Math Objectives

- Students will interpret the variables in the formula for compound interest.
- Students will use the formula for compound interest and understand the effects of changes in the interest rate and the number of compounding periods.
- Students will understand the relationship between compound interest and continuous compounding.
- Model with mathematics (CCSS Mathematical Practice).

Vocabulary

- compound interest
- interest rate
- pay periods
- initial deposit
- continuous compounding

About the Lesson

- This lesson involves exploring the formula for compound interest as a function of the initial deposit, interest rate, and the number of pay periods per year.
- As a result, students will:
    - Learn the relationship between the interest rate and the total amount in the account.
    - Learn the relationship between the number of pay periods and the total amount in the account.
    - Discover the limiting condition as the number of pay periods increases without bound.

TI-Nspire™ Navigator™ System

- Use Screen Capture to investigate the account amounts for various values of n.
- Use Screen Capture to compare graphs of $y = A(t)$ for various values of n.
- Use Teacher Edition computer software to review student work.
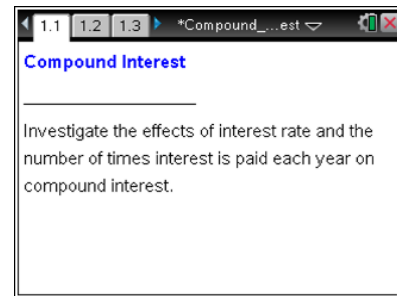
TI-Nspire™ Technology Skills:
- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:
- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing [ctrl] [G]. The entry line can also be expanded or collapsed by clicking on the chevron.

Lesson Materials:
Student Activity
Compound_Interest_Student.pdf
Compound_Interest_Student.doc
TI-Nspire document
Compound_Interest.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** Make sure students execute the program **initialize** on a calculator page. If students experience difficulty dragging a point on a slider bar, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Then press ⌈ctrl⌉ 🖲 to grab the point and close the hand (☜).
>
> Students can animate a slider by right-clicking on the slider (⌈ctrl⌉ ⌈menu⌉) and selecting Animate. Right-click again to select Stop Animate.

Move to page 1.3.

1. Suppose $50,000 is deposited in an account paying 2% ($r = 0.02$) per year ($n = 1$). These values have been entered for P, r, and n on Page 1.3. Move to Page 1.4 to see information about this account. Column A displays the total amount in the account after each interest pay period. Column B displays the amount of interest earned after each pay period. Note: row 1 corresponds to the initial deposit; row 2 corresponds to the first pay period, etc.

    a. Explain why the interest earned after each pay period increases.

        Answer: After each pay period, the account balance is the original deposit, or principal, plus interest. Therefore, interest is paid based on a larger account balance each pay period.

    b. Use Column A to estimate the number of years until the initial deposit doubles. (Hint: Press ⌈ctrl⌉ ⌈3⌉ to page down.)
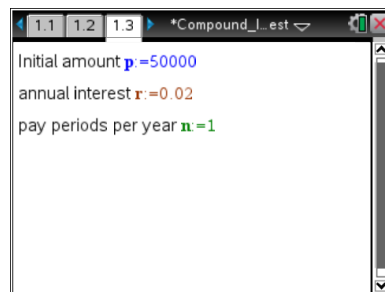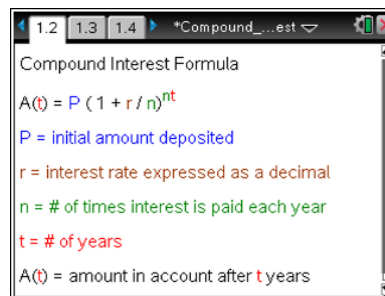
        Answer: The initial deposit doubles after 36 years. Row 37 of the spreadsheet indicates the total amount in the account is $101,994.37.

> **Teacher Tip:** Students might suggest the initial deposit doubles between year 35 and year 36. However, remember that interest is only paid once per year ($n = 1$). We assume no additional interest is earned until the end of the pay period.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 1 at the end of this lesson.

---

c. Go back to Page 1.3, and change the interest rate so that the initial deposit doubles after 15 years.

   Answer: For $r = 0.0473$ (interest rate of 4.73%, approximately 5%), the initial deposit will double after 15 years. Note: Student answers will vary. Consider asking for the smallest interest rate such that the initial deposit doubles after 15 years. Consider asking for an interest rate so that the initial deposit doubles after 15 years, but no earlier.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 2 at the end of this lesson.

---

2. Suppose $10,000 is deposited in an account paying 5% ($r = 0.05$) semi-annually ($n = 2$). Enter the values for P, r, and n on Page 1.3.

   a. Complete the following table to find the amount in the account after two years. Change the value of n as necessary on Page 1.3.

      Answer:

n	2	4	6	12	52
$A(2)$	11,038.13	11,044.86	11,047.13	11,049.41	11,051.18

      As n increases, how do you expect the value of $A(t)$ to change for a fixed value of t?

      Answer: For a fixed value of t the table suggests that as n increases, the amount in the account at time t, $A(t)$, also increases.

b. Explain the meaning of $n = 365$, $n = 365 * 24 = 8760$,
    $n = 365 * 24 * 60 = 525,600$, and $n = 365 * 24 * 60 * 60 = 31,536,000$.

Answer:

$n = 365$: Interest is paid daily.

$n = 8760$: Interest is paid hourly.

$n = 525,600$: Interest is paid every minute.

$n = 31,536,000$: Interest is paid every second.

c. Insert a Calculator page, and complete the following table.

n	365	8760	525,600	31,536,000
$A(2)$	11,051.63	11,051.71	11,051.71	11,051.72

d. As n increases, how would you explain the compounding period? Explain how the
    amount in the account changes for a fixed value of t as n increases.

Answer: As n increases, the number of compounding periods increases, towards
interest being paid continuously, or continuous compounding. This question
suggests that as n increases, the amount in the account at time t, $A(t)$, also
increases.

e. Using your results from Questions 1 and 2, what characteristics would you like in
    an account in order to earn the most interest after every pay period?

Answer: In order to earn the most in an account after every pay period, we should
search for the greatest interest rate and an account with the greatest number of
pay periods.

3. Suppose $25,000 is deposited in an account paying 4%
    ($r = 0.04$) quarterly ($n = 4$). Enter the values for P, r, and n
    on page 1.3. Move to Page 1.5. Column B displays the amount in
    the account, A, after each pay period. Column A contains values
    of the function $c(t) = Pe^{rt}$ for each corresponding pay period,
    where $e \approx 2.71828\ldots$, the base of the natural logarithm. This
    function does not depend upon n. Column C contains the
    difference between the two values for corresponding pay periods.
    Note: row 1 corresponds to the initial deposit, row 2 corresponds to the first pay period,
   etc.

Move or animate the slider on the right side to increase the value of n. Use the slider to change the value of n. As n increases, explain the relationship between $c(t)$ and $A(t)$.

Answer: As n increases, the values of $A(t)$ tend to get closer and closer to $c(t)$, but $A(t) \leq c(t)$ for all values of t.

Move to page 2.1.

4. The graph of $y = c(t)$ is displayed as a solid curve, and the graph of $y = A(t)$ is displayed as a dashed curve. Move or animate the slider to change the value of n.
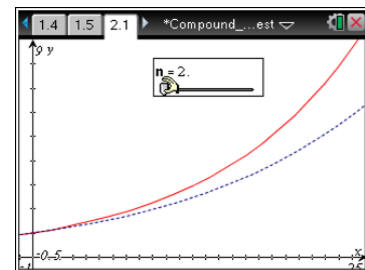
a. Explain the relationship between the two curves as n increases. Is your answer consistent with your response to question 3? If not, why not? Note: you might need to zoom in to examine the relationship between the two curves.

   Answer: As n increases, the graph of $y = A(t)$ tends to get closer and closer to the graph of $y = c(t)$. The graph of $y = A(t)$ does not appear to intersect, or cross, the graph of $y = c(t)$. This answer is consistent with part (a).

b. Can you find values for P, r, and n such that $A(t) > c(t)$ for some value of t?

   Answer: It is not possible to find values for P, r, and n such that $A(t) > c(t)$ for some value of t. For any values of P, r, and n, $A(t)$ is always less than $c(t)$. As n increases, $c(t)$ does not change, and $A(t)$ increases but is always slightly less than $c(t)$.

Wrap Up

Upon completion of this activity, students should be able to understand:

- The relationship between the interest rate and the total amount in the account.
- The relationship between the number of pay periods and the total amount in the account.
- The limiting condition of compound interest as the number of pay periods increases without bound.
- A very basic idea of a limit.
- A very basic idea of continuous compounding, or interest being paid at every instant.

Teacher Notes

1. The graph of $A(t)$ is presented as a smooth curve. In practice, $A(t)$ is a piecewise linear function since interest is paid a discrete periods. Consider a graph of the calculator function

$$ap(t) = P\left(1 + \frac{r}{n}\right)^{\text{floor}(nt)}$$

2. For any fixed value of t, for example t_0, the value $c(t_0) = Pe^{rt_0}$ is the limit of $A(t_0)$ as n increases. This presents a good opportunity for students to discover the idea of a limit.

3. Suppose the initial deposit is \$1 and the interest rate is 100% ($r = 1$). At the end of 1 year, the amount in the account is $A(1) = \left(1 + \frac{1}{n}\right)^n$. Ask students to construct a table of values for $A(1)$ for various values of n. For example:

n	$\left(1+\dfrac{1}{n}\right)^n$
1	2.000000
5	2.488320
10	2.593742
100	2.704814
1000	2.716924
10,000	2.718146
100,000	2.718268
1,000,000	2.718280

*This table might help suggest why the number e is associated with compound interest and appears in the formula for $c(t)$.

TI-Nspire Navigator

Note 1

Question 1b, *Quick Poll (Open Response)*

Tell students that you are going to send a Quick Poll asking for the number of years until the initial deposit doubles.

Note 2

Question 1c, *Quick Poll (Open Response)*

Tell students that you are going to send a Quick Poll, Open Response asking for the interest rate so that the initial deposit doubles after 15 years.