Math Objectives

- Students will investigate the effects parameters *a*, *h*, and *k* have on a given function.
- Students will generalize the effects that parameters *a*, *h*, and *k* have on any function.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- function
- parameter
- vertical stretch and vertical compression
- horizontal translation and vertical translation
- transformation
- scale factor

About the Lesson

- This lesson involves changing the sliders for *a*, *h*, and *k* on each page and observing the effects each has on the graphs of the functions.
- As a result, students will:
  - Use the completed table to make generalizations about the effects of *a*, *h*, and *k* on the graphs of any function.
  - Describe the transformations to a parent function using their generalizations.
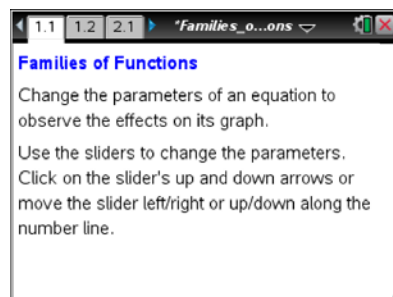
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- Use Class Capture to see if students understand how *a*, *h*, and *k* affect the graph.
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding.

Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Families of Functions

Change the parameters of an equation to observe the effects on its graph.

Use the sliders to change the parameters. Click on the slider's up and down arrows or move the slider left/right or up/down along the number line.

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- Families_of_Functions_ Student.pdf
- Families_of_Functions_ Student.doc

TI-Nspire document

- Families_of_Functions.tns

Discussion Points and Possible Answers

Tech Tip: If students have difficulty moving the point for *h* and *k*, make sure they have moved the cursor (arrow) until it becomes a hand (☜) getting ready to grab the point on the slider. Press ⌈ctrl⌉ ⌈☝⌉ to grab the point and close the hand (☝). Once the point is grabbed, use arrow keys to move it. When finished moving any slider or point, press ⌈esc⌉ to release.

Tech Tip: If students experience difficulty changing the slider for *a*, check to make sure that they are tapping the up or down arrow. If they have difficulty moving the point for *h* and *k*, check to make sure that they are touching the point. If a student double taps off of a slider, the function entry line might open. If that happens, have them minimize the keyboard and tap elsewhere on the screen to close the function entry line.

Teacher Tip: Students should change the sliders for each variable to determine what effects that variable has on each graph. When moving the points for *h* and *k*, the slider for *a* should be set to any value except zero.

| Page	Parent Function (Equation or Type)	Sketch of Parent Function	Effects of Parameter *a*	Effects of Parameter *h*	Effects of Parameter *k*
1.2	Quadratic $f(x) = a \cdot (x-h)^2 + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
2.1	Absolute Value $f(x) = a \cdot |x-h| + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
3.1	Square Root $f(x) = a \cdot \sqrt{x-h} + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
4.1	Exponential $f(x) = a \cdot 2^{x-h} + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
5.1	Logarithmic $f(x) = a \cdot \log(x-h) + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*
6.1	Cubic $f(x) = a \cdot (x-h)^3 + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*

7.1	Periodic (sine) $f(x) = a \cdot \sin(x - h) + k$	Teacher Observation	stretches or compresses the graph vertically	translates the graph left or right depending on the sign of *h*	translates the graph up or down depending on the sign of *k*

Move to page 1.2.

1. Given any function, describe the effects parameter *a* has on its graph when:

   a. |*a*| > 1

   Answer: The graph of the function is stretched vertically by that factor.
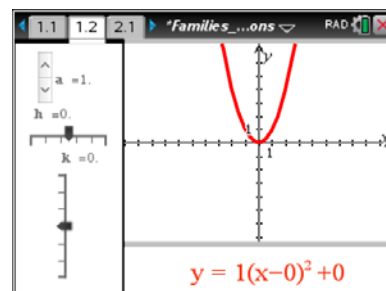
   b. 0 < |*a*| < 1

   Answer: The graph of the function is vertically compressed by that factor.

   c. *a* < 0

   Answer: The graph of the function is reflected over a horizontal line.

   d. *a* = 0

   Answer: The graph of the function becomes a horizontal line.

2. Given any function, describe the effects parameter *h* has on its graph when:

   a. *h* > 0

   Answer: The graph of the function is translated horizontally to the right that number of units.

   b. *h* < 0

   Answer: The graph of the function translated horizontally to the left that number of units.

   c. *h* = 0

   Answer: The graph of the function does not translate horizontally.

3. Given any function, describe the effects parameter k has on its graph when
   a. $k > 0$

   Answer: The graph of the function is translated vertically upward that number of units.

   b. $k < 0$

   Answer: The graph of the function translated vertically downward that number of units.

   c. $k = 0$

   Answer: The graph of the function does not translate vertically.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

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4. Given the following functions, describe the transformations on the parent function, $f(x)$.
   a. $f(x) = x^2$; $h(x) = 3(x - 4)^2 + 2$

   Answer: The graph of $f(x) = x^2$ is vertically stretched by a factor of 3. It is translated horizontally right 4 units and translated vertically up 2 units.

   b. $f(x) = x^3$; $g(x) = -(x - 1)^3$

   Answer: The graph of $f(x) = x^3$ is reflected over the x-axis and translated horizontally to the right 1 unit.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

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5. Given the following transformations, write the equation of the function.
   a. The graph of $f(x) = \sqrt{x}$ is reflected over the x-axis, vertically stretched by a factor of 2, and translated vertically down 1 unit.

   Answer: $g(x) = -2\sqrt{x} - 1$

   b. The graph of $f(x) = |x|$ is translated horizontally to the left 3 units and translated vertically up 5 units.

Answer: $g(x) = |x + 3| + 5$

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The effects the parameters *a*, *h*, and *k* have on the graphs of functions.
- How to describe the transformations on a given parent function.

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Note 1

Question 3, *Class Capture*: After students have explored the effects of all three variables, ask them to display a quadratic that is translated down 3 units and right 5 units. Take a Class Capture when everyone has done so. All quadratics should have a vertex at (5, –3). Some students might change only *h* and *k*, while others might change all three. Discuss why each is correct.

Note 2

Question 4, *Quick Poll*: Use a multiple-choice *Quick Poll* for students to share their answers.

4. a. $h(x) = 3(x - 4)^2 + 2$ has been translated

   A. left 4 units, down 2 units

   B. left 4 units, up 2 units

   C. right 4 units, down 2 units

   D. right 4 units, up 2 units

Students should discuss why choice D is the correct answer.