Introduction to Conics

Math Objectives

- Students will identify how each conic results from slicing cones.
- Students will understand the locus definition of a parabola.
- Students will describe how the values of *a*, *h*, and *k* in the vertex form of the equation of a parabola affect its graph.
- Students will use the locus definition of a parabola to derive the equation of a parabola and will describe the relationships among the focus, the directrix, and the values in the vertex form of a parabola.
- Students will use appropriate tools strategically and make use of structure (CCSS Mathematical Practice).

Vocabulary

- circle
- ellipse
- axis of symmetry
- parabola
- focus
- hyperbola
- directrix

About the Lesson

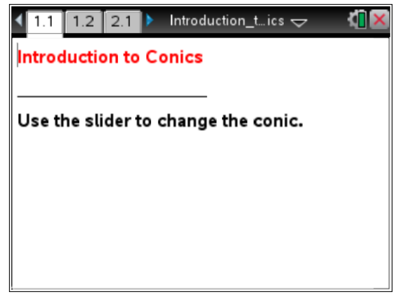
- This lesson involves observing how each of the conic sections is formed and connecting the locus definition of a parabola with the vertex form of a parabola.
- As a result, students will:
  - Explain how each of the conic sections is formed.
  - Manipulate a point on a parabola and the focus of a parabola to discover the locus definition.
  - Manipulate *a*, *h*, and *k* in the vertex form of a parabola to observe the effects of each value.
  - Use the locus definition to derive the equation of a parabola given the focus, directrix, and any point on the parabola.
  - Identify the relationships among the values of the vertex form of a parabola and the focus.

TI-Nspire™ Navigator™

- Use Class Capture to examine patterns that emerge.
- Use Teacher Edition computer software to review student documents.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Introduction_to_Conics_Student.pdf
- Introduction_to_Conics_Student.doc

TI-Nspire document

- Introduction_to_Conics.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand () getting ready to grab the point. Then press ctrl to grab the point and close the hand ().

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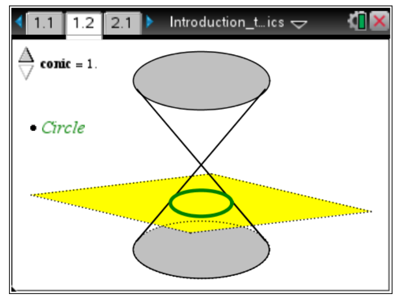

1. Use ▲ and ▼ to scroll through the different conic sections. Briefly describe how each of the conic sections is formed. Complete the table below.

Sample Answers:

Students might think of the picture on page 1.2 as depicting *two* cones placed vertex to vertex. Clarify that the mathematical definition of a **cone** is a surface generated by a line passing through a fixed point (the **vertex of the cone**) at a fixed angle from a line passing through that vertex (the **axis of the cone**). This forms two **nappes** of the cone with a common vertex. So the picture on page 1.2 depicts a *single* mathematical cone, hav.

Conic Section (A cone cut by a plane)	**Description**
Circle	Intersecting a cone with a plane perpendicular to the axis of the cone (Students may say plane parallel to the base.)
Ellipse	Intersecting a cone with a plane not perpendicular to the axis, but passing completely through one nappe. (Students may say plane not parallel to the base.)
Parabola	Intersecting a cone with a plane parallel to the side of the cone.
Hyperbola	Intersecting both nappes of the cone with a plane parallel to the axis (Students may say plane perpendicular to the base, but that is only one possibility.)

> **Teacher Tip:** In the formation of all cases above, the intersecting plane does not pass through the vertex.

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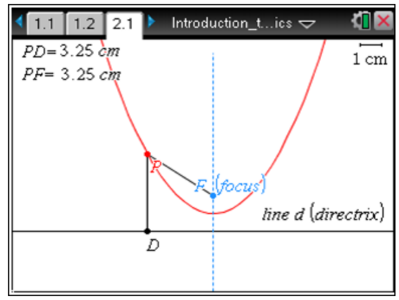

2. Point *F* is called the focus of the parabola. What is the line through point *F* perpendicular to the directrix called?

   <u>Answer:</u> Axis of symmetry

3. Line *d* is called the directrix. What is the relationship between line *d* and the dashed line through point *F*?

   <u>Answer</u>: They are perpendicular.

4. Drag point *P* along the curve. What property seems to be true for all points along the parabola?

   <u>Answer</u>: Point *P* is equidistant from both point *F* and line *d*. Each point on the parabola is equidistant from the focus and the directrix.

> **Teacher Tip:** The locus definition of a parabola is all the points equidistant from a line called the directrix and a point called the focus.

5. Drag point *F* around the screen. Does the property observed in Question 4 remain true? Explain your answer.

   <u>Answer</u>: Yes. The point *P* remains equidistant from the focus and directrix. Students can also make statements about *PF* = *PD*.

TI-Nspire Navigator Opportunity: *Live Presenter/Class Capture*
See Note 1 below.

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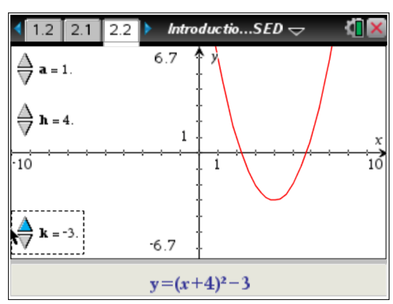

6. The vertex form of the equation for a parabola, $y = a(x - h)^2 + k$, is shown. Use ▲ and ▼ to change the value of *a*. Describe how the value of *a* affects the graph.

   <u>Answer</u>: The *a*-value dilates the parabola.

7. Use ▲ and ▼ to change the value of h. Describe how the value of h affects the graph.

Answer: The h-value changes the horizontal shift.

8. Use ▲ and ▼ to change the value of k. Describe how the value of k affects the graph.

Answer: The k-value changes the vertical shift.

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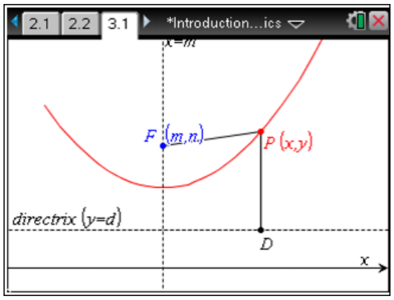

9. Given focus (m, n), directrix $y = d$, and point (x, y) on a parabola, use the distance formula to derive the equation for any parabola function.

   a. Find the distance between the focus and point P.
   Answer: $\overline{FP} = \sqrt{(x-m)^2 + (y-n)^2}$

   b. Find the distance between point P and the directrix.

   Answer: $\overline{PD} = \sqrt{(x-x)^2 + (y-d)^2} = \sqrt{(y-d)^2} = y-d$

   c. Set the two distances you found in Questions 9a and 9b equal to each other and solve for y.
   Answer: $\overline{FP} = \overline{PD}$

$$\sqrt{(x-m)^2 + (y-n)^2} = y-d$$
$$(x-m)^2 + (y-n)^2 = (y-d)^2$$
$$(x-m)^2 + y^2 - 2yn + n^2 = y^2 - 2yd + d^2$$
$$(x-m)^2 = 2yn - 2yd - n^2 + d^2$$
$$(x-m)^2 = 2y(n-d) - (n^2 - d^2)$$
$$(x-m)^2 = 2y(n-d) - (n-d)(n+d)$$
$$(x-m)^2 = (n-d)(2y - (n+d))$$
$$\frac{(x-m)^2}{n-d} = 2y - (n+d)$$
$$\frac{(x-m)^2}{n-d} + (n+d) = 2y$$
$$\frac{(x-m)^2}{2(n-d)} + \frac{(n+d)}{2} = y$$
$$y = \frac{1}{2(n-d)}(x-m)^2 + \frac{(n+d)}{2}$$

10. Use the vertex form of the equation for a parabola, $y = a(x - h)^2 + k$, and the derived equation from Question 9 to answer the following questions.

    a. Explain the relationship among the focus, the directrix, and the value of *a*.

    Answer: The *a*-value is $\frac{1}{2(n-d)}$, the multiplicative inverse of twice the difference between the focus and the directrix.

    b. Explain the relationship between the focus and the value of *h*.

    Answer: The *h*-value is *m*, the *x*-coordinate of the focus. The focus lies on the axis of symmetry. Since changing the value of *h* resulted in a horizontal shift, the axis of symmetry and the focus also shift. It follows that the *h* and *x*-coordinate of the focus are the same. The value *m* is the *x*-coordinate of the vertex.

    c. Explain the relationship among the focus, the directrix, and the value of *k*.

    Answer: The *k*-value is $\frac{(n+d)}{2}$, the *y*-coordinate of the midpoint of the line segment from the focus to the directrix (along the axis of symmetry). Thus, the vertical shift (from the parabola $y = x^2$) is half the distance between the focus and the directrix.

Teacher Tip: It is recommended that students compare and discuss the relationships. To extend this question, students could have a deeper discussion about why these relationships exist.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- How each conic section is formed.
- The locus definition of a parabola.
- The connection between the locus definition and the vertex form of a parabola.

Assessment

Find the equation of the parabola with the focus at (2, 5) and the directrix at $y = -1$.

TI-Nspire Navigator

Note 1

Question 5, Name of Feature: Live Presenter/Class Capture

Choose a student to become the Live Presenter. Have the student demonstrate and explain the relationship they found in Question 4. This will reinforce the locus definition of a parabola.

Turn off Live Presenter. Turn on the Auto-Refresh feature (30-second intervals might be reasonable).

TI-Navigator can be used to monitor student progress. You might want to leave Class Capture projected (without student names). This will enable both you and the students to monitor their progress. Alternatively, you might want to turn off the projector so the student screens are visible only to you.

As you circulate around the classroom, frequently check Class Capture to identify students who need help. Use this information to decide whether to provide individual assistance or bring the class together to address what seems to be a common misunderstanding.
After the class discussion, students should begin Question 6.