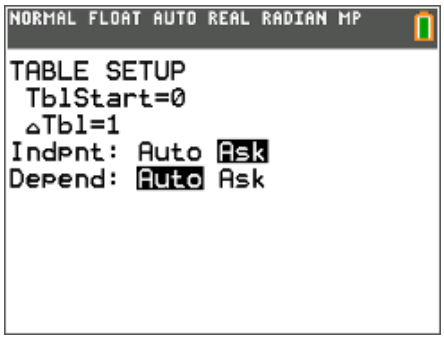
Problem 1 – Periodic Versus Continuous Compounding

Explore what happens to the effective interest rate as the number of compounding periods increases. For this exploration, the nominal interest rate is 20%. Recall the effective rate formula:

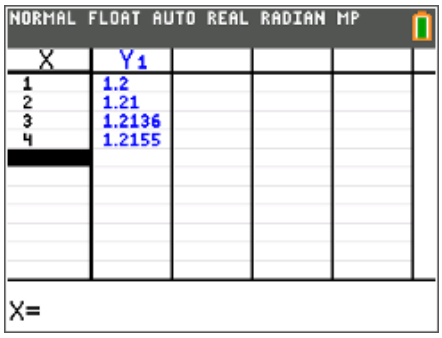
$$\text{effective rate} = \left(1+\frac{r}{n}\right)^{nt}$$

To simplify this problem, let t = 1 year.

Press [y=] and enter the equation **Y1=(1+0.2/x)^x**.
Now, set the table command from **Auto** to **Ask** by pressing [2nd][tablset] and matching the screen to the right.



Now, press [2nd][table] and enter values for x and observe how the y-values change.



1. What is the effective interest rate when there is only one compounding period for the entire year when the nominal rate is 20%?

2. What is the effective interest rate when interest is compounded quarterly?

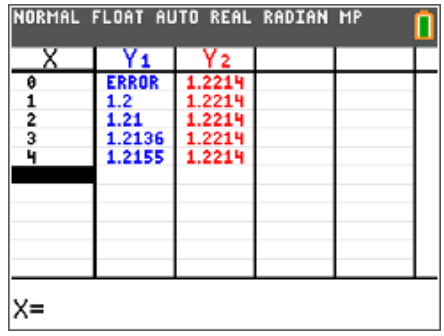
3. What is the effective interest rate when interest is compounded daily?

4. What appears to happen to the effective rate as n, the number of compounding periods in a year, approaches infinity?

Compounding can be performed continuously according to the formula $B = Pe^{rt}$, where B represents balance, P represents principal, and t is time in years.

To keep it simple, still let $t = 1$.

Press [y=] and enter the equation **Y2=*e*^(0.2)**.
Now, press [2nd][table] and enter values for x and observe how the y-values change.



5. How does the value of $\left(1+\frac{r}{n}\right)^{nt}$ compare to the value of e^{rt} when n is very large?

6. After approximately how many compounding periods is periodic compounding virtually the same as continuous compounding? Explain your answer.

The formula $B = Pe^{rt}$ actually is derived from the periodic compounding formula, $B = P\left(1+\frac{r}{n}\right)^{nt}$.

When the limit of the periodic compounding formula is taken as $n \to \infty$, $B = Pe^{rt}$ is obtained.

$$\lim_{n\to\infty}\left(P\left(1+\frac{r}{n}\right)^{nt}\right) = Pe^{rt}$$

Additional Questions to Explore

7. Moneybags Bank offers a 5-year CD at 4.25% compounded continuously to its customers.

a. What is the value of this CD at the end of the 5-year term if $1,000 is invested?

b. How long will it take for the value of the CD to double?

c. How long will it take for the value to reach $3,000?

8. Many store credit cards have an interest rate around 18%. To encourage buying, many stores offer no interest, no payment financing for long periods from 6 months up to 18 months when the consumer uses their credit card service. The "trick" to purchasing this way is to pay the entire balance off in the no-interest period. If the consumer does not pay off the full amount, typically interest is charged for the full amount of purchase over the entire time period.

Let's say that a person purchases an HDTV package complete with home theater and Blu-Ray™ player for $2,459.99 on a 12-month no-interest financing plan. This person does not make any payments over the 12-month period, so the full amount is due. If this customer does not pay the full amount, interest accrued continuously over the 12-month period will be added. Interest will continue to accrue over any additional time needed to pay off the credit card balance.

a. If this consumer cannot pay the full $2,459.99 on time, how much interest will be added for the 12-month period if the store credit card rate is 18%?

b. What do you think stores are expecting when they offer amazing deals on large purchases with special financing offers?

9. A family purchased a home for $55,500 in 1994 and sold the home for $160,000 in 2002. Assuming continuous compounding, what was the annual nominal rate of interest returned on this investment?

Problem 2 – Periodic Versus Continuous Compounding Over a Number of Years

For this exploration, let's assume our nominal interest rate is 15% and we choose to invest $1,000. We can choose between an account that compounds interest at the end of each month (which is common with many interest-earning bank accounts) or an account that compounds interest continuously.

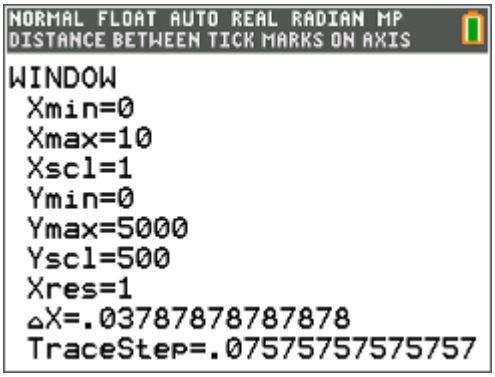
10. Write an expression that will compute the total value of the investment for

a. the account that compounds interest monthly

b. the account that compounds interest continuously

Press [y=] and enter the equation **Y1=** as your expression for Problem 10, part a, above and **Y2=** as your expression for Problem 10, part b, above.
Now, press [2nd][table] and enter values for *x* and observe how the *y*-values change for various years from 1 to 10.

Lastly, adjust your graphing window by pressing [window] and changing your values as shown to the right.
Sketch a graph of the two interest functions by pressing [y=].



11. Assume no deposits (besides interest) or withdrawals are made over a 10-year period of time.

a. How much would the account that compounds interest monthly be worth after 10 years?

b. How much would the account that compounds interest continuously be worth after 10 years?