



About the Lesson

In this activity, students discover the trigonometric ratios through measuring the side lengths of similar triangles and calculating their ratios. The formal definitions of the sine, cosine, and tangent are presented. As a result, students will:

- Apply the sine, cosine, and tangent of angles to find missing side lengths.

Vocabulary

- similar triangles
- hypotenuse
- opposite leg
- adjacent leg
- sine
- cosine
- tangent

Teacher Preparation and Notes

- This activity is designed for use in an Algebra 1 classroom. It can also be used as a review in Algebra 2.
- This activity assumes the students have a basic understanding of similar triangles. Students are asked to use the **Cabri™ Jr** App to measure the lengths of segments and angles, write and evaluate simple formulas, and perform a data capture. If students are unfamiliar with these functions of the handheld, extra time should be taken to explain them.
- Ensure that the **Cabri™ Jr** App is loaded on each student's calculators before beginning the activity.

Activity Materials

- Compatible TI Technologies:

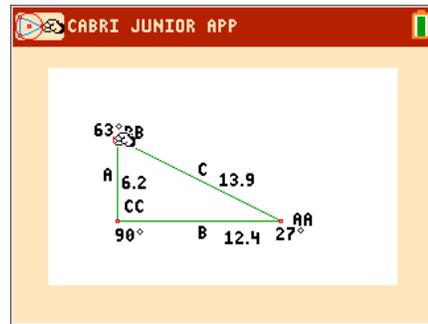
TI-84 Plus*

TI-84 Plus Silver Edition*

 TI-84 Plus C Silver Edition

 TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Trigonometric_Ratios_Student.pdf
- Trigonometric_Ratios_Student.doc
- TRIAN1.8xv
- TRIAN2.8xv
- TRIAN3.8xv



Tech Tip: Before beginning the activity, the files TRIAN1.8xv, TRIAN2.8xv, and TRIAN3.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE Software.

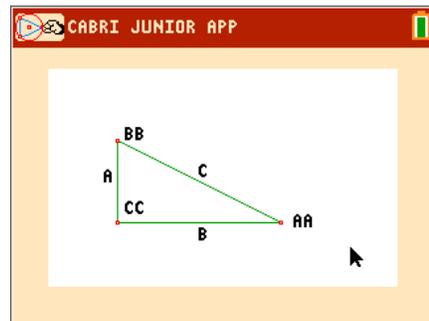
Problem 1 – Exploring a Triangle

To begin the activity, open the **Cabri Jr™** App by pressing **[apps]** and choosing it from the list. Open the file *TRIAN1*.



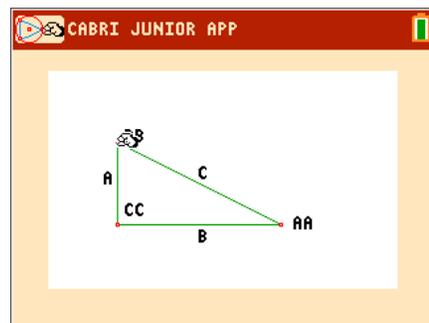
This file shows a triangle with sides A , B , and C .

Angle AA is opposite side A , angle BB is opposite side B , and angle CC is opposite side C .



Drag the point labeled BB . Move your cursor over the point and press **[alpha]** to grab it. Then use the arrow keys to move it. When you are finished, press **[enter]** to let it go.

What happens as you drag point BB ?

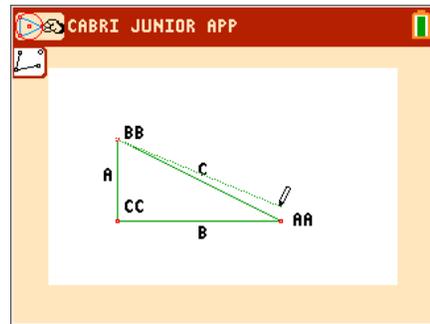


Measure each angle. Press **[graph]** to open the

F5: Appearance Menu. Choose **Measure>Angle**.

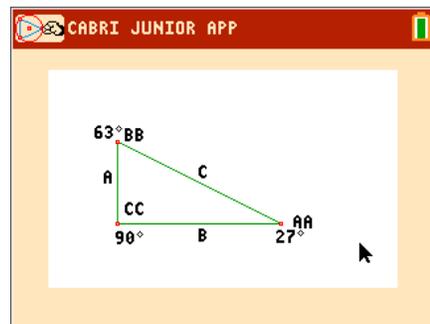


To measure angle AA , click on BB , then point AA , then point CC , in that order.



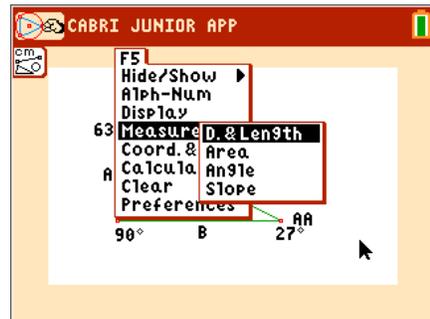
Repeat this process to measure all three angles.

Recall that the longest side of a right triangle is called the hypotenuse and the two shorter sides are called the legs.

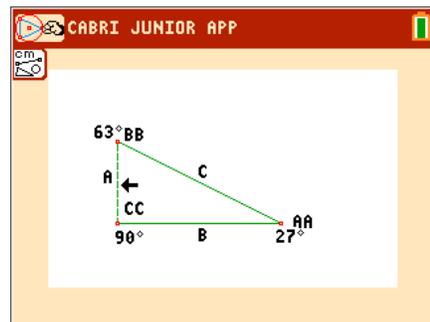


Next measure side lengths A , B , and C . Go to **F5: Appearance>Measure>D. & Length**

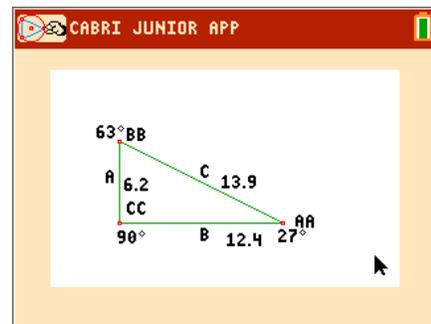
1. What kind of triangle is this?
Answer: It is a right triangle.



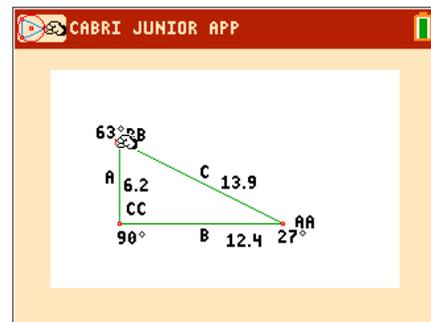
To measure side A , move the cursor over the side and when it flashes press **enter**. (Alternatively, you can also press enter on BB , then CC .)



Repeat to measure all three sides.



Drag point BB by moving your cursor over it and pressing the `alpha` button. You should be able to move the point BB either up or down.



2. Which measurements of the triangle change? Which measurements stay the same?

Answer: The lengths of the three sides, A , B , and C change. The degree measures of the three angles AA , BB and CC do not change.

3. What word describes two triangles that have the same angles but different side lengths?

Answer: similar



4. Drag point BB to complete the first three columns of the table. Calculate the other columns.

Answers:

A	B	C	$\frac{A}{B}$	$\frac{A}{C}$	$\frac{B}{C}$
2.0	4.0	4.5	0.5	0.444	0.889
2.3	4.6	5.1	0.5	0.451	0.902
2.5	5.0	5.6	0.5	0.446	0.893
2.8	5.6	6.3	0.5	0.444	0.889
3.0	6.0	6.7	0.5	0.448	0.896
3.5	7.0	7.8	0.5	0.449	0.897
4.0	8.0	8.9	0.5	0.449	0.899

5. How do the ratios for each of the similar triangles compare?

Answer: They are essentially the same for each triangle.

Note: Due to the calculator's rounding function, slight variations apply to $\frac{A}{C}$ and $\frac{B}{C}$.

6. Write a conclusion about the ratios of the side lengths of similar triangles.

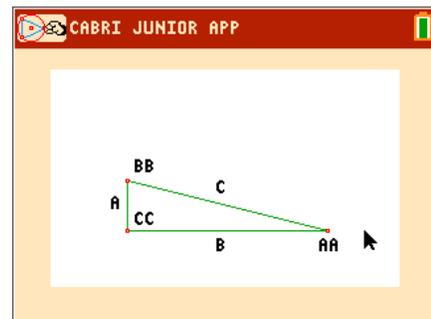
Answer: The ratios of the side lengths of similar triangles will remain the same no matter what size the length of the sides are.

Problem 2 – Exploring Another Triangle

Open the file *TRIAN2*. Repeat the procedure from Problem 1 to measure the three sides A , B , and C and the three angles AA , BB , and CC .

7. Is this triangle a right triangle?

Answer: yes



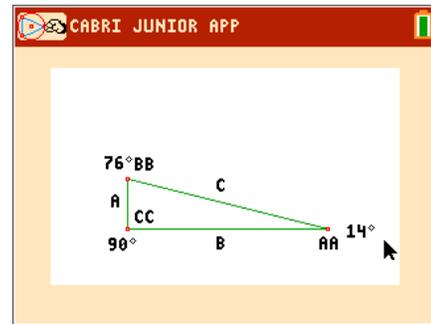


8. Is this triangle similar to the first triangle? Why or why not?

Answer: No, because all three pairs of angles aren't equal to one another.

9. Are all right triangles similar? Why or why not?

Answer: No, because the two acute angles in a right triangle can vary.



10. Drag point *BB* to complete the first three columns of the table. Calculate the other columns.

Answers:

<i>A</i>	<i>B</i>	<i>C</i>	$\frac{A}{B}$	$\frac{A}{C}$	$\frac{B}{C}$
2.0	8.0	8.2	0.25	0.244	0.976
1.8	7.2	7.4	0.25	0.243	0.973
1.6	6.4	6.6	0.25	0.242	0.970
1.4	5.6	5.8	0.25	0.241	0.965

11. How do the ratios of the side lengths of Triangle 2 compare to the ratio of the side lengths of Triangle 1?

Answer: While each ratio of sides of Triangle 2 differ than those of Triangle 1, they are still equal within the triangle.

12. Write a conclusion about the ratios of the side lengths of triangles that are similar and triangles that are not similar.

Answer: The ratios of side lengths of similar triangles are equal, while the ratios of side lengths of non-similar triangles are not.



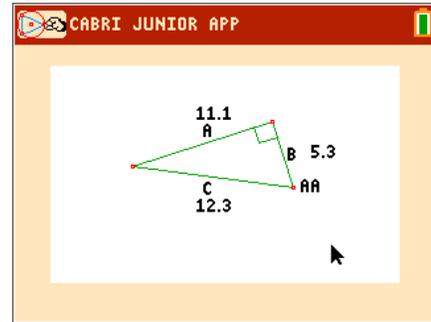
Problem 3 – Introducing the Trigonometric Ratios

Open the Cabri Jr file *TRIAN3*.

13. Measure the side lengths.

$$A = \underline{\hspace{2cm}} \quad B = \underline{\hspace{2cm}} \quad C = \underline{\hspace{2cm}}$$

Answers: $A = 11.1$; $B = 5.3$; $C = 12.3$



Calculate the trigonometric ratios for $\angle AA$.

14. $\text{sine } \angle AA = \frac{A}{C} \approx \underline{\hspace{2cm}}$

Answer: 0.902

15. $\text{cosine } \angle AA = \frac{B}{C} \approx \underline{\hspace{2cm}}$

Answer: 0.431

16. $\text{tangent } \angle AA = \frac{A}{B} \approx \underline{\hspace{2cm}}$

Answer: 2.094

17. What is the measure of angle AA ? $\underline{\hspace{2cm}}$

Answer: $\approx 65^\circ$

Students are asked to use the calculator commands **sin**, **cos**, and **tan** to check their answers. It is important to emphasize to them that the above ratios as well as the angle measure were rounded. Point out that the trigonometric ratios can often be irrational values.



Problem 4 – Calculating the Trigonometric Ratios of a Different Angle

Using the same triangle in Cabri Jr file *TRIANG3*, write and calculate formulas using the side lengths A , B , and C to find sine $\angle BB$, cosine $\angle BB$, and tangent $\angle BB$.

18. sine $\angle BB = \frac{\text{opposite}}{\text{hypotenuse}} \approx \frac{5.3}{12.3}$

Answer: $\frac{5.3}{12.3} \approx 0.431$

19. cosine $\angle BB = \frac{\text{adjacent}}{\text{hypotenuse}} \approx \frac{11.1}{12.3}$

Answer: $\frac{11.1}{12.3} \approx 0.902$

20. tangent $\angle BB = \frac{\text{opposite}}{\text{adjacent}} \approx \frac{5.3}{11.1}$

Answer: $\frac{5.3}{11.1} \approx 0.477$

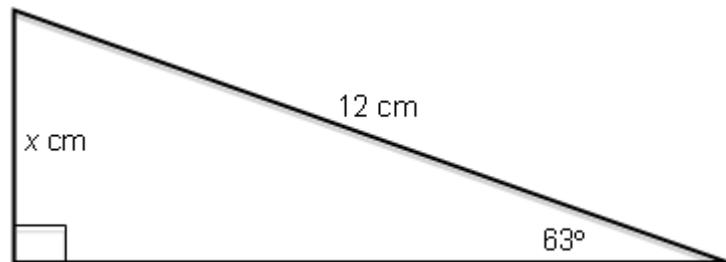
21. What is the measure of angle BB ? _____

Answer: $\approx 25.5^\circ$

Students are again asked to use the calculator commands sin, cos, and tan to check their answers.

Problem 5 – Finding Missing Side Lengths

22. Write a formula that relates 63° , the opposite side, and the hypotenuse. Solve for x .



Answer: $\sin(63^\circ) = \frac{x}{12} \approx 10.692 \text{ cm}$



23. Write a formula that relates 54° , the opposite side, and the adjacent side. Solve for x .

Answer: $\tan(54^\circ) = \frac{10}{x} \approx 7.265 \text{ cm}$

