



Math Objectives

- Students will identify the sample space of an experiment involving repeated independent trials.
- Students will calculate the probability of multiple independent events occurring.
- Students will gain understanding of the distinction of the relationship between theoretical and experimental probability.
- Reason abstractly and quantitatively. (CCSS Mathematical Practice)
- Look for and express regularity in repeated reasoning. (CCSS Mathematical Practice)

Vocabulary

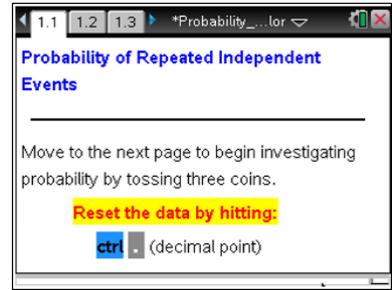
- experiment
- outcomes
- sample space
- event
- independent event
- dependent event
- probability tree diagram

About the Lesson

- This lesson involves an investigation of probability by simulating tossing a coin three times.
- As a result, students will:
 - Understand probability of specific outcomes of repeated independent trials
 - Understand using a probability tree diagram to determine probability

TI-Nspire™ Navigator™ System

- Use File Transfer to share the TI-Nspire™ document.
- Use Class Capture or Live Presenter to monitor student progress or have them share their investigations.
- Use Quick Poll to assess student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire™ document
- Open a document
- Move between pages
- Use a slider

Tech Tips:

- Make sure the font size on your TI-Nspire™ handheld is set to Medium.

Lesson Files:

Student Activity
 Probability_of_Repeated_Independent_Events_Student.pdf
 Probability_of_Repeated_Independent_Events_Student.doc
TI-Nspire document
 Probability_of_Repeated_Independent_Events.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

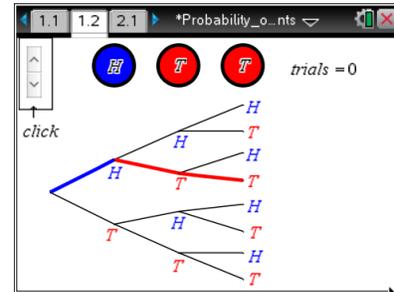


Discussion Points and Possible Answers

TI-Nspire™ Navigator™ Opportunity: *File Transfer*

See Note 1 at the end of this lesson.

1. An experiment is a process, such as tossing a fair coin three times, that gives definite results, called outcomes of the experiment. The sample space, S , of an experiment is the set of all possible outcomes. An event is any subset of the sample space.



- a. If the results of the first two coin tosses are heads, are you more likely to toss a head or a tail on your third toss? Explain.

Answer: You are equally likely to toss a head or a tail. There is a 50% probability of tossing either one.

- b. If an experiment consists of tossing a fair coin three times and recording the results in order, what is the sample space for this experiment? (Use H for heads and T for tails.) How many possible outcomes are there?

Answer: $S = \{HHH, HHT, HTH, HTT, TTH, THT, THH, TTT\}$. There are 8 possible outcomes in the sample space.

- c. What does it mean for a coin toss to be “fair”? Explain.

Answer: It means that there is exactly a 50% chance of getting either heads or tails on any given toss.

Teacher Tip: A coin that is not a fair coin is called “biased” or “unfair.” This might be due to indentations of the coin causing it to be heavier on one side. One mathematician, Persi Diaconis, has even claimed that any coin will land in the orientation from which it was thrown about 51 out of 100 times (<http://news.stanford.edu/news/2004/june9/diaconis-69.html>).



- d. When the occurrence of one event does not affect the probability of another event, the events are independent. Suppose you randomly choose a ball from a bag of 10 colored balls. The first ball is not replaced. Then, you choose another ball. Are your choices independent events or dependent events?

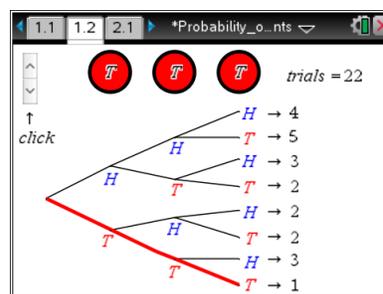
Answer: These events are dependent events. If the first ball were replaced, then they would be independent events.

TI-Nspire™ Navigator™ Opportunity: Class Capture or Live Presenter

See Note 2 at the end of this lesson.

Move to page 1.2.

2. A probability tree diagram is another way to represent the sample space of an experiment. From the trunk, on the left, the first pair of branches represents the possible results of the first coin flip. From each of those branches, two more branches represent the possible results of the second coin flip. Finally, from each of those four branches, two more branches represent the possible results of the third coin flip.



- a. Watch the tree diagram change as you click the arrow in the upper-left corner. Based on the path shown, what is the probability of getting tails on all three tosses?

Answer: The probability of getting tails on all three tosses is $\frac{1}{8}$.

Teacher Tip: Students may have a difficult time understanding how a probability tree diagram works the first time they see it. Allow students the time to explore and see the path of the tree diagram change as they click for new trials.

- b. To find the probability of multiple independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. Here is the rule defined symbolically: $P(A|B) = P(A) \cdot P(B)$. Use the rule to algebraically find the probability of getting tails on all three tosses.

Answer: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Multiply the probability of getting tails for each of the tosses.



- c. In 2010, the New Orleans Saints won the Super Bowl coin toss, becoming the 13th straight NFC team to do so. That seems highly improbable. What is the probability of correctly guessing the outcome of a fair coin toss 13 times in a row?

Answer: $\left(\frac{1}{2}\right)^{13} = \frac{1}{8,192}$

Teacher Tip: Students may try to overcomplicate this problem by treating the team's call and the result of the toss as individual events. While this can be done, because the probability of a win or a loss is always 0.50, they can calculate the probability with a win or loss as a single event.

CCSS Mathematical Practice: Students will reason abstractly and quantitatively.

Mathematically proficient students should understand from part 2c that an outcome is one that is unlikely to be due to chance alone but can only be calculated under randomness and multiplication rules.

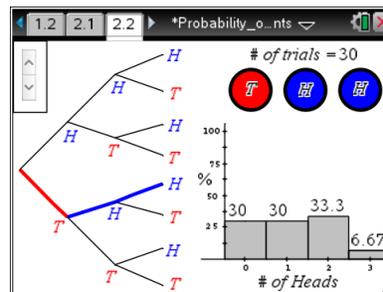
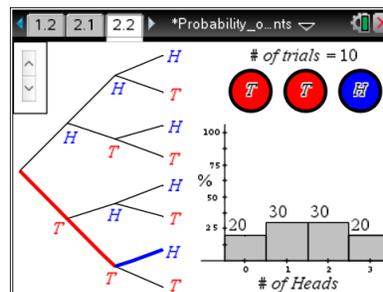
Tech Tip: Press and then to reset the number of trials.

TI-Nspire™ Navigator™ Opportunity: Class Capture

See Note 3 at the end of this lesson.

Move to page 2.1 and then page 2.2.

3. A bar graph records the percentage of trials that have had exactly 0, 1, 2, or 3 heads showing each time the arrow in the upper-left corner of the screen is clicked. Reset the number of trials to zero by pressing . After ten trials, record the percentages in the table below in the first Experimental column. Do twenty additional trials, and then record your data in the next column. Then, use the tree diagram to find the theoretical percentages and record them in the Theoretical column.





Sample answer: See table below. Answers may vary for experimental data.

Number of Heads Showing	Experimental (% after 10 trials)	Experimental (% after 30 trials)	Theoretical (%)
0	0%	10%	$\frac{1}{8} = 0.125 = 12.5\%$
1	50%	43.3%	$\frac{3}{8} = 0.375 = 37.5\%$
2	40%	43.3%	$\frac{3}{8} = 0.375 = 37.5\%$
3	10%	3.33%	$\frac{1}{8} = 0.125 = 12.5\%$

- a. The event “getting one head showing” is a compound event. This indicates that it includes more than one outcome, and the probability that it occurs is the sum of the probabilities of the three separate outcomes. What are all of the possible outcomes that show exactly one head? Use the outcomes to find the theoretical probability.

Answer: *HTT*, *THT*, and *TTH*. The probability of each of these outcomes is $\frac{1}{8}$. To

find the probability of the compound event, just add these together:

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}.$$

- b. Do your percentages in the experimental columns match the percentages in the theoretical column? If not, why?

Answer: The experimental data will not match the theoretical data exactly. They may be somewhat close, or not.

Teacher Tip: Students may have a difficult time understanding why the experimental column and the theoretical column don't match. The experimental results will vary because they are the result of random flips of the coin. The theoretical outcomes are the expected value, which is a prediction of what the outcomes of the experiment will be on average. Try to get your students to verbalize the reason for this disparity.

CCSS Mathematical Practice: Students will look for and express regularity in repeated reasoning.



Mathematically proficient students should understand that random processes as done by this model are simulating experimental data. Using addition and multiplication rules, students should be able to describe theoretical outcomes and the difference from experimental outcomes.

- c. What do you think would happen to the data if you did thousands (or even millions) of trials?

Answer: The data should approach the theoretical values (also called the expected values).

Teacher Tip: This may be a good time to introduce the Law of Large Numbers theorem. It states that the probability that the experimental average is *not* close to the theoretical value will decrease as the number of trials gets larger. In other words, the result of performing the same experiment a large number of times will yield an average of the results that is increasingly likely to be close to the expected value. What this means practically is that if we continue to run repeated independent trials and keep track of the cumulative average, it becomes more and more likely that our values will be close to the theoretical value (but it does not guarantee it).

TI-Nspire™ Navigator™ Opportunity: Quick Poll
See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to use the rule of multiplication to calculate the probability of independent events
- How to use a probability tree diagram to determine the probability

TI-Nspire™ Navigator

Note 1

Entire Document, File Transfer: Use **File Transfer** to efficiently send the .tns file to the students. Using TI-Nspire™ Navigator™ will allow students to receive the file without having to leave their seats or use extra cables.

Note 2

Entire Document, Class Capture or Live Presenter: If students experience difficulty with operation of a file or with a question, use Live Presenter with TI-Nspire™ Navigator™. This can also be used as a way to facilitate student discussion.



Note 3

Page 2.2, Class Capture: Particularly on page 2.2, Class Capture should be used so that students can compare their bar graphs with those of other students in the class.

Note 4

End of lesson, Quick Poll: A Quick Poll can be given at the conclusion of the lesson. Save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students may have.

The following are some sample questions:

1. In this experiment, what is the probability of getting heads on both of the first two coin tosses?

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{3}{8}$

d. $\frac{1}{2}$

2. In this experiment, what is the probability of getting at least two tails on three tosses?

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{3}{8}$

d. $\frac{1}{2}$

3. True/False: In this experiment, the probability of getting all tails is the same as the probability of not getting all tails.

Answer: False

Teacher Tip: Given the probability of an event (e.g., the probability of getting all tails), the probability of its complement (e.g., the probability of *not* getting all tails) can be found by subtracting the given probability from 1.