :: X_2D(34,100) ! 34  = (32/2 + 1)*2
Real :: X(3400)
Equivalence (X_2D, X)
type(DFTI_DESCRIPTOR), POINTER :: My_Desc_Handle
Integer :: Status, L(2)
Integer :: strides_in(3)
Integer :: strides_out(3)
! ...put input data into X_2D(j,k), 1<=j=32,1<=k<=100
! ...set L(1) = 32, L(2) = 100
! ...set strides_in(1) = 0, strides_in(2) = 1, strides_in(3) = 34
! ...set strides_out(1) = 0, strides_out(2) = 1, strides_out(3) = 17
! ...the transform is a 32-by-100
! Perform a real to complex conjugate-even transform
Status = DftiCreateDescriptor( My_Desc_Handle, DFTI_SINGLE,&
DFTI_REAL, 2, L )
Status = DftiSetValue(My_Desc_Handle, DFTI_CONJUGATE_EVEN_STORAGE,&
DFTI_COMPLEX_COMPLEX)
Status = DftiSetValue(My_Desc_Handle, DFTI_INPUT_STRIDES, strides_in)
Status = DftiSetValue(My_Desc_Handle, DFTI_OUTPUT_STRIDES, strides_out)
Status = DftiCommitDescriptor( My_Desc_Handle)
Status = DftiComputeForward( My_Desc_Handle, X )
Status = DftiFreeDescriptor(My_Desc_Handle)
! result is given by the complex value z(j,k) 1<=j<=17; 1<=k<=100 and
! is stored in real matrix X_2D in CCE format.
```

Two-Dimensional REAL Out-of-place FFT (Fortran Interface)

```
! Fortran example.
! 2D and real to conjugate-even
Use MKL_DFTI
Real :: X_2D(32,100)
Complex :: Y_2D(17, 100) ! 17 = 32/2 + 1
Real :: X(3200)
Complex :: Y(1700)
Equivalence (X_2D, X)
Equivalence (Y_2D, Y)
type(DFTI_DESCRIPTOR), POINTER :: My_Desc_Handle
Integer :: Status, L(2)
Integer :: strides_out(3)
```

```
! ...put input data into X_2D(j,k), 1<=j=32,1<=k<=100
! ...set L(1) = 32, L(2) = 100
! ...set strides_out(1) = 0, strides_out(2) = 1, strides_out(3) = 17

! ...the transform is a 32-by-100
! Perform a real to complex conjugate-even transform
Status = DftiCreateDescriptor( My_Desc_Handle, DFTI_SINGLE,&
DFTI_REAL, 2, L )
Status = DftiSetValue(My_Desc_Handle,&
DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX)
Status = DftiSetValue( My_Desc_Handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE )
Status = DftiSetValue(My_Desc_Handle,&
DFTI_OUTPUT_STRIDES, strides_out)

Status = DftiCommitDescriptor(My_Desc_Handle)
Status = DftiComputeForward(My_Desc_Handle, X, Y)
Status = DftiFreeDescriptor(My_Desc_Handle)
! result is given by the complex value z(j,k) 1<=j<=17; 1<=k<=100 and
! is stored in complex matrix Y_2D in CCE format.
```

Three-Dimensional REAL FFT (C Interface)

```
/* C99 example */
#include "mkl_dfti.h"
float x[32][100][19];
float _Complex y[32][100][10]; /* 10 = 19/2 + 1 */
DFTI_DESCRIPTOR_HANDLE my_desc_handle;
MKL_LONG status, l[3];
MKL_LONG strides_out[4];

//...put input data into x[j][k][s] 0<=j<=31, 0<=k<=99, 0<=s<=18
l[0] = 32; l[1] = 100; l[2] = 19;

strides_out[0] = 0; strides_out[1] = 1000;
strides_out[2] = 10; strides_out[3] = 1;

status = DftiCreateDescriptor( &my_desc_handle, DFTI_SINGLE,
DFTI_REAL, 3, l );
status = DftiSetValue(my_desc_handle,
DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX);
status = DftiSetValue( my_desc_handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE );
status = DftiSetValue(my_desc_handle,
DFTI_OUTPUT_STRIDES, strides_out);

status = DftiCommitDescriptor(my_desc_handle);
status = DftiComputeForward(my_desc_handle, x, y);
status = DftiFreeDescriptor(&my_desc_handle);
/* result is the complex value z(j,k,s) 0<=j<=31; 0<=k<=99, 0<=s<=9
and is stored in complex matrix y in CCE format. */
```

Examples of Using Multi-Threading for FFT Computation

The following sample program shows how to employ internal threading in Intel MKL for FFT computation (see case "a" in “Number of user threads”).

To specify the number of threads inside Intel MKL, use the following settings:

set `MKL_NUM_THREADS = 1` for one-threaded mode;

set `MKL_NUM_THREADS = 4` for multi-threaded mode.

Note that the configuration parameter `DFTI_NUMBER_OF_USER_THREADS` must be equal to its default value 1.

Using Intel MKL Internal Threading Mode

```
#include "mkl_dfti.h"

int main ()

{
    float x[200][100];
    DFTI_DESCRIPTOR_HANDLE fft;
    MKL_LONG len[2] = {200, 100};
    // initialize x
    DftiCreateDescriptor ( &fft, DFTI_SINGLE, DFTI_REAL, 2, len );
    DftiCommitDescriptor ( fft );
    DftiComputeForward ( fft, x );
    DftiFreeDescriptor ( &fft );
    return 0;
}
```

The following Example "Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region" and Example "Using Parallel Mode with Multiple Descriptors Initialized in One Thread" illustrate a parallel customer program with each descriptor instance used only in a single thread (see cases "b" and "c" in Number of user threads).

Specify the number of threads for Example "Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region" like this:

set `MKL_NUM_THREADS = 1` for Intel MKL to work in the single-threaded mode (recommended);

set `OMP_NUM_THREADS = 4` for the customer program to work in the multi-threaded mode.

The configuration parameter `DFTI_NUMBER_OF_USER_THREADS` must have its default value of 1.

Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region

Note that in this example, the program can be transformed to become single-threaded at the customer level but using parallel mode within Intel MKL (case "a"). To achieve this, you need to set the parameter `DFTI_NUMBER_OF_TRANSFORMS = 4` and to set the corresponding parameter `DFTI_INPUT_DISTANCE = 5000`.

C code for the example is as follows:

```
#include "mkl_dfti.h"
#include <omp.h>
#define ARRAY_LEN(a) sizeof(a)/sizeof(a[0])
int main ()
{
    // 4 OMP threads, each does 2D FFT 50x100 points
    MKL_Complex8 x[4][50][100];
    int nth = ARRAY_LEN(x);
    MKL_LONG len[2] = {ARRAY_LEN(x[0]), ARRAY_LEN(x[0][0])};
    int th;
    // assume x is initialized and do 2D FFTs
#pragma omp parallel for shared(len, x)
    for (th = 0; th < nth; th++)
    {
        DFTI_DESCRIPTOR_HANDLE myFFT;

        DftiCreateDescriptor (&myFFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len);
        DftiCommitDescriptor (myFFT);
        DftiComputeForward (myFFT, x[th]);
        DftiFreeDescriptor (&myFFT);
    }
    return 0;
}
```

Fortran code for the example is as follows:

```
program fft2d_private_descr_main
  use mkl_dfti
```

```
  integer nth, len(2)
! 4 OMP threads, each does 2D FFT 50x100 points
  parameter (nth = 4, len = (/50, 100/))
  complex x(len(2)*len(1), nth)

  type(dfti_descriptor), pointer :: myFFT
  integer th, myStatus

! assume x is initialized and do 2D FFTs
!$OMP PARALLEL DO SHARED(len, x) PRIVATE(myFFT, myStatus)
  do th = 1, nth
    myStatus = DftiCreateDescriptor (myFFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len)
    myStatus = DftiCommitDescriptor (myFFT)
    myStatus = DftiComputeForward (myFFT, x(:, th))
    myStatus = DftiFreeDescriptor (myFFT)
  end do
!$OMP END PARALLEL DO
end
```

Specify the number of threads for Example "Using Parallel Mode with Multiple Descriptors Initialized in One Thread" like this:

set `MKL_NUM_THREADS` = 1 for Intel MKL to work in the single-threaded mode (obligatory);

set `OMP_NUM_THREADS` = 4 for the customer program to work in the multi-threaded mode.

The configuration parameter `DFTI_NUMBER_OF_USER_THREADS` must have the default value of 1.

Using Parallel Mode with Multiple Descriptors Initialized in One Thread

C code for the example is as follows:

```
#include "mkl_dfti.h"
#include <omp.h>
#define ARRAY_LEN(a) sizeof(a)/sizeof(a[0])
int main ()
{
    // 4 OMP threads, each does 2D FFT 50x100 points
    MKL_Complex8 x[4][50][100];
    int nth = ARRAY_LEN(x);
    MKL_LONG len[2] = {ARRAY_LEN(x[0]), ARRAY_LEN(x[0][0])};
    DFTI_DESCRIPTOR_HANDLE FFT[ARRAY_LEN(x)];
    int th;

    for (th = 0; th < nth; th++)
        DftiCreateDescriptor (&FFT[th], DFTI_SINGLE, DFTI_COMPLEX, 2, len);
    for (th = 0; th < nth; th++)
        DftiCommitDescriptor (FFT[th]);
    // assume x is initialized and do 2D FFTs
#pragma omp parallel for shared(FFT, x)
    for (th = 0; th < nth; th++)
        DftiComputeForward (FFT[th], x[th]);
    for (th = 0; th < nth; th++)
        DftiFreeDescriptor (&FFT[th]);
    return 0;
}
```

Fortran code for the example is as follows:

```
program fft2d_array_descr_main
  use mkl_dfti

  integer nth, len(2)
! 4 OMP threads, each does 2D FFT 50x100 points
  parameter (nth = 4, len = (/50, 100/))
  complex x(len(2)*len(1), nth)

  type thread_data
    type(dfti_descriptor), pointer :: FFT
  end type thread_data
  type(thread_data) :: workload(nth)
```

```
  integer th, status, myStatus

  do th = 1, nth
    status = DftiCreateDescriptor (workload(th)%FFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len)
    status = DftiCommitDescriptor (workload(th)%FFT)
  end do
! assume x is initialized and do 2D FFTs
!$OMP PARALLEL DO SHARED(len, x, workload) PRIVATE(myStatus)
  do th = 1, nth
    myStatus = DftiComputeForward (workload(th)%FFT, x(:, th))
  end do
!$OMP END PARALLEL DO
  do th = 1, nth
    status = DftiFreeDescriptor (workload(th)%FFT)
  end do
end
```

The following Example "Using Parallel Mode with a Common Descriptor" illustrates a parallel customer program with a common descriptor used in several threads (see case "d" in "Number of user threads").

In this case, the number of threads, as well as any other configuration parameter, must not be changed after FFT initialization by the `DftiCommitDescriptor()` function is done.

Using Parallel Mode with a Common Descriptor

C code for the example is as follows:

```
#include "mkl_dfti.h"
#include <omp.h>
#define ARRAY_LEN(a) sizeof(a)/sizeof(a[0])
int main ()
{
    // 4 OMP threads, each does 2D FFT 50x100 points
    MKL_Complex8 x[4][50][100];
    int nth = ARRAY_LEN(x);
    MKL_LONG len[2] = {ARRAY_LEN(x[0]), ARRAY_LEN(x[0][0])};
    DFTI_DESCRIPTOR_HANDLE FFT;
    int th;

    DftiCreateDescriptor (&FFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len);
    DftiSetValue (FFT, DFTI_NUMBER_OF_USER_THREADS, nth);
    DftiCommitDescriptor (FFT);
    // assume x is initialized and do 2D FFTs
#pragma omp parallel for shared(FFT, x)
    for (th = 0; th < nth; th++)
        DftiComputeForward (FFT, x[th]);
    DftiFreeDescriptor (&FFT);
    return 0;
}
```

Fortran code for the example is as follows:

```
 program fft2d_shared_descr_main
  use mkl_dfti

  integer nth, len(2)
! 4 OMP threads, each does 2D FFT 50x100 points
  parameter (nth = 4, len = (/50, 100/))
  complex x(len(2)*len(1), nth)
  type(dfti_descriptor), pointer :: FFT

  integer th, status, myStatus

  status = DftiCreateDescriptor (FFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len)
  status = DftiSetValue (FFT, DFTI_NUMBER_OF_USER_THREADS, nth)
  status = DftiCommitDescriptor (FFT)
! assume x is initialized and do 2D FFTs
!$OMP PARALLEL DO SHARED(len, x, FFT) PRIVATE(myStatus)
  do th = 1, nth
    myStatus = DftiComputeForward (FFT, x(:, th))
  end do
```

```
!$OMP END PARALLEL DO
  status = DftiFreeDescriptor (FFT)
end
```

Examples for Cluster FFT Functions

The following C example computes a 2-dimensional out-of-place FFT using the cluster FFT interface:

2D Out-of-place Cluster FFT Computation

```
DFTI_DESCRIPTOR_DM_HANDLE desc;
MKL_LONG len[2],v,i,j,n,s;
Complex *in,*out;

MPI_Init(...);

// Create descriptor for 2D FFT
len[0]=nx;
len[1]=ny;
DftiCreateDescriptorDM(MPI_COMM_WORLD,&desc,DFTI_DOUBLE,DFTI_COMPLEX,2,len);
// Ask necessary length of in and out arrays and allocate memory
DftiGetValueDM(desc,CDFT_LOCAL_SIZE,&v);
in=(Complex*)malloc(v*sizeof(Complex));
out=(Complex*)malloc(v*sizeof(Complex));
// Fill local array with initial data. Current process performs n rows,
// 0 row of in corresponds to s row of virtual global array
DftiGetValueDM(desc,CDFT_LOCAL_NX,&n);
DftiGetValueDM(desc,CDFT_LOCAL_X_START,&s);
// Virtual global array globalIN is defined by function f as
// globalIN[i*ny+j]=f(i,j)
for(i=0;i<n;i++)
   for(j=0;j<ny;j++) in[i*ny+j]=f(i+s,j);
// Set that we want out-of-place transform (default is DFTI_INPLACE)
DftiSetValueDM(desc,DFTI_PLACEMENT,DFTI_NOT_INPLACE);
// Commit descriptor, calculate FFT, free descriptor
DftiCommitDescriptorDM(desc);
DftiComputeForwardDM(desc,in,out);
// Virtual global array globalOUT is defined by function g as
// globalOUT[i*ny+j]=g(i,j)
// Now out contains result of FFT. out[i*ny+j]=g(i+s,j)
DftiFreeDescriptorDM(&desc);
free(in);
free(out);
MPI_Finalize();
```

1D In-place Cluster FFT Computations

The C example below illustrates one-dimensional in-place cluster FFT computations effected with a user-defined workspace:

```
DFTI_DESCRIPTOR_DM_HANDLE desc;
MKL_LONG len,v,i,n_out,s_out;
Complex *in,*work;

MPI_Init(...);
// Create descriptor for 1D FFT
DftiCreateDescriptorDM(MPI_COMM_WORLD,&desc,DFTI_DOUBLE,DFTI_COMPLEX,1,len);
// Ask necessary length of array and workspace and allocate memory
DftiGetValueDM(desc,CDFT_LOCAL_SIZE,&v);
in=(Complex*)malloc(v*sizeof(Complex));
work=(Complex*)malloc(v*sizeof(Complex));
// Fill local array with initial data. Local array has n elements,
// 0 element of in corresponds to s element of virtual global array
DftiGetValueDM(desc,CDFT_LOCAL_NX,&n);
DftiGetValueDM(desc,CDFT_LOCAL_X_START,&s);
// Set work array as a workspace
DftiSetValueDM(desc,CDFT_WORKSPACE,work);
// Virtual global array globalIN is defined by function f as globalIN[i]=f(i)
for(i=0;i<n;i++) in[i]=f(i+s);
```

```
// Commit descriptor, calculate FFT, free descriptor
DftiCommitDescriptorDM(desc);
DftiComputeForwardDM(desc,in);
DftiGetValueDM(desc,CDFT_LOCAL_OUT_NX,&n_out);
DftiGetValueDM(desc,CDFT_LOCAL_OUT_X_START,&s_out);
// Virtual global array globalOUT is defined by function g as globalOUT[i]=g(i)
// Now in contains result of FFT. Local array has n_out elements,
// 0 element of in corresponds to s_out element of virtual global array.
// in[i]==g(i+s_out)
DftiFreeDescriptorDM(&desc);
free(in);
free(work);
MPI_Finalize();
```

Auxiliary Data Transformations

This section presents code examples for conversion from the Cartesian to polar representation of complex data and vice versa.

Conversion from Cartesian to polar representation of complex data

```
// Cartesian->polar conversion of complex data
// Cartesian representation: z = re + I*im
// Polar representation: z = r * exp( I*phi )
#include <mkl_vml.h>

void
variant1_Cartesian2Polar(int n,const double *re,const double *im,
                         double *r,double *phi)
{
    vdHypot(n,re,im,r);         // compute radii r[]
    vdAtan2(n,im,re,phi);       // compute phases phi[]
}

void
variant2_Cartesian2Polar(int n,const MKL_Complex16 *z,double *r,double *phi,
                         double *temp_re,double *temp_im)
{
    vzAbs(n,z,r);               // compute radii r[]
    vdPackI(n, (double*)z + 0, 2, temp_re);
    vdPackI(n, (double*)z + 1, 2, temp_im);
    vdAtan2(n,temp_im,temp_re,phi); // compute phases phi[]
}
```

Conversion from polar to Cartesian representation of complex data

```
// Polar->Cartesian conversion of complex data.
// Polar representation: z = r * exp( I*phi )
// Cartesian representation: z = re + I*im
#include <mkl_vml.h>

void
variant1_Polar2Cartesian(int n,const double *r,const double *phi,
                         double *re,double *im)
{
    vdSinCos(n,phi,im,re);      // compute direction, i.e. z[]/abs(z[])
    vdMul(n,r,re,re);           // scale real part
    vdMul(n,r,im,im);           // scale imaginary part
}

void
variant2_Polar2Cartesian(int n,const double *r,const double *phi,
                         MKL_Complex16 *z,
                         double *temp_re,double *temp_im)
{
```

```
    vdSinCos(n,phi,temp_im,temp_re); // compute direction, i.e. z[]/abs(z[])
    vdMul(n,r,temp_im,temp_im); // scale imaginary part
    vdMul(n,r,temp_re,temp_re); // scale real part
    vdUnpackI(n,temp_re,(double*)z + 0, 2); // fill in result.re
    vdUnpackI(n,temp_im,(double*)z + 1, 2); // fill in result.im
}
```

CBLAS Interface to the BLAS

D

This appendix presents CBLAS, the C interface to the Basic Linear Algebra Subprograms (BLAS) implemented in Intel® MKL.

Similar to BLAS, the CBLAS interface includes the following levels of functions:

- "Level 1 CBLAS" (vector-vector operations)
- "Level 2 CBLAS" (matrix-vector operations)
- "Level 3 CBLAS" (matrix-matrix operations).
- "Sparse CBLAS" (operations on sparse vectors).

To obtain the C interface, the Fortran routine names are prefixed with `cblas_` (for example, `dasum` becomes `cblas_dasum`). Names of all CBLAS functions are in lowercase letters.

Complex functions `?dotc` and `?dotu` become CBLAS subroutines (void functions); they return the complex result via a void pointer, added as the last parameter. CBLAS names of these functions are suffixed with `_sub`. For example, the BLAS function `cdotc` corresponds to `cblas_cdotc_sub`.

WARNING Users of the CBLAS interface should be aware that the CBLAS are just a C interface to the BLAS, which is based on the FORTRAN standard and subject to the FORTRAN standard restrictions. In particular, the output parameters should not be referenced through more than one argument.

In the descriptions of CBLAS interfaces, links provided for each function group lead to the descriptions of the respective Fortran-interface BLAS functions.

CBLAS Arguments

The arguments of CBLAS functions comply with the following rules:

- Input arguments are declared with the `const` modifier.
- Non-complex scalar input arguments are passed by value.
- Complex scalar input arguments are passed as void pointers.
- Array arguments are passed by address.
- BLAS character arguments are replaced by the appropriate enumerated type.
- Level 2 and Level 3 routines acquire an additional parameter of type `CBLAS_ORDER` as their first argument. This parameter specifies whether two-dimensional arrays are row-major (`CblasRowMajor`) or column-major (`CblasColMajor`).

Enumerated Types

The CBLAS interface uses the following enumerated types:

```
enum CBLAS_ORDER {
   CblasRowMajor=101,  /* row-major arrays */
   CblasColMajor=102};   /* column-major arrays */
enum CBLAS_TRANSPOSE {
   CblasNoTrans=111,     /* trans='N' */
   CblasTrans=112,       /* trans='T' */
   CblasConjTrans=113};  /* trans='C' */
enum CBLAS_UPLO {
   CblasUpper=121,        /* uplo ='U' */
   CblasLower=122};       /* uplo ='L' */
enum CBLAS_DIAG {
   CblasNonUnit=131,      /* diag ='N' */
   CblasUnit=132};        /* diag ='U' */
```

```
enum CBLAS_SIDE {
   CblasLeft=141,         /* side ='L' */
   CblasRight=142};       /* side ='R' */
```

Level 1 CBLAS

This is an interface to "BLAS Level 1 Routines and Functions", which perform basic vector-vector operations.

?asum

```
float cblas_sasum(const int N, const float *X, const int incX);
double cblas_dasum(const int N, const double *X, const int incX);
float cblas_scasum(const int N, const void *X, const int incX);
double cblas_dzasum(const int N, const void *X, const int incX);
```

?axpy

```
void cblas_saxpy(const int N, const float alpha, const float *X, const int incX, float *Y, const int
incY);
void cblas_daxpy(const int N, const double alpha, const double *X, const int incX, double *Y, const int
incY);
void cblas_caxpy(const int N, const void *alpha, const void *X, const int incX, void *Y, const int
incY);
void cblas_zaxpy(const int N, const void *alpha, const void *X, const int incX, void *Y, const int
incY);
```

?copy

```
void cblas_scopy(const int N, const float *X, const int incX, float *Y, const int incY);
void cblas_dcopy(const int N, const double *X, const int incX, double *Y, const int incY);
void cblas_ccopy(const int N, const void *X, const int incX, void *Y, const int incY);
void cblas_zcopy(const int N, const void *X, const int incX, void *Y, const int incY);
```

?dot

```
float cblas_sdot(const int N, const float *X, const int incX, const float *Y, const int incY);
double cblas_ddot(const int N, const double *X, const int incX, const double *Y, const int incY);
```

?sdot

```
float cblas_sdsdot(const int N, const float SB, const float *SX, const int incX, const float *SY, const
int incY);
double cblas_dsdot(const int N, const float *SX, const int incX, const float *SY, const int incY);
```

?dotc

```
void cblas_cdotc_sub(const int N, const void *X, const int incX, const void *Y, const int incY, void
*dotc);
void cblas_zdotc_sub(const int N, const void *X, const int incX, const void *Y, const int incY, void
*dotc);
```

?dotu

```
void cblas_cdotu_sub(const int N, const void *X, const int incX, const void *Y, const int incY, void
*dotu);
void cblas_zdotu_sub(const int N, const void *X, const int incX, const void *Y, const int incY, void
*dotu);
```

?nrm2

```
float cblas_snrm2(const int N, const float *X, const int incX);
double cblas_dnrm2(const int N, const double *X, const int incX);
float cblas_scnrm2(const int N, const void *X, const int incX);
double cblas_dznrm2(const int N, const void *X, const int incX);
```

?rot

```
void cblas_srot(const int N, float *X, const int incX, float *Y, const int incY, const float c, const
float s);
void cblas_drot(const int N, double *X, const int incX, double *Y,const int incY, const double c, const
double s);
```

?rotg

```
void cblas_srotg(float *a, float *b, float *c, float *s);
void cblas_drotg(double *a, double *b, double *c, double *s);
```

?rotm

```
void cblas_srotm(const int N, float *X, const int incX, float *Y, const int incY, const float *P);
void cblas_drotm(const int N, double *X, const int incX, double *Y, const int incY, const double *P);
```

?rotmg

```
void cblas_srotmg(float *d1, float *d2, float *b1, const float b2, float *P);
void cblas_drotmg(double *d1, double *d2, double *b1, const double b2, double *P);
```

?scal

```
void cblas_sscal(const int N, const float alpha, float *X, const int incX);
void cblas_dscal(const int N, const double alpha, double *X, const int incX);
void cblas_cscal(const int N, const void *alpha, void *X, const int incX);
void cblas_zscal(const int N, const void *alpha, void *X, const int incX);
void cblas_csscal(const int N, const float alpha, void *X, const int incX);
void cblas_zdscal(const int N, const double alpha, void *X, const int incX);
```

?swap

```
void cblas_sswap(const int N, float *X, const int incX, float *Y, const int incY);
void cblas_dswap(const int N, double *X, const int incX, double *Y, const int incY);
void cblas_cswap(const int N, void *X, const int incX, void *Y, const int incY);
void cblas_zswap(const int N, void *X, const int incX, void *Y, const int incY);
```

i?amax

```
CBLAS_INDEX cblas_isamax(const int N, const float *X, const int incX);
CBLAS_INDEX cblas_idamax(const int N, const double *X, const int incX);
CBLAS_INDEX cblas_icamax(const int N, const void *X, const int incX);
CBLAS_INDEX cblas_izamax(const int N, const void *X, const int incX);
```

i?amin

```
CBLAS_INDEX cblas_isamin(const int N, const float *X, const int incX);
CBLAS_INDEX cblas_idamin(const int N, const double *X, const int incX);
CBLAS_INDEX cblas_icamin(const int N, const void *X, const int incX);
CBLAS_INDEX cblas_izamin(const int N, const void *X, const int incX);
```

?cabs1

```
double cblas_dcabs1(const void *z);
float  cblas_scabs1(const void *c);
```

Level 2 CBLAS

This is an interface to “BLAS Level 2 Routines”, which perform basic matrix-vector operations. Each C routine in this group has an additional parameter of type CBLAS_ORDER (the first argument) that determines whether the two-dimensional arrays use column-major or row-major storage.

?gbmv

```
void cblas_sgbmv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const int KL, const int KU, const float alpha, const float *A, const int lda, const float *X,
const int incX, const float beta, float *Y, const int incY);
void cblas_dgbmv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const int KL, const int KU, const double alpha, const double *A, const int lda, const double *X,
const int incX, const double beta, double *Y, const int incY);
void cblas_cgbmv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const int KL, const int KU, const void *alpha, const void *A, const int lda, const void *X,
const int incX, const void *beta, void *Y, const int incY);
void cblas_zgbmv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const int KL, const int KU, const void *alpha, const void *A, const int lda, const void *X,
const int incX, const void *beta, void *Y, const int incY);
```

?gemv

```
void cblas_sgemv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const float alpha, const float *A, const int lda, const float *X, const int incX, const float
beta, float *Y, const int incY);
void cblas_dgemv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const double alpha, const double *A, const int lda, const double *X, const int incX, const
double beta, double *Y, const int incY);
void cblas_cgemv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const void *alpha, const void *A, const int lda, const void *X, const int incX, const void
*beta, void *Y, const int incY);
void cblas_zgemv(const enum CBLAS_ORDER order, const enum CBLAS_TRANSPOSE TransA, const int M, const
int N, const void *alpha, const void *A, const int lda, const void *X, const int incX, const void
*beta, void *Y, const int incY);
```

?ger

```
void cblas_sger(const enum CBLAS_ORDER order, const int M, const int N, const float alpha, const float
*X, const int incX, const float *Y, const int incY, float *A, const int lda);
void cblas_dger(const enum CBLAS_ORDER order, const int M, const int N, const double alpha, const
double *X, const int incX, const double *Y, const int incY, double *A, const int lda);
```

?gerc

```
void cblas_cgerc(const enum CBLAS_ORDER order, const int M, const int N, const void *alpha, const void
*X, const int incX, const void *Y, const int incY, void *A, const int lda);
void cblas_zgerc(const enum CBLAS_ORDER order, const int M, const int N, const void *alpha, const void
*X, const int incX, const void *Y, const int incY, void *A, const int lda);
```

?geru

```
void cblas_cgeru(const enum CBLAS_ORDER order, const int M, const int N, const void *alpha, const void
*X, const int incX, const void *Y, const int incY, void *A, const int lda);
void cblas_zgeru(const enum CBLAS_ORDER order, const int M, const int N, const void *alpha, const void
*X, const int incX, const void *Y, const int incY, void *A, const int lda);
```

?hbmv

```
void cblas_chbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const int K,
const void *alpha, const void *A, const int lda, const void *X, const int incX, const void *beta, void
*Y, const int incY);
void cblas_zhbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const int K,
const void *alpha, const void *A, const int lda, const void *X, const int incX, const void *beta, void
*Y, const int incY);
```

?hemv

```
void cblas_chemv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *A, const int lda, const void *X, const int incX, const void *beta, void *Y, const
int incY);
void cblas_zhemv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *A, const int lda, const void *X, const int incX, const void *beta, void *Y, const
int incY);
```

?her

```
void cblas_cher(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const void *X, const int incX, void *A, const int lda);
void cblas_zher(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const void *X, const int incX, void *A, const int lda);
```

?her2

```
void cblas_cher2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *X, const int incX, const void *Y, const int incY, void *A, const int lda);
void cblas_zher2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *X, const int incX, const void *Y, const int incY, void *A, const int lda);
```

?hpmv

```
void cblas_chpmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *Ap, const void *X, const int incX, const void *beta, void *Y, const int incY);
void cblas_zhpmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *Ap, const void *X, const int incX, const void *beta, void *Y, const int incY);
```

?hpr

```
void cblas_chpr(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const void *X, const int incX, void *A);
void cblas_zhpr(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const void *X, const int incX, void *A);
```

?hpr2

```
void cblas_chpr2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *X, const int incX, const void *Y, const int incY, void *Ap);
void cblas_zhpr2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const void
*alpha, const void *X, const int incX, const void *Y, const int incY, void *Ap);
```

?sbmv

```
void cblas_ssbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const int K,
const float alpha, const float *A, const int lda, const float *X, const int incX, const float beta,
float *Y, const int incY);
```

```
void cblas_dsbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const int K,
const double alpha, const double *A, const int lda, const double *X, const int incX, const double beta,
double *Y, const int incY);
```

?spmv

```
void cblas_sspmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const float *Ap, const float *X, const int incX, const float beta, float *Y, const int incY);
void cblas_dspmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const double *Ap, const double *X, const int incX, const double beta, double *Y, const int incY);
```

?spr

```
void cblas_sspr(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const float *X, const int incX, float *Ap);
void cblas_dspr(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const double *X, const int incX, double *Ap);
```

?spr2

```
void cblas_sspr2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const float *X, const int incX, const float *Y, const int incY, float *A);
void cblas_dspr2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const double *X, const int incX, const double *Y, const int incY, double *A);
```

?symv

```
void cblas_ssymv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const float *A, const int lda, const float *X, const int incX, const float beta, float *Y, const
int incY);
void cblas_dsymv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const double *A, const int lda, const double *X, const int incX, const double beta, double *Y,
const int incY);
```

?syr

```
void cblas_ssyr(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const float *X, const int incX, float *A, const int lda);
void cblas_dsyr(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const double *X, const int incX, double *A, const int lda);
```

?syr2

```
void cblas_ssyr2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const float
alpha, const float *X, const int incX, const float *Y, const int incY, float *A, const int lda);
void cblas_dsyr2(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const int N, const double
alpha, const double *X, const int incX, const double *Y, const int incY, double *A, const int lda);
```

?tbmv

```
void cblas_stbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const float *A, const int lda, float *X,
const int incX);
void cblas_dtbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const double *A, const int lda, double
*X, const int incX);
void cblas_ctbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const void *A, const int lda, void *X,
const int incX);
```

```
void cblas_ztbmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const void *A, const int lda, void *X,
const int incX);
```

?tbsv

```
void cblas_stbsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const float *A, const int lda, float *X,
const int incX);
void cblas_dtbsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const double *A, const int lda, double
*X, const int incX);
void cblas_ctbsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const void *A, const int lda, void *X,
const int incX);
void cblas_ztbsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const int K, const void *A, const int lda, void *X,
const int incX);
```

?tpmv

```
void cblas_stpmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const float *Ap, float *X, const int incX);
void cblas_dtpmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N, const double *Ap, double *X, const int incX);
void cblas_ctpmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *Ap, void *X, const int incX);
void cblas_ztpmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *Ap, void *X, const int incX);
```

?tpsv

```
void cblas_stpsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const float *Ap, float *X, const int incX);
void cblas_dtpsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const double *Ap, double *X, const int incX);
void cblas_ctpsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *Ap, void *X, const int incX);
void cblas_ztpsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *Ap, void *X, const int incX);
```

?trmv

```
void cblas_strmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const float *A, const int lda, float *X, const int
incX);
void cblas_dtrmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const double *A, const int lda, double *X, const int
incX);
void cblas_ctrmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *A, const int lda, void *X, const int incX);
void cblas_ztrmv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *A, const int lda, void *X, const int incX);
```

?trsv

```
void cblas_strsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const float *A, const int lda, float *X, const int
incX);
void cblas_dtrsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const double *A, const int lda, double *X, const int
incX);
void cblas_ctrsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
```

```
TransA, const enum CBLAS_DIAG Diag, const int N,const void *A, const int lda, void *X, const int incX);
void cblas_ztrsv(const enum CBLAS_ORDER order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
TransA, const enum CBLAS_DIAG Diag, const int N,const void *A, const int lda, void *X, const int incX);
```

Level 3 CBLAS

This is an interface to "BLAS Level 3 Routines", which perform basic matrix-matrix operations. Each C routine in this group has an additional parameter of type `CBLAS_ORDER` (the first argument) that determines whether the two-dimensional arrays use column-major or row-major storage.

?gemm

```
void cblas_sgemm(const enum CBLAS_ORDER Order, const enum CBLAS_TRANSPOSE TransA, const enum
CBLAS_TRANSPOSE TransB, const int M, const int N, const int K, const float alpha, const float *A, const
int lda, const float *B, const int ldb, const float beta, float *C, const int ldc);
void cblas_dgemm(const enum CBLAS_ORDER Order, const enum CBLAS_TRANSPOSE TransA, const enum
CBLAS_TRANSPOSE TransB, const int M, const int N, const int K, const double alpha, const double *A,
const int lda, const double *B, const int ldb, const double beta, double *C, const int ldc);
void cblas_cgemm(const enum CBLAS_ORDER Order, const enum CBLAS_TRANSPOSE TransA, const enum
CBLAS_TRANSPOSE TransB, const int M, const int N, const int K, const void *alpha, const void *A, const
int lda, const void *B, const int ldb, const void *beta, void *C, const int ldc);
void cblas_zgemm(const enum CBLAS_ORDER Order, const enum CBLAS_TRANSPOSE TransA, const enum
CBLAS_TRANSPOSE TransB, const int M, const int N, const int K, const void *alpha, const void *A, const
int lda, const void *B, const int ldb, const void *beta, void *C, const int ldc);
```

?hemm

```
void cblas_chemm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const int M, const int N, const void *alpha, const void *A, const int lda, const void *B, const int
ldb, const void *beta, void *C, const int ldc);
void cblas_zhemm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const int M, const int N, const void *alpha, const void *A, const int lda, const void *B, const int
ldb, const void *beta, void *C, const int ldc);
```

?herk

```
void cblas_cherk(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const float alpha, const void *A, const int lda, const float beta,
void *C, const int ldc);
void cblas_zherk(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const double alpha, const void *A, const int lda, const double beta,
void *C, const int ldc);
```

?her2k

```
void cblas_cher2k(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const void *alpha, const void *A, const int lda, const void *B, const
int ldb, const float beta, void *C, const int ldc);
void cblas_zher2k(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const void *alpha, const void *A, const int lda, const void *B, const
int ldb, const double beta, void *C, const int ldc);
```

?symm

```
void cblas_ssymm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const int M, const int N, const float alpha, const float *A, const int lda, const float *B, const int
ldb, const float beta, float *C, const int ldc);
void cblas_dsymm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const int M, const int N, const double alpha, const double *A, const int lda, const double *B, const
int ldb, const double beta, double *C, const int ldc);
void cblas_csymm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const int M, const int N, const void *alpha, const void *A, const int lda, const void *B, const int
```

```
ldb, const void *beta, void *C, const int ldc);
void cblas_zsymm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const int M, const int N, const void *alpha, const void *A, const int lda, const void *B, const int
ldb, const void *beta, void *C, const int ldc);
```

?syrk

```
void cblas_ssyrk(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const float alpha, const float *A, const int lda, const float beta,
float *C, const int ldc);
void cblas_dsyrk(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const double alpha, const double *A, const int lda, const double beta,
double *C, const int ldc);
void cblas_csyrk(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const void *alpha, const void *A, const int lda, const void *beta,
void *C, const int ldc);
void cblas_zsyrk(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const void *alpha, const void *A, const int lda, const void *beta,
void *C, const int ldc);
```

?syr2k

```
void cblas_ssyr2k(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const float alpha, const float *A, const int lda, const float *B,
const int ldb, const float beta, float *C, const int ldc);
void cblas_dsyr2k(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const double alpha, const double *A, const int lda, const double *B,
const int ldb, const double beta, double *C, const int ldc);
void cblas_csyr2k(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSP SE
Trans, const int N, const int K, const void *alpha,const void *A, const int lda, const void *B, const
int ldb, const void *beta, void *C, const int ldc);
void cblas_zsyr2k(const enum CBLAS_ORDER Order, const enum CBLAS_UPLO Uplo, const enum CBLAS_TRANSPOSE
Trans, const int N, const int K, const void *alpha, const void *A, const int lda, const void *B, const
int ldb, const void *beta, void *C, const int ldc);
```

?trmm

```
void cblas_strmm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const float
alpha, const float *A, const int lda, float *B, const int ldb);
void cblas_dtrmm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const double
alpha, const double *A, const int lda, double *B, const int ldb);
void cblas_ctrmm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const void
*alpha, const void *A, const int lda, void *B, const int ldb);
void cblas_ztrmm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const void
*alpha, const void *A, const int lda, void *B, const int ldb);
```

?trsm

```
void cblas_strsm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const float
alpha, const float *A, const int lda, float *B, const int ldb);
void cblas_dtrsm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const double
alpha, const double *A, const int lda, double *B, const int ldb);
void cblas_ctrsm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const void
*alpha, const void *A, const int lda, void *B, const int ldb);
void cblas_ztrsm(const enum CBLAS_ORDER Order, const enum CBLAS_SIDE Side, const enum CBLAS_UPLO Uplo,
const enum CBLAS_TRANSPOSE TransA, const enum CBLAS_DIAG Diag, const int M, const int N, const void
*alpha, const void *A, const int lda, void *B, const int ldb);
```

Sparse CBLAS

This is an interface to Sparse BLAS Level 1 Routines, which perform a number of common vector operations on sparse vectors stored in compressed form.

Note that all index parameters, *indx*, are in C-type notation and vary in the range [0..*N*–1].

?axpyi

```
void cblas_saxpyi(const int N, const float alpha, const float *X, const int *indx, float *Y);
void cblas_daxpyi(const int N, const double alpha, const double *X, const int *indx, double *Y);
void cblas_caxpyi(const int N, const void *alpha, const void *X, const int *indx, void *Y);
void cblas_zaxpyi(const int N, const void *alpha, const void *X, const int *indx, void *Y);
```

?doti

```
float cblas_sdoti(const int N, const float *X, const int *indx, const float *Y);
double cblas_ddoti(const int N, const double *X, const int *indx, const double *Y);
```

?dotci

```
void  cblas_cdotci_sub(const int N, const void *X, const int *indx, const void *Y, void *dotui);
void  cblas_zdotci_sub(const int N, const void *X, const int *indx, const void *Y, void *dotui);
```

?dotui

```
void  cblas_cdotui_sub(const int N, const void *X, const int *indx, const void *Y, void *dotui);
void  cblas_zdotui_sub(const int N, const void *X, const int *indx, const void *Y, void *dotui);
```

?gthr

```
void cblas_sgthr(const int N, const float *Y, float *X, const int *indx);
void cblas_dgthr(const int N, const double *Y, double *X, const int *indx);
void cblas_cgthr(const int N, const void *Y, void *X, const int *indx);
void cblas_zgthr(const int N, const void *Y, void *X, const int *indx);
```

?gthrz

```
void cblas_sgthrz(const int N, float *Y, float *X, const int *indx);
void cblas_dgthrz(const int N, double *Y, double *X, const int *indx);
void cblas_cgthrz(const int N, void *Y, void *X, const int *indx);
void cblas_zgthrz(const int N, void *Y, void *X, const int *indx);
```

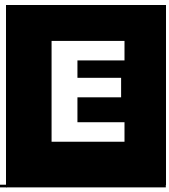
?roti

```
void cblas_sroti(const int N, float *X, const int *indx, float *Y, const float c, const float s);
void cblas_droti(const int N, double *X, const int *indx, double *Y, const double c, const double s);
```

?sctr

```
void cblas_ssctr(const int N, const float *X, const int *indx, float *Y);
void cblas_dsctr(const int N, const double *X, const int *indx, double *Y);
void cblas_csctr(const int N, const void *X, const int *indx, void *Y);
void cblas_zsctr(const int N, const void *X, const int *indx, void *Y);
```

Specific Features of Fortran 95 Interfaces for LAPACK Routines



Intel® MKL implements Fortran 95 interface for LAPACK package, further referred to as MKL LAPACK95, to provide full capacity of MKL FORTRAN 77 LAPACK routines. This is the principal difference of Intel MKL from the Netlib Fortran 95 implementation for LAPACK.

A new feature of MKL LAPACK95 by comparison with Intel MKL LAPACK77 implementation is presenting a package of source interfaces along with wrappers that make the implementation compiler-independent. As a result, the MKL LAPACK package can be used in all programming environments intended for Fortran 95.

Depending on the degree and type of difference from Netlib implementation, the MKL LAPACK95 interfaces fall into several groups that require different transformations (see "MKL Fortran 95 Interfaces for LAPACK Routines vs. Netlib Implementation"). The groups are given in full with the calling sequences of the routines and appropriate differences from Netlib analogs.

The following conventions are used:

```
<interface> ::=  <name of interface> '(' <arguments list>')'
<arguments list> ::=  <first argument> {<argument>}*
<first argument> ::=  < identifier >
<argument> ::=  <required argument>|<optional argument>
<required argument> ::=  ',' <identifier>
<optional argument> ::=  '[,' <identifier> ']'
<name of interface> ::=  <identifier>
```

where defined notions are separated from definitions by `::=`, notion names are marked by angle brackets, terminals are given in quotes, and `{…}*` denotes repetition zero, one, or more times.

<first argument> and each *<required argument>* should be present in all calls of denoted interface, *<optional argument>* may be omitted. Comments to interface definitions are provided where necessary. Comment lines begin with character `!`.

Two interfaces with one name are presented when two variants of subroutine calls (separated by types of arguments) exist.

Interfaces Identical to Netlib

```
GERFS(A,AF,IPIV,B,X[,TRANS][,FERR][,BERR][,INFO])
GETRI(A,IPIV[,INFO])
GEEQU(A,R,C[,ROWCND][,COLCND][,AMAX][,INFO])
GESV(A,B[,IPIV][,INFO])
GESVX(A,B,X[,AF][,IPIV][,FACT][,TRANS][,EQUED][,R][,C][,FERR][,BERR]
[,RCOND][,RPVGRW][,INFO])
GTSV(DL,D,DU,B[,INFO])
GTSVX(DL,D,DU,B,X[,DLF][,DF][,DUF][,DU2][,IPIV][,FACT][,TRANS][,FERR]
[,BERR][,RCOND][,INFO])
POSV(A,B[,UPLO][,INFO])
POSVX(A,B,X[,UPLO][,AF][,FACT][,EQUED][,S][,FERR][,BERR][,RCOND][,INFO])
PTSV(D,E,B[,INFO])
PTSVX(D,E,B,X[,DF][,EF][,FACT][,FERR][,BERR][,RCOND][,INFO])
SYSV(A,B[,UPLO][,IPIV][,INFO])
SYSVX(A,B,X[,UPLO][,AF][,IPIV][,FACT][,FERR][,BERR][,RCOND][,INFO])
HESVX(A,B,X[,UPLO][,AF][,IPIV][,FACT][,FERR][,BERR][,RCOND][,INFO])
HESV(A,B[,UPLO][,IPIV][,INFO])
SPSV(AP,B[,UPLO][,IPIV][,INFO])
HPSV(AP,B[,UPLO][,IPIV][,INFO])
SYTRD(A,TAU[,UPLO][,INFO])
ORGTR(A,TAU[,UPLO][,INFO])
HETRD(A,TAU[,UPLO][,INFO])
UNGTR(A,TAU[,UPLO][,INFO])
SYGST(A,B[,ITYPE][,UPLO][,INFO])
HEGST(A,B[,ITYPE][,UPLO][,INFO])
```

```
GELS(A,B[,TRANS][,INFO])
GELSY(A,B[,RANK][,JPVT][,RCOND][,INFO])
GELSS(A,B[,RANK][,S][,RCOND][,INFO])
GELSD(A,B[,RANK][,S][,RCOND][,INFO])
GGLSE(A,B,C,D,X[,INFO])
GGGLM(A,B,D,X,Y[,INFO])
SYEV(A,W[,JOBZ][,UPLO][,INFO])
HEEV(A,W[,JOBZ][,UPLO][,INFO])
SYEVD(A,W[,JOBZ][,UPLO][,INFO])
HEEVD(A,W[,JOBZ][,UPLO][,INFO])
STEV(D,E[,Z][,INFO])
STEVD(D,E[,Z][,INFO])
STEVX(D,E,W[,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
STEVR(D,E,W[,Z][,VL][,VU][,IL][,IU][,M][,ISUPPZ][,ABSTOL][,INFO])
GEES(A,WR,WI[,VS][,SELECT][,SDIM][,INFO])
GEES(A,W[,VS][,SELECT][,SDIM][,INFO])
GEESX(A,WR,WI[,VS][,SELECT][,SDIM][,RCONDE][,RCONDV][,INFO])
GEESX(A,W[,VS][,SELECT][,SDIM][,RCONDE][,RCONDV][,INFO])
GEEV(A,WR,WI[,VL][,VR][,INFO])
GEEV(A,W[,VL][,VR][,INFO])
GEEVX(A,WR,WI[,VL][,VR][,BALANC][,ILO][,IHI][,SCALE][,ABNRM][,RCONDE][,RCONDV][,INFO])
GEEVX(A,W[,VL][,VR][,BALANC][,ILO][,IHI][,SCALE][,ABNRM][,RCONDE]
[,RCONDV][,INFO])
GESVD(A,S[,U][,VT][,WW][,JOB][,INFO])
GGSVD(A,B,ALPHA,BETA[,K][,L][,U][,V][,Q][,IWORK][,INFO])
SYGV(A,B,W[,ITYPE][,JOBZ][,UPLO][,INFO])
HEGV(A,B,W[,ITYPE][,JOBZ][,UPLO][,INFO])
SYGVD(A,B,W[,ITYPE][,JOBZ][,UPLO][,INFO])
HEGVD(A,B,W[,ITYPE][,JOBZ][,UPLO][,INFO])
SPGVD(AP,BP,W[,ITYPE][,UPLO][,Z][,INFO])
HPGVD(AP,BP,W[,ITYPE][,UPLO][,Z][,INFO])
SPGVX(AP,BP,W[,ITYPE][,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
HPGVX(AP,BP,W[,ITYPE][,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
SBGVD(AB,BB,W[,UPLO][,Z][,INFO])
HBGVD(AB,BB,W[,UPLO][,Z][,INFO])
SBGVX(AB,BB,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,Q][,ABSTOL][,INFO])
HBGVX(AB,BB,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,Q][,ABSTOL][,INFO])
GGES(A,B,ALPHAR,ALPHAI,BETA[,VSL][,VSR][,SELECT][,SDIM][,INFO])
GGES(A,B,ALPHA,BETA[,VSL][,VSR][,SELECT][,SDIM][,INFO])
GGESX(A,B,ALPHAR,ALPHAI,BETA[,VSL][,VSR][,SELECT][,SDIM][,RCONDE][,RCONDV][,INFO])
GGEV(A,B,ALPHAR,ALPHAI,BETA[,VL][,VR][,INFO])
GGEV(A,B,ALPHA,BETA[,VL][,VR][,INFO])
GGEVX(A,B,ALPHAR,ALPHAI,BETA[,VL][,VR][,BALANC][,ILO][,IHI][,LSCALE][,RSCALE][,ABNRM]
[,BBNRM][,RCONDE][,RCONDV][,INFO])
GGEVX(A,B,ALPHA,BETA[,VL][,VR][,BALANC][,ILO][,IHI][,LSCALE][,RSCALE][,ABNRM]
[,BBNRM][,RCONDE][,RCONDV][,INFO])
```

Interfaces with Replaced Argument Names

Argument names in the routines of this group are replaced as follows:

Netlib Argument Name	MKL Argument Name
A	AB
A	AP
AF	AFB
AF	AFP
B	BB
B	BP
K	KL

```
GBSV(AB,B[,KL][,IPIV][,INFO])
!   netlib: (A,B,K,IPIV,INFO)
```

```
GBSVX(AB,B,X[,KL][,AFB][,IPIV][,FACT][,TRANS][,EQUED][,R][,C][,FERR]
[,BERR][,RCOND][,RPVGRW][,INFO])
!   netlib: (A,B,X,KL,AF,IPIV,FACT,TRANS,EQUED,R,C,FERR,
! BERR,RCOND,RPVGRW,INFO)
```

```
PPSV(AP,B[,UPLO][,INFO])
!   netlib: (A,B,UPLO,INFO)

PPSVX(AP,B,X[,UPLO][,AFP][,FACT][,EQUED][,S][,FERR][,BERR][,RCOND][,INFO])
!   netlib: (A,B,X,UPLO,AF,FACT,EQUED,S,FERR,BERR,RCOND,INFO)!

PBSV(AB,B[,UPLO][,INFO])
!   netlib: (A,B,UPLO,INFO)

PBSVX(AB,B,X[,UPLO][,AFB][,FACT][,EQUED][,S][,FERR][,BERR][,RCOND][,INFO])
!   netlib: (A,B,X,UPLO,AF,FACT,EQUED,S,FERR,BERR,RCOND,INFO)!

SPSVX(AP,B,X[,UPLO][,AFP][,IPIV][,FACT][,FERR][,BERR][,RCOND][,INFO])
!   netlib: (A,B,X,UPLO,AF,IPIV,FACT,FERR,BERR,RCOND,INFO)

HPSVX(AP,B,X[,UPLO][,AFP][,IPIV][,FACT][,FERR][,BERR][,RCOND][,INFO])
!   netlib: (A,B,X,UPLO,AF,IPIV,FACT,FERR,BERR,RCOND,INFO)

SPEV(AP,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

HPEV(AP,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

SPEVD(AP,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

HPEVD(AP,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

SPEVX(AP,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
!   netlib: (A,B,W,UPLO,Z,VL,VU,IL,IU,M,IFAIL,ABSTOL,INFO)

HPEVX(AP,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
!   netlib: (A,B,W,UPLO,Z,VL,VU,IL,IU,M,IFAIL,ABSTOL,INFO)

SBEV(AB,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

HBEV(AB,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

SBEVD(AB,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

HBEVD(AB,W[,UPLO][,Z][,INFO])
!   netlib: (A,W,UPLO,Z,INFO)

SBEVX(AB,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,Q][,ABSTOL][,INFO])
!   netlib: (A,B,W,UPLO,Z,VL,VU,IL,IU,M,IFAIL,Q,ABSTOL,INFO)

HBEVX(AB,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,Q][,ABSTOL][,INFO])
!   netlib: (A,B,W,UPLO,Z,VL,VU,IL,IU,M,IFAIL,Q,ABSTOL,INFO)

SPGV(AP,BP,W[,ITYPE][,UPLO][,Z][,INFO])
!   netlib: (A,B,W,ITYPE,UPLO,Z,INFO)

HPGV(AB,BP,W[,ITYPE][,UPLO][,Z][,INFO])
!   netlib: (A,B,W,ITYPE,UPLO,Z,INFO)

SBGV(AB,BB,W[,UPLO][,Z][,INFO])
!   netlib: (A,B,W,UPLO,Z,INFO)

HBGV(AB,BB,W[,UPLO][,Z][,INFO])
!   netlib: (A,B,W,UPLO,Z,INFO)
```

Modified Netlib Interfaces

```
SYEVX(A,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,W,JOBZ,UPLO,VL,VU,IL,IU,M,IFAIL,ABSTOL,INFO)
!   Different order for parameter UPLO, netlib: 4, mkl: 3
!   Absent mkl parameter: JOBZ
!   Extra mkl parameter: Z
```

```
HEEVX(A,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,W,JOBZ,UPLO,VL,VU,IL,IU,M,IFAIL,ABSTOL,INFO)
!   Different order for parameter UPLO, netlib: 4, mkl: 3
!   Absent mkl parameter: JOBZ
!   Extra mkl parameter: Z
```

```
SYEVR(A,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,ISUPPZ][,ABSTOL][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,W,JOBZ,UPLO,VL,VU,IL,IU,M,ISUPPZ,ABSTOL,INFO)
!   Different order for parameter UPLO, netlib: 4, mkl: 3
!   Absent mkl parameter: JOBZ
!   Extra mkl parameter: Z
```

```
HEEVR(A,W[,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,ISUPPZ][,ABSTOL][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,W,JOBZ,UPLO,VL,VU,IL,IU,M,ISUPPZ,ABSTOL,INFO)
!   Different order for parameter UPLO, netlib: 4, mkl: 3
!   Absent mkl parameter: JOBZ
!   Extra mkl parameter: Z
```

```
GESDD(A,S[,U][,VT][,JOBZ][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,S,U,VT,WW,JOB,INFO)
!   Different number for parameter, netlib: 7, mkl: 6
!   Absent mkl parameter: WW
!   Absent mkl parameter: JOB
!   Different order for parameter INFO, netlib: 7, mkl: 6
!   Extra mkl parameter: JOBZ
```

```
SYGVX(A,B,W[,ITYPE][,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,B,W,ITYPE,JOBZ,UPLO,VL,VU,IL,IU,M,IFAIL,ABSTOL,INFO)
!   Different order for parameter UPLO, netlib: 6, mkl: 5
!   Absent mkl parameter: JOBZ
!   Extra mkl parameter: Z
```

```
HEGVX(A,B,W[,ITYPE][,UPLO][,Z][,VL][,VU][,IL][,IU][,M][,IFAIL][,ABSTOL][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,B,W,ITYPE,JOBZ,UPLO,VL,VU,IL,IU,M,IFAIL,ABSTOL,INFO)
!   Different order for parameter UPLO, netlib: 6, mkl: 5
!   Absent mkl parameter: JOBZ
!   Extra mkl parameter: Z
```

```
GETRS(A,IPIV,B[,TRANS][,INFO])
!   Interface netlib95 exists:
!   Different intents for parameter A, netlib: INOUT, mkl: IN
```

Interfaces Absent From Netlib

```
GTTRF(DL,D,DU,DU2[,IPIV][,INFO])
PPTRF(A[,UPLO][,INFO])
PBTRF(A[,UPLO][,INFO])
PTTRF(D,E[,INFO])
SYTRF(A[,UPLO][,IPIV][,INFO])
HETRF(A[,UPLO][,IPIV][,INFO])
```

```
SPTRF(A[,UPLO][,IPIV][,INFO])
HPTRF(A[,UPLO][,IPIV][,INFO])
GBTRS(A,B,IPIV[,KL][,TRANS][,INFO])
GTTRS(DL,D,DU,DU2,B,IPIV[,TRANS][,INFO])
POTRS(A,B[,UPLO][,INFO])
PPTRS(A,B[,UPLO][,INFO])
PBTRS(A,B[,UPLO][,INFO])
PTTRS(D,E,B[,INFO])
PTTRS(D,E,B[,UPLO][,INFO])
SYTRS(A,B,IPIV[,UPLO][,INFO])
HETRS(A,B,IPIV[,UPLO][,INFO])
SPTRS(A,B,IPIV[,UPLO][,INFO])
HPTRS(A,B,IPIV[,UPLO][,INFO])
TRTRS(A,B[,UPLO][,TRANS][,DIAG][,INFO])
TPTRS(A,B[,UPLO][,TRANS][,DIAG][,INFO])
TBTRS(A,B[,UPLO][,TRANS][,DIAG][,INFO])
GECON(A,ANORM,RCOND[,NORM][,INFO])
GBCON(A,IPIV,ANORM,RCOND[,KL][,NORM][,INFO])
GTCON(DL,D,DU,DU2,IPIV,ANORM,RCOND[,NORM][,INFO])
POCON(A,ANORM,RCOND[,UPLO][,INFO])
PPCON(A,ANORM,RCOND[,UPLO][,INFO])
PBCON(A,ANORM,RCOND[,UPLO][,INFO])
PTCON(D,E,ANORM,RCOND[,INFO])
SYCON(A,IPIV,ANORM,RCOND[,UPLO][,INFO])
HECON(A,IPIV,ANORM,RCOND[,UPLO][,INFO])
SPCON(A,IPIV,ANORM,RCOND[,UPLO][,INFO])
HPCON(A,IPIV,ANORM,RCOND[,UPLO][,INFO])
TRCON(A,RCOND[,UPLO][,DIAG][,NORM][,INFO])
TPCON(A,RCOND[,UPLO][,DIAG][,NORM][,INFO])
TBCON(A,RCOND[,UPLO][,DIAG][,NORM][,INFO])
GBRFS(A,AF,IPIV,B,X[,KL][,TRANS][,FERR][,BERR][,INFO])
GTRFS(DL,D,DU,DLF,DF,DUF,DU2,IPIV,B,X[,TRANS][,FERR][,BERR][,INFO])
PORFS(A,AF,B,X[,UPLO][,FERR][,BERR][,INFO])
PPRFS(A,AF,B,X[,UPLO][,FERR][,BERR][,INFO])
PBRFS(A,AF,B,X[,UPLO][,FERR][,BERR][,INFO])
PTRFS(D,DF,E,EF,B,X[,FERR][,BERR][,INFO])
PTRFS(D,DF,E,EF,B,X[,UPLO][,FERR][,BERR][,INFO])
SYRFS(A,AF,IPIV,B,X[,UPLO][,FERR][,BERR][,INFO])
HERFS(A,AF,IPIV,B,X[,UPLO][,FERR][,BERR][,INFO])
SPRFS(A,AF,IPIV,B,X[,UPLO][,FERR][,BERR][,INFO])
HPRFS(A,AF,IPIV,B,X[,UPLO][,FERR][,BERR][,INFO])
TRRFS(A,B,X[,UPLO][,TRANS][,DIAG][,FERR][,BERR][,INFO])
TPRFS(A,B,X[,UPLO][,TRANS][,DIAG][,FERR][,BERR][,INFO])
TBRFS(A,B,X[,UPLO][,TRANS][,DIAG][,FERR][,BERR][,INFO])
POTRI(A[,UPLO][,INFO])
PPTRI(A[,UPLO][,INFO])
SYTRI(A,IPIV[,UPLO][,INFO])
HETRI(A,IPIV[,UPLO][,INFO])
SPTRI(A,IPIV[,UPLO][,INFO])
HPTRI(A,IPIV[,UPLO][,INFO])
TRTRI(A[,UPLO][,DIAG][,INFO])
TPTRI(A[,UPLO][,DIAG][,INFO])
GBEQU(A,R,C[,KL][,ROWCND][,COLCND][,AMAX][,INFO])
POEQU(A,S[,SCOND][,AMAX][,INFO])
PPEQU(A,S[,SCOND][,AMAX][,UPLO][,INFO])
PBEQU(A,S[,SCOND][,AMAX][,UPLO][,INFO])
HESV(A,B[,UPLO][,IPIV][,INFO])
HPSV(A,B[,UPLO][,IPIV][,INFO])
GEQRF(A[,TAU][,INFO])
GEQPF(A,JPVT[,TAU][,INFO])
GEQP3(A,JPVT[,TAU][,INFO])
ORGQR(A,TAU[,INFO])
ORMQR(A,TAU,C[,SIDE][,TRANS][,INFO])
UNGQR(A,TAU[,INFO])
UNMQR(A,TAU,C[,SIDE][,TRANS][,INFO])
GELQF(A[,TAU][,INFO])
ORGLQ(A,TAU[,INFO])
ORMLQ(A,TAU,C[,SIDE][,TRANS][,INFO])
UNGLQ(A,TAU[,INFO])
UNMLQ(A,TAU,C[,SIDE][,TRANS][,INFO])
GEQLF(A[,TAU][,INFO])
```

```
ORGQL(A,TAU[,INFO])
UNGQL(A,TAU[,INFO])
ORMQL(A,TAU,C[,SIDE][,TRANS][,INFO])
UNMQL(A,TAU,C[,SIDE][,TRANS][,INFO])
GERQF(A[,TAU][,INFO])
ORGRQ(A,TAU[,INFO])
UNGRQ(A,TAU[,INFO])
ORMRQ(A,TAU,C[,SIDE][,TRANS][,INFO])
UNMRQ(A,TAU,C[,SIDE][,TRANS][,INFO])
TZRZF(A[,TAU][,INFO])
ORMRZ(A,TAU,C,L[,SIDE][,TRANS][,INFO])
UNMRZ(A,TAU,C,L[,SIDE][,TRANS][,INFO])
GGQRF(A,B[,TAUA][,TAUB][,INFO])
GGRQF(A,B[,TAUA][,TAUB][,INFO])
GEBRD(A[,D][,E][,TAUQ][,TAUP][,INFO])
GBBRD(A[,C][,D][,E][,Q][,PT][,KL][,M][,INFO])
ORGBR(A,TAU[,VECT][,INFO])
ORMBR(A,TAU,C[,VECT][,SIDE][,TRANS][,INFO])
ORMTR(A,TAU,C[,SIDE][,UPLO][,TRANS][,INFO])
UNGBR(A,TAU[,VECT][,INFO])
UNMBR(A,TAU,C[,VECT][,SIDE][,TRANS][,INFO])
BDSQR(D,E[,VT][,U][,C][,UPLO][,INFO])
BDSDC(D,E[,U][,VT][,Q][,IQ][,UPLO][,INFO])
UNMTR(A,TAU,C[,SIDE][,UPLO][,TRANS][,INFO])
SPTRD(A,TAU[,UPLO][,INFO])
OPGTR(A,TAU,Q[,UPLO][,INFO])
OPMTR(A,TAU,C[,SIDE][,UPLO][,TRANS][,INFO])
HPTRD(A,TAU[,UPLO][,INFO])
UPGTR(A,TAU,Q[,UPLO][,INFO])
UPMTR(A,TAU,C[,SIDE][,UPLO][,TRANS][,INFO])
SBTRD(A[,Q][,VECT][,UPLO][,INFO])
HBTRD(A[,Q][,VECT][,UPLO][,INFO])
STERF(D,E[,INFO])
STEQR(D,E[,Z][,COMPZ][,INFO])
STEDC(D,E[,Z][,COMPZ][,INFO])
STEGR(D,E,W[,Z][,VL][,VU][,IL][,IU][,M][,ISUPPZ][,ABSTOL][,INFO])
PTEQR(D,E[,Z][,COMPZ][,INFO])
STEBZ(D,E,M,NSPLIT,W,IBLOCK,ISPLIT[,ORDER][,VL][,VU][,IL][,IU][,ABSTOL][,INFO])
STEIN(D,E,W,IBLOCK,ISPLIT,Z[,IFAILV][,INFO])
DISNA(D,SEP[,JOB][,MINMN][,INFO])
SPGST(A,B[,ITYPE][,UPLO][,INFO])
HPGST(A,B[,ITYPE][,UPLO][,INFO])
SBGST(A,B[,X][,UPLO][,INFO])
HBGST(A,B[,X][,UPLO][,INFO])
PBSTF(B[,UPLO][,INFO])
GEHRD(A[,TAU][,ILO][,IHI][,INFO])
ORGHR(A,TAU[,ILO][,IHI][,INFO])
ORMHR(A,TAU,C[,ILO][,IHI][,SIDE][,TRANS][,INFO])
UNGHR(A,TAU[,ILO][,IHI][,INFO])
UNMHR(A,TAU,C[,ILO][,IHI][,SIDE][,TRANS][,INFO])
GEBAL(A[,SCALE][,ILO][,IHI][,JOB][,INFO])
GEBAK(V,SCALE[,ILO][,IHI][,JOB][,SIDE][,INFO])
HSEQR(H,WR,WI[,ILO][,IHI][,Z][,JOB][,COMPZ][,INFO])
HSEQR(H,W[,ILO][,IHI][,Z][,JOB][,COMPZ][,INFO])
HSEIN(H,WR,WI,SELECT[,VL][,VR][,IFAILL][,IFAILR][,INITV][,EIGSRC][,M][,INFO])
HSEIN(H,W,SELECT[,VL][,VR][,IFAILL][,IFAILR][,INITV][,EIGSRC][,M][,INFO])
TREVC(T[,HOWMNY][,SELECT][,VL][,VR][,M][,INFO])
TRSNA(T[,S][,SEP][,VL][,VR][,SELECT][,M][,INFO])
TREXC(T,IFST,ILST[,Q][,INFO])
TRSEN(T,SELECT[,WR][,WI][,M][,S][,SEP][,Q][,INFO])
TRSEN(T,SELECT[,W][,M][,S][,SEP][,Q][,INFO])
TRSYL(A,B,C,SCALE[,TRANA][,TRANB][,ISGN][,INFO])
GGHRD(A,B[,ILO][,IHI][,Q][,Z][,COMPQ][,COMPZ][,INFO])
GGBAL(A,B[,ILO][,IHI][,LSCALE][,RSCALE][,JOB][,INFO])
GGBAK(V[,ILO][,IHI][,LSCALE][,RSCALE][,JOB][,INFO])
HGEQZ(H,T[,ILO][,IHI][,ALPHAR][,ALPHAI][,BETA][,Q][,Z][,JOB][,COMPQ][,COMPZ][,INFO])
HGEQZ(H,T[,ILO][,IHI][,ALPHA][,BETA][,Q][,Z][,JOB][,COMPQ][,COMPZ][,INFO])
TGEVC(S,P[,HOWMNY][,SELECT][,VL][,VR][,M][,INFO])
TGEXC(A,B[,IFST][,ILST][,Z][,Q][,INFO])
TGSEN(A,B,SELECT[,ALPHAR][,ALPHAI][,BETA][,IJOB][,Q][,Z][,PL][,PR][,DIF][,M][,INFO])
TGSEN(A,B,SELECT[,ALPHA][,BETA][,IJOB][,Q][,Z][,PL][,PR][,DIF][,M][,INFO])
```

```
TGSYL(A,B,C,D,E,F[,IJOB][,TRANS][,SCALE][,DIF][,INFO])
TGSNA(A,B[,S][,DIF][,VL][,VR][,SELECT][,M][,INFO])
GGSVP(A,B,TOLA,TOLB[,K][,L][,U][,V][,Q][,INFO])
TGSJA(A,B,TOLA,TOLB,K,L[,U][,V][,Q][,JOBU][,JOBV][,JOBQ][,ALPHA][,BETA][,NCYCLE][,INFO])
```

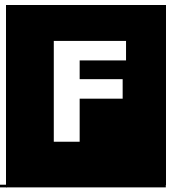
Interfaces of New Functionality

```
GETRF(A[,IPIV][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,IPIV,RCOND,NORM,INFO)
!   Different number for parameter, netlib: 5, mkl: 3
!   Different order for parameter INFO, netlib: 5, mkl: 3
!   Absent mkl parameter: NORM
!   Absent mkl parameter: RCOND
```

```
GBTRF(A[,KL][,M][,IPIV][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,K,M,IPIV,RCOND,NORM,INFO)
!   Different number for parameter, netlib: 7, mkl: 5
!   Different order for parameter INFO, netlib: 7, mkl: 5
!   Absent mkl parameter: NORM
!   Replace parameter name: netlib: K: mkl: KL
!   Absent mkl parameter: RCOND
```

```
POTRF(A[,UPLO][,INFO])
!   Interface netlib95 exists, parameters:
!   netlib: (A,UPLO,RCOND,NORM,INFO)
!   Different number for parameter, netlib: 5, mkl: 3
!   Different order for parameter INFO, netlib: 5, mkl: 3
!   Absent mkl parameter: NORM
!   Absent mkl parameter: RCOND
```

FFTW Interface to Intel® Math Kernel Library



Intel® Math Kernel Library (Intel® MKL) offers FFTW2 and FFTW3 interfaces to Intel MKL Fast Fourier Transform and Trigonometric Transform functionality. The purpose of these interfaces is to enable applications using FFTW (www.fftw.org) to gain performance with Intel MKL without changing the program source code.

Both FFTW2 and FFTW3 interfaces are provided in open source as FFTW wrappers to Intel MKL. For ease of use, FFTW3 interface is also integrated in Intel MKL.

Notational Conventions

This appendix typically employs path notations for Windows* OS.

FFTW2 Interface to Intel® Math Kernel Library

This section describes a collection of wrappers providing FFTW 2.x interface to Intel MKL. The wrappers translate calls to FFTW 2.x functions into the calls of the Intel MKL Fast Fourier Transform interface (FFT interface).

The wrappers correspond to the FFTW version 2.x and the Intel MKL versions 7.0 or higher.

Because of differences between FFTW and Intel MKL FFT functionalities, there are restrictions on using wrappers instead of the FFTW functions. Some FFTW functions have empty wrappers. However, many typical FFTs can be computed using these wrappers.

Refer to chapter 11 "Fourier Transform Functions", for better understanding the effects from the use of the wrappers.

More wrappers may be added in the future to extend FFTW functionality available with Intel MKL.

Wrappers Reference

The section provides a brief reference for the FFTW 2.x C interface. For details please refer to the original FFTW 2.x documentation available at www.fftw.org.

Each FFTW function has its own wrapper. Some of them, which are *not* expressly listed in this section, are empty and do nothing, but they are provided to avoid link errors and satisfy the function calls.

Intel MKL FFT interface operates on both float and double-precision data types.

One-dimensional Complex-to-complex FFTs

The following functions compute a one-dimensional complex-to-complex Fast Fourier transform.

```
fftw_plan fftw_create_plan(int n, fftw_direction dir, int flags);

fftw_plan fftw_create_plan_specific(int n, fftw_direction dir, int flags, fftw_complex
*in, int istride, fftw_complex *out, int ostride);

void fftw(fftw_plan plan, int howmany, fftw_complex *in, int istride, int idist,
fftw_complex *out, int ostride, int odist);

void fftw_one(fftw_plan plan, fftw_complex *in , fftw_complex *out);

void fftw_destroy_plan(fftw_plan plan);
```

Multi-dimensional Complex-to-complex FFTs

The following functions compute a multi-dimensional complex-to-complex Fast Fourier transform.

```
fftwnd_plan fftwnd_create_plan(int rank, const int *n, fftw_direction dir, int flags);
fftwnd_plan fftw2d_create_plan(int nx, int ny, fftw_direction dir, int flags);
fftwnd_plan fftw3d_create_plan(int nx, int ny, int nz, fftw_direction dir, int flags);
fftwnd_plan fftwnd_create_plan_specific(int rank, const int *n, fftw_direction dir, int
flags, fftw_complex *in, int istride, fftw_complex *out, int ostride);
fftwnd_plan fftw2d_create_plan_specific(int nx, int ny, fftw_direction dir, int flags,
fftw_complex *in, int istride, fftw_complex *out, int ostride);
fftwnd_plan fftw3d_create_plan_specific(int nx, int ny, int nz, fftw_direction dir, int
flags, fftw_complex *in, int istride, fftw_complex *out, int ostride);
void fftwnd(fftwnd_plan plan, int howmany, fftw_complex *in, int istride, int idist,
fftw_complex *out, int ostride, int odist);
void fftwnd_one(fftwnd_plan plan, fftw_complex *in, fftw_complex *out);
void fftwnd_destroy_plan(fftwnd_plan plan);
```

One-dimensional Real-to-half-complex/Half-complex-to-real FFTs

Half-complex representation of a conjugate-even symmetric vector of size N in a real array of the same size N consists of $N/2+1$ real parts of the elements of the vector followed by non-zero imaginary parts in the reverse order. Because the Intel MKL FFT interface does not currently support this representation, all wrappers of this kind are empty and do nothing.

Nevertheless, you can perform one-dimensional real-to-complex and complex-to-real transforms using `rfftwnd` functions with *rank*=1.

See Also

Multi-dimensional Real-to-complex/Complex-to-real FFTs

Multi-dimensional Real-to-complex/Complex-to-real FFTs

The following functions compute multi-dimensional real-to-complex and complex-to-real Fast Fourier transforms.

```
rfftwnd_plan rfftwnd_create_plan(int rank, const int *n, fftw_direction dir, int
flags);
rfftwnd_plan rfftw2d_create_plan(int nx, int ny, fftw_direction dir, int flags);
rfftwnd_plan rfftw3d_create_plan(int nx, int ny, int nz, fftw_direction dir, int
flags);
rfftwnd_plan rfftwnd_create_plan_specific(int rank, const int *n, fftw_direction dir,
int flags, fftw_real *in, int istride, fftw_real *out, int ostride);
rfftwnd_plan rfftw2d_create_plan_specific(int nx, int ny, fftw_direction dir, int
flags, fftw_real *in, int istride, fftw_real *out, int ostride);
rfftwnd_plan rfftw3d_create_plan_specific(int nx, int ny, int nz, fftw_direction dir,
int flags, fftw_real *in, int istride, fftw_real *out, int ostride);
void rfftwnd_real_to_complex(rfftwnd_plan plan, int howmany, fftw_real *in, int
istride, int idist, fftw_complex *out, int ostride, int odist);
void rfftwnd_complex_to_real(rfftwnd_plan plan, int howmany, fftw_complex *in, int
istride, int idist, fftw_real *out, int ostride, int odist);
void rfftwnd_one_real_to_complex(rfftwnd_plan plan, fftw_real *in, fftw_complex *out);
```

```
void rfftwnd_one_complex_to_real(rfftwnd_plan plan, fftw_complex *in, fftw_real *out);

void rfftwnd_destroy_plan(rfftwnd_plan plan);
```

Multi-threaded FFTW

This section discusses multi-threaded FFTW wrappers only. MPI FFTW wrappers, available only with Intel MKL for the Linux* and Windows* operating systems, are described in section "MPI FFTW Wrappers".

Unlike the original FFTW interface, every computational function in the FFTW2 interface to Intel MKL provides multithreaded computation by default, with the number of threads defined by the number of processors available on the system (see section "Managing Performance and Memory" in the Intel MKL User's Guide). To limit the number of threads that use the FFTW interface, call the threaded FFTW computational functions:

```
void fftw_threads(int nthreads, fftw_plan plan, int howmany, fftw_complex *in, int
istride, int idist, fftw_complex *out, int ostride, int odist);

void fftw_threads_one(int nthreads, rfftwnd_plan plan, fftw_complex *in, fftw_complex
*out);

...

void rfftwnd_threads_real_to_complex( int nthreads, rfftwnd_plan plan, int howmany,
fftw_real *in, int istride, int idist, fftw_complex *out, int ostride, int odist);
```

Compared to its non-threaded counterpart, every threaded computational function has `threads_` as the second part of its name and additional first parameter *nthreads*. Set the *nthreads* parameter to the thread limit to ensure that the computation requires at most that number of threads.

FFTW Support Functions

The FFTW wrappers provide memory allocation functions to be used with FFTW:

```
void* fftw_malloc(size_t n);

void fftw_free(void* x);
```

The `fftw_malloc` wrapper aligns the memory on a 16-byte boundary.

If `fftw_malloc` fails to allocate memory, it aborts the application. To override this behavior, set a global variable `fftw_malloc_hook` and optionally the complementary variable `fftw_free_hook`:

```
void *(*fftw_malloc_hook) (size_t n);

void (*fftw_free_hook) (void *p);
```

The wrappers use the function `fftw_die` to abort the application in cases when a caller cannot be informed of an error otherwise (for example, in computational functions that return `void`). To override this behavior, set a global variable `fftw_die_hook`:

```
void (*fftw_die_hook)(const char *error_string);

void fftw_die(const char *s);
```

Limitations of the FFTW2 Interface to Intel MKL

The FFTW2 wrappers implement the functionality of only those FFTW functions that Intel MKL can reasonably support. Other functions are provided as no-operation functions, whose only purpose is to satisfy link-time symbol resolution. Specifically, no-operation functions include:

- Real-to-half-complex and respective backward transforms
- Print plan functions
- Functions for importing/exporting/forgetting wisdom
- Most of the FFTW functions not covered by the original FFTW2 documentation

Because the Intel MKL implementation of FFTW2 wrappers does not use plan and plan node structures declared in `fftw.h`, the behavior of an application that relies on the internals of the plan structures defined in that header file is undefined.

FFTW2 wrappers define plan as a set of attributes, such as strides, used to commit the Intel MKL FFT descriptor structure. If an FFTW2 computational function is called with attributes different from those recorded in the plan, the function attempts to adjust the attributes of the plan and recommit the descriptor. Thus, repeated calls of a computational function with the same plan but different strides, distances, and other parameters may be performance inefficient.

Plan creation functions disregard most planner flags passed through the *flags* parameter. These functions take into account only the following values of *flags*:

- FFTW_IN_PLACE

  If this value of *flags* is supplied, the plan is marked so that computational functions using that plan ignore the parameters related to output (*out*, *ostride*, and *odist*). Unlike the original FFTW interface, the wrappers never use the *out* parameter as a scratch space for in-place transforms.
- FFTW_THREADSAFE

  If this value of *flags* is supplied, the plan is marked read-only. An attempt to change attributes of a read-only plan aborts the application.

FFTW wrappers are generally not thread safe. Therefore, do not use the same plan in parallel user threads simultaneously.

Calling Wrappers from Fortran

The FFTW2 wrappers to Intel MKL provide the following subroutines for calling from Fortran:

```
call fftw_f77_create_plan(plan, n, dir, flags)
call fftw_f77(plan, howmany, in, istride, idist, out, ostride, odist)
call fftw_f77_one(plan, in, out)
call fftw_f77_threads(nthreads, plan, howmany, in, istride, idist, out, ostride,
odist)
call fftw_f77_threads_one(nthreads, plan, in, out)
call fftw_f77_destroy_plan(plan)

call fftwnd_f77_create_plan(plan, rank, n, dir, flags)
call fftw2d_f77_create_plan(plan, nx, ny, dir, flags)
call fftw3d_f77_create_plan(plan, nx, ny, nz, dir, flags)
call fftwnd_f77(plan, howmany, in, istride, idist, out, ostride, odist)
call fftwnd_f77_one(plan, in, out)
call fftwnd_f77_threads(nthreads, plan, howmany, in, istride, idist, out, ostride,
odist)
call fftwnd_f77_threads_one(nthreads, plan, in, out)
call fftwnd_f77_destroy_plan(plan)

call rfftw_f77_create_plan(plan, n, dir, flags)
call rfftw_f77(plan, howmany, in, istride, idist, out, ostride, odist)
call rfftw_f77_one(plan, in, out)
call rfftw_f77_threads(nthreads, plan, howmany, in, istride, idist, out, ostride,
odist)
call rfftw_f77_threads_one(nthreads, plan, in, out)
call rfftw_f77_destroy_plan(plan)

call rfftwnd_f77_create_plan(plan, rank, n, dir, flags)
```

```
call rfftw2d_f77_create_plan(plan, nx, ny, dir, flags)
call rfftw3d_f77_create_plan(plan, nx, ny, nz, dir, flags)
call rfftwnd_f77_complex_to_real(plan, howmany, in, istride, idist, out, ostride,
odist)
call rfftwnd_f77_one_complex_to_real (plan, in, out)
call rfftwnd_f77_real_to_complex(plan, howmany, in, istride, idist, out, ostride,
odist)
call rfftwnd_f77_one_real_to_complex (plan, in, out)
call rfftwnd_f77_threads_complex_to_real(nthreads, plan, howmany, in, istride, idist,
out, ostride, odist)
call rfftwnd_f77_threads_one_complex_to_real(nthreads, plan, in, out)
call rfftwnd_f77_threads_real_to_complex(nthreads, plan, howmany, in, istride, idist,
out, ostride, odist)
call rfftwnd_f77_threads_one_real_to_complex(nthreads, plan, in, out)
call rfftwnd_f77_destroy_plan(plan)
call fftw_f77_threads_init(info)
```

The FFTW Fortran functions are actually the wrappers to FFTW C functions. So, their functionality and limitations are the same as of the corresponding C wrappers.

See Also

Wrappers Reference

Limitations of the FFTW2 Interface to Intel MKL

Installation

Wrappers are delivered as source code, which you must compile to build the wrapper library. Then you can substitute the wrapper and Intel MKL libraries for the FFTW library. The source code for the wrappers and makefiles with the wrapper list files are located in the `.\interfaces\fftw2xc` and `.\interfaces\fftw2xf` subdirectory in the Intel MKL directory for C and Fortran wrappers, respectively.

Creating the Wrapper Library

Two header files are used to compile the C wrapper library: `fftw2_mkl.h` and `fftw.h`. The `fftw2_mkl.h` file is located in the `.\interfaces\fftw2xc\wrappers` subdirectory in the Intel MKL directory.

Three header files are used to compile the Fortran wrapper library: `fftw2_mkl.h`, `fftw2_f77_mkl.h`, and `fftw.h`. The `fftw2_mkl.h` and `fftw2_f77_mkl.h` files are located in the `.\interfaces\fftw2xf\wrappers` subdirectory in the Intel MKL directory.

The file `fftw.h`, used to compile libraries for both interfaces and located in the `.\include\fftw` subdirectory in the Intel MKL directory, slightly differs from the original FFTW (www.fftw.org) header file `fftw.h`.

The source code for the wrappers, makefiles, and function list files are located in subdirectories `.\interfaces\fftw2xc` and `.\interfaces\fftw2xf` in the Intel MKL directory for C and Fortran wrappers, respectively.

A wrapper library contains C or Fortran wrappers for complex and real transforms in a serial and multi-threaded mode for one of the two data types (`double` or `float`). A makefile parameter manages the data type.

The makefile parameters specify the platform (required), compiler, and data precision. Specifying the platform is required. The makefile comment heading provides the exact description of these parameters.

Because a C compiler builds the Fortran wrapper library, function names in the wrapper library and Fortran object module may be different. The file `fftw2_f77_mkl.h` in the `.\interfaces\fftw2xf\source` subdirectory in the Intel MKL directory defines function names according to the names in the Fortran module. If a required name is missing in the file, you can modify the file to add the name before building the library.

To build the library, run the `make` command on Linux* OS and Mac OS* X or the `nmake` command on Windows* OS with appropriate parameters.

For example, the command

```
make libintel64
```

builds on Linux OS a double-precision wrapper library for Intel® 64 architecture based applications using the Intel® C++ Compiler or the Intel® Fortran Compiler version 9.1 or higher (compilers and data precision are chosen by default.).

Each makefile creates the library in the directory with the Intel MKL libraries corresponding to the used platform. For example, `./lib/ia32` (on Linux OS and Mac OS X) or `.\lib\ia32` (on Windows* OS).

In the wrapper library names, the suffix corresponds to the used compiler, the letter `"f"` precedes the underscore for Fortran, and the letter `"c"` precedes the underscore for C.

For example,

`fftw2xf_intel.lib` (on Windows OS); `libfftw2xf_intel.a` (on Linux OS and Mac OS X);

`fftw2xc_intel.lib` (on Windows OS); `libfftw2xc_intel.a` (on Linux OS and Mac OS X);

`fftw2xc_ms.lib` (on Windows OS); `libfftw2xc_gnu.a` (on Linux OS and Mac OS X).

Application Assembling

Use the necessary original FFTW (www.fftw.org) header files without any modifications. Use the created wrapper library and the Intel MKL library instead of the FFTW library.

Running Examples

Intel MKL provides examples to demonstrate how to use the MPI FFTW wrapper library. The source code for the examples, makefiles used to run them, and the example list files are located in the `.\examples\fftw2xc` and `.\examples\fftw2xf` subdirectories in the Intel MKL directory for C and Fortran, respectively. To build examples, several additional files are needed: `fftw.h`, `fftw_threads.h`, `rfftw.h`, `rfftw_threads.h`, and `fftw_f77.I`. These files are distributed with permission from FFTW and are available in `.\include\fftw`. The original files can also be found in FFTW 2.1.5 at http://www.fftw.org/download.html.

An example makefile uses the `function` parameter in addition to the parameters that the respective wrapper library makefile uses (see Creating a Wrapper Library). The makefile comment heading provides the exact description of these parameters.

An example makefile normally invokes examples. However, if the appropriate wrapper library is not yet created, the makefile first builds the library the same way as the wrapper library makefile does and then proceeds to examples.

If the parameter `function=<example_name>` is defined, only the specified example runs. Otherwise, all examples from the appropriate subdirectory run. The subdirectory `.\_results` is created, and the results are stored there in the `<example_name>.res` files.

MPI FFTW Wrappers

MPI FFTW wrappers for FFTW 2 are available only with Intel® MKL for the Linux* and Windows* operating systems.

MPI FFTW Wrappers Reference

The section provides a reference for MPI FFTW C interface.

Complex MPI FFTW

Complex One-dimensional MPI FFTW Transforms

```
fftw_mpi_plan fftw_mpi_create_plan(MPI_Comm comm, int n, fftw_direction dir, int flags);
```

```
void fftw_mpi(fftw_mpi_plan p, int n_fields, fftw_complex *local_data, fftw_complex *work);
```

```
void fftw_mpi_local_sizes(fftw_mpi_plan p, int *local_n, int *local_start, int *local_n_after_transform, int *local_start_after_transform, int *total_local_size);
```

```
void fftw_mpi_destroy_plan(fftw_mpi_plan plan);
```

Argument restrictions:

- Supported values of *flags* are FFTW_ESTIMATE, FFTW_MEASURE, FFTW_SCRAMBLED_INPUT and FFTW_SCRAMBLED_OUTPUT. The same algorithm corresponds to all these values of the flags parameter. If any other *flags* value is supplied, the wrapper library reports an error '*CDFT error in wrapper: unknown flags*'.
- The only supported value of *n_fields* is 1.

Complex Multi-dimensional MPI FFTW Transforms

```
fftwnd_mpi_plan fftw2d_mpi_create_plan(MPI_Comm comm, int nx, int ny, fftw_direction dir, int flags);
```

```
fftwnd_mpi_plan fftw3d_mpi_create_plan(MPI_Comm comm, int nx, int ny, int nz, fftw_direction dir, int flags);
```

```
fftwnd_mpi_plan fftwnd_mpi_create_plan(MPI_Comm comm, int dim, int *n, fftw_direction dir, int flags);
```

```
void fftwnd_mpi(fftwnd_mpi_plan p, int n_fields, fftw_complex *local_data, fftw_complex *work, fftwnd_mpi_output_order output_order);
```

```
void fftwnd_mpi_local_sizes(fftwnd_mpi_plan p, int *local_nx, int *local_x_start, int *local_ny_after_transpose, int *local_y_start_after_transpose, int *total_local_size);
```

```
void fftwnd_mpi_destroy_plan(fftwnd_mpi_plan plan);
```

Argument restrictions:

- Supported values of *flags* are FFTW_ESTIMATE and FFTW_MEASURE. If any other value of *flags* is supplied, the wrapper library reports an error '*CDFT error in wrapper: unknown flags*'.
- The only supported value of *n_fields* is 1.

Real MPI FFTW

Real-to-Complex MPI FFTW Transforms

```
rfftwnd_mpi_plan rfftw2d_mpi_create_plan(MPI_Comm comm, int nx, int ny, fftw_direction dir, int flags);
```

```
rfftwnd_mpi_plan rfftw3d_mpi_create_plan(MPI_Comm comm, int nx, int ny, int nz, fftw_direction dir, int flags);
```

```
rfftwnd_mpi_plan rfftwnd_mpi_create_plan(MPI_Comm comm, int dim, int *n, fftw_direction dir, int flags);
```

```
void rfftwnd_mpi(rfftwnd_mpi_plan p, int n_fields, fftw_real *local_data, fftw_real *work, fftwnd_mpi_output_order output_order);
```

```
void rfftwnd_mpi_local_sizes(rfftwnd_mpi_plan p, int *local_nx, int *local_x_start, int
*local_ny_after_transpose, int *local_y_start_after_transpose, int *total_local_size);

void rfftwnd_mpi_destroy_plan(rfftwnd_mpi_plan plan);
```

Argument restrictions:

- Supported values of *flags* are FFTW_ESTIMATE and FFTW_MEASURE. If any other value of *flags* is supplied, the wrapper library reports an error '*CDFT error in wrapper: unknown flags*'.
- The only supported value of *n_fields* is 1.

- Function *rfftwnd_mpi_create_plan* can be used for both one-dimensional and multi-dimensional transforms.
- Both values of the *output_order* parameter are supported: FFTW_NORMAL_ORDER and FFTW_TRANSPOSED_ORDER.

Creating MPI FFTW Wrapper Library

The source code for the wrappers, makefile, and wrapper list file are located in the .\interfaces\fftw2x_cdft subdirectory in the Intel MKL directory.

A wrapper library contains C wrappers for Complex One-dimensional MPI FFTW Transforms and Complex Multi-dimensional MPI FFTW Transforms. The library also contains empty C wrappers for Real Multi-dimensional MPI FFTW Transforms. For details, see MPI FFTW Wrappers Reference.

The makefile parameters specify the platform (required), compiler, and data precision. Specifying the platform is required. The makefile comment heading provides the exact description of these parameters.

To build the library, run the make command on Linux* OS and Mac OS* X or the nmake command on Windows* OS with appropriate parameters.

For example, the command

```
make libintel64
```

builds on Linux OS a double-precision wrapper library for Intel® 64 architecture based applications using Intel MPI 2.0 and the Intel® C++ Compiler version 9.1 or higher (compilers and data precision are chosen by default.).

The makefile creates the wrapper library in the directory with the Intel MKL libraries corresponding to the used platform. For example, ./lib/ia32 (on Linux OS) or .\lib\ia32 (on Windows* OS).

In the wrapper library names, the suffix corresponds to the used data precision. For example,

fftw2x_cdft_SINGLE.lib on Windows OS;

libfftw2x_cdft_DOUBLE.a on Linux OS.

Application Assembling with MPI FFTW Wrapper Library

Use the necessary original FFTW (www.fftw.org) header files without any modifications. Use the created MPI FFTW wrapper library and the Intel MKL library instead of the FFTW library.

Running Examples

There are some examples that demonstrate how to use the MPI FFTW wrapper library for FFTW2. The source C code for the examples, makefiles used to run them, and the example list files are located in the .\examples\fftw2x_cdft subdirectory in the Intel MKL directory. To build examples, one additional file fftw_mpi.h is needed. This file is distributed with permission from FFTW and is available in .\include\fftw. The original file can also be found in FFTW 2.1.5 at http://www.fftw.org/download.html.

Parameters for the example makefiles are described in the makefile comment headings and are similar to the wrapper library makefile parameters (see Creating MPI FFTW Wrapper Library).

The table below lists examples available in the .\examples\fftw2x_cdft\source subdirectory.

Examples of MPI FFTW Wrappers

Source file for the example	Description
`wrappers_c1d.c`	One-dimensional Complex MPI FFTW transform, using `plan = fftw_mpi_create_plan(...)`
`wrappers_c2d.c`	Two-dimensional Complex MPI FFTW transform, using `plan = fftw2d_mpi_create_plan(...)`
`wrappers_c3d.c`	Three-dimensional Complex MPI FFTW transform, using `plan = fftw3d_mpi_create_plan(...)`
`wrappers_c4d.c`	Four-dimensional Complex MPI FFTW transform, using `plan = fftwnd_mpi_create_plan(...)`
`wrappers_r1d.c`	One-dimensional Real MPI FFTW transform, using `plan = rfftw_mpi_create_plan(...)`
`wrappers_r2d.c`	Two-dimensional Real MPI FFTW transform, using `plan = rfftw2d_mpi_create_plan(...)`
`wrappers_r3d.c`	Three-dimensional Real MPI FFTW transform, using `plan = rfftw3d_mpi_create_plan(...)`
`wrappers_r4d.c`	Four-dimensional Real MPI FFTW transform, using `plan = rfftwnd_mpi_create_plan(...)`

FFTW3 Interface to Intel® Math Kernel Library

This section describes a collection of FFTW3 wrappers to Intel MKL. The wrappers translate calls of FFTW3 functions to the calls of the Intel MKL Fourier transform (FFT) or Trigonometric Transform (TT) functions. The purpose of FFTW3 wrappers is to enable developers whose programs currently use the FFTW3 library to gain performance with the Intel MKL Fourier transforms without changing the program source code.

The wrappers correspond to the FFTW release 3.2 and the Intel MKL releases starting with 10.2. **For a detailed description of FFTW interface, refer to www.fftw.org**. For a detailed description of Intel MKL FFT and TT functionality the wrappers use, see chapter 11 and section "Trigonometric Transform Routines" in chapter 13, respectively.

The FFTW3 wrappers provide a limited functionality compared to the original FFTW 3.2 library, because of differences between FFTW and Intel MKL FFT and TT functionality. This section describes limitations of the FFTW3 wrappers and hints for their usage. Nevertheless, many typical FFT tasks can be performed using the FFTW3 wrappers to Intel MKL. More functionality may be added to the wrappers and Intel MKL in the future to reduce the constraints of the FFTW3 interface to Intel MKL.

The FFTW3 wrappers are integrated in Intel MKL. The only change required to use Intel MKL through the FFTW3 wrappers is to link your application using FFTW3 against Intel MKL.

A reference implementation of the FFTW3 wrappers is also provided in open source. You can find it in the `interfaces` directory of the Intel MKL distribution. You can use the reference implementation to create your own wrapper library (see Building Your Own Wrapper Library)

Using FFTW3 Wrappers

The FFTW3 wrappers are a set of functions and data structures depending on one another. The wrappers are not designed to provide the interface on a function-per-function basis. Some FFTW3 wrapper functions are empty and do nothing, but they are present to avoid link errors and satisfy function calls.

This manual does not list the declarations of the functions that the FFTW3 wrappers provide (you can find the declarations in the `fftw3.h` header file). Instead, this section comments particular limitations of the wrappers and provides usage hints:

- The FFTW3 wrappers do not support long double precision because Intel MKL FFT functions operate only on single- and double-precision floating-point data types (`float` and `double`, respectively). Therefore the functions with prefix `fftwl_`, supporting the `long double` data type, are not provided.
- The wrappers provide equivalent implementation for double- and single-precision functions (those with prefixes `fftw_` and `fftwf_`, respectively). So, all these comments equally apply to the double- and single-precision functions and will refer to functions with prefix `fftw_`, that is, double-precision functions, for brevity.
- The FFTW3 interface that the wrappers provide is defined in header files `fftw3.h` and `fftw3.f`. These files are borrowed from the FFTW3.2 package and distributed within Intel MKL with permission. Additionally, files `fftw3_mkl.h`, `fftw3_mkl.f`, and `fftw3_mkl_f77.h` define supporting structures, supplementary constants and macros, and expose Fortran interface in C.
- Actual functionality of the plan creation wrappers is implemented in guru64 set of functions. Basic interface, advanced interface, and guru interface plan creation functions call the guru64 interface functions. Thus, all types of the FFTW3 plan creation interface in the wrappers are functional.
- Plan creation functions may return a `NULL` plan, indicating that the functionality is not supported. So, please carefully check the result returned by plan creation functions in your application. In particular, the following problems return a `NULL` plan:
  - c2r and r2c problems with a split storage of complex data.
  - r2r problems with *kind* values `FFTW_R2HC`, `FFTW_HC2R`, and `FFTW_DHT`. The only supported r2r kinds are even/odd DFTs (sine/cosine transforms).
  - Multidimensional r2r transforms.
  - Transforms of multidimensional vectors. That is, the only supported values for parameter *howmany_rank* in guru and guru64 plan creation functions are 0 and 1.
  - Multidimensional transforms with *rank* > MKL_MAXRANK.
- The `MKL_RODFT00` value of the *kind* parameter is introduced by the FFTW3 wrappers. For better performance, you are strongly encouraged to use this value rather than `FFTW_RODFT00`. To use this *kind* value, provide an extra first element equal to 0.0 for the input/output vectors. Consider the following example:

```
plan1 = fftw_plan_r2r_1d(n, in1, out1, FFTW_RODFT00, FFTW_ESTIMATE);
plan2 = fftw_plan_r2r_1d(n, in2, out2, MKL_RODFT00, FFTW_ESTIMATE);
```

  Both plans perform the same transform, except that the *in2/out2* arrays have one extra zero element at location 0. For example, if *n*=3, *in1*={*x,y,z*} and *out1*={*u,v,w*}, then *in2*={0,*x,y,z*} and *out2*={0,*u,v,w*}.
- The *flags* parameter in plan creation functions is always ignored. The same algorithm is used regardless of the value of this parameter. In particular, *flags* values `FFTW_ESTIMATE`, `FFTW_MEASURE`, etc. have no effect.
- For multithreaded plans, use normal sequence of calls to the `fftw_init_threads()` and `fftw_plan_with_nthreads()` functions (refer to FFTW documentation).
- FFTW3 wrappers are not fully thread safe. If the new-array execute functions, such as `fftw_execute_dft()`, share the same plan from parallel user threads, set the number of the sharing threads before creation of the plan. For this purpose, the FFTW3 wrappers provide a header file `fftw3_mkl.h`, which defines a global structure `fftw3_mkl` with a field to be set to the number of sharing threads. Below is an example of setting the number of sharing threads:

```
#include "fftw3.h"
#include "fftw3_mkl.h"
fftw3_mkl.number_of_user_threads = 4;
plan = fftw_plan_dft(...);
```

- Memory allocation function `fftw_malloc` returns memory aligned at a 16-byte boundary. You must free the memory with `fftw_free`.
- The FFTW3 wrappers to Intel MKL use the 32-bit `int` type in both LP64 and ILP64 interfaces of Intel MKL. Use guru64 FFTW3 interfaces for 64-bit sizes.
- Fortran wrappers (see Calling Wrappers from Fortran) use the `INTEGER` type, which is 32-bit in LP64 interfaces and 64-bit in ILP64 interfaces.

- The wrappers typically indicate a problem by returning a NULL plan. In a few cases, the wrappers may report a descriptive message of the problem detected. By default the reporting is turned off. To turn it on, set variable fftw3_mkl.verbose to a non-zero value, for example:

```
#include "fftw3.h"
#include "fftw3_mkl.h"
fftw3_mkl.verbose = 0;
plan = fftw_plan_r2r(...);
```

- The following functions are empty:
  - For saving, loading, and printing plans
  - For saving and loading wisdom
  - For estimating arithmetic cost of the transforms.
- Do not use macro FFTW_DLL with the FFTW3 wrappers to Intel MKL.
- Do not use negative stride values. Though FFTW3 wrappers support negative strides in the part of advanced and guru FFTW interface, the underlying implementation does not.

Calling Wrappers from Fortran

Intel MKL also provides Fortran 77 interfaces of the FFTW3 wrappers. The Fortran wrappers are available for all FFTW3 interface functions and are based on C interface of the FFTW3 wrappers. Therefore they have the same functionality and restrictions as the corresponding C interface wrappers.

The Fortran wrappers use the default INTEGER type for integer arguments. The default INTEGER is 32-bit in Intel MKL LP64 interfaces and 64-bit in ILP64 interfaces. Argument *plan* in a Fortran application must have type INTEGER*8.

The wrappers that are double-precision subroutines have prefix dfftw_, single-precision subroutines have prefix sfftw_ and provide an equivalent functionality. Long double subroutines (with prefix lfftw_) are not provided.

The Fortran FFTW3 wrappers use the default Intel® Fortran compiler convention for name decoration. If your compiler uses a different convention, or if you are using compiler options affecting the name decoration (such as /Qlowercase), you may need to compile the wrappers from sources, as described in section Building Your Own Wrapper Library.

For interoperability with C, the declaration of the Fortran FFTW3 interface is provided in header file include/fftw/fftw3_mkl_f77.h.

You can call Fortran wrappers from a FORTRAN 77 or Fortran 90 application, although Intel MKL does not provide a Fortran 90 module for the wrappers. For a detailed description of the FFTW Fortran interface, refer to FFTW3 documentation (www.fftw.org).

The following example illustrates calling the FFTW3 wrappers from Fortran:

```
INTEGER*8 plan
INTEGER N
INCLUDE 'fftw3.f'
COMPLEX*16 IN(*), OUT(*)
!...initialize array IN
CALL DFFTW_PLAN_DFT_1D(PLAN, N, IN, OUT, -1, FFTW_ESTIMATE)
IF (PLAN .EQ. 0) STOP
CALL DFFTW_EXECUTE
!...result is in array OUT
```

Building Your Own Wrapper Library

The FFTW3 wrappers to Intel MKL are delivered both integrated in Intel MKL and as source code, which can be compiled to build a standalone wrapper library with exactly the same functionality. Normally you do not need to build the wrappers yourself. However, if your Fortran application is compiled with a compiler that uses a different name decoration than the Intel® Fortran compiler or if you are using compiler options altering the Fortran name decoration, you may need to build the wrappers that use the appropriate name changing convention.

The source code for the wrappers, makefiles, and function list files are located in subdirectories .\interfaces\fftw3xc and .\interfaces\fftw3xf in the Intel MKL directory for C and Fortran wrappers, respectively.

To build the wrappers,

1. Change the current directory to the wrapper directory
2. Run the make command on Linux* OS and Mac OS* X or the nmake command on Windows* OS with a required target and optionally several parameters.

The target, that is, one of {libia32, libintel64}, defines the platform architecture, and the other parameters facilitate selection of the compiler, size of the default INTEGER type, and placement of the resulting wrapper library. You can find a detailed and up-to-date description of the parameters in the makefile.

In the following example, the make command is used to build the FFTW3 Fortran wrappers to MKL for use from the GNU g77 Fortran compiler on Linux OS based on Intel® 64 architecture:

```
cd interfaces/fftw3xf
make libintel64 compiler=gnu fname=a_name__ install_to=/my/path
```

This command builds the wrapper library using the GNU gcc compiler, decorates the name with the second underscore, and places the result, named libfftw3xf_gcc.a, into directory /my/path. The name of the resulting library is composed of the name of the compiler used and may be changed by an optional parameter.

Building an Application

Normally, the only change needed to build your application with FFTW3 wrappers replacing original FFTW library is to add Intel MKL at the link stage (see section *"Linking Your Application with Intel® Math Kernel Library" in the Intel MKL User's Guide*).

If you recompile your application, add subdirectory include\fftw to the search path for header files to avoid FFTW3 version conflicts.

Sometimes, you may have to modify your application according to the following recommendations:

- The application requires

  `#include "fftw3.h"`,

  which it probably already includes.
- The application does not require

  `#include "mkl_dfti.h"` .
- The application does not require

  `#include "fftw3_mkl.h"` .

  It is required only in case you want to use the MKL_RODFT00 constant.
- If the application does not check whether a NULL plan is returned by plan creation functions, this check must be added, because the FFTW3 to Intel MKL wrappers do not provide 100% of FFTW3 functionality.
- If the application is threaded, take care about shared plans, because the execute functions in the wrappers are not thread safe, unlike the original FFTW3 functions. See a note about setting fftw3_mkl.number_of_user_threads in section "Using FFTW3 wrappers".

Running Examples

There are some examples that demonstrate how to use the wrapper library. The source code for the examples, makefiles used to run them, and the example list files are located in the .\examples\fftw3xc and .\examples\fftw3xf subdirectories in the Intel MKL directory. To build Fortran examples, one additional file fftw3.f is needed. This file is distributed with permission from FFTW and is available in the .\include\fftw subdirectory of the Intel MKL directory. The original file can also be found in FFTW 3.2 at http://www.fftw.org/download.html.

Example makefile parameters are similar to the wrapper library makefile parameters. Example makefiles normally build and invoke the examples. If the parameter `function=<example_name>` is defined, then only the specified example will run. Otherwise, all examples will be executed. Results of running the examples are saved in subdirectory `.\_results` in files with extension `.res`.

For detailed information about options for the example makefile, refer to the makefile.

MPI FFTW Wrappers

This section describes a collection of MPI FFTW wrappers to Intel® MKL. The wrappers correspond to the FFTW 3.3 Alpha release and the Intel MKL releases starting with 10.3. For a detailed description of the MPI FFTW interface, refer to www.fftw.org.

MPI FFTW wrappers are available only with Intel MKL for the Linux* and Windows* operating systems.

These wrappers translate calls of MPI FFTW functions to the calls of the Intel MKL cluster Fourier transform (CFFT) functions. The purpose of the wrappers is to enable users of MPI FFTW functions improve performance of the applications without changing the program source code.

Although the MPI FFTW wrappers provide less functionality than the original FFTW 3.3 because of differences between MPI FFTW and Intel MKL CFFT, the wrappers cover many typical CFFT use cases.

The MPI FFTW wrappers are provided as source code. To use the wrappers, you need to build your own wrapper library (see Building Your Own Wrapper Library).

See Also

Cluster FFT Functions

Building Your Own Wrapper Library

The MPI FFTW wrappers for FFTW3 are delivered as source code, which can be compiled to build a wrapper library.

The source code for the wrappers, makefiles, and function list files are located in subdirectory `.\interfaces fftw3x_cdft` in the Intel MKL directory.

To build the wrappers,

1. Change the current directory to the wrapper directory
2. Run the `make` command on Linux* OS or the `nmake` command on Windows* OS with a required target and optionally several parameters.

The target, that is, one of {`libia32, libintel64`}, defines the platform architecture, and the other parameters specify the compiler, size of the default `INTEGER` type, as well as the name and placement of the resulting wrapper library. You can find a detailed and up-to-date description of the parameters in the makefile.

In the following example, the `make` command is used to build the MPI FFTW wrappers to Intel MKL for use from the GNU C compiler on Linux OS based on Intel® 64 architecture:

```
cd interfaces/fftw3x_cdft
make libintel64 compiler=gnu mpi=openmpi INSTALL_DIR=/my/path
```

This command builds the wrapper library using the GNU gcc compiler so that the final user executable can use Open MPI and places the result, named `libfftw3x_cdft_DOUBLE.a`, into directory `/my/path`.

Building an Application

Normally, the only change needed to build your application with MPI FFTW wrappers replacing original FFTW3 library is to add Intel MKL and the wrapper library at the link stage (see section "*Linking Your Application with Intel® Math Kernel Library*" in the *Intel MKL User's Guide*).

When you are recompiling your application, add subdirectory `include\fftw` to the search path for header files to avoid FFTW3 version conflicts.

Running Examples

There are some examples that demonstrate how to use the MPI FFTW wrapper library for FFTW3. The source code for the examples, makefiles used to run them, and the example list files are located in the `.\examples\fftw3x_cdft` subdirectory in the Intel MKL directory.

Example makefile parameters are similar to the wrapper library makefile parameters. Example makefiles normally build and invoke the examples. Results of running the examples are saved in subdirectory `.\_results` in files with extension `.res`.

For detailed information about options for the example makefile, refer to the makefile.

See Also

Building Your Own Wrapper Library

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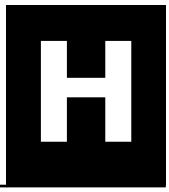
For a reference implementation of BLAS, sparse BLAS, LAPACK, and ScaLAPACK packages (without platform-specific optimizations) visit www.netlib.org

Glossary



A^H	Denotes the conjugate transpose of a general matrix A. *See also* conjugate matrix.
A^T	Denotes the transpose of a general matrix A. *See also* transpose.
band matrix	A general m-by-n matrix A such that $a_{ij} = 0$ for $\|i - j\| > l$, where $1 < l < \min(m, n)$. For example, any tridiagonal matrix is a band matrix.
band storage	A special storage scheme for band matrices. A matrix is stored in a two-dimensional array: columns of the matrix are stored in the corresponding columns of the array, and *diagonals* of the matrix are stored in rows of the array.
BLAS	Abbreviation for Basic Linear Algebra Subprograms. These subprograms implement vector, matrix-vector, and matrix-matrix operations.
BRNG	Abbreviation for Basic Random Number Generator. Basic random number generators are pseudorandom number generators imitating i.i.d. random number sequences of uniform distribution. Distributions other than uniform are generated by applying different transformation techniques to the sequences of random numbers of uniform distribution.
BRNG registration	Standardized mechanism that allows a user to include a user-designed BRNG into the VSL and use it along with the predefined VSL basic generators.
Bunch-Kaufman factorization	Representation of a real symmetric or complex Hermitian matrix A in the form $A = PUDU^{H}P^{T}$ (or $A = PLDL^{H}P^{T}$) where P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D.
c	When found as the first letter of routine names, c indicates the usage of single-precision complex data type.
CBLAS	C interface to the BLAS. *See* BLAS.
CDF	Cumulative Distribution Function. The function that determines probability distribution for univariate or multivariate random variable X. For univariate distribution the cumulative distribution function is the function of real argument x, which for every x takes a value equal to probability of the event A: $X \le x$. For multivariate distribution the cumulative distribution function is the function of a real vector $x = (x_1, x_2, \ldots, x_n)$, which, for every x, takes a value equal to probability of the event $A = (X_1 \le x_1 \ \& \ X_2 \le x_2, \ \& \ \ldots, \ \& \ X_n \le x_n)$.
Cholesky factorization	Representation of a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A in the form $A = U^{H}U$ or $A = LL^{H}$, where L is a lower triangular matrix and U is an upper triangular matrix.
condition number	The number $\kappa(A)$ defined for a given square matrix A as follows: $\kappa(A) = \|\|A\|\| \ \|\|A^{-1}\|\|$.
conjugate matrix	The matrix A^{H} defined for a given general matrix A as follows: $(A^{H})_{ij} = (a_{ji})^{*}$.

conjugate number	The conjugate of a complex number $z = a + bi$ is $z^* = a - bi$.
d	When found as the first letter of routine names, d indicates the usage of double-precision real data type.
dot product	The number denoted $x \cdot y$ and defined for given vectors x and y as follows: $x \cdot y = \Sigma_i x_i y_i$. Here x_i and y_i stand for the i-th elements of x and y, respectively.
double precision	A floating-point data type. On Intel® processors, this data type allows you to store real numbers x such that $2.23*10^{-308} < \lvert x \rvert < 1.79*10^{308}$. For this data type, the machine precision ε is approximately 10^{-15}, which means that double-precision numbers usually contain no more than 15 significant decimal digits. For more information, refer to *Intel® 64 and IA-32 Architectures Software Developer's Manual, Volume 1: Basic Architecture*.
eigenvalue	*See* eigenvalue problem.
eigenvalue problem	A problem of finding non-zero vectors x and numbers λ (for a given square matrix A) such that $Ax = \lambda x$. Here the numbers λ are called the eigenvalues of the matrix A and the vectors x are called the eigenvectors of the matrix A.
eigenvector	*See* eigenvalue problem.
elementary reflector(Householder matrix)	Matrix of a general form $H = I - \tau v v^T$, where v is a column vector and τ is a scalar. In LAPACK elementary reflectors are used, for example, to represent the matrix Q in the QR factorization (the matrix Q is represented as a product of elementary reflectors).
factorization	Representation of a matrix as a product of matrices. *See also* Bunch-Kaufman factorization, Cholesky factorization, LU factorization, LQ factorization, QR factorization, Schur factorization.
FFTs	Abbreviation for Fast Fourier Transforms. *See* Chapter 11 of this book.
full storage	A storage scheme allowing you to store matrices of any kind. A matrix A is stored in a two-dimensional array a, with the matrix element a_{ij} stored in the array element $a(i,j)$.
Hermitian matrix	A square matrix A that is equal to its conjugate matrix A^H. The conjugate A^H is defined as follows: $(A^H)_{ij} = (a_{ji})^*$.
I	*See* identity matrix.
identity matrix	A square matrix I whose diagonal elements are 1, and off-diagonal elements are 0. For any matrix A, $AI = A$ and $IA = A$.
i.i.d.	Independent Identically Distributed.
in-place	Qualifier of an operation. A function that performs its operation in-place takes its input from an array and returns its output to the same array.
Intel MKL	Abbreviation for Intel® Math Kernel Library.
inverse matrix	The matrix denoted as A^{-1} and defined for a given square matrix A as follows: $AA^{-1} = A^{-1}A = I$. A^{-1} does not exist for singular matrices A.
LQ factorization	Representation of an m-by-n matrix A as $A = LQ$ or $A = (L\ 0)Q$. Here Q is an n-by-n orthogonal (unitary) matrix. For $m \le n$, L is an m-by-m lower triangular matrix with real diagonal elements; for $m > n$, where L_1 is an n-by-n lower triangular matrix, and L_2 is a rectangular matrix.

LU factorization	Representation of a general *m*-by-*n* matrix *A* as *A = PLU*, where *P* is a permutation matrix, *L* is lower triangular with unit diagonal elements (lower trapezoidal if *m > n*) and *U* is upper triangular (upper trapezoidal if *m < n*).
machine precision	The number ε determining the precision of the machine representation of real numbers. For Intel© architecture, the machine precision is approximately 10^{-7} for single-precision data, and approximately 10^{-15} for double-precision data. The precision also determines the number of significant decimal digits in the machine representation of real numbers. *See also* double precision and single precision.
MPI	Message Passing Interface. This standard defines the user interface and functionality for a wide range of message-passing capabilities in parallel computing.
MPICH	A freely available, portable implementation of MPI standard for message-passing libraries.
orthogonal matrix	A real square matrix *A* whose transpose and inverse are equal, that is, $A^T = A^{-1}$, and therefore $AA^T = A^TA = I$. All eigenvalues of an orthogonal matrix have the absolute value 1.
packed storage	A storage scheme allowing you to store symmetric, Hermitian, or triangular matrices more compactly. The upper or lower triangle of a matrix is packed by columns in a one-dimensional array.
PDF	Probability Density Function. The function that determines probability distribution for univariate or multivariate continuous random variable *X*. The probability density function $f(x)$ is closely related with the cumulative distribution function $F(x)$. For univariate distribution the relation is $$F(x) = \int_{-\infty}^{x} f(t)dt .$$ For multivariate distribution the relation is $$F(x_1,x_2, \dots ,x_n) = \int_{-\infty}^{x_1}\int_{-\infty}^{x_2}\dots\int_{-\infty}^{x_n} f(t_1,t_2, \dots ,t_n)dt_1dt_2 \dots dt_n$$
positive-definite matrix	A square matrix *A* such that $Ax \cdot x > 0$ for any non-zero vector *x*. Here · denotes the dot product.
pseudorandom number generator	A completely deterministic algorithm that imitates truly random sequences.
QR factorization	Representation of an *m*-by-*n* matrix *A* as *A = QR*, where *Q* is an *m*-by-*m* orthogonal (unitary) matrix, and *R* is *n*-by-*n* upper triangular with real diagonal elements (if $m \geq n$) or trapezoidal (if *m < n*) matrix.
random stream	An abstract source of independent identically distributed random numbers of uniform distribution. In this manual a random stream points to a structure that uniquely defines a random number sequence generated by a basic generator associated with a given random stream.
RNG	Abbreviation for Random Number Generator. In this manual the term "random number generators" stands for pseudorandom number generators, that is, generators based on completely deterministic algorithms imitating truly random sequences.

Rectangular Full Packed (RFP) storage	A storage scheme combining the full and packed storage schemes for the upper or lower triangle of the matrix. This combination enables using half of the full storage as packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels as the full storage.
s	When found as the first letter of routine names, s indicates the usage of single-precision real data type.
ScaLAPACK	Stands for Scalable Linear Algebra PACKage.
Schur factorization	Representation of a square matrix A in the form $A = ZTZ^H$. Here T is an upper quasi-triangular matrix (for complex A, triangular matrix) called the Schur form of A; the matrix Z is orthogonal (for complex A, unitary). Columns of Z are called Schur vectors.
single precision	A floating-point data type. On Intel® processors, this data type allows you to store real numbers x such that $1.18*10^{-38} < \| x \| < 3.40*10^{38}$. For this data type, the machine precision (ε) is approximately 10^{-7}, which means that single-precision numbers usually contain no more than 7 significant decimal digits. For more information, refer to *Intel® 64 and IA-32 Architectures Software Developer's Manual, Volume 1: Basic Architecture*.
singular matrix	A matrix whose determinant is zero. If A is a singular matrix, the inverse A^{-1} does not exist, and the system of equations $Ax = b$ does not have a unique solution (that is, there exist no solutions or an infinite number of solutions).
singular value	The numbers defined for a given general matrix A as the eigenvalues of the matrix AA^H. *See also* SVD.
SMP	Abbreviation for Symmetric MultiProcessing. The MKL offers performance gains through parallelism provided by the SMP feature.
sparse BLAS	Routines performing basic vector operations on sparse vectors. Sparse BLAS routines take advantage of vectors' sparsity: they allow you to store only non-zero elements of vectors. *See* BLAS.
sparse vectors	Vectors in which most of the components are zeros.
storage scheme	The way of storing matrices. *See* full storage, packed storage, and band storage.
SVD	Abbreviation for Singular Value Decomposition. *See also* Singular value decomposition section in Chapter 5.
symmetric matrix	A square matrix A such that $a_{ij} = a_{ji}$.
transpose	The transpose of a given matrix A is a matrix A^T such that $(A^T)_{ij} = a_{ji}$ (rows of A become columns of A^T, and columns of A become rows of A^T).
trapezoidal matrix	A matrix A such that $A = (A_1A_2)$, where A_1 is an upper triangular matrix, A_2 is a rectangular matrix.
triangular matrix	A matrix A is called an upper (lower) triangular matrix if all its subdiagonal elements (superdiagonal elements) are zeros. Thus, for an upper triangular matrix $a_{ij} = 0$ when $i > j$; for a lower triangular matrix $a_{ij} = 0$ when $i < j$.
tridiagonal matrix	A matrix whose non-zero elements are in three diagonals only: the leading diagonal, the first subdiagonal, and the first super-diagonal.
unitary matrix	A complex square matrix A whose conjugate and inverse are equal, that is, that is, $A^H = A^{-1}$, and therefore $AA^H = A^HA = I$. All eigenvalues of a unitary matrix have the absolute value 1.
VML	Abbreviation for Vector Mathematical Library. *See* Chapter 9 of this book.
VSL	Abbreviation for Vector Statistical Library. *See* Chapter 10 of this book.
z	When found as the first letter of routine names, z indicates the usage of double-precision complex data type.

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