Circles—Angles and Arcs

Math Objectives

- Students will define and identify central angles, major and minor arcs, intercepted arcs, and inscribed angles of a circle.
- Students will determine and apply the following relationships:
    - Two inscribed angles intercepting the same arc have the same measure.
    - An inscribed angle measure of 90° results in the endpoints of the intercepted arc lying on a diameter.
    - The measure of an angle inscribed in a circle is half the measure of the central angle that intercepts the same arc.
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

Vocabulary

- central angle
- inscribed angle
- major, minor, and intercepted arc

About the Lesson

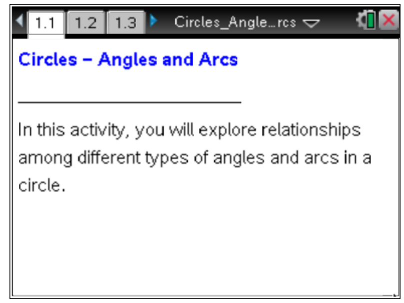
- This lesson involves manipulating endpoints of an arc, manipulating an inscribed angle, and manipulating the vertex of an angle intercepting an arc. As a result students will:
    - Use visualization to understand the definitions of central angle, intercepted arc, and minor and major arcs.
    - Infer that the sum of the measures of minor and major arcs is 360°, that two inscribed angles intercepting the same arc have the same measure, and that the inscribed angle has half the measure of the central angle that intercepts the same arc.
    - Deduce that the opposite angles of a quadrilateral inscribed in a circle are supplementary.

TI-Nspire™ Navigator™

- Use Class Capture to monitor student progress.
- Use Live Presenter to discuss examples as a class.
- Use Quick Poll to assess understanding throughout the lesson.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Circles_Angles_and_Arcs_Student.pdf
Circles_Angles_and_Arcs_Student.doc

TI-Nspire document
Circles_Angles_and_Arcs.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☝) getting ready to grab the point. Press ctrl ☝ to grab the point and close the hand (☝).

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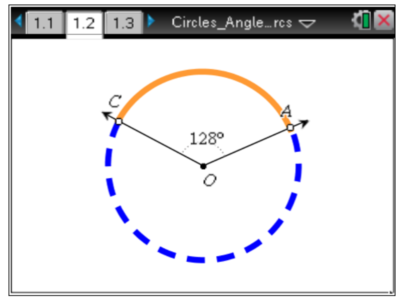

1. Drag point *A* or point *C*. Describe the changes that occur in the figure as you drag the point.

   Answer: The angle measurement changes but is never more than 180°. The solid part of the circle is always in the interior of the angle.

2. Angle *AOC* is called a central angle**.** Why do you think this is so?

   Answer: The vertex is at the center of the circle.

An angle intercepts an arc of a circle if each endpoint of the arc is on a different ray of the angle and the other points of the arc are in the interior of the angle.

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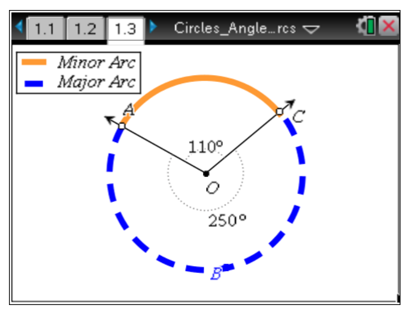

As you move point *A* or point *C*, the central angle ∠*AOC* intercepts a minor arc *AC*. The measure of the minor arc equals the measure of the central angle. The larger remaining arc, *ABC*, is called a major arc.

> **Teacher Tip:** On the TI-Nspire, arcs are measured using Length, not Angle. Therefore, if a student uses the built-in measuring tool, TI-Nspire will report arc length rather than arc measure. Also, sometimes in order to get the exact angle in the chart, both point A and point C may need to be moved.

3. a. Move point *A* or point *C* to help you complete the table.

Sample answer: The completed table is below. The final row of students' tables will vary.

∠*AOC*	arc *AC*	arc *ABC*	arc *AC* + arc *ABC*
50°	50°	**310°**	**360°**
100°	100°	**260°**	**360°**
110°	**110°**	250°	**360°**
(Choose an angle.) **60°**	**60°**	**300°**	**360°**

b. What is true about the measure of arc *AC* +arc *ABC*, the sum of the measures of the minor and major arcs?

Answer: The measure of arc *AC* + arc *ABC* always equals 360°.

4. In a circle, the measure of a central angle ∠*AOC* is *n*°.

a. What is the measure of the minor arc that is intercepted by the central angle? How do you know?

Answer: The minor arc measures *n*° because an intercepted arc has the same measurement as its central angle.

b. What is the measure of the major arc? How do you know?

Answer: The major arc measures (360 – *n*)° because the sum of the measures of the major and minor arcs is 360°.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

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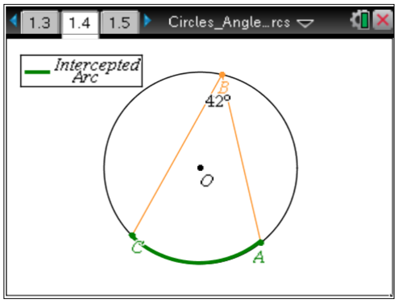

5. Angle *ABC* is called an inscribed angle because $\overline{BA}$ and $\overline{BC}$ are chords of the circle and vertex *B* is on the circle. Drag point *B* around the circle.

   a. As point *B* is moved around the circle, what do you notice about the measure of ∠*ABC*?

      <u>Answer:</u> Angle *ABC* has the same measure until it intercepts the other arc. While point *B* is moved around on that arc, ∠*ABC* will remain the same.

      Teacher Tip: Some students may recognize that the two angles' measures sum to 180°.

   b. Why does *m*∠*ABC* change when point *B* is moved from one arc to the other? Explain your reasoning.

      <u>Answer:</u> The angle measure changes because the intercepted arc changes. The intercepted arc is always in the interior of the inscribed angle.

   c. Move point *A* or point *C* until ∠*ABC* is a right angle. What is special about the arc and $\overline{AC}$?

      <u>Answer:</u> The arc measure is 180° and the arc is a semicircle. $\overline{AC}$ is a diameter.

      Teacher Tip: When students reach the right angle, the diameter should show up as a dotted segment.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

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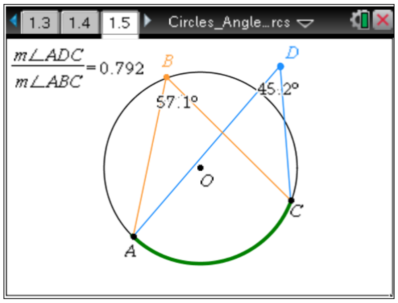


Angle *ABC* intercepts arc *AC*. Drag point *D* to various locations outside the circle, on the circle, inside the circle, and at the center *O*.

6. Place point *D* on the circle so that ∠*ADC* intercepts the same arc as ∠*ABC*.

a. What do you notice about the measures of ∠*ABC* and ∠*ADC*?

Answer: The angle measures are the same. The angles are congruent. The ratio of the angle measurements is 1.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Teacher Tip: Make sure both angles intercept the same arc. Some students may incorrectly place point *D* on the (bold) arc intercepted by ∠*ABC*. Point *D* should "snap" to the circle when it gets close.

b. What happens to the angles if you move point *A* or point *C*?

Answer: The angle measures change. The angles are congruent and the ratio of the measure of ∠*ADC* to the measure of ∠*ABC* is 1 if the angles intercept the same arc. The angles are not congruent and the ratio is not 1 if the angles do not intercept the same arc.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 4 at the end of this lesson.

Teacher Tip: Some students may note that the angle measurements are the same whenever *AD* and *BC* intersect or "criss-cross" and are not the same when they don't. This observation is important but needs to be related to intercepted arcs. Some students may notice that the angles are supplementary when they do not intercept the same arc. This property is addressed in problem 9. If m∠*ABC* equals 90°, then m∠*ADC* equals 90° whether or not they share the same intercepted arc.

7. Place point *D* at the center of the circle. Move point *A* and point *C* so that ∠*ADC* intercepts the same arc as ∠*ABC*.

   a. What is the relationship between the measures of inscribed ∠*ABC* and central ∠*ADC*?

   Answer: The measure of the central angle is double the measure of the inscribed angle. The measure of the inscribed angle is half the measure of the central angle. The ratio of the measure of ∠*ADC* to the measure of ∠*ABC* is 2.

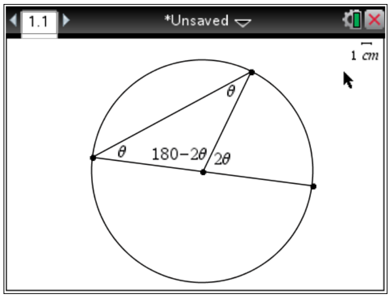
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 5 at the end of this lesson.

b. What happens to the angles if you move point *A* or point *C*?

Answer: The measure of the inscribed angle is half the measure of the central angle (the ratio of the measure of ∠*ADC* to the measure of ∠*ABC* is 2), as long as both angles intercept the same arc.

TI-Nspire Navigator Opportunity: ***Class Capture and/or Live Presenter***
See Note 6 at the end of this lesson.

Teacher Tip: For a proof of the Inscribed Angle Theorem: In the simplest case, one leg of the inscribed angle is a diameter of the circle so it passes through the center of the circle. Since that leg is a straight line, the supplement of the central angle equals 180° − 2*θ*. Drawing a segment from the center of the circle to the other point of intersection of the inscribed angle produces an isosceles triangle, made from two radii of the circle and the second leg of the inscribed angle. Since two angles in an isosceles triangle are equal and since the angles in a triangle sum to 180°, it follows that the inscribed angle equals θ, half of the central angle.



8. Leona said, "Since a central angle can never measure more than 180°, I know an inscribed angle can never measure more than 90°." Do you agree or disagree? Why?

Answer: I disagree because the central angle always intercepts a minor arc, but an inscribed angle can intercept a major arc.

9. Place point *D* on the circle so that *ABCD* is a quadrilateral.

a. What do you notice about the sum of the measures of ∠*ABC* and ∠*ADC*? Check with a classmate to compare.

Answer: The sum of the measures of ∠*ABC* and ∠*ADC* is 180°.

b. What do you notice about the sum of the measures of the angles if you move point *A* or point *C*?

Answer: As long as *ABCD* remains a quadrilateral, the sum of the measures of ∠*ABC* and ∠*ADC* remains 180°.

c. What do you notice about arcs *ABC* and *ADC*?

Answer: One is a major arc and one is a minor arc. Together the arcs make a circle. The measures of the arcs sum to 360°.

d. How does the relationship between arcs *ABC* and *ADC* explain the sum of the measures of inscribed $\angle ABC$ and $\angle ADC$?

Answer: The sum of the measures of the major and minor arcs is 360°. Since the measures of the inscribed angles are half the measures of their intercepted arcs, the angles are supplementary.

Teacher Tip: A quadrilateral inscribed in a circle has the special name "cyclic quadrilateral."

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 7 at the end of this lesson.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students understand:

- Two inscribed angles intercepting the same arc have the same measure.
- An inscribed angle measure of 90° results in the endpoints of the intercepted arc lying on a diameter.
- The measure of the inscribed angle is half the measure of the central angle that intercepts the same arc.

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Note 1

Questions 4a and 4b, *Quick Poll:* Have students enter their answers to the first part of questions 4a and 4b in a *Quick Poll*. Ask students to answer, "How Do You Know?" orally.

Note 2

Question 5c, *Class Capture:* As students complete question 5c, use *Class Capture* to view their screens. Discuss the following:

- Are all the right angles located in exactly the same place?
- What is true about the arc intercepted by the right angle?
- What is the measure of the intercepted arc?
- What is $\overline{AC}$ on each screen when a right angle is shown?

Note 3

Question 6a, *Quick Poll:* Using the Open Response feature of *Quick Poll*, have students enter their answer to 6a.

Note 4

Question 6b, *Class Capture:* Use *Class Capture* to look at students' screens when they complete question 6b. Put the screens together where the angles intercept the same arc and the screens together where the angles do not intercept the same arc. Discuss the results.

Note 5

Question 7a, *Quick Poll:* Using *Quick Poll*, students answer the following:
"The ratio of the measure of $\angle ADC$ to the measure of $\angle ABC$ is ____." Discuss students' responses.

Note 6

Question 7b, *Class Capture:* Use *Class Capture* to discuss students' answers to question 7b. *Class Capture* can also be used to help students answer the extension questions.

Note 7

Wrap Up, *Quick Poll:* Use *Quick Poll* to be sure students understand the three statements listed under Wrap Up. The True/False feature would be a good choice.