Math Objectives

- Students will identify the conditions necessary to build triangles given the lengths of 3 segments.
- Students will use the Triangle Inequality Theorem to determine whether a triangle can be formed when given 3 segments.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

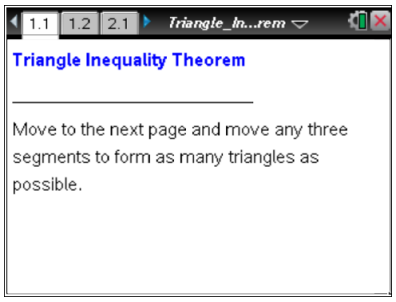
- triangle
- inequality
- segment
- conjecture

About the Lesson

- This lesson involves building triangles from 3 segments.
- As a result students will:
  - Manipulate given segments and make conjectures about the relationships between the lengths of the segments and the possibility of forming a triangle.
  - Use randomly generated side measures and manipulate the corresponding segments to determine whether the combination of measures will form a triangle.
  - Make and justify a conjecture describing the conditions that must be met to build triangles given segments of different length.

TI-Nspire™ Navigator™ System

- Quick Poll
- Class Capture



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity

- Triangle_Inequality_Theorem_Student.pdf
- Triangle_Inequality_Theorem_Student.doc

TI-Nspire document

- Triangle_Inequality_Theorem.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

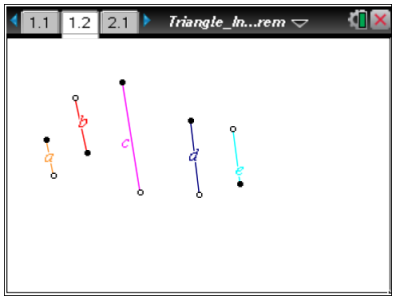
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (✋) getting ready to grab the point. Also, be sure that the word *point* appears, and not *label* or *segment.* Then press ctrl ✋ to grab the point and close the hand (✊).

Be patient when moving the points! It may take the handheld some time to process the move.

Teacher Tip: Page 1.2 of this file is optimized for the software. If students are using page 1.2 on the handheld, you might want to have them try only a few combinations. (See TI-Nspire Navigator Opportunity for question 1 below.)

Move to page 1.2.



1. Move any 3 segments to form as many triangles as possible. You can move the segments by grabbing and dragging the open circles, and you can rotate the segments by grabbing and dragging the closed circles. Record your results below.

Sample Answers:

Successful Combinations	Unsuccessful Combinations
a, b, e; a, b, d; a, e, d	a, d, c; a, e, c
b, e, d; e, d, c; b, d, c	a, b, c; b, e, c

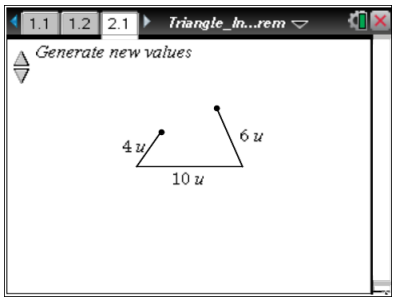
Teacher Tip: You may want to demonstrate how to drag and move the segments and also how to join the segments to form a vertex of an angle.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

2. Make a conjecture about why some of these combinations work and why some of them do not work.

Sample Answer: The sum of the lengths of 2 of the pieces has to be greater than the third to make a triangle. Some of the combinations didn't work because the pieces were not long enough to touch and make the third vertex.

Move to page 2.1.



3. Grab the closed circles at the end of the segments and move the segments to form a triangle. If a triangle can be formed, the inside of the triangle will be shaded.
   a. Use ▲ and ▼ to generate at least 5 sets of sides. Record your results for each set in the table below.

Sample Answer:

Side 1	Side 2	Base	Success? Yes or No
5	8	12	yes
3	5	11	no
5	5	7	yes
4	6	10	no
4	4	4	yes

Teacher Tip: To increase the number of cases students can analyze, it is recommended that students compare and discuss the results. In this activity, the fixed side is the base and is always the longest side. However, students should be reminded that the base does not always have to be the longest side of a triangle.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

   b. Make 2 observations based on your table.

Sample Answer: If the lengths of all 3 sides are 0, a point is formed. If the sum of the lengths of any 2 sides is greater than the length of the third, a triangle is formed. If the lengths of the sides are equal, an equilateral triangle is formed. If the sum of the lengths of 2 sides is equal to the length of the third, a line segment is formed instead of a triangle. If the sum of the lengths of any 2 sides is less than the length of the third, a triangle is not formed.

4. Jesse has 3 pieces of wood. The pieces measure 7 inches, 10 inches, and 14 inches. Tyrone has 3 pieces of wood that measure 6 inches, 11 inches, and 3 inches. Will they each be able to create a triangle? Why or why not?

Answer: Jesse can make a triangle but Tyrone cannot. If Tyrone puts his pieces together, $6 + 3 < 11$, which means the endpoints of the segments would not touch and form a triangle.

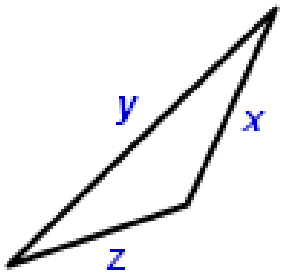
5. Make a conjecture about the relationship of the measures of the sides of a triangle. Justify your conjecture.

Answer: The sum of the lengths of the 2 sides of a triangle must be greater than the length of the third. A typical argument might be indirect considering all the possible cases. The 3 possible cases are: (1) The sum of 2 sides is shorter than the third; therefore, the sides will not meet to form a triangle. (2) The sum of 2 sides is equal to the third; therefore, the sides form a segment equal in length to the third side. (3) The sum of 2 sides is greater than the third forming a triangle. The third case is the *only* case that forms a triangle.

Teacher Tip: Students should share their conjectures and, as a class, decide on the best conjecture. At this point, refer to your textbook to find a more formal statement. Students could compare their conjecture with the text.

TI-Nspire Navigator Opportunity: *Yes/No Quick Poll*
See Note 3 at the end of this lesson.

6. Given a triangle with side lengths x, y, and z, which of the following is true? Justify your reasoning.



a. $x + y > z$:

Answer: True. The sum of any 2 sides of a triangle must be greater than the third.

b. $x + z > y$:

Answer: True. The sum of any 2 sides of a triangle must be greater than the third.

c. $z + y > x$:

Answer: True. The sum of any 2 sides of a triangle must be greater than the third.

Teacher Tip: This question illustrates the 3 possible combinations given the same 3 side lengths. It is important for students to recognize that all 3 of the statements must be true for any triangle.

7. Given the lengths of 2 sides of a triangle, what are the possibilities for the length of the third side?

   a. 4 inches and 7 inches

   Answer: 3 < third side < 11

   b. 10 centimeters and 10 centimeters

   Answer: 0 < third side < 20

   c. 11 inches and 12 inches

   Answer: 1 < third side < 23

   0.5 centimeter and 2.5 centimeters

   Answer: 2 < third side < 3

Teacher Tip: It is recommended that students compare and discuss the results. To extend this question, students could be given side lengths containing variables.

Extension: Students can be asked to extend the idea of the Triangle Inequality Theorem to other polygons.

TI-Nspire Navigator Opportunity: ***Multiple Choice Quick Poll***
See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The conditions that must be met to build triangles given segments of different length.
- How to use the triangle inequality theorem to determine whether a triangle can be formed given 3 segment lengths.

Assessment

On page 1.2, the lengths of the segments you used are as follows: a = 2 units, b = 3 units, e = 3 units, d = 4 units, and c = 6 units. Identify at least 2 sets of segments that can form a triangle and 2 sets that cannot form a triangle.

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Note 1

Question 1, *Screen Capture*: You could assign a different combination to each pair of students and then use Screen Capture for students to share successful and unsuccessful combinations.

Note 2

Question 3, *Screen Capture*: Use Screen Capture so that students can see and discuss more cases of when a triangle can be formed and when a triangle cannot be formed.

Note 3

Question 5, *Yes/No Quick Poll*: Send students a Yes/No Quick Poll.

1. Can a triangle be formed out of sides with lengths 5, 7, and 4? **Answer:** Yes
2. Can a triangle be formed out of sides with lengths 10, 6, and 3? **Answer:** No

Note 4

Question 7, *Multiple Choice Quick Poll*: Send students a Multiple Choice Quick Poll.

1. Two sides of a triangle are 4 and 9. Which of the following cannot be the length of the third side?

   A. 6 B. 8 C. 10 D. 12 E. 14

Answer: E. The third side must be between 5 and 13.