

Objectives

- Explore the relationship between position and velocity
- Explore the relationship between functions and their derivatives
- Connect mathematical relationships to real-world phenomena

Materials

- TI-84 Plus / TI-83 Plus
- Calculator-Based Ranger[™] (CBR[™])

Move My Way—A CBR Analysis of Rates of Change

Introduction

Speed and velocity are concepts that you have observed since early childhood. Because velocity is the derivative of displacement, taking a closer look at position and velocity can develop a deeper understanding of how derivatives and their functions are related.

In this activity, you will use a Calculator-Based Ranger™ (CBR™) data collection device with your graphing handheld to collect motion data with certain characteristics and use that data to draw conclusions about the relationship between position and velocity.

Exploration

A CBR emits an ultrasonic beam that reflects off the closest object and returns to the CBR. Depending on the speed of sound and the time it takes for the sound wave to return, the distance between the object and the CBR is calculated.

 Begin by selecting :CBL/CBR from the APPS Menu. Press any key. Then select 3:RANGER and press ENTER. Select 2:SET DEFAULTS. The CBR will collect distance data for 15 seconds. You will see the graph as it is created and be able to analyze what type of movement causes the graph to increase or decrease.



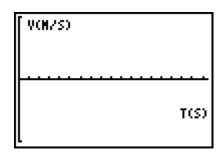
2. Move back and forth in front of the CBR™, or point the CBR at the wall and move back and forth from the wall, changing directions at least two times. The CBR collects data representing the distance from the CBR as a function of time. This data represents the distance between either you and the CBR or the CBR and the wall, depending on your experimental setup.

3.	Make a sketch of your data on the grid
	shown. Write a description of the motion.

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- **4.** The velocity is considered positive when the distance from the CBR is increasing and negative when the distance from the CBR is decreasing. Identify regions on your graph where the velocity is positive and where it is negative. Clearly indicate those regions on the graph.
- 5. The velocity is zero when you stop, even if it is momentary. Remember that in order to change directions you had to first stop, even if you did not think you came to a complete stop. You cannot move forward and then move backward without first stopping. Label all points on your graph where the velocity is zero. What do these correspond to on the position versus time graph?
- 6. From your data, make a sketch of what the velocity versus time graph will look like for your walk. It is easiest to begin by plotting the zero points and then graphing the velocities as positive or negative. Do not worry about estimating the velocity values. The idea is just to sketch what the graph will look like.



7. Instantaneous velocity is the average velocity as the change in time approaches zero:

$$\left(\frac{\Delta d}{\Delta t}\right)$$

In mathematical notation:

$$v = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$

where Δd is the change in distance from the CBRTM and Δt is the elapsed time. To approximate the instantaneous velocity, you can divide the change in distance by the change in time for each set of points taken. This is the average velocity over each small time interval.

Because the time intervals are small, these values will approximate the velocity function. Select **1:Edit** in the **STAT Menu**. Move the cursor to the top of **L3** so that the **L3** is highlighted, and press CLEAR and ENTER to clear the list. Do the same for **L4**. When the lists are clear, calculate the average velocity for each time interval, and place it in **L4** by moving the cursor to the top of **L4** and selecting **7:** Δ **List** from the **LIST OPS Menu**. Calculate Δ **List(L2)**/ Δ **List(L1)**. The Δ List operates by taking the difference between successive items in a list.

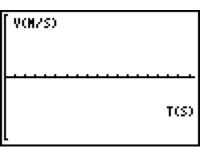
L3		L5	4
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8. If the CBR collected 94 data points, how many values will be in **L4**? In other words, how many differences will be calculated? Explain your answer.

9. Because there will only be 93 values in L4, you will need to have a list of times with only 93 data points to plot. The actual velocity that you calculated will occur at the midpoints of all the time intervals. But because the time intervals are short, you can use the time values from L1 and delete the first one to get a good idea of what the velocity graph looks like. Select 1:Edit from the STAT Menu, and move the cursor to the top of L3 so that the L3 is highlighted. Input L1 and press ENTER to place the values of L1 into L3. Highlight the first value, and press DEL to delete it.

1 2	L4	L5 3
	1217 12018 12018 11985 11985 11985 11985 11985	
L3 =L1		

10. Turn on a stat plot for velocity as a function of time (L3 and L4) as shown in the screen to the right. Sketch the actual velocity graph on the screenshot below. How does it compare with your prediction?

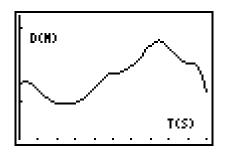


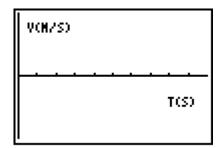
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Xlist:L3 Ylist:L4
Ylist:L4 Mark: □ + ∎

11. What does positive velocity look like on a distance versus time graph?

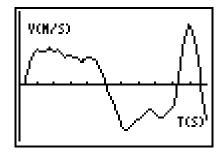
12. What does positive velocity look like on a velocity versus time graph?

13. Make a sketch of the velocity versus time graph for the position versus time graph shown. Again, do not worry about plotting each velocity value exactly. This is just a sketch to show that you understand when the velocity is positive, negative, or zero.





14. Use the RANGER application to collect velocity data so that your graph matches the one shown to the right. Your graph will not match exactly, but you should be able to approximately match each section. After executing prgmRANGER, select
1:SETUP/SAMPLE and set up the collection options as shown. Be sure to set the SMOOTHING to HEAVY. Try to match the graph. Describe your motion.



MAIN MENU	▶START NON
REALTIME:	YES
TIME(S):	15
DISPLAY:	VEL
BEGIN ON:	[ENTER]
SMOOTHING:	HEAVY
UNITS:	METERS

15. Did your starting point affect the graph of velocity versus time? Explain.

16. Make a sketch to show what the position versus time graph would look like for the velocity graph that you matched.

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17. After completing this activity, complete the chart shown:

When the function graph is	The derivative graph is	
Increasing		
Decreasing		
Changing from increasing to decreasing		
Changing from decreasing to increasing		
A constant value		

18. Summarize at least three concepts that were reinforced during this activity.