



Math Objectives

- Students will be able to identify a restricted domain interval and use function translations and dilations to choose and position a portion of the graph accurately in the plane to match a given quadratic graph.
- Students will be able to identify a function and suitable domain interval that will approximate the shape of a given curve in the plane.
- Students will model with mathematics. (CCSS Mathematical Practice)

Vocabulary

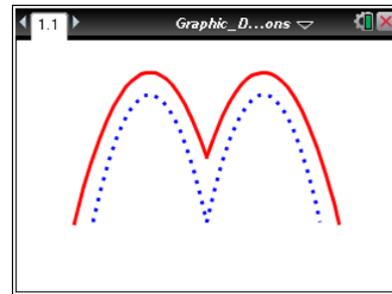
- function
- domain
- transformation

About the Lesson

- This lesson involves creating an image using transformed functions with restricted domains.
- As a result, students will:
 - Graph functions on a TI-Nspire™.
 - Restrict the domain of a function on the TI-Nspire™.
 - Transform a variety of functions.
 - Restrict the domain of a function to achieve a desired result.

TI-Nspire™ Navigator™ System

- Use **File Transfer** to send student documents.
- Use **Live Presenter** to monitor students and allow them to present.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire™ document
- Open a document
- Move between pages
- Graph a function with restricted domain

Tech Tips:

- Make sure the font size on your TI-Nspire™ handheld is set to Medium.
- You can hide the entry line by pressing **ctrl** **G**.

Lesson Materials:

Student Activity

Graphic_Designing_with_Transformed_Functions_Student.pdf

TI-Nspire document

Graphic_Designing_with_Transformed_Functions.tns
Graphic_Designing_with_Transformed_Functions_Emoticon.tns

Visit www.mathnspired.com for lesson updates.



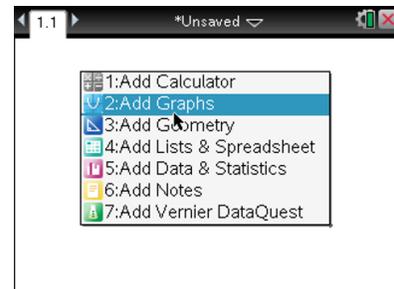
Discussion Points and Possible Answers

Teacher Tip: You may wish to break this activity up over several class periods, depending on the length of your class and/or where you are in the year. Re-creating the letter *M* may be done after transformations of quadratic equations, but designing an emoticon (a computer term for an icon that shows emotion) is better suited for a project at the end of the year to tie together several concepts from the course, including functions, domain, range, zeros, and transformations.

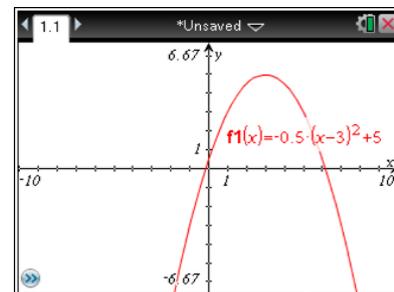
TI-Nspire™ Navigator™ Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Tech Tip: The step-by-step directions for creating the letter *M* are repeated in these Teacher Notes, including the answers to the questions posed to the students. You are encouraged to work through the steps before assigning the activity to students in order to ensure your comfort level with the technology. The TI-Nspire™ document **Graphic_Designing_with_Transformed_Functions.tns** is also available for your reference.

1. Press  on > **New Document > Add Graphs.**

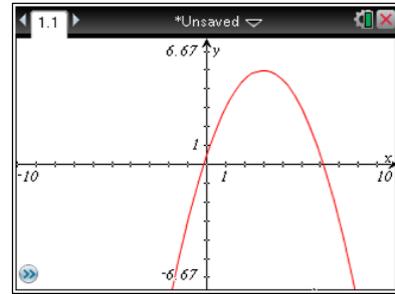


2. Click in the function entry line. In **f1(x)**, enter $-0.5(x - 3)^2 + 5$ to create the right side of a curved letter *M*.





3. The label for the function may appear, depending on the settings of your handheld. To hide the label, move your cursor to it and press **ctrl** **menu** > **Hide**.

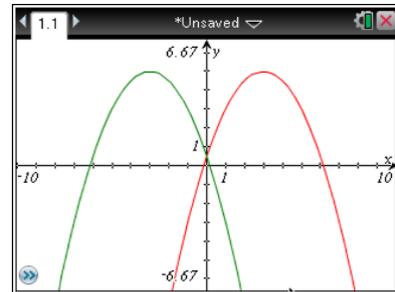


4. What change or changes need to be made to the equation for $f_1(x)$ to create the left half of the letter M ?

Answer: To create the left half of the letter, the equation $-0.5(x - 3)^2 + 5$ for $f_1(x)$ needs to be changed only slightly, to $-0.5(x + 3)^2 + 5$ for $f_2(x)$, to replicate a parabola that is similar but horizontally shifted.

Teacher Tip: This activity is designed to help students better understand horizontal transformations. Many students struggle with this concept, and working with these functions may help enhance understanding. Also, the letter M is symmetrical, which makes working with functions to create its shape easier. You may want to point this out to students.

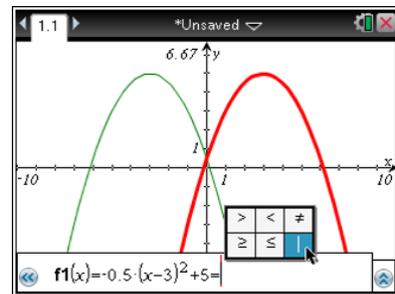
5. Press **ctrl** **G** to get the function entry line to reappear. Use the answer to question 4 to make the left side of the letter, using $f_2(x)$. Does your graph match the picture to the right? If it is incorrect, make corrections to $f_2(x)$ until it does match the picture to the right.



6. Hide the label for $f_2(x)$ as you did before.

Teacher Tip: Students may not initially get the correct formula for $f_2(x)$, but using this approach will help them visualize to learn about their mistake and correct it.

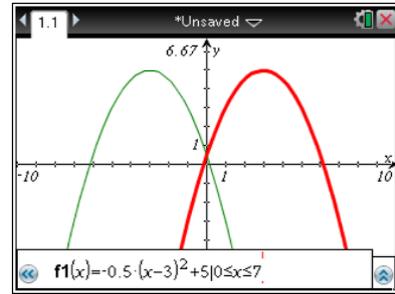
7. Now the shape is beginning to resemble the letter M , but the graphs cross at the y -axis. To make things look better, domain restrictions can be used. Press **ctrl** **G** to make the function entry line reappear. Scroll back up to $f_1(x)$. We want to display the graph of $f_1(x)$ only from 0 to 7 along the x -axis. Press **ctrl** **=** to enter the menu of symbols (shown to the right). Choose the “such that” symbol, $|$, from the menu to begin the restriction.



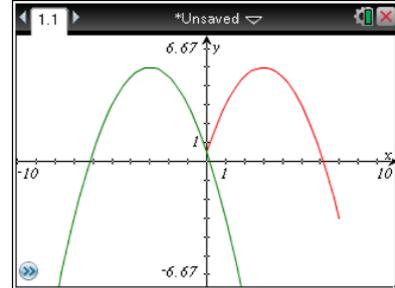


8. Type the restrictions for the domain of $f_1(x)$ as $0 \leq x \leq 7$.

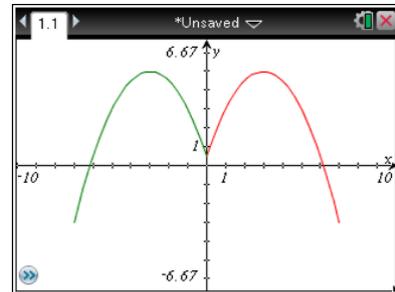
Press **ctrl** **=** to access the inequality symbols. Press **enter**.



9. After the restriction has been added, the function will graph only between the x-values of 0 and 7, inclusive. This will better represent the right side of the letter *M*.



10. Again using what you know about symmetry and functions, restrict the domain of $f_2(x)$ to better represent the left side of the letter *M*.



11. What are the domain restrictions for $f_2(x)$ to make it appear like the picture in step 10?

Answer: The appropriate restriction for the function on the left is $-7 \leq x \leq 0$.

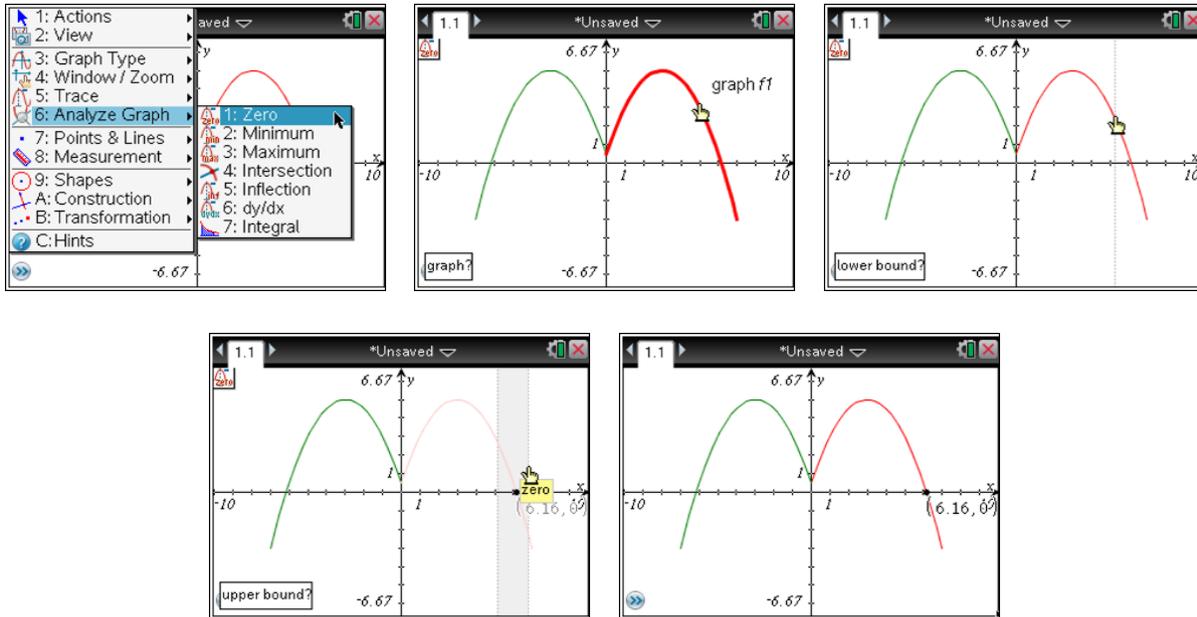
Teacher Tip: This solution may not be readily apparent to some students. Often they mix up negative numbers. If they type the domain restriction in the handheld incorrectly, they will not achieve the desired result. You may want to display a “mistaken” restriction on the domain using Teacher Edition to help students safely see a faulty restriction on the domain.

12. Suppose you wanted the letter *M* to begin on the left at $y = 0$ and end on the right at $y = 0$. What would the domain restrictions for $f_1(x)$ and $f_2(x)$ have to be to make that happen?

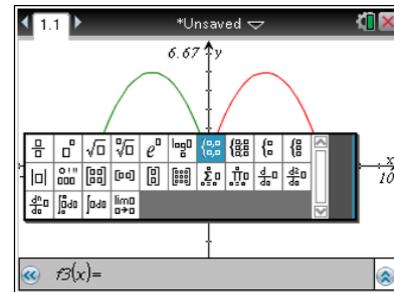
Answer: The quadratic equation on the right should be restricted to between 0 and 6.16 and the one on the left to between -6.16 and 0.



Tech Tip: You may wish to do this using the **Zero** feature. Press **Menu > Analyze Graph > Zero**. Move the cursor to the desired graph and press **enter**. Choose the left and right bounds so that the zero is between them, by pressing **enter** in each location. This will give the zero. Based on symmetry, the zero of the function in $f_2(x)$ should be -6.16 . Encourage students to think about this instead of performing the zero action. You may want to hide the point after finding it.



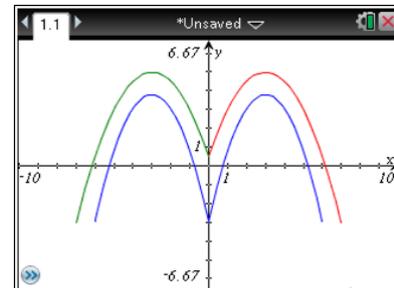
13. Piecewise functions can also be used when the domains are mutually exclusive (i.e., when they don't overlap). Press **ctrl** **G** to make the function entry line reappear. Then press **|x|** to enter the math template. Select the tool for piecewise functions (the seventh icon in the top row) to enter a piecewise function with two conditions.



Note: The next icon to the right is for piecewise functions with the number of conditions defined by the user.

14. Using piecewise functions can be very efficient and can make restricting domains easier. The equation below for $f_3(x)$ represents the beginning of the lower part of the letter *M*.

$$f_3(x) = \begin{cases} -0.75(x+3)^2 + 3.8 & -6 \leq x \leq 0 \\ \text{_____} & \text{_____} \end{cases}$$





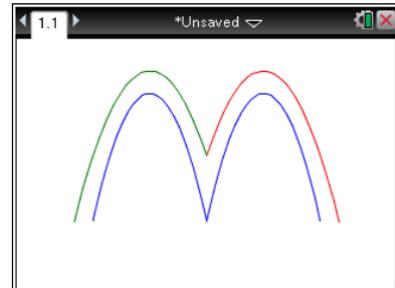
The first part of the piecewise function is for the left portion of the letter. Hide the equation as before.

15. What equation and restriction would need to be added to create the right half of the lower part of the letter? Do your equation and restriction work? Why or why not?

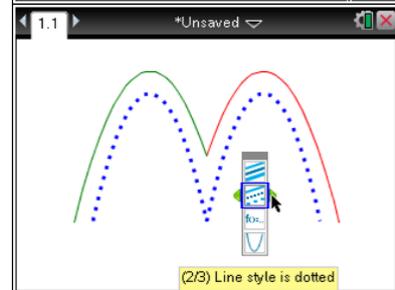
Answer: $-0.75(x - 3)^2 + 3.8$ should be restricted to $0 < x \leq 6$

16. If they do not work, go back and correct them until your shape matches the image in step 14.

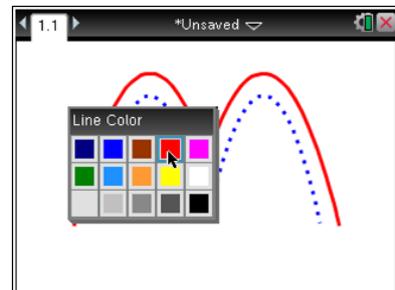
17. To improve the result, hide the coordinate axes. Move the cursor to the axes and press **ctrl** **menu** > **Hide/Show** > **Hide Axes** to keep only the letter *M* on the screen. You can also hide the chevron in the lower-left corner. Move the cursor to the chevron and press **ctrl** **menu** > **Hide Chevron**.



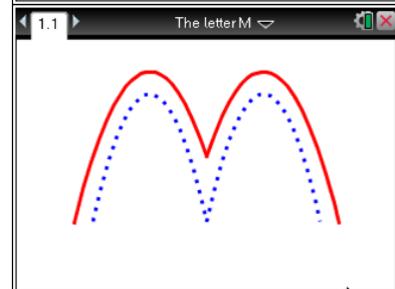
18. If you desire, you can change the graph's attributes including the line weight (thickness) or line style (dotted, dashed, etc.). Move the cursor to the graph and press **ctrl** **menu** > **Attributes**. Select the desired options using **▲** **▼** **▶** **◀**, and press **enter** to accept the changes.



19. If you desire, you can change the graph's color. Move the cursor to the graph and press **ctrl** **menu** > **Color** > **Line Color**.



20. You can save your work by pressing **doc** > **File** > **Save As**. Type in a file name and press **tab** until you are on the **Save** button. Press **enter** to save the file.





21. What change(s) would have to be made to each of the equations for $f_1(x)$, $f_2(x)$, and $f_3(x)$ to make the letter *W* instead of *M*?

Answer: No horizontal shifts would be necessary. The leading coefficients would have to become positive, and the vertical shifts would change to negative. The following would be acceptable answers:

$$f_1(x) = 0.5(x - 3)^2 - 5$$

$$f_2(x) = 0.5(x + 3)^2 - 5$$

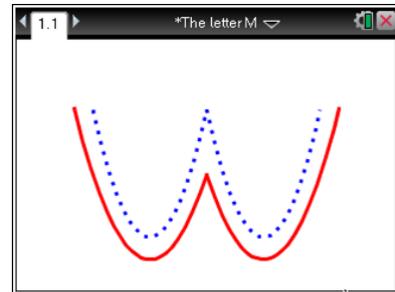
$$f_3(x) = \begin{cases} 0.75(x+3)^2 - 3.8 & -6 \leq x \leq 0 \\ 0.75(x-3)^2 - 3.8 & 0 < x \leq 6 \end{cases}$$

22. What change(s) would have to be made to each of the domain restrictions for $f_1(x)$, $f_2(x)$, and $f_3(x)$ to make the letter *W* instead of *M*?

Answer: No changes to the domains are necessary.

Teacher Tip: Question 22 is designed to drive home the fact that the domain involves the x -values. Changing the transformations of the functions while preserving the domains does not change which portions of the functions are defined. Students may struggle with this, but they should discover it if they attempt to change the domain restrictions. If they do not, then you should point it out to them.

23. Test your answers to questions 21 and 22 to see if you get a picture similar to the one to the right.



Extension: Create Your Own Emoticon Activity

TI-Nspire™ Navigator™ Opportunity: *File Transfer*
See Note 2 at the end of this lesson.

Your teacher will send you a blank TI-Nspire™ document with a place to type your name. Your task is to design your own emoticon using functions with domain restrictions.

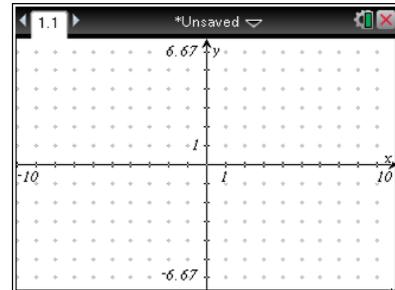


The requirements:

- You must use at least four different types of functions. Selections may include, but are not limited to, linear, quadratic, absolute value, exponential, logarithmic, radical, power, rational, conic, and trigonometric functions.
- Each function must be recorded in the table below. For each function, include its domain and an identification of which part of the figure it creates.
- You must save your work the document name "Emoticon." The teacher will collect it for grading. Be prepared to show your finished product to the class.

Teacher Tip: You may wish to change the requirements to include more functions or to have students list the range. The use of at least four functions is recommended to get students to go beyond linear, quadratic, and polynomial. A sample TI-Nspire™ document (**Graphic_Designing_with_Transformed_Functions_Emoticon_Example.tns**) is available so that you can see a finished product. This activity makes a good project for the end of the course to tie together transformations of several functions, but if you would prefer to do it earlier in the year, it could be done using any subset of functions (e.g., linear and quadratic only).

- Sketch a picture of your emoticon on the blank screen below.



Potential functions or equations students may use are listed below (h represents a horizontal shift, k represents a vertical shift, and r represents the radius of a circle):

Linear: $A(x) = mx + b$

Quadratic: $B(x) = ax^2 + bx + c$ OR $C(x) = a(x - h)^2 + k$

Power: $D(x) = a(x - h)^p + k$

Absolute value: $E(x) = a|x - h| + k$

Exponential: $F(x) = k + a \cdot b^{x-h}$

Logarithmic: $G(x) = a \cdot \log_m(x - h) + k$

Radical: $H(x) = k + a\sqrt[n]{x - h}$

Trigonometric: $I(x) = a \cdot \sin(b(x - h)) + k$ OR $J(x) = a \cdot \cos(b(x - h)) + k$

Polynomial: $K(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + \dots + a_1 \cdot x + a_0$



Conic Sections (will need to be solved for y to graph in the TI-Nspire™):

Circle: $(x - h)^2 + (y - k)^2 = r^2$

Hyberbola: $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

Ellipse: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

Parabola: $(x - h)^2 = 4p(y - k)$ OR $(y - k)^2 = 4p(x - h)$

- 2. Record your equations in the table below along with their domain restrictions. Be sure to identify which part of the image each function creates.

Table with 3 columns: Function Equation, Domain Restriction, What It Creates in the Figure. The table contains 12 empty rows for student input.

Teacher Tip: Students should be given time in class to work if they do not have access to handhelds at home. If students own their own devices, you may consider this a take-home project. The above table should be collected as evidence to support the student work. Grading the results should include comparing the student product to the functions and domain restrictions in the table above.

TI-Nspire™ Navigator™ Opportunity: Live Presenter
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of this project, the teacher should ensure that students understand:

- How to graph a function with and without a restricted domain.
• How to use transformations of various functions to match a desired result.



TI-Nspire™ Navigator™

Note 1

Question 1, *Live Presenter*: Use the **Live Presenter** features of TI-Nspire™ Navigator™ to monitor students' work and to have them share their results at the end of the project. If time allows, you can make each student the presenter so that each may share his or her finished product. Encourage students to identify the functions that they used.

Note 2

Question 23, *File Transfer*: Send the students the document

Graphic_Designing_with_Transformed_Functions_Emoticon.tns using the file transfer feature of TI-Nspire™ Navigator™. This is a document with a place for the students to record their names and to design their own emoticons. Students will open and work with this file. After they have completed their work, have them save the file containing their work. You can then collect it, with their work identified by name.

Note 3

Extension – Question 2, *Live Presenter*: Use the **Live Presenter** features of TI-Nspire™ Navigator™ to allow students to show their finished projects to the class.