



Math Objectives

For the graphs of $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$, students will:

- Infer why the conditions $a > 0$ and $a \neq 1$ are necessary for the function to be logarithmic.
- Determine that for $a > 1$ the function is increasing and for $0 < a < 1$ the function is decreasing.
- Determine the x -intercept, y -intercept, domain, range, and asymptotes.
- Determine that for $a > 1$ the function approaches ∞ as x approaches ∞ and that for $0 < a < 1$ the function approaches $-\infty$ as x approaches ∞ .

Students will construct viable arguments & critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

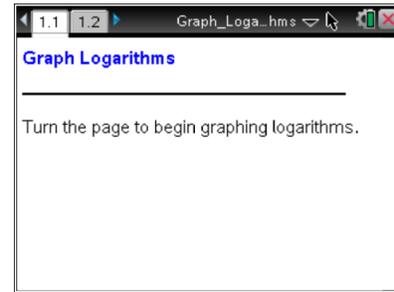
- logarithm function
- end behavior
- intercepts
- domain and range
- asymptotes
- increasing and decreasing functions
- extraneous solution

About the Lesson

- The time varies for this activity depending on whether students create the TI-Nspire document file or use the pre-constructed file.
- Students will investigate the graphs of the family of logarithm functions $f(x) = \log_a x$, by changing the a -value over the interval $0 \leq a \leq 4$.
- As a result, students will:
 - Infer why the conditions $a > 0$ and $a \neq 1$ are necessary.
 - Determine how the value of a affects the increasing or decreasing behavior of the function.
 - Determine the x -intercept, domain, range, and asymptotes.
 - Describe the end behavior.

TI-Nspire™ Navigator™ System

- Use Live Presenter to demonstrate how to use sliders.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing **ctrl** **G**.

Lesson Materials:

Student Activity

Graph_Logarithms_Student.pdf
 Graph_Logarithms_Student.doc

Optional Materials:

Graph_Logarithms_Create.doc
 Graph_Logarithms_Create.pdf

TI-Nspire document

Graph_Logarithms.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

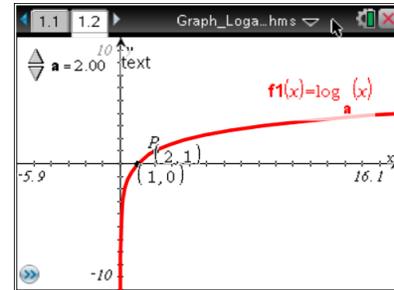


- Use Screen Capture to examine patterns that emerge.
- Use Quick Poll to assess students' understanding throughout the activity.

Discussion Points and Possible Answers

Move to page 1.2.

1. Explore several different a -values by clicking Δ or ∇ .



TI-Nspire Navigator Opportunity: Live Presenter

See Note 1 at the end of this lesson.

- a. Set $a = 1$. Describe the graph.

Answer: It is nonexistent.

- b. By definition, for the logarithmic function $f(x) = \log_a(x)$, a cannot equal 1. What mathematical reason can you give for this restriction?

Answer: When you have $\log_1 x = y$ this is equivalent to $1^y = x$, and 1 to any power is 1.

Therefore, the possible solution is the vertical line $x = 1$; since this is not a function, $x = 1$ is an extraneous solution.

- c. Set $a = 0$. Describe the graph.

Answer: It is nonexistent.

- d. By definition, for the logarithmic function $f(x) = \log_a(x)$, a cannot equal 0. What mathematical reason can you give for this restriction?

Answer: When you have $\log_0 x = y$ this is equivalent to $0^y = x$, and 0 to any power is 0.

Therefore, the possible solution is the vertical line $x = 0$; since this is not a function, $x = 0$ is an extraneous solution.



2. Explore several different a -values by clicking Δ or ∇ .
- a. For what a -values is the function increasing? Why?

Answer: When $a > 1$, $f(x) = \log_a(x)$ is increasing. This is because if $y = \log_a x$ then $a^y = x$. When $a > 1$, as the power increases the resulting value increases. For example, $\log_2 8 < \log_2 16$ because $8 = 2^3$, $16 = 2^4$, and $3 < 4$.

- b. For what a -values is the function decreasing? Why?

Answer: When $0 < a < 1$, $f(x) = \log_a(x)$ is decreasing. This is because if $y = \log_a x$ then $x = a^y$. When $0 < a < 1$, as the power decreases the resulting value decreases. For example, $\log_{0.5} 2 > \log_{0.5} 4$ because $2 = (0.5)^{-1}$, $4 = (0.5)^{-2}$, and $-1 > -2$.

TI-Nspire Navigator Opportunity: Screen Capture
See Note 2 at the end of this lesson.

3. Explore several different a -values by clicking Δ or ∇ .
- a. For each a -value, identify the x -intercept of the function. Interpret your results.

Answer: The x -intercept is always $(1, 0)$ because $\log_a 1 = 0$ for all a , which can be written in exponential form $a^0 = 1$.

- b. When $a > 0$, why is there no y -intercept?

Answer: For there to be a y -intercept, then x would be 0. This implies that $\log_a 0 = y$ or $a^y = 0$, which is impossible.

- c. For each a -value, what part of point P remains the same? Interpret your results.

Answer: For all a -values, the point P has the coordinates $(a, 1)$ since $\log_a a = 1$ can be written in exponential form $a^1 = a$.

TI-Nspire Navigator Opportunity: Screen Capture
See Note 3 at the end of this lesson.

4. Explore several different a -values by clicking Δ or ∇ , such that $a > 1$.
- a. What does $f(x)$ approach as x approaches ∞ ? Explain.



Answer: ∞ ; As you input larger and larger positive values for x , $f(x)$ gets larger and larger.

- b. What does $f(x)$ approach as x approaches 0? Explain.

Answer: $-\infty$; As you input positive x -values that approach 0, $f(x)$ gets smaller and smaller.

- c. What is the equation of the vertical asymptote?

Answer: $x = 0$

TI-Nspire Navigator Opportunity: Screen Capture

See Note 4 at the end of this lesson.

5. Explore several different a -values by clicking Δ or ∇ , such that $0 < a < 1$.

- a. What does $f(x)$ approach as x approaches ∞ ? Explain.

Answer: $-\infty$; As you input larger and larger positive values for x , $f(x)$ get smaller and smaller.

- b. What does $f(x)$ approach as x approaches 0? Explain.

Answer: ∞ ; As you input positive x -values that approach 0, $f(x)$ gets larger and larger.

- c. What is the equation of the vertical asymptote?

Answer: $x = 0$

TI-Nspire Navigator Opportunity: Screen Capture

See Note 5 at the end of this lesson.

6. Find the domain and range for the family of logarithmic functions $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$.

Answer: The domain is $(0, \infty)$, and the range is $(-\infty, \infty)$.

TI-Nspire Navigator Opportunity: Quick Poll and Live Presenter

See Note 6 at the end of this lesson.



7. Gail believes $f(x) = \log_a x$ will eventually intersect the y -axis. Is she correct? Why or why not?

Answer: Gail is incorrect. The function will never cross the y -axis. It will only approach the y -axis.

TI-Nspire Navigator Opportunity: Quick Poll

See Note 7 at the end of this lesson.

8. Judy believes $f(x) = \log_a x$ has a horizontal asymptote. Is she correct? Why or why not?

Answer: Judy is incorrect. When $x > 1$, the function decreases very slowly when $0 < a < 1$ and increases very slowly when $a > 1$.

TI-Nspire Navigator Opportunity: Quick Poll

See Note 8 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that for the graph of $f(x) = \log_a x$:

- The conditions $a > 0$ and $a \neq 1$ are necessary.
- When $a > 1$, the function is increasing and when $0 < a < 1$, the function is decreasing.
- The x -intercept is always $(1, 0)$ and there is never a y -intercept.
- When $a > 1$, the function approaches ∞ as x approaches ∞ , and the function approaches $-\infty$ as x approaches 0 .
- When $0 < a < 1$, the function approaches $-\infty$ as x approaches ∞ , and the function approaches ∞ as x approaches 0 .
- The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
- The function has a vertical asymptote of $x = 0$.

TI-Nspire Navigator

Note 1

Question 1, Live Presenter: You may want to demonstrate how to change the a -values by clicking the arrows using *Live Presenter*.

Note 2

Question 2a and 2b, Screen Capture: Take a *Screen Capture* of page 1.2 where students are on different a -values. As a class, discuss the various cases that occur.



Note 3

Question 3a and 3b, Screen Capture: Take a *Screen Capture* of page 1.2 where students are on different a -values. As a class, discuss the various cases that occur.

Note 4

Question 4a–4c, Screen Capture: Take a *Screen Capture* of page 1.2 where students are on different a -values. As a class, discuss the various cases that occur.

Note 5

Question 5a–5c, Screen Capture: Take a *Screen Capture* of page 1.2 where students are on different a -values. As a class, discuss the various cases that occur.

Note 6

Question 6, Quick Poll (Open Response): Send two *Open Response Quick Polls*, asking students to submit their domain and range. If students struggle to identify the domain and range, consider taking a *Screen Capture* and discussing how, for all of the graphs, the possible x - and y -values are similar.

Note 7

Question 7, Quick Poll (Open Response): Send an *Open Response Quick Poll*, asking students to submit their answer to question 7. For students having difficulty, use Live Presenter and change the a -value to 0.9 and the y -max to 200. Repeatedly zoom in on the “ y -intercept.”

Note 8

Question 8, Quick Poll (Open Response): Send an *Open Response Quick Poll*, asking students to submit their answer to question 8.