Math Objectives

- Students will capture the lengths of a triangle midsegment and the side of the triangle not connected to it.
- Students will analyze the relationship between the lengths using a formula in the spreadsheet.
- Students will measure areas and define them as variables.
- Students will investigate relationships for the measurement data with a spreadsheet.

Vocabulary

- similarity
- similar triangles
- midsegment

About the Lesson

- The estimated time for the student activity is 45 minutes.
- You will need to create or download the *Triangle_Midsegments.tns* file.
- Students should have already studied the definitions of *similarity* and *similar triangles.*
- Students might work independently or with a partner to complete this activity.

TI-Nspire™ Navigator™ System

- Use Class Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he/she used a certain tool.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:
Create Instructions

- Triangle_Midsegments_Create.pdf

Student Activity

- Triangle_Midsegments_Student.pdf
- Triangle_Midsegments_Student.doc

TI-Nspire document

- Triangle_Midsegments.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand (☜). Press
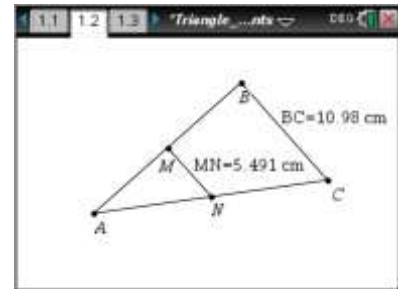> [ctrl] [✥] to grab the point and close the hand (☜).

Move to page 1.2.

Part 1—Find the ratio of the side to the midsegment

A *midsegment* is a segment connecting the midpoints of two sides of a triangle.

Page 1.2 shows ∆*ABC* with midsegment $\overline{MN}$ parallel to side $\overline{BC}$. *MN* and *BC* have been defined as variables.

1.   What can you say about the relationship between ∆*ABC* and ∆*AMN?* How do you know?

     Answer: They are similar. SAS for similar triangles.

You will capture these measurements into the spreadsheet on page 1.3. Drag point *B* or point *C* to form a new triangle. Press [ctrl] [.]. This captures the current lengths for $\overline{MN}$ and $\overline{BC}$. Press [ctrl] ▶ to look at the spreadsheet. The measurements appear in row 1.

Return to page 1.2. Drag point *B* or point *C* to form another triangle. Press [ctrl] [.]. Repeat four more times.

Return to page 1.3. Examine the data in Columns A and B.

2.   What comparison can you make between the two columns?

     Sample Answer: *BC* appears to be twice *MN*.

Move your cursor to the top of Column C (above the diamond row). Name this column *lratio* and press enter .

In the diamond row, enter a formula using the column names for Columns A and B. To do this, move your cursor to the diamond row of Column C and press 📖 . Press var and select *lbc* from the list. Press ÷ . Press var and select *lmn* from the list. Press enter .

3. What does the formula mean?

    Sample Answer: How many times larger $\overline{BC}$ is than $\overline{MN}$; the ratio of the large length to the small length
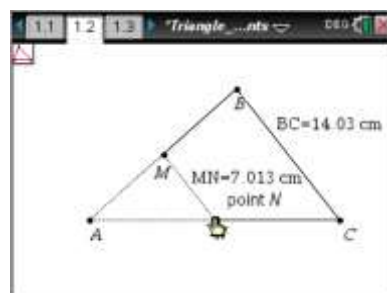
4. What does the result of the formula tell you about side $\overline{BC}$ and the midsegment $\overline{MN}$?

    Answer: $\overline{BC}$ is twice $\overline{MN}$.

Part 2—Finding the ratio of the areas

Draw Δ*AMN*. To do this, return to page 1.2. Press **Menu > Shapes > Triangle**. Click on each of the points *A*, *M*, and *N*. Press esc to exit the **Triangle** tool.

> **Teacher Tip:** In order to measure areas, a closed shape must be created. Triangle *ABC* was originally created from the Shapes menu, but Δ*AMN* was not. So Δ*AMN* must be created at this time.

5. What do you predict the ratio of the areas of Δ*ABC* and Δ*AMN* to be?

    Sample Answer: Students may predict that the ratio is still 2:1, which is a common misconception. The data will disprove this conjecture.

Measure the areas of Δ*AMN* and Δ*ABC*. To do this, press **Menu > Measurement > Area**. Click on Δ*AMN* by hovering over $\overline{MN}$. Move the measurement to a clear area of the screen and click to drop it. Repeat to measure Δ*ABC*. Press esc to exit.

Define each area measure as a variable. To do this, move your cursor to the measurement for △*ABC*. Click once. Press [ctrl] [var]. Then type **aabc** and press [enter]. Repeat for △*AMN,* naming the variable *aamn*.

To analyze the ratio of the areas, capture some of the areas to the spreadsheet. To do this, move to page 1.2. Move your cursor to the top of Column D and name the column *abc*. Name Column E *amn*.
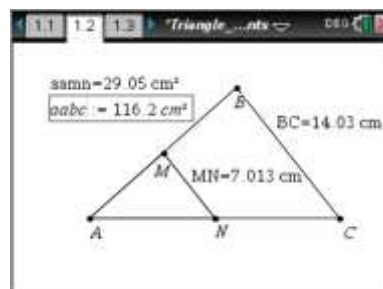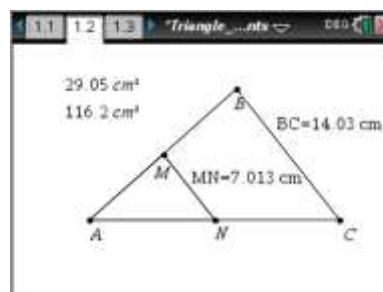
> **Teacher Tip:** The column headings must be different than the variable names used on the sketch. As a column heading, the name will come to represent a set of numbers, whereas the variable name on the sketch represents one discrete value at a time.

Move your cursor to the diamond row of Column D. Press **Menu > Data > Data Capture > Automated Data Capture**. Enter the variable *aabc* (by typing or by using the [var] key as in Column C). Press [enter]. Repeat to define Column E to capture the area of △*AMN*.

Return to page 1.2. Drag point *B* or point *C* for a few seconds. The spreadsheet is rapidly capturing data as you drag.

Return to page 1.3. Use Column F to determine the ratios of the areas. Name Column F *aratio* using a procedure similar to *lratio.*

6. Give and explain a ratio that describes the relationships between the areas of △*ABC* and △*AMN*.

    Sample Answer: Area of △*ABC* / area of △*AMN* is 4 because 4 copies of △*AMN* will fit inside △*ABC*.

7. What is the ratio of the perimeters of the two triangles? How do you know?

    Answer: Perimeter of ∆*ABC* should be twice ∆*AMN* because perimeter is a linear/length measure so it will have the same ratio as the lengths.

    > **Teacher Tip:** Columns A and B were set up for manual data capture so students would see the direct connection between positions of the triangle, measurements appearing on the screen, and the data collected in the spreadsheet. Columns D and E can now be set to capture data automatically, which is a faster process, since students will understand where the data originates. Note that the automatic data capture measures of the areas in columns D and E do not correspond to the manual data capture measures in columns A and B.
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    > If time permits, have students measure perimeters of ∆*ABC* and ∆*AMN* to confirm this. Discuss how all linear ratios will equal the original similarity ratio, but area ratios will equal the (similarity ratio)^2.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students can:

*   Recognize that the ratio between a midsegment of a triangle and the side of the triangle opposite it (or not connected to it) is 1 to 2, or that the side of the triangle is twice the length of the midsegment. This condition is one part of the Triangle Midsegment Theorem. The activity should be extended to establish the parallelism of the two segments.
*   See that the ratio of the area of ∆*ABC* to ∆*AMN* is 4 to 1. If area scale factor has been studied, this fact can be used to reinforce that the scale factor of the areas is the square of the scale factor of the side lengths. If area scale factor has not been studied, this fact can be used to pose the question about how the ratio of the areas relates to the ratio of the lengths.