



Math Objectives

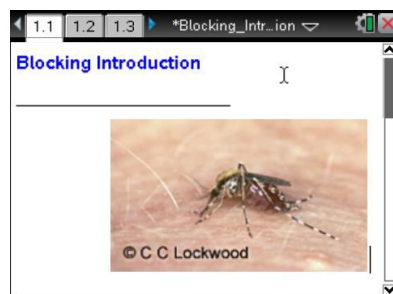
- Students will recognize that blocking can reduce variation.
- Students will describe random allocation of treatments as occurring within each block.
- Students will recognize that the experimental units *within* each block are as similar as possible and as different as possible from the experimental units in the other blocks based on a variable that has the potential to affect the outcome of an experiment.
- Students will look for and make use of structure (CCSS Mathematical Practices).
- Students will use appropriate tools strategically (CCSS Mathematical Practices).

Vocabulary

- experiment
- experimental units
- random allocation
- randomized block design
- simple random sample
- treatments

About the Lesson

- This lesson involves investigating the effectiveness of two mosquito sprays in a large tract of land by using two different experimental designs—one randomized design and one randomized block designs.
- As a result, students will:
 - Select a random sample to be used in an experiment.
 - Randomly allocate treatments and observe the mean number of mosquitoes in the plots after the sprays have been applied.
 - Examine a sampling from a population and observe a pattern in a measured variable regardless of assigned treatments (i.e., more mosquitoes closer to the swamp).
 - Compare the variability in a completely randomized design with a randomized block design.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing **ctrl** **G**.

Lesson Files:

Student Activity

Blocking_Introduction_Student.pdf

Blocking_Introduction_Student.doc

TI-Nspire document

Blocking_Introduction.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.



TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding.

Discussion Points and Possible Answers

Tech Tip: This activity should be used with the TI-NspireCX in order to differentiate between the colors used within the lesson.

Teacher Tip: This activity could be used as a demonstration lesson in PublishView to display multiple windows in the .tns file at one time.

Pages 1.2 and 1.3 contain directions to “seed” your handheld for the activity.

Kate is a realtor who needs to sell 36 lots, some of which are bordered on one side by a swamp. Because of the swamp, the land is known to have a large number of mosquitoes.

Kate has researched different types of mosquito sprays in her quest to decrease the number of mosquitoes and make the land more attractive to buyers. She only has funding to conduct one experiment on six of the 36 plots to determine which of two mosquito sprays would be the most effective in reducing the number of mosquitoes.

This activity will simulate multiple experiments in order to determine whether a completely randomized experimental design or a randomized block design is better at concluding the effectiveness of the mosquito sprays. Knowing this will help Kate determine which experimental design she should use.

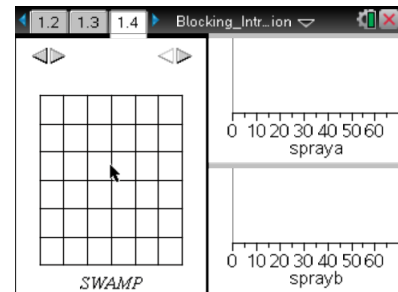
Tech Tip: Page 1.2 gives instructions on how to seed the random number generator on the handheld. Page 1.3 is a Calculator page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.).



Teacher Tip: Once students have seeded their random number generators, they do not have to do it again unless they have cleared all of the memory. But it is important that this be done if the memory has been cleared or with a new device, as otherwise the "random" numbers will all be the same as those on other similarly cleared devices.

Move to page 1.4.

1. The first method Kate used was a completely randomized design. The left work area of the page displays the plots of land. Click the *draw* arrow on the left over the plots of land to randomly select the six plots for the experiment.



Tech Tip: You can press **tab** **enter** to select the arrows, then press **▶** to select the plots instead of using the touchpad.

- a. When selecting a simple random sample of plots, is it possible to select plots that are all on a single row? Explain your reasoning.

Sample Answers: Yes, in a simple random sample all plots of land have an equal chance of being chosen because all samples have an equal chance of being chosen; thus it is possible to randomly select six plots on a single row.

- b. Click the *treat* arrow on the right over the plots of land to randomly allocate treatments (spray A and spray B) to the outlined plots.
 - The number displayed in each plot represents the number of mosquitoes counted one week after the plot was sprayed.
 - The dotplots in the right work area on the page display the number of mosquitoes found in each plot according to which spray was used.
 - The vertical line on each dotplot represents the mean number of mosquitoes in the plots treated with each spray (click on the line to display the value).

Teacher Tip: Students will generate different random samples based on the seeding of their calculators. Students should generate one sample of six plots and use that sample for the rest of the activity. It is possible to have similar boxplots generated from the different experimental designs due to the variation in the plots that are randomly selected.



- c. Record the mean value for Spray A and Spray B in the table below for experiment 1. Based on your random allocation of treatments, does there appear to be a major difference in the mean number of mosquitoes for the two treatments? Explain your reasoning.

Sample Answers: Answers will vary. Since each student is looking at only one random assignment of treatments, some might answer yes because there is a very large difference between the mean number of mosquitoes found in plots using spray A and the plots using spray B. Others might answer no because they see very little difference in the means.

- d. Suppose it were possible to carry out the experiment multiple times on the same plot of land. Click on the *treat* arrow again to generate a different random allocation of treatments. Observe the number of mosquitoes on the land plots and in the dotplots on the right. Record your mean value results in the table above for experiment 2. Write down any observations you have regarding the number of mosquitoes and the means for Sprays A and B. Explain your reasoning.

Sample Answers: It seems there are more mosquitoes near the swamp. Also, spray B seems like it kills more mosquitoes than spray A because the mean number of mosquitoes still alive after spray B is less than the mean for A.

- e. Click the arrow 23 more times (for a total of 25 experiments), each time recording the mean values in the table above and observing the number of mosquitoes on the land plots and the dotplots on the right. Is there a noticeable pattern in the number of mosquitoes in the plots of land? Explain your reasoning.

Sample Answers: There seem to be more mosquitoes in the plots treated with Spray A since most of the means for spray A are larger than those for spray B.

- f. Looking at the plots of land, describe any pattern in the number of mosquitoes based on their geographic location in relation to the swamp. Check your results with another student.

Sample Answers: The plots near the swamp have a larger number of mosquitoes, regardless of the type of spray used, than the plots that are farther away from the swamp.

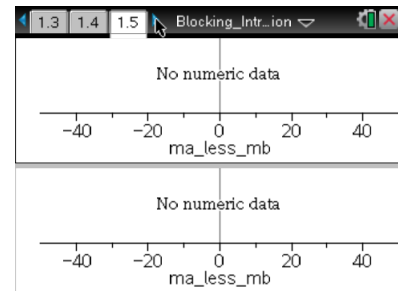


2. Does there appear to be a difference in the effectiveness of the sprays? Explain your reasoning.

Sample Answers: In general, the plots treated with spray B have less mosquitoes than the plots with spray A. As I reallocated treatments, the mean for spray B was almost always below the mean for spray A, showing on average that it killed more mosquitoes.

Move to page 1.5.

3. The distributions on Page 1.5 represent the differences in the mean number of mosquitoes left alive after each random allocation of treatments generated on Page 1.4, $\text{mean}(\text{spray A}) - \text{mean}(\text{spray B})$.
- a. What does a data value of zero mean in context of the problem?



Answer: A data value of zero means that during a simulated experiment, the average number of mosquitoes found alive in plots treated with spray A is equal to the average number of mosquitoes found alive in plots treated with spray B. So, neither spray was more effective in that trial of the experiment.

- b. How will you know whether spray A is more effective than spray B?

Sample Answers: If spray A is more effective than spray B, there will be fewer mosquitoes in the plots treated with spray A. Since there will be fewer living mosquitoes, the difference $\text{spray A} - \text{spray B}$ will be negative. All negative data points show spray A is more effective than spray B.

Teacher Tip: Students might struggle with the interpretation of the graph. A negative number does not necessarily mean a reduction in the number of mosquitoes. Since the graph plots $\text{mean}(\text{spray A}) - \text{mean}(\text{spray B})$, a negative means that there were less mosquitoes in the plots treated with spray A, and therefore A would be more effective at reducing mosquitoes.



- c. Describe what the typical difference between spray A and spray B is in the context of the problem.

Sample Answers: Answers will vary. However, most students should determine that the median is around 5. Thus, typically plots treated with spray B have about 5 fewer mosquitoes than plots with spray A.

Teacher Tip: Due to the random allocation of treatments on page 1.4, it is possible for a student, by chance, to generate trials of the experiment that create a distribution unlike those of their classmates resulting in a different conclusion.

TI-Nspire Navigator Opportunity: Screen Capture

See Note 1 at the end of this lesson.

- d. Describe the variation in the boxplot on Page 1.5.

Sample Answers: Answers will vary. Some students might comment on the range or the interquartile range. For example, the minimum and maximum range from -3.7 to 20, and the first and third quartiles range from 0 to 11.7 meaning that the middle 50% of the data shows that spray B is more effective at eliminating mosquitoes.

- e. Would you consider one mosquito spray to be much more effective at reducing the number of mosquitoes than the other? Explain your reasoning and compare this to your answer in question 2.

Sample Answers: Yes, spray B seems more effective at reducing the number of mosquitoes because the majority of the differences $\text{mean}(\text{spray A}) - \text{mean}(\text{spray B})$ are positive, meaning there are more mosquitoes in spray A's plots than spray B's. This reinforces my conclusion in question 2.

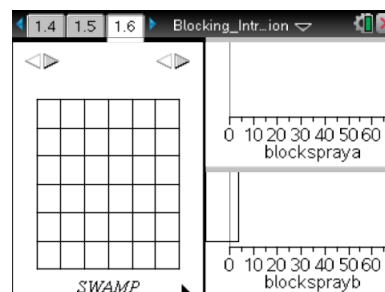
4. If you selected a different random sample of six plots and repeated 25 trials of the experiment with both sprays, would you expect to have the same conclusions as you did in question 3? Explain your reasoning.

Sample Answers: Some students might conclude that you would expect the same conclusions since the experiment was random; however, some might argue that the differences in the number of mosquitoes might have been due to chance.



Move to page 1.6.

Henry thinks there might be another way to design the experiment (besides a completely randomized design) to determine which spray is more effective at reducing the number of mosquitoes. He is required to use the six outlined plots in the experiment. Since there seem to be more mosquitoes near the swamp, he thought it might make sense to pair plots of land based on their distance from the swamp. He wanted to pair the closest two plots to the swamp, the next two closest to the swamp, and so on.



5. a. Think about which plots Henry would pair together. Draw some possible pairings below (be sure to label which plots are paired together).

Sample Answers: Answers will vary depending on the six plots that were randomly selected.

For example:

		1	1		
2					
		2			3
		3			

		1	1		
2					
		3			2
		3			

- b. Click on the *block* arrow in the left corner above the plots. Pairs are highlighted in matching colors. How do these pairings compare to the ones you drew in part a?

Sample Answers: The pairings matched one of my examples. The plots that were the closest to the swamp were paired, then the next two, etc.

TI-Nspire Navigator Opportunity: Screen Capture

See Note 1 at the end of this lesson.

- c. Will Henry be satisfied with the pairings? Explain your reasoning.

Sample Answers: Yes, the two on the lowest rows are paired, the next 2 lowest plots are paired, and the two highest plots were paired.



6. Henry's experimental design is called a randomized block design. Grouping experimental units that are as similar as possible based on a variable that has the potential to affect the experimental results is known as **blocking**. In this case, the distance from the swamp is that variable. Each color pair represents a block.
- a. Look at each block and describe what you see in terms of the experimental units.

Answer: Spray A and spray B were each randomly assigned to one plot within each block.

TI-Nspire Navigator Opportunity: Quick Poll

See Note 2 at the end of this lesson.

- b. Click on the *treat* arrow on the right above the plots to randomly assign treatments (spray A and spray B) within each block. Cursor over the lowest value in the dotplot. Record the value, and write a sentence describing what it means in the context of the problem.

Sample Answers: -2; one block had a difference of two mosquitoes between the two plots within the block. The plot treated with spray A has two fewer living mosquitoes than the plot treated with spray B within a single block.

- c. The vertical line is the mean of the differences in the number of mosquitoes within the three blocks. What does a mean of one (1) represent?

Note: The special case of blocking where the blocks contain two experimental units each is called a *matched pairs design*.

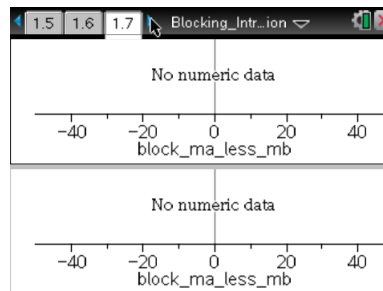
Sample Answers: Each difference (spray A – spray B) in the number of mosquitoes within each block was found. The average of the three differences is 1. On average, within each block the plot treated with spray A has one more living mosquito than the plot treated with spray B.

- d. Suppose it were possible to carry out the experiment multiple times on the same plot of land. Click the arrow 24 more times, to reallocate treatments.



Move to page 1.7.

7. The distributions on Page 1.7 represent the mean differences (spray A – spray B) of the number of mosquitoes for each random allocation of treatments within the blocks you generated on Page 1.6.
- a. Describe the typical difference between spray A and spray B in the context of the problem.



Sample Answers: The center for the differences in the mean number of mosquitoes (spray A – spray B) is about 5, just as before. This means that on average spray B eliminates about 5 more mosquitoes than spray A.

- b. Describe the variation in the boxplot.

Sample Answers: Answers will vary. In my boxplot, the variation is pretty small. The first and third quartile range from -1 to 6. Thus, the majority of the trials of the experiments found a larger average number of mosquitoes in plots with spray A than those with spray B.

- c. Compared to the variation in the plot, does the difference between the median of the plot and zero seem meaningful? Explain your reasoning.

Sample Answers: Since the variation is small, the difference is meaningful. Most of the data values are positive and have a small variation so about 25% of the time I would make the wrong decision about the spray.

- d. Would you consider one mosquito spray to be much more effective at reducing the number of mosquitoes than the other? Does this conclusion differ from the conclusion you made in question 3e? Explain your reasoning.

Sample Answers: Yes, since my boxplot has a small variation and most of the data values are positive. This means more mosquitoes are in the plots treated with spray A on average than those treated with spray B. If there are more mosquitoes with spray A, then spray B must be more effective at reducing the mosquitoes. This answer is the same as my answer for the completely randomized experimental design.

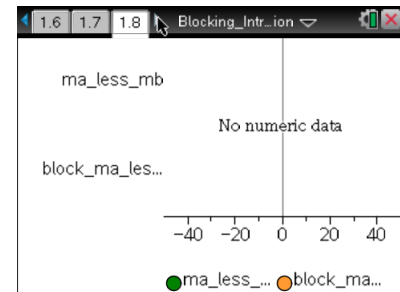
TI-Nspire Navigator Opportunity: Screen Capture

See Note 3 at the end of this lesson.



Move to page 1.8.

8. The top boxplot on this page displays the mean differences between spray A and spray B generated by completely randomized experimental designs. The bottom boxplot displays the mean differences between the sprays generated by randomized block designs.
- a. Compare the centers of the two boxplots. Are they similar or different? Explain your answer based on the type of experimental design.



Sample Answers: The medians of both boxplots are around 5 mosquitoes. Thus, regardless of whether you used a block design or a completely randomized design, it appears that spray B is more effective than spray A at eliminating mosquitoes.

- b. Compare the variation in the two boxplots.

Sample Answers: There is a much larger variation in the upper boxplot from the completely randomized experimental design.

- c. If a researcher can only conduct a single experiment, what advantage does a design that produces a small variation give him?

Sample Answers: A design with a small variation ensures that a researcher is more likely to conclude a typical result since other conclusions are rare due to the small variation. In the completely randomized design, a wrong decision was concluded about 25% of the time.

- d. The goal is to determine which spray is better at reducing the number of mosquitoes. Which method, completely randomized or randomized block design, seems better for determining the answer? Justify your response.

Sample Answers: Randomized block design seems better for determining this answer because it did show a difference in the effectiveness of sprays and had a much smaller variation. It is more likely to have a definitive conclusion and less likely to generate a wrong decision.

- e. What is one benefit of blocking in an experiment? Explain your reasoning.

Sample Answers: Blocking reduces variation as seen in the boxplot generated from blocking the plots of land based on their location to the swamp.



TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- In order to determine the most definitive conclusion to a single experiment, an appropriate experimental design that has the least variation is needed.
- A randomized block experimental design can reduce variation.
- Blocks contain experimental units that are as similar as possible based on a preexisting condition.
- Treatments are randomly assigned within each block.

Assessment

Identify each as always, sometimes or never true. Be ready to justify your reasoning.

1. When blocking, each block should be as similar as possible based on a characteristic that might affect the results of an experiment.

Answer: Always

2. A completely randomized experimental design generates more variation the results of experiments than other experimental designs.

Answer: Sometimes

3. Blocking reduces variation in experiments.

Answer: Sometimes. When blocking is appropriate in a given experiment, it can reduce variation.



TI-Nspire Navigator

Note 1

Question 3c and 5b, Name of Feature: Screen Capture

A Screen Capture can be taken at this point to compare random samples of plots and allocations of blocking. All students should have a different random samples of plots, but all are blocked using the same criteria.

Note 2

Question 6, Name of Feature: Quick Poll

A Quick Poll can be given at this point to ask. True or False. Treatments are randomly assigned within each block.

Note 3

Question 7d, Name of Feature: Screen Capture

A Screen Capture can be taken in order to show that regardless of random sampling and random allocation of treatments, blocking appropriately reduces the variation in the results of the experiments.

Note 4

Name of Feature: Quick Poll

A Quick Poll can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students might have.