About the Lesson

In this activity, students will explore how the measures of angles constructed in a circle are related to the measures of the intercepted arcs. Beginning with central and inscribed angles, students will investigate the angle-arc relationships. Then students will explore a figure that has an angle vertex either inside the circle (angles formed by chords) or outside the circle (angles formed by secants). As a result, students will:

- Conjecture a relationship between the measures of central and inscribed angles with the measures of their intercepted arcs.
- Conjecture a relationship between the measures of angles formed by chords, secants, and tangents with the measures of their intercepted arcs.

Vocabulary

- central angle
- chord
- inscribed angle
- secant

Teacher Preparation and Notes

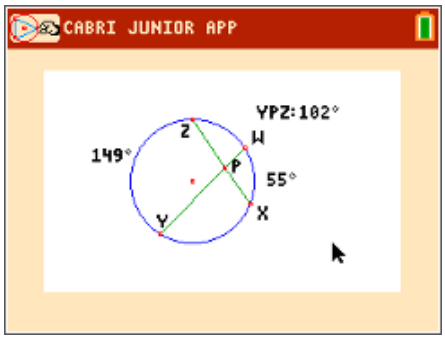
- There is no “arc measure” tool, so the measure of the central angle is used as the measure of the intercepted arc. If the intercepted arc is a major arc, the angle measure will be smaller than 180°, which will not be the correct arc measure. Subtract the angle measure from 360° to correct this issue in the case of a major arc.
- Before beginning the activity, the Cabri Jr. files ANGARC1.8xv and ANGARC2.8xv need to be transferred to the students’ calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

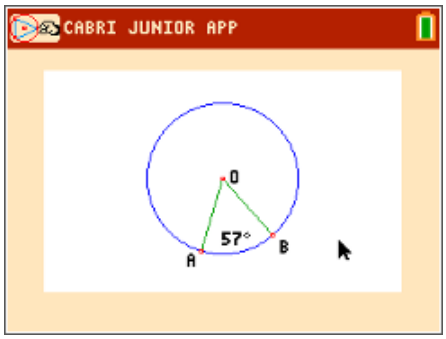
Lesson Files:

- AnglesArcs _Student.doc
- AnglesArcs _Student.pdf
- ANGARC1.8xv
- ANGARC2.8xv

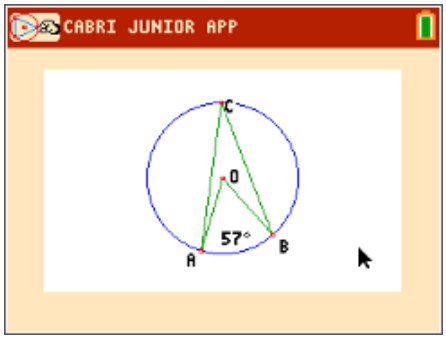
Tech Tip: Before beginning the activity, the Cabri Jr. Files ANGARC1.8xv and ANGARC2.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect or TI-Connect CE.

Problem 1 – Central Angles & Inscribed Angles

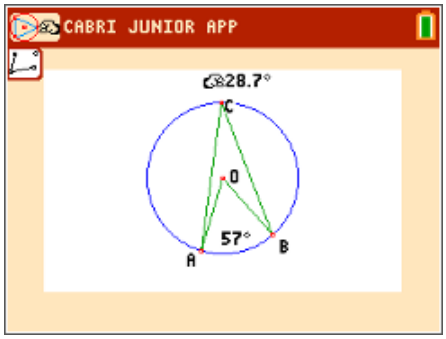
Students begin by opening the ANGARC1 file in the Cabri Jr. app. A central angle $\angle AOB$ has been constructed and measured. Because Cabri Jr. has no "arc measure" tool, the central angle measure is used as the arc measure.



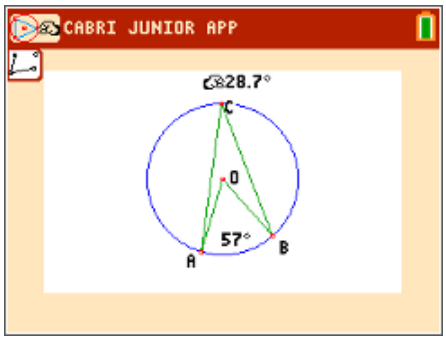
Students should create point *C* on the circle using the **Point On** tool. Next, students should use the **Segment** tool to construct $\overline{AC}$ and $\overline{BC}$.



Students should measure $\angle ACB$. Then, students should drag points *A* or *B* to change the angles and observe the relationships of the angle measures and arc measure.



Next, students should plot point *D* on the circle and create segments to form $\angle ADB$. Students should relate $m\angle ADB$ to the other measures.



1. Record the measurements you made of the central and inscribed angles. Drag points *A* or *B* to change the angles.

Answers:

$m\angle AOB$	$m\angle ACB$	mAB
50°	25°	50°
100°	50°	100°
80°	40°	80°
120°	60°	120°
30°	15°	30°

2. Complete these conjectures:

Answer:

The measure of the inscribed angle is **half** the measure of the central angle.

The measure of the inscribed angle is **half** the measure of the intercepted arc.

3. Drag point *C* instead of *A* or *B.* What do you notice about the measurements? Why do you think this happens?

Answer: The measurements do not change because the intercepted arc does not change.

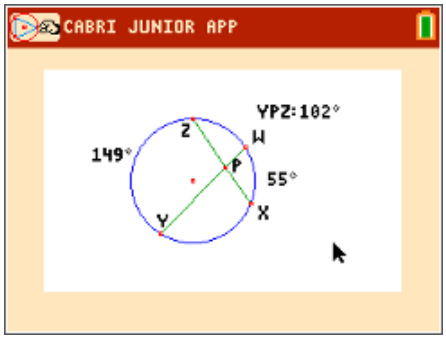
4. Place another point, *D*, on the circle. Construct and measure $\angle ADB$.

What do you notice about the measures of $\angle ADB$ and $\angle ACB$, which are angles that intercept the same arc?

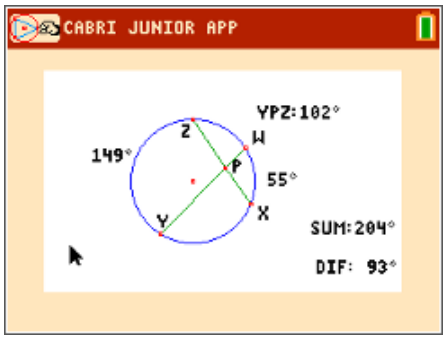
Answer: The two angles have the same measurement

Problem 2 – Angles with Vertex Inside or Outside the Circle

For this problem, students should use the Cabri Jr. file ANGARC2. They should see that $\angle YPZ$ is constructed with its vertical angle $\angle$WPX. The intercepted arcs WX and YZ are measured.
The Calculate tool is used to find the sum and difference of the arc measures.

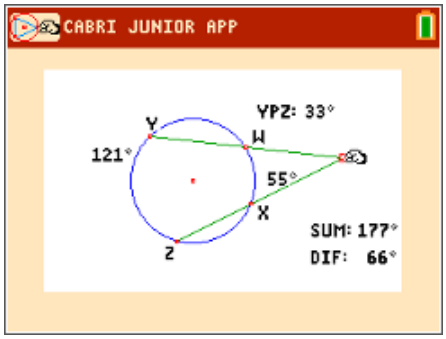


Students should move point *P* to five different locations *inside* the circle and then to five locations *outside* the circle. They can also move points *W* and *Z*.



At each location, students should record the angle measures on their worksheet.

After some examination of the values, they should conclude that when *P* is inside the circle, the measure of $\angle YPZ$ is half of the sum of the arc measures. When *P* is outside the circle, the measure of $\angle YPZ$ is half the absolute value of the difference of the arc measures.



5. Drag point *P* to different locations *inside* the circle. Record the measure of $\angle YPZ$, the sum of the arc measures, and the absolute value of the difference of the arc measures. You can also drag *W* and *X*.

Answers:

$m\angle YPZ$	$mYZ + mWX$	$\lvert mYZ - mWX \rvert$
102°	204°	93°
34°	72°	39°
40°	80°	31°
80°	161°	50°
54°	108°	3°

6. Drag point *P* to different locations *outside* the circle. Record the measure of ∠*YPZ*, the sum of the arc measures, and the absolute value of the difference of the arc measures. You can also drag *W* and *X*.

Answers:

$m\angle YPZ$	$mYZ + mWX$	$\lvert mYZ - mWX \rvert$
28°	167°	56°
41°	194°	82°
16°	142°	32°
8°	94°	17°
20°	240°	40°

7. Based on the location of *P*, when is the measure of ∠*YPZ* related to the sum of the arc measures? When is it related to the difference?

Answers: When *P* is inside the circle, ∠*YPZ* is related to the sum of the arc measures.
When *P* is outside the circle, ∠*YPZ* is related to the difference of the arc measures.

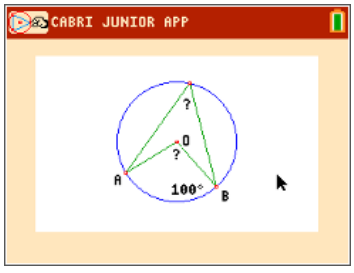
8. What is the relationship between ∠*YPZ* and the sum? between ∠*YPZ* and the difference?

Answer: ∠*YPZ* is half the sum/ difference of the arc measurements.

Extension/Homework

Find the missing measure in each figure.

1.



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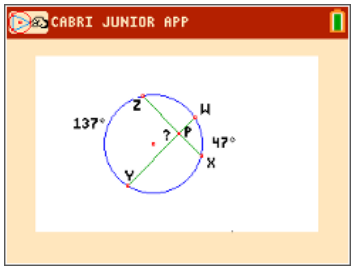

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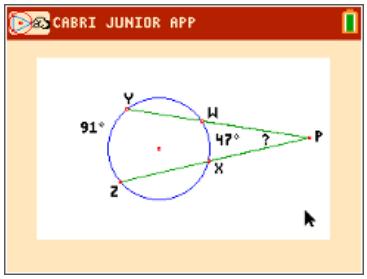

Answers: 50°, 25° 92° 22°