Math Objectives

- Students will recognize that the mean and standard deviation (SD) and the median and interquartile range (IQR) are two ways to measure center and spread.
- Students will recognize that in a skewed distribution, the mean is pulled in the direction of the tail, and the standard deviation is increased; in a distribution with an outlier, the mean is pulled in the direction of the outlier, and the standard deviation is increased.
- Students will be able to illustrate that the median and IQR are resistant to skewness/outliers while the mean and standard deviation are not.
- Students will recognize that the median and IQR are preferred when a distribution is skewed while either mean and SD or median/IQR are acceptable for approximately symmetric distributions.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practices).

Vocabulary

- bimodal
- interquartile range
- mean
- median
- outlier

- resistant
- skewed distribution
- standard deviation
- symmetric distribution
- unimodal

About the Lesson

- As a result of moving points on dot plots, students will:
  - Change the shape of the distribution and notice the effect on the mean and median and on the standard deviation and IQR.
  - Drag points to compare the effects of outliers on the mean/standard deviation and median/IQR.
  - Make and justify a conjecture about which measures are preferred based on the shape of the distribution.

Prerequisites

- Students should understand measures of center and spread
- They should also be familiar with different shapes and graphs of distributions of univariate data.
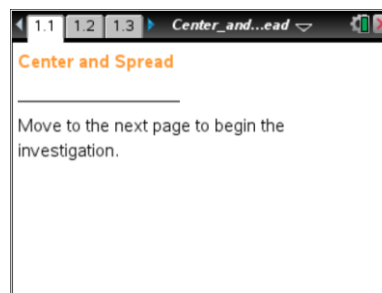
Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- Center_and_Spread_Student.pdf
- Center_and_Spread_Student.doc

TI-Nspire document

- Center_and_Spread.tns

TI-Nspire™ Navigator™

- Use Class Capture to examine patterns that emerge as students create different distributions.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds,
 TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Discussion Points and Possible Answers

> **Teacher Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (✋) getting ready to grab the point. Then press `ctrl` `🖑` to grab the point and close the hand (✊).

> **Tech Tip:** Selecting a vertical line in a plot reveals the associated plot values. Selecting an empty space hides the associated plot values. After moving a point, release it by selecting an empty space before dragging another point. Otherwise, multiple points will be moved.

> **Tech Tip:** Moving the cursor over a box plot reveals the values of the quartiles, extremes, and outliers.

> **Tech Tip:** Selecting the quartiles, extremes, and outliers reveals the associated plot values.

Move to page 1.3.

The graphs show the heights (in inches) of fifteen hypothetical statistics students. The vertical lines in the lower graph represent the mean and the mean plus or minus one standard deviation.

1. a. In the table below, fill in the row for the given dot plot: sketch, mean, standard deviation (SD), median, and interquartile range (IQR).
   b. Keeping the range constant, move points on the dot plot to match one of the other distribution shapes listed in the table. Record your results.
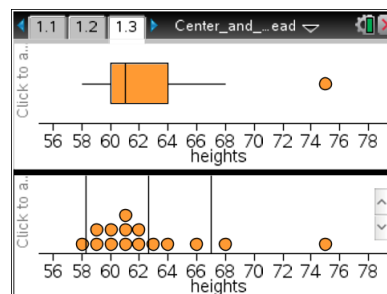
c. Repeat these steps for the other shapes listed in the table.

Sample Answers:

Shape (sketch your dot plot)	Mean	Median	SD	IQR
Skewed right **Sketches will vary; values are** **for one possible sketch.**	62.6 in.	61.0 in.	4.4 in.	4.0 in.
Skewed left	70.0 in.	72.1 in.	4.7 in.	4.4 in.
Symmetric – unimodal	65.7 in.	65.8 in.	3.6 in.	2.9 in.
Symmetric – bimodal	66.5 in.	66.2 in.	7.3 in.	14.7 in.

> **Teacher Tip:** Be sure students recognize that the vertical lines on either side of the vertical line for the mean represent the mean plus or minus one standard deviation, not the value of the standard deviation. One way that students can determine the standard deviation is by subtracting the mean from the value of the mean plus one standard deviation.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

> **Teacher Tip:** The mode is not considered a measure of center.

2. a. What do the mean and median measure?

Sample Answers: The mean and median are two ways to measure center - the arithmetic average and the middle value, respectively.

b. What do the standard deviation and IQR measure?

Sample Answers: They are both ways to measure spread in a distribution; the standard deviation describes the spread around the mean; the IQR is the width of the middle 50% of the data values.

3. Use the information in the table you completed in question 1 to describe the relationship between the shape of a distribution and the relative positions of the measures of center.

    Sample Answers: The median is always the middle value of the data; the mean is approximately the same as the median for symmetric distributions and pulled in the direction of the skew for non-symmetric distributions.

4. Use the information in the table to describe the relationship between the shape of a distribution and the relative sizes of the measures of spread.

    Sample Answers: Both the standard deviation and IQR are smallest for the unimodal, symmetric distribution. The standard deviation seems to increase for skewed and bimodal distributions.

Move to page 1.4.

5. On this page, the graphs show the heights (in inches) of fifteen hypothetical statistics students. Move points in the dot plot to create the four distribution shapes listed in the table below. Use the arrow to reset the dot plot after each shape change. Fill in the rows of the table as you work.
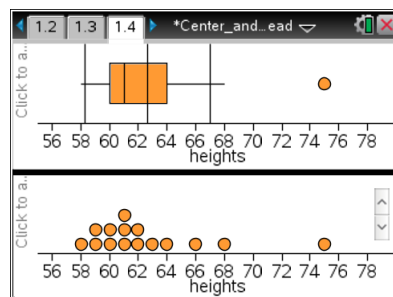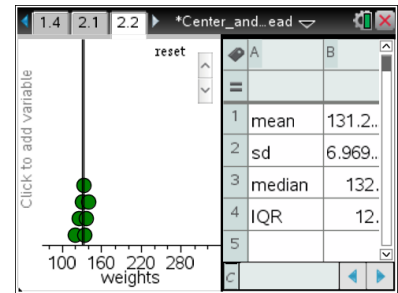
    Sample Answers:

Shape (sketch your dot plot)	Mean__Median <, =, >	Which measure of *center* seems to provide a more accurate description of the data?	Which measure of *spread* seems to provide a more accurate description of the data?
Skewed right **Sketches will vary;**	>	Median	IQR
Skewed left	<	Median	IQR
Symmetric – unimodal	=	Either (mean—used for inference in AP* Statistics)	Either (SD—used for inference in AP* Statistics)
Symmetric – bimodal	=	Neither (each clump should be described separately)	Neither (each clump should be described separately)

*AP is a registered trademark of the College Board, which was not involved in the production of and does not endorse this product. Visit www.collegeboard.org.

Move to page 2.2

6. The graph shows the weights (in pounds) of seven hypothetical statistics students. Measures of center and spread are displayed in the spreadsheet.
   a. Fill in the table below for the "Before" Distribution Shape. Then, move one point to the extreme right of the distribution and fill in the table for the "After" Distribution Shape.

   <u>Sample Answers:</u>

Distribution Shape	Mean, Standard Deviation	Median, IQR
Before	About 131 lbs., 7 lbs.	132 lbs., 12 lbs.
After	About 150 lbs., 60 lbs.	132 lbs., 15 lbs.

   b. Measures that are essentially unchanged by the shape of a distribution are said to be resistant. Which measures appear to be resistant? Explain why.

   <u>Sample Answers:</u> The median and IQR appear to be resistant because they depend on the order of the values. They will be less affected by outliers or skewness.

   c. Would your answer to b change substantially if you moved a different point? Explain.

   <u>Sample Answers:</u> No—the original points are clustered relatively close together. Moving any one of the points to the extreme right of the distribution increases the mean and standard deviation but does not greatly affect the median or the IQR.

7. Based on your work in question 6, give at least one reason why either mean and standard deviation or median and IQR are paired when talking about measures of center and spread.

   <u>Sample Answers</u>: Mean and standard deviation behave the same way with respect to outliers and skewness; they are both sensitive to extreme values. Median and IQR are both more resistant to outliers and skewness.

Wrap Up

Upon completion of the discussion, the teacher should ensure students are able to understand:

- The mean and standard deviation (SD) and the median and interquartile range (IQR) are two ways to measure center and spread.
- In a skewed distribution, the mean is pulled in the direction of the tail, and the standard deviation is increased; in a distribution with an outlier the mean is pulled in the direction of the outlier and the standard deviation is increased.
- The median and IQR are resistant to skewness/outliers while the mean and standard deviation are not.
- The median and IQR are preferred when a distribution is skewed while either mean/SD or median/IQR are acceptable for approximately symmetric distributions.

Assessment

1. In a distribution that is approximately symmetric, is the mean generally less than, greater than, or equal to the median?

   Sample Answers: The mean is generally about equal to the median in an approximately symmetric distribution.

2. In a distribution that is highly skewed to the right, which would you expect to be larger, the mean or the median?

   Sample Answers: In a skewed right distribution, I would expect the mean to be larger than the median.

3. Which measure of spread is more resistant to outliers—the standard deviation or the IQR?

   Answer: The IQR is more resistant to outliers.

TI-Nspire Navigator

Note 1

Question 1, *Class Capture*

Look for and discuss patterns in the mean and median or in the standard deviation and IQR across the different shaped distributions.