How Many?

About the Lesson

Students will be presented a situation in which they must use linear programming to determine the optimum production level to maximize profits. As a result, students will:

- Graph a system of linear inequalities and identify the solution set.
- Apply the minimax theorem to solve problems in linear programming.

Vocabulary

- system of linear inequalities
- fundamental region
- minimax theorem

Teacher Preparation and Notes

- Students should already be familiar with graphing inequalities and be aware that the shaded region is the solution.
- This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators. Although the majority of the ideas and concepts are only presented in this document, be sure to cover all the material necessary for students' total comprehension.
- The student worksheet is intended to guide students through the main ideas of the activity and serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.
- This activity requires the use of the Inequality Graphing application, **Inequalz**. Ensure that the application is loaded onto each student's calculator before beginning the activity.
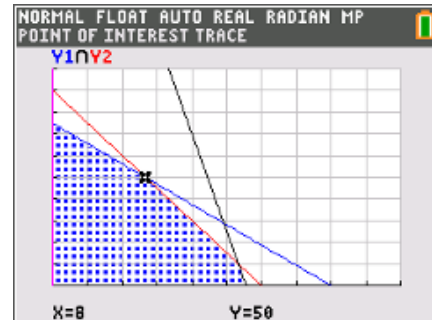
Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  🌐TI-84 Plus C Silver Edition
  🌐TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint^{TM} functionality.

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- How_Many_Student.pdf
- How_Many_Student.doc

Part 1 – Soccer Balls

A company sells two different types of soccer balls: adult and youth. It operates a production level of at most 100 soccer balls a day.

Producing a youth soccer ball costs $5. Producing an adult soccer ball costs $10.

The daily operating budget for the company is $900 a day.
The profit for producing a youth soccer ball is $14 and an adult soccer ball is $20.

How many of each type of soccer ball should the company produce to maximize its profit?

1. What is the profit function if x is the number of youth soccer balls and y is the number of adult soccer balls?

   Answer: The profit function is $P(x, y) = 14x + 20y$.

Next, decide what the constraints are for the problem.

2. Why should you be sure to include $x \geq 0$ and $y \geq 0$ as part of the constraints?
   Answer: The company cannot make a negative number of soccer balls.

3. Write an inequality for the production level.
   Answer: The production level constraint is $x + y \leq 100$.

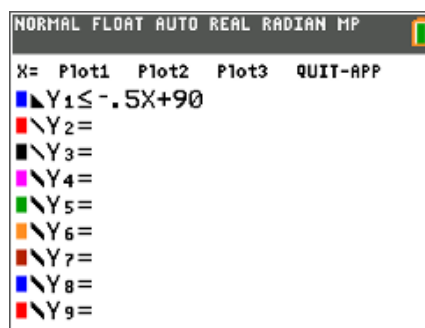4. Write an inequality for the daily operating budget.
   Answer: The daily operating budget constraint: $5x + 10y \leq 900$.

Graph the constraints, or inequalities, using the Inequality Graphing app. To start it, press apps and choose it from the list.
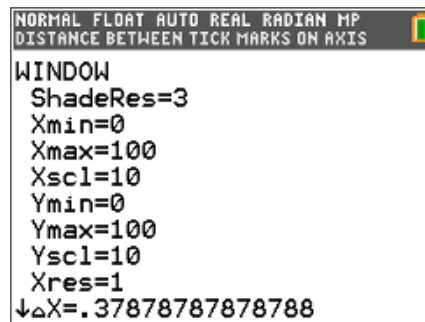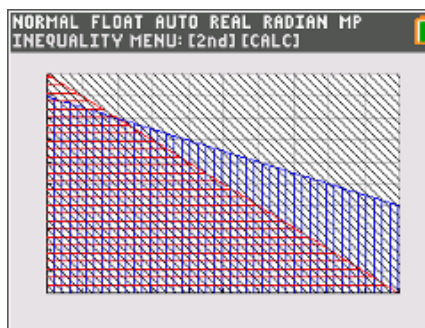
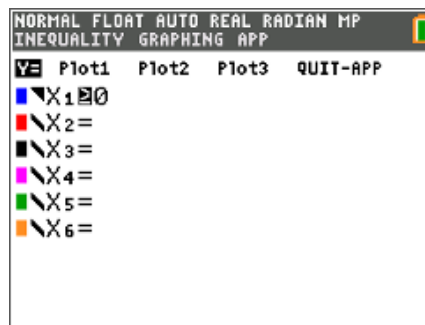Move your cursor over the **Y1** and press enter to bring up the Inequality Selection Menu.

```
NORMAL FLOAT AUTO REAL RADIAN MP
X= Plot1  Plot2  Plot3  QUIT-APP
■▶Y1≤ -.5X+90
■\Y2=
■\Y3=
■\Y4=
■\Y5=
■\Y6=
■\Y7=
■\Y8=
■\Y9=
```

Press window to choose appropriate window settings. Then press graph to view the graph.

```
NORMAL FLOAT AUTO REAL RADIAN MP
DISTANCE BETWEEN TICK MARKS ON AXIS
WINDOW
 ShadeRes=3
 Xmin=0
 Xmax=100
 Xscl=10
 Ymin=0
 Ymax=100
 Yscl=10
 Xres=1
↓ΔX=.37878787878788
```

Enter the other constraints in **Y₂** and **Y₃**.

```
NORMAL FLOAT AUTO REAL RADIAN MP
INEQUALITY MENU: [2nd][CALC]
```

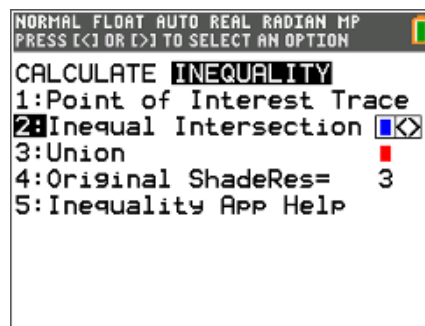To enter the constraint $x \geq 0$, arrow up to the text reading **X=** at the upper left corner of the screen and press enter. This will change the inequality input lines from **Y=** to **X=** form.

```
NORMAL FLOAT AUTO REAL RADIAN MP
INEQUALITY GRAPHING APP
Y= Plot1  Plot2  Plot3  QUIT-APP
■\X1≥0
■\X2=
■\X3=
■\X4=
■\X5=
■\X6=
```

View the graph. Four layers of shading may make the
graph difficult to read. Press 2nd trace and select the
Inequality menu followed by Option 2: **Inequal
Intersection**.

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS [<] OR [>] TO SELECT AN OPTION
CALCULATE INEQUALITY
1:Point of Interest Trace
2:Inequal Intersection
3:Union
4:Original ShadeRes=    3
5:Inequality App Help
```

The set of inequalities overlap to form the fundamental
region. This is where the company production levels
exist.

To find the optimum production level, determine the
coordinates of the vertices of this region.

```
NORMAL FLOAT AUTO REAL RADIAN MP
INEQUAL INTERSECTION
```
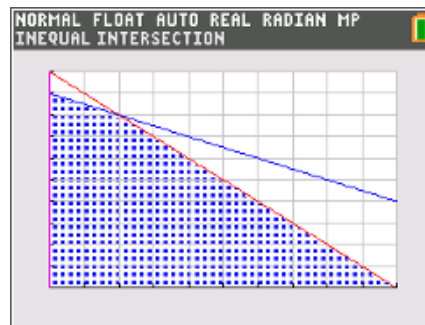
Press 2nd trace and select the **Inequality** menu followed
by Option 1: **Point of Interest Trace**. Use the arrow
keys to move among the 4 vertices.
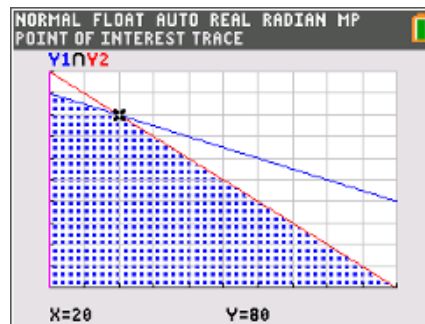
```
NORMAL FLOAT AUTO REAL RADIAN MP
POINT OF INTEREST TRACE
Y1∩Y2



X=20            Y=80
```

Substitute in the coordinates of the vertices into the profit function to determine which one produces
the highest profit. Then find the maximum profit.

5. What production level maximizes the profit?

   Answer: Maximum occurs when 20 youth soccer balls and 80 adult soccer balls are produced
                each day.

6. What is the maximum profit?

   Answer: The maximum profit is $1,880.

Part 2 – Barbeque Catering

Each day, a barbeque catering business maintains a stock of 600 lb. of ribs, 180 lb. of brisket, and 250 lb. of pulled pork.

The business sells two kinds of catering set-ups, the *Tailgate* special and the *Home Viewer* special.

The *Tailgate* special requires 25 lb. of ribs, 10 lb. of brisket, and 15 lb. of pulled pork, and creates a profit of $80.

The *Home Viewer* special requires 8 lb. of ribs, 2 lb. of brisket, and 1 lb of pulled pork and creates a profit of $30.

How many of each special should be sold to maximize profits?

7. What is the profit function if *x* is the *Tailgate* special and *y* is the *Home Viewer* special?

Answer: The profit function is $P(x, y) = 80x + 30y$.

Next, decide what the constraints are for the problem.

8. Why should you be sure to include $x \geq 0$ and $y \geq 0$ as part of the constraints?

Answer: The business cannot cook a negative amount of meat.

9. Write an inequality for each type of smoked meat.

Answer: The smoked meat inequalities are:
Ribs: $25x + 8y \leq 600$
Brisket: $10x + 2y \leq 180$
Pulled Pork: $15x + y \leq 250$

Follow the steps from Part 1 to graph the constraints with the *Inequality Graphing* app, **Inequalz**.

Choose appropriate window settings. Then press [graph] to view the graph.

Again, use the appropriate procedure to choose **Ineq Intersection** to shade only the intersection of the inequalities.

Lastly, use the appropriate procedure to access the **Points of Interest** tool to find the 5 coordinates of the vertices.
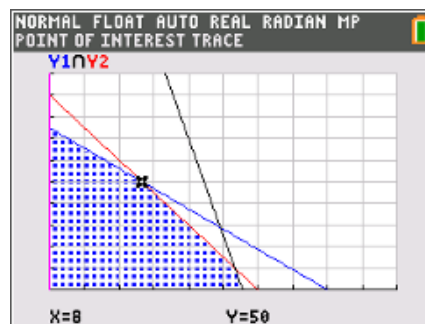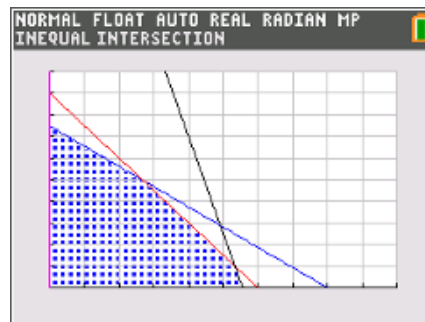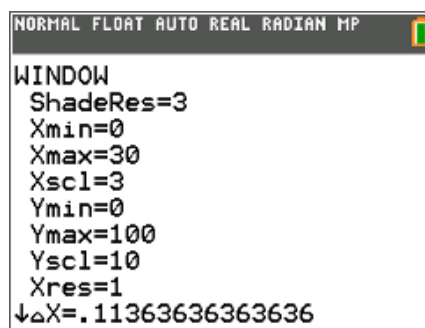
Substitute in the coordinates of the vertices into the profit function to determine which one produces the highest profit. Then find the maximum profit.

10. How many of each special needs to be sold to maximize profit?

 Answer: The maximum profit occurs when 0 *Tailgate* specials and 75 *Home Viewer* specials are sold.

11. What is the maximum profit?

 Answer: The maximum profit is $2,250.

12. Why would this not be a wise decision for the company? (Hint: Think about the quantities of meat it is currently stocking.)

 Answer: The business would be wasting meats in the long run.