

## Trigonometric Identities

ID: 9848

 Time required  
 45 minutes

## Activity Overview

*Students will verify, prove, and explore trigonometric identities symbolically, numerically, and graphically.*

## Topic: Trigonometric Identities

- *Use the Pythagorean Theorem to prove the trigonometric identities  $\sin^2 \theta + \cos^2 \theta = 1$  and  $\sec^2 \theta = 1 + \tan^2 \theta$ .*
- *Verify trigonometric identities by graphing.*

## Teacher Preparation and Notes

- *Prior to beginning the activity, students should download the **VERITRIG** program to their calculators. This investigation has students exploring, proving, and verifying trigonometric identities.*
- *Students should already be familiar with right triangle trigonometric properties, similar triangles, and the Pythagorean Theorem since much of the activity is based on these fundamental theorems.*
- *Students should begin by clearing out any functions from the  $\boxed{Y=}$  screen and turning off all Stat Plots.*
- ***To download the student worksheet and calculator program, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter "9848" in the keyword search box.***

## Associated Materials

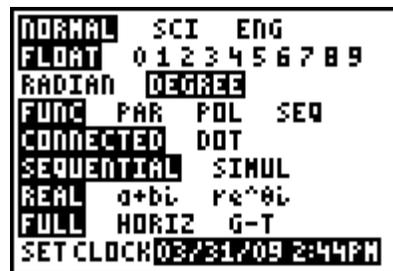
- *TrigIdentities\_Student.doc*
- *VERITRIG.8xp (program)*

## Suggested Related Activities

*To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the quick search box.*

- *Proof of Identity (TI-84 Plus family) — 9846*
- *Sine. It's the Law. (TI-84 Plus family) — 11851*

To start, students will need an empty graph window. They should clear out any functions from the  $\text{Y=}$  screen and turn off all Stat Plots. Make sure they are in degree mode. To do this, press  $\text{MODE}$  and select **DEGREE**.

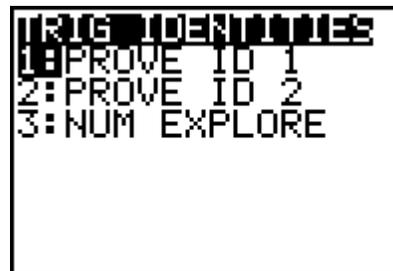


**Problem 1 – Proving  $\cos^2\theta + \sin^2\theta = 1$**

In this problem, students will prove two of the basic trigonometric identities using the Pythagorean Theorem.

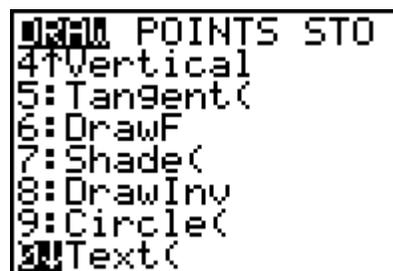
Press  $\text{PRGM}$  to access the **Program** menu.

Then choose the **VERITRIG** program and press  $\text{ENTER}$ . Select **1: PROVE ID 1**.



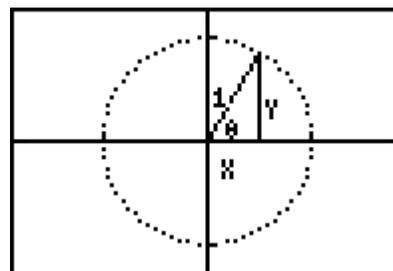
Students will see a circle with a radius of 1 with a right triangle in the first quadrant.

Students will label the right triangle using the **Text** tool. Press  $\text{2nd PRGM}$  and arrow down to select **Text**.



Now students can position the cursor near the middle of the hypotenuse and enter in the radius, 1. Then press  $\text{2nd ALPHA}$  to lock the alpha key on and enter in  $x$ ,  $y$ , and  $\theta$  in their respective places in a right triangle.

If students make a mistake, they can press  $\text{2nd PRGM}$  and select **CirDraw**. Then they need to run the program again.



On the worksheet, students will prove:  $\cos^2\theta + \sin^2\theta = 1$ .

1.  $x^2 + y^2 = 1$
2.  $\cos\theta = \frac{x}{1}$  and  $\sin\theta = \frac{y}{1}$
3. Substituting  $x = \cos\theta$  and  $y = \sin\theta$  into  $x^2 + y^2 = 1$  gives  $\cos^2\theta + \sin^2\theta = 1$ .

When finished, students can press  $\text{2nd QUIT}$  to exit the drawing.

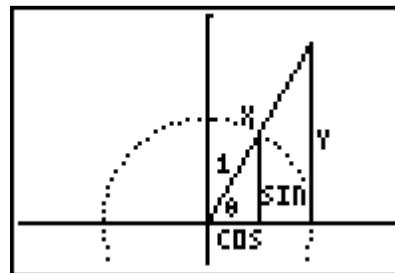
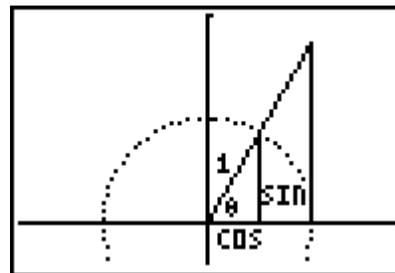
**Problem 2 – Proving  $\sec^2\theta = 1 + \tan^2\theta$**

Students need to run the **VERITRIG** program and select **2: PROVE ID 2**.

Students will see a new version of the unit circle with the original right triangle (from Problem 1) and then a similar triangle drawn to incorporate the original triangle.

Again, they need to use the **Text** tool and label the original triangle.

Students are told on the worksheet that the length of the base of the large triangle is 1, the height is  $Y$  and the hypotenuse is  $X$ . Students need to use the **Text** tool to label the large similar triangle accordingly.



Using the ratio given on the worksheet, students will prove  $\sec^2\theta = 1 + \tan^2\theta$ . You may need to explain to students that in both equations, (**small  $\Delta$** ) and (**large  $\Delta$** ), reference where that part of the ratio came from and is not actually part of the ratio.

$$\frac{1}{\cos\theta} = \frac{X}{1}$$

$$\frac{Y}{\sin\theta} = \frac{1}{\cos\theta}$$

$$1 = (\cos\theta)(X)$$

$$(Y)(\cos\theta) = (\sin\theta)(1)$$

$$X = \frac{1}{\cos\theta} = \sec\theta$$

$$Y = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Substituting  $Y = \tan\theta$  and  $X = \sec\theta$  into the Pythagorean Theorem based on the larger triangle:  $1^2 + Y^2 = X^2 \rightarrow 1 + (\tan\theta)^2 = (\sec\theta)^2 \quad 1 + \tan^2\theta = \sec^2\theta$

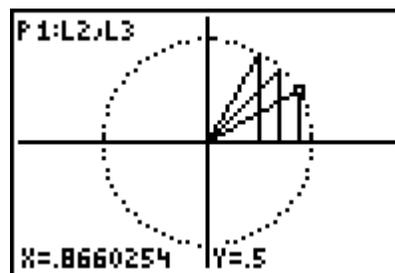
**Problem 3 – Numerical verification**

Now that students have proved the two identities, they will numerically verify the two identities.

Students need to run the **VERITRIG** program and select **NUM EXPLORE**.

Students will see the same circle, but with three triangles in the first quadrant. When they press **TRACE**, they will see the  $x$ -values for cosine and the  $y$ -values for sine for each angle measurement.

Students should observe that the values for cosine and sine repeat but change signs depending on what quadrant the angle is in.



These values are stored in the lists of the calculator with the angle measurements in L1, the cosine values in L2, and the sine values in L3.

To numerically verify  $\cos^2\theta + \sin^2\theta = 1$ ; students are to enter  $L2^2 + L3^2$  at the top of L4.

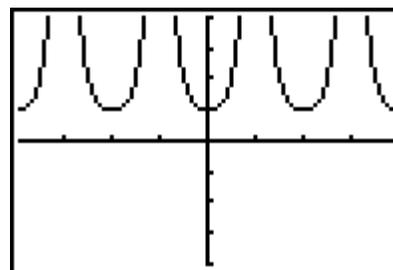
L2	L3	L4	4
1	0	1	
.99619	.08716	1	
.98481	.17365	1	
.96593	.25882	1	
.93969	.34202	1	
.90631	.42262	1	
.86603	.5	1	
$L4 = L2^2 + L3^2$			

Students will discover that they need to enter “ $1/\cos\theta$ ” to get  $\sec\theta$ . The value of  $\sec\theta$  is undefined at 0 and  $\frac{3\pi}{2}$ , and the calculator will not evaluate these values within a list. The same thing will happen to  $\tan\theta$ , which is equal to  $\frac{\sin\theta}{\cos\theta}$ .

```

Plot1 Plot2 Plot3
\Y1=(1/cos(X))^2
\Y2=1+tan(X)^2
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
    
```

However, students should arrive at the fact that they can graph the identities to verify the identity as the screen shots illustrate. They should set the calculator to the trig window by pressing **ZOOM** and select **ZTrig**. (Note: Students will need to use the **ClrDraw** command to erase the unit circle and turn off **Plot1**.)



**Problem 4 – Verifying trig identities using graphing.**

The previous problem leads into utilizing graphing techniques to verify various trig identities.

Students are to determine whether the identity  $\sin^2x = 1 - \cos^2x$  is true by entering  $\sin^2x$  into Y1 and  $1 - \cos^2x$  into Y2. To set the bubble animation for the second graph, students need to press **ENTER** to the left of **Y2 =**.

```

Plot1 Plot2 Plot3
\Y1=sin(X)^2
\Y2=1-cos(X)^2
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
    
```

Students need to select **ZTrig** for a trig window.

If the bubble runs over the same graph as the first, the equation is a trig identity.

Explain to students that they can check any equation using this method.

