Math Objectives

- Students will be able to identify the domain and range of a relation from its graph.
- Students will be able to write symbolic expressions to describe the domain and range of a relation.
- Students will be able to recognize that different relations can have the same domain or the same range.
- Students will create a graph with a given domain and range.
- Students will use appropriate tools strategically. (CCSS Mathematical Practice)

Vocabulary

- domain
- discrete
- function
- range
- continuous
- interval notation
- infinity
- relation

About the Lesson

- This lesson involves finding domain and range by changing the endpoints of a graph.
- As a result, students will:
- Manipulate given segments and make conjectures about the relationships between the lengths of the segments and the possibility of forming a triangle.
- Drag point *P* along the points in a scatter plot. From the ordered pairs, they will determine the domain and range.
- Determine the domain and range of several different relations by dragging a point along the graph.
- Compare domains and ranges that are discrete with those that are continuous over an interval.
- Sketch graphs given a domain and range.

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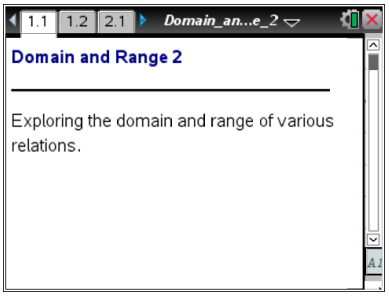
- Send out the *Domain_and_Range_2.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Materials:

Student Activity

- Domain_and_Range_2_Student.pdf
- Domain_and_Range_2_Student.doc

TI-Nspire document

- Domain_and_Range_2.tns

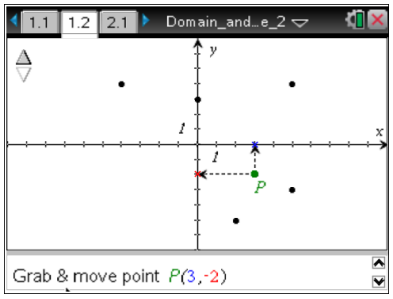
Discussion Points and Possible Answers

TI-Nspire™ Navigator™ Opportunity: *Class Capture* or *Live Presenter*
See Note 1 at the end of this lesson.

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl ☜ to grab point *P* and close the hand (✊). When finished moving the point, press esc to release the point.

Teacher Tip: Students should not begin dragging point *P* until they have recorded the answer to part a in question 1. If they do, the can click on the reset slider and then drag the point.
For pages 2.1–5.1, there is a reset slider to remove the tracing of the domain and range.

Move to page 1.2.



1. Grab and move point *P* to each point on the scatter plot and note the changes.
   a. Name the ordered pair for point *P*. What is the relationship between this ordered pair and the cross mark on each axis?

Answer: Answers may vary for the ordered pair for point *P*. The cross mark on the *x*-axis is at the *x*-coordinate of the ordered pair. If using the TI-Nspire™ CX handheld or the TI-Nspire iPad app, it is marked in blue. The cross mark on the *y*-axis is at the *y*-coordinate of the ordered pair. If using the TI-Nspire CX handheld or the TI-Nspire iPad app, it is marked in red.

b. Grab and drag point *P* to each of the circles on the scatter plot. As you move from point to point, record the coordinates in the table.

Answer: The completed table is below.

x	*y*
–4	4
0	3
2	–5
3	–2
5	–3
5	4

Teacher Tip: The order in which students place the ordered pairs in the table does not matter.

c. State the domain and range of the relation.

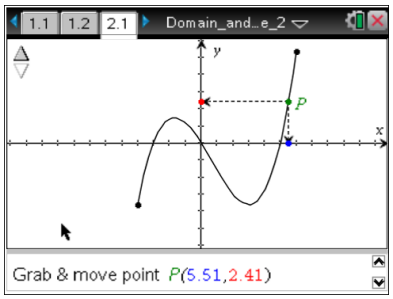
Answer: Domain: {–4, 0, 2, 3, 5}; Range: {–5, –3, –2, 3, 4}

Teacher Tip: It is easier to specify the set if the domain and range are listed from least to greatest, but order is not critical. The values should not be repeated.

d. Where on the graph do you see the domain represented? The range represented?

Answer: The domain of the graph can be seen as the cross marks on the *x*-axis or as the highlighted blue section on the *x*-axis. The range can be seen as the cross marks on the *y*-axis or as the highlighted red section on the *y*-axis.

Move to page 2.1.



2. Move point *P* back and forth along the entire graph.
   a. What does the highlighted portion along the *x*-axis represent?

Answer: The highlighted portion along the *x*-axis represents the domain of the graph. If you are using the TI-Nspire CX handheld or the TI-Nspire iPad App, the highlighted portion will be blue.

b. What does the highlighted portion along the *y*-axis represent?

Answer: The highlighted portion along the *y*-axis represents the range of the graph. If you are using the TI-Nspire CX or the Nspire iPad App, the highlighted portion will be red.

c. How do the domain and range in this problem differ from the domain and range in question 1?

Answer: The domain and range in this graph are continuous over an interval, whereas the domain and range in question 1 are discrete.

d. State the domain as an inequality and in interval notation.

Answer: Domain: $-4 \le x \le 6$; $x \in [-4, 6]$

Teacher Tip: There may be some confusion between interval notation and an ordered pair. To alleviate this confusion, you may want to use "$x \in$," which means "*x* is an element of," before the interval notation.

e. State the range as an inequality and in interval notation.

Answer: Range: $-4 \le y \le 5$; $y \in [-4, 5]$

Teacher Tip: There may be some confusion between interval notation and an ordered pair. To alleviate this confusion, you may want to use "$y \in$," which means "*y* is an element of," before the interval notation.

f. If the endpoints of the graph were open circles, how would the domain and range change? State the new domain and range as inequalities and in interval notation.

Answer: The domain would be between –4 and 6, not including –4 and 6. The range would be between –4 and 5, not including –4 and 5.

Domain: $-4 < x < 6$ Range: $-4 < y < 5$

$x \in (-4, 6)$ $y \in (-4, 5)$

Teacher Tip: Emphasize what open and closed circles mean on a graph, in words, and in an inequality. The endpoint may or may not be part of the domain and range. In this problem, if the circles were open, the inequality symbols would be strictly less than, *not* less than or equal to.

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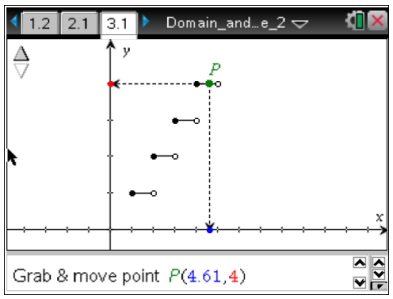

3. Move point *P* back and forth along the entire graph.
   a. State the domain and range of the graph.

      Answer: Domain: $1 \le x < 5$ or $x \in [1, 5)$
      Range: {1, 2, 3, 4}

Teacher Tip: As point *P* is being dragged, make sure that the hand remains close to point *P*. If the hand moves too far away from point *P*, the students will not be able to see the entire domain. Point out all of the representations that are being seen.

   b. How does the domain of this graph compare to the domain of the graph in question 1? Question 2?

      Answer: The domain of this graph is over an interval (continuous) with two defined endpoints, whereas the domain in question 1 is discrete. The domain in question 2 is like the domain in this graph. It is over an interval with two defined endpoints. However, both endpoints are included in the domain in question 2.

   c. How does the range of this graph compare to the range of the graph in question 1? Question 2?

      Answer: The range of this graph is discrete, like the range in question 1. The range in question 2 is over an interval (continuous) with two defined endpoints.

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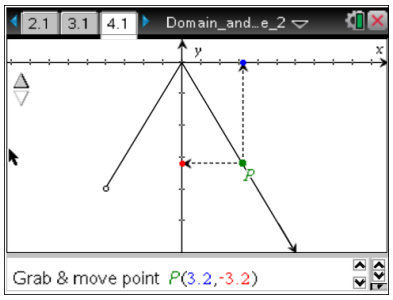

4. Move point *P* back and forth along the entire graph.
   a. State the domain and range as inequalities and in interval notation.

      Answer: Domain: $x > -4$ or $x \in (-4, \infty)$
      Range: $y \le 0$ or $y \in (-\infty, 0]$

b. What is different about the domain and range of this graph compared to the others?

Answer: The domain of this graph is over an interval (continuous) with one defined endpoint. The graph continues infinitely in the positive direction, whereas the domain in question 1 is discrete. The domain in question 2 is like the domain in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

The range of this graph is over an interval (continuous) with one defined endpoint. The graph continues infinitely in the negative direction, whereas the range in question 1 is discrete. The range in question 2 is like the range in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

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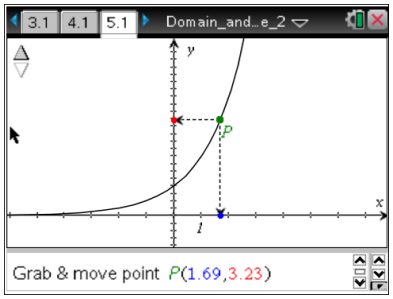

5. Move point *P* back and forth along the entire graph.
   a. State the domain and range as inequalities and in interval notation

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Answer: Domain: $-\infty < x < \infty$ or $x \in (-\infty, \infty)$
Range: $y > 0$ or $y \in (0, \infty)$

b. What is different about the domain and range of this graph compared to the others?

Answer: The domain of this graph is the set of all real numbers. The graph continues infinitely in both directions, whereas the domain in question 1 is discrete. The domain in question 2 is like the domain in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

The range of this graph is the set of all real numbers greater than 0. The graph continues infinitely in the positive direction, whereas the range in question 1 is discrete. The domain in question 2 is like the range in this graph. It is over an interval. However, the graph in question 2 has two defined endpoints.

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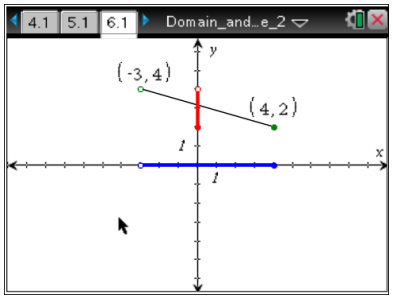

6. Grab and move the endpoints of the line segment to new locations.
   a. What do you notice about the domain and range as you drag the endpoints?

   Answer: As you move the endpoints of the segment, the domain and range change as well to reflect the new domain and range.

TI-Nspire™ Navigator™ Opportunity: *Class Capture* or *Live Presenter*
See Note 2 at the end of this lesson.

   b. Move the open endpoint to (–1, 4) and the closed endpoint to (2, –3). State the domain and range as inequalities and in interval notation.

   Answer: Domain: $-1 < x \leq 2$ or $x \in (-1, 2]$
   Range: $-3 \leq y < 4$ or $y \in [-3, 4)$

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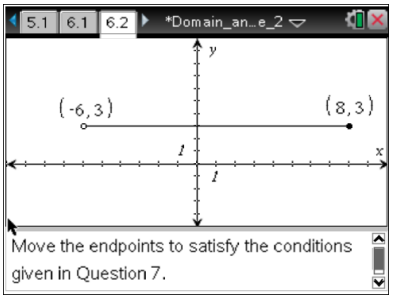


7. Grab and move the endpoints of the line segment to satisfy each of the following conditions:
   a. The open endpoint is (–4, –3) and the closed endpoint is (3, 4). State the domain and range using inequalities and interval notation

   Answer: Domain: $-4 < x \leq 3$ or $x \in (-4, 3]$
   Range: $-3 < y \leq 4$ or $y \in (-3, 4]$

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

b. The domain is [–7, 7) and the range is (–4, 5]. Write the domain and range as inequalities. State the endpoints of the line segment and indicate which endpoint is open.

Answer: Domain: $-7 \le x < 7$
Range: $-4 < y \le 5$
The endpoints are (–7, 5) and (7, –4).
The open endpoint is (7, –4).

Teacher Tip: Students may have difficulty naming the correct endpoints. You may need to redirect their attention to the brackets and parentheses in the domain and range to ensure the correct endpoints are named.

c. Given a domain of $-3 \le x < 8$ and a range of $y = -2$, state the endpoints of the line segment and indicate which endpoint is open.

Answer: The endpoints are (–3, –2) and (8, –2). The open endpoint is (8, –2).

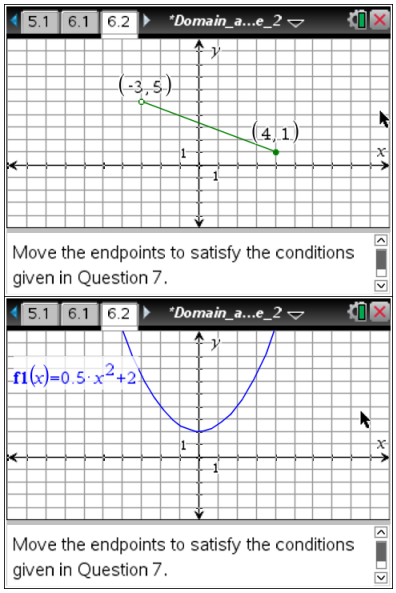
d. The domain is $x = 5$. State the endpoints of the line segment and indicate which endpoint is open. Compare your answers with another student's. Explain how you can have different answers and still both be correct.

Answer: Answers may vary, and either endpoint may be open. However, all ordered pairs should contain 5 as the *x*-coordinate.

8. Sketch a graph and write a description of a function or relation that satisfies the following conditions. Compare your answers with another student's. How are they alike or different?

a. Domain: $-3 < x \le 4$; Range: $1 \le y < 5$

Possible Answer: See example at right.



b. Domain: $(-\infty, \infty)$; Range: $[2, \infty)$

Possible Answer: See example at right.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to determine the domain and range of a relation from its graph.
- The connection between graphical and numerical representations.
- How to read and write symbolic expressions to describe the domain and range of a relation.
- How to sketch a graph with a given domain and range.

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Note 1

Entire Document, *Class Capture* or *Live Presenter*: If students experience difficulty with dragging the point or understanding the results, use Class Capture to show the solution to the students. Highlight the bold areas on the axes that represent the domain and range.

Note 2

Question 6a, *Class Capture* or *Live Presenter*: Use this opportunity to show different students' graphs and point out how the domain and range are affected by changing the endpoints of the segment.

Note 3

Question 7a, *Quick Poll (Multiple Choice or Open Response)*: Use Quick Poll as an opportunity to assess students' understanding of domain and range for a function. Have students identify the domain and range for this function. As a follow-up question, ask students what kind of line will have a domain of $x = 2$.