Handheld Graphing Technology in Secondary Mathematics:

Research Findings and Implications for Classroom Practice

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Table of Contents

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[i] Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2001). *Teacher preparation research: Current knowledge, gaps, and recommendations*. Seattle, Washington: Center for the Study of Teaching and Policy, University of Washington.

Overview

Handheld Graphing Technology at the Secondary Level: Research Findings and Implications for Classroom Practice synthesizes peer-reviewed, published research that addresses questions from five areas related to the use of this technology in teaching and learning secondary mathematics: 1) teacher knowledge and beliefs about handheld graphing technology, 2) nature of student use of the technology, 3) relationship of the technology to student achievement, 4) gains made by students using the technology, and 5) influence of technology on diverse student populations.

From a field of over 180 research reports, the research team for this project chose reports of 43 studies that met the criteria for inclusion in the synthesis. While trying to reduce complex findings from the studies to simple conclusions is very difficult, some important areas for consideration did emerge. A core finding from the research is that the type and extent of gains in student learning of mathematics with handheld graphing technology are a function, not simply of the presence of handheld graphing technology, but of how the technology is used in the teaching of mathematics. Given supporting conditions, the evidence indicates that handheld graphing technology can be an important factor in helping students develop a better understanding of mathematical concepts, score higher on performance measures, and raise the level of their problem solving skills.

The results provide findings in the following areas:

Comprehension The research indicates that students who used handheld graphing technology with curriculum materials supporting its use had a better understanding of functions, variables, solving algebra problems in applied contexts, and interpreting graphs than those who did not use the technology. Students who spent more time using handheld graphing technology showed greater gains than students who had access to the technology for brief interventions or short periods of time.

Equity There is little research on issues of equity. The research on gender issues is mixed with some studies showing no difference in achievement between males and females while others have evidence of increased performance by females. Several studies found that lower achieving students made larger performance gains when using handheld graphing technology than did moderate and high achieving students. The use of handheld graphing technology also seemed to decrease the performance gap between higher and lower achieving students.

Professional Development The findings indicate that simply providing teachers with information about how the technology functions is not likely to result in effective integration in the classroom. Substantial professional development and support is necessary for teachers to make informed decisions about how to best use handheld technology in their classrooms.

Usage There is considerable evidence that students use handheld graphing technology when quick and accurate graphs will aide in their problem solving. Some evidence suggests that handheld technology can be under used, especially when students are not sure how to use the technology as a tool in their work or when they are unsure how much written work is required. Other researchers indicated concern about students' over reliance on the technology, accepting results at face value with little critical thinking.

Approach The research indicates that students with access to handheld graphing technology engaged in problem solving and investigations more often and were more flexible in their solution strategies than students without access.

Mathematical Context The evidence shows that the mathematics and the technology must work together for the outcomes to be most beneficial. Integrating, not simply adding, the use of handheld graphing technology within the context of the mathematics being studied can help students develop essential understandings about the nature, use, and limits of the tool and promote deeper understanding of the mathematical concepts involved.

The results highlight important considerations for teachers. In making informed decisions about how and why they use the technology in their classrooms, for example, teachers should be aware of the need to provide instruction on how to use handheld graphing technology for a particular purpose and emphasize the connections among representations. The report also identifies specific issues regarding the effective use of handheld graphing technology in the classroom that have not yet been adequately investigated. For example, there were no studies on the long-term effects of using handheld graphing technology or about the potential of handheld graphing technology to change the curriculum. There were few studies related to its use in grades 7-9. The results also indicate that additional research is needed with regard to equity issues and the relationship between technology usage and the beliefs held by teachers and students.

Handheld Graphing Technology at the Secondary Level: Research Findings and Implications for Classroom Practice

Executive Summary

Handheld graphing technology in the form of graphing calculators is a part of mathematics teaching and learning in most high school classrooms in the United States. According to data obtained from a national survey, as of 2000, over 80 percent of high school teachers used handheld graphing technology in their classrooms. Yet, questions such as, "What is the nature of the tasks for which the technology is used?" "How do students and teachers choose to use the technology?" "What is the impact of its use on student understanding?" and "Which students benefit from using technology?" are open questions. Research can help us understand how technology may be a positive influence on teaching and learning and how it becomes a barrier.

Differences in how handheld graphing technology is used in classrooms and in how its impact is measured contribute to serious disagreements about the role of graphing calculators in mathematics education, and their effect on students' mathematical understanding, ability to perform routine procedures, and facility with algebraic skills. The purpose of this report is to summarize what rigorous, peer-reviewed research tells us about key issues in the use of handheld graphing technology with content that is traditionally included in secondary mathematics. Research has examined questions about student understanding and achievement, teacher knowledge and beliefs about mathematics and technology, and issues of equity. The results can provide directions for those who are working to improve students' understanding of mathematics with handheld graphing technology. The results also offer some implications for classroom practice that should be considered by teachers who use handheld graphing technology in their classrooms.

We examined more than 180 published research reports about handheld graphing technology and found 43 studies that met our criteria for inclusion in the summary. Reducing the complex findings in these studies to simple conclusions and brief descriptions is both difficult and risky. Statements about student achievement with handheld graphing technology are rarely unencumbered; many variables are involved in the interactions that affect students' mathematical understanding and performance. In addition, the studies differed widely in scope, focus, and design, and in many instances, it was difficult to determine whether there was consistency in the findings. We have chosen to group the studies in terms of their response to the central questions described below and have examined the findings in each group for trends and discrepancies. Individual studies cannot tell us definitively how to proceed – and only occasionally will accumulated work point us in definite directions. While there is clearly a lack of cumulative research in any one area related to handheld graphing technology, the work does allow us to identify some important areas to be considered. The research thus far, while uneven, establishes promising groundwork for rigorous research in the future.

The report addresses five central questions. Overall, research related to the use of handheld graphing technology is relatively sparse, with a disparity in the amount of research on which to draw when discussing each question. Building a more complete picture will take the development of more refined instruments and methods as well as complementary research designs that collect both qualitative and quantitative data.

What Answers Does Research Give to Questions about the Use Of Handheld Graphing Technology?

How do teachers use handheld graphing technology and how is this use related to their knowledge and beliefs about technology, mathematics, and teaching mathematics?

Despite the opportunities offered by technology for teachers to change their teaching practice, researchers report that teachers generally use handheld graphing technology as an extension of the way in which they have always taught. In addition, research indicates that the way teachers use the technology in their classrooms is often related to their beliefs about mathematics. If teachers perceive mathematics as a closed, answer-based domain, graphing calculators are used accordingly in their classes. If, on the other hand, teachers emphasize conceptual understanding, making sense of ideas, and drawing conclusions based on mathematical grounds, their use of the technology tends to reflect these beliefs. Teachers who emphasize connections among representations and sense making in working with both the mathematics and the tool see the results in the performance of their students.

Simply providing teachers with information about how the graphing calculator functions will not lead to significant changes in their teaching practice. Substantial intervention in the form of professional development and support is necessary if teachers are to make informed pedagogical decisions when they and their students have access to handheld graphing technology. Professional developers should expand their focus beyond the functionality of handheld graphing calculators to include investigations into the role technology can play to help teachers achieve their instructional goals and how it can impact the very mathematics being taught.

With what kinds of mathematical tasks do students choose to use handheld graphing technology? How do students use the technology to carry out these tasks?

Research indicates that teachers' beliefs and teaching methods influence how students use technology. Students tend to use the methods that are illustrated and preferred by their teachers. In some cases, teachers leave the development of calculator skills largely to the students; in other cases, student calculator use is strongly shaped by the teacher's decisions and interventions.

Most researchers found students used handheld calculators as a computational tool, to move among different representational forms, and as a visualizing tool. The primary use of handheld graphing technology, however, was to graph. In some cases, researchers found students used the technology to investigate and explore, but on tasks that did not require graphing, their use of handheld calculator was minimal. Research on using handheld calculators to check solutions, varies. There is some evidence that calculators are over-used to the point that students rely on the calculator with little critical analysis of the results. Other evidence suggests that handheld calculators are under-used, especially when students are not sure how to use of the calculator as a tool in their work or when they are unsure how much written work is required. Few studies examined the use of the programming capabilities of handheld graphing technology or to its use in data collection and analysis.

The evidence shows that the mathematics and the technology must work together for the outcomes to be most beneficial. Integrating, not simply adding, the use of handheld graphing technology within the context of the mathematics being studied can help students develop essential understandings about the nature, use, and limits of the tool and promote deeper understanding of the mathematical concepts involved. While some evidence suggested that students use the technology in unexpected ways, most of the studies were designed with specific uses and approaches in mind. Few paid attention to what students might do differently because they have access to the technology.

Learning to use the technology in ways that are useful can be complicated. In particular, studies related to the use of calculators with computer algebra systems (CAS) or symbolic manipulators pointed out that learning to use the tool effectively is extremely complex, need to be mediated by the teacher, and takes considerable time.

What mathematical knowledge and skills are learned by students who use handheld graphing technology? In what ways do students use this knowledge and these skills?

Not surprisingly, students using handheld graphing calculators generally learned what they were taught either implicitly or explicitly. Access seems to make a difference. Students who spent more time learning to solve applied problems did better on those problems, while students who spent time on procedures did better on those problems, with the exception of students who had limited access to the technology. The findings from the studies that met our criteria also indicate that students who use handheld graphing technology have a better understanding of functions, of variables, of solving algebra problems in applied contexts, and of interpreting graphs than those who did not use the technology.[ii] Students who used calculators with computer-assisted algebra systems were better able to apply calculus concepts than those without that experience. No significant differences in procedural skills were found between students who use handheld graphing technology and those who do not. This indicates that extensive use of the technology does not necessarily interfere with students' acquisition of skills.

The evidence that simply introducing handheld graphing technology into the classroom is not enough to make a difference in student learning is relatively strong. In addition to the way in which the technology is used, time spent using the technology is a critical factor. More access to the technology translates into greater impact on student learning. Using graphing calculators with or without CASfor only short periods of time seems to benefit lower-achieving students in terms of skillsand accuracy but not conceptually. The research supports what some might suspect: the curriculum, student teacher interaction, how the tool is used in the classroom, and students' existing mathematical knowledge and beliefs all appear to be significant factors in determining what mathematical knowledge and skillsare learned by students who use handheld graphing technology and how they use this knowledge and these skills.

What is gained mathematically by students using handheld technology that cannot be observed in a non-technology environment? In what ways do students use this knowledge and these skills?

Students with access to handheld graphing technology use graphs and engage in mathematical explorations more often than students without access. They are more flexible in their solution strategies, make conjectures and move among algebraic, numeric and graphical approaches, develop calculator-based strategies to manipulate symbolic expressions, and work comfortably with real data.

The use of handheld graphing technology, however, may further extend students' misconceptions about mathematical concepts, such as increased confusion between rational and real numbers. They may accept visual images without question. They may be often misled by a lack of understanding of scaling and technical details such as the interaction of the pixels with the visual representation. Simply using handheld graphing technology is not likelyto cause students to address these issues. Technical errors that students make, such as syntax errors, may stem from their limited understanding the mathematical concepts involved.

What impact does handheld graphing technology have on the performance of students from different gender, racial, socio-economic status, and achievement groups?

There is little research on issues of equity. Some of the research, particularly those related to closing achievement gaps between groups, investigates differences among those who received the same treatment in an experiment. In studies where researchers examined performance variability within, rather than simply between, the treatment and control groups, the results usually indicated no significant differences in performance could be attributed to gender, race, socio-economic status, or prior knowledge/achievement. However, some studies attributed differences in student performance to one or more of these variables.

The research on gender issues is mixed. Some studies show no difference in achievement between males and females while others provide evidence of increased performance by females using the technology on items where male performance was superior without the use of the tool. The observed interaction between genders and item types raises new questions and offers opportunities to identify additional variables that contribute to differential performance between genders.

With regard to ability level, some studies found that lower-achieving students made larger performance gains when using handheld graphing technology than did moderate and high achieving students who also used handheld graphing technology. More detailed information is needed about the specific contexts in which these differences arose.

Implications for the Classroom

Teachers use technology as an extension of how they already teach which, in turn, is a function of what they know and believe about teaching, learning, and mathematics. If change is desired, teachers need professional development that will help them feel knowledgeable about technology and that focuses beyond the functionality of the tool to incorporate the technology as a means of meeting mathematics learning goals. In addition, professional development should provide opportunities for teachers to reflect upon and discuss their beliefs about mathematics, teaching, and learning in relationship to their knowledge and beliefs about the use of technology in the mathematics classroom. Teachers should be prepared to respond to problems highlighted in the research; for example, the confusion about some features of the calculator such as scaling issues and failure of the technology to accurately represent discontinuities. Teachers should be involved in helping students learn how to use the calculator with full recognition of its constraints and potential. They should also understand various profiles of student behavior in order to design and implement appropriate mathematical activities using handheld graphing technology. Students would benefit from confronting limitations of the technology and considering how to make more effective use of the technology. Attempting to explain these limitations can lead to better mathematical understanding.

Some researchers have pointed out that the use of multiple representations does not ensure that students will make links between representations. Overall, researchers found that reconciling different types of information is not intuitive but needs to be taught. Students learn how to resolve conflicts between symbolic and graphic information.

Mathematical difficulties often point to curricular shortcomings, which may in turn contribute to adverse effects whether or not graphing calculators are used. For example, there is some evidence that students, both with and without the use of handheld graphing technology, are low performing in algebra. This suggests a need to reexamine how the subject is taught. In addition, the ways in which students' use or misuse the calculator can reveal their lack of mathematical understanding that their written work did not. By understanding how students typically use handheld graphing technology teachers can lead students to the best uses in order to develop the desired mathematical knowledge .

Students who owned their own calculators more frequently exhibited a critical awareness of the calculator. Because regular access to the technology seems to have a positive influence on learning, efforts should be made to provide all students with continual access to handheld graphing technology, although, because of the particular learning objectives and nature of understanding desired, some specific tasks may be designed to be independent of the tool. Within classrooms, teachers should pay explicit attention to issues of equitable access. Once equitable access is ensured, teachers should attend to students' patterns of use. If systematic differences are noted, efforts are needed to determine and address the underlying causes. In some cases, addressing this issue may be as simple as providing additional training for students. In other cases, it may require a reconceptualization of the rationale for using handheld graphing technology or a shift in the role that handheld graphing technology plays in instruction.

Future research

The research we studied provides a starting point for efforts to better understand how to effectively use handheld graphing technology in the classroom. Research should be designed both to look across schools and across content areas to support broad generalizations and to take a close look at particular cases. Cases can identify promising variables for inclusion in broad surveys, and surveys can position and help in the interpretation of particular cases. Because one study does not produce definitive results, multiple designs applied over time are necessary to build a knowledge base.

As we move forward, data collected about the use of handheld graphing technology should describe the specific features of the context—including the handheld graphing technology used, the content, and the aspects of use that are being investigated - not merely counts and observations. Better descriptive tools for characterizing student learning with handheld technology and for looking at factors related to this learning are needed.

Research programs should have several characteristics. Design and reporting of research on the use of handheld graphing technology must be explicit about connections to improving student achievement. Programs should include or facilitate comparisons among different ways of using handheld graphing technology as well as between those who use it and those who do not. Research should include within-groups as well as between-groups comparisons of students with and without access to handheld graphing technology to determine if differential effects exist for students from different backgrounds, in various contexts. In addition, research on the use of handheld graphing technology should include the length and nature of access to handheld graphing technology, in particular studying student learning in situations with unlimited access over several years and in a progression of mathematics courses. Finally, research more explicitly informed by a historical perspective would help in sorting out issues that are particular to technology from those that are independent of the technology.

Research on the use of handheld graphing technology is not robust. Individual projects look at specific pieces of the picture, but the pieces do not make a coherent whole and, in fact, often seem unrelated. In addition to recommending that research be coordinated, we recommend research designed to answer questions such as those described below.
Because the use of handheld graphing technology is not a variable that can be isolated but is a part of the complex teaching and learning environment, such research, however, should be done taking other factors in this environment into account.

Recommendations: Research is needed in the areas of:

Teacher knowledge, beliefs, and experiences related to the use of handheld graphing technology

- How do teachers' beliefs about handheld graphing technology explicitly affect their use of graphing calculators in their teaching?

- What experiences inpreservice and inservice education influence teacher beliefs about the use of handheld graphing technology and how they choose to use it in their classrooms?

- What is the relationship between high quality teacher preparation with respect to handheld graphing technology and student achievement?

- What is the relationship between teacher beliefs and use of handheld graphing technology and student beliefs and use?

Curriculum implications related to handheld graphing technology

- What is the role of handheld graphing technology in learning mathematical content that is not part of the traditional mathematics curriculum?

- What is the role of handheld graphing technology in as providing access to mathematics content earlier than would have traditionally been done?

- In what ways does the nature of the curriculum and tasks students are given influence their use of handheld graphing technology?

Student beliefs, understandings, and characteristics

- What are students' attitudes and beliefs about the use of handheld graphing technology and how do these affect student use of the tool?

- How do students who have access to graphing calculators compare in terms of their use with respect to ethnicity, gender, geographic, and socio-economic conditions?

Assessment

- What are the relationships between "high stakes" assessments and the use of handheld graphing technology?

Many educators suggest that handheld graphing technology has the potential to significantly affect mathematics teaching and learning. Research can provide direction and guidance for using handheld graphing technology in ways that support student learning. By conducting rigorous studies of important questions and relating the results to classroom practice, we can work to see that handheld graphing technology makes positive contributions to improved mathematics education.

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[i] This report is modeled on a report by Wilson, S.M., Floden, R.E., & Ferrini-Mundy, J. (2001). Teacher preparation research: Knowledge, gaps, and recommendations. Seattle, WA: Center for the Study of Teaching and Policy, University of Washington.
[ii] For references and study details, see pages 30 - 37 of the full report.

Handheld Graphing Technology at the Secondary Level: Research Findings and Implications for Classroom Practice

> *Look around you in the tree of Mathematics today, and you will see some new kids playing around in the branches. They're exploring parts of the tree that have not seen this kind of action in centuries, and they didn't even climb the trunk to get there. You know how they got there? They cheated: they used a ladder. They climbed directly into the branches using a prosthetic extension of their brains known in the Ed Biz as technology. They got up there with graphing calculators. You can argue all you want about whether they deserve to be there, and about whether or not they might fall, but that won't change the fact that they are there, straddled alongside the best trunk-climbers in the tree -- and most of them are glad to be there."*[1]

The introduction of handheld graphing technology into the secondary school curriculum began in 1986 with the Casio fx-7000G.[2] Opinions differed then - and still do - with regard to the appropriate role of handheld graphing technology in mathematics classrooms. The technology, however, supported the creation of new visions for mathematics education, many of which called for broader access to deeper mathematics for all students.[3] This was especially true of the vision in the 1989 *Curriculum and Evaluation Standards for School Mathematics* produced by the National Council of Teachers of Mathematics (NCTM). The *Standards* asserted that scientific calculators with graphing capabilities should be available to all students at all times.[4] Mathematics educators responded to this challenge in a variety of ways. According to Heid (1997), the technology use that evolved was based on four principles: mathematics classrooms should be learner-centered; students should experience what it is like to be a mathematician, learning should be enhanced through increased opportunities for reflection, and there should be a shift in the locus of authority for learning in the classroom from teacher to student. Technology, including handheld graphing technology, was seen as a way to realize these goals.[5]

This new vision for teaching and learning mathematics in a technologically rich environment suggested the need for changes in mathematics curricula and new expectations for what should take place in mathematics classrooms.[6] In particular, some educators hypothesized that the use of handheld graphing technology would have a significant effect on the way mathematics is taught.[7] Several educators argued that changes should be incremental and that essentially the same mathematics should be studied;[8] others suggested that the nature of the mathematics should be significantly different from that traditionally taught.[9] Some created opportunities to use the technology within traditional courses, for example the Calculator and

Computer Pre-calculus Project (C^2PC).[10] Others created new mathematics curricula, such as the Core Plus Mathematics Project,[1] where the use of graphing technology was integral.[11]

While some educators embraced the technology as a means to improve mathematics education, others expressed concerns about its availability and possible negative impact. One concern was the possibility that the calculator would absorb the attention of individual students to the exclusion of the mathematics.[12] According to Heid (1997), additional concerns were related to:

- the possibility that students may become too reliant on the technology,
- the possibility that technology-infused curricula would simply replace one set of routine behaviors for another rather than promote deeper understanding,
- financial constraints on poor schools and families, and the impact of this on equitable access to the technology,
- questions about gender equity, in particular that the use of the technology would lead to greater benefits for males, and
- the need for teacher preparation and support programs designed to promote effective technology use.

In 1996, the introduction of handheld graphing calculators with computer algebraic systems (CAS)[2] raised additional concerns. Waits and Demana, pioneers in the use of handheld graphing technology, acknowledge both a transformed vision of mathematics classrooms with CAS as well as controversy surrounding the use of such symbolic manipulators.[13] Learning how to use this new technology was unexpectedly complex. In a long-term study on CAS calculator integration into mathematics teaching in France, the IREM[3] team found that teachers and students using the technology faced many situations that have no counterpart in paper-and-pencil environments.[14] This finding raises many concerns about technology-readiness, not simply from the access perspective but also with respect to teacher and student preparation.

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[1] Information about the use of handheld graphing technology in the Core Plus materials can be accessed through the website: http://www.wmich.edu/cpmp/overview.html
[2] CAS calculators can carry out symbolic manipulation.

Teachers who have little experience with handheld graphing technology express varied concerns about its role in instruction and its possible impact on students. In some cases, as mathematics educators' gained more experience with handheld graphing technology, they began to view the tool as a means to improve their practice.[15] Ruthven (1992) reported on a teacher who initially expressed concern that the calculator would increase cognitive demands on students and that it would lead to mindless manipulation. As his experience with handheld graphing technology increased, his appreciation of the technology grew. He later stated that the introduction of the graphical calculator had "revolutionized" his approach to the teaching of many mathematical topics.[16]

Despite instances of successful small-scale implementation and integration, the use of handheld graphing technology on a larger scale was uneven. In 1996, after conducting a survey of the research on the use of calculators, Ruthven concluded that despite a ten year interval since their introduction, "calculators are largely confined to the margins of classroom life; casually used, primarily instrumentally, and often uncritically. Many important issues surrounding calculator use remain poorly conceptualized."[17] In the six years since Ruthven's evaluation of the state of research on calculators, handheld graphing technology has become more available and research on its uses has become more accessible.

The National Research Council found that "instruction that makes productive use of computer and calculator technology has beneficial effects on understanding and learning algebraic representation…"[18] This report takes a comprehensive look at what we do know about the influence of handheld graphing technology on mathematics teaching and learning of mathematical content taught at the secondary level. The critical question was—and continues to be—"What impact does the use of handheld graphing technology have on what students learn and how they learn?"

This is not an easy question to answer. Access to student thinking is difficult in any research, but investigating student use of handheld graphing technology has the added complexity that there is no record

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[3] IREM translates from French as to *Institutes for Research on the Teaching of Mathematics*.

of student work except through their self -reports. Many variables are involved in the interactions that affect students' mathematical understanding and performance. The presence and functional capability of the technology are only two of these variables. Additional variables that a comprehensive study on handheld graphing technology should include relate to:

- Learning to use the tool: Nature and duration of teachers' and students' instruction on the use of handheld graphing technology.
- Frequency of use: Amount of practice with handheld graphing technology.
- Interval between uses: Length of time between activities where handheld graphing technology was used.
- Tasks for which it is used: Mathematical activities for which the use of handheld graphing technology is allowed and actually used.
- Materials: Texts and instructional materials used in courses.
- Instruments: Validity, reliability, and content of instruments used to assess student outcomes.

Within the literature on empirical studies, researchers vary in 1) the ways in which and 2) the degree to which they describe the technology used and the learning context in which it was used. This presents difficulties when we attempt to determine what effect handheld graphing technology has had on student learning. Despite the difficulties inherent in analyzing a data set with these complexities, it is important to make the effort.

According to the *Report of the 2000 National Survey of Science and Mathematics Education,* more than 80 percent of the high school mathematics teachers surveyed use handheld graphing technology in their classrooms.[19] There is also growing evidence of its prevalence in mathematics education other countries.[20] With handheld graphing technology so clearly present in secondary mathematics education in the United States, understanding how this technology is being used and to what effect, will help ensure uses that are consistent with improving mathematics teaching and learning. This report establishes criteria for summarizing research related to the influence of handheld graphing technology with content that is traditionally taught at the secondary level and summarizes the main findings and gaps in the research.

Relevant Background

Any review of handheld graphing technology and its use in mathematics classrooms should begin by considering relevant features of the educational context in which the technology is being used. First, the use of handheld graphing technology is contested. The NCTM *Principles and Standards for School Mathematics* calls for "wide and responsible use "of technology,[21] but there is no common understanding within the mathematics education community of what this means. Attempts to be explicit are often tempered by personal views of mathematics and what it means to learn and to teach mathematics.

Second, a focus of this review is how technology influences what mathematics students are learning and how they are learning it. What constitutes achievement is also a matter of debate and in the views of some educators may not necessarily be aligned with understanding, particularly in terms of scores on large-scale high-stakes tests.[22] The research reviewed in this report attempts to identify the kind of knowledge and understanding that is being considered and to look at factors closely related to questions of student learning such as teacher preparation and background, calculator access, and how what is taught is affected by the technology.

Finally, the nature of handheld graphing technology has changed significantly and rapidly since its introduction in 1986. Early tools gave way to increasingly more sophisticated calculators; calculators with computer algebra systems were introduced in 1996, and in 1998 Flash ROM that allowed software applications to be run on the calculator appeared. These significant changes in the nature of the calculators over this time span add to the complexity of interpreting the relevance and significance of the research literature produced over the last sixteen years. Findings about student use of the first graphing calculators, that had limited list capabilities and fewer choices, may not be related at all to the way students use the current generation of calculators and those with computer algebra capabilities.

Despite these limitations, the available research on handheld graphing technology can point to some overarching themes and conditional results that can assist researchers and educators in their efforts to make the most of what handheld graphing technology has to offer. This report presents summaries of the research since the introduction of handheld graphing technology in 1986 and describes some of the studies more in-depth to illustrate the complexities of addressing questions about the impact of handheld graphing technology on mathematics teaching and learning.

Methods Used

We developed the methods for this report by building on the work of Wilson, Floden, and Ferrini-Mundy (2001) who established a set of criteria to judge the quality of research included in a review of the literature, developed tables for summarizing the included reviewed research, and developed a format for reporting the results. We began by modeling our report on those three dimensions of theirs.[4] In addition, our criteria, stipulated that the research we included was "scientific" as described in the 2001 report from the National Academy of Sciences on scientific inquiry.[23] We added a dimension to the format for reporting results, and made other modifications that were appropriate for the topic of handheld graphing technology at the secondary level.

We identified possible studies by searching databases such as ERIC, First Search, Education Abstracts Full Text, Academic Search Elite, Education Full Text, and WorldCat. We also searched the reference lists of relevant meta-analyses, literature reviews and reports and examined the tables of contents of prominent educational research journals. We contacted researchers and teacher educators for their recommendations, consulted web sites related to the use of technology in education, and reviewed the references cited by other researchers in their work on calculator usage. Scholars reviewed drafts of this report and suggested studies that were missing. When possible, we asked the original researchers of the studies included in this report to verify our summaries and interpretations of their work. We also reviewed what scholars have written about the nature of quality educational research.[24]

Selection Criteria

The following criteria were used to select research studies for review. We sought research that was:

- Published: The studies have been published in a scientific journal that uses independent peer review before accepting research for publication.
- Relevant. The research questions in the study speak directly to the questions framing this report.

[4] Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2001). *Teacher preparation research: Current knowledge, gaps, and recommendations*. Seattle, Washington: Center for the Study of Teaching and Policy, University of Washington.
The model developed by Wilson, Floden, and Ferrini-Mundy was very useful in organizing our approach to the task, and consequently, we used it to frame our results. The report can be found at www.ctpweb.org.

- Evidence based. The studies offered evidence (quantitative, qualitative, or both) for conclusions, rather than merely describing possible uses or potential areas of change.
- Rigorous. The studies included in this review met accepted standards in relevant research traditions.
- Scientific: The design called for direct, empirical investigation of an important question, accounted for the context in which the study is carried out, was aligned with a conceptual framework, reflected careful and thorough reasoning, and disclosed results to encourage debate in the scientific community.[23]

Differences in how research is conducted and reported across continents and within countries made it difficult to synthesize studies from other countries, but where the work was deemed relevant and based on well grounded research, it has been included. One point to note here is that there is no common international consensus about what constitutes secondary mathematics. As a result, some of the studies reviewed consider mathematical content that would not typically be taught in United States high schools. Most of the studies we reviewed were reported in English, although as part of the selection process several were translated into English, then evaluated for possible inclusion.

Based on the above criteria, many articles we located were not included in the final review. We looked carefully at over 180 references, of which 43 studies are included in this review. Three of the studies were reported in multiple articles, making a total of 47 reports summarized in this review. Details on each report can be found in Appendix B. The review includes most of the rigorous empirical studies cited by authors of literature about the use of handheld graphing technology. It does not include meta-analyses and research reviews because the original work in these studies was often incongruent with the criteria established for inclusion in this review. Although there are studies related to the role of visualization in teaching and learning mathematics that focus on the use of computer technology, these studies were not part of this review unless they also addressed at least one of our questions with respect to the use of handheld graphing technology. Of the 43 studies included in this report, 14 were at the post secondary level; three investigated the use of handheld graphing technology in grades 6-9. The studies were done in Great Britain, France, Sweden, South Africa, New Zealand, Australia, the Netherlands, Israel, and the United States. Postsecondary mathematics was the area investigated in 14 of the studies; 20 concentrated on topics from precalculus or calculus. Seventeen of the studies were comparative or quasi-experimental involving some element of randomization in their design.

For the most part, we did not include books, book chapters, or monographs unless there was evidence that their publication was subjected to a review process approximating that of scientific journals. Dissertations were not included because it was impossible to determine the exact nature of the review process and the rigor for each dissertation within the time frame for this project. A final point is that, while a considerable amount of research exists on the use of computers in mathematics teaching and learning, the differences between computers and calculators in areas such as access, screen size, and interaction between user and machine might significantly affect outcomes. For this reason such research was excluded.

Framework for Synthesizing Research on Handheld Graphing Technology

Because the studies varied widely in scope, focus, and design, it was difficult to determine whether there was consistency in the findings. We have chosen to group the studies in terms of their responses to five central questions with a set of sub-questions raised by policy makers, educators, and the public. We then looked across the findings in each group for trends and discrepancies.

Question 1. How do teachers use handheld graphing technology and how is this use related to their knowledge and beliefs about technology, mathematics, and teaching mathematics? What do teachers know and believe about handheld graphing technology and how is this related to their beliefs about mathematics and mathematics education?

Question 2. With what kinds of mathematical tasks do students choose to use handheld graphing technology? How do students use the technology to carry out these tasks?

Question 3. What mathematical knowledge and skills are learned by students who use handheld graphing technology? In what ways do students use this knowledge and these skills?

Question 4. What is gained mathematically by students using handheld technology that cannot be observed in a non-technology environment? In what ways do students use this knowledge and these skills?

Question 5. What impact does handheld graphing technology have on the performance of students from different gender, racial, socio-economic status, and achievement groups?

Structure of the Report

The information relevant to each question is organized in four parts: findings, weaknesses in the studies as reported, gaps in the larger body of literature, and implications for classroom practice.

Findings

This section provides a synthesis of the results of all studies included in the review. The process for selecting the literature included in this section is described in the section on methods.

Weaknesses

This section describes the difficulties that we faced in our attempts to make generalizations based on the literature included in the review. These weaknesses were rooted in the rationale, research design, analysis, or reporting process.

Gaps

This section describes areas of research for which we had difficulty locating literature that met our standards for rigor. There were three main sources for the information in this section. First, when reporting their own research, authors often make calls for future research. These statements were compiled and considered. Second, implications are often stated in studies. These implications are usually not the result of scientific investigation but rather the authors' perceived application of their research to the field. Thus, some implications were interpreted to be calls for future research—they are claims that, while growing out of reported research, have yet to be researched in their own right. And finally, as we reviewed this body of literature, there were questions that we thought should be asked that have not been, as well as questions that were asked but never answered. These too have been interpreted as gaps in the research.

Implications for Classroom Practice

The remarks in this section are a compilation of conclusions made by researchers directly related to classroom practice as well as some of the implications they made about teaching and learning as a result of their work. As in the "gaps" section described above, these implications should be topics of further research but can also provide guidance and framing for the use of handheld graphing technology in classrooms.

Appendices

Appendix A briefly describes policies and practices related to the use of handheld graphing technology in the United States. The discussion addresses the nature and extent of the use of handheld graphing technology in secondary mathematics classrooms and in introductory post-secondary mathematics courses and considers some of the policies that are in place that might affect the extent of use.

Appendix B contains the complete references for the studies summarized in this report and more detail about the findings for each of the five questions.

Appendix C contains the additional references on the use of technology in mathematics classrooms.

Question 1: How do teachers use handheld graphing technology and how is this use related to their knowledge and beliefs about technology, mathematics, and teaching mathematics?

The teacher was pushing students to make sense of the display on their calculator. "Does the calculator always tell the truth?" "To what extent should we believe the calculator?"[24] In a study of a precalculus class, students investigated a decay situation using a small cup of M&M candies, where each candy had an M on one side. The candies were spilled on the table, those with the M removed, the process repeated with the remaining ones until none were left. The students concluded that an exponential function modeled the decay process. One student observed that even though they ended up with zero candies, the model did not attain a zero value since you can divide by two infinitely without getting zero. Another student using the table for the exponential function found that for very large values of x, the function appeared to reach the value zero. The discussions that followed, directed by the teacher, began to help students understand that the calculator was a tool, but that its use depended on their own mathematical understandings. The teacher had great confidence in her ability to use the technology effectively in her classroom. The learning environment she sought to establish emphasized open-ended mathematical investigations and multiple representations for justifying mathematical conjectures. Her beliefs greatly influenced the ways in which and purposes for which technology was used in her classroom.

The above example illustrates the contexts in which researchers have investigated teacher use of handheld graphing technology. We reviewed 14 research articles that reported on 12 studies. The studies fell into three general categories. The first category, exploring teachers' knowledge and beliefs about the use of handheld technology, included eight studies,[25] and the second, also comprised of eight studies, asked questions relative to what teachers do in the presence of handheld graphing technology.[26] The third group, consisting of four crossover studies, looked at the relationship between teachers' roles and their beliefs.[27] In addition, in one study that was reported in two different articles, the authors researched the relationship between teachers' roles and beliefs and students' approaches to learning.[28] Each of the categories described above is organized into three main sections with findings, weaknesses, and gaps for each section. The samples reported in the three categories included studies of individual teachers, a study involving 27 secondary teachers, and a large-scale investigation of 296 students and their teachers divided into two experimental groups and one control group.

What do teachers know and believe about handheld graphing technology and how is this related to their beliefs about mathematics and mathematics education?

Findings

There appears to be a positive relationship between teachers' beliefs about mathematics and their beliefs about the use of handheld graphing technology. One study[29] found that rule-based teachers were less likely to perceive handheld graphing technology as an enhancement to instruction, and more likely to notice the affective aspects of their students' reactions to graphing calculators. On the other hand, non-rule based teachers perceived calculators as integral to instruction and were more likely to focus on the cognitive or conceptual aspects of their students' responses. According to another study,[30] teachers who believed that students needed to know how to do mathematical procedures by hand before using calculators held philosophical orientations focusing primarily on teacher control of the ways students were using technology while those teachers with less rigid beliefs about the necessity of mastery before calculator use were more apt to give students more freedom in how they chose to use the technology.

Weaknesses

The research provides little information about what teachers, in general, know about handheld graphing technology. Some research participants appeared to be well informed whereas others seemed to know very little. Teachers' knowledge of and familiarity with a graphing calculator's functions undoubtedly influence the ways in which and the activities for which they use the tool, but our literature search did not return any reports of studies that adequately explore this relationship. While a number of studies investigated teachers' beliefs, few provided any description of teachers' knowledge.

Gaps

More thorough investigations of teachers' belief systems using methodologies appropriate for understanding beliefs are needed, specifically, studies on teachers' beliefs about handheld graphing technology, mathematics, and mathematics education, and how these three systems interact.

There is also a lack of research that examines the circumstances under which teachers' beliefs about handheld graphing technology shift either towards acceptance or towards rejection. Although research on beliefs often reports teachers' beliefs about the constraints that minimize their use of technology, more research is needed that provides insight as to how teachers come to reject or embrace handheld graphing technology in the presence or absence of such constraints.

How do teachers use handheld graphing technology in their teaching?

Findings

How teachers used graphing technology in their teaching varied extensively. For example, one study found that "the use of the [graphing calculator] was associated with higher levels of discourse in the classroom, including higher-level questioning by the instructor and more active learning behaviors by the students".[31] Another study[32] reported three classroom norms that a teacher developed concerning the graphing calculator: 1) multiple approaches for conjecturing and confirming relationships between variables; 2) requiring that results be justified on mathematical grounds; and 3) interpreting results within problem contexts.

As might be expected but is often overlooked, significant changes in teaching did not necessarily follow when handheld graphing technology was introduced into mathematics classrooms.[33] A study of six 11th and 12th grade teachers found that the teachers "used graphing technology as an extension to the way they always taught".[34] After reviewing teaching practices in classrooms where handheld graphing technology was used, a study of 12 classes of 16 and 17 year old students in the Netherlands concluded that the "adjustment of teaching practices to provide for more problem solving activities will not develop spontaneously" when graphing calculators are introduced into instruction.[35] In response to similar findings, several researchers concluded that a need exists for professional development that provides opportunities for teachers to reflect on their knowledge, beliefs, and philosophies about mathematics, teaching, learning, and technology.[36]

Weaknesses

Some studies that sought to answer the question of how teachers used handheld graphing technology in their teaching relied only on teachers' self-reports of classroom activity. However, because most of the studies also relied on classroom observation, this weakness is not a severe impediment. A larger concern is that there is often no clear coding or observation protocol reported that would enable consistent interpretation and reporting on how handheld graphing technology is being used and its impact on teachers and students.

Gaps

There is an apparent lack of research that provides information not only about the uses of handheld graphing technology but also on the significant aspects of the contexts in which these uses occur. The mathematics education community needs the means to determine which aspects of the context are related to the outcomes they seek, or seek to avoid. In its current state, the body of literature seems to implicitly assert that teachers function in a vacuum–that the use of handheld graphing technology is an artifact of the tool itself rather than the reasons for its use and the content, curricula and students with which it is used. Future research should seek to explore in greater depth the relationships between the use of handheld graphing technology and the classroom norms that give meaning and purpose to those uses. Specific areas in which there is little research include teachers' use of calculator-based probes and use of an overhead projection unit.

What is the relationship between teachers' beliefs about handheld graphing technology and how that technology is used in their teaching?

Findings

Teachers' knowledge, beliefs, and personal philosophies influence how they use handheld graphing technology in their teaching.[37] For example, one study[38] found that teachers with a rule-based view of mathematics were less likely to shift their teaching practices toward investigation and problem solving when the handheld graphing technology was introduced into their classrooms. Although many early advocates of handheld graphing technology expected its introduction to shift teachers' roles toward that of facilitators or consultants, these shifts do not always occur and depend largely on teachers' beliefs and knowledge.[39]

In addition, teacher's beliefs about handheld graphing technology and mathematics education may influence classroom norms for use of the tool. For example, in a study of a teacher with 20 years experience working in two classes of 15 students each,[40] the teacher shared beliefs about the limitations of handheld graphing technology and the importance of understanding the meaning of the numbers in an equation with the class. During instruction, the teacher emphasized meaningful interpretation of calculator output and required that students explain the parameters in the equations based on the original problem context. The students' approach to handheld graphing technology appeared to be influenced by the teacher's beliefs and related actions. They developed reasonable skepticism about calculator-generated results, which led to a norm requiring that conclusions be based on mathematical grounds. Having learned the limitations of various equation models, they were less likely to use regression equations to approximate graphs because they were required to explain the parameters of equations with precision. Influenced by their teacher, they began to use handheld graphing technology flexibly, as a tool that could be used to investigate a range of properties of a function's graph.

Weaknesses

Although researchers have asked questions about teachers' beliefs about technology, the methodologies that have been used thus far do not necessarily support a deep understanding of beliefs. Few of the articles we located provided theoretical descriptions of the term belief and what it means for beliefs to be related to each other or to practice. In addition, researchers in the area of beliefs have explicitly warned against inferring what people believe based solely on what they say.[41] Studies based solely on self-reports or brief classroom observations are not likely to reveal significant insight into the nature of a teacher's beliefs about the use of handheld graphing technology in the teaching and learning of mathematics. In order to develop an understanding of a teacher's beliefs and the relationships those beliefs might have with the role of handheld graphing technology in the classroom, researchers must have a more thorough, triangulated set of data, and analysis should follow rigorous standards.

Gaps

Researchers suggested that professional development programs should pay special attention to certain beliefs in particular, such as beliefs about the necessity of mastering mathematical procedures manually before using technology,[42] teachers' philosophical orientations,[43] beliefs about the nature of problem solving,[44] and about mathematics.[45] In addition, they suggested, "more attention needs to be directed to the inherent mathematical and pedagogical challenges in technology-enhanced classrooms if the goal of a problem-solving and investigative learning environment is to be realized".[46] Studies need to be conducted to test these assertions. In addition, researchers need to address explicitly both the potential and the constraints of how teachers' beliefs relate to their use of handheld graphing technology in their classrooms, including an opportunity to consider the various ways their students may approach the use of the technology.[47] Many of the studies involved teachers who volunteered or were already committed to the use of handheld graphing technology; there is little research on teachers who are skeptical, yet forced by some set of circumstances to incorporate the technology into their classrooms.

Implications for the classroom

Teachers use technology as an extension of how they already teach which, in turn, is a function of what they know and believe about teaching, learning, and mathematics. Unfortunately, the history of educational reform has shown that this implication is not as obvious as it would seem. Allocating large sums of money to place technology into classrooms will not likely result in large changes in classroom instruction. Because the mere availability of handheld graphing technology does not precipitate a change in teaching strategies for all mathematics teachers, professional development should provide teachers the opportunity to explore the role technology can play in helping them achieve their instructional goals as well as how technology can impact not just how but what mathematics is being taught.

Further, if the aim of professional development is to promote improvement (change) in teaching practices, teachers need professional development that will help them feel knowledgeable and comfortable with technology—that focuses beyond the functionality of the tool to incorporate the potential and the constraints of the tool. In addition, an implication that can be inferred in these studies is that teachers' beliefs about the use of handheld graphing technology need to be addressed explicitly in teacher preparation and development programs. Teachers should have opportunities to reflect upon and discuss their beliefs about mathematics, teaching, and learning in relationship to their knowledge and beliefs about the use of technology in the mathematics classroom. Professional development and technical assistance and support need to be ongoing complements to implementing the technology into classroom practice.[48]

Question 2. With what kinds of mathematical tasks do students choose to use handheld graphing technology? How do students use the technology to carry out these tasks?

Sample Task: Given the function $y = \frac{x^3}{x^2 - 1}$, draw the graph and describe the function with as complete reasoning as possible.

This task, from a 1996 Swedish study[49], is an example of the type of tasks that students commonly use a graphing calculator to solve. One of the primary characteristics of tasks for which students choose to use handheld technology are those beginning with the phrase, “Draw a graph.” In addition, the presence of a function or equation within a problem, even without direction to draw its graph, also seems to affect students’ use of a graphing calculator

What do students do in using the calculator as a tool in solving the task? The research described above indicates that teachers’ beliefs and teaching methods have an effect on how students use technology. Students tend to use the methods that are illustrated and preferred by their teachers. In some cases, teachers leave the development of calculator skills largely to the students themselves; in others student calculator use is strongly shaped by teacher decisions and interventions. The Swedish study reported that students used the graphing calculator to find a solution, then copied the figure from the screen to their paper. The students’ errors indicated an incomplete understanding of the information displayed on the calculator screen and included errors such as interpreting the curve as a third degree function, drawing the asymptotes as integrated parts of the curve making it appear continuous, omitting asymptotes altogether, and reporting the points at which the asymptotes were connected to the curve as local maxima and minima. This illustrates that even when a task directly suggests a graphical solution, in order to use the calculator successfully, students need to be familiar with the mathematics surrounding the task at hand and recognize how the limitations of the calculator can inhibit understanding of the mathematics.

Findings

This section describes the tasks used in the research described in this report and how students involved technology to solve them. The study results are not necessarily described; the section is focused on considering the nature of the work students do with handheld graphing technology. Of the fourteen studies we examined, eight are focused on how students' use of the technology might affect their understanding of mathematics or improve their mathematical performance.[50] The others dealt with the students' interaction with the calculator as they used it on a variety of tasks.[51] Three of the studies were comparative with two of the studies comparing two classes of students in advanced high school mathematics while the third study investigated 131 university calculus students in treatment and control classes. The other studies were interpretative studies based on interviews, observations, analyses of student work, and surveys of samples of students ranging from 7 students in one study to 68 in another. The tasks students were asked to perform in the studies had two different purposes: the investigation of students' choice of solution strategy (spontaneous use) and the investigation of students' use of the calculator.

Student choice of solution strategies in the presence of handheld graphing technology

Research about students' choice of solution strategies is dependent on the tasks researchers used in their investigations. The framing of these tasks varied. In some cases, students were given a task that required interpretation of the graphs produced by the calculator, rather than given situations in which they were allowed to choose whether to use the calculator to solve the problem. For example, in four of the tasks presented in one study[52] students were shown graphs that were already in the viewing window and asked to find solutions, making sense of what they saw. To examine students' understanding of function and limit when they have access to a graphing calculator during instruction, another study[53] presented tasks based on problems with graphs entered into the calculator and investigated how the student used the calculator in such instances. A third study[54] provided similar tasks but did not require task solutions. Instead, they asked students for their preference of solution method- graph, table of values, or equation – and compared the preferences of those who had used graphing calculator technology to those who had not.

Using a different approach, a study of 25 Australian 10th and 11th graders[55] presented students with eight different types of problems and held clinical interviews where students were presented with mathematical tasks and asked to think out loud. Students had to sketch graphs, find points of intersections, and interpret calculator displays. The problems encouraged or directed graphing calculator use with instructions like, "you may use the graphics calculator to help you" or "display the graph of the function… on the graphics calculator." Researchers in another study of three classes of ninth grade students in Israel[56] who were investigating students' concept images of linear and quadratic functions administered a questionnaire in which students had to both interpret given graphs or partial graphs and create their own representations.

We found little research on students' spontaneous use (individual choice of solution strategy with or without the technology) of handheld graphing technology. A study[57] examining the work of 404 students on 6 of the 19 questions found on the Tertiary (University) Entrance Exam in Australia reported the number of students they believed used the graphing calculator on each of these problems and described what seemed to be the students' calculator solutions. The researchers selected problems in the study that involved determining limits, finding a bounded area, analyzing functions, and solving equations with complex numbers. The actual problems were listed in the report, along with the correct responses. While the selected problems were, in the opinion of the researchers, problems that would elicit calculator use, they offer no evidence that the actual problems motivated student use of the calculator. However, the study does begin to give us some idea of whether students chose to use the graphing calculator on problems that we believe they will.

We reviewed only one study, an analysis of the responses of 11th and 12th grade students in Sweden to a six-item test[58] that was specifically designed to give students the option of using the graphing calculator. All of the tasks could have been solved without using the calculator. Four of the six problems on a test used in the study were appropriate for graphical solutions and thus, graphing calculator use. Students preferred symbolic methods on one of these problems: $\sin x = .5$. One of the tasks was too trivial (solving a system of simultaneous equations), while another did not elicit any graphing calculator help. The other three problems were tasks for which the graphical solutions were chosen by the students: solve the equation

$x^3 - 3x = \ln x$; given $y = \dfrac{x^3}{x^2 - 1}$, draw the graph and describe the function with as complete a reasoning as possible; and solve the equation

$\sin x + 2\cos x = \dfrac{3}{2}$.

One of the research studies[59] about the use of calculators with computer assisted algebra systems (CAS) provides evidence that such calculators give students, after a limited amount of instruction, a significant advantage on the sorts of questions included in a test for which designers believe such calculators would be an advantage. The researchers selected tasks for which a calculator with a symbolic manipulator would be an advantage. For example, short questions involving factoring, expanding, solving equations, differentiation, integration, and complex numbers can be easily answered using a symbolic calculator, particularly if the tasks are procedural and the students have recently practiced such tasks. Students who had been through four one-hour training sessions and worked through a module to practice solving such calculator positive tasks with a symbolic calculator scored significantly better than their counterparts without the calculator advantage. Thus, they could successfully use the calculator on the calculator friendly tasks. However, when they were not allowed the use of a symbolic calculator, the experimental group did not perform better than the control group on the formal university calculus entrance exam, a calculator neutral test that required intermediate steps in solutions or values from such steps, answers in formats not given by the calculator, the use of variable coefficients, and re-writing given incorrect solutions. Thus, it appears that with the four weeks of instruction, students successfully used symbolic calculators for procedural tasks they had practiced, but such limited use did not transfer to tasks that are calculator neutral or affect their conceptual understanding of the mathematics involved.

How students used handheld technology

Three of the studies we examined classified students' strategies for using handheld graphing technology. In analyzing their data, the researchers in one study[60] categorized five overlapping patterns and modes of graphing calculator use, which the study describes in the following way: the use of a graphing calculator as a 1) computational tool, 2) transformational tool, 3) data collection and analysis tool, 4) visualizing tool, and 5) checking tool. Though the actual tasks are not included in the article, the researchers state that students were expected to create models based on patterns they observed, and they made use of calculator based measurement probes in classroom activities. In a second study[61] researchers developed a model describing different types of student users based on their investigation of how the graphing calculator can be turned into an instrumental tool for solving mathematical tasks. In the model, students used the calculator to mimic calculator strategies they had observed before and memorized, to aide in drawing conclusions through calculator investigations which yield consistent results, to investigate a wide range of imaginative solution strategies, and to verify theoretical solutions. A third study[62] suggested that the graphing calculator encouraged students to use the calculator for exploring mathematical ideas and encouraged them to use flexible solution procedures.

Students used a graphing calculator to produce graphs. When asked for a sketch of a graph, students would use the calculator's graph as a model for their sketch.[63] Three studies[64] reported on how the graphing calculator was used as a visual and transformational tool. Students used graphs to answer questions by tracing the graph or examining a table to find numerical answers. One[65] study described students' in exam conditions where they graphed a logistic function to help determine its inverse, a rational function to produce a sketch of its graph, and given functions to help evaluate requested limits.

There was some evidence that experience with handheld graphing technology influences students' approach to mathematical tasks. One study[66] pointed out that the students who used graphing calculators were more likely to have a graphical preference on both contextualized and non-contextualized problems than students not using technology. A study of student's exam responses and work,[67] however, found that students made minimal use of the calculator for tasks that did not require a graphical response. Technology rich environments seem to affect the way that students think about mathematical tasks and, thus, whether or not they choose to use technology.

A few students used a program or built-in functions that are available on some calculators. In one interpretative study, the teacher was able to help students develop "multiple ways for conjecturing and confirming hypothesized relationships between variables".[68] There was some evidence that students use graphing calculators in ways that we do not expect. In one task, students were to graph $z = rcis\theta$. Some students chose to graph it using polar coordinates or parametric equations on their graphing calculators. The authors stated, "Although these methods are obvious in retrospect, we had not anticipated them".[69]

There is conflicting evidence, however about whether students used handheld technology for checking or confirming algebraic work. Two interpretative studies[70] claimed that students used the graphing calculator for refining answers and the graphs produced by the calculator to support algebraic work. From classroom observations and videotapes, one study[71] concluded that one use of the graphing calculator was as a checking tool. Another based on studying exam scripts and student interviews,[72] however, claimed that the students in their study did not use the graphing calculator to verify algebraic work. There was also conflicting evidence on whether handheld technology is over- or under-used. Some studies claim the graphing calculator was under-used.[73] For example, one study found that students seemed to use traditional algebraic techniques in preference to the calculator; 21 percent did not use a graphing strategy to solve a problem even when there was a time advantage to do so.[74] On the other hand, in a survey of students and their tutors involved in a distance-learning course,[75] the tutors (individuals who monitor, support, and assess student progress in the course) were concerned about calculator dependence.

Several studies also reported on how students misuse the graphing calculator.[76] Misuse or misunderstanding of scaling and zooming was most often described. Cases in which students overlooked a solution since it did not appear in the standard window or mistakenly identified a quadratic function as a linear function because the graphing window only showed a partial picture illustrate typical misinterpretation in graphing calculator use. Another example of calculator misuse was the dependency of some students on the graphing calculator's version of a graph without considering its limitations. Students simply copied the graph as pictured omitting asymptotes or incorrectly connecting various parts of graphs, since that was how the graph appeared in the viewing window. There were also some additional difficulties associated with the use of a symbolic calculator, such as using an incorrect syntax for formula entry leading to an incorrect answer and the difficulties of accessing correct sequences of key presses.

In studying 37 exam responses and student work, researchers[77] found that many students had trouble integrating algebraic and graphical information. When the students encountered a conflict between the two, some disregarded the graphical information in favor of the algebraic while others did the opposite. Some students did not appear to realize that the solution to an early part of a question had any relationship to the graph they had to produce in a later question. Only 14 percent used strategies that linked the two, 27 percent recognized the link but could not integrate the two, and 43 percent did not recognize the connection.

Weaknesses

Students do not use calculators if they have limited experience with the technology. Some studies examined students who had little experience with handheld technology.[78] In one study, not summarized in this question,[79] participant exposure only lasted the duration of the two-week study. In another study, students only had access to graphing calculators during class study sessions.[80]

To understand when students choose to use handheld graphing technology and what they do when they use the tools, it is necessary to see the tasks they are given and the work they do around the tasks. When trying to determine the actual tasks for which students turn to technology as a tool of choice, we are limited to looking at the type of tasks that were created or selected by the researchers, few of which were below the pre-calculus level. We were further limited by the fact that the spontaneous use of handheld technology was seldom the focus of the majority of the research we reviewed. Some studies include the tasks used in their study,[81] but many do not. Other researchers offered generalized descriptions of the mathematical tasks presented to their participants and gave examples of the tasks.[82]

Underutilization of technology can occur if the level of written symbolic reasoning that is expected is unclear to students, especially when involved in high stakes testing. In the study of responses to questions on an Australian university entrance exam,[83] for example, students appeared to be concerned about how much reasoning needed to be evident on their Tertiary (University) Entrance Exam paper in order to gain full marks for a task. Confounding variables such as these are not always addressed as well as they might be in the research design.

When interviews were used as a method of gathering data, protocols were often omitted from the descriptions of the studies. Participant selection methods in the majority of studies about the nature of student use were either not described or were limited to students in intact classes. In some cases the students self-selected the class. Self-selection could imply particular interest in or prior use of the graphing calculator by the student. In addition, some students may have chosen a course because of the time it was offered and other scheduling conflicts. Although self-selection isn't necessarily a problem (unless students at the school are not usually allowed to choose their own class), either of these cases could have an effect on the results. More information about interview protocols and about how students were selected to participate in these studies would add to the validity of the conclusions reported.

Gaps

More information about how students are using technology to perform particular types of tasks would be helpful. Although several studies reported that some students used the graphing calculator in unexpected ways when solving calculus problems on an exam, more knowledge about the strategies students use with technology would be useful. There were some important areas in which we found no studies. For example, we found no studies on how students think about mathematics when they have full access to handheld graphing technology and whether this thinking is different from that of students who do the mathematics without the technology. Among the studies examined for this report few focused primarily on the ways in which students spontaneously used the graphing calculator.

In addition, the bulk of the studies we analyzed reported students using handheld graphing calculators, but only a few addressed the use of calculators with symbolic manipulation capabilities. More information about the use of CAS would improve the body of knowledge about how students choose to use this feature of handheld technology and about how students construct meaning from the tasks they do.

Finally, there were no studies in those reviewed that examined how students used handheld technology associated with plane geometry or statistical tasks and only one study that investigated trigonometry. The research primarily focused on functions and coordinate graphing, and not on the use of the technology to perform simulations, make statistical plots, manipulate data, work with inequalities, or collect and analyze data. In addition, most of the research was about upper level mathematics. Very few studies looked at middle grades students' use of handheld graphing technology, despite the availability of handheld graphing technology specifically designed for those grade levels and the degree of use in mathematics at that level.[5] Knowledge of the role of technology in how students develop initial understandings of mathematical concepts and the issues involved would seem to be desired ground work for the ways in which students use the technology and the mathematical knowledge they have gained with that technology in later grades.

Implications for the classroom

Research on students' use of handheld technology suggests two insights for the classroom. First, tasks and technology used to achieve and assess instructional goals should be aligned. Second, teachers must allot adequate time for instruction on the use of the tool and to ascertain students' competence with it.

Teachers who have gained competence in using the technology to solve mathematical tasks then have a two-fold responsibility. They must take time to teach their students to use the technology, including efficient methods as well as the limitations of the technology, and they must design appropriate tasks with the technology in mind. Guin and Trouche (1999) point out that establishing a relationship with graphing technology takes time, but they note that instruction time lost while working with the calculator will be made up in future activities since students will have a better understanding of when and how to use it. They also argue that taking into consideration the ways in which students typically use the calculator can help teachers lead students to the best uses and understanding of the technology.

When examining how students use the handheld technology to solve mathematical tasks it is clear that both teachers and students need to understand what the calculator can do and what its technical limits are. Some

[5] See Appendix B for detailed information on content and grade use of handheld graphing technology.

researchers[84] offer evidence that a lack of understanding of the technical limits of the technology results in its misuse. If, as one study[85] points out, successful use of the technology requires a basic mathematical knowledge in order to accurately interpret the results obtained from the graphing calculator, teachers need to identify this knowledge for the tasks they give and ensure that students have the appropriate mathematical foundations. The tension students face in resolving the difference in graphic and algebraic approaches to solutions should be addressed in instruction, with students given experience in making links between the representations and dealing with apparent conflicts. In addition, careful examination of student work may reveal students' lack of mathematical understanding by how they use or misuse the calculator in ways that their written work may not reveal.[86]

Finally, teachers must be prepared for multiple and surprising student approaches. For example, in the Swedish study,[87] students were expected to use graphing to solve a system of linear equations problem, but instead, some chose to solve it symbolically and others by using matrices.

Question 3. What mathematical knowledge and skills are learned by students who use handheld graphing technology? In what ways do students use this knowledge and these skills?

We located and reviewed 23 research reports relevant to students' mathematical learning in the presence of handheld graphing technology. Of these, five papers reported findings on students' learning of functions,[88] four on the learning of algebra,[89] six on the learning of pre-calculus,[90] and eight on the learning of calculus.[91] Fifteen studies[92] were experimental or quasi-experimental with learning measured by scores on achievement tests. The studies ranged from 710 students in a U.S. precalculus university class[93] to three classes of ninth graders in Israel.[94] Many of these studies used posttests composed primarily of questions students have traditionally been expected to solve without a calculator. For example,

> The function $P(x) = (x+1)(x-3)$ represents the total daily profit of a high school pizza stand, where x is the number of pizzas sold daily.
>
> a) Graph $P(x)$,
>
> b) What values of x make sense in this problem situation if the profit is to be positive?
>
> c) Find the number of pizzas you must sell daily to obtain a profit of at least $12.00 a day.[95]

Using questions such as these, the researchers investigated the relation between student learning and their ability to use and interpret graphs made with handheld graphing technology.

The remaining seven reports dealt with interpretive studies that provided detailed accounts of student learning and suggested factors that affect what students learn. Together these studies provide macro- and microscopic views of student learning, both of which are necessary to study environments in which students are using handheld graphing calculators.

Findings

Function

The five papers that reported on student learning of function suggest that students' use of the calculator helped them develop their understanding of function. One study[96] found, in general, that students using handheld graphing calculator technology and a text that supported its use had a significantly better understanding of function than students in traditional classes. Specific findings about students' knowledge and skills included the following: students using handheld graphing calculator technology were better at selecting appropriate dimensions for the axes when graphing functions, preferred graphical representations of functions for problem solving, exhibited a dynamic notion of function, and were better able to develop representations of whole graphs of functions from partial graphs.

Algebra

We reviewed four papers that reported on students' learning of algebra. In three large-scale studies,[97] the curricular materials used were written assuming the availability and use of the technology, and the technology was used as a tool in the teaching and learning of mathematics. These studies reported gains in student learning: advanced understanding of variables,[98] improved ability to solve algebraic problems set in realistic contexts,[99] "improved understanding of graphical representations and applications",[100] and no significant differences in procedural skills. The treatment in the remaining study[101] consisted of using handheld graphing technology for two weeks in a class not specifically designed with the use of technology as a tool. Students were shown how to solve algebraic equations graphically using the technology. The researchers reported that students in the treatment group preferred to solve algebraic word problems symbolically. No student used graphing as a solution method. Results here are inconsistent and may be due to the way the technology was used with students. These differing results suggest that the role of handheld graphing technology in the curriculum may have an impact on student learning.

Pre-calculus

Two of the research reports we reviewed on the learning of pre-calculus were large-scale comparative studies. One study[102] found that pre-calculus students using handheld graphing technology with a textbook designed to be used with the technology outperformed control groups on a departmental final exam that included questions involving word problems and the properties of functions, graphs, and equations. The second study[103] compared the performance of students with one year of access to handheld graphing technology with students who had no regular access to technology on a test on graphing polynomials. No differences were found between the two groups' abilities to interpret a given graph; however the treatment group performed significantly better when asked to develop a symbolic model for a given graph.

The remaining studies were interpretive and provided detailed accounts of student learning of vectors, use of symbolic and graphical methods, and modeling. In a study of students' learning of vectors[104] researchers reported two students' understanding of magnitude and direction of a vector was based on their understanding of certain functions of their calculator.[6] In addition the students were able to use their understandings in ways they had not been taught. For example given the direction of a vector and one of its components, one of the students studied was able to use the SOLVE capabilities of the calculator to find the other component. These findings suggest that for some students, their use of a graphing calculator changes the nature of their learning. This is not always the case, however, as a study of three pre-calculus students implies.[105] The students retained a preference for symbolic procedures despite continuous instructional emphasis on graphical methods. Various reasons were given for this finding including emphasis on symbols in previous teaching, current course assessments, and homework. Hence, the tool alone does not appear to be a catalyst for learning. Other factors involved were suggested in the findings of a study about students' development and validation of mathematical models.[106] Four different development and validations strategies were constructed based on students' responses to open ended modeling tasks. Factors identified as influential in students' development and validation of models included mathematical knowledge, association of the problem with student experience, and tool use.

[6] ABS[(I,J)] and ARG[(I,J)]

Not surprisingly, the results of both the comparative and interpretative studies suggest that the tool alone is not enough to make a difference. Student learning is influenced by various factors beyond the tool itself such as the role of the tool in the curriculum, interaction with peers and teacher, assessments used, and the reflective nature of the individual student.

Calculus

Because calculus is commonly a secondary subject in many countries and for some students in the United States, studies about the use of handheld graphing technology in calculus at both the postsecondary and secondary levels have been included in this analysis. Some studies done at the postsecondary level[107] attempted to determine the effect of the calculators with Computer Algebra Systems (CAS) on students' exam performance. These studies compared students using handheld graphing calculator technology with students using CAS. On department final exams, students using CAS did significantly better overall on items that required single and multi-step computations. However, results of students' performance on more complex items were mixed.[7] These results suggest students using CAS for a whole course are advantaged in some ways.

In studies where less time was spent using CAS, the benefit appeared to be restricted to lower achieving students whose scores on computational problems improved. In one such study the treatment condition lasted for only 4 hours.[108] Students' performance on a calculator advantaged (problems were easier to solve with CAS) pre- and posttest showed significant gains for low achievers but no differences in the scores of high achievers. The researchers suggest, however, that the gains of the low achievers were not conceptual. The instruction and use of CAS had no effect on a calculator neutral test or on students' performance on the entrance exam where only standard graphing calculators were allowed.

Three reports[109] of a study designed to investigate calculus students' problem solving strategies seem to confirm the need to spend significant time using CAS in order to see gains in ability to apply calculus

[7] Connor and Snook (2001) and Keller and Russell (1997) both reported gain on more complex problems for the treatment groups in their studies, but Keller and Russell's finding was called into question by their subsequent study (Keller, Russell, & Thompson, 1999)

concepts. Students exposed to three conditions (one year with calculators, two months with calculators, no calculators) were studied. Of the two treatment groups, the group using the graphing calculator all year performed significantly better on a posttest that did not include graphical items than either students who used the calculators for two months or not at all. In addition, "students who used the graphics calculator over a long period used a graphical approach"[110] as well as algorithmic approaches, while students who use the calculator for a short period of time substitute graphic solution strategies for all other approaches taught. The repertoire of solution strategies students used seemed to be enlarged. These studies seem to indicate that experience and length of time with access to the technology is a factor in using handheld graphing technology to aid in learning and doing mathematics.

One study[111] of students' understanding of calculus attempted to link student performance with teachers' actions. Three teachers helped design a curriculum unit they taught with CAS. There were no significant differences between the mean class scores on a posttest; however differences between students' performances on specific items were significant. In particular, students whose teacher illustrated connections between representations and emphasized concepts made much less use of the calculator but with greater success than did students whose teacher focused on technological and algebraic approaches.

The mathematical knowledge and skills learned by calculus students with graphing calculators, based on these studies, included the ability to apply concepts and use problem solving strategies. Time appears to be a factor in the development of these skills.

Summary

The conclusions are not surprising. In general, students using handheld graphing calculators learned what they were taught, either implicitly[112] or explicitly. Access seems to make a difference. Students who spent more time learning to solve applied problems did better on those problems on a problem based assessment while students who spent time on procedures did better on these problems. The exceptions seem to be students who have limited access to the technology.

While it may seem obvious, the outcomes reinforced the fact that student learning of mathematics with handheld graphing calculator technology is not a function of the technology alone. In addition to length of time with access, student teacher interaction, how the tool is used, and existing mathematical knowledge and beliefs of the student all appear to be significant factors.

Weaknesses

One major weakness of many of the studies we found and reviewed was their inadequate or incomplete descriptions of the instruments used, participants, and data analysis procedures. This was a problem for studies that reported using several instruments to collect data. For example one study,[113] noted that students were interviewed after they completed the course, but there was no description of the questions students were given. In general participants were simply described as students enrolled in a mathematics class with no other demographics provided. Few of the studies provided examples of how students' responses or interview data were coded, and the actual responses of the students were not always explicit. For instance in one study,[114] four solution strategies were identified and described, but no examples of student work were provided.

Another weakness of the studies reviewed was their incomplete descriptions of the uses of and access to the technology. Five[115] of the 23 studies reviewed for this question did not specify the type of handheld graphing technology the students used. In general the studies did not provide information about the students' history of calculator use,[116] and only three[117] of the reports were specific about how students used the technology to learn mathematics.

The majority of the instruments used by researchers were self designed and untested with no reliability statistic provided. Studies that used final examinations as posttests attempted to establish content validity by circulating items among course instructors, but no internal and external validity was reported.

Gaps

The majority of the research reporting mathematical knowledge and skills learned was associated with function, algebra, pre-calculus, and calculus. Yet no two studies could be described as studying student learning of the same mathematical concept. The existing research can be used as a base for further work, but a cumulative body of linked research focused on key mathematical concepts is needed. Overall, the research should include a focus across mathematical concepts. In particular, studies on how students' mathematical reasoning and views of proof are affected by the use of handheld graphing technology would be important.

The findings of the studies we summarized indicate that a collection of variables influence students' mathematical learning when they have access to handheld calculators. Studies that consider variables such as time, access, interaction, prior mathematical knowledge and beliefs, and history of calculator use are needed to explain when and how the use of technology affects student learning.

The focus of the research we investigated was on student learning of standard or accepted course content and concepts. No study in our report investigated the potential of handheld graphing technology -what it is possible to learn. Two studies that did so but did not meet our criteria for inclusion, suggest that the use of technology provides opportunities for students to investigate and learn mathematics that they could not without the tool.[8] The mathematics community would benefit from rigorous studies publicly shared that examine students' learning of mathematical concepts traditionally considered inaccessible to high school students because of the complex nature of the mathematics, tedious calculations, or limited time.

Implications for the Classroom

The findings from these studies suggest that teachers should attend to certain factors that can affect what students learn using handheld graphing technology. Students should understand the mathematics independently from the use of the calculator. "Students who do not own the concept of function cannot be

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[8] The two studies not included in this review were Lagrange (1999) and Zarzycki (2000). While they did provide clear examples of how students made us of the technology, the reports did not meet our criteria regarding a discussion of the author's data analysis.

expected to be able to use the graphing calculator to its fullest benefit".[119] Students should have experience over time to gain the most benefit from the use of calculators.

The nature of the curriculum and the assumptions made about the role of handheld graphing technology in the curriculum are important. With or without graphing technology, there seem to be parts of the curriculum on which students do poorly. Students in both experimental and control groups did not do well on multiple representations of algebraic ideas and on understanding function as an entity rather than a process, suggesting that teachers might need to rethink how they approach these ideas. Because students using handheld graphing calculator technology learn to solve problems using multiple methods, teachers should be prepared to help students examine those methods to see when they generalize or what assumptions or limitations might be inherent in a particular method.

As with any tool there are dangers that teachers should recognize in using handheld graphing technology. Students can use the graphing calculator to replace algorithmic strategies. Researchers note this may a problem among low achievers, and teachers should be alert to the fact that low achievers may be especially prone to learning procedures on the calculator without conceptual understanding, suggesting that just as with pencil and paper, procedures may be easily forgotten. Students' knowledge of mathematics may affect how they use the calculator. Traditional tasks such as graphing are easy to perform with the calculator, but teachers have to look beyond and within the task to identify the mathematics students should be learning. Instruction should be designed so the tool can assume more of the computational and representational burden of a given task enabling students to focus on more complex tasks such as a synthesis of results. Some educators make the assumption that using graphing calculators promotes student involvement in the problem-solving process; however more evidence for this claim is needed. Using graphing calculators to motivate student involvement is also an unsubstantiated claim. These suggest that teachers should be alert to making these attributions to their students without any real evidence.

Question 4. What is gained mathematically by students using handheld technology that cannot be observed in a non-technology environment? In what ways do students use this knowledge and these skills?

"Teacher: Can you shift the graph of $y = x^2$ two places to the right?

Johan: $y = x^2 + 2$. (He enters this and presses GRAPH.) Oh no! $y = x^2 + 2x$. GRAPH…that's not right either.

Alex: $y = x^2 + 4x + 4$, no that's not right, the graph shifted two places to the left. So then it is $y = x^2 – 4x + 4$. That's right."[120]

This excerpt from a mathematics lesson illustrates one of the primary uses of graphing calculators in a technology-enhanced environment. This conversation took place with students who were quite familiar with the technology. Would students in a non-technology environment be able to solve the problem posed in the same way? Would their knowledge of transformations of quadratic functions be the same? What does research tell us about the mathematical gains of students in classrooms where handheld technology is used?

Findings

We examined eleven studies that compared students who used handheld graphing technology to those who did not. Overall, the findings indicate that the use of handheld technology had a positive impact on student performance. These studies examined students' conceptual understanding of function,[121] their solution strategies,[122] their ability to link representations and understand the attributes of functions,[123] their performance on a comprehensive final exam,[124] and their use of a symbolic calculator on procedural problems that were deemed calculator friendly.[125] The sample sizes ranged from 710 precalculus university students to 128 community college students studying functions to three classes of ninth graders in Israel.

Ten additional studies that looked more closely at students' mathematical gains in a technological environment were useful in more clearly defining students' gains and the ways in which they used their knowledge and skills. These studies[126] included highly focused case studies, classroom observations, student surveys, and analysis of work of students in upper level mathematics from the United States, Great Britain, New Zealand, France, and the Netherlands.

Mathematical gains

Three studies found improvement in students' conceptual understanding of function when using handheld graphing technology when students were in courses with specific curriculum changes in addition to the introduction of the technology. A study of community college students on their understanding of function[127] reported that using handheld graphing technology with a conceptual change activity designed by the researcher to encourage the students to examine their conceptions before instruction took place significantly improved application of the concepts of domain and range and the selection of appropriate dimensions for the rectangular coordinate system for graphing functions. A large-scale study of 118 ninth and tenth graders in New Zealand[128] reported that students who used handheld graphing technology with a curriculum specifically designed to take advantage of technology[9] exhibited significantly better conceptual understanding of variable than those who did not use the technology. A third study[129] of intermediate algebra students in a large university in the United States found that students who had continuous access to the graphing calculator and studying in a curriculum that was designed with a balance of calculator and traditional problems were more likely to treat functions as objects rather than a process and had a significantly better understanding of functions than the control classes.

Three studies found evidence of an improved ability to link the three (symbolic, graphical, tabular) representations as well as an improved understanding of the attributes of functions. One study[130] found that with extended use of the graphing calculator, students developed a particularly strong relationship between symbolic and graphical forms of functions. A second study[131] reported that students in a function-based curriculum and using graphing technology did better on multiple representations of algebraic ideas requiring representational fluency than the control group, but the mean score on these items for both groups was below 33 percent. In the third phase of a 20-year research and developmental project in Israel,[132] researchers examined junior high school students' concept of function. While students in the second phase of the project, the comparison group, had studied a function-based curriculum, the students in the third phase were the only ones who had access to graphing technology. The researchers reported that the use of multiple representations made possible with a graphing calculator increased the students' ability to justify, report, criticize, and reflect on their own practice. In addition, the graphing calculator students made reference to more examples and linked them to transformations more often than the non-calculator students.

Students with access to handheld graphing technology outperformed those without access on multi-step problems, problems involving applications, and those using real data. Two of the studies that supported these findings examined large secondary curriculum projects in the United States: the University of Chicago Mathematics Project (UCSMP)[133] and Core-Plus Mathematics Project (CPMP) students.[134] Both of these curricula assume technological availability and include activities designed to exploit this availability by asking students to generate graphs, make conjectures and move among algebraic, numeric and graphical approaches. The results indicated that those who spent time practicing symbol manipulation became better at manipulating symbols. Not surprisingly, the students in these studies had learned what they had the opportunity to learn. Researchers investigating intermediate algebra students in a large university[135] also found significant differences in favor of the graphing-approach for students working with real data. In this study the treatment group again used a text in which there was a balance of calculator and traditional problems allowing students to explore, estimate, discover graphically and to approach problems from a multi-representational perspective, while the control group used a text that covered the same topics but emphasized memorization of facts and procedures and becoming proficient with paper and pencil calculations. However, despite the fact that they had not spent time with symbol manipulation strategies, students in CPMP who used handheld graphing technology had learned a variety of alternative, calculator-based strategies for manipulating symbolic expressions.

There is some indication that technology allows different teaching strategies and together with the use of handheld graphing technology can have an impact on student performance.[136] The researchers comparing 710 precalculus students at a large U.S. university found a difference in the overall performance on a comprehensive common final exam between students taught using a graphing calculator with some didactical changes—made possible in part by the graphing tool—and students taught in the traditional way while also using a scientific calculator. Test scores of the experimental groups were considerably higher than those of the control group on functions, graphs, word problems, and equations.

While the development of algebraic skills in a technology rich environment does not seem to be impaired, there is some discrepancy in what the research says. The study of the CPMP students[137] reported that, although not significantly different, treatment students were not as proficient as control students at manipulation of symbolic expressions by hand; the treatment students had, however, learned a variety of alternative, calculator-based strategies for accomplishing the same goals. On the other hand, in the UCSMP study[138] both treatment and control groups performed comparably on items testing algebraic skills as presented on a posttest at the end of the study. Here, however, the test did not include some of the topics in a traditional second year algebra course because they are treated at a later point in the UCSMP curriculum. The New Zealand study of 13 and 14 year olds[139] learning algebra established no significant difference in skills between graphing calculator and non-graphing calculator students. And, no significant difference was found between the scores of treatment and control classes in a study of college intermediate algebra students[140] (although the treatment classes had a slightly higher mean score overall on the departmental exam used to measure traditional algebra skills) or on the multiple choice section of the precalculus final at another university.[141]

We examined four studies related to the use of symbolic calculators.[142] A study of students performance on a university entrance examination in New Zealand reported that students who had been through a four hour training session and worked through modules to practice solving calculator friendly tasks with the calculator scored significantly better than their counterparts without the calculator advantage.[143] However, when parallel tasks that removed the calculator advantage were presented to the students the calculator offered them no advantage over the control group. In addition, they failed to outperform the control group on the formal university exam, where they were not allowed the use of a symbolic calculator.

All three studies found that students gained an awareness of potential and limitations of the calculator. In one study, the researchers observed that the more graphing calculator manipulations were mastered, the more students were able to involve themselves in mathematical work.[144] In the study of two classes of French upper secondary students, researchers observed a real change in most students in their relation to mathematics and their own self-confidence. The researchers classified students according to the way they approached their mathematical work and found for those whose work methods were categorized as mechanical and random, the results were not positive. Depending on their mathematical knowledge, such students had to work harder to adapt to the new machine or they had insufficient knowledge to adapt and often gave up any idea of understanding. Rational and theoretical workers could work in both worlds, working at times independent of the calculator. The random workers, however, were lost without the calculator and could do nothing without their "crutch".

Mathematically, students managed to solve optimization problems in a meaningful way, they showed understanding of the concepts of mathematizing optimization problems and of the strategy of solving them, and the utilization scheme of calculating the zeros of the derivative was managed adequately.[145]

Not surprisingly, there is evidence that students use handheld technology as they are taught.[146] The curriculum and the way the technology is used in instruction seemed to affect the mathematical gains students achieved. Studies that looked at mathematical gains within projects that incorporated graphing technology support the idea that it is not the technology alone that perpetuates such gains.[147]

Use of knowledge and skills gained

The graphing calculator seems to be most often used by students, particularly those for whom the technology has been an integral part of instruction, to produce graphs in an effort to solve a variety of problems. The studies in this section are divided into three areas that illustrate what researchers have observed students doing with handheld graphing technology knowledge and skills.

Students are more likely to use graphical approaches

Students with access to handheld graphing technology used graphical solutions more often than students who did not have technology.[148] Students used handheld graphing technology to produce quick graphs and attain direct feedback,[149] trace a function to find particular values,[150] and create visual approaches when solving problems.[151]

One of the studies we examined of upper secondary students in the Netherlands[152] reported that weaker students seemed to particularly benefit from the availability of technology. Another report of the same experiment[153] confirmed that the graphing calculator increased students' use of graphical strategies. The researchers found that the increase in the use of graphing strategies, however, did not appear to diminish their use of other strategies.

Students tend to be more likely to explore mathematical ideas

Researchers seem to agree that handheld graphing technology stimulates many students to engage in informative exploratory activities, although the mathematical level of the result tends to vary.[154] Case studies about student understanding offer support for this claim.[155] The use of technology also seems to have an impact on the way in which students work. In studying the roles and behaviors of students in technology integrated precalculus classrooms,[156] the researcher noted that students worked together more often when technology was used. They did more than sit, listen, and answer questions, and were more likely to serve as task setters and consultants, in planned and spontaneous instances.

Students utilize additional tools in problem solving situations

Handheld graphing calculators facilitate calculation,[157] aid in working with complex formulas with difficult coefficients, and offset difficulties with drawing and calculating.[158] The technology can lend positive support to the use of realistic data, appears to free the student to construct mathematical models and choose strategies for solving problem and encourages a shift from rigid techniques to more flexible solution procedures.[159]

Mathematical misunderstanding and obstacles

Research also speaks to the problems and mathematical misconceptions involved in using the graphing calculator. In some cases the technology merely aggravates the misconceptions that students already have, for example their lack of understanding of decimal representations of rational and irrational numbers.[160] In other cases it may hinder students' development of functional symbol systems.[161] It seems common for many students to accept the visual image presented in the window of the calculator without considering the context of the problem.[162] Students may, for example, see the view of the graph of a quadratic function that appears to be a straight line and fail to recognize that they are only looking at a portion of a parabola. Some researchers, however, found students developed a more critical attitude toward numerical results.[163]

Several researchers found that difficulties with scale are compounded by the lack of understanding of the technology, particularly how the calculator assigns pixels to graphs.[164] Poor understanding of the zoom function also gave students problems with scale. Most disassociated the zoom operation from any change in the scale of the graphs they saw.

Similarly, research where symbolic calculators were used looked at the difficulties the technology can introduce. A study of French students[165] concluded that while most students with symbolic calculators changed in a positive way in their relation to mathematics—developing deeper mathematical understanding with a powerful problem-solving tool—some students who seemed to depend excessively on the calculator without considering the underlying mathematics involved rarely achieved the same level of benefit from its use. A study of upper secondary students in the Netherlands[166] found that students lacked the ability to decide when and how to use the calculator as a Computer Algebra System when searching for algebraic solutions, particularly ones that were not numerical. For example, many students do not possess the flexible conception of variables and parameters that using a symbolic calculator requires. In another study of upper secondary students in the Netherlands,[167] researchers found students had difficulty understanding the different role of the letters in parametric equations. They concluded CAS often required that students have developed abstractions for the mathematical concepts they are entering into the calculator. Students had problems with commands such as solve (Were answers in decimal form or expressions?), isolating a variable using the technology, and confusing schemes for operations such as substitution. Technical errors that students made, such as syntax errors, seemed to stem from their limitations in the understanding of the mathematical concepts involved. Students also encountered problems in simplifying expressions by the calculator when the calculator expression does not correspond to the expected response.

Weaknesses

Some of the reports of the studies did not include enough detail to make particular aspects of their study clear. For example, one study[168] included a module "Tapping in Algebra", but there is no information about how the teachers used the module or what kinds of tasks were contained in the module. In another study,[169] students worked on tasks in pairs, but while it was clear from the excerpts of student dialog what some of the tasks might be, a representative sample of actual tasks would have improved the reader's understanding of the student responses.

Design questions often arose in reading the reviews. Questions about how participants were specifically selected were left unanswered in some studies, particularly when intact classes were used.[170] Protocols for interviews of teachers were often not included nor were coding methods. Reports of the studies did not always give enough information about specifics of the study to make the exact nature of the study clear to the reader.

There was little attention to the relation between the length of time students had access to handheld graphing technology and the kind of training they had in using the technology and what students did with it. In one study[171] students who had no previous experience with a symbolic calculator received only four intensive one-hour sessions on using the calculator for procedural tasks, before testing them on calculator positive and calculator neutral tasks. Although the researchers point out that this is not the desired approach, short-term studies are only useful in helping understand how students learn to use the technology but not about the impact of the technology on what students will do in the long term.

Gaps

The research covers only a narrow set of mathematics. There is little work on students' understanding of statistics or discrete mathematics or the use of formulas. No attention seems to be focused on how students develop reasoning and approaches to proof using the technology. How these areas are affected by the use of handheld graphing technology remains an open question. Despite the amount of classroom usage at middle grades and the availability of handheld graphing technology specifically designed for those levels, little research exists on middle grades students' use of the technology. Closely related is the lack of longitudinal studies about the cumulative effects when students use the technology through their entire secondary mathematics programs.

The study of the particulars of students' activity, a thorough study of two or three students solving problems, would be beneficial and might help answer questions such as: What are the changes, if any, over time in the ways that students make use of the calculator? What is the effect of handheld technology on students' ways of mathematical thinking? In what ways does handheld technology impact higher level thinking skills?

More studies that take advantage of the symbolic manipulation capabilities would be informative. The research is sparse in helping us understand in what ways the symbolic calculator used as a computer algebra system might affect students' understanding of algebraic concepts or how it would it affect students' algebraic skills.

Implications for the Classroom

Three issues seem to emerge as directly related to practice: over-utilization, under-utilization, and interpretation. Over-utilization occurs when the student begins to see the calculator as a source of authority. Some of the research[172] cautions that a balance between using technology to mediate problem solving and becoming reliant on the tool must be found. Researchers identify this balance as a concern for both calculator designers and teachers. Under-utilization of handheld graphing technology may be attributed to uncertainty on the part of students as to when to use calculators appropriately, and the evidence indicates that instructional time needs to be spent by teachers on exploring the limitations of the calculators.

Many of the researchers[173] argue that teachers should be involved in helping students learn how to use the calculator with full recognition of its constraints and potential. They should also understand various profiles of student behavior in order to design and implement appropriate mathematical activities. They observed that the more graphing calculator manipulations were mastered, the more students were able to involve themselves in mathematical work. It might be useful for teachers to understand the researchers' claim that students learning to use calculators go from an initial orientation phase to an organization phase characterized by a pruning attitude towards first strategies, progressive awareness of effective constraints and potential uses of the calculator, and decreasing trust of the machine's results. Some students construct an efficient relationship with the calculator while keeping certain objectivity with regard to the machine, but others, described as those who may be characterized as mechanical or random in the way they work rarely achieve the same level of understanding of how to use the calculator. Teachers also should understand that the process of becoming accustomed to a calculator is slow and complex because it requires sufficient time to achieve a reorganization of procedures, even for the better students who have established a relationship with the machine. Additional time spent on emphasizing efficient techniques, however, may facilitate access to effective use of the calculator, and, in this way, lost time will probably be made up in future activities.[174]

Mathematical difficulties often point to shortcomings in curriculum, which may contribute to adverse effects whether or not graphing calculators are used. For example, there is probably need for greater emphasis on scale, and students should be given opportunity to explore links between zooming and scale. Students would benefit from confronting limitations of the technology and attempting to explain them—leading to better mathematical understanding and interesting mathematics. Problems highlighted in the research included the confusion that some students experienced using some features of the calculator such as scaling issues and failure of the technology to represent discontinuities accurately. Some researchers have pointed out that the use of multiple representations does not insure that students will make links among representations.

Overall, researchers found that reconciling different types of information is not intuitive but needs to be taught. Conflict between symbolic and graphic information was often unresolved by students. For example, one researcher[175] found interpretation and transcription of graphs to be major areas of difficulty: asymptotic behavior was not recognized, points of discontinuity were not located, the limiting value of functions was believed correct even though the capacity of the calculator to deal with large numbers was exceeded, horizontal asymptotes were omitted, a non-existent turning point was located on an asymptote, and a turning point was not located despite a question suggesting its existence.

Because students who owned their own calculators more frequently exhibited a critical awareness of the calculator's output, it seems regular access to the technology may have a positive influence on linking different representations of functions while other difficulties may lessen.

Topics such as numerical and exact calculations, simplification of formulae and roles of variables and parameters deserve more attention when a CAS is used than when it is not. Teachers should be careful not to leave the student with a feeling of dependence on the technological tool. Without preparation and sufficient background, the top-down character of a CAS, its "black-box style," and its idiosyncrasies can produce obstacles for students.[176]

Question 5. What impact does handheld graphing technology have on the performance of students from different gender, racial, socio-economic status, and achievement groups?

The authors of *The Nation's Report Card: Mathematics 2000* state: "The proper role of calculators in the K-12 curriculum has been and continues to be debated. Calculator use policies vary across schools, and even within the same school, teachers have different opinions about how calculators should be integrated with instruction."[177]

Differing policies and different beliefs on the part of teachers about handheld graphing technology can result in different levels of access and possibly even different learning outcomes for students, who for whatever reason, end up in different courses or in courses with teachers who approach the use of handheld graphing technology in different ways. In this section, we explore what the research from six studies has to say about equity issues in the use of handheld graphing technology. Five of the studies were experimental or quasi-experimental performance comparison studies carried out in New Zealand, England, the Netherlands, and the United States,[178] and the sixth investigated the nature of the errors made by male and females on an Australian college entrance examination.[179]

Findings

In studies where researchers examined performance variability within, rather than simply between, the treatment and control groups, the results usually indicated that there were no significant differences in performance that could be attributed to gender, race, socio-economic status (SES), or prior knowledge/achievement. However, in some studies, differences in student performance were attributed to one or more of these variables.

The research on gender issues, for example, is mixed. Some studies show no difference in achievement between males and females,[180] while a study of 16 year olds in Great Britain found that, "On the symbolization items, use of graphic calculators was associated not only with markedly superior attainment by all students, but with greatly enhanced relative attainment on the part of female students".[181] On these items, female students in the treatment group outperformed the males, while in the control group the males outperformed the females.[182] On graph interpretation items, there was no significant difference between the treatment and control groups, but males in both the treatment group and the control group, outperformed the females in their group.

In the case of ability, researchers studying 17 and 18 year old students from two secondary schools in New Zealand[183] found less within-group variability in the performance of low-, middle-, and high-ability students using handheld graphing technology than they found in the performance of low-, middle-, and high-ability students in the control group. This suggests that the use of handheld graphing technology seemed to decrease the performance gap between higher and lower achieving students. Another study of 16 and 17 year olds in the Netherlands[184] also found that lower achieving students made larger performance gains when using handheld graphing technology than did moderate and high achieving students who also used handheld graphing technology.

Weaknesses

The major weakness in the body of research on equity issues with handheld graphing technology was that when equity issues were examined, they often seemed to have been included as an after-thought rather than as a central issue for investigation. Interaction effects related to equity variables were often described and explained much more briefly than other findings, and were rarely the subject of follow up studies or independent studies designed to test the veracity of the explanations that researchers provided for the interactions they found.

Gaps

Despite finding few actual differences in the performance of students with different backgrounds when using handheld graphing technology, some researchers have expressed concerns about one or more equity issues.[185] Since the production of the *Curriculum and Evaluation Standards for School Mathematics*[186] that recommended the use of technology for all students, many researchers and educators have raised questions about equity issues arising from the cost of technology. Others expressed concern that the technology would only be available to students in certain ability groups. Some were concerned that female students would react negatively, while males would embrace the technology, leading to differential learning outcomes. In *The Nation's Report Card: Mathematics 2000,*[187] an effort was made to describe students' access to calculators and examine the relationship between access in math classes and student performance on the mathematics portion of the National Assessment of Educational Progress (NAEP)—which allows all students to use scientific calculators on certain items. According to this report, 62 percent of 12th grade students reported using graphing calculators in their mathematics courses, and there was a positive relationship between frequency of use and NAEP score (p. 165).[188] These results raise the question, "Who are these students? How do their characteristics vary with regard to gender, race, socio-economic status, and ability level?" Unfortunately, this information was not available. This was a common result in our efforts to locate multivariate equity studies.

A major gap in the body of research on equity issues with handheld graphing technology was that so few studies actually attempted to determine the impact that access to this technology had on different types of students. Although an extensive search for relevant articles was conducted, very few controlled studies on the impact of handheld graphing technology included analyses of its impact on students from different gender, racial, SES, and ability groups. While some claim that the use of handheld graphing technology supports women, minorities, and underachieving students in learning mathematics and having the confidence in their ability to do so, there is little research that actually investigates these claims.

In general, the relationship of primary importance in the research was that between students with access to handheld graphing technology and those without. This type of between-groups comparison was prevalent, but only on rare occasions did researchers attempt to determine whether or not any systematic within-group variability existed. In cases where such attempts were made, researchers were much more likely to look for gender effects than for any other type. There is a need for studies that address the range of equity concerns raised in mathematics education literature. There is also a need for research reports that provide detailed descriptions of the context of the studies. If there are important differences for students with different background characteristics, we need enough contextual information to determine the possible reasons. A simple presentation of the students' characteristics and their treatment group will not suffice if the goal is to address problems, rather than simply report them.

Implications for the Classroom

Within classrooms, teachers should pay explicit attention to issues of equitable access. Once equitable access is ensured, teachers should attend to students' patterns of use. If systematic differences are noted, teachers should make an effort to determine and address the underlying causes. In some cases, addressing these issues may be as simple as providing additional training for students. In other cases, it may require a reconceptualization of the rationale for using handheld graphing technology, or a shift in the role that handheld graphing technology plays in instruction.

Recommendations for Future Research

Our work suggests that the maximum potential for handheld technology has not been explored. More research is needed. The research summarized here can provide a background for this work. In an effort to ensure that future research provides well-grounded findings and offers guidance for both policy and practice, this section includes suggestions for future research. It describes the nature and possible foci of research that can fill the gaps in our knowledge base that prevent us from understanding how to design and implement handheld technology-based mathematics education reforms that are successful on a large-scale and within multiple contexts.

The research we studied provides a starting point for efforts to better understand how to effectively use handheld graphing technology in the classroom. The design of some studies limits their potential use by those in policy and practice. To help in the interpretation of findings and in using them to build toward a cumulative set of knowledge, we make the following recommendations:

- Data collected about the use of handheld graphing technology should describe the specific features of the context—including the handheld graphing technology used, content, and aspect of use that is being investigated not merely report counts and observations. Because teaching and learning are so complex, "… attending to context is critical for understanding the degree to which theories and findings may generalize to other times, places, and populations."[189] Better descriptive tools for characterizing student learning with handheld technology and for looking at factors related to this learning are needed.

- Research programs should include or facilitate comparisons among different ways of using handheld graphing technology as well as between those who use it and those who do not.

- Research should include within-groups as well as between-groups comparisons of students with and without access to handheld graphing technology to determine if differential effects exist for students from different backgrounds, in various contexts.

- Because one study does not usually produce definitive results, multiple methods applied over time are necessary to build a knowledge base. Research should be designed both to look across schools and across content areas to support broad generalization and to take a close look at particular cases. Cases can identify promising variables for inclusion in broad surveys, and surveys can position and help in the interpretation of particular cases.

- Design and reporting of research on the use of handheld graphing technology must be explicit about connections to improving student achievement.

- Research should pay explicit attention to the use of handheld graphing technology in urban and poor rural settings.

- Research designs and analytic methods should control for, or test for, other important variables, systematically ruling out counter-explanations.

- Research on the use of handheld graphing technology should include the length and nature of access to handheld graphing technology, in particular studying student learning in situations with unlimited access over several years and in a progression of mathematics courses.

- Research more explicitly informed by a historical perspective would help in sorting out issues that are particular to technology from those that are independent of the technology.

Areas for Future Research on Handheld graphing technology

Research on the use of handheld graphing technology is not robust. Individual projects look at specific pieces of the picture, but the pieces do not make a coherent whole and, in fact, often seem unrelated. Useful next steps in the research would be those that allow us to answer the question, "How can we encourage teachers to use handheld graphing technology in ways that promote __________?" With greater attention to these details, both in the studies conducted and in the reports produced, we would be able to answer this question regardless of how we finish the sentence. Because the use of handheld graphing technology is not a variable that can be isolated but is a part of the complex teaching and learning environment, such research, however, should be done taking other factors in this environment into account .

In addition to recommending that research be coordinated, we recommend research designed to answer questions in the following areas:

Teacher knowledge, beliefs, and experiences related to the use of handheld graphing technology

- How do teachers' beliefs about handheld graphing technology explicitly affect their use of graphing calculators in their teaching?
- What experiences in preservice and inservice education influence teacher beliefs about the use of handheld graphing technology and how they choose to use it in their classrooms?
- What is the relationship between high quality teacher preparation with respect to handheld graphing technology and student achievement?
- What is the relationship between teacher beliefs and use of handheld graphing technology and student beliefs and use?

Curriculum implications related to handheld graphing technology

- What is the role of handheld graphing technology in learning mathematical content that is not part of the traditional mathematics curriculum?
- What is the role of handheld graphing technology in providing access to mathematics content earlier than would have traditionally been done?
- In what ways does the nature of the curriculum and tasks students are given influence their use of handheld graphing technology?

Student beliefs, understandings, and characteristics

- What are students' attitudes and beliefs about the use of handheld graphing technology and how do these affect student use of the tool?

- How do students who have access to graphing calculators compare in terms of their use with respect to ethnicity, gender, geographic, and socio-economic conditions?

- How is handheld graphing technology used with students in lower secondary grades and how is this use related to the development of their understanding of key mathematical concepts?

- What are the long-term effects of the use of handheld graphing technology on student beliefs, understanding, and achievement?

Assessment

- What are the relationships between "high stakes" assessments and the use of handheld graphing technology?

Many educators feel that handheld graphing technology has the potential to significantly affect mathematics teaching and learning. According to Hiebert (1999), research can document the effectiveness of new ideas, suggest explanations for successes and failures, and inform the discussion. The findings from research can provide direction and guidance for using handheld graphing technology to see that this effect supports student learning. "Research can and should play a critical role in helping educators make informed decisions …".[190] By conducting rigorous studies of important questions and relating the results to classroom practice, we can ensure that handheld graphing technology contributes in positive ways to improved mathematics education.

Endnotes

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Appendix A

What is the nature and extent of the use of handheld graphing technology in secondary mathematics classrooms? What policies are in place that might affect the extent of use?

Findings

There is little research that responds directly to questions of policy and practice. National surveys give a general picture of the extent of the use of handheld graphing technology in the United States. Graphing calculators are well entrenched in U.S. high school classrooms but not quite as common in the middle grades. According to a national probability sample of nearly 6,000 mathematics and science teachers,[1] 39 percent of the teachers surveyed in grades 5-8 used graphing calculators in their most recent lessons while 80 percent of 9-12 teachers did so. Table 1 illustrates the kinds of use described by the mathematics teachers surveyed.

Table 1 Graphing Calculator Use in Most Recent Lessons

Use	% of teachers grades 5-8	% of teachers grades 9-12
Demonstrate mathematical principles	37%	51%
Take a test or quiz	32%	68%
Do simulations	9%	11%
Retrieve or exchange data	8%	9%
Drill and practice	38%	62%

At the university level, the prevalence and use of handheld technology varies significantly from institution to institution. Table 2 shows data from the 1997 and 2002 Conference Board on the Mathematical Sciences *Survey of Undergraduate Programs*[2] related to handheld graphing calculator use in mainstream calculus courses-those for engineers and mathematics majors and non mainstream courses- those for non science and mathematics intending students. Data from the 2002 survey also show that at two-year colleges 20 percent of the sections in elementary algebra, 74 percent of the sections in college algebra, and 83 percent of the sections in precalculus are taught using graphing calculators.

Table 2. Percentage of enrollment in courses in mathematics departments

Year	Type of degree	Mainstream Calculus I in math depts.	Mainstream Calculus II in math depts..	Nonmainstream Calculus I in math depts..	Statistics in Statistics Depts	Statistics in Math Depts	Statistics for preservice teachers	Statistical Literacy in stat depts
1990	PHD	3	3					
	MA	3	1					
	BA	2	2					
	2yr							
1995	PHD	33	27	25				
	MA	44	32	20				
	BA	39	32	34				
	2yr	37		44				
2000	PHD	40	42	27	15	38	0	28
[3]	MA	55	59	66	0	49	0	23
	BA	67	50	63		51		
	2yr	78	74	72		59		

Testing Practices and Policies

While it is possible to distinguish between using hand held graphing technology as a tool in teaching and learning and its use on examinations, in practice the two are closely linked in subtle ways. The National Assessment of Education Progress (NAEP) allows the use of only scientific calculators, and they provide them for students in their national sample. The NAEP data does, however, report on student use of handheld graphing technology. According to the 1996 NAEP survey data, overall 7 percent of the students in 8th grade used graphing calculators; 18 percent of those in algebra, 11 percent in prealgebra, and 10 percent in regular math.[4] The 2000 data show that those students who said they used calculators more often tended to outscore their peers who reported using calculators less frequently.[5] As of this report, specific data are not available about the type and details of calculator use for the 2000 assessment, but 68 percent of the students in grades 9-12 reported using some type of calculator almost every day in their mathematics work.

Calculator policies on examinations related to college entrance are specific about calculator requirements. The SAT I is developed with the expectation that most students are using them, but they are not necessary for the test. Graphing calculators are required on the SAT Mathematics Level IC and IIC.[6] On the Advanced Placement AB and BC tests, some questions need a calculator to answer.[7] The TI 92 Calculator is not allowed on any of these tests because of the qwerty (typewriter) keyboard. The ACT policy is similar.[8] Graphing calculators are allowed, but in addition to those with qwerty keyboards, it also eliminates calculators such as the TI 89 that have built-in computer algebra systems.

Weaknesses
Many of the reports on calculator use mix computers and calculators without specifying details about either. Some reports do not distinguish which calculator was the subject of the study: scientific calculators, graphing calculators, or calculators with computer algebra systems. In addition, much of the data is self - reported by teachers, which can result in different interpretations of what it means to use the calculator and of degrees of implementation.

Gaps
There is a need for research on the relation between policy and practice with respect to the use of graphing calculators. Do policies that restrict calculator use on high stakes tests, for example, have an effect on the use of calculators in classrooms? Questions of how policies on handheld graphing technology affect equity issues are not addressed in the literature. Some educators believe demanding curricula that require the use of such technology will ensure that all students have access. Other educators, as described in Question 5 in this report, are concerned that requiring handheld graphing technology may, in fact, increase the achievement gap between those who have access to the technology and those who do not. Evidence related to these areas would seem to be important for those who make policy regarding access to handheld graphing technology as they consider usage and conditions for usage.

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[1] Weiss et al., 2001.
[2] Loftsgaarden, Rung, & Watkins, 1997; Lutzer, Maxwell, & Rodi, 2002.
[3] Ruthven, 1997.
[4] National Center for Education Statistics, 2000.
[5] http://nces.ed.gov/nationsreportcard/pdf/main2000/2001518.pdf
[6] http://www.collegeboard.com/sat/html/students/prep000.html
[7] http://apcentral.collegeboard.com/
[8] http://www.act.org/aap/taking/calculator.html

Appendix B

Research Summarized in This Report

The studies were divided according to their general methodology: Experimental and quasi-experimental studies must have used random assignment to groups or some form of matching for entering characteristics. "Interpretive" studies were reports that included a description of their data collection process, the analysis, and evidence such as samples of interview responses or detailed descriptions of events in the report.[1]

Question 1: How do teachers use hand-held graphing technology and how is this use related to their knowledge and beliefs about technology, mathematics, and teaching mathematics?

Study	Methodology, Sample Size, Variables, Technology	Findings
Doerr, H.M., & Zangor, R. (1999) "The teacher, the task and the tool: The emergence of classroom norms" *International Journal of Computer Algebra in Mathematics Education, 6: 267-280*	Interpretive (case study) One teacher with 20 years teaching experience and 31 students (ages 15-17) Classroom observations (17 weeks-all class sessions) Classroom videotape transcriptions Teacher interviews (nature and number not given) TI-82 TI-83	The researchers posited three central aspects of the teacher's role, knowledge & beliefs: 1) the teacher's confidence and flexible use of the tool, 2) the teacher's awareness of the limitations of the technology and 3) the teacher's belief in the value of the calculator to support meaningful investigations. These led to the establishment of 3 classroom norms: 1) multiple ways for conjecturing and confirming hypothesized relationships between variables, 2) requiring results to be justified on mathematical grounds and 3) interpreting results in terms of the problem context. The teacher and students developed a flexible use of the graphing calculator as a tool that could be used to investigate the complete view of the properties of a function's graph. The teacher's recognition of the limitations of the graphing calculator led the students to develop a reasonable skepticism about calculator-generated results, which led to a norm that required conclusions to be based on mathematical grounds, not just because the calculator said so: "The graphing calculator was not a tool with some independent role and existence in this classroom. But rather, the teacher's knowledge and beliefs, as reflected in the role she took in the classroom, led to the calculator being used by the students and by the teacher in a particular set of ways that created and then reflected the graphing calculator as tool that supported the mathematics learning in this classroom" (p. 277).

[1] Interpretive studies in this report are defined as in Wilson 2001; the studies try to understand educational experiences from the perspective of those involved, usually some mixture of in-depth interview and observation. Frederick Erickson, "Qualitative Methods in Research on Teaching" (*Handbook of Research on Teaching*, 3rd edition edited by Merle C. Wittrock, pp. 119-61. New York: Macmillan, 1986) describes such studies.

Doerr, H. M., & Zangor, R. (2000). "Creating meaning for and with the graphing calculator" *Educational Studies in Mathematics, 41: 143-163*	Interpretive (case study) One teacher with 20 years teaching experience and 31 students (ages 15-17) Classroom observations (21 weeks-frequency unknown) Classroom videotape transcriptions Teacher interviews (nature and number not given) TI-82 TI-83	The teacher's confidence in her knowledge of the calculator's capabilities led to her flexible use of the tool. Her beliefs about the limitations of the calculator and meaning making "limited the 'black box' use of the graphing calculator" (p. 160). Because of the teacher's knowledge of the limitations of the calculator and her belief that conjectures are proven on the basis of mathematical reasoning or argument, the calculator did not become a source of mathematical authority in this classroom. The role, knowledge, and beliefs of the teacher influenced the emergence of such rich usage of the graphing calculator.
Farrell, A.M. (1996) "Roles and behaviors in technology-integrated precalculus classrooms" *Journal of Mathematical Behavior, 15(1): 35-53*	Mixed Design (comparative & interpretive) Six high school teachers and their pre-calculus students Video taped lesson analysis (10 lessons) Observation tally Calculator type not provided	Three pre-calculus classrooms with access to hand-held graphing technology were compared to three classrooms without such access. The evidence suggested that teachers were holding on to their managerial roles while taking on some new roles (e.g., consultant, fellow investigator) when technology was used. The researchers made it clear, however, that in no way was the report meant to imply that the graphing technology causes the reported differences between technology classrooms and traditional one. It is up to teachers to make use of available tools in ways that support desired student behaviors (p. 51).

Fleener, M. J. (1995a) “The relationship between experience and philosophical orientation: A comparison of preservice and practicing teachers' beliefs About calculators” *Journal of Computers in Mathematics and Science Teaching, 14(3): 359-376*	Comparative (survey research) 233 classroom teachers self-selected taking part in a graphing calculator workshop (11% elementary, 49% intermediate, 40% high school) 78 pre-service elementary teachers Likert scaled survey Calculator type not provided Study divided respondents into two groups based on their opinion of the need for mastery before using calculators and compared their philosophical orientations	The results of the survey indicated that 55% of preservice teachers believed that students needed conceptual understanding (mastery) before using calculators; 45% disagreed or had mixed feelings about the mastery issue. The philosophical orientation of those who expected mastery was mostly aligned with control. The orientation of those who did not expect mastery was more aligned with student autonomy. There was no significant difference ($p=.16$) between the teachers and preservice teachers with regards to the mastery issue.
Fleener, M. J. (1995b) “A survey of mathematics teachers' attitudes About calculators: The impact of philosophical orientation” *Journal of Computers in Mathematics and Science Teaching, 14(4): 481-498*	Comparative (survey research) 94 middle and high school mathematics teachers Likert scaled survey CASIO 7000 Overhead Projectable Graphing Calculator Compared scores for 1) teachers who expect concept mastery before calculator use, 2) teachers who do not expect mastery, 3) teachers who use frequently use calculators in class, 4) teachers who use them infrequently.	There was 97% agreement among the teachers that all students should learn to use calculators, and that students don’t need to show their work on paper when they use a calculator. They agreed that math is easier if a calculator is used to solve problems, that using calculators makes students better problem solvers, and that calculator use is permissible on homework. There was majority disagreement that calculators should only be used to check work once the problem has been worked on paper and that continued use of calculators will cause a decrease in student estimation skills. When the data were examined in relation to frequency of use, there were significant differences in teachers’ views on student access, use on homework, and use during math tests.

Goos , M., Galbraith, P., Renshaw, P., & Geiger, V. (2000) "Reshaping teacher and student roles in technology-enriched classrooms" *Mathematics Education Research Journal, 12(3): 303-320*	Interpretive 11th and 12 grade students taking introductory calculus and statistics (exact number not given) Student questionnaire Classroom observations (# not mentioned) Individual and group interviews (no further details provided) Calculator type not provided	The metaphors of master, servant, partner, and extension of self were applied to both teachers and students: Technology as master: The user is "subservient to the technology and is able to employ only such features as are permitted either by limited individual knowledge or force of circumstance" (p. 307). Technology as servant: "The user may be knowledgeable with respect to the technology but uses it only in limited ways to support preferred teaching methods (Thorpe, 1997)" (p. 307) Technology as partner: The user "has developed an affinity with both the class and the teaching [or learning] resources available" (p. 307). Technology as extension of self: "Powerful and creative use of both mathematical and communications technology forms as natural a part of a [user's] repertoire as do fundamental pedagogical [or learning] and mathematical skills" (p. 308). The authors concluded, "the relationship between technology usage and teaching/learning environments is not one of simple cause and effect" (p. 317). In addition, they found that "even though the graphics calculator is designed as a personal mathematical tool it can facilitate social interaction and sharing of knowledge" (p. 318).
Harskamp, E.G., Suhre, C. J. M., & van Streun, A. (2000) "The graphics calculator and students' solution strategies" *Mathematics Education Research Journal, 12(1): 37-52*	Comparative 12 classes of grade 10 students from 7 Dutch upper secondary schools. 296 students aged 15-18 (Exp Group 1: 3 classes with 74 Exp Group 2: 5 classes with 119 Exp. 3: 4 classes with 103) Teachers had no experience with the technology prior to the study. Classroom observations (2 lessons) TI-81	Teaching – two sample lessons used by the Exp 1 teachers were summarized and the researchers emphasized that the lessons illustrate "the implementation of problem-solving activities and the use of graphs and tables" (p. 44). These results were investigated further using the quantitative data collected. They inferred that the teachers of the two experimental classes spent approximately the same proportion of lesson time to problem-solving activities and instruction using tables and graphs and "considerably" more time than teachers in the control group. Teachers in the experimental group followed the textbook strictly.

Harskamp, E.G., Suhre, C. J. M., & van Streun, A. (1998). "The graphics calculator in mathematics education: An experiment in the Netherlands" *Hiroshima Journal of Mathematics Education, 6:13-31.*	Comparative The students were all 15-18 year old pre-university students. All of the classes were in the Netherlands, used the same text, and were taught the same topics. There were two experimental conditions and one control condition. There were 3 classes in the first experimental group. This group used the graphics calculator throughout the year with all topics. The second experimental group consisted of 5 classes, each of whom used the graphics calculator with only one topic (each topic took about 2 months). The control group, which consisted of 4 classes, did not use the graphics calculator at all. Classroom observations (2 lessons) Classroom videotape Observation tally TI-81	There were no significant differences between the amount of time spent on problem solving in the experimental groups and the control group. There was a significant difference between the experimental and control conditions in the use of graphical means during instruction ($p = .04$). Teachers in the experimental groups were more likely to show graphs of functions than teachers in the control group. Teachers in the experimental groups followed the textbook closely, and only used the graphics calculator on the exercises for which the book indicated that technology was appropriate.
Lloyd, G. M., & Wilson, M. S. (1998) "Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum" *Journal for Research in Mathematics Education, 29(3): 248-274*	Interpretive One secondary teacher with 14 years experience Teacher interviews (2 interviews) Classroom observations (26 lessons) Classroom videotape transcriptions TI-83	The teacher relied on graphs to see the relationship between two things but chose a formal definition as correspondence (each x has one y). The basic framework for understanding was shaped by work in a traditional curriculum. The teacher encouraged students to use a variety of representations and connections among them to investigate real-world occurrences of different families of functions; this impacted his questioning technique. Graphs played a privileged role and were the primary vehicle for descriptions and classifications. The centrality of graphs, however, did not preclude other forms of representation; the teacher developed worksheets to extend the iterative part of the recursive rules using the calculators, used graphs to point out features that distinguished characteristics of families of functions, identified links among the representations and made connections.

Rochowicz, Jr., J. A. (1996). "The impact of using computers and calculators on calculus instruction" *Journal of Computers in Mathematics and Science Teaching,15: 423-435*	Comparative 89 calculus teachers Likert scaled survey Calculator type not provided	"The perceived impact of using computers and calculators on specific topics of calculus appears to shift the focus of learning from symbolic manipulation and skills to more interpretation, approximation, graphing, and modeling of realistic situations" (p. 426). "A large number of the participants in this study agreed that technology use does not replace the teacher; requires more time from the instructor; and requires more meaningful and creative teaching on the part of the teacher…. The teacher becomes a facilitator of learning where more responsibility for learning is placed upon the student…. Only 30% of the subjects of this study agreed that assessment of learning becomes more difficult while 26% expressed uncertainty as to whether assessment of learning becomes more difficult" (p. 429).
Simmt, E. (1997). "Graphing calculators in high school mathematics" *Journal of Computers in Mathematics and Science Teaching, 16(2-3), 269-289*	Interpretive 6 11th and 12th grade teachers Classroom observations (from 3.25 to 7.75 hours) Brief post-observation discussion Teacher interview (1 interview—1-2 hours) Calculator type not provided	Hand-held graphing technology was used to provide images that students could use to investigate transformations of the quadratic function $y=x^2$. The intent of the investigations was for students to "observe, generalize, and abstract the mathematics of the quadratic function" (p. 286). Graphing calculators were also used to verify work and were seldom used to answer "what-if" questions. Students used calculators in guided discovery but were not encouraged to conjecture, prove, or refute ideas. Teachers indicated that calculators saved time in the sense that they allowed the teacher or the student to generate many examples quickly. Teachers also reported that the graphing calculators helped facilitate guided discovery. The way the teachers used the calculators was a function of their personal philosophies of mathematics and mathematics education—"an extension to the way they always taught the course" (p. 287).
Simonson, L. M., & Dick, T. P. (1997). "Teachers' perceptions of the impact of graphing calculators in the mathematics classroom" *Journal of Computers in Mathematics and Science Teaching, 16(2): 239-268*	Interpretive 27 secondary teachers Teacher interview (1 interview—telephone) HP-48S	The teachers' perceptions of the advantages of calculator use were related to instruction while their perceptions about the disadvantages were related to logistics. There may have been a correlation between teachers' fear of calculator dependency and hesitancy to use the calculators, "for it [fear of dependency] was noted least often by those teachers making regular use (more than once a week) of the calculators in the classroom" (p. 252). Also, those who used the calculators least were the ones most concerned with the time it took to learn to use the calculator. Using the calculator seemed to shift instruction toward inquiry and discussion. At the time of the study, students were not permitted to use graphing calculators on the AP exam. This concerned teachers. Teachers who used the calculators frequently allowed students to use them on tests. "More than half of the teachers who mentioned increased mathematical depth also related changes in their test questions" (p. 258). Teachers agreed that there was a need for further professional development.

Slavit. D. (1996). "Graphing calculators in a "hybrid" Algebra II classroom" *For the Learning of Mathematics, 15(1): 9-14*	Interpretive 1 secondary teacher Classroom observations (periodic over 1 year) Calculator type not provided	The teacher, using a traditional algebra II textbook, ended up creating a hybrid classroom that was "a blend of traditional and alternative approaches to the teaching of basic elementary functions" (p. 13). Also, the nature of the mathematics discussed in the teacher's classroom changed because of the nature of the graphing calculator: "The use of the GC was associated with higher levels of discourse in the classroom, including higher-level questioning by the instructor and more active learning behaviors by the students" (p. 14). Also, the number of student initiations of discourse increased when the use of the graphing calculator increased as did the number of analytic questions asked by the teacher (analytic questions were defined as questions that "involve conceptual issues or to allow room for new lines of discourse to emerge" (p. 10). The author explained that the creation of this hybrid classroom was not surprising given that the materials were traditional in that they made "limited use of graphical and numerical situations" (p. 13). Too, the teacher was modifying his teaching practice of 25 years to accommodate the new technology. Because the teacher was excited about graphing calculators, his attitude was not something that hindered his use of them.
Tharp, M. L., Fitzsimmons, J. A., & Ayers, R. L. B. (1997). "Negotiating a technological shift: Teacher perception of the implementation of graphing calculators" *Journal of Computers in Mathematics and Science Teaching, 16(4): 551-575*	Interpretive and Comparative 261 teachers of grades 6-12 mathematics (168), science (72), and other (21) Likert scaled surveys (pre and post) Journal analysis The Wilcoxon signed rank test was used to analyze differences between questionnaires given pre and post intervention (telecourse for teacher) Comparisons between teachers based on their views of mathematics and their views of calculator use TI-82	A significant change occurred toward agreement with each of these statements: "A graphing calculator can be used as a tool to solve problems that I could not solve before," "Using a graphing calculator to teach mathematics or science allows me to emphasize the experimental nature of the subject," and "It is difficult to get funds to buy graphing calculators." There was a significant change toward disagreement with the statement, "Students lack the ability to work with a calculator as complex as a graphing calculator". The authors concluded that "teachers' views of the use of calculators changed significantly in the direction of the use of calculators as an integral part of instruction" (p. 557). There was no significant difference in the non rule-based view of mathematics after participating in the telecourse. There was a significant positive correlation between teachers' views of mathematics and teachers' views on the use of calculators (non-rule based teachers saw calculators as integral to instruction while rule based teachers did not perceive that calculators would enhance instruction—they might even hinder it). Rule-based teachers were more likely to notice affective aspects of their students' reactions to graphing calculators, while non rule-based teachers were more likely to focus on their students' conceptual understanding or the cognitive aspects. No significant difference was found in willingness to at least try either conceptual or procedural approach to learning between rule-based and non rule-based teachers. There was a significantly larger proportion of procedural teaching for rule-based teachers than with non rule-based teachers. Teachers with rule-based views of mathematics still had a rule-based view after the workshop. Non rule-based teachers used more inquiry, and students were free to use the calculator as they wished.

Question 2: With what kind of mathematical tasks do students choose to use hand-held graphing technology?

Study	Methodology, Sample Size, Variables, Technology	Findings
Berger. M. (1998) "Graphic calculators: An interpretive framework" *For the Learning of Mathematics, 18(2): 13-20.*	Interpretive study 68 South African students 20 graphic calculator students, 16 of which participated in data collection, 48 non-graphic calculator students Student interviews Statistical analysis of the task data was performed to corroborate or inform the analysis and interpretation of the qualitative data from the interviews, rather than a stand-alone measurement. Graphing calculator (model not specified)	There was no quantitative data of significant differences between the graphic calculator and non-graphic calculator students. The qualitative evidence of an amplification role (the speed and facility with which the learner operates while using the technology, rather than qualitative changes which may happen as a result of the technology).for the graphic calculator was ample while evidence of its role for cognitive re-organization (effects which may occur as a consequence of using the technology as a systemic change in the consciousness of the learner) was sparse. The socio-cultural context in which the graphic calculator was used is an important element in the interpretation of results.
Boers, M. A,, and Jones, P. L. (1994). "Students' use of graphics calculators under examination conditions" *International Journal of Mathematical Education in Science and Technology, 25: 491-516*	Interpretive study Students in one of two lecture groups on introductory calculus at Swinburne University, Australia Observational data of calculator use from 37 exams including student work and responses Interviews of 7 volunteers were held 2 months after the end of the course. TI-81	Under-utilization of graphics calculator was more the problem than over-use. A lack of confidence in calculator was cited in post-exam interviews. Students seemed to lack depth of understanding and the mathematical judgment to move between the two representations. Calculator can make test more difficult and give examiner a deeper insight into student's understanding. Graphical information is independent of algebraic information.
Dahland, G., and Lingefjard, T. (1996). "Graphing calculators and students'	Interpretive study The participants were a convenience sample	Students see mathematics problems as coming in different types. Some are treated as more suited than others to solution with the use of graphing calculators

interpretations of results" *Nordic Studies in Mathematics Education, 4(2-3): 31-50*	made up of the students of four teachers who were involved as supervisors in the teacher education program at the University of Gothenburg. Responses to a 6-item test based on Swedish core curriculum in levels 11 and 12 Graphing calculator (model not specified)	Students and consequently teachers need to develop a double competence to benefit from the graphing calculator. They need to understand what the graphing calculator can do and what its technical limits are, and they need basic mathematical knowledge for a correct interpretation of results obtained from a graphing calculator. The language is a problem in documenting results-to convert from "screen language" to "pencil and paper language" of traditional graphing techniques, and the behavior of a function
Doerr, H. M., & Zangor, R. (2000) "Creating meaning for and with the graphing calculator" *Educational Studies in Mathematics, 41:143-163*	Interpretive (case study) One teacher and 2 classes of 15 to 17 year old precalculus students, with 14 in one class and 17 in the other. Field notes from classroom observations Transcripts of audio-tapes of the group work Transcripts of the video-taped whole class discussions and interviews Planning sessions It is not clear what was taped and transcribed or when interviews took place or the protocol for the interviews TI—82 TI—83	The teacher's confidence in her knowledge of the calculator's capabilities led to her flexible use of the tool. Her beliefs about the limitations of the calculator and meaning making "limited the 'black box' use of the graphing calculator" (p. 160). The researchers found that the calculator had five different, but overlapping roles when used by the students: computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool. Two constraints and limitations of the calculator were found: (1) "students attempted uses of the device as a 'black box'" (p. 158); (2) students used the tool to pursue individual solutions rather than as a group; this caused problems in group thinking and communication. (Authors call this private or personal use). Teacher and student interactions "created meaning for and with the tool" (p. 159). Teacher encouraged students to use the calculator freely and to question its results. "The calculator did not become a source of mathematical authority in the classroom" (p. 159).
Drijvers, P., & Doorman, M. (1996). The graphics calculator in mathematics education. *Journal of Mathematical Behavior*	Interpretive--Developmental A sixth year class of pre-university (18 year olds) in The Netherlands Experimental instructional material, observations of the lessons, and reflections based on observations were used to analyze	The graphing calculator supports the use of realistic data and seems to stimulate many students towards exploratory activity, though mathematical level tends to be a confounding variable here, Integration of geometric and algebraic models did not occur without a struggle. However, students eventually seemed to accept the intertwining of the two.

	affects of graphing calculator use in a developmental classroom Graphing calculator (model not specified)	Graphing calculators removed the students' inhibitions for making quick sketches and with direct feedback encouraged students to reflect on what they had done Graphing calculators seemed to encourage a shift from rigid techniques to more flexible solutions. A more critical attitude toward numerical results also was developed
Forster, P. A., & Mueller, U. (2001). "Outcomes and implications of students' use of graphics calculators in the public examination of calculus " *International Journal of Mathematical Education in Science and Technology, 32(1)-37-52*	Interpretive study 172 (9%) of the 1882 Australian Tertiary Entrance Exams were studied. The papers were in six randomly assigned bundles and assigned randomly to the authors Australian students taking the 1998 Tertiary Entrance Exam. 404 students described in response to question data. Three female and three male candidates were interviewed in the week following the examination. They were chosen on basis of differing abilities on internal school assessments for calculus for the year. Two teachers of calculus and two examination graders were interviewed Analyzed student responses on six calculator positive problems from 19 problems on the exam. Analysis was informed by discussion with teachers and graders of an acceptable calculator-assisted answer and how students may have used calculators given the nature of their work. Graphing calculator with symbolic processing ability (model not specified) and HP38G	The most prevalent problem area discovered with technology use on this exam is "the interpretation of graphical information and an apparent under-utilization by students of the calculators. This may be because it was not clear when calculator use was appropriate and how much symbolic-manipulation reasoning was required to obtain full marks. Interpretation of graphical information may be a problem for some students Choosing to use a graphing calculator was not associated in general with higher or lower marks than traditional alternatives. Initial data supports no differences between boys and girls in overall performance.

Guin, D., & Trouche, L. (1998). "The complex process of converting tools into mathematical instruments: The case of calculators" *International Journal of Computers for Mathematical Learning, 3(3): 195-227*	Interpretive study Two classes of French students One class of 15/16 year olds and one class of 17/18 year olds Student questionnaires focusing on mathematical tasks and their relationship to the calculators Students' written research reports Observation of groups during the research phase Interviews of selected students	Students gained awareness of potential and limitations of the calculator. A real change in most students in their relation to mathematics and their own self-confidence was observed. Theoretical and rational type students construct a more efficient relationship with the calculator while keeping a certain objectivity with regard to the machine, but those with a mechanical profile who may be particularly attached to the calculator rarely achieve the same level of instrumental genesis. The instrumentation process (becoming skilled at using the calculator effectively) is slow and complex, because it requires sufficient time to achieve a reorganization of procedures, even for the better students who have established a relationship with the machine.
Hennessy, S., Fung, P., & Scanlon, E. (2001). "The role of the graphic calculator in mediating graphing activity" *International Journal of Mathematical Education in Science and Technology, 32: 267-290*	Interpretive/survey and case study 233 questionnaires were mailed to students at Open University in Great Britain with 55 returned; students were enrolled in an entry-level math distance learning course (described as appropriate for students with limited or rusty mathematical knowledge). 67 questionnaires were sent to tutors with 48 returned. (Tutors are individuals who monitor, support, and assess student progress in the course) In the case study three pairs of students were observed; one pair was reported on. One pair of volunteers participated in a follow up case study. TI—80	From the survey Students developed a positive attitude toward the calculator and mathematics. The students and tutors also mentioned visualization, speed, labor-saving, and quick graphs Majority of the students and tutors believed that the calculator played a role in developing links between representations. The two most critical features of this role were visualization and immediate feedback. Tutors expressed concern about calculator dependence. Both students and tutors believed that the graphing calculator plays a role in the students' development of links between representations The calculator encouraged collaboration in problem solving. The survey and the case study support the conclusion that graphing calculators facilitated graphing using visual representation, by making the process less time-consuming, and by encouraging translation.

Hong, Y., Toham, M., and Kiernan, C. (2000). "Supercalculators and university entrance calculus examinations" *Mathematics Education Research Journal, 12(3): 321-336*	Comparative, matched-pairs design (t-test) 17 and 18 year old students from 2 secondary-level schools in New Zealand Experimental students received 4 one-hour training sessions with a researcher on TI-92 calculator procedures. Control students did not. All students were allowed to use a standard graphing calculator on the exam. Participants were also divided into high and low achieving groups based on their pre-test and again compared Scores on pre-test and post-test, and a calculator neutral test composed of items from the 1992 through 1997 Bursary Mathematics with Calculus university entrance exam A correlation of the Bursary Examination scores with the Calculator Test scores was performed. TI-92	Students who are proficient in using calculators like the TI-92 could expect to gain an advantage of approximately 10% on calculus examinations of the type currently set for university entrance in New Zealand. The supercalculator appears to have been most beneficial to students who are lower achieving on standard tests and examinations, but in this study little conceptual gain was evident. It is possible to write a calculator-neutral examination where symbolic calculators have minimal influence. There are some difficulties associated with the use of a complex calculator, such as using an incorrect syntax for formula entry, leading to an incorrect answer, and the difficulties of accessing correct sequences of key presses.
Keller, B. A., & Hirsch, C. R. (1998). "Student preferences for representations of functions" *International Journal of Mathematical Education in Science and Technology, 6: 191-207*	Comparative quasi-experimental (Chi-Square) 131 first semester calculus students at Western Michigan University Treatment: calculator used in class and text supplemented with calculator activities Control: calculator not used Representational Preference test Pretest (precalculus in contextualized and	Most students prefer to use equations on non-contextualized mathematics tasks. On pretest they preferred to use the table, and on the posttest they preferred to use the graph. Students in the treatment group were more likely to have a graphical preference than students in the control group. Use of technology diminished differences in preferences for representations in particular problem contexts.

	non-contextualized settings) Posttest (precalculus and calculus questions in contextualized and non-contextualized settings) Graphing calculator (model not specified)	
Lauten, A. D., Graham, K., & Ferrini-Mundy, J. (1994). Student understanding of basic calculus concepts: Interaction with the graphics calculator *Journal of Mathematical Behavior, 13(2): 225-237*	Interpretive study 5 college calculus students and 2 local high school students randomly selected from volunteers Task based interviews Graphing calculator (model not specified)	Students accepted graphical representation of functions not defined by a rule. When asked to give an example of a function meeting specific conditions, they search for an equation. Students were willing to deal with piecewise functions when aided by the technology. Students tend to use the graphing calculator and the trace function to find specific function values. The dynamic notion of function may be the basis of students' understanding of function and limit.
Mitchelmore, M., & Cavanagh, M. (2000). "Students' difficulties in operating a graphics calculator" *Mathematics Education Research Journal, 12(3): 254-268*	Interpretive study 25 Australian 10th and 11th graders 15 10th graders, 8 girls and 7 boys 10 11th graders, 5 girls and 5 boys 15 were in a year 10 advanced course 5 were in a year 11 three unit course 5 were in the International Baccalaureate Mathematics HL course 3 fifty minute interviews, each two weeks apart where participants attempted at least 7 of 8 tasks If students did not reach one of the critical points during the interview, they were prompted to perform further calculator operations that led to that point.	Technical Difficulties with scale are acerbated by the lack of understanding of the technology—particularly how the calculator assigns pixels to graphs. Poor understanding of the zooming and scaling functions gave students problems with scale. Most disassociated the zoom operation from any change in the scale of the graphs they saw. Mathematical In developing mathematical accuracy and approximation, students demonstrated little understanding of the differences between decimal representations of rational and irrational numbers. Students failed to link graphs and symbolic representations. Many seemed to accept the visual image on the calculator screen without considering the context of the task.

	The students were asked to think aloud and explain the processes they used to interpret the graphs they saw, but errors were not corrected. Casio fx-7400G	
Ruthven, K. (1990). “The influence of graphic calculator use on translation from graphic to symbolic forms “ *Educational Studies in Mathematics, 21(5): 431-450*	Comparative (F-test) A convenience sample of 87 students in two-year advanced level mathematics courses (academic upper secondary) in the U. K. Treatment: 47 in the project with calculator access for 2 years Control: 40 students without calculators (7 students had their own calculators) Two part questionnaire: Part 1 was used to collect background information, and part 2 was a 12-graphic item test Casio graphing calculator fx 7000	Students in the treatment group performed significantly better than control at generating symbolic models for a given graph. On symbolization items, students with graphing calculators outperformed students without them. Females in the treatment group outperformed males in the treatment group. Males in the control group outperformed females. On graph interpretation problems, there were no significant differences in performance between students with calculators and students without them. On these items, males in each group outperformed the females in their group.
Schwarz, B. B., & Hershkowitz, R. (1999) “Prototypes: Brakes or levers in learning the function concept? The role of computer tools” *Journal for Research in Mathematics Education: 30(4): 362-389*	Interpretive and Comparative 3 classes of 9^{th} grade students in Israel Treatment: graphing calculators or multi-representational software used in 1 of 3 weekly lesson throughout school year Control: 2 classes used different curricular materials and no technology MANOVA on test results from three part questionnaire test (items grouped to assess knowledge in prototypes, part-whole reasoning, representations) Prototypes are functions that students	Students in treatment group performed better than the control group on the questionnaire test ($F(1,101)=7.66$, $p<.001$), particularly on the prototype section ($F(1,101) = 7.10$, $p=.009$) Students in the treatment group provided a better interpretation of a given graphical attribute and related it less to particular functions. Students in the treatment group justified significantly more and took the context of the problem into consideration more. Students in the treatment group were able to select appropriate representations among representations and were better at recognizing graphs from partial graphs.

	predict can pass through points plotted on an XY plane (e.g. a line is a prototype). Part-whole reasoning refers to students' ability to generate possible graphs, given a part of the graph. Graphing calculator (model not specified)	

QUESTION 3: What mathematical knowledge and skills are learned by students who use hand-held graphing technology?

Study	Methodology, Sample size, Variables, Technology	Findings
Adams,T. L. (1997). "Addressing students' difficulties with the concept of function: Applying graphing calculators and a model of conceptual change" *Focus on Learning Problems in Mathematics, 19(2): 43-57*	Comparative 128 community college students 2 classes (Casio 7000G and assignment) 2 classes (Casio 7000G only) 2 classes (assignment only) 2 classes control Analysis of Covariance (ANCOVA) on data from: Conceptual Change Assignment (CCA) Domain/Range/Scale Instrument (DRSI). Used as a pre and posttest Identification/Construction/Definition Instrument (ICDI). Used to determine students' concept image and definition of function	There were significant differences between the calculator only group and both the calculator and assignment group and the control group on the DRSI. The use of graphing calculators and the conceptual change assignment significantly improved the students' concept of function regarding application of the concepts of domain and range and the selection of appropriate dimensions for the rectangular coordinate system for graphing functions.
Connors and Snook (2001) "The effects of hand-held CAS on student achievement in a first year college core calculus sequence" *The International Journal of Computer Algebra in Mathematics Education, 8(2)-99-114*	Comparative (z-test) (The data were compared after the study was done) 100 final exams selected from two calculus courses for two successive years selected from exams of all students completing the courses at the United States Military Academy. Treatment TI-89 hand-held Computer Algebra System (CAS) Control used HP48G 20 items on the Departmental Final Exam 6 procedural 8 conceptual 6 application	The mean scores of the treatment group on 8 of the questions/subquestions (2 procedural, 1 conceptual, 5 application) were significantly higher than the control group.

Forster, P. (2000). "Process and object interpretations of vector magnitude mediated by use of the graphics calculator" *Mathematics Education Research Journal, 12(3): 269-285*	Interpretive 1 of 18 women in a Year 11 mathematics class Videotapes of 15 classes Audio tapes of 10 of the 15 lessons HP-38	Author discussed participant's (Jenny) understanding of vector magnitude developed from its introduction in class. Participant used the calculator ABS([I,J]) function and magnitude, M, as equivalent and separated from the Pythagorean procedure that originally linked I, J, and M. Participant frequently used visual methods when problem solving. She interpreted real life problems according to their contexts. Calculator was used for calculation, to check results, and to explore new ideas.
Forster, P. A., & Taylor, P. C. (2000) "A multiple-perspective analysis of learning in the presence of technology" *Educational Studies in Mathematics, 42: 35-59*	Interpretive (case study) 1 of 18 women in a Year 11 mathematics class in Australia Videotapes of 15 classes Audio tapes of 10 of the 15 lessons Student work 3 assessment items HP-38	The authors describe one student's (Katie) concept of the direction of a vector developed using a graphing calculator. Critical factors influencing the development of the concept are also suggested. The student viewed an angle as a rotation but viewed the direction as a generalized equation in her calculator: ARG([I, J]) = θ. The student spontaneously used the solver in her calculator to solve for I given J and θ. Important factors in the student's development were interactions with the teacher, her peers, the technology, and the student's ability to reflect on her own learning.
Graham, A. T., & Thomas, M. O. J. (2000). "Building a versatile understanding of algebraic variables with a graphic calculator" *Educational Studies in Mathematics, 41(3): 265-282*	Comparative (t-test) 189 year 9 and 10 (13-14 year olds) in New Zealand) 147 treatment (6 classes in different schools volunteered) 42 control (2 classes) Treatment taught 'Tapping into Algebra' with graphing calculators 68 question pre-test and post-test TI-82 or Casio FX7700GH	In the two schools with control groups, students who used the graphic calculator and the 'Tapping into Algebra' curriculum had significantly better performance on the post-test. For the controlled classes 9 questions, identified as requiring a higher level of understanding and on which the students with calculators performed significantly better, required an understanding of letter as generalized number or specific unknown. Students in the treatment group did as well in terms of manipulative skills as the control group.

Harskamp, E. G., Suhre, C. J. M., & van Streun,A. (1998). “The graphics calculator in mathematics education: An experiment in the Netherlands” *Hiroshima Journal of Mathematics Education, 6: 13-31*	Comparative 12 classes of pre-university students (age 15-18 years) in the Netherlands 3 classes used the calculator all year 5 classes used the calculator for 1 topic 4 classes did not use the calculator Topics included functions and graphs, change and derivatives, exponential functions and trigonometric functions Pretest about functions Observations of teaching behavior Interviews with students and teachers Posttest on knowledge of functions and graphs, derivatives, exponential, and periodic functions Students’ solution strategies TI-81	Students have a larger repertoire of solution methods when the graphing calculator is used in instruction. Weaker students benefit particularly from using a graphing calculator. Graphing calculator without such a device. Teachers in experimental group 2 (the ones who used the calculator on only one topic) reported that students were, for the most part, unable to balance algebraic solutions and graphical solutions. Teachers in group 1 did not express this concern.
Harskamp, E. G., Suhre, C. J. M., & van Streun,A. (2000). “The graphics calculator and students' solution strategies” *Mathematics Education Research Journal, 12(1): 37-52*	Comparative 12 classes of grade 10 students from 7 Dutch upper secondary schools. 269 students age 15-17 Treatment 1: 3 classes used calculators all year Treatment 2: 5 classes used calculators for 1 topic (2 months) Control: 4 classes did not use calculators Pretest (6 items) Posttest (19 open ended items) Students’ solution strategies Teacher behavior scored every 10 seconds Teacher interviewed on organization and usefulness of the calculator TI-81	There were no significant differences on students’ performance on either the pretest or the posttest. Use of the graphing calculator did not produce improved performance but led to a different distribution of solution strategies. Students in the treatment classes with very low pretest scores made substantial gains on the posttest. Teachers in both treatment groups spent about the same amount of time during lessons on problem-solving activities and instruction using tables and graphs and much more time than the teachers in the control groups Teachers in the control groups followed the textbook strictly

Hollar, J. C., & Norwood, K. (1999). "The effects of a graphing-approach intermediate algebra curriculum on students' understanding of function" *Journal for Research in Mathematics Education*	Comparative quasi-experimental Students in four sections of intermediate algebra at a large state university Treatment: (n = 46) Used calculators and a text that included calculator activities Control: (n = 44) Used a text that did not include calculator activities, and calculators were not used O'Callaghan function test without calculators used as pre- and posttest Departmental final exam (50 questions) Revised Mathematics Attitude Scale before and after treatment TI-82	Treatment classes had significantly better understanding of function than the control classes There was no difference in the mean scores of the two groups on the departmental final exam. There was no difference in the attitudes of the two groups.
Hong, Y., Toham, M., & Kiernan, C. (2000). "Supercalculators and university entrance calculus examinations" *Mathematics Education Research Journal, 12(3): 321-336*	Comparative (matched-pairs design, t-test) 17 and 18 year old students from two secondary-level schools in New Zealand Experimental students received 4 one-hour training sessions with a researcher on TI-92 calculator procedures. Control students did not. All students were allowed to use a standard graphing calculator on the exam. Participants were also divided into high and low achieving groups based on their pre-test and again compared Scores on pre-test and post-test, and a calculator neutral test composed of items from the 1992 through 1997 Bursary Mathematics with Calculus university entrance exam A correlation of the Bursary Examination scores with the Calculator Test scores was performed. TI-92	The treatment group outperformed the control group on the posttest suggesting that students with training in the use of CAS have an advantage on calculator positive exams. The mean scores of low and high achievers' were significantly different on every test indicating that the calculator may help low achieving students answer more questions; however researchers note these students exhibited little conceptual gain. Students demonstrated difficulty with syntax for formula entry leading to incorrect answers.

Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Sangtong, J., & Fey, J. T. (2000). "Effects of standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand" *Journal for Research in Mathematics Education, 31(3): 328-361*	Comparative 6 sites volunteered; each identified a control in their area Approximately 270 control and 320 treatment Treatment: students completing third year of Core-Plus Control: students of comparable ability in traditional advanced algebra courses Teacher interviews Standardized test scores for students as eighth graders 3 paper and pencil tests 1. emphasized contextual problems (calculators allowed) 2. context free symbolic manipulation (no calculators) 3. open-ended contextual problems (calculators permitted) TI-83	Student scores were generally low in both treatment and control groups on all types of algebra items. Treatment did better on algebraic tasks embedded in applied problem contexts when graphing calculators were available, and controls did better on traditional symbol manipulation tasks. CPMP did better on multiple representation of algebraic ideas (tables, graphs symbols) requiring representational fluency, but all items had mean scores below 33%. Suggest that correlation between symbol manipulation and problem solving is weak.
Keller and Hirsch (1998) "Student preferences for representations of functions" *International Journal of Mathematical Education in Science and Technology, 29(1): 1-17*	Comparative (Chi-Square) 131 first semester calculus students at Western Michigan University Treatment: calculator used in class and text supplemented with calculator activities Control: calculator not used	Most students prefer to use equations on non-contextualized mathematics tasks. On pretest they preferred to use the table, and on the posttest they preferred to use the graph. Students in the treatment group were more likely to have a graphical preference than students in the control group.

	Representational Preference test Pretest (precalculus in contextualized and non-contextualized settings) Posttest (precalculus and calculus questions in contextualized and non-contextualized settings) Information interviews Graphing calculator (model not indicated)	Use of technology diminished differences in preferences for representations in particular problem contexts.
Keller, B. A., & Russell, C. (1997). "Effects of the TI-92 on calculus students solving symbolic problems" *International Journal of Computer Algebra in Mathematics Education, 4: 77-97*	Comparative Students in the first and second semester calculus classes at Iowa State University Treatment: Students who self-selected into 1 section Fall (n=32), 2 sections Spring (n=30, n=17) used TI-92 with CAS Control: 8 sections during Fall and 12 sections during Spring. Used HP48G or TI-85 Departmental final exam (10 short answer and 7 long answer questions) TI-92; HP48G ; TI-85	The treatment group outperformed the control group even on problems for which the technology was not helpful. Treatment students were more likely to produce a correct solution when they used a correct strategy. Treatment students attempted more problems. Treatment groups during the second semester asked questions about how the calculator found integrals.
Keller, B. A., Russell, C., & Thompson, H. (1999). "A large-scale study clarifying the roles of the TI-92 and instructional format on students success in calculus" *International Journal of Computer Algebra in Mathematics Education, 6: 191-207*	Comparative 687 College calculus students in the United States with extensive data for 534 students completing the course. 10 treatment and 10 control sections Student performance data was collected from department mandated diagnostic pretest (no calculators), final exam, and course grade. Analyzed drop rates for treatment and control. Treatment instructors used cooperative learning, laboratory experiments, variety of discussion formats; control used lecture-based and "traditional" teaching methods.	"This study replicated a result of the previous study (Keller & Russell, 1997) in that the TI-92 students independent of instructional format performed better on the directive and multiple step questions of the final exam in comparison to students using other calculators. The TI-92 students more successfully completed one-step problems and put combinations of steps together to solve intermediate problems. In particular, on problems with multiple steps, TI-92 students across all groups performed at a greater success rate on eight of nine problems" (p. 203). Use of student centered instructional techniques appears to have had a positive effect on performance. "However, the improved performance of students using a TI-92 on more complex problems found in the previous study was not confirmed in this study" (p. 203).

	All students had graphing calculators; project students had calculators with CAS TI-92, TI-85, or HP-48G	
Kendal, M., & Stacey, K. (1999). "Varieties of teacher privileging for teaching calculus with computer algebra systems" *International Journal of Computer Algebra in Mathematics Education, 6: 233-247*	Comparative/interpretive 3 volunteer calculus teachers and their high school students (59 students, approximately 17 years of age) in Australia. Student questionnaires, problems, and log sheets Written test (items classified as core, symbolic, and option) Students' use of calculator on test Task based interviews (17 students) TI-92 with CAS	Written test scores were similar, but use of technology varied. Students' use of the calculator appeared to depend on how the students were taught. "One class frequently used CAS for algebraic solutions while the second with better algebraic skills preferred by-hand algebraic techniques. The third group of students used CAS more selectively, displayed greater competency with graphical solutions and demonstrated good understanding build from illustrating algebraic ideas graphically" (p. 233).
Lauten, A. D., Graham, K, & Ferrini-Mundy, J. (1994). "Student understanding of basic calculus concepts: Interaction with the graphics calculator" *Journal of Mathematical Behavior, 13(2): 225-237*	Interpretive 5 college calculus students and 2 local high school students randomly selected from volunteers. Study conducted in the United States Task based interviews Graphing calculator (model not specified)	When asked to give an example of a function meeting specific conditions, students search for an equation. Students were willing to deal with piecewise functions when aided by the technology. Students tend to use the graphing calculator and the trace function to find specific function values. The dynamic notion of function may be the basis of students' understanding of function and limit.
Merriweather, M., & Tharp, M. L. (1999). "The effect of instruction with graphing calculators on how general mathematics students naturalistically solve algebraic problems" *Journal of Computers in Mathematics and*	Comparative (matched pairs) 80 eighth grade general math students from a Virginia middle school Treatment: 52 students used calculators to solve algebraic equations for 2 weeks Control: 28 students	Few of the students interviewed used the graphing calculator to solve an algebraic word problem and none used graphing as a solution method. 76% of the control group felt that it was important for everyone to learn how to use a graphing calculator compared with 55% of the treatment group.

Science Teaching, 18(10: 7-22	Pre and post surveys to determine attitudes 16 (12 treatment and 4 control) participated in an in-depth interview TI-82	4% of the control group felt that it is important to know how to do a process and not why it works, compared with 27% of the treatment group.
Quesada, A. R., & Maxwell, M. E. (1994). "The effects of using graphing calculators to enhance college students' performance in precalculus" *Educational Studies in Mathematics, 27(2): 205-215*	Comparative 710 students from a precalculus course at a large university during 3 semesters Treatment: (5 sections) Used textbook designed to be used with a graphing calculator and calculator and a more interactive classroom presentation Control: (8 sections) traditional mode and scientific calculators Survey of students perceptions Final Exam with 10 multiple choice and 10 open-ended questions TI-81 and Casio G-7000	Students who used the graphing calculator believed they had done more exploration and the calculator helped them understand the concepts they studied. Students indicated that the calculator saves time on calculations and helps them check their answers. Their primary concern was that they would become too dependent on it. On the final exam the treatment groups outperformed the control groups. Students in the treatment sections obtained considerably higher scores on four categories of questions (about properties of functions, graphs, word problems, and equations) and only slightly higher scores on the multiple choice questions.
Ruthven, K. (1990). "The influence of graphic calculator use on translation from graphic to symbolic forms" *Educational Studies in Mathematics, 21(5): 431-450*	Comparative A convenience sample of 87 students in two-year advanced level mathematics courses (academic upper secondary) in the U. K. Treatment: 47 in the project with permanent calculator access for 2 years during instruction and on the test. Control: 40 students without calculators (7 students had their own calculators) Two part questionnaire: Part 1 was used to collect background information and part 2 was a 12-	Students in the treatment group performed significantly better than control at generating symbolic models for a given graph. On symbolization items, students with graphing calculators outperformed students without them. Females in the treatment group outperformed males in the treatment group. Males in the control group outperformed females. On graph interpretation problems, there were no significant differences in performance between students with calculators and students without them. On these items, males in each group outperformed the females in their group.

	graphic item test covering topic areas central to any advanced level course and where the use of graphs is normal practice. Graphing calculator fx 7000	
Schwarz, B. B., & Hershkowitz, R. (1999). "Prototypes: Brakes or levers in learning the function concept? The role of computer tools" *Journal for Research in Mathematics Education, 30(4): 362-389*	Comparative 3 classes of 9th grade students in Israel Treatment: calculators 1 of 3 weekly lesson Control: 2 classes used different curricular materials and no technology Three part questionnaire (prototypes, part-whole reasoning, representations) Graphing calculator (model not specified)	Students in the treatment group provided a better interpretation of a given graphical attribute and related it less to particular functions. Students in the treatment group justified significantly more and took the context of the problem into consideration more. Students in the treatment group were able to select appropriate representations among representations and were better at recognizing graphs from partial graphs.
Slavit, D. (1998) "Three women's understanding of algebra in precalculus course integrated with the graphing calculator" *Journal of Mathematical Behavior, 17(3): 303-389*	Interpretive (Case Studies) 3 students in an experimental precalculus course at a Mid-Atlantic University Four task based interviews (45 minutes each) to investigate knowledge of function, representations of functions, and attitude Self-paced tests – symbolically oriented In-class tests and quizzes –graphically oriented	Students think using symbols and given a choice solve problems using symbolic procedures more often than graphic ones despite instructional emphasis on the later. Students used the graphing calculator only on graphical tasks, hindering possible connections between symbolic representations of functions and their graphs.

Thompson, D. R., & Senk, S. L. (2001). "The effects of curriculum on achievement in Second-Year Algebra: The example of the University of Chicago School Mathematics Project" *Journal for Research in Mathematics Education, 32: 58-84*	Comparative Experimental (Matched Pairs) Students in 2nd year algebra at four different volunteer schools Treatment: (n=150) classes used University of Chicago School Mathematics Project (UCSMP) Advanced Algebra Control: (n=156) classes used traditional texts Pretest: algebra and geometry) Posttest 1: multiple choice, 2nd year algebra concepts Posttest 2: problem solving Survey of attitudes Graphing calculator (model not specified)	The use of graphing technology did not negatively affect the ability of the UCSMP students to deal with procedural skills. UCSMP students out performed comparison students on multi-step problems and problems with applications or graphical representations. Groups had comparable performance on items testing algebraic skills.
Van Streun, C., Harskamp, E., & Suhre, C. J. M. (2000). "The effect of the graphic calculator on students' solution approaches: A secondary analysis." *Hiroshima Journal of Mathematics Education*, 8: 27-39.	Comparative 12 classes of grade 10 students from 7 Dutch upper secondary schools. 269 students age 15-17 Treatment 1: 3 classes used calculators all year Treatment 2: 5 classes used calculators for 1 topic (2 months) Control: 4 classes did not use calculators Pretest (6 items) Posttest (19 open ended items) Students' solution strategies Teacher behavior scored every 10 seconds Teacher interviewed on organization and usefulness of the calculator TI-81	Students in both treatment groups used the graphic approach more often than students in the control group, but students who use the handheld graphing technology for the whole year used the graphic approach more than either group. Compared to students in the control group, students who used the calculator for a short period of time developed graphic solutions and showed a decrease in algorithmic and heuristic strategies. However, students who used the calculator for a short period of time did not outperform the control group students on the posttest. "In general, students mathematical achievement improves only after a prolonged period of use" (p. 37) of handheld graphing technology. Short-term use is insufficient to establish understanding.

Zbiek, R. M. (1998). "Prospective teachers' use of computing tools to develop and validate functions as mathematical models" *Journal for Research in Mathematics Education, 29(2): 184-201*	Interpretive One class of 13 pre-service teachers Researcher designed materials and taught course Open ended modeling tasks Instructor's notes Audio tapes of class interactions TI-81	Identified four different strategies for modeling using calculator. Various knowledge and skills were used ranging from interpretation of graphs and integration of algebraic, graphical, mathematical formulas to make predictions. Strategies ranged from complete dependence on the calculator to select and validate a model to ignoring the calculator. Strategy use varied and depended on the mathematical and real world knowledge of the user. Students used their understanding of functions and graphs to adapt models generated by the calculator. In situations where students had personal experience with a problem context, they used their knowledge and the calculator to select a model. Students relied on the goodness of fit values provided by the calculator in most situations.

Question 4: What is gained mathematically by students using hand-held technology that cannot be observed in a non-technology environment?		
Study	Methodology, Sample Size, Variables, Technology	Findings
Adams, T. L. (1997). “Addressing students' difficulties with the concept of function: Applying graphing calculators and a model of conceptual change” *Focus on Learning Problems in Mathematics, 19(2): 43-57*	Comparative 128 community college students 2 classes (Casio 7000G and assignment) 2 classes (Casio 7000G only) 2 classes (assignment only) 2 classes control Analysis of Covariance (ANCOVA) on data from: Conceptual Change Assignment (CCA) Domain/Range/Scale Instrument (DRSI). Used as a pre and posttest Identification/Construction/Definition Instrument (ICDI). Used to determine students’ concept image and definition of function Casio 700G	There were significant differences between the calculator only group and both the calculator and assignment group and the control group on the DRSI. The use of graphing calculators and the conceptual change assignment significantly improve the students' concept of function regarding application of the concepts of domain and range and the selection of appropriate dimensions for the rectangular coordinate system for graphing functions.
Drijvers, P. & Doorman, M. (1996). “The graphics calculator in mathematics education.” *Journal of Mathematical Behavior, 15: 425-440*	Interpretive Several classes of pre-university (18 year old) students in The Netherlands Data collected over several years. Experimental instructional material, observations of the lessons; reflections based on observations were used to analyze affects of graphing calculator use in a developmental classroom	The graphing calculator supports the use of realistic data, and seems to stimulate many students towards exploratory activity, though mathematical level tends to be a confounding variable here. Integration of geometric and algebraic models did not occur without a struggle. However, students eventually seemed to accept the intertwining of the two. Graphing calculators removed the students' inhibitions to making quick sketches and with direct feedback encouraged students to reflect on what they had done.

	Graphing calculator (model not specified)	Graphing calculators seemed to encourage a shift from rigid techniques to more flexible solutions. Students also developed a more critical attitude toward numerical results.
Drijvers, P. (2000). "Students encountering obstacles using a CAS" *International Journal of Computers for Mathematical Learning, 5(3): 189-209*	Interpretive 22 Dutch students in a pre-examination class studying advanced pre-university level mathematics-8 females, 14 males-all about 17 years old. Video taping of the viewscreen of one pair of students who used the calculator in class Classroom observations and interviews Questionnaire (not provided) Pre and post-tests (analysis not described) Data were interpreted and scored by means of classification in different categories, with reliability checked by the three observers. TI—92	Using computer algebra in mathematics education was identified with reference to the theory of Realistic Mathematics Education and shown to have become somewhat of a reality. The top-down character of a CAS, its black-box style and its idiosyncrasies of syntax produced obstacles during the performance of instrumentation schemes and during the interpretation of the results.
Drijvers, P. & van Herwaarden, O. (2000). "Instrumentation of ICT-tools: The case of algebra in a computer algebra environment" *International Journal of Computer Algebra in Mathematics Education, 7(4): 255-276*	Interpretive Two classes of about 25 14-15 year old Dutch students who were preparing for the highest, pre-university level of upper secondary education. Classroom observations and transcribed audio recordings Field notes on the observation forms that had been developed Written work of the students	When reification (treating the isolated expression as an object) has yet to occur, difficulties of student use of isolation-substitution schemes were observed for students using CAS. When parameters were part of functions being used, students without reification had particular difficulty. The computer algebra device often requires that mathematical conceptions have been made abstract by means of vertical mathematization before representations of the concepts can be entered into the technological device. The process of instrumentation, relating mathematical concepts and technical skills, was not easy for many students. The technical errors that students made seem to be related to limitations in the understanding of the mathematical concepts

	Teachers and students evaluation of the teaching sequence Pre-test/Post-test Data were coded according to previously defined categories No further information about the pre/post tests was given TI—89	involved. The use of parameters may complicate this situation.
Farrell (1996) "Roles and behaviors in technology-integrated precalculus classrooms" *Journal of Mathematical Behavior, 15(1): 35-53*	Interpretive Six teachers and their corresponding precalculus classes. Modified Systematic Classroom Analysis Notation (SCAN) Teachers videotaped 10 consecutive non-testing lessons. Two observers and the investigator coded the videotapes using the SCAN instrument. Graphing calculator (model not specified)	No one classroom exhibited on a daily basis the examined behaviors of discussion, non-routine problem solving, open investigation, and student-teacher partnerships. Though time spent in discussion was no different, sophisticated discussion about mathematics did take place, and non-routine problem solving was observed. Some classrooms had carefully guided investigations, others had open investigations when the teacher would follow students leads.
Forster, P. A. (2000). "Process and object interpretations of vector magnitude mediated by use of the graphics calculator" *Mathematics Education Research Journal, 12(3): 269-285*	Interpretive 1 of 18 women in a Year 11 mathematics class Videotapes of 15 classes Audio tapes of 10 of the 15 lessons HP-38	Author discussed participant's understanding of vector magnitude developed from its introduction in class. Participant used the calculator ABS([I,J]) function and magnitude, M, as equivalent and separated from the Pythagorean procedure that originally linked I, J, and M. Participant frequently used visual methods when problem solving. She interpreted real life problems according to their contexts. Calculator was used for calculation, to check results, and to

		explore new ideas.
Graham, A. T., & Thomas, M. O. J. (1998). "A graphic calculator approach to algebra" *Mathematics Teaching, 167: 34-27*	Comparative (t-test) 189 year 9 and 10 (13-14 year olds) in New Zealand) 147 treatment (6 classes in different schools volunteered) 42 control (2 classes) Treatment taught 'Tapping into Algebra' with graphing calculators 68 question pre-test and post-test TI-82 or Casio FX7700GH	In the two schools with control groups students who used the graphic calculator and the 'Tapping into Algebra' curriculum had significantly better performance on the post-test. For the controlled classes 9 questions, identified as requiring a higher level of understanding and on which the students with calculators performed significantly better, required an understanding of letter as generalized number or specific unknown. Students in the treatment group did as well in terms of manipulative skills as the control group.
Guin, D., & Trouche, L. (1998). "The complex process of converting tools into mathematical instruments: The case of calculators" *International Journal of Computers for Mathematical Learning, 3(3): 195-227*	Interpretive Two classes of French students One class of 15/16 year olds and one class of 17/18 year olds Student questionnaires focusing on mathematical tasks and their relationship to the calculators Students' written research reports Observation of groups during the research phase Interviews of selected students	Students gained an awareness of potential and limitations of the calculator. A real change in most students in their relation to mathematics and their own self-confidence was observed. Theoretical and rational type students construct a more efficient relationship with the calculator while keeping a certain objectivity with regard to the machine, but those with a mechanical profile who may be particularly attached to the calculator rarely achieve the same level of instrumental genesis. The instrumentation process (becoming skilled at using the calculator effectively) is slow and complex because it requires sufficient time to achieve a reorganization of procedures, even for the better students who have established a relationship with the machine. .
Harskamp, E. G., Suhre, C. J. M., & van Streun, A. (1998). The graphics calculator in mathematics education: An experiment in the Netherlands *Hiroshima Journal of Mathematics*	Comparative 16-17 year old pre-university students Pretest Posttest	Teaching practices to provide for more problem solving activities will not develop spontaneously Students have a larger repertoire of solution methods when the graphing calculator is used in instruction. Weaker students benefit particularly from using a graphing

Education, 6: 13-31	Teacher observations	calculator.
Harskamp, E. G., Suhre, C. J. M., & van Streun, A. (2000). "The graphics calculator and students' solution strategies" *Mathematics Education Research Journal, 12(1): 37-52*	Comparative quasi-experimental 12 classes of grade 10 students from 7 Dutch upper secondary schools. 269 students age 15-17 Treatment 1: 3 classes used calculators all year Treatment 2: 5 classes used calculators for 1 topic (2 months) Control: 4 classes did not use calculators Pretest (6 items) Posttest (19 open ended items) Students' solution strategies Teacher behavior scored every 10 seconds Teacher interviewed on organization and usefulness of the calculator TI-81	There were no significant differences on students' performance on either the pretest or the posttest. Use of the graphing calculator did not produce improved performance but led to a different distribution of solution strategies. Students in the treatment classes with very low pretest scores made substantial gains on the posttest. Teachers in both treatment groups spent about the same amount of time during lessons on problem-solving activities and instruction using tables and graphs and much more time than the teachers in the control groups Teachers in the control group followed the textbook strictly.
Hennessy, S., Fung, P., & Scanlon, E. (2001). "The role of the graphic calculator in mediating graphing activity" *International Journal of Mathematical Education in Science and Technology, 32: 267-290*	Interpretive/survey and case study 233 questionnaires were mailed to students at Open University in Great Britain with 55 returned; students were enrolled in an entry-level math distance learning course (described as appropriate for students with limited or rusty mathematical knowledge). 67 questionnaires were sent to tutors with 48 returned. (Tutors are individuals who monitor, support, and assess student progress in the course) Low return on student survey attributed to student drop out from course. In the case study three pairs of students were observed; one pair was reported on.	From the survey Students developed a positive attitude toward the calculator and mathematics. The students and tutors also mentioned visualization, speed, and labor-saving, and quick graphs as advantages of the calculator. Majority of the students and tutors believed that the calculator played a role in developing links between representations. The two most critical features of this role were visualization and immediate feedback. The tutors expressed concern about calculator dependence. Both students and tutors believed that the graphing calculator plays a role in the students' development of links between representations. The calculator encouraged collaboration in problem solving.

	One pair of volunteers participated in a follow up case study. TI—80	The survey and the case study support the conclusion that graphing calculators facilitated graphing using visual representation, by making the process less time-consuming, and by encouraging translation.
Hollar, J. C., & Norwood, K. (1999). "The effects of a graphing-approach intermediate algebra curriculum on students' understanding of function" *Journal for Research in Mathematics Education*	Comparative quasi-experimental Students in four sections of intermediate algebra at a large state university Treatment: (n = 46) Used calculators and a text that included calculator activities Control: (n = 44) Used a text that did not include calculator activities, and calculators were not used O'Callaghan function test without calculators used as pre- and posttest Departmental final exam (50 questions) Revised Mathematics Attitude Scale before and after treatment TI-82	Treatment classes had significantly better understanding of function than the control classes There was no difference in the mean scores of the two groups on the departmental final exam. There was no difference in the attitudes of the two groups.
Hong, Y., Toham, M., & Kiernan, C. (2000). "Supercalculators and university entrance calculus examinations" *Mathematics Education Research Journal, 12(3): 321-336*	Comparative, matched-pairs design (t-test) 17 and 18 year old students from 2 secondary-level schools in New Zealand Experimental students received 4 one-hour training sessions with a researcher on TI-92 calculator procedures. Control students did not. All students were allowed to use a standard graphing calculator on the exam. Participants were also divided into high and low achieving groups based on their pre-test and again compared. Scores were collected on pre-test and post-test,	Students who are proficient in using calculators like the TI-92 could expect to gain an advantage of approximately 10% on calculus examinations of the type currently set for university entrance in New Zealand. The supercalculator appears to have been most beneficial to students who are lower achieving on standard tests and examinations, but in this study little conceptual gain was evident. It is possible to write a calculator-neutral examination where calculators such as the TI-92 have minimal influence. There are some difficulties associated with the use of a complex calculator such as the TI-92, such as using an incorrect syntax for formula entry, leading to an incorrect

	and a calculator neutral test composed of items from the 1992 through 1997 Bursary Mathematics with Calculus university entrance exam A correlation of the Bursary Examination scores with the Calculator Test scores was performed. TI-92	answer, and the difficulties of accessing correct sequences of key presses.
Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Sangtong, J., & Fey, J. T. (2000). "Effects of standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand" *Journal for Research in Mathematics Education, 31(3): 328-361*	Comparative quasi-experimental 6 sites volunteered, each identified a control in their area Approximately 270 control and 320 treatment Treatment: students completing third year of Core-Plus Control: students of comparable ability in traditional advanced algebra courses Teacher interviews Standardized test scores for students as eighth graders 3 paper and pencil tests 4. emphasized contextual problems (calculators allowed) 5. context free symbolic manipulation (no calculators) 6. open-ended contextual problems (calculators permitted) TI-83	Student scores were generally low in both treatment and control groups on all types of algebra items. Treatment did better on algebraic tasks embedded in applied problem contexts when graphing calculators were available, and controls did better on traditional symbol manipulation tasks. CPMP did better on multiple representation of algebraic ideas (tables, graphs symbols) requiring representational fluency, but all items had mean scores below 33%. Suggest that correlation between symbol manipulation and problem solving is weak. For specific algebraic skills and problem-solving strategies such as translating problem conditions into symbolic expressions solving equations and interpreting results and on problems that required integration of those specific skills for work on more complex modeling tasks, CPMP outperformed control students. On questions with pure algebraic symbol manipulation testing equivalence of expressions and solving equations and inequalities, control outperformed CPMP. CPMP did better on algebraic tasks embedded in applied problem contexts when graphing calculators were available; controls did better on traditional symbol manipulation tasks. CPMP did better on multiple representation of algebraic ideas

		(tables, graphs symbols) requiring representational fluency, but all items had mean scores below 33%. Suggest that correlation between symbol manipulation and problem solving is weak.
Mitchelmore, M., & Cavanagh, M. (2000). "Students' difficulties in operating a graphics calculator" *Mathematics Education Research Journal, 12(3): 254-268*	Interpretive study 25 10th and 11th graders 15 10th graders, 8 girls and 7 boys 10 11th graders, 5 girls and 5 boys 15 were in a year 10 advanced course 5 were in a year 11 three unit course 5 were in the International Baccalaureate Mathematics HL course 3 fifty minute interviews, each two weeks apart where participants attempted at least 7 of 8 tasks If students did not reach one of the critical points during the interview, they were prompted to perform further calculator operations that led to the point. The students were asked to think aloud and explain the processes they used to interpret the graphs they saw, but errors were not corrected. Casio fx-7400G	Technical Difficulties with scale are acerbated by the lack of understanding of the technology—particularly how the calculator assigns pixels to graphs. Poor understanding of the zooming and scaling functions gave students problems with scale. Most disassociated the zoom operation from any change in the scale of the graphs they saw. Mathematical In developing mathematical accuracy and approximation, students demonstrated little understanding of the differences between decimal representations of rational and irrational numbers. Students failed to link graphs and symbolic representations. Many seemed to accept the visual image on the calculator screen without considering the context of the task.
Porzio, D. (1999). "Effects of differing emphases in the use of multiple representations and technology on students' understanding of calculus concepts" Focus on Learning Problems in Mathematics, 21(3): 1-29	Comparative/Interpretive study Calculus students who self selected the section of calculus—traditional, computer with Mathematica notebook, or graphing calculator. The prerequisites for the course were the same. Students volunteered and from the volunteers, 12 from each section were selected via a stratified random sample.	Viewing multiple representations of a concept does not necessarily help students develop better understanding of the concept. There is evidence that students "behave" as they are taught. This is supported by comparing understanding and representations used by students in each group of student interviews. Building more well-connected internal networks of knowledge

	Classroom observations Posttest Interviews Graphing calculator (make not specified), computers with Mathematica notebook	associated with the concepts imbedded in problems can be helped by having instructors and students use a variety of representations when solving problems (rather than favoring one particular representation). It is important to have graphing calculator students solve well-chosen problems designed to help them reflect and make connections between different representations of concepts that are provided by the technology. Curriculum for calculus should not just tack on multiple representations and technology, but emphasize using and connecting multiple representations of concepts and processes, making appropriate use of technology and providing adequate time and opportunities for reflection for the learner.
Quesada, A. R., & Maxwell, M. E. (1994). "The effects of using graphing calculators to enhance college students' performance in precalculus" *Educational Studies in Mathematics, 27(2): 205-215*	Comparative 710 students from a precalculus course at a large university during 3 semesters Treatment: (5 sections) Used textbook designed to be used with a graphing calculator and calculator and a more interactive classroom presentation Control: (8 sections) traditional mode and scientific calculators Survey of students perceptions Final Exam with 10 multiple choice and 10 open-ended questions TI-81 and Casio G-7000	Students who used the graphing calculator believed they had done more exploration and the calculator helped them understand the concepts they studied. Students indicated that the calculator saves time on calculations and helps them check their answers. Their primary concern was that they would become too dependent on it. On the final exam the treatment groups outperformed the control groups. Students in the treatment sections obtained considerably higher scores on four categories of questions (about properties of functions, graphs, word problems, and equations) and only slightly higher scores on the multiple choice questions.
Ruthven, K. (1990). "The influence of graphic calculator use on translation from graphic to symbolic forms" *Educational Studies in Mathematics, 21(5): 431-450*	Comparative A convenience sample of 87 students in two-year advanced level mathematics courses (academic upper secondary) in the U.K. Treatment: 47 in the project with permanent	Students in the treatment group performed significantly better than control at generating symbolic models for a given graph. On symbolization items, students with graphing calculators outperformed students without them. Females in the treatment group outperformed males in the treatment group. Males in the control group outperformed females.

	calculator access for 2 years during instruction and on the test. Control: 40 students without calculators (7 students had their own calculators) Two part questionnaire: Part 1 was used to collect background information and part 2 was a 12-graphic item test covering topic areas central to any advanced level course and where the use of graphs is normal practice. Graphing calculator fx 7000	On graph interpretation problems, there were no significant differences in performance between students with calculators and students without them. On these items, males in each group outperformed the females in their group.
Schwarz, B. B., & Hershkowitz, R. (1999). "Prototypes: Brakes or levers in learning the function concept? The role of computer tools " *Journal for Research in Mathematics Education, 30(4): 362-389*	Interpretive and Comparative quasi-experimental components MANOVA on test results 3 classes of 9^{th} grade students in Israel Treatment: graphing calculators or multi-representational software used in 1 of 3 weekly lessons throughout school year Control: 2 classes used different curricular materials and no technology Three part questionnaire test (items grouped to assess knowledge in prototypes, part-whole reasoning, representations) Graphing calculator (model not specified)	Students in treatment group performed better than the control group on the questionnaire test ($F(1,101)=7.66$, $p<.001$), particularly on the prototype section ($F(1,101) = 7.10$, $p=.009$) Students in the treatment group provided a better interpretation of a given graphical attribute and related it less to particular functions. Students in the treatment group justified significantly more and took the context of the problem into consideration more. Students in the treatment group were able to select appropriate representations among representations and were better at recognizing graphs from partial graphs.
Slavit, D. (1996). "Graphing calculators in a "hybrid" Algebra II classroom" *For the Learning of Mathematics, 15(1): 9-14*	Interpretive/case study Experienced teacher 18 students most of whom were sophomores Classroom observations over a one year period Graphing calculator (model not specified)	There was an increase in student-initiated discourse and in number of analytic questions. The graphing calculator also made investigations with parameters of functions easier which increased the breadth of the investigation.
Slavit, D. (1998).	Interpretive Case Studies	Students think using symbols and, given a choice, solve

"Three women's understandings of algebra in precalculus course integrated with the graphing calculator" *Journal of Mathematical Behavior, 17(3): 303-389*	3 students in an experimental precalculus course at a Mid-Atlantic University Four task based interviews (45 minutes each) to investigate knowledge of function, representations of functions, and attitude Self-paced tests – symbolically oriented In-class tests and quizzes –graphically oriented Graphing calculator (model not specified)	problems using symbolic procedures more often than graphic ones despite instructional emphasis on the later. Students used the graphing calculator only on graphical tasks hindering possible connections between symbolic representations of functions and their graphs.
Thompson, D. R., & Senk, S. L. (2001). "The effects of curriculum on achievement in second-year algebra: The example of the University of Chicago School Mathematics project" *Journal for Research in Mathematics Education, 32*, 58-84.	Comparative Experimental (Matched Pairs) Students in 2nd year algebra at four different volunteer schools Treatment: (n=150) classes used University of Chicago School Mathematics Project (UCSMP) Advanced Algebra Control: (n=156) classes used traditional texts Pretest: algebra and geometry) Posttest1: multiple choice, 2nd year algebra concepts Posttest 2: problem solving Survey of attitudes Graphing calculator (model not specified)	The use of graphing technology did not negatively affect the ability of the UCSMP students to deal with procedural skills. UCSMP students out performed comparison students on multi-step problems and problems with applications or graphical representations. Groups had comparable performance on items testing algebraic skills.

Question 5:	**Does hand-held graphing technology have similar effects on the performance of students from different gender, socio-economic status and achievement groups?**	
Study	Methodology, sample size, variables, technology	Findings
Forster, P. A., & Mueller, U. (2001). "Outcomes and implications of students' use of graphics calculators in the public examination of calculus" *International Journal of Mathematical Education in Science and Technology, 32*(1), 37-52	Interpretive study 172 (9%) of the 1882 Australian Tertiary Entrance Exams were studied. The papers were in six randomly assigned bundles and assigned randomly to the authors Australian students taking the 1998 Tertiary Entrance Exam. 404 students described in response to question data Three female and three male candidates were interviewed in the week following the examination. They were chosen on basis of differing abilities on internal school assessments for Calculus for the year. Two teachers of calculus and two examination graders were interviewed Analyzed student responses on six calculator positive problems from 19 problems on the exam. Analysis was informed by discussion with teachers and graders about an acceptable calculator-assisted answer and how students may have used calculators given the nature of their work. Graphing calculator with symbolic processing ability (model not specified) and HP38G	There was no significant difference between the performance of male and female students or in the types of errors made. The number of females taking the exam increased 20% between 1995 and 1998, the years when technology use also increased. This raises the question of whether or not there is a relationship.

Harskamp, E. G., Suhre, C. J. M., & van Streun, A. (2000). “The graphics calculator and students' solution strategies” *Mathematics Education Research Journal, 12(1): 37-52*	Comparative quasi-experimental 12 classes of pre-university students age 16-17 in the Netherlands 3 classes used the calculator all year 5 classes used the calculator for 1 topic 4 classes did not use the calculator Pretest about functions Observations of teaching behavior Interviews with students and teachers Posttest on knowledge of functions and graphs, derivatives, exponential, and periodic functions Students’ solution strategies TI-81	On the post-test, students with lower scores on the pre-test made larger gains than students with higher scores on the pre-test. Weaker students benefit more from year-long calculator use than average students.
Hollar, J. C., & Norwood, K. (1999). “The effects of a graphing-approach intermediate algebra curriculum on students’ understanding of function” *Journal for Research in Mathematics Education*	Comparative quasi-experimental Students in four sections of intermediate algebra at a large state university Treatment: (n = 46) Used calculators and a text that included calculator activities Control: (n = 44) Used a text that did not include calculator activities, and calculators were not used O’Callaghan function test without calculators used as pre- and posttest Departmental final exam (50 questions) Revised Mathematics Attitude Scale before and after treatment TI-82	No significant differences in scores for main effects of instructor or gender, or for any interaction among the three variables. No difference in attitude between experimental and control, between different instructors, or between genders.

Hong, Y., Toham, M., & Kiernan, C. (2000). "Supercalculators and university entrance calculus examinations" *Mathematics Education Research Journal 12(3): 321-336*	Comparative, matched-pairs design (t-test) 17 and 18 year old students from 2 secondary-level schools in New Zealand Experimental students received 4 one-hour training sessions with a researcher on TI-92 calculator procedures. Control students did not. All students were allowed to use a standard graphing calculator on the exam. Participants were also divided into high and low achieving groups based on their pre-test and again compared Scores were collected from a pre-test, post-test, and a calculator neutral test composed of items from the 1992 through 1997 Bursary Mathematics with Calculus university entrance exam A correlation of the Bursary Examination scores with the Calculator Test scores was performed. TI-92	Low achieving students who received TI-92 calculator training outperformed low achieving students who did not. TI-92 calculator training did not reduce the performance gap between high and low achievers.

Ruthven, K. (1990). "The influence of graphic calculator use on translation from graphic to symbolic forms" *Educational Studies in Mathematics, 21(5): 431-450*	Comparative A convenience sample of 87 students in two-year advanced level mathematics courses (academic upper secondary) in the U. K. Treatment: 47 in the project with permanent calculator access for 2 years during instruction and on the test. Control: 40 students without calculators (7 students had their own calculators) Two part questionnaire: Part 1 was used to collect background information and part 2 was a 12-graphic item test covering topic areas central to any advanced level course and where the use of graphs is normal practice. Graphing calculator fx 7000	Students in the treatment group performed significantly better than control at generating symbolic models for a given graph. On symbolization items, students with graphing calculators outperformed students without them. Females in the treatment group outperformed males in the treatment group. Males in the control group outperformed females. On graph interpretation problems, there were no significant differences in performance between students with calculators and students without them. On these items, males in each group outperformed the females in their group.
Shoaf-Grubbs, M. M. (1994). "The effect of the graphing calculator on female students' spatial visualization skills and level of understanding in elementary graphing and algebra concepts" In E. Dubinsky, A. Shoenfeld, & J. Kaput (Eds.), *Research in Collegiate Mathematics Education,* 1: 169-194	Comparative Two classes of female elementary college algebra students. Study conducted in the United States. Both groups had the same instructor, same lesson plans, and examined the same number of graphs. Treatment class used graphing calculator. Control class did not. Scores on spatial visualization tasks.	Students who used graphing calculators outperformed students without them on spatial visualization tasks. Students can use graphing calculators to test conjectures, explore ideas and verify that general mathematical laws are true in specific cases. Graphing calculators provide a means of concrete imagery that gives students new control over their learning environment and over the pace of the learning process. Graphing calculators relieve the need to emphasize symbolic manipulation and computational skills, and support an active exploration process of learning and understanding the concepts behind the mathematics.

Appendix C

Additional References
The following pages contain references, not included in this research, that may provide additional information about the use of technology in mathematics classes.

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