



About the Lesson

In this activity, students will observe the slopes of the secant and tangent line as a point on the function approaches the point of tangency. As a result, students will:

- Determine the average rate of change for an interval.
- Determine the average rate of change on a closed interval.
- Approximate the instantaneous rate of change using the slope of the secant line.

Vocabulary

- secant line
- tangent line

Teacher Preparation and Notes

- Students are introduced to many initial calculus concepts in this activity. Students develop the concept that the slope of the tangent line representing the instantaneous rate of change of a function at a given value of x and that the instantaneous rate of change of a function can be estimated by the slope of the secant line. This estimation gets better the closer point gets to point of tangency.

(**Note:** This is only true if $Y_1(x)$ is differentiable at point P .)

Activity Materials

- Compatible TI Technologies:

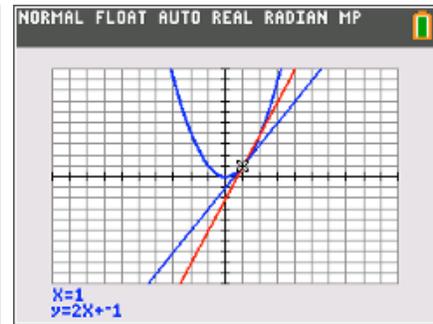
TI-84 Plus*

TI-84 Plus Silver Edition*

 TI-84 Plus C Silver Edition

 TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Secant_Lines_Student.pdf
- Secant_Lines_Student.doc

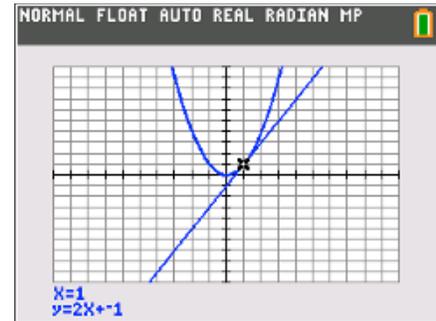


Problem 1 – Graphing Secant and Tangent Lines

Using the **draw** menu, students will have a tangent line drawn at point $P(1, 1)$. The equation of the line will appear on the screen.

Note:

$$\begin{aligned}
 y &= 2(x - 1) + 1 \\
 &= 2x - 2 + 1 \\
 &= 2x - 1
 \end{aligned}$$



1. What is the equation of the tangent line at $(1, 1)$?

Answer: $y = 2x - 1$

2. What is the slope of the line?

Answer: 2

3. What is the slope of the secant line that passes through point $P(1, 1)$ and point $Q(3, 9)$?

Answer: 4

4. Show the work that explains why the secant line through point $P(1, 1)$ and point $Q(3, 9)$ is $y = 4(x - 1) + 1$.

$$y - y_1 = m(x - x_1)$$

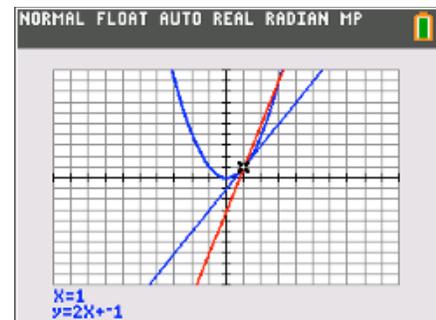
Answer: $y - 1 = 4(x - 1)$

$$y = 4(x - 1) + 1$$

Students will graph the secant line that passes through $(1, 1)$ and $(3, 9)$.

If students are having trouble seeing the points of intersection, have them adjust the window setting to zoom in on the first quadrant.

Discuss with students how the secant and tangent lines compare to each other. (The secant line is steeper.)





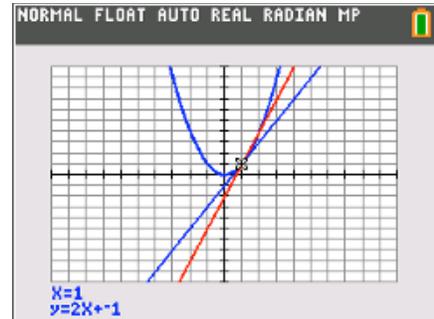
5. What could be done to bring point Q closer to point P(1, 1)? What value does the slope need to approach?

Answer: Point Q can take on an x-value close to 1. These points could be (1.1, 1.1²), (1.01, 1.01²), etc. The value the slope of the secant line needs to approach is 2.

Students are to change the slope of the secant line to 3.5, 3, and 2.5, viewing the graph after each. They will need to re-graph the tangent line using the **draw** menu.

6. As point Q approaches point P, what do you notice about the secant and tangent lines?

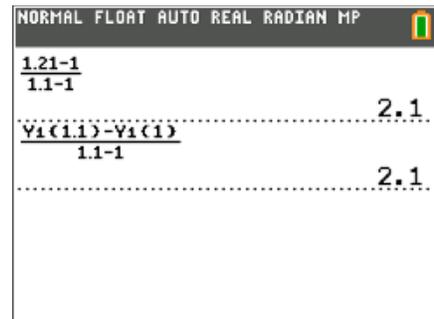
Answer: The slope of the secant line approaches the slope of the tangent line as point Q approaches point P.



Problem 2 – Finding Average Rate of Change

Students are asked to view the table of values of $Y_1(x)$ and use the calculator (Home screen) to find the average rate of change of $Y_1(x)$ on the interval [1, 1.1].

Students will most likely enter the values directly as shown in the first calculation in screenshot to the right. The second calculation is using function notation. Either way, students should arrive at the same answer.



7. Use the table of values of Y_1 to find the average rate of change of $Y_1(x)$ on the interval [1, 1.1].

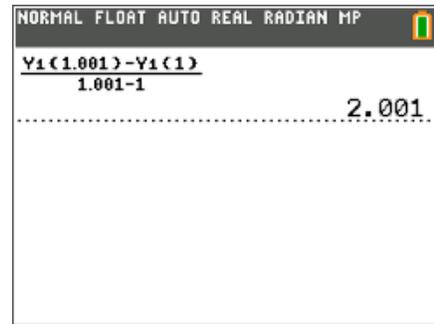
Answer: 2.1

Problem 3 – Finding Instantaneous Rate of Change



Students are asked to write an expression to estimate the instantaneous rate of change of $Y_1(x)$ at $x = 1$ and then use the calculator (Home screen) to evaluate the expression.

The example shown at the right uses the point $(1.001, Y_1(1.001))$.



8. How can you find an even better estimate of the instantaneous rate of change of $Y_1(x)$ at $x = 1$?

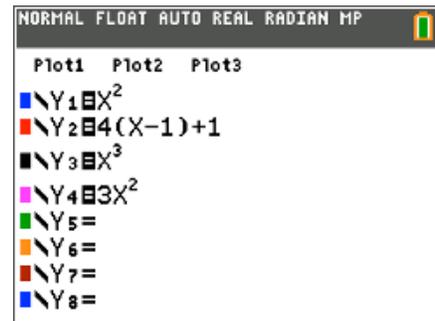
Answer: By using the slope of the secant line containing point P and another point closer to P than $(1.1, 1.21)$.

Problem 4 – Rectilinear Motion Application

Before beginning, students can define the functions to make the calculations easier. This is accomplished by storing the functions in $Y_3(x)$ and $Y_4(x)$.

Introduce the terms average velocity and instantaneous velocity for the first two questions. The answers will have units in meters per second (m/s).

Introduce the terms average acceleration and instantaneous acceleration for the last two questions. The answers will have units in meters per second squared (m/s^2).

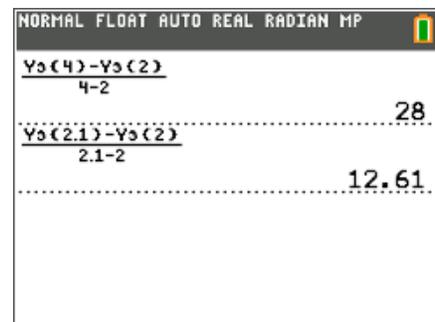


9. What is the average rate of change of the position function from $t = 2$ to $t = 4$?

Answer: 28 m/s

10. Estimate the instantaneous rate of change of the position function at $t = 2$.

Answer: 12.61 m/s



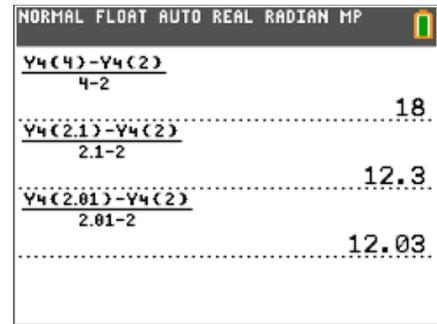


11. What is the average rate of change of the velocity function from $t = 2$ to $t = 4$?

Answer: 18 m/s^2

12. Estimate the instantaneous rate of change of the velocity function at $t = 2$.

Answer: 12.3 m/s^2



The image shows a TI-84 Plus calculator screen with the following text and calculations:

NORMAL FLOAT AUTO REAL RADIAN MP	
$\frac{V_4(4) - V_4(2)}{4 - 2}$	18
$\frac{V_4(2.1) - V_4(2)}{2.1 - 2}$	12.3
$\frac{V_4(2.01) - V_4(2)}{2.01 - 2}$	12.03