## Math Objectives

- Students will obtain the vertex form of a quadratic expression by completing the square.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

## Vocabulary

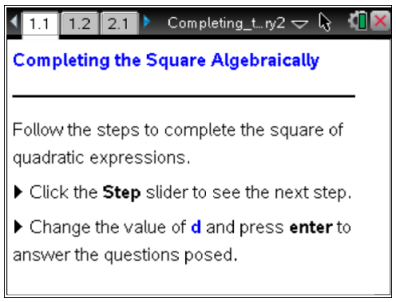
- vertex form of a quadratic expression
- expression
- completing the square
- leading coefficient

## About the Lesson

- This lesson involves students stepping through completing the square algebraically to rewrite a quadratic expression. Students will enter a step number and the answer to that step. If they are correct, a check will show up beside the rewritten expression. There are four steps to completing the square. Once all four steps have been completed, students can click the New slider to get a new expression.
- Problem 2 of the TI-Nspire document may be used to give students additional practice in completing the square of quadratic expressions algebraically. It allows the student to input any quadratic expression and then use the four steps to complete the square algebraically.
- As a result, students will be able to complete the square of any given quadratic expression.

## TI-Nspire™ Navigator™ System

- Use Screen Capture to examine students' generated expressions.
- Use Live Presenter to monitor student understanding.




**TI-Nspire™ Technology Skills:**

- Download a TI-Nspire document
- Open a document
- Move between pages
- Enter information into math boxes

**Tech Tips:**

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- In the *Graphs* application, you can hide the entry line by pressing ctrl G.

**Lesson Materials:**

*Student Activity*
Completing_the_Square_Algebraically_Student.pdf
Completing_the_Square_Algebraically_Student.doc

*TI-Nspire document*
Completing_the_Square_Algebraically.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

## Discussion Points and Possible Answers

**Tech Tip:** Be sure students press Enter after changing the value of d.

**Teacher Tip:** Guide the students step by step through question 1 to make sure that they understand how to manipulate the TI-Nspire document file.

---

**Move to page 1.2.**


Click the Step slider to advance to the next step. Change the value of d by clicking in the math box to answer the prompt.

In Step 2, students are asked to factor out the leading coefficient. This means that the expression has a leading coefficient of 1.

**Tech Tip:** Emphasize that the students should press Enter after changing the value of d. A Math box will go inactive if it is changed and Enter is not pressed. To reactivate a Math box, just press Enter.

If students enter an incorrect value for d in response to the prompt, a <Try Again> message will appear. However, if the value of d has been changed correctly, the message will direct them to continue to the next step.

When the step slider is changed, it shows the step with the previous value of d. Make sure students understand what is going on when this happens. When they enter another value for d, it will change accordingly.

**TI-Nspire Navigator Opportunity: *Screen Capture and Live Presenter***
**See Note 1 at the end of this lesson.**

1. Follow along on your handheld and fill in the table below as your teacher guides you through the first problem, which includes completing the square of the expression $3x^2 + 6x - 4$. Upon completion of Step 4, the completed expression will be in vertex form $a(x - h)^2 + k$.

**Answer:**

| Expression | | $3x^2 + 6x - 4$ | What happened to the original expression? |
|---|---|---|---|
| **Step 1** | Group the $x$-terms. | $(3x^2 + 6x) - 4$ | The $x$-terms were grouped together. |
| **Step 2** | Factor out the leading coefficient. | $3(x^2 + 2x) - 4$ | Factored out the 3 to give $x^2$ a coefficient of 1. |
| **Step 3** | Complete the square. $\left(\frac{b}{2a}\right)^2$ | $3(x^2 + 2x + (1)^2) - 4 - 3$ | Found $\frac{b}{2a}$ and squared it. It was added to the terms inside the parentheses, and it was subtracted from the constant outside the parentheses after it was multiplied by 3. |
| **Step 4** | Factor and simplify. | $3(x + 1)^2 - 7$ | Factored and combined like terms. |

**Teacher Tip:** The answer to be entered in for the value of d in Step 1 is –4, for Step 2 is 3, for Step 3 is 1, and for Step 4 is 1.

a. Refer to the expression created in Step 3. Where does the squared constant term come from?

**Answer:** It is the result of taking $\frac{b}{2a}$ and squaring it.

b. In Step 3, where does the subtracted constant term outside the parentheses come from? Why is it being subtracted?

**Answer:** The number 3 is a result of squaring $\frac{b}{2a}$ and multiplying it by the 3 that is in front of the parentheses. It is being subtracted outside the parentheses because that value was added to the expression inside the parentheses. It keeps the expression balanced.

**Teaching Tip:** Explain that the reason for completing the square is to put a quadratic expression in a form in which the vertex is easily identified.

2. Click the New slider to get a new expression. Continue to click the New slider until you get an expression that has a negative sign at the beginning. Using your handheld, complete the square of the new expression. Change the value of d to answer each prompt.

   **Answer:** Answers will vary. However, students should have similar information in their tables.

| Expression | | $-x^2 - 8x + 6$ | What happened to the original expression? |
|---|---|---|---|
| **Step 1** | Isolate *x*-terms. | $(-x^2 - 8x) + 6$ | The *x*-terms were grouped together. |
| **Step 2** | Factor out the leading coefficient. | $-(x^2 + 8x) + 6$ | Factored out the –1 to give $x^2$ a coefficient of 1. |
| **Step 3** | Complete the square. $\left(\frac{b}{2a}\right)^2$ | $-(x^2 + 8x + (4)^2) + 6 + 16$ | Found $\frac{b}{2a}$ and squared it. It was added to the terms inside the parentheses, and it was subtracted from the constant outside the parentheses after it was multiplied by –1. |
| **Step 4** | Factor and simplify. | $-(x^2 + 4)^2 + 22$ | Factored and combined like terms. |

**TI-Nspire Navigator Opportunity: *Screen Capture and Live Presenter***
**See Note 2 at the end of this lesson.**

a. Refer to the expression created in Step 3. Where does the squared constant term come from?

   **Answer:** It is the result of taking $\frac{b}{2a}$ and squaring it.

b. In Step 3, why is the squared constant term inside the parentheses added to the constant term outside the parentheses instead of subtracted as in question 1?

   **Answer:** The squared constant term appears to be added to the constant term outside the parentheses. It is actually being subtracted. It is the result of squaring $\frac{b}{2a}$ and multiplying it by $-a$. Therefore, for example, 6 – (–16) is 6 + 16.

3. Click the New slider to get a new expression. Using your handheld, complete the square of the new expression. Fill in the table by recording each step.

**Answer:** Answers will vary. However, students should have similar information in their tables.

| Expression | | $3x^2 + 9x + 2$ | What happened to the original expression? |
|---|---|---|---|
| **Step 1** | Isolate $x$-terms. | $(3x^2 + 9x) + 2$ | The $x$-terms were grouped together. |
| **Step 2** | Factor out the leading coefficient. | $3(x^2 + 3x) + 2$ | Factored out the 3 to give $x^2$ a coefficient of 1. |
| **Step 3** | Complete the square. $\left(\frac{b}{2a}\right)^2$ | $3\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+2-\frac{27}{4}$ | Found $\frac{b}{2a}$ and squared it. It was added to the terms inside the parentheses, and it was subtracted from the constant outside the parentheses after it was multiplied by 3. |
| **Step 4** | Factor and simplify. | $3\left(x+\frac{3}{2}\right)^2+\frac{35}{4}$ | Factored and combined like terms. |

a. Refer to the expression created in Step 3. Where does the squared constant term come from?

**Answer:** It is the result of taking $\frac{b}{2a}$ and squaring it.

b. In Step 3, where does the subtracted constant term outside the parentheses come from? Why is it being subtracted?

**Answer:** The number $\frac{27}{4}$ is a result of squaring $\frac{b}{2a}$ and multiplying it by the 3 that is in front of the parentheses. It is being subtracted outside the parentheses because that value was added to the expression inside the parentheses. It keeps the expression balanced.

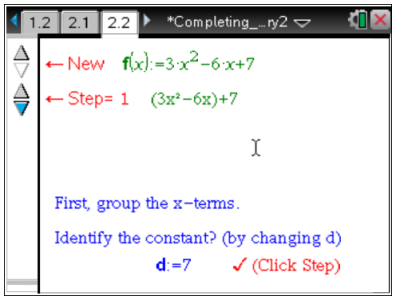
4. On page 2.2, enter f(x) = $-3x^2 + 6x + 4$ by double clicking in the f(x) math box and press **Enter**. Upon completion of Step 4, the completed expression will be in vertex form $a(x - h)^2 + k$. Fill in the missing steps and explanations in the table.

**Answer:**

| Expression | | $-3x^2 + 6x + 4$ | What happened to the original expression? |
|---|---|---|---|
| **Step 1** | Isolate $x$-terms. | $(-3x^2 + 6x) + 4$ | The $x$-terms were grouped together. |
| **Step 2** | Factor out the leading coefficient. | $-3(x^2 - 2x) + 4$ | Factored out the 5 to give $x^2$ a coefficient of 1. |
| **Step 3** | Complete the square. $\left(\frac{b}{2a}\right)^2$ | $-3(x^2 - 2x + (1)^2) + 4 - 3$ | Found $\frac{b}{2a}$ and squared it. It was added to the terms inside the parentheses, and it was subtracted from the constant outside the parentheses after it was multiplied by 5. |
| **Step 4** | Factor and simplify. | $-3(x - 1)^2 + 1$ | Factored and combined like terms. |

**Tech Tip:** Students may use pages 2.2 of the .tns file to complete the square in the following questions.

5. Complete the square of the following expressions. Show your work.




a. $3x^2 - 6x + 7$

**Answer:** $3(x - 1)^2 + 4$

b. $-2x^2 + 6x - 1$

**Answer:** $-2\left(x - \frac{3}{2}\right)^2 + \frac{7}{2}$

c. $2x^2 + x - 4$

**Answer:** $2\left(x + \frac{1}{4}\right)^2 - \frac{33}{8}$

## Wrap Up

Upon completion of the activity, students will be able to algebraically complete the square of a given quadratic expression.

## TI-Nspire Navigator

**Note 1**

**Question 1, *Screen Capture* and *Live Presenter:*** Use *Screen Capture* and *Live Presenter* to guide students through the process of completing the square.

**Note 1**

**Question 2, *Screen Capture* and *Live Presenter:*** You may want to use *Screen Capture* since students may have different expressions that contain a negative leading coefficient. Using *Live Presenter*, you can have several students step through completing the square for their expressions.