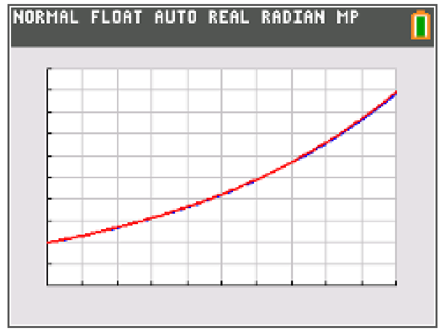
Accelerated Returns

About the Lesson

In this activity, students will compare periodic and continuous compounding and apply continuous compounding to a variety of problem situations. As a result, students will compute using:

- Nominal Rate vs. Effective Rate
- Natural Logarithm Base, e
- Periodic vs. Continuous Compounding of Interest



Vocabulary

- periodically compounded interest
- continuously compounded interest
- natural logarithm
- nominal rate
- effective rate

Teacher Preparation and Notes

- The activity engages students in a graphing activity to explore periodic compounding as $n \to \infty$. It may be helpful to review with students the periodic compounding formula, $B = P(1+\frac{r}{n})^{nt}$, prior to the start of this activity.
- Extension problems are provided on the student worksheet. These problems involve real-world problem applications of continuous compounding. Some basic review of solving exponential equations may be helpful.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

***** *with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

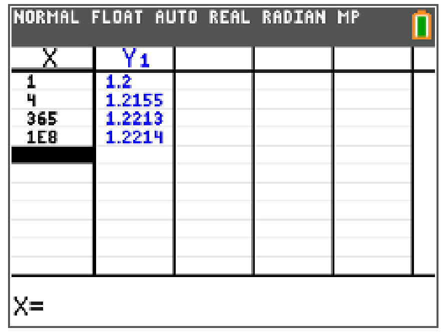
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Accelerated_Returns_Student.pdf
- Accelerated_Returns_Student.doc

Problem 1 – Periodic Versus Continuous Compounding

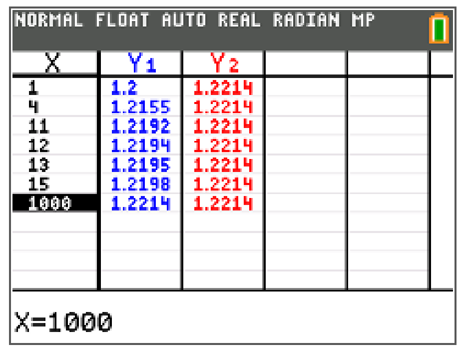
In this problem, students first explore the issue of compounding as $n \to \infty$. This is done by exploring $(1+\frac{r}{n})^{nt}$ with a 20% interest rate, time of 1 year, letting $x = n$, and entering this expression as the equation, $Y_1 = (1+\frac{0.2}{x})^x$. Students will then use the table function to observe data points from the graph of the function for different values of x.



It may be helpful to review with students the meanings of r, n, and t, as well as the related terminology (nominal rate and effective rate).

Students then make a comparison to continuous compounding by inserting a second function, $Y_2 = e^{rt}$ when

$r = 0.2$.



This activity topic provides a great opportunity to teach students mathematics related to finance. Unfortunately, many students learn the impact of buying using credit by making some serious financial mistakes at times when they really cannot afford to be making these mistakes.

If time allows, consider providing students with an assignment involving searching newspaper and internet advertisements for "buy now, pay later" arrangements and have students find the cost to the consumer should the payment terms of the agreement not be met.

1. What is the effective interest rate when there is only one compounding period for the entire year when the nominal rate is 20%?

Answer: 20%

2. What is the effective interest rate when interest is compounded quarterly?

Answer: 21.6%

3. What is the effective interest rate when interest is compounded daily?

Answer: 22.1%

4. What appears to happen to the effective rate as *n*, the number of compounding periods in a year, approaches infinity?

Answer: The function reaches a point at which no further increases will occur—it "levels out" and has a maximum value that will not be exceeded no matter how large *n* is.

5. How does the value of $\left(1+\frac{r}{n}\right)^{nt}$ compare to the value of e^{rt} when *n* is very large?

Answer: The values appear to be the same.

6. After approximately how many compounding periods is periodic compounding virtually the same as continuous compounding? Explain your answer.

Answer: After about 12 compounding periods, there isn't much difference between periodic and continuous compounding. (*Answers will vary*)

7. Moneybags Bank offers a 5-year CD at 4.25% compounded continuously to its customers.

a. What is the value of this CD at the end of the 5-year term if $1,000 is invested?

Answer: $1,236.77

b. How long will it take for the value of the CD to double?

Answer: about 16 years (16.3 years)

c. How long will it take for the value to reach $3,000?

Answer: about 26 years (25.85 years)

8. Many store credit cards have an interest rate around 18%. To encourage buying, stores offer no interest, no payment financing for periods from 6 months up to 18 months when the consumer uses their credit card service. The "trick" to purchasing this way is to pay the entire balance off in the no-interest period. If the consumer does not pay off the full amount, interest is typically charged for the full amount of purchase over the entire time period.

Let's say that a person purchases an HDTV package complete with home theater and Blu-Ray™ player for $2,459.99 on a 12-month no-interest financing plan. This person does not make any payments over the 12-month period, so the full amount is due. If this customer does not pay the full amount, interest accrued continuously over the 12-month period will be added. Interest will continue to accrue over any additional time needed to pay off the credit card balance.

a. This consumer cannot pay the full $2,459.99 on time. How much interest will be added for the 12-month period if the store credit card rate is 18%?

Answer: $485.15 interest will be added (total amount due with interest added is $2,945.14)

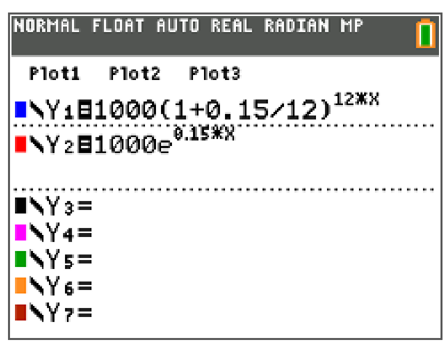
b. What do you think stores are expecting when they offer amazing deals on large purchases with special financing offers?

Answer: Stores are hoping that consumers will take advantage of the special financing offers. They also offer special pricing to attract people to buy on credit, knowing that a number of consumers will not be able to pay in full within the interest-free period. As a result, consumers will pay significant amounts of interest. In this case, the consumer would end up paying nearly $500 more than was initially intended.

9. A family purchased a home for $55,500 in 1994 and sold the home for $160,000 in 2002. Assuming continuous compounding, what was the annual nominal rate of interest returned on this investment?
Answer: 13.24%

Problem 2 – Periodic Versus Continuous Compounding Over a Number of Years

In this problem, students will investigate the difference between compounding interest on a monthly basis versus compounding continuously over a ten-year period. An annual interest rate of 15% and an initial principle of $1,000 are given.



10. Write an expression that will compute the total value of the investment for

a. the account that compounds interest monthly.

Answer: $1{,}000\left(1+\frac{0.15}{12}\right)^{(12)(10)}$

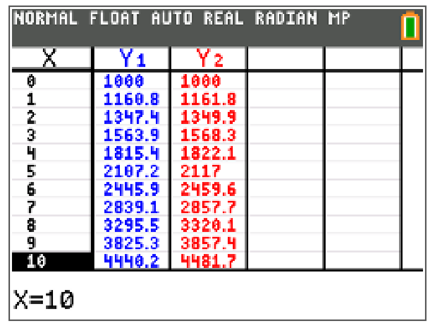
b. the account that compounds interest continuously.

Answer: $1{,}000e^{(0.15)(10)}$

Students can choose to investigate the differences between the two interest models by viewing a table or by looking at a sketch of their graphs.

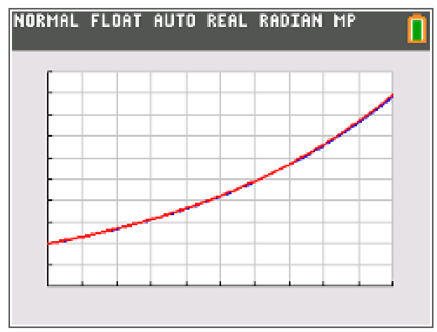
Due to the relatively short time interval that the accounts draw interest, there is little difference between the sketches of the graphs of the two interest models.

Discuss with students what would happen if the two models were analyzed over longer periods of time such as 50 years or 100 years. Verify the students' conjectures by sketching the models within an appropriate window.


NORMAL FLOAT AUTO REAL RADIAN MP

X	Y_1	Y_2
0	1000	1000
1	1160.8	1161.8
2	1347.4	1349.9
3	1563.9	1568.3
4	1815.4	1822.1
5	2107.2	2117
6	2445.9	2459.6
7	2839.1	2857.7
8	3295.5	3320.1
9	3825.3	3857.4
10	4440.2	4481.7

X=10



11. Assume no deposits (besides interest) or withdrawals are made over a 10-year period of time.

a. How much would the account that compounds interest monthly be worth after 10 years?

Answer: $4,440.21

b. How much would the account that compounds interest continuously be worth after 10 years?

Answer: $4,481.69