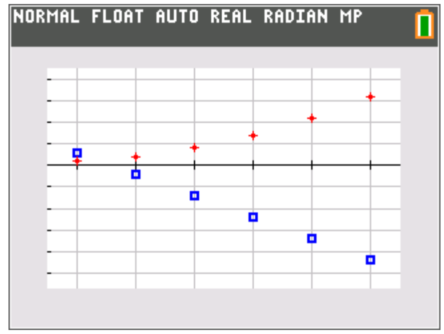
Arithmetic Sequences & Series



About the Lesson

Students use formulas to find the differences of the consecutive terms, plot a scatter plot of each sequence, and determine that sequences with common differences (called arithmetic sequences) have scatter plots whose points form a straight line. They then learn how to find the sum of the first *n* terms of the related series. As a result, students will:

- Given several terms of a sequence, write an algebraic expression that generates the n^{th} term.
- Graph the first *n* terms of a sequence.
- Derive and apply a formula for the first *n* terms of an arithmetic sequence.

Vocabulary

- arithmetic sequence
- common difference
- explicit formula
- arithmetic series

Teacher Preparation and Notes

- It would be beneficial for students to clear all lists and functions. Press $\boxed{\text{2nd}}$ $\boxed{+}$ and select **ClearAllLists**. Press $\boxed{\text{y=}}$, move to any equation that is defined and press $\boxed{\text{clear}}$.
- Students should begin this activity knowing that a sequence is an ordered list of numbers that follows a pattern and that a series is an indicated sum of a sequence. For example, 1, 2, 3, 4 is a sequence and 1 + 2 + 3 + 4 is a series.

Activity Materials

- Compatible TI Technologies:
  - TI-84 Plus*
  - TI-84 Plus Silver Edition*
  - TI-84 Plus C Silver Edition
  - TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

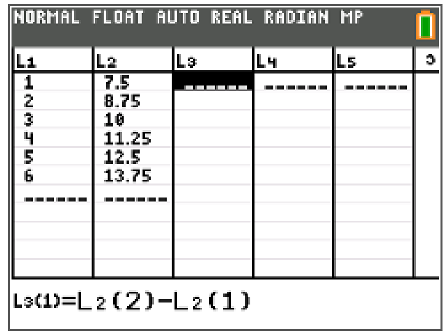
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

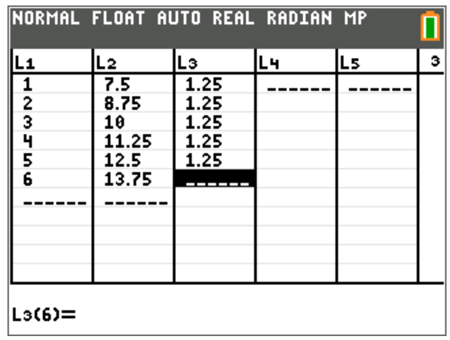
- Arithmetic_Sequences_Series_Student.pdf
- Arithmetic_Sequences_Series_Student.doc

Part 1 – Sequences and Scatter Plots

Students are to create a new data table by pressing $\boxed{\text{stat}}$ and choosing Option 1: Edit under the **EDIT** menu. Instruct them to copy the data shown into L_1 and L_2, find the differences between consecutive terms of the sequence in L_2, and record them in L_3. They can either use the list/row numbers, **L_2(2)–L_2(1)**, or the actual numbers, such as **8.75–7.5.**

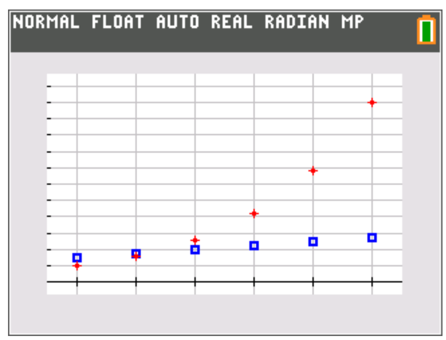


Students will then enter the terms of a sequence into L_4, find the consecutive differences for the L_4 sequence, and record them in L_5.



For each sequence, graph the ordered pairs formed by the domain (the natural numbers listed in L_1) and the terms of the sequence.

Students will plot L_2 in **Plot1** and L_4 in **Plot2**. The Xlist for both plots should be L_1. Have them press $\boxed{\text{zoom}}$ and select **9:ZoomStat** to view the plots. If they have trouble viewing both plots together, have them turn off one plot and then the other.



1. For each sequence, write the differences between the consecutive terms and give a description of the scatter plot.

a. Sequence L2

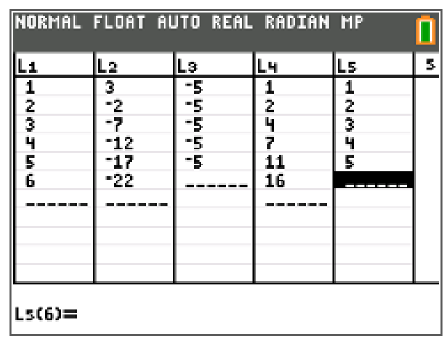
Answer: 1.25, 1.25, 1.25, 1.25, 1.25. Possible answer: The points of the scatter plot form a straight line that slants up to the right.

b. Sequence L4

Answer: 3, 5, 8, 13, 21. Possible answer: The points of the scatter plot form a curve.

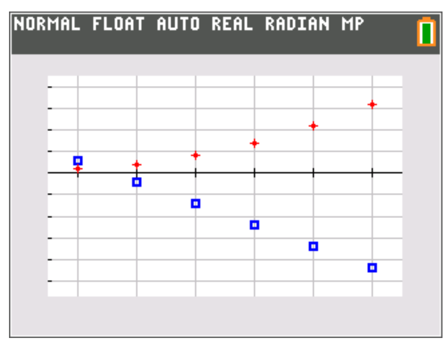
c. Study the graphs and the differences you found in L3 and L5. Make a conjecture.

Answer: Students should make conjectures about the scatter plot of a sequence and the differences between the consecutive terms. They should conjecture that for sequences with a common difference, the points form a straight line.



Now students are to clear the data from L2, L3, L4, and L5 but leave the numbers in L1. To do this, they can arrow up to the top of each list and press clear enter.

After entering the sequences shown at the right into L2 and L4, they will find the differences between consecutive terms, recording them in L3 and L5.



Students can press zoom and select **9:ZoomStat** to view the plots of the sequences.

For each sequence, students are to write the differences between the consecutive terms and give a description of the scatter plot.

2. For each sequence, write the differences between the consecutive terms and give a description of the scatter plot.

a. Sequence L2

Answer: –5, –5, –5, –5, –5. Possible answer: The points of the scatter plot form a straight line that slants down to the right.

b. Sequence L4

Answer: 1, 2, 3, 4, 5. Possible answer: The points of the scatter plot form a curve.

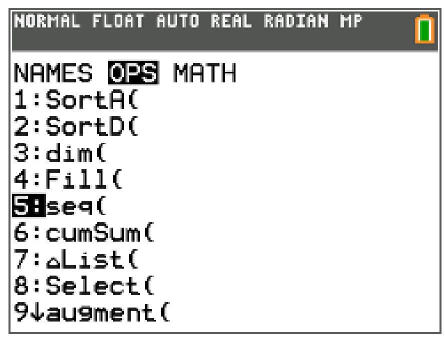
c. How do your observations affect your conjecture about the scatter plot of a sequence and the differences between the consecutive terms? Explain.

Answer: Students should find that the new data reinforces their conjecture that for sequences with a common difference, the points form a straight line.

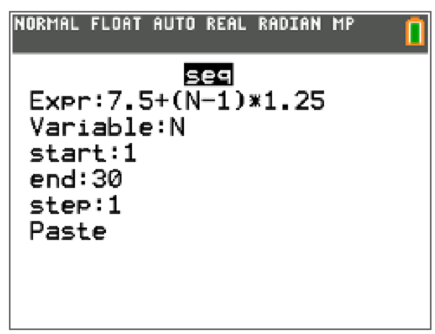
Part 2 – Explicit Formulas and Sums

Students are shown the general explicit formula for an arithmetic sequence.

$$u_n = u_1 + (n-1)d$$



They are to generate a sequence in L2 to display the first 30 terms of $u_n = 7.5 + (n-1) \cdot 1.25$. They will use the sequence command by pressing $\boxed{\text{2nd}}$ $\boxed{\text{stat}}$ [list], arrow over to the OPS menu and select **seq(**.

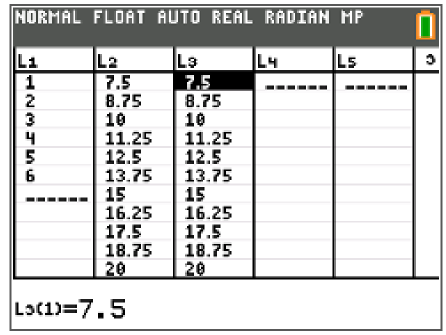


Inside the parentheses, **7.5+(N–1)*1.25** is the explicit formula for this sequence (replace u_1 with 7.5 and d with 1.25 in $u_n = u_1 + (n-1)d$, **N** is the variable, **1** is the first term to display, and **30** is the number of terms to display.

Note: N is selected by using the $\boxed{\text{alpha}}$ key.

Now students are to simplify the formula $u_n = 7.5 + (n-1)\cdot 1.25$ by distributing and combining like terms: $u_n = 1.25n + 6.25$. They need to use this formula and the sequence command to generate 30 terms of this sequence in L3.

Note: It is not necessary that students number L1 all the way to 30.



3. Simplify the formula $u_n = 7.5 + (n-1)\cdot 1.25$ by distributing and combining like terms. Use this formula in the sequence command to generate 30 terms of this sequence in L3.

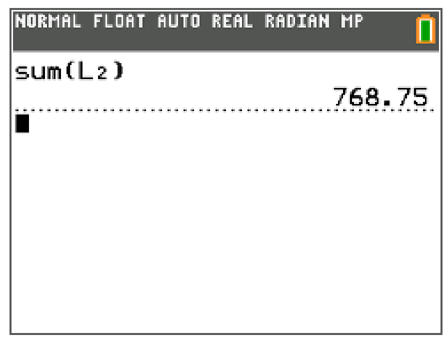
What do you notice about the terms in L2 and L3?

Answer: The terms that appear should be the same as in L2.

Part 3 – Practice Finding the Sum of a Series

Now students are to find the sum of the first 30 terms of the sequence from problem 2. The expression consisting of summing the terms in a sequence is called a series.

On the Home screen students are to enter **sum(L2)**. The sum command can be entered by pressing [2nd] [stat] [list], arrow over to the MATH menu and select **sum(**.

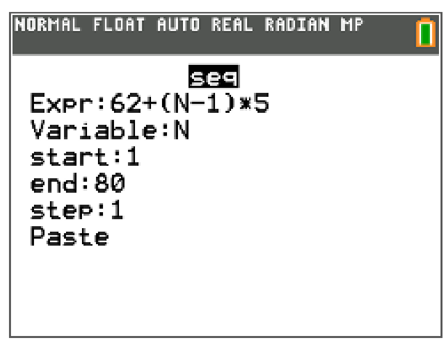


4. What is the sum of the series in L2?

Answer: 768.75

Now they are to find the sum of the first 80 terms of the sequence below, using the **Lists** feature and the **sum()** command.

62, 67, 72, 77, 82…



Students will need to clear their L2 and L3 lists and use the same procedure as before.

5. Now, let's look at another sequence. Find the sum of the first 80 terms of the sequence below, using the **Lists** feature and the **sum()** command.

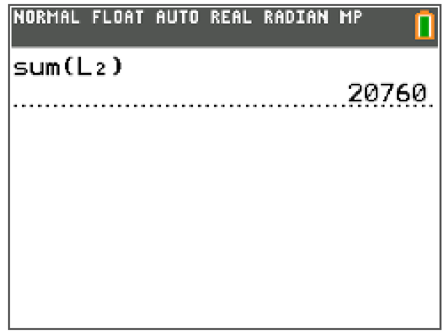
62, 67, 72, 77, 82…

a. Find the explicit formula for this sequence in simplified form.

Answer: $5n + 57$

b. What is the sum of the first 80 terms?

Answer: 20,760



Extension

As an extension, explain to students that the sum of the first *n* terms of an arithmetic series can be found by multiplying the number of terms, *n*, by the average of the first and last terms.

Have students use their calculator to show that this holds true for the sums found in Parts 2 and 3 of this activity.

