

TI-Nspire™ CAS Assisted Solutions

VCE Mathematical Methods Written Sample Examination 2 Section B

Explanatory notes:

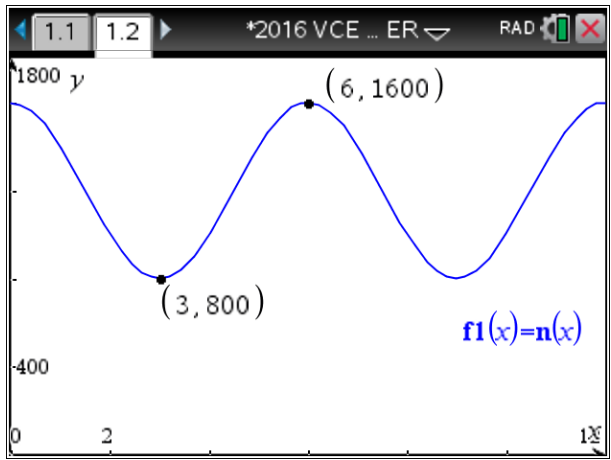
Note that the VCAA only supplies multiple-choice answers to sample papers. Every effort has been made to ensure that these solutions are correct.

The author of these solutions has no affiliation with the VCAA.

SECTION B – Extended response questions

Question 1:

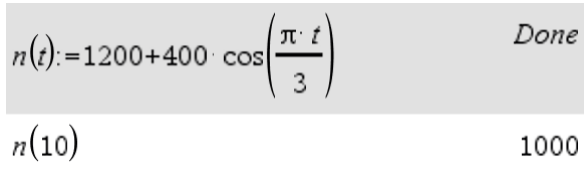
Part (a)

<p>Working</p> $T = \frac{2\pi}{\frac{\pi}{3}}$ $= 6 \text{ (months)}$ $A = 400$	<p>TI-Nspire CAS screenshot(s)</p> 
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Part (b)

<p>Working</p> <p>From the graph above right, $n_{\max} = 1600$ and $n_{\min} = 800$.</p>	<p>TI-Nspire CAS screenshot(s)</p>
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Part (c)

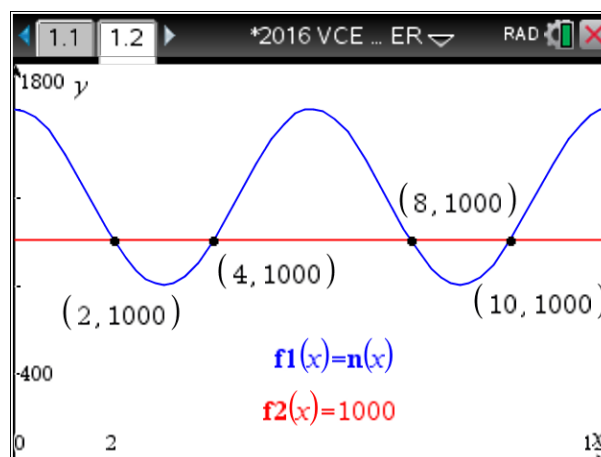
<p>Working</p> $n(10) = 1000$	<p>TI-Nspire CAS screenshot(s)</p> 
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Part (d)

Working

$n < 1000$ for 4 months so the fraction of time is $\frac{1}{3}$.

TI-Nspire CAS screenshot(s)



Question 2:

Part (a)

Working

$$6480 = 5x^2 + 7hx$$

$$7hx = 6480 - 5x^2$$

$$h = \frac{6480 - 5x^2}{7x}$$

TI-Nspire CAS screenshot(s)

$$6480 = 2 \cdot x \cdot h + 2 \cdot h \cdot \frac{5 \cdot x}{2} + 2 \cdot x \cdot \frac{5 \cdot x}{2}$$

$$6480 = 5 \cdot x^2 + 7 \cdot h \cdot x$$

$$6480 - 5 \cdot x^2 = 5 \cdot x^2 + 7 \cdot x \cdot h - 5 \cdot x^2$$

$$6480 - 5 \cdot x^2 = 7 \cdot h \cdot x$$

$$\text{comDenom}\left(\frac{6480 - 5 \cdot x^2}{7 \cdot x} = \frac{7 \cdot h \cdot x}{7 \cdot x}\right)$$

$$\frac{6480 - 5 \cdot x^2}{7 \cdot x} = h$$

Part (b)

<p>Working</p> <p>Solving $V(x) > 0$ for x with $x > 0$ gives $0 < x < 36$.</p> <p>The Notes application can be used to solve max/min problems.</p>	<p>TI-Nspire CAS screenshot(s)</p> <p>Max/min problems</p> $v(x) := \frac{5 \cdot x \cdot (6480 - 5 \cdot x^2)}{14} \rightarrow \text{Done}$ $\text{solve}(v(x) > 0, x) x > 0 \rightarrow 0 < x < 36$
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Part (c)

<p>Working</p> $\frac{dV}{dx} = -\frac{75}{14}x^2 + \frac{16200}{7} \text{ where}$ $a = -\frac{75}{14} \text{ and } b = \frac{16200}{7}$ <p>Author comment: I am uneasy about what working is required for this 3-mark question part when TI-Nspire CAS gives $\frac{dV}{dx}$ directly in its required form.</p>	<p>TI-Nspire CAS screenshot(s)</p> $\frac{d}{dx}(v(x)) \rightarrow \frac{16200}{7} - \frac{75 \cdot x^2}{14}$
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Part (d)

<p>Working</p> <p>Solving $\frac{dV}{dx} = 0$ for x with $x > 0$ gives $x = 12\sqrt{3}$ (cm).</p> <p>Substituting $x = 12\sqrt{3}$ into h gives $h = \frac{120\sqrt{3}}{7}$ (cm).</p>	<p>TI-Nspire CAS screenshot(s)</p> $\frac{d}{dx}(v(x)) \rightarrow \frac{16200}{7} - \frac{75 \cdot x^2}{14}$ $xc := \text{zeros}\left(\frac{d}{dx}(v(x)), x\right) x > 0 \rightarrow \{12 \cdot \sqrt{3}\}$ $h = \frac{6480 - 5 \cdot x^2}{7 \cdot x} x = xc \quad h = \left\{ \frac{120 \cdot \sqrt{3}}{7} \right\}$
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Question 3:

Part (a) (i)

<p>Working</p> $X \sim \text{Bi}\left(20, \frac{5}{8}\right)$ $\Pr(X \geq 10) = 0.9153 \text{ (correct to 4 dp)}$ <p>The Notes application can be used to solve questions involving the binomial distribution.</p>	<p>TI-Nspire CAS screenshot(s)</p> <p>Binomial Probability Calculations</p> <p>$n = 20 \rightarrow 20$</p> <p>$p = \frac{5}{8} \rightarrow \frac{5}{8}$</p> <p>$\text{binomCdf}(n, p, 10, 20) \rightarrow 0.915292$</p>
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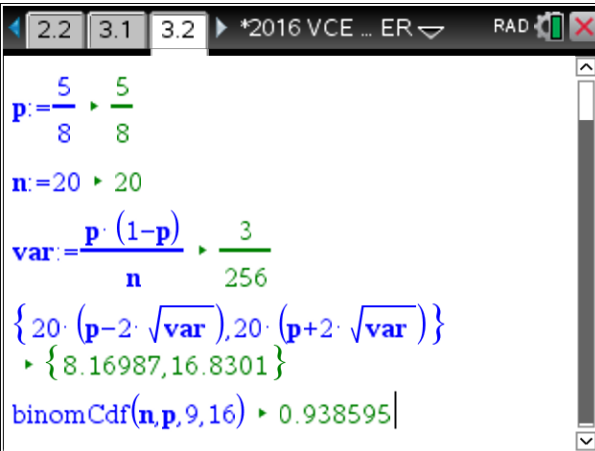
Part (a) (ii)

<p>Working</p> $\Pr(X \geq 15 X \geq 10)$ $= \frac{\Pr(X \geq 15)}{\Pr(X \geq 10)}$ $= 0.195$ <p>(correct to 3 dp)</p>	<p>TI-Nspire CAS screenshot(s)</p> <p>$\frac{\text{binomCdf}(n, p, 15, 20)}{\text{binomCdf}(n, p, 10, 20)} \rightarrow 0.19531$</p>
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Part (a) (iii)

<p>Working</p> $E(\hat{p}) = \frac{5}{8}$ $\text{var}(\hat{p}) = \frac{\frac{5}{8}\left(1 - \frac{5}{8}\right)}{20}$ $= \frac{3}{256}$	<p>TI-Nspire CAS screenshot(s)</p> <p>Sampling distribution calculations</p> <p>$p = \frac{5}{8} \rightarrow \frac{5}{8}$</p> <p>$n = 20 \rightarrow 20$</p> <p>$\text{var} = \frac{p \cdot (1-p)}{n} \rightarrow \frac{3}{256}$</p>
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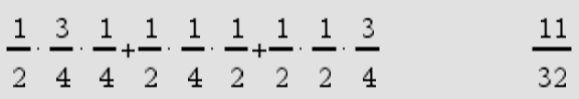
Part (a) (iv)

<p>Working</p> <p>Let X be the number from the sample that complete S in less than 3 mins.</p> $\hat{p} = \frac{X}{20} \Rightarrow X = 20\hat{p} \text{ and}$ $X \sim \text{Bi}\left(20, \frac{5}{8}\right)$ $\Pr\left(20\left(\frac{5}{8} - 2\sqrt{\frac{3}{256}}\right) \leq X \leq 20\left(\frac{5}{8} + 2\sqrt{\frac{3}{256}}\right)\right)$ $= \Pr(9 \leq X \leq 16)$ $= 0.939$ <p>(correct to 3 dp)</p>	<p>TI-Nspire CAS screenshot(s)</p> 
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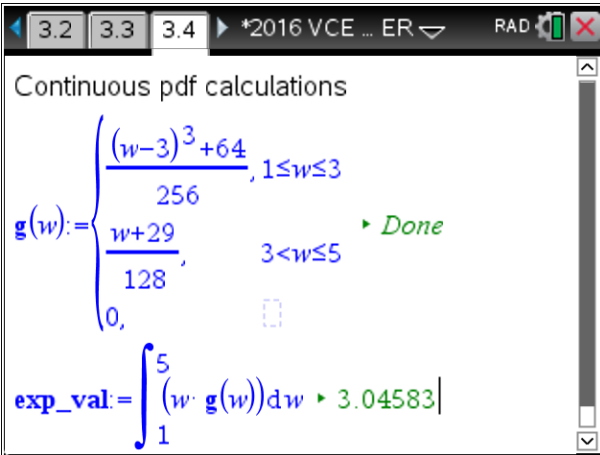
Part (a) (v)

<p>Working</p> $\Pr\left(\hat{p} \geq \frac{3}{4} \mid \hat{p} \geq \frac{5}{8}\right)$ $= \Pr\left(X \geq 20 \times \frac{3}{4} \mid X \geq 15 \times \frac{3}{4}\right)$ $= \Pr(X \geq 15 \mid X \geq 13)$ $= 0.352$ <p>(correct to 3 dp)</p>	<p>TI-Nspire CAS screenshot(s)</p> <p>Binomial Probability Calculations</p> <p>$n = 20$</p> <p>$p = \frac{5}{8}$</p> <p>$\frac{\text{binomCdf}(n, p, 15, 20)}{\text{binomCdf}(n, p, 13, 20)} \rightarrow 0.352001$</p>
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Part (b)

<p>Working</p> $\frac{3}{32} + \frac{1}{16} + \frac{3}{16} = \frac{11}{32}$	<p>TI-Nspire CAS screenshot(s)</p> 
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Part (c) (i)

<p>Working</p> $E(W) = \int_1^3 wg(w)dw + \int_3^5 wg(w)dw$ $= 3.0458$ <p>(correct to 4 dp)</p> <p>The Notes application can be used to solve questions involving continuous probability density functions.</p>	<p>TI-Nspire CAS screenshot(s)</p>  <p>The screenshot shows the TI-Nspire CAS interface. At the top, there are tabs for 3.2, 3.3, and 3.4. The title bar indicates '2016 VCE ... ER' and 'RAD'. The main window is titled 'Continuous pdf calculations'. It defines a piecewise function $g(w)$ as follows:</p> $g(w) = \begin{cases} \frac{(w-3)^3 + 64}{256}, & 1 \leq w \leq 3 \\ \frac{w+29}{128}, & 3 < w \leq 5 \\ 0, & \text{otherwise} \end{cases}$ <p>The function is labeled 'Done'. Below the function definition, the expected value is calculated:</p> $\text{exp_val} = \int_1^5 (w \cdot g(w)) dw \rightarrow 3.04583$
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Part (c) (ii)

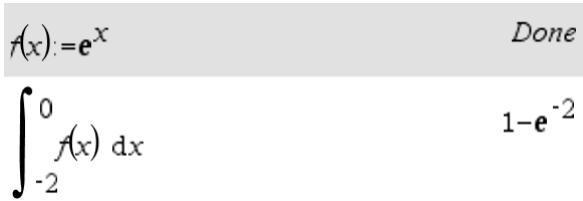
<p>Working</p> $200 \Pr(W > 4) = 52.3438$ <p>So 52 members (nearest integer).</p>	<p>TI-Nspire CAS screenshot(s)</p>  <p>The screenshot shows the TI-Nspire CAS interface. The main window displays the calculation:</p> $200 \cdot \int_4^5 g(w) dw \rightarrow 52.3438$
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Part (d)

<p>Working</p> $0.504 \leq p \leq 0.696$ <p>(correct to 3dp)</p> <p>Author comment: This question part is worth one mark and so we can just write the answer down.</p>	<p>TI-Nspire CAS screenshot(s)</p> <p>Number of successes: $x=60 \rightarrow 60$</p> <p>Sample size: $n=100 \rightarrow 100$</p> <p>Confidence level: $cl=0.95 \rightarrow 0.95$</p> <p>zInterval_1Prop x,n,cl: stat.results</p> <table border="1"> <tbody> <tr> <td>"Title"</td> <td>"1-Prop z Interval"</td> </tr> <tr> <td>"CLower"</td> <td>0.503982</td> </tr> <tr> <td>"CUpper"</td> <td>0.696018</td> </tr> <tr> <td>"p"</td> <td>0.6</td> </tr> <tr> <td>"ME"</td> <td>0.096018</td> </tr> <tr> <td>"n"</td> <td>100.</td> </tr> </tbody> </table>	"Title"	"1-Prop z Interval"	"CLower"	0.503982	"CUpper"	0.696018	"p"	0.6	"ME"	0.096018	"n"	100.
"Title"	"1-Prop z Interval"												
"CLower"	0.503982												
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"ME"	0.096018												
"n"	100.												

Question 4:

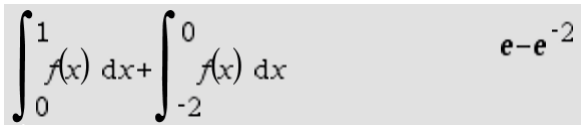
Part (a) (i)

Working	TI-Nspire CAS screenshot(s)
$1 - \frac{1}{e^2}$	

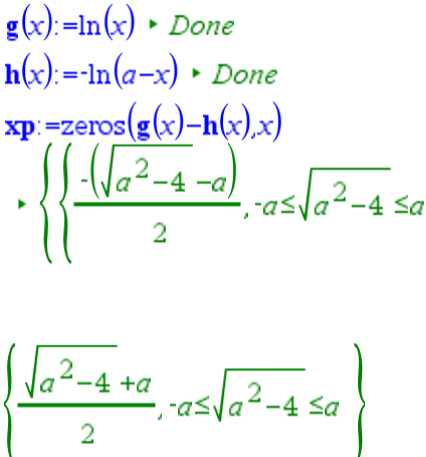
Part (a) (ii)

Working	TI-Nspire CAS screenshot(s)
$1 - \frac{1}{e^2}$	

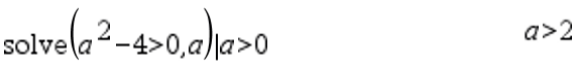
Part (a) (iii)

Working	TI-Nspire CAS screenshot(s)
$e - \frac{1}{e^2}$	

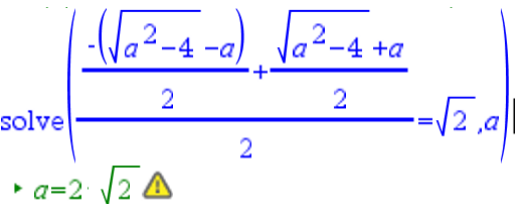
Part (b) (i)

Working	TI-Nspire CAS screenshot(s)
<p>Solving $g(x) = k(x)$ for x gives</p> $x = \frac{a \pm \sqrt{a^2 - 4}}{2}.$ <p>The Notes application can be used to solve equations (use of the zeros command is shown here).</p>	

Part (b) (ii)

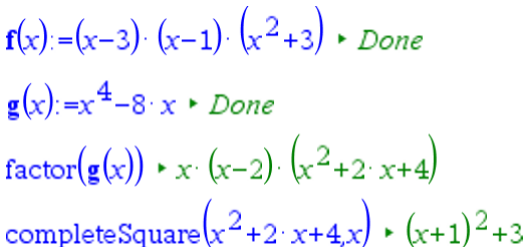
Working $a^2 - 4 > 0 \Rightarrow a > 2 (a > 0)$	TI-Nspire CAS screenshot(s) 
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Part (c)


Working Solving $\frac{a - \sqrt{a^2 - 4}}{2} + \frac{a + \sqrt{a^2 + 4}}{2} = \sqrt{2}$ for a gives $a = 2\sqrt{2}$.	TI-Nspire CAS screenshot(s) 
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Question 5:

Part (a)

Working $g(x) = x(x-2)((x+1)^2 + 3)$ The Notes application can be used to solve extended answer questions testing knowledge of calculus and algebra.	TI-Nspire CAS screenshot(s) 
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Part (b)

Working Translation of 1 unit in the negative direction of the x -axis.	TI-Nspire CAS screenshot(s) 
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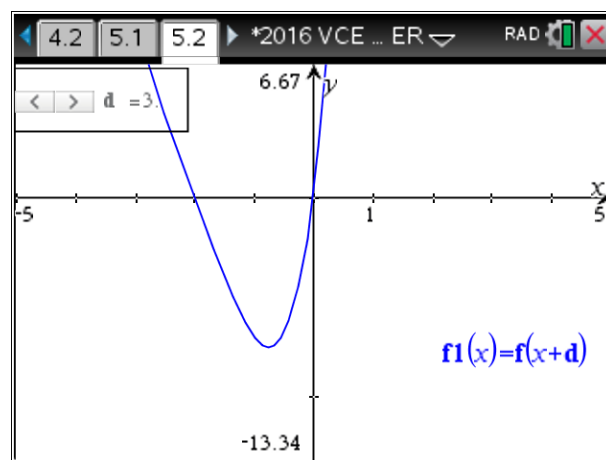
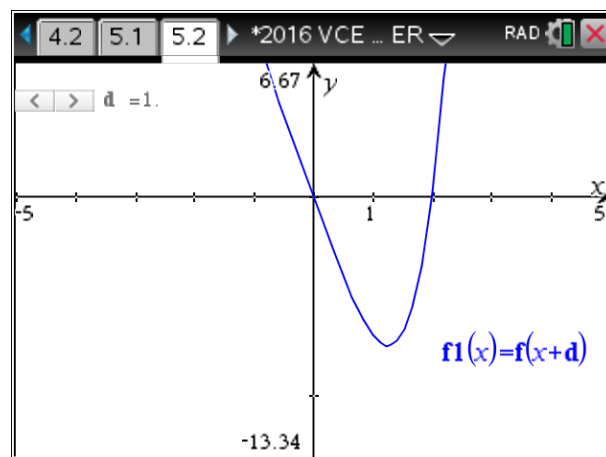
Part (c) (i)

Working

$$1 \leq d < 3$$

If time permits, setting up a slider for the parameter d provides a good insight into the required values of d .

TI-Nspire CAS screenshot(s)

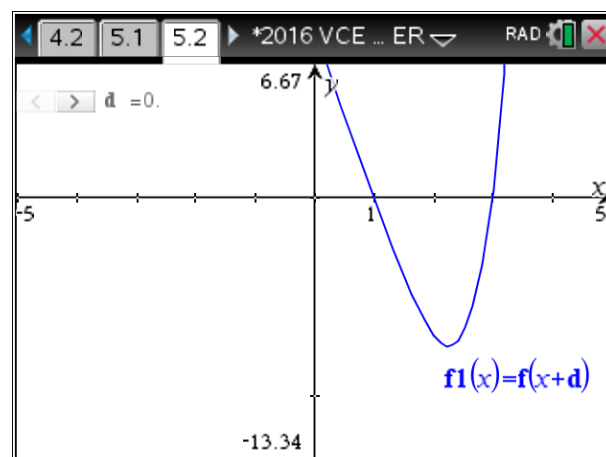


Part (c) (ii)

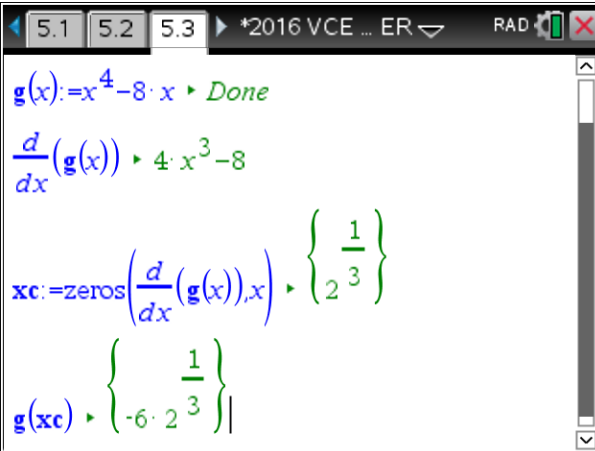
Working

$$d < 1$$

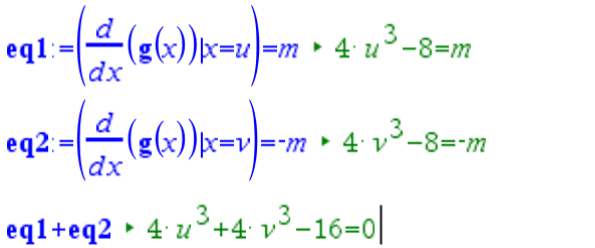
TI-Nspire CAS screenshot(s)



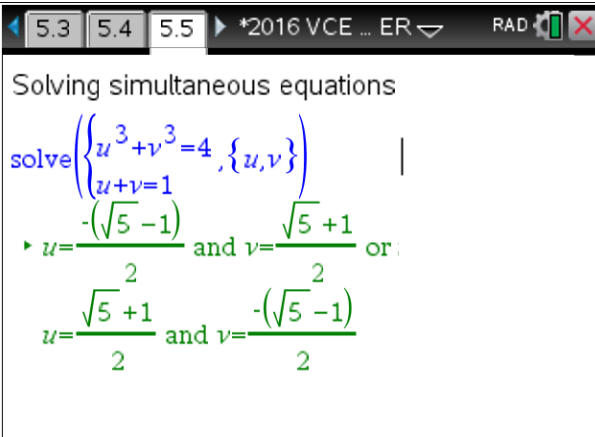
Part (d)

<p>Working</p> <p>Solving $g'(x)=0$ for x gives</p> $x = 2^{\frac{1}{3}}.$ $n = g\left(2^{\frac{1}{3}}\right) = -6\left(2^{\frac{1}{3}}\right)$	<p>TI-Nspire CAS screenshot(s)</p>  <p>The screenshot shows the following steps in the TI-Nspire CAS interface:</p> <ul style="list-style-type: none"> Define $g(x) := x^4 - 8 \cdot x$ (labeled "Done"). Calculate the derivative $\frac{d}{dx}(g(x)) \rightarrow 4 \cdot x^3 - 8$. Solve for the critical point: $xc := \text{zeros}\left(\frac{d}{dx}(g(x)), x\right) \rightarrow \left\{2^{\frac{1}{3}}\right\}$. Evaluate the function at the critical point: $g(xc) \rightarrow \left\{-6 \cdot 2^{\frac{1}{3}}\right\}$.
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Part (e) (i)

<p>Working</p> <p>$g'(u) = m$ and $g'(v) = -m$</p> <p>Adding the two equations and simplifying gives $u^3 + v^3 = 4$.</p>	<p>TI-Nspire CAS screenshot(s)</p>  <p>The screenshot shows the following steps:</p> <ul style="list-style-type: none"> Equation 1: $\left(\frac{d}{dx}(g(x))\right) _{x=u} = m \rightarrow 4 \cdot u^3 - 8 = m$. Equation 2: $\left(\frac{d}{dx}(g(x))\right) _{x=v} = -m \rightarrow 4 \cdot v^3 - 8 = -m$. Adding the equations: $eq1 + eq2 \rightarrow 4 \cdot u^3 + 4 \cdot v^3 - 16 = 0$.
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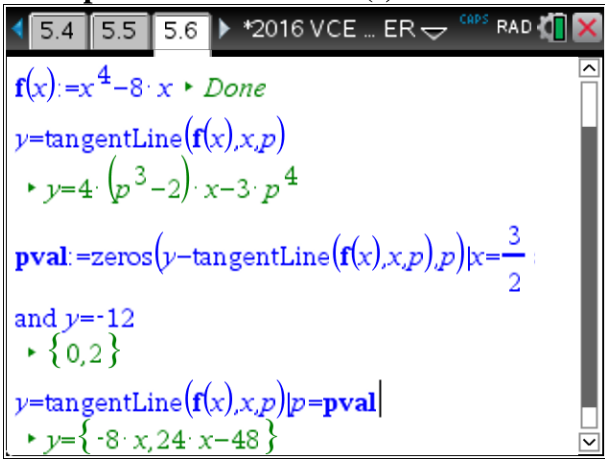
Part (e) (ii)

<p>Working</p> <p>Solving $u^3 + v^3 = 4$ and $u + v = 1$ for u and v with $m > 0$ gives</p> $u = \frac{\sqrt{5}+1}{2} \text{ and } v = \frac{-\sqrt{5}+1}{2}.$	<p>TI-Nspire CAS screenshot(s)</p>  <p>The screenshot shows the following steps:</p> <ul style="list-style-type: none"> Text: "Solving simultaneous equations". Command: $\text{solve}\left(\begin{cases} u^3 + v^3 = 4 \\ u + v = 1 \end{cases}, \{u, v\}\right)$. Result: $u = \frac{-(\sqrt{5}-1)}{2}$ and $v = \frac{\sqrt{5}+1}{2}$ or $u = \frac{\sqrt{5}+1}{2}$ and $v = \frac{-(\sqrt{5}-1)}{2}$.
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Part (f) (i)

<p>Working</p> $y = g'(p)(x - p) + g(p)$ $= 4(p^3 - 2)x - 3p^4$ <p>The Notes application can be used to calculate equations of tangents and normals.</p>	<p>TI-Nspire CAS screenshot(s)</p> <p>Equations of tangents and normals</p> <pre>f(x):=x^4-8·x ▶ Done y=tangentLine(f(x),x,p) ▶ y=4·(p^3-2)·x-3·p^4</pre>
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Part (f) (ii)

<p>Working</p> <p>Solving</p> $4(p^3 - 2)\left(\frac{3}{2}\right) - 3p^4 = -12 \text{ for } p$ <p>gives $p = 0$ or $p = 2$.</p> <p>When $p = 0$, $y = -8x$.</p> <p>When $p = 2$, $y = 24x - 48$.</p>	<p>TI-Nspire CAS screenshot(s)</p>  <pre>f(x):=x^4-8·x ▶ Done y=tangentLine(f(x),x,p) ▶ y=4·(p^3-2)·x-3·p^4 pval:=zeros(y-tangentLine(f(x),x,p),p) x=3/2 and y=-12 ▶ {0,2} y=tangentLine(f(x),x,p) p=pval ▶ y={-8·x,24·x-48}</pre>
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