Teacher Notes

Activity 12

Graphing Relationships

Objectives

- Realize information about a graph based on the first and second derivatives
- Learn that a function's derivative is positive when the funtion increases and negative when the function decreases
- Learn that the second derivative is positive when the graph is concave upward and negative when the graph is concave downward

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 60 minutes

Abstract

In this activity, students will graph functions along with their derivatives and look for relationships. They should recognize that the first derivative is positive when the function increases and negative when the function decreases. They should also observe that the second derivative is positive when the graph of the function is concave upward and negative when the graph is concave downward.

Management Tips and Hints

Prerequisites

Students should be:

- able to take derivatives.
- familiar with the numerical derivative key on the graphing handheld as well as the **CALC Menu**.
- familiar with the terms *relative maximum* and *relative minimum*.

Student Engagement

Students can complete this activity independently but may benefit from the discussion that occurs when working in a small group.

Evidence of Learning

Students should be able to understand how the first derivative provides information about where the function is increasing or decreasing and how the second derivative provides information about the concavity of the graph.

Common Student Errors/Misconceptions

Many students can understand that when a function is increasing, the derivative is positive, but they cannot correctly interpret what that means on a graph. Instead of understanding that this means the graph is always above the *x*-axis, some students may think that the derivative is increasing.

Teaching Hints

The numerical derivative is located in the **MATH Menu** or in the **CATALOG**. It requires three arguments: the function, the variable name, and the value at which the derivative should be found. When graphing the numerical derivative, the arguments are **nDeriv**(function name, X, X). The value at which the derivative should be calculated is X because the derivative is plotted for all values of *x* in the viewing window.

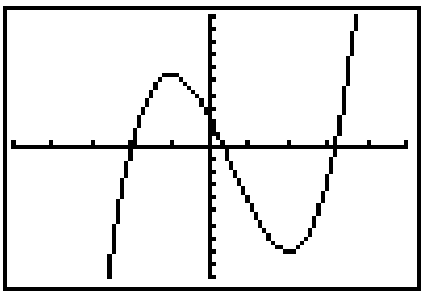
Begin with a graph such as $y = \cos(x)$. Have students examine the derivative or slope of the function at each point as they move through an interval where the function is increasing. Ask students what a graph of those derivatives at each point would look like. This activity will enhance their understanding of the relationship between the first and second derivative and the original function.

Extensions

Students could be asked to sketch a possible function graph for the derivative graph given in Question **20**.

Activity Solutions

1.



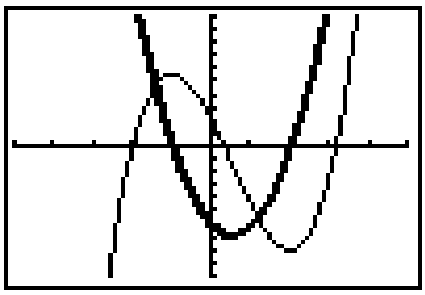
2. The relative maximum occurs at $x = -1$, and the relative minimum occurs at $x = 2$.

3. The function increases over $[-\infty, -1]$ and $[2, \infty]$.

4. The function decreases over $[-1, 2]$.

5. The derivative should have positive values. The derivative is the rate of change or slope at a point, and an increasing portion of a graph has a positive rate of change.

6.



7. The derivative at each relative maximum or minimum is equal to zero.

8. The derivative is positive where the function increases.

9. The derivative is negative where the function decreases.

10. When the derivative crosses the *x*-axis from positive to negative, the function has a relative maximum.

11. When the derivative crosses the *x*-axis from negative to positive, the function has a relative minimum.

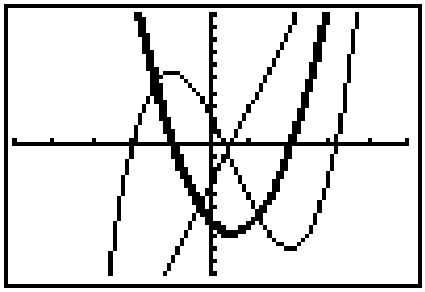
12. The derivative increases from $[0.5, \infty]$.

13. The derivative decreases from $[-\infty, 0.5]$.

14. If y' is increasing, y'' is positive because y'' is the derivative of y'.

15. If y' is decreasing, y'' is negative because y'' is the derivative of y'.

16.



17. Check students' sketches. The second derivative is positive.

18. Check students' sketches. The second derivative is negative.

19.

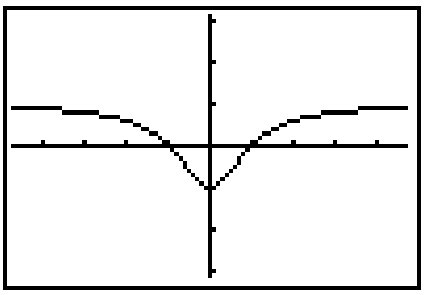
a. The function is increasing from $[-5, -3.425]$, $[-0.860, 0.860]$, and $[3.426, 5]$ within the viewing window.

b. The function is decreasing from $[-3.425, -0.860]$ and $[0.860, 3.426]$.

c. The graph of the function is concave upward from $[-2.289, 0]$ and $[2.289, 5]$.

d. The graph of the function is concave downward from $[-5, -2.289]$ and $[0, 2.289]$.

20.



a. This function increases where the derivative is positive. These intervals are $[-\infty, -1]$ and $[1, \infty]$.

b. The function decreases where the derivative is negative. This interval is $[-1, 1]$.

c. The graph of the function is concave upward when the first derivative is increasing. This interval is $[0, \infty]$.

d. The graph of the function is concave downward when the first derivative is decreasing. This interval is $[-\infty, 0]$.

21. Answers will vary but should include the following:

The derivative is positive when the graph of the function is increasing and negative when the function is decreasing.

The graph of the function is concave upward when the derivative is increasing or second derivative is positive.

The graph of the function is concave downward when the first derivative is decreasing or the second derivative is negative.