## About the Lesson

In this activity, students use the Manual-Fit and Linear Regression commands to find lines of best fit to model data from the Olympic Games. As a result, students will:

- Develop and evaluate inferences and predictions that are based on data.
- Graph a scatterplot of data and determine a function that describes the data.
- Analyze a scatterplot to determine qualitative and quantitative trends in data.
- Determine approximate lines of fit as well as calculated lines of fit to analyze and make predictions about data.


## Vocabulary

- line of best fit
- linear regression
- slope
- $y$-intercept


## Teacher Preparation and Notes

- The lists provided with this activity need to be loaded on all calculators.



## Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI84 Plus family with the latest TI84 Plus operating system (2.55MP) featuring MathPrint ${ }^{\text {TM }}$ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/ calculators/pd/US/OnlineLearning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.


## Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition


## Associated Materials:

- Regressions_of_Olympic_Propor tions_Student.pdf
- Regressions_of_Olympic_Propor tions_Student.doc
- ML1.8XI
- ML2.8XI
- WL3.8XI
- WL4.8XI

Tech Tip: Before students begin the activity, the lists ML1, ML2, WL3, and WL4 need to be transferred to their devices. Men's data are in L1 and $L 2 . L 1$ is the year (where $0=1900$ ) and $L 2$ is the time in seconds. Women's data are in L3 and L4. L3 is the year (where $28=1928$ ) and L4 is the time in seconds.

Tech Tip: On the TI-84 Plus C Silver Edition, turn on the GridLine by pressing [2nd ZOOM to change the [FORMAT] settings. Note that the GridLine feature is unique to the TI-84 Plus C Silver Edition.

## Part 1 - Men's 100-Meter Dash

1. Create a scatterplot of the Men's Olympic 100-Meter Dash data for the given years. If you let the year $1900=$ 0 , then what would be an appropriate window for the plot? Plot the data shown in the table to the right. Make sure to identify the scale and label the axes.

Sample Answer: Answer may vary, but an appropriate viewing window could be $\mathrm{Xmin}=0, \mathrm{Xmax}$ $=100, Y \min =9.8$ and $Y \max =11$. The data are plotted in the image to the right.
First, students graph a scatter plot of the Men's data. They confirm their plot by setting up Plot1 as shown at the right. To access the stat plot, press 2nd [STAT PLOT] ENTER.

After setting up the stat plot, students should press WINDOW and set the window as described above where $\mathrm{Xscl}=20$, $\mathrm{Yscl}=0.2$.
2. What is the general trend in the data?

Sample Answer: There is a negative linear correlation. The times are decreasing as the years increase. As the years pass, the amount of time needed for Olympians to run 100 m becomes less. This means that the runners are getting faster.

| Year | Time (s) |
| :---: | :---: |
| 1900 | 10.8 |
| 1912 | 10.6 |
| 1932 | 10.3 |
| 1960 | 10.2 |
| 1964 | 10.0 |
| 1968 | 9.95 |
| 1988 | 9.92 |
| 1996 | 9.84 |



After the graph is displayed, students will draw a manual-fit line of best fit. To draw the line, from a clear home screen press STAT ALPHA D. When the screen says Manual-Fit, press ALPHA [F4] and select Y1 to store the equation in Y1. Press ENTER to move to the Calculate field and press ENTER to return to the graph screen. Using the cursors, have students move to the location where they want to drop the first point on the line and press ENTER. Then, have students move to where they want the second point on the line to be and press ENTER.


You can use $<$ and to change between the slope of the line and the $y$-intercept and you can change the value of $m$ or $b$ by typing in the number and pressing ENTER.
3. What is the meaning of the slope and $y$-intercept? Include units. Write the line of best fit you determined.

Sample Answer: Answers will vary. A slope $m$ of $-0.01 \mathrm{~s} /$ year means the times are getting faster, or less, at an average rate of $0.01 \mathrm{~s} /$ year. The $y$-intercept represents the time for the 100 m dash (in seconds) when the year was zero, or 1900 . The $y$-intercept of 10.77 s is close to the 10.8 s in the data. These values give the equation $y=-0.01 x+10.77$.
4. Sketch the line on your graph in Question 1.

Answer: Answers will vary. For an example, see the graph to the right.


Once the line is drawn, students can use the left and right arrow keys ( $\boldsymbol{4}$ and $\boldsymbol{\text { ) }}$ ) to select and adjust the $y$-intercept or the slope and type in new values for the equation. When the students have a satisfactory manual fit line, have them record the equation of the line rounded to 2 or 3 decimal places. The equation is stored in the $Y$ editor.

Tech Tip: On the TI-84 Plus C Silver Edition, when using Manual-Fit, students will press GRAPH for the DONE softkey. On the TI-84 Plus, students will press ENTER when finished manually making a line of best fit.

Teacher Tip: Attending to precision is performed as students include units to communicate the meaning of the slope and $y$-intercept, and consider how many digits to record. The data from 1900 to 1964 only have one decimal place. Is it possible that the measuring instruments became more precise after 1964? Students should also consider how their equations and predicted values would change if they round to 2 decimal places instead of 3 decimal places. Encourage students to discuss and defend the number of decimal places that they choose to round to. (Have students consider the precision of the stop watch used to time the runners.)
5. Using the line of best fit, what would you predict the time might be in 4 more years? What might the time be in 12 more years?

Sample Answer: Answers will vary. The expected times should match the equations determined by the students. "4 more" than 1996 is 2000, which means $x=100$. So for example, if $Y_{1}(x)=-0.01 x+$ 10.77, then $Y_{1}(100)=9.77$ seconds and $Y_{1}(108)=9.69$ seconds.

Students can use the manual-fit line $Y_{1}$ and the graph, table, or home screen to answer the given questions. You may want to have small groups of students compare their lines of best fit.
6. Do you think the line can predict the time indefinitely? Why or why not?

Sample Answer: Students should realize that the line will continue to go towards 0 but that runners will never be able to run the race in 0 seconds.

## Part 2 - Women's 100-Meter Dash

Next, students will complete the same exercise with the Women's data. This time, the stat plot should use L3 and L4. Students should store the Manual Fit regression equation in Y3.

If desired, you can have the students use Plot2. Plot1 can be turned OFF so just the Women's data are graphed or both can be turned ON so the data will be graphed at the same time. Graphing both Men's and Women's data on the same screen may require a window adjustment.

The manual-fit equation for the Men's data is still active. Students can turn it off by pressing $Y$ ๒, highlighting the $=$ sign, and then pressing ENTER.
7. Create a scatterplot of the Women's 100 -Meter Dash data for the given years. Choose L3 for the Xlist and L4 for the Ylist. What are the domain and range of the

| NORMAL | FLOAT | JTO | RADİ |  | $\square$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L1 | L2 | L3 | L4 | L5 | 3 |
| $\theta$ | 10.8 | 28 | 12.4 |  |  |
| 12 | 10.6 | 32 | 11.9 |  |  |
| 32 | 10.3 | 48 | 11.9 |  |  |
| 60 | 10.2 | 52 | 11.5 |  |  |
| 64 | 10 | 56 | 11.4 |  |  |
| 68 | 9.95 | 68 | 11.3 |  |  |
| 88 | 9.92 | 64 | 11.2 |  |  |
| 96 | 9.84 | 68 | 11.08 |  |  |
| ------ |  | 72 | 11.07 |  |  |
|  |  | 76 84 | 11.01 10.97 |  |  |
| $L 3(1)=28$ |  |  |  |  |  |
| Normal float auto real radian mp \} |  |  |  |  |  |
| Plot1 Plot2 Plot3 |  |  |  |  |  |
| On Off |  |  |  |  |  |
| Type: |  |  |  |  |  |
|  |  |  |  |  |  |
| Ylist:L4 |  |  |  |  |  |
| Mark: ${ }^{\text {¢ + . }}$ |  |  |  |  |  |
| Color: RED |  |  |  |  |  |

data?
Answer: The data has a domain from 28 to 88, corresonding to 1928 to 1988. The range is12.4 $10.62=1.78 \mathrm{~s}$.

The data are plotted to the right on a window from Xmin $=28$ to $\mathrm{Xmax}=100, \mathrm{Xscl}=4$, and $\mathrm{Ymin}=10.4$, Ymax
$=12.4$ and $\mathrm{Yscl}=0.2$. The $x$-axis is the time (seconds) and the vertical $y$-axis is the year, where $1928=28$.

8. What is the general trend in the data?

Sample Answer: Again, there is a negative linear correlation. The times are decreasing as the years increase. As the years pass the amount of time for women Olympians to run 100 m becomes less. That is, the runners get faster.
9. How does this compare to the Men's data?

Sample Answer: This question will be easier to answer
 if both sets of data are plotted on the same scale.
Students should observe that the women's data does not start as long ago. They may see that the slope of the Women's plot is steeper than the Men's.

The plot to the right is from year 0 to 108 (2008), and the Xscl tick marks every 4 years. $Y$ min $=9.6, Y \max =12.6$, $\mathrm{Yscl}=0.4$.

Students will again find the line of best fit manually by using the Manual-Fit option under STAT, CALC. If students have graphed both sets of data, make sure they use different marks (the last row of the stat plot setup) and/or a different color so they know which points belong in which group.
10. Draw a manual line of best fit for this second set of data. Store this equation in Y3. Write the line of best fit you determine.

Sample Answer: Answers will vary, but should be close to $Y_{3}=-0.03 x+12.90$.
11. Sketch the line on your graph in Question 7.
12. Using the line of best fit, what would you predict the time might be:

In 4 more years?
In 8 more years?

Sample Answer: Sample $Y_{3}(92)=-0.03(92)+12.90=10.14$. So, in four years (1992), the predicted time is $10.14 \mathrm{~s} . \quad Y_{3}(96)=10.50$. In 1996 , the predicted time is 10.02 s .
13. Do you think the Women's time will ever be faster than the Men's time? Explain why or why not.

Sample Answer: Students may notice that the slope is greater for the Women's times, indicating that women are becoming faster each year at a greater rate than men. It is possible that a woman's time could eventually be faster than a man's time.

## Part 3 - Manual Fit vs. LinReg

After students have explored the use of the Manual-Fit to make a line of best fit, you may want to introduce them to the LinReg(ax+b) command. This will find the least-squares regression of best fit.
The key presses for the example at the right are STAT 4 to access LinReg command. A wizard will open. Configure the wizard as shown. To access L1 and L2 press 2nd [L1] and 2nd [L2]. Students should use the arrows to select Store RegEQ and press ALPHA[F4] to select Y2. Press the down arrow to move to Calculate and press ENTER to execute the command.

Have students repeat for L3 and L4 to find the line of best fit for the Women's data. Store the LinReg regression equation in Y4.

Have students press $Y$ to view the regression equations for comparison.

Y1 - Men's manual-fit
Y2 - Men's LinReg
Y3 - Women's manual-fit
Y4 - Women's LinReg

| NORMAL FLOAT AUTO REAL RADIAN MP [] |
| :---: |
| ```LinReg(ax+b) Xlist:Li Ylist:L2 FreaList: Store RegEQ: Y % Calculate``` |
| NORMAL FLOAT AUTO REAL RADIAN MP - |
| LinReg $\begin{aligned} & y=a x+b \\ & a=-.0097076685 \\ & b=10.7109026 \end{aligned}$ |

14．Use the $\operatorname{LinReg}(\mathbf{a x + b})$ command to find a line of best fit for the Men＇s data．From a clear home screen，press STAT and choose LinReg from the CALC menu．A wizard will open．Configure as shown．To access L1 and L 2 press 2nd $[\mathrm{L} 1]$ and 2nd［L2］．Use the arrows to select Store RegEQ：and press ALPHA［F4］and select Y2． Press ENTER to move to Calculate and press ENTER to execute the command．What does the calculator calculate as the line of best fit？

Answer：$Y_{2}=-0.01 x+10.71$

15．How does the calculated line of best fit compare to the equation you found in Question 3？

Answer：The equation for Question 14 should be close to the equation found in Question 3.

16．Calculate the LinReg for the Women＇s data．Store the equation in $\mathbf{Y 4}$ ．

Answer：$Y_{4}=-0.03 x+12.90$ ．

17．How does this compare to the equation you found in Question 10？

Answer：Again，the equation in Question 16 should be close to the equation found in Question 10.

18．What do you think causes the differences in your manual－fit line and the line the calculator find？

Answer：Answers will vary．Possible responses may include that the manual－fit line is based on sight while the calculator＇s uses actual calculation to make the best fit line．Another reason could be the precision of the data；rounding to 2 or 3 decimal places could change the manual and calculated equations．


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Plot1 Plot2 Plot3
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－NY \({ }_{6}=\)
－ \(\mathrm{Y}^{2}=\)
－ Y \(_{8=}=\)
－\(\\) Yg＝
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