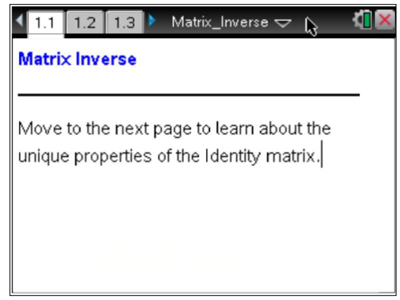
Math Objectives

- Students will Identify the inverse of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ as $\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.
- Students will determine that if the determinant of a matrix is 0, then the matrix does not have an inverse.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a slider
- Modify a math box
- Multiply matrices in Scratchpad

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:

Student Activity
Matrix_Inverse_Student.pdf
Matrix_Inverse_Student.doc

TI-Nspire document
Matrix_Inverse.tns

Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)

Vocabulary

- matrix
- inverse
- determinant
- identity matrix

About the Lesson

- This lesson involves modifying a 2x2 matrix being multiplied by another 2x2 matrix until their product is the identity matrix.
- As a result, students will:
  - Test their newly learned knowledge and determine the inverse of any 2x2 matrix.
  - Determine that, if the determinant of a matrix is 0, then the matrix does not have an inverse.

TI-Nspire™ Navigator™ System

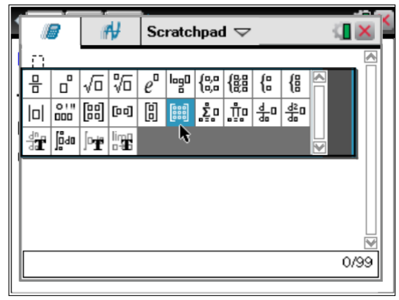
- Use Live Presenter to demonstrate how to perform matrix calculations in the Scratchpad.
- Use Screen Capture to monitor student progress and discuss aspects of the identity matrix and the determinant.

Discussion Points and Possible Answers

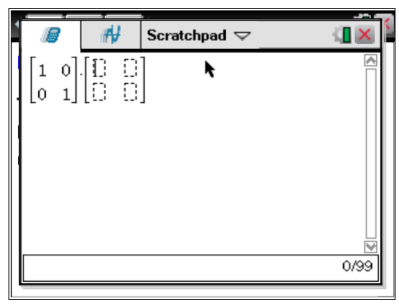
TI-Nspire Navigator Opportunity: ***Live Presenter***
See Note 1 at the end of this lesson.

Teacher Tip: Multiplying an expression by some form of the number 1 results in no change to the expression and is called the multiplicative identity property.

The number 1 is an incredibly powerful number in mathematics, and it can be written in many different ways. In matrix notation, the number 1 is expressed as $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and is called the *identity matrix.*



1. Multiplying 1 by any number results in no change to the number. Test this in matrix notation by multiplying $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by any 2×2 matrix.



- Open the Scratchpad.
- Enter the identity matrix by pressing, selecting the 2×2 matrix template, and entering $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
- Now enter another 2×2 matrix, but choose any element values for the matrix. Press enter.
- When you finish question 1b, press esc to exit the Scratchpad.

a. What is the result of the matrix multiplication?

Answer: The multiplication results in no change to the second matrix.

b. Repeat this two more times using a different second matrix. What do you notice about the results? Will this always happen? Why or why not?

Answer: Yes, this will always happen because of how matrices are multiplied. For example

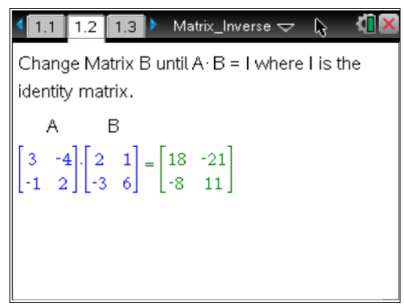
$$\begin{bmatrix} 1(3)+0(5) & 1(2)+0(-3) \\ 0(3)+1(5) & 0(2)+1(-3) \end{bmatrix}.$$

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

Tech Tip: While students modify the matrix, the resulting matrix product calculation will not be displayed. Once the students press enter, the matrix product will reappear.
To copy and paste the matrices in the entry line, highlight the previous entry line, press enter, modify the second matrix, and press enter.

Tech Tip: Most students will be unable to arrive at the identity matrix. Allow students to grapple with the calculation for a couple of minutes, but then discuss the answer to question 2 and move on with the activity.

Move to page 1.2.



2. Attempt to change the element values in matrix B until the product [A][B] is the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Why is it so difficult to find the correct values for matrix B?

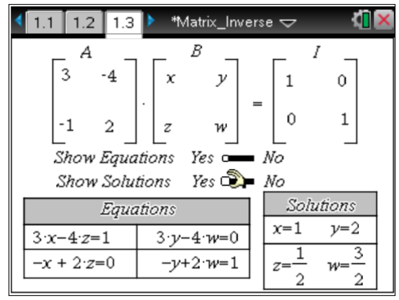
<u>**Answer:**</u> It is difficult due to the fact that in matrix multiplication there is multiplication and addition at every step, which makes it difficult to determine the correct elements in matrix B.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

Tech Tip: If students experience difficulty dragging the slider, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the movable point on the slider. Also, be sure that the slider bar becomes highlighted. Then press ctrl to grab the point and close the hand ( ).

Move to page 1.3.

3. When the product of two matrices is the identity matrix, then the second matrix is the *inverse* of the first matrix. The inverse matrix can be calculated using a system of equations.

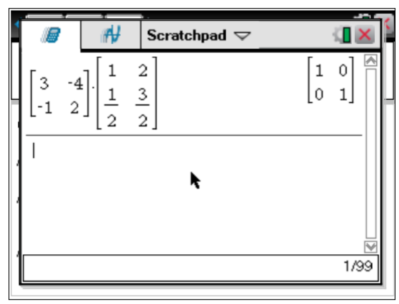


a. Identify the necessary system of equations by multiplying matrices A and B. Write your result below. Confirm your result by moving the slider to *Yes* for *Show Equations.*

Answer:

$3x - 4z = 1, \quad -x + 2z = 0, \quad 3y - 4w = 0, \quad -y + 2w = 1$

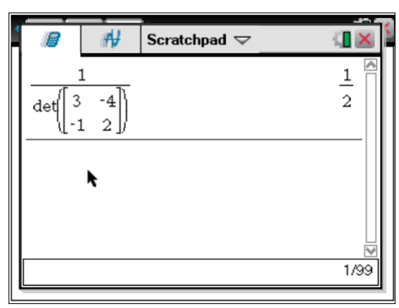
b. Determine the correct element values for matrix B by solving the system of equations. To display the solution to this system, move the slider to *Yes* for *Show Solutions.*



c. Use the Scratchpad to confirm that [A][B] results in the identity matrix. What patterns do you notice between the element values in matrix A and matrix B?

Answer: Students will have a variety of ideas. Some will note that the 3 moved to the bottom right corner but ask where the 2 went. Others might notice that the negative elements are now positive. Many will wonder where the fractions came from. This will lead your students to the next problem.

4. Using the Scratchpad, find the reciprocal of the determinant of matrix A by pressing 1 ÷ D E T (and entering matrix A.



a. Knowing the value of the reciprocal of the determinant, are there other patterns that you now notice between matrix A and matrix B?

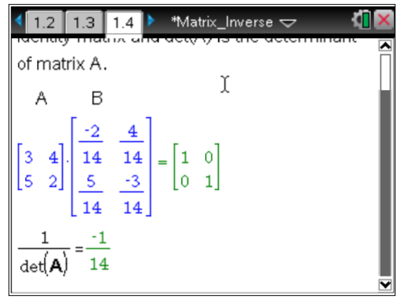
Answer: There will still be a variety of responses from students, but if they rewrite the values of matrix B so each element has a denominator of the determinant, they will see the pattern that every element is multiplied by $\frac{1}{\det(A)}$, the element values of the main diagonal are switched, and the other two values are multiplied by –1.

b. Would you like to change anything you wrote for question 3? Try rewriting the matrix so each element has a common denominator before answering.

Move to page 1.4.

5. Use the calculated determinant to help choose correct values for matrix B so that the product, [A][B], results in the identity matrix.

   <u>Answer:</u> Matrix B is $\begin{bmatrix} -\frac{2}{14} & \frac{4}{14} \\ \frac{5}{14} & -\frac{3}{14} \end{bmatrix}$

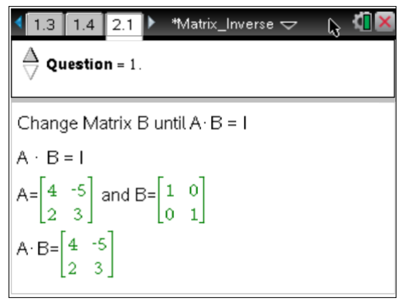


TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 4 at the end of this lesson.

Move to page 2.1.

6. This next page is for practice. Practice finding the correct values for matrix B so that the product, [A][B], is the identity matrix. Click the arrows by the question number to get a new question.

   <u>Answer:</u> The values for each matrix are randomly generated, so there are no provided solutions for the student to check to see when the product of AB is the identity matrix.



TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 5 at the end of this lesson.

7. Amber says that the inverse of $\begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}$ is $\begin{bmatrix} \frac{5}{7} & -\frac{3}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix}$. Is Amber correct? Why or why not?

<u>Answer:</u> Amber is not correct. The correct answer is $\begin{bmatrix} -\frac{5}{7} & -\frac{3}{7} \\ -\frac{1}{7} & -\frac{2}{7} \end{bmatrix}$.

TI-Nspire Navigator Opportunity: ***Quick Polls (Yes/No)***
See Note 6 at the end of this lesson.

8. Sean says every square matrix has an inverse. Is he correct? Explain.

   Answer: No. Sean is almost correct. The only matrices that do not have an inverse are those whose determinant is zero.

TI-Nspire Navigator Opportunity: ***Quick Polls (Yes/No)***
See Note 7 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that given the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse is $\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Assessment

1. Find the inverse of $\begin{bmatrix} 5 & -4 \\ 3 & -2 \end{bmatrix}$.

   Answer: $\begin{bmatrix} -\frac{2}{2} & \frac{4}{2} \\ -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$

2. Does $\begin{bmatrix} 3 & -12 \\ -2 & 8 \end{bmatrix}$ have an inverse? Why or why not?

   Answer: No, the matrix does not have an inverse because its determinant is zero.

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Note 1

Question 1a, *Live Presenter*: Consider using *Live Presenter* to demonstrate how to use the Scratchpad and the matrix templates.

Note 2

Question 1a, *Screen Capture*: Take a *Screen Capture* of the Scratchpad to aid the students in sharing their results. As a class, discuss how multiplying by the identity matrix results in no change to the matrix the students entered and that this is called the multiplicative identity.

Note 3

Question 2, *Screen Capture*: Take a *Screen Capture* of page 1.2 to monitor student progress on this question. As a class, discuss any patterns that develop.

Note 4

Question 5, *Screen Capture*: Take a *Screen Capture* of page 1.4 to monitor student progress. As a class, discuss how being told the value of the determinant was a help.

Note 5

Question 6, *Screen Capture*: Take a *Screen Capture* of page 2.1 to monitor student progress as they work the random practice problems. As a class, discuss how the pattern for finding the values for matrix B is shown in each correct answer.

Note 6

Question 7, *Quick Polls (Yes/No)*: Send a yes/no *Quick Poll* to students asking for their answer to question 6. As a class, discuss why Amber's answer is wrong.

Note 7

Question 8, *Quick Polls (Yes/No)*: Send a yes/no *Quick Poll* to students asking for their answer to question 7. As a class, discuss why Sean was almost correct and how his response could be changed to become correct.