

Impact of Handheld Graphing Calculator Use on  
Student Achievement in Algebra 1

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### Abstract

This study investigated the relationship between instructional use of handheld graphing calculators and student achievement in Algebra 1. Three end-of-course test forms were administered (without calculators) using matrix sampling to 458 high-school students in two suburban school districts in Oregon and Kansas. Questions on two forms were drawn from Texas and Massachusetts publicly-released standardized test items, and the third form was custom-designed to emphasize conceptual understanding and math applications. All classes used Key Curriculum Press's *Discovering Algebra* textbook. Results showed that the more access students had to graphing calculators, and the more instructional time in which graphing calculators were used, the higher the test scores. In addition, scores were significantly higher where teachers reported receiving professional development on how to use a graphing calculator in math instruction.

## Impact of Handheld Graphing Calculator Use on Student Achievement in Algebra 1

The purpose of this research was to investigate the relationship between the use of handheld graphing calculators and student achievement in Algebra 1. A comprehensive review of the research on handheld graphing technology in secondary mathematics instruction (Burrill, Allison, Breaux, Kastberg, Leatham, & Sanchez, 2002) indicated that there is improved student conceptual understanding when students use graphing calculators with curricula specifically designed to take advantage of the technology. “[T]he type and extent of gains in student learning of mathematics with handheld graphing technology are a function, not simply of the presence of handheld graphing technology, but of how the technology is used in the teaching of mathematics” (Burrill, et al., 2002).

A more discriminating review (IESD, Inc., 2003) included only studies in the Burrill, et al. (2002) analysis that met the criteria for scientifically rigorous methodologies as specified in the No Child Left Behind Act. The studies in this smaller set, all of which were conducted in the domain of advanced algebra, found significantly higher algebra achievement for students who had access to graphing calculators during instruction. These results have been found for a broad range of teacher preparation, even in a study where one class was given access to graphing calculators without teachers having experience or receiving guidance on how to use the handhelds (Ruthven, 1990). However, others note that only prolonged use of the graphing calculator may lead to enrichment of students’ solution repertoires and a better understanding of algebraic concepts such as functions (van Streun, Harskamp, & Suhre, 2000).

Thus, previous research found significantly improved conceptual understanding and higher student test performance in algebra classes that had access to graphing calculators during instruction and that used algebra curricula that take advantage of the calculators’ capabilities. Even in classes that are using a textbook that thoroughly integrates graphing calculator use, however, the value of calculator use would logically be influenced by many factors in classroom contexts. The intent of the present study was to identify some of the major sources of variability within classrooms aside from use of highly supportive curricula.

### Research Questions

In previous studies, curricula and instruction in those classes using handheld graphing calculators usually differed from those without the technology, and these variations in instructional context, rather than the use of graphing calculators in particular, could have been responsible for the observed group differences. Algebra textbooks vary widely in the extent to which they explicitly and purposefully integrate graphing calculator use. In the current study, textbook was held constant—all classes used a heavily applications-oriented textbook that fully integrates graphing calculators: Key Curriculum Press’s *Discovering Algebra: An Investigative Approach* (Murdock, Kamischke, & Kamischke, 2002). This allowed the relationship between student use of calculators and student test scores to be examined without gross differences in curriculum contributing to observed group differences.

In order to establish the effectiveness of this textbook, a previous study examined the algebra achievement of students who were using a preliminary version of *Discovering Algebra (DA)*, compared to students using more traditional textbooks (Heller & Paulukonis, 2000). In that study of 674 Algebra 1 students from the U.S. and Hong Kong, 8<sup>th</sup> and 9<sup>th</sup> grade students in classes using *DA* scored significantly higher on tests of algebra skills and concepts than students in classes using other textbooks. These results suggest that the way *DA* relies upon graphing calculator use to communicate algebraic concepts does prepare students at least as well as traditional approaches.

Finally, this study was conducted at the level of Algebra 1 because this course is commonly considered a gateway to further academic preparation and it is essential that we understand the factors that influence its effectiveness, especially with respect to students of average academic ability. The research addresses the question of what factors other than curriculum design influence the impact of graphing calculator use on students’ achievement in Algebra 1. More specifically, when graphing calculators are used with a curriculum designed to incorporate them:

1. **How is graphing calculator use during classroom instruction related to student achievement in Algebra 1?**
2. **How is teacher professional development in graphing calculator use related to student achievement in Algebra 1? and**
3. **How are teacher experience using the *Discovering Algebra* textbook, and extent of textbook use, related to student achievement in Algebra 1?**

## Method

### *Design*

End-of-course tests were administered to Algebra 1 students in two suburban school districts in Oregon and Kansas, and teacher surveys were used to collect information about student demographics, teachers' educational backgrounds, teaching and professional development experience, and instructional uses of graphing calculators. These data were then analyzed to identify predictors of student scores on the algebra tests.

### *Instruments*

Two teacher surveys and an end-of-course algebra test for students were administered in the Spring of 2004. In addition, students' scores from previous mathematics standardized tests were collected.

*Teacher Background Survey.* This 13-item multiple-choice and short-answer survey (see Appendix A) was designed to collect information about teachers' background, training, and experience using graphing technologies in math instruction. Each teacher completed one Teacher Background Survey.

*Classroom Survey.* This survey was used to collect information on classroom instructional context and practices. The 17-item survey includes multiple-choice and short-answer questions about class size, how graphing calculators were integrated into the instruction, demographics of students in the class (e.g., number of students in special education, GATE or honors, receiving free or reduced lunch, and ELL), and degree of student access to and use of algebra textbooks and graphing calculators. Teachers completed this questionnaire for each of their algebra classes participating in the study (see Appendix B).

*End-of-Course Algebra Tests.* Three algebra test forms (*Forms T, M, and C*) were constructed to measure algebra knowledge and skills (see Table 1). If all research sites were in the same state, it would have been possible to use students' math standardized test scores as the end-of-course measure. However, because states use different tests, we could not use actual state test results. To produce assessments in common for all students, we instead used questions that are typical of different tests in actual use—questions were drawn from two states' publicly-released standardized tests, Texas and Massachusetts, and a third form was custom-designed to emphasize conceptual understanding and math applications. By using multiple forms and matrix sampling, it was possible to collect data on a wider range of test items.

1. Form T items were drawn from the 2001 Texas Algebra 1 end-of-course test (available at <http://www.tea.state.tx.us/student.assessment/resources/release/>) (see Appendix C).
2. Form M items were drawn from the 2003 Massachusetts Comprehensive Assessment System (MCAS) 8<sup>th</sup> and 10<sup>th</sup> grade mathematics tests (see Appendix D).
3. Form C items were custom-designed by Heller Research Associates and Key Curriculum Press staff and advisors to emphasize conceptual understanding and math applications (see Appendix E). This test was validated in pilot trials in three San Francisco Bay area high schools, and through cognitive think-aloud interviews with both students and math educators.

Forms C and M have both multiple-choice and open-ended items, while Form T is entirely multiple-choice. The 45-minute tests were administered without graphing calculators so the students who had calculator access during the academic year would not be at an advantage. Otherwise, any differences between students who did and did not use calculators during instruction would be confounded with their access to the calculators for the test itself. Only items with modest computational demands were included.

Test reliability was moderate, in part because the tests proved difficult for the students and included a relatively small number of items.

Table 1  
*Features of Three Algebra Test Forms*

Feature	Test Form		
	C	M	T
Source	Custom-designed for this study	9 items drawn from 13 algebra items on each of 2003 MCAS Grade 8 and Grade 10 Mathematics Examinations	17 items drawn from 40 on 2001 Texas Algebra 1 End-of-Course Examination
Format	<ul style="list-style-type: none"> <li>• 15 multiple-choice</li> <li>• 4 open-ended</li> </ul>	<ul style="list-style-type: none"> <li>• 14 multiple-choice</li> <li>• 4 open-ended</li> </ul>	<ul style="list-style-type: none"> <li>• 17 multiple-choice</li> </ul>
Time allotted	45 minutes	45 minutes	45 minutes
Calculator use	Not permitted during test	Not permitted during test	Not permitted during test
Reliability*	.66	.61	.66

\*Standardized alpha coefficient

As shown in Table 2, the three test forms overlap in their content. Form M and T cover the most categories of test items (12 of 16). Form M is the only form that includes translation from inequalities to a number line, recognition of patterns in realistic data, and evaluation of symbolic expressions. Form T includes fewer categories (9 of 16). All three forms contain items involving translating words to equations, but Form C includes the most items that involve applications of mathematics in authentic situations, including manipulating equations that involve real-world contexts, and interpreting and expressing real world meaning in equations and graphs.

An example of the kinds of applications-rich items included in test Form C is provided in Figure 1. The emphasis of several items in Form C is on conceptual understanding of algebraic equations and relationships, especially in real world contexts. Form C item 7 involves a real context in which a linear relationship would exist: a spring that stretches to different lengths depending on the weight attached to it. The equation for this relationship is given, and the question is what one of the numbers in the equation tells you about this situation. To answer this question, the student must understand the meanings of the algebraic expression.

Test scores for the end-of-course tests were expressed as percent correct. Thus, a student who answered all of the questions incorrectly would receive a score of “0”, and a student who answered all of the questions correctly would receive a score of “100”.

*Mathematics standardized test scores.* The school districts were asked to provide students’ scores on previously administered eighth grade standardized tests in mathematics. Scaled scores from these standardized tests were then used as covariates in the data analysis. Each district used a different standardized test—in Kansas, the quantitative thinking subtest of the Iowa Test of Educational Development (ITED), and in Oregon, the Oregon Statewide Mathematics Assessment.

Table 2  
*Classification of Items on Three Algebra End-of-Course Test Forms*

Category	Subcategory	Test form		
		T	M	C
Translating among representations	Equation or inequality to graph	7		13, 16
	Inequality to number line		5	
	Table or points to equation	2	6, 8, 15	3, 17
	Graph to equation	8, 12	11, 13	15
	Words to equation	3	14	10
Manipulating and Solving equations	Real-world context		17a	8ii, 8iii
	Symbolic only	1, 6, 15	2, 9, 12	14
Identifying characteristics of graphs	Given graph	5, 9	10	1
	Given equation		4, 16	4, 5, 12
Interpreting real-world meaning	From equation			6
	To equation	4, 10, 11		7, 8i, 11
	From graph	14, 17	3	9
Solve and interpret systems of equations		13	17	
Evaluating symbolic expressions	Computation		1	
	Estimation			2
Pattern recognition	Symbolic only		7	

7. If weights are placed on the end of a spring, the spring stretches and becomes longer. The relationship between the spring length and the weight hanging on it is given by the equation  $y = 5 + 3.2x$ , where  $y$  is the length in inches and  $x$  is the weight in ounces.

What does the 5 in the equation tell you?

- a) It tells you that the spring is 5 inches long when no weights are attached to it.
- b) It tells you that the spring is 5 inches long when 3.2 ounces of weight are attached to it.
- c) It tells you that the spring is 3.2 inches long when 5 ounces of weight are attached to it.
- d) It tells you that the spring can hold exactly 5 weights before breaking.
- e) It doesn't tell you anything.

Figure 1: Sample Form C Test Item

### Participants

*School districts.* To be eligible to participate in this study, a district had to have a minimum of 150 high school students enrolled in Algebra 1 in classes in which the textbook, *Discovering Algebra: An Investigative Approach* (Murdock, Kamischke, & Kamischke, 2002) was used. In the Spring of 2004, the researchers contacted 26 districts from a list of sites that had purchased the textbook, which was provided by the publisher, Key Curriculum Press. Fourteen of these school districts indicated that they might be interested in participating in this study and provided additional information about the number of schools, classes, and high-school students using the textbook. Of those, we selected the four largest and most diverse districts to participate in the study.

Unfortunately, two of the larger districts withdrew during the data collection phase for logistical reasons (e.g., district budget cuts and layoffs, and stringent district requirements for parental assent that were not satisfied). The two remaining districts—one in the Northwestern United States and the other in the Midwest—were both suburban with largely white populations. From these districts, 11 teachers and 458 students from 21 class sections in four schools participated in this study (see Table 3).

Table 3  
*Numbers of participating schools, teachers, and students from two districts*

Category	District 1: Oregon	District 2: Kansas	Total
Number of high schools	1	3	4
Number of teachers	4	7	11
Number of classes	7	14	21
9 <sup>th</sup> grade only	0	5	5
Mixed 9 <sup>th</sup> to 11 <sup>th</sup> grades	7	9	16
Number of students	159	299	458

*Classrooms.* The sample included only students in grade 9 or above in order to focus primarily on students of average to low ability who are most in need of educational improvements. Typically, Honors and other students of high academic ability take Algebra 1 in 8<sup>th</sup> grade.

Class sections varied with respect to the grade level of students enrolled, with the majority (approximately three-quarters) containing combinations of 9<sup>th</sup>, 10<sup>th</sup>, and 11<sup>th</sup> graders. Some of the 21 sections of Algebra 1 were taught by the same teacher. Of the 11 teachers in the study, four teachers had one class section in the study; five teachers had two sections in the study; and two teachers had three or more. As shown in Table 4, the average class size was just over 23 students. On average there were 3.4 special education or resource students per class, with a maximum of 7 in any one class. There were almost no GATE or honors students in the sample, and less than one English language learner per class, on average.

*Students.* As shown in Table 5, the majority of students from both districts were white students who were native English speakers. The Oregon sample included a somewhat higher proportion of Asian and Latino students and English language learners.

Table 4  
Average Class Composition

Category	District 1: Oregon ( <i>n</i> = 159)	District 2: Kansas ( <i>n</i> = 299)	Total ( <i>N</i> = 457)
Total number of students in class:			
Mean	27.1	22.4	23.4
( <i>SD</i> )	(3.8)	(3.1)	(4.0)
Range	22 to 31	17 to 26	17 to 31
Special education or resource:			
Mean	3.9	3.2	3.4
( <i>SD</i> )	(2.0)	(1.9)	(1.9)
Range	0 to 6	1 to 7	0 to 7
GATE or honors:			
Mean	0.6	None	0.2
( <i>SD</i> )	(0.5)		(0.4)
Range	0 to 1		0 to 1
English language learners			
Mean	1.0	0.4	0.6
( <i>SD</i> )	(1.4)	(0.6)	(1.0)
Range	0 to 3	0 to 2	0 to 3

Table 5  
Student Demographics

Category	District 1: Oregon ( <i>n</i> = 159)	District 2: Kansas ( <i>n</i> = 299)	Total ( <i>N</i> = 458)
Gender:			
Male	49.1%	46.5%	47.4%
Female	50.9%	53.5%	52.6%
Ethnicity:			
African American	4.4%	4.7%	4.6%
Asian, Filipino, Pacific Islander, Southeast Asian	8.2%	1.7%	3.9%
Latino	5.7%	2.0%	3.3%
White	72.2%	77.6%	75.7%
Other (includes Native American, Mixed)	9.5%	14.0%	12.5%
Age when student learned English:			
Younger than 5	91.1%	96.3%	94.5%
Between 5 and 8	4.4%	2.0%	2.8%
Between 9 and 12	3.8%	1.3%	2.2%
13 or older	.6%	.3%	.4%



*Teachers.* Eleven Algebra 1 teachers from the two school districts participated in the study. One district administrator or teacher agreed to be the site coordinator for each district. These coordinators distributed invitations to participate in the study to teachers, and served as liaisons for all communication with the teachers. Teachers received a stipend of \$75 for each section of Algebra 1 in which they administered the student tests. The coordinators received a stipend of \$175 for their assistance.

As shown in Table 6, most of the 11 teachers in this study were women and all of the teachers identified themselves as white.

Nearly half of the teachers had a BA or BS degree in mathematics. A higher proportion of the Kansas teachers had graduate training in mathematics, and this group had more teaching experience than the Oregon group. Teachers in both districts were experienced in the use of graphing calculators, averaging 7.4 years of use.

#### *Data Collection Procedure*

Coordinators distributed to each teacher one Teacher Background Survey, instructions for student test administration, a Classroom Survey, and a batch of algebra tests for each participating Algebra 1 class. Each test included a cover sheet requesting three items of basic demographic information from students. Matrix sampling of the three test forms was accomplished by pre-sorting each batch of class tests so there were equal numbers of each form. The forms were collated so they would in effect be distributed at random to the students. Thus, if a section had 24 students, then approximately 8 students would have completed Form T, 8 students Form M, and 8 students Form C. The tests were administered in one 45-minute period. Students were not permitted to use graphing calculators because access to calculators varied among students. The teachers and students completed all materials between May 24 and June 3, 2004.

Table 6  
*Teacher Demographics and Background*

Category	District 1: Oregon ( $n = 4$ )	District 2: Kansas ( $n = 7$ )	Total ( $N = 11$ )
Demographics:			
Female	75.0%	100%	90.9%
White	100%	100%	100%
Formal math education:			
Some college mathematics courses	25.0%	0.0%	9.0%
BA or BS in mathematics	50.0%	42.9%	45.5%
Graduate course or degree	25.0%	57.1%	45.5%
Teaching experience:			
Mean no. of years ( $SD$ )	4.00 (2.16)	12.14 (6.39)	9.18 (6.54)
Range	2 to 7 years	3 to 23 years	2 to 23 years
Teaching Algebra 1:			
Mean no. of years ( $SD$ )	2.75 (1.71)	7.29 (6.50)	5.64 (5.61)
Range	1 to 5 years	1 to 20 years	1 to 20 years
Years of experience using graphing calculators:			
Mean no. of years ( $SD$ )	6.25 (4.11)	8.00 (3.92)	7.36 (3.88)
Range	1 to 10 years	4 to 13 years	1 to 13 years

## Analyses

In the analyses that follow, data sets from the two districts were combined, but the test forms were analyzed separately. Teachers' responses on items from the Beginning Algebra Study Classroom Questionnaire were examined in relation to their students' end-of-course algebra test scores to address the research questions. In most of the analyses that follow, rating categories were collapsed so that they could be treated as dichotomous. Collapsing categories was necessary because, with only 11 teachers responding, there were too few responses to compare all of the categories. For example, teachers were asked to choose among four options, such as *Not at all*, *A little*, *Some*, and *A lot*. Depending on how responses were distributed, we combined these original categories in different ways to form just two categories with roughly equal numbers of responses. For example, for some items, students' scores for teachers who responded *A lot* were compared with students' scores for the combined group of teachers who responded either *Not at all*, *A little*, or *Some*, whereas on different items, students' scores where teachers responded either *A lot* or *Some* were compared with those where teachers responded either *Not at all* or *A little*.

The test scores of teachers' students were then analyzed using a two-sample *t*-test to evaluate differences between the two groups. For each *t*-test, the independent variable was the dichotomous distinction between the two groups formed by the particular questionnaire item, and the dependent variable was the end-of-course algebra test scores. For each *t*-test, a corresponding effect size statistic (ESS) was calculated. The ESS statistic used here is the standardized difference between two means, calculated as  $(\bar{X}_1 - \bar{X}_2) / S_{pooled}$ , where  $\bar{X}_1$  and  $\bar{X}_2$  are the means of groups 1 and 2, respectively, and  $S_{pooled}$  is the pooled standard deviation of the two groups. The ESS can be interpreted as the difference between the means in standard deviation units. Thus, an ESS of .5 indicates that the two means are .5 standard deviation units apart.

Finally, regression analyses were performed to examine predictors of student achievement while controlling for student ability and teacher experience.

## Results

### *Student Achievement Measures*

Mean test scores for the end-of-course algebra tests are presented in Table 7. The tests were quite difficult for the students, with the three forms varying somewhat in their overall difficulty. Form M was the most difficult, with a mean of just over 40% correct, and Form T the least difficult, with a mean of approximately 50% correct.

The two districts were similar in their overall mean scores. No consistent pattern was found with respect to differences between the two districts in student performance on these tests. For Forms C and M, on average, students in the Kansas school district had higher scores, whereas for Form T, on average, students in the Oregon school district had higher scores.

The study was designed so that one-third of the students in each class would take each of the three test forms. The numbers in Table 7 confirm that roughly equal number of students took each test—154, 155, and 149 students took Form T, M, and C, respectively.

### ***Research Question 1: How is graphing calculator use during classroom instruction related to student achievement in Algebra 1?***

This question was addressed by examining students' access to graphing calculators in school and at home, and then by looking at the amount and ways that the calculators were used in the classroom.

As shown in Table 8, in all of the classrooms, most or all of the students had access to a graphing calculator for individual use in class. In the majority of classrooms (85%), most or all of the students owned their own graphing calculators so had access to its use at home.

Table 7  
*Students' Mean Percent Correct Scores on Algebra Tests by District, for Test Forms T, M, and C*

Test Form		District 1 Oregon	District 2 Kansas	Total (N = 458)
T	Mean	51.94%	48.86%	49.92%
	(SD)	(19.47)	(18.69)	(18.95)
	n	53	101	154
M	Mean	39.40%	42.77%	41.60%
	(SD)	(16.51)	(16.44)	(16.49)
	n	54	101	155
C	Mean	42.11%	45.74%	44.48%
	(SD)	(17.86)	(16.06)	(16.74)
	n	52	97	149
Total	Mean	44.47%	45.79%	45.33%
	(SD)	(18.67)	(17.24)	(17.74)
	n	159	299	458

*Note.* Data are reported only for students for whom standardized 8th grade test scores were received.

Table 8  
*Student Access to Graphing Calculators (N = 21 Classrooms)*

Survey Question	Teacher Response	Number of Classrooms	Percent
CS7a. Approximately how many students in this class have a graphing calculator for their individual use in class?	None	0	0%
	Some	0	0%
	Half	0	0%
	Most	12	57.0%
	All	9	43.0%
CS7b. Approximately how many students in this class have a graphing calculator for their use at home?	None	0	0%
	Some	2	8.0%
	Half	3	14.3%
	Most	13	61.9%
	All	5	23.8%
CS7c. Approximately how many students in this class own their own graphing calculator?	None	0	0%
	Some	2	9.5%
	Half	1	4.8%
	Most	13	61.9%
	All	5	23.8%

In the majority of classes (61.9%), the teachers reported using graphing calculators for more than half the instructional activities (see Table 9), and more than 80% of the teachers reported using graphing calculators more than is suggested in the textbook. Teachers' reasons for doing so were included in their written responses to the question: *What are the main benefits for your students of using graphing calculators?* The following are representative of teachers' comments:

1. Allows higher-level learning:

"It allows them to take the concepts to a higher level and connect the different concepts."

"The calculator makes graphing easier, allowing students to investigate more complex problems."

2. Efficient, get more work done in the classroom:
  - “They can see the graph quickly and make inferences from the graph.”
  - “It doesn’t take all class period just to get the graph right.”
  - “A lot of work can be done more quickly, more principles can be studied and explored.”
3. Greater accuracy:
  - “Graphs and scatter plots are more accurate.”
  - “Accurate and expedient solutions to graphing a variety of functions.”
  - “Finding intercepts, visualizing the functions.”

Table 9

*Instructional Uses of Graphing Calculators in the Classroom (N = 21 Classrooms)*

Survey Question	Response	Number of Classrooms	Percent
CS8. Approximately what percent of the instructional activities in this class involve a graphing calculator?	10% to 25%	3	14.3%
	25% to 49%	5	23.8%
	50% to 74%	11	52.4%
	75% to 100%	2	9.5%
CS10. How closely does your graphing calculator use correspond to the suggested use in the textbook for this class?	A lot less than suggested	2	9.5%
	About as suggested	2	9.5%
	Somewhat more than suggested	16	76.2%
	A lot more than suggested	1	4.8%
CS17. Are there times that you do not allow students in this class to use graphing calculators?	Yes	15	71.4%
	No	6	28.6%

On the other hand, the teachers reported that there are times they do not allow students to use graphing calculators in over 70% of the classrooms. The reasons teachers gave for limiting access to calculators generally fell into two categories, one corresponding to their efforts to help students develop conceptual understanding and the other to testing that understanding. The following are representative of their written responses to the questions: *Are there times that you do not allow students in this class to use graphing calculators? If so, when and why do you limit the use of graphing calculators in this class?*

To develop conceptual understanding:

“When we are focusing on the algebraic method of finding solutions.”

“Quadratic = roots and vertex ... systems of equations ... linear x- and y-intercepts ... manually plotting points ... finding slopes and deriving equations from graphs.”

“I want students to learn how transformations work without the calculator at first.”

To test conceptual understanding:

“On some exams to ensure conceptual understanding.”

“On tests we have both graphing calculator and non-graphing calculator parts to make sure they have understanding with and without it.”

### *Student Achievement as a Function of Graphing Calculator Access and Use*

As shown in Table 10, the greater the access to and use of graphing calculators during classroom instruction, the higher students scored on end-of-course tests. The mean algebra test score in classrooms where all of the students had graphing calculators for their individual use in class was significantly higher than where some of the students did not have them in class ( $t(153) = 4.60, p = .00, ESS = .76$ ).

Furthermore, the more instructional time spent working with graphing calculators, the higher the test scores. The mean test score for students in classrooms where 50% or more of the instructional activities

reportedly involved graphing calculators was higher than in classrooms where graphing calculators were used in fewer of the activities ( $t(153) = 2.39, p = .02, ESS = .39$ ). However, *not* allowing students to use the calculators some of the time was also important. The mean algebra test score for students in classrooms where the teacher said there were times that students were not allowed to use a graphing calculator was higher than where students always had access to the calculators ( $t(153) = 2.25, p = .03, ESS = .40$ ).

Table 10  
*Significant Differences Between Students' Mean Test Scores in Classes with Different Instructional Access to and Uses of Graphing Calculators*

Survey Question	Student scores*		Form	<i>t</i>	<i>p</i>	<i>ESS</i>
CS7. Approximately how many students in this class have a graphing calculator for their individual use in class?	< All	All	M	4.60	.00	.76
	37.17% (15.74)	48.99% (15.10)				
CS8. Approximately what percent of the instructional activities in this class involve a graphing calculator?	10% - 49%	$\geq 50\%$	M	2.39	.02	.39
	37.83% (15.89)	44.17% (16.47)				
CS17. Are there times that you do not allow students in this class to use graphing calculators?	No	Yes	C	2.25	.03	.40
	37.08% (17.29)	43.50% (15.83)				

Form M:  $n = 155$ ; Form C:  $n = 149$

\*All group differences significant at  $p < .05$

### *Use of Graphing Calculators to Teach Specific Topics*

Teachers were asked about the extent to which they used graphing calculators to teach specific algebra topics in each class. Table 11 lists topics in order of reported graphing calculator use. Teachers indicated that they most frequently used graphing calculators to teach linear equations, finding lines of best fit for data, and systems of equations. Least use of graphing calculators was reported for teaching non-functions, rational equations, and exponential equations. Teachers were mixed in their reported use of graphing calculators to teach linear inequalities and quadratic equations.

### *Student Achievement as a Function of Graphing Calculator Use to Teach Specific Topics*

There were three algebra topics for which student test scores differed significantly (see Table 12). Students' mean test score was significantly higher in classrooms where the teacher reported using graphing calculators for teaching *linear inequalities* "Some or a lot" compared to where the teacher did so "Not at all or a little" ( $t(153) = 3.43, p = .00, ESS = .64$  on Form M), ( $t(153) = 2.44, p = .02, ESS = .46$  on Form T). Similarly, the mean test score where the teacher reported using graphing calculators for teaching *non-functions* "A little, some or a lot" was significantly higher than where the teacher never used graphing calculators to teach *non-functions* ( $t(146) = 2.86, p = .01, ESS = .48$  on Form M). Lastly, students' mean test score was significantly higher where the teacher reported using graphing calculators for teaching *quadratic equations* "Some or a lot" compared to where the teacher reported doing so "Not at all or a little" ( $t(153) = 2.26, p = .03, ESS = .37$  on Form T). No significant differences were found on Form C.

Table 11

*Algebra Topics Taught with Graphing Calculators (N = 21 Classrooms)*

CS14i. To what extent do you use graphing calculators to teach the following topics in this class?	Not at all	A little	Some	A lot
Linear equations	0.0%	14.3%	14.3%	71.4%
Finding lines of best fit for data	9.5%	4.8%	23.8%	61.9%
Systems of equations	0.0%	14.3%	28.6%	57.1%
Absolute value equations	9.5%	14.3%	28.6%	47.6%
Linear inequalities	14.3%	14.3%	38.1%	33.3%
Quadratic equations	23.8%	19.0%	33.3%	23.8%
Exponential equations	33.3%	33.3%	14.3%	19.0%
Rational equations	66.7%	0.0%	14.3%	19.0%
Non-functions	66.7%	9.5%	23.8%	0.0%

Table 12

*Significant Differences Between Students' Mean Test Scores Depending Upon Which Specific Topics Are Taught Using Graphing Calculators*

Survey Question	Test Form	Student scores*		t	p	ESS
CS14c. To what extent do you use graphing calculators to teach linear equations in this class?	M	Not at all or a little 34.76 (15.90)	≥ Some 45.05 (16.03)	3.43	.00	.64
	T	Not at all or a little 43.19 (19.21)	≥ Some 51.91 (18.81)	2.44	.02	.46
CS14i. To what extent do you use graphing calculators to teach non-functions in this class?	M	Not at all 38.75 (15.90)	≥ A little 46.62 (15.97)	2.86	.01	.48
CS14g. To what extent do you use graphing calculators to teach quadratic equations in this class?	T	Not at all or a little 45.99 (18.83)	≥ Some 52.87 (18.61)	2.26	.03	.37

In summary, student scores were significantly higher on Forms M and T for classes in which teachers used graphing calculators to teach specific topics that were relatively infrequently taught with calculators, specifically linear inequalities and non-functions, and quadratic equations.

With respect to the relationship between classroom use of graphing calculators and student achievement, the findings show:

- The more access students have to graphing calculators during instruction, the higher their end-of-course test scores (taken without the use of the calculators).
- The more time that graphing calculators were reportedly used in instruction, the higher were students' test scores.
- Students' scores were higher in classes in which they were sometimes not allowed to use a graphing calculator.
- The more algebra topics that are taught using graphing calculators, the higher the students' test scores.

***Research Question 2: How is teacher professional development in graphing calculator use related to student achievement in Algebra 1?***

Teachers learned to use graphing calculators in a variety of ways (see Table 13). Nearly three-fourths of the teachers participated in a training or workshop on how to use a graphing calculator. Roughly half of the teachers said they learned how to use a graphing calculator in math courses they had taken, and just under half indicated they were self-taught either with or without using a manual. (Note that these percentages do not add to 100% because the response options were not mutually exclusive).

Table 13  
*Teachers' Professional Development Experience (N = 11 Teachers)*

Survey Question	% Yes
TS7. What training have you received in the use of graphing calculators?	
Training/workshop on how to use a graphing calculator	72.7%
Learned how to use in math courses I have taken	54.5%
Self-taught without using manual—explored graphing calculator features on my own	45.5%
Self-taught using manual	45.5%
TS4. Have you done any of the following in the past 4 years?	
Participated in an in-service workshop or course in mathematics or mathematics teaching	100%
Attended a mathematics teacher association meeting	63.3%
Participated in a workshop/training specific to the mathematics text book used in class	63.3%
Participated in a workshop/training on using graphing calculators in math instruction	72.7%
Participated in a workshop/training on using other computerized graphing technology	54.5%
Taught an in-service workshop or course in mathematics or mathematics teaching	27.3%
Other math-related staff development	27.3%

All of the teachers reported participating in professional development related to mathematics or mathematics teaching in the past four years. As shown in Table 13, over 60% of the teachers had attended

a mathematics teacher association meeting and the same proportion had received training specific to the *Discovering Algebra* textbook. Close to three-fourths of the teachers reported having training in the use of graphing calculators in mathematics instruction.

#### *Teacher Familiarity with Various Graphing Calculator Features*

Teachers were asked to rate their own familiarity with specific graphing calculator operations and features (see Table 14). Over 90% of the teachers described themselves as being “very familiar” with the following: graphing a scatter plot, using the WINDOW feature, graphing a function, graphing more than one function on the same screen, and creating a table. Between 70% and 90% indicated that they were very familiar with using the TRACE, ZOOM, INTERSECT, and MAXIMUM & MINIMUM features, as well as with how to graph an inequality. Teachers were least familiar with how to graph a relation, write a program, or connect calculators to other devices.

Table 14

#### *Percent of Teachers “Very Familiar” with Graphing Calculator Features*

Survey Question	% Very Familiar
TS8. How familiar are you with how to do each of the following with a graphing calculator?	
Graph a scatter plot	100%
Use the WINDOW feature	100%
Graph a function	90.9%
Graph more than one function on same screen	90.9%
Create a table	90.9%
Use the TRACE feature	81.8%
Use the ZOOM feature	81.8%
Graph an inequality	72.7%
Use the INTERSECT feature	72.7%
Use the MAXIMUM & MINIMUM features	72.7%
Graph a relation (e.g., $y^2 = x$ )	54.5%
Write a program	27.3%
Connect graphing calculators to motion detectors, computers or other graphing calculators	27.3%

#### *Relationship Between Teacher Professional Development and Student Algebra Test Scores*

Nine items related to teacher professional development that were on the Beginning Algebra Study Teacher Questionnaire were examined to answer the second research question regarding the relationship between teacher professional development and student achievement. Eight of the selected questionnaire items were dichotomous, meaning that teachers could only choose between one of two responses. The ninth item was not originally asked in dichotomous form, and instead was converted to a dichotomous



variable because of low responses in some of the categories. Specifically, for the question “How much time have you spent participating in math professional development programs during the past four years?” teacher responses were collapsed into two categories: “up to eight days” and “nine days or more”.

Table 15 shows mean student test scores in percent correct for teachers with different amounts of reported professional development. The mean score on test Forms M and C for students whose teachers attended a graphing calculator training/workshop was *higher* than the mean score for students whose teachers did not do so ( $t(153) = 2.81$ ,  $ESS = .52$ , and  $t(147) = 2.71$ ,  $ESS = .50$ , both  $p = .01$ ). Workshops that teachers had taken included those offered by Texas Instruments or arranged through schools and school districts. Similarly, the mean algebra test score on Form M, for students whose teachers participated in a workshop/training on using other computerized graphing technology was *higher* than the mean score for those whose teachers who did not take such a workshop ( $t(155) = 2.36$ ,  $p = .02$ ,  $ESS = .38$ ). The mean score was lowest for students whose teachers said they used a manual to teach themselves how to use a graphing calculator ( $t(153) = 2.18$ ,  $p = .03$ ,  $ESS = .36$ ).

Lastly, the mean test score for students whose teachers attended a math teacher association meeting in the previous four years was *higher* than the mean score when teachers did not attend ( $t(153) = 2.08$ ,  $p = .04$ ,  $ESS = .34$ ). For Form T, no statistically significant differences were found. There were no significant differences in student achievement as a function of whether teachers received professional development on using the textbook.

Table 15

*Significant Differences Between Students' Mean Test Scores in Classes Taught by Teachers with Different Professional Development Experience*

Survey Question	Test Form	Yes Mean (SD)	No Mean (SD)	<i>t</i>	<i>p</i>	<i>ESS</i>
TS7. What training have you received in the use of graphing calculators?						
Workshop/training on how to use a graphing calculator	M	43.74 (16.10)	35.42 (16.19)	2.81	.01*	.52
	C	46.68 (16.53)	38.47 (16.01)	2.71	.01*	.50
Self-taught using manual	M	38.19 (16.00)	43.99 (16.49)	2.18	.03*	.36
TS4. Have you done any of the following in the past four years?						
TS4f. Participated in a workshop/training on using other computerized graphing technology	M	44.35 (15.4)	38.16 (17.32)	2.36	.02*	.38
TS4a. Attended a mathematics teacher association meeting	M	43.82 (16.34)	38.26 (16.27)	2.08	.04*	.34

Form M:  $n = 155$ ; Form C:  $n = 149$

\* $p < .05$

In summary, these findings show:

- Student scores were significantly higher for teachers who reported participating in trainings on how to use a graphing calculator, or other computerized graphing technology.
- Students did significantly worse on the test when their teachers reported being self-taught using the graphing calculator manual.

**Research Question 3: How does teacher experience using the *Discovering Algebra* textbook, and extent of textbook use, relate to student achievement in Algebra 1?**

In all of the classrooms, students each had their own copy of the *Discovering Algebra* textbook for use both at home and at school. Teachers had been using the text for one to two years in approximately one-third of the classes, and for three years in about two-thirds of the classes. In nearly half of the classes (47.6%), the teachers said they used the textbook almost exclusively, whereas in the other classes, the teachers said that they supplemented the textbook with other materials and activities.

Table 16 shows mean test scores in relation to teachers' experience with and use of the DA textbook. The mean algebra test score for students in classrooms where the teacher had been using the textbook for two or three years was higher than where the teacher had been using the textbook for one year (means = 46.02 and 39.81, respectively,  $t(147) = 1.97$ ,  $p = .05$ ,  $ESS = .37$  on Form C). In addition, the mean algebra test score for students in classrooms where the teacher used the primary textbook almost exclusively was higher than where the teacher supplemented the use of *Discovering Algebra* with other materials (means = 45.66 and 38.16, respectively,  $t(153) = 2.89$ ,  $p = .00$ ,  $ESS = .47$  on Form M).

In summary, with respect to use of the textbook,

- The more experienced the teacher is with *Discovering Algebra*, the higher the students' scores.
- The truer the teacher is to the *Discovering Algebra* curriculum, the higher the test scores.

Table 16

*Significant Differences Between Students' Mean Test Scores in Relation to Teachers' Experience With and Use of the Discovering Algebra Textbook*

Survey Question	Test Form	Student scores*		<i>t</i>	<i>p</i>	<i>ESS</i>
CS3. To what extent do you have students in this class use the primary textbook, compared to other supplementary materials and activities?	M	Almost exclusively	Sometimes supplement text	2.89	.00*	.47
		45.66 (15.71)	38.16 (16.43)			
CS2. How long have you used the textbook you are currently using?	C	1 <sup>st</sup> year	2 <sup>nd</sup> or 3 <sup>rd</sup> year	1.97	.05*	.37
		39.81 (16.12)	46.02 (16.73)			

Form M:  $n = 155$ ; Form C:  $n = 149$

\*All group differences significant at  $p < .05$

### Regression Analyses

In addition to testing group differences, a regression analysis was performed to identify predictors of students' algebra test scores when controlling for students' math ability and teacher experience. We examined the algebra scores in relation to eight predictor variables:

1. Students' previous math standardized test scores ("Pretest")
2. Total number of years of teaching experience
3. Whether the teacher attended a workshop/training on how to use a graphing calculator (dummy coded as "yes" = 1 versus "no" = 0)
4. Whether the teacher participated in a workshop/training on using other computerized graphing technology (dummy coded as "yes" = 1 versus "no" = 0)

5. How long the teacher has been using the textbook (dummy coded as “1 year” = 0 versus “2 to 3 years” = 1)
6. How exclusively the teacher has the students use the primary textbook (dummy coded as “Use textbook almost exclusively” = 0 versus “Do not use textbook almost exclusively” = 1)
7. What percent of instructional activities involve the graphing calculator (dummy coded as “10% to 49%” = 0 versus “50% to 100%” = 1)
8. How many students have a graphing calculator for their individual use in class (dummy coded as “most” = 0 versus “all” = 1)

The first two variables listed above, namely students’ previous math standardized test scores and number of years of teaching experience, are both quantitative. The remaining variables were treated as dichotomous because there were not enough responses in each category to compare all possible answers or ratings.

For each of the three end-of-course algebra test forms, the eight predictor variables described above were entered into a regression analysis with end-of-course algebra test score as the outcome variable. The predictor variables were entered in two steps—first, the students’ previous standardized math test score was entered as a control variable (Model 1), and then the remaining seven predictor variables were added (Model 2). This allowed us to assess the amount of variance in end-of-course test scores that could be attributed to students’ mathematical ability, and to further assess the amount of additional variance explained by the seven teacher and classroom predictor variables. In other words, by conducting the analysis in the hierarchical manner, we were able to evaluate the amount of variance explained by teacher and classroom variables after controlling for students’ math ability.

As a preliminary step to the regression analyses, zero-order correlation coefficients between each of the eight predictor variables and end-of-course test scores were calculated. These correlation coefficients are presented in Table 17. For all three test forms, the highest correlation coefficient was between students’ previous standardized math test scores and end-of-course scores ( $r = .44, .30$ , and  $.26$  for Forms C, M, and T, respectively). All three of these correlation coefficients were statistically significant.

Table 17  
*Correlations between Predictor Variables and Students’ Test Scores*

Predictor Variable	Form C ( $n = 149$ )	Form M ( $n = 155$ )	Form T ( $n = 154$ )
Students’ previous standardized math test scores	.44**	.30**	.26**
Teachers’ years of teaching experience	.15	.16	-.08
Teacher having attended training-workshop on how to use a graphing calculator	.22*	.22*	-.07
Teacher having participated in a workshop or training on using other graphing technology	.13	.19*	.04
Length of time teacher has been using text	.16*	.14	-.14
How exclusively students use the primary textbook	-.01	-.23**	-.10
Percent of instructional activities that involve graphing calculator	.01	.19*	.14
Number of students who have graphing calculator	.13	.35**	.06

\*\* $p < .005$ . \* $p < .05$ .

The results of the regression analysis for Form C, M, and T are summarized in Tables 18 and 19. The amount of variance in end-of-year test scores explained by pretest scores only (Model 1) was 21%, 09%, and 7% for Forms C, M, and T, respectively (see Table 18). (These percentages are simply the

square of each the corresponding correlation coefficients in the previous table). Adding the seven teacher and classroom variables to the regression equation increases the amount of explained variance in end-of-year test scores (i.e.  $R^2$ -change) by 4%, 15%, and 14%, for Forms C, M, and T, respectively. For Forms M and T, this change is statistically significant ( $F(7,125) = 3.60, p = .00$  and  $F(7,131) = 3.24, p = .004$  for Forms M and T respectively). Thus, for two of the test forms, adding teacher and classroom variables to the regression analysis significantly increased the amount of explained variance in end-of-year test scores.

Table 18

*Amount of Variance in End-of-Year Test Scores Explained by the Student-level, Teacher-level, and Classroom-level Predictor Variables*

Model	Form C				Form M				Form T			
	$R^2$	df1,df2	$F$	$p$	$R^2$	df1,df2	$F$	$p$	$R^2$	df1,df2	$F$	$p$
Pretest Only	.21	1,125	30.43	.00*	.09	1,132	13.20	.00*	.07	1,138	10.08	.00*
Full Model (All 8 predictors)	.24	8,118	4.59	.00*	.24	8,125	5.02	.00*	.21	8,131	4.23	.00*
$R^2$ Change	.04	7,118	.92	.49	.15	7,125	3.60	.00*	.14	7,131	3.24	.00*

\* $p < .01$ .

Individual slope coefficients are presented in Table 19. The pretest was a statistically significant predictor for all three forms of the end-of-year test, with standardized slope coefficients equal to .509, .266, and .474 for Forms C, M, and T, respectively. There were two additional predictor variables that were statistically significant for two test forms each. First, the slope for the extent to which students used the primary textbook (“Use textbook almost exclusively” versus “Do not use textbook almost exclusively”) was statistically significant for Forms C and T (beta = .322 and .318, for Forms C and T, respectively). Second, the slope for how many students in class have a graphing calculator (“most” versus “all”) was statistically significant for Forms M and T (beta = .487, and beta = .324, for Forms M and T). Lastly, there were several predictor variables that were statistically significant for one form only. Specifically, for Form M, three additional predictor variables were statistically significant, namely: (a) years of teaching experience (beta = -.544); (b) whether or not the teacher attended a training/workshop on how to use a graphing calculator (beta = .787); and (c) whether or not the teacher participated in a workshop/training on using other computerized graphing technology (beta = -.580). For Form T, one additional predictor variable was statistically significant, namely number of years the teacher has been using primary textbook (“1 year” versus “2 to 3 years”).

### Summary of Results

**Research Question 1. *How is graphing calculator use during classroom instruction related to student achievement in Algebra 1?*** The more access students had to graphing calculators, and the more graphing calculators were used during algebra instruction, the higher the students’ end-of-course test scores (taken without the use of the calculators). However, *not* allowing students to use the calculators some of the time was also important. Students’ scores were higher in classes in which they were sometimes not allowed to use a graphing calculator than where students always had access to the calculators. In addition, specific uses of graphing calculators during classroom instruction in Algebra 1 were associated with higher student achievement. Students scored significantly higher when their teachers reported using graphing calculators to teach topics that were relatively infrequently taught with calculators, namely linear inequalities, non-functions, and quadratic equations. Put more generally, the more algebra topics that were taught using graphing calculators, the higher the students’ test scores.

**Research Question 2. *How is teacher professional development in graphing calculator use related to student achievement in Algebra I?*** Student scores were significantly higher for teachers who reported participating in trainings on how to use a graphing calculator, or other computerized graphing technology. Conversely, students scored significantly lower when their teachers reported being self-taught using the graphing calculator manual. Furthermore, the mean test score for students whose teachers attended a math teacher association meeting in the previous four years was significantly higher than when teachers did not attend.

Table 19

*Regression Analysis using Specific Student-level, Teacher-level, and Classroom-level Predictor Variables and End-of-Year Forms C, M, and T Algebra Test Scores as the Outcome Variable*

Predictor Variable	Form C		Form M		Form T	
	Standardized beta	<i>t</i>	Standardized beta	<i>t</i>	Standardized beta	<i>t</i>
(Constant)		-2.48*		.06		-1.48
Pretest	.509	5.15**	.266	3.16**	.474	4.96**
Years of teaching experience (TS1)	-.420	-1.79	-.544	-2.52*	-.414	-1.82
Has teacher attended training-workshop on how to use a graphing calculator? (TS7E)	.392	1.31	.787	2.71*	.095	.32
Has teacher participated in a workshop or training using other computerized graphing technology? (TS4F)	-.147	-.62	-.580	-2.68*	-.075	-.33
How long has teacher been using text? (CS2REV)	-.086	-.79	-.132	-1.24	-.249	-2.33*
How exclusively do students use the primary textbook? (CS3REV)	.322	2.03*	.070	.45	.318	2.00*
Percent of instructional activities that involve graphing calculator (CS8REV)	.205	1.32	.245	1.51	.273	1.68
How many students have a graphing calculator (CS7AREV)	.163	1.11	.487	3.72**	.324	2.28*

\*\* $p < .005$ . \* $p < .05$ .

**Research Question 3. *How are teacher experience using the Discovering Algebra textbook, and extent of textbook use, related to student achievement in Algebra I?*** The more experience teachers

had using *Discovering Algebra*, the higher their students' test scores. The more exclusively the *Discovering Algebra* textbook was used in instruction, the higher the students' test scores.

Consistent with these *t*-test results, a regression analysis also showed that, after controlling for pretest scores, statistically significant predictors of students' test scores included:

- (a) how many students in class have a graphing calculator ("most" versus "all");
- (b) whether or not the teacher attended a training/workshop on how to use a graphing calculator or on using other computerized graphing technology; and
- (c) number of years the teacher has been using the *Discovering Algebra* textbook, and how exclusively students used the *Discovering Algebra* textbook.

### Discussion

Previous research found significantly higher student test performance for students who had access to graphing calculators during algebra instruction. The purpose of the current study was to provide additional information about what conditions and factors might be responsible for the observed improvements in student achievement, with a focus on the roles of instructional context and teacher preparation. In the current study, all classes used the same textbook, Key Curriculum Press's *Discovering Algebra*, so the relationship between calculator use and student achievement could be examined without gross differences in curriculum contributing to observed group differences. Results here showed that, in classes that all used graphing calculators with a curriculum that fully integrated their use, the higher the proportion of students who had access to graphing calculators, and the more graphing calculators were used during algebra instruction, the higher the students' end-of-course test scores (taken without the use of the calculators). These findings are compatible with earlier results that only prolonged use of the graphing calculator was associated with improved student understanding of algebraic concepts—basically, "more is better" in this case with respect to access and amount of graphing calculator use.

Still, it appears that some gains in student learning of mathematics with handheld graphing technology are a function of how the technology is used in instruction, even in classes that all use the same textbook. For example, in the current study, students' scores were higher in classes in which they were sometimes not allowed to use a graphing calculator than where students always had access to the calculators. Scores were also significantly higher in classes where graphing calculators were used to teach topics that are relatively infrequently taught with calculators, namely linear inequalities, non-functions, and quadratic equations.

In addition, although all classes used the same textbook, student achievement was higher the more experience teachers had using that textbook, and the more exclusively it was used in instruction. Thus, adding to studies that showed improved student conceptual understanding when students used graphing technology with curricula specifically designed to take advantage of the technology is the finding that the more those curricula are used, the higher the student achievement.

With respect to teacher training in calculator use during instruction, earlier research found that students with access to graphing calculators scored significantly higher than those without access, even with teachers who lacked experience or training on how to use the handhelds. In the present study, all classes had access to graphing calculators so we could examine the relationship between amount and kind of teacher professional development in calculator use and student achievement. Student achievement was significantly higher for teachers who reported participating in trainings on how to use a graphing calculator or other computerized graphing technology, in contrast to those who reported being self-taught using the manual. These findings suggest that students benefit when their teachers receive professional development that is specific to calculator use in math instruction. These findings are consistent with a conclusion that student achievement is enhanced by the use of graphing calculators in Algebra 1 and of the *Discovering Algebra* textbook.

There are limitations to the study that should be considered when interpreting these results. First, there is a possibility that intervening variables account for some of the associations observed here. For example, classes with more students who have access to calculators, or in which students were sometimes

not allowed to use the calculators, may also be higher SES. In addition, because of the last-minute dropout of two urban districts, the sample of students and teachers is of limited diversity. Furthermore, the sample included only high-school students, excluding the often-strongest students who take Algebra 1 in the 8<sup>th</sup> grade. This decision was made deliberately so as to add to the knowledge base regarding educational improvement for more academically challenging students. For all of these reasons, results may not be generalizable to other segments of the population.

Another consideration in interpreting the findings is that the end-of-course algebra tests were administered without graphing calculators in order that the students with more calculator access during the academic year would not be at an advantage. However, this choice prevented measurement of the mathematical performance that the calculator-experienced students might have been capable of when using the technology with which they had become proficient. Test scores therefore may have underestimated these students' algebra knowledge, and their achievement could be even more superior to the less-calculator-experienced group.

Other technical considerations of possible concern are that different results were obtained for the three different test forms but no clear pattern was detected; the tests were quite difficult for the students (post-test scores averaging approximately 45% correct), so test reliability was modest as a result; and differences between subgroups were examined with multiple *t*-tests, not controlling for students' ability levels except for in the regression analyses which removed variance related to pretest scores.

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Appendix A  
Teacher Survey



**National Algebra Curriculum and Instruction Study**  
**Teacher Survey**  
**Spring 2004**

Name \_\_\_\_\_ Date \_\_\_\_\_

School \_\_\_\_\_ School District \_\_\_\_\_

1. How many years of teaching experience do you have? \_\_\_\_\_ years
2. How many years have you taught beginning algebra? \_\_\_\_\_ years
3. Which of the following best describes your formal mathematics education? (Check highest level.)
  - ☐<sub>1</sub> High school mathematics courses
  - ☐<sub>2</sub> Some college mathematics courses
  - ☐<sub>3</sub> B.A. or B.S. degree in mathematics
  - ☐<sub>4</sub> Graduate level coursework in mathematics
  - ☐<sub>5</sub> Graduate level degree in mathematics
4. Have you done any of the following in the past four years? (Check all that apply.)
  - ☐<sub>1</sub> Attended a mathematics teacher association meeting
  - ☐<sub>2</sub> Participated in an in-service workshop or course in mathematics or mathematics teaching
  - ☐<sub>3</sub> Taught an in-service workshop or course in mathematics or mathematics teaching
  - ☐<sub>4</sub> Participated in a workshop/training specific to the mathematics textbook used in this class
  - ☐<sub>5</sub> Participated in a workshop/training on using graphing calculators in math instruction
  - ☐<sub>6</sub> Participated in a workshop/training on using other computerized graphing technology
  - ☐<sub>7</sub> Other math-related staff development (please specify): \_\_\_\_\_
5. Approximately how much time have you spent participating in math professional development programs during the last four years?
  - ☐<sub>1</sub> None
  - ☐<sub>2</sub> Up to 2 days (16 hours or less)
  - ☐<sub>3</sub> 3 to 8 days (17-64 hours)
  - ☐<sub>4</sub> 9 days or more (Please specify approximate number of hours: \_\_\_\_\_ hours)
6. How many years of experience do you have using graphing calculators? \_\_\_\_\_ years  
(If you don't use graphing calculators, write "0.")
7. What training have you received in the use of graphing calculators? (Check all that apply.)
  - ☐<sub>1</sub> None—know very little about how to use
  - ☐<sub>2</sub> Self-taught without using manual—explored graphing calculator features on my own
  - ☐<sub>3</sub> Self-taught using manual
  - ☐<sub>4</sub> Learned how to use in math courses I have taken
  - ☐<sub>5</sub> Training/workshop on how to use a graphing calculator
  - ☐<sub>6</sub> Other: \_\_\_\_\_

8. How familiar are you with how to do each of the following with a graphing calculator?	<b>Not familiar</b>	<b>Somewhat familiar</b>	<b>Familiar</b>	<b>Very familiar</b>
a) Graph a function	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
b) Graph more than one function on the same screen	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
c) Graph an inequality	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
d) Graph a scatter plot	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
e) Graph a relation (e.g., $y^2 = x$ )	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
f) Create a table	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
g) Write a program	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
h) Graph an inequality	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
i) Use the TRACE feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
j) Use the ZOOM feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
k) Use the WINDOW feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
l) Use the INTERSECT feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
m) Use the MAXIMUM and MINIMUM features	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
n) Connect graphing calculators to motion detectors, computers, or other graphing calculators	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

9. What are the main benefits of using graphing calculators for your students?

10. What are the main disadvantages of using graphing calculators for your students?

11. Which of the following best describes the setting of your school?

- |  |  |
|--|--|
| <input type="checkbox"/> <sub>1</sub> Urban    | <input type="checkbox"/> <sub>3</sub> Rural        |
| <input type="checkbox"/> <sub>2</sub> Suburban | <input type="checkbox"/> <sub>4</sub> Other: _____ |

12. Your gender: \_\_\_\_\_

13. Your ethnic identity (optional; check as many as apply):

- |   |  |
|---|--|
| <input type="checkbox"/> <sub>1</sub> American Indian or Alaskan Native       | <input type="checkbox"/> <sub>6</sub> Pacific Islander |
| <input type="checkbox"/> <sub>2</sub> Asian                                   | <input type="checkbox"/> <sub>7</sub> Southeast Asian  |
| <input type="checkbox"/> <sub>3</sub> Black or African American, non-Hispanic | <input type="checkbox"/> <sub>8</sub> White            |
| <input type="checkbox"/> <sub>4</sub> Filipino                                | <input type="checkbox"/> <sub>9</sub> Other: _____     |
| <input type="checkbox"/> <sub>5</sub> Latino, Spanish-Origin, Hispanic        |  |

Appendix B  
Classroom Survey

**Beginning Algebra Study  
Classroom Survey  
June 2004**

Name \_\_\_\_\_ Date \_\_\_\_\_

School \_\_\_\_\_ School District \_\_\_\_\_

Class name \_\_\_\_\_ Class period \_\_\_\_\_

Course type (honors, regular, etc.) \_\_\_\_\_ Grade level(s) \_\_\_\_\_

**Please answer the following questions for this class section.**

1. What textbook do you use in this class?

- ☐ <sub>1</sub> *Focus on Algebra* (Addison-Wesley, Prentice Hall)
- ☐ <sub>2</sub> *Math 1 – Algebra* (College Preparatory Mathematics)
- ☐ <sub>3</sub> *Algebra 1* (Glencoe McGraw-Hill)
- ☐ <sub>4</sub> *Discovering Algebra* (Key Curriculum Press)
- ☐ <sub>5</sub> *IMP* (Key Curriculum Press)
- ☐ <sub>6</sub> *Algebra Structure and Method* (McDougal Littell/Houghton Mifflin)
- ☐ <sub>7</sub> *Foerster Algebra 1: Expressions, Equations, and Applications* (Scott Foresman/Addison Wesley)
- ☐ <sub>8</sub> Other: Title: \_\_\_\_\_  
Publisher: \_\_\_\_\_

2. How long have you used the textbook you are currently using?

- ☐ <sub>1</sub> This is my first year using this book.
- ☐ <sub>2</sub> 2 years
- ☐ <sub>3</sub> 3 years
- ☐ <sub>4</sub> Other: \_\_\_\_\_ years

3. To what extent do you have students in this class use the primary textbook, compared to other supplementary materials and activities? (Please choose one.)

- ☐ <sub>1</sub> I use the primary textbook almost exclusively.
- ☐ <sub>2</sub> I sometimes supplement the primary textbook.
- ☐ <sub>3</sub> I use the primary textbook about half the time.
- ☐ <sub>4</sub> I use the primary textbook less than half of the time.
- ☐ <sub>5</sub> I rarely use the primary textbook.

4. If you supplement the textbook with other materials or activities, what are the main areas of student knowledge and skills focused on in the supplemental materials?

5. How many copies of the primary textbook are available in this classroom? (Choose only one.)

- ☐<sub>1</sub> A single copy of the primary textbook is available for my use.
- ☐<sub>2</sub> Students share a limited number of textbooks in the classroom.
- ☐<sub>3</sub> A class set of the primary textbook is available for each student's use at school only.
- ☐<sub>4</sub> Students each have their own copy of the primary textbook for use both at school and at home.
- ☐<sub>5</sub> Other: \_\_\_\_\_

- | 6. Please estimate how much of class time is spent on: | None                                  | Some                                  | Half                                  | Most                                  | All                                   |
|--|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| a) Teacher presentation or explanation                 | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| b) Whole class discussion                              | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| c) Small group work                                    | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| d) Individual work                                     | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| e) Other: _____  | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |

- | 7. Approximately how many students in this class:   | None                                  | Some                                  | Half                                  | Most                                  | All                                   |
|---|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| a) Have a graphing calculator for their individual use in class?                              | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| b) Share graphing calculators with other students in class?                                   | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| c) Have a graphing calculator for their use at home?  | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| d) Own their own graphing calculator?   | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |
| e) Have access to other computerized graphing technology (e.g., <i>Green Globes</i> program)? | <input type="checkbox"/> <sub>1</sub> | <input type="checkbox"/> <sub>2</sub> | <input type="checkbox"/> <sub>3</sub> | <input type="checkbox"/> <sub>4</sub> | <input type="checkbox"/> <sub>5</sub> |

8. Approximately what percent of the instructional activities in this class involve a graphing calculator?

- ☐<sub>1</sub> None—we do not use graphing calculators.
- ☐<sub>2</sub> 1% to 9%
- ☐<sub>3</sub> 10% to 25%
- ☐<sub>4</sub> 25% to 49%
- ☐<sub>5</sub> 50% to 74%
- ☐<sub>6</sub> 75% to 100%

9. What are the main reasons you use graphing calculators as much (or as little) as you do in this class?

10. How closely does your graphing calculator use correspond to the suggested use in the textbook for this class?

- ☐<sub>1</sub> A lot less than suggested
- ☐<sub>2</sub> Somewhat less than suggested
- ☐<sub>3</sub> About as suggested
- ☐<sub>4</sub> Somewhat more than suggested
- ☐<sub>5</sub> A lot more than suggested

11. Please write the approximate number of students in this class in each of the following categories (if none, write "0"):

- a) Total students in class: \_\_\_\_\_
- b) Special Education or Resource: \_\_\_\_\_
- c) GATE or Honors: \_\_\_\_\_
- d) Free or Reduced Lunch: \_\_\_\_\_
- e) English Language Learner: \_\_\_\_\_

**NOTE: If you do not use graphing calculators in this class, you may skip the remaining questions.  
Thank you for completing the survey.**

12. What model(s) of graphing calculator(s) do you and your students use in this class?

\_\_\_\_\_

13. In terms of how you teach this particular class, how do graphing calculators relate to your curriculum and instruction (not just to the textbook)?

- ☐ <sub>1</sub> *Graphing calculator use is integral to the curriculum:* The unique features of graphing calculators are used to teach core ideas in algebra. Removing them would require fundamental changes in the curriculum and/or instruction.
- ☐ <sub>2</sub> *Graphing calculator use is not integral to the curriculum:* Graphing calculators are used only to supplement instruction and are not used to teach core ideas in algebra. Removing graphing calculators would not require fundamental changes in the curriculum or instruction.

14. To what extent do you use graphing calculators to teach the following topics in this class?

	Not at all	A little	Some	A lot
a) Linear equations	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
b) Absolute-value equations	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
c) Linear inequalities	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
d) Finding lines of best fit for data	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
e) Systems of equations	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
f) Exponential equations	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
g) Quadratic equations	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
h) Rational equations	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
i) Non-functions	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
j) Other: _____	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
k) Other: _____	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

15. How do students in this class use graphing calculators? (Check all that apply.)

- ☐<sub>1</sub> To investigate graphs (e.g., to perform stretches, shifts, reflections)
- ☐<sub>2</sub> To find graphical solutions for different kinds of equations, functions, and relations
- ☐<sub>3</sub> To check answers
- ☐<sub>4</sub> To perform direct manipulations of graphs and numerical data (zooming, scaling, scrolling)
- ☐<sub>5</sub> To create tables
- ☐<sub>6</sub> To find maxima, minima, vertices, x- and y-intercepts, and other points on the graph of a function
- ☐<sub>7</sub> Other: \_\_\_\_\_

16. How familiar are the students in this class with how to do each of the following with a graphing calculator?

	<b>Not familiar</b>	<b>Somewhat familiar</b>	<b>Familiar</b>	<b>Very familiar</b>
l) Graph a function	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
b) Graph more than one function on the same screen	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
c) Graph an inequality	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
d) Graph a scatter plot	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
e) Graph a relation (e.g., $y^2 = x$ )	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
f) Create a table	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
g) Write a program	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
h) Graph an inequality	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
i) Use the TRACE feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
j) Use the ZOOM feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
k) Use the WINDOW feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
l) Use the INTERSECT feature	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
m) Use the MAXIMUM and MINIMUM features	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>
n) Connect graphing calculators to motion detectors, computers, or other graphing calculators	<input type="checkbox"/> <sub>1</sub>	<input type="checkbox"/> <sub>2</sub>	<input type="checkbox"/> <sub>3</sub>	<input type="checkbox"/> <sub>4</sub>

17. Are there times that you do not allow students in this class to use graphing calculators?

- ☐<sub>1</sub> Yes      ☐<sub>2</sub> No

If so, when and why do you limit the use of graphing calculators in this class?

Appendix C

End-of-Course Algebra Test Form T



# End-of-Year Algebra Exam

## Spring 2004

---

### Test Form: T

Student Name (Please print): \_\_\_\_\_

Teacher: \_\_\_\_\_

School: \_\_\_\_\_

Class Period: \_\_\_\_\_

### Student Information:

1. Please check one: ☐<sub>1</sub> Male      ☐<sub>2</sub> Female
2. When did you first learn to speak English? (Please check one)  
☐<sub>1</sub> When I was younger than 5 years old  
☐<sub>2</sub> When I was between 5 and 8 years old  
☐<sub>3</sub> When I was between 9 and 12 years old  
☐<sub>4</sub> When I was 13 years old or older
3. Ethnicity (Please check as many as apply):  
☐<sub>1</sub> American Indian or Alaskan Native  
☐<sub>2</sub> Asian (Chinese, Japanese, Korean, Asian Indian, other Asian)  
☐<sub>3</sub> Black or African American, non-Hispanic  
☐<sub>4</sub> Filipino  
☐<sub>5</sub> Latino, Spanish-Origin, Hispanic  
☐<sub>6</sub> Pacific Islander (Native Hawaiian, Guamanian, Samoan)  
☐<sub>7</sub> Southeast Asian (Cambodian, Laotian, Vietnamese, other Southeast Asian)  
☐<sub>8</sub> White  
☐<sub>9</sub> Other: \_\_\_\_\_

**End-of-Year Algebra Exam**  
**Spring 2004**

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**Test Instructions**

Please answer the test questions as well as you can. There is space on the test pages to show your work or use as scratch paper.

You will have 50 minutes to complete the test.

**You may not use a calculator of any kind during the test.**

**—— Turn the page and begin the test ——**

# End-of-Year Algebra Exam

## Spring 2004

---

**Directions:**

**All questions are multiple choice. Please circle the letter next to your answer.**

- 1 What is the value of  $y$  if  $x$  is 2?

$$y = \frac{x^3}{x^6}$$

**F**  $\frac{1}{8}$

**G**  $\frac{1}{6}$

**H**  $\frac{1}{4}$

**J**  $\frac{1}{2}$

**K** Not Here

- 2 Which equation describes the data in the table?

$x$	$y$
-7	-10
-2	-5
3	0
8	5

**F**  $y = -x - 7$

**G**  $y = x - 3$

**H**  $y = x + 5$

**J**  $y = 2x - 1$

**K**  $y = 3x + 1$

- 3 Mark has \$4.95 in quarters and dimes. He has 3 times as many dimes as quarters. Which system of equations can be used to find  $q$ , the number of quarters, and  $d$ , the number of dimes, that Mark has?

**F**  $d = 3q$

$$0.10q + 0.25d = 4.95$$

**G**  $d = q + 3$

$$0.25q + 0.10d = 4.95$$

**H**  $d = 3q$

$$0.25q + 0.10d = 4.95$$

**J**  $q = 3d$

$$0.25q + 0.10d = 4.95$$

**K**  $q + d = 3$

$$0.25q + 0.10d = 4.95$$

- 4 Paige has started saving for a new television. She saved \$75 last month. She plans to add \$50 each month until she has saved at least \$400. Which inequality can be used to find  $m$ , the number of months it will take Paige to save for her television?

**F**  $50m - 75 \geq 400$

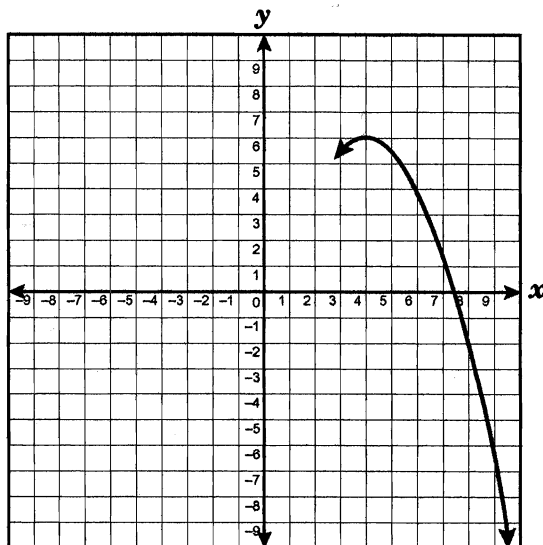
**G**  $75 + 50m \geq 400$

**H**  $50m - 75 < 400$

**J**  $75m + 50 \geq 400$

**K**  $75 + 50m < 400$

- 5 A part of the graph of the equation  $y = -\frac{1}{2}x^2 + 4x - 2$  is shown on the coordinate grid.



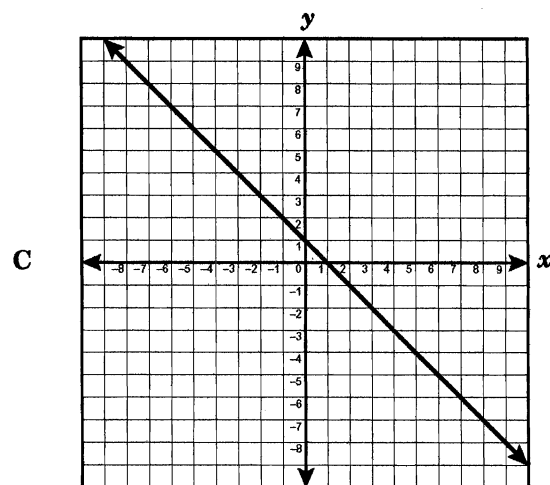
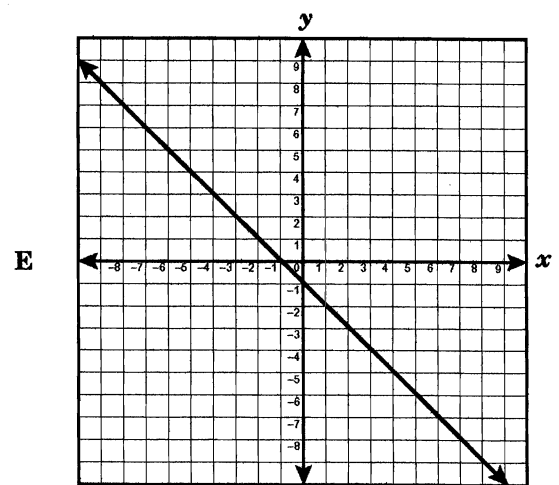
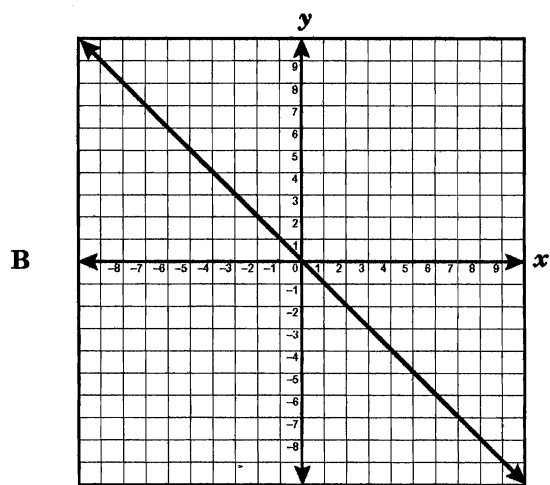
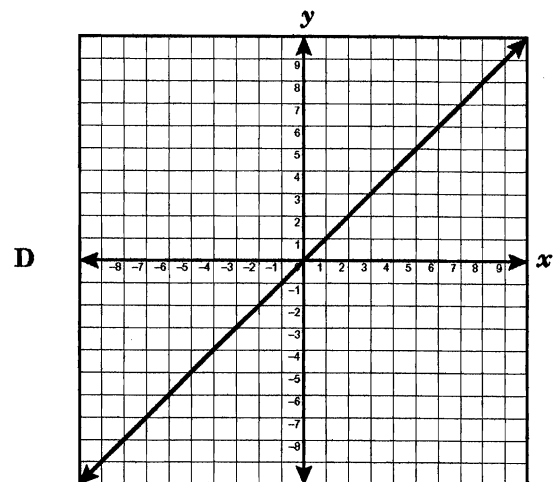
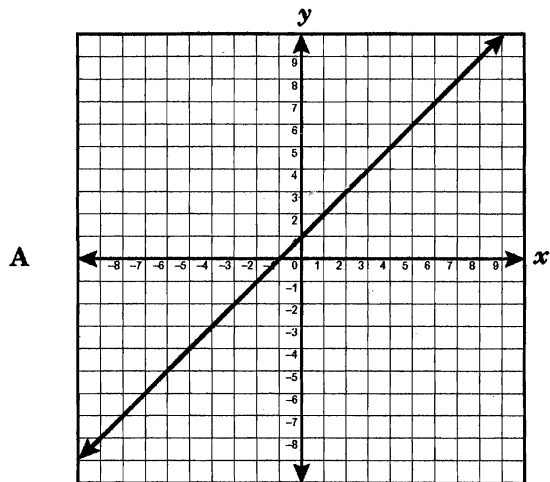
Between which 2 integers will the graph again cross the  $x$ -axis?

- A Between  $-2$  and  $-1$
  - B Between  $-1$  and  $0$
  - C Between  $0$  and  $1$
  - D Between  $1$  and  $2$
  - E Between  $2$  and  $3$
- 6 What is the value of  $x$  in the following equation?

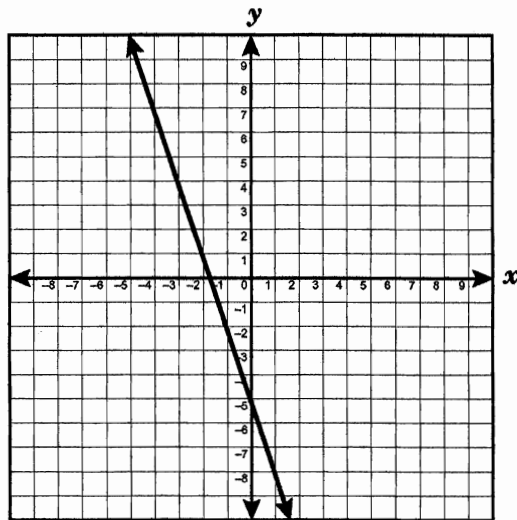
$$2x - (4x - 6) = 0$$

- A  $-8$
- B  $-4$
- C  $-3$
- D  $3$
- E  $4$

7 Which graph is described by  $y = x$ ?



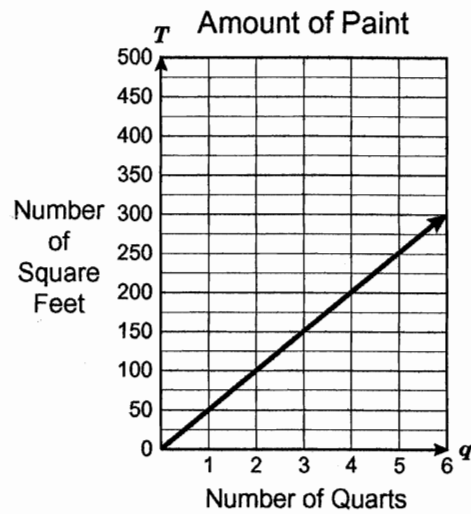
- 8 The graph of a function is shown below.



Which equation best describes the function?

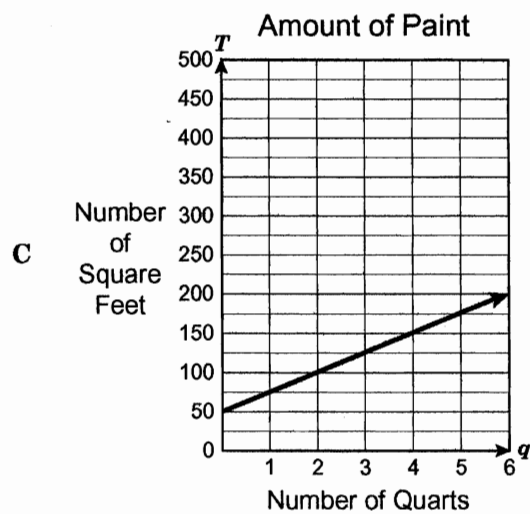
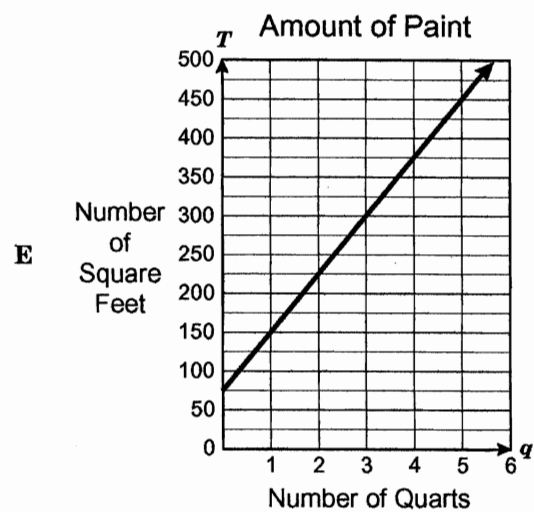
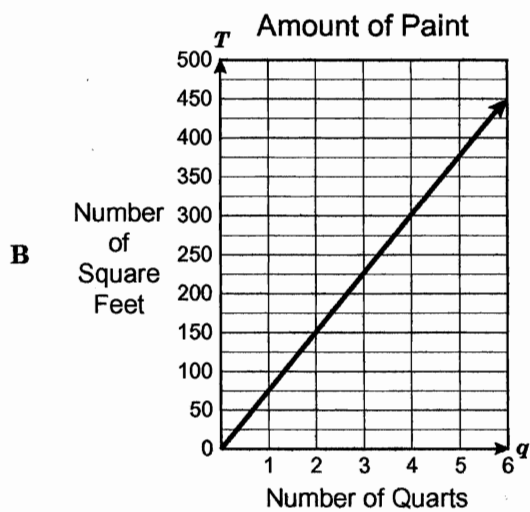
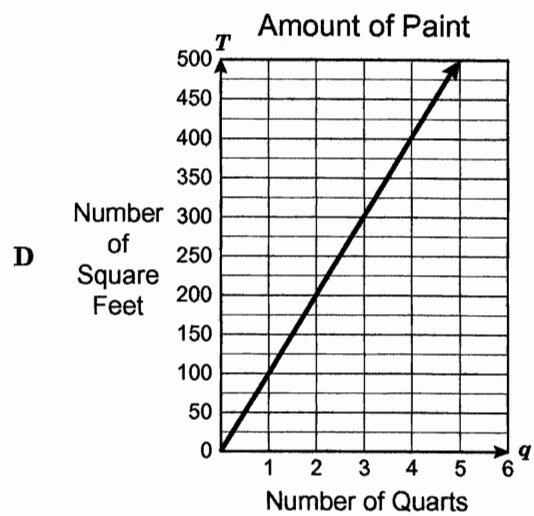
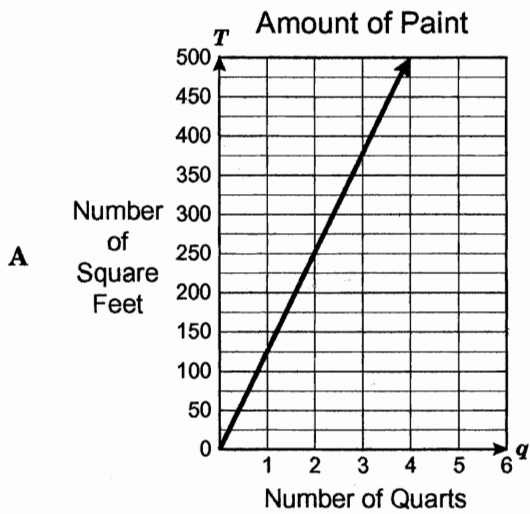
- A  $y = -\frac{1}{3}x - 5$
- B  $y = -\frac{1}{3}x + 5$
- C  $y = -3x - 5$
- D  $y = \frac{1}{3}x - 5$
- E  $y = -3x + 5$

- 9 The graph below shows the total number of square feet of wood surface that can be covered using a particular paint.

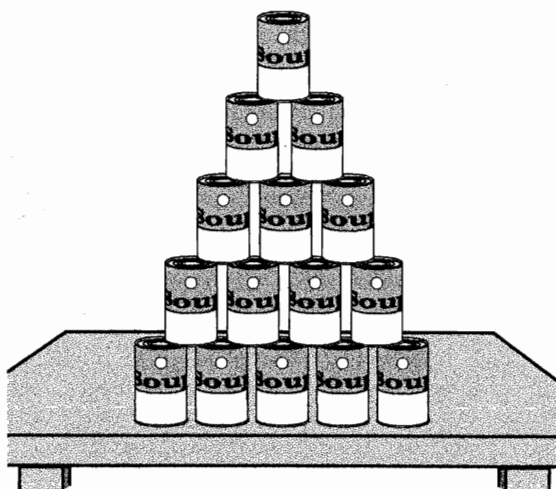


Which graph on the next page best represents the total number of square feet that can be covered by a different paint at a rate of 75 square feet per quart?





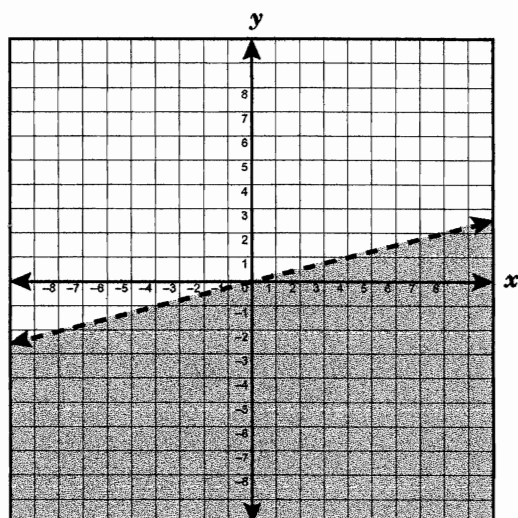
- 10 Martin arranged some cans of soup in a triangular pattern on a table. The top row had 1 can, the second row had 2 cans, the third row had 3 cans, and so on. The arrangement is shown below.



Which equation gives the total number of cans in the arrangement,  $T$ , when the cans are stacked  $n$  rows high?

- A  $T = \frac{n(n-1)}{2}$
- B  $T = 3n$
- C  $T = \frac{2(n+1)}{n}$
- D  $T = \frac{n(n+1)}{2}$
- E  $T = 2n(n-1)$
- 11 The cost of shipping a package overnight from Cedarville to Martindale is \$4.50 plus \$0.85 per ounce. Which equation can be used to find  $C$ , the total cost to ship a package that weighs  $x$  ounces?
- A  $C = 4.50x + 0.85$
- B  $C = 4.50 - 0.85x$
- C  $C = 0.85x + 4.50$
- D  $C = (0.85 + 4.50)x$
- E  $C = x + 0.85 + 4.50$

12 The graph best represents the solutions for which inequality?



A  $y > 4x$

B  $y < \frac{1}{4}x$

C  $y < 4x$

D  $y > \frac{1}{4}x$

E  $y < -\frac{1}{4}x$

13 The equations of 2 lines are given as

$$2x - y = 2$$

$$3x + 4y = 25$$

What are the coordinates of the point of intersection?

F  $(5\frac{1}{2}, 9)$

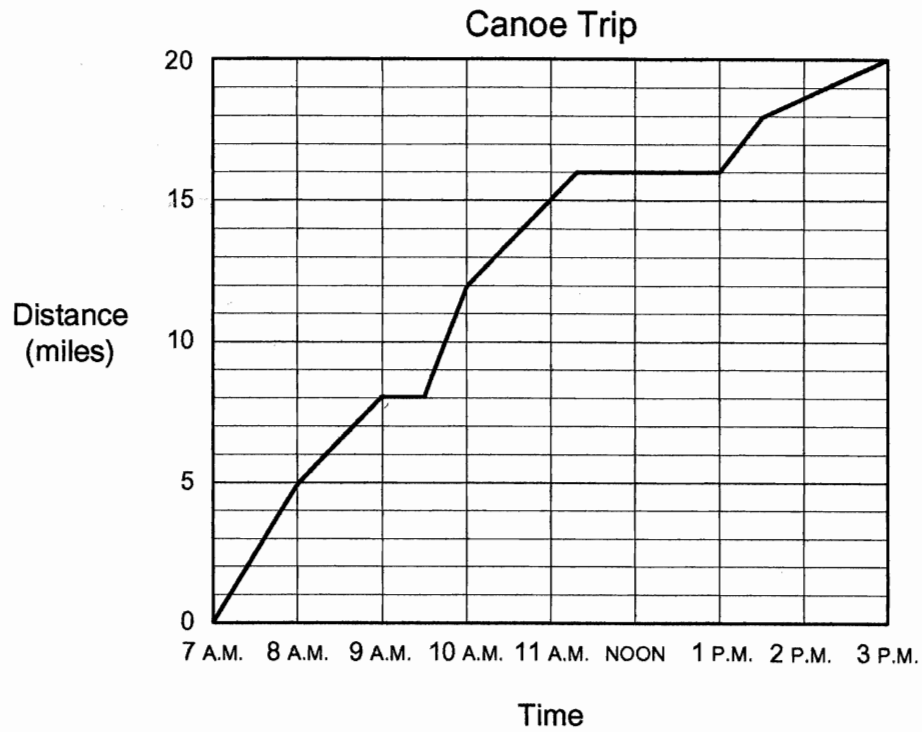
G  $(4, 3)$

H  $(3, 4)$

J  $(3, -4)$

K  $(-1, -4)$

- 14 Mark and his brother went on a canoe trip. The graph shows the relationship between the distance traveled and the time of day.



What was the brothers' average speed in miles per hour from 9 A.M. to 11 A.M.?

- F** 2.5 mph
- G** 3 mph
- H** 3.5 mph
- J** 5 mph
- K** 7 mph

15 Which set of ordered pairs satisfies the function  $y = x^2 + 3x + 4$ ?

**F**

$x$	$y$
1	7
2	14
3	28

**G**

$x$	$y$
1	8
2	10
3	15

**H**

$x$	$y$
1	7
2	14
3	19

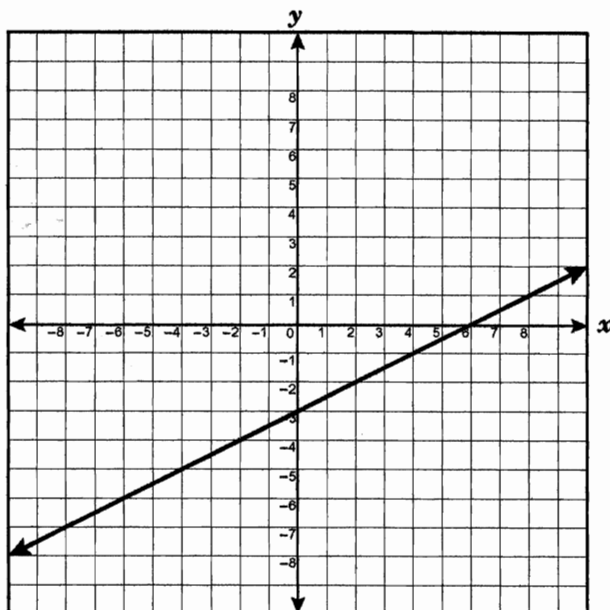
**J**

$x$	$y$
1	8
2	14
3	22

**K**

$x$	$y$
1	9
2	12
3	15

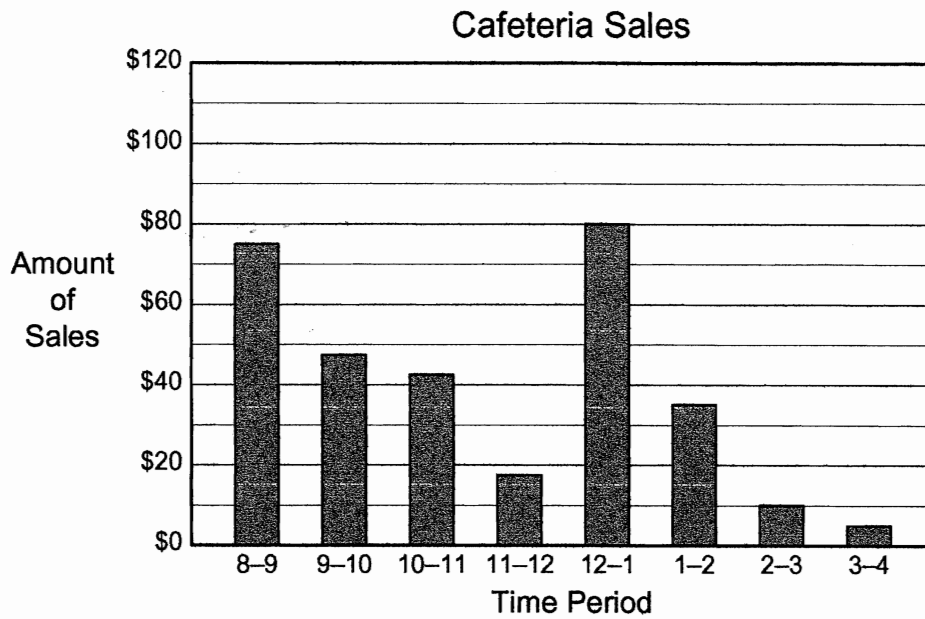
- 16 The graph of the function  $y = \frac{1}{2}x - 3$  is shown below.



If the line is translated 2 units down, which equation will best describe the new line?

- A  $y = \frac{1}{2}x - 1$
- B  $y = x - 3$
- C  $y = \frac{1}{2}x + 1$
- D  $y = x - 5$
- E  $y = \frac{1}{2}x - 5$

- 17 The graph shows the average sales in a small cafeteria in an office building.



Which is a reasonable inference from the information on the graph?

- A Less than 45% of the day's sales are made before 11 A.M.
- B Less than half of the employees in the office building eat in the cafeteria.
- C More than 50% of the day's sales are made from 11 A.M. to 2 P.M.
- D About 80% of the office building's workers leave by 1 P.M.
- E Less than 5% of the cafeteria's sales are made after 2 P.M.

## Appendix D

### End-of-Course Algebra Test Form M



# End-of-Year Algebra Exam

## Spring 2004

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### Test Form: M

Student Name (Please print): \_\_\_\_\_

Teacher: \_\_\_\_\_

School: \_\_\_\_\_

Class Period: \_\_\_\_\_

### Student Information:

1. Please check one: ☐<sub>1</sub> Male      ☐<sub>2</sub> Female
2. When did you first learn to speak English? (Please check one)
  - ☐<sub>1</sub> When I was younger than 5 years old
  - ☐<sub>2</sub> When I was between 5 and 8 years old
  - ☐<sub>3</sub> When I was between 9 and 12 years old
  - ☐<sub>4</sub> When I was 13 years old or older
3. Ethnicity (Please check as many as apply):
  - ☐<sub>1</sub> American Indian or Alaskan Native
  - ☐<sub>2</sub> Asian (Chinese, Japanese, Korean, Asian Indian, other Asian)
  - ☐<sub>3</sub> Black or African American, non-Hispanic
  - ☐<sub>4</sub> Filipino
  - ☐<sub>5</sub> Latino, Spanish-Origin, Hispanic
  - ☐<sub>6</sub> Pacific Islander (Native Hawaiian, Guamanian, Samoan)
  - ☐<sub>7</sub> Southeast Asian (Cambodian, Laotian, Vietnamese, other Southeast Asian)
  - ☐<sub>8</sub> White
  - ☐<sub>9</sub> Other: \_\_\_\_\_

**End-of-Year Algebra Exam**  
**Spring 2004**

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**Test Instructions**

Please answer the test questions as well as you can. There is space on the test pages to show your work or use as scratch paper.

You will have 50 minutes to complete the test.

**You may not use a calculator of any kind during the test.**

—— Turn the page and begin the test ——

# End-of-Year Algebra Exam

## Spring 2004

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### Directions:

Questions 1 - 14 are multiple choice. Please circle the letter next to your answer.

- 1 If  $y + 5$  is an even integer, which of the following could be the value of  $y$ ?

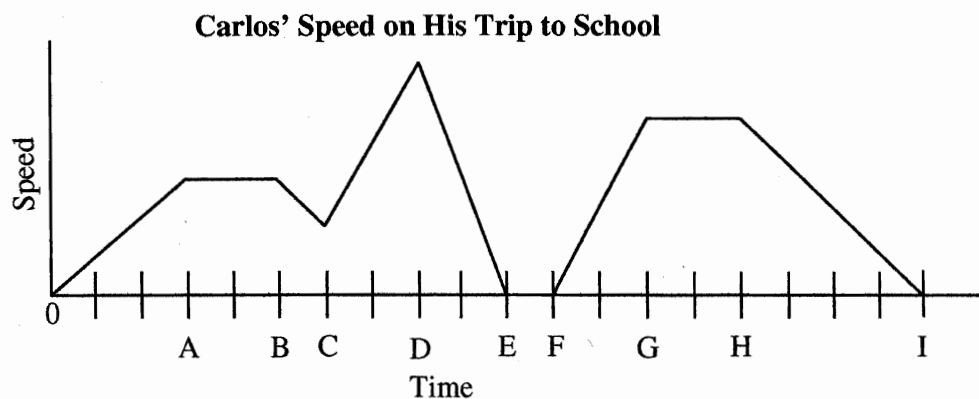
A. -2  
B. -1  
C. 0  
D. 2

- 2 Which steps could be used to solve this equation?

$$\frac{2}{3}x + 9 = 15$$

- A. Subtract 9 from both sides, then multiply both sides by the reciprocal of  $\frac{2}{3}$ .  
B. Subtract 9 from both sides, then divide both sides by the reciprocal of  $\frac{2}{3}$ .  
C. Multiply both sides by the reciprocal of  $\frac{2}{3}$ , then subtract 9 from both sides.  
D. Divide both sides by the reciprocal of  $\frac{2}{3}$ , then subtract 9 from both sides.

- 3 The graph below shows Carlos' speed on his trip to school.



Based on the graph, when is Carlos' speed decreasing most rapidly?

- A. for times between B and C  
B. for times between D and E  
C. for times between E and F  
D. for times between H and I

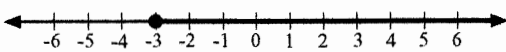
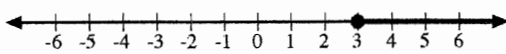
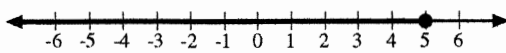
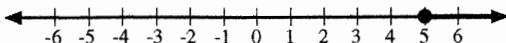
- 4 What is the slope of the line defined by the equation shown below?

$$5x + 2y = 10$$

- A.  $-\frac{2}{5}$   
B.  $-\frac{5}{2}$   
C.  $\frac{5}{2}$   
D.  $\frac{2}{5}$

- 5 Which graph below represents the solution to the inequality below?

$$2(2x - 6) \geq x + 3$$

- A.   
B.   
C.   
D. 

- 6 The input-output table below shows values for  $x$  and  $y$ .

INPUT ( $x$ )	OUTPUT ( $y$ )
-2	6
2	2
3	6
4	12

Which equation could represent a rule for the relationship between  $x$  and  $y$ ?

- A.  $y = x^2 - x$   
B.  $y = x^2 + 2$   
C.  $y = 2x - 2$   
D.  $y = -2x + x$

- 7 Each of the following statements is true.

$$16^x = 4$$

$$9^x = 3$$

$$4^x = 2$$

What is the value of  $3^x$ ?

- A. 1
- B.  $\sqrt{2}$
- C.  $\sqrt{3}$
- D. 9

- 8 The table below shows a linear relationship between  $x$  and  $y$ .

$x$	$y$
-7	$a$
-3	10
-1	6
0	4
5	-6

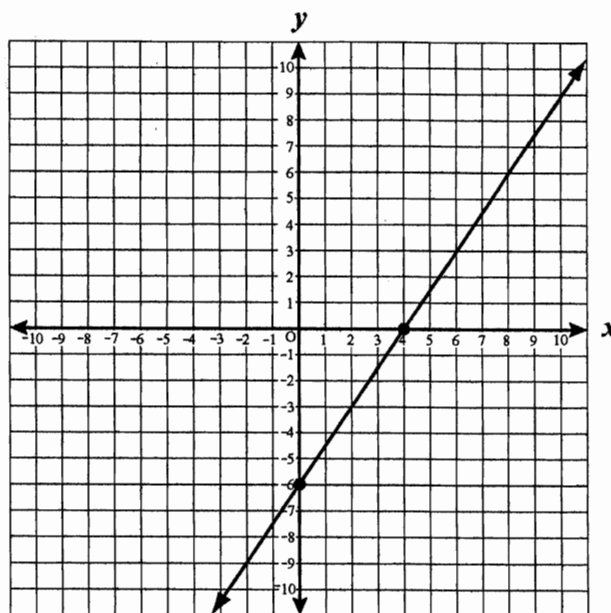
What is the value of  $a$ ?

- A. -18
- B. -14
- C. 14
- D. 18

- 9 If  $5x - 8 = 7$ , what is the value of  $5x + 8$ ?

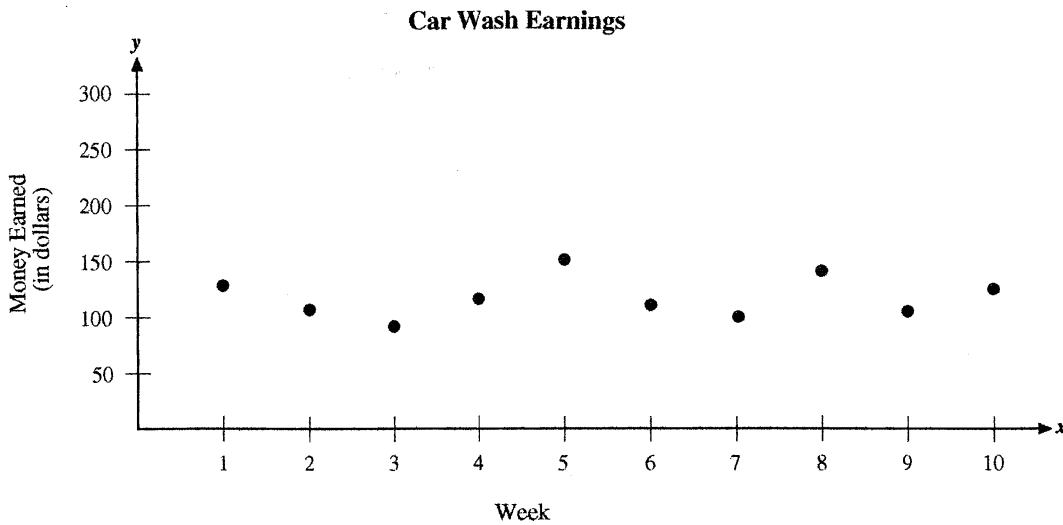
- A. -7
- B. 0
- C. 15
- D. 23

Use the graph below to answer question 18.



- 10 Which statement **best** describes the slope of the line graphed above?
- A. The slope is -6.
  - B. The slope is  $-\frac{2}{3}$ .
  - C. The slope is  $\frac{3}{2}$ .
  - D. The slope is 4.

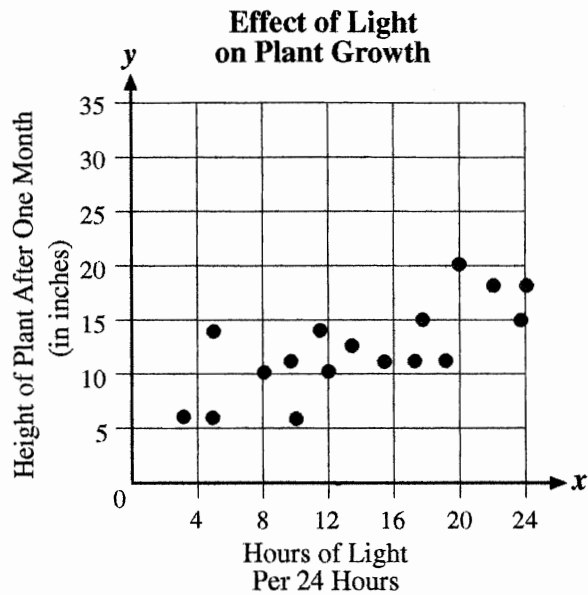
- 11 The students at Albemarle High held a car wash each week for 10 weeks to earn money for the student council. The students made the scatter plot below to represent the amount of the money they earned each week.



Which of the following equations **best** represents the line of best fit for these data?

- A.  $y = 110$
  - B.  $y = 110x$
  - C.  $y = x + 55$
  - D.  $y = -x + 55$
- 12 Tina solved a quadratic equation and found the solutions to be  $-\frac{3}{2}$  and 6. Which of the following is equivalent to the quadratic equation that Tina solved?
- A.  $(x - 6)(3x + 2) = 0$
  - B.  $(x - 6)(2x + 3) = 0$
  - C.  $(x + 6)(2x - 3) = 0$
  - D.  $(x + 6)(3x - 2) = 0$

- 13 Jenny studied the effect of light on plant growth. She graphed a scatterplot to represent her data.



Which of the following **best** represents the equation for the line of best fit for the data shown?

- A.  $y = -0.4x + 5$
- B.  $y = 0.4x + 5$
- C.  $y = -4x + 5$
- D.  $y = 4x + 5$

- 14 Julia and Marcia bought identically priced cans of chili and identically priced jars of salsa to make a dip.

- Julia bought 3 cans of chili and 2 jars of salsa for \$10.07.
- Marcia bought 2 cans of chili and 4 jars of salsa for \$12.98.

Which of the following systems of equations could be used to find  $x$ , the cost of one can of chili, and  $y$ , the cost of one jar of salsa?

- A.  $x + y = 10.07$   
 $x + y = 12.98$
- B.  $10.07x + 12.98y = 11$   
 $x + y = 11$
- C.  $2x + 4y = 10.07$   
 $2x + 3y = 12.98$
- D.  $3x + 2y = 10.07$   
 $2x + 4y = 12.98$

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For questions 15 – 17, follow the directions provided in each problem.

- 15 Write a rule that could be used to show the relationship between  $x$  and  $y$  in the table below.

$x$	$y$
-4	16
-1	1
0	0
3	9
7	49

Answer: \_\_\_\_\_

- 16 What is the  $y$ -intercept of the graph represented by the equation below?

$$y = \frac{4}{5}x - 2$$

Answer: \_\_\_\_\_



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- 17 Larissa plans to select one of the two mobile phone services described in the chart below. Each of the two companies charges a fixed monthly fee plus an additional charge for each minute in excess of the free time allowance.

**Mobile Phone Service**

Company	Monthly Fee	Free Minute Allowance per Month	Cost for Each Additional Minute
Cellco	\$35	300	\$0.08
Firstfone	\$22	400	\$0.15

Larissa plans to use her mobile phone as her only phone and predicts that she will use it between 600 and 900 minutes per month. To find  $t$ , the total monthly charge for each company based on  $m$  minutes of phone use, Larissa wrote the two equations shown below.

$$\text{Cellco: } t = 35 + (m - 300)(0.08) \quad \text{for } m \geq 300$$

$$\text{Firstfone: } t = 22 + (m - 400)(0.15) \quad \text{for } m \geq 400$$

- a. Which is the less expensive plan for 600 minutes of phone use per month? Show or explain how you obtained your answer.
- b. Determine the number of minutes for which the monthly charges for the two companies would be exactly the same amount. Show or explain how you obtained your answer.

Answer: a) \_\_\_\_\_

b) \_\_\_\_\_

Appendix E

End-of-Course Algebra Test Form C

# End-of-Year Algebra Exam

## Spring 2004

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### Test Form: C

Student Name (Please print): \_\_\_\_\_

Teacher: \_\_\_\_\_

School: \_\_\_\_\_

Class Period: \_\_\_\_\_

### Student Information:

1. Please check one: ☐<sub>1</sub> Male      ☐<sub>2</sub> Female
2. When did you first learn to speak English? (Please check one)  
☐<sub>1</sub> When I was younger than 5 years old  
☐<sub>2</sub> When I was between 5 and 8 years old  
☐<sub>3</sub> When I was between 9 and 12 years old  
☐<sub>4</sub> When I was 13 years old or older
3. Ethnicity (Please check as many as apply):  
☐<sub>1</sub> American Indian or Alaskan Native  
☐<sub>2</sub> Asian (Chinese, Japanese, Korean, Asian Indian, other Asian)  
☐<sub>3</sub> Black or African American, non-Hispanic  
☐<sub>4</sub> Filipino  
☐<sub>5</sub> Latino, Spanish-Origin, Hispanic  
☐<sub>6</sub> Pacific Islander (Native Hawaiian, Guamanian, Samoan)  
☐<sub>7</sub> Southeast Asian (Cambodian, Laotian, Vietnamese, other Southeast Asian)  
☐<sub>8</sub> White  
☐<sub>9</sub> Other: \_\_\_\_\_

**End-of-Year Algebra Exam**  
**Spring 2004**

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**Test Instructions**

Please answer the test questions as well as you can. There is space on the test pages to show your work or use as scratch paper.

You will have 50 minutes to complete the test.

**You may not use a calculator of any kind during the test.**

**—— Turn the page and begin the test ——**

# End-of-Year Algebra Exam Spring 2004

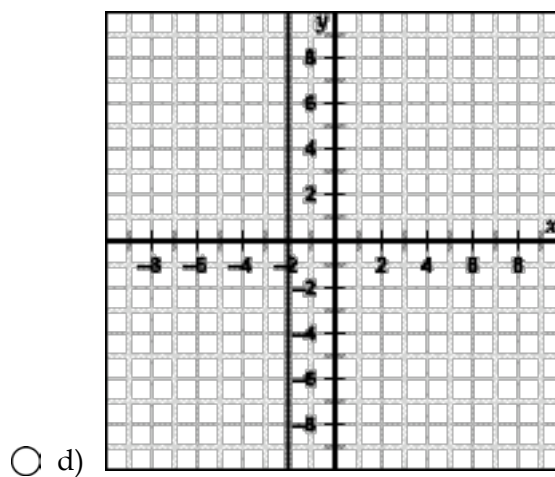
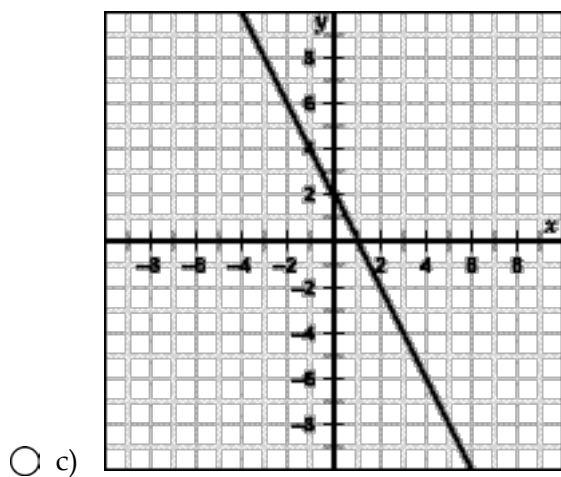
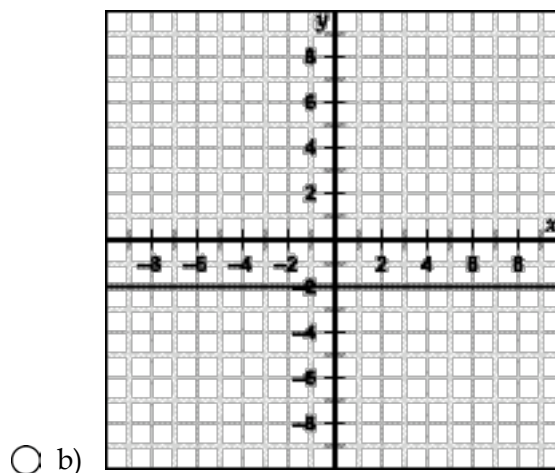
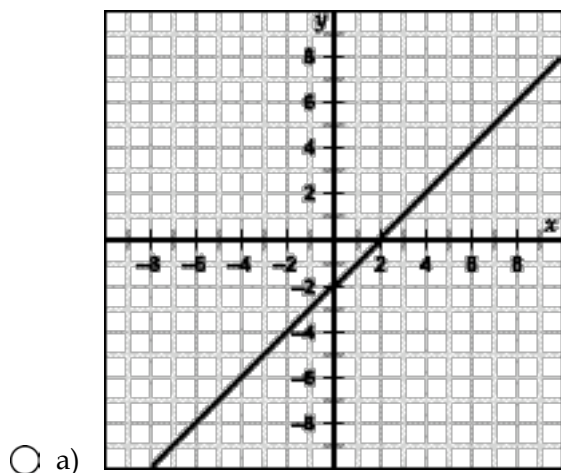
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Directions:

Questions 1 - 13 are multiple choice.

Please fill in the circle next to your answer.

1. Which of the following lines has a negative slope?



## End-of-Year Algebra Exam Spring 2004

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2. Which of the following functions will have the largest value at  $x = 70$ ?

- ☐ a)  $y = 7 + x$
- ☐ b)  $y = 7x$
- ☐ c)  $y = x^2$
- ☐ d)  $y = 7^x$

3. The table shows a set of values for  $x$  and  $y$ .

$x$	-3	-2	1	3	6
$y$	7	5	-1	-5	-11

Which equation best represents this set of data?

- ☐ a)  $y = \frac{1}{2}x + 2$
- ☐ b)  $y = -2x + 1$
- ☐ c)  $y = -3x - 2$
- ☐ d)  $y = 2x + 1$
- ☐ e)  $y = -\frac{1}{2}x - 2$

4. What is the  **$x$ -intercept** of the graph of the function  $2x + 3y = -36$ ?

- ☐ a) -36
- ☐ b) -18
- ☐ c) -12
- ☐ d) 0

## End-of-Year Algebra Exam Spring 2004

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5. Which of the following lines is parallel to the line  $3x + 5y = 7$  ?
- ☐ a)  $5y - 4 = -3x$
  - ☐ b)  $3x + 8y = 1$
  - ☐ c)  $3x - 5y = 7$
  - ☐ d)  $3x = 5y + 7$
6. If weights are placed on the end of a spring, the spring stretches and becomes longer. The relationship between the spring length and the amount of weight hanging on it is given by the equation  $y = 5 + 3.2x$ , where  $y$  is the length in inches and  $x$  is the weight in ounces.

What does the 5 in the equation tell you?

- ☐ a) It tells you that the spring is 5 inches long when no weights are attached to it.
  - ☐ b) It tells you that the spring is 5 inches long when 3.2 ounces of weight are attached to it.
  - ☐ c) It tells you that the spring is 3.2 inches long when 5 ounces of weight are attached to it.
  - ☐ d) It tells you that the spring can hold exactly 5 weights before breaking.
  - ☐ e) It doesn't tell you anything.
7. Tanya keeps a record of her weekly earnings. Last week she worked a total of 6 hours and earned \$51. This week she worked a total of 9 hours and earned \$76.50. Which equation can be used to find  $y$ , the amount she would earn at this rate if she worked  $x$  hours?
- ☐ a)  $y = \frac{2}{17}x$
  - ☐ b)  $y = \frac{2}{3}x$
  - ☐ c)  $y = 12.75x$
  - ☐ d)  $y = 8.5x$

## End-of-Year Algebra Exam Spring 2004

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8. You have a choice of two cell phone companies. Company A charges \$65 for the phone, and \$0.20 per minute for phone calls. Company B offers a free phone, and charges \$0.30 per minute for phone calls.

i. Which equation represents Company A's plan?

- ☐ a)  $y = 65x + 0.20$
- ☐ b)  $y = 0.20x + 65$
- ☐ c)  $y = 0.20x$
- ☐ d)  $y = 0.65x$

ii. The equation for Company B's plan is  $y = 0.30x$ . If you talk for 20 hours during your first month on this plan, how much will it cost?

- ☐ a) \$20
- ☐ b) \$66.67
- ☐ c) \$360
- ☐ d) \$4,000

iii. You decide to choose Company B's plan. If your bill is \$81 the first month, for how many minutes did you talk on the phone?

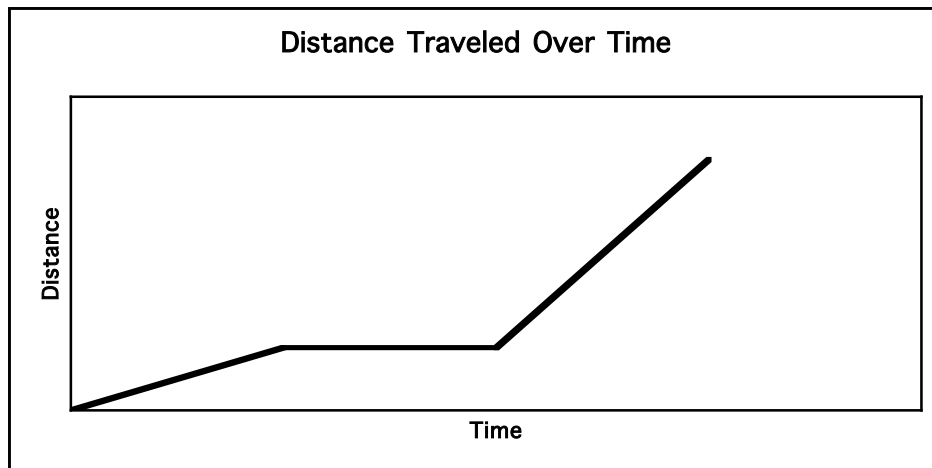
- ☐ a) 24 minutes
- ☐ b) 81 minutes
- ☐ c) 243 minutes
- ☐ d) 270 minutes



## End-of-Year Algebra Exam Spring 2004

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9. Maria recently traveled from home to her cousin's house. She constructed this graph showing the relationship between her travel time and the distance she traveled.



Which of the following best describes her trip?

- ☐ a) Maria drove on a high-speed superhighway, then slowly on a dirt road, and finished her trip on a high-speed superhighway.
- ☐ b) Maria started on a high-speed superhighway. She stopped for lunch just before getting onto a dirt road for the rest of her trip.
- ☐ c) Maria drove slowly on a dirt road, then on a high-speed superhighway, and finished her trip on a dirt road.
- ☐ d) Maria drove slowly on a dirt road. She stopped for lunch just before getting onto a high-speed superhighway for the rest of the trip.

## End-of-Year Algebra Exam Spring 2004

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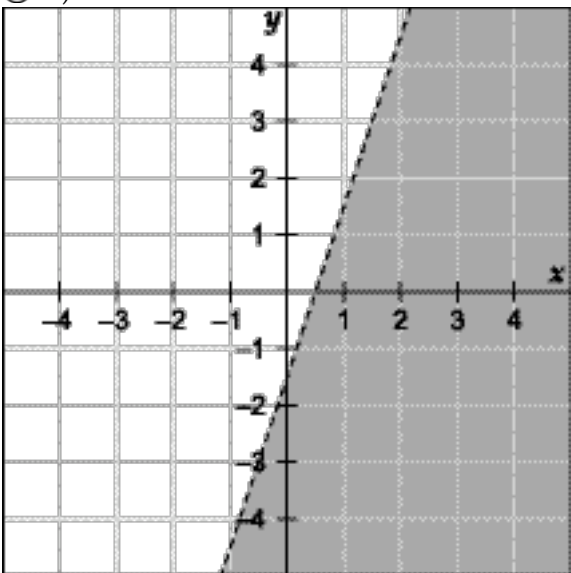
10. Yesterday a total of 24 students were present in Ben's class. There were 3 fewer girls than twice the number of boys. Which system of equations can be used to find  $g$ , the number of girls who were present in Ben's class yesterday, and  $b$ , the number of boys who were present?
- ☐ a)  $24 = g + b$   
 $g = 2b - 3$
- ☐ b)  $g + b = 24$   
 $b = 2g - 3$
- ☐ c)  $24 = g + b$   
 $g = 3 - 2b$
- ☐ d)  $g + b = 24$   
 $b = 3 - 2g$
11. A rabbit population doubles in size each day. There are 400 rabbits today. Which equation correctly describes this situation?
- ☐ a)  $y = (2)(400)x$
- ☐ b)  $y = (400)(2)x$
- ☐ c)  $y = (2)(400)^x$
- ☐ d)  $y = (400)(2)^x$
12. A student solves the equation  $x^2 + 3x - 18 = 0$ , and finds that the solutions are  $x = -6$  or  $x = 3$ . What do these solutions represent in terms of the graph of the equation?
- ☐ a) They tell you the coordinates of the vertex of the parabola.
- ☐ b) They tell you where the parabola crosses the  $x$ -axis.
- ☐ c) They tell you how wide or narrow the parabola is.
- ☐ d) They tell you the coordinates of points in quadrants I and III of the graph.
- ☐ e) They tell you where the parabola crosses the  $y$ -axis.

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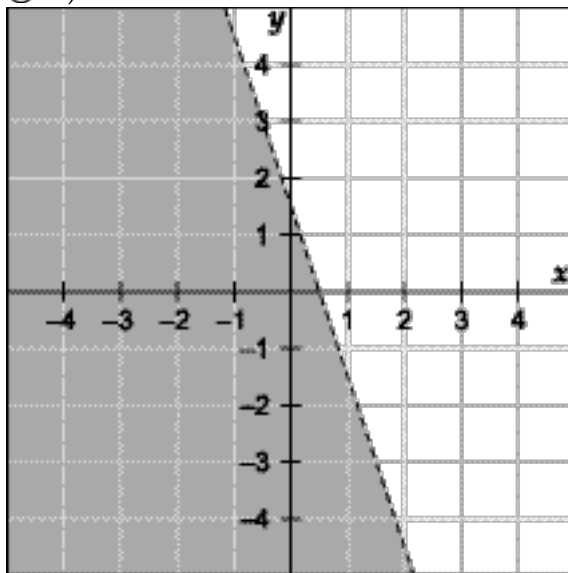
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13. Which graph corresponds with the inequality  $6x + 2y > 3$ ? The gray shaded area is the set of points for which the inequality is true.

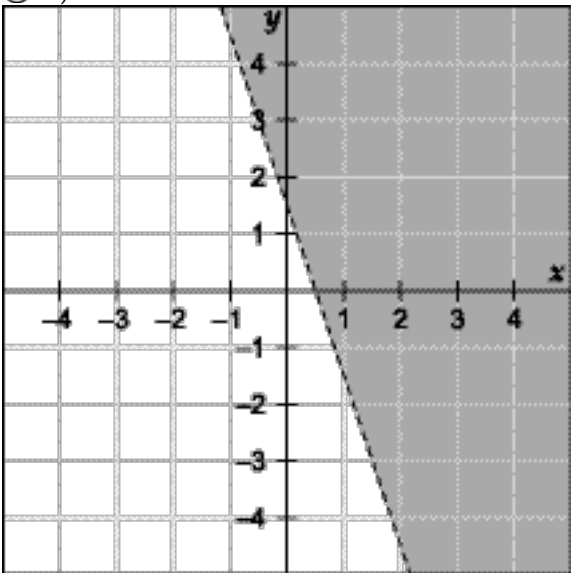
☐ a)



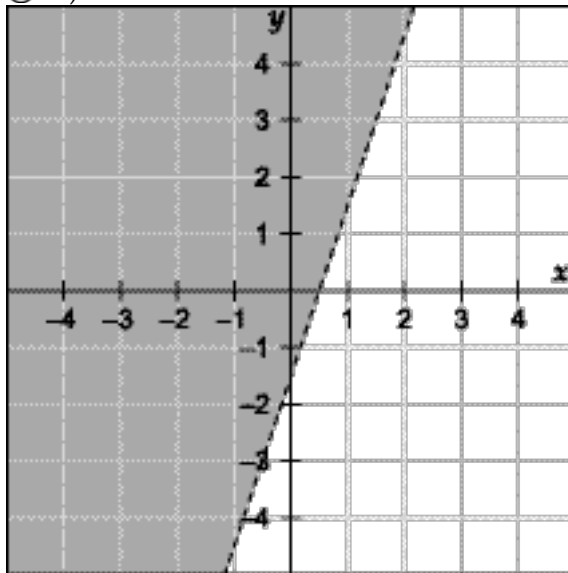
☐ b)



☐ c)



☐ d)



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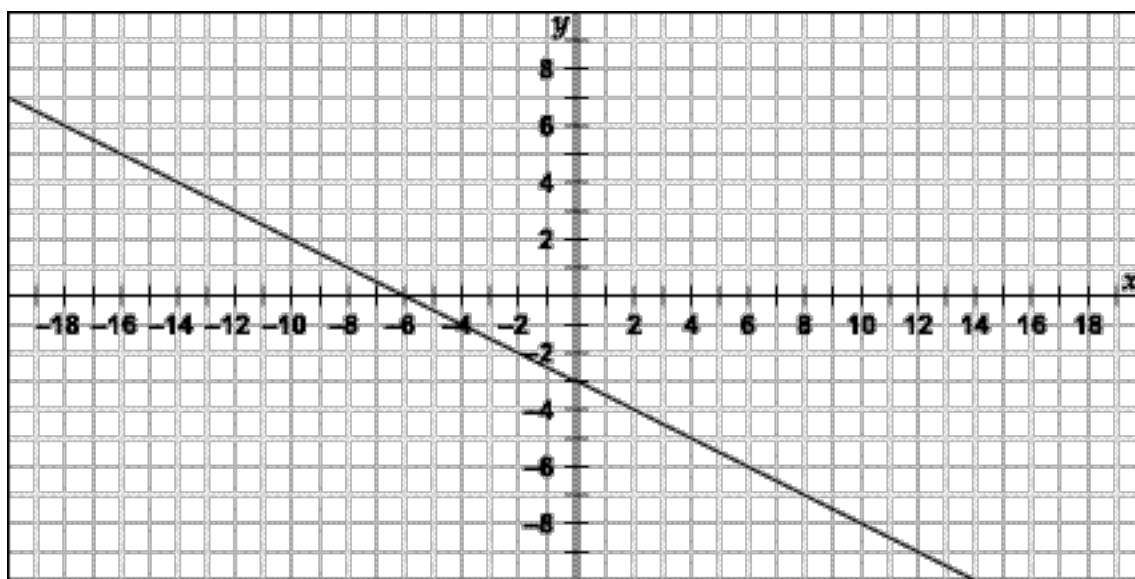
For questions 14 – 17, follow the directions provided in each problem.

14. Solve for  $x$ .

$$\frac{4(x + 7) - 8}{3} = 20$$

$x =$  \_\_\_\_\_

15. Write the equation of the line shown below:

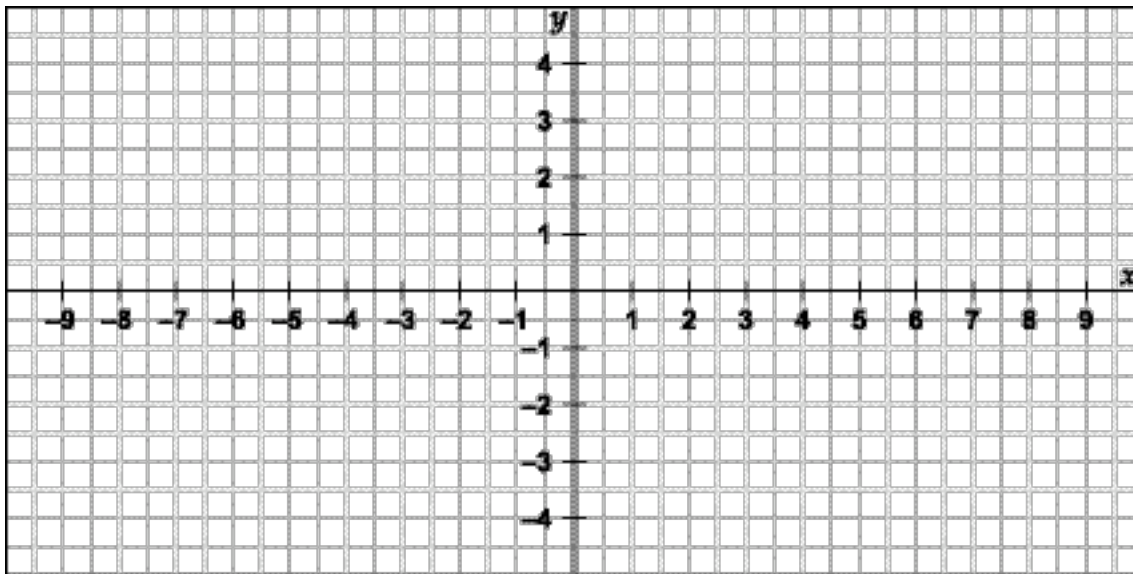


Answer: \_\_\_\_\_

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16. Graph the equation  $x = -3$



17. Write the equation of the line containing the points  $(3,-2)$  and  $(4,5)$ . The equation should be in the form  $y = mx + b$ .

Answer: \_\_\_\_\_

Author Note

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