

About the Lesson

Students will simulate the process that a surveyor would use to measure the width of a river by measuring the length on one side of the river and angles formed by various reference points. They will then prove the Law of Sines and apply it to calculate the river's width, proving that no river is too wide to be measured with trigonometry. As a result, students will:

- Prove and apply the Law of Sines and Law of Cosines to find the unknown sides or angles of a triangle.

Vocabulary

- Law of Sines
- theodolite
- altitude

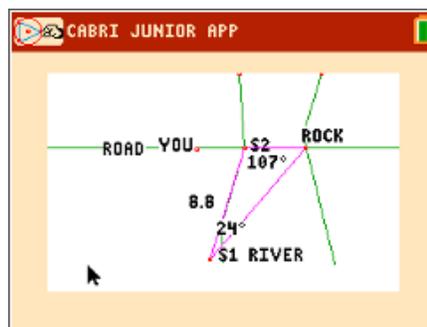
Teacher Preparation and Notes

- This activity is appropriate for an Algebra 2 or Precalculus classroom. Students should have experience calculating basic trigonometric functions and solving simple trigonometric equations.
- This activity is intended to be **student-centered** with brief periods of group instructions. You should seat your students in pairs so they can work cooperatively on their calculators and circulate among them, offering assistance as needed.
- This worksheet provides detailed instructions for the completion of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.

Activity Materials

- Compatible TI Technologies:
 - TI-84 Plus*
 - TI-84 Plus Silver Edition*
 -  TI-84 Plus C Silver Edition
 -  TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Aint_No_River_Wide_Enough_Student.pdf
- Aint_No_River_Wide_Enough_Student.doc
- RIVER.8xv
- LAWSINES.8xv

Tech Tip: Before beginning the activity, the files RIVER.8xv and LAWSINES.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect.

Measuring a River

In this activity, you are part of a team building a new road. To prepare for building a bridge, you need to measure the distance across a river. The river is treacherous and impossible to cross without the right equipment, so you must measure its width *indirectly*, using some stakes to mark points on the bank, a tape measure, and a theolodolite. A theolodolite is a special instrument used by surveyors to measure angles. To measure an angle, the theolodolite must be placed at the vertex.

To see a diagram of the situation, start the **Cabri™ Jr.** app by pressing **APPS**, and then choose it from the list.

Press any key to begin.



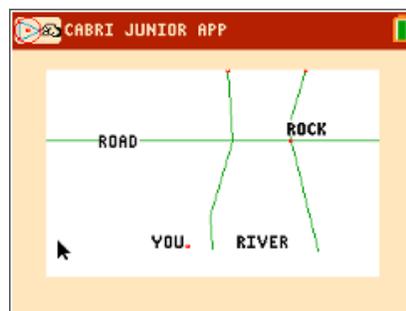
Open the file **RIVER**. Press $\boxed{y=}$ to open the **F1: File** menu, choose **Open...** and choose **RIVER** from the list.



The file shows the river and the proposed path of the road. A point shows your location and another dot represents a rock that lies in the path of the road on the other side of the river. We can use this rock as a point of reference to measure the river indirectly.

1. With your group, brainstorm some ideas how we might measure the width of the river indirectly.

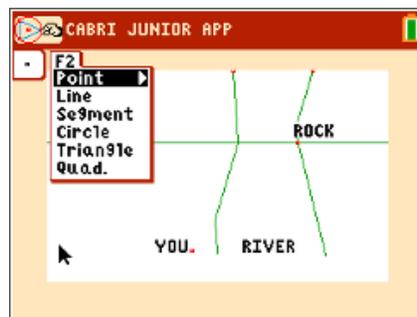
Sample Answer: Allow students to be creative, e.g. throwing a rope across the river, tying branches together and extending them across the river, etc.



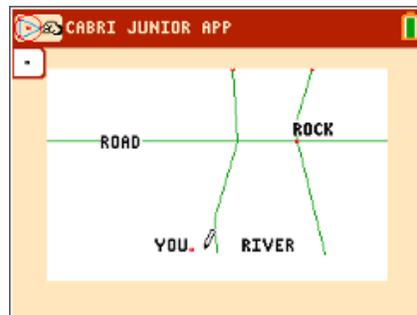
Here is one way to measure the width of the river indirectly using the tools you have on hand.

First, place a stake right next to your current location. In the diagram, we will use points to represent stakes.

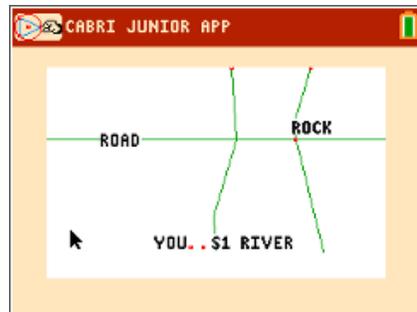
Press **[window]** to open the **F2: Creation** menu and choose **Point**.



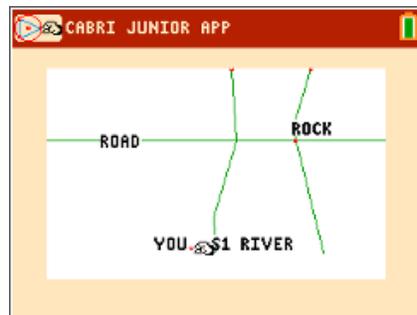
Then move the cursor to a point close to your current location in the diagram and press **[enter]** to place the point. Make sure the point is not on top of your current location or in the river.



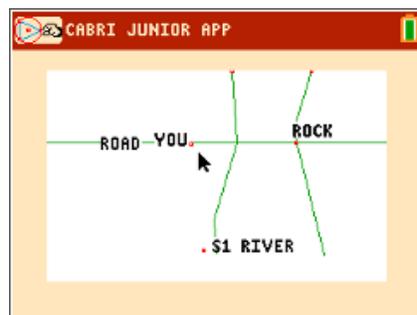
Press **[clear]** to exit the **Point** tool. Label the stake **S1**. Press **[graph]** to open the **F5: Appearance** menu and choose **Alph-Num**. Then move the cursor over the point representing the stake. It will increase in size when you are over top of the point. Press **[enter]** to start typing, type **S1**, and press **[enter]** again.



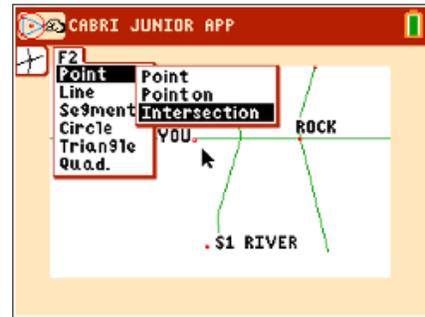
Press **[clear]** to exit the **Alph-Num** tool. Now walk along the bank until you reach the proposed path of the road. Grab the point that represents you by moving your cursor over it and pressing **[alpha]**. The cursor will turn into a closed fist.



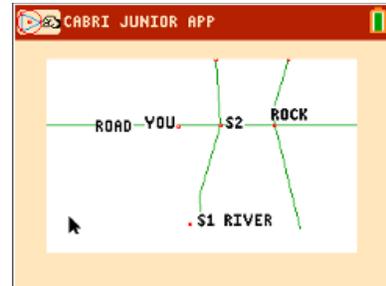
Drag your point along the river until you reach the proposed path of the road. Press **[enter]** to let go of the point.



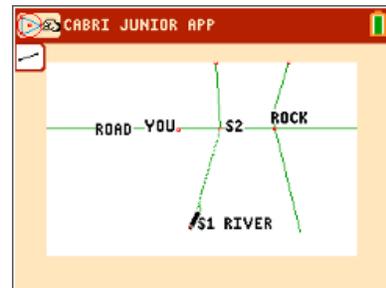
Use the **Intersection** command (found in the **F2: Creation > Point** menu) to drive a stake at the point where the proposed path of the road intersects the riverbank.



Label the stake **S2**.



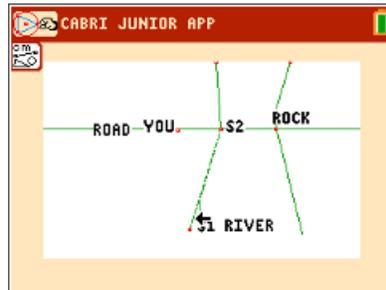
Now you are ready to take some measurements. Use your tape measure to find the distance from S1 to S2. Draw a line segment (found in the **F2: Creation** menu) between the two points.



Use the **D. & Length** tool (found in the **F5: Appearance > Measure** menu) to measure the length of the segment. Be careful to measure the line segment you drew, not the riverbank.

2. One unit on the diagram represents 50 ft. What is the actual distance between the two stakes?

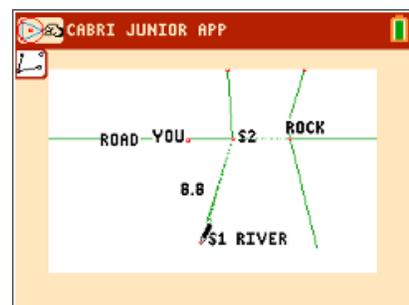
Sample Answer: Approximately $(8.8)(50)$ or 440 feet depending upon the exact placement of **S1**.



Label the segment with its actual length.

Next, set up your theodolite at S2 and measure the angle formed by the rock, S2, and S1. Use the Angle tool (found in the **F5: Appearance > Measure** menu) and choose the rock, S2, and S1, in that order.

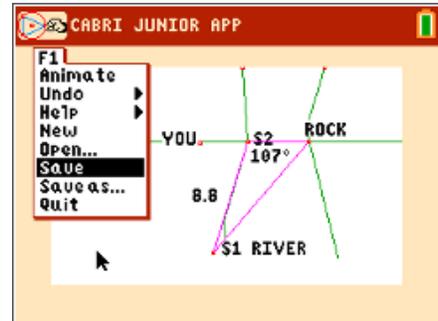
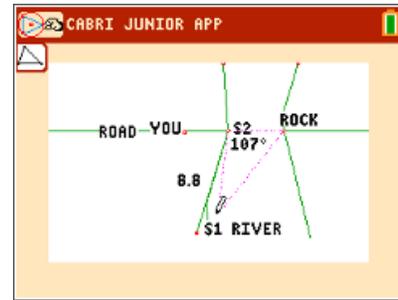
Slight variations in the values of your measurements may occur depending upon the exact placement of **S1**.



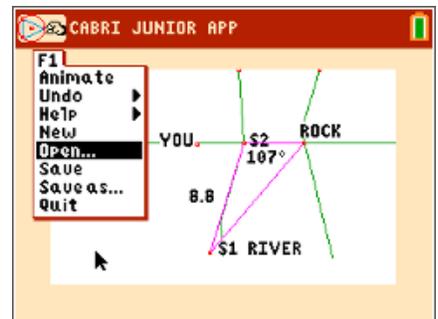
Draw the triangle (**F2: Creation > Triangle**) formed by S1, S2, and the rock.

If this were a right triangle, you would have enough information to calculate the distance from S2 to the rock. Because it is not a right triangle, you will have to take another measurement and use the Law of Sines to find the length of the missing side. Let's examine and prove the Law of Sines, then return to solve the problem.

Save your work on the **Cabri™ Jr.** file **RIVER** by opening the **F1: File** menu and choosing **Save**.

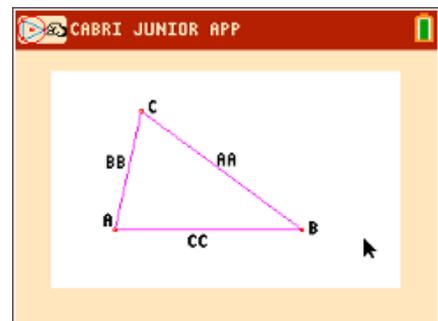


The Law of Sines relates the lengths of the sides of a triangle to its angles. To see the Law of Sines, open the **Cabri™ Jr.** file **LAWSINES**.



The diagram shows a general triangle with angles A , B , and C and side lengths AA , BB , and CC . The Law of Sines states that:

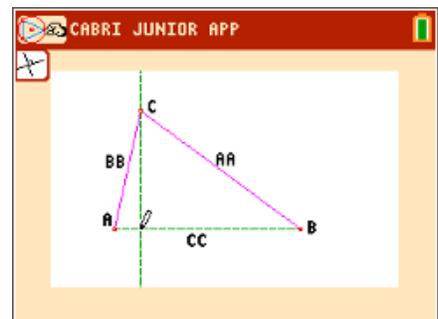
$$\frac{\sin A}{AA} = \frac{\sin B}{BB} = \frac{\sin C}{CC}$$



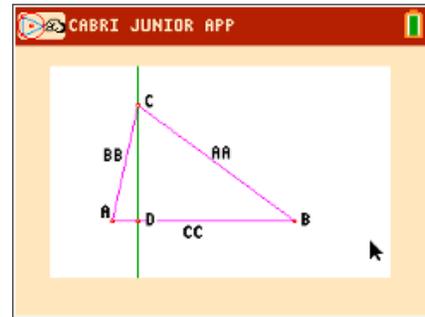
Follow the steps to prove the Law of Sines.

Step 1: Draw an altitude H from C to the opposite side.

An altitude is a straight line through a vertex and perpendicular to the opposite side. Use the **Perp.** tool (found in the **F3: Construction** menu) to draw a line perpendicular to CC through C .

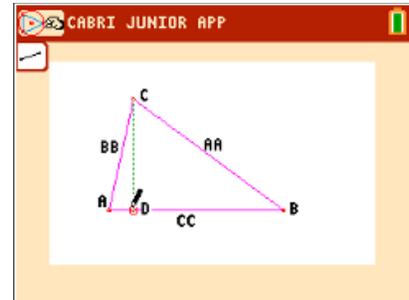


Step 2: Then use the **Intersection** tool to mark the point where this line intersects CC . Then use the **Alpha-Num** tool to label it as point D .



Step 3: Go to **F5: Appearance > Hide/Show > Object** and **Hide** the perpendicular line.

Step 4: Draw segment CD . We shall refer to it as segment H . Note that ACD is a right triangle.



3. Write expressions for $\sin A$ and $\sin B$ using AA , BB , and H .

Answer: $\sin A = \frac{H}{BB}$; $\sin B = \frac{H}{AA}$

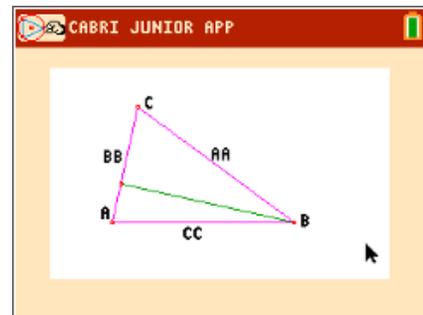
4. Solve these expressions for H and set them equal to each other to make an equation.

Answer: $(\sin A)(BB) = (\sin B)(AA)$

5. Divide this equation by $(AA)(BB)$ to get one half of the Law of Sines.

Answer: $\frac{\sin A}{AA} = \frac{\sin B}{BB}$

Delete H by using **Hide** tool and draw the altitude from B to the opposite side. We'll refer to this segment as G .



6. Write expressions for $\sin A$ and $\sin C$ using AA , CC , and G .

Answer: $\sin A = \frac{G}{CC}$; $\sin C = \frac{G}{AA}$

7. Solve these expressions for G and set them equal to each other to make an equation.

Answer: $(\sin A)(CC) = (\sin C)(AA)$

8. Divide this equation by (AA)(CC) to get the other half of the Law of Sines.

Answer: $\frac{\sin A}{AA} = \frac{\sin C}{CC}$

9. Use the Transitive Property of Equality to complete the proof.

Answer: $\frac{\sin A}{AA} = \frac{\sin B}{BB} = \frac{\sin C}{CC}$

Now you can return to the river problem. Open the **Cabri™ Jr.** file **RIVER**.

Walk back to S1 and set up your theodolite. Measure the angle formed by the rock, S1, and S2.

10. What is the measure of the angle at S1.

Answer: Approximately 24°. *

11. What is the measure of the angle located at the rock?

Answer: The missing angle is $180^\circ - 107^\circ - 24^\circ = 49^\circ$. *

12. Use the Law of Sines to write and solve an equation to find the width of the river.

$$\frac{\sin(49^\circ)}{220} = \frac{\sin(24^\circ)}{x}$$

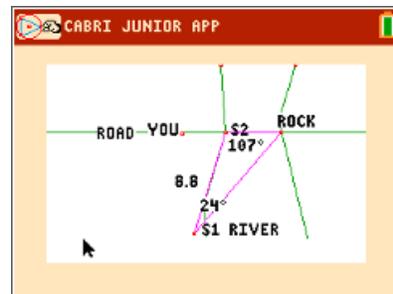
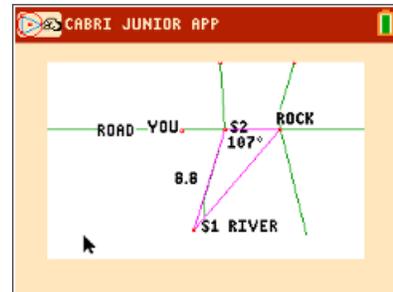
Answer: $x \cdot \sin(49^\circ) = 220 \cdot \sin(24^\circ)$

$$x = \frac{220 \cdot \sin(24^\circ)}{\sin(49^\circ)} \approx 118.56$$

The width of the river is approximately 118.56 feet.

Measure the segment from S2 to the rock.

Apply the scale factor of 1 unit: 50 feet to check your answer. (Choose on S2, then the rock to tell **Cabri™ Jr.** which segment to measure.) Note that the answer you obtained by measuring directly is more accurate.



* Answers will vary slightly based on placement of the first stake, **S1**.