Math Objectives

- Students will graphically explore functions and their inverses.
- Students will find the inverse of a given function algebraically.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

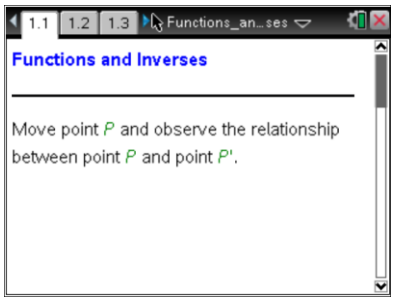
- function
- inverse function
- one-to-one function
- horizontal line test

About the Lesson

- This lesson involves grabbing and dragging point *P* along graphs of different functions to determine the relationship that exists between its ordered pairs and the ordered pairs of point *P′* that change as point *P* changes.
- Students will examine a set of data points. They will then examine a line and will use the ordered pairs to determine the equation of the line containing point *P* and the line containing point *P′*. They will then generate the graphs of two new lines and will write the equations of both. From this, the students will make a conjecture about how to find the inverse of a function algebraically.
- Students will examine the ordered pairs of a graph of a quadratic function and its inverse. They will be given the equation of the quadratic function in vertex form and will find the inverse of it algebraically.
- Students will algebraically find the inverses of three functions.

TI-Nspire™ Navigator™ System

- Use Screen Capture to check student understanding
- Use Quick Poll to ask for the equations.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:
Student Activity

- Functions_and_Inverses_Student.pdf
- Functions_and_Inverses_Student.doc

TI-Nspire document

- Functions_and_Inverses.tns

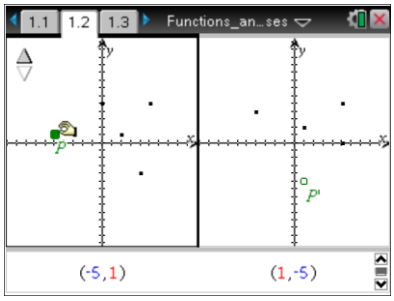
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Then press ctrl ☜ to grab the point and close the hand (✊). When finished moving the point, press esc.

Teaching Tip: Each page has the same type of graph (scatter plot, linear, quadratic etc). Use of the arrow sliders enables students to have different graphs (of the same type). Teachers may want to use this and have students try different ones. The answers are for the first graph given, so encourage students to answer the questions and then move on to other graphs of the same type.

Move to page 1.2.



1. Grab and drag point *P*.
   a. What changes and what remains the same?

   Answer: Point *P* moves and its ordered pairs change as it moves. Point *P′* moves when point *P* moves. The ordered pairs of point *P′* change as it moves. The actual data points do not change.

   b. Record the ordered pairs for point *P* and point *P′* in the tables below.
   Answer: Completed table is below.

Point *P*	
x	***y***
–5	1
0	5
5	5
2	1
4	–4

Point *P′*	
x	***y***
1	–5
5	0
5	5
1	2
–4	4

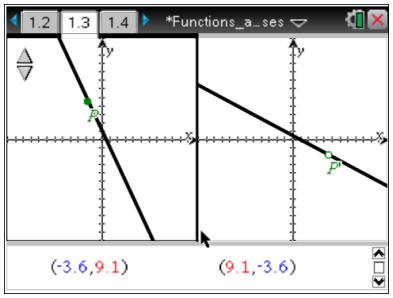
   c. Compare the ordered pairs for point *P* and point *P′*. How are they alike or different?

   Answer: All of the corresponding ordered pairs contain the same numbers. However, the *x*- and *y*-coordinates are interchanged.

Teaching Tip: By clicking on the arrows, other scatter plots will be displayed. Could use this to look at relationships of the points.

Teaching Tip: When dragging point P, dependent on the graph, either a "zero" will show up or a "minimum" might show up. These points might be useful when finding the equations.

Move to page 1.3.



2. Grab and drag point *P*.
   a. What changes and what remains the same?

Answer: Point *P* moves and its ordered pairs change as it is moved. Point *P'* moves when point *P* is moved. The ordered pairs of point *P'* change as it is moved. The lines do not change.

   b. Record some of the ordered pairs for point *P* and point *P'* in the tables below.

Answer: Answers will vary but could include some of the ordered pairs in the tables below.

Point *P*	
x	***y***
4.9	-7.9
-5	12
-1.8	5.6
1	0
3.1	-4.2

Point *P'*	
x	***y***
-7.9	4.9
12	5
5.6	-1.8
0	1
-4.2	3.1

   c. Compare the ordered pairs for point *P* and point *P'*. How are they alike or different?

Answer: All of the corresponding ordered pairs contain the same numbers. However, the *x*- and *y*-coordinates are interchanged.

   d. Write the equation that represents each set of ordered pairs. Show your work. How are the equations alike or different?

Answers:

The equation for the line containing point *P* is y=-2x+2.

The equation for the line containing point *P'* is $y = -\frac{1}{2}x + 1$.

The slopes are reciprocals. The *y*-intercepts are different.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

3. Click the ∇or ∇ on the slider to change the equation.
   a. Is the new graph a function? How do you know?

Answer: Although students may have different graphs of lines, they will all be functions because every *x*-value has exactly one corresponding *y*-value. Students may also say that it passes the vertical line test.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

   b. Is the graph on the right an inverse of the first? Explain how you know.

Answer: Yes. The graph on the right is an inverse of the first graph because the ordered pairs of the inverse have been obtained by interchanging the *x*- and *y*-values of the original ordered pairs.

   c. Write the equations of both graphs. How are the equations alike or different?

Answer: Answers will vary. However, all equations should be linear.

   d. How could you find the inverse of any function algebraically?

Answer: You could find the inverse of any function algebraically by interchanging *x* and *y* in the equation and solving for *y* again.

Teacher Tip: At this point, some students may still have trouble understanding that this is all it would take to find the inverse of a function. If more practice is needed, have students click on the slider in the top-left corner of the screen to generate more examples.
Also of particular interest is any equation that is horizontal (on the left side). Some review might be necessary to determine the equations.

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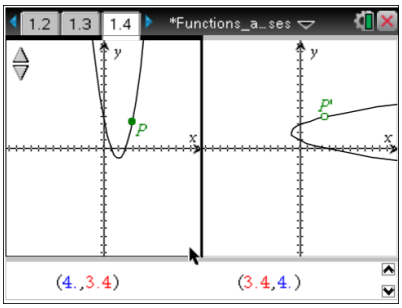

4. Grab and drag point *P*.
   a. What changes and what remains the same?

      Answer: Point *P* moves and its ordered pairs change as it is moved. Point *P'* moves when point *P* is moved. The ordered pairs of point *P'* change as it is moved. The parabolas do not change.

   b. Record some of the ordered pairs for point *P* and point *P'* in the tables below.

      Sample answers: Answers will vary but could include some of the ordered pairs in the tables below.

Point *P*	
x	***y***
.03	3.4
1	0
2	-1.2
3	0
4	3.4

Point *P'*	
x	***y***
3.4	.03
0	1
-1.2	2
0	3
3.4	4

   c. Compare the ordered pairs for point *P* and point *P'*. How are they alike or different?

      Answer: All of the corresponding ordered pairs contain the same numbers. However, the *x*- and *y*-coordinates are interchanged.

   d. Is the graph of the inverse a function? Why or why not?

      Answer: The graph of the inverse is not a function because every *x* has more than one *y*-value. Some students might also say that it fails the vertical line test.

   e. The equation for the graph that includes point *P* is $y = 1.2(x - 2)^2 - 1.2$. Write the equation for the graph containing point *P'*.

      Answer: Students should write the equation for the graph of the inverse. $y = 2 \pm \sqrt{.833x + 1}$

> **Teacher Tip:** If students accidentally grab the tic marks on any of the pages, undo (ctrl Z) if possible. Students can also get out of the file and then reopen it.

5 Find the inverse of each equation algebraically.

a. $y = \frac{3}{5}x - 6$

Answer: $y = \frac{5}{3}x + 10$

b. $y = 2(x-3)^2 + 1$

Answer: $y = 3 \pm \sqrt{\frac{x-1}{2}}$ or $y = 3 \pm \sqrt{0.5x - 0.5}$

c. $y = \frac{1}{2}(x-8)^3 + 3$

Answer: $y = 8 + \sqrt[3]{2x-6}$

6 a. Is $y = 2^x + 2$ the inverse of $y = x^2 - 2$? Why or why not?

Answer: Students should recognize that the inverse of x^2 is $\pm\sqrt{x}$ and, in this particular case, the 2 should be under the radical symbol.

b. Is $y = -3x - \frac{2}{3}$ the inverse of $y = \frac{1}{3}x + \frac{3}{2}$? Why or why not?

Sample answers: Students should recognize that 3 is the inverse of $\frac{1}{3}$ because it is the coefficient of x. When taking the inverse of a function, multiplication or division does not result in a sign change. A common misconception among students is that the inverse of a constant fraction is the reciprocal.

Extension

The TI-Nspire document contains two more pages that contain a cubic and its inverse and an exponential and its inverse. These pages are included in case you would like to have students explore these functions. In addition, every page in the TI-Nspire document contains a slider that will generate new examples if students need more practice.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to find the inverse of a function algebraically.

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Note 1

Question 2d, *Quick Poll*. Use a Quick Poll of equation type to ask for answers to 2d.

Note 2

Question 3a, *Screen Capture*: The teacher may want to use *Screen Capture* to show the students graphs and to discuss the use of the vertical line test to determine whether a graph is a function.