



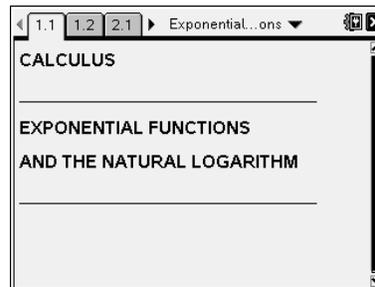
Exponential Functions and the Natural Logarithm

Student Activity

Name _____
Class _____

Open the TI-Nspire document *Exponential_Functions.tns*.

The purpose of this activity is to study the relative growth rate of exponential functions of the form $f(x) = b^x$, to understand their connection to natural logarithms, and to understand more about the natural exponential function.



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Press (ctrl) ► and (ctrl) ◀ to navigate through the lesson.

An exponential function with base b has the form $f(x) = b^x$, where b is any positive real number. If $b = 1$, f is a constant function with $f(x) = 1$ and not very interesting. We will be more interested in exponential functions with $b \neq 1$.

The relative growth rate of any function f at the value x is simply the ratio of its rate of change or slope, $f'(x)$, to its value $f(x)$, that is, $\frac{f'(x)}{f(x)}$. For example, a linear function, $f(x) = mx + b$, has a constant rate of change, $f'(x) = m$ (the slope of the line). The relative growth rate of a linear function at x is given by the expression $\frac{m}{mx + b}$. An exponential function has an amazing and unexpected relative growth rate.

1. For $b = 2$, use the up/down arrows at the lower left of the screen to move the point x along the axis. At the bottom of the page, observe the slope at x , the value of b^x , and the ratio $\frac{\text{slope at } x}{b^x}$.
 - a. What happens to the slope at x and the value of b^x when you move x to the right of 1?
 - b. What happens to the ratio when you move x to the right of 1?
 - c. What happens to the slope at x and the value of b^x when you move x to the left of 1?
 - d. What happens to the ratio when you move x to the left of 1?

2. Use the up/down arrows at the upper left to change the base of the exponential function. Set $b = 3$.
 - a. Move the value x along the axis and observe the ratio. Describe what happens to the relative growth rate as x varies.

 - b. Try other values for b . What happens to the relative growth rate as x varies, for any set value of b ?



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- c. As b increases, what happens to the relative growth rate of the exponential function?
- d. Is the relative growth rate ever 0? If so, for what value(s) of b ?
- e. Is the relative growth rate ever negative? If so, for what value(s) of b ?

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3. For any value of b , the constant relative growth rate of the exponential function $f(x) = b^x$ is called the natural logarithm of b , abbreviated $\ln(b)$. Use the arrow keys to change the value of b and complete the following table of corresponding values for $\ln(b)$.

b	0.1	0.2	0.4	0.5	1	2	2.5	3	5
$\ln(b)$									

- a. Explain what happens to the graph of $y = f(x) = b^x$ as b increases and is greater than 1.
 - b. Explain what happens to the graph of $y = f(x) = b^x$ as b becomes less than 1.
 - c. Explain the value of $\ln(1)$ geometrically.
 - d. A student claims that you can also find $\ln(b)$, the natural logarithm of b , by just looking at the slope of the graph of $y = f(x) = b^x$ at $x = 0$. Is this correct? Why or why not?
4. Find the following natural logarithms using the up/down arrows for these values of b .
 $\ln(2.6) = \underline{\hspace{2cm}}$ $\ln(2.7) = \underline{\hspace{2cm}}$ $\ln(2.8) = \underline{\hspace{2cm}}$
- There is a value b such that $\ln(b) = 1$. The exact value of this special number is labeled e .
5. The value of e lies between two of these values of b : 2.6, 2.7, and 2.8. Which two? Explain your reasoning.



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6. Is e smaller or larger than 2.75? Explain your reasoning.
7. Use this [Calculator](#) page to evaluate the natural logarithm in search of the closest three-decimal-place approximation to e you can make.
 - a. What is your (three-decimal-place) approximation?

$e \approx$ _____

Every exponential function of the form $f(x) = b^x$ has constant relative growth rate $\frac{f'(x)}{f(x)} = \ln(b)$. The natural exponential function has a constant relative growth rate of exactly 1.

- b. What is the base of the natural exponential function $f(x) = b^x$?
- c. What is its derivative $f'(x)$?