## TI-Nspire ${ }^{\text {TM }}$ CAS Assisted Solutions

## VCE Mathematical Methods Written Sample Examination 2 Section B

## Explanatory notes:

Note that the VCAA only supplies multiple-choice answers to sample papers. Every effort has been made to ensure that these solutions are correct.

The author of these solutions has no affiliation with the VCAA.

## SECTION A - Multiple-choice questions

Question 1:
Answer: A

| Working <br> $(4,-3) \rightarrow(4,1) \rightarrow(-4,1)$ | TI-Nspire CAS screenshot(s) |
| :--- | :--- |
| TI-Nspire CAS functionality does not offer |  |
| any assistance here. |  |

## Question 2:

Answer: D

| Working |
| :--- | :--- | :--- | :--- | :--- |
| $f$ is one-to-one |
| $x \in(-2,6]$ |$\quad$ TI-Nspire CAS screenshot(s)



## Question 4:

Answer: E

## Working

Solving $p(-a)=0$ for $a$ with $a \neq 0$ gives $a=2$.

TI-Nspire CAS screenshot(s)

$$
\begin{array}{ll}
p(x):=7 \cdot x^{3}+9 \cdot x^{2}-5 \cdot a \cdot x & \text { Done } \\
\text { solve }(p(-a)=0, a) \mid a \neq 0 & a=2
\end{array}
$$

## Question 5:

Answer: A

| Working $g$ will have an inverse if the domain is restricted so that $g$ is one-to-one. There is a stationary point at $x=-\frac{\pi}{12}$ so the maximum value of $a$ is $\frac{\pi}{12}$. | TI-Nspire CAS screenshot(s) |
| :---: | :---: |
|  |  |
|  | $\mathrm{f} 1(x)=\left\{\sin \left(2 \cdot\left(x-\frac{\pi}{6}\right)\right), \frac{-\pi}{12} \leq x \leq \frac{\pi}{12}\right.$ |
|  | $-\pi$ $\frac{\pi}{12}$ $\frac{\pi}{6}$ <br> 6   |
|  | -1.5 |


| Working | TI-Nspire CAS screenshot(s) |
| :--- | :--- |
| $15-2 \int_{1}^{4} f(x) d x=15-12=3$ | $\int_{1}^{4}(5-2 \cdot f(x)) \mathrm{d} x$ |

## Question 7:

Answer: C

| $\begin{aligned} &=p+\frac{3 p}{5} \\ & \operatorname{Pr}(B)=\operatorname{Pr}(A \cap B)+\operatorname{Pr}\left(A^{\prime} \cap B\right) \\ &=p+p-\frac{1}{8} \\ & \text { Solving }\left(p+\frac{3 p}{5}\right)\left(2 p-\frac{1}{8}\right)=p\end{aligned}$ for $p$ with $p \neq 0$ gives $p=\frac{3}{8}$. |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

TI-Nspire CAS screenshot(s)
solve $\left.\left(\left(p+\frac{3 \cdot p}{5}\right) \cdot\left(2 \cdot p-\frac{1}{8}\right)=p, p\right) \right\rvert\, p \neq 0 \quad p=\frac{3}{8}$

Question 8:
Answer: E

## Working

$f(f(x))=x$ implies that $f$ must be its own inverse i.e.
symmetric in the line $y=x$.
TI-Nspire CAS can be used to confirm the result.

TI-Nspire CAS screenshot(s)

| $f(x):=2-x$ | Done |
| :--- | ---: |
| solve $(f(y)=x, y)$ | $y=2-x$ |
| $f(f(x))$ | $x$ |


| Working <br> $\operatorname{Pr}($ different colours $)$ <br> $=\left(\frac{5}{9} \times \frac{4}{8}\right)+\left(\frac{4}{9} \times \frac{5}{8}\right)$ <br> $=\frac{5}{9}$ | TI-Nspire CAS screenshot(s) |
| :--- | :--- |

## Question 10:

Answer: C
$\left.\begin{array}{|l|l|}\hline \text { Working } & \text { TI-Nspire CAS screenshot(s) } \\ x^{\prime}=1-x \text { and } y^{\prime}=2 y-2 & {\left[\begin{array}{l}x^{\prime} \\ y^{\prime}\end{array}\right]=\left[\begin{array}{cc}-1 & 0 \\ 0 & 2\end{array}\right] \cdot\left[\begin{array}{c}x \\ y\end{array}\right]+\left[\begin{array}{c}1 \\ -2\end{array}\right]}\end{array} \begin{array}{cc}x^{\prime}=1-x \\ y^{\prime}=2 \cdot y-2\end{array}\right]$

## Question 11:

Answer: B

| Working | TI-Nspire CAS screenshot(s) |
| :--- | :--- |
| Solving $a e^{a c}=\frac{a e^{a c}}{c}$ for $c$ gives | solve $\left(\left(\left.\frac{d}{d x}\left(\mathbf{e}^{a \cdot x}\right) \right\rvert\, x=c\right)=\frac{\mathbf{e}^{a \cdot c}-0}{c-0}, c\right) \quad c=\frac{1}{a}$ |
| $c=\frac{1}{a}$. |  |

## Working

Solving the 2 equations gives $x=\frac{8}{a+3}$ and $y=\frac{a-5}{a+3}$.
So no solution for $a=-3$.
Substituting $a=-3$ and attempting to solve with TINspire CAS gives the output 'false'.

The Notes application can be used to solve a system of linear equations.

## TI-Nspire CAS screenshot(s)

solve $(a \cdot x-3 \cdot y=5$ and $3 \cdot x-a \cdot y=8-a, x, y)$ $x=\frac{8}{a+3}$ and $y=\frac{a-5}{a+3}$
solve $(a \cdot x-3 \cdot y=5$ and $3 \cdot x-a \cdot y=8-a, x, y) \mid a=->$
false
solve $(a \cdot x-3 \cdot y=5$ and $3 \cdot x-a \cdot y=8-a, x, y)$

- $x=\frac{8}{a+3}$ and $y=\frac{a-5}{a+3}$


## Question 13:

Answer: A


## Question 14:

Answer: B

## Working

$\hat{p}=0.76, n=574$ and $z=1.65$
(correct to 2 dp )
So $B$.
The Notes application can be used to perform confidence interval calculations.

## TI-Nspire CAS screenshot(s)

Confidence levels and associated $z$-values
cl: $=0.9 \times 0.9$
$\mathrm{z}=$ =invNorm $\left(\frac{1+\mathrm{cl}}{2}\right) \cdot 1.64485$
Working
$f^{\prime}(x)=3 a x^{2}-2 b x+c$
Stationary points occur when
$f^{\prime}(x)=0$ and so
$3 a x^{2}-2 b x+c=0$ has no
solutions for $\Delta<0$.
Solving $\Delta<0$ for $c$ with $a>0$
gives $c>\frac{b^{2}}{3 a}$.

TI-Nspire CAS screenshot(s)

$$
\begin{aligned}
& \frac{d}{d x}\left(a \cdot x^{3}-b \cdot x^{2}+c \cdot x\right) \quad 3 \cdot a \cdot x^{2}-2 \cdot b \cdot x+c \\
& \text { solve }\left((-2 \cdot b)^{2}-4 \cdot 3 \cdot a \cdot c<0, c\right) \mid a>0 \\
& \quad c>\frac{b^{2}}{3 \cdot a} \text { and } a>0
\end{aligned}
$$

## Question 16:

Answer: E

## Working

Solving
$-\int_{0}^{2}\left(x^{2}-4\right) d x=\int_{2}^{a}\left(x^{2}-4\right) d x$ for $a$
with $a>0$ gives $a=2 \sqrt{3}$.

## TI-Nspire CAS screenshot(s)

solve $\left(\int_{0}^{2}\left(x^{2}-4\right) \mathrm{d} x=\int_{2}^{a}\left(x^{2}-4\right) \mathrm{d} x, a\right) \mid a>0$
$a=2 \cdot \sqrt{3}$

Question 17:
Answer: D

## Working

The graph of $y=x^{3}-9 x^{2}+15 x$ has stationary points at $(1,7)$ and $(5,-25)$.
So $x^{3}-9 x^{2}+15 x+w=0$ will have one solution if $w<-7$ or $w>25$.

TI-Nspire CAS screenshot(s)



## Question 18:

Answer: B

## Working

The $x$-intercepts for the graph of $y=f^{\prime}(x)$ correspond to the $x$ coordinates of the stationary points on the graph of $y=f(x)$.

Just to the left of $x=2$,
$f^{\prime}(x)>0$.
At $x=2, f^{\prime}(x)=0$.
Just to the right of $x=2$, $f^{\prime}(x)<0$.

## TI-Nspire CAS screenshot(s)

Given time constraints in an exam setting, TINspire CAS offers inefficient assistance here.

## Question 19:

Answer: D

| Working |
| :--- |
| Solving $-1.4395 \ldots=\frac{90-120}{\sigma}$ for $\sigma$ |
| gives $\sigma=20.8$ (days). |
| So $\sigma$ is closest to 21 days. |
| The Notes application can be used to |
| find the value of normal distribution |

## TI-Nspire CAS screenshot(s)

 TNormal alstribution:

Finding the mean or standard deviation
$\boldsymbol{\mu}=120$ • 120
$\mathrm{t}=90 \times 90$
prob $:=\frac{150}{2000} \times \frac{3}{40}$
solve (invNorm(prob) $=\frac{\mathbf{t}-\boldsymbol{\mu}}{\text { sigma }}$, sigma $) \mid$

- sigma $=20.8401$

| Working |  |
| :--- | :--- |
| $f(g(0))=f(0)=0$ | TI-Nspire CAS screenshot(s) |
| So $(0,0)$ is a point on the graph. | $f(x):=x \cdot(x-1)$ |
| $g(x)<0$ for all other values of $x$ |  |
| and $f(x)>0$ for $x<0$. | $g(x):=-\|x\|$ |
| So $f(g(x))>0$ for all other values <br> of $x$. | Done |
| TI-Nspire CAS could be used if, for |  |
| example, you set $f(x)=x(x-1)$ |  |
| and $g(x)=-\|x\|$ i.e. |  |
| $f(g(x))=\|x\|^{2}+\|x\|$ |  |

