



### About the Lesson

In this activity, students will calculate confidence intervals for estimating the true population mean when the standard deviation is not known. Students will also be introduced to the  $t$  distribution. As a result, students will:

- Use technology to calculate confidence.

### Vocabulary

- $t$  distribution
- normal distribution
- confidence interval
- degrees of freedom

### Teacher Preparation and Notes

- After completing this activity, students will be able to complete free response questions such as AP Stats 2006, Q4.

### Activity Materials

- Compatible TI Technologies:

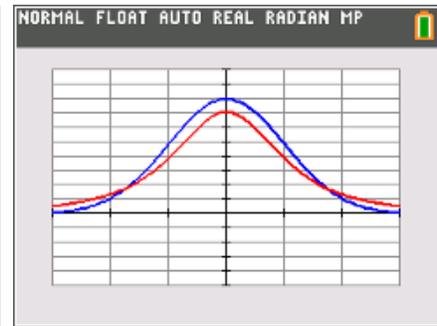
TI-84 Plus\*

TI-84 Plus Silver Edition\*

 TI-84 Plus C Silver Edition

 TI-84 Plus CE

\* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



### Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

### Lesson Files:

- Population\_Mean\_Student.pdf
- Population\_Mean\_Student.doc
- JANTM.8xl



**Tech Tip:** Before beginning the activity, the list JANTM.8xl needs to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE Software.

### Problem 1 – Assumptions

The worksheet introduces students to the assumptions needed to use the  $t$  distribution. Explain to them that, when using the  $t$  distribution, degrees of freedom are used. The definition for degrees of freedom for a collection of sample data is the number of sample values that can vary after certain restrictions have been imposed on all data values

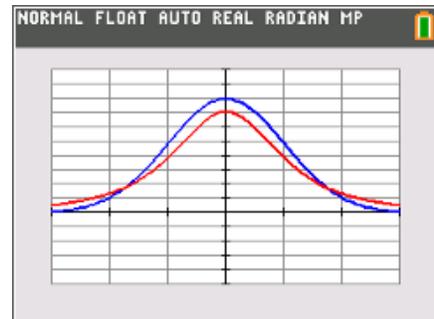
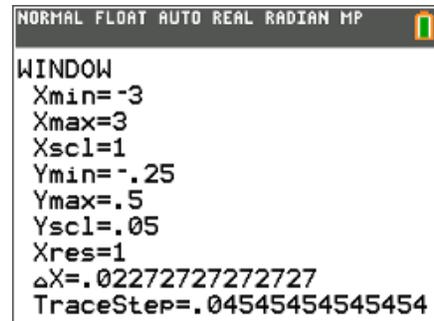
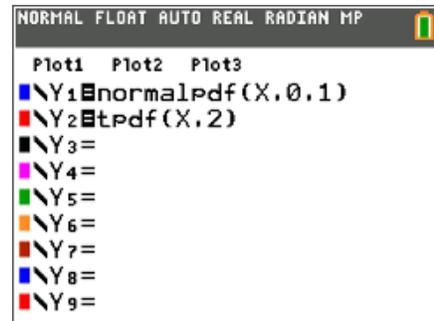
Students will investigate, by graphing, why a  $t$  distribution can be used when estimating the true mean of a population when only the sample mean and sample standard deviation are known.

First students will graph  $Y_1 = \text{normalpdf}(X, 0, 1)$  and  $Y_2 = \text{tpdf}(X, 2)$ .

Both of these functions can be entered by pressing  $\boxed{2\text{nd}} \boxed{\text{prgm}}$  and selecting the appropriate function.

Students are to set an appropriate window. One set of possible settings are shown at the right

Students should see that as the sample size increases the  $t$  distribution more closely resembles the normal distribution. This means that the estimation of the population mean is more accurate as  $n$  increases.



1. What happens as  $n \rightarrow 30$ ?

**Answer:** The  $t$  distribution looks more like the normal distribution.

2. How does the size of the sample play a role in the accuracy of the estimation?

**Answer:** The bigger the size of the sample, the more accurate the estimation.

3. Determine whether to use a normal distribution,  $t$  distribution, or neither.

a.  $n = 50$ ,  $\bar{x} = 10$ ,  $s = 4$ , population is skewed.

**Answer:**  $t$  distribution because  $n > 30$ .

b.  $n = 15$ ,  $\bar{x} = 10$ ,  $s = 4$ , population is normally distributed.

**Answer:**  $t$  distribution because population is normal and  $\sigma$  is not known.

c.  $n = 50$ ,  $\bar{x} = 10$ ,  $\sigma = 4$ , population is very skewed.

**Answer:** Normal distribution because  $\sigma$  is known.

d.  $n = 15$ ,  $\bar{x} = 10$ ,  $s = 4$ , population is skewed.

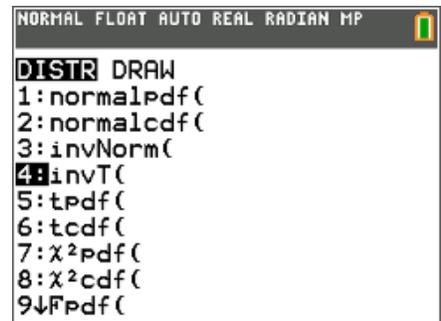
**Answer:** None of the methods learned yet because  $n < 30$ , it is skewed, and  $\sigma$  is not known.

**Problem 2 – Estimating the Interval**

On the worksheet, students are given the formula for calculating the interval and more specifically the error,  $E$ . They will find a 95% confidence interval where  $n = 25$ ,  $\bar{x} = 15$ , and  $s = 0.5$ . Step-by-step instructions are given on the worksheet.

To find the confidence interval, students should choose the **invT**( command from the **DISTR** menu and enter the area and degrees of freedom, as shown.

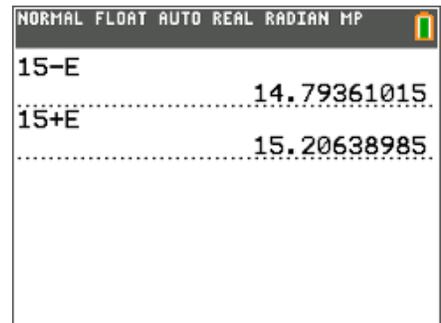
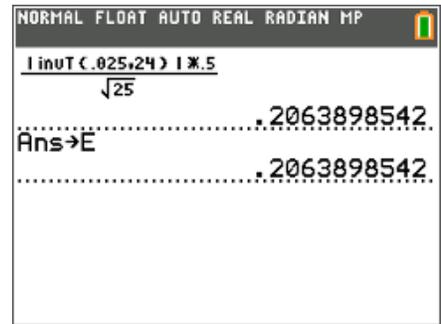
After students use the **Inverse t** command to calculate the  $t$ -value, they should compare this value with the one found using a  $t$  distribution chart.



Students can calculate the value of the formula using the fraction template  $\frac{\square}{\square}$ , the absolute value template  $|\square|$ , and the square root template  $\sqrt{\square}$ .

To stack a fraction  $\frac{\square}{\square}$  and select **n/d**. Use the arrow keys to navigate around and in and out of the templates. To store the answer students found for the error as the variable **E** for later use, students should press  $\text{sto} \rightarrow$   $\frac{\square}{\square}$   $\text{[sin}^{-1}\text{]}$   $\text{[enter]}$ .

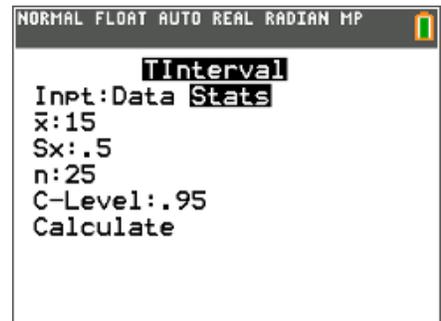
Now students can easily find the interval by recalling **E**.



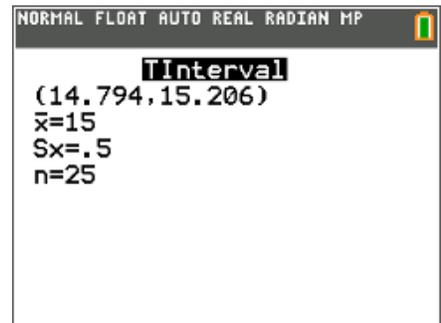
- Find a 95% confidence interval for a sample where  $n = 25$ ,  $\bar{x} = 15$ , and  $s = 0.5$  and the data is normal distributed.

**Answer:** (14.79,15.206)

After all students finish computing the interval in Question 4, demonstrate how to use the built-in command **TInterval** to find the intervals faster. Choose  $\text{[stat]}$ , arrow to the **TESTS** menu, and select **TInterval**. Because there isn't any data associated with the problem, students should choose the **Stats** option and enter the information as shown on the worksheet.



When students press **Calculate**, they will see the screen shown at the right.





### Extension – Using Data

The data on the worksheet gives the normal average January minimum temperature in degrees Fahrenheit of 56 cities.

This data is also stored in the list **JANTM** and can be accessed once transferred to the calculator by pressing `[stat]` and selecting **SetUpEditor**. Then press `[2nd]` `[stat]` selecting **JANTM** from the list. To view the list, press `[stat]` and select **Edit...** under the Edit heading.

JANTM	-----	-----	-----	-----	1
44					
38					
26					
58					
21					
45					
21					
13					
14					
0					
33					

JANTM(1)= 44

Students will use the provided data to construct three confidence intervals. They can use the first method from Problem 2 by finding the error.

NORMAL FLOAT AUTO REAL RADIAN MP					
<b>1-Var Stats</b>					
List: JANTM					
FreqList:					
Calculate					

For this method, they must also find the sample mean,  $\bar{x}$ , and the sample standard deviation, **Sx**, using the **1-Var Statistics** command (found by pressing `[stat]` and arrowing to **CALC** menu).

NORMAL FLOAT AUTO REAL RADIAN MP					
<b>1-Var Stats</b>					
$\bar{x}$ =26.51785714					
$\Sigma x$ =1485					
$\Sigma x^2$ =49225					
Sx=13.3797555					
$\sigma x$ =13.25975527					
n=56					
minX=0					
↓Q1=18.5					

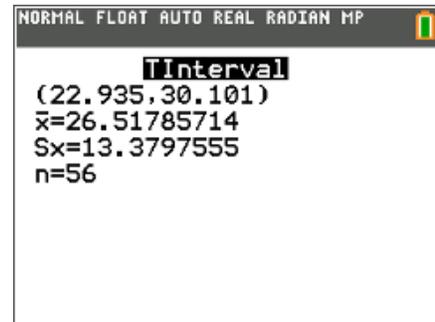
The data list name is found by pressing `[2nd]` `[stat]` and selecting **JANTM** from the list.

Students can use the second method, the **TInterval** command. Because they are given data this time, they should select **Data** rather than **Stats**.

NORMAL FLOAT AUTO REAL RADIAN MP					
<b>TInterval</b>					
Inpt: Data Stats					
List: JANTM					
Freq: 1					
C-Level: .95					
Calculate					



They should enter the information as shown, highlight **Calculate** and press **enter**.



5. Find an interval that estimates the true population mean (average January temperatures for all US cities) with:

a. 90% confidence

**Answer:** (23.5, 29.5)

b. 95% confidence

**Answer:** (22.9, 30.1)

c. 99% confidence

**Answer:** (21.7, 31.3)