

About the Lesson

In this activity, students are to find the solution to a system of linear equations using technology. They are presented with a problem on parking lot fees. As a result, students will:

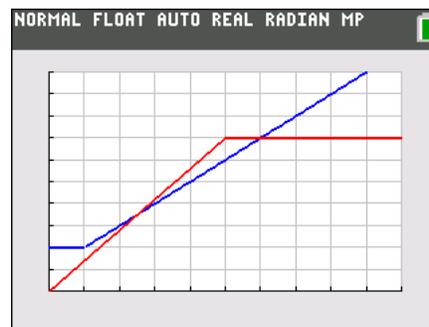
- Graph a linear system in two variables and use technology to find the intersection point(s).

Vocabulary

- linear system
- piecewise function

Teacher Preparation and Notes

- Students should already be familiar with algebraic symbol manipulation, finding linear equations from two points, and properties of piecewise functions.
- This activity is intended to be teacher-led for the introductions, and the problems should be completed by students individually.
- This worksheet provides detailed instructions for the completion of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper
- Students should begin by clearing out any functions from the $Y=$ screen and turning off all Stat Plots.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- WhichGaragelsBetter_Student.pdf
- WhichGaragelsBetter_Student.doc



The cost schedules for two different parking garages are below. The maximum stay is 24 hours.

Blue Street Garage	
Length of time in the garage	Cost
1 hour or less	A flat fee of \$10.
More than 1 hour	\$10 for the first hour plus an additional fee of \$5 for every hour after your first hour in the garage.
Red Street Garage	
Length of time in the garage	Cost
5 hours or less	\$7 per hour
More than 5 hours	A flat fee of \$35

1. Complete the table.

Students may find it easier to complete the table for the Red Street Garage first.

Using the slope, students might build the table for the Blue Street Garage by adding 5 to the previous row of the table.

Length of time in the garage (hours)	Blue Street Garage total cost (dollars)	Red Street Garage total cost (dollars)
0	10	0
1	10	7
2	15	14
3	20	21
4	25	28
5	30	35
6	35	35
7	40	35
8	45	35
9	50	35

2. Joe parked in the Blue Street Garage and Flo parked in the Red Street Garage for the same length of time. After they checked out and paid, they asked each other which garage was cheaper, only to discover they paid the same amount for their stay. What length of time might each have parked in the garage? What would have been their fee? Use the above table to determine the answers to these two questions. Find all possible answers.

Answer: Each could have stayed 6 hours and paid \$35, but that is not the only possible answer.
Red surpasses Blue between 2 and 3 hours.

Length of time	Blue cost	Red cost
2	15	14
3	20	21

Annotations: Arrows show a change of +0.5 for Blue and +2.50 for Red between 2 and 3 hours. A larger arrow shows a change of +3.50 for Red between 3 and 2.5 hours.

Using proportional reasoning, for each half hour the cost of the Blue Street Garage increases by $\frac{1}{2}$ of 5 or \$2.50, and the cost of the Red Street Garage increases by $\frac{1}{2}$ of 7 or \$3.50.

Each could have stayed only 2.5 hours and paid \$17.50. This will be more apparent with the graph.

- Write the piecewise functions that model the cost of staying in each of the garages.

Answer:

$$\text{Blue Street Garage: } B(x) = \begin{cases} 10, & x \leq 1 \\ 5(x - 1) + 10, & x > 1 \end{cases}$$

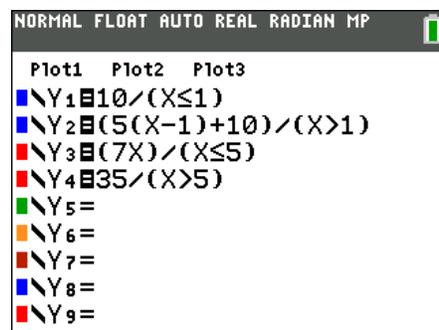
$$\text{Red Street Garage: } R(x) = \begin{cases} 7x, & x \leq 5 \\ 35, & x > 5 \end{cases}$$

Teacher Tip: If your students are new to writing piecewise functions, a scaffolding idea is to give them the structure and have them fill in the rest of it. For example give them the first line and let them figure out the second part.

- To graph a piecewise function, you will need your teacher to lead you through this process. Your graph should look like the one to the right. Graph the functions in the viewing window shown.

Answer:

The screenshots at the right shows how the students are to enter the piecewise functions. The inequality signs are in the TEST menu ($\text{2nd} \rightarrow \text{MATH}$ for [TEST]). These Boolean expressions, like $(x > 5)$ limit the domain because the value of 1 or 'true' is given when $x > 5$ and 0 or 'false' otherwise.



Tech Tip: Use TI-SmartView™ Emulator Software for the TI-84 Plus Family to Show Keypress History.

Y1 and Y2 are for the Blue Street Garage.

Change the graph color for Y2 to blue by pressing the left arrow to highlight the color, press **[ENTER]**, and press the left or right arrow on the popup spinner to select the blue color. Press the down arrow to highlight OK and press **[ENTER]**.

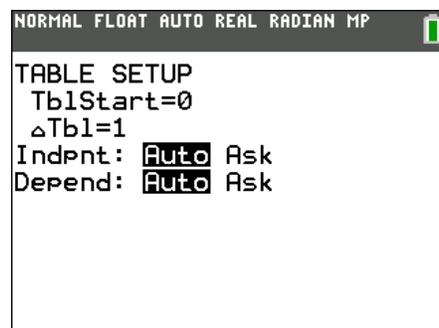
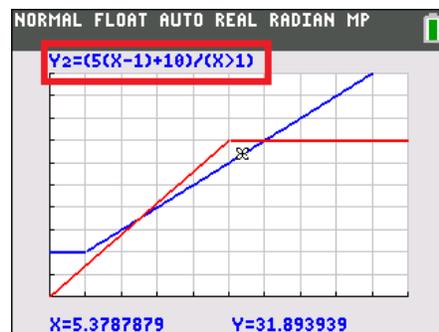
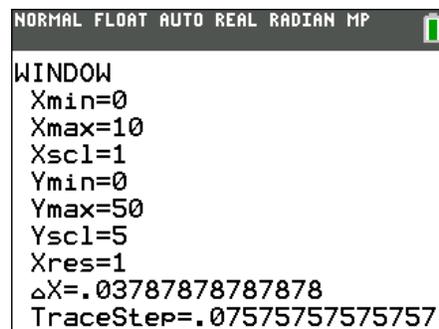
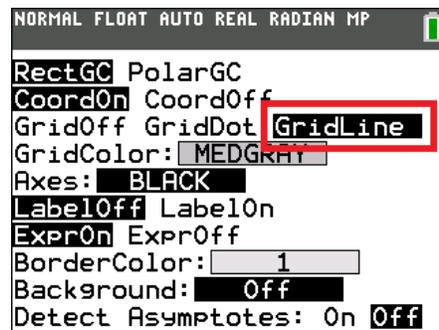
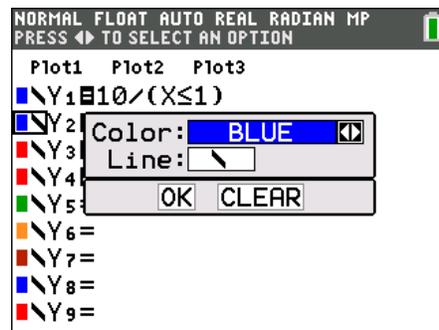
Y3 and Y4 are for the Red Street Garage. Change the graph color for these two lines to red.

Press **[2nd]** **[FORMAT]**, highlight the Gridline, and press **[ENTER]**. Gridlines are a feature only available on the TI-84C, and not on the TI-84.

Remind students to change the window settings.

If students press **[TRACE]** they should use the up and down arrow key to make sure they are on the intended piece of the formula and monitor the expression displayed at the top of the screen. Student can use **[2nd]** **[TRACE]** for **[CALC]** to find the intersection. Again use the up or down arrow to select which segment of the graph.

Students can check their answer using the Table Feature. Press **[2nd]** **[TBLSET]**. Use the settings shown to the right.



Press 2^{nd} [TABLE].

This matches the table built manually in Question 1.

NORMAL FLOAT AUTO REAL RADIAN MP					
PRESS + FOR Δ Tb1					
X	Y1	Y2	Y3	Y4	
0	10	ERROR	0	ERROR	
1	10	ERROR	7	ERROR	
2	ERROR	15	14	ERROR	
3	ERROR	20	21	ERROR	
4	ERROR	25	28	ERROR	
5	ERROR	30	35	ERROR	
6	ERROR	35	ERROR	35	
7	ERROR	40	ERROR	35	
8	ERROR	45	ERROR	35	
9	ERROR	50	ERROR	35	
10	ERROR	55	ERROR	35	

X=0

5. When are the costs for using each garage equal to each other? What equations would you set equal to each other to find when the costs are equal? Solve these equations below.

Answer:

$$7x = 10 + 5(x - 1) \qquad 10 - 5(x - 1) = 35$$

$$7x = 10 + 5x - 5 \qquad 5(x - 1) = 25$$

$$2x = 5 \qquad x - 1 = 5$$

$$x = 2.5 \qquad x = 6$$

6. Use the table, formula, or graph to answer the following:
- Which garage costs less for a short stay? For example, you enter the garage, park, realize you forgot your wallet, and end up having to leave only 15 minutes later.

Students may need to be reminded of units. They can evaluate each function at $x = 0.25$ hours or just use the graph.

Answer: The Red Street Garage is cheaper since the Blue Street Garage is \$10 and the Red Street Garage is \$1.75.

- Suppose after a 2.5-hour movie you decide to go out to a restaurant and stay an additional 2 hours. Which garage will cost less? How much less?



Which Garage is Better?

Answer: If you stay 4.5 hours, the Blue Street Garage (\$25 + \$2.50 = 27.50) is cheaper than the Red Street Garage (4.5 x 7 = \$31.50) by \$4.

You can also use the table.

NORMAL FLOAT AUTO REAL RADIAN MP

TABLE SETUP
 TblStart=0
 Δ Tbl=.5
 Indent: **Auto** Ask
 Depend: **Auto** Ask

NORMAL FLOAT AUTO REAL RADIAN MP
 PRESS + FOR Δ Tbl

X	Y ₁	Y ₂	Y ₃	Y ₄
0	10	ERROR	0	ERROR
.5	10	ERROR	3.5	ERROR
1	10	ERROR	7	ERROR
1.5	ERROR	12.5	10.5	ERROR
2	ERROR	15	14	ERROR
2.5	ERROR	17.5	17.5	ERROR
3	ERROR	20	21	ERROR
3.5	ERROR	22.5	24.5	ERROR
4	ERROR	25	28	ERROR
4.5	ERROR	27.5	31.5	ERROR
5	ERROR	30	35	ERROR

X=4.5

Tech Tip: Students using the TI-84 Plus will need to arrow over to see the other columns.

- c. Suppose you needed to park your car for 12 hours in the garage. Which garage will cost less? How much less?

Answer: If you stay 12 hours, the Red Street Garage (\$35) is cheaper than the Blue Street Garage (\$10 + \$55 = \$65) by \$30.

You can also use the table.

NORMAL FLOAT AUTO REAL RADIAN MP
 PRESS + FOR Δ Tbl

X	Y ₁	Y ₂	Y ₃	Y ₄
3	ERROR	20	21	ERROR
4	ERROR	25	28	ERROR
5	ERROR	30	35	ERROR
6	ERROR	35	ERROR	35
7	ERROR	40	ERROR	35
8	ERROR	45	ERROR	35
9	ERROR	50	ERROR	35
10	ERROR	55	ERROR	35
11	ERROR	60	ERROR	35
12	ERROR	65	ERROR	35
13	ERROR	70	ERROR	35

X=12

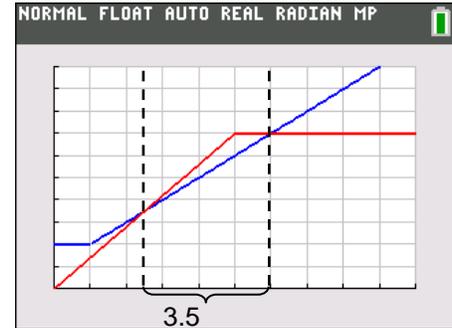
- 7. Use the graph to solve the equation $B(x) < R(x)$. Interpret the solution in real-world practical terms.



$B(x) = R(x)$ if $x = 2.5$ or $x = 6$,
where the graphs intersect.

$B(x) < R(x)$ when the graph of $B(x)$ is *below* the
graph of $R(x)$. Using the graph, we have $2.5 < x < 6$.

Answer: In terms of the context of parking garages,
the Blue Street Garage is cheaper than the Red Street
Garage if you stay more than 2.5 hours but
less than 6 hours.



8. Over the length of a day, what duration is the Blue Street Garage better?
The Red Street Garage?

Answer: The Blue Street Garage is better for a duration of 3.5 hours (when you stay more than 2.5 hours but less than 6 hours). The Red Street Garage is better for the remaining 20.5 hours of the day.