



# Chi Square Distributions

## Student Activity

Name \_\_\_\_\_

Class \_\_\_\_\_

The chi-square distribution is a distribution of sample variances ( $s^2$ ). It has two critical values which are used when constructing confidence intervals to estimate the population variance ( $\sigma^2$ ). The number of degrees of freedom is one less than the sample size. The Chi-Square distribution requires that samples be taken from a normally distributed population.

### Problem 1 – Characteristics of the Chi-Square Distribution

Graph the standard normal distribution:

Y1 = **normalpdf(X, 0, 1)**.

Then graph the chi-square distribution for  $n = 6$ :

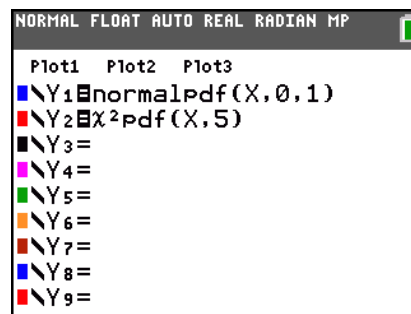
Y2 =  $\chi^2$  **pdf(X, 5)**. The format is (x, degrees of freedom).

The **normalpdf** and  $\chi^2$  **pdf** commands are accessed by pressing **2nd** **[DISTR]**.

Press **[WINDOW]**. Set the values equal to the following.

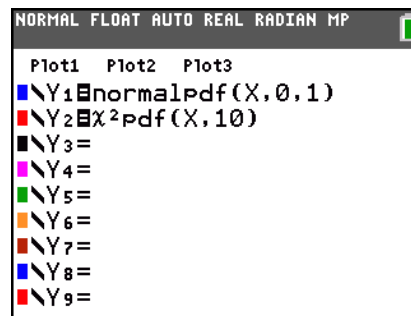
Xmin = -4, Xmax = 16, Xscl = 1, Ymin = 0, Ymax = 0.4, Yscl = 0

1. How does the chi-square distribution for  $n = 6$  (d.f. = 5) compare to the normal distribution?



Press **[Y=]** and change Y2 to a chi-square distribution where  $n = 11$  (degrees of freedom = 10). Press **[GRAPH]**. Repeat for  $n = 26$  and  $n = 51$ . Adjust the viewing window as needed.

2. What happens as  $n$  gets larger?



### Problem 2 – Critical Values for a Chi-Square Distribution

To construct a confidence interval for population variances, or standard deviations, you must find two critical values:  $\chi^2_L$  and  $\chi^2_R$ . The subscripts represent the left and right values. Use a chart, given from your teacher or located in your Statistics book, to find these values.

3. For a 95% confidence interval, find  $\chi^2_L$  and  $\chi^2_R$ , for a sample size of 11 (d.f. = 10).



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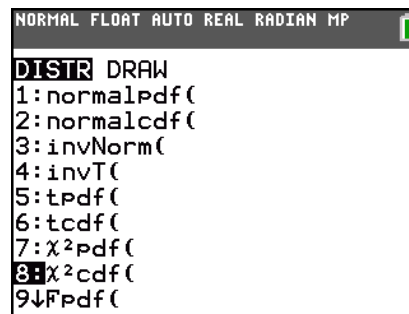
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To check your values, go to the home screen and find the area between the two critical values you listed in Question 3 by entering  $\chi^2$  cdf (from the [DISTR] menu) and then typing the lower bound, upper bound, and degrees of freedom, all separated by commas.

4. What is the area between these two values?



To check that 2.5% is in the lower tail, use  $-1E99$  for the lower bound and  $\chi^2_L$  for the upper bound.

(Type E by pressing [2nd] [EE].)

5. How can you check that 2.5% is in the right tail?

Answer the following without looking at a table of values.

6. For the same value of  $n$ , how will the critical values at the 90% level compare to those at the 80% level? Why?

7. Use a table of values to find  $\chi^2_L$  and  $\chi^2_R$  for  $n = 12$  at both the 80% and 90% levels.

### Problem 3 – Constructing a Confidence Interval

Confidence Interval for  
Population Variance

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

Confidence Interval for  
Population Standard Deviation

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

The weights of certain cereal boxes should have a mean of 482 grams with a standard deviation less than or equal to 6 grams. The weights of 15 randomly selected samples are listed below.

480, 476, 482, 489, 477, 495, 484, 482, 471, 479, 490, 480, 493, 481, 472



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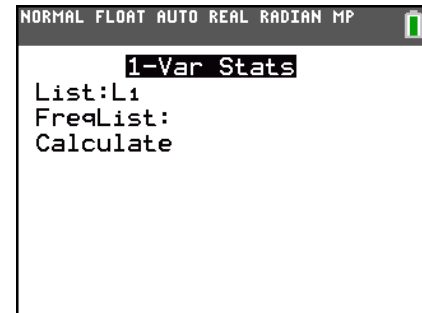
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Press **[STAT]**, choose **Edit** and list the weights in **L1**. Then press **[STAT]** again, arrow to the **CALC** menu and choose **1-Var Stats**. Enter **L1** and press **[ENTER]**.

8. What is the sample standard deviation?



Assume the distribution of weights is normal.

9. Construct a 95% confidence interval of the population standard deviation.
10. Are the differences in the weights of the samples at an acceptable level? Explain.
11. Suppose the standard deviation had to be less than or equal to 5 grams. Would the differences in the weights be considered acceptable? Explain.

### Problem 4 – Practice

12. Forty students at a high school are randomly selected and asked how much time they spend each week on homework. The sample mean and standard deviation were 5.6 hours and 2.4 hours respectively. Construct a 99% confidence interval for the standard deviation of time spent on homework by all the students at the school.
13. Twenty-eight scores are randomly selected from a group of tests. The sample had a standard deviation of 28 points. Construct an 80% and 90% confidence interval for the standard deviation of all the test scores.