Math Objectives

- Students will calculate slopes of secant lines, create tangent lines with the same slope, and note observations about the functions and slopes.

Activity Types

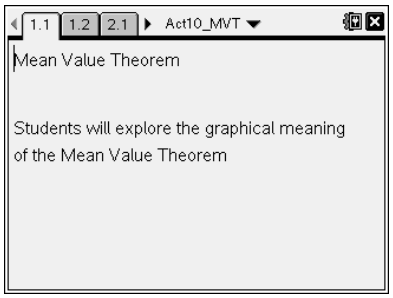
- Teacher Demonstration
- Student Exploration

About the Lesson

- Students will find the slope of a given secant line and a tangent line they place on the graph of a function and try to get their slopes to match. Students should see that the tangent and secant lines are parallel and make a connection leading to the Mean Value Theorem for Derivatives. Following a counterexample, students will also explore Rolle's Theorem.

Directions

- Calculate the slope of the given secant line: **Menu > Measurement > Slope.** Place a tangent line on the graph (**Menu > Points & Lines > Tangent Line**) and find the slope of the tangent line. Move the tangent line so that the slope matches the slope of the secant line. Note any observations about the relationship between the secant and tangent line.
- More detailed instructions are provided to work through the activity on the following pages.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Mean_Value_Theorem_Student.pdf
Mean_Value_Theorem_Student.doc

TI-Nspire document
Mean_Value_Theorem.tns

Visit www.mathnspired.com for lesson updates.

Discussion Points and Possible Answers

Move to page 1.2.

Teacher Notes: Depending on students' capability, it may be beneficial for you to lead the first problem. Detailed directions are provided on the student worksheet.

1. Calculate the slope of the secant line: **Menu > Measurement > Slope.**
   Place the tangent line: Press **Menu > Points & Lines > Tangent.**

   Answer: Slope of the secant line: 4 Slope of the tangent line: 3.99

Record your observations here: When the slope of the secant line is equal to the slope of the tangent line, the lines are parallel.

Move to page 2.1.

2. Repeat the same steps for problem 2 and record your observations.

   Answer: When the slope of the secant line is equal to the slope of the tangent line, the lines are parallel.
   Slope of the secant line: 4.25 Slope of the tangent line: 4.29

Move to page 3.1.

3. Repeat the same steps for problem 3 and record your observations.

   Answer: It is not possible to have the tangent and secant lines equal because the function is the greatest integer function (horizontal lines).
   Slope of the secant line: 0.714 Slope of the tangent line: 0

4. Do your observations still hold? Why or why not?

   Answer: Answers may vary.

CLASS DISCUSSION: Discuss the differences between the functions and whether the students could match the slopes of the secant and tangent lines. Students can copy the Mean Value Theorem from the Notes page.

Notes: Mean Value Theorem (Student can write the MVT here.)

If **f** is differentiable for all values of *x* in the open interval (*a, b*) and **f** is continuous for all values of *x* in the closed interval [*a, b*], then there is at least one number $x = c$ in (*a, b*) such that $\mathbf{f}'(c) = \frac{\mathbf{f}(b) - \mathbf{f}(a)}{b - a}$.

There is a point where the slope of the tangent line equals the slope of the secant line.

Point out to students that the slopes of the secant and tangent lines are equal, but ask students to prove if the function satisfies the conditions of the Mean Value Theorem.

Move to page 5.1.

COUNTEREXAMPLE: Point out to students that the slopes of the secant and tangent lines are equal, but ask students to prove if the function satisfies the conditions of the Mean Value Theorem.

5. Repeat the steps for the previous problems on problem 5.1. Determine if the Mean Value Theorem still holds.

   Record your observations here:

   Answer: Yes, the Mean Value Theorem holds.
   Slope of the secant line: 0.32 Slope of the tangent line: 0.312

Move to page 6.1.

ROLLE'S THEOREM: This problem demonstrates Rolle's Theorem. Follow the same directions as in previous problems.

6. Repeat the steps for the previous problems on problem 6.1. Determine if the Mean Value Theorem still holds.

   Record your observations here: The slope of the secant line equals the slope of the tangent line at the maximum.

   Answer: Yes, the Mean Value Theorem holds.
   Slope of the secant line: 0 Slope of the tangent line: 0

CLASS DISCUSSION: Point out that the difference in conditions between the MVT and Rolle's Theorem is in boldface font. Students should explain what Rolle's Theorem means. The lesson should continue with finding values for *c* algebraically for the examples given in the worksheet.

7. What is the difference in conditions between the MVT and Rolle's Theorem?

   Answer: Rolle's Theorem requires that the endpoint values of the function are equal. This means that the MVT will be satisfied at either the maximum or minimum on that interval.

8. What does Rolle's Theorem tell you must exist on a given interval?

   Answer: Rolle's Theorem guarantees a maximum or minimum on an interval.