Math Objectives

- Students will be able to solve a system of three equations with three unknowns using the elimination method.
- Students will attend to precision (CCSS Mathematical Practice).

Vocabulary

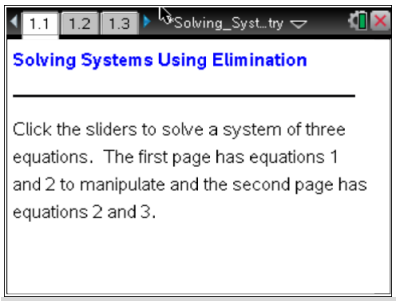
- system of equations
- elimination method

About the Lesson

- This lesson involves solving systems of three equations using the elimination method.
- As a result, students will:
  - Choose which variable to eliminate.
  - Enter the multiplication factor in order to eliminate a variable.
  - Solve the system by solving the final one-variable equation and then substituting back into prior equations.

TI-Nspire™ Navigator™ System

- Use Screen Capture/Live Presenter to show the procedure for using the .tns file and to show different approaches to the solution. Students can be shown that regardless which variable is eliminated, the solution will be the same. Divide the class into three groups and have each group eliminate a different variable first. You can then use Screen Capture to show how all three groups will arrive at the same solution to the system.
- Use Quick Poll to assess the students' readiness for this lesson and their understanding of the concepts at the end of the lesson.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Utilize a slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:

Student Activity

- Solving_Systems_Using_Elimination_Student.pdf
- Solving_Systems_Using_Elimination_Student.doc

TI-Nspire document

- Solving_Systems_Using_Elimination.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

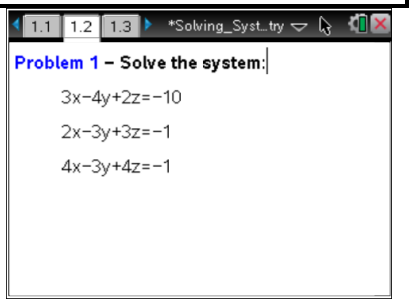
Tech Tip: Students will click the sliders to change the value of the number to multiply each equation.

Teacher Tip: Students may need to be reminded of the elimination method used to solve a system of two equations with two unknowns. Discuss the idea of equivalent equations, the operation of adding equations and using substitution in an equation. Some questions for a pretest are included in the Navigator notes at the end of the lesson.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

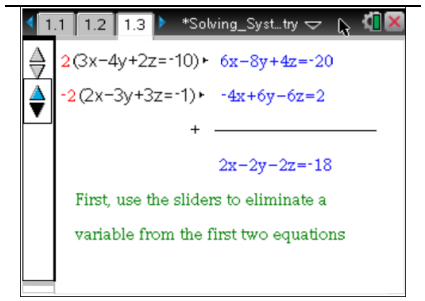
Move to page 1.2.

$$\text{Solve: } \begin{aligned} 3x - 4y + 2z &= -10 \\ 2x - 3y + 3z &= -1 \\ 4x - 3y + 4z &= -1 \end{aligned}$$



Move to page 1.3.

This page has equations 1 and 2. Click the sliders to change the numbers to multiply by each equation so that, when added, the sum of the equations will result in an equation that eliminates one of the variables. You will also be told if you have chosen correctly. Work in groups of three. Each person in your group should choose a different variable to eliminate first.



TI-Nspire Navigator Opportunity: *Screen Capture/Live Presenter*
See Note 2 at the end of this lesson.

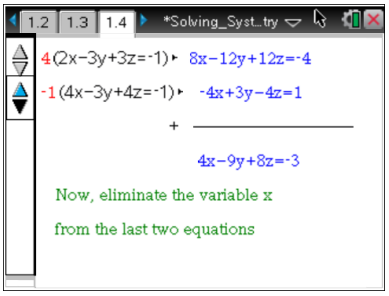
Teacher Tip: Students may be shown that any variable may be eliminated and the solution will still be the same. One group could be assigned to use the .tns file to eliminate *x* then *y*, another group could eliminate *x* then *z*, another group could eliminate *y* then *x*, etc. The solutions should still be the same.

1. Explain the strategy when choosing the factors needed to multiply each equation.
   Sample answer: You want the coefficients of the variable to be the same number; one positive, one negative so that, when added, the new equation will have a zero coefficient for one of the variables.

2. Explain the equation that results (below the line).
   Answer: The equation below the line contains the sum of the two equations that are above the line. This will start as an equation with three variables. The goal of elimination is to have the coefficient of one of the variables in this equation equal to zero so that you have reduced the number of variables to two.

Move to page 1.4.

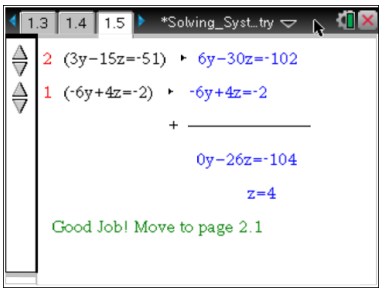
This page has equations 2 and 3. Click the sliders to multiply numbers by each equation so that, when added, the sum of the equations will result in an equation that eliminates the same variable as on page 1.3. The resulting equations will automatically be placed on page 1.5.



3. Explain why you are eliminating the same variable as on page 1.3.
   Answer: It is necessary to eliminate the same variable so that the new system will have the same two variables left to solve. It is possible to solve a system of two equations with two variables, but not a system of two equations with three variables.

Move to page 1.5.

This page has a new system of two equations and two unknowns. Use the same process to eliminate one of the remaining two variables.



Move back to page 1.4 or 1.3.

4. Why do you need to use one of the equations below the line to find the value of another variable? Find that value.

Sample answer: The equations below the line have only two variables in it. One of them is the variable you have just solved for. By substitution you can solve for the second variable.

$3y - 8 = 1$
$\therefore y = 3$

The answer will depend on which variables the student chose to eliminate.

5. Which equation should you substitute into now to completely solve the system? State the solution.

Answer: You need to substitute into one of the original equations in three variables. Now that you have solved for two variables you can use substitution to solve for the last variable.

$2x - 9 + 12 = -1$
$\therefore x = -2$

The solution to the system is {x, y, z} = (–2, 3, 4).

6. Does the solution of the system satisfy the three original equations on page 1.2? Explain.

Answer: The solution will satisfy all three because the steps in the process of elimination use equivalent equations or combinations of equivalent equations. Solving is finding the point of intersection for the three equations.

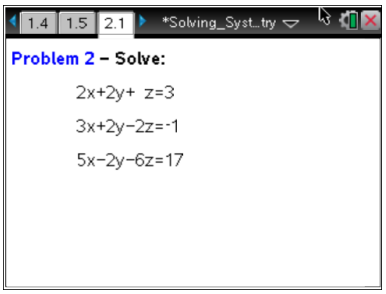
7. Confer with your group. Does it matter which variable you chose to eliminate first? Explain.

Answer: All students should arrive at the same solution. The value of each variable is dependent on the equations in the system. The order in which they are found is not important.

> **Teacher Tip:** This may be a surprise for some students. It should be reassuring that there isn't a wrong choice when deciding which variable to eliminate first, which one second, and so on.

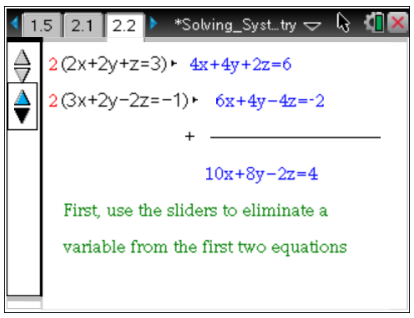
Move to page 2.1.

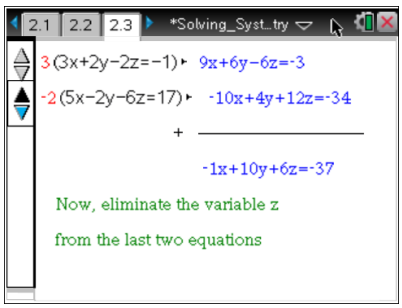
Solve:
2x+2y+z=3
3x+2y-6z=-1
5x-2y-6z=17

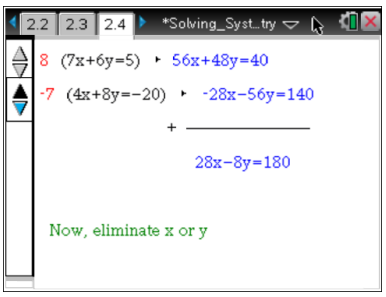


Move to pages 2.2–2.4.

Repeat the process of elimination to solve this system of equations.







8. Jenna came into class late. Explain to her how to solve the system using the method of elimination.
   Sample answer: In the explanation, students should mention the need to eliminate variables to reduce the system to equations that are true in two variables and finally one variable. They should also explain the process for multiplying equations by a scalar so that the coefficients of the chosen variable are the same number; one positive, one negative so that, when added, the equation will have a zero coefficient for that variable.

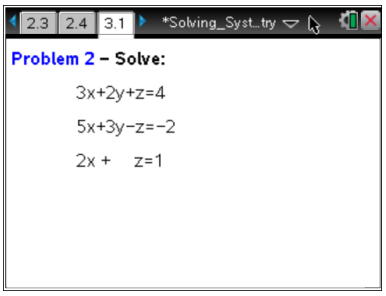
9. State the solution and show that your solution satisfies the three original equations in this system.
   Answer: The solution is {x, y, z} = (5, -5, 3).
   (L.S. stands for the Left Side of the equation to check, R.S. stands for Right Side).

L.S. =2(5)+2(-5)+(3)	*L.S.=3(5)+2(-5)-2(3)*	*L.S.=5(5)-2(-5)-6(3)*
=10-10+3	*=15-10-6*	*=25+10-18*
= 3= R.S.	*=-1= R.S.*	*=17= R.S.*

   The solution satisfies all three equations.

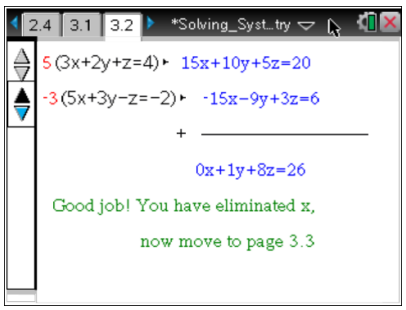
Move to page 3.1.

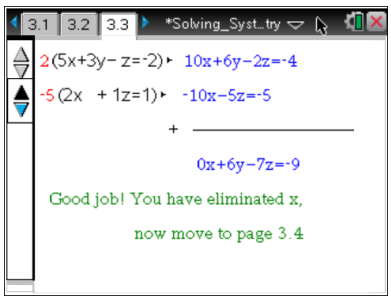
Solve:
3x+2y+z=4
5x+3y-z=-2
2x + z=1

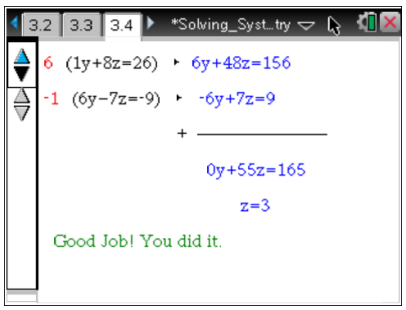


Move to pages 3.2–3.4

Repeat the process of elimination to solve this system of equations.







10. State the solution and show that your solution satisfies the three equations in this system.

    Answer: The solution is {x, y, z}= (–1, 2, 3).

    L.S.=3(-1)+2(2)+(3) *L.S.=5(-1)+3(2)-(3)* *L.S.=2(-1)+(3)*

    =-3+4+3 *=-5+6-3* *=-2+3*

    =4= R.S. *=-2= R.S.* *=1*

    The solution satisfies the three equations.

11. What are some of the benefits of using technology when solving a system of equations?

    Answer: The technology is useful to perform the arithmetic steps. It is easy to make a simple error in sign or calculation near the beginning when solving with paper and pencil. This would result in the entire solution being inaccurate. The calculator can easily handle “messy” solutions where the numbers are rational or decimal. (With larger systems, technology usually solves using matrices.)

12. In a business, efficiency experts need to solve a system with 150 variables. How many equations need to be in the system?

    Answer: You need at least as many equations as variables. You would need at least 150 equations.

> **Teacher Tip:** You may want to mention extending the use of the elimination method for solving systems of four equations with four unknowns or five equations with five unknowns, and so on. It should become apparent that using the technology and matrices may be a more efficient way to proceed. Scientists in NASA routinely solve enormous systems with hundreds of variables.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to use the method of elimination to solve a system of three equations with three unknowns.

Assessment

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Sample Assessment Questions:

1. Which of the following can be used to eliminate a variable in the system $\begin{aligned} 2x+3y-z&=1 \\ -3x+5y+2z&=19 \\ 7x-2y-2z&=-17 \end{aligned}$?

a) multiply equation one by 3 and equation two by 2; equation two by 2 and equation three by 5.

b) multiply equation one by -5 and equation two by 3; equation two by 7 and equation three by 3.

c) multiply equation one by 2 and equation two by 1; equation two by 1 and equation three by 1.

d) multiply equation one by 2 and equation two by 1; equation two by 2 and equation three by 5.

2. Which of the following can be used to eliminate a variable in the system $\begin{aligned} 4x+3y+2z&=-8 \\ -2x-3y+5z&=-11 \\ 7x+2y-2z&=9 \end{aligned}$?

a) multiply equation one by 1 and equation two by 2; equation two by 2 and equation three by 5.

b) multiply equation one by 1 and equation two by 1; equation two by 2 and equation three by 3.

c) multiply equation one by 5 and equation two by -2; equation two by 7 and equation three by 2.

d) multiply equation one by 1 and equation two by 1; equation two by 2 and equation three by 5.

3. The solution to the system $\begin{aligned} 2x+5y+3z&=19 \\ -3x+3y+3z&=6 \\ 4x-2y-7z&=3 \end{aligned}$ is:

a) $\{x,y,z\}=(2,3,0)$

b) $\{x,y,z\}=(3,5,-4)$

c) $\{x,y,z\}=(1,4,-1)$

d) $\{x,y,z\}=(0,2,3)$

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Note 1

Prior to the lesson, *Quick Poll:* You may want to test the students' readiness for this lesson by using questions like the following.

Sample Pretest Questions:

1 Which of the following can be used to eliminate a variable in the system $\begin{array}{l} x+2y=7 \\ x-5y=-14 \end{array}$?

a) multiply equation one by -2 and equation two by 5.

b) multiply equation one by -5 and equation two by 2.

c) multiply equation one by 1 and equation two by -1.

2. Which of the following can be used to eliminate a variable in the system $\begin{array}{l} 3x-2y=5 \\ x+4y=11 \end{array}$?

a) multiply equation one by 3 and equation two by -1.

b) multiply equation one by 2 and equation two by 1.

c) multiply equation one by -2 and equation two by 1.

3. What is eliminated when these two equations are added? $\begin{array}{l} 2x+3y=16 \\ 5x-3y=-2 \end{array}$

a) the variable x

b) the variable y

c) the constant

Note 2

Questions 3 and on, *Screen Capture/Live Presenter*: Demonstrate the procedure for using the file. You could also show various approaches to the problem (eliminating *x* first, then *y*; or eliminating *y* first, then *x*; or any combination of *x*, *y*, and *z*). You can also discuss which factors will work to eliminate a variable.

Note 3

End of Lesson, *Quick Poll*: Test the students' understanding of this lesson by using questions like the ones provided above under assessment.