EXPLORATIONS ™

Activity 12

Graphing Relationships

Objectives

- Realize information about a graph based on the first and second derivatives
- Learn that a function's derivative is positive when the function increases and negative when the function decreases
- Learn that the second derivative is positive when the graph is concave upward and negative when the graph is concave downward

Materials

- TI-84 Plus / TI-83 Plus

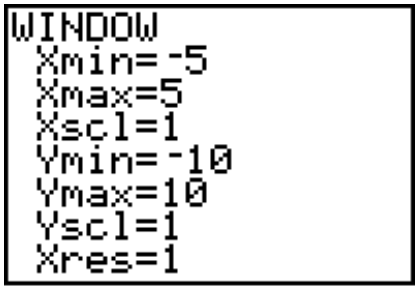
Introduction

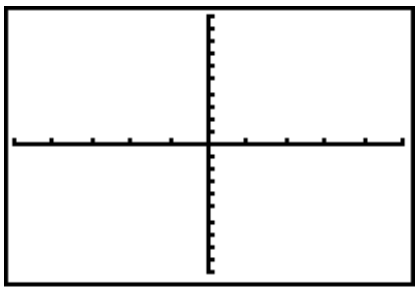
The first and second derivative of a function can provide a great deal of information about the function itself. In this activity, you will examine the graphs of functions along with their derivatives and look for relationships that exist.

Exploration

Round your answers throughout this activity to the nearest hundredth.

1. Input the equation **Y1 = $x^3 - 1.5x^2 - 6x + 2$** into the **Y=** editor. Make sure that the graphing handheld is in **Radian** mode. Set the viewing window as shown. Sketch the graph.





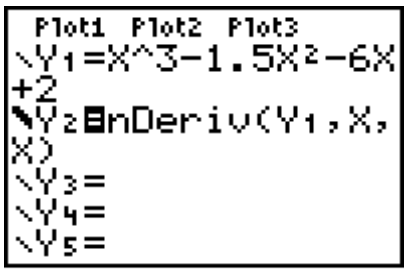
2. At what *x*-values do the relative maximum and relative minimum values of the function occur? Use the **CALC Menu** to find the exact values.

3. Over what intervals of *x* does the function increase?

4. Over what intervals of *x* does the function decrease?

5. What kind of values should the derivative have over an interval where the function increases? Explain.

6. Find the derivative of the function. The operation **nDeriv** is in the **MATH Menu** or in the **CATALOG**. Change the graph style by moving the cursor left of **Y2** and pressing ENTER. This will cause the derivative to be drawn with a thicker line.



7. What are the *x*-values of the derivative where a relative maximum or minimum of the original function occurs?

8. Is the derivative positive or negative over the intervals where the function increases?

9. Is the derivative positive or negative over the intervals where the function decreases?

10. When the derivative crosses the *x*-axis from positive to negative, what happens to the graph of the function?

11. When the derivative crosses the *x*-axis from negative to positive, what happens to the graph of the function?

12. Over what intervals of *x* does the derivative increase?

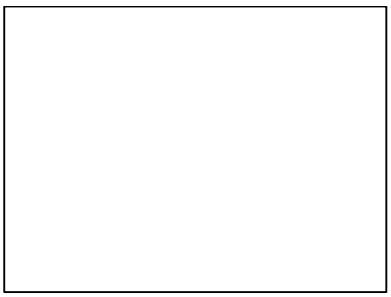
13. Over what intervals of *x* does the derivative decrease?

14. If the first derivative is increasing, is the second derivative positive or negative?

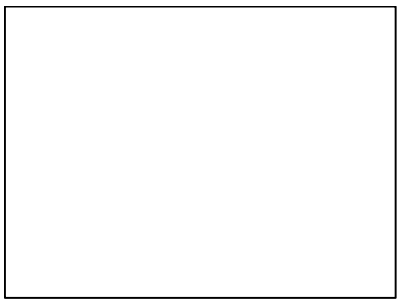
15. If the first derivative is decreasing, is the second derivative positive or negative?

16. Graph the second derivative. Examine it where the first derivative is increasing and decreasing. Does it match your predictions? Explain any differences, and state any additional observations.

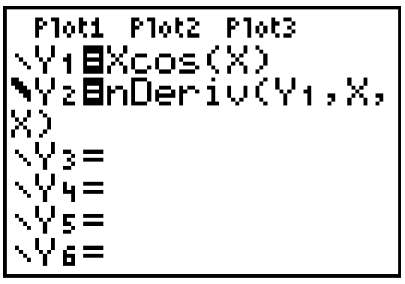
17. The graph of a function is concave upward when the graph of the first derivative is increasing. Sketch a portion of the graph $y = f(x)$ that is concave upward. What is true about the graph of $f'(x)$ where the graph of $y = f(x)$ is concave upward?



18. The graph of a function is concave downward when the graph of the first derivative is decreasing. Sketch a portion of the graph $y = f(x)$ that is concave downward. What is true about the graph of $f'(x)$ where the graph of $y = f(x)$ is concave downward?



19. Examine the equation $Y_1 = x \cos(x)$ over the interval $[-5, 5]$.



a. For what x-values is the function increasing?

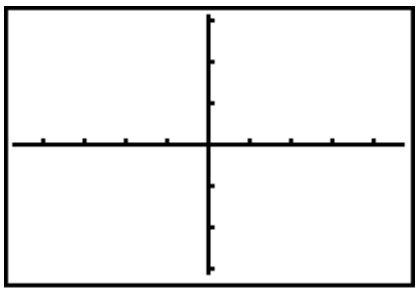
b. For what x-values is the function decreasing?

c. For what x-values is the function concave upward?

d. For what x-values is the function concave downward?

20. Suppose that you have only an equation for the derivative of a function. The derivative of $f(x)$ is:

$$f'(x) = \frac{x^2 - 1}{x^2 + 1}$$



Graph this in the **ZDecimal** viewing window. Sketch the graph.

a. For what x-values does the function $y = f(x)$ increase? Explain.

b. For what x-values does the function $y = f(x)$ decrease? Explain.

c. For what x-values is the function $y = f(x)$ concave upward? Explain.

d. For what x-values is the function $y = f(x)$ concave downward? Explain.

21. Summarize at least three main concepts that you explored in this activity.